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Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique

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Abstract: In this paper, initially a matrix representation of Plithogenic Hypersoft Set (PHSS) is introduced and then with the help of this matrix some local operators for Plithogenic Fuzzy Hypersoft set (PFHSS) are developed. These local operators are used to generalize PFHSS to Plithogenic Fuzzy Whole Hypersoft set (PFWHSS). The generalized PFWHSS set is hybridization of Fuzzy Hypersoft set (which represent multiattributes and their subattributes as a combined whole membership i.e. case of having an exterior view of the event) and the Plithogenic Fuzzy Hypersoft set (in which multi attributes and their subattributes are represented with individual memberships case of having interior view). Thus, the speciality of PFWHSS is its presentation of an exterior and interior view of a situation simultaneously. Later, the PFWHSS is employed in development of multi attributes decision making scheme named as Frequency Matrix Multi Attributes Decision making scheme (FMMADMS). This innovative technique is not only simpler than any of the former MADM techniques, but also has a unique capability of dealing mathematically a variety of human mind psychologies at every level that are working in different environments (fuzzy, intuitionistic, neutrosophic, plithogenic). Besides, FMMADMS also provides the percentage authenticity of the final ranking which in itself is a new idea providing a transparent and unbiased ranking. Moreover, the new introduced idea of frequency matrix handles the ranking ties in the best possible way and has an ability to provide the authenticity comparative analysis of previously developed schemes. Lastly, application of this FMMADMS is described as a numerical example for a case of ranking and selecting the best alternative.

Keywords: Plithogenic Hypersoft set, Exterior view, Plithogenic Whole Hypersoft set, Interior view, Frequency Matrix, Multi Attribute Decision making Scheme, Percentage authenticity.

1. Introduction

The theory of uncertainty in mathematics was initially introduced by Zadeh [26] in 1965 named as fuzzy set theory (FST). A fuzzy set is a set where each element of the universe of discourse X has some degree of belongingness in unit closed interval $[0,1]$ in given set A, where A is subset of universal set X with respect to some attribute say M with imposing condition that the sum of membership and non membership is one unlike crisp set where element from the universe either belong to given set A or does not belong to A. In Fuzzy set theory, elements of set are expressed with one quantity i.e. degree of membership. To represent this degree of membership a notation $\mu_A(x) \in [0,1] \forall x \in X$ was used and to represent the degree of nonmembership a notation $\upsilon_A(x) \in [0,1] \forall x \in X$ was used. The members of fuzzy set are represented by using one quantity i.e. the degree of membership $\{x : \mu_A(x)\}$.

Due to the condition $\mu_A(x) + \upsilon_A(x) = 1 \forall x \in X$ imposed by Zadeh the degree of non membership $\upsilon_A(x)$ to A will be $1 - \mu_A(x)$, where $\upsilon_A(x) \in [0,1] \forall x \in X$.

Further generalization of fuzzy set was made by Atanassov [1] in 1986 which are known as Intuitionistic fuzzy set (IFS). In IFS the natural concept of hesitation in human mind was used in assigning a degree of membership in unit closed interval such that sum of degree of membership, degree of non membership and degree of hesitation should be one. The degree of hesitation or indeterminacy was represented by the notation"ι"now the improved condition is $\mu_A(x) + \upsilon_A(x) + \iota_A(x) = 1 \forall x \in X$. The members of IFS are represented by using two quantities $\mu_A(x)$and $\upsilon_A(x) \{x : (\mu_A(x), \upsilon_A(x))\}$. Later, IFS were further generalized by Smarandache [15]. He considered membership $\mu_A(x)$, nonmembership $\upsilon_A(x)$ and indeterminacy $\iota_A(x)$ as independent quantities or functions in the unit cube, representing three axis of the unit cube in non standard unit interval $]0^-, 1^+[$. Smarandache represented the elements of Neutrosophic set (NS) by using three independent quantities and introduced "Neutrosophy"[16-17] as a new branch of philosophy which studies the origin nature,by considering neutrality and opposite and their interactions with different ideational spectra. Mathematically, a NS is represented by $\{x : (\mu_A(x), \upsilon_A(x), \iota_A(x))\}$ with condition $0 \le \mu_A(x) + \upsilon_A(x) + \iota_A(x) \le 3$. The new defined approach of dealing with human mind consciousness in form of Neutrosophic Set is utilized in MCDM and MADM techniques ([2-7],[9], ,[12], [18] ,[25]).

Furthermore, Smarandache[13] has generalized the Soft set to Hypersoft set by transforming the function F of one attribute into a multi attribute function where $a_1, a_2, \dots, a_n$ for $n \ge 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \dots, A_n$ with $A_i \cap A_j = \varphi$ for $i \ne j$ and $i, j \in \{1, 2, \dots, n\}$ and assigning a combine membership $\mu_{A_1 \times A_2 \times \dots \times A_N}(x)$, non membership $\upsilon_{A_1 \times A_2 \times \dots \times A_N}(x)$ and Indeterminacy $\iota_{A_1 \times A_2 \times \dots \times A_N}(x)$ $\forall x \in X$ with condition and introduced a hybrids of Crisp/ Fuzzy/ Intuitonistic Fuzzy and Neutrosophic Hypersoft set and then generalized Hypersoft set to Plithogenic Hypersoft set (PHSS) by assigning a separate degree of membership, nonmembership and indeterminacy

$\mu_{A_i}(x), \upsilon_{A_i}(x), \iota_{A_i}(x)$ respectively to each attribute value A_i. Thus a Plithogenic Set, as the generalization of Crisp, Fuzzy, Intuitionistic Fuzzy, Picture Fuzzy and Neutrosophic Set was introduced by F.Smarandache in 2017 [14].

In this paper, we have firstly presented to our reader an entirely new concept of looking at a Plithogenic Hypersoft set in a form of a matrix. This matrix representation is further utilized in the emergence of some new local operators such as disjunction, conjunction and averaging operators for Plithogenic Fuzzy Hyper soft sets (PFHSS). In the second stage, we have utilized these local operators to the define a new idea of a Plithogenic Fuzzy Whole Hypersoft Set (PFWHSS). This new PWHSS not only present a deep insight into a Plithogenic decision making environment but also a broader outlook of a situation which clearly is more generalized and precise approach of modelling human mind capabilities. Moreover, the new PWHSS are employed in development of a multi attribute decision making scheme named as Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS).

In most MADM techniques, ranking is achieved by generating a comparison of alternatives with ideal and non ideal solution ([8], [19], [20]) etc. Mostly, comparison are made on the basis of distance, inclusion, and similarity measurements etc. These scheme when studied analytically are actually representing fuzzy behavior of human mind. The ideal solution represents membership and the non ideal solution represents nonmembership behavior of fuzzy environment. Besides, the selection of any input information taken from any background (fuzzy, intuitionistic fuzzy, neutrosophic or any other) the use of ideal and non ideal solution in modelling of different MADM schemes actually drives the entire scheme to a fuzzy environment. So the ranking is based on optimist and pessimist human behavior. In this new FMMADMS, the ranking includes the three behavior of human mind, optimist behavior (represented mathematically by using Max operator employed in construction of local operators which are involved in ranking procedure), pessimist behavior (represented mathematically by using Min operator used in designing local disjunction also used in ranking process) and the neutral behavior (represented mathematically by using averaging operator). The final decision is made by combining the three human mind behaviors in a matrix called Frequency Matrix which gives the ultimate ranking of alternatives. The major advantage of the new scheme is its capacity of indulging many human mind behaviors by introducing variety of operators between Min, Max and averaging operators. Thus, generalizing the scheme from neutrosophic to plithogenic modelling environment [14]. Also, in our scheme at its final stage a ratios authenticity of the ranking operators is provided to guarantee the rightfulness of the final decision.

With a brief introduction of our work in Section 1, we have organized the rest of the paper in following sections: Section 2, is a collection of all the necessary preliminaries required for understanding of this work while in Section 3, we have presented the new concept of representing a

Plithogenic Fuzzy Hyper Soft Set in form of a matrix. Moreover, have introduced some new local operators on this set and constructed a whole membership using these local operators. This whole membership over a PFHSS set gives a birds eye view of the entire situation thus driving to new idea of Plithogenic Fuzzy Whole Hyper Soft Set. Furthermore, the newly defined PFWHSS is used in constructing a new MADM technique called Frequency Matrix Multi Attributes Decision making scheme (FMMADMS). In Section 4, a numerical example is presented to elaborate the new scheme while in Section 5 we give the final Conclusion of this work along with some open problems related to this field.

2. Preliminaries

In this section, we will present some basic definitions of soft set, fuzzy soft set, hypersoft set, crisp hypersoft set, fuzzy hypersoft set, plithogenic hypersoft set, plithogenic crisp hypersoft set and plithogenic fuzzy hypersoft set which are useful in development of our literature.

Definition 2.1 [21] **(Soft Set)**

Let U be the initial universe of discourse, and E is a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U, and $A \subseteq E$ is a set of attributes. Then pair (F, A), where $F: A \longrightarrow P(U)$ is called **Soft Set** over U. In other words, a soft set (F, A) over U is parameterized family of subset of U. For $e \in A$, $F(e)$ may be considered as set of e elements or e approximate elements

$(F, A) = \{(F(e) \in P(U): e \in E, F(e) = \varphi \text{if} e \notin A\}$.

Definition 2.2 [24] **(Soft subset)**

For two soft set (F, A) and (G, B) over a universe U, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subseteq B$, and

(ii) $\forall e \in A, F(e) \subseteq G(e)$

The set of all soft set over U will be denoted by $S(U)$.

Definition 2.3 [26] **(Fuzzy set)**

Let U be the universe . A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping $\mu_X : U \longrightarrow [0,1]$

The vale of $\mu_X(x)$ for the fuzzy set X is called the membership value of the grade of membership of $x \in U$.The membership value represent the degree of belonging to fuzzy set X. Then a fuzzy set X on U can be represented as follows.

$X = \{(\mu_X(x)/x): x \in U, \mu_X(x) \in [0,1]\}$.

Definition 2.4 [9] **(Fuzzy soft set)**

Let U be the initial universe of discourse, $F(U)$ be all fuzzy set over U. E be the set of all parameters or attributes with respect to U and $A \subseteq E$ is a set of attributes. A fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows, $\Gamma_A = \{x, \gamma_A(x): x \in E, \gamma_A(x) \in F(U)\}$
where $\gamma_A: E \longrightarrow F(U)$ such that $\gamma_A(x) = \varphi$ if $x \notin A$

$$\gamma_A(x) = \{\mu_{\gamma_A(x)}(u)/u: u \in U, \mu_{\gamma_A(x)}(u) \in [0,1]\}.$$

Definition 2.5 [13] **(Hypersoft set)**

Let U be the initial universe of discourse $P(U)$ the power set of U and $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1,2, \ldots, n\}$.
Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$ where, $F: A_1 \times A_2 \times \ldots \times A_n \rightarrow P(U)$.
is called a Hypersoft set over U

Definition 2.6 [13] (Crisp Universe of Discourse)

A Universe of Discourse U_C is called Crisp if $\forall x \in U_C$, $x \in 100\%$ to U_C or membership of x $T(x)$ with respect to A in M is 1 denoted as $x(1)$.

Definition 2.7 [13] (Fuzzy Universe of Discourse)

A Universe of Discourse U_F is called Fuzzy if $\forall x \in U_C$ x partially belongs to U_F or membership of x $T(x) \subseteq [0,1]$ where $T(x)$ may be subset , an interval, a hesitant set, a single value set, etc. denoted as $x(T_x)$.

Definition 2.8 [13] (Plithogenic Universe of Discourse)

A Universe of Discourse U_p over a set V of attributes values, where $V = \{v_1, v_2, \ldots, v_n\}$, $n \geq 1$, is called Plithogenic if $\forall x \in U_p$ x belongs to U_p in the degree $d_x^0(v_i)$ with respect to the attribute value v_i, for all $i \in \{1,2, \ldots, n\}$. Since the degree of membership may be Crisp, Fuzzy, Intuitionistic Fuzzy, or Neutrosophic, the Plithogenic Universe of discourse may can be Crisp, fuzzy, Intuitionistic fuzzy, or Neutrosophic.

Definition 2.9 [13] (Crisp Hypersoft set)

Let U_C be the initial universe of discourse $P(U_C)$ the power set of U.

Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1,2, \ldots, n\}$. Then the pair $(F_C, A_1 \times A_2 \times \ldots \times A_n)$ where $F_C: A_1 \times A_2 \times \ldots \times A_n \rightarrow P(U_C)$, is called Crisp Hypersoft set over U_C.

Definition 2.10 [13] (Fuzzy Hypersoft set)

Let U_F be the initial universe of discourse $P(U_F)$ the power set of U_F.

$a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1,2, \ldots, n\}$. Then the pair

$(F_F, A_1 \times A_2 \times \ldots \times A_n)$ where $F_F: A_1 \times A_2 \times \ldots \times A_n \to P(U_F)$, is called Fuzzy Hypersoft set over U_F.

Definition 2.11 [13] (Plithogenic Hypersoft set)

Now instead of assigning combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ $\forall x \in U_c/\ U_F/U_{IF}/U_N$ for Hyper Soft set if each attribute A_j is assigned an individual membership $\mu_{A_j}(x)$, non membership $\upsilon_{A_j}(x)$ and Indeterminacy $\iota_{A_j}(x)$ $j = 1,2,\ldots,n$ in Crisp/Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft set then these generalized Crisp/Fuzzy/Intuitionistic Fuzzy and neutrosophic Hypersoft set are called Plithogenic Crisp/ Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft set.

3. Plithogenic Fuzzy Hyper Soft set, their representation in a Matrix form and generalization to Plithogenic Fuzzy Whole Hypersoft set

In this section, we define initially Crisp Whole Hypersoft set, Fuzzy Whole Hypersoft set, Intuitionistic Fuzzy Whole Hypersoft set, Neutrosophic Whole Hypersoft set.

Definition 3.1 (Plithogenic Crisp/ Fuzzy/ Intuitionistic Fuzzy and neutrosophic Whole Hypersoft set)

Let $U_{pl}(X)$ be the plithogenic universe of discourse and $F: A_1^k \times A_2^k \times \ldots \times A_N^k \to P(U_{pl})$ where $k = 1,2,3,\ldots,M$ represent Numeric values of attributes A_j for each j, k: and A^krepresent sub attributes of the given attributes, can attain different numeric values. Now if in Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hypersoft set all attributes A_j^k have both an individual membership $\mu_{A_j}(x)$, non membership $\upsilon_{A_j}(x)$and indeterminacy $\iota_{A_j}(x)$ where $j = 1,2,\ldots,N$ and a whole combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Omega(x)$, non membership $\upsilon_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Phi(x)$ and Indeterminacy $\iota_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Psi(x)$ then these generalized Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy /Neutrosophic Hypersoft set are called *Plithogenic Crisp/ Fuzzy/Intuitionistic Fuzzy / Neutrosophic Whole Hypersoft set.*

The Plithogenic Whole Hypersoft set is hybridization of Plithogenic Hypersoft set and Hypersoft set. If we are representing our set only with fuzzy memberships say $\mu_{A_j}(x)$ for individual attributes and Fuzzy whole memberships $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ say $\Omega(x)$ for combined attributes then the set under consideration are *Plithogenic Fuzzy Whole Hypersoft set.* Initially the literature is developed only for Plithogenic Fuzzy Hypersoft set and Plithogenic Fuzzy Whole Hypersoft set.

3.1 Plithogenic Fuzzy Whole Hypersoft set and Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS)

For convenience in dealing with plithogenic hypersoft set the data or informations i.e. memberships will be represented in the form of matrix denoted by C_{ij}^{α} for some combination of numeric values of attributes where α represent the given combination of attributes, i represent rows of matrix with respect to objects x_i, j represents columns of matrix with respect to numeric values of attributes A_j. These matrices will be helpful in construction of local Disjunction, Conjunction and Averaging operators. Furthermore, local constructed operators are used for the development of whole memberships denoted by Ω and then these memberships are used to generalize the Plithogenic Hypersoft Set to Plithogenic Whole Hypersoft Set and in development of a multi attributes decision making scheme named as Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS). The speciality of these local operators is that they deal within the matrix constructed by using informations or one can say within one combination of attributes which gives interior view of the event. In this section, we shall be dealing with PFHSS only. Later the idea can be generalized to other environments (intuitionistic, neutrosophic, plithogenic) etc. Let us now formally introduce the steps of FMMADMS. In this scheme, the first four steps are related to the matrix construction of PFHSS and their local operators while in the next three steps PFWHSS are developed using these operators and are utilized in defining the local ranking. Moreover, a final ranking is obtained using a frequency matrix. Also, a percentage authenticity is calculated to guarantee the transparency of the process.

Step 1. Decision of universe: Consider universe of discourse $U_{pl} = \{x_i\}$ $i = 1,2,3,\ldots, M$ and then $T = \{x_i\} \subset U_{pl}$ where i could be chosen between 1 to M. Here x_i represent the objects under consideration.

Step 2. Defining attributes and mapping: Let $A_1^k, A_2^k, A_3^k, \ldots, A_N^k$ be the attributes. Choose some attributes represented by A_j, $j = 1,2,3,\ldots,N$ and then assign k some numeric values can be presented by A_j^k where k and j can take values $1,2,3,\ldots,N$. The data of the numerical values is based on the decision maker's opinion by using the linguistic scales [[10],[11],[23]]. Define $F: A_1^k \times A_2^k \times A_3^k \times \ldots \times A_N^k \rightarrow P(U_{pl})$, where F is a mappings from combination of attributes to some subset of power set of U_{pl}.

Step 3. Matrix representation: Write the data or information (Memberships) in the form of a matrix. Let C_{ij}^{α}; $j = 1,2,3,\ldots,N$ and $i = 1,2,3,\ldots, M$: be the matrix and let α represent the given combination of attributes A_j^k for some j and k.

$$C_{ij}^{\alpha} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_M \end{array} \begin{array}{c} A_1^k \quad\quad A_2^k \quad\quad\quad\quad A_N^k \\ \begin{bmatrix} \mu_{A_1}(x_1) & \mu_{A_2}(x_1) & . & . & . & \mu_{A_N}(x_1) \\ \mu_{A_1}(x_2) & \mu_{A_1}(x_2) & . & . & . & \mu_{A_N}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1}(x_M) & \mu_{A_2}(x_M) & . & . & . & \mu_{A_N}(x_M) \end{bmatrix} \end{array} \quad (3.1)$$

Step 4. Construction of Local operators and Global whole memberships: Now by using individual memberships $\mu_j(x_i)$, for $x_i \in T$ and varying j from 1 to N one can develop a combined whole membership, say $\Omega^t(x_i)$ to x_i in T with respect to given combination of attributes by using different operators on rows of matrices of representation C_{ij}^{α} for Construction of local operators. These operators can be represented by taking different integer values of t i.e. $t = 1$ represent local disjunction operator , $t = 2$ represent local conjunction operator and $t = 3$ represent local averaging operator. The following local operators are constructed. Here, we define some local operators for Plithogenic Fuzzy Hypersoft Set. It is observed that the same operators are applicable for Plithogenic Crisp Hypersoft set but as the results are trivial so we will consider here only the case of Plithogenic Fuzzy Hypersoft set

Local Disjunction Operator for Plithogenic Fuzzy Hypersoft Set :

$$\vee_{loc}(F) = \cup(C_{ij}^{\alpha}) = Max_j(C_{ij}^{\alpha}) = Max_j(\mu_j(x_i)) \quad (3.2)$$

(Choose maximum membership from i_{th} row)

Here $\vee_{loc}, \cup$ are representations for local disjunctions operators for F, $\mu_j(x_i)$ is the membership for j_{th} attribute with respect to i_{th} object.

Local Conjunction Operators for Plithogenic Fuzzy Hypersoft Set :

$$\wedge_{loc}(F) = \cap(C_{ij}^{\alpha}) = Min_j(C_{ij}^{\alpha}) = Min_j(\mu_j(x_i)) \quad (3.3)$$

(Choose minimum membership from i_{th} row amongst j columns) and the result will be a column matrix representation three entities. Here $\wedge_{loc}$ are representations for local conjunctions operators for F, $\mu_j(x_i)$ is the membership for j_{th} attribute with respect to i_{th} object.

Local Averaging Operator for Plithogenic Fuzzy Hypersoft Set :

$$\Gamma(F) = \Gamma(C_{ij}^{\alpha}) = \sum_{j=1}^{N} \frac{\mu_j(x_i)}{N} \quad (3.4)$$

Here Γ represent averaging operator for mapping F for α combination of attributes applied on the given matrix of representation C_{ij}^{α} by taking average of memberships for i_{th} row.

Local Compliment for Plithogenic Fuzzy Hypersoft Set :

$$C_{loc}(F) = c\left(C_{ij}^{\alpha}\right) = \begin{Bmatrix} Max_j\left(1-\mu_j(x_i)\right) \\ Min_j\left(1-\mu_j(x_i)\right) \\ \sum_{j=1}^{N} \frac{\left(1-\mu_j(x_i)\right)}{N} \end{Bmatrix} \quad (3.5)$$

Here C_{loc} represent the local compliment for F mapping forα combination of attributes applied over matrix of representation C_{ij}^{α} by taking compliment of memberships for i_{th} row and then choosing either maximum or minimum or taking average of them. By applying Local disjunction, Local conjunction and Local averaging operators (3.2, 3.3, 3.4) to (3.1) one can develop a combined whole membership, denoted by $\Omega_{\alpha}^{t}(x_i)$.

Note: Here we have not used the compliment operator to develop the whole membership. But the choice is open for reader to work with this operator or any other operator of their choice.

Here $\Omega_{\alpha}^{t}(x_i)$ is representation for whole combined membership for i_{th} object withe respect to α combination of attributes in subset of $P(U_{pi})$

$$\Omega_{\alpha}^{1}(x_i) = \cup_j \left(C_{ij}^{\alpha}\right) = Max_j\left(\mu_j(x_i)\right) \quad (3.6)$$

$\Omega_{\alpha}^{1}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using disjunction operator ($t = 1$) developed in (3.2).

$$\Omega_{\alpha}^{2}(x_i) = \cap_j \left(C_{ij}^{\alpha}\right) = Min_j\left(\mu_j(x_i)\right) \quad (3.7)$$

$\Omega_{\alpha}^{2}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using conjunction operator ($t = 2$) developed in (3.3).

$$\Omega_{\alpha}^{3}(x_i) = \Gamma\left(C_{ij}^{\alpha}\right) = \sum_{j=1}^{N} \frac{\left(\mu_j(x_i)\right)}{N} \quad (3.8)$$

$\Omega_{\alpha}^{3}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using averaging operator Γ ($t = 3$) developed in (3.4).

We shall use $\Omega_\alpha^1(x_i), \Omega_\alpha^2(x_i)$ and $\Omega_\alpha^3(x_i)$ for three different whole memberships of Plithogenic Fuzzy Whole hypersoft set.

Step 5. Matrix representation of Plithogenic Fuzzy Whole Hypersoft set and initial ranking:

Write the data or information (local individual membership and global whole memberships) in the form of an other matrix denoted by $C_{ij}^{\alpha t}$; $j = 1,2,3,\ldots,N$ and $i = 1,2,3,\ldots, M$ and α represents the given combination of attributes and $t = 1,2,3$ represent the local operators used to get the whole combined memberships where $C_{ij}^{\alpha t}$ is the matrix representation for Plithogenic Fuzzy whole Hypersoft set.

$$\begin{array}{cc} & \begin{array}{ccccc} A_1^k & A_2^k & & A_N^k \ldots\ldots & \Omega_\alpha^t \end{array} \\ C_{ij}^{\alpha t} = \begin{array}{c} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_M \end{array} & \begin{bmatrix} \mu_{A_1}(x_1) & \mu_{A_2}(x_1) & . & . & . & \mu_{A_N}(x_1) & \Omega_\alpha^t(x_1) \\ \mu_{A_1}(x_2) & \mu_{A_1}(x_2) & . & . & . & \mu_{A_N}(x_2) & \Omega_\alpha^t(x_2) \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \mu_{A_1}(x_M) & \mu_{A_2}(x_M) & . & . & . & \mu_{A_N}(x_M) & \Omega_\alpha^t(x_M) \end{bmatrix} \end{array}$$

Where in A_j^k, k takes values with respect to given some α combination and in Ω_α^t and in $C_{ij}^{\alpha t}$while i represent rows of matrix and j represent its columns and $C_{ij}^{\alpha t}$; Plithogenic Fuzzy Whole Hypersoft Matrix (PFWHSM). For $t = 1,2,3$ we shall get three PFWHSM's.

In particular, for a fixed t and for some α combination of attributes $A_j, j = 1,2,3,\ldots,N$ we will get an initial ranking for alternatives $T = \{x_i\}$ under consideration in $C_{ij}^{\alpha t}$ from the last column of $C_{ij}^{\alpha t}$ which is the column of whole membership value Ω_α^t . The first position is assigned to an alternative having highest whole membership $\Omega_\alpha^t(x_i)$ [which is the highest numeric value in last column] and the second position to one having second largest membership and so on. If a tie occurs for the position of alternatives in this initial ranking, it will be removed in final ranking. In this step, by varying $t = 1,2,3$ we shall obtain the three types of initial ranking of our alternatives based on three operators see (3.6,3.7 and 3.8). All of these ranking will be utilized in next stage to get the final ranking of alternatives.

It is worth mentioning here the fact that these initial rankings presents three human mind behaviors for three different choices of operators. To be more specific for $t = 1$ the use of Max operator will provide the choice of optimist behavior of human mind. Similarly for $t = 2$ which represent the use of Min operator one can represent the pessimist behavior of human mind. Furthermore, the choice of $t = 3$ i.e., the use of averaging operator will represent the neutral behavior of human mind. Finally in the next step by using the frequency matrix we will combine the three human mind behaviors to provide the final results of the ranking procedure.

Step 6. Construction of frequency matrix F_{qp} for final ranking:

Finally, we have constructed the frequency matrix of positions F_{qp} from initial ranking where $q = 1,2,\ldots M$ is used to represent rows (alternatives) of frequency matrix F_{qp} and $p = 1,2,\ldots,M$ is used to represent columns (positions attained by these alternatives) of frequency matrix F_{qp}.

$$F_{qp} = \begin{matrix} & p_1 & p_2 & \cdot & \cdot & \cdot & p_M \\ x_1 & f_{11} & f_{12} & \cdot & \cdot & \cdot & f_{1M} \\ x_2 & f_{21} & f_{22} & \cdot & \cdot & \cdot & f_{2M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_M & f_{M1} & f_{M2} & \cdot & \cdot & \cdot & f_{MM} \end{matrix}$$

The final frequency matrix F_{qp} of alternatives and positions is a square matrix of order $M \times M$ i.e. number of ordering positions will be equal to the number of alternatives, The selection of first position to any alternative will be made by looking into the first column corresponding to the position 1 i.e. p_1. The alternative having the largest frequency value in this column will be assigned first position. Once first position is decided, the entire row corresponding to this alternative and the first column will be excluded from the process of selection. Next, we shall look into the second column to select the candidate having the largest frequency value to be assigned the second position of ordering. Once done he shall be excluded from the process by excluding his row and the second column from the process. This procedure of selection will continue until all the positions are assigned to the rightful alternative.

In final frequency matrix if two alternatives have the same frequency of position 1 which is a very rare case, then we check their frequency of position 2, the one having higher frequency value in position 2 will be assigned the first position. After this selection the particular alternative and the position 1 will be excluded from selection procedure. Then other competitor will be assigned the second position. In this way all the ties can be fairly handled in this process.

Step 7. Percentage measure of authenticity of ranking: Finally the percentage measure of authenticity can be obtained by using the ratios formula:

Percentage authenticity of p_{th} position for q_{th} alternative $= \frac{\max_q (f_{qp})}{\sum_q f_{qp}} \times 100$, where f_{qp} is the obtained frequency of the p_{th} position for q_{th} alternative and $\sum_q f_{qp}$ is the total frequency of p_{th} position.

4. Numerical Example

Step 1. Decision of universe: let $U = \{x_1, x_2, x_3, x_4, x_5\}$ set of five members of Engineering department and $T = \{x_2, x_3, x_5\} \subset U$ set of three members who have applied for the post of Assistant professor.

Step 2. Defining Attributes and mapping:

Let the attributes be $A_j^k; j = 1,2,3,4$ and k may have any value from 1 to 3

$A_1^k =$ Subject skill area with numeric values, $k = 1,2,3$

$A_1^1 =$ Mathematics, $A_1^2 =$ Physics, $A_1^3 =$ Computer science

$A_2^k =$ Qualification with numeric values, $k = 1,2$

$A_2^1 =$ Higher qualification like Ph.D. or equivalent, $A_2^2 =$ lower qualification like MS or equivalent $A_3^k =$ Teaching experience with numeric values, $k = 1,2$

$A_3^1 =$ Three years or less, $A_3^2 =$ More than three years

$A_4^k =$ Age , with numeric values $k = 1,2,3$

$A_4^1 =$ Age is less than thirty years, $A_4^2 =$ Age is between thirty to forty years $A_4^3 =$Age is greater than forty years

We need to select faculty members.

Let the Function F be given by,

$F: A_1^k \times A_2^k \times A_3^k \times A_4^k \rightarrow P(U)$ for $k = 1,1,1,2$ respectively.

We are interested in ranking of these three candidates for the Engineering department with the following criteria.

1. Subject skill area: Mathematics: $k = 1$
2. Qualification: Higher qualification like Ph.D or Equivalent $k = 1$
3. Teaching experience: Three years or less $k = 1$
4. Age: Age required is between thirty to forty years $k = 2$

$F(A_1^1, A_2^1, A_3^1, A_4^2) = \{x_2, x_3, x_5\}$ let we name $A_1^1, A_2^1, A_3^1, A_4^2$ combination as α

With respect to T =$\{x_2, x_3, x_5\}$ have memberships in PFHSS. Consider the memberships of x_2, x_3, x_5 as $\mu_j(x_i)$ for $i = 2,3,5$ and $j = 1$ to 4 in T with respect to α combination of attributes.

$F(\alpha) = F(A_1^1, A_2^1, A_3^1, A_4^2) = \{x_2(0.3,0.6,0.4,0.5), x_3(0.4,0.5,0.3,0.1), x_5(0.6,0.3,0.3,0.7)\}$

Step 3. Matrix representation: Let C_{ij}^{α} represented in 3.1 is the matrix of representation for the combination of attributes α,in PFHSS. Here rows are representing x_2, x_3, x_5 and columns are representing $A_1^k, A_2^k, A_3^k, A_4^k$.

$$C_{ij}^{\alpha} = \begin{matrix} & A_1^1 & A_2^1 & A_3^1 & A_4^2 \\ x_2 & 0.3 & 0.6 & 0.4 & 0.5 \\ x_3 & 0.4 & 0.5 & 0.3 & 0.1 \\ x_5 & 0.6 & 0.3 & 0.3 & 0.7 \end{matrix}$$

Step 4. Construction of Local operators and Global whole memberships for PFHSS: By using individual memberships $\mu_j^{\alpha}(x_i)$, for $x_i \in T \subset U$ now with respect to α combination of attributes by fixing $i = 2,3$ and 5 and varying j from 1 to 4 in 3.6, 3.7 and 3.8 one can assign a combined (whole) membership, $\Omega_{\alpha}^{t}(x_i)$ to $x_i \in U$ in T with respect to α combination of attributes by using operators developed in 3.6, 3.7 and 3.8 on rows of matrix of representation C_{ij}^{α}. Using (3.1)

$$\Omega_{\alpha}^{1}(x_i) = \cup\,(\mu_j^{\alpha}(x_i)) = Max_j\left(\mu_j^{\alpha}(x_i)\right)$$

$$\Omega_{\alpha}^{2}(x_i) = \cap\left(\mu_j^{\alpha}(x_i)\right) = Min_j\left(\mu_j^{\alpha}(x_i)\right)$$

$$\Omega_{\alpha}^{3}(x_i) = \Gamma\left(\mu_j^{\alpha}(x_i)\right) = \Sigma_{j=1}^{N}\frac{(\mu_j^{\alpha}(x_i))}{N}$$

This membership is used in Generalization of PFHSS to Plithogenic Fuzzy Whole Hyper Soft set.

$\Omega_{\alpha}^{1}(x_2) = \cup\left(\mu_j^{\alpha}(x_2)\right) = Max_j\left(\mu_j^{\alpha}(x_2)\right) = 0.6$ for $i = 2$ and varying j from 1 to 4

$\Omega_{\alpha}^{1}(x_3) = \cup\left(\mu_j^{\alpha}(x_3)\right) = Max_j\left(\mu_j^{\alpha}(x_3)\right) = 0.5$ for $i = 3$ and varying j from 1 to 4

$\Omega_{\alpha}^{1}(x_5) = \cup\left(\mu_j^{\alpha}(x_5)\right) = Max_j\left(\mu_j^{\alpha}(x_5)\right) = 0.7$ for $i = 5$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_2) = \cap\left(\mu_j^{\alpha}(x_2)\right) = Min_j\left(\mu_j^{\alpha}(x_2)\right) = 0.3$ for $i = 2$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_3) = \cap\left(\mu_j^{\alpha}(x_3)\right) = Min_j\left(\mu_j^{\alpha}(x_3)\right) = 0.1$ for $i = 3$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_5) = \cap\left(\mu_j^{\alpha}(x_5)\right) = Min_j\left(\mu_j^{\alpha}(x_5)\right) = 0.3$ for $i = 5$ and varying j from 1 to 4

$$\Omega_\alpha^3(x_2) = \Gamma\left(\mu_j^\alpha(x_2)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^\alpha(x_2)\right)}{4} = 0.45 \text{ for } i = 2, N = 4 \text{and varying } j \text{ from } 1 \text{ to } 4$$

$$\Omega_\alpha^3(x_3) = \Gamma\left(\mu_j^\alpha(x_3)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^\alpha(x_3)\right)}{4} = 0.325 \text{ for } i = 2, N = 4 \text{ and varying } j \text{ from } 1 \text{ to } 4$$

$$\Omega_\alpha^3(x_5) = \Gamma\left(\mu_j^\alpha(x_5)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^\alpha(x_5)\right)}{4} = 0.45 \text{ for } i = 2, N = 4 \text{ and varying } j \text{ from } 1 \text{ to } 4$$

Step 5 Matrix representation of Plithogenic Fuzzy Whole Hypersoft set and initial ranking:

$$C_{ij}^{\alpha 1} = \begin{matrix} & A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_\alpha^1 \\ x_2 & 0.3 & 0.6 & 0.4 & 0.5 & 0.6 \\ x_3 & 0.4 & 0.5 & 0.3 & 0.1 & 0.5 \\ x_5 & 0.6 & 0.3 & 0.3 & 0.7 & 0.7 \end{matrix}$$

For choosing the best one will select the largest value from last column i.e. $x_5 = 0.7$ The initial ranking for $t = 1$, is Position 1: for x_5,Position 2: for x_2 and Position 3: for x_3.

$$C_{ij}^{\alpha 2} = \begin{matrix} & A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_\alpha^1 \\ x_2 & 0.3 & 0.6 & 0.4 & 0.5 & 0.3 \\ x_3 & 0.4 & 0.5 & 0.3 & 0.1 & 0.1 \\ x_5 & 0.6 & 0.3 & 0.3 & 0.7 & 0.3 \end{matrix}$$

For choosing the best one will select the largest value from last column i.e. $x_2 = x_5 = 0.3$. The initial ranking for $t = 2$, is Position 1: could be assigned to both the candidates x_5 and x_2. This tie will be removed in final step of ranking.

$$C_{ij}^{\alpha 3} = \begin{matrix} & A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_\alpha^1 \\ x_2 & 0.3 & 0.6 & 0.4 & 0.5 & 0.45 \\ x_3 & 0.4 & 0.5 & 0.3 & 0.1 & 0.325 \\ x_5 & 0.6 & 0.3 & 0.3 & 0.7 & 0.475 \end{matrix}$$

For choosing the best one will select the largest value from last column i.e. $x_5 = 0.7$ The initial ranking for $t = 3$, is Position 1: for x_5,Position 2: for x_2 and Position 3: for x_3.

Step 6. Construction of frequency matrix F_{qp} for final ranking: Next we construct a frequency matrix to get the final ranking using the data of step 5.

$$F_{qp} = \begin{matrix} & p_1 & p_2 & p_3 \\ x_2 & 1 & 2 & 0 \\ x_3 & 0 & 0 & 3 \\ x_5 & 3 & 0 & 0 \end{matrix}$$

This frequency matrix shows the frequency of getting first position for $x_2$is 1, for $x_3$is 0 and for x_5 is 3, the frequency of getting second position for $x_2$is 2, for $x_3$is 0 and for x_5 is 3,and the frequency of getting third position for $x_2$is 3, for $x_3$is 0 and for x_5 is 0. We can see here the initial ranking for $t = 1$ is $x_5 \rhd x_2 \rhd x_3$, for $t = 2$ is $x_5 = x_2 \rhd x_3$ and ranking for $t = 3$ is $x_5 \rhd x_2 \rhd x_3$ and the final ranking from the frequency matrix F_{qp} is same i.e., $x_5 \rhd x_2 \rhd x_3$ which shows use of frequency matrix increases the authenticity of the ranking and selection of right candidate for the post.

Step 7. Percentage measure of authenticity of ranking:

Percentage authenticity of first position for $x_5 = \frac{\max_q(f_{qp})}{\sum_q f_{qp}} = \frac{f_{31}}{\sum_q f_{q1}} \times 100 = 75\%$

Percentage authenticity of second position for $x_2 = \frac{f_{12}}{\sum_q f_{q2}} \times 100 = 100\%$

Percentage authenticity of third position for $x_3 = \frac{f_{23}}{\sum_q f_{q3}} \times 100 = 100\%$

5. Conclusion

A novice idea of matrix representation of Plithogenic Fuzzy Hypersoft Set (PFHSS) is introduced along with construction of their local operators such as conjunction, disjunction and averaging operators. These local operators are utilized in defining a new concept of Plithogenic Fuzzy Whole Hyper Fuzzy Soft Set (PFWHSS). The PFWHSS deals fuzziness of the data or information as a combined vision (external view) in case of combined membership of a combination of attributes and individually (internal view) as a in case of considering individual memberships. Furthermore, an innovative yet simple MADM technique called Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS) is developed. In this technique, at first stage, we have employed three different PFWHSS to get three initial rankings of alternatives representing decisions made by three different human mind behaviors of being optimist (the case in which whole membership is obtained by using conjunction (Max) operator), pessimist (the case in which whole membership is obtained using disjunction (Min) operator) and the neutral behavior (the case in which whole membership is obtained using averaging operator). In the next stage, we have introduced a new concept of frequency matrix that combines all the three possibilities of human mind behavior to provide with a final ranking decision of alternatives. In many decision making schemes, there are possibilities of ties between ranking alternatives. The use frequency matrix in FMMADMS provides a unique way of handling these ties. It results into a final ranking free of ties. Lastly, the scheme works with a percentage measure to guarantee the authenticity and accuracy of the final ranking. This itself, is entirely a new idea to get to get an authenticity of different ranking schemes which shows that the final decision is transparent and unbiased.

Moreover, this technique is more generalized since it use PFWHSS which deals with not only attributes but also sub attributes at the same time. One of the beauty of this scheme is its simplicity as the user need not to handle with complicated long calculations based operators. Also this new technique have a flexible approach of using wide range of operators that can absorb changes according to the requirement of the provided environment. To be more specific, the selection of three operators represent a neutrosophic behavior which clearly is a special case of plithogenic attitude as mentioned in [14]. Now introducing more operators among these three neutrosophic elemental behaviors (membership, nonmembership, neutrality) we can generalize the model of this scheme in plithogenic environment which may handle more of human mind complexities.

Some of the open problems that could be addressed: This work have vast extensions by developments of some new literature on operators, their properties and applications in different environments like Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic etc. and development of multi attributes decision making techniques in different environments. Moreover, the matrix representation of plithogenic whole hypersoft set opens new dimensions towards development of many operators and MADM techniques.

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