# Explicit Port-Hamiltonian Formulation of Bond Graphs

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Port-Hamiltonian theory is a well-known framework for control design in complex physical systems [2]. The majority of port-Hamiltonian controller and observer design methods base on an *explicit* input-state-output port-Hamiltonian model of the system under consideration. However, the derivation of a port-Hamiltonian system (PHS) is in general a difficult task, especially for large-scale and networked systems. Remedy can be provided by systematic modeling procedures that are based on a graphical description of the system. The authors of [6] use open directed graphs to formulate port-Hamiltonian models for various physical systems. In [3], schematics of analog circuits build the basis for an automated generation of PHSs in form of differential-algebraic equations (DAEs). Besides directed graphs and schematics, bond graphs are a particular promising starting point for the derivation of PHSs [2]. While the *implicit* port-Hamiltonian formulation of bond graphs is well investigated in literature [4], approaches for an *explicit* formulation have been restriced to special case of bond graphs [1, 5]. In particular, algebraic conditions for the existence of an explicit port-Hamiltonian formulation of a bond graph were missing.

In this contribution, we present an automatable and mathematically rigorous method for the derivation of explicit input-state-output PHSs from bond graphs. Fig. 1 illustrates the approach which comprises the following four steps: (i) description of the set of power-continuous bond graph elements by a corresponding set of Dirac structures in implicit representation; (ii) merging of the set of Dirac structures into one single Dirac structure in implicit form; (iii) conversion of the Dirac structure from an implicit into an explicit representation; (iv) derivation of an explicit PHS by merging the Dirac structure obtained from (iii) with the constitutive relations of storages and resistors from the bond graph.

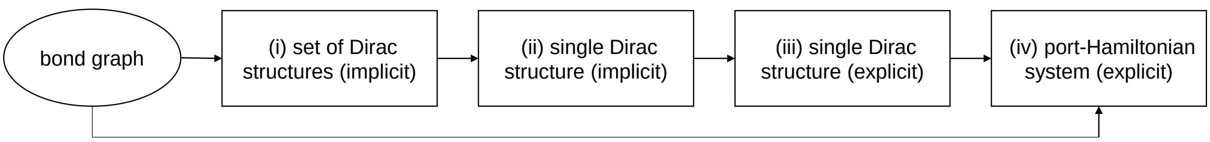


Figure 1: *Flowchart of the modeling approach*

Along with the method of Fig. 1 we provide two algebraic conditions, one necessary and one sufficient, for the existence of an explicit port-Hamiltonian formulation of a bond graph. In the last part of this contribution, we interpret these algebraic conditions by means of bond graph *causality* which allows for some interesting results concerning the form of mathematical models (ODE vs. DAE) that can be obtained from bond graphs.

The methods from this contribution are collected, integrated, and automated in the software tool AMOTO which will be presented in the GMA special session on *software tools for modeling, simulation, identification, and control design.*

## References


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