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*Published in:*  
Fifth Congress of the European Society for Research in Mathematics Education

*Publication date:*  
2008

*Document version*  
Publisher's PDF, also known as Version of record

*Citation for published version (APA):*  
Winsløw, C., & Madsen, L. M. (2008). Interplay between research and teaching from the perspective of mathematicians. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2379-2388). European Society for Research in Mathematics Education .  
Proceedings: Conference for European Research in Mathematics Education

## **INTERPLAY BETWEEN RESEARCH AND TEACHING FROM THE PERSPECTIVE OF MATHEMATICIANS**

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*In this paper, we examine the relation between teaching and research on mathematics in universities. We suggest that this relation can be fruitfully examined from the perspective of mathematicians' praxeologies (organisations of didactical and mathematical practice). We illustrate the approach with data from an interview study involving five top-level mathematicians.*

### **INTRODUCTION**

One of the specificities of higher education is that scientific research is “close by” in several ways: through teachers who are also researchers; as a future career option for students; and because the contents and methods taught can be conceived as “closer” to actual research than pre-university education. The last point is particularly strong in the case of mathematics, where the school subject is essentially based on mathematical knowledge developed before 1900. The distance between “academic mathematics” and “school mathematics” is manifest and the study of their relation is a classical theme in the didactics of mathematics (e.g. Chevallard, 1985; Brousseau, 1986, 33ff). On the other hand, research activity is often used as a kind of ideal for pupils' exploration of school mathematics, both in the mathematics education literature (we return to this in the next section) and in official curricula.

On this background, it is strange that university mathematics courses tend to be rather “didactic”, in the common sense. The learning of students in such courses seems to be very different from learning through research. Burton (2004, p. 198) concludes from a large-scale interview study with British mathematicians' that

the gap between mathematicians' views of mathematical knowing and that encountered by learners is monstrous. It could be said to be an indicator as to why so much teaching in mathematics fails...

What are the reasons for this gap? How do the mathematicians articulate the relations between research and teaching practice? If they also see a “gap”, how do they explain it: is it simply due to norms and choices, in the sense that they do not see it as an ideal that research activity and teaching are closely related? Or does it mainly arise from constraints and necessities, for instance of an institutional nature? In this paper, we look more closely at these questions. Above all, we try – supported by a case study of five mathematicians' views – to state them more precisely. We begin by putting them in into a wider context, in part linked to higher education in general.

## MOTIVATION AND BACKGROUND.

The meaning of the term “university” is one that varies considerably, both historically and geographically. In particular, three very different university models have developed within Europe over the past centuries (cf. Mora, 2001):

- the “Humboldtian” national universities of Northern Europe, democratically governed and serving the ideal of *Einheit von Lehre und Forschung* (unity of teaching and scientific research),
- the “Napoléonic” university of the Mediterranean region, a prestigious institution controlled by the State and entrusted above all to form its élite of civil servants,
- the “Oxbridge” university, with independent and private colleges, which is more similar to the Medieval university in terms of aims and structures.

In Europe, these models still occur as implicitly assumed bases for lay discussions on university politics. But in the last decades, the functioning and functions of universities have been rapidly changing, and there is a tendency of *convergence* towards universities as corporation-like “service providers of the knowledge age” (*ibid.*, 108). To these new knowledge corporations, both education and scientific research are crucial “products” which are being marketed, sold and frequently evaluated. We have several international rankings of universities, based primarily on “measures” of their performance on research and education. In this global competition, top academics may soon be “traded” at the price level of soccer stars...

On this background, it is not surprising that the *relation* between teaching and research gets renewed attention: what will be left of the *Einheit von Lehre und Forschung* in mass universities that compete for funding and students, and where teaching and research compete for the time of its employees? Should they (continue to) co-exist in the same institutions and be delivered by the same people? If so, why?

From the perspectives of the university models mentioned above, these questions may seem almost sacrilegious. The classical models simply assume a general and intrinsic symbiosis, valid in any discipline. In the so-called *higher education* research literature, this hypothesis of a general “nexus” between teaching and research (TR-nexus, for short) has been scrutinised in hundreds of papers. The term “nexus” means a semantic “connection” of phenomena that influence each other, in positive or negative ways. For instance, Neumann (1992) found “a strong belief in a symbiotic nexus between teaching and research” among senior academic administrators in Australia, and identified three levels: (1) the *tangible* nexus, relating to the communication in teaching of the newest knowledge; (2) an *intangible* nexus, referring to effects on the working modes of teachers and students resulting from the fact that the teachers are also researchers; (3) a *global* nexus, where the connection is situated at the institutional rather than at the individual level. A large number of studies have attempted to find evidence for positive or negative correlations of type (3), rather than just beliefs. In a seminal paper, Hattie and Marsh (1996) examined a total of 58 studies from

which they extracted a total of 498 correlation coefficients between measures of quality of research and teaching in institutions of education and research. The weighted average of these coefficients turned out to be within the total variation; after excluding outliers it was a mere .05 (p. 525). Hattie and Marsh concluded that “the relationship between teaching and research is zero, and it would be more useful to investigate ways to increase the relationship” (p. 533). In fact, ten years earlier, Elton (1986) stated that questions about a possible connection between teaching and research “when put on a departmental or institutional scale cannot conceivably be answered through simplistic quantitative methods” (p. 300). After examining several classical arguments, he suggests that

it is necessary to distinguish between three activities - teaching, scholarship and research. It is then likely that at this [*the individual*, auth.] level teaching and research can fertilize each other, but only through the mediation of scholarship (p. 303).

Here, *scholarship* can be seen either as a “common factor” that could fertilise both teaching and research (the position of Elton), or as an overarching concept for “scholarship of discovery”, “scholarship of teaching” etc. (cf. Steen, 2000, 334). So we are led to consider the TR-nexus at the *individual level* ((1) and (2) above) which is where scholarship is enacted. Also we must do so with *disciplinary specificity*, if we are to surpass superficial ideas of scholarship. Then the fruitful question is not whether teaching and research support each other *automatically*, but *what* such a mutual support is, and how it can be furthered. In short we must explore the *nature of* and *conditions for* a positive TR-nexus in university mathematics scholarship.

A significant part of the tertiary mathematics education literature can be said to point in that direction, along the following three lines:

- studies of innovative *teaching* which aims at student activities that are implicitly or explicitly “research like” (e.g. Grenier and Godot, 2004; Legrand, 2001; Mahavier, 1999; Steen, 2000)
- studies of the nature of “advanced” mathematical *thinking and learning* among students (e.g. Tall, 1991; Rasmussen et al., 2005)
- studies of the learning perspective in mathematicians’ research, or *scholarship of discovery* (e.g. Burton, 2004; Misfeldt, 2006).

However in virtually all existing studies, the main object of research is either *teaching and student learning*, or *mathematicians’ research*, rather than the full interaction of the three. In the next section we propose a theoretical framework that takes into account these crucial elements, and we define the nexus in terms of them. Then, as a first illustration of how this model can be used, we consider the TR-nexus from the perspective of five mathematicians.

## AN ANTHROPOLOGICAL APPROACH TO THE TR-NEXUS

As noticed by Rasmussen et al. (2005), mathematical scholarship is an *activity* that involves not just cognitive but also communicative acts. To do this we use the anthropological theory of didactics (we use it freely here and refer readers unfamiliar with it to Chevallard, 1999 or Barbé et al., 2005, sec. 2). Winsløw (2006) used this theory of didactics to propose a framework in which tertiary didactics of mathematics may be considered the study of the interplay between two types of human activity, namely:

- *mathematical organisations* (MO) where tasks (enacted by students, teachers and researchers) are mathematical in the usual sense, associated with mathematical techniques, technology and theories;
- *didactical organisations* (DO) where the tasks (enacted by teachers) are to induce students into enacting a MO, using didactical techniques, which may, in principle, be described and justified using didactical technology and theory.

The didactical transposition is mediated by the DO and can be located in the transition from the regional mathematical organisation  $MO_m$  of the professional mathematician (where tasks are in part research problems), to the local mathematical organisation  $MO_s$  to be enacted by students. In this sense,  $MO_s$  can be thought of as a delimitation (sometimes a drastic reduction) and adaptation to learners of a smaller section of  $MO_m$ , while the DO is the praxis through which  $MO_s$  is constructed and “delivered” (in a sense, by *devolution* in the sense of Brousseau, 1986) to students:

$$(MO_m - DO) \leftrightarrow MO_s$$

Here,  $MO_m$  and DO are the two families of praxeologies enacted by the mathematician, in research and teaching. The nexus is in the interplay between these two, potentially at all four praxeological levels:

- between *tasks* of teaching (DO) and *tasks* of research ( $MO_m$ ) in the material sense that they are distributed in the working time of the mathematician. This corresponds to a *minimal* nexus which arises by material necessity in the activity of the mathematician.
- between DO *techniques*, e.g. for constructing  $MO_s$ -tasks and deliver explanations and presentations (of  $MO_s$ -technology and theory), and  $MO_m$  *techniques*. We call this the *implicit* nexus (close to the *intangible* nexus mentioned above).
- between DO *technology and theory* and the specificities of the knowledge block (*technology, theory*) of  $MO_m$  that could contribute to explain and justify didactical choices, including those involved in preparing  $MO_s$ . We call this the *explicit* nexus (it seems close to the *tangible* nexus mentioned above).

The minimal nexus is always present under the assumption that the mathematician does both teaching and research, which is of course a necessary assumption to study the TR-nexus at the individual level. One might have the minimal nexus without

much retroaction from students' activity to that of the teacher, i.e. in a setting that could be described as  $(MO_m - DO) \rightarrow MO_s$ . In this setting, the mathematician does his two "jobs" independently and sometimes even without much consideration of what the students do. Indeed, DO techniques could be simply inherited from his own experience as a student, without being affected by his subsequent experience as researcher. However, the implicit and explicit nexus clearly require a certain retroaction from the students' praxis of  $MO_s$  on the activity of the teacher. It could be limited, for instance by institutional constraints, to minor adaptations of DO techniques. At the other extreme, it could lead to genuine interaction between  $MO_m$  and  $MO_s$  e.g. when students get involved directly in current research of the mathematician. But most of the time – prior to this last situation, which is likely to be rare, at least in undergraduate teaching – it would be a dangerous illusion to imagine the absence of a DO, i.e. genuine collaborative learning of professors and students.

### WHAT MATHEMATICIANS SAY

The two organisations  $MO_m$  and DO are enacted and to a large extent controlled by mathematicians, and their interaction even more so. The former is difficult to "observe" directly. Indeed existing studies of what corresponds to  $MO_m$ , such as those by Burton (2004) and Misfeldt (2006), are based on mathematicians assertions about it. But to our knowledge it is new to analyse such data with the praxeological model outlined above, and with the explicit intention to investigate the nexus  $MO_m - DO$ .

#### Context and Design of study

Our data come from interviews with five mathematicians (3 full professors, 2 associate professors) at the University of Copenhagen. It is the biggest and oldest university in Denmark – and the only one to figure in the top 100 of international ratings. It is close to the "Humboldtian" tradition: professors' time is, at least in theory, equally divided between research and teaching. As in most European universities of similar status, research is highly valued and the main factor in hiring and promotion.

The interviews were done individually, based on an interview guide, and lasted for about an hour. The interview guide aimed at having respondents explain (1) their current research, (2) their current teaching, with an emphasis on the bachelor level, and (3) the relations they saw between (1) and (2). Questions were adapted to the development of the interview, with the goal of identifying main characteristics of the  $MO_m$  and DO involved, as well as their interplay as it could affect students' work ( $MO_s$ ); for instance, we specifically asked about techniques involved, and their interplay. Our main interest lies in part (3) of the interviews, but the two first parts (on  $MO_m$  and DO separately) were indeed useful to enable concrete references and examples. Interview transcripts (in Danish) can be made available through the authors.

Our purpose here is to illustrate some main points from our preliminary analysis of the data. The interviews with mathematicians form part of a larger study of the TR

nexus, which is currently in process; it involves also interviews with professors of geography.

### Some instances of interplay

It may not be surprising that all five mathematicians cite the supervision of more advanced students – particularly during master thesis work – as an instance of teaching which can draw significantly on their experience and current work as researchers, at least indirectly. Although this seems to be relatively rare, three of them have profited in their own research from having master thesis students work with sub-problems coming from their own research projects, and two have even done joint research with excellent master students (in some cases published by the student alone). Four of them think that this is also possible, at least in principle, at the bachelor level, but they did not have actual experience with this.

Even in a first year course, students may, according to one respondent, get an “experience which is somewhat like that of a researcher”:

We try to do that. (...) We give them relatively open tasks. (...) That is, where it is not just an exercise, with a question and a specific point in the text book to refer to and a unique answer. (...) For instance (...) we ask them to compute  $\pi$  by using the formula for a function, like arcus tangens or something, which gives  $\pi$  in some point, and then use the Taylor series at a point where they know it [*the value of the function*]. That I would say is a completely standard exercise. They also learn to estimate the error. But then we can go on and ask, can you with certainty find the first 100 decimals in  $\pi$ , using this method. Or when can they be certain that they found the first 100 decimals. (...) And that we give them as an open task which we don't even ourselves completely, well of course we could, but we don't even consider beforehand if we know precisely what the perfect solution would be, because we are not after the perfect solution, we are after *them* thinking about what are the problems involved in this task. [*explains the central difficulty of evaluating the error term*] (...) just they explore this *problematique* we feel they have achieved a lot. And that we think reminds us of our own research, as for the processes (...) that type of exercises we give a lot. (...) We take a standard exercise, and open it up a bit (...) to see how far can you go on this type of task. (...) they need to get the experience that here they have to explore a domain by themselves.

Notice that a general idea (in fact, DO-technique) for constructing student tasks is explained here: you expand on a standard type of task from  $MO_s$ , to obtain a task with some of the overall characteristics of a task from  $MO_m$  (as explained). The techniques used by students come, of course, from  $MO_s$ ; in some cases the same task would not have the “research flavour” if attacked by more advanced techniques. And in this way, asking such “twisted”  $MO_s$ -tasks can be in itself a mathematical challenge:

That in a way you could call a kind of research. (...) One thing is to prove a theorem with all available mathematical methods. But to ask yourself, can you prove this theorem by just using the following methods, it is in itself a mathematical question. (...) [*similarly*] we ask ourselves, can we solve this *without* using this or that result.

Another respondent reports on asking a problem (about matrices) in a first year course, to which he didn't know the answer; after a lot of activity among students and teachers, it was completely solved by a student. This, however, seems to be an exceptional case (and an optional exercise for enthusiastic students).

A main component in the DOs of all five mathematicians is *lecturing*, at the bachelor level often to large audiences of students. One respondent cites the preparation of lectures as an activity which is both supported by and inspiring for his research:

My teaching couldn't at all be like it is without my research. (...) When you teach, you learn the material... it's the teacher who learns something from a course. (...) My whole approach to mathematics is shaped by research, how I feel mathematics should be understood. But also the other way: you work through things you thought you knew when you prepare a lecture, and then you discover you didn't know them, you find new things...

Notice that here the strong implicit nexus (DO-MO<sub>m</sub>) is considered very fruitful for the respondent, but it is open how (if) it affects the work space (MO<sub>s</sub>) of students.

Another respondent says that his access to advanced MO<sub>m</sub> knowledge blocks helps him to select topics and perspectives to present, in a way which is relevant in the students' long way towards modern research: "I can tell them things which are not in the text books". This amounts to a explicit nexus with a strong direction *from* MO<sub>m</sub> *to* DO. But to him, there is no way undergraduate teaching could contribute to his work with MO<sub>m</sub>, given that even at master level, MO<sub>s</sub> may not come close to it. Also, he seems to refuse the idea of an implicit nexus (at least for undergraduate DOs).

### **Research-like – what does it mean?**

Overall, the *explicit* nexus seems to occur in two ways: (a) in the teachers' work on DO (presentation and exercise construction) where he makes certain choices in order to let MO<sub>s</sub> approach – possibly from a long distance – certain topics and methods from MO<sub>m</sub>; (b) in the supervision of advanced students' projects, where MO<sub>s</sub> involves directly elements from MO<sub>m</sub> (at the level of technology and theory). These forms of nexus are recognised by all five interviewees. However, (b) seems to be exceptional. About (a) one can say that it does not in itself mean that MO<sub>s</sub> becomes research like, in the sense that students' actual activity involves tasks and techniques similar to that of a researcher. But then, after selection of appropriate topics, could one leave the DO to non-researchers? Here, a certain doubt is expressed:

*Interviewer*: it almost sounds like one could let university researchers devise the "menu" and then let other serve it?

*Mathematician*: I don't believe so... I don't think others can explain it as well as I can. (...) In principle, I could say, I write up the lecture notes and then one could hire a slave worker to deliver them, but that I think would be a very weird policy to have.

The *implicit* nexus occurs in several ways, which are (naturally) harder to categorise. However, one can distinguish two major areas: (a) construction of MO<sub>s</sub> tasks, from



project problems to exam exercises; (b) organisation of the students' work with  $MO_s$ -theory (including modes of presentation). In both cases, the nexus is at the level of techniques, and therefore it is typically more difficult to articulate. As one mathematician puts it,

I think it's fun to make such exercises, one can use a lot of time on it. (...) I just let my imagination run freely, and then I cut down to something the students can manage.

The interplay between  $DO$ - and  $MO_m$ -techniques may not always be conscious:

But, eh, there was one of the students who said to me, after an exam on a course where I had made the exercises, then he said, and I had not thought about that before, "but it's really research to make these exercises", and, it really is, for one has to come up with something they can't have seen anywhere else.

However, to some of the respondents,  $MO_m$ -techniques are explicitly mobilised in constructing  $MO_s$ -tasks as well as in organising students' work with  $MO_s$ -theory:

I think I use my research a lot in my teaching. (...) I like the way, sometimes, when I choose exercises, to elucidate problems I have myself encountered in my research. It often happens, when you work with [*respondent's specialty*] that you end up with some problem on finite-dimensional matrices.

A very difficult task is when people ask you for literature about something you know well, but you have come so far away from books. It's much more fun... The essence you rarely find in a book. So I often give people a book where they read up to a point, and then the rest I give as exercises (...) 'cause often I can't find a book which is suitable.

### **Continuity or rupture?**

A final question posed to respondents used a mathematical metaphor: in your work, do you regard teaching and research as separate parts or is there rather a kind of continuum? Despite the relative vagueness of the question, the previous discussion of their research and teaching seems to have given sufficient basis for the respondents to reply with certainty (4 opted for continuity, 1 for difference). One qualified continuity as follows (the others being essentially on the same track):

There is clearly a continuum. The point where the two things collide is that what my research is about here and now is not often what I try to make the students learn. There is a time conflict. (...) My own research is the funniest, right, because it's me playing, while in the other, I try to make the students play.

The option for a more principal difference is also accompanied by a reservation:

It is to a certain degree dependent on what area of mathematics we talk about. Certain areas are far more problem focused, in sense that the problems are given. (...) Everything has to be motivated by them. (...) But then there are other areas where it looks different, and where people may look differently at questions like the one you ask.

One of the respondents who argue for continuity also mentions that the answer could vary according to the amount of theoretical requirements which are required for even understanding the basic questions in a mathematical specialty. Although more evidence is clearly needed to assess this variation, we doubt that the specialty of the respondents alone can be said to account for the difference in points of view on this issue.

### **Institutional obstacles to continuity**

We have already mentioned the institutional and didactical reasons to look for ways to strengthen the TR-nexus. It is therefore interesting to note that the participants mention several *obstacles* for this, even if many of them would like research to be more present in students' activities. Time constraints and students' lack of interest and knowledge are mentioned by some (but not all). An obstacle which seems important to all respondents is the requirements of the study programme. As one puts it,

In an ideal world, I would like to see [*long silence*] that the students learned the mathematical method. (...) Of course they need some concrete ideas, definitions, to work from, but I would like that going from them... there is a bare field, a beginning to a path, and then they have to pave it. (...) The barrier is that there is a concrete syllabus, they have to learn that and that, and the other method is time demanding. (...) In physics, they require, they have to learn this particular theorem. (...) I think we should put more weight on the working methods, rather than learning mathematics in a mechanical way.

This experience of a pressure to “cover a large amount of topics”, resulting in poor practice blocks of MO<sub>s</sub>, is of course not new. But we think that the need to strengthen the bonds between research and teaching is an important reason – and potential help – to consider it anew, and as a didactic problem. In this light, it is interesting to note that four out of five mathematicians interviewed could in fact imagine that research problems, or problems close to research, could be taken up already in the context of the project work at the end of the B.Sc.-studies.

### **CONCLUSIONS**

The transition from “school like” introductory courses to more “research like” activities (cf. Grenier et al., 2004) is complex and difficult in university mathematics education. The five mathematicians' points of view seem to indicate that even in those introductory courses, a research point of view may in several ways help to improve the quality of students' activities and learning, and that it may be possible to strengthen the bonds between mathematical organisations of students and teachers by promoting those forms of DO that mobilise professors *as researchers*. Giving more institutional legitimacy to such DOs may not be simple, though. Some students and professors may not want them, for different reasons perhaps. And the individual professor has to respect a syllabus which is often rigid and demanding. Nevertheless, there seems to be considerable potential among mathematicians to enact such DOs.

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