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THE ROLE OF THE MATERIAL FRACTURE PROPERTIES IN THE SIZE-EFFECT LAW OF CONCRETE STRUCTURES

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ABSTRACT. The size effect of the nominal stress at failure in concrete structures is dealt with in general. An existence of a rather large fracture process zone in front of crack tip is proved to be the main reason leading to the size effect of the nominal strength. On the basis of the new general size-effect law and numerical results of fracture propagation, a particularly proposed size effect law for beams in bending is developed, in which the role of each material fracture characteristic, especially the shape of the stress - crack opening curve, is elaborated clearly.

1. INTRODUCTION

It has been proved that the fracture process zone (the micro crack zone) existing in front of a crack tip is to be considered as the main factor causing the nonlinear fracture in concrete structures. Furthermore, the crack tip front-blunting phenomenon (Bazant, [2]) has caused deviations of the well-known law of structural size effect from the linear elastic fracture mechanics (LEFM). The size-effect law discovered by Bazant in 1984 is mainly based on the arguments resulting from the strength criterion and the linear elastic fracture theory. In the strength criterion, failure takes place when the calculated stress equals the ultimate tensile strength. In the mean time, the failure criterion based on the critical energy release rate is considered to be constant during the crack propagation. The nominal strength, of a loaded element, obtained from the strength criterion is constant, regardless of the structural size. The LEFM classical solution provides an inverse proportionality between the calculated stress at failure and the structural size. Bazant argued that when the structural size is relative by small failure takes place according to the strength criterion theory. On the other hand, when the structural size is relative by large, the obtained nominal strength is rather in accordance with that derived from ELFM theory. This means that the size-effect curve of the nominal strength approaches a straight inclined line (downward slope of $-1/2$ in the double-logarithmic coordinates) derived from the LEFM solution as the structural size is rather large.

Formally, according to Bazant, the obtained curve describing the size effect of the nominal strength in nonlinear fracture materials has a gradual transitions from a horizontal line determined from the strength theory to an inclined line derived from the strain energy release rate criterion in LEFM as in Fig. 1. Many previous investigators have confirmed this curve by the test data or numerically (see [1, 2, 7 and 14]). Theoretically, Bazant, [2] proposed the following equation:

$$\frac{f_n}{f_t} = \beta\Big(1 + \frac{d}{\lambda_0 d_a}\Big)^{-1/2} \tag{1.1}$$

where f_n is the nominal stress at failure, f_t - ultimate tensile strength, d - beam depth, d_a - width of the fracture process zone, β and λ_0 - empirical coefficients determined from specimens of the similar structural configuration, but differing only in the size. Therefore β and λ_0 (they may be replaced by $\lambda = \lambda_0 d_a$) depend on the structural configuration, the loading condition and, especially, the material fracture properties. Eq (1.1) has been considered to be the size-effect law in the nonlinear fracture mechanics. Bazant, [2] attempted to demonstrate the authenticity of Eq (1.1) with the help of the previous test data sets. However it has been seen that (see Ref. [2]) the dispersion is too big. It was explained by Bazant that the test data sets derived from different laboratories were not available for the demonstration. It may be true due to the following set-backs:

1) The empirical coefficients in Eq. (1.1) depend on many factors, especially on the material's fracture properties. These coefficients are not universal for all concrete compositions. This means that the size-effect law may be only described for loaded elements possessing the same material fracture properties. The data used by Bazant for checking the size-effect law do not possess the same fracture properties (although the authors have attempted to transform them to the non-dimension). The size-effect law is universal for concrete only if the functional correlations between the empirical coefficients and the intrinsic fracture properties are determined.

2) This relates to an assumption that as the structural size is relatively large the size effect of the nominal strength behaves similar to the solution obtained from the LEFM theory (σ_n = const/$b^{1/2}$). This limiting size, however, depends very much on the fracture properties of materials. On the other hand, the expression σ_n = const/$b^{1/2}$ is only true when there is no plastic deformation zone close to the crack tip (it relates to an assumption of K_{Ic} = constant). But for most engineering materials, as the size tends to infinity, the nonlinear fracture zone forms, always, around the crack tip (although its size is small). This means that the downward slope of tangents of the size-effect curve (in double-logarithmic coordinates) seems to be larger than $-1/2$ for all structures. Some expressions obtained using the asymptotic analysis and the Weibull's statistical theory were given by Bazant later, [3] and other investigators (Duda and König, [7], Li and Liang, [10]) have confirmed this observation.

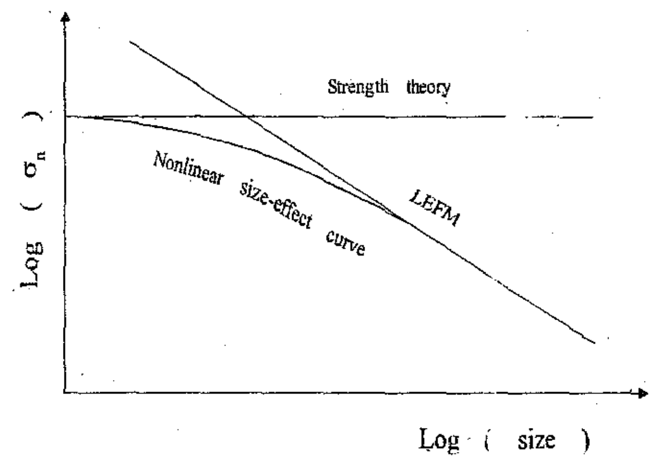

Fig. 1. Assumed size-effect curve on concrete

It is, however, necessary to emphasize that the essential lack of the size-effect law proposed by Bazant, [2] is due to the unknown dependence of the empirical coefficients on the fracture properties of materials as many investigators have mentioned later. In order to derive the general size-effect law for concrete this dependence must be determined. Building up a constitutive relationship between the size effect of the nominal strength and the fracture properties of concrete is the essential topic of this text. The methodology used is numerical analysis combined with dimensional analysis.

A new term known as the shape index (S_T) of the stress - crack opening curve ($\sigma - w$ curve) is briefly described and is used in following the proposed methods of approach. The shape index S_T proposed by Tran Tu and Kasperkiewicz, [14] is one of the parameters describing the $\sigma - w$ curve and determined from the intrinsic fracture properties of materials:

$$S_T = \frac{G_F}{f_t w_c} = \int_0^1 f(x)dx$$
$$y = f(x), \quad y = \frac{\sigma}{f_t}, \quad x = \frac{w}{w_c} \tag{1.2}$$

where σ is the tensile stress in the fracture process zone, f_t - ultimate tensile strength, w_c - critical crack opening displacement, G_F - fracture energy. For elastic deformation materials S_T tends to infinity, for materials with the plastic deformation taking place close to the crack tip, S_T is equal to 1, and $S_T = 0.5$ when the $\sigma - w$ curve is mono-straight.

2. GENERAL SIZE-EFFECT LAW

The size-effect law in concrete structures can be explained more clearly by analyzing the existence of the fracture process zone in front of the crack tip. Let us consider the crack plate under mode-I loading. Within the elastic region, the stress along the axis $y = 0$ is given by:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \tag{2.1}$$

The fracture process zone (FPZ) with the length r_y in front of a crack tip is replaced by a fictitious crack and the action of the cohesive stress $\sigma(x)$, (Fig. 2). The equilibrium condition along the axis $y = 0$ determines that

$$\int_0^{r_y} \frac{K_I}{\sqrt{2\pi r}} dr = \int_0^{r_y} \sigma(x)dx = r_y g(\lambda, b, \ell_{ch}, S_T) f_t \tag{2.2}$$

where $g(\lambda, b, \ell_{ch}, S_T) = S^{1/2}$ is obtained from the average integral principle, that is a function of the structural configuration (λ), of the structural size (b) and of the material fracture properties (ℓ_{ch}, S_T), S_T - shape index of the $\sigma - w$ curve and ℓ_{ch} - characteristic length of materials. After integration, the FPZ length r_y is determined as follows

$$r_y = \frac{2K_I^2}{\pi f_t^2 S} \tag{2.3}$$

On the other hand, when FPZ is not very large the stress intensity factor may be calculated as follows

$$K_I = f(\alpha)\sigma_0\sqrt{\pi(a + r_y)} \tag{2.4}$$

where $f(\alpha)$ is a configuration factor depending on the crack length to plate width ratio. Combining Eqs (2.3) and (2.4), the stress intensity factor is determined as

$$K_I^2 = \frac{f(\alpha)\sigma_0^2 \pi a}{1 - \dfrac{2f(\alpha)\sigma_0^2}{f_t^2 S}} \tag{2.5}$$

From Griffith's postulate, [11], a crack becomes unstable and begins to propagate when K_I is equal to K_{Ic}, σ is then considered as f_n, (it is always assumed for the structures that the crack length is rather small compared to the structural size or the structure is large enough). The following equation is derived:

$$\left(\frac{f_n}{f_t}\right)^2 = \frac{1 - \dfrac{2f(\alpha)}{S}\left(\dfrac{f_n}{f_t}\right)^2}{f(\alpha)} \tag{2.6}$$

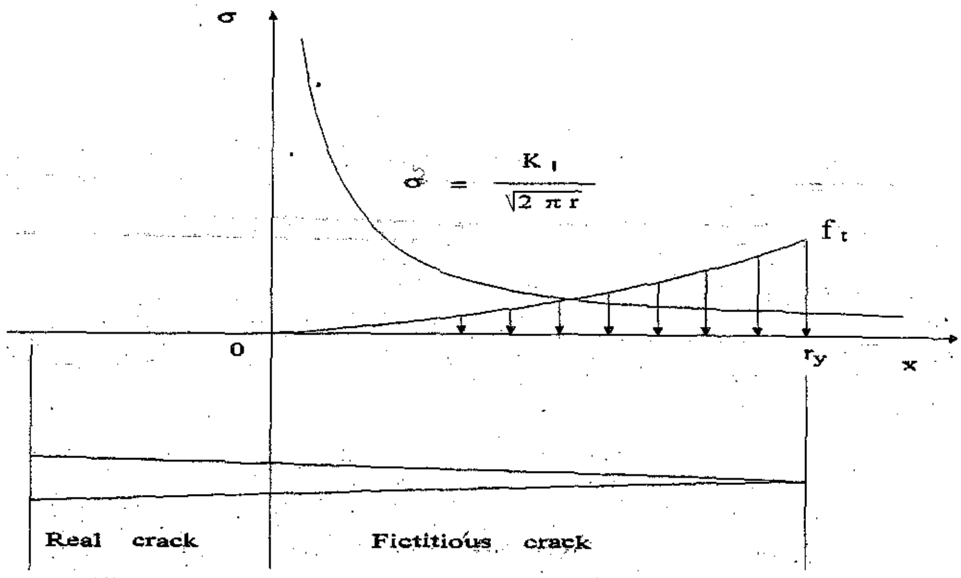

Fig. 2. Stress distribution and the fracture process zone in front of a crack tip

After suitable changes and manipulations we derive the expression similar to that obtained by Bazant:

$$\frac{f_n}{f_t} = A\left[1 + \frac{B}{f(\lambda, b, \ell_{ch}, S_T)}\right]^{-1/2} \tag{2.7}$$

where A and B - coefficients depending on the structural configuration. The function $g(\lambda, b, \ell_{ch}, S_T)$ is determined from Eq. (2.2) having the following properties:

i) Increasing with the increase of ℓ_{ch}, and S_T.

ii) Decreasing with increasing structural size and slender (it is a ratio of structural size normal to the crack growing direction to the remaining structural size).

We can consider Eq. (2.7) under the alternative form depending on the way of choosing the function $g(\lambda, b, \ell_{ch}, S_T)$ by the dimensional analytical method:

$$\frac{f_n}{f_t} = \alpha\left[\gamma + \left(\frac{1}{\lambda}\right)^{n_1}\left(\frac{\ell_{ch}}{b}\right)^{-n_2}\frac{1}{1 - e^{-n_3 S_T}}\right]^{-\beta/2} \tag{2.8}$$

where a, b and γ are coefficients dependent on the structural configuration only. Powers n_1, n_2 and n_3 in Eq. (2.8) are determined by combining the dimensional analysis and the best fit of the test or numerical results.

Eq. (2.8) is considered to be the general constitutive relationship between size-effect of the nominal stresses at failure and the fracture properties of concrete and similar materials. From Eq. (2.8), as well as from Eq. (2.7), we can see that the size effect of the nominal strength of structures made of cohesive materials as concrete is substantial.

3. SIZE EFFECT FOR BEAMS IN BENDING

The numerical analysis of fracture in concrete notched beams in bending (either beams in three-point bending or beams in four-point bending) is carried out here by employing the fictitious crack model, [8] and the finite element method. It is, however, very important for investigating the crack propagation using the fictitious crack model is the creation of the cohesive forces in accordance with the different cases of the $\sigma - w$ relationship. The $\sigma - w$ relation proposed by Tran Tu and Kasperkiewicz, [14] is described by the following equation:

$$\frac{\sigma}{f_t} = (1 - A)(1 - x^k) + A(1 - x)^{1/k} \tag{3.1}$$

where coefficient $A = 0.5$, k depends only on the shape index S_T, [14], the others are indicated above.

The numerical approach with the different shape index of the $\sigma - w$ curve is presented in brief as follows:

1) Calculating the external load and the deflection of a beam in accordance with the LEFM. The criterion of the stress intensity factor is used.

2) Calculating the crack extension, with first assuming the increment of the fictitious crack length to be Δa. The crack extension takes place when the maximum stress at the crack tip equals the tensile strength. The cohesive forces are calculated by steps in accordance with the $\sigma - w$ relation and they are verified so closely that the deviations after two neighboring iterative steps are not greater than 0.01 [N]. The calculation is ended when the total length of the real crack and fictitious crack reaches, approximately, the length of the beam ligament.

The influence of the different shapes of the $\sigma - w$ curve on the load-deflection diagrams for a beam in three-point bending was presented in [14]. It proved that the obtained maximum load from numerical fracture analysis will not be correct if the influence of the shape of the $\sigma - w$ curve is neglected.

The coefficients in (2.8) are determined from the fracture analysis of three hundreds beams with the depth varied from 10 to 1000 [mm], $a/b = 0.20 \div 0.50$, the material fracture properties varied in a broad range as $G_F = 10 - 450[N/m]$, $f_t = 2 \div 5$ [MPa], $E = 20000 \div 45000$ [MPa] and $S_T = 0.1 \div 1$. Based on the best fit of calculated data the powers $n_2 = 1.25$, $n_3 = 0.03$ are obtained, the power n_1 represents the influence of the beam slender on the nominal strength through the crack surface displacement. By the well known equation in LEFM, n_1 equal to 1 is allowed. Therefore Eq (2.8) becomes:

$$\frac{f_n}{f_t} = a_n\left[d_n + W\right]^{-b_n/2} \tag{3.2}$$

$$W = \frac{1}{\lambda}\left(\frac{\ell_{ch}}{b}\right)^{-1.25}\frac{1}{1 - e^{-0.03S_T}}\,,\quad f_n = \frac{6M}{t(b-a)^2}\,,\quad \ell_{ch} = \frac{G_F E}{f_t^2}\,,\quad S_T = \frac{G_F}{f_t w_c}\,,\quad \lambda = \frac{\ell}{b}$$

where M is the bending moment, the coefficients a_n, b_n and d_n depend only on the notched depth given in Table 1 together with the statistical factors estimating the functional correlation.

Table 1

a_0/b	a_n	b_n	d_n	s_y	s_a	s_b
0.20	2.1677	0.4214	0.1937	0.0586	0.0143	0.00163
0.30	2.2555	0.4284	0.2227	0.0559	0.0136	0.00157
0.40	2.3768	0.4298	0.2343	0.0540	0.0130	0.00152
0.50	2.5574	0.4316	0.2392	0.0606	0.0145	0.00171

Eq. (3.2) with the coefficients presented in Table 1 is applied for all beams in bending with different slender and depths. Fig.3 shows the size effect of the nominal strength of beams in bending with the different notched length. To check Eq. (3.2) a series previous data obtained from the test and calculation are presented together with the diagrams representing Eq. (3.2), in this case W changes from 0.01 to 2.5. It is plotted in Fig.4 in the semi-logarithmic coordinates. It proves the good agreement between Eq. (3.2) with the data sets in references, [9, 12, 13]. It is very interesting to note that (3.2) is also true for beams made of metals that the nominal stress at failure is supposed regardless of the beam size. Table 2 shows the results calculated for beams made of metals in three-point bending according to Bucci et al., [6], Zhang and Lin, [15] and those calculated from (3.2).

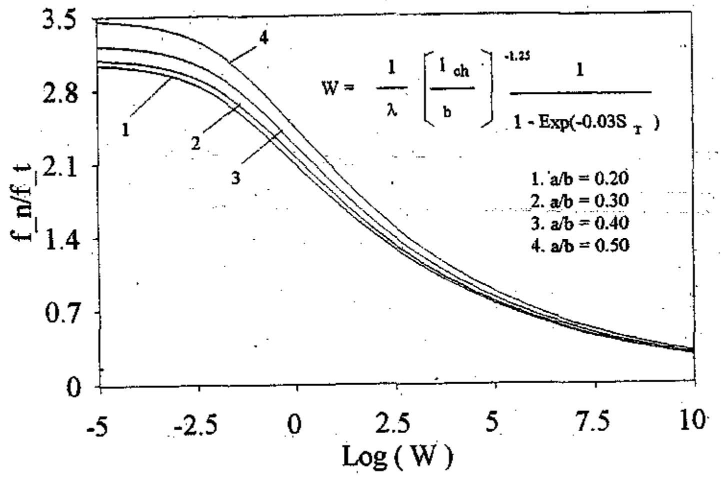

Fig. 3. **General size effect curves for beams in bending**

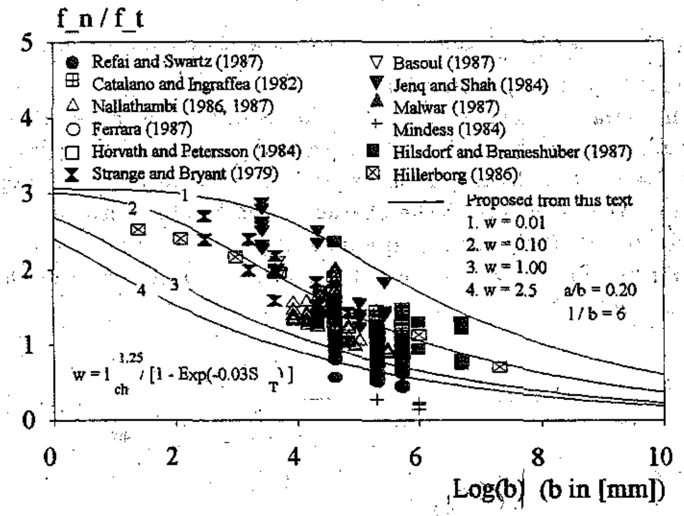

Fig. 4. **Checking the size effect law by test and calculation data**

Table 2 **(for $\lambda = 4$)**

a_0/b	f_n/f_t, [19]	f_n/f_t, [6]	f_n/f_t, Eq. (3.2)
0.20	1.774	2.184	1.957
0.30	1.909	2.184	2.022
0.40	2.020	2.184	2.081
0.50	2.106	2.184	2.165

4. SOME INTERPRETATIONS OF THE PROPOSED SIZE EFFECT LAW

The following presentation is the interpretations of Eq. (3.2). It proves the role of the intrinsic material fracture properties and the notched depth on the size effect law.

Notched sensitivity

Fig. 5 shows the size effect law for the different notched depth. We can see that the notched

sensitivity is rather strong. The larger the notched sensitivity of the nominal strength the smaller the beam size is.

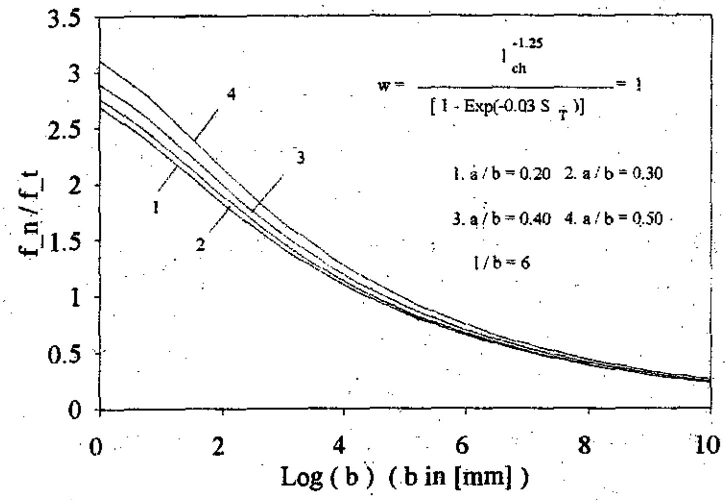

Fig. 5. **Influence of the notched depth on the size effect of the nominal strength**

Role of material fracture properties

The role of the material fracture properties in the size-effect law are dealt with in this chapter. This includes the change of the characteristic length and the shape index. We can clearly see this influence in Fig. 3, in which the change of ℓ_{ch} and S_T are covered by the change of the parameter W. In Fig. 6 plotted is the influence of the characteristic length (with plain concreteit is about 100-400 and may even reach 1000), for a shape index S_T of 0.3. The influence of the shape index S_T is presented in Fig. 6. These figures are self explanatory on the role of the intrinsic fracture properties in the size-effect law.

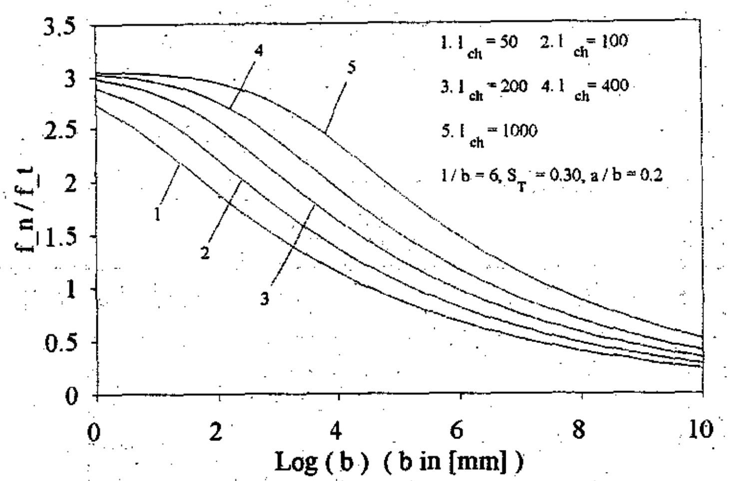

Fig. 6. **The role of ℓ_{ch} in the size effect law**

5. CONCLUSION

1) The size effect of the nominal strength of structures made of concrete like material is explained more clearly than before. It is due to the appearance of large fracture process zone in front of the crack tip.

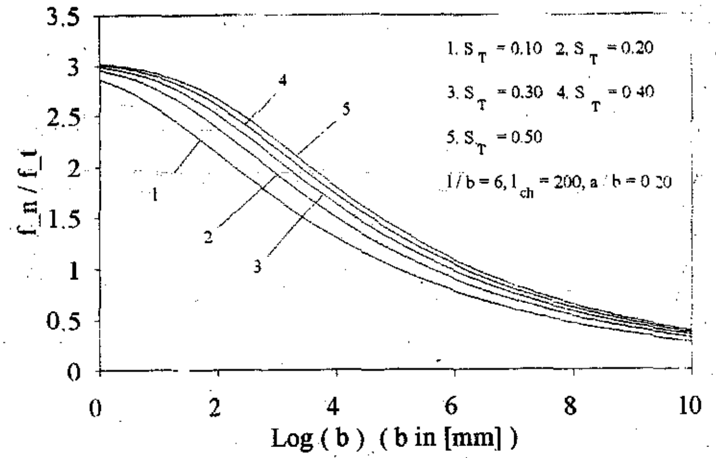

Fig. 7. The role of S_T in the size effect law

2) The size-effect law proposed in this text is a general law for beams in bending made of concrete and similar materials. For other configuration structures the change may be only in the constant coefficients and powers in Eq. (2.8) or it is employed directly from the general equation (2.7).

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VAI TRÒ CỦA CÁC TÍNH CHẤT PHÁ HỦY VẬT LIỆU TRONG QUY LUẬT HIỆU ỨNG KÍCH THƯỚC CỦA KẾT CẤU BÊ TÔNG

Trong bài báo, quy luật hiệu ứng kích thước tổng quát được thành lập trên cơ sở phân tích phá hủy phi tuyến của kết cấu bằng vật liệu bê tông và các vật liệu composite tương tự. Trong công thức tổng quát này, vai trò của các tính chất phá hủy của vật liệu, đặc biệt là hệ số hình dạng của nhánh biến dạng á dòn (Softening branch) đã được thể hiện. Đây là một thành tựu mới hiện nay trong lĩnh vực cơ học phá hủy của vật liệu composite. Bằng phương pháp số tác giả đã áp dụng nghiên cứu cho dầm chịu uốn có tạo vết nứt trước, vật liệu là bê tông hoặc các loại chế tạo từ vữa xi măng. Vai trò của các tính chất phá hủy vật liệu đã được phân tích và minh chứng bằng đồ thị. Những số liệu thực nghiệm của các tác giả ở các phòng thí nghiệm trên thế giới đã xác nhận sự đúng đắn của công thức tổng quát được đưa ra ở bài báo này.

RANDOM OSCILLATION IN SYSTEMS SUBJECT TO ...

(tiếp theo trang 7)

DAO ĐỘNG NGẪU NHIÊN TRONG HỆ CHỊU KÍCH ĐỘNG CỦA MỘT LỚP ỒN MÀU

Trong bài báo phương pháp trung bình hóa ngẫu nhiên cấp cao được phát triển để nghiên cứu sự kích động của một lớp ồn màu có bộ lọc tương ứng chỉ có các giá trị riêng âm hữu hạn và khác nhau. Đã thu được xấp xỉ gần đúng bậc 2 cho hàm mật độ xác suất của hệ Duffing chịu kích động của quá trình ngẫu nhiên tương quan mũ. Các tính toán số đã được tiến hành nhằm xem xét sự phụ thuộc của bình phương trung bình của biên độ theo tham số giải tần của lực kích động.