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Abstract. Gathering the attitudes of the examined respondents would be very significant in some evaluation models. Therefore, a multiple criteria approach based on the use of the neutrosophic set is considered in this paper. An example of the evaluation of restaurants is considered at the end of this paper with the aim to present in detail the proposed approach.

Keywords: neutrosophic set, single valued neutrosophic set, multiple criteria evaluation.

1. Introduction

In order to deal with indeterminate and inconsistent information, Smarandache [1] proposed a neutrosophic set (NS), thus simultaneously providing a general framework generalizing the concepts of the classical, fuzzy [2], interval-valued [3, 4], intuitionistic [5] and interval-valued intuitionistic [6] fuzzy sets.

The NS has been applied in different fields, such as: the database [7], image processing [8, 9, 10], the medical diagnosis [11, 12], decision making [13, 14], with a particular emphasis on multiple criteria decision making [15, 16, 17, 18, 19, 20].

In addition to the membership function, or the so-called truth-membership $T_A(x)$, proposed in fuzzy sets, Atanassov [5] introduced the non-membership function, or the so-called falsity-membership $F_A(x)$, which expresses non-membership to a set, thus creating the basis for the solving of a much larger number of decision-making problems.

In intuitionistic fuzzy sets, the indeterminacy $I_A(x)$ is $1-T_A(x)-F_A(x)$ by default.

In the NS, Smarandache [21] introduced independent indeterminacy-membership $I_A(x)$, thus making the NS more flexible and the most suitable for solving some complex decision-making problems, especially decision-making problems related to the use of incomplete and imprecise information, uncertainties and predictions and so on.

Smarandache [1] and Wang *et al*. [22] further proposed the single valued neutrosophic set (SVNS) suitable for solving many real-world decision-making problems.

In multiple criteria evaluation models, where evaluation is based on the ratings generated from respondents, the NS and the SVNS can provide some advantages in relation to the usage of crisp and other forms of fuzzy numbers.

Therefore, the rest of this paper is organized as follows: in Section 2, some basic definitions related to the SVNS are given. In Section 3, an approach to the determining of criteria weights is presented, while Section 4 proposes a multiple criteria evaluation model based on the use of the SVNS. In Section 5, an example is considered with the aim to explain in detail the proposed methodology. The conclusions are presented at the end of the manuscript.

2. The Single Valued Neutrosophic Set

Definition 1. [21] Let X be the universe of discourse, with a generic element in X denoted by x. Then, the Neutrosophic Set (NS) A in X is as follows:

$$A = \{x < T_A(x), I_A(x), F_A(x) > | \, x \in X\}, \tag{1}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively, $T_A, I_A, F_A : X \to]^-0,1^+[$ and $^-0 \leq T_A(x) + I_A(x) + U_A(x) \leq 3^+$

Definition 2. [1, 22] Let X be the universe of discourse. The Single Valued Neutrosophic Set (SVNS) A over X is an object having the form:

$$A = \{x < T_A(x), I_A(x), F_A(x) > | \, x \in X\}, \tag{2}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the intermediacy-membership function and the falsity-membership function, respectively, $T_A, I_A, F_A : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + U_A(x) \leq 3$.

Definition 3. [21] For an SVNS A in X, the triple $< t_A, i_A, f_A >$ is called the single valued neutrosophic number (SVNN).

Definition 4. *SVNNs*. Let $x_1 =< t_1, i_1, f_1 >$ and $x_2 =< t_2, i_2, f_2 >$ be two SVNNs and $\lambda > 0$; then, the basic operations are defined as follows:

$$x_1 + x_2 =< t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2 > . \tag{3}$$

$$x_1 \cdot x_2 =< t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2 > . \tag{4}$$

$$\lambda x_1 =< 1 - (1 - t_1)^{\lambda}, i_1^{\lambda}, f_1^{\lambda} > . \tag{5}$$

$$x_1^{\lambda} =< t_1^{\lambda}, i_1^{\lambda}, 1 - (1 - f_1)^{\lambda} > . \tag{6}$$

Definition 5. [23] Let $x =< t_x, i_x, f_x >$ be a SVNN; then the cosine similarity measure $S_{(x)}$ between SVNN x and the ideal alternative (point) <1,0,0> can be defined as follows:

$$S_{(x)} = \frac{t}{\sqrt{t^2 + i^2 + f^2}} . \tag{7}$$

Definition 6. [23] Let $A_j =< t_j, i_j, f_j >$ be a collection of SVNSs and $W = (w_1, w_2, \ldots, w_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Average (SVNWA) operator of A_j is as follows:

$$\begin{aligned} SVNWA(A_1, A_2, \ldots, A_n) &= \sum_{j=1}^{n} w_j A_j \\ &= \left(1 - \prod_{j=1}^{n} (1 - t_j)^{w_j}, \prod_{j=1}^{n} (i_j)^{w_j}, \prod_{j=1}^{n} (f_j)^{w_j} \right), \end{aligned} \tag{8}$$

where: w_j is the element j of the weighting vector, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. The SWARA Method

The Step-wise Weight Assessment Ratio Analysis (SWARA) technique was proposed by Kersuliene *et al.* [25]. The computational procedure of the adapted SWARA method can be shown through the following steps:

Step 1. Determine the set of the relevant evaluation criteria and sort them in descending order, based on their expected significances.

Step 2. Starting from the second criterion, determine the relative importance s_j of the criterion j in relation to the previous (j-1) criterion, and do so for each particular criterion as follows:

$$s_j = \begin{cases} >1 & \text{when significance of } C_j \succ C_{j-1} \\ 1 & \text{when significanse of } C_j = C_{j-1} \\ <1 & \text{when significance of } C_j \prec C_{j-1} \end{cases} . \tag{9}$$

By using Eq. (9), respondents are capable of expressing their opinions more realistically compared to the ordinary SWARA method, proposed by Kersuliene *et al*. [25].

Step 3. The third step in the adapted SWARA method should be performed as follows:

$$k_j = \begin{cases} 1 & j = 1 \\ 2 - s_j & j > 1 \end{cases} . \tag{10}$$

where k_j is a coefficient.

Step 4. Determine the recalculated weight q_j as follows:

$$q_j = \begin{cases} 1 & j = 1 \\ q_{j-1} / k_j & j > 1 \end{cases} . \tag{11}$$

Step 5. Determine the relative weights of the evaluation criteria as follows:

$$w_j = q_j \Big/ \sum_{k=1}^{n} q_k , \tag{12}$$

where w_j denotes the relative weight of the criterion j.

4. A Multiple Criteria Evaluation Model Based on the Use of the SVNS

For a multiple criteria evaluation problem involving the m alternatives that should be evaluated by the K respondents based on the n criteria, whereby the performances of alternatives are expressed by using the SVNS, the calculation procedure can be expressed as follows:

The determination of the criteria weights. The determination of the criteria weights can be done by applying various methods, for example by using the AHP method. However, in this approach, it is recommended that the SWARA method should be used due to its simplicity and a smaller number of pairwise comparisons compared with the well-known AHP method.

The determination of the criteria weight is done by using an interactive questionnaire made in a spreadsheet file. By using such an approach, the interviewee can see the calculated weights of the criteria, which enables him/her modify his or her answers if he or she is not satisfied with the calculated weights.

Gathering the ratings of the alternatives in relation to the selected set of the evaluation criteria. Gathering the ratings of the alternatives in relation to the chosen set of criteria is also done by using an interactive questionnaire. In this questionnaire, a declarative sentence is formed for each one of the criteria, thus giving an opportunity to the respondents to fill in their attitudes about the degree of truth, indeterminacy and falsehood of the statement.

The formation of the separated ranking order based on the weights and ratings obtained from each respondent. At this steep, the ranking order is formed for each one of the respondents, based on the respondent's respective weights and ratings, in the following manner:

– the determination of the overall ratings expressed in the form of the SVNN by using Eq. (8), for each respondent;
– the determination of the cosine similarity measure, for each respondent; and
– the determination of the ranking order, for each respondent.

The determination of the most appropriate alternative. Contrary to the commonly used approach in group decision making, no group weights and ratings are used in this approach. As a result of that, there are the K ranking orders of the alternatives and the most appropriate alternative is the one determined on the basis of the theory of dominance [26].

5. A Numerical Illustration

In this numerical illustration, some results adopted from a case study are used. In the said study, four traditional restaurants were evaluated based on the following criteria:

– the interior of the building and the friendly atmosphere,
– the helpfulness and friendliness of the staff,
– the variety of traditional food and drinks,
– the quality and the taste of the food and drinks, including the manner of serving them, and
– the appropriate price for the quality of the services provided.

The survey was conducted via e-mail, using an interactive questionnaire, created in a spreadsheet file. By using such an approach, the interviewee could see the calculated weights of the criteria and was also able to modify his/her answers if he or she was not satisfied with the calculated weights.

In order to explain the proposed approach, three completed surveys have been selected. The attitudes related to the weights of the criteria obtained in the first survey are shown in Table 1. Table 1 also accounts for the weights of the criteria.

Criteria	s_j	k_j	q_j	w_j
C_1		1	1	0.15
C_2	1.00	1.00	1.00	0.15
C_3	1.15	0.85	1.18	0.18
C_4	1.30	0.70	1.68	0.26
C_5	1.00	1.00	1.68	0.26

Table 1. The attitudes and the weights of the criteria obtained on the basis of the first of the three surveys

The attitudes obtained from the three surveys, as well as the appropriate weights, are accounted for in Table 2.

	E_1		E_1		E_1	
	s_j	w_j	s_j	w_j	s_j	w_j
C_1		0.15		0.16		0.19
C_2	1.00	0.15	1.00	0.16	1.00	0.19
C_3	1.15	0.18	1.20	0.20	1.05	0.20
C_4	1.30	0.26	1.10	0.22	1.10	0.22
C_5	1.00	0.26	1.10	0.25	0.95	0.21

Table 2. The attitudes and the weights obtained from the three surveys

The ratings of the alternatives expressed in terms of the SVNS obtained on the basis of the three surveys are given in Tables 3 to 5.

	C_1	C_2	C_3	C_4	C_5
w_j	0.15	0.15	0.18	0.26	0.26
A_1	<0.8,0.1,0.3>	<0.7,0.2,0.2>	<0.8,0.1,0.1>	<1,0.01,0.01>	<0.8,0.1,0.1>
A_2	<0.7,0.1,0.2>	<1.0,0.1,0.1>	<1.0,0.2,0.1>	<1,0.01,0.01>	<0.8,0.1,0.1>
A_3	<0.7,0.1,0.1>	<1.0,0.1,0.1>	<0.7,0.1,0.1>	<0.9,0.2,0.01>	<0.9,0.1,0.1>
A_4	<0.7,0.3,0.3>	<0.7,0.1,0.1>	<0.8,0.1,0.2>	<0.9,0.1,0.1>	<0.9,0.1,0.1>

Table 3. The ratings obtained based on the first survey

	C_1	C_2	C_3	C_4	C_5
w_j	0.16	0.16	0.20	0.22	0.25
A_1	<0.8,0.1,0.4>	<0.9,0.15,0.3>	<0.9,0.2,0.2>	<0.85,0.1,0.25>	<1.0,0.1,0.2>
A_2	<0.9,0.15,0.3>	<0.9,0.15,0.2>	<1.0,0.3,0.2>	<0.7,0.2,0.1>	<0.8,0.2,0.3>
A_3	<0.6,0.15,0.3>	<0.55,0.2,0.3>	<0.55,0.3,0.3>	<0.6,0.3,0.2>	<0.7,0.2,0.3>
A_4	<0.6,0.4,0.5>	<0.6,0.3,0.1>	<0.6,0.1,0.2>	<0.7,0.1,0.3>	<0.5,0.2,0.4>

Table 4. The ratings obtained based on the second survey

	C_1	C_2	C_3	C_4	C_5
w_j	0.19	0.19	0.20	0.22	0.21
A_1	<1.0,0.1,0.1>	<0.9,0.15,0.2>	<1.0,0.2,0.1>	<0.8,0.1,0.1>	<0.9,0.1,0.2>
A_2	<0.8,0.15,0.3>	<0.9,0.15,0.2>	<1,0.2,0.2>	<0.7,0.2,0.1>	<0.8,0.2,0.3>
A_3	<0.6,0.15,0.3>	<0.55,0.2,0.3>	<0.55,0.3,0.3>	<0.6,0.3,0.2>	<0.7,0.2,0.3>
A_4	<0.8,0.4,0.5>	<0.6,0.3,0.1>	<0.6,0.4,0.1>	<0.7,0.1,0.3>	<0.5,0.2,0.4>

Table 5. The ratings obtained from the third of the third survey

The calculated overall ratings obtained on the basis of the first of the three surveys expressed in the form of SVNSs are presented in Table 6. The cosine similarity measures, calculated by using Eq. (7), as well as the ranking order of the alternatives, are accounted for in Table 6.

	Overall ratings	S_i	Rank
A_1	<1.0,0.06,0.07>	0.995	2
A_2	<1.0,0.06,0.06>	0.996	1
A_3	<1.0,0.12,0.06>	0.991	3
A_4	<1.0,0.12,0.13>	0.978	4

Table 6. The ranking orders obtained on the basis of the ratings of the first survey

The ranking orders obtained based on all the three surveys are accounted for in Table 7.

	E_1 S_i	E_2 S_i	E_3 S_i	E_1 Rank	E_2 Rank	E_3 Rank
A1	0.995	0.963	0.985	2	1	1
A2	0.996	0.962	0.966	1	2	2
A3	0.991	0.864	0.867	3	4	4
A4	0.978	0.882	0.894	4	3	3

Table 7. The ranking orders obtained from the three examinees

According to Table 7, the most appropriate alternative based on the theory of dominance is the alternative denoted as A_1.

6. Conclusion

A new multiple criteria evaluation model based on using the single valued neutrosophic set is proposed in this paper. For the purpose of determining criteria weights, the SWARA method is applied due to its simplicity, whereas for the determination of the overall ratings for each respondent, the SVNN is applied. In order to intentionally avoid the group determination of weights and ratings, the final selection of the most appropriate alternative is determined by applying the theory of dominance. In order to form a simple questionnaire and obtain the respondents' real attitudes, a smaller number of the criteria were initially selected. The proposed model has proven to be far more flexible than the other MCDM-based models and is based on the conducted numerical example suitable for the solving of problems related to the selection of restaurants. The usability and efficiency of the proposed model have been demonstrated on the conducted numerical example.

References

[1] F. Smarandache. Neutrosophy: Neutrosophic probability, set, and logic, American Research Press, Rehoboth, USA, 1998.

[2] L. A. Zadeh. Fuzzy Sets. Information and Control, 8, (1965), 338-353.

[3] I. B. Turksen. Interval valued fuzzy sets based on normal forms. Fuzzy sets and systems, 20(2), (1986), 191-210.

[4] I. B. Turksen. Non-specificity and interval-valued fuzzy sets. Fuzzy sets and systems, 80(1), (1996), 87-100.

[5] K. Atanassov. Intuitionistic fuzzy sets. Fuzzy sets and Systems, 20(1), (1986), 87-96.

[6] K. Atanassov and G. Gargov. Interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 31(3), (1989), 343-349.

[7] M. Arora, R. Biswas, and U. S. Pandy. Neutrosophic relational database decompositio. International Journal of Advanced Computer Science & Applications, 1(2), (2011), 121-125.

[8] A. Salama, F. Smarandache, and M. Eisa. Introduction to Image Processing via Neutrosophic Techniques. Neutrosophic Sets and Systems, 5, (2014), 59-64.

[9] J. Mohan, A. T. S. Chandra, V. Krishnaveni, and Y. Guo. Evaluation of neutrosophic set approach filtering technique for image denoising. The International Journal of Multimedia & Its Applications, 4(4), (2012), 73-81.

[10] Y. Guo, and H. D. Cheng. New neutrosophic approach to image segmentation. Pattern Recognition, 42, (2009), 587–595.

[11] S. Broumi, and I. Deli. Correlation measure for neutrosophic refined sets and its application in medical diagnosis. Palestine Journal of Mathematics, 3(1), (2014), 11-19.

[12] Ansari, A. Q., Biswas, R., and Aggarwal, S. (2011). Proposal for applicability of neutrosophic set theory in medical AI. International Journal of Computer Applications, 27(5), 5-11.

[13] A. Kharal. A Neutrosophic Multicriteria Decision Making Method, New Mathematics and Natural Computation, Creighton University, USA, 2013.

[14] J. Ye. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems, 42(4) (2013) 386--394.

[15] J. Chen, and J. Ye. A Projection Model of Neutrosophic Numbers for Multiple Attribute Decision Making of Clay-Brick Selection. NSS, 12, (2016), 139-142.

[16] J. Ye, and F. Smarandache Similarity Measure of Refined Single-Valued Neutrosophic Sets and Its Multicriteria Decision Making Method. NSS, 12, (2016), 41-44

[17] J. J. Peng, and J. Q. Wang. Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems. Neutrosophic Sets and Systems, 10, (2015), 3-17.

[18] K. Mandal, and K. Basu. Hypercomplex Neutrosophic Similarity Measure & Its Application in Multicriteria Decision Making Problem. NSS, 9, (2015), 6-12.

[19] S. Pramanik, P. P. Dey, and B. C. Giri. TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems. NSS, 10 (2015), 88-95

[20] Y. Jun. Another Form of Correlation Coefficient between Single Valued Neutrosophic Sets and Its Multiple Attribute Decision Making Method. NSS, 1, (2013), 8-12.

[21] F. Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

[22] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman. Single valued neutrosophic sets, Multispace and Multistructure, 4, (2010), 410-413.

[23] R. Sahin. Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment. arXiv preprint arXiv:1412.5202, 2014.

[24] J. Ye. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26, (2014), 2459-2466.

[25] V. Kersuliene, E. K. Zavadskas and Z. Turskis. Selection of rational dispute resolution method by applying new step- wise weight assessment ratio analysis (SWARA). Journal of Business Economics and Management, 11(2), (2010), 243-258.

[26] W. K. M. Brauers and E. K. Zavadskas. MULTIMOORA optimization used to decide on a bank loan to buy property. Technological and Economic Development of Economy, 17(1), (2011), 174-188.

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