

# APPLYING THE ATTRIBUTED PREFIX TREE FOR MINING CLOSED SEQUENTIAL PATTERNS

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## ABSTRACT

Mining closed sequential patterns is one of important tasks in data mining. It is proposed to resolve difficult problems in mining sequential pattern such as mining long frequent sequences that contain a combinatorial number of frequent subsequences or using very low support thresholds to mine sequential patterns is usually both time- and memory-consuming. This paper applies the characteristics of closed sequential patterns and sequence extensions into the prefix tree structure to mine closed sequential patterns from the sequence database. The paper uses the parent-child relationship on prefix tree structure and each node on prefix tree is also added fields to determine whether that is a closed sequential pattern or not. Experimental results show that the number of sequential patterns is reduced significantly.

*Keywords:* sequential pattern, closed sequential pattern, prefix tree, sequence database.

## 1. INTRODUCTION

Sequential pattern mining, since it was first introduced by Agrawal [1], has played an important role in data mining tasks with broad applications including market and customer analysis, web log analysis, pattern discovery in protein sequences, and mining XML query access patterns for caching and so on. The sequential pattern mining algorithms proposed so far have a good performance in databases with short frequent sequences [2, 3, 9 - 10, 16]. However, when mining long frequent sequences that contain a combinatorial number of frequent subsequences, such a mining will generate an explosive number of frequent subsequences for long patterns, or when using very low support thresholds to mine sequential patterns, which is prohibitively expensive in both time and space. So, the performance of such algorithms often degrades dramatically. To overcome this difficulty, the problem for mining closed sequential patterns have also been proposed. A sequence  $S$  is called closed if there exists no supersequence of  $S$  with the same support in the database. For example, sequence pattern  $\langle\langle AB \rangle\langle B \rangle\langle BC \rangle\rangle$  is a closed sequence pattern of sequence patterns  $\langle\langle B \rangle\langle BC \rangle\rangle$  and  $\langle\langle AB \rangle\langle B \rangle\rangle$  if they have the same support.

Several studies have been recently proposed to mine closed sequential patterns [4, 11 - 12, 14 - 15]. But these algorithms used the corresponding projected databases of frequent subsequences to find closed sequences. It consumes much time to construct projected databases of frequent subsequences for a set of sequence. In this paper, we propose an efficient algorithm for directly finding closed sequential patterns at the generating sequential patterns process, which is based on the combination of the parent-child relationship and its properties on prefix tree structure. On the prefix tree in our approach, each node stores a sequential pattern and its corresponding support value. Besides, it will be added one field to consider whether this node is a closed sequential pattern (*IsCSP*). Based on the *IsCSP* field added to each node, the algorithm easily determines if a node is a closed sequential pattern so the mining time is reduced significantly. This algorithm also uses join operations over the prime block encoding approach of the prime factorization theory to represent candidate sequences and determine the frequency for each candidate. The experimental results showed that the performance for mining closed sequential patterns in this paper is much better.

The rest of this paper is organized as follows. Section 2 reviews some works related to mine closed sequential patterns. Section 3 presents some problem definitions related to sequential patterns/closed sequential pattern and prefix tree. The algorithm for mining closed sequential patterns is discussed in Section 4. Section 5 presents the experimental results, and conclusion and future work are presented in Section 6.

## **2. RELATED WORK**

Mining sequential patterns with closed patterns may significantly reduce the number of patterns generated in the mining process without losing any information because it can be used to derive the complete set of sequential patterns; so, the number of closed sequential patterns is usually fewer than the number of sequential patterns. Several studies have been recently proposed to mine closed sequential patterns [4, 11 - 12, 14 - 15]. The CloSpan algorithm [15] has been proposed. Like most of the frequent closed itemset mining algorithms CLOSET [6] and CHARM [17], CloSpan algorithm used the candidate maintenance and test approach. It needs to maintain the set of already mined closed sequence candidates for doing the backward subpattern and backward superpattern check to verify if a newly found frequent sequence is promising to be closed or not. Because CloSpan needs to maintain the set of historical closed sequence candidates, when there are many frequent closed sequences, it will consume much memory and lead to huge search space for pattern closure checking. BIDE [14] is another faster closed sequence mining algorithm. Different from CloSpan, it used a novel sequence closure checking scheme called BI-Directional Extension, and pruned the search space more by using the BackScan pruning method and the ScanSkip optimization technique to directly get the complete set of the frequent closed sequence patterns without candidate maintenance. Thus, in most cases, BIDE is more efficient than CloSpan, especially when a database is dense or the minimum support value is low. But to implement the closure check, the BIDE algorithm spends a lot of time on scanning the pseudo-projected database repeatedly to verify the existence of extension of position with a prefix sequence, which costs much time in the mining process. To reduce the time consumed on scanning the pseudo-projected database for verifying in the BIDE algorithm, the FCSM-PD algorithm was proposed by Huang et al. [4]; the positional data was used to reserve the position information of items in the data sequences. In the pattern growth process, the extension of position with a prefix sequence is checked directly and all the position information of the new prefix sequences will be recorded. The FCSM-PD algorithm must store

all the position information of a prefix sequence in the process of pattern growth in advance; so it consumes more memory in this algorithm. In 2013, Pham et al. proposed an algorithm called CloGen algorithm [7]. It used the parent-child relationship on prefix tree structure to find closed sequences and their corresponding generators at the same time. The result of CloGen algorithm has been used to generate non-redundant sequential rules [8]. This paper is using the parent-child relationship on prefix tree structure to find only closed sequences and its result can be used to mine sequential rules.

### 3. PROBLEM DEFINITIONS

A sequence database  $SD$  is a set of sequences  $S = \{s_1, s_2, \dots, s_n\}$  and a set of items  $I = \{i_1, i_2, \dots, i_n\}$ , where each sequence  $s_x$  is an ordered list of itemsets  $s_x = \{x_1, x_2, \dots, x_n\}$ , and  $s_1$  occurs before  $s_2$ , which occurs before  $s_3$ , and so on, such that  $x_1, x_2, \dots, x_n \subseteq I$ . The size of a sequence is the number of itemsets in the sequence. The length of a sequence is the number of instances of items in the sequence. A sequence with length  $l$  is called an  $l$ -pattern sequence. A sequence with size  $k$  is called a  $k$ -sequence.

Given two sequences  $\alpha = \langle a_1 a_2 \dots a_n \rangle$  and  $\beta = \langle b_1 b_2 \dots b_m \rangle$  (where  $a_i, b_i$  are itemsets), sequence  $\alpha$  is called a subsequence of  $\beta$  and  $\beta$  is a supersequence of  $\alpha$ , denoted as  $\alpha \subseteq \beta$ , if there exist integers  $1 \leq j_1 < j_2 < \dots < j_n \leq m$  ( $n \leq m$ ) such that  $a_1 \subseteq b_{j_1}, a_2 \subseteq b_{j_2}, \dots, a_n \subseteq b_{j_n}$ . For example, if  $\alpha = \langle (ab), d \rangle$  and  $\beta = \langle (abc), (de) \rangle$ , where  $a, b, c, d$ , and  $e$  are items, then  $\alpha$  is a subsequence of  $\beta$  and  $\beta$  is a supersequence of  $\alpha$ . The support of a sequence  $\alpha$  (denoted by  $Sup(\alpha)$ ) in a sequence database is the number of sequences in the database containing  $\alpha$ . Sequence  $\alpha$  is a frequent sequence in sequence database  $SD$ , if  $Sup(\alpha) \geq minSup$  where  $minSup$  is the minimum support threshold defined by user. A frequent sequence is called a sequential pattern. A sequential pattern  $\alpha$  is called a closed sequential pattern if and only if  $\nexists \beta$  such that  $\alpha \subseteq \beta$  (i.e.,  $\beta$  contains  $\alpha$ ) and  $Sup(\alpha) = Sup(\beta)$ . Considering the sequence database in table 1 and  $minSup = 50\%$ , sequences  $\langle (AB) \rangle$ ,  $\langle (B)(BC) \rangle$  and  $\langle (B)(B)(BC) \rangle$  are sequential patterns because their support are greater and equal than  $minSup$  ( $\langle (AB) \rangle$  has the support value 4,  $\langle (B)(BC) \rangle$  has the support value 3, and  $\langle (B)(B)(BC) \rangle$  has the support value 3). Sequential pattern  $\langle (B)(B)(BC) \rangle$  is a closed sequential pattern  $\langle (B)(BC) \rangle$ .

Sequence  $\alpha$  is a prefix of  $\beta$  if and only if  $a_i = b_i$  for all  $1 \leq i \leq n$ . After eliminating the prefix part  $\alpha$  of sequence  $\beta$ , the remainder of  $\beta$  is a postfix of  $\beta$ . From the above definition, we know that a sequence of size  $k$  has  $(k-1)$  prefixes. For example, a sequence  $\langle (A)(BC)(D) \rangle$  has 2 prefixes:  $\langle (A) \rangle$  and  $\langle (A)(BC) \rangle$ . Therefore,  $\langle (BC)(D) \rangle$  is the postfix for prefix  $\langle (A) \rangle$ , and  $\langle (D) \rangle$  is the postfix for prefix  $\langle (A)(BC) \rangle$ .

A prefix tree is similar to a lexicographic tree, which starts from the tree root at level 0. In this paper, the prefix tree is started at the root with a null sequence  $\emptyset$ . Each child node stores a sequential pattern, its support value, one field to consider whether this node is a closed sequential pattern ( $IsCSP$ ). At level 1, each node is set with a single frequent item; at level  $k$ , each node is set with a  $k$ -pattern sequence. Recursively, there are nodes at the next level  $(k+1)$  after a  $k$ -pattern sequence is extended with a single frequent item. There are two ways to extend a  $k$ -pattern sequence, namely sequence extension and itemset extension [3]. In sequence extension, a single frequent item from  $I$  is added to the  $k$ -pattern sequence as a new itemset, increasing the size of the sequence. A sequence  $\alpha$  is a prefix of all sequence-extended sequences of  $\alpha$ , and  $\alpha$  is the prefix of all subnodes of the nodes that are sequence-extended in  $\alpha$ . In itemset extension, single frequent item from  $I$  that is greater than all items in the last itemset is added to

the last itemset in the  $k$ -pattern sequence. The size of itemset-extended sequences does not change and  $\alpha$  is an incomplete prefix of all subnodes of itemset-extended nodes in  $\alpha$ .

#### 4. PROPOSED ALGORITHM FOR MINING CLOSED SEQUENTIAL PATTERNS

In this section, the extension of a sequence on the prefix tree by performing a depth-first search and the attribute of closed sequential patterns are used to produce an algorithm for generating all closed sequential patterns. Using the prefix tree, new sequences, which are children nodes  $Cnode$ , can be easily created by appending an item to the last position of a parent node  $Pnode$  as an itemset extension or a sequence extension. When a new node  $Pnode$  is created, if  $Sup(Pnode) = Sup(Cnode)$ , then we set the  $IsCSP$  of  $Cnode$  to false and the  $IsCSP$  of  $Pnode$  to true.

First, the algorithm initializes the prefix tree  $pretree$  with the root node being null and children nodes being sequential 1-patterns with its  $IsCSP$  field as *true*. Each child node  $cn$  on  $pretree$  is considered as a root node for  $EXTEND\_TREE(cn, pretree)$  function to create its children nodes and extend the  $pretree$  tree. In this function, each child node of root node  $P$  is created by itemset extension or sequence extension. To represent candidate sequences as well as determine the frequency for each candidate, it uses the prime block encoding approach and the join operations over the prime blocks in [3]. With each new child node is created  $Pnew$  from  $P$ , if  $Sup(Pnew)=Sup(P)$ , then the value of  $Pnew.IsCSP$  is set to *true* and the value of  $P.IsCSP$  is set to *false*. The  $CHECK\_CLOSED(Pnew, pretree)$  function is called to update closed sequential patterns on the  $pretree$  tree. Finally, the algorithm returns the corresponding  $pretree$  tree with the sequential patterns which have the corresponding  $IsCSP$  values. The details of the proposed algorithm are introduced in Figure 1.

```

Input:  $SD, minSup$ .
Output: Prefix tree that contains Sequential/Closed sequential patterns.
Method:
MCSP-PreTree( $SD, minSup$ )
     $pretree \leftarrow \emptyset$ ;
     $SPs \leftarrow$  all frequent 1-pattern sequences;
    for each pattern  $P$  in  $SPs$ 
        Add  $P$  into  $pretree$  as a child node;
    for each child node  $r$  in  $pretree$ 
         $EXTEND\_TREE(r, pretree)$ ; //extend tree with root node  $r$ 
    return  $pretree$ ;
//.....
EXTEND_TREE( $Root, pretree$ )
     $EXTEND\_ITEMSET(Root, pretree)$ ; //create a new child node of root by itemset extension
     $EXTEND\_SEQUENCE(Root, pretree)$ ; //create a new child node of root by sequence extension
    For each node  $P_i$  that is an itemset extension of  $Root$ 
         $EXTEND\_TREE(P_i, pretree)$ ; // recursively call to extend tree with root node  $P_i$ 
    For each node  $P_s$  that is a sequence extension of  $Root$ 
         $EXTEND\_TREE(P_s, pretree)$ ; // recursively call to extend tree with root node  $P_s$ 
// .....
EXTEND_ITEMSET( $P, pretree$ )

```

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For each pattern  $P_i$  in  $SPs$  with  $P_i \triangleright$  last item in last itemset of  $P$ 
     $P_{new}$  is a new node created by adding  $P_i$  into last position in last itemset of  $P$  and using block encoding
    based on prism factorization in [3] to count the support;
    If ( $sup(P_{new}) \geq minSup$ )
        Set  $P_{new}$  as a closed pattern;
        If ( $sup(P) = sup(P_{new})$ )
            Set  $P$  as a non-closed pattern;
        Else
            CHECK_CLOSED( $P_{new}$ ,  $pretree$ );//check whether there exist subsequences of
             $P_{new}$  in  $pretree$  which are closed sequential patterns.
        Add  $P_{new}$  into  $pretree$  as a itemset-extended child node of  $P$ ;

//.....

EXTEND_SEQUENCE( $P$ ,  $pretree$ )
For each 1-pattern  $P_i$  in  $SPs$ 
     $P_{new}$  is a new node created by adding  $P_i$  as last itemset into  $P$  and using block encoding based on
    prism factorization in [3] to count the support;
    If ( $sup(P_{new}) \geq minSup$ )
        Set  $P_{new}$  as a closed pattern;
        If ( $sup(P) = sup(P_{new})$ )
            Set  $P$  as non-closed pattern;
        Else
            CHECK_CLOSED( $P_{new}$ ,  $pretree$ );//check whether there exist subsequences of  $P_{new}$  in
             $pretree$  which are closed sequential patterns
        Add  $P_{new}$  into  $pretree$  as a sequence-extended child node of  $P$ ;

//.....

CHECK_CLOSED ( $P_{new}$ ,  $pretree$ )
For each node  $P$  in  $pretree$ 
    If  $P_{new}$  is a supersequence of  $P$ 
        If  $P$  is a closed pattern and  $sup(P) = sup(P_{new})$ 
            Set  $P$  as a non-closed pattern;
         $subtree$  = the tree which rooted at  $P$ ;
        CHECK_CLOSED( $P_{new}$ ,  $subtree$ );

```

Figure 1. Algorithm for generating set of closed sequential patterns.

## 5. EXPERIMENTAL RESULTS

Figures 2–4 illustrate the process of mining closed sequential patterns on prefix tree using the proposed algorithm for the sequence database as presented in Table 1, where  $minSup = 50\%$ . First, the root node of prefix tree is null ( $\{\}$ ) with each child node containing: sequential pattern, support value, and  $IsCSP$  field that contains one of two values (true T or false F). Level 1 of the prefix tree contains nodes with sequential 1-pattern, their support values and the values of  $IsCSP$  set to *true* as in Figure 2, for example node  $\langle(B)\rangle:5:T$ . Consider node  $\langle(A)\rangle:4:T$  in Figure 2, the children nodes of  $\langle(A)\rangle:4:T$  are created by itemset extension or sequence extension, which include sequential pattern nodes:  $\langle(A)(B)\rangle:3:T$ ,  $\langle(A)(C)\rangle:3:T$ ,  $\langle(AB)\rangle:4:T$ . The support value of

the child node containing sequential pattern  $\langle\langle AB \rangle\rangle$  is 4, which is same as the support value of the parent node containing  $\langle\langle A \rangle\rangle$ , according to the definition of closed sequential pattern, then  $\langle\langle A \rangle\rangle$  is not a closed sequential pattern, so the *IsCSP* value of  $\langle\langle A \rangle\rangle$  is set to *false* T. After updating nodes in the prefix tree, we have the prefix tree as in Figure 3. The above process is repeated for all children nodes of  $\langle\langle A \rangle\rangle$  and all remaining nodes on the prefix tree. The final experimental results are shown in Figure 4. The experimental results in Figure 4 show that the number of closed sequential patterns is always smaller than the number of sequential patterns, where the complete set of closed sequential patterns consists of only 8 sequences while the whole set of sequential patterns consists of 21 sequences.

Table 1. Sequence database.

SID	Sequence
1	$\langle\langle AB \rangle\rangle(B)(B)(AB)(B)(AC)\rangle\rangle$
2	$\langle\langle AB \rangle\rangle(BC)(BC)\rangle\rangle$
3	$\langle\langle B \rangle\rangle(AB)\rangle\rangle$
4	$\langle\langle B \rangle\rangle(B)(BC)\rangle\rangle$
5	$\langle\langle AB \rangle\rangle(AB)(AB)(A)(BC)\rangle\rangle$

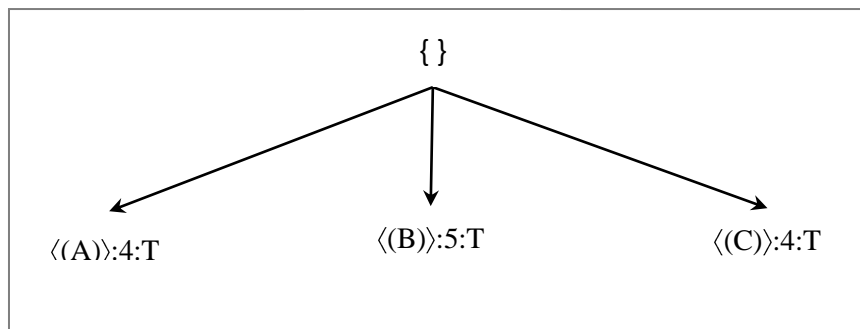


Figure 2. Level 1 of the prefix tree (each node contains: sequential pattern, support, and IsCSP).

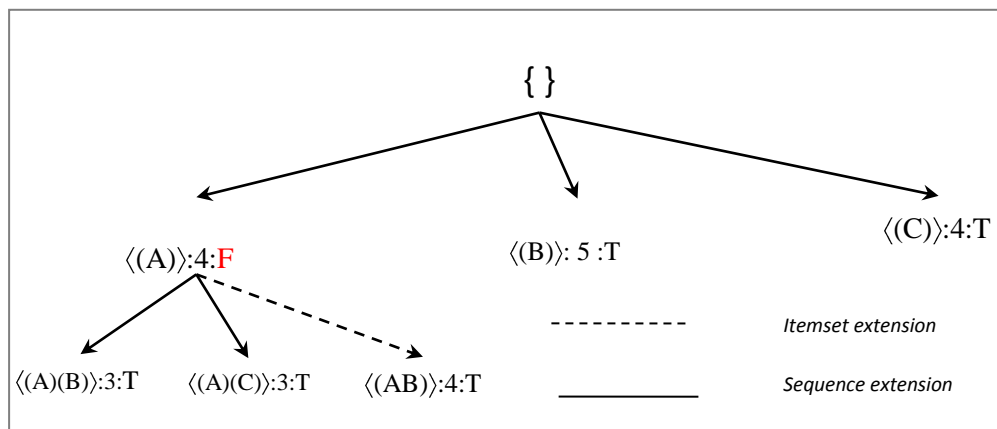


Figure 3. Updating nodes in the prefix tree after the children nodes of  $\langle\langle A \rangle\rangle$  node are created.

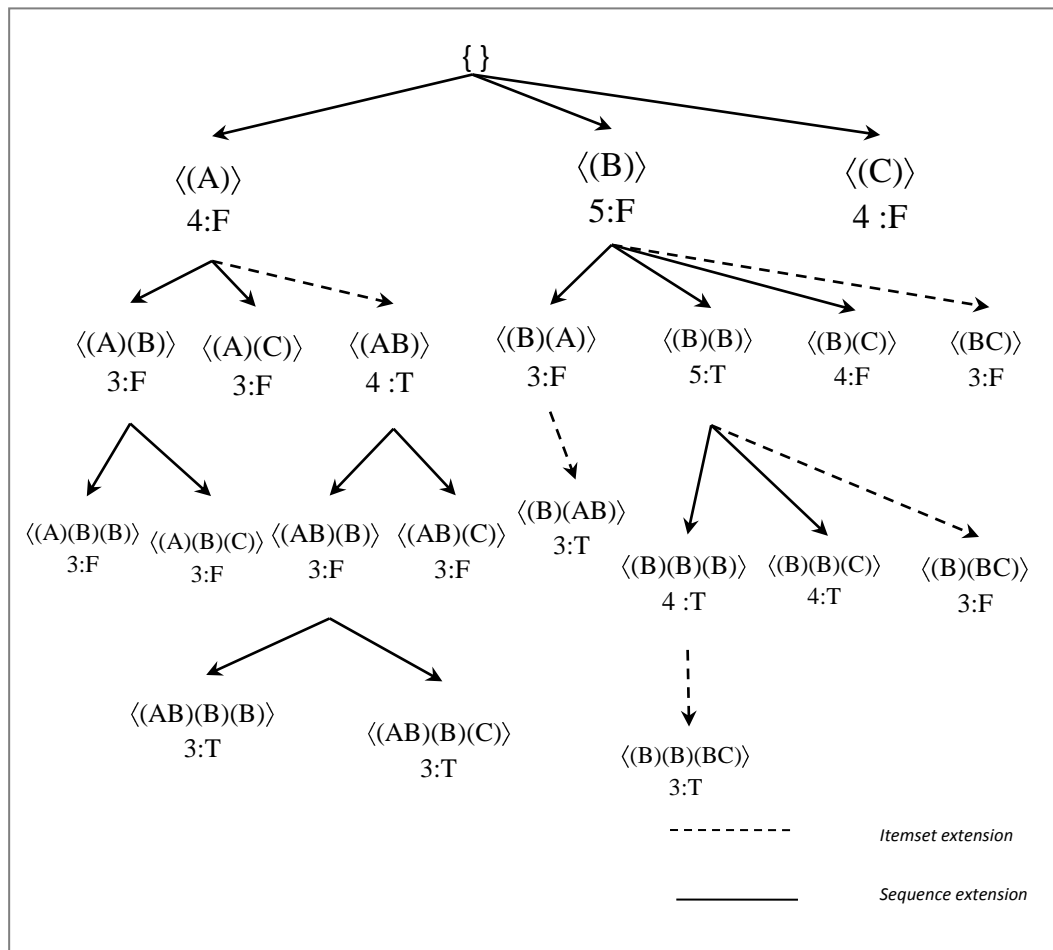


Figure 4. The prefix tree of sequence database in Table 1 with  $minSup = 50\%$ , which has the corresponding  $IsCSP$  values.

### 5. CONCLUSIONS AND FUTURE WORK

This paper proposed an algorithm for mining closed sequential patterns. This algorithm combined the parent–child relationship between nodes on prefix tree and the definition of closed sequential pattern to determine whether a sequential pattern is a closed sequential pattern by adding  $IsCSP$  field into each sequential pattern node on the prefix tree. The algorithm also applied the prime block encoding approach and the join operations over the prime blocks in [3] for generating candidate sequences and determining the frequency for each candidate. Experimental results are examined on the small sequence database also showed that the number of closed sequential patterns is much smaller than that of the sequential patterns.

In future, based on this algorithm, we will perform more experimental results with the larger sequence databases such as synthetic and real databases, namely C6T5S4I4N1kD1k, Chess, and Mushroom and so on. C6T5S4I4N1kD1k was generated using the synthetic data generator developed by IBM to mimic transactions in a retail environment Chess and Mushroom databases were downloaded from <http://fimi.ua.ac.be/data/>. Besides, the lattice-base approach has been proposed for mining association rules and classification association rules in recent

years [5, 13]. We will study how to apply this approach for mining sequential patterns and sequential rules in the future.

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