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Optimum Commodity Taxation with a Non-Renewable Resource*

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Pierre left us in the spring of 2017. He is missed so much.
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Abstract

We examine optimum commodity taxation (OCT), including the taxation of non-renewable resources (NRRs), by a government that needs to rely on commodity taxes to raise revenues. NRRs should be taxed at higher rates than otherwise-identical conventional commodities, according to an augmented, dynamic version of the standard Ramsey rule. This rule is compatible with the variety of observed NRR tax systems. OCT of NRRs distorts developed reserves, which are reduced, and their depletion, which is slowed down. We contradict Chamley-Judd conclusion that capital should not be taxed in the long run. Our formulas can directly be used to indicate how Pigovian taxation of carbon NRRs should be increased in the presence of public-revenue needs, as illustrated in a numerical example. We show that NRR substitutes and complements should receive a particular tax treatment. Finally, in a NRR-importing economy, Ramsey NRR taxes are further increased to allow the capture of foreign rents.

JEL classification: Q31; Q38; H21
Keywords: Optimum commodity taxation; Inverse elasticity rule; Non-renewable resources; Supply elasticity; Carbon taxation

1 Introduction

The theory of optimum commodity taxation (OCT) addresses the following question: How should a government concerned with total welfare distribute the burden of commodity taxation across sectors in such a way as to collect a set amount of tax income while minimizing the deadweight loss? The literature originated with Ramsey's "A Contribution to the Theory of Taxation" published in 1927. It was further developed by Pigou (1947), Baumol and Bradford (1970), Diamond and Mirrlees (1971), Auerbach (1985), and others. The influence, relevance and modernity of Ramsey's approach were recently celebrated in Stiglitz's (2015) "In Praise of Frank Ramsey's Contribution to the Theory of Taxation." The most famous OCT result is the static "inverse elasticity rule" which says that, under simplifying conditions, the tax rate applied on each good should be proportional to the sum of the reciprocals of its elasticities of demand and of supply, where the latter is often neglected, either because supply is considered infinitely elastic in a long run perspective or because profits are assumed entirely taxed away at the outset.

In this paper we reexamine the theory of OCT in presence of natural non-renewable resources (NRRs). Is there any reason to give any particular attention to NRRs in that context? The flow of most energy NRR commodities receives a special tax treatment. To explain this treatment and its extraordinary contribution to governments' income, it is often noted that energy demand is relatively price inelastic—for oil, see, for example, Berndt and Wood (1975), Pindyck (1979) and Hamilton (2009a). On the one hand, Ramsey's framework seems perfectly natural to examine this property. On the other hand, however, it is the supply side that makes NRRs different from other commodities: The long-run supply of a NRR is not infinitely elastic even if marginal extraction costs are constant, because the short-run supply of a NRR consists in allocating the production from a limited stock of reserves over time. This has several important implications for the problem of OCT.

First, NRR reserve limitations generate economic rents which make Ramsey taxes particularly sharp—see Stiglitz (2015) and further details later in this introduction. As an extreme example, a constant-rate commodity tax imposed on a costlessly extracted resource turns out to raise the NRR rent in a non-distortionary fashion—see, e.g., Dasgupta, Heal, and Stiglitz (1981). Second, the non renewability of a natural resource adds a dynamic dimension to the OCT distortion. Third, the OCT problem introduces a revenue maximization component to the government's objective (Boiteux, 1956; Baumol and Bradford, 1970) and, in this context, NRR demand elasticity is conferred a special role, as in Stiglitz's (1976) intertemporal NRR monopoly—see also Lewis, Matthews and Burness (1979)—or in Bergstrom's (1982) NRR tax competition problem.

We find that, for NRRs, the optimal Ramsey-Pigou tax formula is an augmented, dynamic, version of the standard rule, requiring a novel interpretation. It has implications not only for the understanding of existing NRR tax systems, but also on policy prescriptions, e.g., for carbon NRRs as we illustrate in an application of Section 6.

1.1 Related Literature

Surprisingly, the OCT problem was never extended to economies with NRRs despite the appearance of "The Economics of Exhaustible Resources" in 1931 by Hotelling shortly after Ramsey's paper. Our analysis connects with various strands of the economics literature, as we explain in detail in Appendix A for the interested reader. As a matter of fact and despite economists' recommendations—see, for recent examples, Boadway and Keen (2010) and Daubanes and Andrade de Sá (2014)—the ability to use direct rent taxation proved limited in NRR sectors, leaving large NRR rents untaxed. Recent World Bank data suggest that, for instance, economic profits—including rents—from oil extraction worldwide exceeded US$ 609 billion in 2015.[1] In this context, Ramsey commodity taxes are particularly useful as they allow to indirectly tap such untaxed rents (Stiglitz, 2015). In fact, royalties and other linear commodity taxes are dominant forms of resource taxation (Daniel, Keen, and McPherson, 2010). Accordingly, the NRR taxation literature has largely studied the neutrality of, or the distortions caused by, NRR commodity taxes, although it has hitherto ignored governments' revenue needs—see, for influential examples, Dasgupta et al. (1981), Long and Sinn (1985), Sinn (2008), and van der Ploeg and Withagen (2012), as well as Gaudet and Lasserre's (2013) recent synthesis. Our analysis lies precisely at the intersection of the OCT and NRR literatures and allows to examine how the burden of OCT should be optimally spread over NRR sectors. To sum up, this analysis was needed not only because it has been overseen theoretically, but also because the premises of Ramsey-Pigou original

[1] We have used data available at `http://data.worldbank.org/indicator/NY.GDP.PETR.RT.ZS?end=2015&start=1970&view=chart` and at `http://data.worldbank.org/indicator/NY.GDP.MKTP.CD?end=2015&start=1960`.

framework fit particularly well with the characteristics of NRR tax systems. Moreover, most governments imposing special commodity taxes on NRRs are also struggling to meet their debt and budget constraints, especially in the aftermath of the financial crisis of the early century.

This paper is also complementary with the literature on carbon NRR taxation which has mostly disregarded public revenue needs—see, for recent influential examples, Chakravorty, Moreaux, and Tidball (2008), and Golosov, Hassler, Krusell, and Tsyvinski (2014)—although some have examined the ability of a carbon tax to transfer NRR rents—see, among others, Liski and Tahvonen (2004), Dullieux, Ragot, and Schubert (2011), and Fischer and Salant (2017). Besides, some of our results are reminiscent of the double dividend literature addressing OCT in presence of non-distortionary (corrective) taxes, although this literature has hitherto ignored NRRs—see, among other important papers, Sandmo (1975), Bovenberg and de Mooij (1994), Fullerton (1997), and Cremer and Gahvari (2004). At the intersection of these two literatures, Barrage (2014) comes close to our analysis by introducing a NRR sector in her study of OCT with carbon pollution (p. 50), but she left Ramsey dynamic distortions to this sector unexplored.

Our formulas can directly be used to establish by how much carbon taxation should be augmented in presence of a public budget constraint and how this affects the direction prescribed for the resolution of carbon externalities (Withagen, 1994). Relatedly, some of our results concern the treatment of non-carbon substitutes when the non renewability of carbon resources is considered, unlike Sandmo's (1975) time-honored paper on OCT with externality-generating commodities and non-externality-generating substitutes. Since NRRs are a form of capital, our paper also contributes to the capital taxation literature initiated by Judd (1985) and Chamley (1986) that has focused on capitals which are producible without limit, unlike NRRs. Finally, since open economies importing NRRs rely on commodity taxes as substitutes to NRR rent taxation, we will highlight the connection between our results and important results due to Bergstrom (1982).

1.2 Methodology

Except for the dynamic aspects induced by resource extraction and the endogeneity of reserves, our model adheres to the Ramsey-Pigou OCT framework for theoretical and empirical reasons justified above and in more details in Appendix A. Lump-sum tax collection is impossible.[2] Direct taxation is not a controllable option, whether it aims at income,[3] or at pure profits like resource rents.[4] Indirect linear taxes or subsidies can be applied on the final consumption or on the production of any commodity or service;[5] these taxes (or subsidies) may take the form of *ad valorem* taxes or of *unit* taxes, proportional to quantities. The government is not concerned with individual differences; in fact we assume a representative consumer.[6] The optimal supply of public goods is not addressed either; we assume that the government faces exogenous financial needs in order to fulfill its role as a supplier of public goods so that the government's problem is to raise that amount of revenues in the least costly way, given the available tax instruments.

1.3 Structure and Principal Findings

In order to facilitate comparisons with the conventional analysis involving non-resource sectors, we proceed in several steps. In the first step, presented in Section 2, we follow the traditional OCT literature in assuming constant marginal costs of production. This implies that supply is infinitely elastic in non-resource sectors as should be the case in a long-run analysis when no factors are fixed. In the NRR sector, the same assumption on the technology, constant marginal extraction cost, implies that there is no limit to short-run supply; however Hotelling's long-run exhaustibility of the resource retains its central role.

In that setup, the resource should be taxed in priority over producible commodities:

[2] In particular, this means that neither labor endowment nor leisure consumption are taxable (e.g., Auerbach, 1985, p. 89); it is standard to interpret the untaxed numeraire as being leisure.

[3] When income taxation is linear, there is no loss of generality in letting labor income be untaxed (e.g., Atkinson and Stiglitz, 1976). Even with non-linear income taxation, the substantial literature that questions the relevance of Atkinson and Stiglitz's framework shows that commodity taxation remains an issue under sensible assumptions.

[4] As long as direct profit and rent taxes are less than 100 per cent and cannot be adjusted, assuming that profits and rents are not taxed directly at all amounts to rescaling the production cost function and does not involve any loss of generality.

[5] For simplicity, our formulation does not distinguish between a NRR-based final good or service, and the quantity of NRR inputs needed to produce it; this approach does not make any difference when resource transformation technologies are linear and has been shown to be appropriate in situations where the final good is difficult to tax (e.g., transportation; Stiglitz, 2015). Similarly, that distinction was avoided in the original partial-equilibrium formulation of the OCT problem, an option further validated by Baumol and Bradford (1970). In general, Stiglitz and Dasgupta (1971) further showed that production efficiency is not in general required in presence of untaxed profits or rents.

[6] Using a representative agent is a simplification that should not be interpreted to mean that a poll tax is feasible. With heterogeneous consumers and concerns about equity, the standard Ramsey tax formula for one commodity takes into account the social contribution of consumers' incomes (Diamond, 1975; see Belan et al., 2008, for a partial-equilibrium exposition closer to ours).

If government revenue needs are low, they may be satisfied by using the ability of linear royalties to tax the NRR without distortions, thus dispensing from distortionary taxes in the rest of the economy. A similar situation arises, for example, in Sandmo (1975), as well as in Fullerton (1997) and Barrage (2014), where the economy suffers from externalities that may be corrected by Pigovian taxes: If government revenue needs are low, they may be covered by such Pigovian taxes.

When revenue needs are too high to be covered by neutral royalties, distortions are inevitable and should be spread optimally in the economy. In that case, we find that the resource should be taxed at a higher rate than conventional commodities having the same demand elasticity. New results concern the way that the Ramsey tax is conferred a dynamic dimension. The distortion to the NRR sector takes the form of slower extraction at given level of remaining reserves so that the path of reserves over time does not diminish as fast as if the tax was neutral or if there was no public revenue needs.

In the rest of the paper, we examine the role of some basic assumptions affecting resource demand and supply. There are three basic ways to alleviate resource supply limitations. One is to rely on resource substitutes; a second way is to produce reserves for subsequent extraction; a third way is to rely on resource imports. The first two aspects will be addressed in the main text (Sections 3 and 4) in the manner described below, assuming a closed economy. An open-economy extension is provided in Appendix M and discussed in Section 6.

In Section 3, we introduce non-zero cross-price elasticities between the NRR and other commodities. As far as the resource is concerned, exhaustibility retains the central role identified in Section 2. Yet the existence of resource substitutes and complements raises the question of whether they should receive a particular tax treatment as such. Our results differ from Sandmo's (1975) OCT analysis with externalities. Sandmo finds that Ramsey taxes on substitutes for externality-generating commodities should not acquire any Pigovian dimension. In contrast, we find that resource substitutes should be taxed at a different rate than similar commodities that are substitutes for conventional goods. Perhaps surprisingly, that rate should be lower; it also exhibits a dynamic dimension.

In Section 4, we assume that reserves are endogenous; their production is determined by the net-of-tax rents derived during the extraction phase completed by subsidies (negative

Ramsey taxes) that the owner receives towards the production of reserves if any. This means that resource supply is allowed to be elastic not only in the short run as in the first part of the paper, but also in the long run. In that case, NRRs should never be singled out as sole targets for OCT because it is now impossible to avoid distortions, whatever the government's revenue needs. We establish the proper Ramsey rule for that case. It shows how demand elasticity combines with the long-run elasticity of reserves to determine how the taxation burden should be spread across resource and non-resource sectors. As far as the NRR sector is concerned, we show that there exists a continuum of equivalent mixed tax systems, combining subsidies towards reserve supply with taxes on resource extraction, that achieve government's objectives in terms of NRR exploitation and tax revenues. This variety is observed empirically, and includes the polar case of a nationalized extraction sector.

All such optimal combinations of extraction taxes with exploration and development subsidies imply a tax load at least as high on the resource as on conventional commodities having the same demand elasticity. However, the distortion induced by NRR taxes is split between a distortion on the extraction profile extending the inverse demand elasticity rule dynamically, and a distortion on the level of induced reserves, obeying a rule reminiscent of the inverse elasticity rule for commodities of finite supply elasticity.

Section 5 presents a numerical application, for the fossil oil resource, of Section 4's analysis. Our calibration allows to provide orders of magnitude for the level of NRRs' OCT. It also illustrates how the need for public-revenue collection should affect both the development of oil reserves and their rate of exploitation.

Section 6 concludes the analysis, discusses extensions presented in Appendices, and confronts aspects of our findings with existing results on capital taxation, on the taxation of carbon NRRs and their substitutes, and on the capture of foreign rents by an open economy.

Proofs that are economically enlightening are provided in the main text; proofs mostly involving algebraic manipulations are in Appendices.

2 OCT with a Non-Renewable Hotelling Resource

There are n produced commodities or services indexed by $i = 1, ..., n$, one non-renewable resource indexed by s and extracted from a finite reserve stock S_0, and a numeraire which is not taxed and should be interpreted as the negative of labor. We adopt the standard partial-equilibrium restrictions under which Baumol and Bradford (1970) obtain the inverse elasticity rule; that is: All goods or services $i = 1, ..., n$ and s are final-consumption, non-leisure goods. Assuming a single NRR simplifies the exposition without affecting the generality of the results. At each date $t \geq 0$, quantity flows are denoted by $x_t \equiv (x_{1t}, ..., x_{nt}, x_{st})$.[7] Storage is not possible, so that goods and services must be consumed as they are produced. Producer prices $p_t \equiv (p_{1t}, ..., p_{nt}, p_{st})$ are expressed in terms of the numeraire. Goods and services are taxed at unit levels $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{st})$ so that the representative consumer faces prices $q_t = p_t + \theta_t$. In this autarkic economy, as in any situation where production equals consumption, taxes may indifferently be interpreted as falling on consumers or producers, but must be such that they leave non-negative profits to producers. In the case of the NRR, this requires that, at any date, the discounted profits accruing to producers over the remaining life of the mine be non negative. Taxes that meet these conditions will be called feasible.

Since the resource is non renewable it must be true that

$$\int_0^{+\infty} x_{st}\, dt \leq S_0, \tag{1}$$

where S_0 is the initial size of the depletable stock.

In the rest of the paper, a "$\sim$" on top of a variable means that the variable is evaluated at the competitive market equilibrium. For given feasible taxes $\Theta \equiv \{\theta_t\}_{t\geq 0}$, competitive markets lead to the equilibrium allocation $\{\widetilde{x}_t\}_{t\geq 0}$ where $\widetilde{x}_t = (\widetilde{x}_{1t}, ..., \widetilde{x}_{nt}, \widetilde{x}_{st})$. Under the set of taxes Θ, this intertemporal allocation is second-best efficient.

Defining social welfare as the cumulative discounted sum of instantaneous utilities $\widetilde{W}_t$, the OCT problem consists in choosing a feasible set of taxes Θ in such a way as to maximize

[7] Production processes by which resources are transformed into final products are linear in the quantity of pre-transformed resource; hence, there is no need to distinguish the raw extracted resource from its derivative.

welfare while raising a given level of discounted revenue $R_0 \geq 0$:

$$\max_{\Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt \tag{2}$$

$$\text{subject to } \int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0. \tag{3}$$

The fact that all variables are evaluated at the competitive market equilibrium further indicates that the problem is implicitly constrained by the realization of this equilibrium. In particular, that means that the finiteness of NRR reserves as expressed in (1) does not need to be taken into account by the government, as long as it is a constraint integrated by the NRR producers that will be reflected in the market equilibrium, as explained shortly below. It is assumed that the set of feasible taxes capable of collecting R_0 is not empty.

The tax revenue constraint (3) does not bind the government at any particular date because financial markets allow expenditures to be disconnected from revenues. The government accumulates an asset a_t over time by saving tax revenues:

$$\dot{a}_t = ra_t + \theta_t \widetilde{x}_t, \tag{4}$$

where the initial amount of asset is normalized to zero and

$$\lim_{t \to +\infty} a_t e^{-rt} = R_0. \tag{5}$$

Thus the problem of maximizing (2) subject to (3) can be replaced with the maximization of (2) subject to (4) and (5), by choice of a feasible set of taxes.

As in Ramsey (1927, p. 55) and Baumol and Bradford (1970), we assume that the demand $D_i(q_{it})$ for each commodity or service i or s depends only on its own price, with $D_i'(.) < 0$, implying that consumer preferences are quasi linear in the numeraire.[8] Moreover, following Baumol and Bradford (1970) and many others, we assume in this section that supply is perfectly elastic, i.e., that marginal costs of production are constant in terms of the numeraire. Let $c_i \geq 0$ be the marginal cost of producing good or service $i = 1, ..., n$.

In the case of the NRR, the supply is determined by Hotelling's rule under conditions

[8] In Section 3, we introduce non-zero cross-price demand elasticities.

of competitive extraction. Consistently with our assumption of constant marginal costs of production, we assume that the unit NRR extraction cost is constant, equal to $c_s \geq 0$.

However, this does not imply that the producer price of the resource reduces to this marginal cost. Indeed, Hotelling's analysis shows NRR supply to be determined in competitive equilibrium by the so-called "augmented marginal cost" condition:

$$\widetilde{p}_{st} = c_s + \widetilde{\eta}_t, \tag{6}$$

where $\widetilde{\eta}_t$ is the current-value unit Hotelling rent accruing to producers; it depends on the tax and the level of initial reserves, and must grow at the rate of discount over time. In competitive Hotelling equilibrium,

$$\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}. \tag{7}$$

At any date, the net consumer surplus, producer surplus, and resource rents in competitive equilibrium are respectively

$$\widetilde{CS}_t = \sum_{i=1,\ldots,n,s} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,\ldots,n,s} (\widetilde{p}_{it} + \theta_{it})\widetilde{x}_{it}, \tag{8}$$

$$\widetilde{PS}_t = \sum_{i=1,\ldots,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,\ldots,n,s} c_i\widetilde{x}_{it} - \widetilde{\eta}_t\widetilde{x}_{st} \tag{9}$$

and

$$\widetilde{\phi}_t = \widetilde{\eta}_t\widetilde{x}_{st}. \tag{10}$$

Define $\widetilde{W}_t$ in problem (2) as the sum of net consumer surplus, net producer surplus, and resource rents accruing to resource owners.[9,10] The present-value Hamiltonian associated with the problem of maximizing cumulative discounted social welfare (2) under constraints (4) and (5) resulting from the budget requirement of the government is

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t), \tag{11}$$

[9]Although changes in current taxes may affect current tax revenues, the budget constraint of the government applies only over the entire optimization period. The revenue requirements being treated as given over that period, they enter the general problem as a constant and thus no amount of redistributed taxes needs to enter the objective.

[10]This formulation has the advantage of making the value of the resource as a scarce input explicit; it would also apply if producers were not owners of the resource but were buying the resource from its owners at its *in situ* price $\widetilde{\eta}_t$.

where λ_t is the co-state variable associated with a_t while θ_t is the vector of control variables. λ_t can be interpreted as the current unit cost of levying one dollar of present-value revenues through taxes. From the maximum principle, $\dot{\lambda}_t = -\frac{\partial \mathcal{H}}{\partial a_t}$, so that $\lambda_t = \lambda e^{-rt}$, where λ is the present-value unit cost of levying tax revenues. Indeed tax revenues must be discounted according to the date at which they are collected. λ is equal to unity when there is no deadweight loss associated with taxation; it is higher than unity otherwise.

2.1 Optimal Taxation of Conventional Goods

Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on good $i = 1, ..., n$ is

$$[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - c_i]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0. \quad (12)$$

Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = c_i + \theta_{it}$, it is the case that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.)}$. The optimum tax is thus $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}D_i^{-1\prime}(.)$ and the optimum tax rate is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_i}. \quad (13)$$

In this formula, the elasticity of demand $\varepsilon_i \equiv \frac{D_i^{-1}(.)}{x_{it}D_i^{-1\prime}(.)}$ is negative and is further assumed to be decreasing in x_{it}; this standard monotonicity property guarantees that the optimal tax in (13) is unique. As $\lambda \geq 1$, the optimal tax rates on conventional goods $i = 1, ..., n$ are positive in general, lower than unity, and vanish if $\lambda = 1$.

Formula (13) is Ramsey's formula for the optimal commodity tax rate. It provides an inverse elasticity rule for the case of perfectly-elastic supplies. Since market conditions are unchanged from one date to the other, the taxes and the induced tax rates are constant over time.

2.2 Optimal Taxation of the Non-Renewable Resource

The first-order condition for an interior solution to the choice of the resource tax is

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0. \quad (14)$$

However, since NRR supply is determined by condition (6), it follows that $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$, which is different from zero unlike the corresponding expression in (12). Consequently the Ramsey-type formula obtained for conventional goods does not apply.

If $\lambda = 1$, (14) reduces to $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = 0$. This means that the tax should not distort the Hotelling extraction path. Such a non-distortionary resource tax exists (Burness, 1976; Dasgupta et al., 1981); it must grow at the rate of interest to keep the path of consumer prices unchanged:[11] $\theta^*_{st} = \theta^*_{s0}e^{rt}$. Since θ^*_{st} grows at the rate of interest and the resulting $\widetilde{q}_{st}$ generally grows at a lower rate, the neutral tax rate is rising over time. The only exception is when the marginal cost of extraction is zero so that $\widetilde{q}_{st}$ grows at the rate of interest and the resulting optimal tax rate is constant.

As shown earlier, when $\lambda = 1$, commodity taxes on conventional goods are zero. Hence the totality of the tax burden falls on the NRR. Since the tax on the resource is neutral in that case, then a value of unity for λ is indeed compatible with taxing the natural resource exclusively. Consequently, provided the tax on the non-renewable resource brings sufficient cumulative revenues, the government should tax the resource exclusively, and should do so while taxing a proportion of the resource rent that remains constant over time.

The maximum revenue such a neutral NRR tax can raise is the totality of gross cumulative scarcity rents that would accrue to producers in the absence of a resource tax. Since unit rents are constant in present value, any reserve unit fetches the same rent, whatever the date at which it is extracted. The present value of total cumulative NRR rents is thus $\widetilde{\eta}_0 S_0$ and its maximum possible value $\overline{\eta}_0 S_0$ corresponds to the absence of taxation; the maximum tax revenue that can be raised by a neutral resource tax is thus

$$\overline{R}_0 = \overline{\eta}_0 S_0.$$

This maximum is implemented with a tax equal to the unit rent in the absence of taxation: $\theta^*_{st} = \overline{\eta}_0 e^{rt}$. Both $\widetilde{\eta}_0$ and $\overline{\eta}_0$ are determined in Appendix B. If the tax revenues needed by the government are lower than $\overline{R}_0$, the level of the neutral NRR tax θ^*_{st} is set in such a

[11] Their proof goes as follows. Assume $\theta_{st} = \theta_{s0}e^{rt}$, for any θ_{s0} lower than the consumer price exclusive of the marginal cost in the absence of any resource tax. Then $\widetilde{q}_{st} = \widetilde{p}_{st} + \theta_{st} = c_s + \widetilde{\eta}_t + \theta_{st} = c_s + (\widetilde{\eta}_0 + \theta_{0t})e^{rt}$. Therefore, the price with the tax satisfies the Hotelling rule. The exhaustibility constraint must also be satisfied with equality: $\int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt = S_0$. As a result, the extraction path under this tax is the same as in the absence of tax.

way as to exactly raise the required revenue: $\theta^*_{st} = \theta^*_{s0}e^{rt}$ with

$$\theta^*_{s0} = \overline{\eta}_0 - \widetilde{\eta}_0 = \frac{R_0}{S_0}. \tag{15}$$

If $R_0 > \overline{R}_0$, revenue needs cannot be met by neutral taxation of the resource sector and $\lambda > 1$; this case will be discussed further below. The following proposition summarizes our findings when government revenue needs are low in the sense that $\lambda = 1$.

Proposition 1 *(Low government revenue needs) The maximum tax revenue that can be raised neutrally from the non-renewable resource sector is $\overline{R}_0 = \overline{\eta}_0 S_0$ where $\overline{\eta}_0$ is the unit present-value Hotelling rent under perfect competition and in the absence of taxation.*

1. *If $R_0 \leq \overline{R}_0$, then $\lambda = 1$, and government revenue needs are said to be low; if $R_0 > \overline{R}_0$, then $\lambda > 1$, and government revenue needs are said to be high;*

2. *When $R_0 \leq \overline{R}_0$, the optimum unit tax on the non-renewable resource is positive and independent of demand elasticity while the optimum unit tax on produced goods is zero. The resource tax raises exactly R_0 over the extraction period.*

As long as the government's revenue needs are low, Proposition 1 indicates that the archetypical distortionary tax of the OCT literature should not be applied to conventional commodities; taxation should be applied to the sole resource according to a rule that has nothing to do with Ramsey's rule, is independent of the elasticity of demand and does not induce any distortion.[12] Except for a few resource rich economies, this situation of low revenue needs is less realistic than the second-best case studied below.

If the government revenue needs are high in the sense that $R_0 > \overline{R}_0$ and $\lambda > 1$, revenue needs cannot be met by neutral taxation; then we have shown that both the resource and the conventional goods should be taxed. Furthermore, the question arises whether the government can and should collect more NRR revenues by departing from neutral taxation of the resource sector.[13] This possibility was not addressed by Dasgupta, Heal, and Stiglitz

[12]The fact that neutral taxation of the Hotelling commodity is possible does not mean that neutral profits taxation à la Stiglitz and Dasgupta (1971) or capital levy à la Lucas (1990), or some form of resource rent tax à la Boadway and Keen (2010) have been allowed into the model. It should be clear from the formulation that neutral resource taxation is reached by commodity taxation only.

[13]Clearly, at each date, a non-linear tax on the resource extraction rate reaching the level of the maximum

(1981), nor by followers in the NRR taxation literature. Barrage (2014) came closest by introducing a NRR sector in her study of OCT with carbon pollution (p. 50) but she left Ramsey dynamic distortions to this sector unexplored.

The neutral tax that maximizes tax revenues does not leave any resource rent to producers: $\widetilde{q}_{st} = c_s + \theta_{st}$. Assume, as will be seen to be true later on, that the government can maintain its complete appropriation of producers' resource rents while further increasing tax revenues: The condition $\widetilde{q}_{st} = c_s + \theta_{st}$ remains true while θ_{st} is set so as to further extract some of the consumer surplus. This implies that, when $\lambda > 1$, $\widetilde{p}_{st} = c_s$, $\widetilde{\eta}_t = 0$, and $\widetilde{x}_{st} = D_s(c_s + \theta_{st})$. With $\widetilde{\eta}_t = 0$, resource extraction is no longer determined by the Hotelling supply condition (6): Since $\widetilde{p}_{st} = c_s$, producers are indifferent to the quantity they supply so that quantity is determined on the demand side by the condition $\widetilde{q}_{st} = c_s + \theta_{st}$. Consequently, the choice of θ_{st} by the government determines extraction so that the finiteness of reserves, if it turns out to be binding, comes as a constraint faced by the government in its attempt to increase cumulative tax revenues rather than as a constraint faced by producers in maximizing cumulative profits. Thus the government's problem is now to maximize (2), not only subject to (4) and (5), but also subject to

$$\dot{S}_t = -\widetilde{x}_{st}, \tag{16}$$

where S_t denotes the size of the remaining depletable stock at date t.

The Hamiltonian is modified to

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t) - \mu_t\widetilde{x}_{st}, \tag{17}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are defined as before but with $\widetilde{\eta}_t = 0$, and μ_t is the co-state variable associated with the exhaustibility constraint (16). From the maximum principle, $\lambda_t = \lambda e^{-rt}$, as above, and $\mu_t = \mu \geq 0$. If the exhaustibility constraint is binding, that is to say if optimal taxation induces complete exhaustion of the reserves, $\mu > 0$; if optimal taxation leads to incomplete exhaustion, then $\mu = 0$.

constant neutral tax at the Pareto-optimal extraction rate, would achieve such a goal. However such a non-distortionary tax is ruled out in the conventional Ramsey-Pigou optimal taxation analysis. If it was feasible the Ramsey-Pigou problem would be meaningless.

The first-order condition for the choice of the tax on the resource becomes

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}. \tag{18}$$

Since no resource rent is left to producers above the marginal cost of extraction, $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = 0$, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$, and the optimum tax on the resource is thus

$$\theta_{st}^* = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{19}$$

where the elasticity of NRR demand $\varepsilon_s \equiv \frac{D_s^{-1}(.)}{x_{st}D_s^{-1\prime}(.)}$ is negative and is further assumed decreasing for the same reason as for conventional commodities.

Provided the resource is scarce ($\mu > 0$) from the government's point of view, (19) implies that the resource is taxed at a higher rate than would be the case according to (13) for a conventional commodity having the same demand elasticity. Furthermore, while the first term on the right-hand side of (19) is neutral as it rises at the rate of discount, the presence of the second term implies that the tax is not constant in present value, so that it is distortionary in general.

Can the tax revenue collection motive cause the government to assign no scarcity value to a resource that would otherwise be extracted until exhaustion? The answer is negative. For suppose that $\mu = 0$ in (19). This implies that the tax rate is constant over time, so that the extraction rate is also constant and strictly positive, which in turn implies that the exhaustibility constraint must be violated in finite time.

The following proposition summarizes the results on the optimum taxation of the resource when neutral taxation is not sufficient to collect the revenue needs.

Proposition 2 *(High government revenue needs) If $R_0 > \overline{R}_0$, then commodity taxation is distortionary ($\lambda > 1$) and both the NRR sector and conventional sectors are subject to taxation. In that case:*

1. *Taxes on conventional commodities are given by Ramsey's rule (13) and the tax on the NRR is given by (19), where λ is determined by the condition that total tax revenues levied from the non-resource and resource sectors equal R_0;*

2. *The NRR is taxed at a higher rate than a conventional commodity having the same demand elasticity;*

3. *The after-tax resource rent to producers is nil: $\widetilde{\eta}_t = \widetilde{\eta}_0 = 0$;*

4. *The OCT distortion to the NRR extraction is determined by the time path of taxes (19);*

5. *OCT of the NRR causes extraction to slow down, but not reserves to be left unexploited.*

Although the NRR tax rate is higher than the rate on conventional commodities of identical elasticities, its distortionary effect is not higher; it is chosen so as to minimize the total welfare effect of all commodity tax distortions. The distortion slows down extraction, thus working in the same direction as prescribed by Withagen (1994) to deal with cumulative pollution.

Propositions 1 and 2 also have implications on the evolution of the total flow of tax revenues over time. When the government's revenue needs are low, the total flow of tax revenues decreases in present value as the resource unit tax is constant in present value while extraction diminishes. Tax revenues from conventional sectors being nil, total tax revenues decrease in present value and vanish entirely if the resource is exhausted in finite time. When government revenue needs are high, the flow of tax revenues from conventional sectors is constant in current value. If the NRR is exhausted in finite time, the total tax revenue flow is thus lower at and after the date of exhaustion than before exhaustion. In either case, the government's assets accumulated at resource exhaustion must be sufficient to ensure that expenditures taking place after exhaustion can be financed.

When the government cannot avoid the introduction of distortions, as when revenue needs are high, its problem acquires a revenue-maximizing dimension. This confers OCT a resemblance with monopoly pricing as the term $\frac{1}{-\tilde{\varepsilon}_s}$ in (13) is nothing but a monopoly mark-up (for details see Appendix E). The resource monopoly literature has shown that the exercise of market power by a Hotelling NRR monopoly is constrained by exhaustibility. The sharpest example is Stiglitz (1976) who showed that a resource monopoly facing a constant-elasticity demand and zero extraction costs must adopt the same behavior as a competitive firm; such a monopoly cannot increase its profits above the value of the mine under competition by distorting the extraction path. This limitation also applies to the OCT problem. With zero extraction cost and isoelastic demand, the tax defined by (19) is neutral and rises at the discount rate. In that case, OCT requires that no distortion be imposed to the NRR extraction. We prove that result and make use of it in Section 4, where initial reserves are treated as endogenous.

From Propositions 1 and 2, the resource should be taxed in priority whatever its demand elasticity and whatever the demand elasticity of regular commodities. This irrelevance of demand elasticities contrasts sharply with the standard rationalization of OCT but not with Ramsey's original message. The message is "tax inelastic sectors" whether the source of inelasticity is demand or supply. Once it is realized that long-run reserve supply fixity results in reduced short-run resource supply elasticity, it becomes clear that the emphasis should shift from demand to supply in the case of a Hotelling resource.

In Appendix F, we extend the analysis to the case of increasing marginal costs of production and increasing marginal costs of extraction, so that the supply elasticity of conventional goods is no longer infinite. While the inverse elasticity rule then acquires a supply elasticity component, the finiteness of ultimate reserves implies that NRRs should be taxed in priority and at higher rates than otherwise identical conventional commodities. The inelasticity of long-run resource supply dominates other considerations. We also examine the role of resource heterogeneity. Again, the results are altered but not modified in any fundamental way.

In Section 3, we do away with the assumption that demands are independent from each other; a standard assumption under which the inverse elasticity rule is usually derived. This allows us to examine the specific tax treatment that resource substitutes and complements should receive from an OCT perspective.

In Section 4, it is the Hotelling assumption that reserves are exogenously given that is relaxed. Doing away with this assumption introduces the long-run supply elasticity of the resource and also allows us to highlight the distinction between a NRR and conventional capital.

3 OCT with Resource Substitutes or Complements

Assume that conventional goods may be substitutes or complements for the NRR or for each other while their marginal cost of production, as well as the marginal extraction cost, remains constant as previously. For the reasons explained in the previous section, when government revenue needs are low in the sense of Proposition 1, substitutes for, or complements to, the NRR may be left untaxed while the resource alone is taxed. However, high government revenue needs warrant that NRR substitutes and complements be given specific tax treatments.

Assume that the demand $D_j(q_{jt}, q_{kt})$ for a conventional commodity $j \in \{1, ..., n\}$ not only depends on its own price, but also on the price of another commodity $k \in \{1, ..., j-1, j+1, ..., n, s\}$, with $\frac{\partial D_j(.)}{\partial q_j} < 0$, $\frac{\partial D_k(.)}{\partial q_k} < 0$, and $\frac{\partial D_j(.)}{\partial q_k}, \frac{\partial D_k(.)}{\partial q_j} > 0$ (< 0) if the goods are substitutes (complements). The joint consumer surplus arising from that pair of goods is given by the concave money-metric

$$\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}), \text{ with } \frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_j} = \widetilde{q}_j \text{ and } \frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_k} = \widetilde{q}_k. \tag{20}$$

Redefining the consumer surplus (8) accordingly, the constrained maximization of (2) gives first-order conditions that take account of the effect of any tax θ_{jt} on the tax income raised in sector k.

Consider two conventional commodities $j \in \{1, ..., n\}$ and $k \in \{1, ..., n\}$ that are substitutes for or complements to each other. The first-order condition for the tax on j is[14]

$$\left[\frac{\partial\psi(.)}{\partial x_{jt}} - \theta_{jt} - c_j\right]\frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \left[\frac{\partial\psi(.)}{\partial x_{kt}} - \theta_{kt} - c_k\right]\frac{d\widetilde{x}_{kt}}{d\theta_{jt}} - \widetilde{x}_{jt}$$
$$+\lambda\left(\widetilde{x}_{jt} + \theta_{jt}\frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \theta_{kt}\frac{d\widetilde{x}_{kt}}{d\theta_{jt}}\right) = 0, \qquad k \neq s, \tag{21}$$

where $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{jt}} = c_j + \theta_{jt}$ and $\frac{\partial\psi(\widetilde{x}_{jt},\widetilde{x}_{kt})}{\partial x_{kt}} = c_k + \theta_{kt}$. Moreover, $\widetilde{x}_{jt} = D_j(\widetilde{q}_{jt}, \widetilde{q}_{kt})$ and $\widetilde{x}_{kt} = D_k(\widetilde{q}_{kt}, \widetilde{q}_{jt})$ so that $\frac{d\widetilde{x}_{jt}}{d\theta_{jt}} = \frac{\partial D_j(.)}{\partial q_{jt}}$ and $\frac{d\widetilde{x}_{kt}}{d\theta_{jt}} = \frac{\partial D_k(.)}{\partial q_{jt}}$. It follows that the optimum tax on conventional commodity j when its complement or substitute k is not a NRR is

$$\theta^*_{jt} = \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \theta_{kt}\frac{\widetilde{x}_{kt}\widetilde{\varepsilon}_{kj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}}, \ k, \ j \neq s. \tag{22}$$

Obviously, the optimal tax on good $k \neq s$ is given by the same expression where k and j are interchanged. Consequently time does not enter the above tax formula either directly or through θ_{kt}. Therefore, the optimal taxes and induced tax rates for commodities that are neither NRRs, nor substitutes for (or complements to) NRRs, are constant over time.

A similar derivation for a conventional good j when its substitute or complement is a NRR ($k = s$) yields a time dependent optimal tax

$$\theta^*_{jt} = \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \left(\theta_{st} - \frac{1}{\lambda}\mu e^{rt}\right)\frac{\widetilde{x}_{st}\widetilde{\varepsilon}_{sj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}}, \ j \neq s, \ k = s. \tag{23}$$

In both formulas (22) and (23), the own-price elasticity of the demand for good j is now denoted by $\varepsilon_{jj} = \frac{q_{jt}\frac{\partial D_j(.)}{\partial q_j}}{x_{jt}}$ while $\varepsilon_{kj} = \frac{q_{jt}\frac{\partial D_k(.)}{\partial q_j}}{x_{kt}}$ is the cross-price elasticity of the demand for commodity k with respect to the price of commodity j. The former is negative as in Section 2; the latter is positive for substitutes, and negative for complements.

[14]The present-value Hamiltonian associated with the maximization of (2) subject to (4) and (5) takes the same form as in Section 2:

$$\mathcal{H}(a_t, \theta_t, \lambda_t) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t\right)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t).$$

However, the consumer surplus is adjusted to comprise the relation (20):

$$\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t = \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}) + \sum_{i=1,\dots,n,s \ i\neq j,k}\int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\,du - \sum_{i=1,\dots,n,s}(c_i + \theta_{it})\widetilde{x}_{it},$$

where $D_i^{-1}(\widetilde{x}_{it}) - c_i - \theta_{it} = 0$ for conventional goods $i = 1, ..., n$, and $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$ for the NRR. The maximum principle implies $\lambda_t = \lambda e^{-rt}$ as in Section 2.

Finally consider the taxation of the NRR sector. When the resource admits conventional commodity j as a substitute or complement, the first-order condition for the choice of the NRR tax is the same as (21) except that s and j must be interchanged on the left-hand side and that the right-hand side is $\mu e^{rt} \frac{d\widetilde{x}_{st}}{d\theta_{st}}$ rather than zero. Thus the optimum tax on a NRR that has a substitute or complement j is

$$\theta_{st}^{*} = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda} \frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_{ss}} + \theta_{jt} \frac{\widetilde{x}_{jt}\widetilde{\varepsilon}_{js}}{-\widetilde{x}_{st}\widetilde{\varepsilon}_{ss}}. \tag{24}$$

All three tax formulas (22)-(24) are identical to their independent-demand counterparts in Section 2 (see (13) for conventional commodities and (19) for the NRR) except for the last term on the right-hand side of (22)-(24). This new term reflects the change in the fiscal revenues levied on the sector indirectly affected by the tax. The tax θ_{it} can be interpreted as a producer mark-up.[15] The adjustment to the mark-up (to the tax) is positive (negative) when commodities j and k are substitutes (complements) and does not depend on time directly. This applies whether the sector indirectly affected by the tax is a conventional sector as in (22) or a NRR sector as in (24).

When the commodity indirectly impacted by the tax is a resource, the additional term must also be interpreted as a correction accounting for monopoly power on two markets. However the correction in (23) is reduced by the time-dependent term $\frac{1}{\lambda}\mu e^{rt}$ that reflects the scarcity of the resource. Proposition 3 summarizes the results.

[15] When the commodity indirectly impacted is a conventional good, the new term is the same as in the formula giving the price chosen by a firm that holds monopoly power on two commodities with interrelated demands and separable costs—see, e.g., Tirole (1988, p. 70).

Proposition 3 *(Resource substitutes and complements) Assume that the NRR has substitutes or complements.*

1. *If $R_0 \leq \overline{R}_0$, resource substitutes and complements should be left untaxed, while the resource should be taxed positively;*

2. *If $R_0 > \overline{R}_0$,*

   (a) *Other things equal, conventional goods that are substitutes for a NRR should be taxed at lower rates than substitutes for a conventional good;* vice versa *conventional goods that are complements to a NRR should be taxed more;*

   (b) *The optimal unit tax on resource substitutes and complements depends on time.*

The rationale behind the special tax treatment of substitutes or complements is clear. On the one hand, a higher mark-up on any good positively affects the demand for substitutes, thus their tax base. On the other hand, this effect is less pronounced when the impacted substitute or complement is a NRR—compare the right-hand sides of (23) and (22). The reason is that the tax on a NRR substitute shifts demand towards a sector with an inelastic supply (the NRR), unlike the tax on other conventional goods.

This specific treatment differs from the treatment of substitutes for, or complements of, externality-generating commodities in the absence of government financial restrictions. In Sandmo (1975), the "marginal social damage . . . does not enter the formulas for the other [non externality-creating] commodities, regardless of the pattern of complementarity and substitutability" (p. 92). In contrast, financially strapped governments should tax substitutes to fossil NRRs more lightly, and complements more heavily, than commodities unrelated to NRRs.

Proposition 3 further indicates that the optimal tax on the substitutes for, or complements to, a NRR depends on time, unlike the optimal tax on conventional commodities without links to a NRR. The link with the NRR confers a dynamic dimension to the tax on its substitutes and complements, as can be seen by comparing (23) with (22). For a given NRR tax level, the term in μ indicates that the tax on a NRR substitute should fall as the resource scarcity increases; *vice versa* the tax on a NRR complement should rise. The tax on NRR substitutes and complements is also compounded by the motion of the

NRR tax θ_{st} itself: The tax on NRR substitutes should rise as the resource tax increases; *vice versa* for NRR complements.

4 Endogenous Reserves

In order to focus on the role of the long-run supply of reserves, we assume in this section, as in Section 2, that marginal extraction costs are constant, equal to $c_s \geq 0$. This means that the supply of the natural resource is only limited by the availability of reserves. Consistently, we assume that marginal costs of production are constant, equal to $c_i \geq 0$.[16]

The stock of reserves exploited by a mine does not become available without some prior exploration and development investment. Although exploration for new reserves and exploitation of current reserves often take place simultaneously at the industry level—see, e.g., Pindyck (1978) and Quyen (1988)—most exploration is aimed at the discovery of new deposits and deposit-specific exploration becomes limited once the mine is in exploitation. It is thus a meaningful simplification to adopt the micro-economic view that exploration and exploitation take place in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way to model the supply of reserves is particularly adapted to the OCT problem under study[17] because it provides a simple and natural way to distinguish short-run supply elasticity from long-run supply elasticity. It also raises the issue of the government's ability to tax and subsidize, as well as its ability to commit.[18]

Most commonly observed extractive resource tax systems feature royalties and levies based on extraction revenues or quantities, often combined with tax incentives to exploration and development. During the extraction phase, i.e., once reserves are established, these systems let some Hotelling rents accrue to producers, perhaps to compensate firms for the prior production of reserves.[19] On the other hand, state-owned extraction sectors

[16]See Appendix F for the case of rising marginal costs of production.

[17]Firms seldom exploit only one site, so that exploration is an on-going process at the firm level (as opposed to the site level). However linear Ramsey taxes do not need to be industry or firm specific; they can be site specific in theory. In practice they sometimes are. For example, the Albertan taxation of conventional oil and natural gas commits to royalty rate reductions that depend on a well's discovery date (Alberta Royalty Review, 2007); those reductions amount to exploration subsidies that aim at recognizing exploration costs; they are made dependent on discovery date to reflect changing costs of exploration across deposits over time. To the extent that their sum depends on cumulative extraction, they are based on the amount of discovered reserves.

[18]On issues of commitment and regime changes in resource taxation, see Daniel et al. (2010).

[19]Clearly, Ramsey's tax setup rules out the direct taxation of rents, but not their indirect taxation by commodity taxes (Stiglitz, 2015).

are common. A nationalized industry means that no extraction rents are left to private producers.

Thus two situations are common empirically: In the first instance extraction is taxed in such a way that strictly positive rents are left to firms; in the second instance no extraction rents are left to firms. The results from the previous section point to the importance of that distinction. Indeed, when S_0 is given as in Section 2, if the government has high revenue needs in the sense of Proposition 2, it should use the NRR commodity tax to take the totality of extraction rents away from producers. If it did so when S_0 were endogenous, it would tax quasi-rents together with scarcity rents, thus removing incentives for producers to generate reserves in the first place. If the government wants to create a tax environment allowing net extraction profits to compensate firms for the cost of reserve production, it must be able to commit, prior to extraction, to a system of *ex post* extraction taxation that leaves enough rents to producers. Alternatively, if the government taxes away extraction rents, including quasi-rents sunk into them, it must compensate firms by subsidies prior to extraction. In practice, these subsidies often take the form of commitments to reductions in the tax rate applied to future extraction;[20] thus they are formally equivalent to subsidies linear in the quantity developed and put into exploitation. We will show that there exists a continuum of mixed systems, combining subsidies (negative commodity taxes) on reserve production with positive taxes on extraction, that leave some rents in the hands of firms while meeting the government's revenue needs.[21]

For simplicity assume that *ex ante* reserve producers (explorers) are the same firms as *ex post* extractors. Assume that the stock of reserves to be exploited is determined prior to extraction by a supply process that reacts to the sum of the subsidies obtained by the firms for reserve production and the cumulative net present-value rents accruing to resource producers during the exploitation stage; also for simplicity, assume that reserve production is instantaneous.

Express total cumulative present-value rents from extraction as $\eta_0 S_0$. Suppose further that a negative linear tax $-\rho$ may be applied to the production of reserves, for a total

[20] See Footnote 17 for the example of the relatively advanced Albertan system.

[21] These mixed systems are feasible if the government is able to commit to leave firms the prescribed after-tax extraction rent; otherwise, an optimal system relying on reserve supply subsidies while not leaving the firms any extraction surplus can also achieve the same objective.

subsidy of ρS_0. Then the initial stock of reserves may be written as a function of $\eta_0 + \rho$.[22] This function $\mathcal{S}(\eta_0 + \rho)$ can be interpreted as the long-run after-tax supply of reserves as follows. Suppose that reserves can be obtained, via exploration or purchase, at a cost $E(S_0)$. As not only known reserves but also exploration prospects are finite, the long-run supply of reserves is subject to decreasing returns, so that $E'(S_0) > 0$ for any $S_0 > 0$, and $E''(.) > 0$. Then the profit from the production of a stock S_0 of initial reserves is $(\widetilde{\eta}_0 + \rho) S_0 - E(S_0)$. Given ρ and $\widetilde{\eta}_0$, its maximization requires $\widetilde{\eta}_0 + \rho = E'(S_0)$. We define $\mathcal{S}(\widetilde{\eta}_0 + \rho) \equiv E'^{-1}(\widetilde{\eta}_0 + \rho)$, making the following assumption.

Assumption 1 *(Long-run supply) The supply of initial reserves $\mathcal{S}(.)$ is continuously differentiable and such that $\mathcal{S}(0) = 0$, $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$, and $\mathcal{S}'(\eta_0 + \rho) > 0$.*

The property $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$ is introduced because it is sufficient to rule out the uninteresting situation where the demand for the NRR does not warrant the production of any reserves.

4.1 Optimal Resource Taxation with a Strictly Positive Producer Rent

Even when the government can subsidize exploration, i.e., when $\rho > 0$, leaving some positive after-tax extraction rent to producers may be desirable. Two reasons make it interesting to analyze situations in which the government leaves positive extraction rents to producers. First, they are empirically relevant. Second, they will be shown to constitute a general case that includes no-commitment as a limiting case. In this subsection, we assume that ρ is given and is not high enough to remove the need for the government to leave producers positive after-tax extraction rents. Later on, we will analyze the choice of ρ and study whether it is desirable for the government to leave positive extraction rents to producers at all.

Ex post, once reserves have been established, producers face a standard Hotelling extraction problem and the government chooses taxes. Furthermore we assume that the government is committed to leaving the producers a Hotelling rent $\widetilde{\eta}_t > 0$, with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, as defined in (6) and (7), for a total rent commitment of $\widetilde{\eta}_0 S_0$. Clearly, given ρ, the level

[22] Clearly, at the equilibrium, for a given subsidy ρ, $\widetilde{\eta}_0$ will depend on the stock of reserves in the same way as in Section 2.

of initial reserves will be determined *ex ante* by that commitment; it will be denoted $\widetilde{S}_0$, with

$$\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho), \tag{25}$$

and discussed further below.

Thus the government chooses optimal taxes on extraction given $\widetilde{\eta}_0$, or, equivalently, given any positive $\widetilde{S}_0$. The problem is thus identical to the problem with exogenous reserves analyzed in Section 2, except that the government is now subject to its *ex ante* rent commitment. The Hamiltonian is thus (17), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$ rather than $\widetilde{\eta}_t = 0$:

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t\widetilde{x}_t) - \mu_t\widetilde{x}_{st}, \tag{26}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are respectively defined by (8), (9), and (10), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. The control variables are the taxes θ_t.

Suppose, as an assumption to be contradicted, that revenue needs are low ($\lambda = 1$); then, according to Proposition 1, conventional goods are not taxed and a tax is imposed on the NRR during the extraction phase to satisfy revenue needs. This reduces the rent accruing to extracting firms and, by (25), reduces the initial amount of reserves relative to the no-tax situation. Consequently, any attempt to satisfy revenue needs by taxing the resource extraction sector results in a distortion, so that, in contradiction with the initial assumption, λ is strictly higher than unity whatever the revenue needs. It follows that the tax on conventional goods is given by (13) with $\lambda > 1$.

Consider the taxation of the NRR sector now, with $\lambda > 1$. In Appendix H, we show that the optimal extraction tax differs from its value when reserves are exogenous, in that it now depends on the rent that the government is committed to as follows:

$$\theta_{st}^* = \frac{1}{\lambda}(\mu - \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}. \tag{27}$$

The second term on the right-hand side of that expression is the familiar inverse elasticity rule; it appears in the same form as in Formula (19) describing the resource tax when reserves are exogenous. As in that case, the tax rate on the resource thus exceeds the tax rate on a conventional good of identical demand elasticity if and only if the first term is non negative. Such is clearly the case with exogenous reserves when the first term on the right-hand side is $\frac{1}{\lambda}\mu e^{rt}$ but perhaps not so with endogenous reserves as the sign of the first term on the right-hand side of (27) depends on the sign of $(\mu - \widetilde{\eta}_0)$. Intuition suggests that the government would not commit *ex ante* to leaving a unit after-tax rent of $\widetilde{\eta}_0$ to firms if this was not at least equal to its *ex post* implicit valuation μ of a reserve unit. One can validate this intuition by analyzing the choice of $\widetilde{\eta}_0$, which we now turn to.

Let us characterize the *ex ante* choice of the rent $\widetilde{\eta}_0$ left to firms after payment of the extraction taxes, for a given level of ρ.[23] The choice of $\widetilde{\eta}_0$ is dual to the choice of reserves $\widetilde{S}_0$ since (25) must hold. The marginal cost of establishing reserves at a level S_0 is $E'(S_0) = \mathcal{S}^{-1}(S_0)$ implying a total cost of reserves $\int_0^{S_0} \mathcal{S}^{-1}(S)\, dS$. This cost should be deducted from the *ex ante* objective of the government which is given by (2) when reserves are exogenous. The objective should also include as benefit the total subsidy payment to producers ρS_0.

The *ex ante* problem of the government is thus

$$\max_{\widetilde{\eta}_0,\ \Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{28}$$

subject to (25) and subject to the tax revenue constraint, adapted to take account of the additional liability associated with the reserve subsidy:

$$\int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{29}$$

Denote by $V^*\left(\widetilde{S}_0, R; \rho\right)$ the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximized under (29) with respect to Θ given $\widetilde{\eta}_0$; because (25) holds, this value function may be defined indifferently as a function of $\widetilde{S}_0$ or $\widetilde{\eta}_0$. Thus, by definition, $V^*\left(\widetilde{S}_0, R; \rho\right)$ is the value function for the *ex post* problem just analyzed, whose Hamiltonian is (26) and which requires that the optimal tax satisfies (27). The constant co-state variable μ in (26) gives the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of reserves, while $-\lambda$ gives the marginal impact $\frac{\partial V^*}{\partial R}$ of a tightening of the budget constraint. Define $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) \equiv V^*\left(\widetilde{S}_0, R; \rho\right)$, making use of the definition $R = R_0 + \rho \widetilde{S}_0$; we have $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} = \frac{\partial V^*}{\partial \widetilde{S}_0} + \rho \frac{\partial V^*}{\partial R} = \mu - \rho\lambda$.

[23] Clearly the subsidy must be low enough to necessitate the presence of after-tax rents at the extraction stage. This will be addressed further below.

Problem (28) can thus be written as that of maximizing $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right)+\rho\widetilde{S}_0-\int_0^{\widetilde{S}_0}\mathcal{S}^{-1}(S)\,dS$ with respect to $\widetilde{S}_0$. The first-order condition is $\frac{\partial\mathcal{V}}{\partial\widetilde{S}_0}+\frac{\partial\left(\rho\widetilde{S}_0-\int_0^{\widetilde{S}_0}\mathcal{S}^{-1}(S)\,dS\right)}{\partial\widetilde{S}_0}=0$, i.e., $\mu-\rho\lambda+\rho-\mathcal{S}^{-1}(\widetilde{S}_0)=0$ so that, using (25),

$$\mu=\lambda\rho+\widetilde{\eta}_0. \tag{30}$$

Indeed, as hinted earlier, the marginal unit value of reserves for the government in its taxation exercise exceeds the private marginal cost $\rho+\widetilde{\eta}_0$ of developing those reserves by a factor reflecting the cost of raising funds ($\lambda>1$) to finance the subsidy payment.

With $\mu-\widetilde{\eta}_0\geq 0$, it thus follows from (27) and (13) that the tax rate on the NRR is higher than the tax rate on a conventional good with the same demand elasticity. Precisely, the unit tax θ^*_{st} on the resource exceeds the common inverse-elasticity term by ρe^{rt}. This component of the unit tax grows at the discount rate so that, alone, it would leave the extraction profile unchanged. In contrast, the component that is common to the resource tax and the tax on the conventional good[24] causes a distortion to the extraction profile; its value is $\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}$, exactly that of a conventional Ramsey tax. This is stated in Proposition 4.

Proposition 4 *(Optimal extraction taxes; endogenous reserves) When the supply of reserves is elastic and is subsidized at the unit rate $\rho\geq 0$ while the supply of conventional goods or services is infinitely elastic,*

1. *The NRR is taxed at a strictly higher rate than a conventional good or service having the same demand elasticity if $\rho>0$; it is taxed at the same rate if $\rho=0$;*
2. *The resource tax rate is given by (31); it is made up of a non-distortionary component complemented by a generally distortionary Ramsey inverse-elasticity component.*

Substituting (30) into (27) implies

$$\frac{\theta^*_{st}}{\widetilde{q}_{st}}=\frac{\rho e^{rt}}{\widetilde{q}_{st}}+\frac{\lambda-1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_s}, \tag{31}$$

[24] As already mentioned, an exception arises when the demand has constant elasticity and the extraction cost is zero (Stiglitz, 1976), in which case the tax has no effect on extraction, given reserves. More on this further below.

where $\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$. This expression identifies the role of the reserve subsidy on the tax rate at any extraction date explicitly. Any parametric change $\Delta\rho$ exactly compensated by a one-to-one change $\Delta\widetilde{\eta}_0 = -\Delta\rho$ and by a change $\Delta\theta^*_{st} = -\Delta\widetilde{\eta}_0 e^{rt}$ not only leaves $\widetilde{q}_{st}$ unchanged but ensures that (31) remains satisfied without any further adjustment. This is because $\widetilde{\eta}_0 + \rho$ is then unchanged so that the new combination of subsidy and after-tax rent commands the same reserves level; as $\widetilde{q}_{st}$ is unchanged it generates the same extraction path; all constraints remain satisfied. In other words the optimum after-tax rent depends on the *ex ante* subsidy: $\widetilde{\eta}_0 = \widetilde{\eta}_0(\rho)$; similarly $\theta^*_{st} = \theta^*_{st}(\rho)$, with $\frac{d\widetilde{\eta}_0(\rho)}{d\rho} = -1$ and $\frac{d\theta^*_{st}(\rho)}{d\rho} = e^{rt}$. However the optimum level of reserves $\widetilde{S}_0$ and the equilibrium price profile are independent of ρ.

This is true within an admissible range for ρ. Indeed the subsidy must not exceed the threshold level above which it would not be necessary for the government to leave firms a rent during the extraction phase. That threshold can be determined as follows. The unit after-tax extraction rent induced by the optimal policy is $\widetilde{\eta}_0(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) - \rho$. Therefore, the condition ensuring that the after-tax rent $\widetilde{\eta}_0$ remains strictly positive is

$$\rho < \bar{\rho} \equiv \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0), \tag{32}$$

where $\widetilde{S}_0$ must satisfy (25), or $\mathcal{S}^{-1}(\widetilde{S}_0) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) = \bar{\rho}$.

Proposition 5 *(Tax-subsidy mix) For $0 \leq \rho \leq \bar{\rho}$, the optimum initial reserve level and the optimum extraction profile are independent of the combination of tax and subsidy by which they are induced.*

An immediate corollary is that subsidies are not necessary to achieve the optimum if the government can commit to extraction taxes that leave sufficient rents to extractors; *vice versa* commitment is not necessary if the government is willing to subsidize sufficiently, at $\rho = \bar{\rho}$. This subsidy level corresponds to the special case of Section 2 taken with initial reserves at $\widetilde{S}_0$. By Proposition 2, the tax is then given by (19) where $\mu = \lambda\bar{\rho}$ according to (30). Thus the observed variety in NRR taxation systems is compatible with optimum Ramsey taxation. To the extent that commitment is not costly, the government is financially indifferent between the proportion of *ex ante* subsidies and *ex post* rents left to extracting firms in order to finance exploration and development expenditures.

4.2 Distortion to Reserve Production under Optimum NRR Taxes

There is another peculiarity in (31). The usual interpretation of Ramsey's inverse elasticity rule is that goods or services whose demand is relatively less elastic should be taxed at a relatively higher rate because this keeps quantities demanded as close as possible to the Pareto optimum, thus balancing the distortions across sectors in the socially least costly way. Here, this interpretation does not apply. As a matter of fact, a NRR tax may leave the extraction path of a given stock of reserves undisturbed. In that case the distortion affects the initial stock of reserves but not their extraction profile over time. In fact, as underlined by Stiglitz (1976) in his analysis of monopoly pricing in the Hotelling model, confronted with the dilemma of raising the price at some date while increasing supply at some other date, a zero-cost monopoly facing an isoelastic demand ends up choosing the same price as would prevail under competitive equilibrium given the same amount of initial reserves. Under the same cost and demand conditions the Ramsey tax will be neutral for the same reason. More generally, even when it affects current extraction, a Ramsey tax cannot be given the standard interpretation in terms of distortion to current extraction. It must simultaneously be appraised in terms of the distortion that it causes to initial reserves.

Given the subsidy, initial reserves are determined by the optimum level of the unit after-tax rent $\widetilde{\eta}_0$ via (25). Since $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ and $\widetilde{q}_{st} = c_s + \widetilde{\eta}_t + \theta^*_{st}$, $\widetilde{\eta}_0$ also affects the optimum tax rate given by (31). However, it is very difficult in general to isolate its effect because there is a continuum of relationships such as (31)—one at each date—and it is their combined influence over the whole extraction period that determines initial reserves. An exception is the special case just discussed. With an isoelastic demand and zero extraction cost, the optimal tax does not cause any distortion to the extraction profile of a given stock of reserves; this provides the ideal laboratory for the analysis of the distortion to initial reserves.

When the tax is neutral at given initial reserves, it grows at the rate of discount, so that the optimal tax can be characterized at any date by its initial level and alternative tax profiles can be compared by comparing initial levels. A higher initial tax level implies a lower after-tax rent to firms which implies lower initial reserves by (25). In the spirit of Ramsey taxation, one would then expect the optimal initial tax to be inversely related to supply elasticity. This is precisely the message of the following expression established in Appendix K for the optimum long-run resource tax rate:

$$\frac{\theta^*_{s0}}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} + \frac{\lambda - 1}{\lambda} \left[\frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{\zeta}} - \frac{1}{\widetilde{\xi}} \right], \tag{33}$$

where $\widetilde{\zeta} \equiv \frac{\widetilde{\eta}_0}{\widetilde{S}_0 \mathcal{S}^{-1\prime}(.)}$ is the *long-term* elasticity of reserve supply measured using (25) at the resource scarcity rent induced by the tax at the beginning of extraction; and where $\widetilde{\xi} \equiv \left(\frac{d\widetilde{\mathcal{D}}}{dq_{s0}}\right) \frac{\widetilde{q}_{s0}}{\widetilde{\mathcal{D}}}$ is the elasticity of the *cumulative* demand for the resource $\widetilde{\mathcal{D}} \equiv \int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt$ with respect to the initial price q_{s0}, measured over the path of equilibrium prices $\{\widetilde{q}_{st}\}_{t\geq 0}$ induced by the optimal tax.

Keeping in mind that, by Proposition 5, the optimal NRR tax adjusts to changes in the reserve subsidy in such a way that optimal initial reserves are the same for any admissible value of ρ, let us again assume that $\rho = 0$. Then (33) looks similar to the well-known expression for the optimum rate of tax that applies to conventional goods whose supply is not perfectly elastic.[25] Its interpretation is also standard: Tax more when elasticity is lower, whether the source of elasticity is on the supply or the demand side. Hence, to the extent that the supply of conventional commodities is more elastic than the supply of reserves $\left(\widetilde{\epsilon}_i > \widetilde{\zeta}\right)$, (33) implies that the resource is taxed at a higher rate than commodities of identical demand elasticity. There is an important difference between the NRR and conventional goods or services though, having to do with the notions of elasticities involved.

Indeed, in (33), the supply elasticity $\widetilde{\zeta}$ measures the long-run adjustment of the stock of initial reserves relative to the percentage change in the unit producer rent. This stock elasticity depends on how sensitive exploration is to the rent. In usual formulas of the inverse elasticity rule applying to commodities whose supply is not perfectly elastic, the concept of supply elasticity is standard; it measures the instantaneous percentage change in production (a flow) relative to the percentage change in the unit producer price.[26]

[25]This expression involves the sum of the reciprocals of demand and supply elasticities. See for instance Expression (11) in Ramsey (1927); for a formula derived under our notations, see (F.3) in Appendix F.

[26]If the supply elasticity of a conventional good is finite, it must be the case that some input, e.g., the stock of capital, does not fully adjust to price and tax changes, which implies decreasing returns to scale. For a NRR the increasing scarcity of exploration prospects makes decreasing returns unavoidable in the long run.

Similarly, while the elasticity of demand is the standard flow notion in (13), its counterpart in (33) is defined as the elasticity of cumulative resource demand–over the whole extraction period—with respect to the initial resource price. In the current special case, the long-run elasticity of cumulative demand is the same as the standard flow demand elasticity: $\widetilde{\xi} = \widetilde{\varepsilon}_s$.

The results are gathered in the following proposition.

Proposition 6 *(Time profile and initial reserves) When the supply of reserves is elastic and is subsidized at the unit rate $\rho \geq 0$,*

1. *The Ramsey tax profile described by (31) implies distortions in both the time profile of extraction and the level of initial reserves;*
2. *When the demand for the NRR is isoelastic and the extraction cost is zero, the optimal extraction tax is neutral with respect to the time profile of extraction and only affects the level of initial reserves. In that case, the optimal tax rate is given by (33), a rule resembling that for conventional goods and services whose supply is not perfectly elastic.*

When reserves are endogenous, distortions are unavoidable whatever the revenue needs of the government; less reserves are to be developed. Again, if Pigovian policies were in place that penalized the carbon-emitting use of NRRs, the Ramsey tax would go in the same direction as, and would reinforce, the Pigovian tax.

The analogy underlined in Section 2 between Ramsey taxation and monopoly pricing when reserves are exogenous is thus even more pronounced when reserves are endogenous. Whether one considers the tax on the production flow of conventional goods or the extraction flow of a NRR as in (13) and (31), or the long-run formula (33), the optimal tax rate approaches a monopoly mark-up as the factor $\frac{\lambda-1}{\lambda}$ approaches unity, i.e., when the government's revenue needs are at their highest.

5 A Numerical Example: OCT of Fossil Oil

Whether reserves are taken as given or are endogenous, the supply of a NRR is never infinite. As we just demonstrated, when reserves are endogenous, OCT affects both the level of discovered reserves and the flow of extraction that they constrain. We have illustrated how the distortion on the latter vanishes when the demand is isoelastic and the unit extraction cost is zero. In general, however, the tax described by Formula (31) causes both distortions.

The numerical example that we provide now for the case of fossil oil not only illustrates the yield of the tax, but the distortions that it implies on extraction and reserve development. The application follows the steps explained in Section 4 to resolve the OCT problem. Formula (31) gives the optimum NRR tax as a function of the gross resource price

$$\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}, \tag{34}$$

where the unit present-value rent $\widetilde{\eta}_0$ solves the *ex ante* problem (28)-(29). By Proposition 5, the distortions in extraction over time and in the level of reserves discovered are independent of the repartition of the taxes or subsidies between the exploration phase and the extraction period. Thus, we can use Formula (31) without loss of generality while assuming that $\rho = 0$. Under that assumption, tax revenues and subsidies are confined to the extraction phase so that (31) accounts for the full extent of Ramsey taxation applied to fossil oil over both the exploration phase and extraction.[27]

The solution is obtained in two steps: First solve the system that consists of equations (31) and (34) for the trajectories of θ^*_{st} and $\widetilde{q}_{st}$ while treating $\widetilde{\eta}_0$ as parametric. Then, use the level of $\widetilde{S}_0$ from (25) and the trajectory of $\widetilde{x}_t$ implied by $\widetilde{q}_{st}$ via the demand function to establish the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximized with respect to Θ given $\widetilde{\eta}_0$ in the *ex ante* problem (28). Finish the maximization by choosing $\widetilde{\eta}_0$.

Such an application implies a further simplification: treating the unit cost λ of levying one dollar of tax revenues as exogenous, rather than endogenous as in the full analysis of Section 4. This allows our application to focus on the resource sector; indeed, λ summarizes the extent to which the resource sector must contribute to public finance needs, taking into

[27] The analysis has been confined to a closed economy. However the highlighted principles remain valid in an open economy as shown in Appendix M and as explained in the concluding discussion.

account the contribution of the other sectors.

We specify the model as follows. Values are in US dollars ($) of 2015. The short-run extraction cost c_s is assumed to be $35 per barrel of oil; this is the total cost—including development cost—of producing the first barrel developed (e.g., van der Ploeg and Rezai, 2017). Resource costs rise in the long run because the marginal cost of developing additional reserves increases: We assume a reserve supply function $\mathcal{S}(.)$ of constant elasticity $\zeta = 0.5$. Similarly, we assume an isoelastic demand function $D_s(.)$ with $\varepsilon_s = -0.5$.[28] Both elasticities are chosen to be relatively high in absolute values so that the Ramsey resource tax tends to be underestimated—compare with the long-run estimates for the demand and reserve elasticities suggested by Krichene (2005), and Hamilton (2009b). We use a middle of the road value of 3 % for the discount rate (Nordhaus, 2014). As for the social cost of one dollar of public funds, values of $1.1 to $1.2 are considered sensibly low, for developed as well as for developing countries (e.g., Dahlby, 2008, and Auriol and Warlters, 2012): We consider $\lambda = 1$, which represents the baseline case in which no distortionary taxes are needed, as well as $\lambda = 1.1$, and $\lambda = 1.2$.

Given these parameter values, the computations are carried out yearly over a period of one hundred years, and the calibration is made by choosing the shift terms K_D and K_S of the demand and the reserve supply functions. Therefore, the demand for fossil oil is $\widetilde{x}_{st} = K_D \widetilde{q}_{st}^{\,-0.5}$ and the supply of reserves is $S_0 = K_S {\eta_0}^{0.5}$. By choosing $K_D = 260$ and $K_S = 500$, we obtain that the world extraction rate and price are respectively slightly below 35 billion barrels (BB) and slightly below $57 a barrel in 2015 when $\lambda = 1$, i.e., in the absence of any Ramsey taxation. The corresponding cumulative extraction of 2327 BB over one hundred years exceeds the currently-proven world oil reserves of 1662 BB by about 40%.[29]

In Table 1, Column $\lambda = 1$ represents business as usual, as no Ramsey tax is then necessary. The 2015 computed extraction level and producer price (gross of other taxes) approximately match observed values in 2015, while the total reserves to be exploited are

[28] Under these specifications, the finiteness of the time horizon—inherent in the simulation exercise—allows to eliminate some arbitrarily small strictly positive extraction levels from the integration of the consumer surplus. This avoids that the objective in (28) take an infinite value.

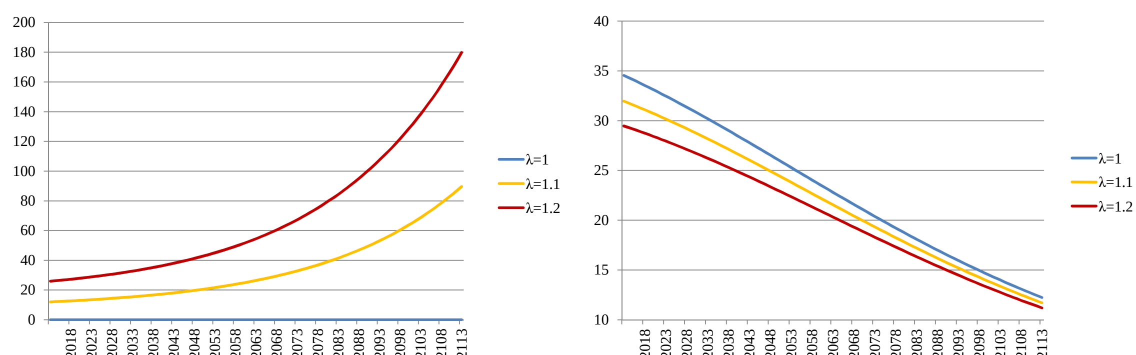
[29] This might be considered conservative given that the fear to run out of reserves has been proven wrong in the past repeatedly. However, mean estimations of undiscovered reserves by the US Geological Survey (Schenk, 2012) were 565 BB in 2012, i.e., 38 % of proven reserves that year.

sensibly higher than official proven reserves. When the cost of $1 of government funds is $\lambda = 1.1$, the Ramsey tax is set at $12 in 2015 so that the producer price is driven to $54 instead of $57 causing extraction to be reduced from 35 BB to 32 BB that year. The corresponding tax yield is $385 billion. If the US were to collect a share of that yield corresponding to its 20% share of world oil consumption, this would correspond to roughly 17% of its $440 billion 2015 Federal deficit. At the world level, cumulative discounted tax revenues over the horizon of 100 years would amount to about $16,000 billion.

Table 1: 2015 OCT of fossil oil

Cost of $1 of tax revenues	$\lambda = 1$	$\lambda = 1.1$	$\lambda = 1.2$
2015 Unit optimal tax ($)	0	12	26
2015 Extraction rate (BB)	35	32	29
2015 Producer price ($)	57	54	52
2015 Tax yield ($ billion)	0	385	765
Total exploited reserves (BB)	2327	2189	2055
Total cumulative discounted tax yield ($ billion)	0	16,002	31,352

Table 1 gives a static reading of the magnitude and impact of the Ramsey oil tax on 2015 variables for alternative levels of λ. It also indicates effects on total exploited reserves and cumulative tax yield. Dynamic aspects are presented in the two graphs of Figure 1, respectively giving trajectories of the Ramsey oil tax, and the corresponding extraction paths.



(a) OCT unit tax of fossil oil

(b) OCT-induced oil extraction

Figure 1: Dynamics of fossil oil OCT

In Figure 1, the extraction trajectories implied by the OCT of the fossil oil resource (right-hand graph) become both flatter and lower as λ increases. Since they do not cross, that also means that exploited reserves become lower with λ, as can be verified in Table 1: Compared with their level of 2327 BB in the absence of Ramsey taxation, initial reserves diminish to 2189 BB; for a more financially constrained government ($\lambda = 1.2$), developed and exploited reserves further diminish to 2055 BB. This illustrates an important result established in Section 4: The need of collecting tax revenues commands the slower extraction of lower reserves.

Since the climate impact of using carbon resources increases both with the total amount of reserves to be exploited and with the speed at which those reserves are consumed, the Ramsey taxation objective of collecting public funds also serves the objective of fighting climate change. As mentioned in the introduction, the Ramsey tax may be imposed on top of other taxes. In the concluding section, we show how the presence of a carbon tax as in Nordhaus (2014) modifies the Ramsey tax and the industry.

6 Conclusion and Final Remarks

The standard Ramsey-Pigou framework used in this paper considers indirect, linear taxes or subsidies on any commodity or service. This includes linear subsidies to the production of natural resource reserves (exploration) as well as linear taxes on extraction and on consumption of the natural resource. In that framework, the objective of the government is to maximize the welfare of producers and consumers while securing a given level of revenues for the production of public goods. The need to secure revenues confers a profit-maximizing dimension to government taxation decisions. Optimum taxes distort consumer prices away from the Pareto optimum towards the monopoly price. This means that results from the NRR monopoly literature are relevant to Ramsey taxation and implies that Ramsey taxation also helps the objective of preventing climate change as the numerical example of Section 5 indicates.

When initial reserves are exogenous, the NRR must be taxed in priority, however elastic the demands for the conventional goods and for the non-renewable resource. Precisely, the resource should be the sole taxed commodity unless the required tax revenue exceeds the totality of the rents that would be generated by the untaxed resource. When the required tax revenue is higher than the maximum that can be generated by neutral resource taxation, conventional producible goods and services should contribute to government revenues, but the resource should be taxed at a higher rate than conventional producible goods having identical elasticities according to a time-dependent tax formula.

When the supply of initial reserves is elastic and determined by the combination of after-tax rents to extraction and *ex ante* subsidies to reserve production, all sectors should be taxed simultaneously whatever the tax revenue needs of the government. In the absence of any subsidies to the production of reserves and provided the government can commit to leaving after-tax rents to firms, the optimum tax rate on resource extraction is determined according to the inverse elasticity rule applying to any conventional good whose supply elasticity is infinite, yet time-dependent despite stationary conditions. Formal similarities in that case hide another crucial difference: Due to the dynamic nature of the extraction problem, a similar rule must hold at all dates during the extraction period. As a result, the distortion to extraction cannot be measured simply according to the tax applying at any particular date, however determined, but also depends on the tax applied at all other dates. If the demand for the non-renewable resource is isoelastic and the marginal extraction cost is zero, this goes as far as implying that the optimal tax, although set according to a standard inverse elasticity rule, does not cause any distortion to the extraction path, given initial reserves. The distortion imposed on the industry then materializes at the level of reserve production rather than the extraction profile. It can be expressed by the standard inverse elasticity rule applying to elastically supplied conventional goods and services, where the elasticities are the long-run notions defined in the paper: Both the supply and demand elasticities relevant to a NRR are elasticities of a stock in response to an after-tax asset price, rather than the flow elasticities encountered in usual Ramsey formulas.

Another noticeable result arising with endogenous reserves is that, although the optimal extraction tax varies according to the reserve subsidy, the optimal amount of initial reserves and the optimal extraction path of these reserves, do not depend on the extraction-tax reserve-subsidy combination. As a result, all the tax-induced distortions just described when subsidies are absent are insensitive to the tax-subsidy combination adopted by the government. In particular, a government that were unable to commit to leaving positive after-tax rents to firms during the extraction period, could finance reserve production by subsidies exclusively and achieve the same objective as a government that were able to commit. Similarly, a government that could not devote subsidies to reserve production could give the same incentives by committing to limit extraction taxes appropriately. As a result, Ramsey taxation is compatible with institutional forms ranging from a nationalized industry, where the entire reserve production effort is subsidized while the total surplus from extraction is taxed away, to a system where firms finance reserve production and are paid back by future extraction rents.

There is little doubt that most governments are financially constrained, often severely. The figures given in our numerical example indicate that this reality has significant implications for the taxation of NRR. They also indicate that Ramsey taxation of NRR would have significant implications for government finances.

The rest of this concluding section connects the above findings with major results in three related literatures. The first one is the literature on the taxation of capital income. The second one is the literature on the taxation of carbon resource. The third one deals with the capture of foreign NRR rents.

6.1 Ramsey Extraction Taxes as a Capital Income Tax

Natural resource reserves are a form of capital while discoveries and extraction are forms of positive and negative investments. While Ramsey taxation rules out the direct taxation of capital and profits, linear indirect commodity taxes considered in this paper have the ability to tax natural resource rents. We found that resource rents should be taxed prior to introducing distortionary commodity taxes when the initial amount of reserves is exogenous, as anticipated by Stiglitz and Dasgupta (1971). When reserves are produced endogenously and resource rents include quasi-rents, the situation is close to that analyzed by Judd (1985) and Chamley (1986) in that the question whether capital should be taxed in the long run arises in a similar fashion. Chamley identified two aspects of capital revenue taxation. In the short run, capital is rigid; this makes it an attractive target for taxation if the objective is to obtain revenues while avoiding distortions. However, when capital is not rigid, its constitution relies on investment, and investment becomes less profitable the more capital is taxed. In the long run, Chamley finds that the latter effect becomes dominant and that the revenue from capital should not be taxed at all if the horizon of the government is long enough. The famous Chamley-Judd result obeys the standard OCT logic: The social cost of capital taxation over the long run is so high that it is impossible to evenly spread distortions across sectors while imposing a positive capital tax.

Our OCT analysis, however, yields a very different result when capital is a NRR rather than Chamley's conventional capital. As *per* Proposition 4, the natural resource should be taxed whatever the horizon of the government, despite the fact that the supply of reserves is responsive to the tax.

The reason is resource scarcity. While Chamley's capital can be constituted—by production and investment—without limit under constant returns to scale, reserves are endogenously produced—by exploration—under conditions of decreasing returns because exploration prospects are not unlimited. Unlike Chamley's capital, the supply of a natural NRR is not infinitely elastic in the very long run. It follows that the distortion associated with the taxation of NRR capital is finite and can be weighted against other distortions. In that sense, the intuition is comparable with other results contradicting Chamley. For instance, Piketty and Saez (2013) departed from Chamley's treatment by introducing inter-generation inheritance in the capital accumulation process, in the spirit of Cremer, Pestieau, and Rochet (2003). This modification limits the elasticity of capital supply as in our framework, and yields a similar conclusion. When the supply of a capital is not infinitely elastic, be it a resource or not, capital should be taxed in the long run.

Besides the supply limitation to investment in new capital, our analysis of NRRs differs from Chamley (1986) by two aspects. First, the use of NRRs is consumptive whereas Chamley's capital is perfectly durable. Second, and relatedly, NRRs are often physically contained in the final good consumed—which justified not distinguishing between the NRR and the final good—whereas physical capital provides a service that is an intermediate input in the production process. These two aspects explain the difference between our formulas and the formulas in Chamley (1986). The systematic comparison of the optimal tax treatment that various resource and non-resource capital goods should receive goes beyond the present paper; this task is left for future research.

6.2 Ramsey-Augmented Taxation of Carbon-Emitting NRRs

NRRs are sources of carbon emissions and should be subject to Pigovian taxation to correct the climate externality that they generate. Whatever the actual Pigovian tax on a NRR—whether it is appropriate or not—our results can directly be used to indicate the OCT tax that should augment it in presence of tax revenue needs.

Introduce such a carbon tax in the analysis of Section 4. Assume, for simplicity, that it is set at each date t at an exogenous level τ_{st} per unit of NRR consumed. In this case, it is straightforward to see that Formula (31) is unchanged, although it now applies to the higher gross resource price

$$\widetilde{q}_{st} = c_s + \tau_{st} + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}, \tag{35}$$

which now includes the carbon tax, unlike (34) in Section 4 and in the application of Section 5. The relevant resource price $\widetilde{q}_{st}$ being adjusted in that way, the obtained level of the OCT tax θ^*_{st} establishes by how much the carbon tax τ_{st} should be augmented, without any reinterpretation of the model.

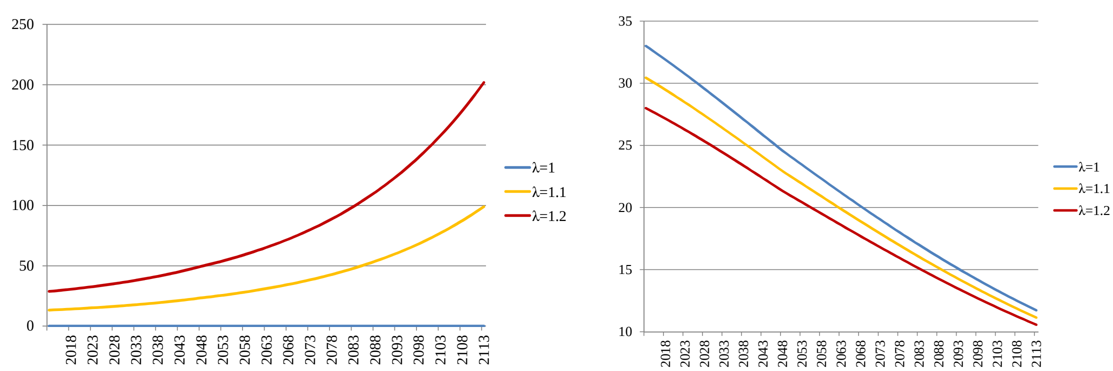
This simple application is consistent with the famous "additionality property," highlighted by Sandmo (1975) in his analysis of OCT with explicit externality-generating commodities. In Sandmo's paper, the OCT and externality taxation problems are resolved simultaneously, unlike the above simplifying assumption that the Pigovian tax component is imposed at the outset while the OCT component endogenously adjusts to it. In general, Sandmo (1975) shows—and Kopczuk (2003) confirms—that in the presence of public-revenue requirements, Pigovian taxes should simply be augmented by a component that corresponds to the Ramsey tax. This result has an important and convenient implication: Establishing by how much carbon taxation should be augmented in presence of revenue needs does not require treating the externality issue explicitly.

Therefore, one can conclude from our previous results that the Ramsey resource tax causes a distortion to the extraction of carbon NRRs that goes further than the Pigovian tax in the direction prescribed for the resolution of carbon externalities: The reserves are lower and so is the speed at which they are exploited.

In the numerical example of Section 5, assume that the Ramsey tax is imposed on top of a carbon tax. The carbon tax is taken from Nordhaus (2014), starting in 2015 at about \$22 per ton of CO2 equivalent or about \$8 per barrel, rising in real terms at an annual rate of 3.1% until 2050, and rising at 2.1% thereafter. Table 2 and Figure 2 show how the results carry over in that case. For $\lambda = 1.1$, the Ramsey tax is set at \$13 and the induced extraction rate is 30 BB, lower than if the Ramsey tax was alone. As a result, its yield is lower than if there was no carbon tax. The yield of the carbon tax is also lower than in the absence of a Ramsey tax, and the more so the higher λ. Nevertheless, the joint yield of the two taxes is higher than if either of them was alone. Discoveries are also lower than if either of the two taxes were present in isolation. Clearly, both contribute to the objectives of increasing revenue and protecting the climate.

Table 2: 2015 OCT of fossil oil on top of a carbon tax

Cost of \$1 of tax revenues	$\lambda = 1$	$\lambda = 1.1$	$\lambda = 1.2$
2015 Unit carbon tax (\$)	8	8	8
2015 Unit optimal tax (\$)	0	13	29
2015 Extraction rate (BB)	33	30	28
2015 Producer price (\$)	54	52	50
2015 Tax yield from carbon tax (\$ billion)	264	243	223
2015 Tax yield from Ramsey tax (\$ billion)	0	404	805
Total exploited reserves (BB)	2185	2043	1905



(a) OCT unit tax of fossil oil

(b) OCT-induced oil extraction

Figure 2: Dynamics of fossil oil OCT on top of a carbon tax

To sum up, public financial hardness does not need to obscure or delay environmental decisions; on the contrary, it calls for policies that go even further than correcting externalities.

6.3 Ramsey Taxes and the Capture of Foreign Resource Rents

More often than not, Ramsey's commodity taxes are applied domestically to an open economy. This has two immediate implications for OCT. First, since domestic supply does not need to meet domestic demand, taxes applied on demand and supply are not equivalent; consequently distinct optimum taxes must be chosen for the domestic supply and the domestic demand of each traded good or service, rather than one tax applying indifferently to demand or supply. Second, prices are formed on international markets, so that the effects of commodity taxes on prices are diluted.[30] The imperfect elasticity of world NRR supply makes these effects all the more relevant.

It is well known that the combination of domestic demand and supply taxes plays the role of a tariff (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). When they are susceptible of affecting international prices, commodity taxes are then capable of pursuing both a tax revenue objective and a rent capture objective as optimum tariffs do.[31] While the rent capture objective has been addressed in a rich literature on optimum tariffs, it has resonated especially loudly in the NRR taxation literature with the famous paper by Bersgtrom (1982) on the capture of foreign NRR rents.

In Appendix M, we extend Section 4's analysis of OCT with endogenous NRR reserves to the case of an open economy. We formally derive in that case the open-economy counterparts of the tax formulas (31) and (33), and we reexamine the optimal distortions to the NRR extraction and to reserve supply.

The main results are the following. Because of the ability of the Ramsey commodity taxes to pursue both a minimum revenue objective and the objective to capture foreign rents by manipulating prices, the distinction between low and high revenue needs emerges in an open economy with endogenous reserves as it does in a closed economy with exogenous reserves (Section 2). Revenue needs are low or high according to whether government needs are covered or not by the amount raised when NRR taxes are set so as to maximize welfare in absence of tax-revenue constraint, that is with the sole purpose of capturing foreign NRR rents.

When revenue needs are low in the sense defined above, our open-economy OCT rule

[30]The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982). On market power on the demand side, see also Liski and Montero (2011).

[31]The OCT problem and the optimum-tariff problem differ only by the constraint for the OCT government to collect a minimum revenue. The latter characterizes an unconstrained optimum while optimum commodity taxes are distortionary: As Boadway et al. (1973) put it "domestic commodity taxes *introduce* a distortion while optimum tariffs *eliminate* a distortion" (p. 397, their italics).

for NRR extends Bergstrom's (1982) analysis to the case of endogenous reserves but does not modify his message that importing countries should tax NRR consumption and that exporting countries should subsidize NRR consumption. As he showed for fixed reserves, the two-country Nash equilibrium is then such that importing countries capture some foreign NRR rents while exporting countries limit the cut to their rents by subsidizing.

When revenue needs are high, the revenue collection constraint becomes active, and the Bergstromian rent capture component is complemented by the two components described in the inverse elasticity formula (33) for the closed economy with endogenous reserves.[32] For an importing country, the result that the NRR should be taxed at a higher rate than commodities of identical demand elasticity comes reinforced because the three terms of the open-economy inverse elasticity formula push in the same direction. In the case of exporters, the Bergstromian component is negative and counteracts the traditional Ramsey components; exporting countries may even subsidize resource consumption despite a pressing fiscal-revenue constraint.

[32]The definitions of the demand and supply elasticities must be adjusted to reflect the fact that there is a home and a foreign sector.

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Online Appendix to

"Optimum Commodity Taxation with a Non-Renewable Resource:"

Literature, Proofs and Extensions

October, 2017

A More on the Related Literature

A.1 Direct Rent Taxation, and its Limitations, Especially in NRR Sectors

Ramsey's original approach rules out the direct taxation of profits, rents, or incomes, leaving it open to the criticism that it ignores the possibility of neutral taxation. Nevertheless, as Sandmo pointed out in 1976 ". . . it seems definitely sensible to admit the unrealism of the assumption that the public sector can raise all its revenue from neutral . . . taxes, and once we admit this we face the second-best problem of making the best of a necessarily distortionary tax system. This is the problem with which the optimal tax literature is mainly concerned." This view was shared by Stiglitz and Dasgupta (1971), who pointed out that "no government imposes 100 per cent taxes on profits and the income of fixed factors, in spite of the desirability of such non-distortionary taxes."[33] Their remark applies more definitely nowadays, where most countries are struggling to meet debt and budget constraints in institutional environments where profits or rents may be only partially taxed but where no increase in the amount raised by profit taxes is considered feasible. According to a recent Ernst and Young's (2015) report, one major feature of the world tax landscape is that "Indirect taxes continue to grow while direct taxes stagnate." In this paper, whatever the amount of profits or rents being taxed away, we assume that governments do not have enough control on direct profit or rent taxes to use them as instruments.

This limitation to the ability to use direct taxes is very apparent in NRR sectors. On the demand side, resource taxes are almost exclusively linear commodity taxes. On the supply side, direct non-linear taxes such as the resource rent tax have been advocated as non distortionary—see, e.g., Boadway and Flatters (1993), and Boadway and Keen (2010). However, they are contested on theoretical grounds—e.g., Gaudet and Lasserre (1986), and Garnaut (2010)—and meet strong opposition in practice for reasons ranging from institutions (property rights) to feasibility (information and agency issues, e.g., Boadway and Keen, 2014). In any case royalties and other linear commodity taxes are dominant forms of resource taxation—see Daniel et al. (2010)—and are the apparently most significant

[33] According to Stiglitz and Dasgupta (1971), "two possible explanations for this limitation suggest themselves. (1) It is difficult if not impossible . . . to separate out pure profits from, say, income to capital, and few if any governments—or national income accountants—have even attempted the task. (2) In at least some western economies, where the rights of private property are considered to be very important, a 100 per cent profits tax would be considered equivalent to nationalization of the fixed factors."

tax instruments available to governments nowadays.[34]

A.2 OCT Analysis in Presence of Untaxed Profits

While Diamond and Mirrlees (1971) showed that Ramsey's results held in a fully specified general equilibrium framework, OCT was dealt a serious theoretical blow by Atkinson and Stiglitz (1976) who showed that, under some conditions, direct income taxation suppresses any need for commodity taxes. The Atkinson-Stiglitz result does not hold unless profits are fully taxed or are absent because of constant returns to scale.[35] While constant returns to scale may be considered a useful simplification for economies limited to conventional goods, doing away with that assumption and with the assumption of 100 per cent profit taxation is an empirical and theoretical necessity in the presence of NRRs:[36,37] Decreasing returns and the presence of untaxed rents are fundamental characteristics of NRR sectors. In this context, as Stiglitz (2015) reminded us, commodity taxes have the ability to indirectly tap profits and rents when they would otherwise be left untaxed. Therefore, while we treat conventional sectors in line with much of the literature by assuming constant returns to scale, hence no profits, we make the rents arising in the NRR sector explicit. These rents arise from the total or partial inelasticity of reserves. As is well known, when reserves are given, commodity taxes have the ability to tax resource rents in a neutral way. Therefore, the optimization of NRR commodity taxes in the model of this paper combines the proceeds of neutral rent taxation with the proceeds of possibly distortionary NRR taxation into government tax revenues.

Ramsey's framework seems perfectly adapted to examine the fact that NRRs usually

[34] For a good practical example of a relatively advanced system, see Alberta Royalty Review (2007, pp. 54-60).

[35] As a matter of fact, Corlett and Hague showed as early as 1953 that uniform commodity taxation may be optimal under specific conditions. As the Atkinson-Stiglitz theorem, this case of uniform OCT relies on the assumption that returns to scale are constant. Under otherwise similar conditions, decreasing returns to scale and the incomplete taxation of the resulting profits would justify differential commodity taxes along the lines explained by Ramsey (1927). Indeed, as recalled by Stiglitz (2015, p. 237), Corlett and Hague's analysis is a "a special application of Ramsey's analysis." For a derivation of Ramsey's and Corlett and Hague's results in the same framework of analysis, see Ley's (1992) note.

[36] This includes, *a fortiori*, Hotelling models when reserves are allowed to be endogenous as in this paper.

[37] Another parallel issue is taxes that interfere with productive efficiency such as taxes on intermediate inputs. In the case of NRR-based final goods or services, this often occurs through linear technologies, that is in a given proportion of the final consumption. In such a situation, a linear tax on a NRR input does not compromise efficiency and does not need to be distinguished from a linear tax on the NRR-based final good or service; that distinction was avoided in the original partial equilibrium formulation of the OCT problem, an option further validated by Baumol and Bradford (1970). Stiglitz and Dasgupta (1971) further showed that production efficiency is not in general required in presence of untaxed profits or rents.

receive a special commodity tax treatment. That is, to the extent that Ramsey taxes can be set at different levels according to the commodity involved.[38] In our paper, a crucial assumption is that NRRs can be (linearly) taxed independently of other commodities. This assumption is satisfied by NRR tax regimes; whether they are applied to the demand or supply sides, they are largely independent of the commodity taxes applied to conventional goods or clusters of conventional goods.

A.3 Taxation of Rents and Capitals

The renewed interest in the profit-capturing dimension of commodity taxes is pervasive in the double dividend literature and in the carbon taxation literature. Barrage (2014, p. 50) shows that absent 100 per cent profits taxes, the optimal tax on carbon resources acquires a Ramsey component. The recent literature on capital taxation—e.g., Piketty (2015)—sees commodity taxation as an alternative to the direct taxation of wealth. As Auerbach and Hassett (2015) put it, consumption taxation has "the ability . . . to hit existing sources of wealth." In our paper, a similar logic applies; the dynamic formulation highlights the role of NRR taxes as taxes on NRR capital and rents.

As a matter of fact, Ramsey's framework explicitly rules out the direct taxation of capital income, whether in the form considered by Chamley (1986), or in a form mimicking profit taxation as with Lucas' (1990) capital levies, or via some form of resource rent taxes as described by Boadway and Keen (2010). However, a NRR is a form of capital and applying a commodity tax to NRR extraction over time is not unlike taxing the income of that capital. We find that taxing NRR extraction is optimal, in apparent contradiction with Chamley who shows that no tax should be applied on the income of capital in the long run. We explain why these results differ despite their similar OCT logic. The elasticity of supply again plays the central role, although in a way completely distinct from that in Piketty and Saez (2013).

A.4 Taxation of Carbon NRRs and their Substitutes

Given recent governmental commitments to penalize CO2 emissions generated by the use of fossil NRRs, a currently important application of our research concerns carbon taxation.

[38]Ramsey himself had worried about possible restrictions to the set of linear taxes that can be imposed on various commodities; the extension of Ramsey's original work with limited groups of commodities was recently undertaken by Belan et al. (2008).

Our formulas can directly be used to calculate by how much carbon taxation should be augmented when the regulator is budget constrained, as well as to qualify the distortion required by public revenue needs.

This is in line with Sandmo's (1975) analysis of commodity taxation in presence of externality-generating goods which highlights the famous "additionality property:" In the presence of public-revenue requirements, Pigovian taxes should simply be augmented by a component that corresponds to the Ramsey tax. Recent applications of this property include, for example, Sandmo (2011), and d'Autume, Schubert and Withagen (2016). The robustness of Sandmo's result has been confirmed by Kopczuk (2003). It implies that establishing by how much carbon taxation should be augmented in presence of revenue needs does not require treating the externality issue explicitly.

We show that public policies facing financial constraints should go further in the direction prescribed for the resolution of carbon externalities (Withagen, 1994) than in the absence of these constraints: Less reserves should be developed and they should be depleted less rapidly. As far as the treatment of non-carbon substitutes to fossil NRRs is concerned, our results importantly differ from Sandmo (1975). He found that public-revenue needs did not warrant a special treatment of substitutes to externality-generating goods, which should be taxed solely according to Ramsey's rule. In contrast, we find that they should receive a favorable tax treatment, not because they are substitutes to carbon-containing goods, but because they are substitutes to NRRs.

A.5 Capture of Foreign Rents by an Open Economy

A NRR importer cannot apply any form of direct resource rent taxation to foreign suppliers; in that sense, Ramsey's assumption that direct taxation is not possible applies to the foreign suppliers of an importing country implacably. However that country can apply commodity taxes to home consumption as substitute for the taxation of foreign resource rents. Since the capture of foreign rents involves the exercise of market power, the OCT problem for a NRR importer connects with the famous result of Bergstrom (1982) on rent capture. Here again, in an extension presented in Appendix M and discussed in Section 6, we find that the OCT tax rate in a NRR importing country is higher than the rate on conventional commodities having the same demand elasticity.

B The Hotelling Rent and the Neutral Tax

A Hotelling resource is a homogenous non-renewable natural asset, such as an oil deposit. As an asset it should provide the same return as any traded asset if it is to be detained. Since a unit of oil underground does not provide any return other than the value realized upon extraction, its return consists of capital gains over time. If oil was traded underground, absent any uncertainty, non arbitrage would thus require its current price to rise at the risk-free rate of interest. The value of such a non-traded asset is known as Hotelling rent and the non-arbitrage rule that it should satisfy is known as Hotelling's rule (Hotelling, 1931; Dasgupta and Heal, 1979, pp. 153-156; Gaudet, 2007).

This appendix defines the Hotelling rent with tax $\widetilde{\eta}_0$ and the Hotelling rent without tax $\overline{\eta}_0$ in competitive equilibrium. In competitive equilibrium with linear taxation, Hotelling's current-value unit rent to producers equals producer price minus marginal cost. At time zero, with constant unit extraction cost, this is $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta_{s0} - c_s$. By Hotelling's rule the rent is constant in present value so that, at any date, its present value is $\widetilde{\eta}_0$; it can be computed as follows.

If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource will be depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$: $\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0$, with $\widetilde{q}_{st} - \theta_{st} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\left(\widetilde{T}-t\right)}$. At time zero, the rent is thus $\widetilde{\eta}_0\left(S_0\right) = \widetilde{q}_{s0} - \theta_{s0} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\widetilde{T}}$. If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define the present-value rent $\widetilde{\eta}_0\left(S_0\right)$ implicitly: $\lim_{T\to+\infty} \int_0^T D_s(\widetilde{\eta}_t + \theta_{st} + c_s)dt = S_0$, where $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. It can be shown that $\widetilde{\eta}_0$ is a positive and decreasing function of S_0.

The maximum value that can be raised from the mine by non-distortionary taxation is its discounted cumulative rent under competitive extraction and in the absence of taxation. That is $\overline{\eta}_0\left(S_0\right) = \widetilde{\eta}_0\left(S_0\right)$, where $\widetilde{\eta}_0$ is computed as above for the values of $\widetilde{q}_{st}$ implied by $\theta_{st} = 0$, $\forall\, t$. The present value of the mine in the absence of tax is thus $\overline{\eta}_0\left(S_0\right) S_0$.

If taxes are neutral, $\theta_{st} = \theta_{s0}e^{rt}$ and part of the unit scarcity rent is captured. The present value of the net-of-tax unit rent earned by the owner of the mine is thus $\widetilde{\eta}_0\left(S_0\right) = \overline{\eta}_0\left(S_0\right) - \theta_{s0}$ and the after-tax present value of the mine is $\widetilde{\eta}_0\left(S_0\right) S_0$.

C Proof of Proposition 1

1. We have shown in the main text that $\lambda = 1$ implies $\theta_i^* = 0$, $i = 1, ..., n$, and $\theta_{st}^* = \theta_{s0}^* e^{rt}$, so that the totality of tax revenues is raised from the resource sector. Moreover, we have argued that, if $\lambda = 1$, it must be the case that $R_0 \leq \overline{\eta}_0 S_0$. The contrapositive of that statement is that if $R_0 > \overline{\eta}_0 S_0$, then $\lambda > 1$. In that case, we have shown in the main text that $\theta_i^* > 0$, $i = 1, ..., n$, and that θ_{st}^* must be set in such a way as to raise more than $\overline{\eta}_0 S_0$ from the resource sector.

There remains to show that $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$. Assume $R_0 \leq \overline{\eta}_0 S_0$ and $\lambda > 1$. Then taxes on conventional goods θ_i^*, $i = 1, ..., n$, raise a strictly positive revenue, causing distortions. Since it is possible to generate $\overline{\eta}_0 S_0 \geq R_0$ without imposing any distortions by taxing the natural resource, this cannot be optimal. Hence, $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$.

2. Shown in the main text, once it is observed that the tax formula given by (15) is independent of demand elasticities. ■

D Proof of Proposition 2

1. As shown in the main text, when $\lambda > 1$, the optimum tax rate on conventional good $i = 1, ..., n$ is θ_{it}^* as given in (13) and depends on λ. The optimum tax on the resource is given by (19), where $\mu > 0$ is determined to satisfy (1) with equality. Together, taxes on conventional goods and the tax on the resource must exactly raise $R_0 > \overline{\eta}_0 S_0$, which requires that $\sum_{i=1,...,n,s} \int_0^{+\infty} \theta_{it}^* \widetilde{x}_{it} e^{-rt}\, dt = R_0$. Substituting for θ_{it}^* implicitly defines λ.

2 − 4. Shown in the main text.

5. The result that OCT of the NRR never induces reserves to be left unexploited is shown in the main text. There remains to show that the optimal NRR tax does not induce a more rapid extraction than in the non-distortive case.

In any non-distortive equilibrium—*a fortiori* in absence of resource tax—the resource price at any date of the exploitation period is

$$\widetilde{q}_{st} = c_s + \overline{\eta}_0 e^{rt}, \tag{D.1}$$

where $\overline{\eta}_0$ is defined in Appendix B. If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource is depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where

$\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$:

$$\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0. \tag{D.2}$$

If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define $\overline{\eta}_0$ implicitly: $\lim_{T \to +\infty} \int_0^T D_s(\widetilde{q}_{st})dt = S_0$.

In the second-best equilibrium where high revenue needs imply that the resource should by taxed at the rate θ^*_{st} given by (19) with $\lambda > 1$, the resource price at any date of the exploitation period is $\widetilde{q}_{st} = c_s + \theta^*_{st}$; indeed, the producer rent $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ must be zero in this case, as explained in the main text. Making use of the presence of $\widetilde{q}_{st}$ on the right-hand side of (19), the resource price may be written as follows:

$$\widetilde{q}_{st} = \frac{c_s + \frac{1}{\lambda}\mu e^{rt}}{1 + \frac{\lambda - 1}{\lambda}\frac{1}{\widetilde{\varepsilon}_s}}. \tag{D.3}$$

In this expression, $\lambda > 1$, and $\mu > 0$ is determined in a way similar as $\overline{\eta}_0$ in the non-distortive case. It can easily be verified that the denominator on the right-hand side of (D.3) is strictly positive. Indeed, rearranging (19) immediately yields

$$\frac{\lambda - 1}{\lambda}\frac{1}{\widetilde{\varepsilon}_s} = \frac{\theta^*_{st} - \frac{1}{\lambda}\mu e^{rt}}{\theta^*_{st} + c_s} < 1.$$

When $c_s = 0$ and the ε_s is constant, as in Stiglitz's particular case, both the second-best equilibrium price (D.3) and the non-distortive equilibrium price (D.1) reduce to a single term that rises at the rate of discount. Their levels are also identical as they are both determined in such a way that (D.2) holds. Indeed, we have established that the optimal NRR tax induces reserves to be entirely depleted. In this case, the second-best equilibrium with $\lambda > 1$ implies the same extraction profile as prevails when no distortions are needed at all.

In all other cases, extraction cost c_s is strictly positive or the NRR demand elasticity $\varepsilon_s < 0$ decreases with q_{st} along the demand (increases in absolute value), and the second-best extraction profile strictly differs from the non-distortive case. In (D.3), the numerator takes the same form as (D.1); it consists of the same constant c_s and of a term rising at the rate of discount. However, as time t goes and price $\widetilde{q}_{st}$ increases, the denominator in (D.3) increases. Clearly, price $\widetilde{q}_{st}$ increases less rapidly in the second-best equilibrium. Since (D.2) must hold, the NRR price must be higher at early dates and lower at more distant dates relative to the non-distortive case.

When there exists a finite choke price $\overline{q}$ for the resource, the resource is depleted at a finite date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$. Since the resource price rises less rapidly in the second-best case, it is immediate that the exhaustion date $\widetilde{T}$ is postponed relative to the non-distortive case. ■

E OCT and Monopoly Pricing

If the need of tax revenues was extreme, that is to say if λ tended towards infinity, the optimum tax rate implied by (19) would be[39] $\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{1}{-\widetilde{\varepsilon}_s}$, corresponding to static monopoly pricing; indeed, $\frac{\theta_{st}}{\widetilde{q}_{st}} = \frac{\widetilde{q}_{st}-c_s}{\widetilde{q}_{st}}$ is the static Lerner index for the resource industry. Under such extreme condition the optimum resource tax rate would be determined by the same inverse elasticity rule as the tax rate applying to other commodities according to (13).

When revenue needs equal total rents ($\lambda = 1$), the second term in the right-hand side of (19) vanishes so that the optimal extraction tax is neutral.

Since $\frac{1}{\lambda}$ and $\frac{\lambda-1}{\lambda}$ sum to unity, the optimum tax on the resource industry given by (19) is a weighted sum of two elements. The first element μe^{rt} can be interpreted as the neutral component of the tax since it rises at the rate of discount as does a neutral Hotelling tax. The second element was just seen to correspond to monopoly pricing.

F Extension to Rising Marginal Costs and Resource Heterogeneity

One may wonder whether the sharp results of Section 2 are not due to the parsimony of the model, in particular the assumption that the supply of all conventional commodities is perfectly elastic, and the assumption that marginal extraction costs are not only constant but independent of the source of resource supply. It will be shown that the basic message—tax the resource more than similar conventional commodities—is not much affected by

[39] Although μ varies as λ changes, this scarcity rent cannot become infinite as $\lambda \to \infty$ so that the first term on the right-hand side of (19) indeed vanishes as required for this statement to be true.

relaxing these assumptions, although several new insights are derived from the analysis.

The assumption of infinite supply elasticity made by so many contributors to the OCT literature may be justified on the ground that they adopt a long-run perspective, where all commodities can be produced at constant marginal costs because all inputs are variable. The natural counterpart of constant marginal production cost for conventional commodities is constant marginal extraction cost. This will be replaced by rising marginal production and extraction costs shortly.

There is another consideration. The conditions of extraction of a non-renewable resource may be quite variable over time, as resources are not necessarily homogeneous; a possibility which is ruled out by the simple Hotellian formulation adopted so far. Even with rising marginal extraction cost, the extraction technology does not provide for resource heterogeneity. Two approaches have been used in the literature to deal with this issue. The Ricardian approach considers a single stock of reserves but assumes that the extraction cost increases with cumulative extraction (see, e.g., Levhari and Liviatan, 1977; Pindyck, 1978); this approach has been criticized because it implicitly assumes that the economically most accessible reserves are used first, which is not always optimal.[40] The second approach consists in modeling the resource as originating from different deposits each with its own cost function and its own stock of reserves. It underlies the manner in which advanced systems such as the Alberta Oil and Gas taxation regime approach resource taxation[41] (see Slade, 1988, for a theoretical formulation, empirical considerations, and references).

We start with introducing rising marginal costs; then we further add multiple deposits. Thus assume that conventional good i is supplied according to the function $S_i(p_{it})$, with $S_i'(.) > 0$, for $i = 1, ..., n$; $S_i^{-1}(x_{it})$ is the increasing marginal cost of producing a quantity

[40]As Slade (1988) put it "The idea that the least-cost deposits will be extracted first is so firmly embedded in our minds that it is an often-made but rarely tested assumption underlying the construction of theoretical exhaustible-resource models." (p. 189). See her references.

[41]Conrad and Hool (1981) pointed at the relevance of deposits' differences for resource taxation: In the ". . . mining problem, . . . differences in the composition of the ore bodies cause differences in response to a given economic change. In part because of this, mineral tax policy in some countries has been negotiated on a mine-by-mine basis. Geological features must therefore be an essential part of any model that is to be used for policy or empirical analysis" (p. 18).

For example in Alberta, royalties depend on the type of resource (conventional oil, gas, oil sands) and the date at which the deposit was discovered, because exploration targets different deposits as extraction technology evolves, as oil prices increase, and as exploration prospects become exploited (Alberta Royalty Review, 2007).

x_{it}. Regarding the non-renewable resource, assume an increasing marginal cost of extraction. For notational simplicity, this marginal cost is denoted by $S_s^{-1}(x_{st})$. However, this does not denote the inverse supply function. In competitive equilibrium, the supply of resource is determined by the "augmented marginal cost" condition:

$$\widetilde{p}_{st} = S_s^{-1}\left(\widetilde{x}_{st}\right) + \widetilde{\eta}_t, \tag{F.1}$$

where the current-value Hotelling's rent $\widetilde{\eta}_t$ grows at the rate of discount.

The OCT problem of maximizing (2) subject to (4) and (5), and the associated Hamiltonian are only modified to the extent that the producer surplus becomes

$$\widetilde{PS}_t = \sum_{i=1,\ldots,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,\ldots,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\,du - \int_0^{\widetilde{x}_{st}} \left(S_s^{-1}\left(u\right) + \widetilde{\eta}_t\right)\,du. \tag{F.2}$$

Given this change, the structure of the analysis is quite similar to that of constant marginal costs. Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on conventional good i is $[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - S_i^{-1}(\widetilde{x}_{it})]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0$. Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = S_i^{-1}(\widetilde{x}_{it}) + \theta_{it}$, it follows that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.) - S_i^{-1\prime}(.)}$. The optimum tax is thus such that $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}\big(D_i^{-1\prime}(.) - S_i^{-1\prime}(.)\big)$. Consequently the optimum tax rate on conventional commodity i is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\left(\frac{1 - \frac{\theta_{it}^*}{\widetilde{q}_{it}}}{\widetilde{\epsilon}_i} - \frac{1}{\widetilde{\varepsilon}_i}\right), \tag{F.3}$$

where $\epsilon_i \equiv \frac{S_i^{-1}(.)}{x_{it}S_i^{-1\prime}(.)}$ is the elasticity of supply, positive by assumption. As before, λ is strictly greater than unity when taxes are distortionary and equals unity if there is a non-distortionary way to collect enough revenues. Formula (F.3) provides an inverse elasticity rule for the case of non-perfectly-elastic supplies (Ramsey, 1927, p. 56).

The first-order condition for an interior tax on the resource is now $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0$. Since resource supply is determined by condition (F.1), it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st}) = \widetilde{\eta}_t$, which is different from zero. If tax revenue needs are low, the other commodities are not taxed at all and the resource is the sole provider of tax revenues; the resource should be taxed in priority even when supply elasticities in the other sectors are not assumed to be infinite.

If the revenues needed cannot be raised neutrally so that λ exceeds unity, all sectors are taxed in such a way that the distortions are spread across sectors; the tax on the resource sector is distortionary as in the previous section. What is new, however, is that the distortion aims at capturing part of the consumer surplus and part of the producer surplus while no producer surplus was available when marginal extraction were assumed to be constant. In that case, as in Section 2, the government's problem is subject to the exhaustibility constraint (16); taxation completely expropriates producers' resource rents, so that $\widetilde{\eta}_t = 0$ and $\widetilde{q}_{st} = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; the first-order condition for the resource tax becomes $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$, where μ is the present-value co-state variable associated with the exhaustibility constraint. The competitive equilibrium allocation satisfies $D_{si}^{-1}(\widetilde{x}_{st}) = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; transforming the first-order condition as for conventional goods yields the optimum tax on the resource

$$\theta^*_{st} = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\left(\frac{\widetilde{p}_{st}}{\widetilde{\epsilon}_s} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right), \tag{F.4}$$

where $\epsilon_s \equiv \frac{S_s^{-1}(.)}{x_{st}S_s^{-1\prime}(.)}$, the reciprocal of the elasticity of marginal extraction costs, can also be interpreted as the elasticity of short-run resource supply. Consequently, the resource should be taxed at a higher rate than conventional commodities having identical elasticities.

Consider now that the resource may be extracted from m deposits using an extraction technology characterized by rising marginal costs, as above but possibly different for each deposit. Each deposit $l = 1, ..., m$ makes a contribution z_{lt} to total production so that consumption of the homogeneous final commodity is $x_{st} = \sum_{l=1,...,m} z_{lt}$. While the consumer price q_{st} is unique, producer prices and scarcity rents typically differ because extraction costs and reserves may differ from one deposit to the next: $p_{lt} = S_l^{-1}(z_{lt}) + \eta_{lt}$, $l = 1, ..., m$. However, since each deposit is homogeneous, the corresponding rent satisfies Hotelling's rule and must grow at the rate of interest so that its supply is determined in competitive equilibrium by $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$ where the Hotelling rent $\widetilde{\eta}_{lt}$ corresponds to the exhaustibility constraint applying to deposit l: $\int_0^{+\infty} z_{lt}\, dt \leq S_{l0}$. We assume that the government has the ability to tax each deposit individually[42] so that $q_{st} = p_{lt} + \theta_{lt}$, $l = 1, ..., m$. Precisely, the tax θ_{st} that could indifferently fall on demand or supply in the previous cases, is replaced with a vector of taxes that fall on the supply of individual deposits; resource demand is not taxed. For any feasible tax trajectory and Hotelling rent, the output from each deposit adjusts in such a way that marginal extraction cost plus rent equals producer price as required.

The government budget constraint is only modified by the increase in the size of the tax vector which becomes $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{n+1\ t}, ..., \theta_{n+m\ t})$ and by the replacement of consumption x_{st} by the vector of supply tax bases $(z_{1t}, ..., z_{mt})$ in the government budget constraint. Except for the increased number of variables the OCT problem is only modified to the extent that producer surplus becomes, instead of $(F.2)$,

$$\widetilde{PS}_t = \sum_{i=1,...,n} \widetilde{p}_{it}\widetilde{x}_{it} + \sum_{l=1,...,m} \widetilde{p}_{lt}\widetilde{z}_{lt} - \sum_{i=1,...,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\, du - \sum_{l=1,...,m} \int_0^{\widetilde{z}_{lt}} \left(S_l^{-1}(u) + \widetilde{\eta}_{lt}\right) du,$$

and the resource rents become, instead of (10),

$$\widetilde{\phi}_t = \sum_{l=1,...,m} \widetilde{\eta}_{lt}\widetilde{z}_{lt}.$$

The first-order conditions for an interior solution to the choice of the taxes on resource extraction are $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt})]\frac{d\widetilde{z}_{lt}}{d\theta_{lt}} - \widetilde{z}_{lt} + \lambda(\widetilde{z}_{lt} + \theta_{lt}\frac{d\widetilde{z}_{lt}}{d\theta_{lt}}) = 0$. Since supply from deposit l is determined by condition $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$, it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt}) = \widetilde{\eta}_{lt}$, $l = 1, ..., m$; the rest of the solution process is as above. If revenue needs are low, a combination of neutral taxes rising at the rate of interest is applied on the extraction of the deposits. If revenue needs are high, the analysis of the single-deposit case applies; denoting by μ_l the present-value co-state variable associated with the exhaustibility constraint of deposit l, one obtains the optimal tax on deposit l

$$\theta_{lt}^* = \frac{1}{\lambda}\mu_l e^{rt} + \frac{\lambda - 1}{\lambda}\left(\frac{\widetilde{p}_{lt}}{\widetilde{\epsilon}_l} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right),\ l = 1, ..., m, \tag{F.5}$$

where $\epsilon_l \equiv \frac{S_l^{-1}(.)}{z_{lt}S_l^{-1\prime}(.)}$. Qualitative results are unchanged.

[42] See Footnote 41.

G Proof of Proposition 3

1. Shown in the main text.

2.(*a*) The result follows from the fact that the tax in (23) is adjusted by a positive term when $\widetilde{\varepsilon}_{sj}, \widetilde{\varepsilon}_{js} > 0$ and by a negative term otherwise.

2.(*b*) Shown in the main text. ■

H Proof of Expression (27)

The Hamiltonian (26) associated with the *ex post* problem is identical to (17). Hence, the application of the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. The first-order condition for the choice of the tax is also (18). However, unlike in Section 2, the first term on the left-hand side is not zero since the government is subject to its *ex ante* commitment, which determines $\widetilde{\eta}_0$ at this stage: $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st}^* - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. Therefore, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Substituting into the first-order condition and rearranging gives (27), where $\varepsilon_s \equiv \frac{q_{st}}{x_{st} D_s^{-1\prime}(.)}$.

I Proof of Proposition 4

1. Shown in the main text.

2. This is a restatement of (31), which is immediately obtained by substituting (30), shown in the main text, into (27), proven in Appendix H. The rest of the proposition summarizes findings established in the text preceding it. ■

J Proof of Proposition 5

The proof is shown in the main text. ■

K Proof of Expression (33)

Expression (33) is established under the assumption that extraction cost is zero, $c_s = 0$, and that the demand for the resource is isoelastic, $\varepsilon_s(q_{st}) = \varepsilon_s$. As mentioned in the main text, substituting $\widetilde{q}_{st} = \widetilde{\eta}_0 e^{rt} + \theta_{st}^*$ into (19) with $\widetilde{\eta}_0 = 0$, or into (27) and into (31) with $\widetilde{\eta}_0 \geq 0$, while using the constancy of $\widetilde{\varepsilon}_s$, immediately shows that the optimal extraction unit tax then grows at the rate of interest:

$$\theta_{st}^{*} = \theta_{s0}^{*} e^{rt}, \tag{K.1}$$

where θ_{s0}^{*} is to be determined.

For a given ρ, the *ex ante* choice of θ_{s0}^{*} is equivalent to the choice of the unit rent $\widetilde{\eta}_0$ it induces, account being taken of (25). The first-order condition for the *ex ante* static maximization of (28) with respect to θ_{s0}^{*} subject to (29), taking the *ex post* solution (K.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta_{s0}^{*} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt} = \left(D_s^{-1}(\widetilde{x}_{st}) - \theta_{s0}^{*} e^{rt}\right) \frac{d\widetilde{x}_{st}}{d\theta_{s0}} e^{-rt} - \widetilde{x}_{st} = \widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st}$ and where $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} \right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta_{s0}^{*} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{S}_0$ and $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$ gives

$$\theta_{s0}^{*} = \rho - \frac{(\lambda - 1)}{\lambda} \frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\theta_{s0}}}. \tag{K.2}$$

In long-run market equilibrium $\mathcal{S}^{-1}(\widetilde{S}_0) = \widetilde{\eta}_0 + \rho$ and $\int_0^{+\infty} D_s(\widetilde{\eta}_t + \theta_{st}^{*})\, dt = \int_0^{+\infty} D_s\big((\widetilde{\eta}_0 + \theta_{s0}^{*})e^{rt}\big)\, dt = \widetilde{S}_0$. It follows by differentiation with respect to θ_{s0} that $\mathcal{S}^{-1\prime}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and $\left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1 \right) \int_0^{+\infty} D_s'(.) e^{rt}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$. Substituting in $\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$, one obtains $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D_s'(.) e^{rt}\, dt}{1 - \mathcal{S}^{-1\prime}(.) \int_0^{+\infty} D_s'(.) e^{rt}\, dt}$. Introducing this expression into (K.2) yields

$$\theta_{s0}^{*} = \rho + \frac{\lambda - 1}{\lambda} \left[\widetilde{S}_0 \mathcal{S}^{-1\prime}(.) - \frac{\widetilde{S}_0}{\int_0^{+\infty} D_s'(.) e^{rt} dt} \right], \tag{K.3}$$

from which (33) is derived after substituting the expressions for $\widetilde{\zeta}$ and $\widetilde{\xi}$ defined in the main text and using the fact that $\widetilde{q}_{st} = (\widetilde{\eta}_0 + \theta_0^{*}) e^{rt} = \widetilde{q}_{s0} e^{rt}$ under (K.1) so that $\frac{d\widetilde{\mathcal{D}}}{dq_{s0}} = \int_0^{+\infty} D_s'(.) e^{rt}\, dt$. Furthermore, the constancy of ε_s implies $\widetilde{\xi} = \varepsilon_s$.

L Proof of Proposition 6

The proposition summarizes findings established in the main text. ■

M Extension to an Open Economy

OCT in an open economy raises a number of issues. In a static, closed economy, commodity taxes applied on the demand side are equivalent to taxes applied on the supply side. In the closed economy taxation during the extraction phase can be interpreted to apply to resource demand while the reserve development subsidy can be interpreted to apply to resource supply. Proposition 5 then means that the equivalence of supply and demand taxation extends to the resource sector, despite the difference in timing between reserve development and resource extraction. In the open economy, domestic consumption generally differs from domestic production so that OCT must be addressed by considering taxes or subsidies on both supply and demand rather than a single tax on demand or supply indifferently. The result of Proposition 5 nonetheless allows us to simplify the taxation of domestic resource supply by focusing on the domestic reserve subsidy rather than on the taxation of domestic extraction, while combining that subsidy with a commodity tax on resource consumption, whether from domestic or foreign origin. That way, much of the model structure used in the previous sections will be preserved.

In fact, the combination of a tax or subsidy on domestic demand and a tax or subsidy on domestic supply can be designed so as to be equivalent to a tariff (Mundell, 1960, p. 96). Consequently, the use of Ramsey's traditional tax instruments in an open economy could achieve the objective pursued by optimum tariffs (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). Since the OCT problem and the optimum-tariff problem then differ only by the constraint to collect a minimum revenue, the latter characterizes an optimum of Pareto from the country's point of view while optimum commodity taxes are distortionary: As Boadway et al. (1973) put it "domestic commodity taxes *introduce* a distortion while optimum tariffs *eliminate* a distortion" (p. 397, their italics).

For reasons that need no explanation, tariffs will not be directly available as tax instruments in the open-economy OCT problem. However, demand and supply commodity taxes will seek the same objective as optimal tariffs and, consequently, their first-best levels (that is, unconstrained by revenue needs) will differ from zero.[43] Besides the obvious difference in domestic versus world surplus, the ability of the government to affect national surplus differs in the closed economy, where the government has the power to affect prices as a monopoly, from the open economy, where the government is competing with other countries much like an oligopolist. Non-renewable resources are very different from conventional goods in that respect; roughly, the supply of conventional goods is elastic while the supply of the Hotelling resource is inelastic in a closed economy. In an open economy, if the country is small and trades the resource competitively, the non-renewable resource behaves just like another commodity; its supply is infinitely elastic and optimal commodity taxes on the non-renewable resource obey the conventional closed-economy inverse elasticity rule.

Consequently, the interesting setup to study Ramsey taxation in an open economy is strategic. The country trades the non-renewable resource and is big enough to affect suppliers' surplus, whether supply is domestic or foreign.[44] In this section we are going to assume that the country has no influence on the prices of other commodities. Three reasons justify this restriction. First it does not affect the generality of the results presented; second it puts the focus on the key difference between non-renewable resources and conventional goods and services: supply elasticity. Third it connects with the literature on rent capture and optimal tariffs in the presence of a non-renewable resource; more on this further below.

M.1 Analysis and Results

The government faces a problem similar to that of Section 2—choose linear commodity taxes to maximize domestic surpluses subject to a minimum tax revenue constraint and to a stock of endogenously supplied mineral reserves. These reserves are located either within the country, or outside, or both but have the same constant unitary extraction cost.[45] The

[43]Since the distortion results from the failure by the country to exert market power, only "large" countries should adopt different domestic taxes when they are open to trade than when they are closed to trade. This is also true when some tariffs are set at suboptimal levels; then, as shown by Dornbusch (1971, p. 1364), domestic taxes are conferred a corrective role. Not surprisingly, if the government can freely use both tariffs and commodity taxes, it can achieve its surplus maximization objective with tariffs and satisfy its revenue collection needs using commodity taxes; then, as Boadway et al. (1973) showed, Ramsey optimal domestic commodity taxes are "the same as in the case of a closed economy" (p. 391).

[44]The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982). On market power on the demand side, see also Liski and Montero (2011).

[45]See Appendix F for generalizations.

non-renewable resource sector is now open to trade. World scarcity rents are equalized by free trade but domestic reserve supply is determined by the sum of the rent and the domestic reserve subsidy. As in our treatment of the closed economy, we simplify and sharpen the analysis by assuming that there is an *ex ante* step where domestic and world reserve stocks are established, followed by an *ex post* extraction phase.

Although the government has less power to affect the resource price than when the economy is closed, its choice of consumption taxes applied during the extraction period and the domestic reserve subsidy applied *ex ante* determine the scarcity rent enjoyed by both foreign producers and domestic ones, if any; they amount to a rent commitment towards the latter. This rent depends on the policies implemented in the rest of the world, which are taken as given in Nash equilibrium by the home government. Unlike the closed economy, the government is restricted to leaving its suppliers a rent at least as high as they would get if the domestic market was taxed to extinction.[46] The rent commitment occurs *ex ante* and is simultaneous with the choice of the reserve subsidy. Given that market power is limited to the non-renewable resource and that the supply of conventional goods is infinitely elastic, no tax or subsidy is applied on the supply of conventional commodities. Trade in these commodities combines with resource trade as in Bergstrom in such a way that the trade balance constraint is satisfied. For simplicity, and with no consequence on the results, it is assumed that there are only two countries.

Unless otherwise mentioned, all variables and functions are redefined so as to refer to the home country. Variables or functions pertaining to the rest of the world will be denoted by the same symbol and identified with the superscript F. Given the absence of rents or taxes on the supply side of conventional goods, surpluses on conventional goods are defined in terms of the (domestic) demands x_{it} as before. In the case of the resource, x_{st} now denotes instantaneous domestic demand while y_t denotes instantaneous domestic supply, and θ_{st} denotes the tax on demand. The resource supply tax or subsidy ρ is applied *ex ante* as in Section 4. Given these remarks and redefinitions, the equilibrium domestic consumer surplus $\widetilde{CS}_t$ is still given by (8), the producer surplus under competitive equilibrium is

[46] As justified above, we do not allow the government to tax domestic extraction. If it would, domestic rents would be allowed to differ from world rents; however the sum of extraction rent and support to exploration could be kept unchanged by adjusting the reserve subsidy, implying identical domestic reserves. Thus our treatment is compatible with a continuum of domestic resource taxation systems of combining extraction taxes and support to exploration as in many observed situations.

identical to (9) except that $\widetilde{y}_t$ replaces $\widetilde{x}_{st}$, and the home producers' total resource rent, formerly (10) becomes $\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{y}_t$.

The analysis replicates that of Section 4. Consider first the *ex post* extraction stage under the *ex ante* commitment to consumption taxes that induce a given unit rent $\widetilde{\eta}_0 > 0$. The choice of $\widetilde{\eta}_0$ and of the supply subsidy ρ will be discussed immediately thereafter. Given that the resource is traded and that its marginal extraction cost is the same in the rest of the world as in the home country, unit rents are equalized: $\widetilde{\eta}_0 = \widetilde{\eta}_0^F$. The relevant supply to the home country is the residual world supply, that is the supply remaining once demand from the rest of the world has been met. At each date, the remaining stock of reserves available for consumption in the home country is thus $\widetilde{S}_t^H \equiv \widetilde{S}_t + \widetilde{S}_t^F - \int_t^{+\infty} \widetilde{x}_{su}^F \, du$ where home and foreign reserves $\widetilde{S}_0$ and $\widetilde{S}_0^F$ are established *ex ante* so that they are given when extraction starts; and where, since $\widetilde{x}_{st}^F = D_s^F(c_s + \widetilde{\eta}_t)$, the remaining foreign demand $\int_t^{+\infty} \widetilde{x}_{su}^F \, du$ is determined by the *ex ante* rent commitment. The exhaustibility constraint relevant to the home government is thus

$$\dot{\widetilde{S}}_t^H = -\widetilde{x}_{st}. \tag{M.1}$$

The Hamiltonian corresponding to this open-economy problem differs from its closed-economy counterpart (26) only by the producer surplus and the resource rent:

$$\mathcal{H}(a_t, \theta_t, \lambda_t, \mu_t) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t \right) e^{-rt} + \lambda_t (r a_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{M.2}$$

where μ_t is now associated with (M.1). From the maximum principle, as in Section 4, $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu \geq 0$, with μ again given by (30); then (see the proof below),

$$\frac{\theta_{st}^*}{\widetilde{q}_{st}} = \rho \frac{e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda} \frac{1}{-\widetilde{\varepsilon}_s} + \frac{1}{\lambda} (1 - \widetilde{\alpha}_t) \widetilde{\eta}_0 \frac{e^{rt}}{\widetilde{q}_{st}}, \tag{M.3}$$

where $\widetilde{\alpha}_t \equiv \frac{d\widetilde{y}_t / d\theta_{st}}{d\widetilde{x}_{st} / d\theta_{st}}$ is the change in domestic resource production relative to the change in domestic consumption, induced by domestic taxation.

Formula (M.3) is the open-economy counterpart of (31) and differs from it by the last term; if $\widetilde{\alpha}_t$ equalled unity, this term would vanish. By the definition of $\widetilde{\alpha}_t$, this happens if any change in domestic consumption is exclusively met by domestic supply. Clearly, this includes the limit case where the rest of the world is negligible as well as situations where the foreign country does not hold any resource. In contrast, $0 < \widetilde{\alpha}_t < 1$ whenever foreign supply to the domestic resource market adjusts to a change in the tax on domestic demand in the same direction as domestic supply does. This reinforces the closed-economy result stated in Proposition 4 that the consumption of the non-renewable resource is taxed at a higher rate than the consumption of a conventional good or service having the same demand elasticity when $\rho \geq 0$.

M.2 Proof of Expression (M.3)

The Hamiltonian associated with the ex post open-economy problem is (M.2). Applying the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. Since the government is subject to its ex ante commitment, $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ is determined at this stage, as well as $\widetilde{x}_{st}^F$, which depends on θ_{st} only via $\widetilde{\eta}_t$. Hence, the first-order condition for the choice of the tax is

$$\left[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s - \widetilde{\eta}_t\right] \frac{d\widetilde{x}_{st}}{d\theta_{st}} + \widetilde{\eta}_t \frac{d\widetilde{y}_t}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt} \frac{d\widetilde{x}_{st}}{d\theta_{st}}.$$

Since $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, where $\widetilde{\eta}_0$ is given, the first term on the left-hand side is zero and $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Inserting into the above condition and rearranging give

$$\theta_{st}^* = \frac{1}{\lambda}(\mu - \widetilde{\alpha}_t \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda} \frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{M.4}$$

where $\widetilde{\alpha}_t = \frac{d\widetilde{y}_t/d\theta_{st}}{d\widetilde{x}_{st}/d\theta_{st}}$ and $\widetilde{\varepsilon}_s \equiv \frac{\widetilde{q}_{st}}{\widetilde{x}_{st} D_s^{-1\prime}(.)}$.

In the open economy, the ex post maximized value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$, $V^*(\widetilde{S}_0^H, R; \rho)$, is a function of the residual reserves available to the home country $\widetilde{S}_0^H \equiv \widetilde{S}_0 + \widetilde{S}_0^F - \int_0^{+\infty} \widetilde{x}_{st}^F\, dt$. The constant co-state variable μ in (M.2) should be interpreted as giving the value $\frac{\partial V^*}{\partial \widetilde{S}_0^H}$ of a marginal unit of residual reserves. By definition of $\widetilde{S}_0^H$ it must be that μ is also the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of domestic reserves. The rest of the reasoning leading to (30) in Section 4 applies.

Substituting (30) into (M.4) yields (M.3). ■

M.3 Resource Consumption Tax in Open Economy

Clearly there is an intertemporal equilibrium where $\widetilde{\alpha}_t$ is time invariant.[47] In that case the last term in (M.3) defines a component of the unit tax θ^*_{st} which is rising at the discount rate; hence, the extra taxation imposed upon resource consumption in the open economy relative to the closed economy is neutral. The second term, the distortionary Ramsey component, is the same as in the closed economy.

Consider now the *ex ante* open-economy problem. Given that the resource consumption taxes must satisfy (M.3) *ex post*, the problem of choosing $\widetilde{\eta}_0$ and ρ is

$$\max_{\widetilde{\eta}_0,\ \rho} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{M.5}$$

subject to

$$\int_0^{+\infty} \theta^*_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{M.6}$$

There is an important difference between this problem and its closed-economy counterpart (28). In the closed-economy problem, the first-order condition with respect to ρ and the expression for the *ex post* tax (31) are linearly dependent. This is why an infinity of *ex post* taxes-*ex ante* subsidy combinations were shown to be optimal and equivalent: In the closed economy the equivalence of demand taxation and supply taxation extends from the static realm of conventional goods to the dynamic framework of resource extraction where ρ is applied prior to θ_{st}. This is not so in the open economy; the first-order condition for ρ in problem (M.5) and expression (M.3) for the optimal extraction tax, are not linearly

[47] This is because in any intertemporal equilibrium domestic and foreign resource supply flows are only determined to the extent that their sum is determined and that domestic and foreign exhaustibility constraints must be met. This can be shown as follows. For any given tax schedule, the rent must rise at the rate of interest: $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. The resource market must clear at each date so that $\widetilde{x}_{st} + \widetilde{x}^F_{st} = \widetilde{y}_t + \widetilde{y}^F_t$. On the demand side, $\widetilde{x}_{st}$ and $\widetilde{x}^F_{st}$ are demanded quantities for the current resource price, uniquely determined at each date by $\widetilde{\eta}_0$, thus giving the world equilibrium supply $\widetilde{y}^W_t = \widetilde{x}_{st} + \widetilde{x}^F_{st}$. On the supply side, however, producers are indifferent about when to extract since $\widetilde{\eta}_t$ rises at the rate of interest. Hence, equilibrium domestic and foreign supplies $\widetilde{y}_t$ and $\widetilde{y}^F_t$ are only determined to the extent that they must fulfill the exhaustibility constraints for established reserves, $\widetilde{S}_0 = \int_0^{+\infty} \widetilde{y}_t\, dt$ and $\widetilde{S}^F_0 = \int_0^{+\infty} \widetilde{y}^F_t\, dt$, as well as the clearing condition $\widetilde{y}_t + \widetilde{y}^F_t = \widetilde{y}^W_t$, where $\widetilde{y}^W_t$ is determined as above.

Clearly, there is an infinity of combined paths of domestic supply $\widetilde{y}_t$ and foreign supply $\widetilde{y}^F_t$ satisfying these two conditions. A simple and natural combination is the one along which relative instantaneous supplies remain constant, so that $\frac{\widetilde{y}_t}{\widetilde{y}^F_t} = \frac{\widetilde{S}_0}{\widetilde{S}^F_0} \equiv \sigma$. For a given rent-commitment $\widetilde{\eta}_0$, foreign consumption $\widetilde{x}^F_{st}$ is given, so that tax changes only affect $\widetilde{x}_{st}$. Hence, the above condition implies that the domestic supply reaction to a change $d\widetilde{x}_{st}$ must be $d\widetilde{y}_t = \frac{\sigma}{1+\sigma} d(\widetilde{y}_t + \widetilde{y}^F_t) = \frac{\sigma}{1+\sigma} d\widetilde{x}_{st}$, which defines $\widetilde{\alpha} \equiv \frac{\sigma}{1+\sigma}$, constant and lower than unity.

dependent; they combine to determine the optimal tax path and the optimal subsidy for any feasible rent-commitment $\widetilde{\eta}_0$ by the government.

Consequently, while Proposition 4 survives almost unscathed the extension from the closed economy to the open economy, Proposition 5, which states that an infinity of tax-subsidy mixes yield the optimum level of reserves and extraction path in a closed economy, does not hold in an open economy.

Consider the afore-mentioned equilibrium in whcih $\widetilde{\alpha}_t$ is time invariant; see Footnote 47 for a detailed description. The comparison of (M.3) with (13) and (31) yields the following proposition.

Proposition 7 *(Resource consumption tax in open economy) When the non-renewable resource is traded, there is an equilibrium such that the Home country and the Rest of the world contribute to world resource supply in the same proportion as they share reserves. Then,*

1. *Domestic resource consumption is taxed at a strictly higher rate than the consumption of conventional goods of the same demand elasticity when supply subsidies in the resource sector are non negative ($\rho \geq 0$).*

2. *The optimal tax rate (M.3) on resource consumption is made up of non-distortionary and distortionary components. The distortionary component is the same as in the closed economy and expresses Ramsey's inverse elasticity rule.*

M.4 Rent Capture and Ramsey Taxation

In the closed economy with endogenous reserves, first-best optimum commodity taxes do not yield any fiscal revenues. In contrast, in the open economy, it is well known that optimal tariffs are not nil, so that a combination of commodity taxes mimicking optimal tariffs produces tax revenues and may meet government needs without involving any distortion. The distinction between low and high revenue needs made in Section 2 with exogenous reserves thus arises again when the economy is open in spite of the endogeneity of reserves. Low and high revenue needs should be defined according to whether government needs are below or above the amount $\overline{\overline{R}}_0$ raised when the resource tax is set so as to maximize welfare in the absence of tax-revenue constraint ($\overline{\overline{R}}_0$ is established in the proof of Proposition 8 below). Call this the rent-capture component of the optimal domestic consumption tax. If $R_0 > \overline{\overline{\overline{R}}}_0$, the rent-capture component of the domestic resource consumption tax is not sufficient to meet revenue needs and it must be true that $\lambda > 1$; only then does the second term in (M.3), the distortionary component of the optimal consumption tax, become positive.

When the taxation of NRRs is distortionary, the distortion may affect both the extraction path and the amount of initial reserves. Consider the international equilibrium where $\widetilde{\alpha}_t$ is time invariant; the first and third terms in (M.3) then rise at the rate of discount while the distortionary component is identical to its counterpart in (31). Stiglitz's (1976) special case of isoelastic domestic demand and zero extraction costs then again implies that the optimal tax on resource demand is neutral and rises at the rate of interest.

An additional interest of Stiglitz's special case is that, when extraction costs are zero, a unit resource consumption tax that is rising at the rate of interest induces the final price $\widetilde{q}_{st}$ to rise at the same rate. Hence, such a tax is tantamount to the constant *ad valorem* tax in Bergstrom (1982). Our open-economy model then differs from Bergstrom's only in the treatment of reserves, exogenous in his paper, endogenous here. Bergstrom's inverse elasticity rule maximizes the country's surplus without any constraint on tax revenues, so that it is equivalent to an optimum tariff. Stiglitz's special case then enables us to investigate how the optimal resource tax of the Ramsey government differs from a commodity tax that would pursue the objective of an optimum tariff.

With θ^*_{st} now equal to $\theta^*_{s0}e^{rt}$, expressions (M.3) are determined at all dates by the initial level of the optimal resource tax. The maximization of (M.5) with respect to θ_{s0} is equivalent to its maximization with respect to the rent $\widetilde{\eta}_0$ induced by θ_{s0}. The resulting optimum tax rate, the open-economy counterpart of (33) is:

$$\frac{\theta^*_{s0}}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} \frac{\widetilde{S}_0 \widetilde{\zeta}}{\widetilde{S}^H_0 \widetilde{\zeta}^H} + \frac{\lambda - 1}{\lambda} \left[\frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{\zeta}^H} + \frac{1}{-\widetilde{\xi}} \right] + \frac{1}{\lambda} \frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{S}^H \widetilde{\zeta}^H} \left[\widetilde{\mathcal{D}} - \widetilde{S}_0 \right], \tag{M.7}$$

where $\widetilde{S}^H_0 \equiv \widetilde{S}_0 + \widetilde{S}^F_0 - \widetilde{\mathcal{D}}^F$ is the residual supply of reserves available for home country consumption, whose elasticity is defined as $\widetilde{\zeta}^H \equiv \left(\frac{d\widetilde{S}^H_0}{d\eta_0} \right) \frac{\widetilde{\eta}_0}{\widetilde{S}^H_0}$. The proof of (M.7) is presented below, in the next subsection.

Expression (M.7) simplifies to (33) when the totality of domestic consumption is met by domestic production.[48] Although complex, it brings up simple and important insights. First it shows the role of resource supply and its elasticity explicitly. It stresses the distinction between domestic production $\widetilde{S}_0$, which may be consumed locally or exported and can be taxed or subsidized in both cases, and foreign supply to the domestic market, which cannot be taxed or subsidized; $\widetilde{S}_0^H$ combines both. For a resource importer $\left(\widetilde{\mathcal{D}} - \widetilde{S}_0 > 0\right)$ that does not tax reserve production ($\rho \geq 0$), the optimum tax rate decreases when the elasticity of residual reserve supply $\widetilde{\zeta}^H$ increases. Indeed, Pigou (1947, p. 113) attempted to extend Ramsey's principles to trading economies. Since the residual supply of internationally-traded commodities presumably has a greater elasticity than total supply, he conjectured that Ramsey's analysis would imply imposing lower tax rates on those commodities.

Second, (M.7) connects neatly with the literature on the capture of resource rents initiated by Bergstrom (1982) and with the question of optimal tariffs in the presence of NRRs. Bergstrom treats reserves as given so he does not envisage a subsidy: $\rho = 0$. Bergstrom does not consider that the government faces any revenue constraint: $\lambda = 1$. Consequently the first and second terms disappear under his setup. Multiplying by $\widetilde{q}_{s0}$, substituting $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta^*_{s0}$, we obtain

$$\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{1}{\widetilde{S}_0^H \widetilde{\zeta}^H} \left(\widetilde{\mathcal{D}} - \widetilde{S}_0\right). \tag{M.8}$$

Since extraction costs are assumed nil, $\frac{\theta^*_{s0}}{\widetilde{\eta}_0}$ is the optimal, constant *ad valorem* tax given by Bergstrom in Expression (32), p. 198. One may wonder why Bergstrom's formula involves countries' demand elasticities and no supply elasticity. The reason is the assumption of exogenous world reserves. A country's residual supply then only depends on other countries' demands and not on the technology of reserve discovery as in this paper. Once $\widetilde{S}_0^H$ and its elasticity are written in terms of resource demands using $\widetilde{S}_0^H = \widetilde{S}_0 + \widetilde{S}_0^F - \widetilde{\mathcal{D}}^F$, we obtain Bergstrom's Expression (32).[49]

This formula is famous for it implies that a net importer should tax the resource, at least to the extent that it holds market power. This is Pareto optimal from that country's

[48] The last term vanishes when $\widetilde{\mathcal{D}} - \widetilde{S}_0 = 0$, and it must then also be the case that $\widetilde{S}_0^F - \widetilde{\mathcal{D}}^F = 0$ so that $\widetilde{S}_0^H = \widetilde{S}_0$, $\widetilde{\zeta}^H = \widetilde{\zeta}$, and the first term reduces to $\frac{\rho}{\widetilde{q}_{s0}}$ as in (33).

[49] This being the two-country case, the summation symbol in Bergstrom disappears, so that, in our notations—we also corrected a typo in Bergstrom—the formula reads $\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{\widetilde{\mathcal{D}} - \widetilde{S}_0}{-\widetilde{\mathcal{D}}^F \widetilde{\xi}^F}$.

point of view and allows it to capture some of the rents otherwise falling into the hands of exporters. When reserves are endogenous this power to capture rents is attenuated: $\widetilde{S}_0^H \widetilde{\zeta}^H$ being higher than its exogenous-reserve counterpart $-\widetilde{\mathcal{D}}^F \widetilde{\xi}^F$, the importer must not tax resource consumption as much: Depriving foreign suppliers of resource rents would reduce their supply of reserves.

Third, the first term in (M.7) shows the arbitrage between *ex ante* reserve subsidization and *ex post* taxation of resource consumption: The consumption tax increases with reserve subsidization by a factor of proportionality equal to the ratio of local production over residual supply to the home country, both weighted by their respective elasticities. This ratio is unity in the closed economy, so that the trade-off between taxing extraction or subsidizing reserves is financially neutral. The trade-off would be financially neutral in a competitive open economy if the coefficient of ρ were $\frac{\widetilde{S}_0}{\widetilde{S}_0^H}$, reflecting the fact that the tax base of domestic production is smaller than the tax base of domestic consumption; the presence of elasticities in the coefficient of ρ makes it plain that the optimum tax-subsidy combination further reflects the ability of the country to manipulate prices by its choice of the tax instruments.

The main results are gathered in the following proposition; see the proof below.

Proposition 8 *(Rent capture and Ramsey taxation) When further to the conditions of Proposition 7, domestic demand is isoelastic, and extraction is costless, the maximum revenue need $\overline{\overline{R}}_0$ compatible with neutral resource taxation is given by (M.13) and the optimum taxes or subsidies on resource consumption and reserve supply are jointly determined by (M.7) and (M.12). More precisely,*

1. *When $R_0 \leq \overline{\overline{R}}_0$, so that (M.7) and (M.12) hold with $\lambda = 1$, OCT is Pareto optimum and fulfills a resource-rent-capture objective. For an importing country, this involves taxing resource consumption while subsidizing domestic production, and* vice versa *for an exporter.*

2. *When government revenue needs are high, (M.7) and (M.12) apply with $\lambda > 1$. Optimum resource taxes are then higher than when $R_0 \leq \overline{\overline{R}}_0$ (reserve subsidies are lower) by an amount that reflects both domestic and foreign demand elasticities, as well as domestic and foreign supply elasticities.*

The formula giving the optimal level of ρ is (M.12); being the sister of Formula (M.7), it can be read and interpreted in much the same way. When revenue needs are low, ρ is always strictly positive for importing countries, as is well understood from the optimum-tariff literature. Sufficiently high revenue needs, however, may reverse the result, implying that it may be optimal to tax reserve production, even in importing countries. Similarly, under sufficiently high revenue needs, exporters may tax consumption according to (M.7).

M.5 Proof of Proposition 8, and of Expressions (M.7) and (M.12)

1. Most of the first part of Proposition 8 is shown in the main text and in the proof of (M.3). Expressions (M.7) and (M.12) remain to be proven. They can be obtained as follows.

Throughout this proof, Stiglitz (1976)'s conditions hold: The elasticity of domestic demand $\varepsilon_s(q_{st})$ is a constant ε_s and marginal extraction cost c_s is zero. Without any further loss of generality, we may then restrict attention to the equilibrium in which $\widetilde{\alpha}_t = \widetilde{\alpha}$ is time invariant.

In this case the optimal extraction unit tax is given by (M.3) multiplied by $\widetilde{q}_{st}$; it rises at the rate of interest. This formula only differs from (31) by its last term, which is, after multiplying by $\widetilde{q}_{st}$, $\frac{1}{\lambda}(1-\widetilde{\alpha})\widetilde{\eta}_0 e^{rt}$. Recalling that the unit tax given by (31) has been shown to rise at the rate of interest in Appendix K, it remains to show that the new term does so, which is immediate since $\widetilde{\alpha}$ is constant. Hence, (K.1) is valid, where θ^*_{s0} is to be determined as follows.

The first-order condition for the *ex ante* static maximization of (M.5) with respect to θ^*_{s0} subject to (M.6), taking the *ex post* solution (K.1) into account, is, as in Appendix K,

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Furthermore, $\frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt} = (D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st}) \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} + \frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$, and $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st} \right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{\mathcal{D}}$, $\int_0^{+\infty} \widetilde{y}_t\, dt = \widetilde{S}_0$, $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}$ and $\int_0^{+\infty} \frac{d\widetilde{y}_t}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$, and rearranging give

$$\theta_{s0}^* = \rho \frac{\frac{d\widetilde{S}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} - \frac{(\lambda-1)}{\lambda} \frac{\widetilde{\mathcal{D}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} + \frac{1}{\lambda} \frac{\frac{d\widetilde{\eta}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{\theta_{s0}}} \left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right]. \tag{M.9}$$

In long-run market equilibrium, $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho)$ and $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \rho)e^{rt}\right) dt = \widetilde{S}_0^H$, where $\widetilde{S}_0^H$ is the residual supply as defined in the main text. It follows by differentiation with respect to θ_{s0} that $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \mathcal{S}'(.)\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and that $\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}} = \left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1\right)\int_0^{+\infty} D_s'(.)e^{rt}\, dt = \frac{d\widetilde{S}_0^H}{d\eta_0}\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}{\frac{d\widetilde{S}_0^H}{d\eta_0} - \int_0^{+\infty} D_s'(.)e^{rt}\, dt}$. Introducing these expressions in (M.9) yields

$$\theta_{s0}^* = \rho \frac{\mathcal{S}'(.)}{\frac{d\widetilde{S}_0^H}{d\eta_0}} + \frac{\lambda-1}{\lambda}\left[\frac{\widetilde{S}_0^H}{\frac{d\widetilde{S}_0^H}{d\eta_0}} - \frac{\widetilde{\mathcal{D}}}{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}\right] + \frac{1}{\lambda}\frac{1}{\frac{d\widetilde{S}_0^H}{d\eta_0}}\left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right], \tag{M.10}$$

from which (M.7) is obtained after substituting $\widetilde{\zeta}$, $\widetilde{\zeta}^H$, $\widetilde{\xi}$. For the latter, we proceed in the same way as described in Appendix K.

The first-order condition for the *ex ante* static maximization of (M.5) with respect to ρ subject to (M.6), taking the *ex post* solution (K.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\rho} e^{-rt}\, dt + \widetilde{S}_0 + \rho\frac{d\widetilde{S}_0}{d\rho} - \mathcal{S}^{-1}(.)\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty} \theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\, dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\rho}e^{-rt} = \left(D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st}\right)\frac{d\widetilde{x}_{st}}{d\rho} + \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$. Substituting and using $S^{-1}(.) = \widetilde{\eta}_0 + \rho$, one has

$$\int_0^{+\infty} \left(\frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}\right) dt + \widetilde{S}_0 - \widetilde{\eta}_0\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty} \theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\, dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0.$$

Integrating as above and rearranging give

$$\rho^* = \theta_{s0}\frac{\frac{d\widetilde{\mathcal{D}}}{d\rho}}{\frac{d\widetilde{S}_0}{d\rho}} - \frac{(\lambda-1)}{\lambda}\frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\rho}} + \frac{1}{\lambda}\frac{\frac{d\widetilde{\eta}_0}{d\rho}}{\frac{d\widetilde{S}_0}{\rho}}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right]. \tag{M.11}$$

In long-run market equilibrium, $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \theta_{s0})e^{rt}\right) dt$ and $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho) =$

$\widetilde{\mathcal{D}}^H$, where $\widetilde{\mathcal{D}}^H \equiv \widetilde{\mathcal{D}} + \widetilde{\mathcal{D}}^F - \widetilde{S}_0^F$, is the residual cumulative demand of the rest of the world, which has to be met by the supply of domestic reserves. It follows by differentiation with respect to ρ that $\frac{d\widetilde{\mathcal{D}}}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho} \int_0^{+\infty} D_s'(.)e^{rt}\, dt$ and $\frac{d\widetilde{S}_0}{d\rho} = \mathcal{S}'(.) \left(\frac{d\widetilde{\eta}_0}{d\rho} + 1 \right) = \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0} \frac{d\widetilde{\eta}_0}{d\rho}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\rho} = \frac{-\mathcal{S}'(.)}{\mathcal{S}'(.) - \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}$. Introducing these expressions into (M.11) yields

$$\rho^* = \theta_{s0} \frac{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}} - \frac{\lambda - 1}{\lambda} \left[\frac{\widetilde{S}_0}{\mathcal{S}'(.)} - \frac{\widetilde{\mathcal{D}}^H}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}} \right] + \frac{1}{\lambda} \frac{1}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}} \left[\widetilde{S}_0 - \widetilde{\mathcal{D}} \right].$$

Using the definition $\widetilde{\xi}^H \equiv \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0} \frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}^H} < 0$ and redefining $\widetilde{\xi} \equiv \frac{d\widetilde{\mathcal{D}}}{d\eta_0} \frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}}$ as well as $\widetilde{\zeta} \equiv \frac{(\widetilde{\eta}_0 + \rho)\mathcal{S}'(.)}{\widetilde{S}_0}$, we obtain

$$\frac{\rho^*}{\widetilde{\eta}_0 + \rho^*} = \frac{\theta_{s0}}{\widetilde{\eta}_0 + \rho^*} \frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H \widetilde{\xi}^H} - \frac{\lambda - 1}{\lambda} \left[\frac{1}{\widetilde{\zeta}} + \frac{1 - \frac{\rho^*}{(\widetilde{\eta}_0 + \rho^*)}}{-\widetilde{\xi}^H} \right] + \frac{1}{\lambda} \frac{1 - \frac{\rho^*}{(\widetilde{\eta}_0 + \rho^*)}}{\widetilde{\mathcal{D}}^H \widetilde{\xi}^H} \left[\widetilde{S}_0 - \widetilde{\mathcal{D}} \right]. \quad \text{(M.12)}$$

When $\lambda = 1$, the second term on the right-hand side, the distortionary Ramsey component of the subsidy, vanishes. If $\theta_{s0} > 0$ and the home country is importing the resource, i.e., $\widetilde{S}_0 - \widetilde{\mathcal{D}} < 0$, ρ^* is non-ambiguously positive. Since $\frac{\widetilde{S}_0 \widetilde{\zeta}}{\widetilde{S}_0^H \widetilde{\zeta}^H} < 1$ and $\frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H \widetilde{\xi}^H} < 1$ by the definitions of S_0^H and $\mathcal{D}^H$, combining (M.12) with (M.7), computed for $\lambda = 1$, yields a strictly positive tax $\theta_{s0}^* > 0$ and a strictly positive subsidy $\rho^* > 0$. The second term on the right-hand side of (M.12) is negative. Therefore, for sufficiently high revenue needs, ρ^* may turn negative, i.e., may become a tax on reserve development.

Symmetrically, if the home country is exporting the resource, i.e., $\widetilde{S}_0 - \widetilde{\mathcal{D}} > 0$, then θ_{s0}^* and ρ^* are strictly negative when $\lambda = 1$; the second term on the right-hand side of (M.7) being positive, θ_{s0}^* may turn positive for sufficiently high revenue needs, i.e., may become a tax on domestic resource consumption.

2. The proof is similar to the Proof of Proposition 1. We know that when $\lambda = 1$, $\theta_i^* = 0$, $i = 1, ..., n$, so that the totality of fiscal revenues is raised from the resource sector. In the context of Proposition 8, $\theta_{st}^* = \theta_{s0}^* e^{rt}$, where θ_{s0}^*, given by (M.7), is jointly determined with ρ^*, given by (M.12). Combining both expressions for $\lambda = 1$ and substituting into

$$\overline{\overline{R}}_0 \equiv \theta_{s0}^* \widetilde{\mathcal{D}} - \rho^* \widetilde{S}_0 \quad \text{(M.13)}$$

defines the net amount raised by the resource sector. Hence, when $\lambda = 1$ it must be the case that $R_0 \leq \overline{\overline{R}}_0$. The contrapositive is that any $R_0 > \overline{\overline{R}}_0$ implies $\lambda > 1$. Moreover, following the reasoning of the Proof of Proposition 1, any $R_0 \leq \overline{\overline{R}}_0$ will be raised without imposing distortion, implying $\lambda = 1$. ∎

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