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Avishek Chakraborty

Said Broumi

Prem Kumar Singh

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Some properties of Pentagonal Neutrosophic Numbers and its Applications in Transportation Problem Environment

Avishek Chakraborty [1, 3], Said Broumi[2*] and Prem Kumar Singh[4]

[1]Department of Basic Science, Narula Institute of Technology, Agarpara, Kolkata-700109, India; Email:avishek.chakraborty@nit.ac.in
[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco; Email: broumisaid78@gmail.com
[3] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, India.
[4]Amity Institute of Information Technology, Amity University, Sector 125-Noida, Uttar Pradesh-201313, India
Email: premsingh.csjm@gmail.com

*Correspondence: Said Broumi; broumisaid78@gmail.com.

Abstract: In this research article we actually deals with the conception of pentagonal Neutrosophic number from a different frame of reference. Recently, neutrosophic set theory and its extensive properties have given different dimensions for researchers. This paper focuses on pentagonal neutrosophic numbers and its distinct properties. At the same time, we defined the disjunctive cases of this number whenever the truthiness, falsity and hesitation portion are dependent and independent to each other. Some basic properties of pentagonal neutrosophic numbers with its logical score and accuracy function is introduced in this paper with its application in real life operation research problem which is more reliable than the other methods.

Keywords: Neutrosophic set, neutrosophic number, Pentagonal Neutrosophic number; Score and Accuracy function.

1. Introduction

Recently, handling the uncertainty and vagueness is considered as one of the prominent research topic around the world. In this regard, mathematical algebra of Fuzzy set theory [1] has provided a well-established tool to deal with the same. Vagueness theory plays a key role to solve problems related with engineering and statistical computation. It is widely used in social science, networking, decision making problem or any kind of real life problem. Motivating from fuzzy sets the Atanassov [2] proposed the legerdemain idea of an intuitionistic fuzzy set in the field of Mathematics in which he considers the concept of membership function as well as non-membership function in case of intuitionistic fuzzy set. Afterwards, the invention of Liu F, Yuan XH in 2007 [3], ignited the concept of triangular intuitionistic fuzzy set, which in reality is the congenial mixture of triangular fuzzy set and intuitionistic fuzzy set. Later, Ye [4] introduced the elementary idea of trapezoidal intuitionistic fuzzy set where both truth function and falsity function are both trapezoidal number in nature instead of triangular. The uncertainty theory plays an influential role to create some interesting model in various fields of science and technological problem.

Smarandache in 1995 (published in 1998) [5] manifested the idea of neutrosophic set where there are three different components namely i) truthiness, ii) indeterminacies, iii) falseness. All the aspect of neutrosophic set is very much pertinent with our real-life system. Neutrosophic concept is a very effective & an exuberant idea in real life. Further, R. Helen [7] introduced the pentagonal fuzzy number and A.

Vigin [8] applied it in neural network. T.Pathinathan [9] gives the concept of reverse order triangular, trapezoidal, pentagonal fuzzy number. Later, Wang et al. [10] invented the perception of single typed neutrosophic set which so much useful to solve any complex problem. Later, Ye [11] presented the concept of trapezoidal neutrosophic fuzzy number and its application. A.Chakraborty [12] developed the conception of triangular neutrosophic number and its different form when the membership functions are dependent or independent. Recently, A.Chakraborty [13] also developed the perception of pentagonal fuzzy number and its different representation in research domain. Christi [14] applied the conception of pentagonal intuitionistic number for solving a transportation problem. Later, Chen [15, 16] solved MCDM problem with the help of FN-IOWA operator and using trapezoidal fuzzy number analyse fuzzy risk ranking problem respectively. Recently, S. Broumi [17-19] developed some important articles related with neutrosophic number in different branch of mathematics in various real life problems. Moreover, Prem [20-25] invented some useful results in neutrosophic arena, mainly associated with computer science engineering problem and networking field. Chakraborty A. [26, 27] applied the idea of vagueness in mathematical model for diabetes and inventory problem respectively. Recently, Abdel-Basset [28-34] introduced some interesting articles co-related with neutrosophic domain in disjunctive fields like MCDM problem; IoT based problem, Supply chain management problem, cloud computing problem etc. K . Mondal [35,36] apply the concept of neutrosophic number in teacher recruitment MCDM problem in education sector. Later, different types of developments in decision making problems, medical diagnoses problem and others in neutrosophic environment [37-49] are already published in this impreciseness arena. Recently, the conception of plithogenic set is being developed by Smarandache and it has a great impact in unceairty field in various domain of research.

1.1 Motivation

The perception of vagueness plays a crucial role in construction of mathematical modeling, engineering problem and medical diagnoses problem etc. Now there will be an important issue that if someone considers pentagonal neutrosophic number then what will be the linear form and what is the geometrical figure? How should we categorize the type-1, 2, 3 pentagonal neutrosophic numbers when the membership functions are related to each other? From this aspect we actually try to develop this research article. Later we invented some more interesting results on score and accuracy function and other application.

1.2 Contribution

In this paper, researchers mainly deal with the conception of pentagonal neutrosophic number in different aspect. We introduced the linear form of single typed pentagonal neutrosophic fuzzy number for distinctive categories. Basically, there are three categories of number will comes out when the three membership functions are dependent or independent among each other, namely Category-1, 2, 3 pentagonal neutrosophic numbers. All the disjunctive categories and their membership functions are addressed here simultaneously.

Researchers from all around the globe are very much interested to know that how a neutrosophic number is converted into a crisp number. Day by day, as research goes on they developed lots of techniques to solve the problem. We developed score and accuracy function and built up the conception of conversion of pentagonal neutrosophic fuzzy number in to a crisp number. In this current era, researchers are very much interested in doing transportation problem in neutrosophic domain. In this phenomenon, we consider a transportation problem in pentagonal neutrosophic domain where we utilize the idea of our developed score and accuracy function for solving the problem.

1.3 Novelties

There are a large number of works already published in this neutrosophic fuzzy set arena. Researchers already developed several formulations and application in various fields. However there will be many interesting results are still unknown. Our work is to try to develop the idea in the unknown points.

- Introduced the distinctive form of pentagonal neutrosophic fuzzy number and its definition for different cases.
- The graphical representation of pentagonal neutrosophic fuzzy number.
- Development of score and accuracy function.
- Application in transportation problem.

1.4 Verbal Phrase on Neutrosophic Arena

In case of daily life, an interesting question often arises: How can we connect the conception of vagueness and neutrosophic theory in real life domain and what are the verbal phrases in case of it?

Example: Let us consider a problem of vote casting. Suppose in an election we need to select some number of candidates among a finite number of candidates. People have different kind of emotions, feelings, demand, ethics, dream etc. So according to their viewpoint it can be any kind of fuzzy number like interval number, triangular fuzzy, intuitionistic, neutrosophic fuzzy number. Let us check the verbal phrases in each different case for the given problem.

Table 1.3.1: Verbal Phrases

Distinct parameter	Verbal Phrase	Information
Interval Number	[Low, High]	Voter will select according to their first priority within a certain range like [2nd, 3rd] candidate.
Triangular Fuzzy Number	[Low, Median, High]	Voter will select according to their first priority containing an intermediate candidate like [1st,2nd,3rd]
Intuitionistic (Triangular)	[Standard,Median,High; Very Low,Poor,Low]	Voters will select some candidate directly and reject others immediately according to their view.
Neutrosophic (Pentagonal)	[VeryLow,Low,Median,High, Very High; VeryLow,Low,Median,High, Very High; VeryLow,Low,Median,High, Very High]	Some Voters will select directly some candidates, some of them are in hesitation in casting vote and some of them directly reject voting according to their own viewpoints.

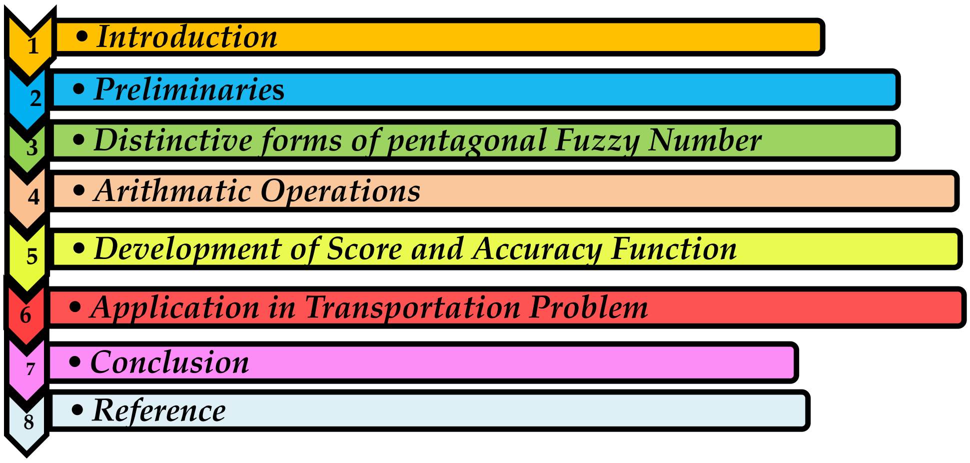
It can be observed that,in 1st coloum of the above table which contains distinct parameters like interval number, triangular fuzzy number, triangular intuitionistic fuzzy number and neutrosophic number, obviously neutrosophic concept gives us a more reliable and logical result since it will contain truth,false as well as the hesitation information absent in the other parameters. Also it is a key question why we take pentagonal neutrosophic instead of triangular or trapezoidal? Now, if we observe the verbal phrase section we can observe that, in case of triangular it will contain only three phrase like low,median,high and trapezoidal contains four like low, semi median, quasi median,high. Suppose some voters choose truth part very strongly and reject the other two sections because these are very low or someone chooses truth part in an average way since he/she thought that rest of the portions are very low. That means we need to develop the verbal phrase such that it will contain much more distict categories. Pentagonal shapes give us atleast five disjunctive verbal categories like very low, low, median, high, very high which is much more logical, strong and it also contains more sensitive cases than the rest sections.

1.5 Need of Pentagonal Neutrosophic Fuzzy Number

The pentagonal neutrosophic fuzzy number stretches us superfluous opportunity to characterize flawed knowledge which leads to construct logical models in several realistic problems in a new way. Pentagonal neutrosophic represents the data and information in a complete way and the truth, hesitation and falsity can be characterized in more accurate and normal technique. The info is reserved throughout the operation and the full material can be utilized by the decision maker for further investigation. It can be finding its applications in different optimization complications, decision making problems and economic difficulties etc. which need fifteen components. In case of transportation problem, if the numbers of variable are five for each of the three components then it is problematic to signify by using Triangular or Trapezoidal neutrosophic Fuzzy numbers. Therefore, pentagonal neutrosophic fuzzy number can invention its dynamic applications in resolving the problem.

1.6 Structure of the paper

The article is developed as follows:



2. Mathematical Preliminaries

Definition 2.1: Fuzzy Set: [1] A set $\widetilde{B}$, defined as $\widetilde{B} = \{(x, \mu_{\widetilde{B}}(x)) : x \in X, \mu_{\widetilde{B}}(X) \in [0,1]\}$ and generally denoted by the pair $(x, \mu_{\widetilde{B}}(x))$, x belongs to the crisp set X and $\mu_{\widetilde{B}}(X)$ belongs to the interval $[0, 1]$, then set $\widetilde{B}$ is called a fuzzy set.

Definition 2.2: Intuitionistic Fuzzy Set (IFS): An Intuitionistic fuzzy set [2] $\tilde{S}$ in the universal discourse X which is denoted generically by x is said to be a Intuitionistic set if $\tilde{S} = \{\langle x; [\tau(x), \varphi(x)]\rangle : x \in X\}$, where $\tau(x): X \rightarrow [0,1]$ is named as the truth membership function which indicate the degree of assurance, $\varphi(x): X \rightarrow [0,1]$ is named the indeterminacy membership function which shows the degree of vagueness.

$\tau(x), \varphi(x)$ parades the following the relation $0 \leq \tau(x) + \varphi(x) \leq 1$.

2.3 Definition: Neutrosophic Set: Smarandache[5] A set $\widetilde{NeA}$ in the universal discourse X, symbolically denoted by x, it is called a neutrosophic set if $\widetilde{NeA} = \{\langle x; [\alpha_{\widetilde{NeA}}(x), \beta_{\widetilde{NeA}}(x), \gamma_{\widetilde{NeA}}(x)]\rangle : x \in X\}$, where $\alpha_{\widetilde{NeA}}(x): X \rightarrow [0,1]$ is said to be the truth membership function, which represents the degree of assurance, $\beta_{\widetilde{NeA}}(x): X \rightarrow [0,1]$ is said to be the indeterminacy membership, which denotes the degree of vagueness, and $\gamma_{\widetilde{NeA}}(x): X \rightarrow [0,1]$ is said to be the falsity membership, which indicates the degree of skepticism on the decision taken by the decision maker.

$\alpha_{\widetilde{NeA}}(x), \beta_{\widetilde{NeA}}(x) \& \gamma_{\widetilde{NeA}}(x)$exhibits the following relation: $-0 \leq \alpha_{\widetilde{NeA}}(x) + \beta_{\widetilde{NeA}}(x) + \gamma_{\widetilde{NeA}}(x) \leq 3+$.

2.4 Definition: Single-Valued Neutrosophic Set: A Neutrosophic set $\widetilde{NeA}$ in the definition 2.1 is said to be a single-Valued Neutrosophic Set $(\widetilde{SNeA})$ if x is a single-valued independent variable. $\widetilde{SNeA} = \{\langle x; [\alpha_{\widetilde{SNeA}}(x), \beta_{\widetilde{SNeA}}(x), \gamma_{\widetilde{SNeA}}(x)]\rangle : x \in X\}$, where $\alpha_{\widetilde{SNeA}}(x), \beta_{\widetilde{SNeA}}(x) \& \gamma_{\widetilde{SNeA}}(x)$ denoted the concept of accuracy, indeterminacy and falsity memberships function respectively.

If there exist three points $a_0, b_0 \& c_0$, for which $\alpha_{\widetilde{SNeA}}(a_0) = 1, \beta_{\widetilde{SNeA}}(b_0) = 1 \& \gamma_{\widetilde{SNeA}}(c_0) = 1$, then the $\widetilde{SNeA}$ is called neut-normal.

$\widetilde{SnS}$is called neut-convex, which implies that $\widetilde{SnS}$ is a subset of a real line by satisfying the following conditions:

i. $\alpha_{\widetilde{SNeA}}\langle \delta a_1 + (1-\delta)a_2 \rangle \geq min\langle \alpha_{\widetilde{SNeA}}(a_1), \alpha_{\widetilde{SNeA}}(a_2) \rangle$
ii. $\beta_{\widetilde{SNeA}}\langle \delta a_1 + (1-\delta)a_2 \rangle \leq max\langle \beta_{\widetilde{SNeA}}(a_1), \beta_{\widetilde{SNeA}}(a_2) \rangle$
iii. $\gamma_{\widetilde{SNeA}}\langle \delta a_1 + (1-\delta)a_2 \rangle \leq max\langle \gamma_{\widetilde{SNeA}}(a_1), \gamma_{\widetilde{SNeA}}(a_2) \rangle$

Where$a_1 \& a_2 \in \mathbb{R}$ and $\delta \in [0,1]$

2.5 Definition: Single-Valued Pentagonal Neutrosophic Number: A Single-Valued Pentagonal Neutrosophic Number $(\tilde{S})$ is defined as $\tilde{s} = \langle [(m^1, n^1, o^1, p^1, q^1); \pi], [(m^2, n^2, o^2, p^2, q^2); \rho], [(m^3, n^3, o^3, p^3, q^3); \sigma] \rangle$, where $\pi, \rho, \sigma \in [0,1]$. The accuracy membership function$(\tau_{\tilde{S}}): \mathbb{R} \to [0, \pi]$, the indeterminacy membership function $(\iota_{\tilde{S}}): \mathbb{R} \to [\rho, 1]$ and the falsity membership function $(\varepsilon_{\tilde{S}}): \mathbb{R} \to [\sigma, 1]$ are given as:

$$\tau_{\tilde{S}}(x) = \begin{cases} \tau_{\widetilde{Sl1}}(x) m^1 \leq x < n^1 \\ \tau_{\widetilde{Sl2}}(x)\, n^1 \leq x < o^1 \\ \mu \quad x = o^1 \\ \tau_{\widetilde{Sr2}}(x)\, o^1 \leq x < p^1 \\ \tau_{\widetilde{Sr1}}(x) p^1 \leq x < q^1 \\ 0 \quad otherwise \end{cases}, \quad \iota_{\tilde{S}}(x) = \begin{cases} l_{\widetilde{Sl1}}(x) m^2 \leq x < n^2 \\ l_{\widetilde{Sl2}}(x)\, n^2 \leq x < o^2 \\ \vartheta \quad x = o^2 \\ l_{\widetilde{Sr2}}(x)\, o^2 \leq x < p^2 \\ l_{\widetilde{Sr1}}(x) p^2 \leq x < q^2 \\ 1 \quad otherwise \end{cases}$$

$$\varepsilon_{\tilde{S}}(x) = \begin{cases} \varepsilon_{\widetilde{Sl1}}(x) m^3 \leq x < n^3 \\ \varepsilon_{\widetilde{Sl2}}(x)\, n^3 \leq x < o^3 \\ \vartheta \quad x = o^3 \\ \varepsilon_{\widetilde{Sr2}}(x)\, o^3 \leq x < p^3 \\ \varepsilon_{\widetilde{Sr1}}(x) p^3 \leq x < q^3 \\ 1 \quad otherwise \end{cases}$$

3. Linear Generalized Pentagonal Neutrosophic number:

In this section, we introduce the linear and generalized neutrosophic number.

3.1 Pentagonal Single Typed Neutrosophic Number of Specification 1: When the quantity of the truth, hesitation and falsity are independent to each other.

A Pentagonal Single typed Neutrosophic Number (PTGNEU) of specification 1 is described as$\tilde{A}_{PtgNeu} = (p_1, p_2, p_3, p_4, p_5; q_1, q_2, q_3, q_4, q_5; r_1, r_2, r_3, r_4, r_5; \tau)$, whose truth membership; hesitation membership and falsity membership are specified as follows:

$$T_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \tau\frac{x-p_1}{p_2-p_1} if p_1 \le x \le p_2 \\ 1-(1-\tau)\frac{x-p_2}{p_3-p_2} if p_2 \le x \le p_3 \\ 1 \quad if x = p_3 \\ 1-(1-\tau)\frac{p_4-x}{p_4-p_3} if p_3 \le x \le p_4 \\ \tau\frac{p_5-x}{p_5-p_4} if p_4 \le x \le p_5 \\ 0 \quad Otherwise \end{cases}$$

$$I_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \tau\frac{q_2-x}{q_2-q_1} if q_1 \le x < q_2 \\ 1-(1-\tau)\frac{q_3-\mathrm{x}}{q_3-q_2} if q_2 \le x \le q_3 \\ 0 \quad if x = q_3 \\ 1-(1-\tau)\frac{x-q_3}{q_4-q_3} \; if q_3 \le x \le q_4 \\ \tau\frac{x-q_4}{q_5-q_4} \; if q_4 \le x \le q_5 \\ 1 \quad Otherwise \end{cases}$$

$$F_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \tau\frac{r_2-x}{r_2-r_1} if r_1 \le x < r_2 \\ 1-(1-\tau)\frac{r_3-\mathrm{x}}{r_3-r_2} if r_2 \le x \le r_3 \\ 0 \quad if x = r_3 \\ 1-(1-\tau)\frac{x-r_3}{r_4-r_3} \; if r_3 \le x \le r_4 \\ \tau\frac{x-r_4}{r_5-r_4} \; if r_4 \le x \le r_5 \\ 1 \quad Otherwise \end{cases}$$

Where $-0 \le T_{\tilde{A}_{PtgNeu}}(x) + I_{\tilde{A}_{PtgNeu}}(x) + F_{\tilde{A}_{PtgNeu}}(x) \le 3+, x \in \tilde{A}_{PtgNeu}$

The parametric form of the above type number is

$$\left(\tilde{A}_{PtgNeu}\right)_{\mu,\vartheta,\varphi} = \begin{bmatrix} T_{PtgNeu1L}(\mu), T_{PtgNeu2L}(\mu), T_{PtgNeu1R}(\mu), T_{PtgNeu2R}(\mu); \\ I_{PtgNeu1L}(\vartheta), I_{PtgNeu2L}(\vartheta), I_{PtgNeu1R}(\vartheta), I_{PtgNeu2R}(\vartheta); \\ F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi) \end{bmatrix}$$

where, $T_{PtgNeu1L}(\mu) = p_1 + \frac{\mu}{\tau}(p_2 - p_1)$ for $\mu \in [0,\tau]$, $T_{PtgNeu2L}(\mu) = p_2 + \frac{1-\mu}{1-\tau}(p_3 - p_2)for\mu \in [\tau, 1]$

$T_{PtgNeu2R}(\mu) = p_4 - \frac{1-\mu}{1-\tau}(p_4 - p_3)for\mu \in [\tau, 1]$, $T_{PtgNeu1R}(\mu) = p_5 - \frac{\mu}{\tau}(p_5 - p_4)for\mu \in [0,\tau]$

$I_{PtgNeu1L}(\vartheta) = q_2 - \frac{\vartheta}{\tau}(q_2 - q_1)for\vartheta \in [\tau, 1]$, $I_{PtgNeu2L}(\vartheta) = q_3 - \frac{1-\vartheta}{1-\tau}(q_3 - q_2)for\vartheta \in [0,\tau]$

$I_{PtgNeu2R}(\vartheta) = q_3 + \frac{1-\vartheta}{1-\tau}(q_4 - q_3)for\vartheta \in [0,\tau]$, $I_{PtgNeu1R}(\vartheta) = q_4 + \frac{\vartheta}{\tau}(q_5 - q_4)for\vartheta \in [\tau, 1]$

$F_{PtgNeu1L}(\varphi) = r_2 - \frac{\varphi}{\tau}(r_2 - r_1)for\varphi \in [\tau, 1]$, $F_{PtgNeu2L}(\varphi) = r_3 - \frac{1-\varphi}{1-\tau}(r_3 - r_2)for\varphi \in [0,\tau]$

$F_{PtgNeu2R}(\varphi) = r_3 + \frac{1-\varphi}{1-\tau}(r_4 - r_3)for\varphi \in [0,\tau]$, $F_{PtgNeu1R}(\varphi) = r_4 + \frac{\varphi}{\tau}(r_5 - r_4)for\varphi \in [\tau, 1]$

Here,$0 < \mu \le 1$, $0 < \vartheta \le 1$, $0 < \varphi \le 1$ and $-0 < \mu + \vartheta + \varphi \le 3+$

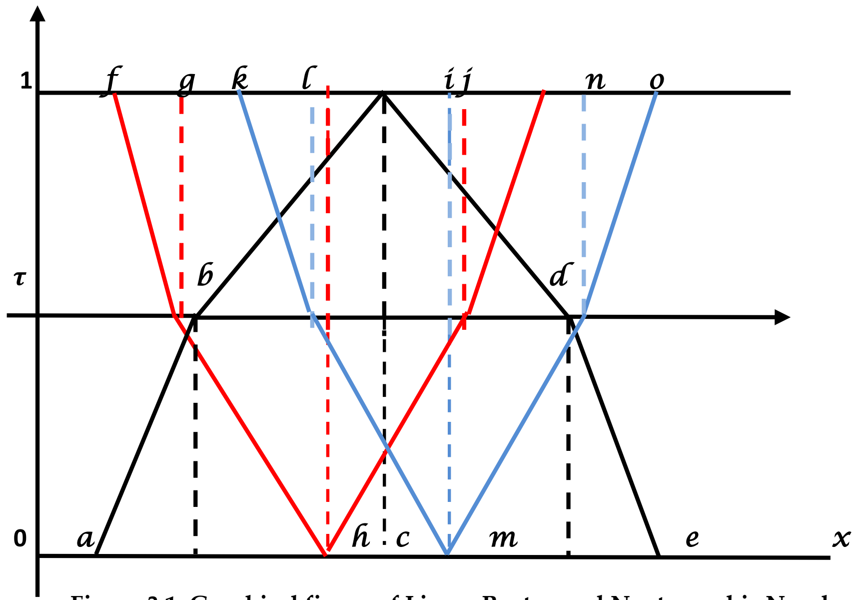


Figure 3.1: Graphical figure of Linear Pentagonal Neutrosophic Number.

Note 3.1 - Description of above figure: In this above figure we shall try to address the graphical representation of linear pentagonal neutrosophic number. The pentagonal shaped black marked line actually indicate the truthiness membership function, pentagonal shaped red marked line denotes the falseness membership function and pentagonal shaped blue marked line pointed the indeterminacy membership function of this corresponding number. Here, τ is a variable which follows the relation $0 \leq \tau \leq 1$. If $\tau = 0\ or\ 1$ then the pentagonal number will be converted into triangular neutrosophic number.

3.2 Pentagonal Single Typed Neutrosophic Number of Specification 2: If the sections of Hesitation and Falsity functions are dependent to each other

A Pentagonal Single Typed Neutrosophic Number (PTGNEU) of specification 2 is described as $\tilde{A}_{PtgNeu} = (p_1, p_2, p_3, p_4, p_5; q_1, q_2, q_3, q_4, q_5; \theta_{PtgNeu}, \delta_{PtgNeu})$ whose truth membership; hesitation membership and falsity membership are specified as follows:

$$T_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \tau \dfrac{x - p_1}{p_2 - p_1} if p_1 \leq x \leq p_2 \\ 1 - (1 - \tau)\dfrac{x - p_2}{p_3 - p_2} if p_2 \leq x \leq p_3 \\ 1 \quad if x = p_3 \\ 1 - (1 - \tau)\dfrac{p_4 - x}{p_4 - p_3} if p_3 \leq x \leq p_4 \\ \tau \dfrac{p_5 - x}{p_5 - p_4} if p_4 \leq x \leq p_5 \\ 0 \quad Otherwise \end{cases}$$

$$I_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \dfrac{q_2 - x + \theta_{PtgNeu}(x - q_1)}{q_2 - q_1} & if\ q_1 \le x < q_2 \\ \dfrac{q_3 - x + \theta_{PtgNeu}(x - q_2)}{q_3 - q_2} & if q_2 \le x < q_3 \\ \theta_{PtgNeu} if x = q_3 \\ \dfrac{x - q_3 + \theta_{PtgNeu}(q_4 - x)}{q_4 - q_3} if q_3 \le x < q_4 \\ \dfrac{x - q_4 + \theta_{PtgNeu}(q_5 - x)}{q_5 - q_4} & if q_4 \le x < q_5 \\ 1 & Otherwise \end{cases}$$

and

$$F_{\tilde{A}_{PtgNeu}}(x) = \begin{cases} \dfrac{q_2 - x + \delta_{PtgNeu}(x - q_1)}{q_2 - q_1} & if\ q_1 \le x < q_2 \\ \dfrac{q_3 - x + \delta_{PtgNeu}(x - q_2)}{q_3 - q_2} & if q_2 \le x < q_3 \\ \delta_{PtgNeu} if x = q_3 \\ \dfrac{x - q_3 + \delta_{PtgNeu}(q_4 - x)}{q_4 - q_3} if q_3 \le x < q_4 \\ \dfrac{x - q_4 + \delta_{PtgNeu}(q_5 - x)}{q_5 - q_4} & if q_4 \le x < q_5 \\ 1 & Otherwise \end{cases}$$

where,$0 \le T_{\tilde{A}_{PtgNeu}}(x) + I_{\tilde{A}_{PtgNeu}}(x) + F_{\tilde{A}_{PtgNeu}}(x) \le 2+, x \in \tilde{A}_{PtgNeu}$

The parametric form of the above type number is

$$\left(\tilde{A}_{PtgNeu}\right)_{\mu,\vartheta,\varphi} = \begin{bmatrix} T_{PtgNeu1L}(\mu), T_{PtgNeu2L}(\mu), T_{PtgNeu1R}(\mu), T_{PtgNeu2R}(\mu); \\ I_{PtgNeu1L}(\vartheta), I_{PtgNeu2L}(\vartheta), I_{PtgNeu1R}(\vartheta), I_{PtgNeu2R}(\vartheta); \\ F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi), F_{PtgNeu1L}(\varphi) \end{bmatrix}$$

$$T_{PtgNeu1L}(\mu) = p_1 + \frac{\mu}{\tau}(p_2 - p_1) \text{for} \mu \in [0, \tau],\ T_{PtgNeu2L}(\mu) = p_2 + \frac{1-\mu}{1-\tau}(p_3 - p_2) \text{for} \mu \in [\tau, 1]$$

$$T_{PtgNeu2R}(\mu) = p_4 - \frac{1-\mu}{1-\tau}(p_4 - p_3) \text{for} \mu \in [\tau, 1], T_{PtgNeu1R}(\mu) = p_5 - \frac{\mu}{\tau}(p_5 - p_4) \text{for} \mu \in [0, \tau]$$

$$I_{PtgNeu1L}(\vartheta) = \frac{q_2 - \theta_{PtgNeu} q_1 - \vartheta(q_2 - q_1)}{1 - \theta_{PtgNeu}} \text{for} \vartheta \in [\tau, 1], I_{PtgNeu2L}(\vartheta) = \frac{q_3 - \theta_{PtgNeu} q_2 - \vartheta(q_3 - q_2)}{1 - \theta_{PtgNeu}} \text{for} \vartheta \in [0, \tau]$$

$$I_{PtgNeu2R}(\vartheta) = \frac{q_3 - \theta_{PtgNeu} q_4 + \vartheta(q_4 - q_3)}{1 - \theta_{PtgNeu}} \text{for} \vartheta \in [0, \tau],\ I_{PtgNeu2L}(\vartheta) = \frac{q_4 - \theta_{PtgNeu} q_5 + \vartheta(q_5 - q_4)}{1 - \theta_{PtgNeu}} \text{for} \vartheta \in [\tau, 1]$$

$$F_{PtgNeu1L}(\varphi) = \frac{q_2 - \delta_{PtgNeu} q_1 - \varphi(q_2 - q_1)}{1 - \delta_{PtgNeu}} \text{for} \varphi \in [\tau, 1], F_{PtgNeu2L}(\varphi) = \frac{q_3 - \delta_{PtgNeu} q_2 - \varphi(q_3 - q_2)}{1 - \delta_{PtgNeu}} \text{for} \varphi \in [0, \tau]$$

$$F_{PtgNeu2R}(\varphi) = \frac{q_3 - \delta_{PtgNeu} q_4 + \varphi(q_4 - q_3)}{1 - \delta_{PtgNeu}} \text{for} \varphi \in [0, \tau],\ F_{PtgNeu2L}(\varphi) = \frac{q_4 - \delta_{PtgNeu} q_5 + \varphi(q_5 - q_4)}{1 - \delta_{PtgNeu}} \text{for} \varphi \in [\tau, 1]$$

Here, $0 < \mu \le 1$, $\theta_{PtgNeu} < \vartheta \le 1$, $\delta_{PtgNeu} < \varphi \le 1$ and $-0 < \vartheta + \varphi \le 1+$ and $-0 < \mu + \vartheta + \varphi \le 2+$

4. Arithmetic Operations:

Suppose we consider two pentagonal neutrosophic fuzzy number as $\tilde{A}_{PtgNeu} = (p_1, p_2, p_3, p_4, p_5; \mu_a, \vartheta_a, \theta_a)$ and $\tilde{B}_{PtgNeu} = (q_1, q_2, q_3, q_4, q_5; \mu_b, \vartheta_b, \theta_b)$ then,

i) $\tilde{A}_{PtgNeu} + \tilde{B}_{PtgNeu} = [p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4, p_5 + q_5; \max\{\mu_a, \mu_b\}, \min\{\vartheta_a, \vartheta_b\}, \min\{\theta_a, \theta_b\}]$

ii) $\tilde{A}_{PtgNeu} - \tilde{B}_{PtgNeu} = [p_1 - q_5, p_2 - q_4, p_3 - q_3, p_4 - q_2, p_5 - q_1; \max\{\mu_a, \mu_b\}, \min\{\vartheta_a, \vartheta_b\}, \min\{\theta_a, \theta_b\}]$

iii) $k\tilde{A}_{PtgNeu} = [kp_1, kp_2, kp_3, kp_4, kp_5; \mu_a, \vartheta_a, \theta_a]\ if\ k > 0,\ = [kp_5, kp_4, kp_3, kp_2, kp_1; \mu_a, \vartheta_a, \theta_a]\ if\ k < 0$

iv) ${\tilde{A}_{PtgNeu}}^{-1} = (1/p_5, 1/p_4, 1/p_3, 1/p_2, 1/p_1; \mu_a, \vartheta_a, \theta_a)$

5. Proposed Score and Accuracy Function:

Score function and accuracy function of a pentagonal neutrosophic number is fully depend on the value of truth membership indicator degree, falsity membership indicator degree and hesitation membership indicator degree. The need of score and accuracy function is to compare or convert a pentagonal neutrosophic fuzzy number into a crisp number. In this section we will proposed a score function as follows.

For any Pentagonal Single typed Neutrosophic Number (PTGNEU)

$$\tilde{A}_{PtgNeu} = (p_1, p_2, p_3, p_4, p_5; q_1, q_2, q_3, q_4, q_5; r_1, r_2, r_3, r_4, r_5)$$

We consider the beneficiary degree of truth indicator part as = $\frac{p_1+p_2+p_3+p_4+p_5}{5}$

Non- beneficiary degree of falsity indicator part as =$\frac{r_1+r_2+r_3+r_4+r_5}{5}$.

And the hesitation degree of indeterminacy indicator as = $\frac{q_1+q_2+q_3+q_4+q_5}{5}$

Thus, we defined the score function as $SC_{PtgNeu} = \frac{1}{3}\left(2 + \frac{p_1+p_2+p_3+p_4+p_5}{5} - \frac{q_1+q_2+q_3+q_4+q_5}{5} - \frac{r_1+r_2+r_3+r_4+r_5}{5}\right)$,

Where, $SC_{PtgNeu} \in [0,1]$ and the Accuracy function is defined as $AC_{PtgNeu} = \left(\frac{p_1+p_2+p_3+p_4+p_5}{5} - \frac{r_1+r_2+r_3+r_4+r_5}{5}\right)$,

Where, $AC_{PtgNeu} \in [-1,1]$, Now we conclude that

If $A_{PtgNeu} =< (1,1,1,1,1; 0,0,0,0,0; 0,0,0,0,0) >$ then, $SC_{PtgNeu} = 1$ and $AC_{PtgNeu} = 1$

If $A_{PtgNeu} =< (0,0,0,0,0; 1,1,1,1,1; 1,1,1,1,1) >$ then, $SC_{PtgNeu} = 0$and $AC_{Sp_{neu}} = -1$

5.1 Relationship between any two pentagonal neutrosophic fuzzy numbers:

Let us consider any two pentagonal neutrosophic fuzzy number defined as follows

$A_{PtgNeu1} = \left(T_{PtgNeu1}, I_{PtgNeu1}, F_{PtgNeu1}\right)$ and $A_{PtgNeu2} = \left(T_{PtgNeu2}, I_{PtgNeu2}, F_{PtgNeu2}\right)$ if,

1) $SC_{A_{PtgNeu1}} > SC_{A_{PtgNeu2}}$, $then\ A_{PtgNeu1} > A_{PtgNeu2}$

2) $SC_{A_{PtgNeu1}} < SC_{A_{PtgNeu2}}$, $then\ A_{PtgNeu1} < A_{PtgNeu2}$

3) $SC_{A_{PtgNeu1}} = SC_{A_{PtgNeu2}}$, $then$

i) $AC_{A_{PtgNeu1}} > AC_{A_{PtgNeu2}}$, $then\ A_{PtgNeu1} > A_{PtgNeu2}$

$ii)$ $AC_{A_{PtgNeu1}} < AC_{A_{PtgNeu2}}$, $then\ A_{PtgNeu1} < A_{PtgNeu2}$

$iii)$ $AC_{A_{PtgNeu1}} = AC_{A_{PtgNeu2}}$, $then\ A_{PtgNeu1} \sim A_{PtgNeu2}$

6. Application in Neutrosophic Transportation Environment:

6.1 Mathematical Formulation of Model-I

In this section we consider a transportation problem in pentagonal neutrosophic environment where there are "p" sources and "q" destinations in which the decision makers need to choose a logical allotment such that it can start from "m"th source and it will went to "n"th section in such a way where the cost become the minimum once in presence of uncertainty, hesitation in transportation matrix. We also consider the available resources and required values are real number in nature.

Assumptions;

'm' is the source part for all m=1,2,3……p

'n' is destination part for all n=1,2,3…….q

$x_{mn}$is amount of portion product which can be transferred from m-th source to n-th destination.

$\widetilde{N_{mn}}$is the unit cost portion in neutrosophic nature which can be transferred from m-th source to n-th destination.

$u_m$is the total availability of the product at the source m.

$v_n$is the total requirement of the product at the source n.

Here, supply constraints $\sum_{n=0}^{q} x_{mn} = u_m$ for all m.

Demand constraints $\sum_{m=0}^{p} x_{mn} = \vartheta_n$ for all n.

Also, $\sum_{n=0}^{q} \vartheta_n = \sum_{m=0}^{p} u_m$,$x_{mn} \geq 0$

So, the mathematical formulation is, $Min\ Z = \sum_{n=0}^{q} \sum_{m=0}^{p} x_{mn} . \widetilde{N_{mn}}$, Subject to the constrain, $\sum_{n=0}^{q} x_{mn} = u_m$, $\sum_{m=0}^{p} x_{mn} = \vartheta_n$, Where, $x_{mn} \geq 0$ for all m, n.

Proposed Algorithm to find out the optimal solution of Model-I:

Step-1: Conversion of each pentagonal neutrosophic numbers into crisp using our proposed score functions and creates the transportation matrix into crisp system.

Step-2: Calculate the non-negative difference for each row and column between the smallest and next smallest elements row and column wise respectively.

Step-3: Take the highest difference and placed the availability or demand into the minimum allocated cell of the matrix. In case of tie in highest difference take any one arbitrarily.

Step-4: The process is going on unless and until the final optimal matrix is created. Lastly, check the number of allocated cells in the final matrix; it should be equal to row+column-1.

Step-5: Now, calculate the minimum total cost using the allocated cells.

Illustrative Example:

A company has three factories A, B, C which supplies some materials at D, E and F on monthly basis with pentagonal neutrosophic unit transportation cost whose capacities are 12,14,4 units respectively and the transportation matrix is defined as below and the requirements are 9, 10, 11 respectively. The problem is to find out the optimal solution and the minimum transportation cost.

	A	**B**	**C**	**Available**
D	<(10,15,20,25,30; 0,3,5,7,10; 0,1,2,3,4)>	<(1,1,1,1,1; 0,0,0,0,0; 0,0,0,0,0)>	<(10,20,30,40,50; 1,4,7,8,10; 0,1,1.5,2,2.5,3)>	12
E	<(2,3,4,7,9; 0,0.5,1,1.5,2; 0,0,0,0,0)>	<(5,10,15,20,25; 1,2,3,4,5; 1,1.5,2,2.5,3)>	<(0,0.5,1,1.5,2.5; 0,0.5,1,1.5,2; 0,0,0,0,0)>	14
F	<(5,9,11,12,13; 0,1,2,2.5,4.5;	<(10,15,20,25,30; 0,2,4,6,8;	<(15,20,25,30,50; 0,3,7,10,15;	4

	0,0.5,1,1.5,2)>	0,0,0,0,0)>	1,2,4,5,8)>	
Required	9	10	11	

Step-1

Table-1: We convert this pentagonal neutrosophic transportation problem in to a crisp model using the concept of score and accuracy function.

	A	B	C	u_m
D	5	1	8	12
E	2	4	0	14
F	3	6	7	4
v_n	9	10	11	

Step-2

Table-2:

	A	B	C	u_m(Penalty)
D	5	1	8	12(4)
E	2	4	0	14(2)
F	3	6	7	4(3)
v_n (Pen-alty)	9 (1)	10 (3)	11 (7)	

Step-3

Table-3: After processing the same operations finite number of times finally we get the final optimal solution matrix as, here $number\ of\ allocation = row + column - 1 = 5$

	A	B	C	u_i
D	[2] 5	[10] 1	8	12
E	[3] 2	4	[11] 0	14
F	[4] 3	6	7	4
v_j	9	10	11	

Thus, the total cost of this transportation problem is $Min\ Z = \sum_{n=0}^{3} \sum_{m=0}^{3} x_{mn} \cdot \widetilde{N_{mn}}$

=2*(10,15,20,25,30;0,3,5,7,10;0,1,2,3,4)+3*(1,2,3,6,8;0,0.5,1,1.5,2;-3,-2,1,0,1)+

4*(1,3,5,7,9;-5,-4,0,1,3;-5,-3,0,1,2)+10*(1,1,1,1,1;0,0,0,0,0;0,0,0,0,0)+11*(1,0,1,2,3;

0, 0.5, 1, 1.5, 2; 0, 0, 0, 0, 0)

$= < (26,58,90,128,163; -20,-3,24,39,60; -20,-16,1,10,19) >$

= 25.4 Units.

6.2 Mathematical Formulation of Model-II

Mathematical formulation is, $Min\ \check{Z} = \sum_{n=0}^{q} \sum_{m=0}^{p} \widetilde{x_{mn}} \cdot \widetilde{N_{mn}}$

Subject to the constrain, $\sum_{n=0}^{q} \widetilde{x_{mn}} = \widetilde{u_m}$

$$\sum_{m=0}^{p} \widetilde{x_{mn}} = \widetilde{\vartheta_n}$$

Also, $\sum_{n=0}^{q} \widetilde{\vartheta_n} = \sum_{m=0}^{p} \widetilde{u_m}$, Where, $\widetilde{x_{mn}} \geq 0$ for all m, n.

Here $\widetilde{N_{mn}}, \widetilde{u_m}, \widetilde{\vartheta_n}$ are all pentagonal neutrosophic numbers.

In formulation of Model-II with the help of pentagonal neutrosophic number cost, demand and supply formulated in the following table 1,First, we calculate the score value of individual neutrosophic cost to get crisp cost and consider the rest terms that is demand and supply as it is in neutrosophic nature. Now, for the allocation we first consider the score values of availability and demand and take the minimum value for the allocation. Then, we use the arithmetic operations in pentagonal neutrosophic domain to run the iteration process. Following the same above algorithm finally we get the optimal solution table where number of allocation=row+column-1 and finally we need to compute the final cost.

Table-1:

	A	B	C	Available
D	<(10,15,20,25,30; 0,3,5,7,10; 0,1,2,3,4)>	<(1,1,1,1,1; 0,0,0,0,0; 0,0,0,0,0)>	<(10,20,30,40,50; 1,4,7,8,10; 0,1,1.5,2,2.5,3)>	<(20,30,40,50,60; 3,5,6,10,12; 5,10,15,20,25)>
E	<(2,3,4,7,9; 0,0.5,1,1.5,2; 0,0,0,0,0)>	<(5,10,15,20,25; 1,2,3,4,5; 1,1.5,2,2.5,3)>	<(0,0.5,1,1.5,2.5; 0,0.5,1,1.5,2; 0,0,0,0,0)>	<(15,20,25,30,35; 5,10,15,20,30; 2,4,6,8,10)>
F	<(5,9,11,12,13; 0,1,2,2.5,4.5; 0,0.5,1,1.5,2)>	<(10,15,20,25,30; 0,2,4,6,8; 0,0,0,0,0)>	<(15,20,25,30,50; 0,3,7,10,15; 1,2,4,5,8)>	<(10,20,30,40,50; 4,6,8,10,12; 1,4,7,10,13)>
Required	<(30,40,50,60,70; 4,8,11,17,26; 4,8,12,16,20)>	<(10,20,30,40,50; 4,6,8,10,12; 3,6,9,12,15)>	<(5,10,15,20,25; 4,7,10,13,16; 1,4,7,10,13)>	

Step-1

Table-2:

	A	B	C	u_m
D	**5**	**1**	**8**	<(20,30,40,50,60; 3,5,6,10,12; 5,10,15,20,25)>
E	**2**	**4**	**0**	<(15,20,25,30,35; 5,10,15,20,30;

				2,4,6,8,10)>
F	**3**	**6**	**7**	<(10,20,30,40,50; 4,6,8,10,12; 1,4,7,10,13)>
v_n	<(30,40,50,60,70; 4,8,11,17,26; 4,8,12,16,20)>	<(10,20,30,40,50; 4,6,8,10,12; 3,6,9,12,15)>	<(5,10,15,20,25; 4,7,10,13,16; 1,4,7,10,13)>	

After the iteration process according to the proposed algorithm finally we get the allocations in the allocated cell as,

$$a_{11} =< (-30,-10,10,30,40;\ -9,-5,-2,4,8;-10,-2,6,14,22) >$$

$$a_{21} =< (-10,0,10,20,30;\ -11,-3,5,13,26;-11,-6,-1,4,9) >$$

$$a_{12} =< (10,20,30,40,50;4,6,8,10,12;3,6,9,12,15) >$$

$$a_{23} =< (5,10,15,20,25;4,7,10,13,16;1,4,7,10,13) >$$

$$a_{31} =< (-40,-10,30,70,110;-30,-9,8,25,46;\ -27,-10,7,24,41) >$$

Thus, the optimal solution of this model-II system is, $Min\ \check{Z} = \sum_{n=0}^{3}\sum_{m=0}^{3} \widetilde{x_{mn}} . \widetilde{N_{mn}}$

$$= < (-30,-10,10,30,40;\ -9,-5,-2,4,8;-10,-2,6,14,22) > \times < (10,15,20,25,30;0,3,5,7,10;0,1,2,3,4) > +$$

<(-10,0,10,20,30; -11,-3,5,13,26;-11,-6,-1,4,9)>×<(2,3,4,7,9;0,0.5,1,1.5,2;0,0,0,0,0)> +

$< (10,20,30,40,50;4,6,8,10,12;3,6,9,12,15) >$×<(1,1,1,1,1;0,0,0,0,0;0,0,0,0,0)> +

$< (5,10,15,20,25;4,7,10,13,16;1,4,7,10,13) >$×<(0,0.5,1,1.5,2.5;0,0.5,1,1.5,2;0,0,0,0,0)> +

<(-40,-10,30,70,110;-30,-9,8,25,46; -27,-10,7,24,41)>×<(5,9,11,12,13;0,1,2,2.5,4.5;0,0.5,1,1.5,2)>

$$= < (-510,-215,615,1800,3012.5;\ 0,-22,21,129.5,371;\ 0,-7,19,78,170) >$$

=263.53 units.

6.3 Discussion: In section 6.1, in model -I we observe that if we take pentagonal neutrosophic fuzzy number as a member of feasible solution then we get the $Min\ Z = 25.4$ units, whereas, if we take crisp number in this computation procedure then we get from table 3, Min Z= (2×5) + (3×2) + (4× 3) + $(10 \times 1) = (11 \times 0) = 38\ units$. Thus we can observe that pentagonal neutrosophic number give us better results. So, we follow the same technique in section 6.2 where we consider both availability and the demand as a pentagonal neutrosophic number.

The conception of pentagonal neutrosophic number is totally a new idea and till now, in this domain anyone doesn't considered the transportation problem so fur. Thus in future study, we can compare our work with the other established methods. Also, we can do comparative analysis in pentagonal neutrosophic arena whenever researchers from different section could develop some interesting and useful algorithm in this transportation domain.

7. Conclusion

In this current era, the conception of neutrosophic number plays a paramount role in different fields of research domain.There is a proliferating popularity for the conundrum concept of neutrosophic number presenting before the world a vibrant spice of logic and innovation to reach the zenith of excellence. The world is driven into a paradigm of brilliance as well as expertise with the formation of the corresponding number which assists the researcher dealing with uncertainty and also with the transportation problem.

Neutrosophic set conception is a generalization of intuitionistic fuzzy set which actually contains truthiness, falseness and indeterminacy concept. In this article, we developed a new concept of pentagonal neutrosophic fuzzy number, introduced its graphical representation and its properties. We also invented logical score and accuracy function which has a strong impact in conversion and ranking in this domain of research. Transportation problem is a very important application in operation research domain and we build up two different models in this article within neutrosophic environment. We also employed the arithmetic operations to find the solution which gives us better result than the general conception. Thus, it can be concluded that the approach for taking the pentagonal neutrosophic single-valued number is very helpful for the researchers who are involved in dealing the mathematical modelling with impreciseness in various fields of sciences and engineering. It reveals very realistic results in both mathematical points of view. There is still a massive amount of work in this field; hence much spectacular study can be explored with pentagonal neutrosophic parameters. Further, we can compare our research work with other established methods in pentagonal neutrosophic domain related with transportation problem.

In future, this article can be extended into multi criteria decision making problem. Also, researchers can apply this conception in various fields like engineering problem, pattern recognition problem, mathematical modeling etc.

Reference

1. Zadeh LA; Fuzzy sets. Information and Control, 8(5): (1965),338- 353.
2. Atanassov K ; Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20: (1986);87-96.
3. Liu F, Yuan XH ;Fuzzy number intuitionistic fuzzy set. Fuzzy Systems and Mathematics, 21(1): (2007); 88-91.
4. Ye J ; prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multi criteria decision making, Neural Computing and Applications, 25(6): (2014);1447-1454.
5. Smarandache, F. A unifying field in logics neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth. 1998.
6. D. Dubois, H. Prade, Operations on fuzzy numbers, Internat. J. Systems Sci. 9 (1978) 613-626.
7. R.Helen and G.Uma, A new operation and ranking on pentagon fuzzy numbers, Int Jr. of Mathematical Sciences & Applications, Vol. 5, No. 2, 2015, pp 341-346.
8. A.Vigin Raj and S.Karthik, Application of Pentagonal Fuzzy Number in Neural Network, International Journal of Mathematics And its Applications, Volume 4, Issue 4 (2016), 149-154.
9. T.Pathinathan and K.Ponnivalavan, Reverse order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers, Annals of Pure and Applied Mathematics, Vol. 9, No. 1, 2015, 107-117.
10. H. Wang, F. Smarandache, Q. Zhang and R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure**4** (2010), 410–413.
11. Yun Ye, Trapezoidal neutrosophic set and its application to multiple attribute decision-making, Neural Comput&Applic (2015) 26:1157–1166 DOI 10.1007/s00521-014-1787-6.
12. A.Chakraborty, S.P Mondal, A.Ahmadian, N.Senu,S.Alam and S.Salahshour, Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their Applications, Symmetry, Vol-10, 327; doi:10.3390/sym10080327.
13. A.Chakraborty, S.P Mondal, A.Ahmadian, N.Senu, D.Dey, S.Alam, S.Salahshour, The Pentagonal Fuzzy Number: Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problem, Symmetry,Vol-11(2), 248; doi:10.3390/sym11020248.
14. M.S. Annie Christi and B.Kasthuri, Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers Solved Using Ranking Technique and Russell's Method, Int. Journal of Engineering Research and Applictions, Vol. 6, Issue 2, 2016,pp 82-86.
15. Chen, S. J. and Chen, S. M.. A new method for handling multi criteria fuzzy decision making problems using FN-IOWA operators, Cybernetic andSystems, 34: (109-137.(2003)
16. Chen, S. J. and Chen, S. M. Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, Applied intelligence, 26: 1-11.(2007)

17. S. Broumi, M Murugappan, M. Talea, A Bakali, F. Smarandache, Prem Kumar Singh and A. Dey, Single valued (2n+1) sided polygonal neutrosophic numbers and Single-valued (2n) sided polynomial neutrosophic numbers, Neutrosophic Sets and Systems, vol 25, April 2019, pp. 54-65,
18. S. Broumi, A. Bakali, M. Talea, Prem Kumar Singh, F. Smarandache, Energy and Spectrum Analysis of Interval-valued Neutrosophic graph Using MATLAB, *Neutrosophic Set and Systems*, vol. 24, Mar 2019, pp. 46-60.
19. S. Broumi, M. Talea, A. Bakali, F.Smarandache, Prem Kumar Singh, M. Murugappan, and V. Venkateswara Rao, Neutrosophic Technique Based Efficient Routing Protocol For MANET Based On Its Energy And Distance. Neutrosophic Sets and Systems, vol. 24, pp. 61-69, Mar 2019.
20. P. K. Singh, Three-way n-valued neutrosophic concept lattice at different granulation, International Journal of Machine Learning and Cybernetics, November 2018, Vol 9, Issue 11, pp. 1839-1855
21. P. K. Singh, Three-way fuzzy concept lattice representation using neutrosophic set, *International Journal of Machine Learning and Cybernetics,* Year 2017, Vol 8, Issue 1, pp. 69-79
22. P. K. Singh, Interval-valued neutrosophic graph representation of concept lattice and its (α, β, γ)-decomposition, *Arabian Journal for Science and Engineering,* Year 2018, Vol. 43, Issue 2, pp. 723-74
23. P. K. Singh, Medical diagnoses using three-way fuzzy concept lattice and their Euclidean distance, Computational and Applied Mathematics, July 2018, Vol. 37, Issue 3, pp. 3282-3306.
24. P. K. Singh, Complex neutrosophic concept lattice and its applications to Air quality analysis, *Chaos, Solitons and Fractals,* Year 2018, Vol 109, pp. 206-213.
25. S. Broumi, P. K. Singh, M. Talea, A. Bakali, F. Smarandache and V.Venkateswara Rao, Single-valued neutrosophic techniques for analysis of WIFI connection, Advances in Intelligent Systems and Computing Vol. 915, pp. 405-512,DOI: 10.1007/978-3-030-11928-7_36.
26. Mahata A., Mondal S.P, Alam S, Chakraborty A., Goswami A., Dey S., Mathematical model for diabetes in fuzzy environment and stability analysis- Journal of of intelligent and Fuzzy System, doi: https://doi.org/10.3233/JIFS-171571.
27. S. Maity, A.Chakraborty, S.K De, S.P.Mondal, S.Alam, A comprehensive study of a backlogging EOQ model with nonlinear heptagonal dense fuzzy environment,Rairo Operations Research, https://doi.org/10.1051/ro/2018114, 2018.
28. Abdel-Basset, M., Manogaran, G., Gamal, A., &Smarandache, F. (2019). A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, *43*(2), 38.
29. Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., &Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise Information Systems*, 1-21.
30. Nabeeh, N. A., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Neutrosophic multi-criteria decision making approach for iot-based enterprises. *IEEE Access*, *7*, 59559-59574.
31. Abdel-Baset, M., Chang, V., &Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry*, *108*, 210-220.
32. Abdel-Basset, M., Saleh, M., Gamal, A., &Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, *77*, 438-452.
33. Abdel-Baset, M., Chang, V., Gamal, A., &Smarandache, F. (2019). An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. *Computers in Industry*, *106*, 94-110.
34. Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. Future Generation Computer Systems, 86, 12-29.
35. K. Mondal, and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34. [130]
36. K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.
37. J. R. San Cristóbal. Multi-criteria decision-making in the selection of a renewable energy project in Spain: The VIKOR method. Renewable Energy, 36 (2011), 498-502.
38. J. Ye. Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers. Journal of Intelligent and Fuzzy System 30 (2016), 1927–1934.
39. J. Ye. Similarity measures between interval neutrosophic sets and their applications in multi criteria decisionmaking. Journal of Intelligent and Fuzzy Systems, 26 (2014), 165–172.

40. X. Zhao. TOPSIS method for interval-valued intuitionistic fuzzy multiple attribute decision making and its application to teaching quality evaluation. Journal of Intelligent and Fuzzy Systems, 26 (2014), 3049–3055.
41. H. Pouresmaeil, E. Shivanian, E. Khorram, and H. S. Fathabadi. An extended method using TOPSIS and VIKOR for multiple attribute decision making with multiple decision makers and single valued neutrosophic numbers. Advances and Applications in Statistics, 50 (4) (2017), 261.
42. P. Liu, L. Zhang. The extended VIKOR method for multiple criteria decision making problem based on neutrosophic hesitant fuzzy set, (2014).
43. J. Hu, L. Pan, and X. Chen. An interval neutrosophic projection-based VIKOR method for selecting doctors. Cognitive Computation, 9(6) (2017), 801-816.
44. K. Selvakumari, and M. Ajitha Priyadharshini. VIKOR method for decision making problem using octagonal neutrosophic soft matrix. International Journal of Latest Engineering Research and Applications, 2 (7) (2017), 41-45.
45. Chakraborty, A.; Mondal, S. P.; Alam, S.; Ahmadian, A.; Senu, N.; De, D. andSalahshour, S.; Disjunctive Representation of Triangular Bipolar Neutrosophic Numbers, De-Bipolarization Technique and Application in Multi-Criteria Decision-Making Problems,Symmetry,11(7), (2019).
46. S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. Neutrosophic Sets and Systems, 17 (2017), 20-28.
47. S. Ye, and J. Ye. Dice similarity measure between single valued neutrosophic multi sets and its application in medical diagnosis. Neutrosophic Sets and System, 6 (2014), 49-54.
48. I.M. Hezam, M. Abdel-Baset, and F. Smarandache. Taylor series approximation to solve neutrosophic multiobjective programming problem. Neutrosophic Sets and Systems, 10 (2015), 39-45.
49. S. Broumi, and F. Smarandache. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems, 6 (2014), 42-48.
50. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., &Smarandache, F. (2019). A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. *Symmetry*, *11*(7), 903.

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