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Prediction of a Non-Abelian Fractional Quantum Hall State with f-Wave Pairing of Composite Fermions in Wide Quantum Wells

W. N. Faugno,[1] Ajit C. Balram,[2] Maissam Barkeshli,[3] and J. K. Jain[1]
[1]Department of Physics, 104 Davey Lab, Pennsylvania State University, University Park, Pennsylvania 16802, USA
[2]Niels Bohr International Academy and the Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark
[3]Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20472 USA

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We theoretically investigate the nature of the state at the quarter filled lowest Landau level and predict that, as the quantum well width is increased, a transition occurs from the composite fermion Fermi sea into a novel non-Abelian fractional quantum Hall state that is topologically equivalent to f-wave pairing of composite fermions. This state is topologically distinct from the familiar p-wave paired Pfaffian state. We compare our calculated phase diagram with experiments and make predictions for many observable quantities.

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The Moore-Read (MR) Pfaffian model [1] for the even-denominator fractional quantum Hall effect (FQHE) at filling factor $\nu = 5/2$ [2] predicts Majorana excitations that are neither fermionic nor bosonic but obey non-Abelian braid statistics [3]. This follows most directly from the understanding that the MR wave function represents a topological chiral p-wave "superconductor" of composite fermions [3], which themselves are emergent particles formed from the binding of electrons and quantized vortices [4,5]. Quasiparticle tunneling [6], quasiparticle interference [7,8], and thermal Hall [9,10] experiments have sought to measure the Majorana excitations, but the observations are not fully consistent with the predictions arising from either the Pfaffian [1] or its hole conjugate, called the anti-Pfaffian [11,12]. Realization of other non-Abelian states will therefore not only be fundamentally interesting in its own right, but can help provide an unambiguous demonstration of non-Abelian anyons. We predict in this Letter that the FQHE state observed at $\nu = 1/4$ in wide quantum wells (WQWs) [13–16] provides a realization of a new type of non-Abelian state [17,18] that is topologically distinct from the (anti-)Pfaffian state. We make detailed predictions for several topological properties of this state that are measurable by currently available experimental techniques.

The $\nu = 1/4$ state of our interest belongs to a large class of states appearing within the parton theory of the FQHE [17,18]. Here one divides each electron into k fictitious particles called partons, places each species of parton into an integer quantum Hall effect (IQHE) state with filling n_λ, and then glues the partons back together to recover the physical electrons. This leads to candidate "$n_1 n_2 \dots n_k$" FQHE states [17,18]

$$\Psi^{n_1 n_2 \cdots n_k} = \mathcal{P}_{\mathrm{LLL}} \prod_{\lambda=1}^{k} \Phi_{n_\lambda}(\{z_i\}). \tag{1}$$

Here Φ_n is the wave function of the IQHE state with n filled Landau levels (LLs), $\{z_i = x_i - iy_i\}$ are electron coordinates, and $\mathcal{P}_{\mathrm{LLL}}$ represents projection into the lowest LL (LLL). Negative values of n are denoted as $\bar{n}$, with $\Phi_{-n} = \Phi_{\bar{n}} \equiv [\Phi_n]^*$ being the wave function of the $|n|$ filled LL state in a negative magnetic field. To ensure equal area for each parton, the charge of the parton species labeled by λ is given by $e_\lambda = \nu/n_\lambda$ in units of the electron charge, with $\sum_{\lambda=1}^{k} e_\lambda = 1$. The candidate wave function $\Psi^{n_1,n_2,\dots,n_k}$ represents an incompressible state at filling factor $\nu = [\sum_{\lambda=1}^{k} n_\lambda^{-1}]^{-1}$. Remarkably, even though the partons themselves are unphysical, they leave their footprints in the physical world; for example, an excited parton in the factor Φ_{n_λ} produces a charge e_λ excitation in the physical state. A field theoretical description of these states was constructed by Wen and co-workers [19–22].

The familiar wave functions of the composite-fermion (CF) theory $\Psi_{\nu=n/(2pn+1)} = \mathcal{P}_{\mathrm{LLL}} \Phi_n \Phi_1^{2p}$ and $\Psi_{\nu=n/(2pn-1)} = \mathcal{P}_{\mathrm{LLL}} \Phi_{\bar{n}} \Phi_1^{2p}$ are obtained as $n11\cdots$ and $\bar{n}11\cdots$ states. The parton theory contains states beyond the CF theory. Wen showed [21] that the Jain parton states of the form $\Psi^{nn\cdots}_{\nu=n/k} = [\Phi_n]^k$ with $n \geq 2$ and $k \geq 2$ are non-Abelian. For these states, because all k partons are identical, the theory must be invariant under an SU(k) rotation within the internal parton space. Imposing this constraint through a non-Abelian gauge field and integrating out the partons leads to an SU$(k)_n$ Chern-Simons theory, which implies that the underlying state hosts non-Abelian quasiparticles.

Wen showed [21] that the $[\Phi_n]^k$ state has a chiral central charge $c = n(kn+1)/(k+n)$. In particular, the bosonic 22 state at $\nu = 1$ has $c = 5/2$. Other states that contain factors of $[\Phi_n]^k$ also support non-Abelian quasiparticles for the same reason. The electron states 221 at $\nu = 1/2$ and 22 111 at $\nu = 1/4$, described by $\mathrm{U}(1) \times \mathrm{SU}(2)_2$ Chern-Simons theory, also are non-Abelian with $c = 5/2$.

All wave functions in Eq. (1) are, in principle, valid candidates for FQHE, but the important question is which ones occur for realistic interactions. Extensive work has shown that the LLL primarily stabilizes composite fermions for narrow quantum wells. For states beyond the CF theory, one must therefore look to higher LLs, to monolayer or bilayer graphene, or to systems in WQWs, all of which have different Coulomb matrix elements than purely two-dimensional electrons in the LLL. The simplest non-Abelian parton state, namely, the 221 state at $\nu = 1/2$ [17,18,21,23,24], is not a satisfactory candidate for the $\nu = 1/2$ FQHE in the second LL, i.e., the 5/2 FQHE, because exact diagonalization does not produce an incompressible state at the corresponding "shift" [25]. Recently, Balram *et al.* [26] have shown the surprising result that the seemingly more complicated $\bar{2}\bar{2}111$ state provides a rather good representation of the Coulomb ground state at $\nu = 5/2$, although the $\bar{2}\bar{2}111$ state happens to lie in the same universality class as the anti-Pfaffian state. There are indications that the 221 state may be relevant to 1/2 FQHE in bilayer graphene for appropriate parameters [23] and to the $n = 3$ LL of monolayer graphene [27].

We now come to FQHE at $\nu = 1/4$ in WQWs. Which state occurs as the ground state is an energetic question. We consider the following candidate states:

$$\Psi^{\mathrm{CFFS}} = \mathcal{P}_{\mathrm{LLL}} \Phi^{\mathrm{Fermi\ sea}} \Phi_1^4, \tag{2}$$

$$\Psi^{\mathrm{Pf}} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\Phi_1^4, \tag{3}$$

$$\Psi^{\mathrm{Pf}_{l=3}} = \mathrm{Pf}\left(\frac{1}{(z_i - z_j)^3}\right)\Phi_1^4, \tag{4}$$

$$\Psi^{22111} = \mathcal{P}_{\mathrm{LLL}}\Phi_2\Phi_2\Phi_1^3 \sim \frac{[\mathcal{P}_{\mathrm{LLL}}\Phi_2\Phi_1^2]^2}{\Phi_1} = \frac{\Psi_{2/5}^2}{\Phi_1}, \tag{5}$$

$$\Psi^{\bar{2}\bar{2}11111} = \mathcal{P}_{\mathrm{LLL}}[\Phi_2^*]^2\Phi_1^5 \sim [\mathcal{P}_{\mathrm{LLL}}\Phi_2^*\Phi_1^2]^2\Phi_1 = \Psi_{2/3}^2\Phi_1. \tag{6}$$

These represent the compressible CF Fermi sea (CFFS) [28,29], the MR Pfaffian state [1], an $l = 3$ pairing Pfaffian state, the 22 111 state, and the $\bar{2}\bar{2}11111$ state. (We assume throughout this work that the magnetic field B is large enough to freeze the spin degree of freedom.) The Pfaffian of an antisymmetric matrix $M_{i,j}$ is defined as $\mathrm{Pf}(M_{i,j}) \sim \mathcal{A}(M_{1,2}M_{3,4}, \ldots, M_{N-1,N})$ with $\mathcal{A}$ representing antisymmetrization. The 22111 and $\bar{2}\bar{2}11111$ states are projected into the LLL as shown above; this form allows the Jain-Kamilla projection [30,31] to obtain the LLL states for up to 40 and 36 particles, respectively, in the spherical geometry [32] (see Supplemental Material [33] accompanying this Letter). Particle-hole (PH) symmetry implies that the anti-Pfaffian state has the same energy as the Pfaffian state. We do not consider the so-called PH-Pfaffian state [38] because its wave function $\mathcal{P}_{\mathrm{LLL}}\mathrm{Pf}[1/(z_i^* - z_j^*)]\Phi_1^4$ [26,39–41] is not amenable to calculations for large systems, precluding a reliable thermodynamic limit for the energy.

The 22 111 state is topologically distinct from the Pfaffian, anti-Pfaffian, and PH-Pfaffian states, which have chiral central charges of $c = 3/2$, $-1/2$, and $1/2$, respectively. Nevertheless, while Ψ^{Pf} and $\Psi^{\mathrm{Pf}_{l=3}}$ represent CF pairing in an obvious manner, the 22 111 and $\bar{2}\bar{2}11111$ states are also paired states of composite fermions [3,11,12,26,38], which can be seen as follows. At filling fraction $\nu = 1/4$, a natural set of FQHE states to consider correspond to attaching four vortices to each electron to obtain a composite fermion that sees zero magnetic field on average. In the parton construction, this amounts to writing the electron operators as $\wp = b\psi$, where b is a boson that forms a $\nu_b = 1/4$ bosonic Laughlin FQHE state, while ψ is the composite fermion. If we specialize to the case where ψ forms a paired state, we can consider any odd ℓ angular momentum pairing. This leads to wave functions of the form $\Psi_\ell^{\mathrm{CF-paired}} = \Psi_\ell^{\mathrm{paired}}\Phi_1^4$, where $\Psi_\ell^{\mathrm{paired}}$ is the wave function of an angular momentum ℓ paired superconductor of spinless fermions. $\Psi_\ell^{\mathrm{CF\text{-}paired}}$ describes a state with chiral central charge $c = \ell/2 + 1$. The edge theory consists of ℓ chiral Majorana modes, together with a charge mode described by a single chiral boson. Because there is a unique topological quantum field theory with Ising quasiparticles for a given chiral central charge [42], it follows that the 22 111, Pfaffian, PH-Pfaffian, and anti-Pfaffian (or $\bar{2}\bar{2}11111$) states are topologically equivalent, respectively, to $\ell = 3$, 1, -1, and -3 paired states of composite fermions (Table I). In particular, the 22 111 state corresponds to an f-wave superconductivity of composite fermions [26]. The 22 111 state and $\Psi^{\mathrm{Pf}_{l=3}}$ represent two different choices for the f-wave pair wave functions; while possibly topologically equivalent, these two states are microscopically very different, as seen below.

Since FQHE at $\nu = 1/4$ is seen only in a WQW, it is crucial to incorporate into the calculation the variation in the interaction due to transverse wave function $\xi(x)$ of the electrons in a realistic fashion, where x is the transverse coordinate. We determine $\xi(x)$ via the local density approximation (LDA) [43] for a given width and electron density at zero magnetic field [see examples in Fig. 1(a)]. This results in a modified effective interaction given by $V_{\mathrm{eff}}(r) = \int dx_1 \int dx_2 \{[|\xi(x_1)|^2|\xi(x_2)|^2]/[\sqrt{r^2 + |x_1 - x_2|^2}]\}$, where r is the distance between the electrons within the plane. All energies are quoted in units of $e^2/\epsilon l$, where ϵ is

TABLE I. This table gives the pairing channel ℓ, the shift $\mathcal{S}$ on the sphere, the electron and quasiparticle tunneling exponents α_{e} and α_{qp} (defined so that the tunnel current behaves as $I \sim V^{\alpha}$), and the chiral central charge c for several states at $\nu = 1/4$. The chiral central charge and the shift are related to the thermal Hall conductance and the Hall viscosity. Dots indicate that the quantity is not expected to be quantized to a universal value due to the edge theory not being fully chiral or the bulk being gapless.

State	ℓ	Shift $\mathcal{S}$	α_{e}	α_{qp}	Central charge c
CFFS	$\cdots$	4	$\cdots$	$\cdots$	$\cdots$
22 111	3	7	9	$-1/8$	$5/2$
Pfaffian ($l = 3$)	3	7	9	$-1/8$	$5/2$
Pfaffian	1	5	9	$-5/8$	$3/2$
PH-Pfaffian	-1	3	$\cdots$	$\cdots$	$1/2$
$\bar{2}\bar{2}$11111	-3	1	$\cdots$	$\cdots$	$-1/2$

the dielectric constant of the background host material and $l = \sqrt{\hbar c/eB}$ is the magnetic length.

The thermodynamic limits for the energies of various candidate states as a function of density are plotted in Fig. 1(a) for a quantum well of width 60 nm (see Supplemental Material [33] for details). (The energy of $\Psi^{\mathrm{Pf}_{l=3}}$ is much higher than that of other candidate states, typically by 0.1 $e^2/\epsilon l$, and is not shown.) From similar calculations at other quantum well widths, we obtain the phase diagram presented in Fig. 1(b). For small widths and small densities, the CFFS state dominates, but when the width and density are made large enough, the 22 111 state becomes the ground state. The Pfaffian, anti-Pfaffian, or $\bar{2}\bar{2}$ 11111 states are not realized in any part of the parameter space we have studied. Figure 1(b) also shows (solid squares), for two quantum well (QW) widths, the densities where the 1/4 FQHE has been first seen to appear in experiments [13–15].

A sufficiently wide quantum well can behave as a bilayer, which raises the question whether a two-component FQHE state could also be competitive [44]. The following considerations point to a single-component state. (i) The experimental onset of the 1/4 FQHE with increasing width or density agrees well with the phase boundary obtained in our single-component calculation [Fig. 1(a)]. (ii) The competition between one- and two-component states depends sensitively on the gap Δ_{SAS} separating the symmetric and the antisymmetric (SAS) bands. A large Δ_{SAS} favors a one-component state. From the LDA calculation, the value of Δ_{SAS} at the phase boundary in Fig. 1(b) is $\sim$0.1, 0.08, and 0.06 $e^2/\epsilon l$, respectively, for QWs of widths 50, 60, and 70 nm. While seemingly small, Δ_{SAS} is large compared to typical Coulomb energy differences between competing states [e.g., the Coulomb energy differences are <0.005 $e^2/\epsilon l$ in Fig. 1(a)]. For another two-component system, namely, spinful electrons in a single layer, the system in the vicinity of $\nu = 1/2$ becomes fully polarized (i.e., single component) when $E_{\mathrm{Z}} \gtrsim 0.01 e^2/\epsilon l$ for WQWs [45,46], where the Zeeman splitting E_{Z} is analogous to Δ_{SAS}. It is therefore likely that two-component states are not relevant for $\Delta_{\mathrm{SAS}} \sim 0.05$–$0.10 e^2/\epsilon l$. (iii) In the vicinity of $\nu = 1/4$, the FQHE states and the CFFS of spinful electrons are predicted to be single component (i.e., fully spin polarized) even for $E_{\mathrm{Z}} = 0$ [47,48]. (iv) Finally, we have considered an ideal bilayer system of two two-dimensional systems separated by a distance d. We have studied a total of 11 compressible and incompressible liquid states (Table II) and 24 different crystal states (Supplemental Material [33]). The crystal labeled BG$(2p, m)$ refers to a "bilayer graphene" crystal of composite fermions with $2p$ attached vortices, with m interlayer zeros; CS$(2p, m)$ refers to an analogous "correlated square" crystal [49]. For $d = 0$ the

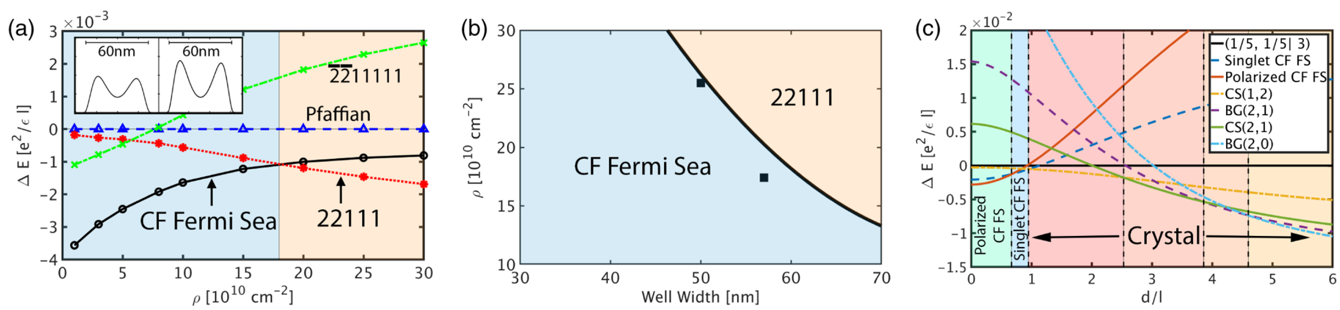

FIG. 1. (a) Energies of several candidate states at $\nu = 1/4$ as a function of density ρ for a quantum well of width 60 nm. The different states are labeled as shown on the figure. All energies are thermodynamic values, measured relative to the energy of the Pfaffian state. Only the CFFS and 22 111 states become ground states for the parameters studied. (Inset) The electron density as a function of transverse position for densities 1.5×10^{11} and 2.0×10^{11} cm^{-2} as given by LDA for a quantum well of width 60 nm. (b) The calculated phase diagram at $\nu = 1/4$ as a function of the quantum well width and density. In the region of parameter space shown in the figure only the CFFS and 22 111 states are realized. We also include experimental results, shown by black squares, taken from Refs. [13,14]. (c) Energies of several bilayer states as a function of the layer separation d/l. We have studied 11 liquid states (Table II) and 24 crystal states (Supplemental Material [33]). Here we omit the high energy states (see Supplemental Material for more complete results) and show the energies of the $(1/5, 1/5|3)$ state, the pseudospin singlet CFFS, the pseudospin polarized CFFS, and several crystal states (notation explained in Supplemental Material [33]). All energies are measured relative to the $(1/5, 1/5|3)$ state. No FQHE is stabilized.

TABLE II. Candidate liquid state wave functions at $\nu = 1/4$ in a bilayer system. The coordinates $z^{\uparrow}$ and $z^{\downarrow}$ denote different layers, while z denotes all coordinates. The terms singlet and polarized refer to "layer polarization".

Bilayer states at $\nu = 1/4$	
State	Wave function
$(1/8_{\rm CFFS}, 1/8_{\rm CFFS}\vert 0)$	$\Psi^{\rm CFFS}_{1/8}(z^{\uparrow})\Psi^{\rm CFFS}_{1/8}(z^{\downarrow})$
$(1/7, 1/7\vert 1)$	$\Phi^7_1(z^{\uparrow})\Phi^7_1(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)$
$(1/6_{\rm CFFS}, 1/6_{\rm CFFS}\vert 2)$	$\Psi^{\rm CFFS}_{1/6}(z^{\uparrow})\Psi^{\rm CFFS}_{1/6}(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^2$
$(1/5, 1/5\vert 3)$	$\Phi^5_1(z^{\uparrow})\Phi^5_1(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^3$
Singlet CFFS	$\Psi^{\rm CFFS}_{1/4}(z^{\uparrow})\Psi^{\rm CFFS}_{1/4}(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^4$
$(1/4_{\rm Pf}, 1/4_{\rm Pf}\vert 4)$	$\Psi^{\rm Pf}_{1/4}(z^{\uparrow})\Psi^{\rm Pf}_{1/4}(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^4$
${\rm Pf} \times (1/6, 1/6\vert 2)$	${\rm Pf}[1/(z_i - z_j)]\Phi^6_1(z^{\uparrow})\Phi^6_1(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^2$
$(1/6_{\rm Pf}, 1/6_{\rm Pf}\vert 2)$	$\Psi^{\rm Pf}_{1/6}(z^{\uparrow})\Psi^{\rm Pf}_{1/6}(z^{\downarrow})\Pi_{i,j}(z^{\uparrow}_i - z^{\downarrow}_j)^2$
Polarized CFFS	$\Psi^{\rm CFFS}_{1/4}(z)$
Polarized Pf	$\Psi^{\rm Pf}_{1/4}(z)$
Singlet $2_{\uparrow\downarrow}2111$	$\mathcal{P}_{\rm LLL}\Phi_1(z^{\uparrow})\Phi_1(z^{\downarrow})\Phi_2(z)\Phi^3_1(z)$

system is formally equivalent to that of spinful particles in a single layer with zero Zeeman energy. Here, as mentioned above, the ground state is a fully pseudospin polarized CFFS, which has lower exchange energy than the pseudospin singlet CFFS because of exchange effects. We find, unexpectedly, that as d is increased, a transition occurs into a pseudospin singlet CFFS, which is followed by a sequence of correlated CF crystals at larger d/l [see Fig. 1(c)]. We thus predict that no FQHE will occur at $\nu = 1/4$ in a bilayer system. This is consistent with current experiments in gallium arsenide double QW systems [50] and can be tested more accurately in double-layer graphene where a plethora of FQHE states have recently been observed [51,52].

These considerations make it plausible that the single-component 22 111 state is stabilized in WQWs. Nonetheless, a decisive confirmation requires further experimental evidence, and in the remainder of this Letter we outline certain experimental consequences of the 22 111 state.

The thermal Hall conductance of a FQHE sate is given by $c[\pi^2 k_B^2/(3h)]T$, where c is the chiral central charge [53]. It can thus decisively distinguish between different candidate states at $\nu = 1/4$, as they have different values of c (see Table I). An advantage of thermal Hall conductance is that it is robust against edge reconstruction.

We next come to tunneling exponents. We first consider quasiparticle tunneling at a quantum point contact (QPC) separating two edges of the same quantum Hall fluid. (See Supplemental Material [33] for the properties of various quasiparticles.) This tunneling is expected to be dominated by the minimally charged quasiparticle carrying charge 1/8, given by the operator $\sigma e^{i\varphi/2\sqrt{\nu^{-1}}}$. For $\ell = 1$, σ is the usual Ising spin field; for general ℓ, it is the primary field that changes the sign of the boundary condition of each of the chiral Majorana fermions and has scaling dimension $\ell/16$. The chiral boson φ carries the charge. For $\ell > 0$, the quasiparticle operator has scaling dimensions $(h, \bar{h}) = [(\nu/8) + (\ell/16), 0]$, where h and $\bar{h}$ are the left and right scaling dimensions. This implies that, at a QPC, the backscattering tunneling current would be (with $\nu = 1/4$) [22]

$$I \propto V^{4(h+\bar{h})-1} = V^{(\ell+2)/4-1} = V^{(2\ell-7)/8}. \tag{7}$$

For $\ell > 0$, since the edge theory is fully chiral, these exponents are quantized and universal due to charge 1/8 quasiparticles (in the absence of edge reconstruction). For $\ell < 0$, this operator has scaling dimensions $(h, \bar{h}) = [(\nu/8), (\ell/16)]$. In the unperturbed edge theory, we would therefore expect $I \propto V^{4(h+\bar{h})-1} = V^{(2|\ell|-7)/8}$. However, since the theory is not fully chiral, there are marginal perturbations of the edge theory that can modify the scaling dimensions. Therefore, for $\ell < 0$, we expect these exponents to be non-universal and thus not quantized. In particular, we can consider perturbations $\delta L = i\alpha_{ij}\partial\varphi\eta_i\partial\eta_j$, for coupling constants α_{ij}, where η_i for $i = 1, \ldots, \ell$ are the chiral Majorana fermions. These perturbations are marginal, having scaling dimension 2, and can change the exponents of the quasiparticle and the electron operators.

We next consider tunneling of an electron between two distinct adjacent FQHE fluids. The edge theory has ℓ chiral Majorana fermions, η_i, for $i = 1, \ldots, \ell$. We therefore have ℓ different types of electron operators: $\Psi_{e;i} \propto \eta_i e^{i\sqrt{\nu^{-1}}\varphi}$ and, in general, can consider a linear combination of the above operators: $\Psi_e = e^{i\sqrt{\nu^{-1}}\varphi}\sum_{i=1}^{\ell} a_i\eta_i + \cdots$ where the a_i are some constant coefficients for the expansion of the electron in terms of long wavelength field operators, and $\cdots$ indicate higher order (less relevant) operators in the expansion. For $\ell > 0$, this operator has scaling dimensions $(h, \bar{h}) = [(1/2\nu) + (1/2), 0]$, where h and $\bar{h}$ are the left and right scaling dimensions. The tunneling current behaves as

$$I \propto V^{4(h+\bar{h})-1} = V^9. \tag{8}$$

For $l < 0$, this operator has scaling dimensions $(h, \bar{h}) = [(1/2\nu), (1/2)]$. Naively, we would still get the same tunneling exponent, namely, $I \propto V^{4(h+\bar{h})-1} = V^9$. However, as before, for $\ell > 0$, the exponent is quantized and universal (assuming no reconstruction), but not for $l < 0$.

One can similarly consider tunneling of an electron from an external Fermi liquid [54–56]. In this case, the tunneling current becomes $I \propto V^{2(h+\bar{h})} = V^5$.

Finally, we note that the Hall viscosity is conjectured to be quantized at $\eta_H = \hbar\rho\mathcal{S}/4$, where $\mathcal{S}$ is the shift in the spherical geometry [57] and ρ is the density. The shifts for different candidate states are shown in Table I.

In summary, we have presented extensive calculations that suggest that the $\nu = 1/4$ FQHE in wide quantum wells is the realization of a new kind of single-component non-Abelian state that is topologically equivalent to f-wave pairing of composite fermions. We have listed many experimental consequences of this state. If confirmed, it will provide a convenient new platform for creating and studying non-Abelian anyons.

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