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Explanation and Prediction: Strategies for Extending Scientific Realism to Mathematics

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EXPLANATION AND PREDICTION:

STRATEGIES FOR EXTENDING SCIENTIFIC REALISM TO MATHEMATICS

by

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ABSTRACT

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One central question in the philosophy of mathematics concerns the ontological status of mathematical entities. Platonists argue that abstract, mathematical entities exist, while nominalists argue that they do not. Scientific realism is the position that science is (roughly) true and the objects it describes exist. There are two major competing arguments for platonism on the basis of scientific realism: *Indispensability* and *Explanation*. In this paper I consider which argument the platonist ought to prefer by comparing their motivations and results. I conclude that, given the current role of mathematics in our best scientific theories, *Explanation* does not support platonism. Thus, *Indispensability* is preferable.

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Introduction

One central question in the philosophy of mathematics concerns the ontological status of mathematical entities. Platonists argue that abstract, mathematical entities exist, while nominalists argue that they do not. It is a virtue of a theory if it commits us to fewer kinds of entities.[1] If platonism is to be accepted even though it expands our ontology, then, it must be motivated. One source of independent support for platonism has been through scientific realism. A central commitment of contemporary scientific realism is naturalism, the claim that scientific practice ought to determine our ontological commitments. If scientific practice includes mathematical entities in ontologically committing roles that non-mathematical (nominalistic) entities cannot play, then naturalism supports the existence of mathematical entities. In this paper I consider two major competing accounts of what this role might be and thus the basis on which scientific realism supports platonism.

The traditional argument for platonism from scientific realism uses the Quine-Putnam indispensability thesis (*Indispensability*). This thesis asserts the existence of all the entities that are indispensable to our best scientific theories. Platonists argue that mathematical entities are indispensable to our best scientific theories and thus, using *Indispensability*, that they exist. A newer platonist alternative narrows the ontological focus through the explanatory indispensability thesis (*Explanation*). According to *Explanation*, not all scientific roles, even if they are indispensable, are ontologically committing. Only appropriately *explanatory* roles garner ontological commitment. The explanatory platonist argues that mathematical entities play such roles in our best scientific theories (and do so indispensably). I will argue against this claim. In our current best theories, mathematical entities are not required to play explanatory roles of the kind

[1] See, e.g., Quine (1948): 23.

required by explanatory platonism. This means that *Explanation* is not compatible with naturalist platonism. The scientific realist platonist, therefore, should endorse *Indispensability*.

I begin by laying out scientific realism and the principles it is based on. I then show how *Indispensability* develops from scientific realism and how *Explanation* develops as a reaction to perceived problems for *Indispensability*. After laying out these two competing ontological conditions, I show how the explanatory platonist uses *Explanation* and examples of mathematical explanation to argue for the existence of mathematical entities. I then argue that, in order for mathematical explanations to explain, they require a supplementary principle, *Determination*. Assuming *Determination*, I lay out two accounts, one platonist (Direct Mathematical Explanation – DME) and one nominalist (Indirect Mathematical Explanation – IME), of the role that mathematics plays in mathematical explanations. I argue that, as the debate stands, the nominalist account (IME) is a better explanation. I then introduce the prediction criterion, a tool for assessing explanations, and argue that it offers the best hope for explanatory platonism. I conclude with the state of scientific realist platonism, given current scientific evidence.

Scientific Realism

Scientific realism is, roughly, the thesis that scientific theories aim at being true and that to accept a theory is to accept that it is (at least approximately) true.[2] According to Hilary Putnam, the defining positive motivation for scientific realism "is that it is the only philosophy that doesn't make the success of science a miracle."[3] This is the no-miracles argument. One central tenet of scientific realism which enshrines this desire to avoid miracles is *inference to the best explanation* (IBE).[4] According to IBE, we should infer the truth of whatever the best explanation of a given

[2] Putnam (1975).
[3] Putnam (1975): 73.
[4] Liston (2016): 15.

phenomenon is (so long as there is one). IBE only requires inference in cases where there is at least one explanation which meets a minimal explanatory threshold. The best explanation is whichever theory explains the most and meets the other theoretical virtues we value. Science aims at producing the best theories, and IBE supports realism towards those theories.

This realist drive to avoid miracles is what motivates existentially committing to various kinds of unobservable physical entities, like neutrinos or electrons.[5] We can see an example of scientific realist thinking in the molecular explanation of Brownian motion. Small, observable particles suspended in a fluid do not sink; they move about quickly and haphazardly without coming to rest.[6] Why? The explanation (from Albert Einstein[7] and Jean Perrin[8]) is that fluids themselves consist of unobservable molecules. When a fluid is heated, its molecules receive an energy increase and move about. The motion of those molecules causes the motion of the small observables.[9] That this explanation generally accords with the phenomena would not be enough to make its success miraculous if these molecules did not exist. More is needed to confirm the explanation.

This confirmation came using a formula developed by Einstein[10] which predicts the distance traveled by observable particles (the mean square displacement) as a function of (a) observable properties of the setup and the suspended particles and (b) constants derived from the molecular-kinetic theory of gases, chiefly among them Avogadro's number N (the number of atoms in a gram molecule of a gas).[11] Perrin later experimentally verified that precisely this

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[5] Field (1980): 16.
[6] Perrin (1913): 83. This phenomenon was discovered by Robert Brown in 1827.
[7] Einstein (1905).
[8] Perrin (1913).
[9] Perrin (1913): 86. In Perrin's words, "Every granule suspended in a fluid is being struck continually by the molecules in its neighborhood and receives impulses from them that do not in general exactly counterbalance each other; consequently it is tossed hither and thither in an irregular fashion".
[10] Einstein (1905): 7.
[11] Maiocchi (1990): 263-265.

formula holds for Brownian motion.[12] Perrin also showed that the values of Avogadro's number derived from Einstein's equation and his own Brownian motion experiments are highly consistent with the values determined by the observations of other phenomena with molecular explanations.[13] He concludes that, "*Such decisive agreement can leave no doubt as to the origin of the Brownian movement*";[14] and thus that unobservable molecules exist. The realist argues, along with Perrin, that if the unobservable molecules found in this explanation do not in fact exist, then the success of the molecular explanation of Brownian motion and its agreement with other scientific explanations built on molecular theory is rendered a coincidence and a miracle. The traditional argument for platonism, based on the Quine-Putnam indispensability thesis, seeks to apply this scientific realist reasoning to mathematical entities.

Indispensability

The indispensability argument:[15]

1. We ought to be existentially committed to all and only those entities that are indispensable[16] to our best scientific theories – *Indispensability*.
2. Mathematical entities are indispensable to our best scientific theories.
3. Therefore, we ought to be existentially committed to mathematical entities.

The second premise is an empirical matter, and subject to actual scientific practice.[17] The first premise, *Indispensability*, rests on two general commitments: (1) Quinean naturalism, which is the claim that science is the first and best (or only) arbiter of ontological commitment; and (2) confirmation holism,[18] which is the claim that "the various posits of a theory can only be confirmed

[12] Maiocchi (1990): 269.
[13] Perrin (1913): 105.
[14] Perrin (1913): 105.
[15] Something like this argument is scattered through Quine's writings. See, e.g., Quine (1981): 149-150.
[16] An entity is theoretically dispensable if a "reasonably attractive" alternate theory without quantification over that entity can explain the same phenomena (Field 1980: 8.).
[17] Here I assume this premise.
[18] Putnam offers an alternative argument for indispensability that requires a less strict standard than confirmation holism. E.g., Putnam (1975): 74-75.

in toto, and distinctions between the precise role of different posits do not matter."[19] Naturalism ensures that scientific practice is the proper domain for determinations of ontological commitment, and confirmation holism ensures that all scientific posits are treated ontologically equally.

The thesis follows from the principle of IBE. The best scientific theories are best because they are most explanatory and meet the other theoretical virtues we value. Because of confirmation holism, a theory can only be confirmed as a whole and so its explanatory success can also be only assessed as a whole. For a theory to be best, it must best explain the phenomena. Therefore, IBE can be applied and we can infer the truth of the theory and the existence of its posits.

In platonism motivated by the indispensability thesis (Quinean platonism), mathematical entities play an organizing role which is no different from the role played by any other entity. If an entity indispensably appears in all our best scientific theories, whatever the context, the indispensability thesis commits us to its existence. Looking back to the molecular explanation, according to the indispensability thesis we are not only committed to the existence of the unobservable molecules, but also to any other indispensable posits in the explanation. For example, if Avogadro's number is indispensable in the Brownian motion explanation, then the indispensability thesis commits us to the existence of that number.

Explanation

Not all of those attracted to scientific realism and the existence of unobservable scientific entities (electrons, quarks, etc.), however, accept this argument. Scientific realist objections to *Indispensability* focus on two claims: (a) that confirmation holism is not justified; and (b) that the *Indispensability* is too strong and commits us to the existence of more entities than is desirable. Penelope Maddy, for instance, argues that holism is insufficiently supported and may contradict

[19] Baker (2005): 224.

certain aspects of scientific and mathematical practice.[20]

Nominalists who reject confirmation holism and the Quinean platonist ontology must offer an alternative to *Indispensability* which maintains the no-miracles motivation and explains why the motivation does not require commitment to mathematical entities. This requires a principled distinction between the roles in scientific theories that require existence to be successful and the roles that do not. Nominalists often point to explanation and causation as serving to distinguish these sorts of roles.[21] Here is an argument for how that might follow from the scientific realist principles I have introduced.

One scientific role which I have already identified as bearing special ontological importance is explanation. I introduced IBE as a central scientific realist confirmational tool. Under *Indispensability*, only whole theories can be confirmed. This means that IBE can only apply to theories as a whole. Without the framework of confirmation holism, however, IBE instead applies specifically to the explanatory elements of a theory. If only some of the indispensable posits in a theory are playing explanatory roles, then IBE only applies to those explanatory entities. Scientific realism and Quinean platonism presuppose IBE, so using IBE to guide ontology does not require further or extra-scientific commitments from the nominalist.

But mathematics does appear in explanations (such as Avogadro's number), sometimes indispensably. The condition 'appearing indispensably in explanations' is not sufficiently fine-grained to return the ontological results nominalists are looking for. Nominalists must therefore offer a more limiting condition, but one that still focuses on explanation and is thus supported by IBE. Without IBE, ontological commitment on the basis of the condition would not follow from scientific realism alone.

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[20] See Maddy (1992).
[21] E.g., Armstrong (1989); Melia (2000).

The nominalist must then distinguish still further between the ways in which different posits appear in explanations. This further distinction too should follow from scientific realist principles. One way to make this distinction involves looking back to no-miracles. On this strategy, the nominalist must claim that only certain posits play explanatory roles that require them to exist. The existence of explanatory entities only serves to avoid miracles, and is thus only genuinely explanatory, if those entities themselves ensure the existence of the phenomena they explain. The standard model for how entities might ensure the existence of phenomena is causation. On this picture, then, entities can only explain phenomena that they causally determine;[22] e.g., the existence of unobservable molecules in a fluid only explains the motion of an observable particle if the molecules themselves cause that motion. Mathematics is assumed to be causally inert, and so if scientific realism only commits us to the existence of causal entities, then mathematics need not be included in our ontology.

'Explanatory' roles that do not involve causation are, on this account, not genuinely explanatory and therefore not ontologically committing. However, we have assumed that mathematics appears indispensably in many of our best explanations. On this account, any sort of explanation or explanatory role which does not involve causal relations can only explain derivatively, by standing proxy for (or indexing) some real explanation.[23] For example, Avogadro's number is used in the Brownian motion example to compare different molecular phenomena. The number serves to indicate an equality of size relation between certain collections of molecules. That size relation does not hold between the number and the molecules, nor does the number itself determine any facts about those molecules. The number only *describes* the molecules. While the molecules must exist in order to bear a causal relation (and thus avoid making

[22] Or, in a statistical explanation, causally support the phenomena.
[23] See Colyvan (2010): 299.

that relation – and the explanation that depends on it - miraculous), the number need not exist in order for the description to hold. The non-existence of the number is not miraculous. In general, mathematics is used in scientific explanations merely to index facts about causal entities, including the presence of determination relations. This indexical role, even if it is indispensable, is not itself explanatory. Only a causal role can be explanatory.

This account can be formalized as an alternative to the indispensability thesis. The Explanatory Indispensability Argument:

1. We ought to be existentially committed to all and only those entities that play an indispensable explanatory role in our best scientific theories – *Explanation*.[24]
2. Only causal entities can have explanatory power – *Causation*.
3. Mathematics is causally inert – *Inert*.
4. Therefore, we ought not to be existentially committed to mathematical posits.

I will refer to nominalists who endorse this thesis or something like it as explanatory nominalists.

Explanatory Platonism

Explanatory platonism seizes upon this account of no-miracles and the resources of the explanatory thesis as a way to support and further specify scientific realist platonism. It is motivated by similar concerns to those of explanatory nominalists. Explanatory platonists, such as Mark Colyvan,[25] want to further support platonism by either hedging against the Quinean indispensability thesis or denying the thesis entirely. They worry that the indispensability thesis, built as it is on confirmation holism, may be inconsistent with scientific practice or may commit us to unjustified or undesirable nonmathematical ideal posits (e.g., frictionless planes and ideal centers of mass). Explanatory platonists thus accept *Explanation* and argue for the existence of mathematical entities on explanatory grounds.

[24] *Explanation* is a restatement of the first premise of Alan Baker's "enhanced" indispensability thesis (Baker 2009): 613.
[25] See, e.g., Colyvan (1998), Colyvan (2001), Lyon and Colyvan (2008).

If *Explanation* is to support the existence of mathematical entities, then mathematical entities must be able to play explanatory roles. Given *Inert* and *Causation*, they are excluded from such roles. The explanatory platonist must therefore deny one of these premises. Typically, they accept *Inert*, and so must deny *Causation*, that the causal model is the best model for explanation.[26] Colyvan does this, not by suggesting an alternative model for explanation, but by side-stepping the question. He argues that science does not limit itself to causal explanations and that scientific practice produces non-causal explanations.[27] If so, the causal explanation model cannot be correct, since the ontological debate should be responsive to scientific practice. To motivate this claim, here are two proposed examples of how mathematics can be used to explain physical phenomena.[28]

Examples of Mathematical Explanation

The first is an explanation of why honeycombs are hexagonal.[29] Bees build their honeycombs with a hexagonal structure. Biologists, beginning with Darwin,[30] have hypothesized that the explanation of this fact has to do with efficiency. Natural selection pressures bees to favor the arrangement that most efficiently uses energy and resources. Bees which use less wax in building combs will be more successful and will be selected for. The dominance of the hexagonal honeycomb strategy suggests the honeycomb conjecture: *a hexagonal grid represents the best way to divide a surface into regions of equal area with the least total perimeter*.[31] This conjecture was

[26] See Baker (2009) for more.
[27] Colyvan (1998).
[28] These are two of the most commonly discussed examples, probably because they are so easily described. As some of the most familiar cases, they have also faced many objections and may not be the strongest examples available. Here I assume that some explanations of this kind go through. For a collection of additional examples, see Mancosu (2008) and Bangu (2017).
[29] This example comes from Lyon and Colyvan (2008).
[30] Darwin (1998): 350. "[...] that individual swarm which thus made the best cells with least labour, and least waste of honey in the secretion of wax, having succeeded best, and having transmitted their newly-acquired economical instincts to new swarms, which in their turn will have had the best chance of succeeding in the struggle for existence."
[31] Lyon and Colyvan (2008): 2-3.

only recently proved, by Thomas Hales.[32] Because of this mathematical proof, we know that the most efficient arrangement for honeycombs will be hexagonal. Because of the evolutionary factor, we know that any bees that happen upon this arrangement will continue to build their combs with it, *ceteris paribus*. This is the best explanation science has produced for the phenomenon of bees in general building their combs with this particular arrangement.

Another common example in the literature seeks to explain why certain species of cicada of the genus *Magicicada* in North America have certain particular life-cycle periods.[33] Three of these species have life-cycle periods of 13 or 17 years. During this period, they remain as nymphs underground, only to emerge together when the cycle is ending. The particular length of these periods is explained, as in the honeycomb case, by a combination of evolutionary and mathematical factors. First, it is advantageous for the emergence of the cicadas to intersect with the presence of the fewest predators and the fewest subspecies with different periods. This is achieved by having periods which intersect with the smallest number of other possible periods. The numbers with the fewest factors are prime, meaning that prime life-cycle periods will intersect with the least number of possible periods for other organisms. Therefore, cicadas ought to have a prime life-cycle period. Various other evolutionary factors in the development of the cicada show that having a life cycle between 12 and 18 years long is beneficial. The only prime numbers between 12 and 18 are 13 and 17. This explains why, given a few factors, these specific life-cycle periods are optimal.

These explanations are not causal, undercutting *Causation*. The explanandum in both cases is generic, ranging over all the members of the species of bee or cicada. There are no candidate causal explanations for these explananda, because of their wide scope. While there could be a

[32] Hales (2001).
[33] This example was first given in Baker (2005) but is often discussed.

causal explanation (or disjunctive manifold of causal explanations) for the behavior of every individual bee or cicada who follows these patterns, listing the physical and chemical causes of their behavior, the fact that all these causal explanations result in the same phenomena cannot be causally explained.[34] That these facts need explanation can be denied, but doing so is inconsistent with naturalism, because these explanations come from science, and with no-miracles, because denying the explanation of a phenomenon makes it miraculous.[35]

As in explanatory nominalism, these sorts of examples in which mathematical entities play a distinctly explanatory role are supposed to be distinguished from explanations in which mathematics appears but plays a merely representational, descriptive, or indexing role. For example, the appearance of Avogadro's number in the explanation of Brownian motion still seems to be merely indexical.

Determination

The explanatory platonist position is that without the mathematics in explanations like honeycomb and cicada examples, the phenomena they explain are inexplicable. With no competitor explanation, the mathematical explanation[36] is trivially the best, and therefore (if the explanation meets the threshold requirement) *Explanation* and IBE justify commitment to the mathematical entities doing the explaining. There is, however, a problem with this account. The purpose of explanatory platonism is to provide a stronger and more specific argument for the

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[34] For more on this claim, see Lyon (2012). Lyon also argues, following Jackson and Pettit (1990), that many explanations (not all mathematical) are of this kind. Denial of this kind of explanation thus significantly reduces the available explanatory resources.

[35] This argument can be resisted. Sorin Bangu, for example, argues that the explananda of the cicada explanation is not purely physical but is mixed physical/mathematical fact (Bangu 2008). As such, the naturalist point might not apply.

[36] Unless otherwise noted, I use 'mathematical explanation' to refer to scientific explanations of physical phenomena in which mathematics appears indispensably. 'Extra'-mathematical explanations are referred to as such to distinguish them from intra-mathematical explanations – mathematical explanations of mathematical phenomena – such as the proof of the honeycomb conjecture itself.

existence of mathematical entities than the argument from *Indispensability*. Simply presenting non-causal explanations in which mathematics appears, as Colyvan and Baker do, does not clarify the distinction between truly mathematical explanations like the cicada case and those that employ mathematics on other (indexing, representational, or organizational) grounds. Nor does presenting such explanations justify making an ontological distinction between such entities on the basis of no-miracles. Just as the explanatory nominalist used *Causation* to support the determination account of no-miracles, the explanatory platonist must show how the mathematical entities in the explanations determine the phenomena being explained. This requires committing to an alternative to *Causation*, which identifies the relevant determination relation grounding these explanations.

Causal relation is able to serve the function required of it in grounding explanations because it is a species of natural necessity relation, and thus able to reduce explanation to determination relations. But any relation capable of determining or necessitating should be able to serve that function. Expressing this generically gives us *Determination*: Only causal or nomologically determining entities can have explanatory power.[37] Mathematical explanations, as I stated above, cannot be causal. They must then rely on alternative accounts of explanation, like the deductive-nomological model, and use something like laws of nature to provide the underlying fundamental necessity relations required by the determination no-miracles account. If a mathematical explanation is genuinely explanatory, then it must present some entities or properties which nomologically determine its explanandum. I now develop two different possible accounts of which entities bear the nomological relations required if the proposed mathematical explanations are to be explanatory - Direct Mathematical Explanation and Indirect Mathematical Explanation. Given *Determination*, whether *Explanation* supports explanatory nominalism or

[37] Here, I retain causation for the sake of generality. See Bangu (2017) for a discussion of causal eliminativism in this context.

explanatory platonism is at stake.

Direct Mathematical Explanation

Mathematical entities are incapable of bearing the causal relations required for explanation under *Causation*, but they are potential bearers of the nomological relations required for explanation under *Determination*. This allows the possibility of Direct Mathematical Explanation (DME): In mathematical explanations, mathematical entities nomologically determine physical facts. Nora Berenstain is an explanatory platonist proponent of this sort of explanation.[38] She argues that, for mathematical explanations to be explanatory, the nomological relations involved must relate mathematical entities themselves to the physical phenomena. She additionally argues that the best candidate for this relation is instantiation, where a mathematical structure which has explanatory features is instantiated in the phenomena being explained.[39] To see how this looks in application, consider the explanation of the fact that honeycomb has the same structural arrangement. On this account, the (mathematical) hexagonal grid is instantiated in the physical honeycomb. Because the hexagonal grid has the property of being the most efficient way to divide a plane into regions of equal area, the honeycomb will also have that property. The honeycomb which has this efficiency property will be the optimal honeycomb strategy and will be selected for. Under this model, the mathematical grid itself *directly* determines that the hexagonal honeycomb is the most efficiently arranged honeycomb.

It looks like DME might be able to account for the explanatory power of mathematical explanations. If DME is the best explanation of this explanatory power, then IBE justifies inferring the existence of the nomological relations it posits between mathematical entities and the phenomena being explained. If mathematical entities are nomologically active then, according to

[38] Berenstain (2016). I have simplified the account.
[39] Berenstain (2016): 12-15.

Explanation and *Determination*, mathematical entities exist and explanatory platonism is supported. For IBE to apply, however, DME must offer the *best* explanation. If the only explanation of mathematical explanation is that the mathematics itself nomologically determines the physical phenomena, then it is the best. But if there is an alternative, DME must be argued for. The task, then, is to assess whether there is another way to non-miraculously ground mathematical explanation that does not require mathematical entities to directly determine physical phenomena.

Nominalism and Mathematical Explanation

In developing an indexical alternative to DME, I must first develop some indexing resources. The first place to look is to those who have a vested interest in avoiding DME: nominalists. David Liggins[40] and Mary Leng[41] are both nominalists who accept that some phenomena like those in the honeycomb and cicada examples should be explained. They each offer alternative accounts of the mathematical explanations of such phenomena. I will argue that neither of these accounts successfully grounds mathematical explanations nomologically, as required by *Explanation* and *Causation*, but that they lay the groundwork for a nominalistic competitor to DME.

Liggins

Liggins seeks to generate a nominalist account of mathematical explanations. He argues that even platonists ought not to accept accounts, like DME, on which mathematics is difference-making.[42] Because he rejects DME-type explanations, on his account the explanatory resources of mathematical explanations are very limited and do not rely on mathematics.

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[40] Liggins (2014).
[41] Leng (2012).
[42] Liggins (2014): 4. He argues for this point on the basis that difference-making is not part of our pre-theoretic idea of what math does.

Liggins argues that if there is a relation between mathematics and physical phenomena in a putative mathematical explanation, that relation should obtain in virtue of a nominalistic property of the physical entities involved. In the cicada explanation, this concerns what he calls the "*has-life-cycle-in-years-of*"[43] relation between the numbers, 13 and 17, and the cicadas whose life-cycle periods are being explained. If a cicada bears the *has-life-cycle-in-years-of* relation to the number 13 or the number 17, it should be in virtue of the fact that the cicadas have a nominalistic property like *has-life-cycle-period-in-years-of-13* or *has-life-cycle-period-in-years-of-17*. Otherwise, the mathematics itself would be making a difference and play an unjustifiably strong role. Because the nominalistic property grounds the mathematical property, whatever is explanatory about the mathematical property is also grounded by the nominalistic property. This means, according to Liggins, that whatever is genuinely explanatory about mathematical explanations is not dependent upon mathematics. Therefore, the mathematical aspects of mathematical explanations can be disposed of without any loss in explanatory power. As a result, Liggins' version of the cicada explanation is that these species of cicada have the life-cycle periods they do because of the brute fact that they have some physical property. This method of trivially rewriting mathematical explanations is meant to be universally applicable and allow him to easily nominalize explanations.

This account is not capable of generating explanations of equal strength to those of DME, which accepts the difference-making power of mathematics. There are two primary reasons for this. The first is that, because the account involves rewriting mathematical explanations, its explanations do not involve mathematical entities at all. It cannot, therefore, appeal to the underlying system of mathematical relations that make the platonist explanation an explanation. In the cicada case, the mathematical explanation explains by using facts about the relations

[43] Liggins (2014): 4.

between certain numbers. Prime numbers have only themselves and 1 as factors. The numeric structure which entails that there are certain specific numbers with this feature is what allows the cicada explanation to be explanatory. In DME, because the number is instantiated in the life-cycle period, these facts about the number determine facts about the life-cycle period. In DME, then, mathematical properties of the number itself determine that that particular life-cycle period is optimal. For Liggins, on the other hand, the explanation cannot appeal to mathematical entities. This means he cannot explain why it should be some nominalistic property that explains the phenomena rather than any other. Moreover, it means he cannot identify nominalistically which property is serving as the explanans in any given rewritten mathematical explanation. Nothing in his account of the cicada explanation allows for the identification of the property *has-life-cycle-period-in-years-of-13*, because that property is only understood as a replacement for the mathematical relation. While he can posit that there must be some such property, he has refused himself the resources to characterize such an explanans mathematically and offers no new resources for identifying it nominalistically. Without the explanans, there is no explanation. If a nominalist account is to compete with DME, it must solve this problem.

The second problem is that Liggins' version of the explanation, even if there were an explanans, does not meet the condition set for explanation by *Determination*. He gives no account of determination or nomological relations between the unspecified nominalistic property and the cicada life-cycle period such that one can explain facts about the other. This explanation is thus a miracle, given the determination account of no-miracles. Liggins clearly has not offered us a similarly explanatory alternative to DME.

His account does, however, point us in the right direction. His nominalistic property replacements for mathematical properties offer a non-mathematical candidate to bear the nomological relations required by *Determination*. If an explanation can be generated with a nominalistic property like *has-life-cycle-period-in-years-of-13* as the explanans, then IBE applied to the explanation again justifies the assertion of a nomological relation. But, as the first problem has not been addressed, there does not yet seem to be such an explanation.

Leng

Leng's account focuses on addressing the first problem I noted with Liggins account. She notes that certain mathematical explanations in science are *structural*.[44] These explain through the structural characteristics of a given mathematical structure instantiated in a particular physical system. Liggins' account did not allow him to take advantage of these features in explanation, and so he could not generate a convincing alternative to DME. Leng's account, on the other hand, is specifically designed with the structural features of putative mathematical explanantia in mind. This requires explaining how these structural features can be attributed to physical objects without committing to the mathematical objects they putatively belong to.

Leng describes how this is supposed to work:

> We can think of a mathematical structure as characterized by axioms. A physical system instantiating that structure is one where those axioms are true when interpreted as about that physical system. A structural explanation will explain a phenomenon by showing (a) that the phenomenon occurs in a physical system instantiating a general mathematical structure, and (b) the existence of that phenomenon is a consequence of the structure characterizing axioms once suitably interpreted.[45]

Here, Leng tries to access the intra-mathematical explanatory power of mathematical proofs by making the 'instantiation' of a mathematical structure merely metaphorical (it is *as if* the physical structure instantiates the mathematical structure) and follow from a nominalistically acceptable interpretation. Consider the honeycomb example. The mathematical portion of the explanation of

[44] Leng (2012): 988.
[45] Leng (2012): 989.

the honeycomb's physical structure is based on the honeycomb theorem – the proof in discrete geometry that a hexagonal grid is the most efficient way to divide up a plane, with certain caveats. Leng claims that only a limited portion of mathematics is relevant to theorems like this and that the axioms characterizing that portion are sufficient to construct the proof. Moreover, only this proof is needed for the sake of the explanation, so only the area of mathematics which is relevant to it needs to be characterized. If the axioms which characterize the hexagonal grid and the portion of geometry relevant to it can be interpreted as about the structure of the honeycomb, then the honeycomb will 'instantiate' the hexagonal grid and proofs about the grid will be able to describe facts about the honeycomb.

If Leng's account succeeds, then we have a non-miraculous scientific realist account of how mathematical resources can be applied to physical phenomena without ontological commitment to those resources. But her account falls short as an indexical account of mathematical explanation. While she addresses how mathematics can be employed indispensably in an explanatory context without playing a DME-type determination role, she does not offer an alternative candidate for the determination role. There needs to be an account of what the underlying nominalistic nomological relations are. Without such an account, there is no explanation under *Determination*. It is not yet clear what the mathematics in these explanations is indexing.

Indirect Mathematical Explanation

Combining the virtues of these two theories, we can hopefully develop a plausible alternative to DME. Leng describes how the mathematical resources used to prove that a mathematical structure necessarily possesses a certain property relevant to scientific explanation can be applied to a physical entity without ontological commitment to that mathematical structure, through a suitable interpretation of the axiomatization of the structure. In order to use Leng's argument to support an alternative to DME, we need a way of characterizing the 'set of axioms which when suitably interpreted are true of a physical system' in a way that is nominalistic and capable of bearing a nomological dependence relation.

Liggins' account offers a strategy we can use to do this. If a mathematical axiomatization can be interpreted as being about an entity, it can be asserted that it is because that entity has a nominalistic property. This is the property of being such that the appropriate mathematical metaphor can be interpreted as about that entity. In the honeycomb example, this would be the nominalistic property of *having-the-honeycomb-structure-of-hexagonal-grid*. If this property is nomologically related to the similarly nominalistic property of *using the fewest resources*, then in every structure in which the first property appears (whenever the suitable interpretation of the honeycomb theorem axioms is possible) the second property (*using the fewest resources*) will also appear. The mathematics in the explanation describes this determination relation the nominalistic properties bear to one another without itself bearing any such nomological relations. Making an analogous move to Berenstain's, if this account is the best explanation of how mathematical explanation is possible, then inference to that nomological relation is justified. I call this account Indirect Mathematical Explanation (IME): In mathematical explanations, mathematics serves only to index nomological relations between nominalistic entities.

DME v. IME

To recap: The question at hand is whether the scientific realist platonist ought to endorse *Indispensability* or *Explanation* in order to ground her ontological commitment to mathematical entities. If the proponent of *Explanation* is to offer a more limited ontology than that of the Quinean platonist and maintain no-miracles, then she should endorse some principle like *Determination* to account for certain mathematical explanations. I have presented two accounts of the role mathematics plays in these explanations – DME and IME.

Both DME and IME seem equally capable of showing how mathematical explanations can fulfill the requirements for explanation under *Determination*, with some potential costs. Only under DME, however, do *mathematical* entities bear the nomological relations required for explanation under *Determination*. Given *Explanation*, that means that mathematical entities only exist if DME is true. IME is a nominalist account of mathematical explanation.[46] Platonists who endorse both *Explanation* and *Determination* must endorse DME, if they are to remain platonists and retain mathematical entities in their ontology. However, recall that commitment to DME or IME must be through IBE in order to follow from scientific realist principles. IBE only applies to the *best* explanation, if there is one. If the explanatory platonist is going to endorse DME on scientific realist grounds, then DME must (1) offer a *better* explanation than IME and (2) meet a minimal explanatory threshold.

DME does not yet meet these conditions. Following the criterion of ontological simplicity, we should only accept mathematical entities into our ontology if they serve a function which nominalistic entities cannot. DME proposes that the role mathematics plays by bearing determination relations in mathematical explanation is such a function. But, since IME is able to give an account of the determination relations in such explanations, nominalistic entities can play all the ontologically committing roles available under *Explanation* + *Determination*. The simplicity of IME means that it is a better explanation than DME, and so IBE cannot be applied to

[46] The platonist proponent of *Explanation* could also try to support platonism under IME. Mathematics under IME only indexes determination relations, and so this would require replacing *Determination* with something like *Indexing*: Only entities which are causal, nomologically active, or index such entities can have explanatory power. On this account, even indexing relations are miraculous if the indexing entities do not exist. I see two problems with this: 1. It is not clear that the existence of indexing entities helps to avoid miracles, and so *Indexing* is unsupported by scientific realism. 2. *Explanation* and *Indexing* do not leave any room for non-ontologically committing indispensable roles, and so they entail *Indispensability*. This means that if the aim of explanatory platonism is to offer a better-supported alternative to *Indispensability*, then *Indexing* is a poor prospect.

DME.[47] If the platonist accepts DME and the existence of the relations and entities it entails without showing that DME is justified under IBE, then the commitment no longer follows from scientific realism.[48] The failure of DME to justify IBE inference means that there are no ontologically committing explanatory roles for mathematical entities in the kinds of explanations we have seen, and thus that *Explanation* (given current scientific practice) does not support platonism.

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[47] I leave it an open question here whether IME blocks DME merely because it is better than DME, or whether IME meets the minimal threshold requirement for inference under IBE. At stake is whether explanatory nominalism can account for mathematical explanations. IME also requires some significant work. If this work is not done, then it is not clear whether IME meets the minimal threshold either, and *Explanation* cannot account for mathematical explanations. E.g., IME:

1. **Leaves the relation between mathematics and the nominalistic relations it indexes inexplicable.** IME cannot explain why some portion of mathematics indexes a given relation, or else a determination relation would have to hold between the two and *Explanation + Determination* would ontologically commit us to the mathematics. That this is unexplained is a cost to scientific realism because of no-miracles.
2. **Requires more work to demonstrate feasibility.** For IME to go through it should give conditions for the interpretation of axioms and answer questions like whether all mathematics is readily axiomatic, whether we can determine the required axioms, and whether we only ever need a limited set of axioms to account for a given explanation.

[48] The explanatory platonist could still argue that the way nominalistic entities in IME play the explanatory roles in mathematical explanations infringes more upon scientific realist principles and our other theoretical virtues than mathematical entities and their role in DME. The prospects for this argument are poor. Here are a few additional problems DME faces:

1. **Too strong.** DME posits an unfamiliarly strong metaphysical role for mathematical entities, opposed to our pre-theoretic idea of what mathematics does.
2. **Ad hoc.** Any particular choice about how mathematical difference-making occurs is insufficiently supported. Berenstain argues that we should view the relation between mathematical entities and physical phenomena as instantiation, but it's not clear how that can be justified.
3. **Expanded mathematical ontology.** Part of the motivation for explanatory platonism was to limit ontological commitments. But extra-mathematical explanations (scientific explanations of physical phenomena, like the cicada and honeycomb explanations) are not the only explanations that feature mathematical entities. Mathematicians treat a whole range of intra-mathematical claims/theories as explanatory (such as the honeycomb conjecture). Scientific realists (including Quine and explanatory platonists) standardly treat intra-mathematical explanations as extra-scientific and therefore not ontologically relevant. But DME has trouble maintaining this distinction. Mathematical entities which we are committed to on the basis of extra-mathematical explanations are not limited to appearances in extra-mathematical explanations. These entities also appear in intra-mathematical explanations (e.g., the hexagonal grid appears in the honeycomb explanation and the proof of the honeycomb conjecture), in which other mathematical posits explain their mathematical features and in which they explain other mathematical facts about other posits. The explanatory platonist must either accept all the other mathematical posits which bear explanatory relations to the mathematical entities they say exist (a significant expansion to their ontology and to the Quinean mathematical ontology) or deny that intra-mathematical explanations explain (contra mathematicians).

The explanatory platonist must resolve these at least these difficulties in favor of DME in order to justify IBE.

The problem for the explanatory platonist is that, by endorsing *Determination*, they have accepted a limitation on the set of features relevant to IBE. Under *Determination*, all it is to explain something is to bear a determination relation to it. But mathematical entities are not more capable of bearing such relations than nominalistic entities. This means that more general tools for theoretical assessment, like simplicity, must be employed. If *Explanation* is to remain a viable option, then the platonist needs to offer a comparison tool on which DME explanations can fare better than IME explanations.

The Prediction Criterion

A plausible candidate for such a tool is the prediction criterion, which comes from a strategy developed by Heather Douglas for the assessment and comparison of competing explanations. Douglas says that we ought to judge explanations by their "ability to generate new [successful] predictions".[49] Given two equivalent explanations of the same phenomenon, the one that successfully predicts the occurrence of new phenomena is better. One way to justify the use of this criterion for scientific realists is by appeal to the no-miracles argument.[50] An explanation's ability to make successful novel predictions is miraculous unless the explanation is true and the nomological relations which determine the explanandum also determine in some way the predicted phenomena.

We can see an example of how this could be used to support DME using the honeycomb case. On the DME account, the spatial efficiency of the mathematical hexagonal grid determines the efficiency of the physical honeycomb, because the hexagonal grid is instantiated in the honeycomb. We can extrapolate that other properties of the hexagonal grid should also determine

[49] Douglas (2009): 445.
[50] For independent support of this kind of no-miracles argument, see Worrall (1989). There, John Worrall argues that the only kind of theoretical success relevant to no-miracles is novel predictive success.

properties of the honeycomb. This means that if we discover through mathematical proof that the hexagonal grid has some property x which, if had by the honeycomb, would have empirical consequences which have not yet been observed, we can predict that those consequences will be observed. Observation of the consequences would confirm the prediction.

Such a prediction could not be made using the IME version of the honeycomb explanation. On that explanation, there is a nomological relation between the nominalistic property *having-the-honeycomb-structure-of-hexagonal-grid* and the efficiency property. But there is no nomological relation between the mathematical posits and any of the physical entities or properties involved in the explanation. This means that, while any new properties of the honeycomb can be incorporated post hoc into a new explanation (the nominalist can say that there is an additional nomological relation between *having-the-honeycomb-structure-of-hexagonal-grid* and property x), the explanation of property x could not have been predicted by the IME efficiency explanation. This is because the explanation of property x is in no way determined by the nomological relations present in the efficiency relation. Given the prediction criterion (assuming that there is some property x), the fact that DME supports the new prediction and IME cannot is a point strongly in favor of DME.

Even a successful prediction like this would not necessarily justify an IBE inference to the truth of DME and the existence of mathematical entities, however. Ontological simplicity will still always be in favor of IME. Also, some factors of the prediction itself are relevant to the strength of the evidence it offers. The novelty and generality of the prediction may hold some weight in deciding how miraculous the prediction would be if it were not supported by a nomological relation. A lucky guess is not a miracle. Also, even IME can support mere extensions of an explanation to new entities. In the IME account of a mathematical explanation, the nominalistic property serving as explanans is understood as the property of being such that some set of mathematical axioms can be interpreted as about that entity. Thus the explanation holds not only for some particular entity which has the nominalistic property, but also for any physical entities about which that set of axioms can be interpreted. IME too, then, can explain the success of predictions that extend an explanation to appropriately similar entities. The prediction criterion does not therefore necessarily decide IBE in favor of DME (unlike ontological simplicity for IME), but it does make it possible for DME to compete.

The Naturalist Upshot

The prediction criterion is consistent with scientific realism and *Explanation*, as it follows from no-miracles. For that reason alone, it ought to be accepted by the explanatory nominalist. But the criterion also helps *Explanation* address a broader worry it faces as a naturalist thesis and helps it to compete with *Indispensability*. This worry is that the ontological condition set by *Explanation + Determination* is scientifically underdetermined.

The central commitment of contemporary scientific realism is naturalism, so scientific realist ontological commitments should follow from scientific practice. But whether an entity is explanatory and bears certain nomological or causal relations can be scientifically ambiguous. The original versions of the mathematical explanations presented by scientific practice (prior to the consideration of DME and IME) do not specify which entities are playing explanatory roles. This means that assessing which entities are supported under *Explanation* may require extra-scientific tools and conditions. This is in tension with naturalism.

Indispensability and confirmation holism, on the other hand, do support univocal naturalism. Holism focuses ontological commitment on an unambiguous feature of fully fleshed out scientific theories (the entities quantified over). Scientific practice itself addresses which entities must be quantified over and so settles which entities cannot be dispensed with. No interpretive work outside of that used within science to produce theories is required to see which entities are quantified over. The supporter of *Explanation* aims to deny confirmation holism and reduce the commitments required for scientific realism, but denying confirmation holism does not further naturalism if doing so requires us to institute new extra-scientific standards.

The prediction criterion offers a way of assuaging this worry for *Explanation*. The criterion is a tool that can reliably and naturalistically guide arbitration between explanations which appeal to different entities (and different nomological relations). And it does so from explicit scientific practice – predictions made and confirmed. Realists can expect that only true explanations, which feature the right entities in nomological and causal roles, will exhibit novel predictive success and thus meet the prediction criterion.

The prediction criterion brings great benefit and it, or something like it, ought to be accepted by proponents of *Explanation* on both sides of the mathematical realism debate. This means that the criterion should be available for explanatory platonists to use to support DME. The only problem is that there are no successful novel predictions from DME explanations, let alone predictions that decide IBE in favor of DME. The supporter of DME has two ways to look for such evidence. The first is to examine the historical scientific record, identify mathematical explanations, and see if successful predictions have been made on the basis of mathematical features of those explanations. The aim would be to either justify a positive induction for the future of mathematical explanation or to develop strategies for future mathematical prediction. The second way is to consider our current best theories, identify mathematical explanations in them, and generate new predictions from those explanations which have empirical consequences. This could be done by tabulating the additional properties of the mathematical entities involved (meaning those that bear nomological relations) which would have testable consequences if had by the physical entities involved. While it is not clear that we have any reason to expect there to be evidence of this kind of success for mathematical explanations, it is the case that mathematical explanations themselves (of the kind subject to DME interpretation) have not consistently been recognized as such through the history of science.

Conclusion

Unless and until successful predictions can be produced that decide the debate conclusively in favor of DME, DME is more ontologically profligate than IME and therefore a worse explanation. Given the current state of our scientific evidence, *Explanation* is thus not a viable platonist ontological condition. The scientific realist platonist must seek alternative justification. *Indispensability* offers that justification. Mathematical entities are indispensable to our best scientific theories, and so *Indispensability* does support platonism. Mathematical entities play a variety of indispensable organizing roles in our best scientific theories. Scientific realist platonists ought to endorse *Indispensability* then, if they are to remain platonists.

Works Cited

Armstrong, D.M. (1989). *A Combinatorial Theory of Possibility*. Cambridge: Cambridge University Press.

Baker, A. (2005). Are there genuine mathematical explanations of physical phenomena? *Mind*, 114(454), 223–238.

Baker, A. (2009). Mathematical explanation in science. *British Journal of Philosophy of Science*, 60, 611-633.

Bangu, S. (2008). Inference to the Best Explanation and Mathematical Realism. *Synthese*, Vol. 160, No. 1, 13-20.

Bangu, S. (2017). Indispensability, causation and explanation. *Theoria*. 33/2 (2018): 219-232

Berenstain, N. (2016). The applicability of mathematics to physical modality. *Synthese*.

Colyvan, M. (1998). Can the Eleatic Principle be Justified? *Canadian Journal of Philosophy*, Vol. 28, No. 3 (September 1998), 313–336.

Colyvan, M. (2001). *The indispensability of mathematics*. New York: Oxford University Press.

Colyvan, M. (2010). No Easy Road to Nominalism. Mind, Vol. 119. 474. 285-306.

Colyvan, M. (2012). Road Work Ahead: Heavy Machinery on the Easy Road. *Mind*, Vol. 121, No. 484, 1031-1046.

Darwin, C. (1859). *The Origin of Species*. New York: Random House. (Reprinted 1998).

Douglas, H. (2009). "Reintroducing Prediction to Explanation." *Philosophy of Science*, 76 (4): 444–63.

Einstein, Albert (1905). On the Movement of Small Particles Suspended in a Stationary Liquid Demanded by the Molecular-Kinetic Theory of Heat. In R. Furth (ed.) *Investigation on the Theory of the Brownian Movement*, Dover Publications (Reprinted 1956).

Field, H. (1980). *Science without Numbers: A Defence of Nominalism*. Oxford: Blackwell.

Hales, T. (2001). The honeycomb conjecture. *Discrete and Computational Geometry*, 25, 1–22.

Jackson, F. & Pettit, P. (1990) Program Explanation: A General Perspective. *Analysis*, 50: 107-117.

Leng, M. (2013). Taking it Easy: A Response to Colyvan. *Mind*, 121, 983-96.

Liggins, D. (2014). Grounding and the indispensability argument. *Synthese*.

Liston, M. (2016). Scientific Realism and Antirealism. *Internet Encyclopedia of Philosophy*, URL = <https://www.iep.utm.edu/sci-real/>.

Lyon, A., & Colyvan, M. (2008). The explanatory power of phase spaces. *Philosophia Mathematica*, 16(2), 227–243.

Lyon, A. (2012). Mathematical explanations of empirical facts, and mathematical realism. *Australasian Journal of Philosophy*, 90(3), 559–578.

Maddy, P. (1992). Indispensability and Practice, *Journal of Philosophy*, 89(6): 275–289.

Maiocchi, R. (1990). The Case of Brownian Motion. *The British Journal for the History of Science*, Vol. 23, No. 3, 257-283.

Mancosu, P. (2008). Mathematical explanation: Why it matters. In P. Mancosu. *The philosophy of mathematical practice*, 134-50. Oxford: Oxford University Press.

Melia, J. (2000). Weaseling Away the Indispensability Argument, *Mind* 109/435: 455–79.

Perrin, J. (1913). *Atoms*, (trans. D.L. Hammick), Woodbridge, CT: Ox Bow Press. (Reprinted 1990).

Putnam, H. (1979). What is mathematical truth? *Mathematics, Matter and Method: Philosophical Papers* (pp. 60-78). Cambridge: Cambridge University Press.

Quine, Willard V.O. (1948) On What There Is. *Review of Metaphysics*. Reprinted in 1953 *From a Logical Point of View*. Cambridge, MA: Harvard University Press.

Quine, W.V. (1981). *Theories and Things*. Cambridge, MA: The Belknap Press of Harvard University Press.

Worrall, J. (1989). “Structural Realism: The Best of Both Worlds?” *Dialectica*, 43 (1–2): 99–124.