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# New Neutrosophic Crisp Topological Concepts

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## Abstract

In this paper, we introduce the concept of "neutrosophic crisp neighborhoods system for the neutrosophic crisp point". Added to, we introduce and study the concept of neutrosophic crisp local function, and construct a new type of neutrosophic crisp topological space via neutrosophic crisp ideals. Possible application to GIS topology rules are touched upon.

**Keywords:** Neutrosophic Crisp Point, Neutrosophic Crisp Ideal; Neutrosophic Crisp Topology; Neutrosophic Crisp Neighborhoods

## 1 INTRODUCTION

The idea of "neutrosophic set" was first given by Smarandache [14, 15]. In 2012 neutrosophic operations have been investigated by Salama et al. [4-13]. The fuzzy set was introduced by Zadeh [17]. The intuitionistic fuzzy set was introduced by Atanassov [1, 2, 3]. Salama et al. [11] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [16]. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts. Here we shall present the neutrosophic crisp version of these concepts. In this paper, we introduce the concept of "neutrosophic crisp points" and "neutrosophic crisp neighbourhoods systems". Added to we define the new concept of neutrosophic crisp local function, and construct new type of neutrosophic crisp topological space via neutrosophic crisp ideals.

## 2 TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [14, 15], and Salama et al. [4 -13].

### 2.1 Definition [13]

Let  $X$  be a non-empty fixed set. A neutrosophic crisp set (NCS for short)  $A$  is an object having the form  $A = \langle A_1, A_2, A_3 \rangle$  where  $A_1, A_2$  and  $A_3$  are subsets of  $X$  satisfying  $A_1 \cap A_2 = \emptyset$ ,  $A_1 \cap A_3 = \emptyset$  and  $A_2 \cap A_3 = \emptyset$ .

### 2.2 Definition [13].

Let  $X$  be a nonempty set and  $p \in X$ . Then the neutrosophic crisp point  $p_N$  defined by  $p_N = \langle \{p\}, \emptyset, \{p\}^c \rangle$  is called a neutrosophic crisp point (NCP for short) in  $X$ , where NCP is a triple ( $\{$ only one element in  $X$ ), the empty set,  $\{$ the complement of the same element in  $X$ ).

### 2.3 Definition [13]

Let  $X$  be a nonempty set, and  $p \in X$  a fixed element in  $X$ . Then the neutrosophic crisp set  $p_{NN} = \langle \emptyset, \{p\}, \{p\}^c \rangle$  is called "vanishing neutrosophic crisp point" (VNCP for short) in  $X$ , where VNCP is a triple (the empty set,  $\{$ only one element in  $X$ ),  $\{$ the complement of the same element in  $X$ ).

### 2.4 Definition [13]

Let  $p_N = \langle \{p\}, \emptyset, \{p\}^c \rangle$  be a NCP in  $X$  and  $A = \langle A_1, A_2, A_3 \rangle$  a neutrosophic crisp set in  $X$ .

(a)  $p_N$  is said to be contained in  $A$  ( $p_N \in A$  for short) iff  $p \in A_1$ .

(b) Let  $p_{NN}$  be a VNCP in  $X$ , and  $A = \langle A_1, A_2, A_3 \rangle$  a neutrosophic crisp set in  $X$ . Then  $p_{NN}$  is said to be

(b) Let  $p_{NN}$  be a VNCP in  $X$ , and  $A = \langle A_1, A_2, A_3 \rangle$  a neutrosophic crisp set in  $X$ . Then  $p_{NN}$  is said to be contained in  $A$  ( $p_{NN} \in A$  for short) iff  $p \notin A_3$ .

### 2.5 Definition [13].

Let  $X$  be non-empty set, and  $L$  a non-empty family of NCSs. We call  $L$  a neutrosophic crisp ideal (NCL for short) on  $X$  if

- i.  $A \in L$  and  $B \subseteq A \Rightarrow B \in L$  [heredity],
- ii.  $A \in L$  and  $B \in L \Rightarrow A \vee B \in L$  [Finite additivity].

A neutrosophic crisp ideal  $L$  is called a  $\sigma$ -neutrosophic crisp ideal if  $\{M_j\}_{j \in N} \leq L$ , implies  $\bigcup_{j \in J} M_j \in L$  (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set  $X$  are  $\{\phi_N\}$  and the NSs on  $X$ . Also,  $NCL_f$ ,  $NCL_c$  are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic subsets having finite and countable support of  $X$  respectively. Moreover, if  $A$  is a nonempty NS in  $X$ , then  $\{B \in NCS : B \subseteq A\}$  is an NCL on  $X$ . This is called the principal NCL of all NCSs, denoted by  $NCL \langle A \rangle$ .

### 2.1 Proposition [13]

Let  $\{L_j : j \in J\}$  be any non - empty family of neutrosophic crisp ideals on a set  $X$ . Then  $\bigcap_{j \in J} L_j$  and

$\bigcup_{j \in J} L_j$  are neutrosophic crisp ideals on  $X$ , where

$$\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j1}, \bigcap_{j \in J} A_{j2}, \bigcup_{j \in J} A_{j3} \right\rangle \text{ or}$$

$$\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j1}, \bigcup_{j \in J} A_{j2}, \bigcup_{j \in J} A_{j3} \right\rangle \text{ and}$$

$$\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j1}, \bigcup_{j \in J} A_{j2}, \bigcap_{j \in J} A_{j3} \right\rangle \text{ or}$$

$$\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j1}, \bigcap_{j \in J} A_{j2}, \bigcap_{j \in J} A_{j3} \right\rangle.$$

### 2.2 Remark [13]

The neutrosophic crisp ideal defined by the single neutrosophic set  $\phi_N$  is the smallest element of the ordered set of all neutrosophic crisp ideals on  $X$ .

### 2.1 Proposition [13]

A neutrosophic crisp set  $A = \langle A_1, A_2, A_3 \rangle$  in the neutrosophic crisp ideal  $L$  on  $X$  is a base of  $L$  iff every member of  $L$  is contained in  $A$ .

## 3. Neutrosophic Crisp Neighborhoods System

### 3.1 Definition.

Let  $A = \langle A_1, A_2, A_3 \rangle$ , be a neutrosophic crisp set on a set  $X$ , then  $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$ ,  $p_1 \neq p_2 \neq p_3 \in X$  is called a neutrosophic crisp point

An NCP  $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$ , is said to be belong to a neutrosophic crisp set  $A = \langle A_1, A_2, A_3 \rangle$ , of  $X$ , denoted by  $p \in A$ , if may be defined by two types

- i) **Type 1:**  $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$  and  $\{p_3\} \subseteq A_3$
- ii) **Type 2:**  $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$  and  $\{p_3\} \subseteq A_3$

### 3.1 Theorem

Let  $A = \langle \langle A_1, A_2, A_3 \rangle \rangle$ , and  $B = \langle \langle B_1, B_2, B_3 \rangle \rangle$ , be neutrosophic crisp subsets of  $X$ . Then  $A \subseteq B$  iff  $p \in A$  implies  $p \in B$  for any neutrosophic crisp point  $p$  in  $X$ .

### Proof

Let  $A \subseteq B$  and  $p \in A$ . Then two types  
**Type 1:**  $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$  and  $\{p_3\} \subseteq A_3$  or  
**Type 2:**  $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$  and  $\{p_3\} \subseteq A_3$ . Thus  $p \in B$ .  
 Conversely, take any  $x$  in  $X$ . Let  $p_1 \in A_1$  and  $p_2 \in A_2$  and  $p_3 \in A_3$ . Then  $p$  is a neutrosophic crisp point in  $X$ . and  $p \in A$ . By the hypothesis  $p \in B$ . Thus  $p_1 \in B_1$ , or Type 1:  $\{p_1\} \subseteq B_1, \{p_2\} \subseteq B_2$  and  $\{p_3\} \subseteq B_3$  or Type 2:  $\{p_1\} \subseteq B_1, \{p_2\} \supseteq B_2$  and  $\{p_3\} \subseteq B_3$ . Hence.

$A \subseteq B$ .

### 3.2 Theorem

Let  $A = \langle A_1, A_2, A_3 \rangle$ , be a neutrosophic crisp subset of  $X$ . Then  $A = \bigcup \{p : p \in A\}$ .

### Proof

Since  $\bigcup \{p : p \in A\}$ , may be two types

**Type 1:**

$$\langle \cup\{p_1 : p_1 \in A_1\}, \cup\{p_2 : p_2 \in A_2\}, \cap\{p_3 : p_3 \in A_3\} \rangle \text{ or}$$

**Type 2:**

$$\langle \cup\{p_1 : p_1 \in A_1\}, \cap\{p_2 : p_2 \in A_2\}, \cap\{p_3 : p_3 \in A_3\} \rangle. \text{ Hence}$$

$$A = \langle A_1, A_2, A_3 \rangle$$

**3.1 Proposition**

Let  $\{A_j : j \in J\}$  is a family of NCSs in X. Then

$$(a_1) \ p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in \bigcap_{j \in J} A_j \quad \text{iff } p \in A_j \text{ for each } j \in J.$$

$$(a_2) \ p \in \bigcup_{j \in J} A_j \quad \text{iff } \exists j \in J \text{ such that } p \in A_j.$$

**3.2 Proposition**

Let  $A = \langle A_1, A_2, A_3 \rangle$  and  $B = \langle B_1, B_2, B_3 \rangle$  be two neutrosophic crisp sets in X. Then

a)  $A \subseteq B$  iff for each  $p$  we have

$$p \in A \Leftrightarrow p \in B \text{ and for each } p \text{ we have}$$

$$p \in A \Rightarrow p \in B.$$

b)  $A = B$  iff for each  $p$  we have

$$p \in A \Rightarrow p \in B \text{ and for each } p \text{ we}$$

$$\text{have } p \in A \Leftrightarrow p \in B.$$

**3.3 Proposition**

Let  $A = \langle A_1, A_2, A_3 \rangle$  be a neutrosophic crisp set in X.

Then

$$A = \cup \langle \{p_1 : p_1 \in A_1\}, \{p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\} \rangle.$$

**3.2 Definition**

Let  $f : X \rightarrow Y$  be a function and  $p$  be a neutrosophic crisp point in X. Then the image of  $p$  under  $f$ , denoted by  $f(p)$ , is defined by

$$f(p) = \langle \{q_1\}, \{q_2\}, \{q_3\} \rangle, \text{ where } q_1 = f(p_1), q_2 = f(p_2) \text{ and } q_3 = f(p_3).$$

It is easy to see that  $f(p)$  is indeed a NCP in Y, namely  $f(p) = q$ , where  $q = f(p)$ , and it is exactly the same meaning of the image of a NCP under the function  $f$ .

**4 4. Neutrosophic Crisp Local functions**

**4.1 Definition**

Let  $p$  be a neutrosophic crisp point of a neutrosophic crisp topological space  $(X, \tau)$ . A neutrosophic crisp neighbourhood (NCNBD for short) of a neutrosophic crisp point  $p$  if there is a neutrosophic crisp open set (NCOS for short)  $B$  in  $X$  such that  $p \in B \subseteq A$ .

**4.1 Theorem**

Let  $(X, \tau)$  be a neutrosophic crisp topological space (NCTS for short) of  $X$ . Then the neutrosophic crisp set  $A$  of  $X$  is NCOS iff  $A$  is a NCNBD of  $p$  for every neutrosophic crisp set  $p \in A$ .

**Proof**

Let  $A$  be NCOS of  $X$ . Clearly  $A$  is a NCBD of any  $p \in A$ . Conversely, let  $p \in A$ . Since  $A$  is a NCBD of  $p$ , there is a NCOS  $B$  in  $X$  such that  $p \in B \subseteq A$ . So we have  $A = \cup\{p : p \in A\} \subseteq \cup\{B : p \in A\} \subseteq A$  and hence  $A = \cup\{B : p \in A\}$ . Since each  $B$  is NCOS.

**4.2 Definition**

Let  $(X, \tau)$  be a neutrosophic crisp topological spaces (NCTS for short) and  $L$  be neutrosophic crisp ideal (NCL, for short) on  $X$ . Let  $A$  be any NCS of  $X$ . Then the neutrosophic crisp local function  $NCA^*(L, \tau)$  of  $A$  is the union of all neutrosophic crisp points (NCP, for short)  $P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$ , such that if  $U \in N((p))$  and  $NA^*(L, \tau) = \cup\{p \in X : A \wedge U \notin L \text{ for every } U \text{ nbd of } N(P)\}$ ,  $NCA^*(L, \tau)$  is called a neutrosophic crisp local function of  $A$  with respect to  $\tau$  and  $L$  which it will be denoted by  $NCA^*(L, \tau)$ , or simply  $NCA^*(L)$ .

**4.1 Example**

One may easily verify that.

If  $L = \{\phi_N\}$ , then  $NCA^*(L, \tau) = NCcl(A)$ , for any neutrosophic crisp set  $A \in NCSs$  on  $X$ .

If  $L = \{\text{all NCSs on } X\}$  then  $NCA^*(L, \tau) = \phi_N$ , for any  $A \in NCSs$  on  $X$ .

**4.2 Theorem**

Let  $(X, \tau)$  be a NCTS and  $L_1, L_2$  be two topological neutrosophic crisp ideals on  $X$ . Then for any neutrosophic crisp sets  $A, B$  of  $X$ . then the following statements are verified

- i)  $A \subseteq B \Rightarrow NCA^*(L, \tau) \subseteq NCB^*(L, \tau)$ ,
- ii)  $L_1 \subseteq L_2 \Rightarrow NCA^*(L_2, \tau) \subseteq NCA^*(L_1, \tau)$ .
- iii)  $NCA^* = NCcl(A^*) \subseteq NCcl(A)$ .

- iv)  $NCA^{**} \subseteq NCA^*$ .
- v)  $NC(A \cup B)^* = NCA^* \cup NCB^*$ .
- vi)  $NC(A \cap B)^*(L) \subseteq NCA^*(L) \cap NCB^*(L)$ .
- vii)  $\ell \in L \Rightarrow NC(A \cup \ell)^* = NCA^*$ .
- viii)  $NCA^*(L, \tau)$  is neutrosophic crisp closed set.

### Proof

- i) Since  $A \subseteq B$ , let  $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in NCA^*(L_1)$  then  $A \cap U \notin L$  for every  $U \in N(p)$ . By hypothesis we get  $B \cap U \notin L$ , then  $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in NB^*(L_1)$ .
- ii) Clearly,  $L_1 \subseteq L_2$  implies  $NCA^*(L_2, \tau) \subseteq NCA^*(L_1, \tau)$  as there may be other IFSs which belong to  $L_2$  so that for GIFFP  $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in NCA^*(L_1)$  but P may not be contained in  $NCA^*(L_2)$ .
- iii) Since  $\{\phi_N\} \subseteq L$  for any NCL on X, therefore by (ii) and Example 3.1,  $NCA^*(L) \subseteq NCA^*(\{O_N\}) = NCcl(A)$  for any NCS A on X. Suppose  $P_1 = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in NCcl(A^*(L_1))$ . So for every  $U \in NC(P_1)$ ,  $NC(A^*) \cap U \neq \phi_N$ , there exists  $P_2 = \langle \{q_1\}, \{q_2\}, \{q_3\} \rangle \in NCA^*(L_1) \cap U$  such that for every V NCNBD of  $P_2 \in N(P_2)$ ,  $A \cap U \notin L$ . Since  $U \wedge V \in N(P_2)$  then  $A \cap (U \cap V) \notin L$  which leads to  $A \wedge U \notin L$ , for every  $U \in N(P_1)$  therefore  $P_1 \in NC(A^*(L))$  and so  $NCcl(NCA^*) \subseteq NCA^*$  While, the other inclusion follows directly. Hence  $NCA^* = NCcl(NCA^*)$ . But the inequality  $NCA^* \subseteq Ncl(NCA^*)$ .
- iv) The inclusion  $NCA^* \cup NCB^* \subseteq NC(A \cup B)^*$  follows directly by (i). To show the other implication, let  $p \in NC(A \cup B)^*$  then for every  $U \in NC(p)$ ,  $(A \cup B) \cap U \notin L$ , i.e.,  $(A \cap U) \cup (B \cap U) \notin L$ . then, we have two cases  $A \cap U \notin L$  and  $B \cap U \in L$  or the converse, this means that exist  $U_1, U_2 \in N(P)$  such that  $A \cap U_1 \notin L$ ,  $B \cap U_1 \notin L$ ,  $A \cap U_2 \notin L$  and  $B \cap U_2 \notin L$ . Then  $A \cap (U_1 \cap U_2) \in L$  and  $B \cap (U_1 \cap U_2) \in L$  this gives  $(A \cup B) \cap (U_1 \cap U_2) \in L$ ,  $U_1 \cap U_2 \in N(C(P))$  which contradicts the hypothesis. Hence the equality holds in various cases.
- vi) By (iii), we have  $NCA^{**} = NCcl(NCA^*)^* \subseteq NCcl(NCA^*) = NCA^*$   
Let  $(X, \tau)$  be a NCTS and L be NCL on X. Let us define the neutrosophic crisp closure operator

$NCcl^*(A) = A \cup NC(A^*)$  for any NCS A of X. Clearly, let  $NCcl^*(A)$  is a neutrosophic crisp operator. Let  $NC\tau^*(L)$  be NCT generated by  $NCcl^*$   
i.e  $NC\tau^*(L) = \{A : NCcl^*(A^c) = A^c\}$  now  $L = \{\phi_N\} \Rightarrow NCcl^*(A) = A \cup NCA^* = A \cup NCcl(A)$  for every neutrosophic crisp set A. So,  $N\tau^*(\{\phi_N\}) = \tau$ . Again  $L = \{all \text{ NCSs on } X\} \Rightarrow NCcl^*(A) = A$ , because  $NCA^* = \phi_N$ , for every neutrosophic crisp set A so  $NC\tau^*(L)$  is the neutrosophic crisp discrete topology on X. So we can conclude by Theorem 4.1.(ii).  $NC\tau^*(\{\phi_N\}) = NC\tau^*(L)$  i.e.  $NC\tau \subseteq NC\tau^*$ , for any neutrosophic ideal  $L_1$  on X. In particular, we have for two topological neutrosophic ideals  $L_1$ , and  $L_2$  on X,  
 $L_1 \subseteq L_2 \Rightarrow NC\tau^*(L_1) \subseteq NC\tau^*(L_2)$ .

### 4.3 Theorem

Let  $\tau_1, \tau_2$  be two neutrosophic crisp topologies on X. Then for any topological neutrosophic crisp ideal L on X,  $\tau_1 \leq \tau_2$  implies  $NA^*(L, \tau_2) \subseteq NA^*(L, \tau_1)$ , for every  $A \in L$  then  $NC\tau^*_1 \subseteq NC\tau^*_2$

### Proof

Clear.

A basis  $NC\beta(L, \tau)$  for  $NC\tau^*(L)$  can be described as follows:

$NC\beta(L, \tau) = \{A - B : A \in \tau, B \in L\}$ . Then we have the following theorem

### 4.4 Theorem

$NC\beta(L, \tau) = \{A - B : A \in \tau, B \in L\}$  Forms a basis for the generated NT of the NCT  $(X, \tau)$  with topological neutrosophic crisp ideal L on X.

### Proof

Straight forward.

The relationship between  $NC\tau$  and  $NC\tau^*(L)$  established throughout the following result which have an immediately proof.

### 4.5 Theorem

Let  $\tau_1, \tau_2$  be two neutrosophic crisp topologies on X. Then for any topological neutrosophic ideal L on X,  $\tau_1 \leq \tau_2$  implies  $NC\tau^*_1 \subseteq NC\tau^*_2$ .

### 4.6 Theorem

Let  $(X, \tau)$  be a NCTS and  $L_1, L_2$  be two neutrosophic crisp ideals on X. Then for any neutrosophic crisp set A in X, we have

i)  $NCA^*(L_1 \cup L_2, \tau) = NCA^*(L_1, NC\tau^*(L_1)) \wedge NCA^*(L_2, NC\tau^*(L_2))$  ii)

$NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1))^* \cap (NC\tau^*(L_2))^*$

**Proof**

Let  $p \notin (L_1 \cup L_2, \tau)$ , this means that there exists  $U \in NC(P)$  such that  $A \cap U_p \in (L_1 \cup L_2)$  i.e. There exists  $\ell_1 \in L_1$  and  $\ell_2 \in L_2$  such that  $A \cap U \in (\ell_1 \vee \ell_2)$  because of the heredity of  $L_1$ , and assuming  $\ell_1 \wedge \ell_2 = O_N$ . Thus we have  $(A \cap U) - \ell_1 = \ell_2$  and  $(A \cap U_p) - \ell_2 = \ell_1$  therefore  $(U - \ell_1) \cap A = \ell_2 \in L_2$

and  $(U - \ell_2) \cap A = \ell_1 \in L_1$ . Hence  $p \notin NCA^*(L_2, NC\tau^*(L_1))$  or  $p \notin NCA^*(L_1, NC\tau^*(L_2))$ , because  $p$  must belong to either  $\ell_1$  or  $\ell_2$  but not to both. This gives

$NCA^*(L_1 \cup L_2, \tau) \supseteq NCA^*(L_1, NC\tau^*(L_1)) \cap NCA^*(L_2, NC\tau^*(L_2))$ .

To show the second inclusion, let us assume  $p \notin NCA^*(L_1, NC\tau^*(L_2))$ . This implies that there exist  $U \in N(P)$  and  $\ell_2 \in L_2$  such that  $(U_p - \ell_2) \cap A \in L_1$ . By the heredity of  $L_2$ , if we assume that  $\ell_2 \subseteq A$  and define  $\ell_1 = (U - \ell_2) \cap A$ . Then we

have  $A \cap U \in (\ell_1 \cup \ell_2) \in L_1 \cup L_2$ . Thus,

$NCA^*(L_1 \cup L_2, \tau) \subseteq NCA^*(L_1, NC\tau^*(L_1)) \cap NCA^*(L_2, NC\tau^*(L_2))$

and similarly, we can get  $NCA^*(L_1 \cup L_2, \tau) \subseteq NCA^*(L_2, \tau^*(L_1))$ .

This gives the other inclusion, which complete the proof.

**4.1 Corollary**

.Let  $(X, \tau)$  be a NCTS with topological neutrosophic crisp ideal L on X. Then

i)  $NCA^*(L, \tau) = NCA^*(L, \tau^*)$  and  $NC\tau^*(L) = NC(NC\tau^*(L))^*(L)$

ii)  $NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1)) \cup (NC\tau^*(L_2))$

**Proof**

Follows by applying the previous statement.

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