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Abstract

We define an evolutionary process of "economic Darwinism" for playing-the-field, symmetric games. The process captures two forces. One is "economic selection": if current behavior leads to payoff differences, behavior yielding lowest payoff has strictly positive probability of being replaced by an arbitrary behavior. The other is "mutation": any behavior has at any point in time a strictly positive, very small probability of shifting to an arbitrary behavior. We show that behavior observed frequently is in accordance with "evolutionary equilibrium", a static equilibrium concept suggested in the literature. Using this result, we demonstrate that generally under positive (negative) externalities, economic Darwinism implies even more under-(over-) activity than does Nash equilibrium.

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1 Introduction

An interesting view in economics asserts that the foundations for behavioral principles such as profit maximization or behavior in accordance with Nash equilibrium should be found in evolutionary selection against those who do not behave in accordance with these principles. Such a view was, e.g., underlying the defense of the assumption of profit maximization by Alchian (1950), Enke (1951) and Friedman (1953). Informally the idea is that economic agents, e.g. owners and managers of firms, in their behavior are guided by simple rules of thumb and markets select against behavior that leads to relatively low payoffs. This happens, for instance, by capital seeking to the more profitable firms. Hence, badly performing behavior will disappear and remaining behavior will perform well vis-a-vis other remaining behavior in accordance with profit maximization and Nash equilibrium. The economist can therefore safely analyze *as if* active individuals obey these principles although no one does so consciously.

This paper studies an evolutionary process intended to formalize the above idea in the context of strategic interactions (games) and investigates what kind of behavior it supports. The situation we have in mind is that a symmetric game is played recurrently by individuals, and in each round the game is played "all against all", such that participating individuals are "playing the field" as opposed to a situation with "pair-wise contests". Any participant is locked at a fixed strategy, its rule of thumb, except for random shifts. We intend to identify the behavioral implications of two forces: economic selection and mutation.

Economic selection should capture the elimination of the unfit: if current behavior implies payoff differences between individuals, then (individuals with) behavior yielding relatively low payoffs have positive probability of being displaced by (incoming individuals with) an arbitrary behavior. Displaced behavior is not necessarily replaced by a more successful behavior that already occurs, but by an *arbitrary* possible behavior. This means that the kind of selection we have in mind is not appropriately mimicked by imitation. In a biological context it may be natural to assume that badly performing, displaced individuals are replaced by the offspring of more successful individuals, and the offspring *inherits* the more successful behavior of its parents, so that the resulting evolutionary selection is *as if* more successful behavior is imitated. In an economic context the idea of inherited behavior is not as natural. For instance, a fired manager is not replaced by a clone of another and more successful manager, but rather by a new candidate with own "fresh" ideas. Hence, if one wants to study the mere implications of elimination of the unfit in an economic context, a process of imitation will not generally be an appropriate metaphor. It is to distinguish the kind of Darwinian selection studied here from a biological one that we refer to it as "economic". The fact that the evolutionary process we study is not one of imitation, is a main novel feature of this paper.

The second force, *mutation*, should capture that economic agents experiment with alternative behavior: for any individual there is in any round of play a small positive probability that current behavior is replaced by an arbitrary behavior.

By *economic Darwinism* we understand the combined force of economic selection and mutation.

To be more specific we formulate an explicit dynamic process as follows. In each round of play the actions of the individuals participating in the play of a symmetric, n-player game are listed in the current "state". A *basic dynamic process*, a Markov chain on the state space, captures economic selection: if the current state implies payoff differences, actions giving maximal payoff are not changed, while for actions yielding lowest payoff, and possibly for other actions yielding less than maximal payoff, there is positive probability that the actions are replaced by arbitrary actions. A *perturbed process* captures economic selection as well as rare mutation: in addition to the transitions between states as governed by the basic process, every individual has in any round a small independent probability of switching to an arbitrary behavior. As the probability of mutation becomes very small, only particular states will be observed frequently. These are the so-called *stochastically stable states* and the particular patterns of behavior associated with these are the patterns supported by economic Darwinism.

To characterize these patterns of behavior we make use of a static solution concept originally suggested by Schaffer (1989) and further studied by Possajennikov (2003) and Alós-Ferrer and Ania (2002). Let "wide spread" behavior be represented by a common strategy used by everybody, so that everybody obtains the same payoff and the situation therefore is stable with respect to economic as well as biological selection. For such wide spread behavior to be stable also with respect to mutation there should be no alternative strategy such that a single player deviating to this strategy obtains strictly larger payoff than the non-deviating players, since a mutation to such a strategy would be successful and displace existing behavior. Following Schaffer and Possajennikov we refer to a strategy fulfilling this criterion as a *symmetric evolutionary equilibrium*.[1] It is well-known that such an equilibrium is equivalent to a symmetric Nash equilibrium of a modified game where payoffs are defined relatively, such that a player's payoff in the modified game is the excess of his payoff from the original game over the lowest payoff of a player.

Our first main result is that for a large class of games, any stochastically stable state involves behavior is as in a symmetric evolutionary equilibrium (given such an equilibrium exists). As a corollary, if there is a unique symmetric evolutionary equilibrium, the two solution concepts coincide. These results are useful for applications because evolutionary equilibrium is relatively tractable.

One implication is that economic Darwinism does not generally support Nash equilibrium since Nash equilibria in relative payoffs only accidently coincide with the original Nash equilibria. Hence, the view described in the first paragraph is not generally supported by our formal results.

We use the equivalence between stochastically stable states and evolutionary equilibrium when the latter is unique to prove our second main result: in typical economic situations where externalities are either overall positive or overall negative, the outcomes of economic Darwinism are even worse than Nash equilibrium outcomes with respect to efficiency, as they involve even more under- or over-activity.

The contributions of the literature most closely related to this paper study evolutionary processes of *imitation* and mutation. Originally Vega-Redondo (1997) suggested such a process and studied it in symmetric, n-player Cournot oligopoly games. Vega-Redondo's process can be viewed either as one of conscious imitation or as a metaphor for evolutionary (biological) selection. Vega-Redondo found that behavior in the unique stochastically stable state according to his process of imitation and mutation corresponds exactly to the competitive or price-taking, Wal-

[1]Schaffer (1989) studies the implications of symmetric evolutionary equilibrium in Cournot duopoly games and finds equivalence with competitive (price taking) equilibrium. Possajennikov (2003) and Alós-Ferrer and Ania (2002) study more general games and establish equivalence results between (symmetric) evolutionary equilibrium and so-called aggregate-taking equilibrium, see Section 2 below.

rasian outcome. The results of Vega-Redondo have been generalized and extended by Schenk-Hoppé (2000), who also studies a process of imitation and mutation in Cournot oligopoly. Furthermore, Schipper (2003) and Stegeman and Rhode (2004) study processes of imitation and mutation in more general settings.[2]

According to these imitative processes, there is in each round for each individual positive probability that the current strategy is revised into the strategy of another individual who earns at least as much payoff. Since, whenever a type of behavior is revised, it is changed into a behavior that is already being used, the set of behaviors can only shrink (except for mutation). According to economic selection relatively poorly performing behavior is replaced by arbitrary behavior, so that the set of strategies used can increase even without mutation. Thus, our process of economic Darwinism is different from imitation and mutation, also in a mathematical sense. As a consequence, the proofs of the basic equivalence theorems as well as their underlying assumptions are essentially different between the contributions mentioned and this one: economic selection allows more generality than imitation, as will be explained in detail in Section 4 below.

Section 2 defines symmetric games and evolutionary as well as other static equilibrium concepts, and states some well-known equivalence results. Section 3 defines the dynamic process of economic Darwinism and stochastic stability with respect to it. In Section 4 we state our result on the relation between evolutionary equilibrium and stochastic stability with respect to economic Darwinism, and explain how our theorem and its proof differ from similar theorems and proofs for processes based on imitation. In Section 5 we use our characterization to identify the outcomes of economic Darwinism in some games of economic interest, and Section 6 states our result on the general efficiency implications of economic Darwinism. Section 7 offers some concluding remarks.

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[2] Stegeman and Rhode formulate very explicitly the idea that imitation can (should) be seen as capturing Darwinian selection: "Our strategy selection process is imitative, but it is useful to view imitation dynamics as a special case of Darwinian dynamics: in a finite population of fixed size, Darwinian selection requires that individuals playing poorly-performing strategies switch to strategies that are performing better". As we have argued above, this may be a natural view for biological selection, but less natural in an economic context.

2 Symmetric games and evolutionary equilibrium

We consider a game with a fixed set of players, $N = \{1, ..., n\}$. The non-empty strategy set of each player i is $S \subseteq \mathbb{R}$. The set of strategy profiles is $\Omega \equiv S^n$. A profile (element in Ω) will be denoted by ω or $(s_1, ..., s_n)$ or (s_i, s_{-i}). Payoff functions are $u_i : \Omega \to \mathbb{R}$, $i = 1, ..., n$.

We focus on symmetric games and assume that the payoff obtained by player i, when player i uses strategy s_i and the other players use a constellation of strategies s_{-i}, depends in the same way on s_i and s_{-i} independently of i, and is the same independently of how the strategies of s_{-i} are distributed among the players different from i:

Assumption 1. *There is a common payoff function $u : \Omega \to R$, such that $u_i(s_1, ..., s_i, ..., s_n) = u(s_i, s_{-i})$ for $i = 1, ..., n$, and u has the property that $u(s_i, s_{-i}) = u(s_i, s'_{-i})$ for any permutation s'_{-i} of the strategies in s_{-i}.*

Assumption 1 implies that whenever two players take the same action they receive the same payoff.

Contributions such as Alós-Ferrer and Ania (2002), Possajennikov (2003), Schipper (2003), and Leininger (2004) assume that there is a common utility function, $v : \mathbb{R}^2 \to \mathbb{R}$, and an aggregator function, $A : \Omega \to \mathbb{R}$, such that $u_i(s_i, s_{-i}) = v(s_i, A(s_i, s_{-i}))$ for all players i, and A fulfils $A(\omega) = A(\omega')$ for all permutations ω' of ω. When the payoff functions have this special structure we say that the game has "the aggregator property".[3] The aggregator property implies Assumption 1 and is featured by many economic examples, but Assumption 1 allows for more (see Section 5 below).

The static concept of evolutionary equilibrium formulated by Schaffer (1989) and meant to capture the elimination of the least fit when mutations occur is motivated as follows. A first requirement for evolutionary stability is that all players obtain the same payoff, since if there were payoff differences a behavior yielding lowest payoff would tend to die out. Since the aim is to establish support for certain types of "common, widespread behavior", the focus is on symmetric strategy profiles where all players take the same action and thus automatically obtain the same payoff.

[3] A game has the aggregator property in our terminology if it is an aggregative game in the sense of Corchón (1994) *and* is symmetric in the sense of Assumption 1.

Second, there should be stability with respect to mutations, such that if a single player mutates to a new behavior, then this behavior will be displaced because it obtains least payoff in the constellation of behavior after the mutation. These considerations lead to the following definition.

Definition 1 *A strategy profile* $(s_1^*, ..., s_n^*) \in \Omega$ *is a symmetric evolutionary equilibrium if* $s_i^* = s^*$ *for all* i*, and for all* i*,* j *and* s_i*:* $u_i(s_i, s_{-i}^*) \leq u_j(s_i, s_{-i}^*)$*. It is strict if furthermore for all* i*,* j *and* $s_i \neq s^*$*:* $u_i(s_i, s_{-i}^*) < u_j(s_i, s_{-i}^*)$*.*

Alternatively, s^* is the strategy of an evolutionary equilibrium if there is no i and s_i such that $u_i(s_i, s_{-i}^*) > u_j(s_i, s_{-i}^*)$ for $j \neq i$, and it is strict if furthermore there is no i and $s_i \neq s^*$ such that $u_i(s_i, s_{-i}^*) \geq u_j(s_i, s_{-i}^*)$ for $j \neq i$. In terms of the common payoff function u, the requirement of Definition 1 is that for all s: $u(s, s^*, ..., s^*) \leq u(s^*, s^*, ..., s^*, s)$.

As observed by Schaffer (1989), it is straightforward that symmetric evolutionary equilibria can be characterized as symmetric Nash equilibria of a modified game with payoffs defined relatively:

Proposition 1 *Let* $\hat{u}_i(\omega) \equiv u_i(\omega) - \min_{j \neq i} u_j(\omega)$*. Then* $\omega^* = (s^*, ..., s^*)$ *is a (strict) symmetric evolutionary equilibrium in the game under consideration if and only if* ω^* *is a (strict) Nash equilibrium of the modified game where the set of players and the strategy set are the same and the payoff functions are* $(\hat{u}_i), i = 1, ..., n$*.*

Proposition 1 is useful for finding or characterizing evolutionary equilibria as in Sections 5 and 6, and it can be used for demonstrating existence of symmetric evolutionary equilibrium for some games by standard fixed point arguments, e.g.:

Proposition 2 *If* S *is compact and convex and payoff functions are such that* $u_i(s_i, s_{-i}) - \min_{j \neq i} u_j(s_i, s_{-i}))$ *is continuous in* (s_i, s_{-i}) *and quasi concave in* s_i *for all* i*, then a symmetric evolutionary equilibrium exists. If, furthermore,* $u_i(s_i, s_{-i}) - \min_{j \neq i} u_j(s_i, s_{-i})$ *is strictly quasi concave, then all symmetric evolutionary equilibria are strict.*

For games with the aggregator property, Possajennikov (2003), Alós-Ferrer and Ania (2002), and Schipper (2003) define a symmetric aggregate-taking equilibrium as a symmetric profile where each player has maximized payoff neglecting any influence on the aggregator A:

Definition 2 *A strategy profile* $(s^a, ..., s^a)$ *is a symmetric aggregate-taking equilibrium if for all* $s \in S$*:* $v(s^a, A(s^a, ..., s^a)) \geq v(s, A(s^a, ..., s^a))$*. It is strict if furthermore for all* $s \neq s^a$*:* $v(s^a, A(s^a, ..., s^a)) > v(s, A(s^a, ..., s^a))$*.*

Assuming differentiability, the first order condition for s^* being a symmetric aggregate-taking equilibrium is:

$$\frac{\partial}{\partial s}\left[v(s, A(s^*))\right] \equiv v_1 = 0.$$

Furthermore, according to Proposition 1, $(s^*, ..., s^*)$ is an evolutionary equilibrium if and only if s^* maximizes $u_i(s_i, s^*_{-i}) - \min_{j \neq i} u_j(s_i, s^*_{-i})$ with respect to s_i for each i. For games with the aggregator property this maximization problem is the same as:

$$\max_{s \in S}\left[v(s, A(s, s^*_{-i})) - v(s^*, A(s, s^*_{-i})\right],$$

for which the first order condition is:

$$\frac{\partial}{\partial s}\left[v(s, A(s, s^*_{-i})) - v(s^*, A(s, s^*_{-i}))\right] = v_1 + v_2\frac{\partial A}{\partial s} - v_2\frac{\partial A}{\partial s} = v_1 = 0.$$

The two first order conditions are thus equal. This proves:[4]

Proposition 3 *If each of symmetric evolutionary equilibrium and symmetric aggregate-taking equilibrium is unique and characterized by first order conditions then they coincide.*

3 The process of economic Darwinism

For our explicit dynamic analysis we have, for technical reasons, to assume that the strategy set S is finite. We assume furthermore that the finite game considered is such that equal payoffs for everybody can only be obtained if everybody uses the same strategy, that is, in addition to Assumption 1 we assume:

Assumption 2. *The strategy set* S *is finite and the common payoff function* u *is such that if* $u(s_i, s_{-i})$ *takes the same value for for all* i*, then* $s_1 = \cdots = s_n$*.*

[4] The equivalence between evolutionary equilibria and aggregate-taking equilibria is investigated more thoroughly by Possajennikov (2003) for the differentiable case and by Alós-Ferrer and Ania (2002) for the general case. Proposition 3 is a version of Possajennikov's result.

Given finiteness, Assumption 2 is fulfilled generically: Consider a given finite grid S. Assume that the payoff function u were such that for some strategy profiles with strategy differences all players got the same payoff. Then almost any (small) perturbation of u would imply payoff differences for the same profiles. Hence, given finiteness, Assumption 2 is little restrictive.

In a game with a large, but finite number of strategies in S, for instance a price-setting oligopoly where prices have to be integers, appealing to the genericity argument for "only equal payoffs at equal strategies" as we do, implies appealing that very small payoff differences can have evolutionary consequences. However, in the present context an appropriate interpretation of a strategy is perhaps not "set price equal to 55 cents", but rather a general rule of thumb such as "set price equal to the double of short run average cost" or "set price equal to the double of long run marginal cost". There may well be relatively few relevant such rules even though there are truly many possible different prices.

The n-player game considered is assumed to be played recurrently among individuals. The outcome in each round is described by a state that lists the actions taken by each of the n individuals. Hence, a state has the structure of a strategy profile and will be denoted by $\omega = (s_1, ..., s_n) \in \Omega$. The state space Ω is finite.

On this state space we define a dynamic process capturing the two essential forces of economic selection and mutation.

According to economic selection individuals who perform badly in terms of payoff, the unfit, are sometimes displaced by other individuals who take up a random action. We express economic selection dynamics by a basic (unperturbed) Markov chain on Ω with a matrix Π^0 of transition probabilities, $\pi^0_{\omega\omega'}$ ($0 \leq \pi^0_{\omega\omega'} \leq 1$ for all (ω, ω'), and $\sum_{\omega'} \pi^0_{\omega\omega'} = 1$ for all ω), requiring:

(1) If $\omega = (s_1, ..., s_n)$ is such that all players obtain the same payoff ($u_i(\omega) = u_j(\omega)$ for all i, j), then $\pi^0_{\omega\omega} = 1$.

(2) If $\omega = (s_1, ..., s_n)$ is such that there are payoff differences ($u_i(\omega) \neq u_j(\omega)$ for some i, j), then for any state $\omega' = (s'_1, ..., s'_n)$: (a) if $\pi^0_{\omega\omega'} > 0$, it must be for all $i \in \arg\max_{j \in N} u_j(\omega)$, that $s'_i = s_i$, and (b) if $s'_i = s_i$ for all $i \notin \arg\min_{j \in N} u_j(s_j, s_{-j})$), then $\pi^0_{\omega\omega'} > 0$.

The restrictions on Π^0 expressing economic selection are that (1) if everybody gets the same payoff in ω, then the next round's state will also be ω for sure, and (2) if there are payoff differences at ω then (a) strategies yielding highest payoff are not changed, while (b) any successor where at least one of the players who got minimal payoff has shifted to another strategy, has positive probability, implying that strategies with lowest payoff disappear with positive probability and are taken over by arbitrary other strategies.

We have expressed economic selection generally in terms of restrictions on the transition probabilities, $\pi^0_{\omega\omega'}$. Several explicit selection processes will imply our general assumptions (1) and (2). Assume, for example, that in each round individuals who receive a "selection draw" are replaced by individuals whose strategies are picked according to a given probability distribution with full support on S. One possible process is defined by everybody with lowest payoff in the last round, and no other, having a given, strictly positive (and independent) probability of receiving a selection draw. Another possibility is that everybody who did not get the highest payoff in the last round, and no other, has a given (independent) probability of receiving a selection draw. There are many possibilities in between, for instance where the probability of receiving a selection draw depends negatively on payoff in "softer", more monotone ways than in the two cases just described.

The mutation process is added as follows: From the state chosen in a given round by the economic selection dynamic, Π^0, there is for each player and for each $s \in S$ (independently) a small probability $\varepsilon > 0$ of switching to s within the round. Each such switch is referred to as a mutation.[5] This defines a modified Markov chain Π^ε, where the transition probability from ω to ω'' is defined as follows: For a pair of states ω', ω'', let $k(\omega', \omega'')$ be the number of players behaving differently in the two states, i.e., $k(\omega', \omega'') = \#\{i \,|\, s'_i \neq s''_i\}$. Thus, $k(\omega', \omega'')$ is the number of mutations involved in the transition from ω to ω'' via ω'. Then, $\pi^\varepsilon_{\omega\omega''} = \sum_{\omega' : \pi^0_{\omega\omega'} > 0} \pi^0_{\omega\omega'} \varepsilon^{k(\omega', \omega'')}$.

It is obvious that Π^ε is ergodic.[6] Furthermore, Π^ε is a regular perturbation of Π^0 in the sense of Young (1993), i.e., Π^ε is ergodic, $\Pi^\varepsilon \to \Pi^0$ as $\varepsilon \to 0$, and for each

[5] The mutation process is that each individual mutates (independently) with probability $\varepsilon \cdot \#S$, and in case of mutation a random strategy is picked from S according to the uniform probability distribution. It is well known that one can be much more general with respect to the mutation process without affecting results. For instance different players could mutate with different probabilities and different strategies could have different probabilities when mutations occur.

[6] It is a finite Markov chain and *irreducible* (since from any state one can go to any other state in one step by appropriate mutation), and *aperiodic* (since for any state one can stay in that state by appropriate mutation).

transition $\omega\omega''$ for which $\pi^0_{\omega\omega''} = 0$, there is a well-defined order by which $\pi^\varepsilon_{\omega\omega''} \to 0$ as $\varepsilon \to 0$. Here this order is given by the minimal number of mutations required to go from ω to ω'', i.e., by $\min_{\omega':\pi^0(\omega,\omega')>0} k(\omega', \omega'')$. This number (which is equal to zero when $\pi^0_{\omega\omega''} > 0$) is referred to as the resistance in the transition from ω to ω''.

We state some standard results from the theory of Markov chains needed for our purposes. The results mentioned can all be found in Young (1993) or Freidlin and Wentzel (1984), see also Young (1998).

Since Π^ε is ergodic it has a unique invariant (stationary) distribution μ^ε, i.e., a probability distribution over Ω fulfilling $\mu^\varepsilon\Pi^\varepsilon = \mu^\varepsilon$. In the long run the relative frequencies by which states are visited converge with probability one to the probabilities of μ^ε. Since our interest is in the process with economic selection and mutation *for small mutation probability*, we will be interested in the limit distribution, $\mu^0 = \lim_{\varepsilon\to 0} \mu^\varepsilon$. This limit distribution exists and is invariant for Π^0 ($\mu^0\Pi^0 = \mu^0$). Thus, the states that have strictly positive probability according to μ^0 are the only states that will be observed frequently when mutations are rare. These states are termed stochastically stable states.

Definition 3 *A state ω is stochastically stable with respect to the process of economic Darwinism if and only if $\mu^0(\omega) > 0$.*

It follows from the existence of μ^0 that stochastically stable states exist. The remainder of this section presents a characterization of stochastically stable states.

An absorbing set is a subset M of Ω, which is closed with respect to finite chains of transitions with positive probability according to Π^0, that is, for all $\omega \in M$, $\omega' \notin M$, one has $\pi^0_{\omega\omega'} = 0$, and for all $\omega, \omega' \in M$, there are states $\omega_1, ...\omega_m \in M$, such that $\pi^0_{\omega\omega_1} > 0$, $\pi^0_{\omega_1\omega_2} > 0, ..., \pi^0_{\omega_m\omega'} > 0$. In our case, the absorbing sets are particularly simple, being exactly all the singleton sets of form $\{(s, s, ..., s)\}$, where s is an arbitrary strategy in S:

Lemma 1 *Given Assumption 1 and Assumption 2, for any $s \in S$, the set $\{(s, s, ..., s)\}$ is absorbing, and there are no other absorbing sets.*

Proof. In a state $\omega = (s, s, ..., s)$ all players obtain the same payoff by Assumption 1. Hence $\pi^0_{\omega\omega} = 1$, and $\{\omega\}$ is absorbing.

Consider a state $\omega = (s_1, ..., s_n)$, where players do not all use the same strategy. By Assumption 2, they do not all obtain the same payoff. With positive probability according to Π^0, all the players with minimal payoff switch to one of the strategies in $\{s_i\}_{i \in N}$ that did not yield minimal payoff, so in the resulting state fewer different strategies are used. If not all players use the same strategy in the resulting state, then the argument can be repeated. It therefore has positive probability according to Π^0 that in a finite number of rounds a state of the form $(s, s, ..., s)$ is reached, and such a state is absorbing. Hence, no state with action differences can be in an absorbing set and therefore no other absorbing sets than the singleton sets of form $\{(s, s, ..., s)\}$ exist. $\square$

We will talk simply of absorbing states (not sets). An invariant distribution for Π^0 can only attach positive probability to absorbing states. Hence, only absorbing states can be stochastically stable. This leaves us with many candidates, but stochastically stable states can be further characterized.

Above we defined the resistance in a transition $\omega \rightarrow \omega''$ from one state to another as the integer number $\min_{\omega' : \pi^0(\omega, \omega') > 0} k(\omega', \omega'')$, i.e., as the minimal number of mutations required to go from ω to ω'' in one step. Define the resistance in a transition from one absorbing state, ω, to another, ω'', as the minimal total resistance (minimal number of mutations) required to go from ω to ω'', possibly indirectly over other states ω' which do not have to be absorbing.

For any absorbing state ω, define an ω-tree as a directed graph on the set of absorbing states, such that for any absorbing state $\omega' \neq \omega$ there is exactly one path in the graph leading from ω' to ω. If there are k absorbing states, then any ω-tree contains $k - 1$ arcs. For any given ω-tree define its total resistance as the sum of all the resistances over the arcs in the tree. For any absorbing state ω define the stochastic potential as the minimal total resistance over all ω-trees.

Young (1993) proves that *the stochastically stable states are exactly the absorbing states with minimal stochastic potential.* We use this in the next section to establish a very close relation between evolutionary equilibrium and stochastic stability with respect to our process of economic Darwinism.

4 Economic Darwinism implies evolutionary equilibrium

In this section we (still) impose everywhere Assumption 1 and Assumption 2. This means that any symmetric evolutionary equilibrium must be strict, since a deviating player cannot obtain the same payoff as the other players and must therefore obtain strictly less. Given a symmetric evolutionary equilibrium, $(s^*, ..., s^*)$, we say that the absorbing state $\omega^* \equiv (s^*, ..., s^*)$ corresponds to the evolutionary equilibrium.

Theorem 1 *Given Assumption 1 and Assumption 2, if a symmetric evolutionary equilibrium exists, then any stochastically stable state with respect to the evolutionary process of economic Darwinism corresponds to a symmetric evolutionary equilibrium.*

To prove this note that by assumption there is at least one symmetric evolutionary equilibrium, so there is at least one state, $\omega^* = (s^*, ..., s^*)$, corresponding to an evolutionary equilibrium. The proof proceeds by showing, in two lemmas, that the resistance in the transition from such an ω^* to any other absorbing state is at least two, while from any absorbing state, $\omega = (s, ..., s)$, that does not correspond to a symmetric evolutionary equilibrium, the resistance in the transition from ω to any state ω^* that does correspond to a symmetric evolutionary equilibrium is one.

Lemma 2 *The resistance in a transition from an absorbing state, ω^*, corresponding to a symmetric evolutionary equilibrium to any other absorbing state, ω, is at least two.*

Proof. We show that it takes at least two mutations to go from an absorbing state $\omega^* = (s^*, ..., s^*)$ corresponding to a symmetric evolutionary equilibrium to a different absorbing state. If there is only one mutation from ω^*, then the resulting state has (at most) one player using a strategy $s \neq s^*$, while all the remaining players still use s^*. By Assumption 1, all the players using s^* get the same payoff. Furthermore, since $(s^*, ..., s^*)$ is a strict symmetric evolutionary equilibrium, and different strategies are now used, it follows by Assumption 2 that the player who plays s gets *strictly* less payoff than the other players. So, according to Π^0 it has probability one that in the next state all the other players still play s^*, while the last player plays some s', and it has positive probability that $s' = s^*$. If $s' \neq s^*$, then the same happens again with positive probability of reaching ω^* in the next step. As long as ω^* has not yet been reached there is in each round positive probability of reaching ω^* in the next round. Let the minimal of these probabilities (over all states where the last player has not yet come to play s^*) be $\pi^{\min}$. Then over T rounds, the probability of not reaching ω^* is at most $(1 - \pi^{\min})^T$, which goes to zero as T goes to infinity. It thus has probability one according to Π^0 to eventually return to ω^*. One cannot get from ω^* to another absorbing state by just one mutation, so the resistance from ω^* to any other absorbing state ω is at least two. □

Lemma 3 *The resistance in a transition from an absorbing state, ω, that does not correspond to a symmetric evolutionary equilibrium, to any ω^*, that does corresponding to a symmetric evolutionary equilibrium, is one.*

Proof. We show that from an absorbing state $\omega = (s, ..., s)$ not corresponding to a symmetric evolutionary equilibrium, there is a single mutation leading to a state from which it has positive probability according to Π^0 to go to any $\omega^* = (s^*, ..., s^*)$ corresponding to a symmetric evolutionary equilibrium. Since $(s, ..., s)$ is not an evolutionary equilibrium, there is a deviation, s', that will bring the deviator a strictly larger payoff than the other players, who by Assumption 1 get the same payoff. Assume that player 1 mutates and plays s'. Now with positive probability according to Π^0, the process reaches in the next round a state $(s', s^*, ...s^*)$, where player 1 still plays s' while all other players play the strategy s^*. In this new state the players $2, ..., n$ all get the same payoff by Assumption 1 and player 1 obtains strictly less, because $(s^*, ..., s^*)$ is a strict evolutionary equilibrium. With positive probability according to Π^0 the process will therefore in the next round reach the state $(s^*, ...s^*)$. This means that the resistance from ω to any ω^* is one. □

Proof of Theorem 1. Consider an ω-tree, where ω does not correspond to a symmetric evolutionary equilibrium. Let ω^* be a an absorbing state that does correspond to a symmetric evolutionary equilibrium. Change the ω-tree in the following way: remove the transition out of ω^* (by Lemma 2 this has resistance at least 2), and add a transition from ω to ω^* (by Lemma 3 this has resistance 1). Thereby an ω^*-tree with strictly lower resistance than the ω-tree has been constructed. This shows that the stochastic potential of ω^* is strictly lower than the stochastic potential of ω, so that ω cannot be stochastically stable. Thus, any stochastically stable state must correspond to an evolutionary equilibrium. □

Theorem 1 does not say that in games with more than one evolutionary equilibrium all states corresponding to evolutionary equilibria are stochastically stable,[7] but for games with a unique evolutionary equilibrium we have:

Corollary 1 *Given Assumption 1 and Assumption 2, if the game has a unique evolutionary equilibrium* $(s^*, ...s^*)$*, then the unique stochastically stable state is* $\omega^* = (s^*, ...s^*)$*.*

If a game has exactly one symmetric evolutionary equilibrium, then only behavior as in this equilibrium will be observed frequently according to the dynamics of economic Darwinism.

Theorem 1 and its corollary are our basic results characterizing behavior resulting from economic selection and mutation. A result similar to Corollary 1 for processes of imitation and mutation is found in Schipper (2003), who extends the analysis of Vega-Redondo (1997) to a class of games more general than Cournot oligopoly, namely games which have the aggregator property, and where the payoff function v fulfils a particular assumption of "quasi-submodularity" with respect to a player's own strategy and the aggregate. Schipper shows that under these assumptions, if the game has a unique aggregate-taking equilibrium, then there is a unique stochastically stable state with respect to the process of imitation and mutation and this state corresponds to the aggregate-taking equilibrium. Each of these characterizing results, Schipper's and our, essentially follows from three basic features corresponding to our lemmas 1, 2, and 3.

First, *absorbing sets are singletons where all individuals use the same strategy.* In our model this follows by Assumption 2, and this is the crucial role played by Assumption 2. In Schipper's and in Vega-Redondo's settings it follows from a particular feature of their imitation process: different individuals who all obtain maximal payoff, but use different strategies, imitate each other with positive probability. This is not an essential difference, however, since if one imposed Assumption 2 in an imitation setting one would be able to do without this particular assumption.

[7] Section 5.3 provides an example of a game with several evolutionary equilibria out of which exactly one is stochastically stable.

Second, *it takes more than one mutation to escape an equilibrium*. This follows by more or less the same argument for the two types of processes, since a single mutation from either an evolutionary or an aggregate-taking equilibrium will bring the mutant less payoff than the non-mutants and hence one is led back to the equilibrium with positive probability either by economic selection or by imitation.

Third, *it takes just one mutation to reach an equilibrium*. In our setting of economic selection, if play is stuck at an absorbing state that does not correspond to an evolutionary equilibrium, then a mutation exists that gives the mutant strictly more payoff than the non-mutants. After such a mutation economic selection will with positive probability bring all the non-mutants to play the strategy of a particular symmetric evolutionary equilibrium, and in this new situation the mutant obtains strictly less payoff than the non-mutants, so economic selection will with positive probability take play all the way to the equilibrium.[8] In a setting of imitation a similar argument would not work: it would still be true that from an absorbing state not corresponding to an (aggregate-taking) equilibrium, there would be a single mutation giving the mutant more payoff than the non-mutants (since under the considered assumptions an aggregate-taking equilibrium is an evolutionary equilibrium), but the mutating strategy would not have to be an aggregate-taking equilibrium strategy and therefore imitation of the successful mutation would not necessarily take play all the way to an equilibrium. This is why sub-modularity is assumed. For games with the aggregator property, sub-modularity ensures that if all players use one and the same strategy that is not an aggregate-taking equilibrium strategy, then a deviation directly to the aggregate-taking equilibrium strategy (assumed to be unique) will yield the deviator higher payoff than the non-deviating players. Hence, from any non-equilibrium absorbing state a single mutation to the equilibrium strategy gives the mutant higher payoff than the non-mutants and then imitation will with positive probability bring all players to the equilibrium strategy.

Summing up, with respect to how the basic equivalence results are obtained, the essential difference between the imitation based and the economic selection based approaches, and the one intimately linked to the difference between imitation and economic selection, is how the property that *it takes just one mutation to reach an*

[8] In the second step of this argument it is used that the evolutionary equilibrium in question is strict, which is implied by Assumption 2. However, for this purpose one could have imposed a direct assumption of strictness of equilibrium.

equilibrium is ensured. As explained, for the process based on economic selection an assumption like submodularity is not required, exactly because economic selection allows for new strategies to emerge without mutation.

Our results thus apply also for games that are not submodular. This is an important extension because some games of economic interest are not submodular, e.g., the price-setting oligopoly with strategic complementarity considered in Section 5.2.

Furthermore, our results apply for symmetric games that do not have the aggregator property. In Schipper's contribution, as well as in Vega-Redondo's, the intention is to relate stochastic stability (with respect to the process of imitation and mutation) to aggregate-taking equilibrium, for which purpose the aggregator property is, of course, needed. Relating to evolutionary rather than to aggregate-taking equilibrium allows us the generality of avoiding the assumption of the aggregator property. Should this be fulfilled, evolutionary and aggregate-taking equilibrium coincide under relevant assumptions, Possajennikov (2003), Alós-Ferrer and Ania (2002) and Proposition 3 above.

5 Examples

The examples of this section illustrate the usefulness of the equivalences established above, that is, Theorem 1 or Corollary 1 in combination with Proposition 1 or 3. We consider a game with the aggregator property that is submodular (Section 5.1) and a game with the aggregator property that is not submodular (Section 5.2), as well as a game without the aggregator property (Section 5.3). The examples also illustrate the economic consequences of economic Darwinism under externalities as dealt with more generally in Section 6.

5.1 Cournot oligopoly

Each of n firms sets a quantity $q_i \in S = [0, \infty)$ and obtains payoff:

$$u_i (q_1, ..., q_n) = P \left(\sum_{h=1}^{n} q_h \right) q_i - C (q_i) ,$$

where the inverse demand curve, $P(Q)$, and the cost curve, $C(q_i)$, are assumed to be differentiable with $P' < 0$, $C' > 0$, and $C'' \geq 0$. This game has the aggregator property with $\sum_{h=1}^{n} q_h$ as the aggregator. We assume $P(\infty) < C'(0) < P(0)$, which means that there is a unique "competitive equilibrium", $q_i = q^c$ for all i and $P(nq^c) = C'(q^c)$.

We do not have to conduct any formal analysis to find the outcome of economic Darwinism in this example. A unique symmetric evolutionary equilibrium is equivalent to symmetric Nash equilibrium in relative payoffs (Proposition 1) as well as to symmetric aggregate-taking equilibrium (Proposition 3), and it is well-known that each of these coincide with the unique competitive equilibrium under appropriate conditions, Possajennikov (2003) and Vega-Redondo (1997). It follows that under assumptions ensuring a unique symmetric evolutionary equilibrium this must be the competitive equilibrium. For a version of the game where the strategy set is finite and $q^c \in S$ and Assumption 2 is fulfilled, the state $\omega^c = (q^c, ..., q^c)$ is by Theorem 1 the unique stochastically stable state.

The intuition for this result is instructive: Start from a Cournot-Nash equilibrium given by $q_i = q^{ne}$ for all i, and $\partial u_i / \partial q_i = 0$, or $P'(nq^{ne})q^{ne} + P(nq^{ne}) = C'(q^{ne})$. Price, $P(nq^{ne})$, is above marginal cost, $C'(q^{ne})$. A unilateral increase of production from q^{ne} by one oligopolist will (of course) imply a loss of profit for the deviator, but the other oligopolists will loose more (strictly at q^{ne} the deviating player's loss is of second order while the others' are of first order). All oligopolists' payoffs are reduced to the same extent by a decreased price, but as long as price is above marginal cost the deviator is partly compensated by the positive marginal profit earned on the increased production (given the price). Hence, all the way up to the competitive equilibrium where price is equal to marginal cost, increases in production will imply increased relative profit for the deviator, so only at the competitive equilibrium no increase in relative profit can be obtained. For the particular game of Cournot oligopoly the possibilities for relative payoff increases are exhausted exactly when price equals marginal cost.

In the Cournot example economic Darwinism leads to higher activity than does Nash equilibrium (now assumed to be unique), $q^{ne} < q^c$, as can easily be seen from the conditions $P(nq^c) = C'(q^c)$ and $P'(nq^{ne})q^{ne} + P(nq^{ne}) = C'(q^{ne})$. Nash equilibrium in turn involves higher activity than in the social optimum (the monopoly outcome), where $q_i = q^o$ for all i, and q^o maximizes $P(nq)\,q - C(q)$: Assuming $P''(Q)Q/P'(Q) > -2$, the function $P(nq)\,q - C(q)$ is concave in q and there is a unique social optimum, $0 < q^o < \infty$, given by $P'(nq^o)nq^o + P(nq^o) = C'(q^o)$. Comparison to the first order condition for Nash equilibrium shows that $q^o < q^{ne}$. All in all $q^o < q^{ne} < q^c$, and the example suggest that this will be a general property of situations with negative externalities, and vice versa for positive externalities.

Our next example demonstrates primarily the usefulness of our established equivalences and the bite of economic Darwinism in games that are not submodular. It also shows that competitive equilibrium, or price equal to marginal cost, is not a general implication of economic Darwinism.

5.2 A differentiated product Bertrand oligopoly with strategic complementarity

Consider a market with n price-setting firms and zero maginal costs, where the payoff functions are:

$$u_i(p_i, p_{-i}) = \left(1 - p_i + \frac{1}{2}\frac{\sum_{h=1}^n p_h}{n}\right) p_i.$$

This game has the aggregator property with the average price, $A = \left(\sum_{h=1}^n p_h\right)/n$, as the aggregator.

The symmetric unique Nash equilibrium of this game is $p_i = p^{ne}$ for all i, where:

$$p^{ne} = \frac{2n}{3n-1}.$$

Taking the aggregate as given, best responses are:

$$p_i = \frac{1}{2}\left(1 + \frac{1}{2}A\right).$$

Thus, the optimal p_i is increasing in A, i.e., there is strategic complementary between the aggregate A and the individual price p_i, which implies that quasi submodularity is not fulfilled.

The unique symmetric aggregate-taking equilibrium is $p_i = p^* = 2/3$ for all i, and by Proposition 3 this is also the unique symmetric evolutionary equilibrium. In an appropriate finite version of the game where $2/3 \in S$, Theorem 1 implies that the unique stochastically stable state is $\omega^* = (p^*, ..., p^*)$.

The symmetric price that maximizes the firms' common payoff, $u_i(p, ..., p) = \left(1 - p + \frac{1}{2}p\right)p$, is $p^o = 1$. Hence $p^* < p^{ne} < p^o$.

In our examples so far the outcome of economic Darwinism has corresponded to "optimization neglecting one's influence on the aggregator". We now consider a game where this is not the case, because the game does not have the aggregator property.

5.3 A learning spill-over game

Each of $n \geq 4$ firms produces output from labor input $\ell_i \in [0, \infty)$. The wage rate is normalized to one. The firms sell output at a common price $p > 0$. A productive externality, a learning spillover, implies that a firm's production depends positively on the labor input of the firm among the other firms that uses the largest labor input and hence has the highest production (each firm learns from the one opponent there is most to learn from). The payoff functions are:

$$u_i(\ell_1, ..., \ell_n) = 4\ell_i^{\frac{1}{2}} \left[\max_{j \neq i} \ell_j\right]^{\frac{1}{4}} p - \ell_i.$$

This game does not have the aggregator property.

The best reply of firm i, given by

$$\frac{\partial u_i}{\partial \ell_i} = 2\ell_i^{-\frac{1}{2}} \left[\max_{j \neq i} \ell_j\right]^{\frac{1}{4}} p - 1 = 0,$$

is $\ell_i = \left[\max_{j \neq i} \ell_j\right]^{\frac{1}{2}} (2p)^2$. Hence, the unique symmetric Nash equilibrium is $\ell_i = \ell^{ne} \equiv (2p)^4$ for all i.

The Nash equilibrium is not efficient. The best symmetric outcome is found by maximizing $4\ell^{\frac{3}{4}} p - \ell$ with respect to ℓ. The first order condition is $3\ell^{-\frac{1}{4}} p - 1 = 0$, giving the symmetric social optimum, $\ell_i \equiv \ell^o = (3p)^4$ for all i. In Nash equilibrium too little effort is exerted from a social point of view, because each firm does not take the positive externality of its production into account.[9]

To find the symmetric evolutionary equilibria by use of Proposition 1, we find the following right and left hand derivatives at symmetric points ($\ell_h = \ell$ for all h):

$$\left.\left(\frac{\partial u_i}{\partial \ell_i} - \frac{\partial u_j}{\partial \ell_i}\right)_{\partial \ell_i > 0}\right|_{\ell_h = \ell} = 2\ell_i^{-\frac{1}{2}} \left[\max_{j \neq i} \ell_j\right]^{\frac{1}{4}} p - 1 - \ell_j^{\frac{1}{2}} \ell_i^{-\frac{3}{4}} p = \ell^{-\frac{1}{4}} p - 1,$$

[9] The efficient symmetric outcome is not overall efficient: a higher total payoff can be obtained in asymmetric situations where only one or two firms exert a lot of effort.

$$\left(\frac{\partial u_i}{\partial \ell_i} - \frac{\partial u_j}{\partial \ell_i}\right)_{\partial \ell_i < 0}\Bigg|_{\ell_h = \ell} = 2\ell_i^{-\frac{1}{2}}\left[\max_{j \neq i} \ell_j\right]^{\frac{1}{4}} p - 1 = 2\ell^{-\frac{1}{4}} p - 1.$$

By Proposition 1, the conditions:

$$\left(\frac{\partial u_i}{\partial \ell_i} - \frac{\partial u_j}{\partial \ell_i}\right)_{\partial \ell_i > 0}\Bigg|_{\ell_h = \ell} \leq 0, \text{ and } \left(\frac{\partial u_i}{\partial \ell_i} - \frac{\partial u_j}{\partial \ell_i}\right)_{\partial \ell_i < 0}\Bigg|_{\ell_h = \ell} \geq 0,$$

characterize interior evolutionary equilibrium. These conditions are equivalent to $p^4 \leq \ell \leq (2p)^4$. Thus, there is a continuum of symmetric evolutionary equilibria, $\ell_i = \ell$ for all i, namely one for each ℓ from p^4 up to the Nash equilibrium $(2p)^4$.

In an appropriate finite version of the game, where $(2p)^4 \in S$ and Assumption 2 is fulfilled, the unique stochastically stable state is $\omega^* = \left((2p)^4, ..., (2p)^4\right)$ corresponding to the evolutionary equilibrium that is also Nash equilibrium. To see this first note that by Theorem 1, only states corresponding to evolutionary equilibria can be stochastically stable. Consider any state $(\ell, ..., \ell)$ where ℓ is an evolutionary equilibrium and $\ell \neq (2p)^4$. From this state simultaneous mutation by two players to $(2p)^4$ will result in a state ω' where the mutants get the Nash-equilibrium payoff (since they can utilize the learning spill-over from each other), whereas the non-mutants get a strictly lower payoff (facing the same learning spill-over as the mutants, but exerting too little effort). Thus there will be positive probability according to the unperturbed process to go from ω' to ω^*. From ω^*, on the other hand, two mutations do not suffice to escape, since any two mutants will obtain strictly lower payoff than the remaining (at least two) players who still use $(2p)^4$: two mutants who both mutate to $\ell < (2p)^4$, will (as the non-mutants) utilize the spill-over from $(2p)^4$, but their effort levels will be inferior.

This example illustrates the bite of economic Darwinism in situations where the aggregator property is not fulfilled and in situations where evolutionary equilibrium does not give a sharp prediction.

6 Economic implications of economic Darwinism

The examples of Section 5 suggest that externalities imply that economic Darwinism generally leads to socially too high or low activity to a degree at least as bad as, and sometimes worse than, Nash equilibrium. This is indeed a general result as will now be shown.

We impose some simplifying assumptions in addition to Assumption 1. First we assume that the strategy set, S, is a convex and closed subset of $\mathbb{R}_+$, and that the payoff functions, u_i, are differentiable (until we "finitize" the game to be able to use the relation between evolutionary equilibrium and stochastically stable states).

Second, we consider situations where externalities are overall positive, $\partial u_i / \partial s_h > 0$ for all $i \neq h$. Results for the case of overall negative externalities, $\partial u_i / \partial s_h < 0$ for all $i \neq h$, follow analogously.

Third, we assume that each of the concepts symmetric social optimum, symmetric Nash equilibrium and symmetric evolutionary equilibrium is unique and that the latter two are interior and fully characterized by first order conditions (considered below) and furthermore, these properties are insensitive to small perturbations of payoff functions (the latter property being fulfilled generically).

Welfare, $W(s)$, at a common strategy, s, is the common utility of all players at the strategy profile $(s, ..., s)$, that is, $W(s) \equiv u_i(s, ..., s)$ for any i. The symmetric social optimum is: $s_i = s^o$ for all i, where s^o maximizes $W(s)$.

Our final assumption is that W is concave. The derivative of $W(s)$ is:

$$W'(s) = \sum_{h=1}^{n} \frac{\partial u_i}{\partial s_h} (s, ..., s),$$

where, from the assumed symmetry, i can again be any player.

Define the "marginal product", $m(s) \equiv \frac{\partial u_i}{\partial s_i}(s, ..., s)$, from a symmetric profile. Again, i can be any player in this definition. The unique symmetric Nash equilibrium, $s_i = s^{ne}$ for all i, is given by $m(s^{ne}) = 0$. From our assumptions, $m(s)$ has to be strictly decreasing at s^{ne}: First, $m(s)$ has to intersect strictly with the s-axis at $s = s^{ne}$, since otherwise a small perturbation of payoff functions could imply non-existence of Nash equilibrium. Second, $m(s)$ must be decreasing by the second order condition.

Since, at the symmetric Nash equilibrium, $m(s^{ne}) = \frac{\partial u_i}{\partial s_i}(s^{ne}, ..., s^{ne}) = 0$, one has:

$$W'(s^{ne}) = \sum_{h \neq i} \frac{\partial u_i}{\partial s_h} (s^{ne}, ..., s^{ne}),$$

which is strictly positive because of positive externalities. Hence, $s^o > s^{ne}$, and obviously $W(s^{ne}) < W(s^o)$. This is just a restatement of the well-known result that with positive externalities, Nash equilibrium implies socially too little activity.

A symmetric evolutionary equilibrium, $s_i = s^*$ for all i, is given by the first order condition: $\frac{\partial u_i}{\partial s_i}(s^*, ..., s^*) - \frac{\partial u_j}{\partial s_i}(s^*, ..., s^*) = 0$, where the first term is $m(s^*)$, and, again because of symmetry, i and j can be any two (different) players. Hence,

$$m(s^*) = \frac{\partial u_j}{\partial s_i}(s^*, ..., s^*) .$$

Positive externalities thus imply $m(s^*) > 0$. Since $m(s^{ne}) = 0$, and $m(s)$ is decreasing at $s = s^{ne}$ and $m(s)$ only has the one intersection with the axis at s^{ne}, one has that $s^* < s^{ne}$. Hence, the evolutionary equilibrium is further away from the social optimum than the Nash equilibrium implying, from the concavity of W, that $W(s^*) < W(s^{ne})$.

Imposing an appropriate further assumption of finiteness where all of s^o, s^{ne} and s^* are in S, and Assumption 2, symmetric social optimum, symmetric Nash equilibrium, and symmetric evolutionary equilibrium are unchanged and the latter coincide with the unique stochastically stable state. This suffices for:

Theorem 2 *Under the assumptions of this section, if externalities are overall positive (negative), symmetric social optimum implies "higher" ("lower") actions and higher welfare than does symmetric Nash equilibrium, and symmetric Nash equilibrium implies "higher" ("lower") actions and higher welfare than does the outcome of economic Darwinism.*

7 Concluding remarks

Our first main results, Theorem 1 and Corollary 1, establish a close connection between frequently observed, so-called stochastically stable states and evolutionary equilibrium, in fact an equivalence when the latter exists uniquely. One implication is that economic Darwinism does not in general support Nash equilibrium behavior.

Theorem 1 and Corollary 1 hold under relatively general conditions. By considering an evolutionary process based on economic selection rather than on imitation we have obtained a process that in our eyes captures 'survival of the fittest in economics contexts' more reasonably *and* at the same time gives more generality in the basic characterization results: we have avoided underlying assumptions of the aggregator property and sub-modularity.

Our second main result, Theorem 2, establishes what we consider to be the most important economic implication of economic Darwinism: in the presence of overall positive or negative externalities, outcomes arising from economic Darwinism are even worse than Nash equilibrium outcomes.

It is not just, as with Nash equilibrium, that each individual's behavior is insensitive to the way it affects other people's payoffs. For the outcomes of economic Darwinism it is true that even if an increase in "effort" would benefit the individual undertaking it, the increase will *not* be undertaken unless it improves the individual's *relative* position, that is, unless it benefits the individual more than it benefits other individuals. And even if it hurts the individual, it *will* be undertaken if it hurts other people more. This gives an increased tendency (as compared to Nash equilibrium behavior) to contributing too little in the presence of positive spill-overs, and to exploiting too much in the presence of negative spill-overs.

It is often argued that social institutions such as legal systems or customary norms and conventions are rooted in the fact that traditional selfish (Nash equilibrium) behavior would create socially too bad outcomes in standard social environments. In so far as unchecked behavior is more guided by evolutionary forces than by selfish "rationality", the argument in favor of social institutions stands even stronger.

8 References

Alchian, Armen (1950): "Uncertainty, evolution and economic theory", *Journal of Political Economy* **58**, 211-221.

Alós-Ferrer, Carlos and Anna B. Ania (2002): "The Evolutionary Logic of Feeling Small", Working Paper 0216, Department of Economics, University of Vienna.

Corchón, L. (1994): "Comparative Statics for Aggregative Games. The strong Concavity Case", *Mathematical Social Sciences* **28**, 151-165

Enke, Stephen (1951): "On maximizing profits: A distinction between Chamberlin and Robinson", *American Economic Review* **41**, 566-578.

Freidlin, Mark I. and Alexander D. Wentzel (1984): *Random Perturbations in Dynamical Systems*, New York: Springer.

Friedman, Milton (1953): "The Methodology of Positive Economics" in *Essays in Positive Economics*, Chicago: Chicago University Press, 3-45.

Leininger, Wolfgang (2004): "Fending off one means fending off all: Evolutionary Stability in Submodular Games", Discussion Papers in Economics No. 04-03, Universität Dortmund.

Possajennikov, Alex (2003): "Evolutionary Foundations of Aggregate-Taking Behavior", *Economic Theory* **21**, 921-928.

Schaffer, Mark. E. (1989): "Are Profit-Maximizes the Best Survivors? A Darwinian Dynamics of Economic Natural Selection", *Journal of Economic Behavior and Organization* **12**, 29-45.

Schenk-Hoppé, Klaus Reiner (2000): "The Evolution of Walrasian Behavior in Oligopolies", *Journal of Mathematical Economics* **33**, 35-55.

Schipper, Burkhard C. (2003): "Submodularity and the Evolution of Walrasian Behavior", *International Journal of Game Theory* **32**, 471-477.

Stegeman, Mark and Paul Rhode (2004): "Stochastic Darwinian Equilibria in Small and Large Populations", *Games and Economic Behavior* **49**, 171–214.

Vega-Redondo, Fernando (1997): "The Evolution of Walrasian Behavior", *Econometrica* **65**, 375-384.

Young, H. Peyton (1993): "The Evolution of Conventions", *Econometrica* **61**, 57-84.

Young, H. Peyton (1998): *Individual Strategy and Social Structure,* Princeton

NJ: Princeton University Press.