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Helset, Andreas; Trott, Michael

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Andreas Helset and Michael Trott

Niels Bohr International Academy and Discovery Centre, Niels Bohr Institute,
University of Copenhagen, Blegdamsvej 17, Copenhagen, DK-2100 Denmark

E-mail: ahelset@nbi.ku.dk, michael.trott@cern.ch

ABSTRACT: We discuss interference in the limit $\hat{m}_W^2/s \to 0$ in the Standard Model Effective Field Theory (SMEFT). Dimension six operators that contribute to $\bar{\psi}\psi \to \bar{\psi}'_1 \psi'_2 \bar{\psi}'_3 \psi'_4$ scattering events can experience a suppression of interference effects with the Standard Model in this limit. This occurs for subsets of phase space in some helicity configurations. We show that approximating these scattering events by $2 \to 2$ on-shell scattering results for intermediate unstable gauge bosons, and using the narrow width approximation, can miss interference terms present in the full phase space. Such interference terms can be uncovered using off-shell calculations as we explicitly show and calculate. We also study the commutation relation between the SMEFT expansion and the narrow width approximation, and discuss some phenomenological implications of these results.

Contents

1 Introduction

When physics beyond the Standard Model (SM) is present at scales larger than the Electroweak scale, the SM can be extended into an Effective Field Theory (EFT). This EFT can characterize the low energy limit (also known as the infrared (IR) limit) of such physics relevant to the modification of current experimental measurements. Assuming that there are no light hidden states in the spectrum with appreciable couplings in the SM, and that a $SU_L(2)$ scalar doublet with hypercharge $y_h = 1/2$ is present in the IR limit of a new physics sector, the theory that results from expanding in the Higgs vacuum expectation value $\sqrt{2 \langle H^\dagger H \rangle} \equiv \bar{v}_T$ over the scale of new physics $\sim \Lambda$ is the Standard Model Effective Field Theory (SMEFT).

When the SMEFT is formulated using standard EFT techniques, this theoretical framework is a well defined and rigorous field theory that can consistently describe and characterize the breakdown of the SM emerging from experimental measurements, in the presence of a mass gap ($\bar{v}_T/\Lambda < 1$). For a review of such a formulation of the SMEFT see ref. [1]. The SMEFT is as useful as it is powerful as it can be systematically improved, irrespective of its UV completion, to ensure that its theoretical precision can match or exceed the experimental accuracy of such measurements.

Calculating in the SMEFT to achieve this systematic improvement can be subtle. Well known subtleties in the SM predictions of cross sections can be present, and further

subtleties can be introduced due to the presence of the EFT expansion parameter $\bar{v}_T/\Lambda < 1$. Complications due to the combination of these issues can also be present. As the SMEFT corrections to the SM cross sections are expected to be small $\lesssim \%$ level perturbations, it is important to overcome these issues with precise calculations, avoiding approximations or assumptions that introduce theoretical errors larger than the effects being searched for, to avoid incorrect conclusions. For this reason, although somewhat counterintuitive, rigour and precise analyses on a firm field theory footing are as essential in the SMEFT as in the SM.

In this paper we demonstrate how subtleties of this form are present when calculating the leading interference effect of some $\mathcal{L}^{(6)}$ operators as $\hat{m}_{W/Z}^2/s \to 0$. We demonstrate how this limit can be modified from a naive expectation formed through on-shell calculations due to off-shell contributions to the cross section. Furthermore, we show[1] how to implement the narrow width approximation in a manner consistent with the SMEFT expansion.

These subtleties are relevant to recent studies of the interference of the leading SMEFT corrections in the $\hat{m}_{W/Z}^2/s \to 0$ limit, as they lead to a different estimate of interference effects than has appeared in the literature when considering experimental observables.

2 CC03 approximation of $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$

The Standard Model Effective Field Theory is constructed out of $\text{SU}_C(3) \times \text{SU}_L(2) \times \text{U}_Y(1)$ invariant higher dimensional operators built out of SM fields. The Lagrangian is given as

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \qquad \text{for} \quad d > 4. \quad (2.1)$$

We use the Warsaw basis [4] for the operators ($Q_i^{(6)}$) in $\mathcal{L}^{(6)}$, that are the leading SMEFT corrections studied in this work. We absorb factors of $1/\Lambda^2$ into the Wilson coefficients below. We use the conventions of refs. [1, 5] for the SMEFT; defining Lagrangian parameters in the canonically normalized theory with a bar superscript, and Lagrangian parameters inferred from experimental measurements at tree level with hat superscripts. These quantities differ (compared to the SM) due to the presence of higher dimensional operators. We use the generic notation $\delta X = \bar{X} - \hat{X}$ for these differences for a Lagrangian parameter X. See refs. [1, 5] and the appendix for more details on notation.

Consider $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ scattering in the SMEFT with leptonic $\bar{\psi}\psi$ and quark $\bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ fields. The differential cross section for this process in the SM can be approximated by the CC03 set of Feynman diagrams,[2] where the $W^\pm$ bosons are considered to be on-shell. This defines the related differential cross section $d\sigma(\bar{\psi}\psi \to W^+W^-)/d\Omega$, which is useful to define as an approximation to the observable, but it is formally unphysical as the $W^\pm$ bosons decay. The lowest order results of this form were determined in refs. [6–13] and the CC03 diagrams are shown in figure 1. The amplitude for $\bar{\psi}\psi \to W^+W^- \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$

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[1]For past discussions see refs. [1–3].

[2]So named as CC indicates charged current.
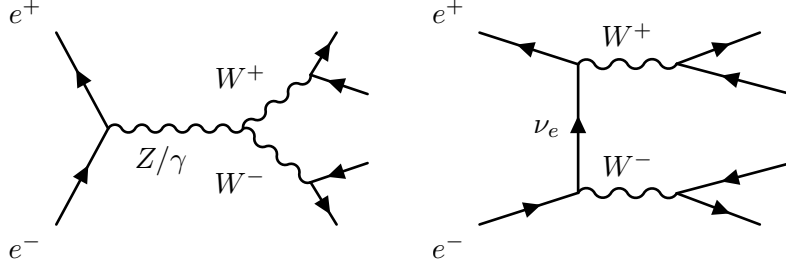
Figure 1. The CC03 Feynman diagrams contributing to $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ with leptonic initial states.

in this approximation is defined as

$$\sum_{\substack{X=\{\nu,A,Z\} \\ \lambda^{\pm}=\{+,-\}}} M_X^{\lambda^{\pm}} = \bar{D}_W(s_{12})\bar{D}_W(s_{34})\mathcal{M}_X^{\lambda_i}\mathcal{M}_{W^+}^{\lambda_{12}}\mathcal{M}_{W^-}^{\lambda_{34}}, \quad \bar{D}_W(s_{ij}) = \frac{1}{s_{ij} - \bar{m}_W^2 + i\bar{\Gamma}_W\bar{m}_W + i\epsilon},$$

(2.2)

where a constant s-independent width for the $W^{\pm}$ propagators $\bar{D}_W(s_{ij})$ is introduced[3] and

$$\mathcal{M}_\nu^{\lambda_i} = \mathcal{M}_{ee\to WW,\nu}^{\lambda_{12}\lambda_{34}\lambda_+\lambda_-}\delta_{\lambda_+}^+\delta_{\lambda_-}^-, \qquad \mathcal{M}_V^{\lambda_i} = \mathcal{M}_{ee\to WW,V}^{\lambda_{12}\lambda_{34}\lambda_+\lambda_-},$$
$$\mathcal{M}_{W^+}^{\lambda_{12}} = \mathcal{M}_{W^+\to f_1\bar{f}_2}^{\lambda_{12}}, \qquad \mathcal{M}_{W^-}^{\lambda_{34}} = \mathcal{M}_{W^-\to f_3\bar{f}_4}^{\lambda_{34}},$$

where $V = \{A,Z\}$. Here λ_{12} and λ_{34} label helicities of the intermediate $W^{\pm}$ bosons with four momenta s_{12}, s_{34}, and $\lambda_{\pm}$ label helicities of the $\bar{\psi}\psi$ initial state fermions. Transversely polarized massive vector bosons are labeled as $\lambda_{12/34} = \pm$ and the remaining polarization (in the massless fermion limit) is labeled as $\lambda_{12/34} = 0$. The individual sub-amplitudes are taken from ref. [2] where the complete SMEFT result was reported (see also refs. [14–24]). The total spin averaged differential cross section is defined as

$$\frac{d\sigma}{d\Omega\,ds_{12}ds_{34}} = \frac{\sum|M_X^{\lambda^{\pm}}|^2}{(2\pi)^2\,8s}, \qquad \sum|M_X^{\lambda^{\pm}}|^2 = |\bar{D}_W(s_{12})\bar{D}_W(s_{34})|^2 \sum_{\substack{X=\{\nu,A,Z\} \\ \lambda^{\pm}=\{+,-\}}} M_X^{\lambda^{\pm}}(M_X^{\lambda^{\pm}})^*,$$

(2.3)

where $d\Omega = d\cos\theta_{ab}\,d\phi_{ab}\,d\cos\theta_{cd}\,d\phi_{cd}\,d\cos\theta\,d\phi$, with θ, ϕ the angles between the W^+ and ℓ^- in the center of mass frame. The remaining angles describing the two body decays of the $W^{\pm}$ are in the rest frames of the respective bosons. The integration ranges for $\{s_{12}, s_{34}\}$ are $s_{34} \in \left[0, (\sqrt{s} - \sqrt{s_{12}})^2\right], s_{12} \in [0, s]$. It is instructive to consider the decomposition of the general amplitude in terms of helicity labels of the initial state fermions, and the intermediate $W^{\pm}$ bosons in the limit $\hat{m}_{W/Z}^2/s \to 0$ [10, 12, 19, 25–27]. Note that the results we report below are easily mapped to other initial and finals states, so long as these states are distinct.

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[3]We have checked and confirmed that the novel interference effects we discuss below persist if an s dependent width is used.

$\lambda_{12}\lambda_{34}\lambda_+\lambda_-$	$\sum_X M_X^{\lambda^\pm}/4\pi\hat\alpha$
$00-+$	$\frac{\sin\theta}{2\sqrt{s_1 s_3}}\left[\frac{1}{c_{\hat\theta}^2} + \left(\delta\kappa^{Z\alpha} - \delta F_2^{Z\alpha}\right)y\right]$
$\pm\pm-+$	$-\sin\theta\left[\frac{x^2}{c_{\hat\theta}^2} + \frac{y\,\delta\lambda^{Z\alpha}}{2} + \left(\delta g_1^{Z\alpha} - \delta F_2^{Z\alpha} - (s_1+s_3)\frac{\lambda_{Z\alpha}}{2} + \frac{\delta\lambda_Z}{2\,c_{\hat\theta}^2}\right)y\,x^2\right]$
$\pm 0-+$	$-\frac{(1\pm\cos\theta)\,x}{\sqrt{2}s_3}\left[\frac{1}{c_{\hat\theta}^2} + \frac{y}{2}\left(\delta g_1^{Z\alpha} - 2\delta F_2^{Z\alpha} + \delta\kappa^{Z\alpha} + s_3\,\delta\lambda^{Z\alpha}\right)\right]$
$0\pm-+$	$\frac{(1\mp\cos\theta)\,x}{\sqrt{2}s_1}\left[\frac{1}{c_{\hat\theta}^2} + \frac{y}{2}\left(\delta g_1^{Z\alpha} - 2\delta F_2^{Z\alpha} + \delta\kappa^{Z\alpha} + s_1\delta\lambda^{Z\alpha}\right)\right]$
$00+-$	$\frac{\sin\theta}{2\sqrt{s_1 s_3}}\left[\left(\frac{s_{\hat\theta}^2 - c_{\hat\theta}^2}{2\,c_{\hat\theta}^2 s_{\hat\theta}^2}\right) + \frac{s_1+s_3}{2s_{\hat\theta}^2} + \left(\delta\kappa^{Z\alpha} - \frac{\delta\kappa_Z}{2s_{\hat\theta}^2} + \frac{2\,\delta\bar g_W^\ell}{s_{\hat\theta}^2} - \delta F_2^{Z\alpha}\right)y\right]$
$\pm\pm+-$	$-\frac{\sin\theta}{2}\left[\left(1 - \frac{1}{2s_{\hat\theta}^2}\right)\delta\lambda_Z - \delta\lambda_\alpha\right]y$
$\pm 0+-$	$\frac{(1\mp\cos\theta)\,x}{2\sqrt{2}s_3}\left[\left(\frac{s_{\hat\theta}^2-c_{\hat\theta}^2}{c_{\hat\theta}^2 s_{\hat\theta}^2}\right) + \frac{s_1}{s_{\hat\theta}^2} + \frac{s_3}{s_{\hat\theta}^2}\frac{1\pm 2+3\cos\theta}{1+\cos\theta} - y\frac{\left(\delta g_1^z + \delta\kappa_z + s_3\,\delta\lambda_z\right)}{2s_{\hat\theta}^2}\right.$ $\left. -y\left(\delta F_1^{Z\alpha} - \frac{4\,\delta\bar g_W^\ell}{s_{\hat\theta}^2} - \left(\delta g_1^{Z\alpha} + \delta\kappa^{Z\alpha} + s_3\,\delta\lambda^{Z\alpha}\right)\right)\right]$
$0\pm+-$	$-\frac{(1\pm\cos\theta)\,x}{2\sqrt{2}s_1}\left[\left(\frac{s_{\hat\theta}^2-c_{\hat\theta}^2}{c_{\hat\theta}^2 s_{\hat\theta}^2}\right) + \frac{s_3}{s_{\hat\theta}^2} + \frac{s_1}{s_{\hat\theta}^2}\frac{1\mp 2+3\cos\theta}{1+\cos\theta} - y\frac{\left(\delta g_1^z + \delta\kappa_z + s_1\,\delta\lambda_z\right)}{2s_{\hat\theta}^2}\right.$ $\left. -y\left(\delta F_1^{Z\alpha} - \frac{4\,\delta\bar g_W^\ell}{s_{\hat\theta}^2} - \left(\delta g_1^{Z\alpha} + \delta\kappa^{Z\alpha} + s_1\,\delta\lambda^{Z\alpha}\right)\right)\right]$
$\pm\mp+-$	$\frac{(\mp 1+\cos\theta)\sin\theta}{2\,s_{\hat\theta}^2(1+\cos\theta)}$

Table 1. Expansion in $x, y < 1$ for the near on-shell region of phase space of the CC03 diagrams approximating $\bar\psi\psi \to \bar\psi'_1\psi'_2\bar\psi'_3\psi'_4$. For exactly on-shell intermediate $W^\pm$ bosons $s_1 = s_3 = 1$. We have used the notation $\delta F_{Z\alpha}^i = (\delta F_i^Z + \delta F_i^\alpha)/4\pi\hat\alpha$, $\delta\lambda^{Z\alpha} = \delta\lambda_Z - \delta\lambda_\alpha$, $\delta\kappa^{Z\alpha} = \delta\kappa_Z - \delta\kappa_\alpha$ and $\delta g_1^{Z\alpha} = \delta g_1^Z - \delta g_1^\alpha$.

2.1 Near on-shell phase space

First, consider the near on-shell region of phase space for the $W^\pm$ bosons defined by

$$\textbf{Case 1:} \qquad s_{12} = s_1\,\bar m_W^2, \qquad s_{34} = s_3\,\bar m_W^2. \qquad (2.4)$$

This expansion is limited to the near on-shell region of phase space for the intermediate $W^\pm$ bosons ($s_1 \sim s_3 \sim 1$) by construction. Introducing $x = \hat m_W/\sqrt{s}$ and $y = s/\Lambda^2$ an expansion in $x, y < 1$ can be performed by expressing the dimensionful parameters in terms of these dimensionless variables, times the appropriate coupling constant when required. The δX parameters were rescaled to extract these dimensionful scales as $x^2 y\,\delta X = \bar X - \hat X$ where required. This gives the results shown in table 1.

Table 1 shows an interesting pattern of suppressions to $\mathcal{L}^{(6)}$ operator corrections dependent upon the helicity configuration of the intermediate $W^\pm$ polarizations. This result is consistent with recent discussions in refs. [19, 25–27]. In the near on-shell region of phase space a relative suppression of interference terms by x^2 for amplitudes with a $\pm$ polarized $W^\pm$ compared to the corresponding case with a 0 polarization is present. These results for the $\lambda_{12}\lambda_{34}\lambda_+\lambda_- = \pm\pm+-$ and $\pm\pm-+$ helicity terms (which correspond to initial state left and right handed leptons respectively) involve an intricate cancellation of a leading SM contribution between the CC03 diagrams as

$$\frac{\mathcal{A}_{\pm\pm-+}}{4\pi\hat{\alpha}} \simeq -\sin\theta \left[\left(1+\delta\lambda_\alpha\,\frac{y}{2}\right)_\alpha - \left(1+\delta\lambda_Z\,\frac{y}{2}\right)_z\right] + \cdots,$$

$$\simeq -\frac{\sin\theta}{2}\,(\delta\lambda_\alpha - \delta\lambda_Z)\,y, \tag{2.5}$$

$$\frac{\mathcal{A}_{\pm\pm+-}}{4\pi\hat{\alpha}} \simeq -\sin\theta \left[\left(1+\delta\lambda_\alpha\,\frac{y}{2}\right)_{\alpha\text{ pole}} - \left(\left(1-\frac{1}{2\,s_{\hat{\theta}}^2}\right)\left(1+\delta\lambda_Z\,\frac{y}{2}\right)\right)_{z\text{ pole}} - \left(\frac{1}{2\,s_{\hat{\theta}}^2}\right)_\nu\right] + \cdots$$

$$\simeq \frac{\sin\theta}{2}\left[\left(1-\frac{1}{2s_{\hat{\theta}}^2}\right)\delta\lambda_Z - \delta\lambda_\alpha\right] y. \tag{2.6}$$

Here we have labeled the contributions by the internal states contributing to $M_X^{\lambda^\pm}$. The $\{\nu, \alpha, Z\}$ contributions to the scattering events populate phase space in a different manner in general. These differences are trivialized away in the near on-shell limit, leading to the cancellation shown of the leading SM contributions in the expansion in x, but can be uncovered by considering different limits of s_{12}, s_{34} and considering off-shell phase space.

2.2 Both $W^\pm$ bosons off-shell phase space

For example, consider the off-shell region of phase space defined through

$$\textbf{Case 2:} \qquad s_{12} = s_1\, s, \qquad s_{34} = s_3\, s, \tag{2.7}$$

with $s_1 \lesssim 1, s_3 \lesssim 1$. In this limit, one finds the expansions of the CC03 results

$$\mathcal{A}_{\pm\pm-+}^{s_1,s_3} \simeq -4\pi\hat{\alpha}\,\sin\theta\,\sqrt{\tilde{\lambda}(s_1,s_3)}\left[\left(1+\delta\lambda_\alpha\,\frac{y}{2}\right)_\alpha - \left(1+\delta\lambda_Z\,\frac{y}{2}\right)_z\right] + \cdots, \tag{2.8}$$

$$\mathcal{A}_{\pm\pm+-} \simeq -4\pi\hat{\alpha}\,\sin\theta\sqrt{\tilde{\lambda}(s_1,s_3)}\left[\left(1+\delta\lambda_\alpha\,\frac{y}{2}\right)_{\alpha\text{ pole}} - \left(\left(1-\frac{1}{2\,s_{\hat{\theta}}^2}\right)\left(1+\delta\lambda_Z\,\frac{y}{2}\right)\right)_{z\text{ pole}}\right],$$

$$+ \left[\left(\frac{4\pi\hat{\alpha}\,\sin\theta}{2\,s_{\hat{\theta}}^2\,\sqrt{\tilde{\lambda}(s_1,s_3)}}\right)\left(1+\frac{-(s_1+s_3)+(s_1-s_3)(s_1-s_3\mp\sqrt{\tilde{\lambda}(s_1,s_3)})}{1-s_1-s_3+\sqrt{\tilde{\lambda}(s_1,s_3)}\cos\theta}\right)\right]_{\nu\text{ pole}}. \tag{2.9}$$

Here we have defined $\sqrt{\tilde{\lambda}(s_1,s_3)} = \sqrt{1 - 2\,s_1 - 2s_3 - 2s_1\,s_3 + s_1^2 + s_3^2}$. In the case of left handed electrons, the differences in the way the various t and s channel poles populate phase

space are no longer trivialized away, and a SM contribution exists at leading order in the x expansion. This SM term can then interfere with the contribution due to a $\mathcal{L}^{(6)}$ operator correction in the SMEFT. The complete results in this limit for the helicity eigenstates are reported in table 2.

2.3 One $W^\pm$ boson off-shell phase space

One can define the region of phase space where one $W^\pm$ boson is off-shell as

$$\text{Case 3a:} \qquad s_{12} = s_1 s, \qquad s_{34} = s_3 \bar{m}_W^2,$$
$$\text{Case 3b:} \qquad s_{12} = s_1 \bar{m}_W^2, \qquad s_{34} = s_3 s,$$

with $s_1 \lesssim 1, s_3 \sim 1$ for Case 3a, and $s_1 \sim 1, s_3 \lesssim 1$ for Case 3b. In these limits, the expansions of the CC03 results are as follows. In Case 3a one has $\mathcal{A}_{\pm\pm-+}^{s_1,0}$ and

$$\mathcal{A}_{\pm\pm+-} \simeq -4\pi\,\hat{\alpha}\,\sin\theta\sqrt{\tilde{\lambda}(s_1,0)}\left[\left(1+\delta\lambda_\alpha\frac{y}{2}\right)_{\alpha\text{ pole}} - \left(\left(1-\frac{1}{2\,s_{\hat\theta}^2}\right)\left(1+\delta\lambda_Z\frac{y}{2}\right)\right)_{z\text{ pole}}\right]$$
$$+\left[\left(\frac{4\pi\,\hat{\alpha}\,\sin\theta}{2\,s_{\hat\theta}^2\sqrt{\tilde{\lambda}(s_1,0)}}\right)\left(1+s_1-\frac{2\,s_1(1-s_1\pm\sqrt{\tilde{\lambda}(s_1,0)})}{1-s_1+\sqrt{\tilde{\lambda}(s_1,0)}\cos\theta}\right)\right]_{\nu\text{ pole}}. \quad (2.10)$$

While in Case 3b one finds $\mathcal{A}_{\pm\pm-+}^{0,s_3}$ and

$$\mathcal{A}_{\pm\pm+-} \simeq -4\pi\,\hat{\alpha}\,\sin\theta\sqrt{\tilde{\lambda}(0,s_3)}\left[\left(1+\delta\lambda_\alpha\frac{y}{2}\right)_{\alpha\text{ pole}} - \left(\left(1-\frac{1}{2\,s_{\hat\theta}^2}\right)\left(1+\delta\lambda_Z\frac{y}{2}\right)\right)_{z\text{ pole}}\right]$$
$$+\left[\left(\frac{4\pi\,\hat{\alpha}\,\sin\theta}{2\,s_{\hat\theta}^2\sqrt{\tilde{\lambda}(0,s_3)}}\right)\left(1+s_3-\frac{2\,s_3(1-s_3\mp\sqrt{\tilde{\lambda}(0,s_3)})}{1-s_3+\sqrt{\tilde{\lambda}(0,s_3)}\cos\theta}\right)\right]_{\nu\text{ pole}}. \quad (2.11)$$

Again, the SM term for left handed initial states does not vanish and can interfere with the contribution due to a $\mathcal{L}^{(6)}$ operator correction in the SMEFT in these regions of phase space. The complete results in this limit for the helicity eigenstates are reported in table 3, 4.

These results make clear that non-interference arguments based on on-shell simplifications of the kinematics of decaying $W^\pm$ bosons get off-shell corrections for an LHC observable that includes off-shell intermediate $W^\pm$ kinematics. (Admittedly a somewhat obvious result.) Such kinematics are parametrically suppressed by the small width of the unstable gauge boson, but are generically included in LHC observables due to realistic experimental cuts.[4]

3 Mapping to past results

The results in table 1, 2, 3, 4 are input parameter scheme independent, and can be applied to more than one basis for $\mathcal{L}^{(6)}$. Specializing to the Warsaw basis of operators, and the

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[4]In some cases, off-shell effects are not relevant for physical conclusions. For example, ref. [28] used helicity arguments similar to those employed here to study the approximate holomorphy of the anomalous dimension matrix of the SMEFT [29]. Ref. [28] was focused on the cut-constructable part of the amplitude related to logarithmic terms and the corresponding divergences. As noted in ref. [28] such reasoning does not apply to finite contributions, which can come about due to off-shell effects.

λ_i	$\sum_X M_X^{\lambda^\pm}/4\pi\hat\alpha$
$00-+$	$\frac{\sqrt{\tilde\lambda}\,\sin\theta}{2\sqrt{s_1 s_3}}\left[\frac{1}{c_{\hat\theta}^2}(1+s_1+s_3)+\left(\delta\kappa^{Z\alpha}-\delta F_2^{Z\alpha}(1+s_1+s_3)+\delta g_1^{Z\alpha}(s_1+s_3)\right)y\right]x^2$
$\pm\pm-+$	$-\sin\theta\sqrt{\tilde\lambda}\left[\frac{x^2}{c_{\hat\theta}^2}+\frac{y\,\delta\lambda^{Z\alpha}}{2}+\left(\delta g_1^{Z\alpha}-\delta F_2^{Z\alpha}+\frac{\delta\lambda_Z}{2c_{\hat\theta}^2}\right)y\,x^2\right]$
$\pm 0-+$	$-\frac{(1\pm\cos\theta)}{\sqrt{2s_3}}\left[\frac{x^2}{c_{\hat\theta}^2}+\frac{y\,s_3\,\delta\lambda^{Z\alpha}}{2}+\frac{y\,x^2}{2}\left(\delta g_1^{Z\alpha}-2\delta F_2^{Z\alpha}+\delta\kappa^{Z\alpha}+s_3\,\delta\lambda^{Z\alpha}\right)\right]$
$0\pm-+$	$\frac{(1\mp\cos\theta)}{\sqrt{2s_1}}\left[\frac{x^2}{c_{\hat\theta}^2}+\frac{y\,s_1\,\delta\lambda^{Z\alpha}}{2}+\frac{y\,x^2}{2}\left(\delta g_1^{Z\alpha}-2\delta F_2^{Z\alpha}+\delta\kappa^{Z\alpha}+s_1\delta\lambda^{Z\alpha}\right)\right]$
$00+-$	$-\frac{\sin\theta\,\sqrt{\tilde\lambda}}{4\sqrt{s_1 s_3}\,s_{\hat\theta}^2}\left[1+s_1+s_3-\frac{1}{\tilde\lambda}\left(1-(s_1-s_3)^2-\frac{8\,s_1 s_3}{1-s_1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right)\right]$
$\pm\pm+-$	$\frac{\sin\theta\,\sqrt{\tilde\lambda}}{2\,s_{\hat\theta}^2}\left[1-\frac{1}{\tilde\lambda}\left(1+\frac{-(s_1+s_3)+(s_1-s_3)(s_1-s_3\mp\sqrt{\tilde\lambda})}{1-s_1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right)-s_{\hat\theta}^2 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$\pm 0+-$	$-\frac{(1\mp\cos\theta)\sqrt{\tilde\lambda}}{2\sqrt{2s_3}\,s_{\hat\theta}^2}\left[1-\frac{1}{\tilde\lambda}\left(1-s_1+s_3-\frac{2\,s_3(1+s_1-s_3\mp\sqrt{\tilde\lambda})}{1-s_1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right)-s_{\hat\theta}^2 s_3 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$0\pm+-$	$\frac{(1\pm\cos\theta)\sqrt{\tilde\lambda}}{2\sqrt{2s_1}\,s_{\hat\theta}^2}\left[1-\frac{1}{\tilde\lambda}\left(1+s_1-s_3-\frac{2\,s_1(1-s_1+s_3\pm\sqrt{\tilde\lambda})}{1-s_1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right)-s_{\hat\theta}^2 s_1 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$\pm\mp+-$	$\frac{(\mp 1+\cos\theta)\sin\theta}{2\,s_{\hat\theta}^2\left(1-s_1-s_3+\sqrt{\tilde\lambda}\cos\theta\right)}$

Table 2. Expansion in $x,y<1$ for the off-shell region of phase space of the CC03 diagrams in when $s_{12}=s_1\,s, s_{34}=s_3\,s$. Here we have used a short hand notation $\tilde\lambda=\tilde\lambda(s_1,s_3)$ and $F_3(\lambda_\alpha,\lambda_Z)=\left(\left(\frac{2s_{\hat\theta}^2-1}{2s_{\hat\theta}^2}\right)\delta\lambda_Z-\delta\lambda_\alpha\right)$ to condense results.

electroweak input parameter scheme $\{\hat\alpha_{ew},\hat m_Z,\hat G_F\}$ the (re-scaled) $x^2 y\,\delta X$ parameters are given by

$$\frac{\hat m_W^2}{\Lambda^2}\,\delta g_1^\alpha=0, \qquad\qquad \frac{\hat m_W^2}{\Lambda^2}\,\delta\kappa_\alpha=\frac{1}{\sqrt{2}\hat G_F}\frac{c_{\hat\theta}}{s_{\hat\theta}}C_{HWB},$$

$$\frac{\hat m_W^2}{\Lambda^2}\,\delta\lambda_\alpha=6s_{\hat\theta}\frac{\hat m_W^2}{\sqrt{4\pi\hat\alpha}}C_W, \qquad \frac{\hat m_W^2}{\Lambda^2}\,\delta\lambda_Z=6s_{\hat\theta}\frac{\hat m_W^2}{\sqrt{4\pi\hat\alpha}}C_W,$$

$$\frac{\hat m_W^2}{\Lambda^2}\,\delta F_{1,2}^\alpha=0,$$

and

$$-\frac{\hat m_W^2}{\Lambda^2}\frac{\delta F_1^Z}{4\pi\hat\alpha}=\delta\bar g_Z\,(g_L^\ell)_{ss}^{\rm SM}-\frac{1}{2\sqrt{2}\hat G_F}\left(C_{H\ell\atop ss}^{(1)}+C_{H\ell\atop ss}^{(3)}\right)-\delta s_\theta^2,$$

$$-\frac{\hat m_W^2}{\Lambda^2}\frac{\delta F_2^Z}{4\pi\hat\alpha}=\delta\bar g_Z\,(g_R^\ell)_{ss}^{\rm SM}-\frac{1}{2\sqrt{2}\hat G_F}C_{He\atop ss}-\delta s_\theta^2,$$

λ_i	$\sum_X M_X^{\lambda^\pm}/4\pi\hat\alpha$
$00-+$	$\frac{\sqrt{\tilde\lambda}\sin\theta}{2\sqrt{s_1 s_3}}\left[\frac{1}{c_{\hat\theta}^2}(1+s_1)+\left(\delta\kappa^{Z\alpha}-\delta F_2^{Z\alpha}(1+s_1)+\delta g_1^{Z\alpha}s_1\right)y\right]x$
$\pm\pm-+$	$-\sin\theta\sqrt{\tilde\lambda}\left[\frac{x^2}{c_{\hat\theta}^2}+\frac{y\,\delta\lambda^{Z\alpha}}{2}+\left(\delta g_1^{Z\alpha}-\delta F_2^{Z\alpha}+\frac{\delta\lambda_Z}{2\,c_{\hat\theta}^2}-\frac{\delta\lambda^{Z\alpha}(1+s_1)s_3}{2\,\tilde\lambda}\right)y\,x^2\right]$
$\pm 0-+$	$-\frac{(1\pm\cos\theta)\sqrt{\tilde\lambda}\,x}{\sqrt{2}s_3}\left[\frac{1}{c_{\hat\theta}^2}+\frac{y}{2}\left(\delta g_1^{Z\alpha}-2\delta F_2^{Z\alpha}+\delta\kappa^{Z\alpha}+s_3\,\delta\lambda^{Z\alpha}\right)\right]$
$0\pm-+$	$\frac{(1\mp\cos\theta)\sqrt{\tilde\lambda}}{\sqrt{2}s_1}\left[\frac{x^2}{c_{\hat\theta}^2}+\frac{y\,s_1\,\delta\lambda^{Z\alpha}}{2}+\frac{y\,x^2}{2}\left(\delta g_1^{Z\alpha}-2\delta F_2^{Z\alpha}+\delta\kappa^{Z\alpha}+\frac{s_1\delta\lambda_Z}{c_{\hat\theta}^2}-\frac{s_1\,s_3\,(1+s_1)}{(1-s_1)^2}\,\delta\lambda^{Z\alpha}\right)\right]$
$00+-$	$-\frac{\sin\theta}{\sqrt{s_1\,s_3}\,\sqrt{\tilde\lambda}\,s_{\hat\theta}^2}\,\frac{s_1(s_1^2-1)}{4\,x}$
$\pm\pm+-$	$\frac{\sin\theta\,\sqrt{\tilde\lambda}}{2\,s_{\hat\theta}^2}\left[1-\frac{1}{\tilde\lambda}\left(1-\frac{s_1(1-s_1\pm s_1\sqrt{\tilde\lambda})}{1-s_1+\sqrt{\tilde\lambda}\cos\theta}\right)-s_{\hat\theta}^2 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$\pm 0+-$	$-\frac{s_1(s_1-1)(1\mp\cos\theta)}{2\,\sqrt{2\,s_3}\,s_{\hat\theta}^2\,\sqrt{\tilde\lambda}}\,\frac{1}{x}$
$0\pm+-$	$\frac{(1\pm\cos\theta)\sqrt{\tilde\lambda}}{2\sqrt{2s_1}\,s_{\hat\theta}^2}\left[1-\frac{1}{\tilde\lambda}\left(1+s_1-\frac{2\,s_1(1-s_1\pm\sqrt{\tilde\lambda})}{1-s_1+\sqrt{\tilde\lambda}\cos\theta}\right)-s_{\hat\theta}^2 s_1 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$\pm\mp+-$	$\frac{(\mp 1+\cos\theta)\sin\theta}{2\,s_{\hat\theta}^2\left(1-s_1+\sqrt{\tilde\lambda}\,\cos\theta\right)}$

Table 3. Expansion in $x,y<1$ for the off-shell region of phase space of the CC03 diagrams. Here we have used a short hand notation $\tilde\lambda = \tilde\lambda(s_1,0)$.

$$\frac{\hat m_W^2}{\Lambda^2}\,\delta g_1^Z = \frac{1}{2\sqrt{2}\hat G_F}\left(\frac{s_{\hat\theta}}{c_{\hat\theta}}+\frac{c_{\hat\theta}}{s_{\hat\theta}}\right)C_{HWB}+\frac{1}{2}\delta s_\theta^2\left(\frac{1}{s_{\hat\theta}^2}+\frac{1}{c_{\hat\theta}^2}\right),$$

$$\frac{\hat m_W^2}{\Lambda^2}\,\delta\kappa_Z = \frac{1}{2\sqrt{2}\hat G_F}\left(-\frac{s_{\hat\theta}}{c_{\hat\theta}}+\frac{c_{\hat\theta}}{s_{\hat\theta}}\right)C_{HWB}+\frac{1}{2}\delta s_\theta^2\left(\frac{1}{s_{\hat\theta}^2}+\frac{1}{c_{\hat\theta}^2}\right),$$

with $\delta\bar g_Z, \delta s_\theta^2$ defined in the appendix. The left and right handed couplings are $(g_L^\ell)_{ss}^{\rm SM} = -1/2+s_{\hat\theta}^2$, and $(g_R^\ell)_{ss}^{\rm SM} = s_{\hat\theta}^2$. Here $s=\{1,2,3\}$ is a flavour index labeling the initial state leptons. The results in table 1 can be more directly compared to refs. [19, 25–27, 30] using this procedure, finding agreement in the subset of terms that were reported in these works. This comparison also utilizes the naive narrow width limit to simplify the amplitudes as follows. In the sense of a distribution over phase space, the following replacement is made

$$|\bar D_W(s_{12})\bar D_W(s_{34})|^2\,ds_{12}\,ds_{34} \to \frac{\pi^2}{\bar m_W^2\,\bar\Gamma_W^2}\,\delta(s_{12}-\bar m_W^2)\,\delta(s_{34}-\bar m_W^2)\,ds_{12}\,ds_{34}. \quad (3.1)$$

The result of this replacement is a factorizing of the diboson production mechanism $d\sigma(\bar\psi\psi \to W^+W^-)/d\Omega$ and the branching ratios of the $W^\pm$ decays into specified final states as $s_1=s_3=1$ is fixed in table 1. This approximation holds up to $\mathcal{O}(\Gamma_W/M_W)$ corrections to eq. (3.1). The corrections in tables 2, 3, 4 are present and should not be overlooked by the

λ_i	$\sum_X M_X^{\lambda\pm}/4\pi\hat\alpha$
$00-+$	$\dfrac{-\sqrt{\tilde\lambda}\sin\theta}{2\sqrt{s_1 s_3}}\left[\dfrac{1}{c_{\hat\theta}^2}(1+s_3) - \left(\delta\kappa^{Z\alpha} + \delta F_2^{Z\alpha}(1+s_3) + \delta g_1^{Z\alpha}s_3\right)y\right]x$
$\pm\pm-+$	$-\sin\theta\sqrt{\tilde\lambda}\left[\dfrac{x^2}{c_{\hat\theta}^2} + \dfrac{y\,\delta\lambda^{Z\alpha}}{2} + \left(\delta g_1^{Z\alpha} - \delta F_2^{Z\alpha} + \dfrac{\delta\lambda_Z}{2c_{\hat\theta}^2} - \dfrac{\delta\lambda^{Z\alpha}(1+s_3)s_1}{2\tilde\lambda}\right)y\,x^2\right]$
$\pm0-+$	$\dfrac{-(1\pm\cos\theta)\sqrt{\tilde\lambda}}{\sqrt2 s_3}\left[\dfrac{x^2}{c_{\hat\theta}^2} + \dfrac{y\,s_3\,\delta\lambda^{Z\alpha}}{2} + \dfrac{y\,x^2}{2}\left(\delta g_1^{Z\alpha} - 2\delta F_2^{Z\alpha} + \delta\kappa^{Z\alpha} + \dfrac{s_3\delta\lambda_Z}{c_{\hat\theta}^2} - \dfrac{s_1 s_3(1+s_3)}{(1-s_3)^2}\delta\lambda^{Z\alpha}\right)\right]$
$0\pm-+$	$\dfrac{(1\mp\cos\theta)\sqrt{\tilde\lambda}x}{\sqrt2 s_1}\left[\dfrac{1}{c_{\hat\theta}^2} + \dfrac{y}{2}\left(\delta g_1^{Z\alpha} - 2\delta F_2^{Z\alpha} + \delta\kappa^{Z\alpha} + s_1\,\delta\lambda^{Z\alpha}\right)\right]$
$00+-$	$-\dfrac{\sin\theta}{\sqrt{s_1 s_3}\,\sqrt{\tilde\lambda}\,s_{\hat\theta}^2}\dfrac{s_3(s_3^2-1)}{4\,x}$
$\pm\pm+-$	$\dfrac{\sin\theta\sqrt{\tilde\lambda}}{2\,s_{\hat\theta}^2}\left[1 - \dfrac{1}{\tilde\lambda}\left(1 - \dfrac{s_3(1-s_3\pm s_3\sqrt{\tilde\lambda})}{1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right) - s_{\hat\theta}^2 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$\pm0+-$	$\dfrac{-(1\mp\cos\theta)\sqrt{\tilde\lambda}}{2\sqrt2 s_3 s_{\hat\theta}^2}\left[1 - \dfrac{1}{\tilde\lambda}\left(1+s_3 - \dfrac{2\,s_3(1-s_3\pm\sqrt{\tilde\lambda})}{1-s_3+\sqrt{\tilde\lambda}\cos\theta}\right) - s_{\hat\theta}^2 s_3 F_3(\lambda_\alpha,\lambda_Z)y\right]$
$0\pm+-$	$\dfrac{s_3(s_3-1)(1\pm\cos\theta)}{2\sqrt{2s_1}\,s_{\hat\theta}^2\,\sqrt{\tilde\lambda}\,x}\dfrac{1}{x}$
$\pm\mp+-$	$\dfrac{(\mp1+\cos\theta)\sin\theta}{2\,s_{\hat\theta}^2\left(1-s_3+\sqrt{\tilde\lambda}\,\cos\theta\right)}$

Table 4. Expansion in $x, y < 1$ for the off-shell region of phase space of the CC03 diagrams and $\tilde\lambda = \tilde\lambda(0, s_3)$.

construction of a simplified high energy expansion, that is formally unphysical. It is not advisable to extrapolate the limited phase space results of table 1 to the full phase space.

Another key difference between more recent studies of interference in the SMEFT in the high energy limit, compared to the past studies of interference of higher dimensional operators in the high energy limit for gluonic operators [31, 32], is the presence of an unstable massive gauge boson. Such massive gauge bosons have been studied using the narrow width approximation. However, a too naive version of the narrow width approximation does not commute with the SMEFT expansion.

This non-commutation can be seen as follows. Expanding the propagator of the intermediate W boson in the SMEFT

$$\frac{1}{(p^2 - \bar{m}_W^2)^2 + \bar\Gamma_W^2\bar m_W^2} = \frac{1}{(p^2 - \hat m_W^2)^2 + \hat\Gamma_W^2\hat m_W^2}(1 + \delta D_W(p^2) + \delta D_W(p^2)^*) \quad (3.2)$$

one has

$$\delta D_W(p^2) = \frac{1}{p^2 - \hat m_W^2 + i\hat\Gamma_W\hat m_W} \times \left[\left(1 - \frac{i\hat\Gamma_W}{2\hat m_W}\right)\delta m_W^2 - i\hat m_W\delta\Gamma_W\right]. \quad (3.3)$$

By first doing the narrow width approximation, and then doing the SMEFT expansion,

one obtains

$$\frac{dp^2}{(p^2 - \bar{m}_W^2)^2 + \bar{\Gamma}_W^2 \bar{m}_W^2} \rightarrow \frac{\pi dp^2}{\bar{\Gamma}_W \bar{m}_W} \delta(p^2 - \bar{m}_W^2)$$

$$= \frac{\pi dp^2}{\hat{\Gamma}_W \hat{m}_W} \left(1 - \frac{\delta m_W^2}{2\hat{m}_W^2} - \frac{\delta\Gamma_W}{\hat{\Gamma}_W} \right) \delta(p^2 - \bar{m}_W^2). \qquad (3.4)$$

Reversing the order of operations, we square the expanded propagators and then do the narrow width approximation. For a general function $f(p^2)$, we find that after integrating

$$\frac{f(p^2)dp^2}{(p^2 - \hat{m}_W^2)^2 + \hat{\Gamma}_W^2 \hat{m}_W^2} \left(1 + \delta D_W(p^2) + \delta D_W(p^2)^* \right)$$

$$\rightarrow \frac{f(\hat{m}_W^2)\pi}{\hat{\Gamma}_W \hat{m}_W} \left(1 - \frac{\delta m_W^2}{2\hat{m}_W^2} - \frac{\delta\Gamma_W}{\hat{\Gamma}_W} \right) + \frac{f'(\hat{m}_W^2)\pi}{\hat{\Gamma}_W \hat{m}_W} \delta m_W^2. \quad (3.5)$$

In a naive version of the narrow width approximation, we simply replace $\bar{m}_W$ by $\hat{m}_W$ in eq. (3.4). The operations of expanding in the SMEFT and doing the naive narrow width approximation don't commute in general. The reason is that the naive narrow width approximation assumes that the part of the integrand that is odd in its dependence on the invariant mass cancels out in the near on-shell region. With the SMEFT corrections, this is no longer the case, as the real part of δD_W gives a finite contribution to this part of the integrand. This difference is proportional to the shift of the mass of the $W^\pm$ boson. The correct way to implement the narrow width approximantion in the SMEFT is to use eq. (3.4) and expand the general fuction $f(p^2)$ in the SMEFT expansion after integration. We then obtain eq. (3.5), and see that the commutation property is restored. Furthermore, we note that the x expansion parameter itself can be chosen to be $\hat{m}_W/\sqrt{s}$ or $\bar{m}_W/\sqrt{s}$ when studying the high energy limit (we choose the former expansion parameter). This is another ambiguity that can be introduced into studies of this form, when using a $\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme.

4 Single charge current resonant contributions (CC11)

It is well known in the SM literature, that the CC03 diagrams, with $W^\pm$ bosons fixed to be on-shell, are an insufficient approximation to a $\bar{\psi}\psi \rightarrow \bar{\psi}'_1 \psi'_2 \bar{\psi}'_3 \psi'_4$ cross section to describe the full phase space of scattering events [33–39]. Such scattering events need not proceed through the CC03 set of diagrams, so limiting an analysis to this set of diagrams is formally unphysical. This issue can be overcome using the standard techniques of expanding around the poles of the process [40–42] and including more contributions to the physical scattering process due to single resonant or non-resonant diagrams. Including the effect of single resonant diagrams allows one to develop gauge invariant results for such scattering events [33–38] when considering the full phase space (so long as the initial and final states are distinct). Including the single resonant diagrams is frequently referred to as calculating the set of CC11 diagrams in the literature. Some of the additional diagrams required are shown in figure 2.[5]

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[5]Note that the CC03 diagrams are a (gauge dependent) subset of the CC11 diagrams [13] which can be seen considering the differences found in CC03 results comparing axial and R_ξ gauges.
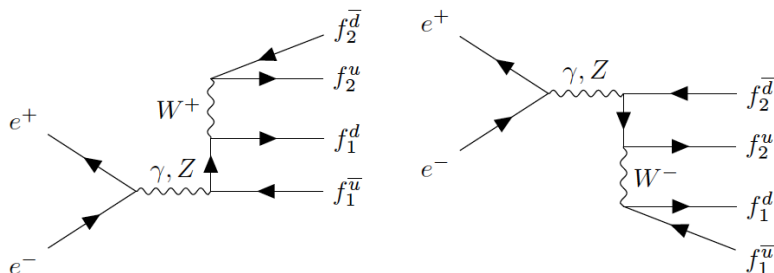
Figure 2. A subset of the CC11 Feynman diagrams contributing to $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ with leptonic initial states.

Considering the results in the previous sections, it is of interest to check if single resonant diagrams contribute to the physical $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ observables in a manner that potentially cancels the contributions for the off-shell phase space results in tables 2, 3, 4. We find this is not the case, as can be argued on general grounds, and demonstrated in explicit calculations which we report below.

In general, an expansion of a SM Lagrangian parameter with a SMEFT correction is generically considered to be a correction of the form

$$\bar{X} = \hat{X} + x^2\, y\, \delta X \tag{4.1}$$

in the high energy limit considered, and one expects the SMEFT shifts to enter at two higher orders in the x expansion compared to a SM result. In addition the SMEFT can introduce new operator forms that directly lead to high energy growth and scale as a y correction to the amplitude, such as the effect of the operator $\mathcal{Q}_W$ in $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ scattering.

The CC03 diagram results are quite unusual due to the intricate cancellation present between the leading terms in the x expansion in the SM, at least in some regions of phase space. This leads to the SM and SMEFT terms occurring in some cases at the same order in x, contrary to the expectation formed by eq. (4.1). Conversely, the CC11 diagram contributions[6] follow the expectation in eq. (4.1).

4.1 Single charge current resonant contributions — the SM

We use the results of refs. [33–38], in particular ref. [34], for the SM results of the CC11/CC03 diagrams. We neglect contributions suppressed by light fermion masses. The generic SM amplitude is defined to have the form

$$i\,\mathcal{M}^{\sigma_a\,\sigma_b\,\sigma_c\,\sigma_d\,\sigma_e\,\sigma_f}_{V_1\,V_2}(p_a,p_b,p_c,p_d,p_e,p_f) = -4i\,\bar{e}^4\,\delta_{\sigma_a,-\sigma_b}\delta_{\sigma_c,-\sigma_d}\delta_{\sigma_e,-\sigma_f}\bar{g}^{\sigma_b}_{V_1\bar{f}_a f_g}\bar{g}^{\sigma_b}_{V_2\bar{f}_g f_b}\bar{g}^{\sigma_d}_{V_1\bar{f}_c f_d}\bar{g}^{\sigma_f}_{V_2\bar{f}_e f_f}$$

$$\times \frac{\bar{D}_{V_1}(p_c+p_d)\bar{D}_{V_2}(p_e+p_f)}{(p_b+p_e+p_f)^2}A_2^{\sigma_a,\sigma_c,\sigma_e}(p_a,p_b,p_c,p_d,p_e,p_f). \tag{4.2}$$

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[6]Modulo the CC03 diagrams which we indicate with CC11/CC03.

We have adopted the conventions of ref. [34], and the initial and final states are labelled as $a\,b \to c\,d\,e\,f$. See the appendix for more notational details. The functions $A_2^{\sigma_a,\sigma_c,\sigma_e}$ are given in terms of spinor products as [34, 43],

$$A_2^{+++}(p_a, p_b, p_c, p_d, p_e, p_f) = \langle p_a p_c \rangle \langle p_b p_f \rangle^* \left(\langle p_b p_d \rangle^* \langle p_b p_e \rangle + \langle p_d p_f \rangle^* \langle p_e p_f \rangle \right),$$

(4.3)

and satisfy [34, 43]

$$A_2^{++-}(p_a, p_b, p_c, p_d, p_e, p_f) = A_2^{+++}(p_a, p_b, p_c, p_d, p_f, p_e), \qquad (4.4)$$

$$A_2^{+-+}(p_a, p_b, p_c, p_d, p_e, p_f) = A_2^{+++}(p_a, p_b, p_d, p_c, p_e, p_f), \qquad (4.5)$$

$$A_2^{+--}(p_a, p_b, p_c, p_d, p_e, p_f) = A_2^{+++}(p_a, p_b, p_d, p_c, p_f, p_e), \qquad (4.6)$$

$$A_2^{-,\sigma_c,\sigma_d}(p_a, p_b, p_c, p_d, p_e, p_f) = \left(A_2^{+,-\sigma_c,-\sigma_d}(p_a, p_b, p_c, p_d, p_e, p_f) \right)^*. \qquad (4.7)$$

The CC11/CC03 results are

$$\mathcal{M}^{\sigma_+,\sigma_-,\sigma_1,\sigma_2,\sigma_3,\sigma_4} = \sum_{V=A,Z} [\mathcal{M}_{VW}^{-\sigma_1,-\sigma_2,\sigma_+,\sigma_-,-\sigma_3,-\sigma_4}(-k_1,-k_2,p_+,p_-,-k_3,-k_4),$$

$$+ \mathcal{M}_{VW}^{-\sigma_3,-\sigma_4,\sigma_+,\sigma_-,-\sigma_1,-\sigma_2}(-k_3,-k_4,p_+,p_-,-k_1,-k_2),$$

$$+ \mathcal{M}_{WV}^{-\sigma_1,-\sigma_2,-\sigma_3,-\sigma_4,\sigma_+,\sigma_-}(-k_1,-k_2,-k_3,-k_4,p_+,p_-),$$

$$+ \mathcal{M}_{WV}^{-\sigma_3,-\sigma_4,-\sigma_1,-\sigma_2,\sigma_+,\sigma_-}(-k_3,-k_4,-k_1,-k_2,p_+,p_-)\,]. \quad (4.8)$$

As the final state fermions couple to one $W^\pm$ boson, and fermion masses are neglected, $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} = \{- + -+\}$. We denote the amplitude by the helicities of the incoming fermions, $\mathcal{M}^{\sigma_+,\sigma_-,\sigma_1,\sigma_2,\sigma_3,\sigma_4} = \mathcal{M}^{\sigma_+,\sigma_-}$ and find using [34] in the $x < 1$ limit for Case 1 and right handed electrons

$$\mathcal{M}^{-+} = \frac{\hat{e}^4 Q_l \sin\theta \sin\tilde{\theta}_{12} \sin\tilde{\theta}_{34}}{4 s_{\hat\theta}^2 c_{\hat\theta}^2 s\, x^2} \left[\frac{Q_{f_1} - I_{f_1}^3 - Q_{f_2} + I_{f_2}^3}{s_3 - 1 + i\hat\gamma_W} + \frac{Q_{f_4} - I_{f_4}^3 - Q_{f_3} + I_{f_3}^3}{s_1 - 1 + i\hat\gamma_W} \right], \quad (4.9)$$

and for left handed electrons

$$\mathcal{M}^{+-} = \frac{\hat{e}^4 \sin\theta \sin\tilde{\theta}_{12} \sin\tilde{\theta}_{34}}{4 s_{\hat\theta}^4 c_{\hat\theta}^2 s\, x^2}$$

$$\left[\frac{[Q_{f_1} s_{\hat\theta}^2 (Q_l - I_l^3) + I_{f_1}^3 (I_l^3 - Q_l s_{\hat\theta}^2)] - [Q_{f_2} s_{\hat\theta}^2 (Q_l - I_l^3) + I_{f_2}^3 (I_l^3 - Q_l s_{\hat\theta}^2)]}{s_3 - 1 + i\hat\gamma_W} \right.$$

$$\left. + \frac{[Q_{f_4} s_{\hat\theta}^2 (Q_l - I_l^3) + I_{f_4}^3 (I_l^3 - Q_l s_{\hat\theta}^2)] - [Q_{f_3} s_{\hat\theta}^2 (Q_l - I_l^3) + I_{f_3}^3 (I_l^3 - Q_l s_{\hat\theta}^2)]}{s_1 - 1 + i\hat\gamma_W} \right].$$

(4.10)

Here $\hat{\gamma}_W = \hat{\Gamma}_W / \hat{m}_W$, Q_{f_i} is the electric charge and $I^3_{f_i} = \pm 1/2$ is the isospin of the fermion f_i. Similarly for Case 2 we find using [34] the results for right handed electrons

$$\mathcal{M}^{-+} = \frac{4\hat{e}^4 Q_l}{s_{\hat{\theta}}^2 c_{\hat{\theta}}^2 s} \left[\frac{I^3_{f_1} - Q_{f_1}}{s_3(1 - s_1 + s_3 - \tilde{\lambda}\cos\tilde{\theta}_{12})} R_1 - \frac{I^3_{f_2} - Q_{f_2}}{s_3(1 - s_1 + s_3 + \tilde{\lambda}\cos\tilde{\theta}_{12})} R_2 \right.$$
$$\left. + \frac{I^3_{f_3} - Q_{f_3}}{s_1(1 + s_1 - s_3 - \tilde{\lambda}\cos\tilde{\theta}_{34})} R_3 - \frac{I^3_{f_4} - Q_{f_4}}{s_1(1 + s_1 - s_3 + \tilde{\lambda}\cos\tilde{\theta}_{34})} R_4 \right], \quad (4.11)$$

and for left handed electrons

$$\mathcal{M}^{+-} = \frac{-4\hat{e}^4}{s_{\hat{\theta}}^4 c_{\hat{\theta}}^2 s} \left[\frac{Q_{f_1} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_1}(I^3_l - Q_l s_{\hat{\theta}}^2)}{s_3(1 - s_1 + s_3 - \tilde{\lambda}\cos\tilde{\theta}_{12})} L_1 - \frac{Q_{f_2} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_2}(I^3_l - Q_l s_{\hat{\theta}}^2)}{s_3(1 - s_1 + s_3 + \tilde{\lambda}\cos\tilde{\theta}_{12})} L_2 \right.$$
$$\left. + \frac{Q_{f_3} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_3}(I^3_l - Q_l s_{\hat{\theta}}^2)}{s_1(1 + s_1 - s_3 - \tilde{\lambda}\cos\tilde{\theta}_{34})} L_3 - \frac{Q_{f_4} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_4}(I^3_l - Q_l s_{\hat{\theta}}^2)}{s_1(1 + s_1 - s_3 + \tilde{\lambda}\cos\tilde{\theta}_{34})} L_4 \right].$$
$$(4.12)$$

The functions R_i, L_i, $i = 1, \ldots, 4$ are given in the appendix, along with additional definitions. For Case 3a one finds for right handed electrons

$$\mathcal{M}^{-+} = \frac{\hat{e}^4 Q_l \sin\tilde{\theta}_{34}}{4 s_{\hat{\theta}}^2 c_{\hat{\theta}}^2 s x^2 (s_3 - 1 + i\gamma_W)} \left[(Q_{f_1} - I^3_{f_1}) - (Q_{f_2} - I^3_{f_2}) \right] \quad (4.13)$$
$$\times \left(\sin\theta \sin\tilde{\theta}_{12}(1 + s_1) + \sqrt{s_1} e^{-i\tilde{\phi}_{12}}(1 - \cos\theta)(1 + \cos\tilde{\theta}_{12}) + \sqrt{s_1} e^{i\tilde{\phi}_{12}}(1 + \cos\theta)(1 - \cos\tilde{\theta}_{12}) \right),$$

and for left-handed electrons

$$\mathcal{M}^{+-} = \frac{\hat{e}^4 \sin\tilde{\theta}_{34}}{4 s_{\hat{\theta}}^4 c_{\hat{\theta}}^2 s x^2 (s_3 - 1 + i\gamma_W)} \quad (4.14)$$
$$\times \left[(Q_{f_1} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_1}(I^3_l - Q_l s_{\hat{\theta}}^2)) - (Q_{f_2} s_{\hat{\theta}}^2 (Q_l - I^3_l) + I^3_{f_2}(I^3_l - Q_l s_{\hat{\theta}}^2)) \right]$$
$$\times \left[\sin\theta \sin\tilde{\theta}_{12}(1 + s_1) - \sqrt{s_1} e^{-i\tilde{\phi}_{12}}(1 + \cos\theta)(1 + \cos\tilde{\theta}_{12}) - \sqrt{s_1} e^{i\tilde{\phi}_{12}}(1 - \cos\theta)(1 - \cos\tilde{\theta}_{12}) \right],$$

and finally for Case 3b one finds for right-handed electrons

$$\mathcal{M}^{-+} = \frac{\hat{e}^4 Q_l \sin\tilde{\theta}_{12}}{4 s_{\hat{\theta}}^2 c_{\hat{\theta}}^2 s x^2 (s_1 - 1 + i\gamma_W)} \left[(Q_{f_4} - I^3_{f_4}) - (Q_{f_3} - I^3_{f_3}) \right] \quad (4.15)$$
$$\times \left(\sin\theta \sin\tilde{\theta}_{34}(1 + s_3) - \sqrt{s_3} e^{-i\tilde{\phi}_{34}}(1 - \cos\theta)(1 - \cos\tilde{\theta}_{34}) - \sqrt{s_3} e^{i\tilde{\phi}_{34}}(1 + \cos\theta)(1 + \cos\tilde{\theta}_{34}) \right),$$

and for left-handed electrons

$$\mathcal{M}^{+-} = \frac{\hat{e}^4 \sin\tilde{\theta}_{12}}{4 s_{\hat{\theta}}^4 c_{\hat{\theta}}^2 s x^2 (s_1 - 1 + i\gamma_W)} \quad (4.16)$$
$$\times \left[(Q_{f_4} s_{\hat{\theta}}^2 (I^3_l - Q_l s_{\hat{\theta}}^2) + I^3_{f_4}(I^3_l - Q_l s_{\hat{\theta}}^2)) - (Q_{f_3} s_{\hat{\theta}}^2 (I^3_l - Q_l s_{\hat{\theta}}^2) + I^3_{f_3}(I^3_l - Q_l s_{\hat{\theta}}^2)) \right]$$
$$\times \left[\sin\theta \sin\tilde{\theta}_{34}(1 + s_3) + \sqrt{s_3} e^{-i\tilde{\phi}_{34}}(1 + \cos\theta)(1 - \cos\tilde{\theta}_{34}) + \sqrt{s_3} e^{i\tilde{\phi}_{34}}(1 - \cos\theta)(1 + \cos\tilde{\theta}_{34}) \right].$$
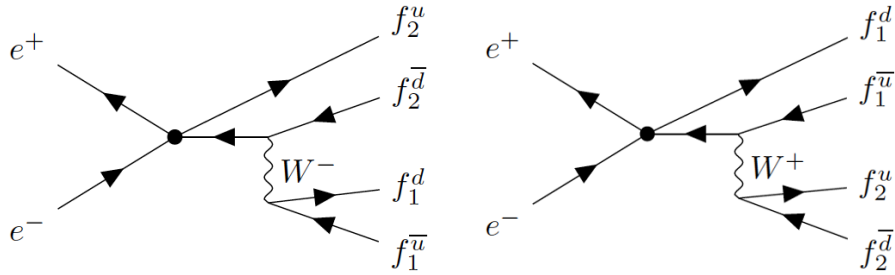
Figure 3. A subset of ψ^4 operator insertions contributing to $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ scattering.

4.2 Single resonant contributions — the SMEFT

The SMEFT corrections to the single resonant charged current contributions to $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$, follow directly from the results in the previous section. These corrections follow the scaling in x expectation formed by eq. (4.1), and the spinor products are unaffected by these shifts. As the charges of the initial and final states through neutral currents are fairly explicit in the previous section, it is easy to determine the coupling shifts and the SMEFT corrections to the propagators ($\delta D_{W,Z}$) by direct substitution.

We find that the single resonant contributions are distinct in their kinematic dependence compared to the novel interference results we have reported in section 2. The direct comparison of the results is somewhat challenged by the lack of a meaningful decomposition of the single resonant diagrams into helicity eigenstates of two intermediate charged currents, when only one charged current is present. Furthermore, we also note that the angular dependence shown in the single resonant results in eqs. (4.11)–(4.16) reflects the fact that the center of mass frame relation to the final state phase space in the case of the CC03 diagrams is distinct from the CC11/CC03 results. This is the case despite both contributions being required for gauge independence in general [13].

To develop a complete SMEFT result including single resonant contributions, it is also required to supplement the results in the previous section with four fermion diagrams where a near on-shell charged current is present. For diagrams of this form see figure 3. These contributions introduce dependence on $\mathcal{L}^{(6)}$ operators that are not present in the CC03 diagrams, and once again the angular dependence in the phase space is distinct from the CC03 results.

5 Conclusions

In this paper, we have shown that off-shell effects in CC03 diagrams contributing to $\bar{\psi}\psi \to \bar{\psi}'_1\psi'_2\bar{\psi}'_3\psi'_4$ observables lead to interference between the SM and $\mathcal{L}^{(6)}$ operators in the high energy limit. These effects can be overlooked when studying a simplified limit of these scattering events, as defined by the CC03 diagrams and the narrow width approximation. We have determined the results of the CC03 diagrams in several novel regions of phase space, compared to recent SMEFT literature, and have shown that single resonant diagrams do not change these conclusions when included into the results. We have also illustrated how to make the narrow width approximation consistent with the SMEFT expansion.

The off-shell phase space of the CC03 diagrams considered, and the phase space of the single resonant diagrams, is parametrically suppressed in an inclusive $\bar{\psi}\psi \to \bar{\psi}'_1 \psi'_2 \bar{\psi}'_3 \psi'_4$ observable. The full phase space is dominated by the near on-shell contributions of the CC03 diagrams which can be parametrically larger by $\sim (\hat{\Gamma}_W \hat{m}_W / \bar{v}_T^2)^{-1}$ or $\sim (\hat{\Gamma}_W \hat{m}_W / p_i^2)^{-1}$ where p_i^2 is a Lorentz invariant of mass dimension two. The exact degree of suppression that the off-shell region of phase space experiences strongly depends on the experimental cuts defining the inclusive $\bar{\psi}\psi \to \bar{\psi}'_1 \psi'_2 \bar{\psi}'_3 \psi'_4$ observables, which should be studied in a gauge independent manner including all diagrams that contribute to the experimental observable, i.e. including all CC11 diagrams.

In some sense, our results coincide with the overall thrust of the discussion of ref. [25], which emphasizes that searching for the effects of $\mathcal{L}^{(6)}$ operators interfering with the SM in tails of distributions (i.e. in the $\hat{m}_W^2/s \to 0$ limit) can be challenged in some helicity configurations, by the smallness of such interference effects. Arguably, this encourages prioritizing SMEFT studies on "pole observables" and makes such LHC studies a higher priority compared to pursuing such suppressed "tail observables".[7] At the same time, we stress that the results of this work indicate that the strong statements on non-interference of the SM and $\mathcal{L}^{(6)}$ operators, in subsets of phase space, and for some helicity configurations, are tempered by finite width effects, in addition to perturbative corrections [25, 32] and finite mass suppressions [25]. Finally, our results also demonstrate that a careful examination of historical and rigorous SM results, in the well developed SM literature, are an essential foundation to precise and accurate SMEFT studies.

A Conventions and notation

We use the generic notation $\delta X = \bar{X} - \hat{X}$ for the differences for a Lagrangian parameter X [1, 5] due to $\mathcal{L}^{(6)}$ corrections in the SMEFT and define

$$\delta G_F = \frac{1}{\sqrt{2}\hat{G}_F}\left(\sqrt{2}C_{Hl}^{(3)} - \frac{C_{ll}}{\sqrt{2}}\right), \tag{A.1}$$

$$\delta m_Z^2 = \frac{1}{2\sqrt{2}}\frac{\hat{m}_Z^2}{\hat{G}_F}C_{HD} + \frac{2^{1/4}\sqrt{\pi\hat{\alpha}}\,\hat{m}_Z}{\hat{G}_F^{3/2}}C_{HWB}, \tag{A.2}$$

$$\delta \bar{g}_Z = -\frac{\delta G_F}{\sqrt{2}} - \frac{\delta m_Z^2}{2\hat{m}_Z^2} + \frac{s_{\hat{\theta}}c_{\hat{\theta}}}{\sqrt{2}\hat{G}_F}C_{HWB}, \tag{A.3}$$

$$\delta g_1 = \frac{\hat{g}_1}{2c_{2\hat{\theta}}}\left[s_{\hat{\theta}}^2\left(\sqrt{2}\delta G_f + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) + c_{\hat{\theta}}^2 s_{2\hat{\theta}}\bar{v}_T^2 C_{HWB}\right],$$

---

[7]For a recent discussion on a systematic SMEFT pole program see ref. [5]. One of the comparative strengths of the pole observable program is that observables can be optimized so that interference suppression effects *enhance* theoretical control of a process for SMEFT studies.

$$\delta g_2 = -\frac{\hat{g}_2}{2c_{2\hat{\theta}}}\left[c_{\hat{\theta}}^2\left(\sqrt{2}\delta G_f + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) + s_{\hat{\theta}}^2 s_{2\hat{\theta}}\bar{v}_T^2 C_{HWB}\right], \tag{A.4}$$

$$\delta s_\theta^2 = 2c_{\hat{\theta}}^2 s_{\hat{\theta}}^2\left(\frac{\delta g_1}{\hat{g}_1} - \frac{\delta g_2}{\hat{g}_2}\right) + \bar{v}_T^2\frac{s_{2\hat{\theta}}c_{2\hat{\theta}}}{2}C_{HWB}, \tag{A.5}$$

$$\frac{\hat{m}_W^2}{\Lambda^2}\delta\bar{g}_W^\ell = \frac{1}{2\sqrt{2}\hat{G}_F}\left(C_{H\ell}^{(3)} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}C_{HWB}\right) - \frac{1}{4}\frac{\delta s_\theta^2}{s_{\hat{\theta}}^2}. \tag{A.6}$$

$$R_1 = \left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2} - \sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_1}\left[e^{-i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + \gamma_{12}^+\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\cos\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\sin\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.- \sqrt{s_3}\left[-e^{i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.7}$$

$$R_2 = \left[\gamma_{12}^-\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_1}\left[\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} - \gamma_{12}^+ e^{-i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.+ \left[\gamma_{34}^+\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} - e^{i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.8}$$

$$R_3 = \left[\gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2} - \sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_3}\left[e^{-i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^-\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.+ \sqrt{s_1}\left[\gamma_{12}^-\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.9}$$

$$R_4 = \left[\gamma_{34}^+\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} - e^{i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_3}\left[\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^- e^{-i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.+ \left[\gamma_{12}^-\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.10}$$

$$L_1 = \left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2} + \cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_1}\left[e^{-i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} - \gamma_{12}^+\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\cos\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\sin\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.- \sqrt{s_3}\left[e^{i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.11}$$

$$L_2 = \left[-\gamma_{12}^-\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]$$
$$\left\{\sqrt{s_1}\left[\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + \gamma_{12}^+ e^{-i\tilde{\phi}_{12}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\left[\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$
$$\left.- \left[\gamma_{34}^+\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} + e^{i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\right\}e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\gamma_{12}^+\gamma_{34}^- \tag{A.12}$$

$$L_3 = \left[\gamma_{34}^+ e^{i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]$$

$$\left\{-\sqrt{s_3}\left[e^{-i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^-\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} + \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$

$$\left. + \sqrt{s_1}\left[-\gamma_{12}^-\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\right\} e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\,\gamma_{12}^+\gamma_{34}^- \tag{A.13}$$

$$L_4 = \left[\gamma_{34}^+\cos\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} + e^{i\tilde{\phi}_{34}}\sin\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^+ e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]$$

$$\left\{-\sqrt{s_3}\left[\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^- e^{-i\tilde{\phi}_{34}}\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{34}}{2}\right]\left[-\gamma_{12}^- e^{i\tilde{\phi}_{34}}\cos\frac{\tilde{\theta}_{12}}{2}\cos\frac{\tilde{\theta}_{34}}{2} - \gamma_{34}^+ e^{i\tilde{\phi}_{12}}\sin\frac{\tilde{\theta}_{12}}{2}\sin\frac{\tilde{\theta}_{34}}{2}\right]\right.$$

$$\left. - \left[-\gamma_{12}^-\cos\frac{\theta}{2}\cos\frac{\tilde{\theta}_{12}}{2} + e^{i\tilde{\phi}_{12}}\sin\frac{\theta}{2}\sin\frac{\tilde{\theta}_{12}}{2}\right]\right\} e^{-i(\tilde{\phi}_{12}+\tilde{\phi}_{34})}\sqrt{s_1 s_3}\,\gamma_{12}^+\gamma_{34}^-. \tag{A.14}$$

A.1 Phase space

The four momenta are defined as $p_+^\mu = \frac{\sqrt{s}}{2}(1, \sin\theta, 0, -\cos\theta)$, $p_-^\mu = \frac{\sqrt{s}}{2}(1, -\sin\theta, 0, \cos\theta)$ with $s = (p_+ + p_-)^2$ and $s_{ij} = (k_i + k_j)^2$ while the final state momenta (boosted to a common center of mass frame) are

$$\frac{2k_1^\mu}{\sqrt{s_{12}}} = \left(\gamma_{12,0} - \gamma_{12}\cos\tilde{\theta}_{12}, -\sin\tilde{\theta}_{12}\cos\tilde{\phi}_{12}, -\sin\tilde{\theta}_{12}\sin\tilde{\phi}_{12}, \gamma_{12,0}\cos\tilde{\theta}_{12} + \gamma_{12}\right), \tag{A.15}$$

$$\frac{2k_2^\mu}{\sqrt{s_{12}}} = \left(\gamma_{12,0} + \gamma_{12}\cos\tilde{\theta}_{12}, \sin\tilde{\theta}_{12}\cos\tilde{\phi}_{12}, \sin\tilde{\theta}_{12}\sin\tilde{\phi}_{12}, \gamma_{12,0}\cos\tilde{\theta}_{12} + \gamma_{12}\right), \tag{A.16}$$

$$\frac{2k_3^\mu}{\sqrt{s_{34}}} = \left(\gamma_{34,0} - \gamma_{34}\cos\tilde{\theta}_{34}, \sin\tilde{\theta}_{34}\cos\tilde{\phi}_{34}, \sin\tilde{\theta}_{34}\sin\tilde{\phi}_{34}, \gamma_{34,0}\cos\tilde{\theta}_{34} - \gamma_{34}\right), \tag{A.17}$$

$$\frac{2k_4^\mu}{\sqrt{s_{34}}} = \left(\gamma_{34,0} + \gamma_{34}\cos\tilde{\theta}_{34}, -\sin\tilde{\theta}_{34}\cos\tilde{\phi}_{34}, -\sin\tilde{\theta}_{34}\sin\tilde{\phi}_{34}, -\gamma_{34,0}\cos\tilde{\theta}_{34} - \gamma_{34}\right). \tag{A.18}$$

We use the definitions $\lambda = s^2 + s_{12}^2 + s_{34}^2 - 2ss_{12} - 2ss_{34} - 2s_{12}s_{34}$

$$\gamma_{12} = \frac{\sqrt{\lambda}}{2\sqrt{ss_{12}}}, \qquad \gamma_{12,0} = \frac{s + s_{12} - s_{34}}{2\sqrt{ss_{12}}},$$

$$\gamma_{34} = \frac{\sqrt{\lambda}}{2\sqrt{ss_{34}}}, \qquad \gamma_{34,0} = \frac{s + s_{34} - s_{12}}{2\sqrt{ss_{34}}},$$

$$\gamma_{12}^\pm = \gamma_{12,0} \pm \gamma_{12}, \qquad \gamma_{34}^\pm = \gamma_{34,0} \pm \gamma_{34}.$$

Useful identities are $\gamma_{12,0}^2 - \gamma_{12}^2 = \gamma_{12}^+\gamma_{12}^- = 1$ and $\gamma_{34,0}^2 - \gamma_{34}^2 = \gamma_{34}^+\gamma_{34}^- = 1$. A phase convention choice on $\phi_{12,34}$ in the spinors is required to be the same in the CC03 and CC11 results.

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