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# Beliefs, parental investments, and intergenerational persistence: A formal model

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[journals.sagepub.com/home/rss](http://journals.sagepub.com/home/rss)**Anders Hjorth-Trolle**

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**Abstract**

Empirical research documents persistent socioeconomic and race gaps in parental investments in children. This article presents a formal model that describes the process through which parents' beliefs about the returns on investments in children evolve over time in light of new information that they receive regarding the outcomes of past investments. The model, which is based on Bayesian learning, accounts for how parents of low socioeconomic status may come to underinvest in their children because they have false low beliefs about the returns on investments. Moreover, the model describes how beliefs are transmitted across generations, thus creating dynasties of underinvesting parents who reproduce inequalities in children's socioeconomic outcomes. Finally, this article uses National Longitudinal Survey of Youth data to provide illustrative empirical evidence on key aspects of the proposed model. The main contribution of this article is to integrate parents' beliefs about returns on investments into existing models of intergenerational transmissions.

**Keywords**

Beliefs, formal modeling, intergenerational transmission of resources, parental investments, rational choice theory, utility function

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## Introduction

Practitioners of empirical analyses based on Rational Choice Theory have long been aware that when individuals face various choices, they are rarely fully informed about what options they have and the returns on their choices, but rather base their decisions on personal beliefs and attitudes (Breen, 1999; Morgan, 2005; Piketty, 1995). The concept of beliefs has been subject to a large body of research in various contexts. For example, Calargo (2014), Lareau (2003), and Weininger and Lareau (2009) document how parents of high and low socioeconomic status (SES) differ in their beliefs about parenting practices and desired school behaviors. Similarly, Morgan (2005) addresses how young people form beliefs about their educational opportunities and how these beliefs vary across racial groups. On a theoretical level, Breen (1999) presents a formal model which assumes that students hold beliefs about the relative returns on effort versus ability with respect to school outcomes and shows how students in theory can have stable, but false beliefs about these returns. Furthermore, recent research shows that parental resources and their investments in children have substantial impacts on educational, labor market, and health outcomes regardless of gender, race, and SES (Alwin and Thornton, 1984; Björklund and Salvanes, 2011; Cunha and Heckman, 2007; Englund et al., 2004; Smyth et al., 2010; Yeung et al., 2002; Zhan and Sherraden, 2003). To date, however, no study has presented a comprehensive formal model analyzing how parents' investments in their child are shaped by the beliefs they (the parents) have about the returns on those investments.

Parents' monetary investments in buying toys and employing high-quality day care, as well as their time investments in reading and taking their child to museums, have been shown to generate large returns with respect to the child's educational attainment, earnings, and behavioral problems (Björklund and Salvanes, 2011; Cunha and Heckman, 2007; Deslandes et al., 1997; Smyth et al., 2010; Todd and Wolpin, 2007). For example, Yeung et al. (2002) find that access to cognitively stimulating materials, such as newspapers and books, improves a child's cognitive skills and reduces behavioral problems. Jæger and Breen (2016) argue that parents actively use activities, such as museum visits, to transfer cultural knowledge to their children, thereby increasing academic performance and long-term success. Similarly, Englund et al. (2004) show significant returns on parental involvement in schools as well as the instructional skills of the parents, a finding which Deslandes et al. (1997) and Smyth et al. (2010) corroborate. Englund et al. (2004) also show that parents' involvement and educational expectations for their children increase when the child performs well in school. Some research also points to mixed evidence of the uniformly positive returns to parental involvement in the home and suggests that certain parent behaviors can

adversely affect the child (Pomeranz et al., 2007). The overall picture, though, is that parental investments in children under most circumstances increase cognitive skills, improve educational attainment, and reduce behavioral problems (Björklund and Salvanes, 2011; Cunha and Heckman, 2007; Englund et al., 2004; Smyth et al., 2010).

However, mounting empirical evidence also indicates that low SES and non-White parents tend to invest less in their children, not only in terms of monetary investments but also in terms of engaging their children in stimulating activities (Mayer, 1997; Pomeranz et al., 2007; Todd and Wolpin, 2007; Yeung et al., 2002). Consequently, differences in parental investments by SES and race, for example, may be key factors in explaining the persisting inequalities in children's outcomes. A wide range of theoretical models exist that account for SES and race gaps in parental investments in children. These models can, broadly speaking, be classified into socio-cultural reproduction models and rational choice models (Smyth et al., 2010).

Socio-cultural reproduction models attribute SES and/or race differences in parental investments and children's achievements to differences in norms, values, and behavior and often emphasize the role of cultural resources or cultural capital (e.g. Bourdieu and Passeron, 1990; Fordham and Ogbu, 1986; Kohn, 1977; Lareau, 2003). Intergenerational transmission of SES, in these models, happens when parents (in some cases, unwittingly) imbue their children with norms, values, and/or behaviors that the educational system rewards. Rational choice models such as those presented by Becker and Tomes (1979, 1986), Cunha and Heckman (2007), Boudon (1974), Breen and Goldthorpe (1997), and Morgan (2005) instead conceptualize children and their parents as forward-looking rational agents who attempt to maximize their utility in light of their available information and preferences. In these models, intergenerational transmission of human capital happens through parents' investments, which in turn depend on the child's endowments and the parents' resources.

Although socio-cultural reproduction models differ from rational choice models in their behavioral assumptions as well as their implied mechanisms, they also share several aspects. Pertinent to this article, both types of models have parents behaving in certain ways with respect to their children dependent on their SES. While this article presents a new rational choice model, it also acknowledges findings from research based on socio-cultural reproduction models, which document differences in beliefs and values concerning parental behavior across SES groups and races (Fordham and Ogbu, 1986; Lareau, 2003; Morgan, 2005).

Models such as the ones mentioned above have greatly expanded our understanding of why SES and race differences in parental investment and child outcomes exist and how they persist over time. However, each class of

models also has its shortcomings, which the model presented in this article attempts to amend. Socio-cultural reproduction models are rarely formalized, and their assumptions and mechanisms often remain implicit and difficult to test (for a notable exception, see Jæger and Breen (2016)). This article formalizes the role of SES and race for parental investments and makes explicit how these factors interact with parents' beliefs about the returns on their investments in their children. Rational choice models typically operate on the assumption that parents are fully informed about the returns on their investments (e.g. Becker and Tomes, 1986; Cunha and Heckman, 2007). While such assumptions reduce the complexity of the model, they can be relaxed by explicitly incorporating parents' beliefs about the returns on investments into the theoretical model. This provides a richer and more realistic account of the causes and consequences of parental investments as parents are unlikely to possess full information about the actual returns on their investments in their children. However, they do have beliefs about these returns that may or may not be consistent with the truth. It then follows that utility-maximizing parents invest in their children to the extent that they believe their investments will have a positive effect on child outcomes. Regardless of whether investments actually yield a return, parents who do not believe that investments matter, or who do not derive utility from their child's outcomes, would not be expected to invest time and money in their children but rather to increase their personal consumption (Becker and Tomes, 1986).

Finally, Morgan (2005) indicates that there is little reason to believe that beliefs are static but rather that they change as individuals receive new information, echoing the theoretical notion of beliefs presented in Breen (1999). In the case of returns on parental investments, if parents believe that their investments yield low returns with respect to their child's school performance, they may invest little time and/or money in their child. However, if they experience that their child performs poorly in school, this may cause them to shift their beliefs toward higher returns on investments when they see that they did not invest much and their child performed poorly (Quadlin, 2014). This article models this process using Bayesian learning, which allows parents to gradually change their beliefs in light of new information while still taking into account prior beliefs (Anwar and Loughran, 2011; Breen, 1999; Breen and García-Peñalosa, 2002; Morgan, 2005; Piketty, 1995).

This article extends existing theoretical models of intergenerational transmissions in three respects. First, it proposes a formal model that uses Bayesian learning to explicitly incorporate parents' beliefs about the returns on investments into the modeling framework. This model argues that parents are rational actors who, while having their children's best interests at heart, may come to systematically underinvest in their children if they believe that the returns on their investments are lower than what they actually are.

Second, the article models how beliefs about returns on investments depend on SES and race, and it provides a theoretical explanation of the persistent SES and racial gaps in parental investments that previous research has documented. Third, the model incorporates an account of how beliefs about the returns on investments are transmitted across generations. If beliefs help to explain investment gaps, the intergenerational persistence of these gaps is conditional on children who, to some degree, inherit their parents' beliefs. The literature that addresses the intergenerational transmission of beliefs normally assumes that children inherit their parents' exact beliefs (Breen, 1999; Piketty, 1995). While this assumption is both convenient and useful, the process through which beliefs are transmitted across generations may be modeled more completely by relaxing it.

Furthermore, this article uses data from the National Longitudinal Survey of Youth 1979–Children and Young Adults (NLSY79-CYA) to test key features of the model. In particular, the empirical results illustrate that parents modify their investments over time based on prior investments and on their children's academic performance. Many of the theoretical models presented in this article greatly owe their formulations and conceptualizations to Breen (1999), who presented similar models. While part of this article's contribution comes from applying the framework of Breen (1999) to a new field—that of parental investments—the article also contributes beyond mere re-application of existing models. Apart from including an empirical test of some of the assumptions of the model, the article also offers a formalized mechanism for intergenerational transmission of beliefs and shows how systematic underinvestment due to low, stable beliefs may itself differ by SES and race.

The following section presents the formal model of how parents' beliefs about the returns on investments evolve during their children's childhoods and how beliefs partly determine parents' level of investments. The subsequent section addresses intergenerational transmission of beliefs and demonstrates that even when relaxing the assumption that children inherit their parents' exact beliefs, they still tend toward doing so. The third section presents an empirical illustration of a set of the model's predictions. The final section discusses the article's findings and reflects on the limitations of the research as well as the prospects of future research.

## **A formal model of beliefs in returns on parental investments**

This section proposes a formal model of the formation and evolution of beliefs regarding returns on parental investments. The section first outlines the processes leading to child academic performance and argues that this performance serves as a signal to parents about the returns on

their investments. Second, the section addresses parents' decision-making when determining how much to invest in their child by jointly considering the utility derived from both their child's academic success and their personal consumption. Third, the section argues that the evolution of parents' beliefs about the returns on their investments can be modeled using Bayesian learning models, and moreover, it shows how parents update beliefs in different ways depending on their prior beliefs. Fourth, the section shows that the model predicts the existence of two different sets of stable beliefs: one which leads to optimal parental investments in their child and one which, due to false beliefs, leads to underinvestment.

### *Child academic outcome as an imperfect signal*

Following Burton et al. (2002), Cunha and Heckman (2007), and Jæger and Breen (2016), this article conceptualizes childhood as consisting of a series of time periods denoted as  $t \in \{0, 1, 2, \dots, T-1\}$ , with period  $T$  marking the transition from childhood to adulthood. The initial time period,  $t_0$ , is the first period in which the child experiences an academic outcome, which typically occurs at enrollment in compulsory education.<sup>1</sup> During each period, the child experiences academic outcomes such as grades, test scores, successful course completion, or some other outcome that (re)occurs during childhood. This article treats academic success as a binary state: the child either succeeds or fails in a given time period, which is denoted by  $A$  (success) or  $\neg A$  (failure), respectively. Moreover, the model assumes that all parents are equally capable of recognizing success and failure. While this assumption can be relaxed, this is not possible within the scope of this article. For each period, parents observe their child's academic outcome (success or failure) and use this information to update their beliefs about the likely returns on their investments. Finally, upon reaching adulthood in period  $T$ , the child forms her own beliefs about returns on parental investments based on her parents' beliefs.

The probability that a child experiences academic success at time  $t$  is denoted by  $P(A)_t$  and depends on parental investments at time  $t$  and other factors. These other factors, such as race, SES, child's ability, school quality, teacher biases, and luck, are labeled  $X$  and are considered to be outside the parents' immediate control. Parental investments are denoted by  $I_t$  and represent the amount of resources that parents invest to increase the probability of their child's success, and moreover, these are resources the parents could have used for personal consumption. Resources include economic investments, for example, buying study materials and paying for tutoring, or time investments, such as helping the child with homework and arranging extracurricular activities. This article makes no distinction between the different types of investments. Both  $X$  and  $I_t$  range from 0 to 1, and  $X$  is

assumed to be constant over time. The article assumes that each family has one child.

Thus, at any given time  $t$ , a child experiences academic success with probability  $P(A)_t$ , as determined by parental investments,  $I_t$ , and other factors,  $X$ . This section addresses the returns on  $I_t$  and  $X$  and explicitly details the conceptualization of beliefs with respect to returns. Following Breen (1999), consider two possible states of the world,  $s'$  and  $s$ . In state  $s'$ , parental investments are *more* important for academic success than other factors,  $X$ . The returns on parental investments and other factors are denoted by  $\theta'$  and  $\pi'$ , respectively, with  $\theta' > \pi'$ . Conversely, in state  $s$ , parental investments are *less* important than other factors and the respective returns are denoted by  $\theta$  and  $\pi$ , with  $\theta < \pi$ . All returns are constrained to be strictly greater than 0 and less than 1, which means that, regardless of the actual state of world, both parental investments and other factors positively affect academic success to some extent.<sup>2</sup> The only difference between the two states is the relative importance of  $I_t$  and  $X$ , and the two states are assumed to be symmetrical such that  $\theta' - \theta = \pi - \pi'$  (Breen, 1999). Accordingly, the two states reflect fundamentally different states of the world. It then follows that the probability that the child experiences academic success in state  $s'$  can be written as

$$P(A|s')_t = \theta'I_t + \pi'X$$

And conversely, the probability that the child experiences success in state  $s$  is

$$P(A|s)_t = \theta I_t + \pi X$$

The world exists in one and only one of these states. That is, either parental investments are more important than other factors, or conversely, the opposite holds. Parents are unaware of the true state of the world but have subjective beliefs about the relative importance of their investments.<sup>3</sup> These beliefs are denoted by  $\tilde{\theta}$  and  $\tilde{\pi}$  and can be expressed as a weighted average of  $\theta$  and  $\theta'$  and  $\pi$  and  $\pi'$ , respectively. Parents do not necessarily believe completely in one of either opposing state but may have a stronger belief in one state than in the other. Belief in state  $s'$ , denoted by  $z_t$ , ranges from 0 to 1 and may change over time. Parents' belief in state  $s$  is defined by  $(1 - z_t)$ . Thus, parents' beliefs in the importance of parental investments and other factors,  $X$ , can be summed up by  $z_t$ , such that

$$\tilde{\theta}_t = z_t\theta' + (1 - z_t)\theta$$



and

$$\tilde{\pi}_t = z_t \pi' + (1 - z_t) \pi$$

As  $z_t$  approaches 1, parents' belief in the returns on their investments increases relative to their belief that other factors affect educational success. At  $z_t = 1$ , parents fully believe that state  $s'$  is true, which means that they place more importance on parental investments than on other factors.<sup>4</sup> In model terms, belief  $z_t = 1$  leads to  $\tilde{\theta}_t = \theta'$  as parents' belief in the returns on investments corresponds with the true returns, given that state  $s'$  is the true state of the world.<sup>5</sup> The prototypical parent who holds this belief might be characterized as having a strong sense of control over their own life, a notion that they themselves play the greatest role in their child's academic outcomes.<sup>6</sup> The opposite belief,  $z_t = 0$ , represents parents who fully believe state  $s$  to be true, that is, that other factors,  $X$ , matter more than parental investments. These parents might be thought of as believing that their child's academic outcome is, for the most part, not within their control but mostly determined by factors such as child's innate ability, luck, and race.<sup>7</sup>

Combining the notation above, parents' belief in the probability that their child succeeds in school is written as

$$\tilde{P}(A)_t = \tilde{\pi}_t X + \tilde{\theta}_t I_t$$

Thus, while parents' beliefs in the probability of their child succeeding academically,  $\tilde{P}(A)_t$ , as well as the actual probability,  $P(A)_t$ , depend on parental investments and other factors,  $X$ , neither are completely determined by these factors. Given that both  $\tilde{\pi}_t$  and  $\tilde{\theta}_t$  lie between 0 and 1,  $\tilde{P}(A)_t$  and  $P(A)_t$  will always be less than 1 regardless of the state of the world and the exact values of the parameters. Parents, however, do not observe the actual probability of academic success but only the binary realization of  $P(A)_t$ , that is, the child either succeeds or fails. Accordingly, for two reasons, child academic success or failure is an imperfect signal of the returns on parental investments. First, it is not possible for parents to disentangle the returns on  $X$  and  $I_t$  in a single time period, even if they observe both, because they do not know their precise returns. In other words, parents cannot know with absolute certainty whether the reason their child succeeded or failed was due to their own investments or to other factors such as having high or low SES. Second, the child's academic outcome is partly stochastic. This means that even if parents invest heavily in a child and the child has a value of  $P(A)_t$  close to 1, the child will eventually experience academic failure as  $P(A)_t$  is always less than 1. In this situation, parents most likely respond by updating their beliefs about the returns on their investments.

In summary, the model proposes that the probability that a child is academically successful depends on both parental investments,  $I_t$ , and other factors outside the parents' immediate control,  $X$ . Parents use information about their child's academic outcomes as signals regarding the returns on their investments. However, this signal is imperfect because the probability that a child succeeds is the product of both parental investments and other factors. Furthermore, child academic success is partly stochastic, as by construction the probability that the child is successful will always be less than 1. Parents, not knowing the actual importance of investments and other factors, have beliefs about their respective returns that they rely on when deciding their level of investment.

### *Parental utility and level of investment*

Models of intergenerational transmission and human capital accumulation often argue that parents face a trade-off between investments in children and personal consumption (i.e. Ayalew, 2005; Becker and Tomes, 1986; Behrmann, 1997). It is demonstrated herein how parents, all of whom possess a limited stock of resources, make decisions about this trade-off based on their beliefs about the probability that their child will succeed academically. If all parents were perfectly informed about this probability, differences in investments would be the sole product of differences in resources and altruism, as in Becker and Tomes (1986). However, the utility function presented herein incorporates the role of parental beliefs not previously included in economic models of intergenerational transmissions. Moreover, the model illustrates how parents determine the level of investment that will maximize their utility.

The utility function includes two components: child academic success and personal consumption. These are interdependent as resources can be used either for parental investment, thereby increasing the probability that the child is academically successful, or for personal consumption, but not for both. The model assumes that while all parents have a preference for their child's academic success, they also have some preference for personal consumption. Parents weigh these two preferences against each other by maximizing their joint utility,  $U_t^p$

$$U_t^p = a \ln\left(\tilde{P}(A)_t U_A - (1 - \tilde{P}(A)_t) U_{-A}\right) + (1 - a) \ln(R - I_t) \quad (1)$$

where  $a$  is an altruism parameter that captures parents' preference for their child's academic success relative to their own consumption.  $U_A$  and  $U_{-A}$

are the utilities of their child succeeding or failing, respectively, and are subject to  $U_A > U_{-A}$ , which means that the child's academic success yields more utility and their academic failure yields disutility.<sup>8</sup> Note that while  $U_{-A}$  is a positive quantity, it is understood as a disutility. That is, the larger  $U_{-A}$  is, the more disutility parents get from experiencing child academic failure.  $R$  denotes the parents' total stock of resources, standardized between 0 and 1 with  $R > I_t$ , as parents cannot invest more resources than they possess. The budget constraint,  $R - I_t$ , implicitly states that parents who possess more resources will, all other things being equal, invest more in their child than parents who possess fewer resources as their own consumption more easily reaches a level where the marginal returns on personal consumption become negligible compared to increasing the probability that the child is academically successful (Becker and Tomes, 1986).

The utility function also assumes that both expected utilities,  $U_A$  and  $U_{-A}$ , remain constant over time as parents are assumed not to receive new information concerning the value of education until they can observe their adult child at time  $T$ .<sup>9</sup> Moreover, utilities are constant for all parents, which implies that although beliefs in the returns on investments may differ across parents, their expected returns on the child's academic success do not differ.<sup>10</sup> The utility that parents derive from their child's academic success is based on their belief in the probability that their child succeeds,  $\tilde{P}(A)_t$ , which is determined partly by  $I_t$ , thus forcing parents to decide between investing in their child and increasing their personal consumption. Log-transforming both sources of utility reflects diminishing returns on both child academic success and personal consumption.

The utility function above describes how parents derive utility from both child academic success and their personal consumption, and that they do not know exactly how much their child benefits from their investments. This lack of full information constrains parents to invest according to their beliefs about the returns on their investments,  $\tilde{\theta}_t$ , and on other factors,  $\tilde{\pi}_t$ . In model terms, parents maximize equation (1) by choosing their optimum level of investments,  $I_t$ , defined by

$$I_t = \frac{U_{-A} \left( 1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t \right) + U_A \left( Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X) \right)}{\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)} \quad (2)$$

Appendix 1 shows the derivation of equation (2).  $I_t$  increases uniformly in  $R$  and decreases in both  $X$  and  $\tilde{\pi}_t$  (for proofs, see Appendix 2). The model thus predicts that the higher the level of parents' resources, the more they will invest in their children, and conversely, the larger they believe the

returns to other factors are relative to investments, the less they will invest. Furthermore, the higher the level of their other factors,  $X$ , the less they will invest as these other factors substitute investments in the production of academic success. Regarding altruism,  $a$ , the utility of academic success,  $U_A$ , and the disutility of academic failure,  $U_{-A}$ , these do not have a uniformly positive or negative impact on parental investments. Rather, the direction of their influence depends on the model's other parameters. For example, if a set of parents are highly altruistic, and furthermore believe that other factors have large returns relative to investments, then it stands to reason that increasing their investments might not be their optimal choice. Appendix 2 explicates the conditions under which parental investments increase and decrease in these parameters. Finally, parents' investments increase in their belief about the returns to these investments. Note that this condition requires assuming that  $(U_{-A}/U_A) < (\tilde{\pi}_t X)/(1 - \tilde{\pi}_t X)$ , which is only violated when both  $\tilde{\pi}_t$  and  $X$  are small and the difference between  $U_{-A}$  and  $U_A$  is also small. Appendix 2 details the derivation of this condition.

### *Bayesian learning in the evolution of beliefs*

This article has argued that beliefs play a crucial part in shaping parental investments. Moreover, beliefs may change over time. It is now proposed that parents are Bayesian learners who update their beliefs in light of new information. Bayesian learning allows parents to change their beliefs based on the signaling value of their child's academic success or failure while acknowledging that new beliefs do not emerge in a vacuum. Rather, parents have a prior belief before receiving a signal from their child. They then update their prior belief in accordance with the new information provided by the signal as they form their posterior belief. This subsequent belief then becomes their new prior as parents receive even more signals over time. In this way, both prior beliefs and new signals contribute to the evolution of parents' beliefs about returns on investments in their child.

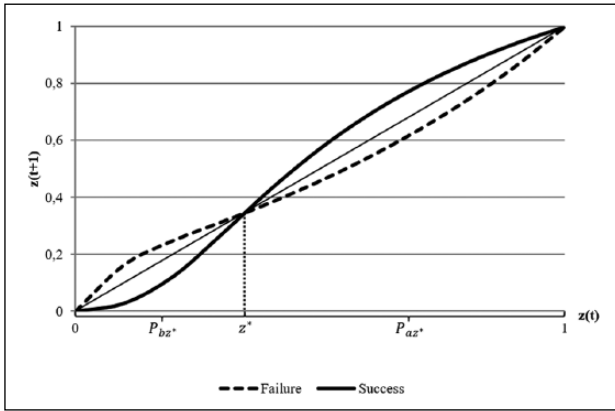
At time 0, parents have a belief,  $z_0$ , about the relative importance of their investments versus the importance of other factors. They inherit this belief from their parents, as described in detail in this section.  $z_0$  serves as the starting point of the evolution of their beliefs and is referred to as the initial belief. As previously illustrated, parents then optimize their joint utility by choosing their optimal level of investments,  $I_t$ . At time  $t$ , they receive a signal from their child, that is, whether he or she succeeded or failed academically. This signal provides them with new information about the returns on their investments and allows them to update their beliefs in time period  $t+1$ . This model proposes that parents update beliefs following Bayes' theorem, such that

$$z_{(t+1)} = \frac{z_t P(A) | s', z_t}{z_t P(A) | s', z_t + (1 - z_t) P(A) | s, z_t}$$

This is a standard application of Bayesian learning (Breen, 1999; Breen and García-Peñalosa, 2002; Piketty, 1995). Note that this method of updating requires that parents consider the signal, whether it comes in the shape of grades, test scores, or comments from teachers, to be valid information, to which they should react. For simplicity, this article assumes that parents accept the signal at face value, although a more realistic elaboration of the model could incorporate different levels of parents' trust in the signal. Ultimately, though, as long as parents trust the information somewhat (and do not act counter to it), the model's properties and predictions hold.

A vital property of this learning model is that because parents are conditioned on their prior beliefs, different parents may react in different ways to the same new information. For example, a child's academic success can cause parents to either increase or decrease their belief in state  $s'$ , depending on their previous beliefs. The intuition behind this process is as follows. For the remainder of this article, suppose, without loss of generality, that state  $s'$  is the true state of the world. This means that parental investments are more important in producing academic success than are other factors,  $X$ . This belief, responding to  $z_t = 1$ , is henceforth referred to as the true belief.<sup>11</sup> Parents holding this belief know the true returns on their investments and invest accordingly. Furthermore, suppose that two types of parents exist,  $Par_{az}$  and  $Par_{bz}$ , who are characterized by having initial beliefs,  $z_0$ , above and below, respectively, a certain threshold belief,  $z^*$ .  $Par_{az}$  parents have a strong belief in the returns on investments and invest heavily in their child. Their child then experiences either academic success or failure and they, in turn, update their beliefs. In the case of academic success, the parents' belief that their investments yield large returns is re-affirmed, and thus, they invest even more in the next period until they attain the level corresponding to the true belief. Conversely, if their child experiences academic failure, parents will reason that investing has lower returns than they expected and they will decrease their belief in their returns. The other parental group,  $Par_{bz}$  parents, exhibit low investments due to their low belief in their returns. If their child experiences academic success, their low beliefs in parental investments will be re-affirmed, and they will further decrease their belief that state  $s'$  is true. On the contrary, should their child experience academic failure, they will increase their belief in the importance of investments. Figure 1 illustrates this process.

Figure 1 shows parental belief,  $z_t$  on the  $x$ -axis and updated belief  $z_{t+1}$  on the  $y$ -axis. The figure illustrates how beliefs change depending on both



**Figure 1.** The evolution of belief.

initial belief and information received from child outcomes. The solid and dashed lines indicate how beliefs change if the child experiences academic success or failure, respectively, and both adhere to the pattern as described. In other words, depending on whether the prior belief  $z_t$  lies above or below a certain threshold,  $z^*$ , academic success or failure may either increase or decrease parental belief in returns on investments. In this way,  $Par_{az^*}$  and  $Par_{bz^*}$  parents may be identical in all other parameters but may still, conditioned only on prior beliefs, draw different conclusions from the same piece of information. The thin solid line represents no change in beliefs, which is a possible outcome only when  $z_t$  equals 0, 1, or  $z^*$ , cases that are treated separately in the next section.

### Steady state beliefs

As presented in the previous section, this model does not treat beliefs as static. Parents continuously receive signals from their child and use these signals to make inferences about the returns on their investments in their child. Accordingly, the proposed model leads to two steady-state beliefs (or equilibria) in which new signals about academic success or failure do not change parents' beliefs. These beliefs lead to underinvestment in the child, which then causes investment gaps that are conditioned on prior beliefs and on other factors, such as SES and race.

In a system of Bayesian learning, steady-state beliefs are possible at two different values (Piketty, 1995; Smith and Sørensen, 2000). One such value is at the true belief,  $z_t = 1$ , and the other is at some intermediate belief,  $z^*$ .

The latter belief is false as it does not place full weight on the true state of the world; however, it is still stable. In the true belief state, parents place full weight on state  $s'$ , and thus, they know the actual returns on their investments. According to the terminology used in the model, having this stable, true belief means that  $\tilde{\pi} = \pi'$  and  $\tilde{\theta} = \theta'$ , that is, parents' beliefs about returns are in accordance with the actual returns. This belief is stable since all new information will tend toward confirming the existing belief (Breen, 1999). It is also the most desirable belief for all parents to hold as knowing the true state of the world allows them to invest optimally and increase the probability that their child will be successful.

Figure 1 also shows that there exists a certain value of  $z_i$  that is less than 1 and where beliefs do not change regardless of success or failure. This belief, denoted by  $z^*$ , occurs when the solid and the dashed lines cross and represent a stable, but false belief. Put differently, the crossing of these lines means that, at this belief, parents are unable to gain any new information from their child's academic success or failure as both signals provide equal support for states  $s$  and  $s'$  being true. Thus,  $P(A|s) = P(A|s')$ , and this belief is stable as all new information confirms the existing belief. This is a central feature of the model. In this way,  $z^*$  represents a steady-state misconception, which is a stable, low false belief regarding the returns on parental investments that parents cannot escape through Bayesian learning. Persistent investment gaps between disadvantaged and advantaged groups, with respect to SES, race, and so on, that have been identified in previous research can be conceptualized as systematic differences in the steady-state beliefs that these groups come to hold. If disadvantaged groups tend toward false beliefs with greater probability than their advantaged counterparts, they will then consistently underinvest in their children, thereby reproducing their low social status.

However, before making the above claim, the mechanism of the model that generates  $z^*$  must be clarified. From this point forward, consistent with previous research, this stable, false belief is labeled the confounded learning equilibrium (CLE) (Breen, 1999; Breen and García-Peñalosa, 2002). Unlike the true belief, which occurs at  $z_i = 1$  for all parents, the CLE takes on a specific value for each set of parents depending on several individual specific factors. In a given family, parents' CLE occurs at their value of  $I_i$ , dubbed as  $I^*$ , which satisfies

$$\theta I^* + \pi X = \theta' I^* + \pi' X \quad (3)$$

that is, the level of parental investment that makes child academic success equally likely, regardless of whether the world is in state  $s$  or  $s'$ . Isolating  $I^*$  in equation (3), plugging it into equation (2), and solving for  $z$  lead to

$$z^* = \frac{U_A(\pi X(1-a) + \theta(X - Ra)) + U_{-A}(\pi X(1-a) + \theta(X(1-2a) + Ra) + a - 1)}{a\Delta(R - X)(U_A - U_{-A})} \tag{4}$$

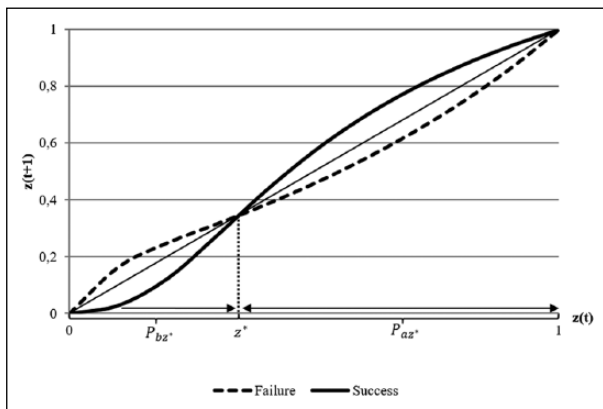
where  $\Delta = \theta' - \theta = \pi - \pi'$ , as per previous assumptions. Equation (4) shows that the value of the CLE varies across parents conditioned on their value of  $X$ , such as SES or race, as well as on their resources and altruism.<sup>12</sup> This is a key feature of the CLE. Compare, for example, a high SES and a low SES family. If both of these families have adopted the CLE and thus hold low, false beliefs regarding their returns on their investments, low SES parents will still invest even less than the high SES parents, as the latter have a higher valued CLE.<sup>13</sup> In this way,  $z^*$  is not an objective value, unlike in the true belief, but it is rather dependent on parents' value of  $X$ . Thus, different values of  $z^*$  represent degrees of misconception. While having a CLE close to the true belief will result in an underinvestment, the underinvestment is less than when the CLE is close to 0.

Before considering the role of other factors,  $X$ , with respect to the CLE, the process leading to parents moving toward either the CLE or the true belief must be explored. This process is contingent upon both the value of  $z^*$  for the parents in question and on their initial belief of  $z_0$  being above or below  $z^*$ .

The arrows parallel to the  $x$ -axis in Figure 2 indicate the possible patterns of convergence. Converging with  $z^*$  is contingent on initial beliefs as follows:  $Par_{bz^*}$  parents, holding initial beliefs below their CLE, will always shift upward toward  $z^*$  over time, which implies that it is impossible for them to reach the true belief. The certainty of upward movement comes from the fact that holding a truly false belief, that is,  $z_t = 0$ , is not a steady state (Breen, 1999; Piketty, 1995), and thus, the CLE is their only steady-state option. In other words, as long as parents receive signals from their child, their beliefs will evolve in one direction or the other until they reach a stable belief, the CLE. At this point, parents will cease to update their beliefs although they are underinvesting, as  $z^* < 1$ . On the contrary,  $Par_{az^*}$  parents, defined as holding initial beliefs above the CLE, may shift either upward toward the true belief or downward toward their CLE, as indicated by the double arrow in Figure 2. They risk converging downward to  $z^*$  with probability  $p$

$$p = \frac{z_0 - z^*}{z_0(1 - z^*)} \tag{5}$$





**Figure 2.** The evolution of beliefs and pattern of convergence.

Thus, it follows that movement upward to  $z_t = 1$  has a probability of  $1 - p$  (Breen, 1999). The intuition is as follows. The further  $z^*$  is away from the initial belief, the lower the probability of moving toward the CLE. Thus, while belonging to the  $Par_{az^*}$  group does not ensure convergence to the true belief, it remains a possibility.  $Par_{bz^*}$  parents, on the contrary, have no chance of converging with the true belief, but they will eventually reach the CLE. Accordingly, the model demonstrates that gaps in parental investments may be due to sets of parents holding different stable beliefs. Parents in the true belief state will always invest more than CLE parents, thus resulting in an investment gap as illustrated in Figure 3.

In addition to the investment gap illustrated in Figure 3, another gap is possible if the CLE varies systematically between advantaged and disadvantaged social groups. From equation (4), it is possible that  $z^*$  is dependent on other factors,  $X$ , such that

$$\frac{\partial z^*}{\partial X} > 0$$

See Appendix 3 for full proof and the conditions under which this holds true. Thus, the CLE assumes a higher value for parents with favorable other factors,  $X$ , for example, high SES parents, which suggests that advantaged groups will invest more heavily in their children than will disadvantaged groups, even if both hold low false beliefs. Figure 4 illustrates this point.

Figure 4 presents two sets of success and failure trajectories, one each for advantaged and disadvantaged parents (the failure trajectories are dimmed

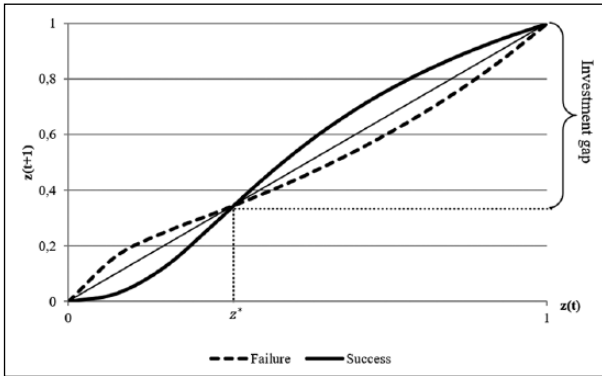


Figure 3. The evolution of beliefs and the investment gap.

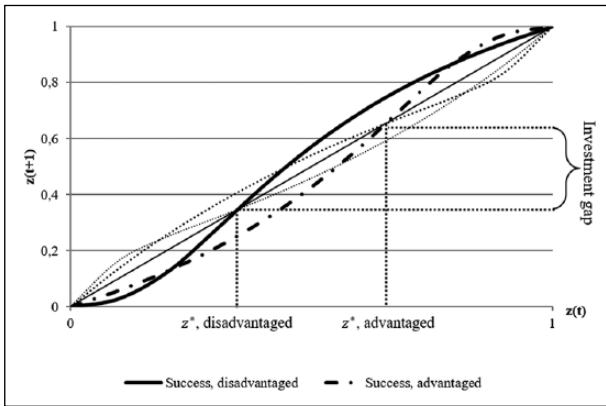


Figure 4. The evolution of beliefs and investment gap with false belief.

for ease of understanding). As with previous figures, the CLE occurs where the success and failure trajectories simultaneously cross the diagonal, thus indicating no change in belief. The figure further indicates that investment gaps between advantaged and disadvantaged groups are present even when both groups are at their CLE.

A final aspect of the relationship between other factors and the CLE to consider is that, for  $Par_{az}$  parents, the higher the CLE, the higher the risk of moving toward the CLE rather than moving toward the true belief. In other words, the closer the CLE is to the initial belief, the stronger its pull. Consider two sets of  $Par_{az}$  parents, identical in all factors except for SES, that is, one

set is high SES parents and the other is low SES parents. They hold the same initial belief,  $z_0$ , but due to a higher value of  $X$ , the high SES parents' CLE also has a higher value. This higher value not only makes the CLE less severe in terms of underinvestment but also makes the parents more likely to converge toward the CLE (cf. equation (5)). Thus, compared to low SES parents, high SES parents are at a higher risk of holding a false belief, albeit the consequences are less severe. Intuitively, imagine two low SES parents who believe strongly that their investments will yield a high return. When their child experiences academic success, these parents may be less likely than high SES parents to attribute this success to other factors, such as their own low SES, knowing that they are at a comparative disadvantage. Thus, having a high CLE is not always an advantage, as it also increases the risk of downward convergence rather than adopting the true belief.

Finally, a remark upon the CLE, which discusses its status as an equilibrium. Unlike previous research employing this concept (Breen, 1999; Breen and García-Peñalosa, 2002; Piketty, 1995), this article recognizes that the CLE is not "trembling hand perfect" (Carbonell-Nicolau, 2011; Selten, 1975). That is, the CLE is only an equilibrium as long as parents update their beliefs and choose their level of investments without any error or confusion. Should a pair of parents at the CLE invest above their expected level for stochastic reasons, they might escape the CLE and converge to the true belief.<sup>14</sup> However, as proposed by Breen (1999), though never formalized, updating beliefs might also impose a cost on the parents, making them less likely to update their beliefs. As such, the equilibrium property of the CLE might be compromised in opposing directions: If parents are prone to stochastic variation in their level of investments, the CLE is not necessarily stable. On the contrary, if there is a high enough cost to updating beliefs, every belief between 0 and 1 is a potential equilibrium (though not a CLE) as parents weigh the cost of updating against the benefits of having an updated belief. Concurrently, as long as both (or none) of these forces are in play, the CLE has the possibility to exist as a stable equilibrium to the degree that they cancel each other out. In the interest of space, this article does not formalize the circumstances under which this balance occurs, but implicitly assumes that it does. Future research, both theoretical and empirical, should address these potential limitations of the CLE.

In conclusion, this article has now proposed a formal model that conceptualizes parental investment gaps as arising from systematic differences in stable beliefs about the returns on investments and has identified two types of possible gaps. First, a possible gap exists between parents holding the true belief and parents at their CLE, as illustrated in Figure 3. Second, a possible gap exists between advantaged and disadvantaged parents both at

their CLE. This gap between false beliefs is a product of disadvantaged parents' CLE being systematically lower than that of advantaged parents, as presented in Figure 4.

## Intergenerational transmission of beliefs

It has been argued herein that parents continuously receive imperfect signals from their child and use these signals to make inferences about the returns on their investments and that over time parents shift toward one of two stable beliefs: the CLE or the true belief. Which belief parents adopt is determined by an interplay between parents' initial beliefs and other factors, such as SES and race. The model argues that unfavorable factors outside of the parent's immediate control, for example, low SES, increase the detrimental effect of having low initial beliefs in the returns on parental investments by lowering the parents' CLE, which in turn results in underinvestment. This section expands the model to consider how beliefs are transmitted from one generation to the next and, moreover, to examine the long-run consequences of parents' beliefs in the reproduction of SES. In doing so, the model explains how parents form their initial beliefs.

There is strong evidence that children inherit values and beliefs from their parents (Bisin et al., 2011; Dahl et al., 2014; Hitlin, 2006; Kohn, 1977; Weininger and Lareau, 2009). Research that has used Bayesian learning models argues that children inherit beliefs from their parents in a direct way, which suggests that their initial beliefs are identical to the steady-state beliefs of their parents (Breen, 1999; Piketty, 1995). Introducing a generation subscript  $g \in \{c, p\}$  for child and parents, respectively, this type of transmission mechanism (Breen, 1999; Piketty, 1995) is formalized as

$$z_{0c} = z_{Tp} \tag{6}$$

and

$$z_{Tp} \in \{z_p^*; 1\}$$

This simple model of belief transmission means that a child's initial beliefs can only be either the true belief or his or her parents' CLE, assuming that parents have had sufficient time to adopt a steady state at time  $T$ . The main limitation in assuming perfect transmission of beliefs is that, over sufficient generations, all parents' beliefs would eventually converge with the truth. This can be inferred as all children who inherit their parents' true beliefs will remain at this steady state, and all children who inherit their parents' CLE have the possibility to move upward if  $z_{0c} = z_p^* > z_c^*$ .

However, relaxing the assumption of perfect transmission of beliefs breaks this deterministic pattern by introducing a stochastic error term,  $\varepsilon < 1 - z_p^*$ , in the transmission process. The nature of the error term is such that while information is not perfectly transmitted, the noise is never great enough for the child to hold initial beliefs in another steady state than that of his or her parents. Introducing  $\varepsilon$  in equation (6) leads to the following two models of transmission, which are conditioned on the parents' steady state

$$z_{0c} = 1 - \varepsilon \mid z_{Tp} = 1$$

and

$$z_{0c} = z_p^* \pm \varepsilon \mid z_{Tp} = z_p^*$$

Considering equation (5), the probability that a child converges with the CLE if his or her parents held the true belief at time  $T$  is

$$p_c \left( z_{Tp} = 1 \right) = \frac{1 - \varepsilon - z_c^*}{(1 - \varepsilon)(1 - z_c^*)}$$

Conversely, a child whose parents had adopted the CLE will, when he or she enters adulthood, also adopt the CLE with probability

$$p_c \left( z_{Tp} = z_c^* \right) = \frac{z_p^* \pm \varepsilon - z_c^*}{(z_p^* \pm \varepsilon)(1 - z_c^*)}$$

as long as  $z_p^* \pm \varepsilon < z_c^*$ . If the child's initial belief is lower than  $z_c^*$ , convergence with the CLE is assured, as evidenced in Figure 2. In this model, the intergenerational transmission of beliefs is non-deterministic. The probability that the child adopts either the CLE or the true belief is by no means independent of the parental steady state as parental beliefs affect this process in a probabilistic manner. In other words, steady states are steady within a generation but not necessarily across generations. Nonetheless, when considering several generations, subsequent beliefs about the returns on parental investments is not a random walk from one generation to the next, and the probability that a child adopts his or her parents' steady state remains higher than the probability that he or she does not.

The model suggests that intergenerational transmission of beliefs has clear implications for the persistence of parental investment gaps as

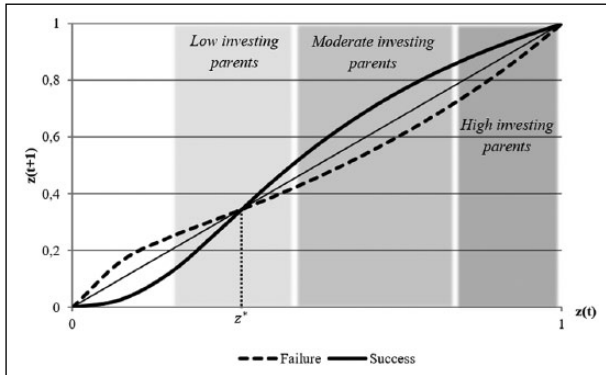
underinvesting parents at the CLE are likely to pass on this false belief to their child. Accordingly, the intra- and intergenerational dynamics in the evolution of beliefs reveal that the level of investments is established in one generation and then passed on to the next. Thus, the model leads to two types of dynasties: one in which parents underinvest due to false beliefs and one in which parents invest in accordance with the true state of the world. In addition, as previously shown, the CLE of disadvantaged groups occurs at a lower value than it does for advantaged groups, thus placing low SES families at a double disadvantage. First, they tend to transmit their false beliefs in the low returns on investments to their children. Second, this false belief has a more detrimental effect on their investments even when compared to the false beliefs of high SES families, as the latter have a higher value of CLE. In this manner, the model helps to understand how investment gaps persist across generations, which then leads to gaps in education, income, and health (Ayalew, 2005; Burton et al., 2002; Card, 1999; Reynolds and Ross, 1998).

## **Empirical illustration**

An empirical illustration of key aspects of the proposed formal model is necessary. As previously stated by Breen (1999), it is challenging to test Bayesian learning models such as these, as doing so in this case requires longitudinal and reliable measures of beliefs, parental investments, and child academic success or failure. Absent such a data source, this article uses an empirical measure of parents' investments as a proxy for parents' beliefs about the returns on investments. While not ideal, such a framework may provide provisional evidence on the usefulness of the model. Following Breen (1999), the model's predictions are analyzed by evaluating "testable propositions about how [parents' beliefs] should change conditional on its previous value and on the outcome of the event in question" (p. 475). Specifically, the hypothesis that parents with either an initial low or high level of investment are likely to adopt a stable belief, the CLE or the true belief, respectively, is tested. Thus, these groups should not make adjustments to their investments in light of new information about child academic success. On the contrary, parents with an initial level of investment in the middle of the distribution of investments should change their level of investment when observing child academic success or failure (cf. Figure 5).

## ***Data and variables***

The empirical illustration uses data from NLSY-CYA. These data are the best-suited available data for this purpose as they are longitudinal, measured every second year, and have detailed information on parental investment,



**Figure 5.** Illustration of the empirical hypothesis.

child academic achievement, and a multitude of parent and child background factors. The data include observations only if they have valid information for all variables for two consecutive periods between 1988 and 2010, which is the span of the data that include parental investment measures. This sampling frame leads to an analysis sample of 5434 children aged 6–14 years with 75% of children aged 6–10 years.

The model uses changes in parental investments over time as the dependent variable. While ideally the analysis would instead focus on changes in parents' beliefs, to my knowledge, no such longitudinal data exist. Using actual investments as a proxy is not perfect, but the assumption that actual investments are linked to beliefs about their importance is supported by the theoretical model and also tentatively by the data. When the mothers in the data are first surveyed about their child, they report whether they believe children learn best on their own or whether their parents should teach them.<sup>15</sup> Mothers who believe that parents should teach their children also tend to exhibit higher levels of investments, lending some credence to this assumption.

This article measures parental investments using a series of variables from the HOME-SF battery of questions pertaining to both money spent on and time spent with the child, providing a single variable that captures overall parental investments (Cunha and Heckman, 2007; US Bureau of Labor Statistics, 2015). The battery includes the number of books the child has, whether the child has a musical instrument, whether he or she is enrolled in extra lessons or extracurricular activities, how often the child is taken to a museum or a performance, how often parents assist with homework, and how often parents discuss television programs with the child.<sup>16</sup> Appendix 4 describes the wording and distributions of all items.

The main explanatory variable in the analysis is the child's academic success or failure as measured by changes in the Peabody Individual Achievement Test (PIAT) of mathematics and reading. This test is considered a highly reliable and valid assessment of academic ability, its results correlate strongly with other cognitive measures (US Bureau of Labor Statistics, 2015), and it is often used as a measure of academic ability (Fryer and Levitt, 2004; Rosenzweig and Wolpin, 1994; Todd and Wolpin, 2007; Waldfogel et al., 2002). However, it is important to note that this test score is not viewed directly by the parents and only serves as a signal insofar as it correlates with other measures that parents observe, such as grades, SAT scores, or passing classes. Thus, the variable is inherently a weak signal and is likely to underestimate the changes in investments that would occur from stronger signals. In light of this circumstance, the analysis code for test scores is divided into three groups: success, failure, or no signal. Parents whose child's changes in test scores fall in the upper or lower quartile of the distribution of test score changes receive a signal of child academic success or failure, respectively.<sup>17</sup> The empirical model assumes that test score changes in the middle part of the distribution are too weak a signal for parents to respond in a meaningful way. All models also control for race and sex of the child, mother's education, and changes over time in household disposable income. The models do not control for child age as the sample has little variation in child age, and performing the analysis separately for age groups does not alter the results substantially (results not shown). Table 1 summarizes these variables.

Furthermore, Table 2 shows how parents' initial level of investment varies by their observable characteristics. Higher income and higher maternal education are both associated with substantially increased parental investment. White parents invest more on average than both Hispanic and Black parents do, and girls tend to benefit from a slightly higher level of investments than boys do. These differences follow the expected patterns considering both the formal model and previous findings and suggests that they should be included as control variables in the analysis.

### *Model illustration: heterogeneous responses to signals from the child*

To test the hypothesis presented herein, this section proposes a model that exploits variations over time in parental investments and child test scores. The model regresses the change in parental investments on changes in the child's reading and math test scores, representing success, failure, or no signal, as described above, net of a vector of controls. By estimating a



**Table 1.** Descriptive statistics.

Constant variables		
Child race		
White	0.489 (0.500)	
Hispanic	0.205 (0.404)	
African American	0.306 (0.461)	
Child gender = female	0.504 (0.500)	
Mother's years of education	12.658 (2.484)	
N	5434	
Time-variant variables		
	t0	t1
Parental investment	-0.002 (0.759)	0.019 (0.768)
Net household income	47,426 (76,407)	50,651 (70,667)
PIAT Math test score	30.788 (14.041)	43.574 (12.131)
PIAT Reading test score	30.676 (13.050)	42.030 (12.179)
N	5434	5434

PIAT: Peabody Individual Achievement Test.

Standard deviations in parentheses.

total of three regression models for parents with a low, intermediate, or high level of prior investments, the model evaluates whether these three groups react differently to their child's signals of academic success or failure. As changes in both test scores and parental investments occur simultaneously over a period of 2 years, causality does not only flow from changes in test scores to changes in investments but also from changes in investments to changes in test scores. Accordingly, over the course of the 2 years, multiple updates may occur. The model is descriptive and seeks to describe heterogeneity in responses conditioned on initial levels of investments rather than to evaluate the causal link between test scores and parental investments. The model uses ordinary least squares for estimation and takes the following form

$$\Delta inv | inv0 = \beta_0 + \beta_1 \Delta test + \beta_2 X + e$$

where the dependent variable,  $\Delta inv$ , represents changes in parental investments from period 0 to 1 when measured continuously and standardized for ease of interpretation. Similar to changes in test scores,  $\Delta test$ , initial investments,  $inv0$ , is a vector of three categories, specifically, the lowest quartile (the reference category), the highest quartile, and the intermediate quartiles;  $X$  is a vector of the control variables described above,  $\beta_0$  is a constant

**Table 2.** Initial level of parental investment by parent characteristics.

	HOME score (standardized)	No. of observations
Child race		
White	0.156 (0.654)	2659
Hispanic	-0.184 (0.761)	1112
African American	-0.134 (0.857)	1663
Child gender		
Male	-0.021 (0.760)	2696
Female	0.016 (0.759)	2738
Household income		
1st quartile	-0.285 (0.798)	1361
2nd quartile	-0.107 (0.743)	1355
3rd quartile	0.079 (0.719)	1361
4th quartile	0.303 (0.641)	1357
Mother's years of education		
<12	-0.320 (0.818)	1213
12-15	-0.008 (0.704)	3326
≥16	0.450 (0.641)	895

Standard deviations in parentheses.

term and all other  $\beta$ s are regression coefficients where bold font denotes a vector, and  $e$  is a random error term that is estimated robustly within family clusters to avoid autocorrelation between siblings. The central term of the model is vector  $\beta_1$ , which reflects the associations between changes in test scores and parental investments. The main hypothesis is that this vector is either larger or smaller than 0 for the middle group between the steady states. For the highest and lowest quartiles of initial investments, the expectation is that the vector is 0 as these two groups are more likely to hold a stable belief, whether it is the CLE or the true belief. Table 3 presents the empirical results for vector  $\beta_1$  conditioned on  $inv0$  and using test scores from reading and mathematics, respectively:

Overall, the empirical results follow expectations as parents in the top and the bottom quartiles of the initial investment distribution do not adjust their investments due to changes in child test scores. By contrast, parents whose initial investments are in the two intermediate quartiles increase their investments significantly in light of positive changes in test scores, and vice versa. These results support the hypothesis that investments only change for parents who have not yet adopted one of the two stable beliefs. Compared to using math test scores, effect sizes are larger and p-values are smaller when using reading test scores, perhaps because the ability to read and write is a larger component of the everyday interactions that parents

**Table 3.** Effects of test scores on changes in investments—conditional on initial investments.

	Low initial inv.	Moderate initial inv.	High initial inv.
Changes in PIAT reading test scores			
Lowest quartile (ref. cat.)	–	–	–
2nd quartile	–0.005 (0.066)	0.085 <sup>†</sup> (0.051)	0.027 (0.071)
3rd quartile	0.041 (0.072)	0.141 <sup>**</sup> (0.049)	–0.008 (0.071)
Highest quartile	–0.015 (0.077)	0.151 <sup>**</sup> (0.049)	0.118 (0.073)
Controls	Yes	Yes	Yes
N	1360	2717	1357
Changes in PIAT math test scores			
Lowest quartile (ref. cat.)	–	–	–
2nd quartile	0.050 (0.070)	0.017 (0.049)	0.029 (0.068)
3rd quartile	0.045 (0.074)	0.107 <sup>*</sup> (0.051)	0.023 (0.073)
Highest quartile	0.111 (0.072)	0.108 <sup>*</sup> (0.051)	0.059 (0.073)
Controls	Yes	Yes	Yes
N	1360	2717	1357

PIAT: Peabody Individual Achievement Test.

All models controlled for child sex and race as well as mother's years of education and changes over time in household income. Standard errors clustered within mothers. Standard errors in parentheses.

<sup>†</sup> $p < 0.010$ ; <sup>\*</sup> $p < 0.005$ ; <sup>\*\*</sup> $p < 0.010$ ; <sup>\*\*\*</sup> $p < 0.001$ .

have with their child. In conclusion, the empirical results are consistent with a central assumption in the proposed formal model. That is, parents respond differently to the same signals of child academic performance when they have different prior levels of investments. This finding cautiously points to the feasibility of conceptualizing parents as Bayesian learners and supports the notion that multiple stable beliefs exist. At the very least, any conceptualization of parents learning the returns on their investments should include the possibility of holding a stable, but false belief. Despite the returns on parental investments presented in previous literature works (Cunha and Heckman, 2007; Todd and Wolpin, 2007; Yeung et al., 2002), these parents maintain a low level of investments regardless of their child's academic failure or success. Thus, these parents represent the prototype of parents who believe that they do not have a substantial influence on their child's academic success when compared to other factors, such as race, luck, teacher bias, school quality, or child ability. Conversely, parents with high levels of investment do not respond to the signals they receive from their child because they are already convinced (rightly) that they know the actual returns on their investments.

## Discussion

The positive effects of parental investments in children, such as reading to them or providing high-quality day care, are well documented in recent research, as are the socioeconomic gradients in these investments (Cunha and Heckman, 2007; Todd and Wolpin, 2007; Yeung et al., 2002). Highly educated, financially stable, and/or White parents systematically invest more in their children, which offers an explanation for the empirically observed gaps in academic success between SES and race groups (e.g. Fryer and Levitt, 2004). This article proposes a formal model to explain the mechanisms through which these gaps in parental investments come to be and how they persist across generations. The main contribution of the article is to propose a formal model that clarifies the role of parental beliefs in the returns on their investments, a concept traditionally missing from models of intergenerational transmission of resources and human capital accumulation (e.g. Becker and Tomes, 1986; Cunha et al., 2006). Rather than assuming that all parents know the actual returns on their investments, this article presents a model in which parents have a belief about the returns on their investments, and this belief may or may not be consistent with the truth. Conceptually, the model argues that if parents do not believe that their investments are an important factor contributing to their child's outcomes, they will spend more of their resources on personal consumption compared to those parents who believe their investments contribute significantly to the success of their child.

Consistent with previous research, this article models the process of belief formation and evolution by using Bayesian learning (Breen, 1999; Morgan, 2005; Piketty, 1995). The model indicates that parents can have two different stable beliefs about the returns on their investments. They can either hold the true belief knowing the actual returns on their investments or hold a false belief, known as the CLE (Breen, 1999; Breen and García-Peñalosa, 2002; Piketty, 1995), which causes them to underinvest. While the true belief is the same for all parents, the CLE varies due to other factors, such as race and SES. Thus, comparing, for example, high SES and low SES parents both at the CLE, the disadvantaged parents will underinvest even more so than their advantaged counterparts. This provides two explanations for the parental investment gaps that arise from the systematic differences in holding true or false beliefs as well as from disadvantaged parents who suffer even more severe consequences due to false beliefs than do advantaged parents. The model also formalizes the mechanism through which parents transmit beliefs to their child and concludes that children are more likely to inherit their parents' stable beliefs, thus reproducing intergenerational gaps in parental investments.

The empirical section of the article uses American data on parental investments to test key features of the model. The results offer tentative support for the existence of two stable beliefs, one true and one false, by showing that parents adjust their investments differently depending on their prior investments. Parents with either a high or a low level of prior investments do not change their investments in light of their children's academic success or failure, a finding that the model interprets as them having already committed to the true belief or the CLE, respectively. On the contrary, parents with an intermediate level of prior investments react to child academic outcomes by changing their subsequent level of investments.

### *Suggestions for future research*

Models such as the one presented in this article need not be restricted to explaining educational outcomes. Previous empirical research has shown differentials in parental investment behavior whether the outcome is child health or child education (Ayalew, 2005). Just as the model presented in this article draws on Breen's (1999) work on student beliefs in effort versus ability, future models could utilize this framework with different outcomes or different inputs. This type of formal modeling presents a flexible way to conceptualize and explain a wide range of processes regarding the adjustment of beliefs in light of new information and the transmission of these beliefs across generations.

In addition, this article holds several prospects for future empirical research pertaining to the role of beliefs in parental investments. In particular, an analysis of longitudinal data sets with direct measures of beliefs would be useful with respect to the further investigation of the usefulness of the model. Similarly, studies of the links between, on one hand, beliefs in the returns on parental investments and, on the other, parents' actual investment levels would be valuable.

### *Limitations and assumptions*

The model presented in this article assumes that parent learning is completely endogenous, that is, that parents only update their beliefs according to the signals they receive from their child. This assumption is strong, and research on identity formation finds that although parents are the primary source of values and beliefs, peer groups and neighborhoods also play an important role (Bisin et al., 2011; Fordham and Ogbu, 1986). Furthermore, research on the importance of information and beliefs about college costs reveals that external sources of learning, such as instruction, can change not only the beliefs but also the behaviors of students (Barone et al., 2017;

Bettinger et al., 2010; Loyalka et al., 2013). However, as observed by Breen (1999), there are several challenges associated with learning from others as, for example, it is difficult for parents to obtain precise information on many aspects of other parents, such as SES, child's ability, and level of investments. Considering this, it seems unlikely that parents will change beliefs based on information that they may deem unreliable. However, if parents are able to observe a case about which they have information regarding most of the variables, such as family or close friends, they may then be able to learn from this information. For policy purposes, an example of such a case would be to provide parents with prolonged exposure to strong role models, a strategy that has previously proven effective (Evans, 1992; Ssewalama et al., 2012; Whiting, 2006). In regard to this approach, future research should seek to formalize learning from others as a component in the model.

Another limitation of the formal model in its current form is the assumption of one-child families. While this simplification substantially eases the derivation and interpretation of the model, it is unlikely to hold for obvious reasons. Parents more often than not have more than one child, and it is highly plausible that they use lessons learned from their first child to inform their investments in the second child. The possibilities to theorize about these learning processes are many. For example, do parents hold separate beliefs about returns on investment for each child, or do they update a single belief for all children? If so, what implications does this have for the first-born child as parents receive new information? Questions such as these should be considered as contributions in themselves and as promising avenues for future research.

### *Policy implications*

Although the formal presented in this article is theoretical and the empirical illustration not a strict causal analysis, this article may cautiously inspire a new supplementary focus for policy intervention. For example, research on college education indicates that it is possible to change people's beliefs about costs and opportunities by providing them with high quality information (Barone et al., 2017; Bettinger et al., 2010; Loyalka et al., 2013). On the contrary, though, interventions targeted at increasing parental involvement show mixed results when it comes to changing parents' behavior (Pomeranz et al., 2007). If, however, persistent parental investment gaps are indeed partially rooted in false beliefs about low returns, policy-makers could provide disadvantaged groups with reliable information and attempt to change their beliefs rather than directly affect their behavior. Through campaigning and the use of role models, it may be possible to shift low investing parents' beliefs toward the optimal stable belief, thus prompting them to invest more

in their child. Such policy interventions are relatively inexpensive compared to targeted monetary transfers. Thus, they may serve to activate untapped parent resources already present in disadvantaged racial or social groups, thereby narrowing or closing parental investment gaps. It bears repeating, however, that this article in itself does not provide sufficient evidence to base any policy intervention on; rather the article presents a theoretical framework for further empirical analyses that may bolster or test these recommendations.

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### Notes

1. Prior to enrollment in compulsory education, parents may still attempt to make inferences about the returns on their investments by, for example, observing their child's cognitive development. The model presented herein only accounts for signals of academic performance, but the framework can be extended to include preschool investments as well.
2. Note that this assumption, that parental investments always have positive returns, runs somewhat counter to the argument put forth by Pomeranz et al. (2007) in which they suggest that certain types of parental involvement may adversely affect children. This type of involvement includes, for example, increased parental control and negative affect toward the child, which may be unproductive. Further work extending the model could formalize this mechanism, but that remains out of scope for this article.
3. The origins of parental beliefs will be further discussed at a later stage.
4. These parents do not consider other factors completely irrelevant since  $\pi' > 0$ . However, they place greater importance on parental investments since  $\theta' > \pi'$ .
5. Also,  $z_i = 1$  leads to  $\tilde{\pi}_i = \pi'$ , with  $\pi' < \pi$  as stated above.
6. Although it not formalized within this article, it is easy to imagine that some parents may believe that their investments have larger returns than they truly

- do (theoretically,  $\tilde{\theta}_t > \theta'$ ). While these investments are not unproductive, they are sub-optimal with respect to parents' utility since they could spend these resources on consumption instead. This article then assumes that parents do not overinvest and notes that overinvestment is not an equilibrium state.
7. As per a previous footnote, these parents do not consider parental investments completely irrelevant as  $\theta > 0$ , but they do place greater importance on other factors.
  8. This utility function is only defined within the condition of  $\tilde{P}(A)_t U_A - (1 - \tilde{P}(A)_t) U_{-A} > 0$ , that is, under  $\tilde{P}(A)_t U_A > (1 - \tilde{P}(A)_t) U_{-A}$ .
  9. Furthermore, the model assumes time-invariant altruism as well as resources, the latter reflecting parents rationally expecting a stable level of resources regardless of transitory shocks, such as unexpected unemployment.
  10. This assumption, which implies that all parents place the same amount of importance on academic achievement, is arguably heroic and contrasts with previous theories in which high socioeconomic status (SES) parents derive more utility from their children's education than low SES parents do (e.g. Breen and Goldthorpe, 1997). A more realistic model could reflect that some parents might have preferences for optimizing their children's employability or steering them toward a specific career. This elaboration is out of scope for the current paper, though.
  11. Truth, in this sense, is not to be interpreted in any ontological sense, but merely as a convenient turn of phrase.
  12. All other factors are assumed to be constant.
  13. This section formalizes this point below.
  14. Naturally, if parents at the confounded learning equilibrium (CLE) accidentally invest below their expected level, they could also escape the CLE but would eventually converge back to it (cf. Figure 2) making this case uninteresting.
  15. The phrasing of the question is, "Some parents spend time teaching their children new skills while other parents believe children learn best on their own. Which of the following most closely describes your attitude?" and options range from "Parents should always spend time teaching their children" to "Parents should always allow their children to learn on their own" with two intermediate options.
  16. All parental investment items are measured identically across child age with the exception of number of books owned. The possible answers for this question are adjusted for older age groups to reflect accumulation of books throughout childhood.
  17. The model's estimates do not differ significantly when employing quintiles and combining the three intermediate groups or when splitting the sample into three groups of equal size.

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## Appendix I

### Deriving equation (2)

Proof that parents choose their level of investments according to

$$I_t = \frac{U_{-A} \left( 1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t \right) + U_A \left( Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X) \right)}{\tilde{\theta}_t \left( U_A + U_{-A} - 2U_{-A}a \right)}$$

Recall the utility function of the parents

$$\begin{aligned}
 U_t^p &= a \ln \left( \tilde{P}(A)_t U_A - (1 - \tilde{P}(A)_t) U_{-A} \right) + (1-a) \ln(R - I_t) \\
 &= a \ln \left( U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} (1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t)) \right) + (1-a) \ln(R - I_t)
 \end{aligned}$$

Parents maximize their utility by

$$\begin{aligned}
 &\frac{\partial}{\partial I_t} \left( a \ln \left( U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} (1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t)) \right) \right. \\
 &\quad \left. + (1-a) \ln(R - I_t) \right) \\
 &= \frac{\partial}{\partial I_t} \left( a \ln \left( U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} (1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t)) \right) \right) \\
 &\quad + \frac{\partial}{\partial I_t} \left( (1-a) \ln(R - I_t) \right)
 \end{aligned}$$

Solving the right summand

$$\frac{\partial}{\partial I_t} \left( (1-a) \ln(R - I_t) \right) = -\frac{1-a}{R - I_t}$$

To solve the left summand, first define

$$F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) = \ln \left( U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} (1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t)) \right)$$

So the left summand is written as  $a \cdot F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t)$ .

This means that

$$\begin{aligned}
 &\frac{\partial}{\partial I_t} \left( a \cdot F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) \right) = \left( \frac{\partial}{\partial I_t} a \right) \cdot F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) + \\
 &a \cdot \left( \frac{\partial}{\partial I_t} F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) \right) = a \cdot \left( \frac{\partial}{\partial I_t} F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) \right)
 \end{aligned}$$

since  $a$  is constant w.r.t.  $I_t$ .

We now have that

$$\begin{aligned}
& \frac{\partial}{\partial I_t} F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t) \\
&= \frac{\partial}{\partial I_t} \left( \ln \left( U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} \left( 1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t) \right) \right) \right) \\
&= \frac{\tilde{\theta}_t (U_A - U_{-A})}{U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} \left( 1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t) \right)}
\end{aligned}$$

So by

$(\partial / \partial I_t)(a \cdot F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t)) = a \cdot ((\partial / \partial I_t)F(U_A, U_{-A}, X, I_t, \tilde{\pi}_t, \tilde{\theta}_t))$ ,  
the left summand becomes

$$\frac{a\tilde{\theta}_t (U_A - U_{-A})}{U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} \left( 1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t) \right)}$$

Adding the right summand then yields

$$\frac{\partial}{\partial I_t} = \frac{a\tilde{\theta}_t (U_A - U_{-A})}{U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} \left( 1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t) \right)} - \frac{1-a}{R-I_t}$$

Solving this equation for  $I_t$  yields

$$\begin{aligned}
0 &= \frac{a\tilde{\theta}_t (U_A - U_{-A})}{U_A (\tilde{\pi}_t X + \tilde{\theta}_t I_t) - U_{-A} \left( 1 - (\tilde{\pi}_t X + \tilde{\theta}_t I_t) \right)} - \frac{1-a}{R-I_t} \Leftrightarrow \\
I_t &= \frac{Ra\tilde{\theta}_t (U_A - U_{-A}) - (1-a)(\tilde{\pi}_t X (U_A + U_{-A}) - U_{-A})}{a\tilde{\theta}_t (U_A - U_{-A}) + \tilde{\theta}_t (1-a)(U_A + U_{-A})} \Leftrightarrow \\
I_t &= \frac{U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X))}{\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)}
\end{aligned}$$

Q.E.D.

## Appendix 2

### Partial derivatives from equation (2)

Parents choose their level of investment,  $I_t$ , according to

$$I_t = \frac{U_{-A} \left( 1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t \right) + U_A \left( Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X) \right)}{\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)}$$

To show how  $I_t$  increases or decreases in the equation's parameters, take partial derivatives according to the quotient rule so that for a given parameter,  $V$ ,

$$\frac{\partial I_t}{\partial V} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

where

$$f(x) = U_{-A} \left( 1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t \right) + U_A \left( Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X) \right)$$

and

$$g(x) = \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)$$

As per usual,  $I_t$  increases in a given parameter,  $V$ , if  $(\partial I_t / \partial V) > 0$  and decreases if  $(\partial I_t / \partial V) < 0$ . The following derivations show if, and under which assumptions,  $I_t$  increases or decreases in each of the equation's parameters. In interest of space, the derivations are not fully annotated or expanded. Full proofs with annotation are available upon request.

*Parents' resources, R.* Proof that  $(\partial I_t / \partial R) > 0$ .

$$\frac{\partial I_t}{\partial R} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = U_A a \tilde{\theta}_t - U_{-A} a \tilde{\theta}_t = a \tilde{\theta}_t (U_A - U_{-A})$$

$$g'(x) = 0$$

$$[g(x)]^2 = [\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\frac{\partial I_t}{\partial R} = \frac{[a \tilde{\theta}_t (U_A - U_{-A})][\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]}{[\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2} = \frac{a(U_A - U_{-A})}{U_A + U_{-A} - 2U_{-A}a}$$

By assumption,  $U_A > U_{-A}$ , and  $0 < a < 1$ , so  $U_A + U_{-A} > 2U_{-A}a$ , and  $a(U_A - U_{-A}) > 0$ , so  $(\partial I_t / \partial R) > 0$ . Thus,  $I_t$  always increases in parents' resources,  $R$ .

Q.E.D.

*Parents' altruism, a.* Proof of the conditions under which  $(\partial I_t / \partial a) > 0$ .

$$\frac{\partial I_t}{\partial a} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = U_{-A}(\tilde{\pi}_t X - R\tilde{\theta}_t) + U_A(R\tilde{\theta}_t + \tilde{\pi}_t X)$$

$$g'(x) = -2\tilde{\theta}_t U_{-A}$$

$$[g(x)]^2 = [\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned} & [(U_A + U_{-A})(\tilde{\pi}_t X - R\tilde{\theta}_t)][\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)] \\ & - [U_{-A}(1 + \tilde{\pi}_t a X - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X))] \\ \frac{\partial I_t}{\partial a} = & \frac{[-2\tilde{\theta}_t U_{-A}]}{[\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2} \end{aligned}$$

$$\text{So } \frac{\partial I_t}{\partial a} > 0 \text{ if } \frac{\left[ (U_A + U_{-A})(\tilde{\pi}_t X - R\tilde{\theta}_t) \right] \left[ \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a) \right] - \left[ U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) \right] \left[ -2\tilde{\theta}_t U_{-A} \right] + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X))}{\left[ \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a) \right]^2} > 0$$

Algebraically, this expression reduces to

$$\frac{U_{-A}^2 (2 - 2a - \tilde{\pi}_t X - R\tilde{\theta}_t) + U_A^2 (\tilde{\pi}_t X - R\tilde{\theta}_t)}{U_A U_{-A}} > 2R\tilde{\theta}_t (1 + 2a)$$

$I_t$  increases in parents' altruism if

$$(U_{-A}^2 (2 - 2a - \tilde{\pi}_t X - R\tilde{\theta}_t) + U_A^2 (\tilde{\pi}_t X - R\tilde{\theta}_t)) / (U_A U_{-A}) > 2R\tilde{\theta}_t (1 + 2a).$$

Otherwise,  $(\partial I_t / \partial a) < 0$ .

Q.E.D.

*Utility of academic success,  $U_A$ .* Proof of the conditions under which  $(\partial I_t / \partial U_A) > 0$ .

$$\frac{\partial I_t}{\partial U_A} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X)$$

$$g'(x) = \tilde{\theta}_t$$

$$[g(x)]^2 = [\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned} \frac{\partial I_t}{\partial U_A} &= \frac{\left[ Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X) \right] \left[ \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a) \right] - \left[ U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X)) \right] \tilde{\theta}_t}{\left[ \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a) \right]^2} \\ &= \frac{(Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X))(U_A + U_{-A} - 2U_{-A}a) - (U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X)))}{\tilde{\theta}_t \left[ (U_A + U_{-A} - 2U_{-A}a) \right]^2} \end{aligned}$$



Since  $U_A + U_{-A} - 2U_{-A}a > 0$  and  $\tilde{\theta}_t > 0$ ,  $\tilde{\theta}_t[(U_A + U_{-A} - 2U_{-A}a)^2] > 0$ , then  $(\partial I_t / \partial U_A) > 0$  when

$$\begin{aligned} & (Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X))(U_A + U_{-A} - 2U_{-A}a) > U_{-A} \\ & (1 + \tilde{\pi}_taX - a - \tilde{\pi}_tX - Ra\tilde{\theta}_t) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X)) \end{aligned}$$

Algebraically, this expression reduces to  $a(R\tilde{\theta}_t + \tilde{\pi}_tX) > (1/2)$ .

$I_t$  increases in the utility of academic success if  $a(R\tilde{\theta}_t + \tilde{\pi}_tX) > (1/2)$ . Otherwise,  $(\partial I_t) / (\partial U_A) < 0$ .  
Q.E.D.

*Disutility of academic failure,  $U_{-A}$*  Proof of the conditions under which  $(\partial I_t / \partial U_{-A}) > 0$ .

$$\frac{\partial I_t}{\partial U_{-A}} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = 1 + \tilde{\pi}_taX - a - \tilde{\pi}_tX - Ra\tilde{\theta}_t$$

$$g'(x) = \tilde{\theta}_t - 2\tilde{\theta}_ta$$

$$[g(x)]^2 = [\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned} & [1 + \tilde{\pi}_taX - a - \tilde{\pi}_tX - Ra\tilde{\theta}_t][\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)] \\ & - [U_{-A}(1 + \tilde{\pi}_taX - a - \tilde{\pi}_tX - Ra\tilde{\theta}_t) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X))] \end{aligned}$$

$$\frac{\partial I_t}{\partial U_{-A}} = \frac{[\tilde{\theta}_t - 2\tilde{\theta}_ta]}{[\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2}$$

Since  $[\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2 > 0$ , then  $(\partial I_t / \partial U_{-A}) > 0$  when

$$\begin{aligned}
 & \left[ 1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t \right] \left[ \tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a) \right] \\
 & > \left[ U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X)) \right] \\
 & \left[ \tilde{\theta}_t - 2\tilde{\theta}_t a \right]
 \end{aligned}$$

Algebraically, this expression reduces to  $a(R\tilde{\theta}_t + \tilde{\pi}_t X) < (1/2)$  (see note below).

$I_t$  increases in the disutility of academic failure if  $a(R\tilde{\theta}_t + \tilde{\pi}_t X) < (1/2)$ . Otherwise,  $(\partial I_t / \partial U_{-A}) < 0$ .

Q.E.D.

Note that the above derivation implicitly assumes that  $f(x) = [U_{-A}(1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X))] > 0$ . However, this assumption does not need to hold to reach the final result since I multiply by  $f(x)$  twice and thus flip the inequality sign either twice (if  $f(x) < 0$ ) or never (if  $f(x) > 0$ ). The only formally required assumption for the derivation regarding  $f(x)$  is  $f(x) \neq 0$ .

*Parents' belief in the returns to investments,  $\tilde{\theta}_t$ .* Proof of the conditions under which  $(\partial I_t / \partial \tilde{\theta}_t) > 0$ .

$$\frac{\partial I_t}{\partial \tilde{\theta}_t} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = -RaU_{-A} + RaU_A = Ra(U_A - U_{-A})$$

$$g'(x) = U_A + U_{-A} - 2U_{-A}a$$

$$[g(x)]^2 = [\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned}
 & [Ra(U_A - U_{-A})][\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)] \\
 & - [U_{-A} (1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A (Ra\tilde{\theta}_t + \tilde{\pi}_t (aX - X))]
 \end{aligned}$$

$$\frac{\partial I_t}{\partial \tilde{\theta}_t} = \frac{[U_A + U_{-A} - 2U_{-A}a]}{[\tilde{\theta}_t (U_A + U_{-A} - 2U_{-A}a)]^2}$$

Since  $[\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2 > 0$ ,  $(\partial I_t / \partial \tilde{\theta}_t) > 0$  when

$$\begin{aligned} & Ra\tilde{\theta}_t(U_A - U_{-A})(U_A + U_{-A} - 2U_{-A}a) \\ & - \left[ U_{-A}(1 + \tilde{\pi}_t aX - a - \tilde{\pi}_t X - Ra\tilde{\theta}_t) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t(aX - X)) \right] \\ & [U_A + U_{-A} - 2U_{-A}a] > 0 \end{aligned}$$

Algebraically, this expression reduces to  $\tilde{\pi}_t X(U_{-A} + U_A) - U_{-A} > a(\tilde{\pi}_t X(U_{-A} + U_A) - U_{-A})$ .

This expression is always true when  $\tilde{\pi}_t X(U_{-A} + U_A) > U_{-A}$  since  $1 > a$  and always false otherwise. So,  $I_t$  increases in parents' belief in the returns to investments if  $\tilde{\pi}_t X(U_{-A} + U_A) > U_{-A}$ , an expression which equals both  $\tilde{\pi}_t X > ((U_{-A}) / (U_{-A} + U_A))$  and  $(U_{-A} / U_A) < ((\tilde{\pi}_t X) / (1 - \tilde{\pi}_t X))$ . Otherwise,  $(\partial I_t / \partial \tilde{\theta}_t) < 0$ .  
Q.E.D.

*Parents' other factors, X.* Proof that  $(\partial I_t / \partial X) < 0$ .

$$\frac{\partial I_t}{\partial X} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\begin{aligned} f'(x) &= (\tilde{\pi}_t aU_{-A} - \tilde{\pi}_t U_{-A}) + U_A(Ra\tilde{\theta}_t + \tilde{\pi}_t aX - \tilde{\pi}_t X) \\ &= \tilde{\pi}_t(a-1)(U_{-A} + U_A) \end{aligned}$$

$$g'(x) = 0$$

$$[g(x)]^2 = [\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned} \frac{\partial I_t}{\partial X} &= \frac{[\tilde{\pi}_t(a-1)(U_{-A} + U_A)][\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]}{[\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2} \\ &= \frac{\tilde{\pi}_t(a-1)(U_{-A} + U_A)}{\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)} \end{aligned}$$

Since  $0 < a < 1$ , this means that  $a - 1 < 0$ . In addition,  $0 < \tilde{\pi}_t < 1$  and  $U_{-A} + U_A > 0$ , so  $\tilde{\pi}_t(a - 1)(U_{-A} + U_A) < 0$ .

Thus, by  $\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a) > 0$ ,  $(\partial I_t / \partial X) < 0$ .  $I_t$  always decreases in parents' other factors,  $X$ .

Q.E.D.

Parents' belief in the returns to other factors,  $\tilde{\pi}_t$ . Proof that  $(\partial I_t / \partial \tilde{\pi}_t) < 0$ .

$$\frac{\partial I_t}{\partial \tilde{\pi}_t} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = (U_{-A}aX - U_{-A}X) + (U_A(aX - X)) = X(U_{-A}a - U_{-A} + U_Aa - U_A)$$

$$g'(x) = 0$$

$$[g(x)]^2 = [\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2$$

$$\begin{aligned} \frac{\partial I_t}{\partial \tilde{\pi}_t} &= \frac{[X(U_{-A}a - U_{-A} + U_Aa - U_A)] [\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]}{[\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)]^2} \\ &= \frac{X(U_{-A}a - U_{-A} + U_Aa - U_A)}{\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a)} \end{aligned}$$

By assumption,  $U_A > U_{-A}$ , so  $U_A + U_{-A} > 2U_{-A}$ , so  $\tilde{\theta}_t(U_A + U_{-A} - 2U_{-A}a) > 0$ . In addition,  $0 < a < 1$ , so  $U_Aa < U_A$  and  $U_{-A}a < U_{-A}$ , and  $0 < X < 1$ , so  $[X(U_{-A}a - U_{-A} + U_Aa - U_A)] < 0$ . Thus,  $(\partial I_t / \partial \tilde{\pi}_t) < 0$  and  $I_t$  always decreases in parents' belief in the returns to other factors,  $\tilde{\pi}_t$ .

Q.E.D.

### Appendix 3

*Proof that the confounded learning equilibrium (CLE) increases in other factors,  $X$*

We have

$$z^* = \frac{U_A(\pi X(1-a) + \theta(X - Ra)) + U_{-A}(\pi X(1-a) + \theta(X(1-2a) + Ra) + a - 1)}{a\Delta(R - X)(U_A - U_{-A})}$$

As per Appendix 2, we use the quotient rule to prove the conditions under which  $(\partial z^* / \partial X) > 0$

$$\frac{\partial z^*}{\partial X} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\begin{aligned} f(x) &= U_A(\pi X(1-a) + \theta(X - Ra)) \\ &+ U_{-A}(\pi X(1-a) + \theta(X(1-2a) + Ra) + a - 1) \\ &= U_A\pi X - U_A\pi Xa + U_A\theta X - U_A\theta Ra + U_{-A}\pi X - aU_{-A}\pi X \\ &+ U_{-A}\theta X - 2U_{-A}\theta Xa + U_{-A}\theta Ra + U_{-A}a - U_{-A} \end{aligned}$$

$$\begin{aligned} f'(x) &= U_A\pi - U_A\pi a + U_A\theta + U_{-A}\pi - aU_{-A}\pi \\ &+ U_{-A}\theta - 2U_{-A}\theta a = (\pi + \theta)(U_A + U_{-A}) - a(2U_{-A}\theta - \pi(U_A - U_{-A})) \end{aligned}$$

$$g(x) = a\Delta(R - X)(U_A - U_{-A}) = U_A a\Delta R - U_A a\Delta X - U_{-A} a\Delta R + U_{-A} a\Delta X$$

$$g'(x) = -U_A a\Delta + U_{-A} a\Delta = a\Delta(U_{-A} + U_A)$$

$$[g(x)]^2 = [a\Delta(R - X)(U_A - U_{-A})]^2$$

$$[(\pi + \theta)(U_A + U_{-A}) - a(2U_{-A}\theta - \pi(U_A - U_{-A}))]$$

$$[a\Delta(R - X)(U_A - U_{-A})] - \left[ \begin{array}{l} U_A(\pi X(1-a) + \theta(X - Ra)) \\ + U_{-A} \left( \begin{array}{l} \pi X(1-a) + \theta \left( \begin{array}{l} X(1-2a) \\ + Ra \end{array} \right) \\ + a - 1 \end{array} \right) \end{array} \right]$$

$$\frac{\partial z^*}{\partial X} = \frac{[a\Delta(U_{-A} + U_A)]}{[a\Delta(R - X)(U_A - U_{-A})]^2}$$

$$\text{So } \frac{\partial z^*}{\partial X} > 0 \text{ if } \frac{\left[ \begin{aligned} &[(\pi + \theta)(U_A + U_{-A}) - a(2U_{-A}\theta - \pi(U_A - U_{-A}))] \\ &[a\Delta(R - X)(U_A - U_{-A})] \\ & - \left[ \begin{aligned} &U_A(\pi X(1 - a) + \theta(X - Ra)) \\ & + U_{-A} \left( \begin{aligned} &\pi X(1 - a) + \theta(X(1 - 2a) + Ra) \end{aligned} \right) \\ & + a - 1 \end{aligned} \right] \end{aligned} \right]}{[a\Delta(U_{-A} + U_A)]} > 0 > 0$$

Algebraically, this expression reduces to

$$R(U_A(\pi + \theta) + \pi U_{-A}) + \frac{1 - 2a}{1 - a} \theta U_{-A} (R(1 - U_A a) - X(1 - U_A)) > U_{-A}$$

Knowing that  $R(U_A(\pi + \theta) + \pi U_{-A}) > 0$  and  $\theta U_{-A} > 0$ , we assume  $R(U_A(\pi + \theta) + \pi U_{-A}) > U_{-A} \Leftrightarrow (U_A / U_{-A}) \geq ((1 - R\pi) / (R(\pi + \theta)))$ . Then  $(\partial z^* / \partial X) > 0$  when  $((1 - 2a) / (1 - a))(R(1 - U_A a) - X(1 - U_A)) > 0$ , that is, when both  $(1 - 2a) / (1 - a)$  and  $(R(1 - U_A a) - X(1 - U_A))$  are either positive or negative at the same time.

Now, by

$$\frac{1 - 2a}{1 - a} > 0 \Leftrightarrow a < 0.5$$

and

$$\frac{1 - 2a}{1 - a} < 0 \Leftrightarrow a > 0.5$$

as well as

$$R(1 - U_A a) - X(1 - U_A) > 0 \Leftrightarrow R > X \frac{1 - U_A}{1 - U_A a}$$

and

$$R(1 - U_A a) - X(1 - U_A) < 0 \Leftrightarrow R < X \frac{1 - U_A}{1 - U_A a}$$

we conclude that, under the assumption that  $(U_A / U_{-A}) \geq ((1 - R\pi) / R(\pi + \theta))$ ,  $(\partial z^* / \partial X) > 0$  if

$$(1) \quad a < 0.5 \text{ and } R > X((1 - U_A) / (1 - U_A a))$$

or if

$$(2) \quad a > 0.5 \text{ and } R < X((1 - U_A) / (1 - U_A a)).$$

In other words,  $\partial z^* / \partial X$  is only negative if a high level of altruism,  $a$ , is combined with a high level of resources,  $R$ , compared to other factors,  $X$ , or in the case of low altruism,  $a$ , combined with a low level of resources,  $R$ , compared with other factors,  $X$ . While not always the case, given that  $(1 - U_A) / (1 - U_A a)$  increases in  $a$ , a high level of altruism will be likely to satisfy both the condition  $R < X((1 - U_A) / (1 - U_A a))$  as well as  $a > 0.5$  and vice versa.

Regarding the assumption that  $R(U_A(\pi + \theta) + \pi U_{-A}) > U_{-A} \Leftrightarrow (U_A / U_{-A}) \geq ((1 - R\pi) / R(\pi + \theta))$ , this is only violated under a low level of parents' resources combined with a small difference between  $U_A$  and  $U_{-A}$ . However, as  $U_A / U_{-A}$  goes toward one,  $\partial z^* / \partial X$  may become negative for low resource parents.  
Q.E.D.

**Appendix 4.** Parental investment—individual items and their distributions.

Phrasing of question	Distribution of answers	
About how many books does child have?	6–9 years old	
	None	1.09%
	1 or 2	3.58%
	3–9	11.47%
	≥10	83.84%
	10–14 years old	
	None	2.09%
	1 or 9	18.28%
Is there a musical instrument (e.g. piano, drum, guitar) that your child can use here at home?	Yes	50.99%
	No	49.01%
Does your child get special lessons or belong to any organization that encourages activities such as sports, music, art, dance, and drama?	Yes	40.82%
	No	59.18%
How often has any family member taken or arranged to take your child to any type of museum (children's, scientific, art, historical, etc.) within the past year?	Never	25.77%
	Once or twice	38.90%
	Several times	26.77%
	About once a month	7.18%
	About once a week or more often	1.37%
	How often has a family member taken or arranged to take your child to any type of musical or theatrical performance within the past year?	Never
Once or twice		37.66%
Several times		17.91%
About once a month or more		3.87%
About once a week or more		0.93%
When your family watches TV together, do you or your child's father or father-figure discuss TV programs with him or her?		No
	Yes	80.80%
	Do not have a TV	0.38%