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Abstract. Here This paper focuses on solving the transportation problems with neutrosophic data for the first time. The indeterminacy factor has been considered in Transportation Problems (TP). The two methods of linear programming – Fuzzy Linear Programming (FLP) and Crisp Linear Programming (CLP) are discussed with reference to neutrosophic transportation problems. The first method uses the membership, non-membership and indeterminacy degrees separately to find the crisp solution using the Fuzzy Programming Technique and then the optimal solution is calculated in terms of neutrosophic data with the help of defined cost membership functions. The satisfaction degree is then calculated to check the better solution. The second method directly solves the TP to find crisp solution considering a single objective function. The cost objective function is taken as neutrosophic data and the methods have been used as such for the first time. Both the methods have been illustrated with the help of a numerical example and these are then applied to solve a real life multi - objective and multi-index transportation problem. Finally the results are compared.

Keywords: Neutrosophic Transportation Problem; Fuzzy Linear Programming; Crisp Linear Programming; Fuzzy Programming Technique; indeterminacy degree

1 Introduction

The basic transportation problem was originally developed by Hitchcock [1]. There are several classical methods to solve such transportation problems where data is given in a precise way. But in real life transportation problems, data may not be known with certainty. In such cases, the imprecise data can be considered as interval valued or fuzzy data. Fuzzy set theory was introduced by Zadeh [2]. Zimmermann [3] introduced fuzzy linear programming (LP) problems. Zimmermann [4] considered LP with fuzzy goal and fuzzy constraints and used linear membership function and min operator as an aggregator of these functions. Thus Fuzzy Linear Programming (FLP) problem was formulated. Further, Fuzzy set theory was applied to solve LPP with several objectives functions. The fuzzified constraint and objective functions were used to solve the multi-objective linear programming problems. Chanas [5] focused on Fuzzy Linear Programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas and Kuchta [6] developed an algorithm for the optimal solution of TP with fuzzy coefficients which are expressed as L-R fuzzy numbers. Chanas and Kuchta [7] developed an algorithm for solving integer fuzzy transportation problem with fuzzy supply and demand. Bit and Biswal [8] applied the fuzzy programming technique with linear membership function to solve Multi-objective transportation problem (MOTP). Bit and Biswal [9] proposed an additive fuzzy programming model that considered weights and priorities for all non equivalent objectives for the transportation planning problems. Li and Lai [10] developed a fuzzy compromise programming method to obtain a non-dominated compromise solution to the MOTP in which various objectives were synthetically considered with marginal evaluation for individual objectives and the global evaluation for all objective functions. A real life multi-index multi-objective transportation problem was solved by Kour, Mukherjee and Basu in [11],[12],[13],[14] and [15] using different approaches. Intuitionistic fuzzy sets (IFS) were introduced as generalization of fuzzy set (FS). Here membership and non-membership degree were used instead of exact numbers. Intuitionistic fuzzy sets (IFSs) were introduced by Atanassov [16]. Atanassov & Gargov [17] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) as a further generalization of that of IFSs. Atanassov [18] also defined some operational laws of IVIFSs. Angelov [19] reformulated the optimization problems in an intuitionistic fuzzy environment. Several works have been done taking the triangular and trapezoidal intuitionistic fuzzy number. Gani and Abbas [20] proposed a new method for intuitionistic fuzzy transportation problem using triangular intuitionistic fuzzy number. Hussain and Kumar [21] applied the fuzzy zero point method to find optimal solution of intuitionistic fuzzy transportation problems Antony [22] also developed the VAMs method for TP for triangular intuitionistic fuzzy number. Aggarwal and Gupta [23] solved the TP for generalized trapezoidal intuitionistic fuzzy number by ranking method. P. P. Angelov first introduced the Intuitionistic fuzzy optimization (IFO) in his paper [19] and solved the transportation problem with crisp data by this method. The concept of Neutrosophic set was introduced as a generalization of crisp, fuzzy, intuitionistic, interval valued Intuitionistic Fuzzy number by Smarandache[24]. The Indeterminacy function (I) was added to the two available parameters: Truth (T) and Falsity (F) membership functions. In Neutrosophic Set, the indeterminacy is quantified explicitly and truth - membership, indeterminacy membership and false - membership are completely independent. In intuitionistic fuzzy sets, the indeterminacy is 1- $T(x)$ - $F(x)$ (i.e. hesitancy or unknown degree) by default. In Neutrosophy, the indeterminacy membership (I(x)) is introduced as a new subcomponent so as to include the degree to which the decision maker is not sure. This type of treatment of the problem was out of scope of intuitionistic fuzzy sets. Wang et al. [25] introduced the concept of single valued neutrosophic set (SVNS).

The present paper presents the solution of transportation problems with neutrosophic data using linear programming methods. It deals with cost objective function as neutrosophic data and the Neutrosophic TP has been solved using two methods. In the first method, fuzzy linear programming (FLP) has been extended for the neutrosophic data and the second method uses the crisp linear programming method (CLP).

The formulations and solutions are illustrated with the help of solved example and then the results are compared. The uncertainties of the real life problems are considered in the form of neutrosophic data. In transportation problems, the cost of transportation, the demand and the supply may not be known exactly as crisp numbers. Thus the uncertainties can be considered in terms of their degrees of acceptance, degrees of indeterminacy and degrees of rejection. That is, neutrosophic fuzzy numbers can be used for representing the imprecise data of cost of transportation or demand or supply or all in a transportation problem. This can be explained with the help of an example. If the transportation cost is taken in terms of the neutrosophic fuzzy number (0.8,0.1,0.2), that means the degree of acceptance of the available cost is 0.8 , degrees of indeterminacy is 0.1 while the degree of rejection of the available cost is 0.2.

Finally the methods are applied to solve a real life multi-objective and multi-index neutrosophic transportation problem for the first time. The problem is solved to optimize the three objectives simultaneously namely, transportation cost, deterioration rate and underused capacity with neutrosophic data. The paper presents a better application of the method for multi-objective transportation problems.

2 Preliminaries

2.1 Single Valued Neutrosophic Set (SVNS)

An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ϵ [0, 1]. (Wang et al.[25]

When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{\langle T_A(x), I_A(x), F_A(x)\rangle}{x}, x\in X$$

When X is discrete, an SVNS A can be written as

$$A=\sum_{i=1}^{n} \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle}{x_i}, x_i\in X$$

3 Problem description and methodology

3.1 Problem Description

The transportation problem is meant for minimization of transportation cost from different sources to different destinations.

- **Classical transportation problems:**
  In the classical transportation problem cost objective function and the constraints are considered as crisp values. Therefore it is required to calculate the optimal solution which minimizes the cost objective functions and satisfies all the constraints.

Minimize $f(x)$

Subject to $g_j(x)\le 0,\ j=1,2,.....q$

(1)

- **Fuzzy transportation problem:** Later on the fuzzy transportation problem was introduced [5,6,7] and used in further works[11-15,22,26]. The degree of satisfaction of the objective function and the constraints is maximized to find the optimal solution.
- **Intuitionistic fuzzy transportation problem:** Then the intuitionistic fuzzy transportation problem (IFTP) was considered [19, 20, 21, 23]. In such case the degree of rejection $\upsilon_i(x)$ is also considered along with the degree of acceptance $\mu_i(x)$ of the cost objective function and the constraints. The degree of acceptance is maximized and the degree of rejection is minimized to find the optimal solution in such problems.

- **Neutrosophic transportation problem:** In a transportation problems with neutrosophic data , the indeterminacy factor has been considered for the first time. The degree of indeterminacy $r_i(x)$ was also considered along wtith the two available parameters, degree of acceptance $\mu_i(x)$ and degree of rejection $\upsilon_i(x)$ of the cost objective function and the constraints. The problem is to maximize degree of acceptance and minimize the degree of rejection and indeterminacy.

Model 1. Fuzzy Linear Programming Model (for Neutrosophic data):
For single-objective TP

Maximize $Z_1 = \sum\sum \mu_{ij} x_{ij}$

Minimize $Z_2 = \sum\sum r_{ij} x_{ij}$

Minimize $Z_3 = \sum\sum \upsilon_{ij} x_{ij}$

subject to $0 \le \mu(x), r(x), \upsilon(x) \le 1,$

$\sum_j x_{ij} = S_i$ where S_i denotes the supply of source i,

$\sum_i x_{ij} = D_j$ where D_j denotes the demand of destination j,

$x_{ij} \ge 0$

(2)

For Multi-objective TP, we obtain a set of similar three equations for each of the objective functions

Model 2. Crisp Linear Programming Model (for Neutrosophic data)

For Single objective transportation problems, the model is

Maximize $Z = \sum\sum (\mu_{ij} - \upsilon_{ij} - r_{ij}) x_{ij}$

subject to $0 \le \mu(x), r(x), \upsilon(x) \le 1,$ and other constraints mentioned in Equation(2) (3).

For Multi-objective transportation problems, we obtain a set of similar equations for each objective function.

3.2 Methodology

3.2.1 Fuzzy Linear Pogramming

The Transportation problem with neutrosophic data has been formulated as a multi-objective transportation problem as in Model 1 and has been solved by Fuzzy Linear Programming Technique (Das[26], Zimmermann [3]).

Extended Fuzzy Programming Technique

Step 1: Solve the multi-objective transportation problem as a single objective transportation problem using each time only one objective and ignoring others.

Step 2: From the results of Step 1, determine the corresponding values for every objective at each solution derived.

Then find the lower and upper bounds , Z_k^{L}, and Z_k^{U} (k=1,2,3,….,K).

Step 3 **: Linear Membership Function**

A Linear membership function $\mu_{k(x)}$ corresponding to k^{th} objective for the minimization problem is defined as

$$\mu_k(x) = \begin{cases} 1 & if \quad Z_k \le Z_k^L, \\ 1 - \dfrac{Z_k - Z_k^L}{Z_k^U - Z_k^L} & if Z_k^L < Z_k < Z_k^U \\ 0 & if \quad Z_k \ge Z_k^U \end{cases}$$

(4)

Similarly, a linear membership function can be defined for maximization problem as

$$\mu_k(x) = \begin{cases} 0 & if \quad Z_k \le Z_k^L, \\ 1 - \dfrac{Z_k - Z_k^U}{Z_k^U - Z_k^L} & if Z_k^L < Z_k < Z_k^U \\ 1 & if \quad Z_k \ge Z_k^U \end{cases}$$

(5)

The linear programming problem can further be simplified as in Model 3:

Model 3:

Maximize λ

subject to $Z_k + \lambda(Z_k^U - Z_k^L) \le Z_k^U$ (6)

for minimization problem and

$$Z_k + \lambda(Z_k^U - Z_k^L) \ge Z_k^L ,$$

for maximization problem

(7)

with the given constraints and non-negativity restriction as in Model 1 and $\lambda \ge 0$ (8)

Thus the Step 3 gives the values of the three objective functions, Z_1 , Z_2 and Z_3 as in Model 1.

Step4:

$$Z = Z_1 - Z_2 - Z_3$$

(9)

This provides the crisp optimal value for the objective functions. Then using the definition of cost membership function, the satisfaction, indeterminacy and rejection degrees of membership function of the solution are obtained as Single Valued Neutrosophic Set (SVNS)

3.2.2 Crisp Linear Pogramming

The Model 1 can further be formulated as a single-objective linear programming problem as in Model 2 and is solved as usual by standard software. The solution gives the optimal value of cost objective function Z as a crisp value. For multi-objective transportation problems, it forms a set of objective functions in equations which can be solved by fuzzy programming technique and the optimal solution can be obtained as crisp value for each objective function.

4 Numerical Illustration

4.2.1 Example 1

The problem is taken as a neutrosophic transportation problem (NTP) in which each transportation cost is taken as neutrosophic data representing the degree of acceptance, degree of indeterminacy and degree of rejection of the cost as in Table 1. The demand and the capacity are considered as crisp values.

	Market 1	**Market 2**	**Market 3**	**Market 4**	**Capacity (S_i)**
Port 1	(0.6,0.1,0.2)	(0.7,0.2,0.1)	(0.3,0.3,0.1)	(0.8,0.1,0.1)	400
Port 2	(0.5,0.2,0.3)	(0.4,0.1,0.1)	(0.5,0.3,0.1)	(0.3,0.3.0.2)	150
Port 3	(0.4,0.3,0.2)	(0.3,0.2,0.2)	(0.6,0.3,0.1)	(0.7,0.3,0.2)	300
Demand (D_j)	200	200	100	350	

(6)

Table 1: Data for NFTP .

The objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $\upsilon_o(x)$ of the cost function defined as follows:

$$\mu_o(x) = \begin{cases} 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 200 \\ \dfrac{(350 - \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij})}{150}, & 200 \le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350 \\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350 \end{cases}$$

(10)

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}<250\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-250)^2}{10000}, & 250\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}>350\end{cases} \quad (11)$$

$$\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}<200\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-200)^2}{22500}, & 200\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}>350\end{cases} \quad (12)$$

where costs are considered in terms of thousand dollars.

4.2 Solution

The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods and the results are obtained.

Solution with data from Table 1 by the method based on FLP

Substituting the neutrosophic data from Table 1 in the Model 1, we get three different objective functions as

Maximize

$$Z_1 = 0.6x_{11} + 0.7x_{12} + 0.3x_{13} + 0.8x_{14} + 0.5x_{21} + 0.4x_{22} + 0.5x_{23} + 0.3x_{24} + 0.4x_{31} + 0.3x_{32} + 0.6x_{33} + 0.7x_{34}$$

Minimize

$$Z_2 = 0.1x_{11} + 0.2x_{12} + 0.3x_{13} + 0.1x_{14} + 0.2x_{21} + 0.1x_{22} + 0.3x_{23} + 0.3x_{24} + 0.3x_{31} + 0.2x_{32} + 0.3x_{33} + 0.3x_{34}$$

Minimize

$$Z_3 = 0.2x_{11} + 0.1x_{12} + 0.1x_{13} + 0.1x_{14} + 0.3x_{21} + 0.1x_{22} + 0.1x_{23} + 0.2x_{24} + 0.2x_{31} + 0.2x_{32} + 0.1x_{33} + 0.2x_{34}$$

subject to

$$\sum_j x_{ij} = S_i$$

where S_i denotes the supply of source i given in Table 1

$$\sum_i x_{ij} = D_j$$

where D_j denotes the demand of destination j given in Table 1

$x_{ij} \ge 0$

(13)

Step 1 : The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

$Z_1 = 565$, $Z_2 = 180$, $Z_3 = 140$,

Step 2 : For each solution set , the values for the other two objective functions are obtained as:

$Z_1 = 565$ $Z_1 = 505$ (for Z_2 solution set), $Z_1 = 515$ (for Z_3 solution set)

$Z_2 = 140$, $Z_2 = 180$ (for Z_1 solution set), $Z_2 = 150$ (for Z_3 solution set)

$Z_3 = 105$, $Z_3 = 140$ (for Z_1 solution set), $Z_3 = 110$ (for Z_2 solution set)

For each objective, the best and worst values are given as $Z_1^U = 565$, $Z_1^L = 505$, $Z_2^U = 180$, $Z_2^L = 140$, $Z_3^U = 140$, $Z_3^L = 105$,

Step 3 : Using the values obtained in Step 2 in the Equations (6) and (7) obtained from Model 3 of Section 4.2, the final solution is obtained as

$\lambda = 0.9090909$, $Z_1 = 508.64$, $Z_2 = 143.64$, $Z_3 = 108.18$

Step 4: Using the values obtained in Equation (9), $Z = 256.82$

Also the degree of acceptance, indeterminacy and rejection of cost objective functions are obtained using Equations (10),(11) and (12) as

$$\mu_o = 0.62, r_o = 0.0047, \upsilon_o = 0.14$$

i.e. $(\mu_o, r_o, \upsilon_o) = (0.62, 0.0047, 0.14)$

Solution with data from Table 1 by the method based on CLP

Substituting the neutrosophic data from Table 1in the Model 2, we get

Minimize

$$Z = 0.3x_{11} + 0.4x_{12} - 0.9x_{13} + 0.6x_{14} + 0x_{21} + 0.2x_{22} + 0.1x_{23} - 0.8x_{24} - 0.1x_{31} - 0.1x_{32} + 0.2x_{33} + 0.2x_{34}$$

subject to all the constraints in Equation (13)

The transportation problem is solved as single objective TP by crisp linear programming as in Model 2 and the crisp optimal solution is obtained as $Z = 260$.

The degree of acceptance, indeterminacy and rejection of cost objective functions are obtained as

$$\mu_o = 0.6, r_o = 0.01, \upsilon_o = 0.16$$

i.e. $(\mu_o, r_o, \upsilon_o) = (0.6, 0.01, 0.16)$

5 Real life Illustration

5.1 Real life multi-objective multi-index transportation problem

To illustrate the application of the proposed approach for a real life multi-objective multi-index transportation problem, following numerical example from Kour, Mukherjee and Basu [11] is considered, previously taken as approximate past records from DSP Plant, Durgapur,West Bengal, INDIA.

The problem deals with the solution of the multi-objective multi-index real life transportation problem focusing on the minimization of the transportation cost, deterioration rate and underused capacity of the transported raw materials like coal, iron ore, etc from different sources to different destination sites at Durgapur Steel Plant (DSP) by different transportation modes like train, trucks, etc. The problem is formulated by taking different parameters in the objective function as neutrosophic data and supply and demand as crisp numbers.

Consider a problem in which we have three raw materials (m=3) i.e. q=1(Coal), 2(Iron -ore), 3(Limestone).These raw materials are transported from different i^{th} sources to j^{th} destination sites by different transportation modes 'h' where h=1(train), 2(truck) as per Table 2.

Raw materials	Sources	Destinations	Mode of transportation
q=1	i=1,2	j=1,2	h=1
q=2	i=1,2,3	j=1,2	h=1,2
q=3	i=1,2,3,4,5	j=1	h=1

Table 2 Number of raw materials, sources, destinations and mode of transportation

Then we are considering the problem as a neutrosophic transportation problem (NTP) in which each transportation cost is taken as neutrosophic data representing the degree of acceptance, indeterminacy and rejection of the cost. The demand and the supply are considered as crisp numbers. i.e. Transportation Cost functions as C_{ijh}^q (C_{ij1}^1 , C_{ij1}^2 C_{ij2}^2, C_{ij1}^3), and other objective functions for Deterioration rate and underused capacity in matrix form. The data are given below.

$$C_{ij1}^1 = \begin{bmatrix} [0.8,0.01,0.15] & [0.6,0.1,0.3] \\ [0.5,0.02,0.3] & [0.9,0.01,0.01] \end{bmatrix}_{h=1}$$

$$C_{ij1}^2 = \begin{bmatrix} [0.5,0.01,0.4] & [0.75,0.01,0.02] \\ [0.55,0.02,0.3] & [0.4,0.1,0.2] \\ [0.8,0.01,0.1] & [0.9,0.02,0.03] \end{bmatrix}_{h=1}$$

$$C_{ij2}^{2}=\begin{bmatrix}[0.4,0.1,0.3] & [0.5,0.2,0.3]\\ [0.6,0.02,0.25] & [0.6,0.1,0.3]\\ [0.8,0.01,0.12] & [0.9,0.01,0.01]\end{bmatrix}_{h=2}$$

$$C_{ij1}^{3}=\begin{bmatrix}[0.85,0.01,0.02]\\ [0.78,0.02,0.2]\\ [0.8,0.01,0.1]\\ [0.7,0.01,0.15]\\ [0.5,0.02,0.3]\end{bmatrix}_{h=1}$$

$$R_{ij1}^{1}=\begin{bmatrix}[0.6,0.1,0.2] & [0.5,0.1,0.3]\\ [0.8,0.01,0.1] & [0.55,0.02,0.2]\end{bmatrix}_{h=1}$$

$$R_{ij1}^{2}=\begin{bmatrix}[0.6,0.1,0.3] & [0.9,0.01,0.1]\\ [0.8,0.1,0.2] & [0.7,0.01,0.2]\\ [0.4,0.3,0.1] & [0.6,0.2,0.1]\end{bmatrix}_{h=1}$$

$$R_{ij2}^{2}=\begin{bmatrix}[0.6,0.01,0.2] & [0.95,0.01,0.02]\\ [0.9,0.02,0.1] & [0.8,0.01,0.2]\\ [0.62,0.02,0.2] & [0.7,0.1,0.2]\end{bmatrix}_{h=2}$$

$$R_{ij1}^{3}=\begin{bmatrix}[0.9,0.02,0.1]\\ [0.4,0.1,0.3]\\ [0.6,0.02,0.2]\\ [0.75,0.1,0.1]\\ [0.8,0.2,0.1]\end{bmatrix}_{h=1}$$

$$U_{ij1}^{1}=\begin{bmatrix}[0.7,0.1,0.2] & [0.6,0.1,0.2]\\ [0.4,0.2,0.2] & [0.8,0.02,0.1]\end{bmatrix}_{h=1}$$

$$U_{ij1}^{2}=\begin{bmatrix}[0.3,0.1,0.2] & [0.5,0.02,0.2]\\ [0.45,0.1,0.2] & [0.4,0.02,0.3]\\ [0.6,0.1,0.3] & [0.7,0.03,0.2]\end{bmatrix}_{h=1}$$

$$U_{ij2}^{2}=\begin{bmatrix}[0.4,0.02,0.1] & [0.5,0.01,0.2]\\ [0.7,0.01,0.3] & [0.8,0.02,0.2]\\ [0.6,0.02,0.3] & [0.7,0.1,0.2]\end{bmatrix}_{h=2}$$

$$U_{ij1}^{3}=\begin{bmatrix}[0.5,0.03,0.3]\\ [0.8,0.02,0.1]\\ [0.7,0.01,0.2]\\ [0.4,0.02,0.3]\\ [0.6,0.01,0.2]\end{bmatrix}_{h=1}$$

The data in the form of crisp numbers for supply and demand are as follows:

q	**h**	**I**	S_{iq}
1	1	1	182.5
1	1	2	107.5
2	1	1	59
2	1	2	40
2	1	3	30.5
2	2	1	77
2	2	2	89.5
2	2	3	51.25
3	1	1	78.05
3	1	2	47.75
3	1	3	122.5
3	1	4	147.5
3	1	5	120

Table 4 Supply data

Q	**h**	**j**	D_{jq}
1	1	1	90
1	1	2	195
2	1	1	50
2	1	2	81
2	2	1	88
2	2	2	129
3	1	1	49

Table 4 Demand data

The data for supply S_{iq}, $\forall \quad i, q$ are given in the Table 3. The data for demand D_{jq}, $\forall \quad j, q$ are given in the Table 4. The neutrosophic objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $\upsilon_o(x)$ of the three objective functions defined as follows:

For Transportation Cost:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 200\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij})}{150}, & 200\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (14)$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 210\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-200)^2}{19600}, & 210\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (15)$$

$$\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 200\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-200)^2}{22500}, & 200\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (16)$$

For Deterioration Rate:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 150\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij})}{200}, & 200\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (17)$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 155\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-155)^2}{38025}, & 155\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (18)$$

$$\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 150\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-155)^2}{40000}, & 150\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350\end{cases} \quad (19)$$

ForUnderusedcapacity:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 100\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij})}{200}, & 100\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 400\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 400\end{cases} \quad (20)$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 150\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-155)^2}{90000}, & 150\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 400\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 400\end{cases} \quad (21)$$

$$\upsilon_O(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<100\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-100)^2}{40000}, & 100\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 400\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>400\end{cases}\quad (22)$$

The given problem is first written in the form of the formulated model, Model 4 as:

Minimize $$Z_1=\sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p}C_{ijh}^{q}X_{ijh}^{q} \quad (23)$$

Minimize $$Z_2=\sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p}R_{ijh}^{q}X_{ijh}^{q} \quad (24)$$

Minimize $$Z_3=\sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p}U_{ijh}^{q}X_{ijh}^{q} \quad (25)$$

Subject to $$\sum_{j}\sum_{h}X_{ijh}^{q}\ge S_{iq},\ \forall\ i,q \quad (26)$$

$$\sum_{i}\sum_{h}X_{ijh}^{q}\le D_{jq},\ \forall\ j,q \quad (27)$$

$$X_{ijh}^{q}\ge 0. \quad (28)$$

where q = type of raw material; m= number of raw materials;

n = number of sources;

o = number of destination sites;

h = transportation modes; p = number of transportation modes.

X_{ijh}^{q} = Quantity to be transported of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';

C_{ijh}^{q} =Transportation cost (in billion rupees per metric tonne) of transportation of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h' as neutrosophic set

R_{ijh}^{q} = Deterioration rate (in tonnes per million metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h'; as neutrosophic set

U_{ijh}^{q} = Underused capacity (in tonnes per thousand metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h'; as neutrosophic set

S_{iq} = Supplied quantity of q^{th} raw material from i^{th} source (Availability) (in million metric tonnes)

D_{jq} = Demand of q^{th} raw material at j^{th} destination (Requirement) (in million metric tonnes)

Z_1, Z_2, Z_3 are the minimal values of the neutrosophic Transportation Cost, Detrioration rate and Underused capacity.

5.2 Solution

The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods.

Solution by the method based on FLP:

Substituting the above neutrosophic data in the Model 1, three different objective functions for each of the Equations (23), (24) and (25) are obtained. Then the problem can be solved using Extended neutrosophic fuzzy programming technique.

Step 1: The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

1. $Z_{11} = 419.66$ 2. $Z_{12} = 21.43$ 3. $Z_{13} = 166.02$
4. $Z_{21} = 411.94$ 5. $Z_{22} = 76.64$ 6. $Z_{23} = 158.32$
7. $Z_{31} = 326.51$ 8. $Z_{32} = 34.66$ 9. $Z_{33} = 229.94$

Step 2 : For each solution set , the values for the other objective functions can be obtained. The best and worst values for each objective are obtained as:

$Z_{11}^L = 364.58$ $Z_{11}^U = 828.49$ $Z_{11}^U - Z_{11}^L = 463.91$

$Z_{12}^L = 21.43$ $Z_{12}^U = 42.13$ $Z_{12}^U - Z_{12}^L = 20.7$

$Z_{13}^L = 103.13$ $Z_{13}^U = 216.79$ $Z_{13}^U - Z_{13}^L = 113.66$

$Z_{21}^L = 376.66$ $Z_{21}^U = 843.75$ $Z_{21}^U - Z_{21}^L = 467.09$

$Z_{22}^L = 35.43$ $Z_{22}^U = 85.42$ $Z_{22}^U - Z_{22}^L = 49.99$

$Z_{23}^L = 64.12$ $Z_{23}^U = 191.88$ $Z_{23}^U - Z_{23}^L = 127.76$

$Z_{31}^L = 326.51$ $Z_{31}^U = 714.85$ $Z_{31}^U - Z_{31}^L = 388.28$

$Z_{32}^L = 31.22$ $Z_{32}^U = 56.14$ $Z_{32}^U - Z_{32}^L = 24.92$

$Z_{33}^L = 91.65$ $Z_{33}^U = 255.72$ $Z_{33}^U - Z_{33}^L = 164.07$

Step 3 : Corresponding to the three objective functions, a linear membership function can be defined. Then the problem can be solved using Equations(6)and(7) from Model 4 of Section 4.2 and the final solution is obtained as

$\lambda = 0.8365686$,

$Z_{11} = 328.75$, $Z_{12} = 0$,

$Z_{13} = 75.63$ $Z_{21} = 284.38$, $Z_{22} = 35.45$,

$Z_{23} = 85$ $Z_{31} = 263.25$, $Z_{32} = 0$, $Z_{33} = 105.25$

Step 4 : Using the values obtained in Equation (12), $Z_1 = 253.12, Z_2 = 163.93, Z_3 = 158$

The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as

- Transportation cost : $\mu_o = 0.65, r_o = 0.095, \upsilon_o = 0.13$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.65.0.095,0.13)$

- Deterioration rate : $\mu_o = 0.93, r_o = 0.0021, \upsilon_o = 0.005$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.93,0.0021,0.005)$

- Underused capacity : $\mu_o = 0.8, r_o = 0.001, \upsilon_o = 0.037$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.8,0.001,0.037)$

Solution by the method based on CLP:

Substituting the neutrsophic data in the Equation (3) in the Model 2, a set of three similar equations is obtained which form a multi-objective transportation problem and thus can be solved by fuzzy programming technique. This gives the optimal solution for each objective function. The final crisp optimal solution is obtained as

$\lambda = 0.4075449$, $Z_1 = 217.665$, $Z_2 = 329.12$ $Z_3 = 169.355$

. The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as

- Transportation cost : $\mu_o = 0.88, r_o = 0.003, \upsilon_o = 0.014$

  i.e. (μ_o, r_o, υ_o) $= (0.88.0.003, 0.014)$

- Deterioration rate : $\mu_o = 0.104, r_o = 0.7, \upsilon_o = 0.8$

  i.e. (μ_o, r_o, υ_o) $= (0.104, 0.7, 0.8)$

- Underused capacity : $\mu_o = 0.77, r_o = 0.0059, \upsilon_o = 0.053$

  i.e. (μ_o, r_o, υ_o) $= (0.77, 0.0059, 0.053)$

6. Results and Discussions

- The two methods are introduced for neutrosophic transportation problems and illustrated by an example in Section 4. The method is then applied for a real life multi-objective and multi-index neutrosophic transportation problem in Section 5 for the first time.
- The optimal solution for the neutrosophic transportation problems in Section 4 is obtained by the above two methods, i.e. by FLP and the other by CLP. The crisp optimal solution for the cost objective function of the given neutrosophic fuzzy transportation problem in Section 4 is obtained by FLP method using linear membership function as 256.82 (thousand dollars) as in Table 5. The degree of acceptance, indeterminacy and rejection of the obtained solution is calculated as (0.62, 0.0047, 0.14). Thus the satisfaction degree of the solution is 0.62 which means the solution is 62% acceptable 0.4% indeterminant (not known) and 76% rejectable.

λ	Z_1	Z_2	Z_3	Z	(μ_o, r_o, υ_o)
0.909	508.64	143.6	108.2	256.8	(0.6,0.0047,0.14)

Table5: Solution of Example in Section 4 using Linear membership function.

The degree of satisfaction of the optimal solution depends upon the respective defined membership, indeterminacy and non-membership function in the given problems. The degree of satisfaction and the degree of rejection need not be complement to each other. The crisp optimal solution for the cost objective transportation problem of the given neutrosophic transportation problem is obtained by crisp linear programming method as 260 (thousand dollars).The satisfaction degree of this solution is 0.6 which means the solution is 60% acceptable 0.01% indeterminant (not known) and 0.16% rejectable.

	Using Fuzzy linear programming	Using Crisp linear programming
Z	(0.62,0.0047,0.14	(0 .6,0.01,0.1 6)

Table 6 : Comparison of the obtained neutrosophic solutions using FLP and CLP methods.

The crisp optimal solution for the different objective functions – transportation cost, deterioration rate and underused capacity of the given real life neutrosophic transportation problem in Section 5 is obtained by FLP method using linear membership function as 253.12, 163.93 and 158 as in Table 8. The degree of acceptance, indeterminacy and rejection of the obtained solution for transportation cost, deterioration rate and underused capacity is calculated as (0.65,0.095,0.13), (0.93,0.0021,0.005) and (0.8,0.001,0.037). Thus the satisfaction degree of the three solutions are 0.65,0.93 and 0.859 which means the first transportation cost solution is 65% acceptable, 9% indeterminant and 13% rejectable. The second deterioration rate solution is 93% acceptable, 0.2% indeterminant and 0.5% rejectable and the third underused capacity solution is 80% acceptable, 0.1% indeterminant and 3.7% rejectable.

λ	Z_1	Z_2	Z_3	Z	(μ_o, r_o, υ_o)
0.909	328.8	0	75.63	253.1	(0.65,0.09 ,0.13)
	284.38	35.45	85	163.9	(0.93,0.00 2,0.005)
	263.25	0	105.2	158	(0.8,0.001 ,0.04)

Table7: Solution of real life example in Section 5 using Linear membership function.

The crisp optimal solution for the transportation cost, deterioration rate and underused capacity is calculated as 217.67, 329.12 and 169.355 by CLP method. The satisfaction degree of this solution for transportation cost, deterioration rate and underused capacity is calculated as 0.88, 0.104 and 0.77.

	Using Fuzzy linear programming	Using Crisp linear programming
Transportation cost	(0.65.0.09 5,0.13)	(0.88,0.003,0.014)
Deterioration rate	(0.93,0.00 21,0.005)	(0.104,0.7,0.8)
Underused capacity	(0.8,0.001 ,0.037)	(0.77,0.0059,0.053)

Table 8 : Comparison of the obtained neutrosophic solutions using FLP and CLPmethods.

Thus the FLP method appears to be better method as it gives more optimal solution as compared to the crisp linear programming method.

7. Conclusions

- In this paper, the Neutrosophic Transportation Problem (NTP) is solved by two methods- FLP method and CLP method.
- The first method, FLP method gives the solution as crisp and then as SVNS which represent the degree of acceptance, indeterminacy and rejection of the solution obtained from the defined membership function for a particular problem.
- The second method, i.e., CLP method gives the solution as crisp number only. Then the degree of the acceptance, indeterminacy and rejection is calculated
- The FLP method can be seen as a better method and it gives more optimal solution.
- The SVNS data can represent real life uncertainties and so depicts more practical solutions of the problem as it helps to determine the degree of acceptance, indeterminacy and rejection of the obtained solution. Row 1, Col 2 Row 1, Col 3
- A real life multi-objective and multi-index Neutrosophic transportation problem has also been solved in Section 5 other than the numerical example in Section 4 to illustrate the two proposed methods. The results and comparisons of the large scale problem are shown in the Table 5, Table 6, Table 7 and Table 8. The results obtained are compared and the FLP method proves to give better solution compared to the CLP method for most of the circumstances.

- The solution obtained by the proposed approaches has not been compared with any of the existing approaches for NTPs, as no work has been done for neutrosophic transportation problem. It is a new type of problem.

- The application of the methods to a real life multi-objective and multi- index neutrosophic transportation problem is also a new field itself.

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