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Theory and Application

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Competitive Auctions: Theory and Application

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Theory and Application[*]

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Abstract

The theory of competitive auctions offers a coherent framework for modelling coordination frictions as a non-cooperative game. The theory represents an advancement over cooperative approaches that make exogenous assumptions about how output is divided between buyers and sellers and about the forces that bring buyers and sellers into local markets. Moreover, unlike price posting models, which fix the terms of trade prior to matching, competitive auction models have a bidding process that allocates the good (or service) to the highest valuation bidder at a price equal to the second highest valuation. Therefore, the competing auction model is more robust to problems in which there are heterogenous valuations. This paper develops the theory of competitive auctions and applies it to a number of practical problems in microeconomics, labor economics, industrial organization, investment theory and monetary economics.

1 Introduction

At the heart of every economic theory is a description of how people exchange. Economists generally choose between one of two possible extremes: Walrasian or random matching. In the Walrasian extreme the cost of communication between buyers and sellers is assumed to be zero. Therefore,

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[*]The goal of this survey is to introduce my research to students and professors at the University of Copenhagen with the hope that this exchange of knowledge will aid research based teaching initiatives. I also hope that this survey will prove to be of interest to the wider academic community, because the methods described are quite general and have broad application.

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buyers and sellers in a Walrasian economy need only report their characteristics to a central mechanism designer and a set of transfers are then carried out using this information.[1] At the opposite extreme is random matching. In random matching models the cost of communication is assumed to be almost insurmountable. Instead, buyers and sellers are brought together by an exogenous matching technology and only then can they communicate with their potential trading partner(s).

Recent research has developed models in which trade is neither Walrasian nor random matching. These so-called directed search models allow a range of communications that falls within the extremes of Walrasian and random matching environments. For example, in a directed search model we might assume that a set of similar sellers can easily communicate their locations to buyers, but we might also assume that buyers cannot communicate with each other over which seller to visit. This type of model has equilibrium mixed strategies for buyers concerning their location decision over sellers. These mixed strategy equilibrium are a useful method to describe the difficulties of coordination in large decentralized markets. Thus the early applications of these models have been to the theory of unemployment.

Directed search models confront a number of modelling issues that are not found in either Walrasian or random matching environments. The basic premise is that the selling mechanism of each seller matters for both price and probability of trade with buyers. This result is not assumed as in models of monopolistic competition, but is instead derived as the outcome of a market with coordination frictions. For example, in this paper I derive a simple static model of competitive auctions with sellers setting reserve prices. In a small market, the sellers set positive reserve prices. However, in the limit as the market is made large, I show that the equilibrium reserve price at each auction is equal to zero. In other words competition leads to a very simple selling mechanism. The rest of this survey of competitive auctions is dedicated to showing that this basic model has many applications.

A useful application of this framework is a dynamic model of the labor market in which the reserve price of sellers - workers - is equal to their outside option. This model offers a very tractable alternative to Pissarides (2000) of the labor market. The model also avoids making assumptions that are foreign to conventional general equilibrium theory. In particular, Pissarides assumes that wages are determined by an exogenous Nash bargaining rule and that the arrival rate of meetings is determined by an exogenous matching technology. These elements are endogenized in a competitive auction

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[1] This central figure is often referred to as the Walrasian auctioneer.

framework.

A surprisingly robust feature of competitive auction equilibrium is efficiency. The basic premise that auctions generate efficiency in random matching games was first advanced by Mortensen (1982). The Mortensen rule, roughly stated, is that the surplus of a match should go to the iniator of a match. I show how the Mortensen rule can be stated as a set of axioms that give outcomes equivalent to competitive auctions without reserve prices. I also show that this axiomatic rule gives efficiency in a set of matching games for which the well known Hosios rule fails. These games include markets of finite size, where the matching technology displays decreasing returns to scale and markets with heterogenous buyers where the matching technology contains more than to arguments. These examples illustrate that competitive auctions are not subject to a holdup problem and they also introduce the possibility of efficient technology dispersion.

Competitive auction theory offers a very simple framework in which to study endogenous job destruction. This problem is difficult to study in an alternative frameworks such as price posting models with coordination frictions, because the posted prices always leave on-the-job searchers never completely satisfied – they are always looking for more. The auction mechanism gives them what they want - everything - but only if they have multiple offers in the type of job they are currently in. Therefore, competitive auctions offers a much more tractable theory of on-the-job search. To illustrate I extend the basic dynamic model to have (i) heterogenous jobs that are distinguished by their productivity and capital cost and (ii) heterogenous job searcher who are distinguished by their employment status. On-the-job search leads to wage changes as workers move into higher productivity jobs. The equilibrium is also constrained efficient.

Many models of competitive auction treat the identity of buyers and sellers as exogenous. This is not an innocuous assumption. Suppose that there are two types of agents - red and blue - that are to be matched. The question is what characteristics of red and blue agents causes one type to be the seller. I identify two separate causal elements. First, I show red agents are likely to be buyers if they are more numerous than blue agents. Second, I show that red agents are likely to be buyers if they are heterogenous and blue agents are homogenous. Therefore, endogenizing the choice of who is the seller leads to a market in which heterogenous buyers search over homogenous sellers.

I also evaluate closely related models of coordination frictions with price posting. These model works well if (i) buyers are homogenous and (ii) buyer-seller relationships are stable by assumption. In this case, the auction

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and price posting models are equivalent. However, many difficulties arise if buyers are heterogenous. In particular, even in a static model, in order to gain equivalence with efficient competitive auctions, a very complicated discriminatory price posting structure must be adopted by sellers. Moreover, if valuations are determined ex post by nature, then the predetermined terms of trade dictated by a posted prices are inevitably inefficient.

I also consider the wage implications of competitive auction models. The solution of a competitive auction model yields an expression for the present value of a worker at a job. However, it is also straightforward to derive the implications of this present value for wages. Therefore, it is easy to show that a model without on-the job search does not yield much wage dispersion. Moreover, it can also be shown that the model can explain much wage dispersion if on-the-job search is permitted. The other issue about wages is the outcome of competitive auctions in markets with small frictions. I show that removing frictions from the model leads to a Walrasian outcome in which workers are paid their marginal product. This exercise of comparing the limiting properties of models with frictions to Walrasian outcomes was advanced by Rubinstein and Wolinsky (1985) and Gale (1986). Gale argues that this exercise is an important test of how reasonable the assumptions are of the matching game specified.

The theory of competitive auctions implies a shift in focus from imperfect information about buyers to imperfect information about sellers. On the one hand, the problem of informational inefficiency concerning buyers is solved by the auction and thus the high valuation buyer is always rewarded the good for sale at a price equal to the second highest valuation. On the other hand, the sellers' problem of attracting buyers gives them much incentive to advertise themselves as good - a classic lemons problem. Therefore, I consider the effects of exogenously lifting the veil of ignorance about the quality of sellers. This application shows that third party information is used efficiently but that there are important distributional considerations. I show that buyers are always made worse off by small additions to their information set, but can be benefited by a sufficiently large addition.

There are a number of other applications of competitive auctions. I attempt to briefly summarize a number of these applications. The list is by no means exhaustive and is meant to offer suggestions about how the theory might be applied.

I caution that the present paper does not attempt to advance the theory of competitive search by Shimer (1996) and Moen (1997). The theory of competitive search derives a number of very similar results as competitive auction theory, but for a very different reason. In particular, the theory

of competitive search obtains efficiency in matching by the assumption of middlemen who oversee frictional submarkets and set the terms of trade in each. This concept runs into difficulties if buyers are hetergeneous. For example. in a price posting model, Shi (2004) shows that, if buyers are heterogenous, the optimal equilibrium decision of sellers requires a vector of ex ante prices - with each price corresponding to the unique valuation of each type of buyer. Moreover, he shows that these pricing announcements need not be monotonic. In particular, sellers may post lower prices for high valuation buyers than for low valuation buyers. The competitive auction model is a much simpler framework, because prices are determined ex post and there is no need to solve a complicated multi-dimensional ex ante pricing game. It also seems more realistic, because the sale of unique goods and services, such as specialized labor or specialized jobs, is rarely done by posted price (ref: Gautier and Moraga-Gonzalas 2004).

The paper is organized into 8 analytical sections that demonstrate a number of the main results. I then provide a section on further applications where I discuss a number of other contributions to the theory and I attempt to link these results to ideas presented in the preceding sections. The final section offers concluding remarks.

2 Competitive Auctions

This section considers a simple game of competitive auctions described in Julien, Kennes and King (2000).[2] This model introduces the optimal auction of each seller as a choice of reserve price. The coordination frictions arise in the mixed strategy equilibrium of this game. The model illustrates an important result that competition among sellers tends to reduce their reserve prices. In the game presented here, the reserve price is driven to zero in the limit as the number of buyers and sellers is large. The fact that prices cease to play a role in a large market is an important simplification, because this it implies that decentralized trade with auctions can be modelled as an ex post pricing game. Many of the models of this survey use this simplification.

2.1 The model

There are N identical sellers and M identical buyers, all spatially separated. Each seller has one good for sale worth y to any buyer and worth 0 to the

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[2]The seminal contributions to competitive auctions are Wolinsky (1988) and McAfee (1993). Peters (1984) and Montgomery (1991) introduce the basic problem as a price posting game.

seller. All agents are risk neutral and maximize expected income. Buyers can choose the location of only one seller. The sequence of events within the period is as follows. First, each seller announces a reserve price to induce visits from buyers. Buyers then decide which seller to approach. Sellers then auction the good to the highest bidder.

2.2 The bidding game

We start with the bidding game for, and given each seller's announced reserve price r_i where $i \in \{1, 2, ..., N\}$ is used to denote sellers. Let $m_i \in \{0, 1, 2, ..., M\}$ denote the number of buyers bidding at seller i and let $w(r_i, m_i)$ denote the equilibrium price obtained by seller i. As is standard in an ascending-bid auction with homogenous buyers and complete information, the seller's price is given by

$$w(r_i, m_i) = \begin{cases} 0 & \text{if } m_i = 0 \\ r_i & \text{if } m_i = 1 \\ y & \text{if } m_i > 1 \end{cases} \tag{2.1}$$

Clearly, if no buyer approaches the seller, his price is zero. If only one buyer approaches, then the candidate receives his reserve price r_i. If more then one buyer approaches then Bertrand competition between buyers drives the price up to the point where the seller receives all the gains that the buyer makes from owning the good, y.

2.3 Buyers choice of seller to bid for

Having observed the sellers' reserve price announcement vector, buyers decide which seller to bid for. Since we will be focusing on symmetric equilibria, for notational convenience we will assume that all sellers other than i choose the same reserve price r. Let $p_i(r_i, r)$ denote the probability that a particular buyer bids for seller i. Thus, for any buyer, the probabilities must sum to one, and given m_i sellers at seller i, the probability that seller i will accept any offer is given by $\Pr\{i \text{ accepts}\} = 1/m_i$. Once the buyer decides to locate at seller i, given m_i, the expected payoff to this buyer is $R_i = (y - w(r_i, m_i)) \Pr\{i \text{ accepts } w(r_i, m_i)\}$, which is $y - r_i$ if $m_i = 1$ and 0 if $m_i > 1$. In a symmetric equilibrium $(1 - p_i(r_i, r))^{M-1}$ is the probability that the buyer will be alone in his offer to candidate i, and $[1 - (1 - p_i(r_i, r))^{M-1}]$ is the probability that at least one other will make this seller an offer. Hence, before knowing m_i, the buyers expected payoff if she makes an offer to seller i is

$$\Pi_i(r_i, r) = (1 - p_i(r_i, r))^{M-1}(y - r_i). \tag{2.2}$$

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In a symmetric mixed strategy equilibrium, each buyer chooses $p_i(r_i, r), i = 1, 2, ..., N$, so that $\Pi_i(r_i, r) = \Pi$. Let $p(r_i, r)$ denote the symmetric mixed strategy probability assigned to all other sellers, the the constraint that the location probabilities sum to one implies $p(r_i, r) = (1 - p_i(r_i, r))/(N - 1)$. Using this constraint and (2.2) one obtains

$$p_i(r_i, r) = 1 - \frac{N - 1}{1 + (N - 1)\left(\frac{y - r_i}{y - r}\right)^{1/M - 1}} \tag{2.3}$$

2.4 Sellers' reserve price choice

Sellers choose their reserve price to maximize their expected payoffs in a simultaneous moves game with other sellers. Let $q_{i0}(r_i, r) = (1 - p_i(r_i, r))^M$ and $q_{i1}(r_i, r) = Mp_i(r_i, r)(1 - p_i(r_i, r))^{M-1}$ denote the probabilities that seller i will receive zero offers and one offer, respectively. The expected payoff function for seller i is therefore given by

$$V_i(r_i, r) = q_{i1}(r_i, r)r_i + (1 - q_{i0}(r_i, r) - q_{i1}(r_i, r))\, y. \tag{2.4}$$

Since sellers choose their reserve prices simultaneously, an equilibrium array of reserve prices is found by a standard Nash argument, $r_i^* = \arg\max_{r_i} V_i(r_i, r^*)$. The symmetric Nash Equilibrium is

$$r_i^* = r^* = \frac{(M - 1)}{(M - 1) + (N - 1)^2} y \tag{2.5}$$

2.5 A large market

We now consider the properties of this equilibrium, as the scale of the market become large. To do this, we hold the ratio of buyers to sellers constant, $\phi = M/N$, and examine the case where N is very large, but finite number. In this type of environment the economy can be closely approximated by the limit economy here N→ ∞. In the large economy, the reserve price becomes zero.[3]

$$r^* = 0 \tag{2.6}$$

The matching technology in a large market is given by

$$x(N, M) = N(1 - e^{-\phi}), \tag{2.7}$$

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[3] McAfee and McMillan (1985) derive a similar result in a model of endogenous buyer entry with a monopoly auction.

which is the familiar urn ball matching function. The expected payoff to the representative buyer is $\Pi(\phi) = e^{-\phi} y$, which states that the buyer is paid y if she is alone in her offer, which occurs with frequency $e^{-\phi}$. Likewise, the expected payoff of the representative seller is $V(\phi) = \left(1 - e^{-\phi} - (\phi)e^{-\phi}\right) y$, which states that the seller gets y if he has multiple offers, which occurs with frequency $1 - e^{-\phi} - \phi e^{-\phi}$.

3 Dynamics

This section derives a dynamic model of competitive auctions in which workers auction their services to firms. Here, we assume that the reserve price of each worker's auction is equal to their outside option, even though this assumption can be derived explicitly as an equilibrium outcome as is done in Julien, Kennes and King (2000). The model is similar to the basic model of Pissarides (2000). However, we do not need to specify exogenous 'sharing rules' or 'matching functions'. These aspects of the model are derived endogenously. The only friction in the competitive auction model is the length of time between offer rounds.

3.1 The Model

Consider a simple economy in which N identical workers face an infinite horizon, perfect capital markets and a common discount factor β. At the start of each period $t = 0, 1, 2, ..$, there are $N - E_t$ displaced workers of productivity $y_0 = 0$ and E_t workers employed in jobs of productivity $y_1 = y > 0$. The ratio of M_t job vacancies to job searchers (i.e. displaced workers) is

$$\phi_t = \frac{M_t}{(N - E_t)} \tag{3.1}$$

Each of the vacant jobs carries a capital cost k per job per period. Each worker can work at most one job and one job can employ at most one worker. Furthermore, any match in the current period may dissolve in the subsequent period with probability ρ. The job vacancies are randomly assigned to job searchers. Therefore, the net addition of H_t workers is given by the following matching technology: $H_t = (N - E_t)(1 - e^{-\phi_t})$, where $1 - e^{-\phi_t}$ is the probability the worker obtains at least one offer. The exogenous separations at the end of the period imply that the supply employed workers at the start of next period is given by $E_{t+1} = (1 - \rho)(E_t + H_t)$.

The labor market is decentralized with each worker using a second price auction for their labor services. Within each period, the order of play is as follows. At the beginning of the period, given the state, new vacancies enter. Once the number of entrants has been established, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, wages are determined through the auction mechanism. Let Λ_i denote the expected discounted surplus of a match between an unemployed worker and a job of productivity y_i at the start of the period (where a job of productivity $y_0 = 0$ is of course the unemployed state - home production). A second price auction implies that the workers share W_i^j of the total surplus Λ_i is equal to the surplus Λ_j of the worker's second best available job offer. Thus

$$W_i^j = \Lambda_j \tag{3.2}$$

The randomness of the number of jobs at each displaced worker implies that the present value of a displaced worker is given by

$$V_t = p_t^0 \Lambda_{0t} + p_t^1 \Lambda_{1t} \tag{3.3}$$

where $p_{1t}^0 = e^{-\phi_t} + \phi_t e^{-\phi_t}$ is the probability the worker has either one or no job offers in the current period and $p_t^1 = 1 - e^{-\phi_t} - \phi_t e^{-\phi_t}$ is the probability of multiple offers in which case the workers second best offer is a job of productivity y_1. The supply of firms is determined by free entry such that firms earn zero profits in equilibrium. Thus the expected profit Π_t of a firm opening a job is given by

$$\Pi_t = \max\{q_t^0(\Lambda_{1t} - \Lambda_{0t}) - k, 0\}. \tag{3.4}$$

where $q_t^0 = e^{-\phi_t}$ is the probability the buyer of labor (the firm) is alone in its offer to the worker in the current period. The value of a displaced worker that does not find a job is given by

$$\Lambda_{0t} = \beta V_{t+1} \tag{3.5}$$

and the value of a worker that is employed is given by $\Lambda_{1t} = y + \beta[\rho V_t + (1-\rho)y] + \beta^2(1-\rho)[\rho V_{t+1} + (1-\rho)y] + ...$which in a steady state is given by

$$\Lambda_1 = \frac{y + \beta\rho V}{1 - \beta(1 - \rho)} \tag{3.6}$$

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3.2 Equilibrium

In equilibrium, the supply of job vacancies is given by

$$k = \frac{y}{1 - p^0(1 - \rho)\beta} e^{-\phi} \qquad (3.6)$$

This equations states thate the vacancy cost must be offset by the probability the seller is alone in its offer times an appropriately discounted flow of returns equal to y each period. The equilibrium unemployment rate is given by $u = \rho e^{-\phi} / \left(1 - (1 - \rho)e^{-\phi}\right)$.

4 Efficiency

Julien, Kennes and King (2003) apply an axiomatic approach to the efficiency of markets in which the participants in local markets are determined by coordination frictions. This approach is taken from Mortensen (1982) who uses it in a different context. Mortensen's axioms are useful here, because the outcome of these axioms is an auction without reserve price. Therefore, we can use apply these axioms to the game presented previously and to its natural extensions to heterogenous buyers. In both cases, we will show that the Mortensen rule gives efficiency. Moreover, in these games, the well known axioms of Hosios (1990) do not apply. This section will also illustrate why the competitive auction framework is not subject to a hold up problem and why endogenous technology dispersion - the endogenous heterogenous valuation of buyers - is efficient.

4.1 Efficient entry

There are N identical sellers and M identical buyers, all spatially separated. Each seller has one good for sale worth y to any buyer and worth 0 to the seller. All agents are risk neutral and maximize expected income. Buyers are randomly allocated to sellers. Therefore, the expected number of matches is given by

$$x(N, M) = N(1 - (1 - 1/N)^M) \qquad (4.1)$$

The matching function has decreasing returns to scale, but in the limit where N and M are large, it has the function form given by equation 2.7, which displays constant returns to scale.

Let $m \in \{0, 1, 2, ..., M\}$ denote the number of buyers bidding at a seller. A local market contains one seller S and a set of identical buyers, $B =$

$(B_1, B_2, ..., B_m)$ if $m > 1$ and no buyers if $m = 0$.[4] The surplus of a match between the seller and any particular buyer B_i is given by

$$\Lambda(S, B_i) = V(S, B_i) - d_s(B) - d_i(B) \qquad (4.2)$$

where $V(S, B_i)$ is the total value of the match, $d_s(B)$ is the threat point of the seller, and $d_i(B)$ is the threat point of the buyer. The total valuation of a match is given by $V(S, B_i) = y$. The disagreement point of the buyer is zero, because once inside the local market, the buyer can trade only with the seller. The disagreement point of the seller is given by $\max V(S, B_{-i})$ - the maximum total value of the good to the seller and the set of other buyers. This definition of the sellers threat point assumes that each player has a conservative assessment about how well their opponent will be rewarded in the event of a disagreement.

The surplus of a match is divided by the Mortensen rule. The axioms are as follows:

- **Axiom 1** (local efficiency) The pair of local market participants with the highest $V(S, B_i)$ form a match if the surplus $\Lambda(S, B_i)$ is positive.

- **Axiom 2** (initiator of the match) The surplus of the match $\Lambda(S, B_i)$ is rewarded to the initiator of the match - i.e. the buyer.

Axiom 1 of the Mortensen rule is also common to Nash's solution concept. However, the second axiom is simpler than Nash's other axiom, because it presumes that the identity of the match initiator is known. The Mortensen bargaining rule is equivalent to an auction without reserve price by the seller. In particular, the seller obtains a price y if there are multiple buyers at his local market and a price of zero otherwise.

The Mortensen rule has important implications for efficiency in matching. Suppose that the number of buyers is determined by free entry, with each additional buyer to the market paying a capital cost, k. The marginal social benefit of an extra buyer is the extra number of matches created minus this capital cost. It is easy to verify that

$$x(N, M) - x(N, M - 1) = \left(\frac{N - 1}{N}\right)^{M-1} \qquad (4.3)$$

where the right hand side is the probability the buyer is the sole buyer at his chosen local market. Therefore, the marginal social benefit of the extra buyer

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[4]The idea of local markets was advanced by Lucas and Prescott (1974). King and Grouge (1996) work out the dynamics of this model.

is equal to the private return to the extra buyer. Thus the Mortensen rule gives efficient entry in this economy even though the matching technology does not display constant returns to scale!

4.2 Efficient technology dispersion

Consider a simple matching game with a large number of N sellers, M_1 bad buyers and M_2 good buyers. The good buyers have valuation of the sellers good equal to y_2, which is greater than the valuation y_1 of bad buyers. Buyers are randomly allocated to sellers. Therefore, if the best matches are always consummated, the expected value of all the matches is given by

$$S = N \left[(1 - e^{-\phi_2}) y_2 + (1 - e^{-\phi_1}) e^{-\phi_2} y_1 \right] \tag{4.4}$$

where $\phi_1 = M_1/N$ and $\phi_2 = M_2/N$. It should be noted that the Hosios rule cannot be applied to this matching game, because the matching technology has more than two arguments.

Let m_1 and m_2 denote the number of bad and good buyers at the local market of a seller. Let the set of buyers with low valuation be $L = \{B_1, B_2, .., B_{m_1}\}$ if $m_1 \geq 1, \emptyset$ otherwise} and the set of buyers with high valuation be $H = \{(B_{m_1+1}, B_{m_1+2}, ..., B_{m_1+m_2}) $ if $m_2 \geq 1, \emptyset$ otherwise}. In a local market defined by L and H, the surplus of a match between the seller S and any particular buyer B_i is given by $\Lambda(S, B_i) = V(S, B_i) - d_s(L, H) - d_i(L, H)$,.where the total value of the match is

$$V(S, B_i) = \begin{cases} y_2 & \text{if } i \leq m_1 \\ y_1 & \text{if } m_1 + 1 \leq i \leq m_1 + m_2 \end{cases} \tag{4.5}$$

the disagreement point of the buyer $d_i(L, H)$ is zero and the disagreement point of the seller is $\max V(S, B_{-i})$ - the maximum total valuation of the good to the seller and the set of other buyers in the local market.

The Mortensen rule can also be applied to this game. Suppose we assume free entry of buyers, where the capital cost of a bad buyer is k_1 and the capital cost of a good buyer is k_2. If we assume the technological opportunity set displays positive but diminishing returns: $y_1/k_1 > y_2/k_2$ and $y_2 - k_2 > y_1 - k_1$ The decentralized equilibrium under the Mortensen rule is given by

$$e^{-\phi_1} e^{-\phi_2} y_2 + e^{-\phi_1} (1 - e^{-\phi_2})(y_2 - y_1) = k_2 \tag{4.6}$$

and

$$e^{-\phi_1} e^{-\phi_2} y_1 = k_1 \tag{4.7}$$

are both positive. In other words, the decentralized equilibrium has technology dispersion. It is easy to verify that equilibrium is efficient. A social planner faced with the problem of choosing the number of high and low valuation buyers to maximize S less the cost of buyers obtains the same solution as equations (4.6) and (4.7). Therefore, the decentralized economy has efficient technology dispersion.[5]

4.3 Efficient job creation

The matching problem in the dynamic model of section 3 can be also formulated as a social planning problem. The social planner maximizes

$$S = \max_{E_t, E_{t+1}} \sum \beta^t \{y(E_t + H_t) - kM_t\} \tag{4.8}$$

such that $H_t = (N - E_t)(1 - e^{-\phi_t})$ and $E_{t+1} = (1 - \rho)(E_t + H_t)$. It is easy to verify that the solution of this simple dynamic programming problem is the same as the decentralized economy.

5 On-the-Job Search

The dynamic model in section 3 has employer-employee relationships of random but exogenous duration. The duration of relationships can be made endogenous in a number of ways: economic progress that improves the quality of new jobs, jobs distinguished by the opportunities to create specific and general skills, good and bad jobs leading to on-the-job search in the latter. The important issue is to incorporate these factors into the asset equations of the dynamic model. This section considers a simple model of on-the-job search by Julien, Kennes and King (2001), which extends the discussion of technology dispersion in the last section. This model has directed search, because on-the-job searchers - workers in bad jobs - receive fewer offers than workers that are unemployed.[6]

5.1 The Model

A large number of N identical risk neutral workers face an infinite horizon, perfect capital markets, and a common discount factor β. Each worker

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[5] Acemoglu and Shimer (1999) derive related results using a model with non-sequential search, see also Burdett and Judd (1983). Jansen (1999) considers investments by sellers and shows that there is no hold up problem, but also no technology dispersion.

[6] Price posting is considered by Delacroix and Shi (2003).

has one indivisible unit of labor to sell. At the start of each period, $t = 0, 1, 2, 3, ...$, there exist E_{0t} unemployed workers, of productivity $y_0 = 0$, and E_{it} workers in jobs of productivity $y_i > 0$ where $i \in \{1, 2\}$. The ratio of good and bad job vacancies to displaced workers at the start of each period is given by,

$$\phi_{it} = \frac{M_{it}}{(N - E_{1t} - E_{2t})} \tag{5.1}$$

and the ratio of good job vacancies to on-the-job searchers (i.e. workers in bad jobs is given by

$$\widehat{\phi}_{2t} = \frac{\widehat{M}_{2t}}{E_{1t}} \tag{5.2}$$

A vacant job has a capital cost of k_i such that $y_i \geq y_j$ and $k_i \geq k_j$ for all $i \geq j$. A match in any period may dissolve in the subsequent period with fixed probability $\rho \in (0, 1)$. The job vacancies are randomly assigned to job searchers. Therefore, the number of new hires into good and bad jobs is given by $H_{2t} = (N - E_{1t} - E_{2t})p_{2t} + E_{1t}\widehat{p}_{2t}$ and $H_{1t} = (N - E_{1t} - E_{2t})p_{1t} - E_{1t}\widehat{p}_{2t}$ where $p_{2t} = (1 - e^{-\phi_{2t}})$, $p_{1t} = (1 - e^{-\phi_{1t}})e^{-\phi_{2t}}$ and $\widehat{p}_{2t} = (1 - e^{-\phi_{2t}})$. The fraction ρ of all jobs dissolve in the next period, therefore, the supply of worker of each type evolves according to the following transition equations: $E_{it+1} = (1 - \rho)(E_{it} + H_{it})$ $\quad i \in \{1, 2\}$

The labor market is decentralized with each worker using a second price auction for their labor services. Within each period, the order of play is as follows. At the beginning of the period, given the state, new vacancies enter. Once the number of entrants has been established, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, wages are determined through the auction mechanism. Let Λ_{it} denote the expected discounted value of a match between an unemployed worker and a job of productivity y_i. The auction implies that the workers share W_{it}^j is equal to the value of the worker's second best available job offer:

$$W_{it}^j = \Lambda_{jt} \tag{5.3}$$

where Λ_{jt} is the expected discounted value of a match between an unemployed worker and the workers second best available job. The value of a displaced worker is given by

$$V_t = p_t^0 \Lambda_{0t} + p_t^1 \Lambda_{1t} + p_t^2 \Lambda_{2t} \tag{5.4}$$

where $p_t^0 = (1 + \phi_{1t} + \phi_{2t})e^{-\phi_{1t}}e^{-\phi_{2t}}$ is the probability that a worker has one or fewer offers, $p_t^1 = e^{-\phi_{2t}}(1 - \phi_{1t}e^{-\phi_{1t}} - e^{-\phi_{1t}}) + \phi_{2t}e^{-\phi_{2t}} - e^{-\phi_{2t}}$ is

the probability of multiple offers only one of which is possibly good $p_t^2 = 1 - e^{-\phi_{2t}} - \phi_{2t}e^{-\phi_{2t}}$ is the probability of multiple good offers. The profits from introducing good and bad vacancies directed at unemployed workers are given by

$$
\begin{aligned}
\Pi_{1t} &= \max\{(\Lambda_{1t} - \Lambda_{0t})q_t^0 - k_1, 0\} & (5.5) \\
\Pi_{2t} &= \max\{(\Lambda_{2t} - \Lambda_{0t})q_t^0 \\
&\quad + (\Lambda_{2t} - \Lambda_{1t})q_t^1 - k_2, 0\} & (5.6)
\end{aligned}
$$

where $q_t^0 = e^{-\phi_{1t}}e^{-\phi_{2t}}$ is the probability a displaced worker receives no other offer and $q_t^1 = (1 - e^{-\phi_{1t}})e^{-\phi_{2t}}$ is the probability a good firm faces only a competing bad firm in its offer to an unemployed worker. The profits from introducing good vacancies directed at workers in bad jobs is given by

$$
\widehat{\Pi}_{2t} = \max\{(\Lambda_{2t} - \Lambda_{1t})\widehat{q}_t^1 - k_2, 0\} \tag{5.7}
$$

where $\widehat{q}_t^1 = e^{-\widehat{\phi}_{2t}}$ is the probability a worker is not raided by another good firm. The assumption of free entry ensures that these values are equal to zero. The value of a worker that does not find a match this period is

$$
\Lambda_{0t} = \beta V_{t+1}. \tag{5.8}
$$

The value of a high productivity job

$$
\begin{aligned}
\Lambda_{2t} &= y_2 + \beta[\rho V_{t+1} + (1 - \rho)y_2] + & (5.9) \\
&\quad \beta^2(1 - p)[\rho V_{t+1} + (1 - \rho)y_2] + \dots
\end{aligned}
$$

The value of a low productivity job

$$
\begin{aligned}
\Lambda_{1t} &= y_1 + \beta[\rho V_{t+1} + (1 - \rho)X_{t+1}] + & (5.10) \\
&\quad \beta^2(1 - p)[\rho V_{t+1} + (1 - \rho)X_{t+2}] + \dots
\end{aligned}
$$

where X_{t+1} summarizes three possibilities: the employed worker is not recruited, the worker is recruited by one good job, and the worker is recruited by multiple high productivity jobs.

5.2 Equilibrium

It is straight forward to show that the equilibrium has the following properties: 1. the equilibrium is unique,.2. vacant good jobs are directed at workers employed in bad jobs if the present value of the sequence of returns equal to the productivity difference between good and bad jobs discounted by $\beta(1-\rho)$ is greater than the cost of a good job vacancy (i.e. $k_2 < y_2/(1 - \beta(1 - \rho))$, 3. vacant bad jobs are directed at unemployed workers if the cost of bad job vacancies if sufficiently low,.4. unemployed workers receive more good offers on average than workers in bad jobs.and 5. the equilibrium is constrained-efficient.

6 What Makes a Seller?

This section considers the choice of agents to become either buyers or sellers. Suppose that agents on one side of the market are called reds and that agents on the other side of the market are called greens. What factors influence who buys and who sells? There are two factors to consider - (i) relative quantities and (ii) relative heterogeneity. Suppose that there are N red agents and $M = \phi N$ are green agents. The appropriate substititions into the matching technology given by equation 2.7 reveals

$$N(1 - e^{-\phi}) \overset{>}{\underset{<}{-}} M(1 - e^{-1/\phi}) \quad \text{if } \phi \overset{>}{\underset{<}{-}} 1 \qquad (7.1)$$

where the left hand side of the implication is the number of matches if green agents are sellers and the right side is the number of matches if red agents play this role. Thus, other things equal, buyers should be more scarce than sellers for efficiency. A second reason that we can consider is heterogeneity. Consider the case where $\phi = 1$. Suppose that red agents are heterogenous - x good red and (1-x) bad red. Using equation (4.4) it is easy to show

$$M(1 - e^{-x})y_2 + M(1 - e^{-x})e^{-x}y_1$$

$$> \max_{1 \geq z \geq 0} N(1 - e^{-z/x})y_2 + N(1 - e^{-(1-z)/(1-x)})y_1 \qquad (7.2)$$

where $y_2 > y_1$. Thus the efficient outcome is heterogeneous buyers and homogenous sellers, rather than the other way around.

7 Wages

The value of jobs in the dynamic model is given as an asset price. This was done, because the requisite object up for bidding was the present value of the returns to the match. However, it is straightforward to derive equilibrium wages. For example, in the model with one type of job, there are two possible wages given by

$$\Lambda_j = \frac{w^j + \beta\rho V}{1 - \beta(1 - \rho)} \tag{8.1}$$

where Λ_j is the present value of the workers second best offer at the time the wage is negotiated. It is straightforward to extend this exercise to the model of on-the-job search. The model without on-the-job search is unable to produce much wage dispersion, if attention is limited to an economy that has an unemployment rate of 4 or 5% and jobs of significant durability - a separation rate of 4% per month. However, Julien, Kennes and King (2001) show that the model with on-the-job search can easily account for the extreme wage disparity in the United States while maintaining the extreme assumption that all workers are identical. Moreover, as implied by the theory, this extreme wage dispersion is efficient!

There was a major debate about the properties of matching models in markets where the frictions become infinitesimally small (ref: Rubinstein and Wolinsky (1985) and Gale (1985)). This question can be easly studied in the dynamic competitive auction model by reducing the length of time between offer rounds. A consistent shift of the length of offer rounds, $\triangle t$, say from two weeks to one, requires us to appropriately scale the other parameters of the model. This scaling is achieved by adjusting the parameters of the model as follows. In each period, the present value of output produced and the capital cost of each job - vacant or filled - are given by

$$y(\triangle t) = \int_t^{t+\triangle t} y e^{-(\sigma+r)t} dt \tag{8.2}$$

and

$$k(\triangle t) = \int_t^{t+\triangle t} k e^{-rt} dt. \tag{8.3}$$

Likewise, the discount rate and separation rates are given by $\beta(\triangle t) = e^{-r\triangle t}$ and $\delta(\triangle t) = 1 - e^{-\sigma\triangle t}$. Besides being able to illustrate how the Beveridge curve shifts right or left as the size of the friction falls, we can also look at the limiting properties of this economy. In the limit as $\triangle t \to 0$, there is a finite ratio of job searchers to job vacancies, the number of job searcher

approach zero. Moreover, the wage to a worker is equal to the output of the economy less the capital cost and jobs are paid their capital cost. Therefore, the frictionless economy is Walrasian.[7]

8 Price Posting

Hosios (1990) considers a simple model of job entry, based upon Peters model of price posting with capacity constraints. He shows that the equilibrium wage has the following property

$$w = X_1/X \tag{9.1}$$

where the function X is the matching technology given in equation 2.7. A neat implication is the equivalence of auctions and posted prices (e.g. Kultti 1999). Coles and Eeckhout (2001) show the use of auctions or price posting is indeterminate in a simple game with homogenous buyers and sellers.[8]

The price posting model runs into difficulty with heterogenous buyers. In this case, Shi (2004) shows that if there are two types of buyers then the equilibrium must have each seller posting two posted prices - one for each type of buyer. Besides the uncertain practical relevance of such pricing structures, the model is especially complicated to solve. In other words the apparent simplicity of price posting is limited by the existence of heterogenous buyers. A similar criticism can be made of model of competitive search (e.g. Moen 1997).

Perhaps the ultimate reason for the existsence of markets with competitive auctions is ex post heterogeneity driven by nature. Thus the early applications of competitive auctions by Wolinsky (1987) and McAfee (1990) have this feature. Ex post uncertainty is conceptually simple to introduce although it can be mathematically taxing because we must keep track of the statistics for the first and second highest valuations at each local market as a function of the heterogenous number of bidders as is implied by the exponential matching function.[9] These results are slightly beyond the scope of the present discussion and serve mainly to connect the results of competitive auctions more closely to the existing auction literature.[10] However, the

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[7]See Sakovics et al (2001) consider a slightly different analysis, although their matching technology is not exactly the same as under investigation here.

[8](see also Julien, Kennes and King 2001

[9]Papers by Peters and Severinov (1997) and Peters (1997a,b) are highly recommended introductions to these topics.

[10]A simple approach to the decision problem created by endogenous entry of buyers is given by McAfee and McMillan (1985).

goal of this paper has been to be more focused on the literature on matching, which usually treats heterogeneity quite simply, and which we believe is benefited most by the realism that competing auction theory provides.

9 Imperfect Information

The theory of competitive auctions implies a shift in focus from imperfect information about buyers to imperfect information about sellers. On the one hand, the problem of informational inefficiency concerning buyers is solved by the auction and thus the high valuation buyer is always rewarded the good for sale at a price equal to the second highest valuation. On the other hand, the sellers' problem of attracting buyers gives them much incentive to advertise themselves as good - a classic lemons problem. Kennes and Schiff (2003) consider the equilibrium effects of exogenously lifting the veil of ignorance about the quality of sellers. However, there are many applications in which information about sellers is revealed endogenously. Examples of such market mechanisms include reputation systems, accreditation services, and guidebooks,

9.1 The model

Suppose that sellers are separated by some mechanism into two quality differentiated submarkets. Let q_l and q_h denote the expected quality levels of sellers in the two submarkets, and let α denote the fraction of sellers that are allocated to the submarket with expected quality q_h. Without loss of generality we assume $q_h > q_l$. Buyers are informed of q_l, q_h and α. If sellers are separated in such a manner, the average quality of sellers across submarkets cannot change, thus $\widetilde{q} = \alpha q_h + (1 - \alpha) q_l$..In addition, the number of buyers is fixed, so market tightness for each submarket is related to overall market tightness as follows: $\Phi = \alpha \phi_h + (1 - \alpha) \phi_l$,where ϕ_l and ϕ_h denote the buyer-seller ratios of the two submarkets. The division of sellers into submarkets leads to two basic types of equilibrium distribution of buyers across the submarkets. These distributions depend upon the average quality of sellers in each submarket, their relative numbers, and the overall ratio of buyers to sellers. These conditions are summarized by what we call the *exclusion constraint*:

$$q_h e^{-\Phi/\alpha} \geq q_l. \tag{10.1}$$

The left hand side of the exclusion constraint is the expected utility of a buyer if all buyers locate in the high quality submarket. The right hand

side of this constraint is the expected quality of sellers' products in the low quality submarket. If (10.1) is satisfied, a buyer is better off to locate in the high quality submarket even though if he located in the low quality submarket he would not have to compete with any other buyers and could obtain a payoff of q_l with certainty. Thus if the partition of sellers into submarkets satisfies (6.1), all buyers locate in the high quality submarket so that $\phi_h = \Phi/\alpha$ and $\phi_l = 0$. If the partition of sellers into submarkets does not satisfy (10.1), buyers locate in both the high quality and low quality submarkets. In a mixed strategy equilibrium where (10.1) is not satisfied, we must have

$$q_h e^{-\phi_h} = q_l e^{-\phi_l}. \tag{10.2}$$

That is, the expected utility to buyers must be the same from locating in either submarket. The behavior of buyers can therefore be expressed as a function of the distribution of sellers over the submarkets. This function depends crucially on (10.1). From $\Phi = \alpha\phi_h + (1 - \alpha)\phi_l$ and (10.2), market tightness in the high and low quality submarkets are given by

$$\phi_h = \begin{cases} \Phi/\alpha & \text{if EC} \\ \Phi + (1 - \alpha)\ln\left(\frac{q_h}{q_l}\right) & \text{otherwise} \end{cases}. \tag{10.3}$$

and

$$\phi_l = \begin{cases} 0 & \text{if EC} \\ \Phi - \alpha\ln\left(\frac{q_h}{q_l}\right) & \text{otherwise} \end{cases}. \tag{10.4}$$

If (10.1) holds, sellers in the low quality submarket are excluded, a buyer's utility in any period is simply $U = e^{-\Phi/\alpha}q_h$. If (EC) does not hold, buyers visit both submarkets with strictly positive probability and from (10.2), a buyer's utility in any period is $U = e^{-\phi_h}q_h$. Substituting in $\Phi = \alpha\phi_h + (1 - \alpha)\phi_l$, for any distribution of sellers across submarkets, the expected payoff of a buyer when search is guided is given by

$$U = \begin{cases} e^{-\Phi/\alpha}q_h & \text{if EC} \\ e^{-\Phi}q_h^\alpha q_l^{1-\alpha} & \text{otherwise} \end{cases}. \tag{10.5}$$

The utility of buyers is a linear function of the expected quality of sellers in the high submarket if sellers in the low submarket are excluded. If no sellers are excluded, the utility of buyers is a Cobb-Douglas function of the expected qualities of the sellers' products in each submarket with the weights being the fraction of sellers in each submarket.

This model has some surprising results for the distribution benefits of being better informed. Suppose that buyers are made measurably better

informed about seller qualities, but not so well informed as to induce total exclusion of these sellers. In this case, we can compare welfare, with and without guided search as follows

$$
\begin{aligned}
U - U_0 &= e^{-\Phi} q_h^\alpha q_l^{1-\alpha} - e^{-\Phi} \widetilde{q} \\
&= e^{-\Phi} \left[q_h^\alpha q_l^{1-\alpha} - (\alpha q_h + (1-\alpha) q_l) \right] \\
&< 0 \text{ for all } q_h \neq q_l \text{ and } \alpha \in (0,1). \quad (6.5)
\end{aligned}
$$

To see the inequality, note that in general, $x^\alpha y^{1-\alpha} < \alpha x + (1-\alpha) y$ for $\alpha \in (0,1)$ and $x \neq y$. Taking the log of the left-hand side, $\log \left(x^\alpha y^{1-\alpha} \right) = \alpha \log x + (1-\alpha) \log y < \log \left(\alpha x + (1-\alpha) y \right)$ since log is a concave function, and log monotone implies $x^\alpha y^{1-\alpha} < \alpha x + (1-\alpha) y$. Thus the provision of information about sellers hurts buyers since it forces them to compete more intensely for high quality products - thus, in equilibrium, the price of high quality goods increases more than the probability of trade with such sellers.[11] However, Kennes and Schiff (2003) also show that making buyers better informed about scarce good sellers, such that buyers exclude bad sellers, can make them better off.

Another feature of the decentralized economy in this model is that information is used efficiently. This can be seen by setting up the social planning problem and determining the social planner preferred values of ϕ_h, ϕ_l. Therefore, this model is especially useful for the study of reputation systems - algorithms that transfer information across periods. In particular, it means that we concentrate our attention on the specific implications of alternative reputation systems, without adding the assumption about other sources of unrelated inefficiency.

10 Further Applications

This section discusses a number of applications of the theory of competitive auctions. This section is meant to be suggestive of the kind of research that can be pursued. Most of the research described in this section is either very recent, works in progress, or in some cases, speculative.

Albrecht,Gautier and Vroman (2003) consider a model that combines elements of wage posting and bidding for Labor. They assume, like Burdett Shi and Wright (2001) that firms post wages. They also assume that workers apply to more than one job. This model is useful for the purpose of studying

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[11] Rubinstein and Wolinsky (1987) derive essentially the opposite result, because search in their model is undirected.

two sided search. An interesting outcome of the model is that the posted wages of firms are driven to zero if they must bid for workers that apply to multiple positions. Therefore, the simple bidding for labor model presented in section 3 is the equilibrium outcome of their more elaborate game.

The problem of assortative matching was studied by Shimer and Smith (2000) in a random matching model. The basic idea is that individuals are not always matched to their best jobs. This question can be easily addressed with the model of on-the-job search if we simply assume firms can distinguish between alternative types of labor (e.g. sex, race, schooling, etc.). Shimer (2001) also considers a related competitive auction model where heterogenous workers compete for heterogenous jobs. Moreover, Coles and Eeckhout (2000) show an interesting limiting property that perfect coordination can be achieved by sufficient heterogeneity on both sides of the market.[12]

Jovanovic and Rosseau (2004) consider an application in which the assembly of the firm is done by managers who take part in auctions. The model explains movements in product and technology variety since 1990. They argue that rise in the firm specificity of capital explains why (i) labor turnover has declined dramatically, (ii) firms now get a larger fraction of rents, (iii) Tobin's Q is positively related to the skill premium, (iv) stock prices often rise with no accompanying rise in productivity and (v) management and venture capitalism play more important roles in running the firm.

Burdett, Shi and Wright (2001) consider the possibility that some firms may have capacity to hire more than one worker. They show that this feature influence the structure of the equilibrium matching technology. The idea of giving more capacity than the other side of the market - the workers - is closely related to the idea that sellers are more scarce than buyers presented in section 7. Thus one explanation for why firms sell jobs is that firms are more scarce than workers. Of course the fact firms sell multiple units limits the importance of auctions - McAfee and McMillan (1987) suggest that auctions are best used where the seller has a single unit. Therefore capacity choices could explain why firms do not auction their jobs as in Shimer (1996, 1999, 2001).

Kennes and Schiff consider the modelling of reputation systems. In the competitive auction model, information is used efficiently. This outcome is useful for the study of reputation systems that economize on the amount of information transferred across periods in order to maximize transparency.

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[12] This result requires the public observation of all the relevant characteristics of sellers, in addition to their pricing mechanisms.

Thus Kennes and Schiff consider alternative reputation systems that report only a simple metric (eq recommended =1 or not recommended=0). The problem in choosing a simple reputation system is that a stand must be taken on what is important for the determination of the 1's and 0's. There are several alternative reputation systems that could be used. For example, in a market with many anonymous buyers and sellers, one type of reputation system could track whether a seller was honest in the past. Therefore, dishonestly - she is dirty rotten liar - would count as a strike against the seller yielding a 0 while honesty would yield a 1. Alternatively, a reputation system might report something about the quality of the past transaction. For example, a bad transaction - she sold me a piece of crap - would count as a strike against the seller yielding a 0 while the alternative would yield a 1. How do the two simple reputation systems differ in practice? On the one hand, a reputation system that screens for honesty allows good sellers to do the right thing and signal their type in the relatively rare events that they have low quality products to sell. Thus the market introduces information about seller quality at very early periods in a seller's tenure. However, the potential downside of a reputation system for honesty is that bad sellers may almost mimic this strategy in order to delay the public realization that they are bad type. If this bait and switch strategy by bad sellers is optimal, the gains from signalling are reduced and a reputation system for honesty is potentially inefficient.

In the section on imperfect information, we also argued that there are redistributional effects of third party information, which may hurt buyers. This raises questions about how information is sold. In Kennes and Schiff (2003c) it is assumed that there are two methods of information distribution: guidebooks and accreditation services. The assumption is also made that guidebook and accreditation services have a number of independent costs. On the one hand, a guidebook service must exclude some buyers in order to be commercially viable. In particular, the exclusion of third party information from some buyers in favor of others requires the costly creation and distribution of guidebooks. However, such costly exclusion of information is not necessary for the operation of an accreditation service. On the other hand, an accreditation service bears the costs of marketing itself to the sellers and creating an atmosphere of trust. It is not necessary for a third party to convince sellers of its worth in the case of guidebooks. Thus a guidebook service avoids such costs.Kennes and Schiff show that the market structure is an important determinant of the optimal method of guided search. In particular, market, where buyers as a group are hurt by information are likely to be served by an accreditation service. On the other hand, a market

where buyers are benefits as a group are likely to be served by a guidebook.

Filges, Kennes, Larsen and Tranæs (2004) use an auction model to study labor market policy. One observation that they make is that cross country spending on labor market policies is independent of the unemployment rate. This observation presents difficulties for any theory that attempts to paint the use of labor market policy with a broad brush theory with homogenous workers and firms. Therefore, Filges et al use an auction model to work out labor market policy in labor market with heterogenous workers and opportunities for training. The model is able to explain the strong correlation between active and passive subsidies in the data. The basic mechanism is that increases in passive policies weaken the incentive compatibility constraint on training subsidies. Therefore higher quality training programs can be implemented without leading advantaged to also choose these programs.

The dynamic adjustment of unemployment and vacancies can be easily modelled by a Markov process for the productivity of jobs (ref: Kennes 2004). If on-the-job search is introduced into this model as I did in section 5, one might expect that the model can account for the unexplained low variation in vacancies that exist's in Shimer's (2004) calibration of a search model without on-the-job search.

The sale of another form of asset, money, might also be modelled by the assumption that trade is a competitive auction. For example, Rocheteau and Wright (2004).present a model of money with competitive search. As discussed, such a model assumes buyers are homogenous. Thus the competitive auction model could introduce this type of heterogeneity into their framework. This might matter for efficiency, because we have argued that heterogenous agent should be buyers rather than sellers, other things equal.

Finally, recent developments by Eaton and Kortum (2002) are highly suggestive of a theory of competitive auctions applied to international trade. In their model, countries receive technology draws - valuations - in other countries. In equilibrium, the country with the lowest cost draw produces. It seems possible to introduce uncoordinated investments into this framework as in Jovanovic and Rosseau (2003). Such a model could endogenize investments as a function of ex ante observable differences across countries.

11   Conclusions

Having written this paper in Denmark, I feel at liberty to cite Hans Christian Andersen. Two of his stories come to mind: "The Emperor's New Clothes" and "The Ugly Ducking". The price posting model of coordina-

tion frictions fails for the simple reason that its main assumption is false. In the real world, the sellers of specialized labor do not make wage demands in their resumes. Nor do the sellers of specialized jobs set wages in their job advertisements. Thus the emperor has no clothes. Likewise, the theory of competitive auctions has been criticized, or even ignored, because it appears too complex. Therefore, price posting models have been advanced for their simplicity, despite their questionable assumptions. However, these same models quickly become intractable, or yield questionable results, when they are applied to key problems such as on-the-job search, and worker and firm heterogeneity. By contrast, the theory of competitive auctions offers a tractable alternative with generally sensibe results. Thus this ugly ducking of the matching literature may yet prove to be a beautiful swan.

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