STABILIZATION CONTROL OF THE DIFFERENTIAL MOBILE ROBOT USING LYAPUNOV FUNCTION AND EXTENDED KALMAN FILTER

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ABSTRACT

This paper presents the design of a novel control model to navigate the differential mobile robot to reach the desired destination from an arbitrary initial pose. The designed model is divided into two stages: the state estimation and the stabilization control. In the state estimation, an extended Kalman filter is employed to optimally combine the information from the system dynamics and measurements. Two Lyapunov functions are constructed that allow a hybrid feedback control law to execute the robot movements. The asymptotical stability and robustness of the closed loop system are assured. Simulations and experiments are carried out to validate the effectiveness and applicability of the proposed approach.

Keywords: robot stabilization control, Kalman filter, Lyapunov function, mobile robot control

1. INTRODUCTION

Reliable navigation is the key problem in autonomous mobile robotics and it can be split into two categories corresponding to indoor and outdoor environments [1 - 5]. Success in navigation requires success at the four building blocks of navigation: perception, the robot must interpret its sensors to extract meaningful data; localization, the robot must determine its position in the environment; cognition, the robot must decide how to act to achieve its goals; and motion control, the robot must modulate its motor outputs to achieve the desired trajectory [6].

Of these four components, motion control has received great research attention due to the challenge in robot model. In general, the dynamics and kinematics of the mobile robot are nonlinear and nonholonomic (a system whose state depends on the path taken to achieve it) and consist of uncertainty parameters. The design of controller, therefore, requires nonlinear and statistical approaches. A number of methods to stabilize a nonholonomic system via feedback control have been proposed in the literature [7 - 12]. Most papers, however, assumed ideal condition in which there are no disturbances on the mobile robot system. In [12], e.g., feedback laws that globally exponentially stabilize the mobile robot with no input disturbances and measurement noises were proposed. In [7] globally stabilizing time-varying feedbacks for

nonholonomic systems were derived via introducing chain form systems to model the kinematics of the mobile robot. Although a discussion of their convergence properties was performed, the case was, however for ideal conditions. In practice, the noises arisen from the system kinematics and measurement devices are unavoidable so the robustness issue against actuator disturbances and measurement noises deserves further attention. During the past decade, several methods have been proposed to study the robust stabilization of the nonholonomic system by using the Lyapunov stability theory [13 - 16]. In [15], an adaptive sliding-mode dynamic controller for wheeled mobile robots was designed and implemented. It is worth to note that, the authors of [13] and [14] have introduced the navigation variables, that were transformed from configuration variables; namely: the distance from the robot frame to the target frame, the angle between the robot-to-target vector and the target frame, and the angle between the robot-to-target vector and the current vehicle orientation. With those definitions, a stabilization control method that provides a fast and natural performance path could be conducted.

We have developed a multi sensor robot with use of an Extended Kalman Filter for the purpose of localization [17, 18]. It becomes interesting to apply navigation variables, following the approach of [13, 14], to improve the stabilization of our robot and the preliminary simulation results of such investigation have been reported in [19]. In the present paper, after introducing with more details the kinematics equations we aim to present stabilization research by both simulation and experiment. In order to make the paper more complete, we arrange it as follows. Details of the control problem are described in Section II. The algorithm for state estimation using EKF is explained in Section III. Section IV introduces the design and implementation of the stabilization controller. Simulations and experiments are presented in section V. The paper concludes with an evaluation of the system, with respect to its strengths.

2. PROBLEM FORMULATION

Consider the scenario shown in Fig.1a, with an arbitrary position and orientation of the robot and a predefined goal position and orientation. The actual pose error vector between the initial and the final configuration given in the robot reference frame is e $= {}^{R}[x, y, \theta]^{T}$ with x, y and θ being the goal coordinates of the robot.

The task of the controller layout is to find control laws, if it exists, of the translational and the angular velocity such that the error e is driven toward zero, $\lim_{t\to\infty} e(t) = 0$
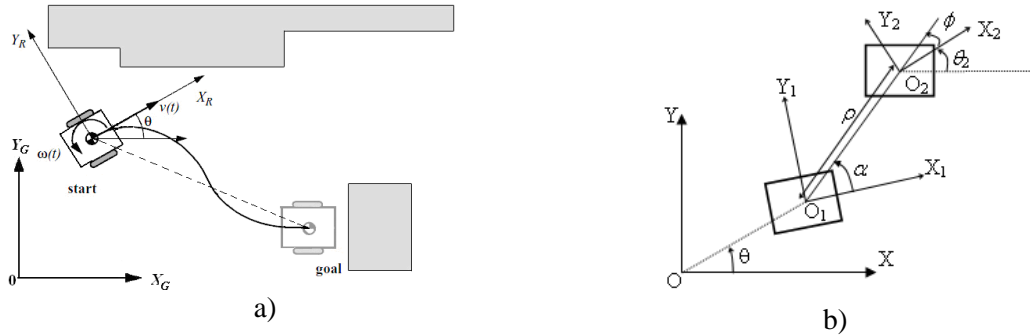

Figure 1. (a) The goal of the controller; (b) The robot poses and parameters

To formulate the problem in more details, we consider the two wheeled, differential-drive mobile robot with non-slipping and pure rolling. The kinematics of the described robot is given as (1). The kinematic equations in the navigation variables domain (ρ, α, ϕ) are written as (2).

$$\dot{X} = v\cos\theta$$
$$\dot{Y} = v\sin\theta \qquad (1)$$
$$\dot{\theta} = \omega$$

$$\dot{\rho} = -v\cos\alpha$$
$$\dot{\alpha} = -\omega + v\frac{\sin\alpha}{\rho} \qquad (2)$$
$$\dot{\phi} = v\frac{\sin\alpha}{\rho}$$

where ω and v are the control inputs which are, respectively, the rotational angular and the translational speed of the robot; X, Y, θ are the coordinates and the orientation of the robot in the global coordinate frame *OXY*.

Let ($\hat{X}, \hat{Y}, \hat{\theta}$) and ($X, Y, \theta$) be, respectively, the estimate and the real poses of the robot. Let (ε_X, ε_Y, ε_θ) be the estimate noises of the robot pose (X, Y, θ). The estimate values of the position ($\hat{X}, \hat{Y}$) and orientation $\hat{\theta}$ are defined as follows: $\hat{X} = X + \varepsilon_X$, $\hat{Y} = Y + \varepsilon_Y$, $\hat{\theta} = \theta + \varepsilon_\theta$ where $|\varepsilon_X| \leq \|\varepsilon_X\|$, $|\varepsilon_Y| \leq \|\varepsilon_Y\|$, $|\varepsilon_\theta| \leq \|\varepsilon_\theta\|$ are the absolute maximum values of the measurement noises of the position ($\hat{X}, \hat{Y}$) and orientation $\hat{\theta}$, respectively. Let $\varepsilon_\rho, \varepsilon_\alpha, \varepsilon_\phi$ denote the state feedback disturbances of the navigation variables (ρ, α, ϕ):

$$\varepsilon_\rho = \sqrt{\left(X_2 - \hat{X}\right)^2 + \left(Y_2 - \hat{Y}\right)^2} - \sqrt{\left(X_2 - X\right)^2 + \left(Y_2 - Y\right)^2}$$

$$\varepsilon_\phi = \sqrt{\left(X_2 - \hat{X}\right)^2 + \left(Y_2 - \hat{Y}\right)^2} - \sqrt{\left(X_2 - X\right)^2 + \left(Y_2 - Y\right)^2} \qquad (3)$$

$$\varepsilon_\alpha = \varepsilon_\phi - \varepsilon_\theta$$

The estimate values of the navigation variables (ρ, α, ϕ) are:

$$\hat{\rho} = \sqrt{\left(X_2 - \hat{X}\right)^2 + \left(Y_2 - \hat{Y}\right)^2}$$

$$\hat{\phi} = \operatorname{atan2}\left(Y_2 - \hat{Y}, X_2 - \hat{X}\right) - \theta_2 \qquad (4)$$

$$\hat{\alpha} = \operatorname{atan2}\left(Y_2 - \hat{Y}, X_2 - \hat{X}\right) - \hat{\theta}$$

The input disturbances of the translational and angular velocities are defined by ε_v, ε_ω; $\varepsilon_v \leq \|\varepsilon_v\|, \varepsilon_\omega \leq \|\varepsilon_\omega\|$, where $\|\varepsilon_v\|, \|\varepsilon_\omega\|$ are the absolute maximum of the input disturbances. With the existing of input disturbances, (2) is rewritten as follows:

$$\dot{\rho} = -\left(v + \varepsilon_v\right)\cos\hat{\alpha}$$

$$\dot{\alpha} = -\left(\omega + \varepsilon_\omega\right) + \left(v + \varepsilon_v\right)\frac{\sin\hat{\alpha}}{\rho} \qquad (5)$$

$$\dot{\phi} = \left(v + \varepsilon_v\right)\frac{\sin\hat{\alpha}}{\rho}$$

without loss of generality, we assume that the goal desired configuration of the system is $(X_d, Y_d, \theta_d) = (0, 0, 0)$ which can also be expressed by $(\rho_d, \alpha_d, \phi_d) = (0, 0, 0)$. The aim of the paper

is to establish a stabilization control law for the mobile robot that is robust against input disturbances and measurement noises.

3. SYSTEM STATE ESTIMATION

Our approach for the proposed problem is the development of a closed-loop controller in which the feedback state are estimated by using an extended Kalman filter (EKF). The control law is then derived by constructing appropriate Lyapunov functions with constraints that asymptotically stabilize the system. Fig. 2 shows the diagram of the controller.
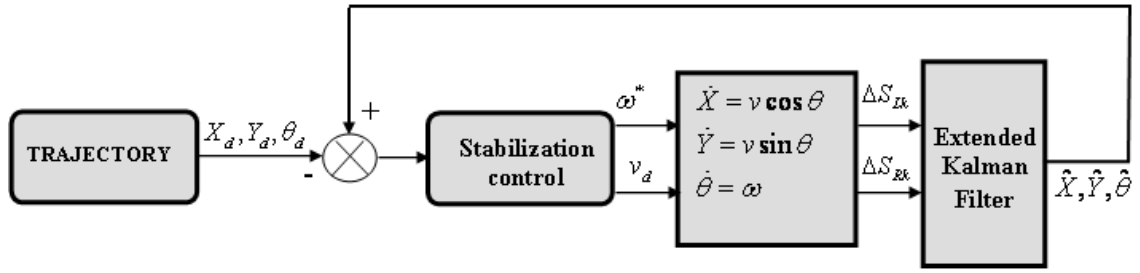

Figure 2. The control model

The design of the stabilization control block will be discussed in the next section. In this section, details of the state estimation using EKF is presented.

During one sampling period Δt, the rotational speed of the left and right wheels ω_L and ω_R create corresponding increment distances Δs_L and Δs_R traveled by the left and right wheels of the robot, respectively:

$$\Delta s_L = \Delta t R \omega_L \qquad \Delta s_R = \Delta t R \omega_R \tag{6}$$

These can be translated to the linear incremental displacement of the robot's center Δs and the robot's orientation angle $\Delta\theta$:

$$\Delta s = \frac{\Delta s_L + \Delta s_R}{2} \qquad \Delta\theta = \frac{\Delta s_R - \Delta s_L}{L} \tag{7}$$

The coordinates of the robot at time $k+1$ in the global coordinate frame can be then updated by:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} \Delta s_k \cos\left(\theta_k + \Delta\theta_k / 2\right) \\ \Delta s_k \sin\left(\theta_k + \Delta\theta_k / 2\right) \\ \Delta\theta_k \end{bmatrix} \tag{8}$$

Let $\mathbf{x} = [x\, y\, \theta]^T$ be the state vector. This state can be observed by some absolute measurements, $\mathbf{z}$. These measurements are described by a nonlinear function, h, of the robot coordinates and a measurement noise, $\mathbf{v}$. Denoting the function (8) as f, with an input vector $\mathbf{u}$, the robot can be described by:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k)$$
$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k)$$

(9)

where the random variables $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the process and measurement noise, respectively. They are assumed to be white noises, independent to each other and with normal probability distributions: $\mathbf{w}_k \sim \mathbf{N}(0, \mathbf{Q}_k)$ $\mathbf{v}_k \sim \mathbf{N}(0, \mathbf{R}_k)$ $E(\mathbf{w}_i \mathbf{v}_j^T) = 0$

The steps to calculate the EKF are then realized as below:

1. Prediction step with time update equations:

$$\hat{\mathbf{x}}_k^- = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_{k-1}, \mathbf{0})$$

(10)

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{W}_k \mathbf{Q}_{k-1} \mathbf{W}_k^T$$

(11)

where $\hat{\mathbf{x}}_k^- \in \Re^n$ is the *priori* state estimate at step k given knowledge of the process prior to step k, $\hat{\mathbf{P}}_k^-$ denotes the covariance matrix of the state-prediction error, $\mathbf{A}_k$ is the Jacobian matrix of partial derivates of f to x; $\mathbf{W}_k$ is the Jacobian matrix of partial derivates of f to w. $\mathbf{Q}_{k-1}$ is the input-noise covariance matrix which depends on the standard deviations of noise of the right-wheel rotational speed and the left-wheel rotational speed.

2. Correction step with measurement update equations:

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

(12)

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k \left(\mathbf{z}_k - \mathbf{h}\left(\hat{\mathbf{x}}_k^- \right) \right)$$

(13)

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-$$

(14)

where $\hat{\mathbf{x}}_k \in \Re^n$ is the *posteriori* state estimation at step k given measurement $\mathbf{z}_k$, $\mathbf{K}_k$ is the Kalman gain, $\mathbf{H}$ is the Jacobian matrix of partial derivates of h to x, $\mathbf{R}_k$ is the covariance matrix of measurement noise.

From (12), (13), (14), the estimated state is better than the raw measurement data and the variation in the estimation is reduced in each step to reach a stationary state. This estimation is the input for equations (5) and is essential for the controller design.

4. CONTROLLER DESIGN

Let $\Omega = \{(X, Y, \theta) : \rho, \alpha, \phi \in R\}$ be the set of all accessible configurations of the robot in the configuration space. Let $\Omega_l = \left\{ (X, Y, \theta) : \rho(X, Y) < \varepsilon_P \cap |\phi(X, Y) - \alpha(X, Y, \theta)| < \varepsilon_\theta \right\}$ be defined as the local configuration set of the robot close to the goal configuration. Let $\Omega_g = \Omega - \Omega_l$ be the global configuration set of the robot distant from the goal configuration. In this section, we derive the control law for the motion control in the global and local configurations.

4.1. Stable control in the global configuration

Let the Lyapunov function for the global configuration set be given by

$$V_g = V_{g1} + V_{g2} = \frac{\rho^2}{2} + \frac{\left(\alpha^2 + h\phi^2\right)}{2} > 0 \tag{15}$$

From (15), V_g is always positive. If we find the constraint of the inputs $v(t)$ and $\omega(t)$ so that $\dot{V}_g = \dot{V}_{g1} + \dot{V}_{g2}$ is always negative then the system is asymptotically stable and the control law successfully drives the robot to the destination.

Let $v = k_v \rho \cos\alpha$, the term $\dot{V}_{g1}$ becomes

$$\begin{aligned}
\dot{V}_{g1} = \rho\dot{\rho} &= -k_v\rho^2 \cos^2\alpha + k_v\rho^2\varepsilon_\alpha \cos\alpha\sin\alpha - \rho\varepsilon_v \cos\alpha \\
&\leq -k_v\rho^2 \cos^2\alpha + k_v\rho^2 \|\varepsilon_\alpha\|.|\cos\alpha\sin\alpha| + \rho\|\varepsilon_v\|.|\cos\alpha|
\end{aligned} \tag{16}$$

We can choose a sufficiently large gain K_v so that $k_v\rho^2 \cos^2\alpha$ is much more dominant than the terms $k_v\rho^2 \|\varepsilon_\alpha\|.|\cos\alpha\sin\alpha|$ and $\rho\|\varepsilon_v\|.|\cos\alpha|$. (16) become $\dot{V}_{g1} \leq 0$ in the region of Ω_g which implies that the term V_{g1} converges to a nonnegative finite limit. Consider the term $\dot{V}_{g2}$:

$$\dot{V}_{g2} = \alpha\left[-\omega - \varepsilon_\omega + \left(\frac{k_v\rho\cos\alpha\sin\alpha}{\alpha} + \frac{k_v\rho\varepsilon_\alpha \cos^2\alpha}{\alpha} + \frac{\varepsilon_v \sin\alpha}{\alpha}\right)\frac{(\alpha + h\phi)}{\rho + \varepsilon_\rho}\right] \tag{17}$$

Let $\omega = k_\alpha\alpha + \frac{k_v\rho\cos\alpha\sin\alpha}{\alpha}\frac{(\alpha + h\phi)}{\rho}$, the term $\dot{V}_{g2}$ becomes:

$$\dot{V}_{g2} = -k_\alpha\alpha^2 - \alpha\varepsilon_\omega - \frac{k_v(\alpha + h\phi)\varepsilon_\rho}{2(\rho + \varepsilon_\rho)}\sin 2\alpha + \frac{k_v\rho(\alpha + h\phi)\varepsilon_\alpha}{\rho + \varepsilon_\rho}\cos^2\alpha + \frac{\varepsilon_v(\alpha + h\phi)\sin\alpha}{\rho + \varepsilon_\rho} \tag{18}$$

The term $k_\alpha\alpha^2$ is much more dominant than the terms $|\alpha\varepsilon_\omega|$, $\left|\frac{k_v(\alpha + h\phi)\varepsilon_\rho}{2(\rho + \varepsilon_\rho)}\sin 2\alpha\right|$,

$\left|\frac{k_v\rho(\alpha + h\phi)\varepsilon_\alpha}{\rho + \varepsilon_\rho}\cos^2\alpha\right|$ and $\left|\frac{\varepsilon_v(\alpha + h\phi)\sin\alpha}{\rho + \varepsilon_\rho}\right|$ so $\dot{V}_{g2} \leq 0$.

We have shown that when the robot is in the configuration set Ω_g, the derivative of the Lyapunov function $\dot{V}_g \leq 0$ becomes semi-definite negative. As a result, by using the control law (19), the robot, which initially starts from the global configuration set Ω_g, will be rendered to the local configuration set Ω_l. The control law in the global configuration set Ω_g is rewritten as follows:

$$v = k_v\rho\cos\alpha \qquad \omega = k_\alpha\alpha + \frac{k_v\rho\cos\alpha\sin\alpha}{\alpha}\frac{(\alpha + h\phi)}{\rho} \tag{19}$$

4.2. Stable control in the local configuration

The control law (19) is asymptotically stable in the global configuration Ω_g. It, however, is not stable in the local configuration Ω_l. This can be proven as follows.

Assume that the navigation variable ρ goes to small parameters $\varepsilon_p > \|\varepsilon_v\|/k_v$. The variables (α, ϕ) go to their small disturbances ($\varepsilon_\alpha, \varepsilon_\phi$). The system kinematics (5) becomes:

$$\dot{\rho} = -k_v \varepsilon_P + \varepsilon_v$$

$$\dot{\alpha} = \left[-k_\alpha - \frac{k_v h \phi}{\alpha} - \frac{\varepsilon_\omega}{\alpha} + \frac{-k_v \varepsilon_\rho + \varepsilon_v}{(\varepsilon_P + \varepsilon_\rho)} + \frac{(k_v \varepsilon_P + \varepsilon_v)\varepsilon_\alpha}{(\varepsilon_P + \varepsilon_\rho)\alpha} \right] \alpha \tag{20}$$

$$\dot{\phi} = (k_v \varepsilon_P + \varepsilon_v) \frac{\alpha + \varepsilon_\alpha}{(\varepsilon_P + \varepsilon_\rho)}$$

Because $\varepsilon_P > \|\varepsilon_v\|/k_v$, we get

$$\dot{V}_{g1} = -k_v \rho^2 \cos^2 \alpha + k_v \rho^2 \varepsilon_\alpha \cos \alpha \sin \alpha - \rho \varepsilon_v \cos \alpha \le -k_v \varepsilon_P^2 + \varepsilon_P \|\varepsilon_v\| \le 0 \tag{21}$$

In (21), V_{g1} is bounded; thus, ρ is also bounded. However, when $\left(-k_\alpha - \dfrac{k_v h \phi}{\alpha} - \dfrac{\varepsilon_\omega}{\alpha} + \dfrac{-k_v \varepsilon_\rho + \varepsilon_v}{(\varepsilon_P + \varepsilon_\rho)} + \dfrac{(k_v \varepsilon_P + \varepsilon_v)\varepsilon_\alpha}{(\varepsilon_P + \varepsilon_\rho)\alpha} \right) > 0$ then α diverges from zero causing the system to be unstable. In the rest of the paper, we will re-design the control law to obtain the robustness property of the closed loop system.

For the local configuration set Ω_l, let a Lyapunov function be given as:

$$V_l = \frac{\rho^2}{2} + \frac{(\theta - \theta_d)^2}{2} > 0 \tag{22}$$

Let $\theta_e = \theta - \theta_d$ and $\dot{\theta}_e = \dot{\theta} - \dot{\theta}_d = \omega$, the derivative of V_l becomes:

$$\dot{V}_l = \rho \dot{\rho} + \theta_e \dot{\theta}_e = v \rho \cos \alpha + \theta_e \omega \tag{23}$$

Let the control law of the for the local configuration set Ω_l be given as follows:

$$v = -k_v \rho \cos \alpha \qquad \omega = -k_\theta \theta_e \tag{24}$$

where k_v, k_θ is positive gains. The term $\dot{V}_l$ becomes:

$$\dot{V}_l = -k_v \rho^2 \cos^2 \alpha - k_\theta \theta_e^2 \le 0 \tag{25}$$

This implies that the configuration of the robot will not escape to the larger values of ($\varepsilon_p, \varepsilon_\alpha, \varepsilon_\phi$) when the robot configuration is in the set Ω_l and the system is again stable.

5. SIMULATION AND EXPERIMENTS

To evaluate the functioning operation of the EKF-based state estimation and the stabilization controller, several simulations and experiments have been conducted.

5.1. Simulation results

Simulations were carried out in MATLAB in which the parameters were extracted from the real system [18]. The behaviors of the control law derived from the Lyapunov function in both configuration sets Ω_g and Ω_l were investigated. In the simulations, the initial configuration of the robot is *(0,0,0)* and the goal configurations are *(2,2,30⁰)*, *(2,2,60⁰)* and *(2,2,90⁰)*, correspondingly. The absolute maximum values of measurement noises and input disturbances are as follows: $\varepsilon_p = \varepsilon_\phi = \varepsilon_\alpha = 0.001$; $\varepsilon_v = \varepsilon_\omega = 0.001$; the parameters for the controller are set as: $k_v = 10$, $k_\phi = 100$. Figure 3 shows the simulation results in which the final configurations of the robot are converged to the position (2,2) from three different directions. This implies the success of the controller.
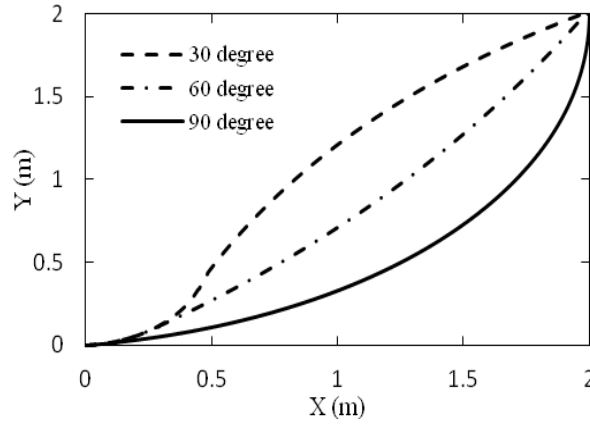

Figure 3. Simulation trajectories with the assumption of random measurement noises and input disturbances

5.2. Experimental results

A) Experimental setup

Experiments on a real mobile robot are implemented in a rectangular shaped flat-wall environment constructed from several wooden plates surrounded by a cement wall. The robot is a two wheeled, differential-drive mobile robot. Its wheel diameter is 10 cm and the distance between two drive wheels is 60 cm. The drive motors are controlled by microprocessor-based electronic circuits. Due to the critical requirement of accurate speed control, the PID algorithm is implemented. The stability of motor speed checked by a measuring program written by LABVIEW is ± 5 %. In case of straight moving, the speed of both wheels is set to 0.3 m/s. In turning, the speed of one wheel is reduced to 0.05 m/s in order to force the robot to turn to that wheel direction. The sensors employed as measurements include a compass sensor and a laser range finder. The compass sensor has the accuracy of 0.1⁰. The LRF has the accuracy of 30 mm in distance and 0.25⁰ in deflect angle. The sampling time ΔT of the EKF is 100 ms.

B) State estimate evaluation

The state estimation was experimentally evaluated in our previous work [18]. Different configurations of the EKF were implemented and it was concluded that the EKF algorithm improved the robot localization and the combination of all available sensors gave the optimal result.

C) Stablization control

In this experiment, we evaluate the applicability of the proposed controller in a real autonomous navigation application. The goal is to navigate the mobile robot from the starting point $(0,0,0)$ to, respectively, reach the following destinations: $(2,2,30^0)$, $(2,2,60^0)$, $(2,2,90^0)$. Figure 4 describes the trajectories of the robot and Fig. 5 presents the tangent and angular velocities of the robot during the operation. The physically reaching the destinations (Fig. 4) and the convergence of tangent and angular velocities toward zero (Fig. 5) in all experiments imply the asymptotic stabilization of the designed controller. The matching between the experiment (Fig. 4) and simulation (Fig. 3) trajectories emphasizes the correctness of our approach. It is also noted that the convergence time among experiments are almost identical (around 70 seconds). This implies the reliability of the system operation.
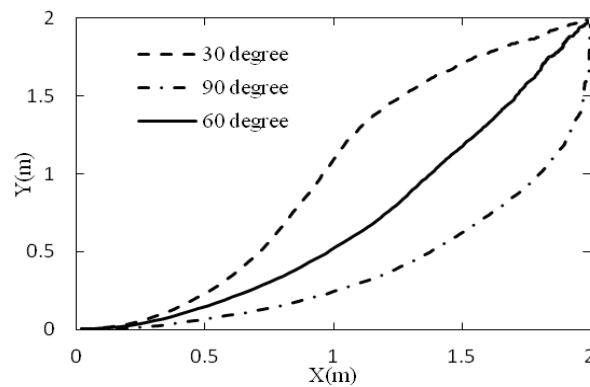

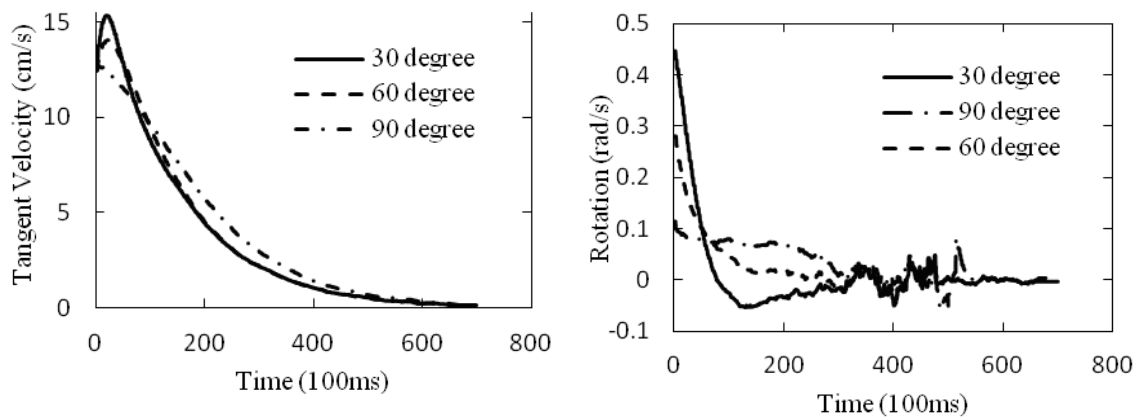*Figure 4.* Operation trajectories of the robot

Figure 5. Operation variation in tangent velocity (left) and direction (right)

In experiments, the real trajectory of the robot depends on the determination of control parameters α, λ, h. Consequently they can be tuned to be suitable to specific applications. In addition, the design of a hybrid controller in combination with an EKF-based estimator could ensure the robustness of the system in noise conditions. This is also the main contribution of our work.

6. CONCLUSION

In this paper, a new controller for the stabilization problem of mobile robot in the presence of system noise and measurement disturbances is proposed. A state estimation algorithm using EKF is implemented in which the knowledge of the system dynamics and the measurement information are combined in an optimal manner. Two Lyapunov functions corresponding to each subset configuration of the mobile robot is constructed and the control law is derived. The asymptotical stabilization of the system is theoretically analyzed and proven. Simulations and experiments confirm the validity of the proposed approach.

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TÓM TẮT

ĐIỀU KHIỂN ỔN ĐỊNH CHO ROBOT DI ĐỘNG HAI BÁNH VI SAI SỬ DỤNG HÀM LYAPUNOV VÀ BỘ LỌC KALMAN MỞ RỘNG

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Bài báo trình bày một mô hình bộ điều khiển mới để điều khiển ổn định cho robot di động có hai bánh xe vi sai di chuyển đi đến một tọa độ đích mong muốn, từ một vị trí và hướng ban đầu bất kỳ của robot. Mô hình bộ điều khiển được chia thành hai giai đoạn: ước tính trạng thái và điều khiển ổn định. Trong giai đoạn ước tính trạng thái, một bộ lọc Kalman mở rộng được áp dụng để tổng hợp một cách tối ưu các thông tin từ hệ thống động lực và các phép đo. Hai hàm Lyapunov sau đó được thiết kế để xây dựng luật điều khiển phản hồi điều khiển chuyển động của robot. Sự ổn định tiệm cận của hệ thống điều khiển vòng kín được chứng minh về mặt lý thuyết. Nhiều mô phỏng và thực nghiệm đã được thực hiện để chứng minh tính hiệu quả và khả năng ứng dụng của phương pháp đã đề xuất.

Từ khóa: điều khiển ổn định Robot, bộ lọc Kalman, hàm Lyapunov, robot di động.