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ABSTRACT

Recently CMS Collaboration measured mixed-harmonic four-particle azimuthal correlations, known as symmetric cumulants $SC(n, m)$, in pp and p+Pb collisions, and interpreted the non-zero $SC(n, m)$ as evidence for long-range collectivity in these small collision systems. Using the PYTHIA and HIJING models which do not have genuine long-range collectivity, we show that the CMS results, obtained with standard cumulant method, could be dominated by non-flow effects associated with jet and dijets, especially in pp collisions. We show that the non-flow effects are largely suppressed using the recently proposed subevent cumulant methods by requiring azimuthal correlation between two or more pseudorapidity ranges. We argue that the reanalysis of $SC(n, m)$ using the subevent method in experiments is necessary before they can used to provide further evidences for a long-range multi-particle collectivity and constraints on theoretical models in small collision systems.

1. Introduction

Measurements of two-particle angular correlation in small collision systems, such as pp or p+A, have revealed the ridge phenomena [1–5]: enhanced production of pairs at small azimuthal angle separation, $\Delta\phi$, extended over wide range of pseudorapidity separation $\Delta\eta$. The azimuthal structure of the ridge is often characterized by a Fourier series $dN_{pair}/d\Delta\phi \sim 1 + 2\Sigma v_n^2 \cos(n\Delta\phi)$, and studied as a function of charged particle multiplicity N_{ch}. The v_n denotes the anisotropy coefficients for single particle distribution, with v_2 being the largest followed by v_3. The ridge reflects multiparton dynamics at early time of the collision and has generated significant interests in high-energy physics community. One key question concerning the ridge is the timescale for the emergence of the long-range multi-particle collectivity, whether it reflects initial momentum correlation from gluon saturation effects [6] or it reflects a final-state hydrodynamic response to the initial transverse collision geometry [7].

More insights about the ridge is obtained via multi-particle correlation technique, known as cumulants, involving four or more particles [8–11]. The multi-particle cumulants probe the event-by-event fluctuation of v_n, $p(v_n)$, as well as the correlation between v_n of different order, $p(v_n, v_m)$. For example, four-particle cumulant $c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$ constrains the width of $p(v_n)$ [8], while four-particle symmetric cumulants $SC(n, m) = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$ quantifies the lowest-order correlation between v_n and v_m [10].

The main challenge in the study of azimuthal correlations in small systems is how to distinguish long-range ridge correlations from "non-flow" correlations such as resonance decays, jets, or dijet production. In A+A collisions, non-flow is naturally suppressed due to large particle multiplicity, i.e. non-flow contribution scales as $1/N_{ch}$ and $1/N_{ch}^3$ for the two- and four-particle cumulants, respectively [12]. In small systems, however, non-flow can be large due to their much smaller N_{ch} values, and one has to emploly new methods that explicitly exploit the long-range nature of the collectivity in η: For two-particle correlations, the non-flow is suppressed by requiring a large $\Delta\eta$ gap and a peripheral subtraction procedure [2–4,13–15]. For multi-particle cumulants, the non-flow can be suppressed by requiring correlation between particles from different subevents separated in η, while keeping the genuine long-range multi-particle correlations associated with the ridge. This so-called subevent method [11] has been shown to be necessary to obtain a reliable $c_n\{4\}$ [16], while the $c_2\{4\}$ based on the standard cumulant method [15,17] are contaminated by non-flow correlations over the full N_{ch} range in pp collisions and the low N_{ch} region in p+A collisions.

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Recently CMS Collaboration also released measurements of $SC(2, 3)$ and $SC(2, 4)$ in pp and $p+Pb$ collisions, based on the standard cumulant method [18]. However, since these observables have much smaller signal than $c_2\{4\}$, they are expected to be even more susceptible to non-flow effects. Therefore, more precise study of the influence of non-flow effects to these observables is required before any interpretation of the experimental measurements. Event generators such as PYTHIA8 [19] and HIJING [20], which contain only non-flow correlations, are perfect test-ground for estimating the influence of non-flow to symmetric cumulants in small systems, which is the focus of this paper. Using a PYTHIA8 simulation of pp collisions and HIJING simulation of $p+Pb$ collisions, we demonstrate that $SC(n, m)$ based on the standard method is dominated by non-flow in pp collisions, and is contaminated by non-flow in $p+Pb$ collisions. We show that reliable $SC(n, m)$ measurements can be obtained using three-subevent or four-subevent methods, which therefore should be the preferred methods for analyzing multi-particle correlations in small systems.

2. Symmetric cumulants

The framework for the standard cumulant is described in Refs. [9,10], which was recently extended to the case of subevent cumulants in Ref. [11,21]. The four-particle symmetric cumulants $SC(n, m)$ are related to two- and four-particle azimuthal correlations for flow harmonics of order n and m, $n \neq m$ as:

$$\langle\{4\}_{n,m}\rangle = \left\langle e^{in(\phi_1-\phi_2)+im(\phi_3-\phi_4)}\right\rangle,$$

$$\langle\{2\}_n\rangle = \left\langle e^{in(\phi_1-\phi_2)}\right\rangle, \quad \langle\{2\}_m\rangle = \left\langle e^{im(\phi_1-\phi_2)}\right\rangle, \quad (1)$$

$$SC(n, m) = \langle\!\langle\{4\}_{n,m}\rangle\!\rangle - \langle\!\langle\{2\}_n\rangle\!\rangle\langle\!\langle\{2\}_m\rangle\!\rangle = \langle\!\langle e^{in(\phi_1-\phi_2)+im(\phi_3-\phi_4)}\rangle\!\rangle$$
$$- \langle\!\langle e^{in(\phi_1-\phi_2)}\rangle\!\rangle\langle\!\langle e^{im(\phi_1-\phi_2)}\rangle\!\rangle. \quad (2)$$

One firstly averages all distinct quadruplets or pairs in one event to obtain $\langle\{4\}_{n,m}\rangle$, $\langle\{2\}_n\rangle$ and $\langle\{2\}_m\rangle$, then average over an event ensemble to obtain $\langle\!\langle\{4\}_{n,m}\rangle\!\rangle$, $\langle\!\langle\{2\}_n\rangle\!\rangle$, $\langle\!\langle\{2\}_m\rangle\!\rangle$ and $SC(n, m)$. In the absence of non-flow correlations, $SC(n, m)$ measures the correlation between event-by-event fluctuations of v_n and v_m:

$$SC(n, m)_{flow} = \left\langle v_n^2 v_m^2\right\rangle - \left\langle v_n^2\right\rangle\left\langle v_m^2\right\rangle \quad (3)$$

In the standard cumulant method, all quadruplets and pairs are selected using the entire detector acceptance. To suppress the non-flow correlations that typically involve particles emitted within a localized region in η, the particles can be grouped into several subevents, each covering a non-overlapping η interval. The multi-particle correlations are then constructed by correlating particles between different subevents, further reducing non-flow correlations.

Specifically, in the two-subevent cumulant method, the entire event is divided into two subevents, labeled as a and b, for example according to $-\eta_{max} < \eta_a < 0$ and $0 < \eta_b < \eta_{max}$. The symmetric cumulant is defined by considering all quadruplets comprised of two particles from each subevent, or pairs comprised of one particle from each subevent:

$$SC(n, m)_{2-sub} = \langle\!\langle e^{in(\phi_1^a-\phi_2^b)+im(\phi_3^a-\phi_4^b)}\rangle\!\rangle$$
$$- \langle\!\langle e^{in(\phi_1^a-\phi_2^b)}\rangle\!\rangle\langle\!\langle e^{im(\phi_1^a-\phi_2^b)}\rangle\!\rangle, \quad (4)$$

where the superscript or subscript a (b) indicates particles chosen from the subevent a (b). The two-subevent method suppresses correlations within a single jet (intra-jet correlations), since each jet usually emits particles to one subevent.

Similarly for the three-subevent and four-subevent methods, the $|\eta| < \eta_{max}$ range is divided into three or four equal ranges, and are labelled as a, b and c or a, b, c and d, respectively. The corresponding symmetric cumulants are defined as:

$$SC(n, m)_{3-sub} = \langle\!\langle e^{in(\phi_1^a-\phi_2^b)+im(\phi_3^a-\phi_4^c)}\rangle\!\rangle - \langle\!\langle e^{in(\phi_1^a-\phi_2^b)}\rangle\!\rangle\langle\!\langle e^{im(\phi_1^a-\phi_2^c)}\rangle\!\rangle$$
$$(5)$$

$$SC(n, m)_{4-sub} = \langle\!\langle e^{in(\phi_1^a-\phi_2^b)+im(\phi_3^c-\phi_4^d)}\rangle\!\rangle - \langle\!\langle e^{in(\phi_1^a-\phi_2^b)}\rangle\!\rangle\langle\!\langle e^{im(\phi_1^c-\phi_2^d)}\rangle\!\rangle$$
$$(6)$$

Since the two jets in a dijet event usually produce particles in at most two subevents, the three-subevent and four-subevent method further suppresses inter-jet correlations associated with dijets. Furthermore, four-subevent suppresses possible three-jet correlations, although such contributions are expected to be small. To enhance the statistical precision, the η range for subevent a is also interchanged with that for subevent b, c or d, which results in three independent $SC(n, m)_{3-sub}$ and three independent $SC(n, m)_{4-sub}$. They are averaged to obtain the final result.

3. Model setup

To evaluate the influence of non-flow to $SC(n, m)$ in the standard and subevent method, the PYTHIA8 and HIJING models are used to generate pp events at $\sqrt{s} = 13$ GeV and $p+Pb$ events at $\sqrt{s_{NN}} = 5.02$ TeV, respectively. These models contain significant non-flow correlations from jets, dijets, and resonance decays, which are reasonably tuned to describe the data, such as p_T spectra, N_{ch} distributions. Multi-particle cumulants based on the standard method as well as subevent methods are calculated as a function of charged particle multiplicity N_{ch}. To make the results directly comparable to the CMS measurement [18], the cumulant analysis is carried out using charged particles in $|\eta| < \eta_{max} = 2.5$ and several p_T ranges, and the N_{ch} is defined as the number of charged particles in $|\eta| < 2.5$ and $p_T > 0.4$ GeV.

The symmetric cumulants are calculated in several steps using charged particles with $|\eta| < 2.5$, similar to Refs. [11,16]. Firstly, the multi-particle correlators $\langle\{2k\}\rangle$ with $k = 1, 2$ (indexes n and m are dropped for simplicity) in Eq. 1 are calculated for each event from particles in one of the two p_T ranges, $0.3 < p_T < 3$ GeV and $0.5 < p_T < 5$ GeV, and the number of charged particle in this p_T range, N_{ch}^{sel}, is calculated. Note that N_{ch}^{sel} is not the same as N_{ch} defined earlier due to different p_T ranges used. Secondly, $\langle\{2k\}\rangle$ are averaged over events with the same N_{ch}^{sel} to obtain $\langle\!\langle\{2k\}\rangle\!\rangle$ and $SC(n, m)$. The $SC(n, m)$ values calculated for unit N_{ch}^{sel} bin are then combined over broader N_{ch}^{sel} ranges of the event ensemble to obtain statistically significant results. Finally, the $SC(n, m)$ obtained for a given N_{ch}^{sel} are mapped to given $\langle N_{ch}\rangle$ to make the results directly comparable to the CMS measurements [18].

To further study the influence of non-flow fluctuations associated with multiplicity fluctuations, several other p_T ranges, different from those used for $\langle\{2k\}\rangle$, are also used to calculated N_{ch}^{sel}. The results from this study are discussed in Appendix A.

4. Results

First we calculate the $SC(2, 4)$ and $SC(2, 3)$ from PYTHIA and HIJING using the standard cumulant method and compare them with the CMS pp and $p+Pb$ data for charged particles. The same p_T selection, $0.3 < p_T < 3$ GeV, is used to calculate the cumulants as well as to select the event class N_{ch}^{sel}.

The comparison is shown in Fig. 1. The results from models are non-zero and they decrease as a function of N_{ch} similar to the
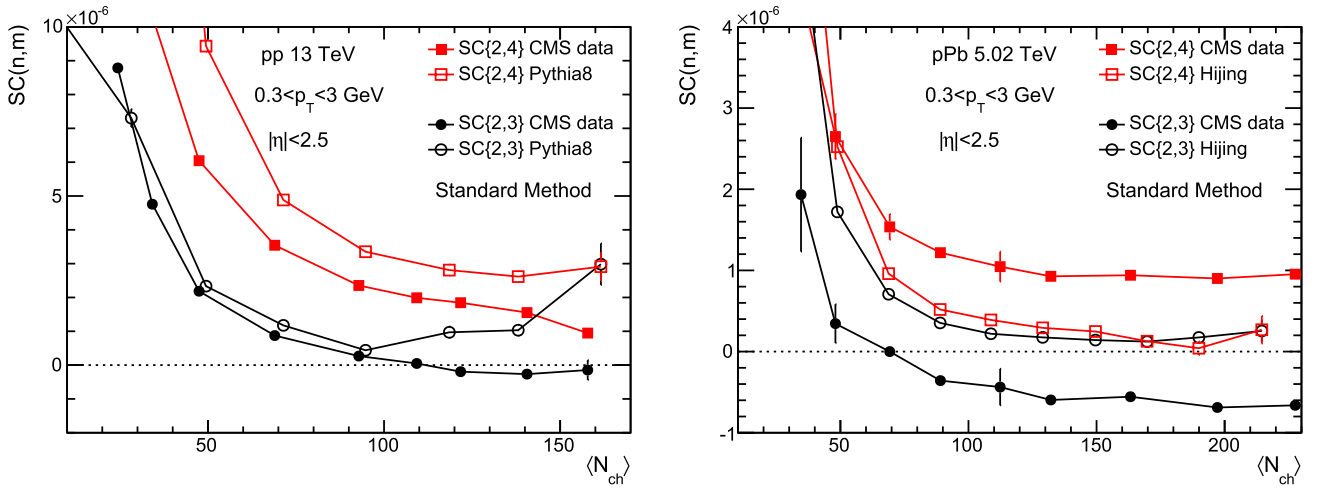
Fig. 1. The $SC(n, m)$ calculated for charged particles with $0.3 < p_T < 3$ GeV with the standard cumulant method in 13 TeV *pp* collisions (left panel) and 5.02 TeV *p+Pb* collisions (right panel) compared between data (solid symbols) and Monte Carlo models (open symbols).
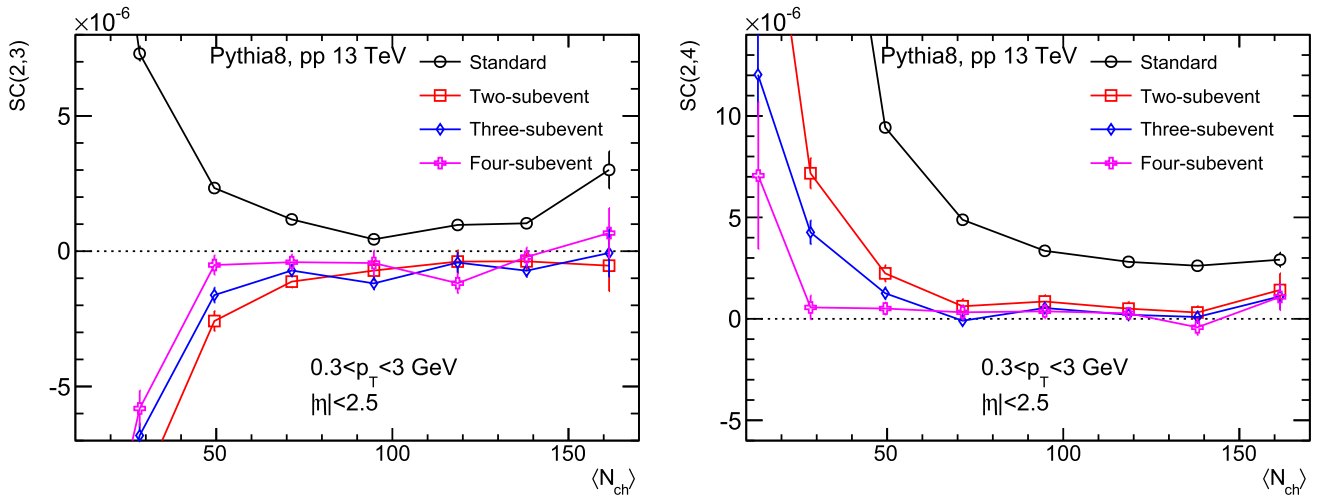

Fig. 2. The $SC(2, 3)$ (left panel) and $SC(2, 4)$ (right panel) in $0.3 < p_T < 3$ GeV and $|\eta| < 2.5$ as a function of N_{ch} obtained from 13 TeV *pp* PYTHIA 8 simulations using the standard cumulant, two-subevent, three-subevent and four-subevent methods.
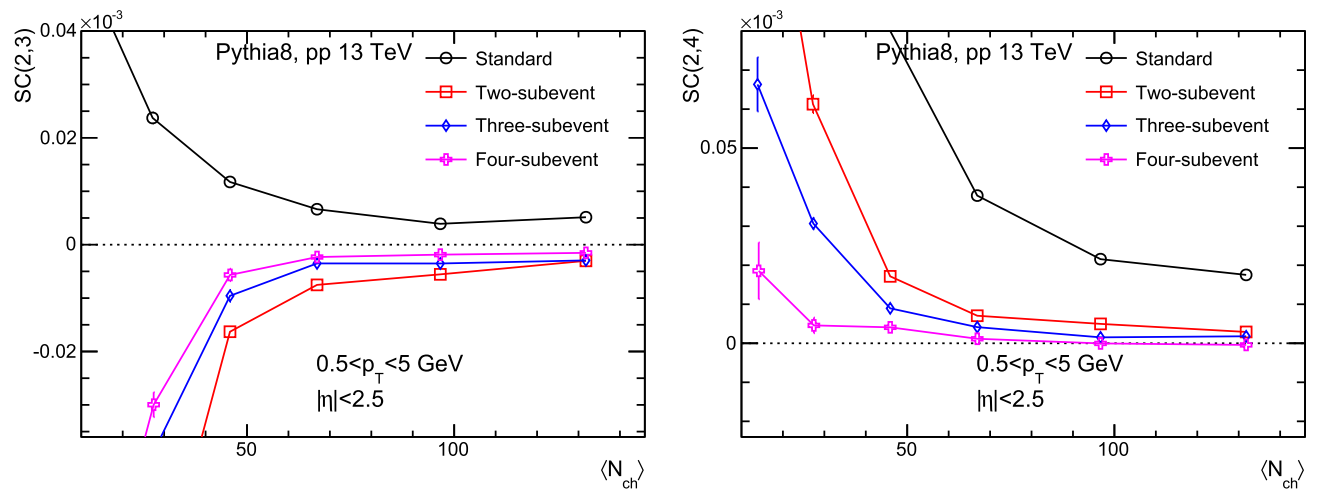

Fig. 3. The $SC(2, 3)$ (left panel) and $SC(2, 4)$ (right panel) for charged particles in $0.5 < p_T < 5$ GeV and $|\eta| < 2.5$ as a function of N_{ch} obtained from 13 TeV *pp* PYTHIA 8 simulations using the standard cumulant, two-subevent, three-subevent and four-subevent methods.
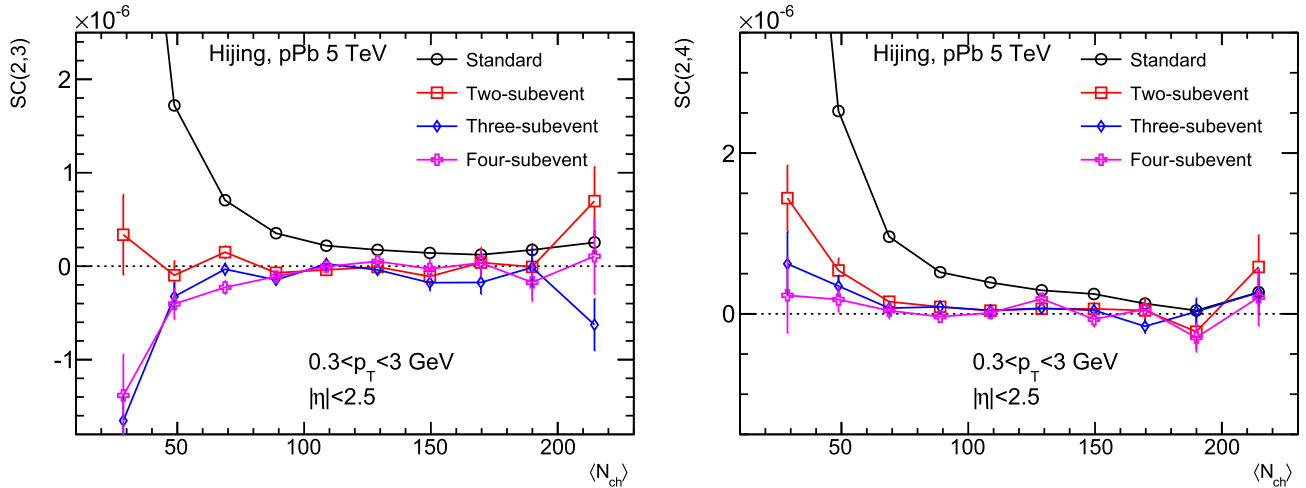
Fig. 4. The SC(2, 3) (left panel) and SC(2, 4) (right panel) in $0.3 < p_T < 3$ GeV and $|\eta| < 2.5$ as a function of N_{ch} obtained from 5.02 TeV p+Pb HIJING simulations using the standard cumulant, two-subevent, three-subevent and four-subevent methods.
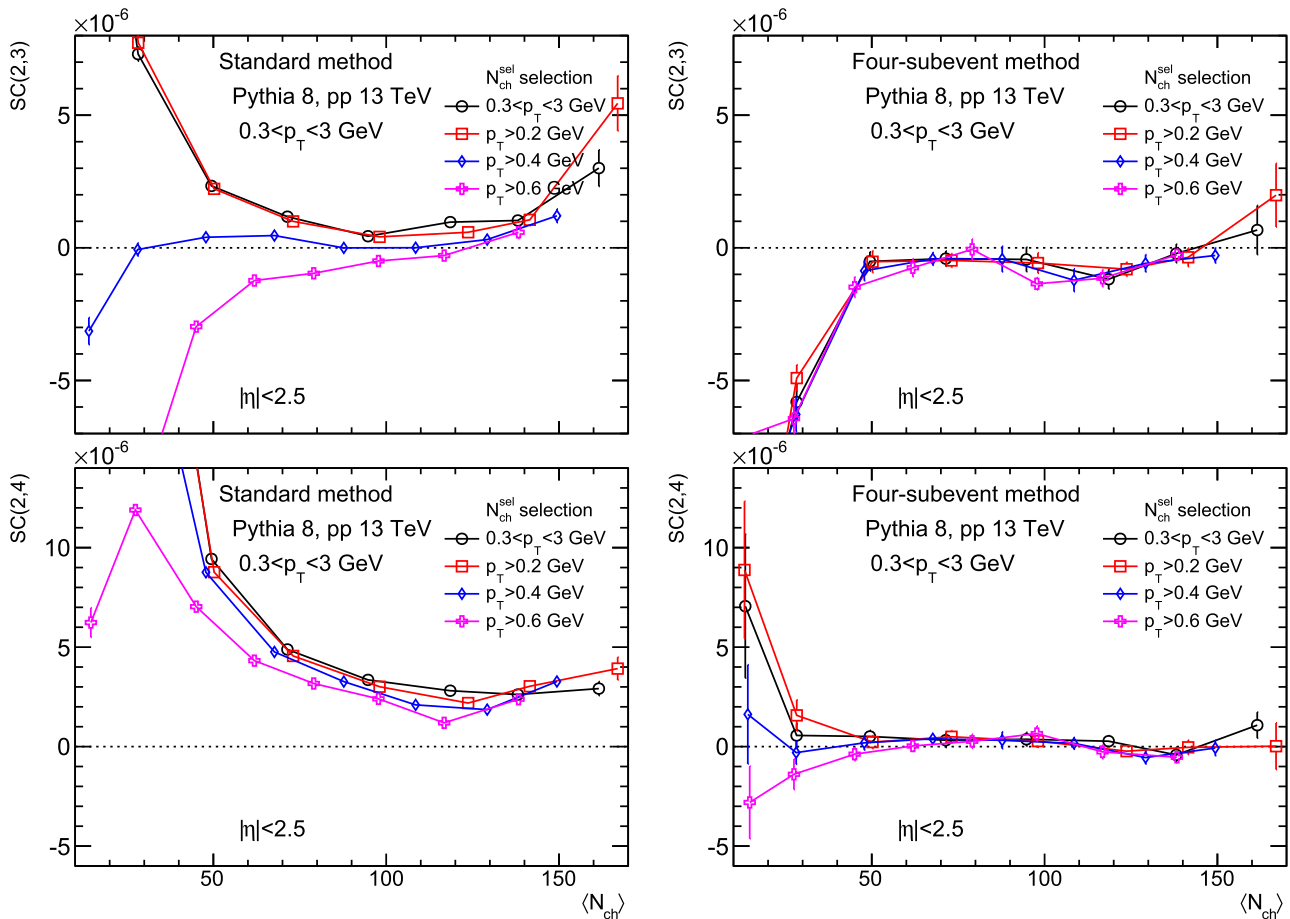

Fig. 5. The SC(2, 3) (top row) and SC(2, 4) (bottom row) calculated for charged particles in $0.3 < p_T < 3$ GeV and several N_{ch}^{sel}. They are obtained using the standard cumulant method (left column) and four-subevent method (right column) in pp collisions generated with PYTHIA 8 model.

data, indicating that the data may have significant non-flow contributions. In pp collisions as shown in the left panel, both SC(2, 4) and SC(2, 3) from the PYTHIA8 model are larger than the data, suggesting that either PYTHIA8 overestimates the non-flow contribution in SC(n, m) or the flow correlation signals are negative. In p+Pb collisions as shown in the right panel, SC(n, m) from the HI-JING model are larger than (for SC(2, 3)) or roughly comparable

(for SC(2, 4)) with the data for $N_{ch} < 70$, but their magnitudes are much smaller than the data for $N_{ch} > 100$. This implies that the influence of the non-flow is subdominant in p+Pb collisions, about 20% or less, at large N_{ch} region, but it still dominates the small N_{ch} region.

The comparison shown in Fig. 1 suggests that the symmetric cumulants measured with the standard method are strongly bi-
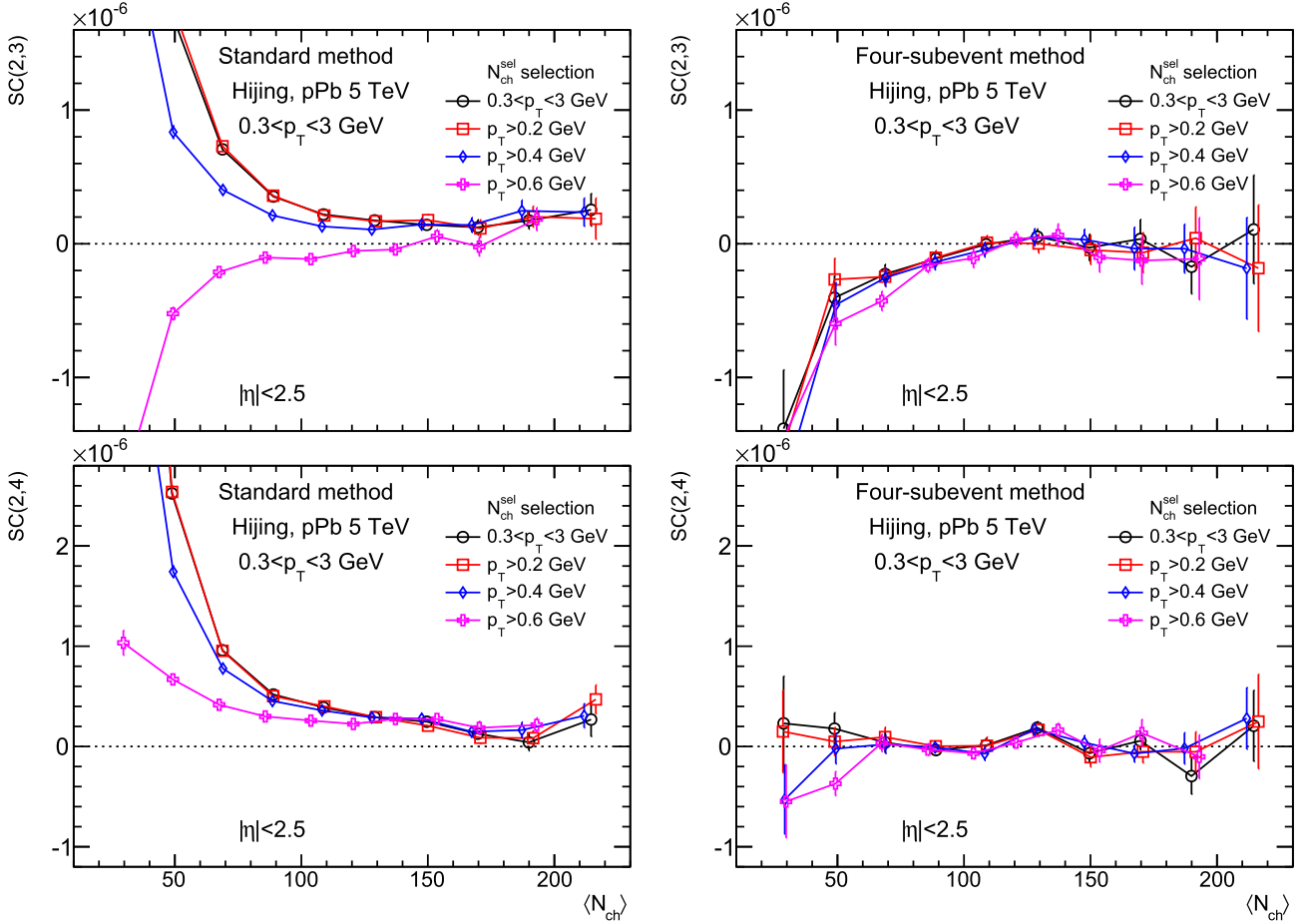
Fig. 6. The SC(2, 3) (top row) and SC(2, 4) (bottom row) calculated for charged particles in $0.3 < p_T < 3$ GeV and several N_{ch}^{sel}. They are obtained using the standard cumulant method (left column) and four-subevent method (right column) in $p+$Pb collisions generated with HIJING model.

ased by non-flow correlations in pp collisions over the full N_{ch} range and in $p+$Pb collisions at low N_{ch} region. On the other hand, the non-flow correlations are expected to be greatly suppressed in the subevent methods. Figs. 2 and 3 show SC(n, m) obtained from various methods in pp collisions for charged particles in $0.3 < p_T < 3$ GeV and $0.5 < p_T < 5$ GeV, respectively. The same p_T selections are used to calculate the cumulants as well as to select the event class.

Figs. 2 and 3 show that the values of SC(n, m) from subevent methods are much smaller than those from the standard method. In particular, the four-subevent method gives results that are closest to 0, followed by the three-subevent method and then the two-subevent method. Since non-flow contributions are known to increase with p_T, such hierarchy between different methods are more clearly revealed in Fig. 3 than in Fig. 2. It is also interesting to note that the values of SC(2, 3) is negative in the subevent methods, and can't be fully suppressed to zero even in the four-subevent method. The sign-change of SC(2, 3) between the standard and two-subevent can be understood as the interplay between the inter-jet and intra-jet correlations: while the inter-jet correlation gives a positive contribution to SC(2, 3), the intra-jet correlation from dijets gives a negative contribution. The SC(2, 3) in standard method is positive because the inter-jet correlation dominates over the intra-jet contribution. However since the dijet contributions are further suppressed in the three-subevent and four-subevent methods, the residual negative SC(2, 3) in these two methods suggest the existence, in PYTHIA8 and HIJING, of a small

long-range non-flow source that correlate between the 2nd and 3rd harmonics.

Similar observations are found in $p+$Pb collisions as shown in Fig. 4, i.e. results from the subevent methods are closer to zero than those from the standard method. However, due to a much smaller non-flow in $p+$Pb collisions ($\sim$ ten times smaller than pp at comparable N_{ch} for $N_{ch} > 100$), the precision of the simulation does not allow a clear separation between different subevent methods. This also implies that we can already obtain reliable SC(n, m) as soon as the subevent method is applied.

5. Summary

Multi-particle azimuthal correlation between different flow harmonics v_n and v_m, known as symmetric cumulants SC(n, m), has been used to study the nature of the long-range ridge in pp and $p+$Pb collision. Using the PYTHIA and HIJING models which contains only non-flow correlations, we show that recently measured SC(2, 4) and SC(2, 3), by the CMS Collaboration via the standard cumulant method, are likely contaminated by non-flow associated with jet and dijets. By requiring azimuthal correlation between multiple pseudorapidity η ranges, we show that calculations using the recently proposed subevent cumulant methods are much less sensitive to these non-flow sources. Although the subevent methods can suppressed SC(2, 4) to nearly zero in high-multiplicity pp and $p+$Pb collisions, the SC(2, 3) from subevent methods still shows a small but negative correlation in these collisions. These studies suggest that the measurements of SC(n, m) need to be re-

done with the subevent methods, before any physics conclusion related to long-range collectivity can be drawn.

Appendix A. Sensitivity to event class definition

Another way to quantify the influence of the non-flow in the cumulant method is to study the sensitivity of $SC(n, m)$ on the choice of N_{ch}^{sel}. Previous studies shows that different N_{ch}^{sel} leads to drastically change the nature of the non-flow fluctuations, leading to different cumulant results. Following the example of Ref. [11, 16], the impact of non-flow fluctuations to $SC(n, m)$ are probed by varying the p_T requirements used to define N_{ch}^{sel} as follows: When $\langle\{2k\}\rangle$ is calculated in the range $0.3 < p_T < 3$ GeV, N_{ch}^{sel} is evaluated in four different track p_T ranges: $0.3 < p_T < 3$ GeV, $p_T > 0.2$ GeV, $p_T > 0.4$ GeV and $p_T > 0.6$ GeV. When $\langle\{2k\}\rangle$ is calculated in $0.5 < p_T < 5$ GeV, N_{ch}^{sel} is evaluated in four different track p_T ranges: $0.5 < p_T < 5$ GeV, $p_T > 0.2$ GeV, $p_T > 0.4$ GeV and $p_T > 0.6$ GeV. The $SC(n, m)$ values obtained for a given N_{ch}^{sel} are mapped to given N_{ch}, so that $SC(n, m)$ obtained for different N_{ch}^{sel} can be compared using a common x-axis defined by N_{ch}.

The results of this study are shown in Fig. 5 and 6 for pp and p+Pb collisions, respectively. A strong sensitivity of $SC(n, m)$ on N_{ch}^{sel} is observed in the standard method. But such sensitivity is greatly reduced in the subevent method.

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