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# The Neolithic Revolution from a price-theoretic perspective

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## Abstract

The adoption of agriculture during the Neolithic period triggered the first demographic explosion in history. When fertility returned to its original level, agriculturalists were more numerous, more poorly nourished, and worked longer hours than their hunter-gatherer ancestors. We develop a dynamic price-theoretic model that rationalizes these events. In the short run, people are lured into agriculture by the increased labor productivity of both adults and children. In the long run, the growth in population overrides the productivity gains, and the later generations of agriculturalists end up being worse off than the hunter-gatherers. Counter-intuitively, the increase in the labor productivity of children causes the long-term reduction in welfare. In the long run, the increase in adult labor productivity only contributes to population growth.

**Keywords:** Neolithic Revolution; hunter-gatherers; child labor; Thomas Malthus.

## 1 Introduction

The shift from hunting and gathering to agriculture, a transition known as the Neolithic Revolution, was followed by a sharp increase in fertility (Bocquet-Appel 2002; Ashraf and Galor 2008). In the

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course of a few centuries, typical communities grew from about 30 individuals to 300 or more, and population densities increased from less than one hunter-gatherer per square mile to 20 or more agriculturalists (Johnson and Earle 2000, pp. 43, 125, 246).

This demographic explosion has been attributed to two main causes. First, food production per unit of land increased (Price and Gebauer 1995). Second, having children was less costly for early agriculturalists than for hunter-gatherers. This was in part because caring for children interfered more with the nomadic lifestyle of hunting and gathering than it did with the sedentary life of agriculturalists (Locay 1983, pp. 60–98). More importantly, the children of agriculturalists contributed substantially more to food production than did the children of hunter-gatherers (Kramer and Boone 2002). In most hunter-gatherer societies, children and adolescents contribute almost nothing to family subsistence (Draper 1976; Kent 2006). Among the !Kung, for example, people do not work until they marry. This takes place at about 20 years of age for males and between 15 and 20 years of age for females (Lee 1965). The children of Gambian primitive agriculturalists, in contrast, work the land from the age of 10 (Ulijaszek 1993).

Although compared to hunter-gatherers early agriculturalists produced food in larger quantities, agriculturalists were more poorly nourished and suffered from poorer health (see, for example, Armelagos et al. 1991; Cohen and Armelagos 1984; Cohen and Crane-Kramer 2007).<sup>1</sup> For example, the archeological record of the Eastern Mediterranean region reveals that male hunter-gatherers who reached adulthood had a life expectancy of 33 years, while females who reached adulthood had a life expectancy of 29 years. After the Neolithic Revolution, the life expectancy of adult males dropped to 32 years, and the life expectancy of adult females dropped to 25 years. Average statures also diminished, from 177 to 165 centimeters among males and from 165 to 152 centimeters among females (Angel 1975).

To make matters worse, the average daily working time increased upon the arrival of agriculture (Harlan 1995; Sahlins 1972). Ethnographic studies suggest that hunter-gatherers worked less than six hours per day, whereas primitive horticulturists worked seven hours on average, and intensive agriculturalists worked nine (Sackett 1996, pp. 338–42).

In this paper, we develop a dynamic price-theoretic model that rationalizes the stylized facts of the transition to agriculture. These include a short-term increase in fertility and long-term reductions in consumption, leisure, and welfare.

The model follows the history of a tribe of hunter-gatherers. The adult members of the tribe derive utility from food consumption, leisure, and childbearing. When the story begins, the tribe is in demographic equilibrium; given his family income and the cost of childbearing, each adult in the tribe chooses to reproduce at the rate of replacement.

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<sup>1</sup>Hutchinson et al. (2007) and Eshed et al. (2010) provide new archeological evidence that contradicts previous findings.

At some point in its history, the tribe discovers agriculture. This new food production technology has two advantages over hunting and gathering. First, agriculture has higher total factor productivity than hunting and gathering; *ceteris paribus*, its higher total factor productivity translates into a higher family income. Second, the children of agriculturalists are more productive than are the children of hunter-gatherers. Because the cost of a child is offset by the food he produces, childbearing is less costly for agriculturalists than for hunter-gatherers.

The sudden increase in the family income and the lower cost of childbearing triggers a demographic explosion among the early agriculturalists. The growth in population progressively reduces the family income of the subsequent generations. Eventually, the family income is so low that the adults once again choose to reproduce at the rate of replacement. When that happens, the population restabilizes at a higher level. In the long run, the family income is even lower than before the onset of agriculture; childbearing carries such low cost for agriculturalists that the only way to persuade them to restrain their fertility is to give them less income than the level obtained by hunter-gatherers. The impoverished agriculturalists also consume less food and have to work more, as compared to their ancestors.

The increase in the labor productivity of children drives the results of the model. In the long run, the increase in the labor productivity of adults only contributes to population growth.

Three conditions must hold for the stylized facts to emerge as implications of the model. The first condition is that children must be normal goods. That is, fertility must be an increasing function of income. The second condition is that the implicit wage rate must decrease with population size. Together, conditions one and two correspond to the classical Malthusian-Ricardian assumptions. The third condition is that consumption and leisure must be normal goods and gross complements of each other.

The remainder of this introduction covers the related literature. We present our model in Section 2. Finally, in Section 3, we offer some conclusions.

## 1.1 Related literature

A variety of theories has been advanced to explain the emergence of agriculture. These range from excessive hunting (Smith 1975) to warfare (Rowthorn and Seabright 2008) and climate change (Dow et al. 2009). Theories on the adoption of agriculture have been extensively surveyed by Weisdorf (2005), who also surveys the main hypotheses forwarded by anthropologists and archaeologists. Therefore, we limit this review to previous explanations for the loss of welfare that followed the Neolithic Revolution.

According to Locay (1989), the fall in the price of children motivated early agriculturalists to substitute fertility for consumption and leisure. He maintains that these changes were *welfare-enhancing*, as the fall in the price of children expanded the agriculturalists' feasible choice set.

Weisdorf (2003) develops a model in which early agriculturalists exchange their leisure time for goods produced by an emerging class of non-food-producing specialists (e.g., craftsmen and bureaucrats). Marceau and Myers (2006) model the fall in consumption and leisure as a tragedy of the commons. Locay (1989), Weisdorf (2004), and Marceau and Myers (2006) assume that the population remains constant during the transition to agriculture. Demography plays no role in their models.

Weisdorf (2009) incorporates Malthusian-Ricardian population principles into a model with two sectors; hunting and agriculture. In his model, the higher productivity of agriculture motivates its adoption, but the subsequent population growth overrides the productivity gain. Weisdorf implicitly assumes that people's only desire in life is to reproduce and that they will work as many hours as it takes to maximize their fertility. He also assumes that food consumption is an increasing function of the energy spent at work, which explains why agriculturalists have to work more than hunter-gatherers.

Robson (2008) develops a model with two goods, children and children's health. In Robson's model, agriculture leads to a higher population density. Infectious diseases, in turn, become more prevalent as the population density increases. This makes the health of children more expensive for agriculturalists than for the less populous hunter-gatherers. Agriculturalists respond by having more children and investing less in their health than did their predecessors. Dalgaard and Strulik (2010) propose a similar model to explain the decline in human body size. They provide some empirical evidence that supports their explanation.

Finally, Lagerlöf (2009) develops a model in which population pressure spurs the development of agriculture. When a society transits to agriculture, a leisurely elite seizes power and enslaves the rest of the population. Slavery reduces the income of the working class below the level they would earn in an egalitarian society of hunter-gatherers.

## 2 A model of agriculture adoption

### 2.1 Model setup

A tribe of hunter-gatherers is on the verge of adopting agriculture.

Time is discrete, indexed by  $t \in \mathbb{N}$ . In time  $t$ , the tribe has  $N_t \in (0, \infty)$  identical adult members. Adults live for one period. When they die, they are replaced by their children.

A representative time- $t$  adult chooses food consumption  $c_t \in [0, \infty)$ , leisure  $l_t \in [0, \infty)$ , and the number of his children  $n_t \in [0, \infty)$  to maximize the following utility function:

$$u_t = \left[ \gamma^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} l_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma\theta}{\sigma-1}} n_t^{1-\theta}, \quad (1)$$

where  $\theta, \gamma \in (0, 1)$ , and  $\sigma \in [0, \infty)$  is the elasticity of substitution between consumption and leisure.<sup>2</sup> Note that the adult does not care about the wellbeing of his children. In Appendix A.2.1 we extend the model to account for this possibility.

There is no labor market. Families produce their own food using three inputs: adult labor, child labor, and a common resource  $Z \in (0, \infty)$  shared by all families in the tribe (e.g., land or water). A family's food production is given by a production function with constant returns to labor:

$$q_t = \frac{AZ}{N_t} (T - l_t + \beta n_t), \quad (2)$$

where  $A \in (0, \infty)$  measures the total factor productivity (TFP),  $Z/N_t \in (0, \infty)$  is the family's share of the common resource,  $T \in (0, \infty)$  is the adult's available time, and  $\beta \in [0, \infty)$  measures child labor productivity in man-hours. Without loss of generality, we normalize  $Z$  to 1.

The representative adult is subject to the following budget constraint:

$$q_t \geq c_t + \kappa n_t \quad (3)$$

where  $\kappa \in [0, \infty)$  represents the food requirements of a child.

Combining equation (2) with inequality (3), and rearranging, we can restate the budget constraint as follows:

$$I_t \geq c_t + w_t l_t + p_t n_t,$$

where  $I_t$  is the implicit total income,  $w_t$  is the implicit hourly wage rate (which is also the price of leisure), and  $p_t$  is the implicit price of children:

$$I_t = \frac{AT}{N_t} = w_t T, \quad (4)$$

$$w_t = \frac{A}{N_t}, \quad (5)$$

$$p_t = \kappa - \frac{\beta A}{N_t} = \kappa - \beta w_t. \quad (6)$$

Note that total income and wages decrease with population, whereas the price of children increases with population. In addition, note that the price of a child is equal to the food he consumes minus the food he produces.

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<sup>2</sup>The assumption that the adult chooses the number of his children may seem unrealistic, but it is not. There is ample evidence that pre-modern peoples controlled their fertility. The methods they used included abstinence, celibacy, prolonged breast-feeding, abortion, and infanticide (Douglas 1966; Cashdan 1985).

Finally, the following equation governs the population dynamics:

$$N_{t+1} = \frac{n_t}{n^*} N_t, \quad (7)$$

where  $n^* > 0$  is the *exogenous* replacement fertility rate.<sup>3</sup>

## 2.2 Solution

The representative time- $t$  adult solves the following problem, taking his total income and the prices as given:

$$\max_{\{c_t, l_t, n_t\}} \left[ \gamma^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} l_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma\theta}{\sigma-1}} n_t^{1-\theta},$$

$$\text{subject to } I_t \geq c_t + w_t l_t + p_t n_t,$$

$$c_t, l_t, n_t \in [0, \infty).$$

Standard calculations yield the adult's demands for consumption, leisure, and children:

$$c_t = \frac{\theta \gamma I_t}{\gamma + (1-\gamma) w_t^{1-\sigma}}, \quad (8)$$

$$l_t = \frac{\theta (1-\gamma) w_t^{-\sigma} I_t}{\gamma + (1-\gamma) w_t^{1-\sigma}}, \quad (9)$$

$$n_t = \frac{(1-\theta) I_t}{p_t}. \quad (10)$$

These demand functions have two important properties. First, the demands are downward-sloping; that is, consumption, leisure, and children are ordinary goods. Second, the demands increase with income; that is, consumption, leisure, and children are normal goods.<sup>4</sup>

Inserting the demand functions into the utility function, we obtain an expression for the indirect utility function (i.e., utility as a function of income and prices):

$$u_t = \frac{\theta^\theta (1-\theta)^{1-\theta} I_t}{p_t^{1-\theta} [\gamma + (1-\gamma) w_t^{1-\sigma}]^{\frac{\theta}{1-\sigma}}}. \quad (11)$$

**Proposition 1** *In the long run, fertility  $n_t$  converges to the rate of replacement  $n^*$ , and population  $N_t$  reaches a unique positive steady state. The steady state is given by*

$$N^* = \frac{[(1-\theta)T + \beta n^*] A}{\kappa n^*}, \quad (12)$$

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<sup>3</sup>One can think of  $n^*$  as equivalent to  $\pi^{-1}$ , where  $\pi \in (0, 1]$  is the probability that a child will survive to reach adulthood.

<sup>4</sup>Empirical evidence indicates that children are normal goods. See, for instance, Lee (1997).

and it will be stable if and only if  $\beta n^* < (1 - \theta)T$ .

**Proof.** Combining equations (4), (6), (7), and (10), we form a dynamic system for the tribe's population:

$$N_{t+1} = f(N_t) = \frac{(1-\theta)AT}{\kappa - \beta \frac{A}{N_t}} N_t.$$

Setting  $N_t = N_{t+1} = N^*$  and solving the system for  $N^*$ , we obtain the positive steady state given in equation (12). In Appendix A.2.1 we prove the stability of the steady state. ■

Inserting the steady-state value of  $N_t$  into equations (4), (5) and (6), we obtain the the steady-state income and prices:

$$I^* = \frac{\kappa n^* T}{(1 - \theta)T + \beta n^*}, \quad (13)$$

$$w^* = \frac{\kappa n^*}{(1 - \theta)T + \beta n^*}, \quad (14)$$

$$p^* = \frac{(1 - \theta) \kappa T}{(1 - \theta)T + \beta n^*}. \quad (15)$$

Because  $I_t$ ,  $w_t$ , and  $p_t$  are all constant in the steady state, consumption, leisure, and fertility remain constant as well.

### 2.3 The adoption of agriculture

Assume that, before the onset of the agricultural era, the tribe has already reached its steady state. This means that all adults chose to reproduce at the exogenous rate of replacement, causing the population and all other model variables to remain constant.

At a certain time  $\tau$ , the tribe discovers agriculture; for instance, through interaction with a neighboring tribe of agriculturalists. This new production technology has two advantages over hunting and gathering. First, agriculture has a higher total factor productivity. Second, the children of agriculturalists are more productive than the children of hunter-gatherers. In the language of our model,  $A$  and  $\beta$  are *permanently* higher for agriculture than they are for hunting and gathering. Figure 1 shows the trajectories of  $A$  and  $\beta$ .

In this section, we will explore the short-term effects of the adoption of agriculture; that is, the changes in the time- $\tau$  variables induced by the increases in  $A$  and  $\beta$ . We will find that consumption and fertility rise in the short run and that leisure time rises if and only if it is a gross complement of consumption. Finally, we will prove that the time- $\tau$  adults will adopt agriculture because their utility increases when they become agriculturalists.



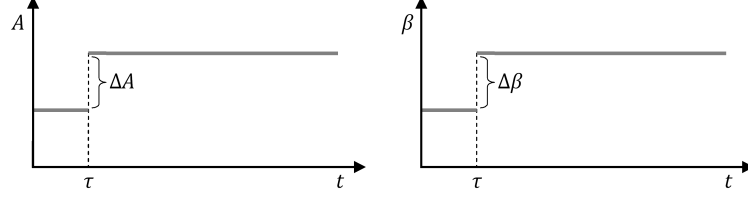


Figure 1: Trajectories of total factor productivity ( $A$ ) and child productivity ( $\beta$ ).

Henceforth, we will use linear expansions to approximate percent changes in variables:

$$\frac{\Delta x}{x} = \frac{\partial \ln x}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln x}{\partial \ln \beta} \frac{\Delta \beta}{\beta},$$

where  $\Delta x$  is the change in any given variable  $x$ . Needless to say, this approximation is more exact the smaller are  $A$  and  $\beta$ .

**Proposition 2** *In time- $\tau$ , the total income and the wage rise, and the price of children falls:*

$$\begin{aligned} \frac{\Delta I_\tau}{I_\tau} &= \frac{\Delta A}{A} > 0, \\ \frac{\Delta w_\tau}{w_\tau} &= \frac{\Delta A}{A} > 0, \\ \frac{\Delta p_\tau}{p_\tau} &= -\frac{\beta w_\tau}{p_\tau} \frac{\Delta A}{A} - \frac{\beta w_\tau}{p_\tau} \frac{\Delta \beta}{\beta} < 0. \end{aligned}$$

**Proof.** See Appendix A.2.2. ■

The intuition is this. The population is fixed at time- $\tau$ . Therefore, a time- $\tau$  family has access to the same amount of the common resource as before the adoption of agriculture. This implies that the increase in TFP translates into a short-term increase in the total income. The wage also rises because each hour of labor is more productive. Finally, the price of children falls in the short run because each child supplies more man-hours ( $\beta$  is higher), and each man-hour pays a higher wage.

**Proposition 3** *In time- $\tau$ , consumption and fertility rise:*

$$\begin{aligned} \frac{\Delta c_\tau}{c_\tau} &= \frac{c_\tau + \sigma w_\tau l_\tau}{c_\tau + w_\tau l_\tau} \frac{\Delta A}{A} > 0, \\ \frac{\Delta n_\tau}{n_\tau} &= \frac{\kappa}{p_\tau} \frac{\Delta A}{A} + \frac{\beta w}{p_\tau} \frac{\Delta \beta}{\beta} > 0. \end{aligned}$$

*Leisure rises if and only if it is a gross complement of consumption:*

$$\frac{\Delta l_\tau}{l_\tau} = \frac{(1 - \sigma) c_\tau}{c_\tau + w_\tau l_\tau} \frac{\Delta A}{A} \begin{cases} > 0 & \text{if } \sigma < 1, \\ = 0 & \text{if } \sigma = 1, \\ < 0 & \text{if } \sigma > 1. \end{cases}$$

**Proof.** See Appendix A.2.3. ■

The intuition behind these results is the following. Consumption is a normal good; thus, when the total income rises, consumption tends to rise as well. If consumption is a gross substitute to leisure ( $\sigma > 1$ ), the increase in the price of leisure (i.e., the hourly wage rate) also tends to increase consumption. Because the income effect and the substitution effect act in the same direction, consumption must rise. If, on the other hand, consumption is a gross complement of leisure ( $\sigma < 1$ ), the increase in the price of leisure tends to reduce consumption. In the particular case of our model, the positive income effect always overrides the substitution effect. Therefore, consumption unambiguously rises in the short run.

Because children are ordinary and normal goods, the reduction in their price and the increase in the total income induce a short-term increase in fertility.

The change in leisure is ambiguous. Because leisure is a normal good, the increase in the total income will act to increase leisure. However, leisure is also an ordinary good. Consequently, the increase in its price will act to reduce leisure. If the complementarity between consumption and leisure is sufficiently high (i.e., if  $\sigma < 1$ ), then the income effect will override the price effect; the adults will increase their leisure time to better enjoy their extra consumption.

Up to this point, we have asked ourselves what would happen if the time- $\tau$  hunter-gatherers became agriculturalists. The next proposition proves that the time- $\tau$  hunter-gatherers will indeed become agriculturalists because the career switch improves their utility.

**Proposition 4** *In time- $\tau$ , utility rises:*

$$\frac{\Delta u_\tau}{u_\tau} = \left( \frac{c_\tau + p_\tau n_\tau}{I_\tau} + \frac{(1 - \theta) \beta w_\tau}{p_\tau} \right) \frac{\Delta A}{A} + \frac{(1 - \theta) \beta w_\tau}{p_\tau} \frac{\Delta \beta}{\beta} > 0.$$

**Proof.** See Appendix A.2.4. ■

The economics behind this result is straightforward; using the same amount of resources, a family of agriculturalists always produces more food than a family of hunter-gatherers.

## 2.4 Long-term effects of the adoption of agriculture

To determine the long-term effects of agriculture, we will compare the steady states of the model before and after time  $\tau$ . We will find that the population rises in the long run, whereas consumption and utility fall. Leisure falls in the long run if and only if it is a gross complement of consumption.

**Proposition 5** *The population increases in the long run:*

$$\frac{\Delta N^*}{N^*} = \frac{\Delta A}{A} + \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} > 0.$$

**Proof.** See Appendix A.2.5. ■

The chain of causality goes like this. As proved in the previous section, agriculture induces a short-term increase in the total income and the wage and a short-term reduction in the price of children. The time- $\tau$  agriculturalists respond to this change in incentives by increasing their fertility above replacement. The demographic equilibrium breaks down, and the population begins to grow.

As the population grows, each family receives an ever-smaller share of the common resource. Therefore, the wage and the total income gradually fall from their time- $\tau$  levels. At the same time, the price of children gradually rises because each hour of child labor pays progressively less.

In response to the gradual reduction in the total income and the gradual increase in the price of children, fertility declines through the generations. Eventually, fertility returns to the rate of replacement. When this happens, the population restabilizes at a higher level than before the adoption of agriculture.

Figure 2 illustrates this process.

**Proposition 6** *The total income, the wage, and the price of children fall in the long run:*

$$\begin{aligned} \frac{\Delta I^*}{I^*} &= -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0, \\ \frac{\Delta w^*}{w^*} &= -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0, \\ \frac{\Delta p^*}{p^*} &= -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0. \end{aligned}$$

**Proof.** See Appendix A.2.6. ■

The economic rationale behind proposition 6 is the following. Equations (4) and (6) relate the population to the total income and to the price of children. Combining these equations we obtain

$$p_t = \kappa - \frac{\beta I_t}{T}. \quad (\text{Technological constraint})$$

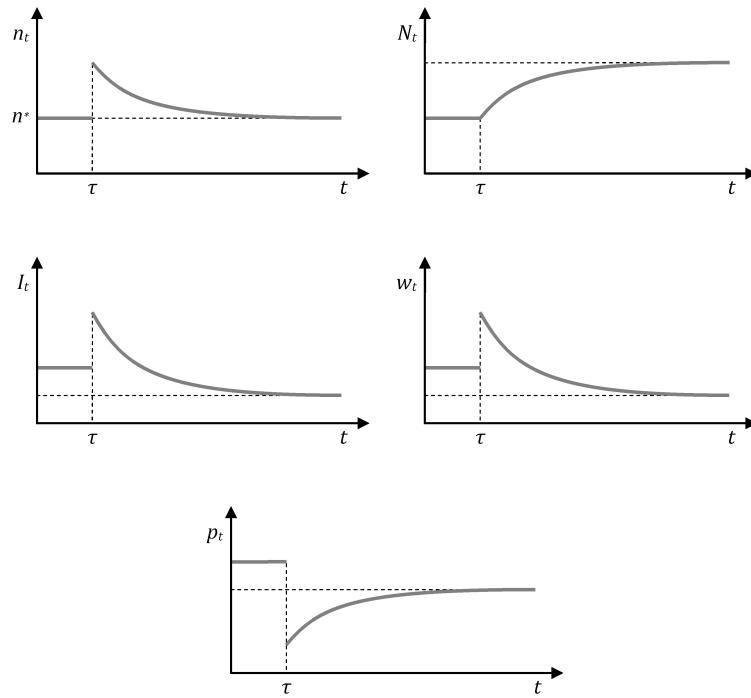


Figure 2: Trajectories of fertility ( $n_t$ ), population ( $N_t$ ), the family income ( $I_t$ ), the wage ( $w_t$ ), and the price of children ( $p_t$ ).

The above equation is a technological constraint because it derives from the budget constraint, which in turn derives from the production function. The technological constraint tells us that when income is high, children are cheap because they are very productive. This constraint must hold at all times, including the steady state.

Equation (10), which we reproduce here, relates fertility to the total income and to the price of children:

$$n_t = \frac{(1 - \theta) I_t}{p_t}.$$

In words, the representative adult will increase his fertility when the total income rises and will reduce his fertility when the price of children increases. In the steady state, fertility must be equal to the rate of replacement,  $n^*$ . Setting  $n_t = n^*$  in the above equation and rearranging, we obtain the following steady-state condition:

$$p^* = \frac{(1 - \theta) I^*}{n^*}. \quad (\text{Steady-state condition})$$

This condition tells us that, to keep the population constant,  $I^*$  and  $p^*$  must move in the same direction. If  $p^*$  falls and  $I^*$  remains the same, the adult will demand more than  $n^*$  children. The only way to persuade the adult to restrain fertility is to reduce  $I^*$  whenever  $p^*$  falls.

Panel A of Figure 3 depicts the technological constraint and the steady-state condition. The steady-state values of  $I_t$  and  $p_t$  lie at the intersection of the two lines.

When  $\beta$  increases, the line that represents the technological constraint rotates inwards. Panel B of Figure 3 represents this situation. As the technological constraint rotates, the equilibrium moves from E to E<sub>2</sub>, the total income falls from  $I^*$  to  $I_2^*$ , and the price of children falls from  $p^*$  to  $p_2^*$ .

Note that this result is driven entirely by the increase in child labor productivity. The increase in TFP plays no role in the determination of  $I^*$  and  $p^*$ . In the long-term, the increase in TFP only contributes to population growth.

**Proposition 7** *Consumption falls in the long run:*

$$\frac{\Delta c^*}{c^*} = -\frac{c^* + \sigma w^* l^*}{c^* + w^* l^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0.$$

*Leisure falls if and only if it is a gross complement of consumption:*

$$\frac{\Delta l^*}{l^*} = -\frac{(1 - \sigma) c^*}{c^* + w^* l^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} \begin{cases} < 0 & \text{if } \sigma < 1, \\ = 0 & \text{if } \sigma = 1, \\ > 0 & \text{if } \sigma > 1. \end{cases}$$

**Proof.** See Appendix A.2.7. ■

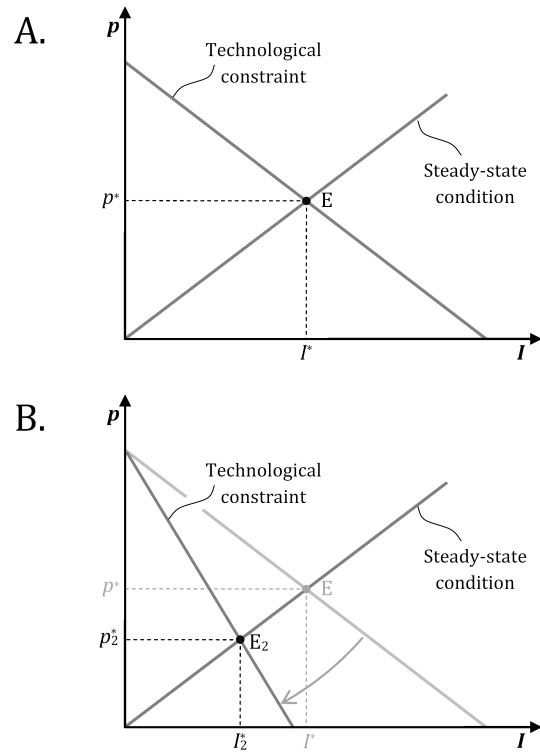


Figure 3: Determination of the steady-state equilibrium values of  $I_t$  and  $p_t$  (panel A). When the child labor productivity rises, the technology constraint rotates inwards, and both  $I^*$  and  $p^*$  fall (panel B).

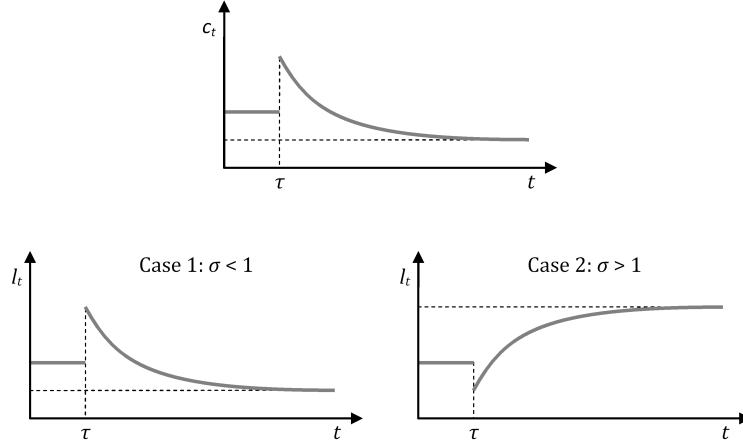


Figure 4: Trajectories of consumption ( $c_t$ ) and leisure ( $l_t$ ).

The above results are a direct consequence of the long-term reductions in the total income, the wage, and the price of children.

Consumption is a normal good; thus, when total income falls, consumption tends to fall as well. If consumption is a gross substitute to leisure, the reduction in the price of leisure (i.e., the hourly wage rate) also tends to reduce consumption. Because the income effect and the substitution effect act in the same direction, consumption must rise. If, on the other hand, consumption is a gross complement of leisure, the reduction in the price of leisure tends to increase consumption. In the particular case of our model, the negative income effect always overrides the substitution effect. Therefore, consumption unambiguously falls in the long run.

The change in leisure is ambiguous. Because leisure is a normal good, the reduction in the total income will act to reduce leisure. However, leisure is also an ordinary good. Therefore, the reduction in its price will act to increase leisure. If the complementarity between consumption and leisure is sufficiently high (i.e., if  $\sigma < 1$ ), then the income effect will override the price effect; the adults will reduce their leisure time because leisure becomes less interesting when consumption falls.

Figure 4 displays the trajectories of consumption and leisure.

**Proposition 8** *Utility falls in the long run:*

$$\frac{\Delta u^*}{u^*} = -\frac{c^*}{I^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0.$$

**Proof.** See Appendix A.2.8. ■

In the long run, fertility always returns to the exogenous rate of replacement. Therefore, the long-term gain or loss of utility is entirely determined by the changes in consumption and leisure.

Proposition 8 states that it does not matter whether consumption and leisure are gross complements, gross neutral, or gross substitutes; agriculturalists always end up being worse-off than hunter-gatherers.

Why should utility fall in the long run?

From equation (10) it follows that a typical adult will destine a fraction  $1 - \theta$  of his income to buy children, leaving the remaining fraction to buy consumption and leisure. It follows that an adult who lives in a steady state faces the following budget constraint:

$$w^* \tilde{T} \geq c_t + w^* l_t,$$

where  $\tilde{T} = \theta T$ . We know that, in the steady state, the adult chooses  $n_t = n^*$ . Therefore, the steady-state adult problem can be restated as follows:

$$\begin{aligned} \max_{\{c_t, l_t\}} \quad & \phi \left[ \gamma^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} l_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma\theta}{\sigma-1}}, \\ \text{subject to} \quad & w^* (\tilde{T} - l_t) \geq c_t \\ & c_t, l_t \in [0, \infty), \end{aligned}$$

where  $\phi = (n^*)^{1-\theta}$  is a constant. The above problem is the classical consumer problem with CES preferences. It is well known that the utility of the consumer increases when the hourly wage increases. Because the wage is lower for steady-state agriculturalists than for steady-state hunter-gatherers, the utility of the agriculturalists must also be lower.

Panel A of Figure 5 shows the trajectory of utility.

Although agriculture reduces welfare in the long run, agriculturalists will not revert to hunting and gathering. For any given population size, families can always produce more by farming than by hunting and gathering. From equation (2), it follows that

$$\underbrace{\frac{A + \Delta A}{N_t} [T - l_t + (\beta + \Delta\beta) n_t]}_{\text{Production of a family of agriculturalists}} > \underbrace{\frac{A}{N_t} (T - l_t + \beta n_t)}_{\text{Production of a family of hunter-gatherers}},$$

for all values of  $N_t$ ,  $l_t$  and  $n_t$ . This explains why people will never abandon agriculture.

Panel B of Figure 5 shows the utility of agriculture and the utility of hunting and gathering for each period. Note that the utility of hunting and gathering always runs below the utility of agriculture.



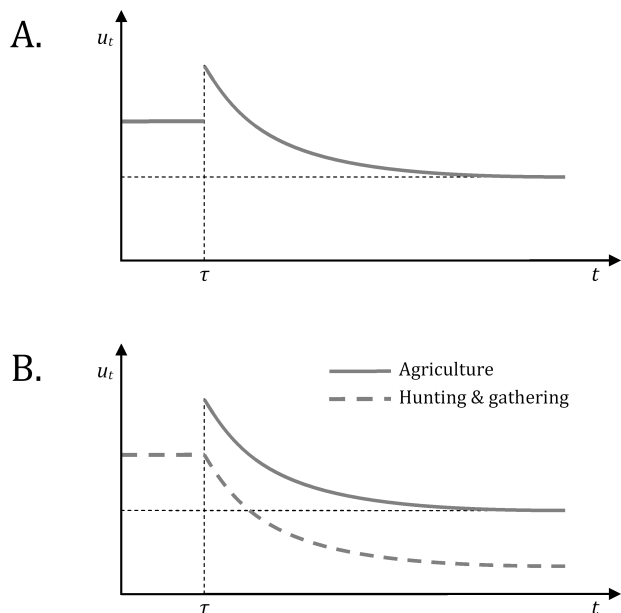


Figure 5: Trajectory of utility (panel A). The utility of hunting and gathering always runs below the utility of agriculture (panel B).

### 3 Conclusion

Our model sheds some light on a central puzzle in the literature: did agriculture emerge out of opportunity, or were our ancestors forced into farming? Traditional scholarship regards agriculture as highly desirable: once humans discovered agriculture and recognized the potential productivity gains associated with it, beginning to farm was an obvious decision (Trigger 1989). In recent times, this idea has been called into question, largely because it has been shown that taking up agriculture reduced the standard of living of our ancestors (Harlan 1992; Sahlins 1974). The reduction in the standard of living seems to suggest that our ancestors did not willingly become agriculturalists. Instead, they may have been forced into agriculture by some external pressure [e.g., overpopulation (Binford 1968; Flannery 1969) or by the extinction of mammoths and other game (Smith 1975)]. Diamond (1987) has called agriculture “the worst mistake in the history of the human race.”

We have shown herein that the adoption of agriculture is perfectly consistent with economic rationality. People were lured into agriculture by the immediate productivity gains that resulted from its development; however, the demographic explosion that ensued overrode the gains and impoverished later generations of agriculturalists. The later generations were forced to remain

agriculturalists for the same reason that agriculture was originally adopted, which is that agriculture is more productive than hunting and gathering for any given level of population.

Some external factors may have influenced our ancestors' decision to become and remain agriculturalists. Their decision, however, is hardly an economic puzzle. It can be explained within the standard framework of price theory.

## A Appendix

### A.1 A forward-looking version of the model

For the sake of simplicity, we will assume that the adults supply all their available time as labor and that children do not work. We will continue to assume that children are cheaper for agriculturalists than for hunter-gatherers; for example, because it is easier for adults to care for children when the tribe is sedentary.

We will show that the tribe will adopt agriculture only if its members care little about the welfare of future generations, if the TFP of agriculture is high relative to the TFP of hunting and gathering, and if children are not too cheap for agriculturalists. We will also show that, if the tribe adopts agriculture, the population will rise in the long run, and consumption and utility will fall.

A representative time- $t$  adult maximizes

$$u_t = \frac{c_t^\theta n_t^{1-\theta} u_{t+1}^\delta}{\theta^\theta (1-\theta)^{1-\theta}}.$$

where  $\theta \in (0, 1)$  and  $\delta \in [0, 1)$ . Parameter  $\delta$  is a discount factor. If  $\delta > 0$ , the adult cares about the welfare of his children. Taking logarithms, we reformulate the utility function as follows:

$$v_t = \ln u_t = \ln \frac{c_t^\theta n_t^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}} + \delta v_{t+1}. \quad (16)$$

The adults do not leave inheritances. Only through its fertility decisions does one generation affect the welfare of those to follow. A higher fertility in the present generation leads to a lower income of future generations. Because each adult is infinitesimal, he cannot by himself affect the future population levels. Therefore, from the point of view of an adult,  $v_{t+1}$  is a constant.

The representative adult is subject to the following budget constraint:

$$\frac{A_t T}{N_t} \geq c_t + \kappa_t n_t.$$

Standard calculations yield

$$c_t = \frac{\theta A_t T}{N_t}, \quad (17)$$

$$n_t = \frac{(1 - \theta) A_t T}{\kappa_t N_t}. \quad (18)$$

Combining equations (16), (18) and (17), we obtain the adult's indirect utility function:

$$v_t = \ln \frac{A_t T}{N_t \kappa_t^{1-\theta}} + \delta v_{t+1}. \quad (19)$$

The following equation governs the population dynamics:

$$N_{t+1} = \frac{n_t}{n^*} N_t. \quad (20)$$

Replacing equation (17) into equation (20), we obtain

$$N_{t+1} = \frac{(1 - \theta) A_t T}{\kappa_t n^*} \quad (21)$$

Generation- $t$  of the tribe has a chief. He must choose between agriculture and hunting and gathering. Let  $s_t \in \{0, 1\}$  denote the chief's decision. If  $s_t = 0$ , generation  $t$  will hunt and gather. If  $s_t = 1$ , generation  $t$  will practice agriculture. The TFP and the cost of children depend on the chief's choice of production technology:

$$A_t = s_t A^A + (1 - s_t) A^H, \quad (22)$$

$$\kappa_t = s_t \kappa^A + (1 - s_t) \kappa^H, \quad (23)$$

where  $A^A > A^H > 0$ , and  $0 < \kappa^A < \kappa^H$ .

The time- $t$  chief maximizes the utility of the adults, assuming that they will respond optimally to incentives. He also takes into account the fact that all future chiefs will make optimal decisions. This gives rise to a recursive maximization problem. Using equations (19), (22) and (23), we form the chief's value function:

$$v(N, s) = \ln \frac{[s A^A + (1 - s) A^H] T}{N [s \kappa^A + (1 - s) \kappa^H]^{1-\theta}} + \delta v[N'(s), s^*(N'(s))]. \quad (24)$$

Population  $N$  is the only state variable, and  $N'(s)$  is the next period's population, and Finally,  $s^*(N)$  is the chief's optimal decision when the population level is  $N$ . From equations (21), (22) and

(23), we get

$$N'(s) = \frac{(1-\theta)[sA^A + (1-s)A^H]T}{[s\kappa^A + (1-s)\kappa^H]n^*}. \quad (25)$$

Note that  $N'(s)$  does not depend on  $N$ .

The chief's optimal decision is given by

$$s^*(N) = \begin{cases} 1 & \text{if } v(N, 1) > v(N, 0), \\ 0 & \text{otherwise.} \end{cases}$$

**Proposition 9** *Regardless of the level of population, the chief will choose agriculture if and only if*

$$\delta < \frac{\ln(A^A/A^H)}{\ln(\kappa^H/\kappa^A)} + 1 - \theta.$$

That is,

$$s^*(N) = \begin{cases} 1 & \text{if } \delta < \frac{\ln(A^A/A^H)}{\ln(\kappa^H/\kappa^A)} + 1 - \theta, \\ 0 & \text{otherwise.} \end{cases}$$

**Proof.** The chief will adopt agriculture if and only if  $v(N, 1) - v(N, 0) > 0$ . However, from equation (24), we know that

$$v(N, 1) - v(N, 0) = \ln \frac{A^A}{A^H} \left( \frac{\kappa^H}{\kappa^A} \right)^{1-\theta} - \delta \{v[N'(1), s^*(N'(1))] - v[N'(0), s^*(N'(0))]\}$$

No term in the above equation depends on  $N$ . This implies that the chief's decision will not depend on  $N$ . In other words,  $s^*(N) = s^*$ , where  $s^*$  is a constant.

Iterating the value function once more, we obtain

$$v(N, s) = \ln \frac{[sA^A + (1-s)A^H]T}{N[s\kappa^A + (1-s)\kappa^H]^{1-\theta}} + \delta \left( \ln \frac{[s^*A^A + (1-s^*)A^H]T}{N'(s)[s^*\kappa^A + (1-s^*)\kappa^H]^{1-\theta}} + \delta v[N'(s^*), s^*] \right).$$

From this equation, it follows that

$$v(N, 1) - v(N, 0) = \ln \left[ \frac{A^A}{A^H} \left( \frac{\kappa^H}{\kappa^A} \right)^{1-\theta} \right] + \delta \ln \frac{N'(0)}{N'(1)}.$$

Combining this equation with equation (25), we obtain

$$v(N, 1) - v(N, 0) = \ln \left[ \frac{A^A}{A^H} \left( \frac{\kappa^H}{\kappa^A} \right)^{1-\theta} \right] - \delta \ln \frac{\kappa^H}{\kappa^A}.$$

Therefore,  $v(N, 1) - v(N, 0) > 0$  if and only if

$$\ln \left[ \left( \frac{A^A}{A^H} \right) \left( \frac{\kappa^H}{\kappa^A} \right)^{1-\theta} \right] - \delta \ln \frac{\kappa^H}{\kappa^A} > 0,$$

or, equivalently, if and only if

$$\delta < \frac{\ln(A^A/A^H)}{\ln(\kappa^H/\kappa^A)} + 1 - \theta. \quad (26)$$

■

Proposition 9 tells us that the chief will never change his mind. If he chooses agriculture, he will stick to this decision. From equations (10), (21) and (22), we obtain the steady-state level of population:

$$N(s^*) = \frac{(1-\theta)[s^*A^A + (1-s^*)A^H]T}{[s^*\kappa^A + (1-s^*)\kappa^H]n^*}. \quad (27)$$

Let  $N^A$  be the steady-state level of population if the chief chooses agriculture, and let  $N^H$  be the steady-state level of population if he chooses hunting and gathering. From equation (27) it follows that

$$N^A - N^H = \frac{(1-\theta)(A^A\kappa^H - A^H\kappa^A)T}{\kappa^A\kappa^Hn^*} > 0,$$

which is positive because  $A^A > A^H$  and  $\kappa^A < \kappa^H$ . In the long run, the population is higher if the tribe adopts agriculture.

Combining equations (18), (22), (23), (27), we obtain the steady-state level of consumption:

$$c(s^*) = \frac{\theta[s^*\kappa^A + (1-s^*)\kappa^H]n^*}{1-\theta}. \quad (28)$$

Let  $c^A$  be the steady-state level of consumption if the chief chooses agriculture, and let  $c^H$  be the steady-state level of consumption if he chooses hunting and gathering. From equation (28) it follows that

$$c^A - c^H = \frac{\theta(\kappa^A - \kappa^H)n^*}{1-\theta} < 0.$$

In the long-term, agriculturalists enjoy a lower level of consumption.

Setting  $v_t = v_{t+1} = v(s^*)$  in equation (19), solving for  $v(s^*)$ , and combining the result with equations (22), (23) and (27) we obtain the steady-state level of utility:

$$v(s^*) = \frac{1}{1-\delta} \ln \frac{n^* [s^*(\kappa^A)^\theta + (1-s^*)(\kappa^A)^\theta]}{1-\theta},$$

In the long run, utility will be lower for agriculturalists:

$$v^A - v^H = \frac{\theta}{1-\delta} \ln \frac{\kappa^A}{\kappa^H} < 0.$$

## A.2 Proofs

### A.2.1 Proof of Proposition 1

To prove the stability of the positive steady state, we resort to the Hartman-Grobman theorem. According to this theorem, the steady state  $N^*$  will be stable if  $|f'(N^*)| < 1$ . Differentiating  $f(N_t)$ , and evaluating the derivative at  $N^*$ , we get

$$f'(N^*) = \frac{-(1-\theta)\beta A^2 T}{n^*(\beta A - \kappa N^*)^2} = -\frac{\beta n^*}{(1-\theta)T},$$

which will be less than one in absolute value if and only if  $\beta n^* < (1-\theta)T$ .

### A.2.2 Proof of Proposition 2

By definition,

$$\frac{\Delta I_\tau}{I_\tau} = \frac{\partial \ln I_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln I_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (29)$$

Log-differentiating equation (4), we obtain

$$\frac{\partial \ln I_\tau}{\partial \ln A} = 1, \quad (30)$$

$$\frac{\partial \ln I_\tau}{\partial \ln \beta} = 0. \quad (31)$$

Combining equations (29), (30), and (31), it follows that

$$\frac{\Delta I_\tau}{I_\tau} = \frac{\Delta A}{A} > 0.$$

By definition,

$$\frac{\Delta w_\tau}{w_\tau} = \frac{\partial \ln w_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln w_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (32)$$

Log-differentiating equation (5), we obtain

$$\frac{\partial \ln w_\tau}{\partial \ln A} = 1, \quad (33)$$

$$\frac{\partial \ln w_\tau}{\partial \ln \beta} = 0. \quad (34)$$

Combining equations (32), (33), and (34), it follows that

$$\frac{\Delta w_\tau}{w_\tau} = \frac{\Delta A}{A} > 0.$$

By definition,

$$\frac{\Delta p_\tau}{p_\tau} = \frac{\partial \ln p_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln p_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (35)$$

Log-differentiating equation (6), we obtain

$$\frac{\partial \ln p_\tau}{\partial \ln A} = -\frac{\beta w_\tau}{p_t}, \quad (36)$$

$$\frac{\partial \ln p_\tau}{\partial \ln \beta} = -\frac{\beta w_\tau}{p_t}. \quad (37)$$

Finally, combining equations (35), (36), and (37), it follows that

$$\frac{\Delta p_\tau}{p_\tau} = -\frac{\beta w_\tau}{p_\tau} \frac{\Delta A}{A} - \frac{\beta w_\tau}{p_\tau} \frac{\Delta \beta}{\beta} < 0.$$

### A.2.3 Proof of Proposition 3

By definition,

$$\frac{\Delta c_\tau}{c_\tau} = \frac{\partial \ln c_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln c_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (38)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln c_\tau}{\partial \ln A} = \frac{\partial \ln c_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln A} + \frac{\partial \ln c_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial A} + \frac{\partial \ln c_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln A}, \quad (39)$$

$$\frac{\partial \ln c_\tau}{\partial \ln \beta} = \frac{\partial \ln c_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln \beta} + \frac{\partial \ln c_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial \ln \beta} + \frac{\partial \ln c_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln \beta}. \quad (40)$$

On the other hand, the income and price elasticities for  $c_t$  are given by

$$\frac{\partial \ln c_t}{\partial \ln I_t} = 1, \quad (41)$$

$$\frac{\partial \ln c_t}{\partial \ln w_t} = \frac{(\sigma - 1) w_t l_t}{c_t + w_t l_t}, \quad (42)$$

$$\frac{\partial \ln c_t}{\partial \ln p_t} = 0, \quad (43)$$

for all  $t$ . Finally, combining equations (30), (31), (33), (34), and (38) to (43), it follows that

$$\frac{\Delta c_\tau}{c_\tau} = \frac{c_\tau + \sigma w_\tau l_\tau}{c_\tau + w_\tau l_\tau} \frac{\Delta A}{A} > 0,$$

which we set out to prove.

By definition,

$$\frac{\Delta n_\tau}{n_\tau} = \frac{\partial \ln n_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln n_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (44)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln n_\tau}{\partial \ln A} = \frac{\partial \ln n_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln A} + \frac{\partial \ln n_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial A} + \frac{\partial \ln n_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln A}, \quad (45)$$

$$\frac{\partial \ln n_\tau}{\partial \ln \beta} = \frac{\partial \ln n_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln \beta} + \frac{\partial \ln n_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial \ln \beta} + \frac{\partial \ln n_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln \beta}. \quad (46)$$

On the other hand, the income and price elasticities for  $n_t$  are given by

$$\frac{\partial \ln n_t}{\partial \ln I_t} = 1, \quad (47)$$

$$\frac{\partial \ln n_t}{\partial \ln w_t} = 0, \quad (48)$$

$$\frac{\partial \ln c_t}{\partial \ln p_t} = -1, \quad (49)$$

for all  $t$ . Combining equations (30), (31), (36), (37), and (44) to (49), we get

$$\frac{\Delta n_\tau}{n_\tau} = \left( \frac{p_\tau - \beta A}{p_\tau} \right) \frac{\Delta A}{A} + \frac{\beta A}{p_\tau} \frac{\Delta \beta}{\beta}.$$

However, from equation (6), we know that  $p_\tau - \beta A = \kappa$ . Using this identity with the above equation, we obtain

$$\frac{\Delta n_\tau}{n_\tau} = \frac{\kappa}{p_\tau} \frac{\Delta A}{A} + \frac{\beta A}{p_\tau} \frac{\Delta \beta}{\beta},$$

which we set out to prove.

By definition,

$$\frac{\Delta l_\tau}{l_\tau} = \frac{\partial \ln l_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln l_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (50)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln l_\tau}{\partial \ln A} = \frac{\partial \ln l_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln A} + \frac{\partial \ln l_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial A} + \frac{\partial \ln l_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln A}, \quad (51)$$

$$\frac{\partial \ln l_\tau}{\partial \ln \beta} = \frac{\partial \ln l_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln \beta} + \frac{\partial \ln l_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial \ln \beta} + \frac{\partial \ln l_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln \beta}. \quad (52)$$



On the other hand, the income and price elasticities for  $l_t$  are given by

$$\frac{\partial \ln l_t}{\partial \ln I_t} = 1, \quad (53)$$

$$\frac{\partial \ln l_t}{\partial \ln w_t} = -\frac{\sigma c_t + w_t l_t}{c_t + w_t l_t}, \quad (54)$$

$$\frac{\partial \ln l_t}{\partial \ln p_t} = 0, \quad (55)$$

for all  $t$ . Finally, combining equations (30), (31), (33), (34), and (50) to (55), we get

$$\frac{\Delta l_\tau}{l_\tau} = \frac{(1 - \sigma) c_\tau}{c_\tau + w_\tau l_\tau} \frac{\Delta A}{A},$$

which is positive if  $\sigma < 1$ , zero if  $\sigma = 1$ , and negative if  $\sigma > 1$ . That completes the proof.

#### A.2.4 Proof of Proposition 4

By definition,

$$\frac{\Delta u_\tau}{u_\tau} = \frac{\partial \ln u_\tau}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln u_\tau}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (56)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln u_\tau}{\partial \ln A} = \frac{\partial \ln u_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln A} + \frac{\partial \ln u_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial A} + \frac{\partial \ln u_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln A}, \quad (57)$$

$$\frac{\partial \ln u_\tau}{\partial \ln \beta} = \frac{\partial \ln u_\tau}{\partial \ln I_\tau} \frac{\partial \ln I_\tau}{\partial \ln \beta} + \frac{\partial \ln u_\tau}{\partial \ln w_\tau} \frac{\partial \ln w_\tau}{\partial \ln \beta} + \frac{\partial \ln u_\tau}{\partial \ln p_\tau} \frac{\partial \ln p_\tau}{\partial \ln \beta}. \quad (58)$$

On the other hand,

$$\frac{\partial \ln u_t}{\partial \ln I_t} = 1, \quad (59)$$

$$\frac{\partial \ln u_t}{\partial \ln w_t} = -\frac{w_t l_t}{I_t}, \quad (60)$$

$$\frac{\partial \ln u_t}{\partial \ln p_t} = -(1 - \theta), \quad (61)$$

for all  $t$ . Combining equations (30), (31), (33), (34), (36), (37), and (56) to (61), we get

$$\frac{\Delta u_\tau}{u_\tau} = \left( 1 - \frac{w_\tau l_\tau}{I_\tau} + \frac{(1 - \theta) \beta w_\tau}{p_\tau} \right) \frac{\Delta A}{A} + \frac{(1 - \theta) \beta w_\tau}{p_\tau} \frac{\Delta \beta}{\beta}.$$

However, from the budget constraint, we know that  $w_\tau l_\tau = I_\tau - c_t - \kappa p_t$ . Using this identity with the above equation, we obtain

$$\frac{\Delta u_\tau}{u_\tau} = \left( \frac{c_\tau + \kappa p_\tau}{I_\tau} + \frac{(1-\theta)\beta w_\tau}{p_\tau} \right) \frac{\Delta A}{A} + \frac{(1-\theta)\beta w_\tau}{p_\tau} \frac{\Delta \beta}{\beta} \geq 0,$$

which we set out to prove.

### A.2.5 Proof of Proposition 5

By definition,

$$\frac{\Delta N^*}{N^*} = \frac{\partial \ln N^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln N^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta}.$$

Differentiating equation (12), we get

$$\begin{aligned} \frac{\partial \ln N^*}{\partial \ln A} &= 1, \\ \frac{\partial \ln N^*}{\partial \ln \beta} &= \frac{\beta n^*}{(1-\theta)T + \beta n^*}. \end{aligned}$$

Combining the three equations above, we obtain

$$\frac{\Delta N^*}{N^*} = \frac{\Delta A}{A} + \frac{\beta n^*}{(1-\theta)T + \beta n^*} \frac{\Delta \beta}{\beta}. \quad (62)$$

On the other hand, from equation (14), we know that

$$w^* = \frac{\kappa n^*}{(1-\theta)T + \beta n^*}.$$

Rearranging the above expression, we obtain

$$\frac{\beta w^*}{\kappa} = \frac{\beta n^*}{(1-\theta)T + \beta n^*}. \quad (63)$$

Finally, inserting equation (63) into equation (62), we obtain

$$\frac{\Delta N^*}{N^*} = \frac{\Delta A}{A} + \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} > 0,$$

which we set out to prove.

### A.2.6 Proof of Proposition 6

By definition,

$$\frac{\Delta I^*}{I^*} = \frac{\partial \ln I^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln I^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta} \quad (64)$$

Log-differentiating equation 13, we obtain

$$\frac{\partial \ln I^*}{\partial \ln A} = 0, \quad (65)$$

$$\frac{\partial \ln I^*}{\partial \ln \beta} = -\frac{\beta w^*}{\kappa}. \quad (66)$$

Combining equations (64), (65), and (66) we get

$$\frac{\Delta I^*}{I^*} = -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0,$$

as desired.

By definition,

$$\frac{\Delta w^*}{w^*} = \frac{\partial \ln w^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln w^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta} \quad (67)$$

Log-differentiating equation (14), we obtain

$$\frac{\partial \ln w^*}{\partial \ln A} = 0, \quad (68)$$

$$\frac{\partial \ln w^*}{\partial \ln \beta} = -\frac{\beta w^*}{\kappa}. \quad (69)$$

Combining equations (67), (68), and (69), we get

$$\frac{\Delta w^*}{w^*} = -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0,$$

as desired.

By definition,

$$\frac{\Delta p^*}{p^*} = \frac{\partial \ln p^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln p^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta} \quad (70)$$

Log-differentiating equation (15), we obtain

$$\frac{\partial \ln p^*}{\partial \ln A} = 0, \quad (71)$$

$$\frac{\partial \ln p^*}{\partial \ln \beta} = -\frac{\beta w^*}{\kappa}. \quad (72)$$

Combining equations (70), (71), and (72), we get

$$\frac{\Delta p^*}{p^*} = -\frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0,$$

as desired.

### A.2.7 Proof of Proposition 7

By definition,

$$\frac{\Delta c^*}{c^*} = \frac{\partial \ln c^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln c^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (73)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln c^*}{\partial \ln A} = \frac{\partial \ln c^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln A} + \frac{\partial \ln c^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial A} + \frac{\partial \ln c^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln A}, \quad (74)$$

$$\frac{\partial \ln c^*}{\partial \ln \beta} = \frac{\partial \ln c^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln \beta} + \frac{\partial \ln c^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial \ln \beta} + \frac{\partial \ln c^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln \beta}. \quad (75)$$

Combining equations (41), (42), (43), (65), (66), (68), (69), (73), (74), and (75), we get

$$\frac{\Delta c^*}{c^*} = -\frac{c^* + \sigma w^* l^*}{c^* + w^* l^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta} < 0,$$

which we set out to prove.

By definition,

$$\frac{\Delta l^*}{l^*} = \frac{\partial \ln l^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln l^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (76)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln l^*}{\partial \ln A} = \frac{\partial \ln l^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln A} + \frac{\partial \ln l^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial A} + \frac{\partial \ln l^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln A}, \quad (77)$$

$$\frac{\partial \ln l^*}{\partial \ln \beta} = \frac{\partial \ln l^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln \beta} + \frac{\partial \ln l^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial \ln \beta} + \frac{\partial \ln l^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln \beta}. \quad (78)$$

Combining equations (53), (54), (55), (65), (66), (68), (69), (73), (74), and (75), we get

$$\frac{\Delta l^*}{l^*} = -\frac{(1 - \sigma) c^*}{c^* + w^* l^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta},$$

which is negative if  $\sigma < 1$ , zero if  $\sigma = 1$ , and positive if  $\sigma > 1$ . This completes the proof.

### A.2.8 Proof of Proposition 8

By definition,

$$\frac{\Delta u^*}{u^*} = \frac{\partial \ln u^*}{\partial \ln A} \frac{\Delta A}{A} + \frac{\partial \ln u^*}{\partial \ln \beta} \frac{\Delta \beta}{\beta}. \quad (79)$$

Applying the chain rule, we obtain

$$\frac{\partial \ln u^*}{\partial \ln A} = \frac{\partial \ln u^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln A} + \frac{\partial \ln u^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial A} + \frac{\partial \ln u^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln A}, \quad (80)$$

$$\frac{\partial \ln u^*}{\partial \ln \beta} = \frac{\partial \ln u^*}{\partial \ln I^*} \frac{\partial \ln I^*}{\partial \ln \beta} + \frac{\partial \ln u^*}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial \ln \beta} + \frac{\partial \ln u^*}{\partial \ln p^*} \frac{\partial \ln p^*}{\partial \ln \beta}. \quad (81)$$

Combining equations (59), (60), (61), (65), (66), (68), (69), (71), (72), (79), (80), and (81), we get

$$\frac{\Delta u^*}{u^*} = -\frac{\theta I^* - w^* l^*}{I^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta}. \quad (82)$$

On the other hand, from equations (8) and (9), we obtain

$$c_t + w_t l_t = \theta I_t,$$

for all  $t$ . Therefore,

$$\theta I^* - w^* l^* = c^*.$$

Using the above identity with equation (82), we obtain

$$\frac{\Delta u^*}{u^*} = -\frac{c^*}{I^*} \frac{\beta w^*}{\kappa} \frac{\Delta \beta}{\beta},$$

which we set out to prove.

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