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The Effects of the Geometer's Sketchpad Software on Achievement of Geometric Knowledge of High School Geometry Students

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THE EFFECTS OF THE GEOMETER'S SKETCHPAD SOFTWARE ON ACHIEVEMENT OF GEOMETRIC KNOWLEDGE OF HIGH SCHOOL GEOMETRY STUDENTS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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CHAPTER I

The Research Problem

Statement of the Problem

The *Geometer's Sketchpad* (Jackiw, 1994) is a "dynamic" software tool of instruction for teaching geometry to high school students. The software tool was designed to assist the user in learning geometry through observation and creation of "dynamic" changes on geometric objects. The term, "dynamic", refers to the capacity of the software to transform geometric sketches on the computer screen.

The first type of "dynamic" transformation of geometric sketches is to manipulate changes in: (a) position, (b) size, and (c) shape of geometric sketches. These changes are observed while the relationships defined in the original sketches remain preserved. A second type of a "dynamic" transformation is to set geometric objects in motion to show the sequence of steps followed in completing a construction or to show a path of a function operating in a sketch. A third type of "dynamic" transformation is to observe the effect of changing measurements on geometric objects in a sketch. Measurements of objects are simultaneously recorded in a chart on the computer screen as the size and shape of objects are made smaller or larger. These visualization techniques assist the learner in developing an understanding of geometric concepts as well as in developing inductive reasoning skills essential for discovering properties of Euclidean geometry.

What are some consequences of this kind of a software tool on learning geometric knowledge? Can this type of software tool extend cognitive capacities for inductive reasoning and problem-solving skills by sharing cognitive operations with its user? How can this sharing of cognitive operations with The *Geometer's Sketchpad* advance achievement of geometric knowledge? When the computer user is engaged as an "intellectual partner" how are cognitive operations extended to facilitate learning? This study investigated the effect on acquiring geometric knowledge of using *The Geometer's Sketchpad* (Jackiw, 1994) as a "dynamic" tool of instruction engaging the user in an "intellectual partnership" to extend cognitive capacities.

The present study addressed the problem of whether *The Geometer's Sketchpad* (Jackiw, 1994) computer program improved learning geometry. If learning can be enhanced through intellectual partnerships whereby cognitive operations are shared between the *Sketchpad* and the user, then a field experience to investigate the potential for improving achievement of geometric knowledge is needed.

Purpose of the Study

The purpose of this study was to address the problem high school students have in learning geometric knowledge. *The Geometer's Sketchpad* (Jackiw, 1994) was the software tool used for instruction to conduct a quasi-experimental study to investigate its capabilities for improving achievement of high school geometric knowledge.

Jackiw (1994) claimed that the *Geometer's Sketchpad* is a powerful, software tool for improving instruction of high school geometry. The present study measured the effectiveness of the *Geometer's Sketchpad* program as a "dynamic" tool for instruction versus using a textbook, *Discovering Geometry: An Inductive Approach* (Serra, 1993) and traditional geometry tools for instruction.

Forty-seven high school geometry students participated in the study. Subjects were placed in one of two levels of instruction. Subjects in each level of instruction were from two intact class groups. One geometry class consisting of twenty students was assigned to participate in the experimental treatment. The experimental treatment group participated in a cognitive technology-based inductive method of instruction in geometry. A second geometry class consisting of twenty-seven students was assigned to participate as the control group. The control group participated in a textbook-based inductive method of instruction in geometry.

Significant changes for effective use of cognitive technologies to expand cognitive capacities to improve achievement of geometric knowledge may be suggested from the study. A software tool qualifies as a cognitive technology, if it provides a "...medium that helps transcend the limitations of the mind, such as memory, in activities of thinking, learning, and problem-solving" (Pea, 1985, p. 168). *The Geometer's Sketchpad* (Jackiw, 1994) is a" dynamic" software program providing the user with a cognitive tool to participate in an "intellectual partnership" with the computer to share cognitive operations.

Definition of Terms

Technical terms on (a) learning theory, (b) instructional methodology, and (c) software design are defined as follows as they were applied in this study:

1. Cognitive Tool: Tools are cognitive insofar as "...they serve to aid students in their own constructive thinking, allowing them to transcend their cognitive limitations and engage in cognitive operations they would not have been capable of otherwise" (Salomon, 1993b, p. 180).

2. Cognitive model of instruction: A cognitive model for instruction is designed with instructional strategies directed to stimulating information processes of the mind operating during learning tasks.

3. Cognitive Science: Cognitive science "attempts to integrate research efforts from psychology, philosophy, linguistics, neuroscience, and artificial intelligence...cognitive science makes greater use of methods such as computer simulation of cognitive processes and logical analysis ....(Anderson, 1990, p. 10).

4. Cognitive Technology: " A cognitive technology is ... any medium that helps transcend the limitations of the mind, such as memory, in activities of thinking, learning, and problem-solving" (Pea, 1985, p. 168).

5. Conjecture: "Geometric conjectures have three key parts: the relationship described in the conjecture, the set of objects for which the relationship holds, and the quantifier that determines the members of the set of objects for which the relationship holds" (Yerushalmy, 1993, p. 58).

6. Constructivism: Constructivism defines learning as a constructive mental process in which the learner builds an internal representation of knowledge based upon the individual's personal interpretation of a given experience.

7. Deductive Reasoning: Deduction is a process of proving "...statements by reasoning from accepted postulates, definitions, theorems, and given information" (R. Jurgensen, Brown, and Jurgensen, 1992, p. 45).

8. Distributed Cognitions: Cognitions become distributed when the computer tool and its user *think jointly* to produce a product (Salomon, 1993b).

9. Dynamic : Dynamic refers to the power of the software tool, *Geometer's Sketchpad,* to transform geometric sketches on the computer screen. Geometric objects are manipulated by changing position, size, and shape of objects, while relationships defined in the original sketches are preserved.

10. Generalization: Generalization in geometry involves three processes: "...formation of samples of examples to serve as a data base for conjectures, manipulations on the samples, and analysis of ideas in order to form more general ideas" (Yerushalmy, 1993, pp. 81-82).

11. Inductive reasoning: Inductive reasoning is a process involving "... observing data, recognizing patterns, and making generalizations from ... observations" (Serra, 1993, p. 39).

12. Intellectual Partnership: An intellectual partnership is formed when tasks are shared between the student and the computer. For example, the computer performs computation, construction, recording, and replaying operations, while the user performs thinking and reasoning tasks on data provided by the computer.

13. Internalization: Salomon (1988) defines internalization as the process whereby computer-tools designed with particular attributes are internalized as cognitive tools and share cognitive operations with the user.

14. Pedagogic Tool: The software program performs as a pedagogic tool when the cognitive *effects of* an intellectual partnership between the user and the computer results in improved solo abilities that can be used in the absence of the software program (Salomon, 1993b).

15. Performance Tool: The software program performs as a performance tool when cognitive *effects with* an intellectual partnership between the user and the computer results in improving joint performance in producing a product (Salomon, 1993b).

16. Procedure-Capturing: Procedure-capturing is the capability of *The Geometer's Sketchpad* (Jackiw, 1994) to capture a sequence of actions which can then be displayed as a script. The script is an

automatically generated program recording steps of constructions. Scripts can be edited and incorporated into other scripts.

17. Reify: To reify an abstract idea, for example, an action or strategy operating while solving a problem, is to treat it as a concrete object to be analyzed through data recorded on the computer screen.

18. Solo Cognitive Abilities: Solo cognitive abilities are intellectual operations of an individual person. For example, higher order reasoning skills such as analysis applied to solving geometric problems.

19. Solo Cognitive Residues: Solo cognitive residues refer to skills of an individual acquired as a result of an intellectual partnership with the computer and applied in the absence of the software program.

Background and Need for the Study

Intelligent software programs are designed for sharing cognitive operations to implement powerful uses of technology-based instruction. These programs guide learners to take efficient routes to attain instructional objectives. Such programs help learners "...reach better understanding of the material, to have a better grasp of whatever has been taught, to better overcome their intuitive notions and replace them with more formal and desirable ones" (Salomon, 1988, p. 124).

Salomon claimed software programs implemented as technology tools can engage the user as an intellectual partner to share cognitive operations. When cognitive operations are shared between the user and the software program, the computer serves as a cognitive tool. This sharing allows "...learners [to] internalize computers' intelligent tools and use them as cognitive ones" (Salomon, 1988, p. 123).

When technology tools become internalized they can extend cognition to accomplish operations beyond the limitation of the mind's capacity. For example, the computer as a cognitive tool can perform mathematical computation with greater efficiency and speed better than most students. Consequently, the use of the computer as a cognitive tool for instruction can influence learning in powerful ways. Further research on cognitive technologies by Salomon, Perkins, and Globerson (1991) indicated,

> *Effects with* technology can redefine and enhance performance as students work in partnership with intelligent technologies--those that undertake a significant part of the cognitive processing that otherwise would have to be managed by the person. Moreover, *effects of* technology can occur when partnership with a technology leaves a cognitive residue, equipping people with thinking skills and strategies that reorganize and enhance their performance even away from the technology in question. (p. 8)

As cognitive technology becomes more available in the field of secondary school mathematics, there is a need for researchers to conduct field experiments to determine the effectiveness of intelligent software programs on learning. According to Kaput (1992) "Technologies based on dynamic interactive electronic media embody fundamental attributes that distinguish them from traditional static media in ways likely to have tremendous long-term impact on mathematics education" (p. 525).

There is a need for both educators and researchers to investigate cognitive benefits of "dynamic" software designs for the purpose of improving learning. As Pea (1985) stated "The urgency of updating education's goals and methods recommends an activist research paradigm: to simultaneously create and study changes in processes and outcomes of human learning with new cognitive and educational tools" (p. 167). The present study applies this research paradigm to an investigation of the effect of cognitive technology-based instruction on acquiring geometric knowledge.

Developments in applying an inductive methodology are challenging the traditional deductive methodology currently being used in teaching high school geometry. Reasoning skills required for developing formal proofs are taught deductively through memorization of definitions, theorems, and postulates, and then applied to writing formal two-column proofs of theorems. The paper and pencil method of constructing geometric figures for use in formal proof limits examples to one or two static one-dimensional illustrations to be completed in any one class period. For example, students are given instructions to construct a triangle and its medians. Students are then asked to prove a theorem to demonstrate logical deductions concluded from defined relationships, postulates, and previously proven theorems.

Construction of geometric figures is the medium through which students visualize relationships they are trying to discover. Visualization is essential to enable students to analyze relationships and formulate conjectures based on observations embedded in the construction of figures. Time, energy, and interest factored into one class period limits construction of geometric sketches to one or two static illustrations.

For example, to formulate conjectures based on sketches in order to write conjectures about relationships that exist among the medians of a triangle, students need to construct several different kinds of triangles to observe all cases before generalizing to a conclusion. Such a task is tedious, time consuming, and students are likely to lose interest before completing the task. Construction of sketches drawn with a compass and ruler are constrained by time, dimension, and student motivation.

Euclidean high school geometry has been taught using the classic geometry tools: compass, ruler, and protractor for most of its history in high school education up to the present time. The validity of teaching students formal proofs of geometry using a deductive approach, i. e., beginning with an abstract concept and then reasoning to a concrete representation of a concept, has been questioned by high school geometry teachers.

This deductive approach for teaching geometry has been challenged by the lack of success and a lack of student interest in geometry. "The National Assessment of Educational Progress found in 1982 that doing proofs was the least liked mathematics topic of 17-year-olds, and less than 50% of them rated the topic as important" (Bennett, 1993, p. 1).

The National Council of Teachers of Mathematics (NCTM) in 1986 recognized a need to address the issue of how best to teach high school geometry as well as other issues concerning the teaching of mathematics. A commission on Standards for School Mathematics comprised of math educators, classroom teachers, and supervisors created a document entitled, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The document suggested new approaches for better success in mathematical learning. One new approach suggested was to change the teaching of geometry from a formal deductive approach to an inductive approach.

A similar process was used by the National Council of Teachers of Mathematics (NCTM) to publish a document on *Professional Standards for Teaching Mathematics (NCTM, 1991)* . This document provided guidelines for teachers to implement the NCTM *Standards* of 1989. New standards for teaching and learning geometry were spelled out in both publications.

The authors articulated changes in geometry instruction to include decreasing attention to formal two-column proofs while increasing attention to the use of technology-based programs capable of manipulations of two and three dimensional figures. The Standards (NCTM, 1989) for secondary level mathematics suggested

> Developing fluency with symbols and other abstract entities, which can be geometric, algebraic, or algorithmic, [these] must be a central aim of secondary school mathematics. Students should learn that, in mathematics, reasoning is the standard of truth. They should experience the power of its application. (Mathematical Sciences Education Board, 1991, p. 11)

Goals for student performance in geometry set by the Mathematical Sciences Education Board in 1991, were to develop the "...ability to discern relationships, reason logically, and use a range of mathematical methods to solve a wide variety of non-routine problems" (p 5). New technologies offer software designs to support these goals by taking advantage of the cognitive benefits of intelligent programs for computer-assisted instruction.

Educators of mathematics have investigated intelligent software programs which offer a "dynamic" visual approach to teaching and learning high school geometry. Computer software designed as intelligent tools offers capabilities to carry out an inductive "dynamic" approach for geometry instruction aligned with goals set by the Standards (NCTM, 1989).

Emerging technologies offer computer-based explorations of two and three dimensional objects capable of being transformed on the computer screen. While pencil and paper provides one example of a diagram, computer software can create limitless numbers of constructions under varying conditions. Pea (1985) wrote, "The consequences for math education and for what mathematical thought requires that result from these new cognitive technologies are remarkable" (p. 175).

Two examples of these new cognitive technology designs to teach geometry are *The Geometric Supposers* (Schwartz, 1985) and *The Geometer's Sketchpad.* (Jackiw, 1994). Emphasis is placed on "dynamic" visualization of geometric constructions, analysis of problems, and investigations using an inductive reasoning approach to discover patterns for formulating conjectures to solve problems.

Sketchpad (Jackiw, 1994) enables students to observe relationships to constructions and dynamically transforms and manipulates geometric figures. Students can observe multiple cases of one construction under several conditions providing visual evidence for students to analyze and formulate conjectures. These skills are essential for the study of Euclidean geometry. By definition Euclidean geometry is the study of properties that remain the same under varying conditions.

Experimental research is needed to investigate the effectiveness of software supporting an inductive approach to teaching and learning geometry as a means of implementing the Standards (NCTM,1989) by the year 2000. In response to this challenge, it is imperative for researchers to investigate new designs of software programs in geometry to discover how these programs respond to new goals set by the Standards (NCTM, 1989).

In the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) one of the goals for instruction was the use of technology in learning mathematics, "Computer software can be used effectively for class demonstrations and independently by students to explore additional examples, perform independent investigations, generate and summarize data as part of a project or complete assignments" (p. 128).

In 1985, Judah Schwartz and Michal Yerushalmy developed software to implement an inductive approach to teaching geometry. The *Geometer Supposers* are examples of software using a guided inquiry approach for discovering properties of geometric figures leading to formulating conjectures about geometric figures. The *Supposer* (Schwartz, 1985) is a microcomputer software series consisting of a preSupposer (for middle school students), and a series of Triangles, Quadrilaterals, and Circles (for high school students). Yerushalmy (1990) designed the software believing

> ...that geometry instruction would be more effective if, rather than teaching definitions and theorems as given and concentrating on proofs, it were to give students an opportunity to experiment with the entire domain of geometric elements and move back and forth between the particular experience and the general theorems. (p. 24)

The *Geometer Supposer* (Schwartz, 1985) software allows the student to construct geometric figures, for example, a triangle or a rectangle. Steps of the construction along with measurement of elements (angles and line segments) are recorded as a procedure. The procedure can be repeated on other examples of the same shape for students to analyze and make conjectures based on problem data.

For example, the Supposer includes an option for the user to draw a figure either by random selection or to draw a self-constructed shape. Students then make conjectures, collect data via constructions, and through analysis of data formulate generalizations.

The *Geometer's Sketchpad* (Jackiw, 1994) is another software tool for teaching geometry whose goals are aligned with recommendations of the NCTM *Standards* (1989). The *Geometer's Sketchpad* is an example of an intelligent software tool with unique capabilities that go beyond those of the *Geometer Supposer* (Schwartz, 1985).

Key features of the *Supposer* (Schwartz, 1985) included in *Sketchpad* (Jackiw, 1994) are: (a) construction capabilities of drawing, labeling and measuring sketches; and (b) duplication of multiple representation capabilities and recording procedures. In addition to these features *Sketchpad* (Jackiw, 1994) also includes: (a) tools for creating geometric figures, (b) buttons for dynamically transforming sketches, (c) scripts for recording step-by-step procedures, (d) tables for displaying measurement data, and (e) buttons for animating sketches, adding sound, or making a film for demonstrating construction procedures of sketches.

The *Geometer's Sketchpad* (Jackiw, 1994) qualifies as a cognitive tool. It engages the user as an intellectual partner by sharing cognitive operations in the following ways: (a) geometric figures are constructed by tools in the program, (b) geometric quantities are measured and recorded in tables, (c) geometric construction steps are recorded in scripts (descriptions of constructions) that can be replayed and modified.

The present study investigated the effect on acquiring geometric knowledge resulting from the *Geometer's Sketchpad's* (Jackiw, 1994) capabilities of sharing cognitive operations by: (a) constructing and dynamically transforming geometric objects, (b) computing and recording data in tables, and (c) capturing and replaying scripts of procedures. The study measured the effect on achieving geometric knowledge by implementing the *Geometer Sketchpad* program as a cognitive tool for instruction.

As an intellectual partner in cognition, the program assists in constructing, computing, and recording geometry tasks in partnership with the user. This shared partnership frees the user to use cognitive operations for higher order reasoning skills required by geometric problem-solving tasks. Skolnic and Smith (1993) defined

> ...higher order thinking [as a] means to move up into an area where the student has to think and reason and put together some subjective material and make some kind of conclusion. What higher order thinking skills do is focus on the practical application of reasoning and using the knowledge that you gain to abstract it to another application as opposed to the one that's right in front of you. (p. 6)

Another capability of *The Geometer's Sketchpad* (Jackiw, 1994) is that it places the control of learning in the hands of the user to create sketches and dynamically transform them using animation and sound. Kaput (1992) stated this "dynamic" quality of electronic media impacts mathematical learning in the following way: "One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance-to see what stays the same-one must have variation. *Dynamic media inherently make variation easier to achieve* " (p. 525). *Sketchpad* (Jackiw and Bennett, 1993) allows the user to manipulate any figure to demonstrate every possible example of that figure, while recording data simultaneously as sketches are changed on the screen.

Figure 1 is a sample of a "Sketch Window" from *Sketchpad* (Jackiw, 1994). The sketch window contains (a) a display of tools to draw, label, and transform sketches; (b) a menu bar to access pull-down commands, (c) a title bar to show the name of the document; (d) a sketch plane to draw sketches, and (e) a pointer tool to show location of operations.

Sample sketch window. [1]

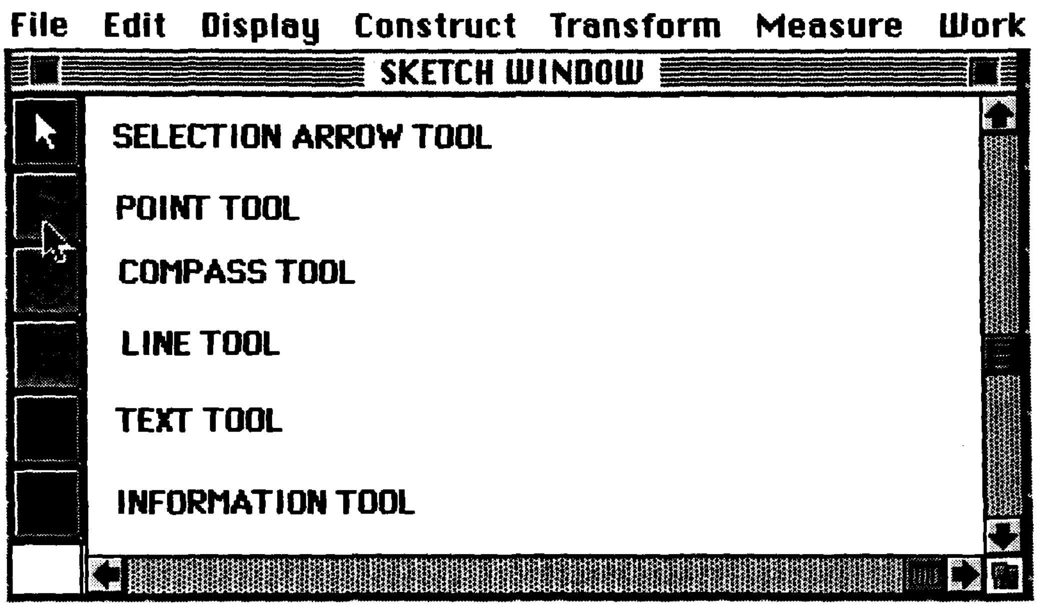

Figure 1. Sketch Window screen from *Geometer's Sketchpad* (Jackiw, 1994) showing the electronic tools of construction.

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[1] From *The Geometer's Sketchpad* used with permission from "The Geometer's Sketchpad, Key Curriculum Press, P. O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH."

The Geometer's Sketchpad (Jackiw, 1994) has another window called the Script Window as shown in Figure 2. The script window contains (a) a control deck for recording and playing back sketches, (b) a status pane for showing current script, (c) a comment pane for showing information about the script, and (d) a script pane showing the script itself (*The Geometer's Sketchpad User Guide and Reference Manual*, 1994, p. 11).

The capture and replay feature of scripts influences geometric learning in new ways. Access to recorded data provides the option to re-examine data for the purpose of formulating conjectures. Claudia Giamati (1995) comments, "The most useful aspect of scripting one's constructions is that students can test whether their constructions work in general or whether they have discovered a special case" (p. 456).

Sample script window. [2]

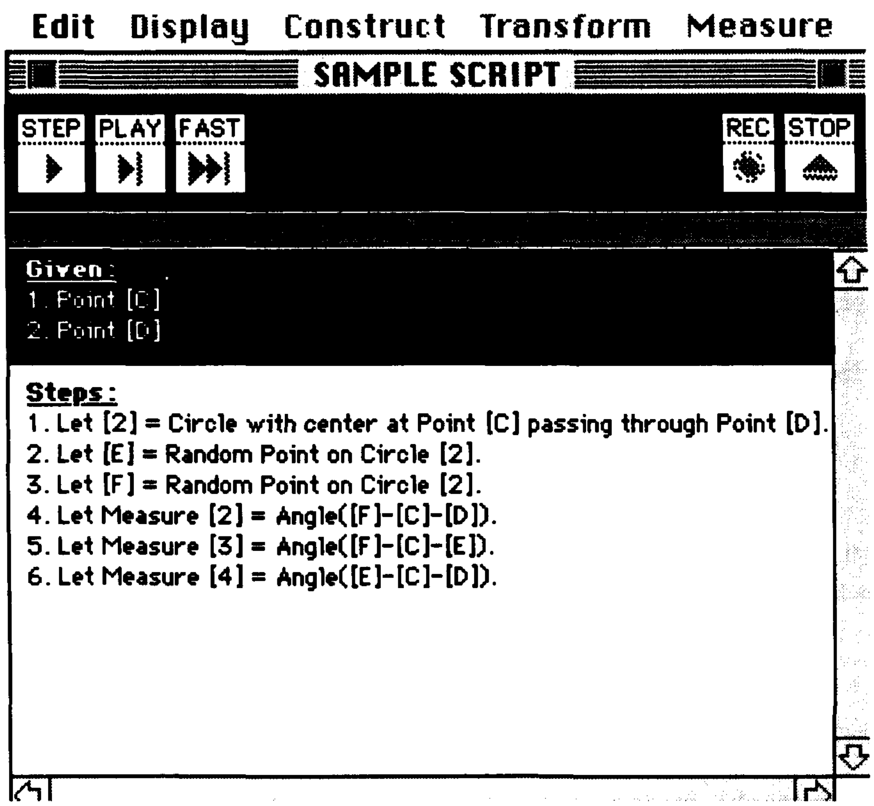


Figure 2. Script Window screen from *Geometer's Sketchpad* (Jackiw, 1994) showing sample scripts of construction.

[2] From *The Geometer's Sketchpad* used with permission from "The Geometer's Sketchpad, Key Curriculum Press, P. O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH."

Kaput (1992) stated, "The ability to record and conveniently display and replay a sequence of one's prior actions provides new means for reifying that most ephemeral and elusive thing called 'strategy.' Once reified, it can be discussed and improved" (p. 533).

If achieving geometric knowledge is improved with the computer as a partner to extend cognitive operations, then this result may lead to further integration of computers as cognitive partners for mathematical learning and instruction. At the present time the computer is not widely implemented in schools as a cognitive tool to transcend limitations of the intellect. Pea (1985) stated,

> ...a primary role for computers is changing the tasks we do by reorganizing our mental functioning, not only by amplifying it.... the predominant use of computers in education today is with software that aims to make more efficient long-familiar drill and practice activities in basic skills, especially in math.... (p. 168)

The consequences of these features measured in the present study may result in integration of this type of software into math education programs. The *Geometer's Sketchpad* (Jackiw, 1994) is commended by many in the field of mathematics, but there is a lack of experimental research data on its effectiveness as a cognitive tool of instruction. The current study compared achievement of geometric knowledge using the *Geometer's Sketchpad* (Jackiw, 1994) as a "dynamic" tool for geometric constructions versus static diagrammatic representations of constructions using traditional geometry tools.

Theoretical Rationale

Computer-assisted instruction designed to extend cognitive operations through an intellectual partnership between user and the computer has the potential to improve classroom instruction and learning. Salomon's theory (1993a) provides a rationale on how powerful intellectual partnerships between the user and the computer can extend cognition when the computer is used as a tool of instruction.

When computers are used as cognitive tools they are "...capable of offering their users an *intellectual partnership* whereby the cognitive burden of carrying out an intellectual task becomes *shared*" (Salomon, 1993b, p. 182). If learning can be improved using software programs as tools for creating intellectual partnerships whereby cognitive operations of the user are extended, then these programs need to be investigated to improve computer-assisted instruction.

The problem is that not all software programs are designed as cognitive tools to engage the user as an intellectual partner with the computer. There are wide varieties of goals, purposes, and activities which determine the design of software programs. For example, a game type of entertainment software program is not designed as a tool for sharing cognitive operations and for engaging the user in an intellectual partnership.

Salomon (1993b) described two effects resulting from sharing cognitive operations through intellectual partnerships created between the user and the computer program. The distinction between these effects lies in the level of shared cognitive operations resulting from the kind of cognitive effect the computer activity has on the user.

As a cognitive tool the computer program engages the user as an intellectual partner on two levels as described by Salomon (1993b): cognitive *effects with* the software program and cognitive *effects of* the software program. On one level, the computer acts as a performance tool; cognitive *effects with* the intellectual partnership result in improved joint performance in producing a product while using the computer. On a second level, the computer acts as a pedagogic tool; cognitive *effects* *of* the intellectual partnership result in improved solo abilities the user applies in the absence of the computer.

When the computer acts as a performance-oriented tool, the effect is a type of distributed cognition with the computer as an intellectual partner. The goal of the performance is the product produced as a result of a joint partnership. "Cognitions become 'distributed' in the sense that the tool and its human partner *think jointly*"(Salomon, 1993b, p. 182). As a performance-oriented tool, achievement of *effects with* the computer as a cognitive tool upgrades *joint performance* using distributed powers of both software program and computer user. The *Writing Partner* is an example of a software program where distributed cognitions are shared in an intellectual partnership of joint performance.

The *Writing Partner* offers assistance through techniques of cueing, prompting, and guiding the user throughout the program. The software design suggests creative avenues to pursue for developing a writing project. The main effect of distributed cognitions is through "...*guided stimulation* -or better, *qualitative scaffolding,* whereby one partner [the computer program] activates, provides meaning to, and possibly directs the cognitive activity of the other [the user] and thereby qualitatively changes the activity" (Salomon, 1993a, p. 133).

When the computer shares cognitive operations as a pedagogical tool, the cognitive *effect of* the intellectual partnership is one of sharing intellectual operations as a division-of-tasks between the user and the software program. Salomon (1993a) stated, "The totality of the cognitive activity, to an extent, is a matter of *division of labor* : The computer does the computation while the user provides the inputs; the list does the remembering while the person does the shopping and so on." (p. 132). The cognitive *effect of* the intellectual partnership results in generalizable skills leaving cognitive residues that can be applied when the computer is not available.

The *Geometric Supposer* (Schwartz, 1985) is an example of a software program which engages the user in intellectual partnership whereby cognitive activities, for example, drawing geometric constructions and computing measures, are off-loaded onto the computer. Salomon stated at this level of cognitive sharing "...changes that take place [are] in the individual [solo abilities], changes that are attributed to the partnership and may result from it, but are nevertheless considered those of the individual. In the latter case the [computer] tool is of the *pedagogic* kind" (Salomon, 1993b, p. 182). Cognitive *effects of* the tool improves solo abilities, and cognitive changes remain in the individual. Cognitive *effects of* the intellectual partnership result in generalizable skills leaving cognitive residues to apply in the absence of the computer. Salomon (1993b) explained:

> ... the partnership ought to be designed such that it leaves the individuals with solo cognitive residues (e.g., improved skill mastery) that would improve their autonomous higher order thinking as well as affect their subsequent partnerships with the tool....They should be designed in a way that turns *effects with* them into more lasting *effects of* them. (p. 184)

The *Geometer's Sketchpad* (Jackiw, 1994) is designed to engage the user as an intellectual partner as a performance tool to upgrade intellectual achievement of geometric knowledge and as a pedagogic tool to improve solo skills and strategies.

Sketchpad (Jackiw, 1994) software can be used as tool for implementing an inductive discovery approach to learning geometry. For example, geometric properties are discovered through observation of patterns and through experimentation. Three distinguishing features of the *Geometer's Sketchpad* (Jackiw, 1994) program for exploring geometric properties are: (a) The program computes and records measures of lengths and angles, (b) the program creates and animates "dynamic" transformations on objects of construction, and (c) the program captures and replays recorded actions of problem-solving procedures.

The *Geometer's Sketchpad* (Jackiw, 1994) provides an opportunity for shared, distributed cognitions of the division-of-labor kind (pedagogic tool) through recording, constructing, and replaying techniques implemented by the user. According to Salomon (1993a),

> To the extent that a tool *shares* the intellectual burden of the learner, it does so only to facilitate higher order thinking by means of freeing the learner from tedious, labor and memory intensive lower level processes that often block higher order thinking. (p. 181)

The *Geometer's Sketchpad* (Jackiw, 1994) qualifies as a cognitive tool of instruction. As a pedagogical tool the program provides opportunities for developing geometric knowledge leaving cognitive residues for applications in the absence of the computer program. The *Sketchpad* is "...providing the knowledge and intelligence to guide learning, it [is]... providing the facilitating structure and tools that enable students to make maximum use of their own intelligence and knowledge (Scardamalia, M., Bereiter, McLean, Swallow, and Woodruff, 1989, p. 54). The *Geometer's Sketchpad* (Jackiw, 1994) qualifies as both a pedagogic and a performance tool with capabilities to turn cognitive *effects with* and *effects of* the program into improved learning of geometric knowledge.

Summary

When the computer is used as a cognitive tool of instruction, a joint intellectual partnership between the user and the computer can be created to share cognitive operations. This sharing extends the cognitive operations of the user, thus improving cognitive capacities for learning. If cognitive effects are embedded in the *Geometric Sketchpad* program, then both solo geometry skills and intellectual achievement of geometric knowledge should be facilitated.

If it is true that *Sketchpad* as a cognitive tool of instruction has the capability to create an intellectual partnership with the user, such that through this partnership cognitions are shared through "a Vygotskian-like process of internalization" (Salomon, 1993b, p. 184), then as a pedagogic tool, solo learning of geometry skills should improve as a result of cognitive *effects of* the software program. If it is true that *Sketchpad* as a performance tool has the capability to improve skills and strategies, then *effects with* the tool upgrades student performance during the partnership.

If the software program *Geometer Sketchpad* is used to extend cognitive operations through an intellectual partnership as a performance tool to produce a joint product with the user and as a pedagogical tool to improve solo abilities, then geometric knowledge can be increased and acquired skills can be applied in the absence of the computer tool.

If *Sketchpad* as a cognitive tool extends cognition by sharing tasks of constructing, computing, and recording with ease of use and perfect accuracy, then the learner freed from these tasks can use cognitive operations to perform other intellectual tasks. Intellectual tasks required for solving geometry problems are inductive reasoning skills applied to data in order to formulate generalizations.

If *Sketchpad* produces charts to record data and transformations to test multiple cases of a construction, then these features provide the stimulus and data for the learner to employ skills for analyzing, synthesizing, and formulating generalizations that can be used in the absence of the computer. The research questions for the study were informed by these if--then statements.

Research Questions

1. What is the cognitive effect on achieving geometric knowledge by instructional use of the software program, *The Geometer's Sketchpad* (Jackiw, 1994) when used as a pedagogical tool to improve subjects'

solo geometry skills, and as a performance tool to upgrade concept development in producing problem solutions?

2. What is the cognitive effect of *The Geometer's Sketchpad* 's (1994) capability of dynamically manipulating, transforming, recording and upgrading data on the quality of conjectures written after completing investigation of sketches?

Research Hypotheses

Theoretical Hypothesis 1. If the *Geometer's Sketchpad* (Jackiw, 1994) is used as a pedagogic tool of instruction for creating intellectual partnerships of sharing cognitive operations, then the cognitive *effects of* improving solo abilities should improve achievement of geometric knowledge. Subjects who receive instruction using the *Geometer's Sketchpad* (Jackiw, 1994) program will achieve a higher mean score on a test of geometric knowledge and construction than subjects who receive instruction using a textbook and traditional tools.

Operational Hypothesis 1. The mean score on a written posttest on geometric knowledge and construction of subjects using the *Geometer's Sketchpad* (Jackiw, 1994) will be higher than the mean score of subjects using a textbook and traditional tools for instruction.

Statistical Hypothesis 1. On a test measuring achievement of geometric knowledge and construction (GK), the mean score will be higher for students using *The Geometer's Sketchpad* (GS) than for those students using a textbook for instruction (TI) when using third quarter geometry grades as a covariate.

$$H_1 : \overline{GK}_{GS} > \overline{GK}_{TI}$$

$$\alpha = .05, \quad N = 47, \quad n_1 = 20, n_2 = 27$$

.....in which α is a one-tailed type I error risk, N is the number of subjects in the study, n_1 is the number of observations generating the means in the experimental group, and n_2 is the number of observations generating the means in the control group.

Theoretical Hypothesis 2. If the *Geometer's Sketchpad* (Jackiw, 1994) is used as a cognitive tool, then the quality of generalizations students formulated when producing a solution to a given problem will be positively influenced. Subjects using *The Geometer's Sketchpad* will formulate generalizations in the form of conjectures indicating higher geometric concept development than those subjects who receive instruction using a textbook.

Operational Hypothesis 2. The mean score on a test measuring the quality of generalizations formulated in written conjectures for those using *Sketchpad* will be higher than the mean score on the same test for those subjects using a textbook for instruction.

Statistical Hypothesis 2. A test to measure the concept level of generalizations formulated in written conjectures (COG) using the *Geometer's Sketchpad* (GS) the mean score will be greater than the mean score on the same test for those subjects using a textbook for instruction.

$$H_2: \overline{COG}_{GS} > \overline{COG}_{TI}$$

$$\alpha = .05, \quad N = 47, \quad n_1 = 20, \quad n_2 = 27$$

.....in which α is a one tailed type I error risk, N is the number of subjects in the study, n_1 is the number of observations generating the means in the experimental group, and n_2 is the number of observations generating the means in the control group.

CHAPTER II

Review of Related Literature

Within the last twenty-five years the pedagogy of mathematics has been revolutionized by discoveries of cognitive science on intelligence theories and their application to computer technology. Cognitive science has provided new theories on how knowledge is represented in the brain. Designers of computer programs have provided cognitive software tools for instruction and learning congruent with new theories on intelligence.

These discoveries initiated reforms of instruction and learning of mathematics described in publications by the National Council of Teachers of Mathematics (NCTM; Mathematics Framework for California Public Schools Kindergarten through Grade Twelve, 1992; Counting on You: Actions Supporting Mathematics Teaching Standards, 1991; Professional Standards for Teaching Mathematics, 1991; Curriculum and Evaluation Standards for School Mathematics, 1989). These publications describe a new vision for instruction and learning mathematics within a technology-based learning environment.

A description of this vision was captured by Schifter (1996):

> Teaching mathematics was reconceived as the provision of activities designed to encourage and facilitate the constructive process. The mathematics classroom was to become a community of inquiry, a problem-posing and problem-solving environment in which developing an approach to thinking would be valued more highly than memorizing algorithms and using them to get right answers. (p. 495)

Constructivist perspectives on instruction and learning underlie this new vision of a classroom environment. The design of the current study was informed by research on a constructivist approach to learning mathematics. The choice of *The Geometer Sketchpad* as the software tool for implementation of instruction relied on research literature on the design of intelligent software for facilitating learning.

In the first section of this chapter literature in three areas of research on instruction and learning mathematics are reviewed. The three subdivisions of literature are:

1. Applications of cognitive science discoveries to learning theory.
2. Applications of cognitive theory to software design.
3. Applications of constructivist theory to instructional design.

Literature from these areas informed the theoretical rationale, variables, hypotheses, and research questions of the current study.

In the second section of this chapter literature on empirical research studies is reviewed. The two subdivisions of literature are:

1. Research Studies on *The Geometer Supposers* (Schwartz and Yerushalmy, 1985).

2. Research Studies on *The Geometer Sketchpad* (Jackiw, 1994).

These studies included four essential components applied to the instructional model of the current study:

a. The structure of the learning environment reflected constructivist perspectives on learning and instruction in geometry.

b. The method of learning geometry was through an inductive reasoning approach to Euclidean geometry.

c. The implementation of computer-assisted instruction was through a guided-inquiry approach to discover geometric properties.

d. The achievement of geometric knowledge was measured according to the van Hiele stages for developing geometric concepts.

Results reported from these studies supported the implementation of the research design of the current study.

Section One

Application of Cognitive Science Discoveries to Learning Theory

Within the last twenty-five years cognitive scientists have discovered new theories on how knowledge is constructed and processed in the human brain. In the comprehensive text on *Cognitive Processes in Education* Farnham-Diggory (1992) suggested "Not until the 1970's did we begin constructing the types of psychological theories that were adequate for the study of educational processes" (p. 16).

From the early 1930s to 1970 behaviorist theory dominated instructional practice in schools. Principles of behaviorism "...reflect [ed] an emphasis on research that examines [ed] how instructional variables such as reinforcement, feedback, practice, and measurable objectives directly contribute[ed] to student achievement" (Clark, 1984, p. 2).

Behaviorism as a foundational learning theory has been challenged by information processing theories on the construction and representation of knowledge in the human mind. "Within four decades, it [cognitive science] has transformed our view of human minds and has provided a new foundation for education" (Farnham-Diggory, 1992, p. xi).

A distinguishing difference between a behaviorist and a cognitivist approach to education is their point of view on how information is represented in the human mind. Behaviorists "...insisted that observable behavior was the only legitimate object of scientific study...."(Putnam, Lampert, and Peterson, 1990, p. 65). Cognitive scientists claimed it was possible to study mental representations and to trace information processing strategies of the mind. Mental representations referred to were "...described as the entire working memory program for the [learning] task--the goals that were established, the cues that were noted, the knowledge that was retrieved, the actions that were emitted, and the feedback that was processed" (Farnham-Diggory, 1992, p. 73).

Implications from cognitive theory on tracing information processing strategies of the mind led to the image of the computer as a metaphor of the human mind. Putnam et al. (1990) claimed,

> The mind receives information from the environment throughout the senses and processes and transforms that information. This function is similar to that performed by computers, which also process information through complex structures. The power of the computer metaphor for human thought is its leading to precise hypotheses about how information is represented and processed in the mind. (p. 68)

The computer-mind metaphor was applied to the design of cognitive-based software for learning environments. DeStefano and Gordon

(1986) pointed out "... the cognitive approach to education assumes that if we can specify in enough detail the processes underlying thinking skills, we can find methods to teach students to master these skills" (p. 174). The idea of possibly matching information processing strategies to instructional design of intelligent software tools led to transformational changes in computer programs.

Applications of Cognitive Theory to Software Design

The computer-mind metaphor was applied to designing computer programs to mimic cognitive processing strategies of the mind. A new branch of cognitive science, *artificial intelligence,* developed. Intelligent software programs attempting to simulate human processing strategies were designed. This type of software was identified as Intelligent Tutoring Systems (ITS) or was sometimes referred to as Intelligent Computer-Assisted Instruction (ICAI).

Intelligent computer-assisted instruction (ICAI) is a computer program modeled on an intelligent and responsive "human" tutor. The program has three components: an expert module, a student module , and an instructional module. The expert module demonstrates to the user how its own reasoning processes work. It judges student responses, generates multiple answers, measures and keeps track of answers. It provides multiple paths for achieving instructional goals. It suggests various non-linear subgoals to achieve efficient problem solving techniques. The student module analyzes the student's knowledge and tracks inconsistencies in responses. It provides cues to direct reasoning along efficient problem solving paths. The instructional module stores information on instructional strategies. This information is retrieved to adjust instructional strategies to the appropriate user levels. The advantage of ICAI design is the capacity to interact with the user in ways analogous to the structures and processes of the mind's cognitive strategies.

One contribution artificial intelligence programs made to cognitive science was "...observing how we could analyze the intelligent behavior of a machine has largely liberated us from our inhibitions and misconceptions about analyzing our own intelligence" (Anderson, 1990, p. 9). New theories of intelligence influenced profound changes in instructional design of intelligent software tools for education.

The goal of intelligent tutoring systems was to create learning environments where "...it becomes possible to transform a student's conceptual flounderings and misconceptions into profound and efficient learning experiences--ones rooted in his own actions and hypotheses" (Sleeman and Brown, 1982, p. 2). This basic notion underlies the design of *The Geometer Sketchpad* software used for instruction for the current study.

One purpose of the study was to use an intelligent software tool for instruction to investigate how computer-generated dynamic representations of geometric figures on the screen affected the mind's information processing of visual images. Although the mind's processing strategies could not be examined empirically, the product of the processed information was evaluated. For example, a geometric construction or a problem solution would be products or outcomes of the mind's information processing strategies that could be examined.

The current study investigated whether or not the dynamic transformative capabilities of the tool stimulated changes in information processing strategies to affect more efficient avenues for achieving problem solutions. It further sought to find whether or not the visualization capabilities augmented a deeper understanding of geometric concepts.

The convergence of instructional technology with cognitive processing strategies suggests a way to optimize cognitive capacities. As White and Collins (1983) suggested, "The most effective way to enhance the quality of the product is to understand the process which produces it" (p. 237).

Researchers of cognitive-based instruction have asked the question, "What ...would be the impact on student cognitive learning processes as a result of the use of computer versus some other medium?" (Clark, 1984, p. 3). For the current study the two mediums of instruction compared were traditional compass and ruler tools and cognitive software tools.

Instructional use of intelligent software tools suggested a new paradigm for using the computer as a more powerful tool for facilitating cognitive development by stimulating efficient operations of the mind's cognitive structures. The instructional theory informing this new paradigm reflects a constructivist approach to teaching and learning.

Applications of Constructivist Theory to Instructional Design

Constructivism reflects a cognitivist perspective on how information is processed by the human brain. Constructivist principles on learning and instruction have developed these theories further "...information-processing technologies have spawned the computer metaphor of the mind as an information processor. Constructivism has added that this information processor must be seen not as just shuffling data, but wielding it flexibly during learning-making hypotheses, testing tentative interpretations,...." (Perkins, 1992, p. 51).

The constuctivist viewpoint on learning is best understood when contrasted with the behaviorist viewpoint:

> From the behaviorist perspective, an individual's learning is determined by the responses he or she makes to environmental stimuli; thus learning can be made more efficient by carefully structuring those environmental stimuli so that the learner makes responses that are gradually shaped toward the target behavior. (Putnam, et al., 1990, p. 87)

From the behaviorist perspective the learner is a passive receiver of information. Knowledge is inert and separated from real world experiences.

In contrast, from the perspective of a constructivist, the learner is an active receiver of knowledge. "Rather than passively receiving and recording incoming information, the learner actively interprets and imposes meaning through the lenses of his or her existing knowledge structures, working to make sense of the world" (Putnam, et al., 1990, p. 87). The learner builds his/her own mental representations and interpretations of knowledge to create new knowledge.

Perkins (1992) described a constructivist portrait of a learner in his statement:

> Central to the vision of constructivism is the notion of the organism as "active" - not just responding to stimuli, as in the behaviorist rubric, but engaging, grappling, and seeking to make sense of things....They [learners] make tentative interpretations of experience and go on to elaborate and test those interpretations. (p. 49)

Implications can be drawn from this portrait to transact transformational changes on instruction and learning. Bednar, Cunningham, Duffy, and Perry, (1992) reflected, "...the implications of constructivism for instructional design are revolutionary rather than evolutionary" (p. 30).

Seymour Papert (1993) labeled his version of constructivism as constructionism. His viewpoint on teaching was captured in the following statement:

> ...[that] every act of teaching deprives the child of an opportunity for discovery is not a categorical imperative against teaching, but a paradoxically expressed reminder to keep it in check. The constructionist attitude to teaching is not at all dismissive because it is minimalist--the goal is to teach in such a way as to produce the most learning for the least teaching. (p. 139)

He commented "...constructionism, my personal reconstruction of constructivism, has as its main feature the fact that it looks more closely than other educational -isms at the idea of mental construction" (Papert, 1993, pp. 142-143). According to Papert (1993), learning is viewed as a constructive mental process in which the learner builds an internal representation on knowledge based upon an individual's personal interpretation of a given experience.

"Creating a teaching practice guided by constructivist principles requires a qualitative transformation of virtually every aspect of mathematics teaching" (Schifter, 1996, p. 497). The methodology for teaching high school geometry in the 1990s has changed dramatically as a result of the constructivist theory of instruction.

Instruction has shifted from the traditional deductive approach to an inductive inquiry approach. Deductive pedagogy was teacher-centered learning through memorization of definitions, postulates, and theorems. Inductive pedagogy is student-centered learning through exploration of relationships, properties, and conjectures.

New teacher-student roles are built on the constructivist viewpoint on instruction and learning. A constructivist defines learning as an active process of constructing knowledge from "...sharing of multiple perspectives and the simultaneous changing of our internal representations in response to those perspectives" (Bednar et al., 1992, p. 21).

Constructivist instructional theory has also been adapted to intelligent design of software programs. Intelligent software designed as cognitive tools of instruction provide the technology to implement this type of learning in computer-based environments.

Cognitive software provides powerful tools for paving efficient pathways to guide the learner to problem solutions. Menu driven commands provide tools to produce dynamic geometric constructions on-screen that can be replicated, measured, and recreated by a click of a mouse. "Given a supportive context, this new way of teaching and learning places teachers and students on the same side and gives them a rich and powerful set of tools with which to become codiscoverers of knowledge" (Wilson, 1993, p. 22).

Constructivist strategies for instruction are applied by teachers in the research studies on both *The Sketchpad* and the *Supposers* reviewed in section two of this chapter. Yerushalmy's (1986) guided-inquiry approach to teaching and learning reflects a constructivist point of view of knowledge representation. She commented "...the major factors in changing geometry learning are the teachers, and their belief in the students' need to learn by being active and free to create" [their own learning] (p. 61).

A constructivist view of knowledge construction presents some difficulties for teachers engaged in a guided-inquiry approach. Gordon (1993) points out the following challenges this approach presents to teachers. Teachers must shift their perspective from deductive to an inductive mode of thinking. This new methodology requires new techniques for evaluating and assessing what has been learned.

Some serious problems arising from a constructivist approach are questions on just how much structure should be imposed versus how much freedom should be allowed. Another concern is "...a constructivist approach is the enemy of coverage" (Gordon, p. 237, 1993). Learning through a discovery approach takes much more time than memorization of ready-made solutions.

Decisions require good judgment on management of time and productivity within a period of time in a guided-inquiry learning environment. To take advantage of inquiry methodology and technology tools is not an easy task. Restructuring classroom learning and teaching within a technology environment requires preparation time for rethinking and redesigning mathematics curriculum and assessment. "Like technology and teachers, neither geometry by itself nor even mathematics by itself is up to the task [of reforming mathematics education] ....The *Supposer* is not a clear model for how to bring about such change in education across the board, but the experience does shed light on the challenge and the opportunity" (Gordon, 1993, p. 240).

Prawat (1992) commented on the practical challenges the constructivist theory poses for teachers, "Being provided with a new set of theoretical or conceptual 'lenses' can be empowering for teachers, but it also complicates their lives....most agree that it [constructivist theory] involves a dramatic change in the *focus* of teaching, putting the students' own efforts to understand at the center of the educational enterprise" (p. 357).

Within the constructivist perspective there exists a wide spectrum of interpretations on learning and instruction. For the purpose of this review essential notions of constructivism applied to the design of the current study are relevant. The constructivist perspective is particularly well suited for transforming a computer-assisted mathematical classroom into a learning environment congruent with the vision of mathematical reforms defined by the National Council of Teachers of Mathematics (NCTM) in the *Professional Standards for Teaching Mathematics* (1991).

Section Two

The preliminary sources consulted manually for section two of this review were: Psychological Abstracts Index, ERIC (educational resources information center) Index, CIJE (current index of journals in education), and the Handbook of Research on Teaching. Computer-assisted searches were conducted through the CDROM Indexes at the University of San Francisco. These searches accessed sources from the following databases: ERIC (Educational Resources Information Center), Science and Technology Indexes, Psychological Abstracts, and Dissertation Abstracts International.

The electronic search of Dissertation Abstracts International produced six dissertations on the *Geometer Sketchpad*. Four of the six dissertations reviewed included the following elements relevant to the present study: (a) using *Sketchpad* for instruction in geometry, (b) exploring skill-acquisition for conjecturing, (c) measuring achievement of geometric concepts by the van Hiele scale, and (d) comparing cognitive effects of software capabilities.

One dissertation not reviewed was an investigation of secondary mathematics preservice teachers' preference for teaching strategies using *Sketchpad*. The results of the study focused on Myers-Briggs personality types of teachers rather than on *Sketchpad* as a teaching tool. These results did not relate to the purpose of the current study. A second dissertation not reviewed was on the van Hiele levels as a measure of achievement in geometry. The dissertation was written in 1982. Journal articles on the van Hiele levels written later than 1982 were included in this review.

An electronic search of Info Trac 2000 via the Internet searched the Expanded Academic ASAP database. Eight journal articles on *Sketchpad* were found. One article was a review of the software program and was not included in this review. Three of the articles are cited in this review of literature. One article not reviewed was an evaluation of using *Sketchpad* to examine circles in Poincare plane geometry. This geometry is a version of Bolyai-Lobachevsky plane geometry. The article was not applicable to the current study on Euclidean geometry.

There are numerous high school geometry classes using the *Sketchpad* software, but articles on these projects have not been published. Only a few experimental studies on the effectiveness of Sketchpad in classrooms have been published at this time.

Ken Koedinger at Carnegie Mellon University has developed a prototype of an intelligent tutor for *Sketchpad.* Experimental research on this software has not yet been published.

One dissertation on *The Geometer Supposers* was found through the electronic search of Dissertation Abstracts International. Three technical reports on the Geometric Supposers were found through an electronic search of ERIC databases. All four studies are reviewed in this section of the chapter under the topic, "Experimental Redearch Studies on *The Geometric Supposers*". Thirteen journal articles were found on *Geometer Supposers* by using the electronic search of CIJE (current index of journals in education). Eight articles are included in this review. Five of the articles not reviewed were generated from dissertations included in the review. An additional article found was an evaluation of the latest version of *The Geometric superSupposer,* an improved version of the original *Supposers.*

Research studies on the effectiveness of intelligent software tools for implementing inductive learning through guided inquiry are reviewed in this section of the chapter. Findings on two software programs *The Geometer Sketchpad* (Jackiw, 1994) and *The Geometer Supposers* (Schwartz, 1985 -1988) are presented. Research available on *The*

Geometer Sketchpad is limited due to the fact that it was first copyrighted in 1991. There are more studies on *The Geometer Supposers* since they have been used in classrooms since 1985.

The studies reviewed include data relevant to the current study on the effectiveness of: (a) an inductive methodology for teaching geometry, (b) the van Hiele model for measuring conjecturing ability, and (c) the dynamic capabilities of *Sketchpad* contrasted to the static capabilities of *The Supposers.*

Experimental Research Studies on The *Geometric Supposers*

The *Geometric Supposers* developed by Judah Schwartz and Michal Yerushalmy (1985-1988) have been utilized as software tools for over a decade. The *Supposers* are a series of four software tools. The *Presupposer* contains problems on points and lines. Problems on quadrilaterals, triangles and circles are contained in the other three programs for high school geometry. The design of pedagogy for implementing the software tools is an inductive discovery approach. Construction of geometric knowledge is facilitated by a methodology of guided-inquiry.

Studies by Yerushalmy, 1993; Yerushalmy 1990; Shepard and Wiske, 1989; Yerushalmy, Wiske and Houde, 1988; Yerushalmy, Chazan, and Gordon 1987; and Yerushalmy 1986 have investigated implementation of a guided-inquiry inductive approach for instruction in geometry. Over a dozen research studies on the use of the *Supposer* addressed the following issues: "... student learning, teacher attitudes and behaviors, school contexts, and implementation" (Gordon, 1993, p. 229).

Supposer Research Studies on Learners

Yerushalmy (1986) conducted a pilot research study during the school year 1984-1985 on student learning using the *Supposers.* Fifty subjects participated in the study. Results showed the *Supposer* facilitated student formulation of conjectures and ability to write generalizations. This pilot study was followed-up by a research study the following year.

Yerushalmy conducted a year-long study of teachers and students during the school year 1985-1986. Two groups of subjects in 10th grade geometry classes participated in the study. One group learned by traditional deductive pedagogy while the other group learned by an inductive pedagogy. Data on inductive learning generated from the study revealed "Rich numerical and visual data tend to motivate a certain level of generalization....Test problems and work with the *Supposer* showed that the same collection of data brings different students to different levels of generalization...." (p. 134). In contrast, Yerushalmy found traditional methodology limited the progression to higher levels of generalization.

Conclusions on factors influencing student's conjecturing processes are captured in the following statement:

> The inductive work with the Supposer offered many options for students to be involved in and to understand geometry. It also exposed us to the variety of methods that could be used in promoting the understanding of geometry, and heightened our awareness of the diversity of methods of representation that affect students and motivate better ideas. (Yerushalmy, 1986, p. 190)

Further conclusions on the effect of an inductive approach to learning geometry confirmed its use as a valid alternative to deductive methodology. "The appreciation of the use of data and information that students had developed while working inductively throughout the year prompted them to look for dynamic visual information as their first step in the analysis of the problems on the test" (p. 195).

Later studies on the *Supposers* by Yerushalmy Chazan, Gordon, & Houde (1987, 88, 89, and 91) found further evidence indicating "...students using the Supposer [with an inductive approach] understand diagrams and their limitations better than students in traditional classrooms portrayed in the research literature" (Yerushalmy and Chazan, 1993, p. 53).

Yerushalmy, Chazan, and Gordon (1987) conducted a year long study for the purpose of assessing the guided-inquiry methodology while using the *Supposers*. Subjects participating in the study were from three Boston area schools. The pedagogy of instruction for the experimental groups was a guided-inquiry inductive approach using *The Supposers.* The pedagogy of instruction for the comparison groups was the traditional deductive approach using a textbook. Yerushalmy et al. (1987) described the difference between these two approaches:

> In traditional geometry instruction students operate on an abstract level only: they are taught axioms and theorems in order to use them to prove other results using deductive reasoning. Using the *Supposer* brings an empirical dimension to the geometry experience in which students can construct, manipulate, and measure particular geometric objects. (p. 52)

Midway through the study *Supposer* students "..were no longer bound by diagrams; they were now able to visualize and manipulate relationships in their heads" (Yerushalmy, Chazan, and Gordon, 1987, p.

15). Results of a chi-square analysis of solutions to posttest questions demonstrated two statistically significant performance differences between the *Supposer* and comparison groups:

1. *Supposer* students produced higher level generalizations on two out of the three posttest questions.

2. *Supposer* students produced more arguments on the posttest abstract question.

Further research on an inductive reasoning approach to teaching geometry using the *Supposers* was conducted by McCoy (1991). She conducted a study on the effect of tool software on high school achievement in geometry. Subjects were both male and female from two intact classes of college-bound geometry students. Each class contained 29 tenth graders. Both classes used the same textbook, but only one class used the *Geometric Supposer*. The study took place over a period of one year. Once every two weeks *Supposer* problems were solved by the experimental group. The pedagogy for the control class used traditional geometry tools with pencil and paper to solve problems.

Results of the Analysis of Covariance for total geometry achievement score, controlling for initial mathematical ability scores, showed the experimental group scored significantly higher on the total posttest

($F(1,57) = 34.24$, $p < .01$). Findings showed the *Supposer* class achieved significantly higher scores on Higher Level problems ($F(1,57) = 33.64$, $p < .01$) and on Application problems ($F(1,57) = 22.35$, $p < .01$).

Higher achievement scores on geometry problems requiring higher level thinking skills was an important finding of this study. The method of inductive learning had a positive effect on developing higher-order skills for analyzing and organizing data.

Supposer Research Studies on Teachers

Wiske and Houde (1988) studied the effect on teachers' use of the *Geometer Supposer* as a tool for implementing a guided-inquiry methodology. Five geometry teachers from three different high schools in Massachusetts participated in the study for a period of two years. Teachers conducted some classes in the computer lab using the *Geometric Supposers* . Other class sessions were held in regular classrooms where discussions of problems were conducted along with presentations by the teachers on topics from the textbook.

To integrate technology and a guided-inquiry approach into one's methodology of teaching was a challenging task. It required a shift from a teacher-dominated lecture approach to student-centered guided-inquiry approach. The teacher's role was to (a) lead students to become responsible for their own learning, (b) to facilitate discovery through guided questioning, and (c) to motivate on-task collaboration through problem-solving activities.

This teaching methodology requires skill in creating discussion, generating ideas, and managing multiple explorations simultaneously. This methodology also requires restructuring the classroom, lesson plans, and assessment tools. Wiske and Houde (1988) found "The construction paradigm and the process of guided-inquiry pose major intellectual, emotional, and moral challenges as well as technological and practical ones for teachers in classrooms" (p. 22).

"In shifting from one paradigm toward the other, teachers do not suddenly and totally transform their knowledge, behaviors, and beliefs" (Wiske and Houde, 1988, p. 14). The results of Wiske and Houde's (1988) study demonstrated the importance of professional teacher training programs to prepare teachers with skills required to implement this new constructivist paradigm.

Magdalene Lampert (1993) observed these same teachers who participated in Wiske and Houde's (1988) study. She conducted a substudy on the *Supposer* as a tool for changing methodology of teachers. The teachers who used the *Supposer* experienced "...an interactive process of empowerment: students taking charge and teachers trusting them to do so, because they recognized capacities they did not know were there" (Lampert, 1993, p. 160). The teachers recognized the technology tool empowered the students to "do mathematics". This was motivating and satisfying for both students and teachers.

To implement an innovative technology tool in the classroom environment requires that many issues be addressed. As Gordon (1993) commented on the results of over one dozen research studies done on the *Supposer* software "The difficulties derive from the changed and expanded demands on teachers, the dilemmas that confront teachers, and the deep shifts in thinking about themselves and their subject that face teachers who attempt to implement this new approach to mathematics education" (p. 235).

Cumulatively *Supposer* studies demonstrated positive effects on the development of inductive reasoning skills as a result of using the *Supposer* within the classroom environment where a constructivist approach to teaching and learning prevailed. Learners became geometers constructing new knowledge through exploration of figures and discovering new dimensions beyond width, length, and height. Transformations yielded discoveries in depth of understanding concepts visible only through the power of the technology tool. Learning mathematics was viewed as a way of creating geometric knowledge for conceptual understanding versus traditional mechanical recitation of definitions, theorems, and postulates without conceptual understanding or meaningfulness.

Experimental Research Studies on *The Geometer Sketchpad*

The *Geometric Supposers* were designed as tools for constructing and understanding geometric knowledge. *The Geometer Sketchpad* achieves the same purpose with the added feature of producing dynamic changes of relationships on the computer screen. *The Supposer* repeats constructions to observe patterns to test conjectures, while the *Sketchpad* changes relationships dynamically on the screen. This feature provides more powerful visualization of changing and unchanging relationships supplying concrete data for testing conjectures.

Foletta (1994) conducted a case study on four subjects to investigate the nature of student thinking while using *The Geometer Sketchpad* (1991 version of the software program). Subjects selected by the teacher to participate in the study were two male subjects of high and low ability level and two female subjects of average ability.

The Geometer Sketchpad was used in conjunction with the textbook *Discovering Geometry* (Serra, 1993). Problems were ".... often solved with paper and pencil first and then results were transferred to *Sketchpad* [italics added] medium for reporting purposes" (Foletta, 1994, p. 124). The subjects used Sketchpad as a tool to construct sketches and to verify conjectures. It was also a means for communicating about investigation of problems. She observed the subjects frequently "...engaged in discourse by thinking aloud, explaining or justifying possible solutions, asking for clarification, or resolving conflicts" (p. 125).

The present study also used the text *Discovering Geometry* in conjunction with *Sketchpad*, but not in the same way as Foletta described in her study. The latter subjects did not use *Sketchpad* as a tool to discover mathematical ideas through the transformation of sketches. This may be due to instructional design, since subjects relied on teacher-direction which did not go beyond following specified steps of investigation. If subjects were encouraged to explore transformations of constructions on the computer screen, this factor may have influenced the results of the study.

Foletta (1994) found "...students do not appear to automatically make connections between mathematical concepts and the tool capabilities based on these concepts" (p. 169). Subjects used *Sketchpad* as a production tool rather than a thinking tool for developing understanding of geometric concepts. They had difficulty, for example measuring the area of a rectangle using *Sketchpad.*, "It seemed that the connection between algorithmically computing the area of a figure and the concepts of its polygonal region was lacking" (Foletta, p. 169). According to Foletta (1994) the inability to make connections from concept to construction and vice versa might point to better preparation of students with the mathematical knowledge underlying the concepts demonstrated on the screen.

It is the opinion of the investigator of the current research study that students do not learn from simply observing sketches showing visualization of changing phenomenon on geometric objects. Understanding of geometric concepts is deepened when connections are made between concrete representations on the screen and abstract ideas in the mind. The purpose of guided inquiry is to supply students with the knowledge they need to make discoveries on their own. The methodology is not intended for students to explore without knowledge and purpose to guide their explorations. The power of the technology tool is to activate cognitive skills and to provide cues to assist students to make connections between mathematical concepts and problem solutions. The role of the teacher is to direct student explorations by supplying knowledge and suggestions through guided inquiry.

Another study investigating the effectiveness of *Sketchpad* as a tool for instruction was conducted by Frerking (1994). She conducted a 24-week study on male and female high school geometry students. An inductive methodology was used for the two treatment groups of 24 subjects. Both treatment groups used either the *Geometric Supposer, Geometer's Sketchpad*, or traditional tools of compass and straightedge.

In contrast to the treatment groups, a deductive methodology was used for the control group of 24 subjects. They used *Geometer's Sketchpad* along with the compass and straightedge for drawing figures. The purpose for using the computer was to offset some of the Hawthorne Effect caused by the fact that the investigator taught all three groups.

The purpose of the study was to investigate "...the effects of the students' use of conjecturing on van Hiele levels and abilities to justify statements or write proofs" (p. 16). Findings on the use of the van Hiele levels in relationship to conjecturing lend support for their use in the current study where they were used to measure achievement of geometric concepts indicated by written conjectures.

Frerking (1994) found no significant differences among the three groups tested on the van Hiele levels between those who were taught conjecturing by an inductive method versus those taught by a deductive method. The mean scores of subjects in the two treatment groups were slightly higher than the mean score of the control group. Therefore, the inductive method might be said to be more effective than the deductive method for these subjects. Since the two treatment groups used both the *Sketchpad* and the *Supposer* versus the control group using the *Sketchpad,* conclusions cannot be separated as attributable to computer use or to be an effect of one or the other software programs.

No significant differences were found on the measures of achievement on proof writing between those subjects using either an inductive or a deductive methodology. The ANOVA results were ($\underline{F}$ (2, 69) = .235, p = .791). Frerking (1994) attributed failure to find statistical significance on this measure was due to the fact that more than half of the subjects did not attempt and/or complete the problems on proofs. One reason students may not have completed the proofs was a time factor or they may have been satisfied with the "...visual arguments they find on the computer screen are enough proof for them" (Frerking, 1994, p. 99). Frerking (1994) suggests this finding from her study may contribute to developing a pedagogy for enabling students to understand the value and need for proofs in geometry.

A fourth hypothesis of the study compared effects of inductive and deductive approaches to conjecturing on achievement of geometry objectives. The results of the analysis of variance on the posttest (F (2,69) = 0.062, p = .940) showed no significant difference. However, the standard deviation of 7.77 for the *Geometer Sketchpad* group was a much smaller deviation than the standard deviation of 13.48 for the *Supposer* group and 11.22 for the control group on the posttest measuring proof-writing abilities and achievement of geometry objectives.

For the *Sketchpad* group on the GEMS (Gwinnett Educational Management System Mathematics Test) posttest a mean of 73.75 with a standard deviation of 7.77 indicates that within one SD, 68.26% of the scores ranged between 65.98 and 81.52. For the *Supposer* group a mean of 73.21 with a standard deviation of 13.48 indicates that within one standard deviation, 68.26% of the scores ranged between 59.73 and 86.69. For the control group a mean of 74.33 with a standard deviation of

11.22, that within one standard deviation 68.26% of the scores ranged between 63.11 and 85.55. Therefore the *Geometer Sketchpad* groups' scores were higher and not as widespread from the mean as the Supposer and the control group.

The fifth hypothesis tested the relationships between van Hiele levels and subject's ability to make conjectures, write proofs, and achieve curriculum objectives. No relationship between student's van Hiele levels and their ability to make conjectures, write proofs for conjectures or achievement on curriculum objectives was found.

Yet significant correlation coefficients at level $p < .01$ were found between van Hiele level scores and ability to write proofs, between students' proof-writing abilities and achievement of geometry objectives, and between proof-writing achievement and ability to write conjectures or justifications. Coefficients were 0.35, 0.51, and 0.54 respectively.

Further research is needed to support the statement made by Frerking (1994), "The use of dynamic geometry software is beneficial in the area of conjecturing since it provides the student an easier, faster, and often more accurate method of exploring ideas for conjecturing" (p. 102). Since the first experimental group was taught by an inductive approach using both *Sketchpad* and Supposer, and the second treatment group was taught by an inductive approach using only *Sketchpad*, and the third control group was taught by a deductive approach using only *Sketchpad,* conclusions on the effectiveness of *Sketchpad* alone cannot be compared between any two of the groups. The study might have been improved by having one treatment group use only the *Supposer* so conclusions about the effect of either dynamic or static software on conjecture-making ability might have been more clearly delineated.

Elchuck (1992) classified the *Geometric Supposer* as a static tool in the following statement, "...[it] is to be interpreted as software that allowed students to create original geometric figures (such as triangles, quadrilaterals, etc.) but did not allow the physical manipulation of such figures" (p. 6).

The effectiveness of *Sketchpad 's* capabilities as a dynamic tool versus a static tool was investigated by Elchuck (1992). For example, the *drag* tool of *Sketchpad* allows users to select any part of a geometric sketch on the screen and dynamically change its shape, dimension, and measure. The original relationships among geometric objects, for example points and lines, are preserved when one component is *dragged.* Dragging provides dynamic visualization of relationships remaining unchanged or changed. Comparison of patterns observed validates application of a conjecture to a particular problem.

Previous studies on *The Geometer's Sketchpad* did not include the independent variables Elchuck's study considered as important factors affecting *conjecture-making* abilities. These were: (a) math achievement scores, (b) locus of control scores, (c) independent time on-task, (d) van Hiele levels, and (e) spatial visualization. The dependent variable was the subject's score on a conjecture-making test. Conjecture-making ability was treated as a continuous variable.

One hundred fifty-seven subjects were randomly assigned to one of two treatment groups either to a static version or a dynamic version of *Sketchpad.* Subjects were from six grade nine academic math classes from two schools in Nova Scotia. Descriptive statistics were reported on 150 subjects who completed all instruments of the study. The length of treatment was for 20 class periods with an additional two class periods for testing.

Similar to the present study Elchuck (1992) examined the effects of dynamic capabilities of tool software in contrast to static tool software. The length of treatment was identical to the length of the current study. This is of particular interest for purposes of comparison of results achieved within the 20 class periods.

Results indicated no statistically significant differences on relationships between the dynamic capabilities of the software tool and conjecture-making ability. Also, no statistically significant differences were found on relationships between spatial visualization skills, locus of control, or the van Hiele levels and *conjecture-making* ability. Elchuck found mathematics achievement and time of independent investigation to be statistically significant factors of conjecture-making ability.

Multiple regression analysis tests were used by Elchuck (1992) to examine data collected from the study. A post hoc regression analysis demonstrated that the type of software was a predictor of conjecture-making ability, when the subjects' schools were included in the analysis as a concomitant variable.

Elchuck's findings achieved data supporting the assumptions of the current study. An assumption of the current study was that the dynamic capability of *Sketchpad* will positively influence conjecture-making ability as well as achievement of geometric knowledge and construction skills. The current study also assumed spatial visualization skills affect achievement of geometry construction and conjecturing skills.

Elchuck (1992) found the dynamic capability of the software was a significant factor for improving *conjecture-making* ability as a result of a post hoc analysis using subjects' schools in the regression analysis as an effect-coded variable, the school variable was statistically significant as well as the software type (dynamic versus static). These results indicated a relationship between the school the subject was attending and the use of the dynamic software. Those subjects attained a higher score conjecturing than those in the static group. The adjusted value of R^2 changed from 26.5% to 48.5%. This finding implies further research might find significant relationships between dynamic software and conjecturing.

Elchuck (1992) also found subjects with high spatial visualization skills did not score high on conjecturing skills. Upon examination of the spatial visualization test he found "... the test did not differentiate between subjects well" (p. 125). The mean score of 62.41 on a 80 point test measured by the nonverbal battery of the Canadian Cognitive Abilities Test (CCAT) indicated a possible "ceiling effect" may have occurred. This finding implies further research needs to be conducted exploring the relationship between spatial visualization skills and conjecturing skills.

Elchuck (1992) suggested for future research to carefully consider "...(i) improved criteria for subject selection and (ii) selection of appropriate measurement tools may uncover other factors, including the type of software environment, that may statistically influence this [conjecture-making] important inductive reasoning ability" (p. 143).

Key conclusions of Elchuck's (1992) study pointed to the need for further research in the following two areas: (a) to explore further the effects on conjecturing of dynamic versus static software tools with different populations, and (b) to examine further the relationship between the van Hiele levels and conjecturing skills.

Summary

A constructivist theory of instruction suggests a new paradigm for learning. Cognitive software tools are designed to create efficient avenues for solving mathematical problems. These tools implemented within a constructivist perspective create environments where learning is optimized. The current study modeled on this paradigm may further classroom use of the computer as a powerful tool to facilitate cognitive development.

Literature from cognitive science, cognitive theory and constructivist theory was reviewed in section one. This literature represented three levels of research impacting mathematical reforms of instruction and learning in the 1990s. These areas of literature presented an instructional design compatible with cognitivist and constructivist perspectives. The vision for instructional practice linked to cognitive theory, and constructivist perspectives linked to cognitive software tools connect together as links in a chain to produce a synergistic change for extending the mind's cognitive capacities.

Research studies on the effectiveness of cognitive software tools in geometry for implementing an inductive guided-inquiry methodology using either *The Supposers* and/or The *Sketchpad* were reviewed in section two. The research studies on *The Supposer* software tool demonstrated research results on: (a) inductive approach versus a deductive approach, (b) *Supposer* use versus textbook use, (c) teacher attitudes and behaviors implementing an inductive approach, and (d) student achievement of geometry conjecturing skills.

Research studies on the *Geometer Sketchpad* software tool demonstrated research results on: (a) effect of making mathematical connections between geometric concepts and constructions, (b) effect of inductive versus deductive method on conjecturing-making abilities, (c) effect of van Hiele measurement levels on ability to write conjectures and proofs, and (c) effect of dynamic versus static use of *Sketchpad.*

The current study added to this research literature on the effectiveness of cognitive tools on instruction in geometry. The current study investigated the effect of a cognitive software tool on achievement of geometric knowledge, construction, and conjecture. This study explored further the effects on conjecturing abilities of dynamic versus static software tools with a female sample of the population .

CHAPTER III

Methodology

Purpose of the Study

A quasi-experimental research design was used in conducting an investigation to determine whether there was a difference in achievement of geometric knowledge between two groups of female high school geometry students engaged in two levels of instruction in geometry. This study investigated the effects of using a technology tool versus using a textbook with classic geometry tools: compass, protractor, ruler, and straightedge for instruction of high school geometry.

The Geometer's Sketchpad (Jackiw, 1994) was the technology tool used for instruction. *Discovering Geometry : An Inductive Approach* (Serra, 1993) was the text used for classroom instruction. This study hypothesized if The *Geometer's Sketchpad* was used as a pedagogical tool sharing an intellectual partnership between the subject and the computer for improving solo geometry skills, then geometry skills would improve. Also, when used as a performance tool for upgrading intellectual performance subjects would achieve a higher mean score on a test measuring geometric knowledge and geometric construction than those subjects using a textbook along with classic tools for instruction.

This study further hypothesized that subjects in the experimental group using *The Geometer's Sketchpad* (Jackiw, 1994) would achieve a higher mean score on the level of concept development indicated by the quality of written conjectures on properties of geometric sketches than those subjects in the control group using the textbook, *Discovering Geometry : An Inductive Approach* (Serra, 1993) and using classic geometry tools for problem investigations.

Description of Design

The research model for the study was a posttest-only control-group quasi-experimental design for investigating two levels of instruction. Subjects in the experimental group received technology-based instruction, while subjects in the control group received instruction using a textbook and classic geometry tools. Third-quarter geometry grades of subjects participating in the study were used as the covariate to account for individual differences existing up until the time of the treatment (see Appendix H).

A non-randomized selection of subjects from two intact high school geometry classes participated in this study. Subjects in the experimental treatment group belonged to one intact geometry class. Class meetings were held in one self-contained classroom. The classroom was equipped with one Macintosh computer for each pair of subjects. These subjects used *The Geometer's Sketchpad* for investigations of geometric sketches to solve problems. Class meetings were held in the same self-contained classroom for subjects in the control group at a different class period during the school day. These subjects used the text *Discovering Geometry: An Inductive Approach* (Serra, 1993) and classical geometry tools for investigation of geometric sketches to solve problems.

The study was conducted from April 24, 1995 to May 19, 1995. The total length of the study extended over a period of 20 class days. Since each geometry class met four out of five days each week, the total number of class meetings for each group was 16. Thirteen class days were devoted to lessons on the Properties of Circles, two days were devoted to completion of the posttest, and one day was devoted to taping interviews with subjects from the experimental group.

Subjects in both groups were asked not to discuss class lessons and investigations with one another during the length of the study (see Appendix K for sample written instructions to subjects). The same lesson procedures were followed for subjects in both groups (see Appendix L for sample lesson plans). The same instructor taught geometry to subjects in both the experimental and control groups. An inductive approach was the instructional methodology for teaching geometry to all subjects participating in the study. Subjects in both the experimental and control groups worked with a partner and collaborated with each other while investigating problem solutions.

The text *Discovering Geometry: An Inductive Approach* (Serra, 1993) and *The Geometer's Sketchpad* (Jackiw, 1994) were used by the instructor for geometry class. Both the text and the software were designed to be used either independently of each other or to complement each other. During the first semester of the school year in which the study was conducted, the instructor had used *Sketchpad* only a few times with both geometry classes. During the third-quarter of the school year, the students used Sketchpad for some geometry investigations. Students were taught how to use *Sketchpad* and knew how to use the software before the study was conducted during the fourth quarter of the school year.

The contrasting difference between the two levels of instruction for the experimental group and the control group was the use of *The Geometer's Sketchpad* (Jackiw, 1994) as a dynamic tool for visualizing and manipulating geometric sketches on the computer screen, versus the use of classic geometry tools for static representation of geometric sketches on paper. The treatment of subjects in the experimental group consisted in their using the *Geometer's Sketchpad* (Jackiw, 1993) to explore investigations of problem-solving activities on the computer. In contrast to the treatment of the experimental group, the control group completed investigations of problem-solving activities using classic geometry tools: compass, protractor, pencil, and straightedge.

After completing the unit lessons on The Circle, all subjects were given a 5-part posttest (see Appendix A for a copy of the posttest). The posttest measured achievement on three dependent variables: geometric knowledge, geometric construction, and geometric conjectures. Points were given for each correct answer on the geometric knowledge section of the posttest Parts 1 - 4. Points were given for each correct construction on the geometric construction section of the posttest Part 5. Points were given for the concept level indicated by written conjectures on the geometric conjecture section of the posttest Part 1. The posttest on geometric conjectures was scored on a 4-point rating scale described in the Instrumentation section of this chapter.

Dependent Variables

In this study there are three dependent variables: geometric knowledge, geometric constructions, and geometric conjectures. All three are measured by scores on the posttest. A score on the posttest on geometric knowledge was a measure representing points scored on problem solutions in Parts 1 - 4. A score on the posttest on geometric constructions was a measure representing points scored on construction problem solutions in Part 5. A score on the posttest on geometric conjectures was a measure representing points scored on a 4-point rating scale on levels of thinking indicated by conjectures written as reasons supporting problem solutions Part 1 of the posttest.

The rating scale was based on the research of van Hiele and van-Hiele-Geldof (Van Hiele, 1986). Their research established a model specifying five levels of thinking for the development of concepts of geometric knowledge. These levels are sometimes combined and specified as only four levels, while at other times five levels are specified. For this study Levels 0 to 3 were applicable based on class lesson presentations.

Van Hiele and van Hiele-Geldof (1986) defined the following characteristics of each level of geometric thought involved in learning geometric concepts.

1. Level 0 was identified as visualization. Visualization "...is defined by the Gestalt-like ability to recognize differences of forms....For example, the student can distinguish and reproduce triangles, angles, and parallel lines, ...on the basis of the figures [perceived] as wholes" (Farnham-Diggory, 1992, p. 405).

2. Level 1 was identified as analysis. Analysis is characterized by a "new perception of geometric forms as being constructed of particular properties...." (Farnham-Diggory, 1992, p. 405). Students perceive geometric forms as created from relationships of parts to properties of the whole.

3. Level 2 was identified as informal deduction. Informal deduction is explained as a level of thinking in which "...students become able to produce informal logical arguments in support of the relations they have observed among properties" (Farnham-Diggory, 1992, p. 405).

4. Level 3 was identified as formal deduction. In applying formal deduction students are mindful "...that arguments themselves can be viewed as objects or entities, and they have particular properties" (Farnham-Diggory, 1992, p. 406).

5. Level 4 was identified as rigorous proof. Students at this level can formulate generalizations to create formal deductive proofs of theorems.

The framework for classroom lessons for all subjects participating in the present study was based on the van Hiele model for developing geometric concepts in high school students. Each lesson presentation built geometric knowledge starting with an activity at Level 0, (visualization).

For example, students might be asked to construct a sketch of a geometric figure. The lesson might then proceed to a Level 1 (analysis) activity. Subjects might be engaged in a process for analyzing geometric sketches. At this level subjects might generate ideas about properties of geometry sketches and might begin to formulate conjectures about relationships of objects observed in the sketches they constructed.

At Level 2 (informal deduction) subjects engaged in discussion activities, which might lead to writing possible conjectures that apply to sketches they have observed. At Level 3 (formal deduction) subjects formulated deductions derived from conjectures. A property might be discovered and stated as a generalization that can be applied to different types of geometric figures.

At Level 4 formal geometric rigorous proof might be applied to theorems. This level was not included in this study. Lessons and investigations did not include the teaching of writing formal deductive proofs of theorems.

Instrumentation

After the treatment, a posttest was administered to subjects in both groups to measure geometric knowledge, geometric constructions, and geometric conjectures. The posttest was a modified version of The Chapter Six Test on Circles from *Discovering Geometry: An Inductive Approach Teacher's Resource Book* (1990, pp. 48, 49 and 50) a test designed to accompany the text (see Appendix A for copy of the Posttest). Additional problems were added to the instrument by members of the validation panel. These problems were added to ensure that all concepts in the lessons on the Circle Properties were included in the posttest.

The posttest instrument measured the three dependent variables: (a) a measure of achievement of geometric knowledge, (b) a measure of geometric constructions, and (c) a measure assigned to written conjectures based on van Hiele levels of geometric concept development. The posttest on Geometric Knowledge consisted of 15 problems to be solved in Parts 1 - 4. The highest score possible was 75. The posttest on Geometric Constructions consisted of 2 construction problems to be solved in Part 5. The highest possible score was 25. Part 5 required the construction of 2 sketches with a written explanation of the steps taken to complete the sketches.

The posttest on Geometric Conjectures consisted of 8 problems in Part 1. For this part of the test, subjects were required to write the conjectures and properties applied to find the correct solution to the problems. The conjecture part of the posttest was scored on a 4-point rating scale indicating levels of concept development according to the van Hiele model (van Hiele 1986).

Posttest Validity

A modified version of the test in geometry on Circle Properties to accompany the text *Discovering Geometry : An Inductive Approach*

(Serra, 1993) was used to measure the dependent variables. A panel consisting of two university mathematics professors and two mathematics consultants established the face validity of the posttest used for the study. Two members of the validity panel were mathematics professors who authored the Mathematics Diagnostic Testing Project for the State of California.

Members of the panel also examined the 4-point scoring scale used for determining levels of geometric concept development indicated by written conjectures (see Appendix C and D for qualifications of panel members and validation form for the posttest).

To establish content validity, the posttest was revised four times. Members of the panel discussed and revised the test changing and adding problems, changing instructions, and directions until the final version was acceptable. These discussions and revisions provided the basis on which test validity was established and the instrument accepted for measuring the three dependent variables of the study.

Posttest Reliability

The posttest used in this study was a well established test written to accompany the textbook, *Discovering Geometry: An Inductive Approach*

(Serra, 1993). The scoring of the Conjecture posttest relied on the van Hiele model established in 1955. The research model of the five levels of thought that can be discerned in the development of geometric concepts according to van Hiele are:

> First level: the visual level
> Second level: the descriptive level
> Third level: the theoretical level; with logical relations, geometry generated according to Euclid
> Fourth level: formal logic; a study of the laws of logic
> Fifth level: the nature of logical laws. (van Hiele, 1986, p. 53)

Although the van Hiele scale is a widely used and respected one, a sample of five posttests from each of the control and experimental groups was given to two members of the validation panel as an interrater reliability measure to examine the scoring of the conjecture posttest to ascertain verification for the researcher on the scores assigned to each answer according to the levels explained above.

The 4-point rating scale used to determine scores on written conjectures to measure the level of geometric concept development were sent to two of the panel members. To each conjecture answer points were assigned as follows:

1. A score of 0 was assigned to an answer where there was no statement of a conjecture.

2. A score of 1 was assigned to a written conjecture containing descriptions of relationship of parts of the sketch to the whole figure.

3. A score of 2 was assigned to a written conjecture explaining properties of geometry that applied to the sketch.

4. A score of 3 was assigned to a written conjecture applying a generalization of a property/conjecture to a geometric sketch producing the correct problem solution.

The overall results of their scoring was congruent with that of the researcher. A detailed example of a problem and sample solutions for each level are described in chapter 5.

Independent Variable

The experimentally manipulated independent variable is the method of investigation of geometric sketches. This study compared two levels of investigation. The first level is cognitive technology-based investigation using the electronic tools of *The Geometer's Sketchpad.* (Jackiw, 1994). The second level of the independent variable, is a textbook approach using *Discovering Geometry: An Inductive Approach* (Serra, 1993), pencil, ruler, protractor, and compass to explore sketches.

Classroom Lesson Sample

For the purpose of understanding the procedure of methodology followed for the two levels of the independent variable an example of Lesson 6.4 is given below. The van Hiele levels on concept development from 0 to 3 form a framework for the lesson plans followed for both levels of instruction.

The geometry topic chosen for the study was the Properties of Circles. Classroom lessons were taken from Chapter 6, sections 6.1 to 6.8, in *Discovering Geometry : An Inductive Approach* (Serra, 1993). Investigations were taken from Chapter 6 on Circles in *Exploring Geometry with The Geometer's Sketchpad* (Bennett, 1993).

Classroom lesson sample on the measure of an inscribed angle and its relationship to the measure of its intercepted arc from *Discovering Geometry : An Inductive Approach* (Serra, 1993, p. 274).

Chapter 6: Circles -Lesson topic- 6.4 - Arcs and Angles

Lesson Objectives:

- To explore inscribed angles, intercepted arcs, measurement of an inscribed angle, and measurement of an intercepted arc.
- To discover relationships between an inscribed angle of a circle and its intercepted arc
- To develop skills for writing conjectures from observations of relationships discovered as a result of constructing and analyzing geometric sketches (Serra, 1990).

Classroom Lesson Procedure:

1. Review of prerequisite definitions of terms to know.

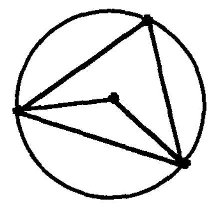
(a) circle, (b) radius, (c) diameter, (d) center, (e) chords of a circle, (f) central angle, and (g) inscribed angle.

2. Sketch inscribed angles and their intercepted arcs (visualize).

Subjects in the experimental group used the electronic tools of *The Geometer's Sketchpad* (Jackiw, 1994) for construction of sketches. Subjects in the control group used classic geometry tools: compass, ruler, protractor and the text *Discovering Geometry: An Inductive Approach* (Serra, 1993).

Subjects in both the experimental and control groups were given a handout with a step-by-step procedure for constructing sketches for use with the text, *Discovering Geometry: An Inductive Approach* (Serra, 1993) and for use with *Sketchpad (Jackiw, 1994)*.

<u>Sample Procedure for Construction of a Sketch</u>



Step 1. Construct a circle.

Name the center A.

Step 2. Locate three points on the circle.

Name the points C, B, and D.

Step 3. Draw and measure an inscribed angle BCD.

Step 4. Draw and measure the central angle of the intercepted arc BD.

Step 5. Measure arc length BD.

Step 6. Repeat Steps 3, 4, and 5 and measure inscribed angles BDC, and DBC.

3. Explore sketches with assigned partners (analyze).

Subjects in the experimental group were stationed at work tables with one computer supplied for each pair of subjects to complete investigation of sketches. Subjects in the control group were stationed at two desks that were placed next to each other to facilitate their investigation of sketches.

During an assigned period of time, subjects discussed and completed an investigation sheet on arcs and angles. Each subject was given an investigation sheet to record written conjectures (see Appendix B for a sample investigation sheet).

4. Produce conjectures with assigned partners (formalize).

Subjects formulated conjectures on: (a) the relationships between the measures of an inscribed angle and its intercepted arc, and (b) the relationships between the measure of a central angle and the measure of its intercepted arc.

5. Collection of investigation sheets

Investigation sheets were collected by the researcher at the end of the class period. All classroom work, tests, and homework papers were collected and graded by the researcher.

Instruction procedures on lessons for subjects in both the experimental and control groups were identical. The difference in the treatment method was that subjects in the experimental group completed lesson investigations using *The Geometer's Sketchpad* (Jackiw, 1994) and subjects in the control group completed lesson investigations using classic geometry tools. *The Geometer's Sketchpad* (Jackiw, 1994) has capabilities for (a) constructing a figure and recording each step of the construction in a script (see Figure 2, chapter one, p. 22), (b) drawing and measuring arcs and angles (see Figure 3), and (c) measuring angles and recording updated measurements in charts as the sketch is transformed (see Figure 4).

Figure 3 represents a sample screen from *Sketchpad* (Jackiw, 1994) showing a sketch of a circle and measurement data on arc lengths, inscribed angles, and circumference of a circle.

Sample screen on measurement of arcs and angles.[3]

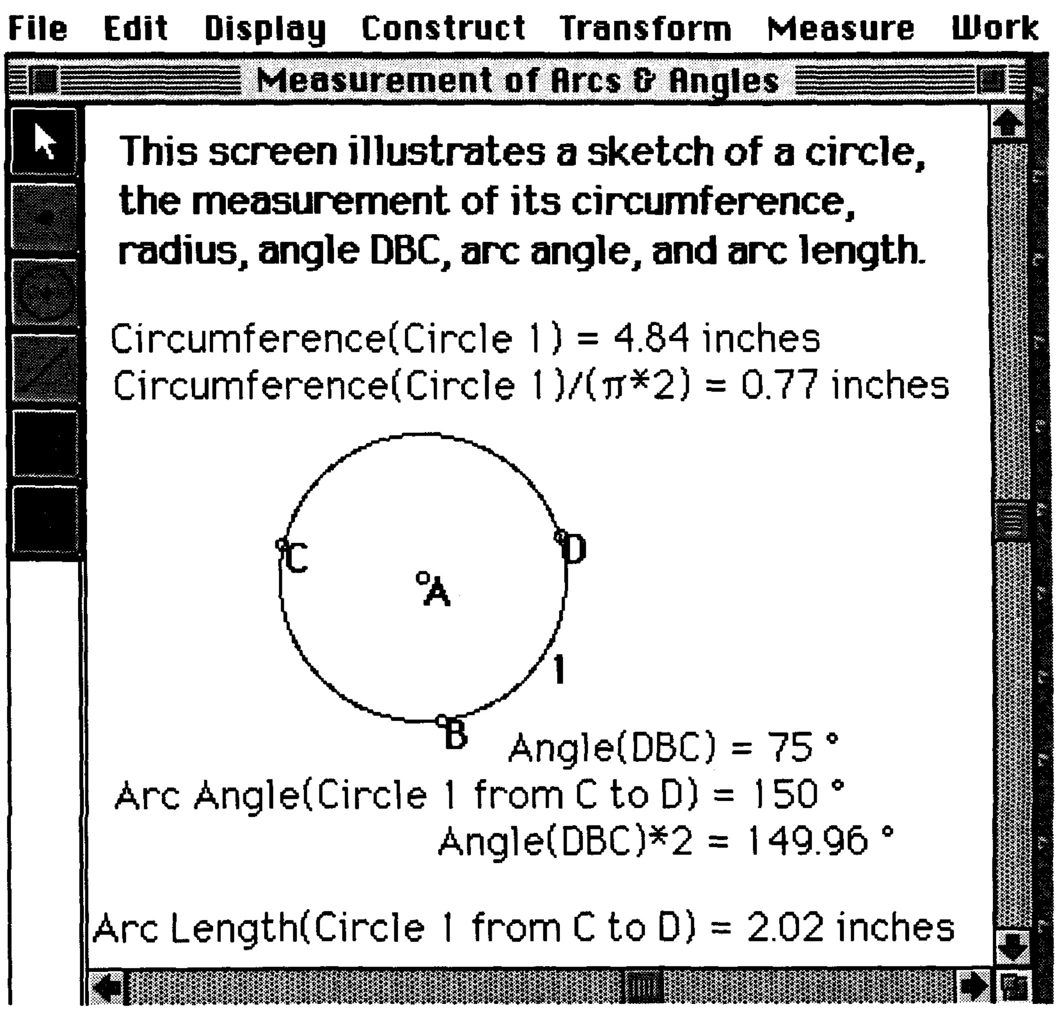


Figure 3. Screen display of *Geometer's Sketchpad* (Jackiw, 1994). Showing measurement of arc length and inscribed angle measure.

[3] From *The Geometer's Sketchpad* used with permission from "The Geometer's Sketchpad, Key Curriculum Press, P. O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH."

Figure 4 displays a sketch of a quadrilateral with measurements of its angles updated and recorded in charts as the sketch is transformed.

Sample screen[4]

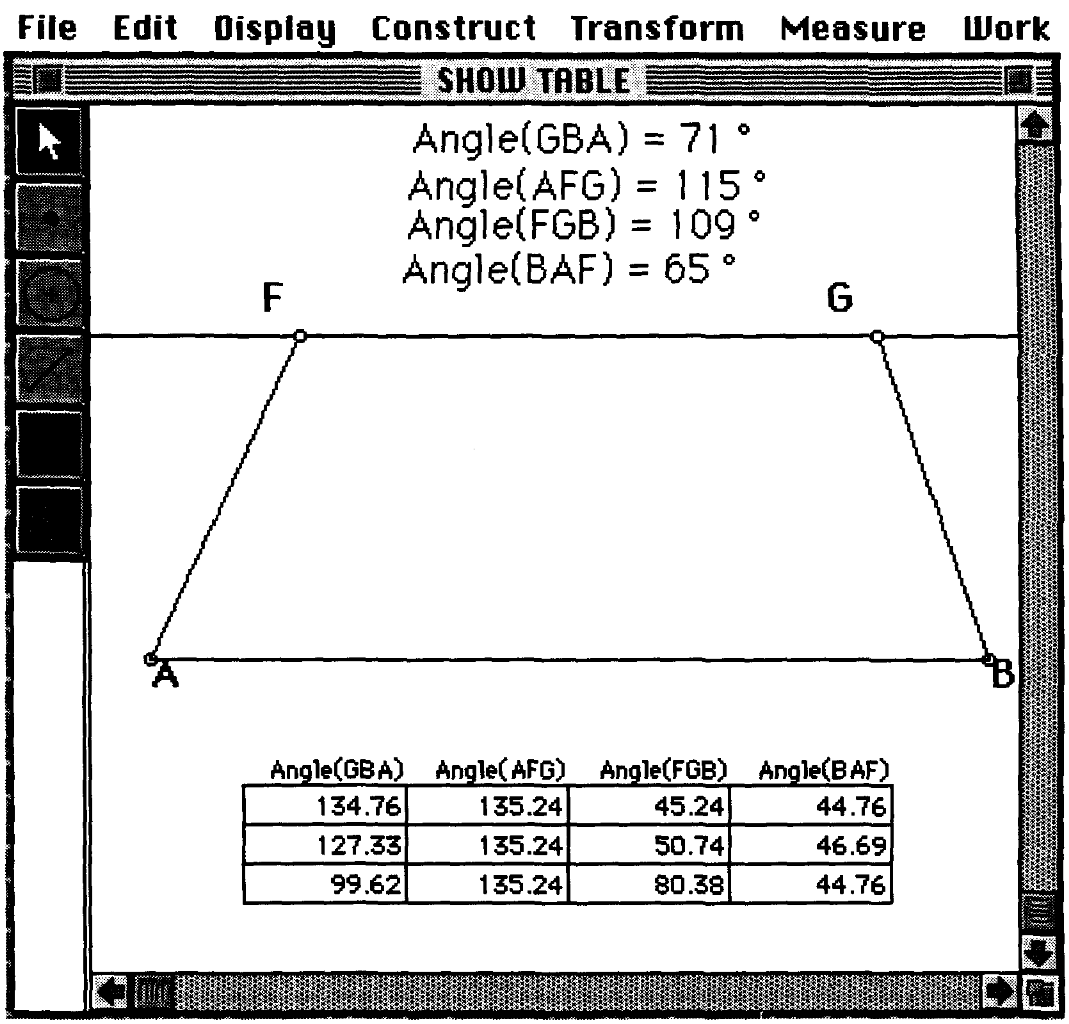


Figure 4 Screen display of measurements in charts.

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[4] From *The Geometer's Sketchpad* used with permission from "The Geometer's Sketchpad, Key Curriculum Press, P. O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH."

Recorded data on the measurement of figures can be tabulated in charts (see Figure 4). As the sketch is changed with the drag tool, measurement data are changed and upgraded.

These capabilities may effect cognitive changes in the user to produce a better understanding of the geometric concept being studied. The user is engaged as an intellectual partner with the computer. The software program acts as a cognitive tool, sharing operations with the user by drawing the sketch (with the help of the user), measuring indicated arc lengths and angles, and recording those measures accurately and efficiently.

Description of Subjects

The subjects in this study were 47 female high school geometry students. These subjects attended a Catholic high school in the city of Oakland, California. The school has a total population of 325 female students. The high school draws from the diverse community of the East Bay, students are from varied socio-economic and ethnic backgrounds.

Twenty subjects in one intact class were assigned to the experimental condition. They participated in technology-based instruction using *The Geometer's Sketchpad* (Jackiw, 1994). Twenty-seven subjects in the second group were assigned to the control condition. They participated in textbook-instruction using *Discovering Geometry: An Inductive Approach* (Serra, 1993).

Subjects selected for the study represented at least six ethnic backgrounds. Table 1 displays numbers and percentages of ethnic backgrounds represented by all subjects who participated in the study. Subjects were not evenly distributed between experimental and control groups according to ethnic background.

Table 1

Ethnic Groups Represented by All Subjects in the Study

Ethnicity	Number of Students	% of Total Sample
Asian	2	4
Black	19	40
Filipino	2	4
Hispanic	4	9
Mixed	6	13
White	14	30
Total	47	100

Subjects participating in the study ranged in age from 13 to 18. Table 2 presents numbers and percentages of ages of all subjects. The number of subjects according to age were represented in both groups.

Table 2

Age of Subjects in the Control and Experimental Groups

Age	Number of Students	% of Total
13	1	2
14	2	4
15	2	4
16	31	66
17	7	15
18	4	9
Totals	47	100

Subjects were not randomly assigned to the experimental and control groups for this study. Subjects selected for the study were from two intact geometry classes. Subjects in each intact class were assigned to one of two instructional conditions, an experimental condition or a control condition.

Subjects in the two comparison groups were not matched on mathematical ability. Within each intact class first semester grades in geometry indicated subjects with similar ranges of mathematical achievement within each group. First semester grades for the subjects selected for the study are listed in Tables 3 and 4.

Semester grades for both the experimental and control groups of subjects ranged from A to F. Sixty-six percent of the first semester grades of subjects in the experimental group were grades of B or C. Seventy-one percent of the first semester grades of subjects in the control group were grades of B or C. Five percent of subjects in the experimental group received a grade of A, while one percent of subjects in the control group received a grade of A. One percent of the subjects in the experimental group received an F grade, while seven percent of subjects in the control group received an F grade. This data indicated subjects in the study were similarly matched on mathematical ability.

Table 3 is a summary of the distribution of first semester geometry grades of subjects in the experimental group.

Table 3

First Semester Geometry Grades of Students in the Experimental Group

(n = 20)

Letter Grade	Number of Students	% of Total
A	01	05
B	11	52
C	05	24
D	03	14
F	01	05
Totals	21	100

Table 4 is a summary of the distribution of first semester geometry grades of subjects in the control group.

Table 4

First Semester Geometry Grades of Students in the Control Group

(n = 27)

Letter Grade	Number of Students	% of Total
A	01	01
B	08	30
C	11	41
D	05	19
F	02	07
Totals	27	100

Since the study took place immediately after third-quarter grades were computed, they were used as the covariate for data analysis instead of using first semester geometry grades as was originally planned. An analysis of covariance (ANCOVA) was the statistical test in which third-quarter geometry grades were used as the covariate to determine differences in achievement of geometric knowledge of students at the beginning of the study (see Appendix H for third-quarter geometry grades).

Description of Procedures

A letter was written to the Superintendent of Schools of the Diocese of Oakland School Department to obtain permission to conduct the study in a high school in the Archdiocese of Oakland (see Appendix M for a copy of the letter granting permission for the study to be conducted). Since there was no interruption in conducting the lessons required by the curriculum it was deemed unnecessary for a letter to be signed by each student to participate in the study.

Two weeks prior to the beginning of the study the instructor asked students whether or not they would be willing to participate in the study. At the beginning the students were informed about the procedures of the study. They were told there would be an observer in the room to conduct an investigation on two levels of instruction in geometry. Students were told one group would be using technology as a tool for instruction while a second group would be using classic geometry tools for instruction. Students were told they would be working with a partner during study on the Unit on The Circle Properties. A sampling of commentaries written by the investigator during each class day and a sampling of lesson plans used during the study are found in Appendixes K and L.

In one self-contained classroom, the experimental group of 20 female geometry students participated in a method of instruction using *The Geometer's Sketchpad* (Jackiw, 1994) software program to learn geometry. Subjects in the experimental group belonged to the C period class. This class met on Tuesdays and Wednesdays at 10:35 a.m., on Thursdays at 9:15 a.m., and Fridays at 10:15 a.m.

In the same self-contained classroom, the control group of 27 female geometry students participated in a method of instruction using the text, *Discovering Geometry: An Inductive Approach* (Serra, 1993) along with classic geometry tools. Subjects in the control group belonged to the F period class. This class met on Mondays, Tuesdays, Wednesdays, and Thursdays at 1: 05 p.m. each day.

The instructor used an inductive methodology to teach high school Euclidean geometry. Subjects were taught lessons on the topics on The Properties of Circles. These topics were new to all the subjects in both groups who participated in the study.

All subjects participating in the study were in geometry class since September of 1994. They used *Discovering Geometry: An inductive Approach* (Serra, 1993) for a text and used *The Geometer's Sketchpad* (Jackiw, 1994) for investigations during the third quarter of the school year. Students learned the basic skills required for using *Sketchpad* during the second and third quarters of this school year. During the study, subjects in the experimental group were taught how to create charts for reporting their findings and saving their sketches on disks. They had not been taught these skills previously.

Both groups of subjects participating in the study were taught by the same classroom geometry teacher. Topics of instruction were the same for both groups of subjects. There were many concepts on the properties of circles that were difficult for students to understand. Topics requiring visualization on relationships between arcs and angles of a circle and their measurements proved to be difficult for some students to understand.

For example, one geometric figure could include as many as four different kinds of angles including: (a) inscribed angles formed by two chords, (b) angles formed by the intersection of two chords, (c) angles formed by two tangent lines to a circle, and (d) angles formed by a tangent line and a secant line drawn to the circle. Properties of angle measurement related to the measurement of the intercepted arcs of those angles represented in the figure presented a learning challenge to many students. Problem-solving activities to sort out the many kinds of angles and their intercepted arcs in one complex sketch were simplified by the *Sketchpad* tool.

The *Sketchpad* (Jackiw, 1994) tool assisted the user in drawing the diagram described. The step-by-step construction procedures were recorded so they could be played back when needed. All data related to the sketch were tabulated in a chart, so information did not have to be held in memory. Sketches on properties of a circle provided powerful demonstrations on how an intellectual partnership between the computer and user created a learning experience for sharing and extending cognitive operations.

This study was conducted for two geometry classes that met four out of five class days, Monday through Friday in one week. The C period class was assigned to the experimental treatment. The F period class was designated as the control group. For a total of 16 class days for each group, this study was conducted during each geometry class session of 50 minutes each, over a period of four weeks.

On 13 class days the instructor presented a new lesson on properties of the Circle to subjects in both groups. All were given a class investigation activity on the lesson topic. This activity was to be completed during the class period. The topics presented on the Circle were on new material, topics not previously studied by the subjects.

After the lesson presentation by the teacher, subjects in the experimental group worked with a partner and used *The Geometer's Sketchpad* (Jackiw, 1994) to complete their lesson using investigation activity sheets from *The Geometer's Sketchpad User Guide and Reference Manual* (Bennett, Rassmussen, and Meyers, 1994). There was one Macintosh computer for each pair of students to use to explore investigations together. There were 11 Macintosh computers in the classroom. There were six Macintosh SE's, three Macintosh Classics, and two Macintosh LC's. *The Geometer Sketchpad* (Jackiw, 1994) software program was loaded onto each computer.

Subjects in the control group completed their investigation activity using the text, *Discovering Geometry, An Inductive Approach* (Serra, 1993), along with the classical tools of geometry: paper, pencil, compass, protractor, and ruler. After the lesson presentation, students placed two desks together to enable them to work with a partner to complete the lesson investigations exploring construction of geometric sketches. Subjects were supplied with classic geometry tools: compass, protractor, ruler, and straightedge.

After the lesson presentation was completed, each subject completed an investigation of geometric sketches and recorded conjectures formulated from her observations on investigation sheets. The researcher collected investigation sheets from each subject at the end of each class session.

The difference between the treatment of the experimental group and the control group was in the procedures of lesson investigations following lesson presentations. Subjects in the experimental group used electronic tools to complete investigation activities. Subjects in the control group used classical geometry tools: paper, pencil, compass, protractor, ruler, and straightedge to complete investigation activities.

Data Collection

Data Sources

1. Classroom observations. The investigator wrote commentaries on the lessons and class activities for each day of the study. These were reviewed every few days by the teacher and the investigator together.

2. Student work. All work on *Sketchpad* was saved on disks by subjects in the experimental group. All written assignments were collected and placed in student folders on each day of the study. All investigations, assignments, and tests were collected and scored by the researcher during the study (see Appendix B for copy of a sample investigation sheet).

3. Teacher and Investigator Meetings. At the end of each day both the teacher and the investigator met to discuss and adjust lesson plans and planned for the next day's activities when necessary.

4. Student Interviews. At the end of the study the investigator interviewed each of the subjects in the experimental group (see Appendix J for interview questions and student responses).

5. Sources for Data Analysis. There were three measures collected for data analysis. Before the study, third quarter grades in geometry of all subjects were collected for use as a covariate for the analysis of

covariance (ANCOVA) test. After the treatment, a posttest on geometric knowledge and scored on the three dependent variables: geometric knowledge, geometric constructions, and geometric conjectures for both the control and experimental groups (see Appendix A for copy of test).

Data Analysis

Quantitative data from posttest results on Hypotheses 1 and 2 were analyzed using the statistics software programs Excel and Statistics Program for Social Sciences (SPSS). Hypothesis 1 predicted a higher mean for the experimental group compared to the control group on posttest scores for the dependent variables geometric knowledge and geometric constructions. Neither were found to be statistically significant at alpha level $p < .05$. Hypothesis 2 predicted a higher mean for the experimental group compared to the control group on posttest scores for the dependent variable geometric conjectures. Findings on this dependent were statistically significant at alpha level $p < .05$. Since findings on Hypothesis 2 were statistically significant, further qualitative analysis of data on Geometric Conjectures posttest was reported in chapter 4.

A Pearson Product-Moment Correlation Coefficient was used to measure the magnitude of the relationship between third quarter geometry grades and posttest scores on the three dependent variables. Since r was moderately correlated with all three dependent variables, it served as a useful covariate for the ANCOVA test conducted on posttest data on the three dependent variables.

A univariate analysis of covariance (ANCOVA) was the statistical test for significance of difference for Hypotheses 1 and 2. Third quarter geometry grades were used (see Appendix H for a table listing third quarter grades of experimental and control groups) as a covariate to control for possible differences in previous achievement compared to posttest scores on achievement in geometry on the dependent variables. A $p < .05$ level of significance was used as the criterion for statistical significance.

Since the posttest on Geometric Conjectures was significant, the effect size (d) was computed for Hypothesis 2. The number of subjects in the experimental and control groups were unequal, therefore the pooled standard deviation was used for computing effect size (d).

To answer Research Question 1, on the cognitive effect on achieving geometric knowledge of the software program, *The Geometer's*

Sketchpad (Jackiw, 1994), when used as a pedagogical tool to improve subjects' solo geometry skills, and as a performance tool to upgrade concept development in producing problem solutions, percentages, means, and standard deviations were computed on posttest part 1 on geometric knowledge and posttest part 2 on geometric construction.

To answer Research Question 2, on the cognitive effect of *Sketchpad's* (1994) capability of dynamically manipulating, transforming, recording and upgrading data on the quality of conjectures written after completing investigation of sketches, frequency distributions, percentages, means, and standard deviations were computed on the posttest conjecture test items one through eight.

Taped interviews between the investigator and subjects in the experimental group were conducted the day after the posttest was completed. Statements made by the subjects corroborated findings of statistical data from the perspective of the subjects participating in the study (see Appendix J for transcription of student interviews).

CHAPTER IV

Results

Overview of Design and Variables

This chapter presents the findings of a quasi-experimental study conducted to investigate the effectiveness of a software program, *The Geometer's Sketchpad* (Jackiw, 1994), on achievement of geometric knowledge. Findings resulting from the statistical analysis of data are described in two of the sections of this chapter. Section one presents an analysis of quantitative data of posttest results on Hypotheses 1 and 2. Section two presents a further analysis of data on Geometric Conjectures from posttest results on Hypothesis 2.

A non-randomized selection of subjects from two intact high school geometry classes participated in this study. The research model for the study was a posttest-only control-group design. This study proposed to answer the following research questions:

1. What is the cognitive effect on achieving geometric knowledge of instructional use of the software program, The *Geometer's Sketchpad* (Jackiw, 1994), designed as a pedagogical tool to improve solo geometry skills, and as a performance tool to upgrade the development of concepts to produce problem solutions?

2. What is the cognitive effect of *The Geometer's Sketchpad's* (1994) capability of dynamically manipulating, transforming, recording and upgrading data on the quality of conjectures written after completing investigation of sketches?

The experimentally manipulated independent variable was the instructional method used to solve problems in geometry. The first level of the independent variable was an instructional method using technology-based investigations to solve problems using the software program, *Geometer's Sketchpad* (Jackiw, 1994). The second level of the independent variable, was an instructional method using textbook-based investigations to solve problems using the text, *Discovering Geometry: An Inductive Approach* (Serra, 1993) along with classic geometry tools. The three dependent variables used to measure achievement were posttest scores on (a) Geometric Knowledge, (b) Geometric Constructions, and (c) Geometric Conjectures.

Quantitative Analysis of Statistical Data

A Pearson Product-Moment Correlation Coefficient was used to measure the magnitude of the relationship between third quarter geometry grades and posttest scores on the three dependent variables.

As might be expected, test results indicated moderate correlations (see Table 5) between each of the three dependent variables and third quarter grades. The relationship between third quarter grades and the dependent variable, Geometric Knowledge, showed the highest correlation $r = .66$. The relationship between third quarter grades and the dependent variable, Geometric Constructions, was somewhat smaller than the other two correlations $r = .42$. The relationship between third quarter grades and the dependent variable, Geometric Conjectures, was a moderate correlation $r = .60$. Since r was moderately correlated with all three dependent variables, then it served as a useful covariate for the ANCOVA test conducted on posttest data on the three dependent variables.

Table 5

Correlation Coefficients between Third Quarter Grades and Posttest Scores

	Subjects (n = 47)	
Dependent Variables	Coefficients	P-Values
Geometric Knowledge	.66	$p < .01$
Geometric Constructions	.42	$p < .01$
Geometric Conjectures	.60	$p < .01$

Operational Hypothesis 1

The mean on a written posttest on Geometric Knowledge will be higher for the experimental group using *The Geometer's Sketchpad* (Jackiw, 1994) than the mean for the control group using the textbook *Discovering Geometry: An Inductive Approach* (Serra, 1993) with traditional geometry tools for investigations. The mean on a written posttest on Geometric Constructions will be higher for the experimental group using *The Geometer's Sketchpad* (Jackiw, 1994) than the mean for the control group using a textbook *Discovering Geometry: An Inductive Approach* with traditional geometry tools for investigations.

<u>Results for Hypothesis 1</u>

The first hypothesis predicted the mean of the experimental group on a posttest on the dependent variables Geometric Knowledge and Geometric Constructions would be higher than the mean of the control group on the dependent variables Geometric Knowledge and Geometric Constructions on the same posttest (see Appendix A for copy of posttest). The posttest on Geometric Knowledge consisted of 15 problems to be solved. The highest score possible was 75. The highest score for individual subjects in both the experimental and control groups was 62 points (see Appendix F for a table listing posttest scores on Geometric Knowledge).

The posttest on Geometric Constructions consisted of one problem requiring two constructions. Each construction required: (1) a written explanation of the steps followed in drawing the construction, and (2) a diagram of each construction. The highest score possible was a total of 25 points (see Appendix G for a table listing posttest scores on Geometric Constructions).

Descriptive findings related to Hypothesis 1 are shown in Tables 6 and 7. Findings shown in these Tables display data indicating minimal differences on posttest scores between the experimental and control groups on the two dependent variables Geometric Knowledge and Geometric Constructions.

Table 6

Mean Scores and Standard Deviations (SD) for Hypothesis 1

DV: Geometric Knowledge		
Experimental group		
n = 20		
	M	33.35
	SD	11.88
Control group		
n = 27		
	M	31.20
	SD	14.73

Table 7

Mean Scores and Standard Deviations (SD) for Hypothesis 1

DV: Geometric Constructions		
Experimental group		
n = 20		
	M	22.00
	SD	3.24
Control group		
n = 27		
	M	20.15
	SD	6.14

The mean score on Geometric Knowledge for the experimental group was 33.35 compared to a mean of 31.20 for the control group. The mean on Geometric Constructions was 22 for the experimental group compared to a mean of 20.15 for the control group. The difference between the means for Geometric Knowledge equals 2.15. The difference between the means for Geometric Constructions equals 1.85. Although the mean for the experimental group was higher for both dependent variables, the differences were not statistically significant.

For the experimental group on the dependent variable, Geometric Knowledge, a mean of 33.35 with a SD of 11.88 indicates that within one SD, approximately 68% of the scores ranged between 21.47 and 45.23. For the control group on the dependent variable, Geometric Knowledge, a mean of 31.20 with a SD of 14.73 indicates that within one SD, approximately 68% of the scores ranged between 16.47 and 45.93.

For the experimental group on the dependent variable, Geometric Constructions, a mean of 22.00 with a SD of 3.24 indicates that approximately 68% of the scores within one SD ranged between 18.76 and 25.24. For the control group, a mean of 20.15 with a SD of 6.14 indicates that approximately 68% of the scores within one SD ranged between 14.01 and 26.29.

A univariate analysis of covariance (ANCOVA) was the statistical technique for testing significance of difference for Hypotheses 1. Third quarter geometry grades were used (see Appendix H for a table listing third quarter grades of experimental and control groups) as a covariate to control for possible differences in previous achievement compared to posttest scores on achievement in geometry on the dependent variables.

The effect of the treatment was not statistically significant on the dependent variables, Geometric Knowledge, and Geometric Constructions, at alpha level of .05 reported by the Analysis of Covariance (ANCOVA) test results shown in Table 8. Neither a p--value of .61 on data on Geometric Knowledge, nor a p--value of .49 on Geometric Constructions were significant, therefore Hypothesis 1 was not supported.

Table 8

Univariate Analysis of Covariance for Hypothesis 1

DV: Geometric Knowledge

Source	SS	DF	MS	F	P
Class	28.13	1	28.13	.26	.61
Covariate	3643.86	1	3643.86	34.27	.00
Within	4677.82	44	106.31		

Univariate Analysis of Covariance for Hypothesis 1

DV: Geometric Constructions

Source	SS	DF	MS	F	P
Class	10.90	1	10.90	.49	.49
Covariate	190.57	1	190.57	8.48	.006
Within	988.84	44	22.47		

Operational Research Hypothesis 2

The mean on written posttest on Geometric Conjectures will be higher for the experimental group using the *Geometer's Sketchpad* (Jackiw, 1994) than the mean of the control group using the textbook *Discovering Geometry: An Inductive Approach* (Serra, 1993) for investigations.

Results for Hypothesis 2

The second hypothesis predicted that the mean score of the experimental group on a posttest on the dependent variable, Geometric Conjectures, would be higher than the mean of the control group on the dependent variable, Geometric Conjectures, on the same posttest. The posttest on Geometric Constructions consisted of eight problems to be solved using written Conjectures as reasons to support problem solutions. The highest score possible was 24 points (see Appendix I for a table listing posttest Conjecture scores for experimental and control groups).

Descriptive findings related to Hypothesis 2 are shown in Table 9. These findings confirm a large difference in achievement scores between the experimental and control groups on the dependent variable, Geometric Conjectures.

Table 9

Mean Scores and Standard Deviations (SD) for Hypothesis 2

DV: Geometric Conjectures

Experimental group		
n = 20	M	14.45
	SD	5.29
Control group		
n = 27	M	9.74
	SD	6.17

The mean on Geometric Conjectures for the experimental group was 14.45 compared to a mean of 9.74 for the control group. The difference between the means equals 4.71, a much higher mean for the experimental group compared to the control group.

For the experimental group on the dependent variable, Geometric Conjectures, a mean of 14.45 with a standard deviation of 5.29 indicates that within one standard deviation approximately 68% of the scores ranged between 9.16 and 19.74.

For the control group on the dependent variable, Geometric Conjectures, a mean of 9.74 with a standard deviation of 6.17 indicates that within one standard deviation approximately 68% of the scores ranged between 3.57 and 16.44.

The effect of the treatment was statistically significant on the dependent variable, Geometric Conjectures, below the alpha level of .05 reported by the Analysis of Covariance (ANCOVA) test results as shown in Table 10. Therefore, Hypothesis 2 was supported.

Table 10

Univariate Analysis of Covariance for Hypothesis 2

DV: Geometric Conjectures

Source	SS	DF	MS	F	P
Class	121.87	1	121.87	5.23	.03
Covariate	496.96	1	496.96	21.33	.00
Within	1025.17	44	23.30		

Since the posttest on Geometric Conjectures was significant, the effect size (d) was computed for Hypothesis 2. The number of subjects in the experimental and control groups were unequal, therefore the pooled standard deviation was used for computing effect size (d) as shown in Figure 5.

Formula for effect size (d):

$$d = \frac{\overline{X_1} - \overline{X_2}}{Sp}$$

$$d = 0.81$$

Figure 5. Formula for computing effect size on data for Hypothesis 2.

The difference in the means, 4.71, divided by the pooled standard deviation, 5.798, yields an ES of .81. Effect sizes (percent of common within group standard deviation) greater than .80 are considered large effects. Therefore, the calculated effect size ratio of .81 indicates a large practical significant difference between the two groups on the dependent variable, Geometric Conjectures.

Further Analysis of Student Conjectures

Since statistical findings on data recorded for the dependent variables, Geometric Knowledge and Geometric Constructions were not significant, this section does not examine data related to Hypothesis 1.

Findings on development of geometric concepts indicated by written Geometric Conjectures used to solve problems in geometry related to Hypothesis 2 are examined in this section.

The posttest on Geometric Conjectures required written statements of conjectures as reasons to support problem solutions. The rating scale for scoring problems corresponded to the van Hiele levels of concept development from 0 to 3. Advancement from one level to the next requires the subject to progress successively through each level from 0 to 3. Each level has specific attributes of concept development related to geometric thought:

1. At concept Level 0 the learner recognizes figures as geometric.
2. At concept Level 1 the learner identifies relationships between and among integral parts of figures.
3. At concept Level 2 the learner interrelates properties between and among objects and figures.
4. At concept Level 3 the learner applies generalizations in the form of conjectures as reasons supporting written solutions to problems.

These levels were used as criteria for determining the number of points scored for each problem on Conjectures in the posttest.

The criteria for assigning the number of points to each problem solution were:

1. A point of 0 was assigned to either an answer stating an incorrect recognition of geometric objects and/or geometric figures in each problem, or to an answer left blank.

2. A point of 1 was assigned to an answer describing correct relationships of geometric objects as they relate to the whole geometric figure in each problem.

3. A point of 2 was assigned to an answer expressing a correct application of properties of geometric figures and objects in relation to the solution of each problem.

4. A point of 3 was assigned to an answer applying a correct statement of conjectures (generalizations) to the solutions of each problem.

A comparison of results between the control and experimental groups of the number of subjects attaining concept Level 3 in their problem solutions was supported by the theoretical rationale of this study. Findings confirmed the software program *The Geometer Sketchpad* (Jackiw, 1994) improved geometric achievement for subjects in the experimental group. Results of analysis of data on Hypothesis 2 indicated higher levels of concept development found in written conjectures of subjects in the experimental group.

The software program, *The Geometer Sketchpad* (Jackiw, 1994), used as a cognitive tool of instruction extends the cognitive operations of the user, thus increasing intellectual capacities for learning. According to the theory of Gavriel Salomon (1993b), when the software program is used as a pedagogic tool, cognitive *effects of* the software program result in improving solo abilities. When the software program is used as a performance tool, *cognitive effects with* the software program result in improving joint performance between the user and the program in producing a product. In this case, the product was application of conjectures to problem solutions.

Results on the Conjecture posttest indicated high performance scores of subjects in the experimental group on solo (individuals' own skills) abilities, when used in the absence of the software program. For the posttest on Conjectures, subjects did not use the computer program during this part of the test. High scores on written conjectures indicated an improved performance through applications of conjectures to produce problem solutions.

Graphs displaying the distribution of posttest scores on Geometric Conjectures for both the experimental and control groups are found in Appendix N. The graphs show total scores of subjects in the experimental group are higher than total scores of subjects in the control group. The totals on the Conjecture test for subjects in the experimental group range from 5 to 24. The totals on the Conjecture test for subjects in the control group range from 0 to 22.

For the purpose of analyzing levels of concept development indicated by posttest scores on Conjectures a frequency distribution of total scores of subjects in the control group are displayed in Table 11. A frequency distribution of total scores of subjects in the experimental group on the Conjecture posttest are displayed in Table 12. The highest number of points an individual could achieve was 24. On the frequency chart, if one looks at the number of individuals who scored in the upper 50% range of the possible 24 points only 8 subjects scored 12 points or above which is approximately 30% of the subjects in the control group. Subjects in the experimental group scoring in the upper 50% range of 12 points or above was 14 or 70% of the subjects. These percentages indicate a much higher achievement of subjects in the experimental group compared to subjects in the control group.

Table 11

<u>Frequency Distribution of Posttest Scores on Conjectures for Control Group.</u>

(n =27)

X (raw Score)	f (frequency of occurrence)
0	1
2	1
3	2
5	2
6	3
7	1
8	3
9	3
10	2
11	1
12	3
15	1
18	1
23	3

Highest possible score = 24

Table 12

Frequency Distribution of Posttest Scores on Conjectures for Experimental Group.

(n =27)

X (raw Score)	f (frequency of occurrence)
5	1
8	1
9	4
11	1
12	3
14	2
15	1
16	1
17	2
18	1
21	1
22	1
23	1
24	1

Highest possible score = 24

Table 13 displays scores received on the eight conjecture problems by subjects in the experimental group. Table 14 shows the scores received on the eight conjecture problems by subjects in the control group. The total number of solutions indicating a concept of development at Level 3 for the control group was 61, or 28% of all student solutions. The total number of solutions indicating a concept of development at Level 3 for the experimental group was 80, or 50% of all student solutions. These percentages indicate a much higher achievement in concept development of geometric conjectures of the subjects in the experimental group compared to subjects in the control group.

Table 13

Number of Points Assigned to Each Conjecture Problem of Subjects in the Experimental Group

(n = 20)

Achievement of Concept Level 3 Applied to Conjecture Problems

Problem Number	Frequency of Occurrence of Level 3
1	9
2	5
3	18
4	4
5	14
6	10
7	11
8	9
Total Number of Solutions	$\frac{80}{160} = 50\%$

Table 14

Number of Points Assigned to Each Conjecture Problem of Subjects in the Control Group

(n = 27)

Achievement of Concept Level 3 Applied to Conjecture Problems

Problem Number	Frequency of Occurrence of Level 3
1	7
2	5
3	14
4	1
5	11
6	5
7	9
8	9
Total Number of Solutions	$\frac{61}{216} = 28\%$

Summary

Analysis of the statistical data of this study conducted to investigate the effectiveness of a software program, *The Geometer's Sketchpad* (Jackiw, 1994), on achievement of geometric knowledge established different statistical results for Hypotheses 1 and 2.

Descriptive findings for Hypothesis 1, which predicted a higher mean for the experimental group compared to the control group on posttest scores for the dependent variables Geometric Knowledge and Geometric Constructions were not statistically significant at alpha level .05. Posttest results indicated the experimental group achieved only a slightly higher mean than the control group mean. Therefore, Hypothesis 1 was not supported.

Descriptive findings for Hypothesis 2, which predicted a higher mean for the experimental group compared to the control group on posttest scores for the dependent variable Geometric Conjectures was statistically significant at an alpha level of .05. The calculated effect size ratio of .81 indicated a large practical significant difference between the experimental and control groups demonstrated by posttest results on Conjectures.

Further analysis of posttest results on conjectures revealed data supporting the theoretical rationale on which the study was based. The theoretical rationale suggested that the software program, The *Geometer's Sketchpad* (Jackiw, 1994), engages the learner as an "intellectual partner" in two ways:

1. As a performance tool the program is designed to upgrade intellectual achievement of geometric knowledge.

2. As a pedagogic tool the program is designed to improve geometry skills and strategies.

Posttest scores related to Hypothesis 2 demonstrated results supporting these statements. Data on Conjecture posttest scores indicated higher achievement levels of concept development for the experimental group using the program, *Sketchpad* (Jackiw, 1994), as compared to achievement levels of concept development for subjects in the control group using the textbook, *Discovering Geometry: An Inductive Approach* (Serra, 1993).

On the day after the posttest was completed, the investigator conducted interviews with the subjects in the experimental group. Subjects were asked for responses related to the research questions of the study. The interviews were taped and transcribed by the investigator (see Appendix J for transcription of the interviews). Data from these interviews lend insight on the treatment of the study from the perspective of the participants. Their comments corroborated findings on the experience of sharing an intellectual partnership with *Sketchpad* (1993) to extend cognitive capacities to optimize learning.

The major cognitive effects of the use of the *Sketchpad* program on student learning indicated by subjects during their interviews with the investigator were the following:

1. Constructing and transforming geometric figures was made easier with *Sketchpad* tools.

2. Measuring and recording data was made visible through charts and labeling with *Sketchpad* tools.

3. Verifying accuracy of data through observation of multiple cases of circle properties assured the user of geometric knowledge.

4. Visualizing transformations of figures deepened students' understanding of circle concepts.

5. Observing geometric figures supported confidence in subjects for reasoning to conclusions and conjectures.

6. Transferring solo geometry skills when applied in the absence of the computer to the posttest problems proved difficult for some subjects.

These cognitive effects of the *Sketchpad* program are supported by the following statements of subjects transcribed from the taped interviews between the subjects in the experimental group and the investigator.

Student #10.

Question: How did the fact that you could manipulate objects on the screen and visualize them while recording and tabulating data assist you in learning geometric concepts?

Answer: The computer gave me a lot of options on the computer, what I could do, what I could manipulate and putting [sic] in another inscribed angle or another chord or something on the circle. It just made things easier and I could see what I was doing. What mistakes I made. I could see what I was doing to help me make the conjectures.

Student #12.

Question: How did the fact that you could manipulate objects and observe their changing measures affect your understanding of geometric concepts and reasoning to conjectures?

Answer: Everything was easier. I couldn't make as many mistakes as I have if I did it myself. The computer really didn't let you make mistakes. When I go home to do homework, it wasn't that easy away from the computer. The conjectures we stayed on the computer. When I went home I didn't have the conjectures with me. I'm not sure I learned better. It [the computer] was at school and I didn't have it at home. It made things clearer, when I had the computer, but when I worked at home it was confusing.

Student #14.

Question: What was the effect of the *Sketchpad's* capability on your being able to observe manipulation of geometric objects? How did this affect your learning geometric concepts?

Answer: I thought with the *Sketchpad* it was easier. I have [sic] the tools and make [sic] sure it was exact measurements. When I am drawing sometimes it might be off and I am not able to find the conjecture. When I am able to use the computer, my conjecture comes easier to me and I am able to find it [conjectures] much easier. I liked the final product which was perfect and I was really proud of the final sketch. When I am away from the computer some of the conjectures I was able to apply to my homework easily.

CHAPTER V

Summary, Limitations, Discussion, and Recommendations

Summary

This study addressed the problem of improving achievement of geometric knowledge through instructional use of the software program *The Geometer's Sketchpad* (Jackiw, 1994). This program was used as a cognitive tool for instruction and learning high school geometry. The tools of the program enabled the user to (a) construct geometric sketches, (b) demonstrate transformations of geometric properties on sketches, and (c) produce dynamic visualization of changes in measurement, shape, and kind of geometric figures.

The software provided capabilities for extending cognitive skills of users by sharing construction, transformation, and measurement tasks between the student and the computer. The program allowed users to produce visible images to demonstrate how relationships can be changed on geometric constructions. Observation of changed relationships provided the learner with data to analyze and validate conjectures. This software was used as an instructional tool to deepen levels of understanding concepts of Euclidean geometry.

The experimentally manipulated independent variable was the methodology of instructional use of the computer versus the use of classic geometry tools for problem investigations. The dependent variables were the measures of the effects of these two levels of instruction on achievement of geometric knowledge and construction, and geometric conjectures. The experiment was controlled by holding classroom conditions constant for both the experimental and control groups with the exception of the treatment of the independent variable.

Classes for both groups were held in the same physical classroom at different class periods during the day. Both groups had been in geometry class since September and had used the same textbook. The instructional methodology was an inductive approach to learning the properties of geometry by the same instructor for both groups. Lesson presentations were on the same topics for both groups (see Appendix L for lesson plans). Subjects in both groups worked with partners while exploring problem investigations.

Descriptive statistics including frequencies, percentages, means, and standard deviations were computed on posttest results. A Pearson product-moment correlation coefficient measured the magnitude of the relationship between third-quarter geometry grades and posttest scores on Geometric Knowledge, Construction, and Conjecture. These test results showed a moderate correlation coefficient between third-quarter grades and posttest scores on the three dependent variables.

Since third quarter grades were moderately correlated with all three dependent variables, they served as a useful covariate for the ANCOVA test conducted on posttest data. Posttest results on Geometric Knowledge and Geometric Construction were not statistically significant (Hypothesis 1). Posttest results on Geometric Conjecture were statistically significant at an alpha level of .05 (Hypothesis 2).

Limitations

The first limitation of the study was the method of selection of the sample. A non-randomized sample of forty-seven female subjects participated in the study. They were selected from two intact geometry classes. This method of selection limited generalizability of results of statistical data as estimators of a larger population. Generalizations might be applicable to populations of female high school geometry classes of students with characteristics similar to those of subjects who participated in the study.

As a result of the non-randomized method of selection, subjects were not matched on the following subject variables: ethnicity, age level, or mathematical ability. Subjects from six ethnic groups were represented in both the experimental and control groups (see Figures 6 and 7). Since the number of subjects representing each ethnic group was small, inferences from data could not be generalized as characteristic of any one of the ethnic groups represented.

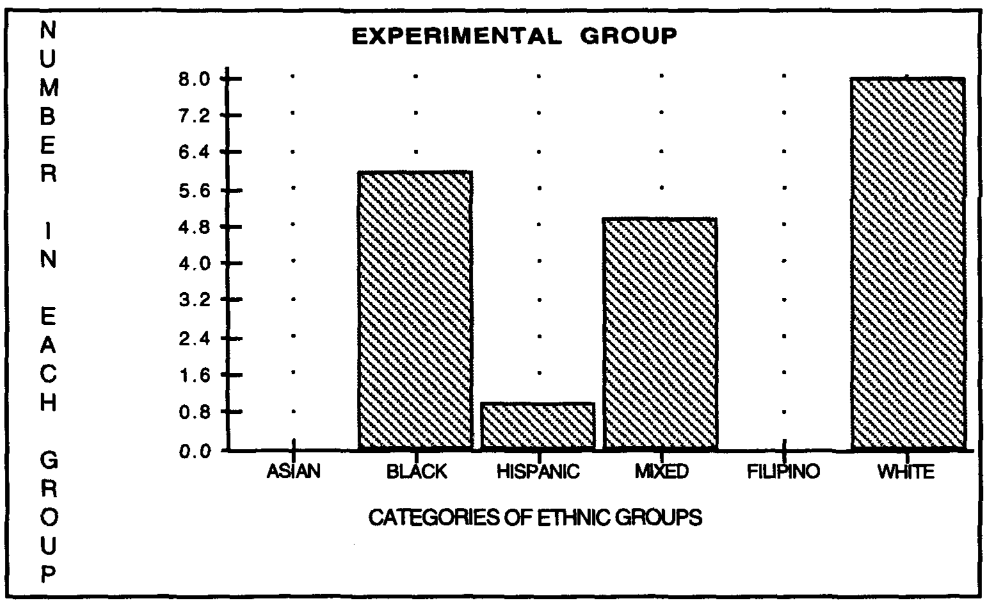


Figure 6. The number of subjects in the experimental group from each of the ethnic groups represented in the study.

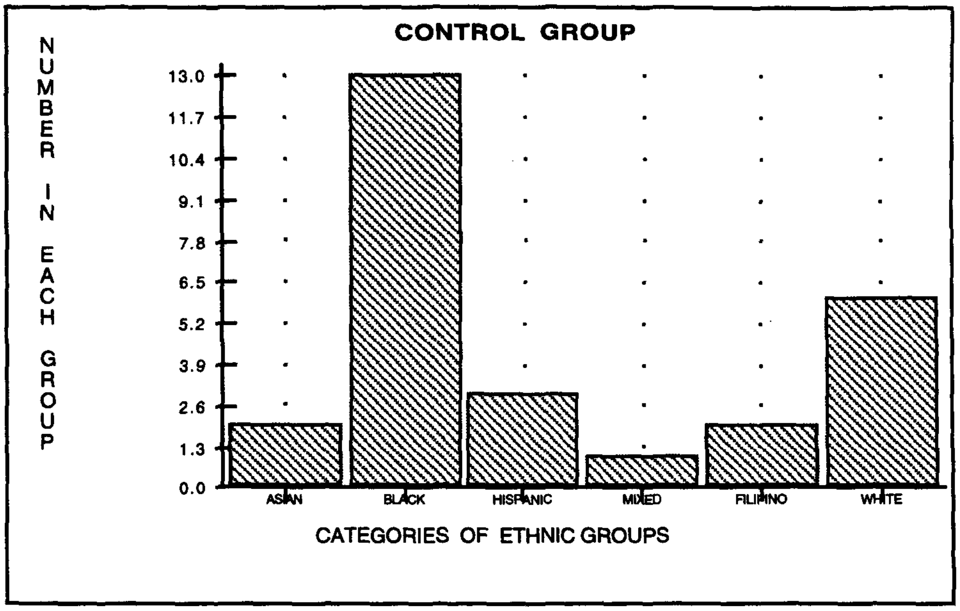


Figure 7. The number of subjects in the control group from each of the ethnic groups represented in the study.

Subjects in all six ethnic categories were represented in the control group. Only four ethnic categories were represented in the experimental group. There were no Asian or Filipino students in the experimental group. There were seven more black students in the control group than in the experimental group. There were six white students in the control group and eight white students in the experimental group. There were three students of mixed ethnicity in the control group and five students in the experimental group.

Subjects in both groups, ranging in ages from thirteen to eighteen, were represented in both the experimental and control groups. Data from posttest results could not be generalized to subjects of a particular age group to compare data between the experimental and control groups as shown in Table 15.

Table 15.

Number of Subjects in Each Age Group in the Experimental and Control Groups.

Years in Age	Control Group	Experimental Group	Totals
13	1	0	1
14	2	0	2
15	0	2	2
16	17	14	31
17	5	2	7
18	2	2	4
Totals	27	20	47

Third quarter geometry scores indicated similar ranges of ability for subjects in both the experimental and control groups (see Appendix H). The range of scores for subjects in the control group were from a low of 60.84 to a high of 89.45. The range of scores for the experimental group were from a low of 62.12 to a high of 90.76. Since subjects were not matched on ability level, inferences drawn from posttest results could not be applied as characteristic of any specific mathematical ability level.

The second limitation of the study was the length of this study. The study took place over a period of a total of 16 class days. There were 13 class days for lessons, 2 days for testing, and 1 day for interviews with subjects in the experimental group. In this short time-period, subject matter content was limited to one topic on "The Circle Properties" of geometry. This factor limited the scope of applicability of results to only specific topics in geometry. Although the inductive reasoning skills applied to one topic only, those same skills operate in similar ways when applied to other topics in geometry.

The third limitation of the study was the time of the school year when the study was conducted. The study was scheduled during the spring semester of the school year. This study was conducted during geometry classes, Monday through Friday, from April 24, 1995 to May 19, 1995.

Concentration on subject matter might have been negatively influenced by warm weather. Also, subjects were distracted by end of the school year activities. The following events took place during the time of the study: distribution of yearbooks, the junior-senior prom, and the school play. For these reasons subjects may not have been motivated by a high degree of discipline toward achieving their best in geometry class.

A fourth limitation of the study was the time of day when the class for the control group was held. This class period occurred at 1:05 p. m. each day. This was the period following lunch. Subjects may have been negatively affected by the time of day their class met. In contrast, the class period for the experimental group was during the morning hours of the day, which might have been a better time for learning geometry.

A fifth limitation of the study was evidence of resentful feelings on the part of subjects in the control group. Some subjects in the control group would have preferred to have used technology tools instead of classic geometry tools. Using technology tools was highly motivating for subjects in the experimental group. Using classic geometry tools demanded more effort on the part of some subjects in the control group.

Despite the limitations imposed by the number of days, the time of the day, and time of the year this study was held, results showed improvement of achievement levels of thinking applied to conjecturing ability for subjects in the experimental group. Subjects in the experimental group achieved a statistically significant difference on Conjecture posttest scores.

Considering the short-period of time subjects had to spend on the computer using the software program, Conjecture posttest results revealed a positive impact on inductive reasoning ability indicated by application of conjectures to solving problems on the posttest. Computer time was limited to 13 class sessions during the study. Subjects did not have access to the computer program at home or during other class-periods at school.

Conclusions

Restatement of the Purpose

The purpose of this study was to investigate the effectiveness of *The Geometer's Sketchpad* (Jackiw, 1994) as a cognitive tool for instruction and learning Euclidean geometry. A quasi-experimental study was conducted to explore the capabilities of *The Sketchpad* for improving achievement of geometric knowledge of high school geometry students. The current study compared investigation of problems in geometry using computer tools to textbook-based investigations using classic geometry tools.

Forty-seven high school geometry students participated in the study. *Discovering Geometry: An Inductive Approach* (Serra, 1993) was the textbook for instruction. An inductive reasoning approach was the pedagogy for discovering geometric properties. The experimentally manipulated independent variable was the two levels of investigation for solving problems in geometry. The *Geometer's Sketchpad* (Jackiw, 1994) was the software tool used by subjects in the experimental group. Classic geometry tools: ruler, pencil, protractor, and compass were used by subjects in the control group.

The research questions addressed by the study were:

Research Question 1

1. What is the cognitive effect on achieving geometric knowledge of the software program, *The Geometer's Sketchpad* (Jackiw, 1994) designed as a pedagogical tool to improve solo geometry skills and as a performance tool to upgrade concept development in producing problem solutions?

Research Question 2

2. What is the cognitive effect on achieving geometric concepts of the tools of the software program allowing the user to dynamically manipulate, transform, record and upgrade data on the quality of conjectures written after completing investigation of sketches?

The first hypothesis was formulated from the first research question. Hypothesis 1 predicted a higher mean for the experimental group compared to the control group on posttest scores *geometric* knowledge and construction. Posttest results indicated the experimental group achieved only a slightly higher mean than the control group mean. Therefore, Hypothesis 1 was not supported.

A plausible reason for not finding a statistically significant difference between the experimental and control groups on posttest scores on the dependent variables geometric knowledge and construction might have been due to the length of time of the study. The time period on the use of the *Sketchpad* program by the subjects in the experimental group was for only 13 class days. If the study were conducted over a longer period of time allowing additional time on the computer, then the results might have shown higher scores for subjects in the experimental group.

Another reason why Hypothesis 1 was not supported was that solo geometry skills of the individual did not transfer when applied in absence of the computer program. When subjects solved the problems on the posttest on geometric knowledge, some subjects in the experimental group found the skills they applied when using the computer did not transfer to problems on the posttest. On the taped interviews some subjects indicated they did not understand nor were they capable of making the connection between the investigations completed on *Sketchpad* and their application to the geometric knowledge problems on the posttest. To remedy this problem a further study might be conducted using *Sketchpad* as a pedagogical tool to improve the transfer of solo geometry skills of subjects by conducting investigation of problems on the computer and then completing applications of those investigations to problems in the absence of the computer.

The second hypothesis was formulated from research question number two. Quantitative analysis of data on the Conjecture posttest results indicated higher achievement levels of concept development for subjects in the experimental group using the program *Geometer Sketchpad* (Jackiw, 1994) than those in the control group indicated by written conjectures on the posttest.

The second research question asked, "What is the cognitive effect of *The Geometer's Sketchpad's* (1994) capability of dynamically manipulating, transforming, recording and upgrading data on the quality of conjectures written after completing investigations of sketches?

An effect size ratio of .81 sigma between the groups was calculated on the Conjecture posttest results for Hypothesis 2. This ratio indicated a large practical difference between the experimental and control groups on achievement of geometric concepts indicated by written statements of conjectures on the posttest. This was an important finding of this study.

In the experimental group 70% of the subjects scored in the upper 50% range of 12 or above out of 24 possible points on the Conjecture Posttest. In the control group only 30% of the subjects scored in the upper range of 12 or above out of 24 possible points on the Conjecture Posttest. According to the van Hiele levels of progression of geometric thought these findings indicated higher levels were achieved by more subjects in the experimental group than in the control group.

Achievement of higher levels of thought suggests achievement of higher levels of understanding of geometric concepts. This data indicated subjects using the *Sketchpad* achieved higher levels of conceptual understanding of geometric concepts than those subjects in the control group. Therefore, the use of the *Sketchpad 's* tools for producing dynamic visualization of transformation of sketches on the screen made a difference on achievement of deeper levels of concept development.

Given the limitations of this study, it is worthwhile to look at conclusions from data on Hypothesis 2 measuring concept development indicated by written conjectures on the posttest. Results showed a statistically significant difference in achievement on applications of conjectures to problem solutions for subjects in the experimental group.

Significance

Evidence supporting the second research question is best illustrated by examples of solutions written by the subjects in the study. According to Van Hiele (1986) levels of thought involved in the development of geometric concepts, each level can be identified through observations of students' problem solving activities and in written work of students' solutions to problems.

Conjecture Posttest Sample Problem

Specific attributes of each level from 0 to 3 were identified in the answer statements on the Conjecture posttest. The following examples illustrate answers corresponding to the first four van Hiele Levels of thinking involved in the development of solutions to problems on the Conjecture posttest.

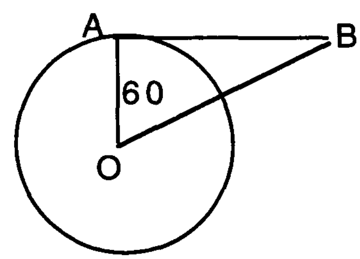
Sample problem two from the Conjecture posttest is shown to illustrate the correct solutions to the problem at each of the four levels from 0 to 3 on the rating scale (see Figure 8). Actual student responses to the same problem at each of the four van Hiele levels are also shown. Sample solutions illustrating criteria for identifying each conjecture level are shown in Figures 9, 10, 11, and 12).

Conjecture Posttest Directions

The verbatim instructions given to the subjects for completing the solutions to the problems on the Conjecture posttest were: "For each of the eight problems in Part 1, find the solution to each problem. Write the correct multiple choice answer on the line provided. Write the statements of each of the conjectures and/or properties you applied to find the solution to each of the problems on the lines provided."

Problem Number Two.

In the figure, line AB is tangent to the circle with center O. If the radius of the circle is 12, then find the length of AB = ___



(A) $12\sqrt{2}$ (B) $12\sqrt{3}$ (C) 6 (D) 8π (E) 1

Answer (3 pts.): ________:

Write the Conjectures on the following lines:

____

____

____

____

Figure 8. An illustration of sample problem number two from the posttest on Conjectures.

*For problem number two the correct answer is B.

Conjecture Posttest Sample Solutions

A Four-Point Scale was used for scoring the van Hiele Levels from 0 to 3. The examples shown in Figures 9, 10, 11, and 12 list *objective* criteria for scoring problem number two at each level from 0 to 3 respectively. Each example is followed by a solution given by a subject on the posttest.

Level 0 solution. Subjects identified geometric objects in the figure. For problem number two the geometric objects are:

1. The center of the circle is point O.
2. The radius of the circle is AO.
3. The line AB is a tangent to the circle.
4. An angle is formed by segments AO and OB.

Multiple Choice Answer : This subject did not write any multiple choice solution.

Conjecture Statement:

The radius of the circle is segment OA.

Student Sample Solution Scored at Level 0.

The two objects were identified in this answer were the circle, and the radius.

Figure 9. An illustration of sample problem solutions at Level 0.

Level 1 solution. Subjects identified relationships between and among geometric objects in the figure.

For this problem the relationships are:

1. Radius OA is perpendicular to the tangent AB.
2. Segments AB and OA meet to form a right angle.
3. Right angle AOB is formed by perpendicular lines AB and OA.
4. Triangle AOB is a right triangle containing exactly one right angle.

Multiple Choice Answer: (A) The wrong multiple choice answer

Conjecture Statement: *Segment AB is a tangent to the circle which makes angle BAO 90 degrees. Segment AB is the longest leg of the triangle then which makes it* $12\sqrt{2}$.

Student Sample Solution Scored at Level 1. The subject identified the relationship of perpendicularity between the tangent to the circle and the radius of the circle to form a right triangle.

Figure 10. An illustration of sample problem solutions at Level 1.

Level 2 solution. Subjects informally interrelated relationships between the geometric objects to previously learned conjectures and applied them to the problem solutions.

For problem number two conjectures related to the solution are:

1. The Tangent Conjecture: "A tangent to a circle is perpendicular to the radius drawn to the point of tangency" (Serra, 1990, p. 277).

2. The 30-60 Right Triangle Conjecture: "In a 30-60 right triangle, if the shorter leg has length x, then the longer leg has length $x\sqrt{3}$ and the hypotenuse has length $2x$" (Serra, 1990, p. 279).

Multiple Choice Answer: (A) The wrong multiple choice answer.

Conjecture Statement:

Since AO is equal to 12 and you're [sic] *trying to find the other length of the triangle then since a is segment AO and OB is segment c then you know AB is b and with the formula of Pythagorean Theorem....*

Student Sample Solution Scored at Level 2.

The subject informally interrelated relationships between the geometric objects in the figure as forming a right triangle, but unsuccessfully applied the Pythagorean Theorem to the problem.

Figure 11. An illustration of sample problem solutions at Level 2.

Level 3 solution. Subjects formally established an application of conjectures and properties to find the solution to the problem. For problem number two the correct answer might read:

1. The tangent AB is perpendicular to the radius OA.
2. The radius OA is 12 units in length.
3. The tangent line AB is perpendicular to the radius OA at the point of tangency (point A).
4. A right angle OAB is formed by the radius and the tangent to the circle and is equal to 90 degrees.
5. A right triangle AOB contains exactly one right angle.
6. The measure of angle AOB is 60 degrees.
7. The measure of the acute angle ABO is 30 degrees.
8. The shorter leg of the triangle opposite the 30 degree angle is the radius OB.
9. The longer leg of the triangle opposite the 60 degree angle is AB.
10. The 30-60 Right Triangle Conjecture states: if the shorter leg has length 12, then the longer leg is length $12\sqrt{3}$.

(figure continues)

Multiple Choice Answer: (B) The correct multiple choice answer.

Conjecture Statement. *In a 30-60-90 degree right triangle the longest leg is always the shortest leg times $\sqrt{3}$. In this case the shortest leg is 12 so the longest leg is $12\sqrt{3}$.*

Student Sample Solution Scored at Level 3.

The subject formally established the correct application of conjectures and properties to find the solution to the problem.

Figure 12. An illustration of sample problem solutions at Level 3.

These examples illustrate how answers were scored according to objective distinctions within each of the four thinking levels of progression. Results of the analysis of data showed statistically significant higher levels of concept development on written conjectures for subjects in the experimental group than those subjects in the control group (see Appendix I).

Chapter 6 Test — Form A Answer Sheet

My Name is ____________________ Period ____ Date __________

Part A (2 points each)

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Part B (4 points each)

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________

Part C (6 points each)

1. $a =$ ______
2. $b =$ ______
3. $f =$ ______
4. ________
5. $r =$ ______
6. ________

Part D (7 points each)

1. ______
2. ______

Part E (7 points each)

1.
2.

Also, subjects in the experimental group were interviewed after the study. Their statements revealed insights into the effects of the dynamic features of the software program affecting cognitive changes on their ability to reason to conjectures as they applied to problem solutions. The transcription of the interviews conducted by the investigator are found in Appendix J.

Previous Research Studies and The Present Study

Empirical studies on the *Geometer Sketchpad* (Jackiw, 1994) as an instructional tool are few in number. The study conducted by Elchuck (1992) found important data on the effectiveness of the *Sketchpad* as a dynamic tool of instruction. Subjects using the dynamic tool attained higher scores on conjecturing than subjects in the control group using a static version of *Sketchpad.* Elchuck found the variables of mathematical achievement and time of investigation of sketches to be significant factors contributing to conjecturing ability skills.

Foletta (1994) conducted a case study on four subjects of varying abilities. She found *Sketchpad* was an effective tool for construction of geometric sketches. In her study, *Sketchpad* was limited to use as a construction tool and not as tool sharing cognitive operations. Subjects failed to make logical connections between the concepts demonstrated on the screen and their application to conjectures.

A third study conducted by Frerking (1994) investigated the effectiveness of *Sketchpad* as a tool for instruction. Since the experimental groups used both the *Supposer* and *Sketchpad* it was not possible to separate the effect of using *Sketchpad* alone. The control group also used *Sketchpad,* but the instructional methodology was a deductive approach in contrast to an inductive approach used by the two experimental groups. Frerking (1994) found subjects taught by the inductive approach achieved higher mean scores than those subjects taught by the deductive approach.

The current study furthered research supporting the effectiveness of the dynamic quality of *Sketchpad* on improving conjecturing ability. This study demonstrated that subjects using *Sketchpad* achieved higher levels of thought measured by the van Hiele scale. Investigations by subjects using *Sketchpad* went beyond the use of the tool just for construction as Foletta (1994) focused on in her study. In this study the *Sketchpad* was used as a pedagogical tool and a thinking tool for developing inductive reasoning skills through observations of transformation of figures on the screen. Statistical data showed conjecturing ability achieved by subjects in the experimental group was higher than those subjects in the control group. Higher levels of thinking measured by the van Hiele scale were achieved by subjects in the experimental group (see Appendix I).

There was no research found on studies conducted on *Sketchpad* exploring its potential as an intelligent software tool to extend cognitive skills of users by sharing cognitive operations. An important component of the research of this study was the investigation of the software tool based on the use of the software as a pedagogical tool to improve solo geometry skills, and as a performance tool to upgrade concept development in producing problem solutions. Another component of the research adding to the literature on technology tools was the influence of both cognitivist and constructivist perspective on the process of learning applied to lesson procedures during the study.

Theoretical Implications on Conjecturing

Putnam, Lampert, and Peterson (1990) capture what is essential for learning mathematics in the statement "...understanding mathematics means having internalized powerful symbols and systems for representing mathematical ideas and being able to move fluently within and between them" (p. 67). The classroom environment and instructional design of the current study provided ingredients for optimizing learning within this framework.

First, the learner visualized concrete representations of geometric concepts necessary for acquiring correct knowledge for cognitive structures provided by the software tools. Second, the learner engaged interactively with the computer as an intellectual partner sharing cognitive operations for integrating new knowledge structures with previous knowledge. Third, the learner worked with a partner providing the opportunity for social interaction for sharing mathematical ideas through conversation. These three components are central to both a constructivist and cognitivist view of mathematical learning.

In order to achieve the highest level of reasoning required by rigorous proof of theorems in geometry, the conjecturing skills must first be acquired. This study focused on improving conjecturing skills through the process of inductive reasoning. This is an essential step toward developing reasoning skills required by formal proofs in geometry. Putnam, et al. (1990) clarified the role conjecturing plays in geometry:

> The simplest way to make the distinction between justifying conjectures and justifying theorems [through formal proofs] is to assert that conjectures are the result of induction; that is they are the result of observing patterns in a phenomenon, and with good reason, asserting that the pattern will continue in a way that leads to some general truth. (p. 116)

The statistical data of this study demonstrated a statistically significant difference on achievement of conjecturing ability by examining statements justifying written conjectures of subjects in the experimental group. This finding supported the potential cognitive software tools have for improving conjecturing abilities.

Recommendations for Future Research

Implications from the study suggested significant changes in mathematics education for effective use of cognitive technologies to expand cognitive capacities to improve achievement of geometric knowledge. A software tool qualifies as a cognitive technology, if it provides a "...medium that helps transcend the limitations of the mind, such as memory, in activities of thinking, learning, and problem-solving" (Pea, 1985, p. 168). *The Geometer's Sketchpad* (Jackiw, 1994) is a dynamic software program providing the user with a cognitive tool to participate in an "intellectual partnership" with the computer to share cognitive operations.

This study applied intelligent software design to instruction for the purpose of improving achievement of geometric knowledge. Results from the study indicated an increase in achievement levels of geometric thought using *The Geometer's Sketchpad* (Jackiw, 1994). Higher level scores on the Conjecture posttest imply subjects found efficient avenues for finding solutions to geometry problems.

Results from this study indicated positive effects of teaching geometry through an inductive approach versus a deductive approach. Results may indicate students learn geometry skills more efficiently by observing dynamic visualization of geometric objects. Through dynamic manipulation of objects on geometric sketches, students may attain a better understanding of concepts underlying structures and properties of Euclidean geometry.

Recent research on information processing theory confirmed if the learner is actively engaged in his/her own learning process, then the greater is the effect on stimulation of cognitive operations. Software programs designed for engaging the learner in an intellectual partnership, like *The Geometer's Sketchpad* (Jackiw, 1994), might have a potential for redefining learning and instruction of high school geometry. If this technology can extend the mind's learning capacities by sharing cognitive operations, then consequences of this sharing need to be further investigated.

Society is dominated by powerful technologies. There are software designs with great capabilities for improving learning and instruction. The challenge to educators today is to empower students through instructional use of intelligent software designs that: (a) place students in control of and responsible for their own learning, and (b) stimulate cognitive operations extending learning capacities to their highest potential.

Conclusions from this study might contribute to development of a greater awareness of how technology can empower the learner and may lead to further research on how technology can extend cognitive capacities of the mind. Further studies need to be conducted based on the hypotheses of this study without the limitations of the current study. For example, studies conducted with geometry classes that meet only during the morning hours of the school day might reveal significantly different results on the hypotheses of the current study.

Studies on the training of geometry teachers in skills for using technology and in skills for using inductive reasoning approaches are needed. As Gordon (1993) recommended from the results of his study, teachers need to be trained to lead students in learning to discover through guided inquiry inductive approaches to solving problems in geometry.

An additional suggestion for future research is to delve ever deeper into cognitive technologies and their design for improving learning of geometry students. Visualization is the key to a deeper understanding of geometric concepts. The power of visualization for understanding mathematical concepts is stated by Hanson, Munzer and Francis (1994)

> Mathematical visualization is the art of creating tangible experiences with abstract mathematical objects and their transformations. While this process has been a cornerstone of mathematical reasoning since the time of ancient geometers, interactive computer graphics systems have opened a new era in the visualization of pure geometry. (p. 73)

Embedded in cognitive technologies are designs for developing deeper understanding of geometric concepts. What is needed is to train teachers to use these technologies in their teaching to empower student learning.

Another area of research needed is to explore motivation factors embedded in the design of *Sketchpad.* Specific factors to explore are its ease of use for: (a) constructing, (b) measuring, and (c) transforming. Subjects in the control group felt their work would have been much easier if they used *Sketchpad* (see Appendix J on student interviews).

There is a need for additional research studies to be conducted on cognitive software tools in geometry on a larger randomized selection of subjects from the population. More research is needed to answer the following questions:

1. What are some further consequences of cognitive software tools on learning geometric knowledge?

2. Can intelligent software tools extend cognitive capacities for inductive reasoning and problem-solving skills by sharing cognitive operations between the computer and the user?

3. What are some additional ways technology tools can share cognitive operations to extend cognitive operations and facilitate learning?

The purpose of the study was to explore ways to assist students to learn geometry with ease, enjoyment, and efficiency. The investigator of the study, as a geometry teacher for 20 years, experienced students having difficulty with learning reasoning skills and applying them to problem solving tasks. Perhaps with an adaptation of a new paradigm for learning through conversation generated through partnerships in a technology classroom environment where the methodology of instruction is through discovery, students will find learning geometry fun, exciting, and intellectually satisfying.

Recommendations for Future Practice

At the point where information processing strategies of both cognition and technology converge, they combine with powerful brain potential to activate optimization of learning. Combining information processing research on the brain with cognitive technology research design of software programs holds the potential for creating learning environments for both teacher and learner to extend cognitive capacities for optimizing learning. The key to empower learning is to unlock information processing strategies of the brain by connecting them to powerful computer processing strategies to stimulate embedded layers of cognitive capacities making efficient connections for effective learning.

Dissemination of this learning paradigm could be accomplished by integrating this knowledge into the curriculum of teacher education programs. Another place to begin implementation of this paradigm would be workshops for in-service and pre-service geometry teachers providing information and tools for implementing:

1. Cognitive-constructivist theory and practice in curriculum.
2. Intelligent computer-assisted instruction of the use of software.
3. Inductive guided-inquiry approaches for teaching geometry.
4. Questioning methods for discovery approaches to learning.
5. Partnership methods for learning through conversational exchange of ideas.

Vision for Future Geometry Environments

Three components essential for creating an environment to optimize mathematical learning are: (a) restructuring of classrooms for use of technology, (b) redesigning curriculum for integrating software into subject area, and (c) retraining teachers in leadership skills to conduct discussions, to facilitate discovery, and to advance guided--inquiry learning.

Restructuring provides the expectations and the organizational conditions for learning. Active learning combined with adventurous teaching defines purpose and direction for innovations. Technologies act as a support and catalyst for the redesign of instruction and learning.

Advances in microcomputer technology together with intelligent software design means there are few constraints educational software cannot accomplish. Technology tools alone cannot create an engaging learning environment. The teacher is the model leader for motivating, guiding, and learning along with students. The prepared teacher equipped with knowledge on the use of intelligent computer tools, with the student actively engaged in partnership with the computer, working together hold the potential for creating a synergistic effect on creating environments where learning is optimized.

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APPENDIXES

APPENDIX A

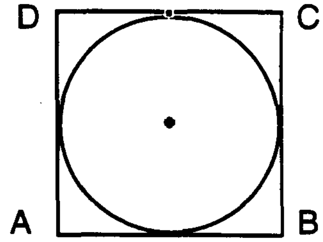
POSTTEST

Name ________________________________ May 18, 1995 Total Score _______

GEOMETRY PROBLEMS FOR POSTTEST PART 1

Please complete the following problems. For each of the problems 1 - 10 write in your own words the conjecture (s), used to solve the problems.

1. In the figure shown to the right, the radius of the inscribed circle is 5. What is the area of the square ABCD?



(A) 10π (B) 25π (C) 20

(D) 25 (E) 100

ANSWER (1 pt): ____________________

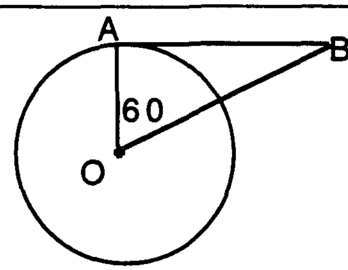
CONJECTURE (4 pts): ____

____

____

____

2. In the figure shown to the right, AB is tangent to the circle with center O. If the radius of the circle is 12, then AB =



(A) $12\sqrt{2}$ (B) $12\sqrt{3}$ (C) 6

(D) 8π (E) 16π

ANSWER (3 pts): ____________________

CONJECTURE (2 pts): ____

____

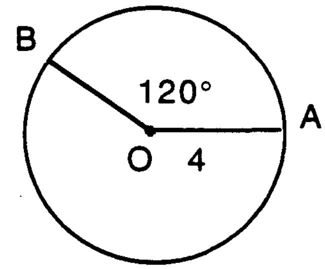
____

____

____

Score on this page________________

3. The circle shown to the right has radius 4 and center 0. If the measure of angle AOB is 120°, what is the length of the minor arc AB?



(A) $\frac{2\pi}{3}$ (B) $\frac{8\pi}{3}$ (C) $\frac{16\pi}{3}$

(D) 8π (E) 16π

ANSWER (1pt): ____________________

CONJECTURE (4 pts): ____

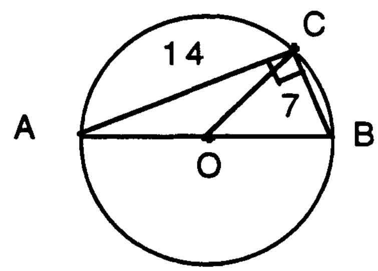
____

____

____

____

4. In the circle with center O and diameter AB, as shown in the figure to the right, OC =



(A) 7 (B) $7\sqrt{2}$ (C) $7\sqrt{3}$

(D) $\frac{7\sqrt{3}}{2}$ (E) $\frac{7\sqrt{5}}{2}$

ANSWER (3pts): ____________________

CONJECTURE (2pts): ____

____

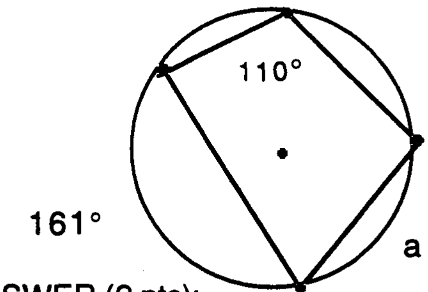
____

____

____

Score on this page ________________

5. Find ‘a’



ANSWER (2 pts): ____________________

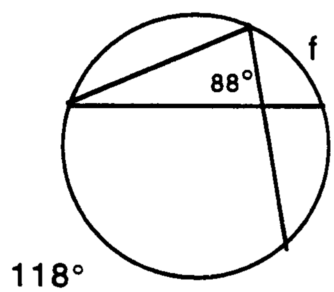
CONJECTURE (3 pts):____

____

____

____

6. Find f



ANSWER (3 pts): f = ______________

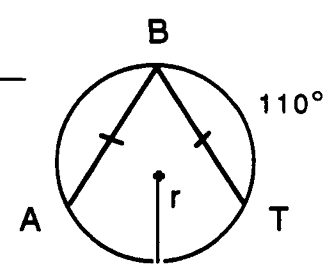
CONJECTURE (2pts):____

____

____

7. r = 36 cm. The arc length of AB is

ANSWER (1pt): ____________________



CONJECTURE (4pts):________________________

____

____

Score on this page______________

8. What is the radius of a circle that has an arc with a degree measure of 180 and an arc length of 90π ?

ANSWER (3 pts): ____________________

CONJECTURE (2pts): ____

____

____

GEOMETRY PROBLEMS FOR POSTTEST - PART 2

TELL WHETHER EACH OF THE STATEMENTS IS TRUE ALWAYS, SOMETIMES, OR NEVER AND DEFEND YOUR REASONING.

1. Every chord is a diameter. Answer (1 pt): ____________________

Reasoning (4pts): ____

____

____

____

____

2. Every radius is a chord. Answer (1 pt): ____________________

Reasoning (4 pts): ____

____

____

____

____

____

Score on this page________________

GEOMETRY PROBLEMS FOR POSTTEST - PART 3

COMPLETE EACH CONJECTURE AND EXPLAIN YOUR REASONING (Hint: draw figures).

1. Every angle inscribed in a semicircle is a (n) (1pt):_______________.

Reasoning (4pts):____

____

____

____

2. Tangents drawn to a circle from a point outside the circle are (1 pt) __________.

Reasoning (4 pts):____

____

____

____

____

3. The opposite angles of a quadrilateral inscribed in a circle are (1pt)___________.

Reasoning (4 pts) ____

____

____

____

____

Score on this page_______________

GEOMETRY PROBLEMS FOR POSTTEST - PART 4

1. What is the relationship between the circumference and the diameter of a circle? Use this information to find the diameter if the circumference is 31 cm write in terms of π.

Reasoning (4 pts)____

Diameter of the circle whose circumference is 31 cm. = (1 pt)________________.

2. How many radii can be marked off along the circumference? Explain why. (Hint: start and end with a formula)

Score on this page________________

GEOMETRY PROBLEMS FOR POSTTEST - PART 5

Instructions for Control Group Subjects: Construct and explain in writing how to construct a rhombus and its inscribed and circumscribed circle.

Part A: Using your geometry tools: compass, straightedge, and protractor first construct a rhombus inscribed in a circle with radius r:
Explain in writing the steps you followed to construct your sketch.

r

Part B: Then construct a circle inscribed in the rhombus.
Explain in writing the steps you followed to construct your sketch.

Use the opposite side of this paper for your construction. Write your explanation in the space below.

Explanation for Part A:

Explanation for Part B:

GEOMETRY PROBLEMS FOR POSTTEST - PART 5

Instructions for Experimental Group Subjects: Construct and explain in writing how to construct a rhombus and its inscribed and circumscribed circle.

Part A: Using *Sketchpad*: first construct a rhombus inscribed in a circle with radius r.
Explain in writing the steps you followed to construct your sketch.

Part B: Then construct a circle inscribed in the same rhombus.
Explain in writing the steps you followed to construct your sketch.

Save your construction on your disk and label it test construction. Write your explanation in the space below.

Explanation for Part A:

Explanation for Part B:

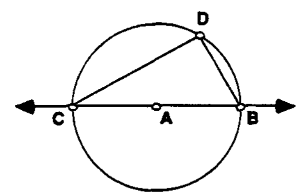
APPENDIX B

INVESTIGATION SHEET SAMPLES

Investigation: More on Circles, Angles, and Arcs

You already know about some relationships among central angles, inscribed angles, and the arcs they intercept. In this activity, you'll discover more relationships that follow from the ones you already know. As you discover them, think about why your conjectures must be true in terms of what you already know about arcs.

Sketch



Step 1: Construct circle AB.

Step 2: Construct $\overleftrightarrow{AB}$.

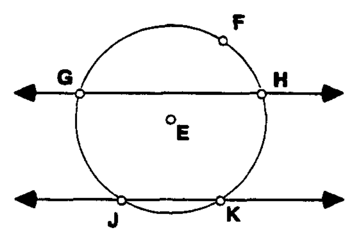
Step 3: Construct $\overline{CD}$, where C is the other point of intersection of $\overleftrightarrow{AB}$ and the circle and D is a point on the circle.

Step 4: Construct $\overline{DB}$.

Investigate: Measure $\angle CDB$ and move point D around the circle. What can you say about any angle inscribed in a semicircle?

Conjecture: Write your conjectures below.

Sketch



Step 1: Construct circle EF.

Step 2: Construct $\overleftrightarrow{GH}$, where G and H are on the circle.

Step 3: Construct point J on the circle and a line through J, parallel to $\overleftrightarrow{GH}$.

Step 4: Construct point K, the other point of intersection of the parallel line with the circle.

Investigate: Measure arcs GJ and HK. Move points G, H, J, and F. What can you say about arcs intercepted by parallel lines?

Conjecture: Write a conjecture below.

Present Your Findings: Discuss your results with your partner or group. To present your findings you could print a captioned sketch with several circles with central and inscribed angles. Show measures that illustrate your conjectures.

Explore More

Construct a circle and inscribe a quadrilateral in it. Measure the four angles of the quadrilateral. Make a conjecture about opposite angles of a quadrilateral inscribed in a circle.

Investigation: More on Circles, Angles, and Arcs

Student Audience: High School

Prerequisites: Students should know basic relationships among **central angles**, **inscribed angles**, and the **arcs** they intercept.

Sketchpad Proficiency: Beginner

Example Sketch: **More Angles and Arcs** (Mac) or **6circles\angsarcs.gsp** (Windows)

Class Time: 20-30 minutes. You might want to do this investigation in the same class period as Circles and Angles.

Construction Tips

The first construction is a simple construction of a triangle inscribed in a semicircle

Investigate/Conjecture

Students should conjecture:

Any angle inscribed in a semicircle is a right angle.

Construction Tips

The second construction is a simple construction of two parallel lines intercepting a circle.

Investigate/Conjecture

Students should conjecture:

Arcs intercepted by parallel lines are congruent.

Explore More

The opposite angles in an inscribed quadrilateral intercept two arcs that make up the entire circle. Therefore, the sum of the arcs they intercept is 360°, so their sum must be 180°. Hence, opposite angles in a quadrilateral inscribed in a circle are supplementary. (This should reveal to students what type of quadrilaterals can be circumscribed. Such quadrilaterals are called cyclic.)

APPENDIX C

POSTTEST VALIDATION PANEL MEMBERS

Titles, Position, and Qualifications

Panel Member 1

Title:

Thomas J. Lester

Director of NSF Math Matters, a National Science
Foundation Project in Staff Development in Mathematics
California Department of Education

Lecturer, Mathematics Department
Sacramento State University
Sacramento, CA.

Qualifications:

Author of five mathematics' books:

Calculus, Trigonometry, Plexers(2), Investigation Mathematics, An Interactive Approach

Member on Joint CSUC-UC Workgroup on Diagnostic
Testing in Mathematics (MDTP)

Co-Director of the Caltrans Transportation Demand Management
Project

Panel Member 2

Title:

Wallace Etterbeek, Ph. D.

Professor of Mathematics
Mathematics Department
Sacramento State University
Sacramento, CA.

Qualifications:

Table Leader for the Advancement Placement Mathematics Examination

Instructor in Mathematics Program for Gifted and Talented Students Mathematics

Math Consultant to San Juan Unified School District

Member on Joint CSUC-UC (California State University at Sacramento University of California) workgroup on Diagnostic Exams in Precalculus Mathematics

Panel Member 3

Title:

Patricia Duckhorn

Regional Coordinator of NSF Math Matters, a National Science Foundation Project in Staff Development in Mathematics
California Department of Education
Sacramento, CA.

Qualifications:

Math Coach: Middle School Demonstration Project.
John Still Center for the Performing Arts.
Sacramento City Unified School District

Presenter of Workshops for K-8 teachers, administrators, and school board members with emphasis on the changes in math education as outlined by the *California Math Framework and the Curriculum and Evaluation Standards* of the NCTM

Panel Member 4

Title:

Nancy Aaberg

Acting Director of the Northern California Mathematics Project (K-12)
University of California at Davis

Mathematics Coordinator (K-12)
Yuba City Unified School District

NSF Math Matters Leadership Coordinator
California Department of Education

Instructor, Yuba Community College

Qualifications

Mathematics Workshop Leader (K-12) on the following topics:

Participate in California Math A planning and implementation.

Present content specific and developmentally appropriate ideas for elementary, middle grade, high school, and college teachers.

Prepare leadership teams in decision making skills related to mathematics curriculum, teaching, and student outcomes.

APPENDIX D

POSTTEST VALIDATION PANEL FORM

Posttest Validation Panel Form

The questions on this form apply to the posttest and to the van Hiele (1986) scoring scale for the level of concept development indicated by statements of conjectures.

1. Face Validity

   Does this test appear to measure what it is intended to measure with all items relating clearly and obviously to the purpose?

2. Concept Validity

   Are all concepts on the Properties of the Circle included in test items?

3. Content Validity

   Are all topics of Chapter 6 on the Circle included in the test items?

4. Item analysis

   a. Are there problems that should be eliminated?

   b. Are there problems that should be modified?

   c. Are there additional problems that should be included?

5. Format

   a. Are instructions clearly stated?

   b. Are formatting modifications required?

   c. Are diagrams clearly marked?

APPENDIX E

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3 March 1995

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St. Paul Convent
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San Francisco, CA 94131

Dear Sr. Lynn,

This note grants you permission to include up to six screen captures from The Geometer's Sketchpad in your dissertation proposal. Our credit should read, "The Geometer's Sketchpad, Key Curriculum Press, P.O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH." Please do *not* omit the phone number—we'd like to be as accessible as possible to anyone interested in ordering materials from us.

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APPENDIX F

POSTTEST SCORES ON GEOMETRIC KNOWLEDGE OF SUBJECTS IN THE EXPERIMENTAL AND CONTROL GROUPS

Posttest Scores on Geometric Knowledge of Subjects in the Experimental and Control Groups

Experimental (n = 20)	Control (n = 27)
30	62
43	60
24	50.5
52	44.5
44	44.5
27.5	45.5
62	40
12	40.5
34	37.5
35.5	32.5
21	35.5
39	35

(table continues)

Experimental (n = 20)	Control (n = 27)
41	27
15	28
38	27
28	14.5
28	27.5
33	33.5
26	32
34	27.5
	22
	18
	14.5
	6.5
	16.5
	16
	4

APPENDIX G

POSTTEST SCORES ON GEOMETRIC CONSTRUCTIONS OF SUBJECTS IN THE EXPERIMENTAL AND CONTROL GROUPS

Posttest Scores on Geometric Constructions of Subjects in the Experimental and Control Groups

Experimental (n = 20)	Control (n = 27)
25	25
25	25
21	22
25	25
21	25
15	25
25	22
23	20
25	25
22	20
22	25

(table continues)

Experimental (n = 20)	Control (n = 27)
20	22
25	23
19	23
20	22
19	16
15	14
25	18
23	22
25	25
	19
	22
	11
	6
	0

APPENDIX H

THIRD QUARTER GEOMETRY GRADES OF SUBJECTS

IN THE

EXPERIMENTAL AND CONTROL GROUPS

Third Quarter Grades of Subjects in the Experimental and Control Groups

Experimental (n = 20)	Control (n = 27)
79.21	89.45
74.29	88.75
73.16	88.61
90.76	73.01
87.41	77.97
85.72	85.83
98.42	89.45
72.44	85.86
82.36	75.63
77.03	63.39
68.67	83.42

(table continues)

Experimental (n = 20)	Control (n = 27)
83.25	80.62
82.08	85.53
62.12	77.15
79.99	84.24
82.72	70.03
83.25	83.81
84.34	62.62
77.50	71.12
90.25	77.84
	65.17
	77.84
	75.90
	70.64
	68.27
	60.84
	70.83

APPENDIX I

POSTTEST SCORES ON CONJECTURES OF SUBJECTS IN THE EXPERIMENTAL AND CONTROL GROUPS

Posttest on Conjectures Scores of Subjects in the Experimental and Control Groups

Experimental (n = 20)	Control (n = 27)
15	23
17	6
9	8
24	18
22	2
12	10
23	6
9	7
9	5
16	6
12	8
14	9

(table continues)

Experimental (n = 20)	Control (n = 27)
14	12
5	9
18	12
12	23
17	8
11	3
9	23
21	9
	3
	10
	15
	0
	11
	5
	12

APPENDIX J

TRANSCRIPT OF TAPED INTERVIEWS WITH SUBJECTS IN THE EXPERIMENTAL GROUP

Statements that follow are from the taped interviews between the investigator and subjects from the experimental group on the use of the cognitive software tool, *Geometer Sketchpad* (1994). The answers are direct quotes from the taped interviews. Grammar was not changed in order to keep the responses reflective of authentic student language. When the translated dialogue was unclear the investigator added words in brackets to help understand the meaning of student answers.

Transcripts Interviews with Subjects from the Experimental Group

Sophomore Student #1

Question:

How did the Sketchpad assist you in learning concepts and properties of the circle?

Answer

I felt like it was easy to do when you had to do more than one circle or when you had to pull the tangents around to see how relationships were- [affected]. It [*Sketchpad*] was easier because you didn't have to draw a whole new circle or anything, but when I got to the test, I felt even though I understood the material I didn't understand how to do the test, so it was kind of bad, so I don't know whether or not I liked using the *Sketchpad*.

Sophomore Student #2

Question:

When you manipulated objects on the screen, and tabulated measurements how did that enable you to understand the circle properties? For example, changing arcs and angles to show the relationship of the measures of the arc to the angle?

Student Answer

I think it was pretty good. It was easy, I didn't have to draw a whole new circle when I wanted to make a change. It was easy to tabulate the solutions and have all my conclusions in neat little squares and boxes.

Sophomore Student #3

Question: As a tool for learning, what capabilities of *Sketchpad* helped you to understand the concepts of the circle? How did measuring, drawing, or tabulating help you to understand concepts about the circle?

Student Answer

I don't think they did. I don't learn by computers. I am not used to computers so it wasn't very good for me. I'm used to the book. I like to write things down and keep going over them. I did not like the experience. It didn't help me.

Sophomore Student #4

Question:

What problem solving skills did you share with the *Sketchpad* and what features of the *Sketchpad* helped you to learn the properties of the circle?

Student Answer (Sophomore).

You could take a point on the circle wherever it is and move it around to check to verify angles and how you could measure the angles-- that helped me because when we were making sure to see if like angles and things worked out you could move it around and it helped me.

Sophomore Student #5

Question:

How did using the *Sketchpad* help you to reason to conclusions and write conjectures from your observations?

Student Answer (Sophomore).

Those were real simple to do because it's all right in front of you, it's like doing it on paper.

Sophomore Student #6

Question:

Did you find the *Sketchpad* easier to use than using paper, pencil and the geometry textbook?

Student Answer (Sophomore)

Overall, no, because you had the *Sketchpad* here at school and the textbook at home but it doesn't help to do homework from the text. The computer helped me. It gave me more skills. I could use the information at home. It will be perfect when computers are everywhere.

Sophomore Student #7

Question

As a tool for learning what capabilities of Sketchpad helped you learn geometric concepts?

Student Answer (Sophomore)

The dragging features helped when you drew the tangents to the circle. It also helped when you had the choice of all the tools. It helped a lot. Like how you could measure segments. Like how you could measure arc length and arc measure and slopes. You select all points and you can do the chart and tabulate all measures in a chart-- that helped by dragging. When writing conjectures you can look at it [the sketch] and know the measures were correct . You could see how they [tools] measured of lengths and angles all fit together and then just write down the conjecture. I do it better if I write it out. It helped getting conjectures better from the computer rather than doing it by hand. I had the skills in mind when I did the written work. In other chapters I didn't always get the conjectures I would get like two out of ten. When I did it written, on the computer you could see it and visualize it more so it was pretty much easier.

Sophomore Student #8

Question

What problems solving skills did you share with Sketchpad while completing construction on the *Sketchpad*?

Student Answer (Sophomore)

[When I used the computer] I found the computer a lot harder to remember what I did on the computer on the test. It would have been much easier to remember if I have [had] done it on paper. When I done [sic] it with my own hands when I write on paper because you are more involved when working with your own hands. On the computer it goes a lot quicker but like you write a word on paper you remember it better by writing it rather than just seeing it. It's a lot easier. It is better for me to write down on paper. Skills applied on the computer were not remembered when doing the test.

Student #9

Question

How do you think the *Sketchpad* helped you to reason and find solutions to the problems on the explorations and investigations you did on the circle?

Student Answer (Junior)

Well, it was easier on the computer-- to make the constructions and instead of all the pencil marks on the paper we could see what we did and what we did and how we constructed it. For me it was easier I could find the lengths on the computer, that helped me a lot.

Student #10

Question

How did the fact that you could manipulate objects on the screen and visualize them while recording and tabulating data on the screen assist you in learning geometric concepts?

Answer

The computer gave me a lot of options on the computer, what I could do, what I could manipulate and putting in another inscribed angle or another chord or something on the circle. It just made things easier and I could see what I was doing. What mistakes I made. I could see what I was doing to help me make the conjectures.

Student #11

Question

When you did exercises without assistance from the computer, did you think the skills you learned on the computer helped you to solve the problems.

Answer

Yeah, to me it was easier to learn on the computer than it was to do the work on pencil and paper. To me it was boring, things were not clicking in my mind, but when I see it visually on the computer I can learn more, it gets to my head more. It stays there better, instead of boring teacher-student textbook way. I like learning on the computer.

Student #12 (Sophomore)

Question

As a tool what capabilities of the *Sketchpad* helped you learn the circle concepts that you studied?

Answer

The way it gave you the measurements they figured it out for you. It was easier to see it.

Question

How did the fact that you could manipulate objects and observe their changing measures affect your understanding of geometric concepts?

Answer

I understood it a lot better. I could move the objects around. It was clear on the computer.

Question

How did it help you reason to conjectures?

Answer

Everything was easier. I couldn't make as many mistakes as I would have if I did it myself. The computer really didn't let you make mistakes. When I got home to do homework, it wasn't that easy away from the computer. The conjectures we had stayed on the computer. When I went home I didn't have the conjectures with me. I'm not sure I learned better. It [the computer] was at school and I didn't have it at home. It made things clearer, when I had the computer, but when I worked at home it was confusing.

Student #13 (Sophomore)

Question

Did you find it easier to analyze data when using the Sketchpad program?

How do you think that your ability to manipulate data on the computer screen helped you understand the circle concepts?

Answer

It made it easier because instead of having to keep redraw the sketches I get confused if I have to keep redrawing over and over. It was just to move a button to see the difference it made instead of having to keep on changing it by drawing it over. It made it easier to understand.

Student #14 (Sophomore)

Question

When you did your work away from the computer, were you able to apply the skills you learned in the absence of the computer?

Answer

It made it a lot easier to apply some of the conjectures helped and the worksheets with the building of the constructions helped too. What would you like to say about your learning experience. It was easier to learn.

Some of the stuff like the arcs I didn't understand that part.

Student #14 (Sophomore)

Question

What was the effect of the *Sketchpad's* capability on you being able to observe manipulations of geometric objects? How did that affect your learning of geometric concepts?

Answer

I thought with the Sketchpad it was easier I have the tools and make sure it was exact measurements. When I am drawing sometimes it might be off and I am not able to find the conjecture. When I am able to use the computer, my conjecture comes easier to me and I am able to find it [conjectures] much easier. I liked the final product which was perfect and I was really proud of the final sketch. When I am away from the computer some of the conjectures I was able to apply it to my homework easily.

APPENDIX K

INSTRUCTIONS AND LESSON COMMENTARY FOR SUBJECTS IN THE EXPERIMENTAL AND CONTROL GROUPS

Lesson Commentary

Control Group F- Class - Day One

Date: Monday, April 24, 1995 Time: 1:05 PM

Lesson Title: Topic 6-1 Defining Circles

Text: Discovering Geometry: An Inductive Approach (Serra, 1993)

Pages: 261-265

Teacher's Guide and Answer Key:

Discovering Geometry (Key Curriculum Press, 1989, 1990)

Pages: 80-81

1. Students had been told there would be an observer in the room collecting data on the lessons in Chapter 6. Students were also told that one group would be called an experimental group and the other would be a control group.

2. Students chose a partner to work with during the time of the study.

3. Students were given a syllabus of the work to be accomplished for the first week of the study.

4. Students in the control group were told to bring geometry tools to class each day.

5. Lesson introduction:
Students were asked to tell what they knew about a circle.

Some responses were:

A circle has 360 degrees and is continuous.
π is used to calculate the circumference.
Two names for π are 3.14 and 22/7.
π is a constant number.

6. Definitions of a circle, a chord, a diameter, and circumference were formulated with the teacher leading the discussion. Students responded by editing, adding to, and 'cleaning up' suggestions made by other students.

The teacher also described (on the overhead projector) congruent radii, congruent circles and concentric circles.

7. Responses from the students were elicited for characteristics of a good definition:

A good definition is: (a) reversible, (b) precise, and (c) classifies. Using these qualities students, were asked to choose a partner to work with and formulate the definitions for terms discussed in class.

8. Working together in partners, the students wrote the definitions of the following terms on topic 6-1 and completed exercise Set A: 1-6.

9. The homework assignment for today is to complete the definitions begun in class and do exercise Set B 1-14 on pp. 264 and 265; and Set D 1 and 3.

Lesson Commentary

Control Group F- Class - Day Two of Study

Date: Tuesday., Apr. 25, 1995 Time: 1:05 PM

Lesson Title: Topic 6-1 Defining Circles Continued

Text: Discovering Geometry: An Inductive Approach (Serra, 1993)
Pages: 261-265

Teacher's Guide and Answer Key:

Discovering Geometry (Key Curriculum Press, 1989, 1990)
Pages: 80-81

Objectives:

To define a circle and learn the related vocabulary
To practice creating definitions
To identify the parts of a circle
To review construction skills
To develop writing skills and cooperative behavior

Terms defined:

Circle	Radius	Center of Circle
Congruent Circles	Concentric Circles	Chord
Diameter	Secant	Tangent
Inscribed Angle	Central Angle	Arc of a circle
Semicircle	Minor Arc	Major Arc

Class Activity:

Exercise set A p. 262 (1-6):

Class began with a discussion of definitions of each geometric term learned the day before. The teacher led the class discussion. Students contributed the definitions they had formulated. Agreement was made on a common set of definitions and those definitions were added to students' list in their notebooks.

Homework Assignment Sheet:
Exercise Set B p. 264 (1-14) was handed in to the teacher.

Class Activity Work:

Exercise Set C p. 265 - (1-5) Do on a separate piece of paper in class and complete constructions for homework.

Lesson Procedure:

1. New definitions were written on the board. Responses from students on a final consensus of a good definition of secant, tangent, inscribed angle, and central angle were discussed.

2. Visualization of definitions were provided by pictures drawn on the board and on the overhead projector.

3. Construction tool were used for construction of sketches. Constructions were drawn and the teacher used the overhead projector to show what the sketches of the students should look like.

Exercise #1 was to draw two overlapping circles intersecting in two points, and joining these points, and joining the two centers of the circle to form a rhombus.

Exercise #2 was to construct two tangent circles and measure the distance between the two centers (distance = 2 times the radius).

Exercise #3 was to construct 2 concentric circles.

Homework assignment is to complete constructions 4 & 5 .

APPENDIX L

LESSON PLANS FOR

EXPERIMENTAL AND CONTROL GROUPS

LESSON PLAN

Control Group F- Class - Day One of Study

Date: Monday, April 24, 1995 Time: 1:05 PM

Lesson Title: Topic 6-1 Defining Circles

Text:
Discovering Geometry: An Inductive Approach (Serra, 1993)

Pages: 261-265

Teacher's Guide and Answer Key:

Discovering Geometry (Key Curriculum Press, 1989, 1990)

Pages: 80-81

Objectives:

To define a circle and learn its related vocabulary
To practice creating definitions
To identify the parts of a circle
To review construction skills
To develop writing skills and cooperative behavior

Presentation of **Terms** to be defined:

Circle	Radius	Center of Circle
Congruent Circles	Concentric Circles	Chord
Diameter	Secant	Tangent
Inscribed Angle	Central Angle	Arc of a circle
Semicircle	Minor Arc	Major Arc

Class Activity:

<u>Exercise set A p. 262 (1-6):</u>

Write a definition of each geometric term. Discuss your definitions with others in your group. Agree on a common set of definitions and add them to your definition list. Draw and label a picture to illustrate each definition. Hand in assignment at the end of the class period.

Homework Assignment:

Name

____

<u>Exercise Set B p. 264 (1-14)</u>: See diagrams on p. 264. Write solutions on the form below:

1. ________ ________ ________

2. ________ ________ ________ ________

3. ________

4. ________ ________

5. ________ ________ ________ ________ ________

6. ________ ________ ________ ________ ________

7. ________ ________

8. ________ ________

9. ________ ________

10. ________ ________ ________

11. ________ ________

12. ________

13. ________

14. ________ ________

Exercise Set C p. 265 - (1-5) Do on a separate piece of paper.

Control Group F- Class - Day Two of Study

Date: Tuesday, Apr. 25, 1995 Time: 1:05 PM

Lesson Title: Topic 6-1 Defining Circles Continued

Text:

Discovering Geometry: An Inductive Approach (Serra, 1993)
Pages: 261-265

Teacher's Guide and Answer Key:

Discovering Geometry (Key Curriculum Press, 1989, 1990)

Pages: 80-81

Objectives:

- To define a circle and learn its related vocabulary
- To practice creating definitions
- To identify the parts of a circle
- To review construction skills
- To develop writing skills and cooperative behavior

Terms defined:

Circle	Radius	Center of Circle
Congruent Circles	Concentric Circles	Chord
Diameter	Secant	Tangent
Inscribed Angle	Central Angle	Arc of a circle
Semicircle	Minor Arc	Major Arc

Class Activity:

Exercise set A p. 262 (1-6):

Discussion of definitions of each geometric term. Class discussion with teacher leading and students contributing. Agreement was made on a common set of definitions and they were added to students' definitions list in their notebooks.

Homework Assignment Sheet:
Exercise Set B p. 264 (1-14) was handed into the teacher.

Class Activity Work:

Exercise Set C p. 265 - (1-5) Do on a separate piece of paper in class and complete constructions for homework.

Lesson Procedure:

1. Definitions were written on the board using responses from students for final consensus of a good definition of secant, tangent, inscribed angle, and central angle.

2. Visualization of definitions were provided by pictures drawn on the board and on the overhead projector.

3. Construction tools: compass and straightedge were tools used for construction of sketches. Constructions were drawn and the teacher used the overhead projector to show what the picture should look like.

EXERCISE # 1 was to draw two overlapping circles intersecting in two points, and joining these points and the two centers to form a rhombus.

EXERCISE# 2 was to construct two tangent circles and measure the distance between the two centers (distance = 2 times the radius).

EXCERCISE# 3 was to construct 2 concentric circles.

Homework assignment is to complete constructions 4 & 5 .

LESSON PLAN

Experimental Group C-Class - Day One & Two of Study

Date: Tuesday, April 25, 1995 Time: 10:35

Lesson Title: Topic 6-1 Defining Circles

{The lesson presentation for the Control Group can be used for the Experimental Group- What follows is the additional information needed for using the Sketchpad]

Exploring Geometry with the Geometer's Sketchpad Blackline Masters for Use with The Geometer's Sketchpad

1. Exploration: Chords in a Circle p. 191 and 192.
2. Exercise Set C page 265- Discovering Geometry Investigation -Introductory Circle Constructions
3. Use Circle by Center + Radius from Construct menu. Measure AB/PQ in problem
4. What is this constant?

Presentation of Terms to be defined for Sketchpad::

1. Definition of arc measure in a circle: arc measure is called ArcAngle by Sketchpad to distinguish it from arc length.
2. Terms to know: central angle, arc and chord.

Sketchpad Proficiency: Beginner

Example Sketch: Congruent Chords Step 3. Use the command Circle+ Radius in the Construct menu.

Investigate/Conjecture: Students should make the following conjectures:

1. Congruent chords intercept congruent arcs.
2. The arcs between parallel chords are congruent.
3. Chords in a circle that are closest to the center are longest. The longest chord in a circle is a diameter.
4. The perpendicular bisector of any chord in a circle goes through the center.
5. The measure of an inscribed angle is 1/2 the measure of the arc it intercepts.
6. If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.
7. The perpendicular bisectors of chords intersect at the center of the circle. If a circle's center is hidden, it can be found by constructing two non-parallel chords and their perpendicular bisectors. The point of intersection of these chords is the circle's center.

Exercises: Set B p. 264, (1 - 14), Set C p. 265, (1-5), Set D p.165, (1-3)

Geometer's Sketchpad: exercise Set C p. 265 Introductory Circle

Constructions Use Circle by Center + Radius from Construct menu.

Measure AB/PQ in Problem 4. What is this constant?

Homework Assignment:
Name

____

Exercise Set B p. 264 (1-14): See diagrams on p. 264. Write solutions on the form below:

1. _______ _________ __________

2. ________ ________ ________ ________ ________

3. ________

4. ________ ________

5. ________ ________ ________ ________ ________

6. ________ ________ ________ ________ ________

7. ________ ________

8. ________ ________

9. ________ ________

10. ________ ________ ________

11. ________ ________

12. ________

13. ________

14. ________ ________

Exercise Set C p. 265 - (1-5) Do on a separate piece of paper.

APPENDIX M

LETTER OF PERMISSION TO CONDUCT THE STUDY

through christ our lord

DIOCESE OF OAKLAND
SCHOOL DEPARTMENT

2900 LAKE SHORE AVENUE • OAKLAND, CALIFORNIA, 94610-3697 • 510/893-4711
FAX: 510/451-6516 Connect ID: DioOaklandSD

April 25, 1995

Sister M. Lynn Lester, BVM
323 - 29th Street
San Francisco, CA 94131

Dear Sister Lynn:

Thank you for your letter of April 18 regarding your research. I wish you great success in your studies.

I would love to have a copy of the results upon your completion.

Best wishes,

Ann Manchester

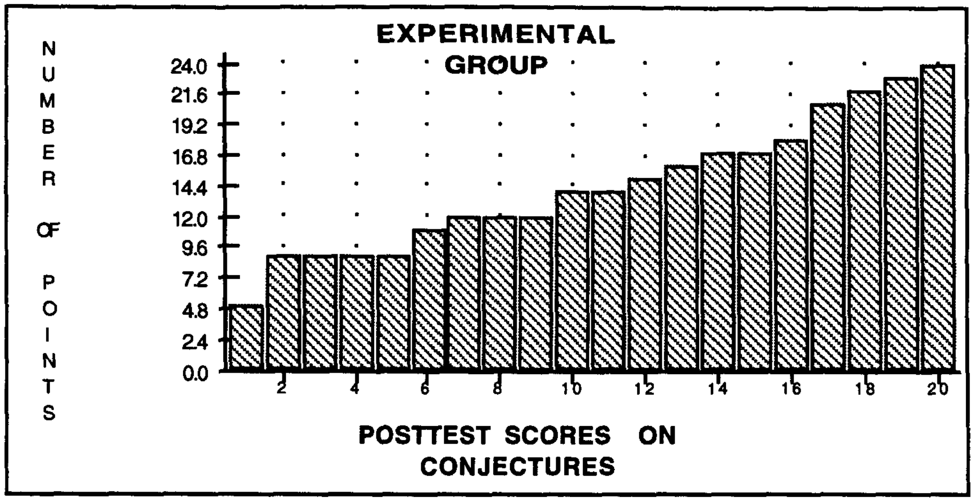
Ann Meyers Manchester, Ed.D.
Superintendent of Schools

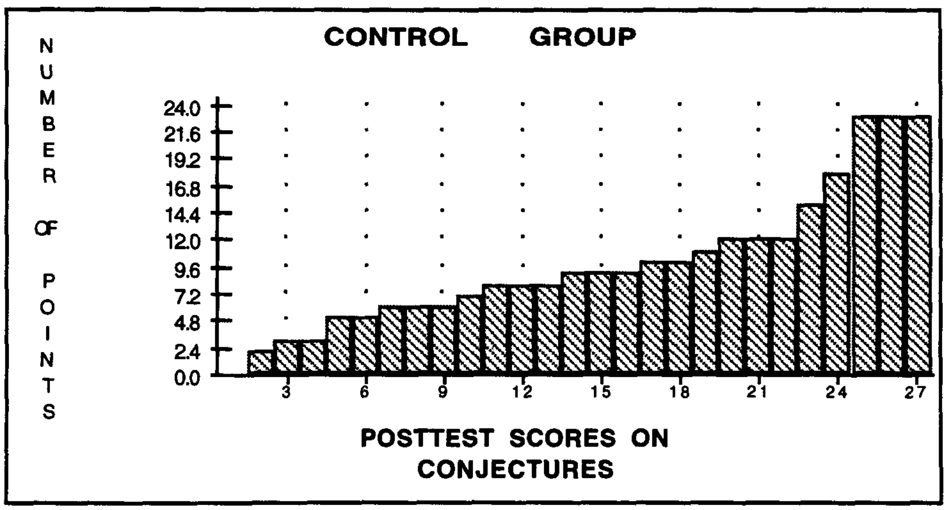
AMM:so

APPENDIX N

POSTTEST SCORES ON CONJECTURES FOR THE EXPERIMENTAL AND CONTROL GROUPS

Graphs of posttest scores on Conjectures for the experimental and control groups.





Note. The number of subjects in the experimental group was 20. The number of subjects in the control group was 27.

APPENDIX O

COPYWRIGHT PERMISSION FOR

CHAPTER SIX TEST

KEY CURRICULUM PRESS
Innovators in Mathematics Education

2512 Martin Luther King Jr. Way
P.O. Box 2304 · Berkeley · California 94702

7 March 1995

Sr. Lynn Lester
St. Paul Convent
323 29th Street
San Francisco, CA 94131

Dear Sr. Lynn,

This note grants you permission to include (1) the Chapter 6 Tests and Quizzes Form A (combined as a post-test) from *Discovering Geometry: An Inductive Approach Teacher's Resource Book* and (2) pp 199 and 201 from *Exploring Geometry with The Geometer's Sketchpad* in your dissertation proposal. Our credit should read, "The Geometer's Sketchpad, Key Curriculum Press, P.O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH." Please do *not* omit the phone number—we'd like to be as accessible as possible to anyone interested in ordering materials from us.

Thank you very much for your interest in Key Curriculum Press.

Sincerely,

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Chapter 6 Test — Form A

To complete this test you will need the Chapter 6 Test answer sheet. You may also need scratch paper. Do not write on this test. Put your answers on your Chapter 6 Test answer sheet. Attach any scratch paper that you may have used to your answer sheet.

Part A

Identify each statement as true or false

1. If A is (0, 0); B is (2, 3); C is (4, 8); and D is (7, 6), then $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.
2. If ΔDOG is congruent to ΔCAT, then $\overline{DG}$ is congruent to $\overline{CT}$.
3. The degree measure of an arc is equal to one-half the measure of its central angle.
4. The ratio of the diameter divided by the circumference of a circle is π.
5. A chord is a segment connecting the center of a circle to any point of the circle.
6. Two circles are congruent if they have the same radius.

Part B

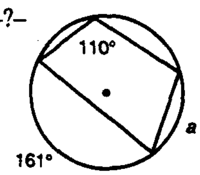
Complete each conjecture.

1. Tangent segments to a circle from a point outside the circle are —?—.
2. Every angle inscribed in a semicircle is a(n) —?—.
3. The arc length equals the degree measure of the arc divided by 360, times —?—.
4. A tangent to a circle is —?— to the radius drawn to the point of tangency.
5. The measure of a(n) —?— angle equals half the measure of the intercepted arc.
6. The opposite angles of a quadrilateral inscribed in a circle are —?—.

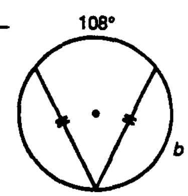
Part C

Use your new conjectures to solve each problem.

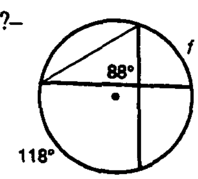
1. a = –?–



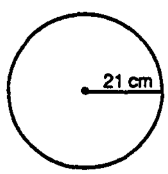
2. b = –?–



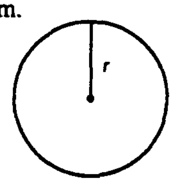
3. f = –?–



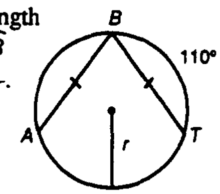
4. Use 22/7 for π. What is the circumference?



5. Circumference is 24π m. r = –?–



6. r = 36 cm. The arc length of $\overset{\frown}{AB}$ is –?–.



Part D

Use your new conjectures to solve each word problem.

1. What is the measure of the angle formed by the hands of a clock at 9:40?
2. What is the diameter of a circle that has an arc with a degree measure of 80 and an arc length of 88π cm?

Part E

1. Construct an acute scalene triangle ΔABC. Construct the circumscribed circle.
2. Construct a rhombus. Construct the inscribed circle.

Chapter 6 Test — Form A Answer Sheet

My Name is ____________________ Period ____ Date __________

Part A (2 points each)

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Part B (4 points each)

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________

Part C (6 points each)

1. $a =$ ______
2. $b =$ ______
3. $f =$ ______
4. ________
5. $r =$ ______
6. ________

Part D (7 points each)

1. ______
2. ______

Part E (7 points each)

1.
2.