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THE EFFECTS OF HANDS-ON EQUATIONS ON MATH ACHIEVEMENT OF NINTH GRADE STUDENTS

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Dedication

This is dedicated to my parents, Laura Ann and the late Timothy E. Scott. Thank you for teaching me how to be the best person I can be. To my mother, I really appreciate everything you have done for me. I want you to know that my love for you will never end.

To my God parents, Evelyn and Danny Burke, I want to thank you for exposing me to what this world has to offer. You have taken me to places that a little boy from a small town probably would have never seen. To my other dad, Rev. Melvin Allen Tukes, when my father passed away my senior year of high school, you were there to help pick up the pieces, and you have never left my side. For that, I am extremely grateful. To my brother and sister, Champ and Vette, you have been very supportive of me as I matriculated through high school and college.

I would also like to extend this dedication to my children, the late Chauncey A. Scott II, Jasmine, Cassidy, Deja (Brian), and a handsome grandbaby Maddox. You all bring me joy and happiness and without your smiles and hugs through this process it would have been very difficult. Lastly, to my loving wife, you have been so supportive throughout this process. You kept pushing me to stay the course and finish the race. Grace, I simply love and adore you!

ABSTRACT

Each year, ninth-grade students across the United States of America fail to meet the national standards in mathematics. Ninth grade students with disabilities, especially in the southeastern region of the United States, consistently fail the math portion of the Georgia Milestones Test. As a response to this problem in Georgia, Hands-On Equations by Henry Borenson represents a possible solution for many students failing to meet the standards in ninth grade mathematics. The purpose of this quantitative causal-comparative study was to examine the difference between the mathematics achievement of ninth-grade students with and without disabilities in a high school in Southeastern Georgia who received instruction with Hands-On Equations versus those who received instruction without the use of Hands-On Equations. The data used were historical data from the 2016 school year. One group of students participated in instruction using Hands-On Equations while another group received traditional teaching methods without the use of the Hands-On Equations. The participating schools were urban schools located in the Southeastern part of Georgia. Most of the students were African American, and the students in these schools received 100% free lunch. An analysis of covariance (ANCOVA) generated comparative data. The results related to Hypothesis $H_0 1$ and $H_0 3$ indicated that there was a significant difference in the mathematics achievement scores for ninth-grade students with or without disabilities who received instruction with Hands-on Equations. However, results related to hypothesis $H_0 2$ indicated that there was no statistically significant difference in mathematics achievement scores for ninth-grade students who did or did not receive instruction using Hands-on Equations.

***Keywords*:** active learning, cognitive, cooperative learning, Hands-On Equations, higher-order thinking, mathematics

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CHAPTER 1: INTRODUCTION

Overview

This chapter provides an introduction to the research topic for this study, focusing on the use manipulatives in mathematics among students with and without disabilities. The chapter will include the following key sections: (a) background of the problem, (b) problem statement, (c) purpose statement, (d) significance of the study, (e) research questions, (f) null hypotheses, and (g) definitions of terms. This introduction will serve as the foundation of the proposed study, which will be further expanded on in the succeeding chapters.

Background

Every year, educators across the United States of America take on the daunting task of improving achievement in mathematics among students with and without disabilities (Bouck, Joshi, & Johnson, 2013; Fuchs, Fuchs, & Compton, 2012). It is the role of the educator to ensure all students regardless of their disabilities become productive citizens in life (Albers &Goblirsch, 2013; Meyen, 2015). Teachers in schools across the nation are ever striving toward a high level of student success and achievement, especially in mathematics (Minsoo, 2012).For example, in Georgia many of the students with disabilities struggle to meet the standards in ninth grade math because of the algebra portion of the test (Georgia Department of Education, 2015).Students with disabilities struggle to keep up with the standards because of the pressure to meet expectations comparable to those of their classmates without disabilities (Yell, Katsiyannis, Collins, & Losinski, 2012). Therefore, educators must find a solution to the dilemma of effectively instructing students with disabilities to meet the school standards.

Historical Context

As a response to the Sputnik launching by the Soviets in 1957, the U.S. began financing education programs in math and science (Pinder, 2013). The Elementary and Secondary Education Act (ESEA) of 1965 was passed to emphasize high standards and accountability from schools (Standerfer, 2006). In 1983, the *A Nation at Risk* report created a well-publicized perception of educational reform in the U.S. as imperative (Lund & United States National Commission on Excellence in Education, 1993). The result was the expansion of the federal government into education, which had previously been up to the individual states (Johanningmeier, 2010). The expansion opened the door for more federal legislation. In 1990, the Excellence in Mathematics, Science, and Engineering Education Act was passed to promote excellence in American mathematics, science, and engineering education as well as stimulate the professional development of scientists and engineers (International Labor Organization, 2014). In 2001, public schools in the United States were required to follow guidelines mandated in the No Child Left Behind (NCLB) Act (Shelly, 2012). The NCLB is a direct reauthorization of a modified ESEA (Standerfer, 2006). This legislation mandated assessments in reading and mathematics. As a result, reading and mathematics became the focus of instruction (Miller, 2010).

The impact of NCLB could be seen in the improved math scores for every grade level, especially for the 9th-grade students (Dee, Jacob, & Schwartz, 2013). Due to NCLB requirements and some evidence of the legislation’s effectiveness, restructuring how math is taught has become a component of improvement in many classrooms. The Organization for Economic Cooperation and Development (OECD) (2012) reported a weakness of students in the U.S. is they are not able to create a mental model to show their understanding of math in real-world situations. This model requires a firm understanding of what is being asked and knowing how to apply the appropriate mathematical thinking to solve the problem. Students in the U.S. are not able to interpret real-world situations and apply mathematical concepts. Other weaknesses include reasoning and a lack of focus in higher-order activities relating to the real world (OECD, 2012).

Within the past few decades, the education system in the United States of America has gone through several changes. The traditional classroom setting separating the general education students and the special education students has been replaced with a more progressive diverse classroom setting including both the special education students and the general education students in an inclusive classroom (Morningstar, Shogren, Lee, & Born, 2015; Santos, Sardinha, & Reis, 2016). This change is due to the Education Act for All Handicapped Children (EAHCA) (Civic Impulse, 2017), a law passed by congress in 1975 requiring local schools to provide education to all disabled children. The EAHCA identified some of the issues to be addressed, including nondiscriminatory placement in special education, an Individualized Education Program (IEP) to report goals and objectives for students with disabilities, and the establishment of special education services in the least restrictive environment (LRE). It also allowed for services without partiality or preconception as students worked towards exact educational goals and objectives (Timberlake, 2014; Yell, Conroy, Katsiyannis, & Conroy, 2013).

The movement toward inclusion of students with disabilities in general education classrooms has caused confusion among some special education teachers about the roles and responsibilities of regular educators in providing appropriate education for all students in United States public schools (Ajuwon, Lechtenburger, Zhou, & Mullins, 2012; McLeskey, Landers, Williamson, & Hoppey, 2012). It has been argued that the most important factor in inclusive education is the teacher, and the success of inclusive education is dependent upon the teacher's positive attitude towards inclusion and ability to teach subjects proficiently, especially for fundamental areas, such as mathematics (Savolainen, Engelbrecht, Nel, & Malinen, 2012; Seçer, 2010). It has been widely accepted that the mathematics achievement gap still exists within classrooms, including students with disabilities, which poses utmost concern among educators and school administrators (Schulte & Stevens, 2015; Stevens, Schulte, Elliott, Nese, & Tindal, 2015).

Social Context

In this current era of world economic competition and globalization, it is essential that all students are well prepared with knowledge and skills in mathematics to successfully compete in this 21st-century global economy and society (Mundia, 2010). Adults who are highly skilled in mathematics are twice as likely to be employed and three times as likely to earn above-median salaries (Mundia, 2010). All over the world, economies continue to be negatively impacted by learning difficulties in mathematics (Mundia, 2010). For example, in England and Wales, the economy is at a tremendous disadvantage because adults lack sufficient numeracy skills. As a result, their federal governments are currently providing a wide range of business training in numeracy to enable adults to manage budgets, to use discretion in obtaining credit, and to maintain good health (Mundia, 2010). Several studies document poor math performance of students around the globe (Ali, 2011; Bingolbali, Akkoc, Ozmantar, & Demir, 2011; Ciltas & Tartar, 2011; Mundia, 2010).

Students, regardless of their capacities, have the same areas of concerns and needs. Four general principles basic to all children are: (a) non-discrimination; (b) the right to life, survival and development; (c) the right to be listened to and taken seriously; and (d) the right to pursue their best interests (United Nations Committee on the Rights of the Child, 2016). From the perspectives of individuals with disabilities, educators, policy-makers, and planners should be asking the following questions regarding these principles: Are educational activities providing an equal opportunity for students with special needs along with other students? Are the activities resulting in exclusion? Are activities exposing children to prejudice and stigma? Are activities and actions threatening children's human dignity? (Pijl & Hamstra, 2005).

DiGennaro Reed, McIntyre, Dusek, and Quintero (2011) suggested teaching students with disabilities in inclusive settings is a multifaceted task requiring a team of mutually supporting players who provide the best practices for all students. Professional preparation of school personnel is essential. Teachers must learn new teaching strategies and understand how to work cooperatively with other teachers. Without proper planning and support, successful inclusive placements are difficult (Wade, 2000). Inclusion is the meaningful participation of students with disabilities in general education classrooms (Ajuwon et al., 2012). To practice inclusion successfully, the educators involved must understand the history, terms, and legal requirements involved as well as have the necessary levels of support and commitment.

Theoretical Context

This study was based upon the theoretical frameworks put forth by Piaget (1965), Bruner (1977), and Dienes (1973). Each of these theorists proposed children's interaction with their environments creates new experiences building on their prior knowledge. Piaget (1965) introduced four stages of development to explain the nature and development of human intelligence. Bruner (1977) offered a theory for discovery learning. Dienes (1973), who stressed the importance of processes for learning mathematics through interaction with one's environment, developed six stages of learning. These theoretical frameworks are pertinent to the current study because they help to explain how manipulatives may be helpful to students, with and without disabilities, in understanding and solving abstract equations in mathematics.

Piaget (1965) theorized children should understand concrete symbols and concepts when the symbols and concepts are introduced to them on the concrete level. Many children absorb and retain what they learn in life when they can touch and feel what they are doing or experiencing. When they touch, feel, take apart, put together, and manipulate a concrete object and its different pieces, they begin to develop a clear mental picture in their minds (Raphael & Wahlstrom, 1989).

Bruner (1977) proposed students learn through discovery. Learning through discovery occurs when students interact with their environments (Bruner, 1977). Bruner (1977) also proposed students engage in discovery learning when they struggle with concepts and questions, when they develop and manipulate objects, and when they answer questions by testing and verifying hypothesis. According to Bruner (1977), students should initially use objects they can manipulate to gain an understanding of mathematical concepts, and teachers should support students in their efforts to create different models, carry out experiments, and revise or validate their models.

Dienes (1973) developed a theory to explain how students learn mathematical concepts. Dienes' (1973) theory consists of six stages: (a) free play, (b) playing by the rules, (c) comparison, (d) representation, (e) symbolization, and (f) formalization. During the first stage, free play, students use trial and error to figure out a problem or phenomenon they seek to solve. The second stage, playing by the rules, refers to following rules or principles to solve a problem. The comparison stage occurs when students discuss, evaluate, and compare the processes and products of their peers (Dienes, 1973). The representation stage occurs when the student identifies abstract content and invents a representation or maps the math concept. The symbolization stage occurs when the student is able to describe properties through the use of conventional symbolic language (Dienes, 1973). During the sixth stage, formalization, rules are applied, and the inductive and deductive reasoning processes are used to describe mathematical concepts, such as axioms, theorems, and proofs (Dienes, 1973). Dienes' theory is important to this study because it asserts students should interact with their environment as they learn mathematical concepts.

The theories of Piaget (1965), Bruner (1977), and Dienes (1973) made up the theoretical framework of this study. These theories were used to provide a rationale for the findings of the study. These theories were used in providing context for examining the problem, summarizing the information, and preparing the reader for the research problem.

Problem Statement

Ninth grade students with disabilities in Georgia consistently fail the math portion of the high stakes test required by the NCLB (Georgia Department of Education, 2015). It is unknown if Hands-on Equations will increase mathematics achievement among ninth-grade students with disabilities as measured by the state-mandated Georgia Milestones. Studies have shown the use of manipulatives have been effective in improving math achievement (Carbonneau, Marley, & Selig, 2013; Gurbuz, 2010; Sherman & Bisanz, 2009) and specifically Hands-On Equations (Barber & Borenson, 2008; Brown, 2011; Jimenez, 2011; Liendenbach & Raymond, 1996; Skaggs, 2007). However, existing studies often used students without disabilities as subjects in determining the effectiveness of manipulatives, such as Hands-On Equations. Research has shown students with disabilities often fail to reach mathematics standards set forth by the school (Garderen, Scheuermann, & Jackson, 2012; Schulte & Stevens, 2015; Stevens & Schulte, 2017), making them a good focus for a study on the effectiveness of Hands-On Equations in improving their achievement.

Hennessey, Higley, and Chesnut (2012) concluded effective instruction in math should include a constructivist philosophy where problem solving is incorporated into active learning. The teaching of these skills in math is becoming more important because it contributes to the development of countries through innovation and discovery (Juan & IGI Global, 2011; Li, Silver, & Li, 2014). The use of manipulatives is effective in improving the academic achievement of students so they can learn how to think in a rapidly changing world (Golafshani, 2013; Moyer-Packenham, 2016). The problem is the lack of information regarding the difference between the mathematics achievement of ninth-grade students with and without disabilities who are exposed to instruction with Hands-On Equations versus those who receive instruction without the use of Hands-On Equations.

Purpose Statement

The purpose of this quantitative causal-comparative study was to examine the difference between the mathematics achievement of ninth-grade students with and without disabilities in a high school in Southeastern Georgia who received instruction with Hands-On Equations versus those who received instruction without the use of Hands-On Equations. Two groups of participants were considered for the study. One group received instruction using Hands-On Equations while the other group received instruction without the use of Hands-On Equations. Each group consisted of students with and without disabilities. The independent variables were the type of instruction (i.e., Hands-on Eqquations and without Hands-on Equations) and the type of students (i.e., with and without disabilities). The dependent variable was the mathematics achievement scores. The difference between the groups on the basis of the mathematics achievement scores was analyzed through analysis of a two-way ANCOVA.

Significance of the Study

Improving math education has been a focus in the United States over the past 50 years. However, many high schools in Georgia struggle to make gains on the College and Career Readiness Performance Index (CCRPI) because of the performance of the ninth-grade students in mathematics (Georgia Department of Education, 2015). One of the indicators for the CCRPI is achievement on the Algebra I I-Ready test. Equations make up 30% of the Algebra I I-Ready test, and this area is where most students struggle (Georgia Department of Education, 2015).

There are many methods of teaching that allow for active engagement in math, one of which is the use of Hands-On Equations. Hands-On Equations has the potential to increase the achievement scores of the students in the equations portion of the I-Ready test. Henry Borenson (1987) indicated Hands-On Equations could assist any school or district searching for interventions to improve student achievement. This study examined whether the use of Hands-On Equations would make an impact on the mathematics achievement of students, especially those with disabilities.

Differentiated instruction and Hands-On Equations could address the needs of individual students with disabilities in mathematics (Kablan, 2016). Differentiated instruction with the use of Hands-On Equations could transform the way the information is being presented by the teacher and received by the student (Golafshani, 2013; Young, 2013). In using differentiated instruction and Hands-On Equations, teachers could adapt knowledge, concepts, and skills in mathematics to the students' interests, and reduce the rigor of instruction without reducing the information. Then, students with disabilities could become self-motivated to learn mathematics knowledge, concepts, and skills.

Studies have shown the use of manipulatives could potentially improve student achievement in mathematics (Carbonneau et al., 2013; Gurbuz, 2010; Sherman &Bisanz, 2009). Specifically, research on Hands-On Equations generally indicates positive results in improving the math achievement of students (Barber & Borenson, 2008; Brown, 2011; Jimenez, 2011; Liendenbach& Raymond, 1996; Skaggs, 2007). However, little research has been done on the effectiveness of Hands-On Equations in improving mathematics achievement of students with disabilities. Students with disabilities have different needs and capabilities compared to students without disabilities, which supports the need to investigate the effectiveness of Hands-on Equations among students with disabilities.

This study also investigated whether using Hands-On Equations effectively increased the understanding and improved the mathematical achievement of students with disabilities. Moreover, the findings of this study may be helpful to school administrators and teachers in identifying effective strategies to increase mathematics achievement. Lastly, the insights of this study will further expand the knowledge about the use of manipulatives in improving student achievement.

Research Questions

RQ1: Is there a difference in mathematics achievement scores for ninth-grade students with and without disabilities in Southeastern Georgia who received instruction using Hands-On Equations, as measured by the Algebra I I-Ready test?

RQ2: Is there a difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia who did or did not receive instruction using Hands-On Equations, as measured by the Algebra I I-Ready test?

RQ3: Is there a difference in mathematics achievement scores for ninth-grade students with disabilities and those without disabilities, in Southeastern Georgia who did or did not receive instruction with Hands-On Equations, as measured by the Algebra I I-Ready test?

Definitions

- *Abstract to Concrete Mathematical Concepts* – The abstract to concrete mathematical concepts refers to the act of computing mathematical concepts without physical objects (abstract) and with physical objects (concrete) (Ding & Li, 2014).
- *Accountability* – Accountability refers to the act of being responsible (GDOE, 2015).
- *Georgia Milestones Test* – Georgia Milestones Test is a state-mandated assessment used in the state of Georgia to measure the knowledge of students in grades one through eight in reading, English/language arts (ELA), social studies, science and mathematics (GDOE, 2015).
- *Disability* – Disability refers to a physical or mental problem preventing someone from functioning at a normal rate (Americans with Disabilities Act, n.d.)
- *Hands-On Equations* – Hands-On Equations is a visual and kinesthetic system developed by Dr. Henry Borenson for introducing and teaching students essential algebraic concepts (Borenson, 1987).
- *Individual Education Plan (IEP)* – The IEP is a document that delineates the special education services for a student with disabilities. The IEP will outline educational and/or

behavioral goals and lists the types of services that are to be given to the student with disabilities (GDOE, 2015).

- *Inclusion* – Inclusion is an arrangement where students with disabilities receive services that are listed in their Individual Education Plan in the same classroom as their non-disabled peers (Ajuwon et al., 2012).
- *Learning Style* – Learning style refers to a facet of a student's learning profile. It refers to the personal and environmental factors that may affect learning (Desmedt & Valcke, 2004).
- *Mathematics Anxiety* – Mathematics anxiety refers to a state of mind that causes discomfort and adverse bodily effects when presented with mathematical problems or tasks (Lyons & Beilock, 2012).
- *Manipulative* – Manipulatives are concrete objects used to help students understand abstracts concepts (Burns & Hamm, 2011).

CHAPTER 2: LITERATURE REVIEW

Overview

The purpose of this quantitative causal-comparative study was to examine the difference between the mathematics achievement of students with and without disabilities who received instruction with Hands-On Equations versus those who did not receive instruction using Hands-On Equations. The review was grounded by a theoretical framework consisting of Piaget's (1965) cognitive development theory, Bruner's (1977) theory of development, and Dienes' (1973) theory of learning mathematics. The empirical evidence supporting the use of manipulatives will also be discussed in this review. Related literature, such as the importance of mathematics, abstract thinking in algebra, characteristics of students with learning disabilities, the use of mathematics manipulatives, role of manipulatives in mathematics, Hands-On Equations, and the Common Core Math Standards will be discussed. The chapter will conclude with a summary of the key themes from the literature, including the gap in previous studies.

Theoretical Framework

The theoretical framework guiding this study was Piaget's (1965) cognitive development theory, Bruner's (1977) theory of development, and Dienes' (1973) theory of learning mathematics. Each of these theorists proposed children's interaction with their environments creates new experiences building on their prior knowledge. The selected theories in the theoretical frameworks are pertinent to the current study because they help to explain how and why manipulatives may be helpful to students, with and without disabilities, in understanding and solving many abstract equations in mathematics. Each of these theories will be discussed in the following sub-sections.

Piaget's Cognitive Development Theory

Piaget (1965) theorized children understand abstract symbols and concepts when the symbols and concepts are introduced in concrete formats. Many children absorb and retain what they have learned in life when they can touch and feel what they are doing or experiencing. When children touch, feel, take apart, put together, and manipulate a concrete object, a mental picture begins to develop in their minds (Raphael & Wahlstrom, 1989). Piaget (1965) introduced four stages of cognitive development: (a) the sensorimotor stage, (b) the preoperational stage, (c) the concrete operational stage, and (d) the formal operational stage.

The sensorimotor stage is the first stage of Piaget's stages of operational development. The sensorimotor stage typically occurs when children are from birth to two years old. During the sensorimotor stage, children become aware of their immediate surroundings through their senses. For the most part, their behaviors are a reaction to stimuli. Also, during the sensorimotor stage, children accomplish object permanence or understanding objects continue to exist even though they cannot be seen or heard (Paget, 1965).

The second stage, the preoperational stage, occurs when a child is about two years old until about seven years of age. Children at this stage lack conservation and cannot reverse operations. They can perform some mathematical tasks, such as comparing physical objects and assigning numeric values to objects when counted, but children at this stage have difficulties with concepts, such as length, area, weight, and volume. During the preoperational stage, children engage in symbolic play and learn to manipulate symbols. However, they do not understand concrete concepts. Language begins to develop at the preoperational stage (Piaget, 1965). In addition, Piaget (1965) noted during the preoperational stage, children have a challenged understanding of people and objects dissimilar to themselves. They also have difficulty understanding different situations and dissimilar points of view, which is deemed egocentrism.

The concrete operational stage is the third stage of Piaget's theory of cognitive development. This stage occurs when children are from seven to eleven years old. During the concrete operational stage, children become more logical in their thinking, but they are still challenged with understanding abstract ideas. They become more engaged in inductive logic, which is a mental understanding of concepts from specific to general principles. Reversibility is also introduced in the preoperational stage. Reversibility is related to the child's awareness of the order of relationships between mental categories (Paget, 1965).

The fourth stage of Piaget's operational development is the formal operational stage. The formal operational stage starts at twelve years of age and continues into adulthood. Abstract thought and hypothetical reasoning occur during this stage. Also, during the formal operational stage, individuals rely on previous experiences to make sense of their current situations and consider possible outcomes and consequences of their actions (Paget, 1965).

During the fourth stage, individuals engage in deductive reasoning. Individuals can solve problems systematically through logic, and concrete models are no longer needed to understand abstract ideas. They can separate and control variables, make assumptions contrary to fact, and test hypotheses (Paget, 1965).

The cognitive development theory has been used in education to support instructional methods and strategies (Siegler, 2016). When the cognitive development stages are applied in the use of manipulatives, Piaget (1965) suggested children need many experiences with concrete materials and drawings for learning to occur because they do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone (Piaget, 1965).

Piaget’s stages of operational development are often endorsed by educators because it supports the use of manipulatives.

The use of manipulatives is related to Piaget’s (1965) cognitive development theory because of the assertion that children’s interaction with their environments creates new experiences building on their prior knowledge. The cognitive development theory provides a framework in explaining how manipulatives may be helpful to students, with and without disabilities, in understanding and solving many abstract equations in mathematics. This study may potentially advance the cognitive development theory by providing empirical support to the main assertions of the theory regarding how manipulatives can help teachers bridge the gap between concrete and abstract mathematical concepts.

Bruner’s Theory of Development

Bruner’s (1977) theory of development proposed students learn through discovery. Learning through discovery occurs when students interact with their environments (Bruner, 1977). Bruner (1977) also proposed students engage in discovery learning when they struggle with concepts and questions, when they develop and manipulate objects, and when they answer questions by testing and verifying hypothesis. According to Bruner (1977), students should initially use objects they can manipulate to gain an understanding of mathematical concepts, and teachers should support students in their efforts to create different models, carry out experiments, and revise or validate their models.

Bruner (1977) presented three levels of thinking that occur when students learn mathematical concepts. First, learners are enactive, which is to say they manipulate objects directly. The second level of thinking is the iconic level. The iconic level involves the use of images or other visuals to represent concrete objects (Bruner, 1977). The third level of thinking is the symbolic level. In the symbolic level, students manipulate abstract representations.

The theory of development has been used in education to support instructional methods and strategies in mathematics (Kitta & Kapinga, 2015; Krummheuer, 2013). Bruner (1977) asserted when teaching math, teachers should move students through the levels, from the enactive level, iconic level, and the symbolic level. Krummheuer (2013) used the theory of development as a framework for understanding the mathematics thinking of young children through exposure to diagrammatic and narrative argumentations. Kitta and Kapinga (2015) noted the intent of the theory of development is to emphasize the role of the interaction between the environment and the individual learning of children.

The use of manipulatives is related to Bruner's (1977) theory of development because of the assertion children learn through discovery. Bruner's (1977) theory is relevant to the current study because it suggests manipulatives can aid students in their understanding of abstract concepts by building on mental images. Through the three stages of discovery, manipulatives can be used as a tool to facilitate the learning of complex mathematical concepts. The theory of development provides a framework in explaining how manipulatives may be helpful to students in understanding and solving many abstract equations in mathematics. This study may potentially advance the theory of development by providing empirical support to the main assertions of the theory regarding how manipulatives can help teachers facilitate students' learning of complex mathematical concepts through discovery.

Dienes' Theory of Learning Mathematics

Similar to Piaget and Bruner, Dienes (1973) also promoted students' active engagement during the process of learning mathematical concepts. Dienes (1973) authored several articles that advocated the use of manipulatives in teaching elementary mathematics. Dienes is considered the inventor of algebraic manipulatives (Lesh & Sriraman, 2007). Many of Dienes' theories and creations are used in classrooms to teach students about mathematical concepts.

Dienes (1973) provided an explanation of how students learn mathematical concepts. Dienes understood one of the keys to retaining the basics of anything is to make the process fun. Making the learning of mathematics fun can be exemplified in the use of algebraic manipulatives (Dienes, 1973).

Dienes (1973) developed a theory to explain how students learn mathematical concepts. His theory is important to this study because it asserts students should interact with their environment as they learn mathematical concepts. Dienes' (1973) theory consists of six stages. These stages are: (a) free play, (b) playing by the rules, (c) comparison, (d) representation, (e) symbolization, and (f) formalization.

During the first stage, free play, students use trial and error to figure out a problem or phenomenon they seek to solve. For example, a student who wishes to solve a puzzle may randomly try to unscramble the pieces to construct the image of the picture seen on the box. After some form of regularity begins to emerge, a more systematic problem-solving method is applied. The second stage, playing by the rules, refers to following rules or principles to solve a problem. After engaging in problem solving through free play, the rules may be applied until the condition becomes satisfied (Dienes, 1973). The comparison stage occurs when students discuss, evaluate, and compare the processes and products of their peers (Dienes, 1973). The representation stage occurs when the student identifies abstract content and invents a representation or maps the math concept. The symbolization stage occurs when the student can describe properties through the use of conventional symbolic language (Dienes, 1973).

Furthermore, a symbol system is developed to describe the properties of the concepts. At the sixth stage, formalization, rules are applied, and the inductive and deductive reasoning processes are used to describe mathematical concepts, such as axioms, theorems, and proofs (Dienes, 1973).

The use of manipulatives is related to Dienes (1973) theory of learning mathematics because of the assertion children's active engagement during the process of learning can be crucial in the learning of mathematical concepts. The theory of learning mathematics provides a framework in explaining how manipulatives may be helpful to students, with and without disabilities, by stimulating their active engagement in solving many abstract equations in mathematics. This study may potentially advance the theory of learning mathematics by providing empirical support to the main assertions of the theory regarding how teachers can use manipulatives as tools to engage students in learning abstract mathematical concepts.

Related Literature

Literature related to the importance of mathematics, characteristics of students with learning disabilities, the use of mathematics manipulatives, abstract thinking in algebra, the role of manipulatives in mathematics, Hands-On Equations, and the Common Core Math Standards is relevant to the research problem and purpose of this study. A review of this literature provided an in-depth background for the research problem and purpose.

The Importance of Mathematics

In this current era of world economic competition and globalization, it is essential all students are well-prepared with knowledge and skills in mathematics to successfully compete in this 21st-century global economy and society (Mundia, 2010). Adults who are highly skilled in mathematics are twice as likely to be employed and three times as likely to earn above-median salaries (Mundia, 2010). All over the world, economies continue to be negatively impacted by learning difficulties in mathematics (Mundia, 2010 For example, in England and Wales the economy is at a tremendous disadvantage because adults lack sufficient numeracy skills. As a result, their federal governments are currently providing a wide range of business training in numeracy to enable adults to manage budgets, to use discretion in obtaining credit, and to maintain good health (Mundia, 2010). Several studies document poor math performance of students around the globe (Ali, 2011; Bingolbali et al., 2011; Ciltas& Tartar, 2011; Mundia, 2010).

Ali (2011) explained students in Pakistan struggled with the mathematics curriculum, complex languages, mathematical concepts, and real-life connections to mathematics. Further, Ali (2011) explicated higher-order thinking skills are needed for understanding linear algebra. Moreover, the author asserted there are no simple computational procedures in linear algebra; one must have the ability to think abstractly. Therefore, learning difficulties in mathematics classrooms are inevitable when students struggle with fundamental math skills (Ali, 2011). Ciltas and Tartar (2011) also discussed how high school students encountered difficulties with abstract concepts of algebra, particularly in solving equations with inequalities containing concepts with absolute value. Their findings showed 90% of 170 ninth-grade students in Turkey answered math questions of this nature similarly and incorrectly (Citas & Tartar, 2001).

Broadway (2010) asserted accountability and standardization has placed our public schools under intense scrutiny and review. As a result, schools are being held increasingly responsible for the success and academic achievement of their students. Political and business leaders, community members, parents, and other school stake holders are demanding our schools prepare all of our students to meet the challenges of the 21st century world (Broadway, 2010).

School accountability and standardization has mandated the implementation of effective academic interventions and supports for students to ensure all students are ready to enter college or the career world upon completion of high school (Broadway, 2010). Many schools are addressing the call to improve student achievement by providing their teachers with high quality professional development opportunities and providing students with effective academic interventions in core subject areas in the hopes of improving student achievement. Mathematics is currently considered the main subject which leads to success in life and is a key to successfully competing in a global economic world (Broadway, 2010).

The National Assessment of Educational Progress (NAEP; 2011) documents 82% of U.S. students only demonstrated *partial* mastery of mathematics content based on their achievement scores. Perpetuated mathematics achievement gaps have continued over decades as the NAEP (2011) showed gains in performance among Hispanic students at the *Proficient* and *Advanced* levels between 2009 and 2011; however, Black and American/Indian/Alaska Native students demonstrated below basic level mastery on the 2011 NAEP. Even with the disparity in mathematics achievement among ethnic groups in the United States*, the majority of the nation's students are demonstrating only partial mastery (*NAEP, 2011*).*

With the focus on academic accountability in math, many school districts have instituted academic programs to improve student performance and to increase the number of students who meet or exceed established standards on standardized achievement tests (Mundia, 2010). The No Child Left Behind Act of 2001 specifically requires the implementation of only scientifically-based research instructional activities and programs, which are rigorous, systematic, and objective procedures to obtain reliable and valid results (Mundia, 2010).

Abstract Thinking in Algebra

Abstract thinking in algebra consists of the nontangible aspects of completing an algebraic problem. Rainbolt and Gallian (2010) suggested in their book, *Abstract Algebra with GAP*, algebra concepts weigh heavily on abstract thinking. Traditionally algebra has been taught through the lecture style of teaching. The abstract way of thinking with algebra creates difficulty for some students, and thus their appreciation for the subject area is diminished (Connely-Fukawa, 2012). Many students have difficulty with abstract thinking because it is intimidating to them and some learn better by performing an actual action. Connely-Fukawa (2012) suggested the abstract way of thinking and teaching algebra is one of the main reasons undergraduate students change their majors away from mathematics.

Donohue, Gfeller, and Schubert (2013) conducted a research experiment where they used teaching abstract algebra as the basis of their study. Their article, “Using Group Explorer in Teaching Abstract Algebra,” suggests one of the most overwhelming aspects of teaching and learning algebra is the abstract nature of the concepts (Donohue et al., 2013). Many teachers relied heavily on the traditional style of teaching where the concepts are conveyed via the lecture, which may not always be effective when teaching abstract subjects, such as mathematics (Donohue et al., 2013).

Piaget and other behavioral theorists have linked abstract thought of an individual to the concrete or physical attributes to which they can relate. When a person works with information, not in pictorial or concrete form, it is abstract thinking (Hawker & Cowley, 1997). Many students have difficulty with mathematics because of the abstract thinking involved. Many educators and students have failed to correctly comprehend algebraic concepts in mathematics because of its abstract nature (Coquin-Viennot & Moreau, 2007).

Students with Learning Disabilities

According to the United States Department of Education (2013), almost two million students have been identified as having a learning disability. Students with learning disabilities can be categorized as those who: (a) have difficulty analyzing and processing information, (b) have extremely low levels of motivation and self-esteem, (c) experience repeated academic failure, (d) fail to meet challenges with reasoning and problem-solving, (e) are reluctant to try new tasks, and (f) who have computational deficits (Strickland & Maccini, 2010). Teachers have a responsibility as teachers to facilitate instruction, leading to success for all students, including those with learning disabilities. Students who have learning disabilities often struggle to achieve at the levels of their peers who do not have learning disabilities, especially in mathematics (Steele, 2010). The inferior academic performance in mathematics of students with learning disabilities compared to students who do not have learning disabilities is a major concern of educators throughout the United States (Steele, 2010).

Approximately 5-8% of K-12 students have been identified as having a math-related learning disability (United States Department of Education, 2013). The two areas which are most difficult among students with math-related difficulties are computations and problem solving (Bottge, Rueda, Grant, Stephens, & LaRoque, 2010). Students with learning disabilities, especially experience difficulty with basic concepts and procedures (Cortiella, 2011). Procedures related to algorithms, which are steps used to find solutions to problems (Bottge et al., 2010; Cortiella, 2011). Students who have difficulty following procedures also often have difficulty memorizing, paying attention, and organizing (Bottge et al., 2010; Cortiella, 2011).

Teachers are not always cognizant of the reasons why some students do not understand mathematical concepts easily. Wang (2013) added five categories of why students, including those with learning difficulties, have difficulty understanding math concepts. They are mathematics content, cognitive gap, teaching issues, learning matters, and transition knowledge. In order for students with learning disabilities to advance their understandings of mathematics, teachers must use effective, research-based instruction (Wang, 2013).

Geary (2011) noted students with disabilities often lack self-confidence and therefore expect to fail and often do not make connections between existing and new information. They are also unable to distinguish between relevant and irrelevant information and understand the difference between concrete and abstract concepts. When Geary (2011) tracked students' achievement in identifying and combining quantities associated with Arabic numerals, he found students with disabilities demonstrated inferior performance, and their achievement fell behind their peers at least one year. Therefore, it is important educators identify and utilize the most effective practices to increase the math performance of students with learning disabilities.

Steele (2010) wrote due to No Child Left Behind and the Individuals with Disabilities Act (IDEA), students with learning disabilities are required to take algebra in general education classrooms. Nonetheless, students with disabilities may be at a disadvantage, mainly with problems requiring reading and thought processing, which may impede their ability to complete equations and understand the processes required for problem solving, such as finding the square root, writing geometric proofs, and finding angles. Students with disabilities may have difficulty understanding, interpreting, and explaining more simple tasks, such as plotting points on a grid, constructing graphs, and drawing parallel and vertical lines. The inability to complete such tasks makes it difficult for this populace of students to grasp more complex mathematical concepts (Steele 2010).

Strickland and Maccini (2010) offered eight instructional strategies for teaching mathematics concepts to students with disabilities: (a) general problem-solving strategies, (b) self-monitoring strategies, (c) peer-assisted learning, (d) concrete-representation-abstract instructional sequence, (e) teaching prerequisite skills, (f) explicit instruction, (g) technology, and (h) using graphic organizers. According to Servilio (2009), students with disabilities learn mathematical concepts when their teachers: (a) build on prior knowledge students bring into the classroom, (b) build on concepts by providing examples and practice, (c) integrate metacognitive skills into the math; (d) use formative assessments regularly, and (e) monitor students' progress. Researchers also contend an effective strategy for teaching students with disabilities is the use of manipulatives, which are tangible simple or complex objects used to model or to demonstrate math problems (Allensworth, Nomi, Montgomery, & Lee, 2009; Brodesky & Gross, 2009; Burns, 2010; Ellis, 2009; Gersten et al., 2009; Strickland &Maccini, 2010).

The Use of Math Manipulatives

Manipulatives are objects used to present students with opportunities to physically interact with materials while learning math concepts (Carbonneau et al., 2013; Gurbuz, 2010; Sherman &Bisanz, 2009). Urban and Wagnor (2009) noted manipulatives have been used to solve mathematical problems since 3000 B.C. when the abacus was used in China. Boggan, Harper, and Whitmire (2010) added individuals from many civilizations used physical objects to help them solve everyday math problems. For example, in Southwest Asia, individuals used counting boards, or wooden and clay trays covered by a thin layer of sand to draw symbols and to tally. Boggan et al. (2010) gave credit to the Romans as being the creators of the first abacus, building on the concept of the counting board. The Romans made abacus from beans and stones (Boggan et al., 2010).

The Chinese adopted the use of the Roman abacus and used it centuries later (Boggan et al., 2010). The Mayan and Aztec Indians used corn kernels, whereas the Incas used knotted string called quipu (Boggan et al., 2010). Then, during the latter part of the 1800s, manipulatives began to be applied more formally in teaching mathematical concepts (Boggan et al., 2010). In the early 1900s, Montessori designed several manipulatives to help teachers and students explain and learn basic concepts at the elementary school level (Boggan et al., 2010). Currently, the National Council of Teachers of Mathematics recommends the use of manipulatives at all grade levels (Boggan et al., 2010).

Sinclair and Chorney (2012) wrote Montessori stressed the importance of concrete learning experiences during the 20th century. Montessori noted students should learn through self-directed exploration by using manipulatives and believed they demonstrated increased success when they used manipulatives. Roberts (2014) wrote calculators were used as manipulatives in classrooms at the beginning of the 21st century. Advanced graphing calculators were available to students who enrolled in advanced math courses, such as calculus and statistics. Roberts (2014) also noted during the 21st Century, computers became a new type of manipulative called a virtual manipulative.

D'Angelo and Iliev (2012) asserted manipulatives are the basis for understanding mathematical concepts. The researchers also contended math manipulatives may be used as early as preschool to help students assign values to numbers through symbols. Following students’ understanding values, teachers can then teach their students to use basic principles of mathematics. Also, according to D'Angelo and Iliev (2012), manipulatives provide students with opportunities to actively engage in understanding the processes of mathematics.

Authors Blenenky and Nokes (2009) discussed the potential outcomes when using manipulatives in the article “Examining the Role of Manipulatives and Metacognition on Engagement, Learning, and Transfer.” Teachers often explore different ways to engage their students in math and manipulatives improve engagement (Belenky & Nokes, 2009). When students are engaged in productive activities with learning materials, they become more interactive and learn the concept being taught (Belenky & Nokes, 2009). Manipulatives give students hands-on experience, which concretizes their knowledge of the concept and solidify their problem-solving skills (Belenky and Nokes, 2009).

Boggan et al. (2010) contended manipulatives can be used in teaching a wide variety of topics in mathematics, such as reasoning, estimation, measurement, and problem solving. Manipulatives may also be used to teach place-value, fractions, addition, subtraction, and order of operations. For example, fraction strips may be used to show equivalent fractions. Pattern blocks can be used to assist students with basic algebra concepts, and geoboards can be used to teach geometric shapes. While the numbers of ways teachers can use manipulatives to teach math are limitless, they must be used correctly. More importantly, students should develop a thorough understanding of the theories behind the mathematical concepts being taught (Boggan et al., 2010).

According to Boggan et al. (2010), “the effective use of manipulatives can help students connect ideas and integrate their knowledge, so they gain a deep understanding of mathematical concepts” (p. 4). Smith (2009) stated manipulatives are used in various grade levels and in different countries because achievement improves when manipulatives are used effectively

Manipulatives vary from simple household items, such as colored clothespins, to specifically designed items, such as unifix cubes. Graham (2013) reported three types of concrete manipulatives used for teaching mathematics. The first types are everyday objects, such as buttons, dice, coins, popsicle sticks, and beads. The second type of concrete manipulative are those which are commercially manufactured and serve other purposes besides mathematical conceptualization. They are items such as jigsaw puzzles, Legos, and building blocks. The third type of concrete manipulatives are designed specifically for teaching mathematics. They are base ten blocks, Cuisenaire rods, geoboards, tangrams, color tiles, attribute blocks, pattern blocks, and unifix cubes. While the literature by Fraser (2013) offers concrete examples for aligning manipulatives with common core standards, the examples provided by the researcher tend to focus on kindergarten through second-grade students with disabilities and do not take into account complex processes requiring mastery.

Boggan et al. (2010) focused on the importance and benefits of math manipulatives. The authors reviewed several research articles which gave foundation to the use of manipulatives. Boggan et al. indicated the German educator Friedrich Froebel designed the educational play material known as Froebel Gifts. This type of instruction is given credit for the foundation for the manipulatives considered by Italian educator Maria Montessori as essential in teaching mathematics at the elementary school level (Boggan et al., 2010).

Boggan et al. (2010) provided a more precise definition of manipulatives, defining manipulatives as physical objects used as teaching tools to engage students in hands-on learning of mathematics. Manipulatives can be made with common household materials or can be store-bought items, such as blocks or cubes. A manipulative is deemed effective if it is able to bridge the gap between informal math and formal math (Boggan et al., 2010).

Boggan et al. (2010) also indicated five of the National Council of Teachers of Mathematics (NCTM) standards could be taught using manipulatives: problem solving, communicating, reasoning, connections, and estimation. When addressing the notion of correct manipulatives usage, Boggan et al., (2010) explained the importance of clear expectations of the purpose of the lesson and the manipulative used within the lesson. The focus must be on the concept, not the manipulative. The math manipulative should be appropriate for the student and chosen to meet the specific goals and objectives of the lesson (Boggan et al., 2010).

When introducing a new manipulative, it is important to allow exploration time for students (Boggan et al., 2010). Boggan et al. (2010) stated when given the opportunity to work with a material with open-ended objectives having no specific preset goal, the students have time to explore their own questions and generate a variety of answers. Additionally, the research found teachers need support in making decisions regarding manipulative use, including when and how to use manipulatives to help them and their students think about mathematical ideas more closely (Boggan et al., 2010).

It must also be noted using manipulatives does not solve the problem of understanding complex math concepts. The students' understanding of the concepts while using the manipulatives is directly related to teachers' knowledge of the concepts being taught and teachers' knowledge of the use of the mathematical manipulatives (Raphael & Wahlstrom, 1989).

Many teachers fail to use mathematical manipulatives because they feel manipulatives are difficult to use and are a difficult concept to teach. Other teachers have expressed outside challenges to using manipulatives in the classroom, such as classroom management, lack of resources, improper professional development on the use of manipulatives, and assessing the use of the manipulatives (Kim, 1993). As with any teaching technique, however, the teacher must lead the instruction in the classroom.

Moyer (2001) conducted a study on the use of manipulatives in mathematics in the middle school. The study included 10 middle school teachers in a yearlong project on the use of manipulatives. The results indicated although the teachers used the manipulatives, student achievement was not high in all classes. The interviews indicated some teachers did not understand the purpose of using the manipulatives, and thus, the effective use of manipulatives was not taught in the classroom. Other teachers indicated they were ineffective in teaching with the use of manipulatives because they did not understand how to use manipulatives themselves (Moyer, 2001).

Moreover, some teachers viewed the use of manipulatives as a waste of time (Moyer & Jones, 1998). Teachers felt manipulatives were extra work or secondary to using the abstract form of math. The use of math manipulatives should not be considered as the sole solution for understanding mathematical concepts. The achievement levels of the students could be correlated directly to the experience and expertise of the teachers who teach the use of manipulatives (Sowell, 1989).

Hands-on Equations

Many students struggle to understand basic mathematics concepts. Solving simple linear equations can be challenging for many students and especially for those students with disabilities who already struggle in mathematics. Hands-on Equations, by Henry Borenson (1987), was developed to curtail the lack of achievement in mathematics. Hands-On Equations utilizes manipulatives to assist the students with understanding mathematics equations. This manipulative system changes abstract linear equations to concrete linear equations. This change gives many students, who learn better with tangible objects, the opportunity to learn the concepts needed to be successful in mathematics (Agency for Instructional Technology, 2003).

Hands-On Equations teaches students the basic concepts of mathematics (Borenson, 1987). Hands-On Equations turns solving mathematics equations into a game in which the students can move chess-like pieces and number cubes to solve the equations (Agency of Instructional Technology, 2003). The system teaches students how to solve mathematics word problems and how to add and subtract integers. The students also learn the addition and subtraction properties of equality, the concept of variable, and the basic concepts associated with zero.

Liendenbach and Raymond (1996) conducted an action research study using Hands-On Equations. The researchers utilized the traditional method of teaching mathematics during the first nine weeks of the school year. Then they introduced Hands-On Equations and taught all 26 lessons of the system. Once the lessons were completed, Liendenbach and Raymond (1996) returned to the traditional way of teaching. The results indicated a higher level of achievement in the students in the classroom when Hands-On Equations was being taught (Liendenbach& Raymond, 1996).

At the end of the year, when the state-mandated tests were given to the students, their results exceeded the researchers' expectations. The students had bridged the gap between the concrete concepts of Hands-On Equations with the abstract form of the questions on the test. The students were positive when they used the manipulatives versus the traditional form of teaching (Liendenbach& Raymond, 1996).

Barber and Borenson (2008) summarized the effect of utilizing the Hands-On Equations module on the learning of algebra by fourth and fifth graders of Broward County public schools in a recent study. This research was designed to determine whether fourth- and fifth-grade students can successfully solve equations normally taught in the ninth grade. The samples in this research included six fourth-grade regular classes, three fifth-grade regular classes, and five gifted and talented fifth-grade classes. This learning module included games pieces and a pictorial notation piece. Each teacher received 3 weeks of training on how to use the Hands-On Equations learning module. The research study conducted by Barber and Borenson (2008) had the following results:

- Students in the fifth grade had an average of 42.8% on the pretest, 84.7% on the posttest, and 79.3% on the retention test. The t-test conducted found the increase to be significant at a value of 3.88.
- The fourth-grade students from Broward County had an average of 30% on the pretest, 84% on the posttest, and 88% on the retention test. A t-test was done between the Lesson 6 posttest scores and the Lesson 7 posttest scores. The t-test was significant, with a t value of 2.86.

Skaggs (2007) conducted a qualitative study that sought to examine the perceptions of high school graduates who experienced the mathematics education materials from Hands-On Equations when the students were in the sixth grade. The study also included the perspectives of students who did not participate in the system. Participants of the study attended school and graduated from high school in Kansas, and of the 19 students who were interviewed, 10 had experienced 21 lessons using Hands-On Equations when they were in the sixth grade in January 1997. Ten of the students were male, and nine were female. The data consisted of the interviews conducted with these students in 2005, solutions to six one-variable linear equations completed by each student, and GPA and ACT information for each student. The results indicated students who participated in Hands-On Equations favored mathematics more than the non-Hands-On Equations students. Additionally, the Hands-On Equations group had both a lower mean GPA and lower mean ACT mathematics score compared to the non-Hands-On Equations group; however, the students in the Hands-On Equations group solved the six one-variable linear equations with more success (72% accuracy) than did the non-Hands-on Equations group (59% accuracy).

Brown (2011) conducted a quantitative correlational study to determine the impact of Borenson's Hands-on Equations on the math achievement of ninth-grade students in South Carolina. The participants were ninth grade regular and special education students who were either gifted and talented students or who had been identified as having a learning disability. Brown (2011) used an eight-question pretest and eight-question posttest to measure different skills when solving linear equations. The pretest was used to analyze prior knowledge about solving linear equations. The posttest was used to measure what the students had learned since receiving instruction on how to use the Borenson's module to solve linear equations. The posttest was also used to measure whether the students scored significantly higher using the module than those students who did not. Results of the study indicated while 80% of the students did not score above 70 percent on the Solving Linear Equations pretest, on the posttest, 60% of the students scored above 70%. Moreover, the students who utilized the module were able to retain slightly more skills than the students who did not utilize the module. As a result of the study, Brown (2011) recommended future research be conducted to analyze the effects of the module in relation to its impact on the achievement of students from diverse racial backgrounds.

Jimenez (2011) investigated the effectiveness of Hands-On Equations on the math achievement of 9th and 10th-grade students. Jimenez (2011) used pretests, posttests, retention tests, and benchmark tests to evaluate the academic growth of students in two set groups. The collected data was analyzed by conducting t-tests and an ANOVA. The results of the study indicated Hands-On Equations was effective with solving linear equations and was a positive factor in students' success with linear equations. However, analysis of the data also revealed the program was not as effective six weeks after the intervention in terms of performance in the retention test or the benchmark test.

Borenson (2009) examined the performance of 195 gifted third graders on specific verbal problems pretest and posttests. The pretests were provided after the students had completed Level I of Hands-On Equations, but prior to receiving instruction on how to apply the equations to solve verbal problems. All of the students had completed Level I of Hands-On Equations and were provided with six verbal problem lessons to be solved using the Hands-On Equations approach to word problems. At the conclusion of the six lessons, the students were provided with a post-test, which consisted of six verbal problems similar to those provided on the pre-test. A t-test was conducted to determine if there was a significant increase in performance from the pre-test to the post-test. The results of the study indicated a significant gain by these gifted third graders in solving the specific problems after instruction. However, the researcher, suggested additional studies be conducted with other students in grades 3-12 to see if this method of instruction also leads to successful learning by those students.

The studies reviewed in this section indicated Hands-On Equations can be effective in improving the academic achievement of students in mathematics (Borenson, 2009; Brown, 2011; Jimenez, 2011; Skaggs, 2007). These researchers also recommended more studies be conducted on the effectiveness of Hands-On Equations because of the suggestion that the benefits of the technique may not be sustainable (Jimenez, 2011). Most of the studies reviewed also focused on students who had no learning disabilities or difficulties (Jimenez, 2011; Skaggs, 2007).

Empirical Evidence

This study examined the effects of using Hands-On Equations in instructing ninth-grade students to determine if this form of instruction has a positive impact on the achievement of students with disabilities and other low performing students. In this quantitative study, a causal-comparative design was used to examine the effectiveness of Hands-On Equations, a program developed to provide a hands-on approach to presenting algebraic concepts to elementary and middle school students. In this section, the empirical evidence supporting the use of manipulatives in teaching mathematics was provided. Gurbuz (2010), Sherman and Bisanz (2009), and Carbonneau et al. (2013) agreed the use of manipulatives is an effective approach to improved student achievement in mathematics.

A research study was conducted by Suydam and Higgins (1977) on the use of physical manipulatives in mathematics. These researchers examined the effects of mathematical manipulatives on the achievement of students in elementary and middle school. The results indicated who utilized physical manipulatives had a higher level of achievement than their counterparts who did not.

Sowell (1989) compiled a meta-analysis of 60 different studies and found manipulatives used in mathematics were effective in increasing the students' overall knowledge in the targeted subject matter. The results indicated when the instruction was completed over a period of a school year, the students' retained most of the knowledge, concepts, and skills when manipulatives were used. The study also indicated there was no significant increase in knowledge when the manipulatives were used over a shorter period of time (Suydam& Higgins, 1977).

The research performed by Sowell (1989) was the first documented research synthesis on the use of manipulatives. Since the time of Sowell's research in 1989, there has been more research on the use of manipulatives, indicating though Sowell's research had limitations, there was validity to the research in the arena of mathematical manipulatives (Carbonneau et al., 2013). The study by Sowell (1989) indicated although the manipulatives contributed to the achievement of the students on the mathematical portion of the test, the amount of time spent on teaching the students how to use the manipulative was an important factor as well. The study also indicated students had to be able to go from the concrete to the abstract and vice versa.

Another research study indicated there was a significant increase in achievement when students used mathematical manipulatives. Parham (1983) indicated the students who used manipulatives during their course of study in mathematics scored in the 85th percentile on the mathematical portion of the California Achievement Test. The students who did not use mathematical manipulatives scored in the 50th percentile on the California Achievement Test (Parham, 1983). Clearly, there is a difference in achievement when the students can make physical representations.

The previous studies reviewed primarily involved students without disabilities. The literature on the effects of manipulatives in the mathematics learning of students with disability is less researched. One study conducted by Marsh and Cooke (1996) found support for the effectiveness of manipulatives among students with disabilities. The researchers conducted a research study on the effects of using mathematical manipulatives with third-grade students with a learning disability in mathematics. This study focused on using Cuisenaire rods during teaching. The results indicated using manipulatives significantly increased student achievement.

The Cuisenaire rods increased the students' ability to correctly identify the proper procedures to solve the problems (Marsh & Cooke, 1996).

Miller and Mercer (1997) conducted a study on the effects of the three types of instruction, including concrete, semi-concrete, and abstract instruction of students with learning disabilities. The concrete type of instruction consisted of learning with concrete manipulatives. The semi-concrete type of instruction consisted of learning when students used pictures or pictorial representations. The abstract type of instruction consisted of learning when the students used only numbers. The study concluded students with mathematical learning disabilities performed better with concrete instruction as opposed to abstract instruction (Miller & Mercer, 1997).

Research on Hands-On Equations generally indicates positive results in improving the math achievement of students. Liendenbach and Raymond (1996) found a higher level of achievement in the students in the classroom when Hands-On Equations was being taught. Barber and Borenson (2008) found an increase among students who engaged with the system. Skaggs (2007) found students who participated in Hands-on Equations favored mathematics more than the non-Hands-on Equations students. Additionally, the Hands-on Equations group had both a lower mean GPA and lower mean ACT mathematics score; however, the students in the Hands-on Equations group solved the six one-variable linear equations with more success (72% accuracy) than did the non-Hands-on Equations group (59% accuracy). Brown (2011) also found positive results among students who used the system.

Jimenez (2011) investigated the effectiveness of Hands-on Equations on the math achievement of ninth and tenth-grade students and concluded Hands-on Equations was effective with solving linear equations. Borenson (2009) examined the performance of 195 gifted third graders on specific verbal problems pretest and posttests using Hands-On Equations found a significant gain by these gifted third graders in solving the specific problems under instruction.

Common Core State Standards

The Common Core State Standards were created to ensure all K-12 students have the skills needed to be successful in post-secondary education and in a career. Common Core State Standards were adopted by the National Governors Association Center for Best Practices and by the Council of Chief State School Officers (Hill, 2013: Porter, McMaken, Hwang, & Yang 2011). Educational stakeholders also collaborated with these agencies to create Common Core State Standards for K-12 mathematics (Moursund & Sylvester, 2013). The Common Core Standards Initiative of 2010 was developed to expand from individual state standards to national standards, with the expectation all students in the United States would learn the same content and develop the same skills within these two domains (Hill, 2013; Moursund& Sylvester, 2013).

Moursund and Sylvester (2013) noted the Common Core Standards were first implemented in 2010 and adopted by 40 states. The Common Core Standards were adopted by the District of Columbia in 2011 and by six additional states in 2012 (Moursund& Sylvester, 2013). As a result of the Common Core Standards, students at each grade level are assigned specific domains and key topics that they must master (Moursund& Sylvester, 2013). For students in kindergarten through second grade, students must master: (a) number names and sequencing of numbers and identifying geometric shapes; (b) addition and subtraction, understanding of place value, interpreting data, and geometric reasoning; (c) multiplication, measurement of objects in units, working with money; and (d) representing and interpreting data. Fourth-grade students must master concepts related to: (a) multi-digit multiplication and multi-digit dividends; (b) fraction equivalence, addition and subtraction of fractions, and multiplication of fractions by whole numbers; and (c) the analysis of geometric figures and their properties. By the end of the fifth grade, students are expected have a plethora of mathematical concepts and procedures to help them move into more advanced applications (Common Core Standards Initiative, 2013). Under the Common Core Standards Mathematics Initiative, students must also foster their problem solving, critical thinking, and reasoning skills.

In the state of Georgia, Common Core Standards in math are presented by grade level from kindergarten to ninth grade. The standards for mathematics were arranged into three different categories: (a) domains, (b) standards, and (c) clusters. The mathematics content is organized into four domains, with standards specific to each domain. Clusters are combinations of related standards (Common Core Standards Initiative, 2013). The four domains for grade four are outlined as: (a) operation, (b) algebraic thinking, (c) numbers and operations in base ten, and (d) geometry. Moreover, there are critical areas for each grade level in addition to the standards (Common Core Standards Initiative, 2013). In grade eight, there are five critical areas: (a) ratios and proportional relationships, (b) number system, (c) expressions and equations, (d) geometry, and (e) statistics and probability (Common Core Standards Initiative, 2013; Ediger, 2011).

Ratios and proportional relationships focus on students' understanding of ratio concepts and the use of ratio reasoning to solve problems (Common Core Standards Initiative, 2013). The number system focuses on the application of previous understandings of multiplication and division to divide fractions by fractions (Common Core Standards Initiative, 2013). This area also focuses on math computations with multi-digit numbers and finding common factors and multiples. Additionally, students must be able to apply and extend previous understandings of numbers to the system of rational numbers. Expressions and equations focus on students' application and the extension of their previous understandings of arithmetic to algebraic expressions. This area also focuses on one-variable equations and inequalities (Common Core Standards Initiative, 2013). In this area, students must also analyze quantitative relationships between dependent and independent variables. Geometry requires students to solve real-world and mathematical problems involving area, surface area, and volume (Common Core Standards Initiative, 2013). Statistics and probability require students to develop an understanding of statistical variability standards (Common Core Standards Initiative, 2013). The current study focused on the effects of Hands-On Equations on the math achievement of ninth-grade students with learning disabilities. (See Appendix A for complete details of the 2015-2016 Common Core Standards for eighth-grade students who attend schools in the state of Georgia). Given ninth grade is the first year of high school, students should be able to demonstrate the standards from the previous year.

Summary

According to the United States Department of Education (2013), two million students have been identified as having a learning disability. Students with learning disabilities have computational deficits and difficulty analyzing and processing information. They fail to meet challenges with reasoning and problem solving. These students generally have low levels of motivation and low self-esteem as they have experienced repeated academic failure. They are reluctant to try new tasks (Geary, 2011; Strickland &Maccini, 2010). Given these limitations and vulnerabilities, students with learning disabilities need to be exposed to teaching strategies that assist in the development of their ability to solve complex math problems (Geary, 2011; Strickland &Maccini, 2010).

The use of manipulatives has been proposed as a strategy that can help students with disabilities improve their ability to learn and understand math concepts. The use of manipulatives is an effective approach to improve student achievement in mathematics (Carbonneau et al., 2013; Gurbuz, 2010; Sherman &Bisanz, 2009). The limitation of the studies reviewed is they primarily involved students who had no disability (Carbonneau et al., 2013; Gurbuz, 2010; Sherman &Bisanz, 2009). The effectiveness of manipulatives among students with special needs is less established compared to the literature on students without disabilities (Jimenez, 2011; Skaggs, 2007).

Hands-On Equations was developed by Borenson (1987) to curtail the lack of achievement in mathematics. Research has generally shown Hands-On Equations is effective in improving the math achievement of students (Barber & Borenson, 2008; Brown, 2011; Liendenbach& Raymond, 1996; Jimenez, 2011). Hands-on Equations is particularly effective in improving the ability of students to solve linear equations (Jimenez, 2011; Skaggs, 2007). The literature on the application of Hands-On Equations within the special education population is limited. To address the gap in the literature, this study examined how Georgia educators in a struggling school are implementing The Hands-On Equations, using math manipulatives, to achieve the state-mandated standard in mathematics among their special education population and low performing students.

CHAPTER 3: METHODOLOGY

Overview

The purpose of this quantitative causal-comparative research study was to examine the difference between the mathematics achievement of ninth-grade students with and without disabilities in high school in Southeastern Georgia who were exposed to instruction with Hands-on Equations versus those who received instruction without the use of Hands-on Equations. This chapter includes the following key sections of the methodology; (a) design, (b) research questions, (c) null hypotheses, (d) participants and settings, (e) instrumentation (f) procedures, and (g) data analysis. The detailed discussion of the methodology was instrumental in demonstrating the step-by-step procedure of the study.

Design

This study employed a quantitative method with a causal-comparative research design. Quantitative methods measure variables or data numerically and objectively and make use of statistical techniques to analyze the underlying relationship between and among these variables or data (Mustafa, 2011). Quantitative methods deduce insights from numerically measured and statistically tested data in the hope of generalizing the findings to a larger population (Allwood, 2012). Thus, a quantitative methodology allowed the determination of differences in mathematics achievement between students with and without disabilities and between students who received instruction using Hands-on Equations and those who did not. In other words, the study attempted to ascertain the extent of differences between two groups based on a criterion variable, which for this study was mathematics achievement.

Furthermore, this study employed a causal-comparative research design. A causal-comparative design is a research design used to determine the cause or consequences of differences already existing between or among groups of individuals (Gall, Gall, & Borg, 2010). A causal-comparative design is often used when there is a need to compare two groups defined by categorical variables in terms of one or more quantified dependent variables to assess causation (Cohen, Manion, & Morrison, 2013). This study used test result data from two classes (independent variable), one utilized Hands-on Equations in instruction and one did not use Hands-on Equations, to measure the difference in terms of a mathematics achievement (dependent variable), which makes a causal-comparative design appropriate.

Research Questions

The research questions and hypotheses that guided this research are as follows:

RQ1: Is there a difference in mathematics achievement scores for ninth-grade students with and without disabilities in Southeastern Georgia who received instruction using Hands-on Equations, as measured by the Algebra I I-Ready test?

RQ2: Is there a difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia who did or did not receive Hands-on Equations instruction, as measured by the Algebra I I-Ready test?

RQ3: Is there a difference in mathematics achievement scores for ninth-grade students with disabilities and those without disabilities, in Southeastern Georgia who did or did not receive instruction using Hands-on Equations, as measured by the Algebra I I-Ready test?

Null Hypotheses

$H_0$1: There is no statistically significant difference in mathematics achievement scores for ninth-grade students with or without disabilities in Southeastern Georgia who received instruction using Hands-on Equations, as measured by the Algebra I I-Ready test.

$H_0$2: There is no statistically significant difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia, who did or did not receive instruction using Hands-on Equations, as measured by the Algebra I I-Ready test.

$H_0$3: There is no statistically significant difference in mathematics achievement scores for ninth-grade students with or without disabilities, in Southeastern Georgia, who did or did not receive instruction using Hands-on Equations as measured by the Algebra I I-Ready test.

Participants and Setting

The data studied was the result of a trial use of Hands-on Equations with groups of high school students enrolled in two schools located in South Eastern Georgia during the spring semester of 2016-2017 school year. The first selected high school had approximately 600 students enrolled, around 60 of which were students with disabilities. The student population in this high school was 90% African American, 5% Caucasian, and 5% other races. There were approximately 120 staff workers in the school, including administration, teachers, custodial, and lunchroom workers. Additionally, there was one public safety officer and one parole officer. The school had two self-contained special education classes, with the remainder of the classes being inclusive. The school is located in an urban community and is also 100% free lunch. There were approximately 160 students who could participate in the study in this high school. The second selected high school had approximately 600 students enrolled, around 50 of which were students with disabilities. The student population in this high school consisted of 95% African American, 2% Caucasian, and 3% other races. There were approximately 120 staff workers in the school, including administration, teachers, custodial, and lunchroom workers. Additionally, there was one public safety officer. The school had four self-contained special education classes, with the remainder of the classes being inclusive. The school was located in an urban community and is also 100% free lunch. There were approximately 150 students who could participate in the study in this high school.

A power analysis was conducted to determine the minimum required sample size. Four factors were considered in conducting the power analysis including the power of the test, effect size, significance level, and statistical technique. The level of significance refers to the probability of rejecting a null hypothesis given it is true, which is commonly referred to as the Type I error (Haas, 2012). The power of test refers to the probability the test correctly rejects a false null hypothesis thus accepting the alternative hypothesis (Haas, 2012). In most quantitative studies, an 80% power of test is used. The effect size is an approximated measurement of the magnitude of the relationship between the dependent and independent variables (Cohen, 1988). Berger, Bayarri, and Pericchi (2013) asserted effect sizes in quantitative studies could be categorized according to small, medium, and large, where medium is generally used to denote a balance between being too strict (small) and too lenient (large). The level of significance is usually denoted with an alpha and in most quantitative studies is set at 95% (0.05) (Creswell, 2012).

Instrumentation

The difference between the groups of students whose data were utilized for this study lies in the use of Hands-on Equations. One of the groups received instruction with Hands-on Equations, and the other did not. In addition, the same tests were used for both groups, but the pretest was different from the posttest. Pretest and posttest data were gathered from I-Ready. I-Ready is a diagnostic test for reading and mathematics intended for K-12 students (Curriculum Associates, 2017). The test can determine the learning needs of students by monitoring their progress for every skill. The diagnostic test provides information about the achievement of students in both reading and mathematics at the end of the school year based on pre-determined targets.

Curriculum Associates (2017) reported I-Ready was informed by best practices in assessment development, calibration, and testing supported by a large and diverse population. As of 2017, I-Ready was administered to more than 500,000 students in the United States and has been state-approved for student growth measure (through achievement scores) and Common Core curriculum in states such as Utah, Colorado, Oklahoma, Georgia, Ohio, Virginia, and New York. The Educational Research Institute of America reported I-Ready has strong correlations to the 2013 New York Assessment, which has correlation coefficients ranging from .77 to .85 across grades and subjects – thus, I-Ready predicted individual student proficiency on the CCSS.

Given that the I-Ready Diagnostic is a computer-adaptive assessment that does not have a fixed form, some traditional reliability estimates such as Cronbach's alpha are not an appropriate index for quantifying consistency or inconsistency in student performance. The I-Ready Diagnostic is often used as an interim assessment, and students can take the assessment multiple times a year. The test-retest reliability estimate is appropriate to provide stability estimates for the same students who took two diagnostic tests. The Pearson correlation coefficient for the diagnostic test was .97 (National Center on Intensive Intervention).

Historical data consisting of Algebra I mathematics achievement scores were retrieved from the I-Ready database. In this data set, one class was taught using Hands-on Equations at a high school in southeastern Georgia, and another class was at a different high school in southeastern Georgia with similar demographics. Each class was taught the same information from the same teachers; however, the delivery method was different. One class utilized Hands-on Equations, and the other class received traditional instruction without the use of Hands-on Equations. The data set spanned for one whole semester or 18 weeks across two classes.

Procedures

The data collection procedures commenced once the approval from both the Liberty University's Institutional Review Board (IRB), and the selected high schools was secured. For the IRB, an application form was submitted detailing the proposed research, including how the participants would be protected and the ethical procedure of the study. For the selected schools, the Liberty University template was completed to request permission to request the data.

The data retrieved for this study were from ninth-grade students attending the target high schools. The data retrieved were the test results (specifically on mathematics) of the students following the use and non-use of Hands-on Equations for classroom instruction. The groups were (a) students who were exposed to Hands-on Equations and (b) students who were exposed to traditional instruction without Hands-on Equations. The school administrators identified students who scored 50% or lower on the pre-test. The assessment determined the prior knowledge of the students. From this student population, the students were assigned randomly to each of the two groups.

In each of the two high schools, the instruction for both groups focused on the same standard, for example, "given ax + 3 = 7, solve for x," but each group received different forms of instruction from two different teachers. Each of the teachers obtained their Bachelor of Arts degree in business education. They both collaboratively planned with the remaining teachers in the math department.

The teacher who implemented Hands-on Equations received training before utilizing it in the classroom with the group of students who received instruction with the system. After a semester, a post-test was given to both to assess the knowledge, concepts, and skills the students gained. Data from the pre-tests and post-tests were collected and analyzed, as will be discussed in the succeeding sections.

Information relevant to the study variables, such as the mathematics achievement scores, were collected from the student records housed in the target high schools. Specifically, the mathematics achievement scores, as well as the information about whether manipulatives were used, were gathered to compare data for pretest and posttest. The researcher sought help from the administrators of the schools to identify the data of the students who met the eligibility criteria of ninth-grade students with and without disabilities.

All data collected from the database were imported into an Excel spreadsheet, and coded for analysis into SPSS 22.0 software, which was the software used for statistical analysis (Arora, 2014). SPSS is a computer program used for statistical analysis. In-depth access and preparation, graphics, modeling, and analytical reporting are possible through this program.

Data Analysis

All information gathered from the student participants' records were coded to Microsoft Excel for preprocessing. Occurrences of missing data were addressed before data analysis was conducted (Dong & Peng, 2013). Schlomer, Baum, and Card (2010) asserted more than ten percent of missing data on a data set could render research ineffective and powerless. For simplicity, the researcher ensured only those participants with complete information were included in the study. Once a complete data set was achieved, the data from Microsoft Excel was transferred to a working sheet in SPSS. A participant ID (e.g., P01 for Participant) was assigned to each participant to link the data from a survey in the Microsoft Excel spreadsheet to the SPSS working sheet. Specifically, SPSS Version 22 was used for this study.

Two types of statistical techniques were used, descriptive statistics and inferential statistics. The descriptive statistics provided basic information, such as the frequency and percentages of the demographic information (such as gender, race, etc.), while the mean and standard deviation were used for continuous variables (such as the mathematics achievement score). Specifically, a two-way ANCOVA was used as the inferential statistics to analyze the differences in the means (mathematics achievement) among identified groups (students with and without disabilities and students exposed and not exposed to Hands-on Equations) and pretest as a covariate.

RQ1, RQ2, and RQ3 were addressed using ANCOVA. The objective of an ANCOVA is to determine whether there was a statistically significant difference between two dependent/independent populations based on a dependent variable in the presence of a covariate (Gamst, Meyers, & Guarino, 2008; Pandis, 2016). The two dependent populations in this study were the (a) students with and without disabilities and (b) students exposed and not exposed to Hands-on Equations, and the dependent variable was the mathematics achievement score while the pretest is the covariate. The test results were based on an F-statistic distributed on an F-distribution (Christensen, 2016). If a significant difference exists between the two groups the test statistic will exceed a critical value from the F-distribution (Hoaglin, Mosteller, & Tukey, 1991). The sign of the test statistic (positive or negative) indicated whether the control or intervention group had a tendency to score higher or lower on the dependent variables. All tests followed a significance level of 0.05. The following is the information reported from the ANCOVA analysis: Number (N), Number per cell (n), Degrees of freedom (df within/ df between), Observed F value (F), Significance level (p), and Effect size and power. If a significant interaction effect is found, additional analysis will be needed.

Since ANCOVA is a parametric test, there is a need to examine first whether the data gathered adheres to the statistical assumptions of these tests. Particularly, the level of measurement, sampling, normality, linearity, bivariate normal distribution, homogeneity-of-slope, and homogeneity assumptions was tested and ensured. The level of measurement for the dependent variable should be in interval or ratio form. The achievement scores for this study were measured in interval form. Groups were formed through random sampling. The normality assumption assumes the distribution of the test is normally distributed with a mean of zero, one standard deviation, and an asymmetric bell-shaped curve (Goodwin & Goodwin, 2013). A normal probability plot was generated to examine if a violation of the normality assumption exists. The assumption of linearity indicates the relationship between variables (i.e., the independent and dependent variables) follows a straight line (Bücher, Dette & Wieczorek, 2011). A scatter plot with standard regression output was generated to examine if a violation of the linearity assumption exists. To test the bivariate normal distribution assumption, a series of scatter plots between the pre-test variable and post-test variable for each group was used (Goodwin & Goodwin, 2013). If the plot exhibits the classic “cigar shape” then it is said to follow the bivariate normal distribution. The assumption of homoscedasticity refers to the equal variance of all values of the independent variables around the regression line (Goodwin & Goodwin, 2013). A residual scatter plot was generated to examine if a violation of the linearity assumption existed.

CHAPTER 4: FINDINGS

Overview

This chapter will present analytical findings from the test scores received by the two test groups of students; those who received instruction with Hands-on Equations, and those whose instruction did not include Hands-On Equations. These analyses allowed testing the null hypothesis and accepting or rejecting it accordingly. In the first section of this chapter, the general demographic results of the study sample are presented. The later section of this chapter provides an analysis of means and covariances in the target groups.

Research Questions

RQ1: Is there a difference in mathematics achievement scores for ninth-grade students with or without disabilities in Southeastern Georgia who received instruction using Hands-on Equations, as measured by the Algebra I I-Ready test?

RQ2: Is there a difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia who did or did not experience Hands-on Equations, as measured by the Algebra I I-Ready test?

RQ3: Is there a difference in mathematics achievement scores for ninth-grade students with disabilities and those without disabilities, in Southeastern Georgia who did or did not experience Hands-on Equations, as measured by the Algebra I I-Ready test?

Null Hypotheses

$H_0$1: There is no statistically significant difference in mathematics achievement scores for ninth-grade students with or without disabilities in Southeastern Georgia who received instruction using Hands-on Equations, as measured by the Algebra I I-Ready test.

$H_0$2: There is no statistically significant difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia, who did or did not experience Hands-on Equations, as measured by the Algebra I I-Ready test.

$H_0$3: There is no statistically significant difference in mathematics achievement scores for ninth-grade students with or without disabilities, in Southeastern Georgia, who did or did not experience Hands-on Equations, as measured by the Algebra I I-Ready test.

Descriptive Statistics

This section discusses the demographics of the study sample to generate a better understanding of the targeted population. Gender was considered, and though it proved impractical, initial efforts were made to make the study gender-neutral. The racial orientation was also identified with representation from African-American, White, and Native Hawaiian or Other Pacific Islanders. Finally, ethnicity was also included as a demographic factor.

Gender

Efforts were made to make the study gender-neutral by including participants from both genders in the study. However, a complete equivalence could not be achieved due to the different male to female student ratios in the surveyed schools. Table 1 depicts a summary of the outcome related to the gender demographics for the students who received instruction using Hands-on Equations.

Table 1: Gender of Students Receiving Instruction using Hands-on Equations

	Gender	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	female	44	53.0	53.0	53.0
	male	39	47.0	47.0	100.0
	Total	83	100.0	100.0	

The gender demographics for the ninth-grade students exposed to the instruction based on Hands-on Equations depict that out of the 83 participants, 44 (53.0%) were females, while 39 (47.0%) constituted the male participants. Gender composition was fairly balanced among the participants.

Table 2 highlights a summary of the demographic results attributed to the group of participants who did not receive instruction using the Hands-On Equations system.

Table 2: Gender for Students Not Receiving Instruction Using Hands-on Equations

	Gender	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	female	19	63.3	63.3	63.3
	male	11	36.7	36.7	100.0
	Total	30	100.0	100.0	

For the ninth-grade students who did not receive instruction using Hands-On Equations, 30 complete observations were made, with 19 (63.3%) being females and 11 (36.7%) being males. The stated gender demographics depict that the female ninth-grade students outnumbered the males in the sample constituent of the students who did not receive instruction using Hands-on Equations.

Race

Table 3 depicts the racial composition of the participants who received instruction using Hands-on Equations

Table 3: Race of Students Receiving Instruction with Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Black or African American	79	95.2	95.2	95.2
	Native Hawaiian or Other Pacific Islander	1	1.2	1.2	96.4
	White	3	3.6	3.6	100.0
	Total	83	100.0	100.0	

The majority of ninth-grade students who received instruction using Hands-on Equations were Black or African American (95.2%) followed by the White students (3.6%) and the Native Hawaiian or Other Pacific Islanders students (1.2%).

The racial composition of the students who did not receive instruction using Hands-on Equations is illustrated in table 4.

Table 4: Race of Students Not Receiving Instruction Using Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Black or African American	26	86.7	86.7	86.7
	White	4	13.3	13.3	100.0
	Total	30	100.0	100.0	

Similarly, for the students who did not experience the use of the manipulative (Hands-on Equations), a majority of them were Black or African American (86.7%) followed by White (13.3%). The implication is that the sample composition in the two population groups was dominated by the ninth-grade students who were of the Black (African-American) descent.

Ethnicity

In addition to the sample composition based on race, the researcher also inquired for ethnicity (Hispanic or Latino) in the observed group. The ethnicity demographics related to the students who were using Hands-on Equations is depicted in table 5 below.

Table 5: Ethnicity Demographics for Students Who Received Instruction Using Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	N	80	96.4	96.4	96.4
	Y	3	3.6	3.6	100.0
	Total	83	100.0	100.0	

The majority of the ninth-grade students exposed to Hands-on Equations were not classified as Hispanic or Latino (96.4%). The insinuation is that only 3.6% of the ninth-grade students were identified as Hispanic or Latino.

The ethnicity demographic composition of the students who did not receive instruction using Hands-on Equations is illustrated in Table 6 below.

Table 6: Ethnicity Demographics for Students Not Receiving Instruction Using Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	N	29	96.7	96.7	96.7
	Y	1	3.3	3.3	100.0
	Total	30	100.0	100.0	

Similarly, the demographic results for the ninth-grade students who did not receive instruction using Hands-on Equations depict that the majority of them were not Hispanic or Latino (96.7%). This means that only 3.3% of the participants who did not receive instruction using Hands-on Equations were considered as Hispanic or Latino.

Special Education Status

The analysis also ensured that in each of the two groups (students who used and did not use hands-on Equations) were equally represented in terms of the special education needs status. Table 7 depicts the results of the participants' special education needs status from the group of ninth-grade students that received instruction using Hands-on Equations.

Table 7: Special Needs Status for Students Receiving Instruction Using Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	N	68	81.9	81.9	81.9
	Y	15	18.1	18.1	100.0
	Total	83	100.0	100.0	

The results show that the majority of the ninth-grade students using Hands-on Equations did not require special education (81.9%). The insinuation is that only 18.1% of the group of students using Hands-on Equations needed special education.

The outcome on the students' special education needs status from the group that did not receive instruction using Hands-on Equations is illustrated in Table 8 below.

Table 8: Special Needs Status for Students Not Receiving Instruction Using Hands-on Equations

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	N	26	86.7	86.7	86.7
	Y	4	13.3	13.3	100.0
	Total	30	100.0	100.0	

Similarly, for the ninth-grade students who were not using Hands-on Equations, the majority (86.7%) did not need any special education services. However, a slightly lower percentage (13.3%) of the group that did not receive instruction using Hands-on Equations was considered in need of special education. The outcome based on the special needs education status depicts that in both groups, the ninth-grade students with a disability were dominated by the students that did not require any special education services.

Results

This section presents the results obtained from the surveys of the data conducted in the study. The results are grouped according to the three researched hypotheses. Specifically, the results of the ANCOVA analysis, which depicts a comparison of the adjusted means (mathematics scores) controlling for the effect of the pretest results, are discussed in this section of the paper.

Hypotheses

The three hypotheses tested the individual and combined differences between the special education need and the use of Hands-on Equations with the dependent variable being the mathematics achievement (scores) represented by the diagnostic overall scale score-2 from the Algebra I I-Ready test.

Hypothesis 1 ($H_0 1$). Mathematics Achievement in Special Education Group.

The first null hypothesis of the study stated there is no significant difference in mathematics achievement scores for ninth-grade students with or without disabilities in Southeastern Georgia who were exposed to instruction using the Hands-on Equations system as measured by the Algebra I I-Ready test. In order to test the hypothesis, a one-way ANCOVA analysis was employed to assess the covariance in the two groups of students (those with special education needs and those that were not in need of special education). The dependent variable was the Diagnostic Overall Scale Score-2, as measured by the Algebra I I-Ready test. The independent explanatory factor variable was the ‘Special Education’ categorical variable. The factor covariate, which controls for the effect of disability on the students’ mathematics achievement scores, is specified as the pretest scores (Diagnostic Overall Scale Score-1) as measured by the Algebra I I-Ready test. SPSS v. 22 was employed to conduct the ANCOVA analysis.

Table 9 illustrates a summary of the estimated adjusted marginal means related to the mathematics scores for the ninth-grade students with disabilities (in need of special education) and those without any form of disability (do not require special education).

Table 9: Estimated Adjusted Marginal Means: Mathematics Scores of Students with and without Disabilities

Dependent Variable: Diagnostic Overall Scale Score-2

Special Education	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
N	481.827[a]	2.414	477.023	486.631
Y	466.517[a]	5.903	454.770	478.265

a. Covariates are evaluated at the following values: Diagnostic Overall Scale Score-1 = 474.193.

As can be seen in Table 9 with respect to the Diagnostic Scale Score-2, the ninth-grade students who did not require special education had a higher adjusted mean score ($\mu = 481.83$; $\sigma = 2.41$) when assessed against the adjusted mean scores of the ninth-grade students that were in need of special education ($\mu = 466.52$; $\sigma = 5.90$). The one-way ANCOVA analysis was conducted to determine whether the difference in the adjusted mean mathematical scores was statistically significant or not. Table 10 highlights the outcome of the one-way ANCOVA analysis to generate answers for research question 1.

Table 10: ANCOVA Analysis Outcome for Research Question 1

Tests of Between-Subjects Effects

Dependent Variable: Diagnostic Overall Scale Score-2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	58440.850[a]	2	29220.425	81.050	.000	.670
Intercept	5587.499	1	5587.499	15.498	.000	.162
DiagnosticOverallScaleScore1	27372.414	1	27372.414	75.924	.000	.487
Special Education	1859.536	1	1859.536	5.158	.026	.061
Error	28841.849	80	360.523			
Total	19135676.000	83				
Corrected Total	87282.699	82				

a. R Squared = .670 (Adjusted R Squared = .661)

The outcome of the one-way ANCOVA analysis depicts that both the covariate and the independent variable were significant at $\alpha_{0.05}$. This means that both the pretest scores and the

Special Education needs status of the ninth-grade students exposed to Hands-on Equations had a substantial influence on the mathematics scores as measured by the Diagnostic Scale Score-2.

The partial Eta squared column illustrates that the pretest scores (Diagnostic Overall Scale Score 1) (0.487) explain 48.7% of the changes in the mathematics scores as measured by the Diagnostic Scale Score-2. In addition, the Special Education need status (0.061) expounds 6.1% of the movement in the mathematics scores as defined by the Diagnostic Scale Score-2. Therefore, based on the significance of the 'Special Education' independent factor variable [$F(1, 83) = 5.16$; $\rho = 0.026 < 0.05$)], the analysis rejects the null hypothesis 1 (***$H_0$1***), which means that there is a significant difference in the mathematics scores of the students with a disability and those without a disability among the group of participants exposed to instruction using Hands-on Equations.

Hypothesis 2 ($H_0$2). Mathematics Achievementusing the Hands-On Equations

The second null hypothesis of the study stated that there is no statistically significant difference in mathematics achievement scores for ninth-grade students in Southeastern Georgia, who did or did not receive instruction using Hands-on Equations as measured by the Algebra I I-Ready test. A one-way ANCOVA analysis was used to examine the covariance in the two groups of students (participants using Hands-on Equations and the students not subjected to the manipulative). Similarly, in this case, the dependent variable was the Diagnostic Overall Scale Score-2. The independent factor variable was the "Instruction with Hands-on Equations," which is also a categorical variable. The covariate, which controls for the effect of exposure to the Hands-on Equations on the mathematics achievement scores is defined as the pretest scores (Diagnostic Overall Scale Score-1). The ANCOVA analysis was conducted in SPSS v. 22.

Table 11 presents the estimated adjusted marginal means related to the mathematics scores for the ninth-grade students using and not using Hands-on Equations.

Table 11: Estimated Adjusted Marginal Means: Mathematics Scores of Students Exposed and Not Exposed to Hands-on Equation Instruction

Estimates

Dependent Variable: Diagnostic Overall Scale Score-2

Exposure to Hands-on Equations	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
N	476.516[a]	3.614	469.355	483.678
Y	480.849[a]	2.164	476.561	485.138

a. Covariates are evaluated at the following values: Diagnostic Overall Scale Score-1 = 476.558.

From Table 11, the mathematics scores (Diagnostic Scale Score-2) of the ninth-grade students who were using Hands-on Equations had a higher adjusted mean score ($\mu = 480.85$; $\sigma = 2.16$) compared to the adjusted mean scores of the ninth-grade students that did not receive instruction using the manipulative ($\mu = 476.52$; $\sigma = 3.61$). Similarly, the one-way ANCOVA analysis was also conducted to determine whether the difference in the adjusted mean scores was substantial or not. The outcome of the one-way ANCOVA analysis to generate solutions related to the research inquiry (question 2) is delineated in Table 12.

Table 12: ANCOVA Analysis Outcome for Research Question 2

Tests of Between-Subjects Effects

Dependent Variable: Diagnostic Overall Scale Score-2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	57409.549[a]	2	28704.774	74.207	.000	.574
Intercept	6061.929	1	6061.929	15.671	.000	.125
DiagnosticOverallScaleScore1	57281.944	1	57281.944	148.084	.000	.574
Exposure to Hands-on Equations	406.618	1	406.618	1.051	.307	.009
Error	42550.221	110	386.820			
Total	26102530.000	113				
Corrected Total	99959.770	112				

a. R Squared = .574 (Adjusted R Squared = .567)

The ANCOVA analysis results depict that only the covariate, pretest scores, was significant at the $\alpha = 0.05$ level. However, the independent factor variable, Exposure to Hands-on Equations, was not significant at $\alpha_{0.05}$. The explanation is that only the pretest scores (Diagnostic Scale Score-1) had a considerable influence on the mathematics scores as measured by the Diagnostic Scale Score-2. However, exposure to the Hands-on Equations did not have any substantial effect on the succeeding students' mathematics scores as measured by the Diagnostic Scale Score-2.

Based on the partial Eta squared, the pretest scores (Diagnostic Overall Scale Score 1) (0.574) expounds 57.4% of the fluctuations that occur in the student mathematics scores as measured by the Diagnostic Scale Score-2. However, the exposure to Hands-on Equations (0.009) accounts for 1% of the changes in the mathematics scores (Diagnostic Scale Score-2). The outcome of the ANCOVA analysis concurs with the null hypothesis defined for the research question 2. Specifically, given that the 'Exposure to Hands-on Equations' independent variable [$F(1, 113) = 1.05$; $\rho = 0.31 > 0.05$)] is non-significant, the null hypothesis 2 (H_{0}***2)*** cannot be rejected. This means that there is no significant difference in the mathematics scores of the ninth-grade students exposed and not exposed to instruction using Hands-on Equations.

Hypothesis 3 (H_{03}). Mathematics Achievement for Hands-On Equations in the Special Education Group

The third null hypothesis of the study stated there is no statistically significant difference in mathematics achievement scores for ninth-grade students with or without disabilities, in Southeastern Georgia, who did or did not receive instruction using Hands-on Equations. The implication is that there are two independent factor variables that influence the dependent factor variable (mathematics score as measured by the Diagnostic Scale Score-2). The two independent factor variables include the 'Special Education' and 'Exposure to Hands-on Equations.' The covariate, which controls for the effect of 'Exposure to the Hands-On Equations' and 'Special Education' is the pretest scores (Diagnostic Overall Scale Score-1). The two-way ANCOVA analysis was conducted in SPSS v. 22. The two-way ANCOVA analysis is specified because there are two explanatory variables ('Special Education' and 'Exposure to Hands-on Equations') that influence the student mathematics scores.

Table 13 presents the estimated adjusted marginal means related to the mathematics scores for the ninth-grade students for both the 'Special Education' group and for the participants exposed and not exposed to 'Hands-on Equations.'

Table 13: Estimated Adjusted Marginal Means: Mathematics Scores for the Interaction Effect of Exposure to Hands-on Equations and Special Education

Exposure to Hands-on Equations * Special Education

Dependent Variable: Diagnostic Overall Scale Score-2

Exposure to Hands-on Equations	Special Education	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
N	N	476.020[a]	3.820	468.448	483.592
	Y	484.664[a]	9.619	465.598	503.730
Y	N	483.735[a]	2.375	479.027	488.443
	Y	466.456[a]	5.898	454.764	478.147

a. Covariates are evaluated at the following values: Diagnostic Overall Scale Score-1 = 476.558.

Based on Table 13, the mathematics scores (Diagnostic Scale Score-2) of the ninth-grade students who were subjected to Hands-on Equations and did not require special education had a higher adjusted mean score ($\mu = 483.74$; $\sigma = 2.38$) compared to the adjusted mean scores of the ninth-grade students that received instruction using Hands-on Equations and required special education ($\mu = 466.46$; $\sigma = 5.90$). On the other hand, mathematics scores of the ninth-grade students who were not exposed to Hands-on Equations but required special education had a higher adjusted mean score ($\mu = 484.66$; $\sigma = 9.62$) when assessed against the adjusted mean scores of the students that were not using Hands-on Equations and did not require special education ($\mu = 476.02$; $\sigma = 3.82$). The two-way ANCOVA analysis was also conducted to determine whether the difference in the adjusted mean scores was considerable or not. The results of the two-way ANCOVA analysis to answer the research inquiry (question 3) are outlined in Table 14.

Table 14: ANCOVA Analysis Outcome for Research Question 3

Tests of Between-Subjects Effects

Dependent Variable: Diagnostic Overall Scale Score-2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	60331.687[a]	4	15082.922	41.106	.000	.604
Intercept	8357.436	1	8357.436	22.777	.000	.174
Diagnostic Overall ScaleScore1	29032.968	1	29032.968	79.125	.000	.423
Exposure to Hands-on Equations	284.384	1	284.384	.775	.381	.007
Special Education	167.123	1	167.123	.455	.501	.004
Exposure to Hands-on Equations * Special Education	1758.506	1	1758.506	4.793	.031	.042
Error	39628.083	108	366.927			
Total	26102530.000	113				
Corrected Total	99959.770	112				

a. R Squared = .604 (Adjusted R Squared = .589)

The outcome based on the two-way ANCOVA analysis indicate that only the covariate (pretest scores) [F (1, 113) = 79.13; $\rho = 0.000 < 0.05$)] and the interaction effect (Special Education* Exposure to Hands-on Equations) [F (1, 113) = 4.79; $\rho = 0.031 < 0.05$)] were significant at the $\alpha = 0.05$ level. However, the two independent factor variables, 'Special Education' [F (1, 113) = 0.46; $\rho = 0.501 > 0.05$)] and the 'Exposure to Hands-on Equations' [F (1, 113) = 0.78; $\rho = 0.381 > 0.05$)] were not significant at $\alpha_{0.05}$. The insinuation is that only the pretest scores (Diagnostic Scale Score-1) and the interaction cross factor variables had a substantial influence on the mathematics scores as defined by the Diagnostic Scale Score-2. However, when incorporated separately, the exposure to Hands-on Equations and Special Education did not have any substantial effect on the mathematics scores as measured by the Diagnostic Scale Score-2.

From the partial Eta squared, the pretest scores (0.423) explain 42.3% of the changes in the student mathematics scores as measured by the Diagnostic Scale Score-2. The interaction effect (0.042) expounds 4.2% of the changes in the mathematics scores (Diagnostic Scale Score-2). However, when accounted independently, ‘Special Education’ and ‘Exposure to Hands-on Equations’ explain less than 1% of the variations in the mathematics scores (Diagnostic Scale Score-2). The insight from the ANCOVA analysis contrasts the null hypothesis associated with the research question 3. Precisely, given that the interaction effect (“Special Education” and “Instruction using Hands-on Equations”) [$F(1, 113) = 4.79$; $\rho = 0.031 < 0.05$)] is significant, the initial hypothesis 3 (H_0***3)*** is discarded. The implication is that there is a significant difference in the mathematics scores of the ninth-grade students with and without disability in Southeastern Georgia for those who received and did not receive instruction using Hands-on Equations.

Table 15: Hypotheses Mean Scores

H_01		
Special Education		Mean
N		481.827
Y		466.517
H_02		
Exposure to Hands-on Equations		Mean
N		476.516
Y		480.849
H_03		
Exposure to Hands-on Equations	Special Education	Mean
N	N	470.02
	Y	484.664
Y	N	483.735
	Y	466.456

Testing the Assumptions for ANCOVA Analysis

The validity and accuracy of the ANCOVA analysis depend on the attainment of several sets of assumptions. This section reports tests used to assess the assumptions related to the normality, linearity, and the homogeneity (constant residual variance), which are the primary foundations that guide the ANCOVA analysis.

Normality Test:

The ANCOVA analysis presumes that the mathematics scores data for the students exposed to Hands-on Equations and those who did not receive the manipulative instruction have a normal distribution with a mean of nil and a standard deviation (variance) of 1. Table 16 depicts the results of the normality test to assess whether the mathematics scores of the students who received instruction using Hands-on Equations have a normal distribution or not.

Table 16: Normality Test for the Mathematics Scores of the Students Using Hands-on Equations

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
HoE-Diagnostic Overall Scale Score-2	.163	83	.000	.917	83	.000

a. Lilliefors Significance Correction

The outcome from both the K-S normality test (ρ = 0.000< 0.05) and the Shapiro-Wilk normality test (ρ = 0.000< 0.05) illustrate that the assumption of normal distribution in the student mathematics score data does not hold at the α = 0.05. The visual analysis from the histogram also confirms that the stated data is not normally distributed.

Figure 1: Diagnostic Overall Scale Score

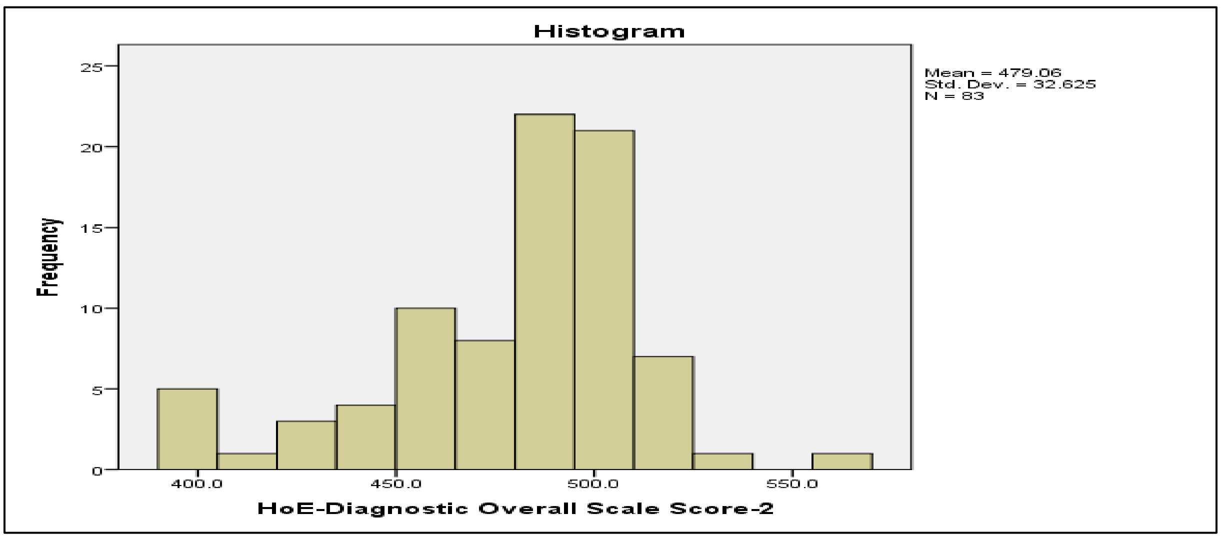


Table 17 illustrates the outcome of the normality test to assess whether the mathematics scores of the students who did not receive instruction using Hands-on Equations is normally distributed or not.

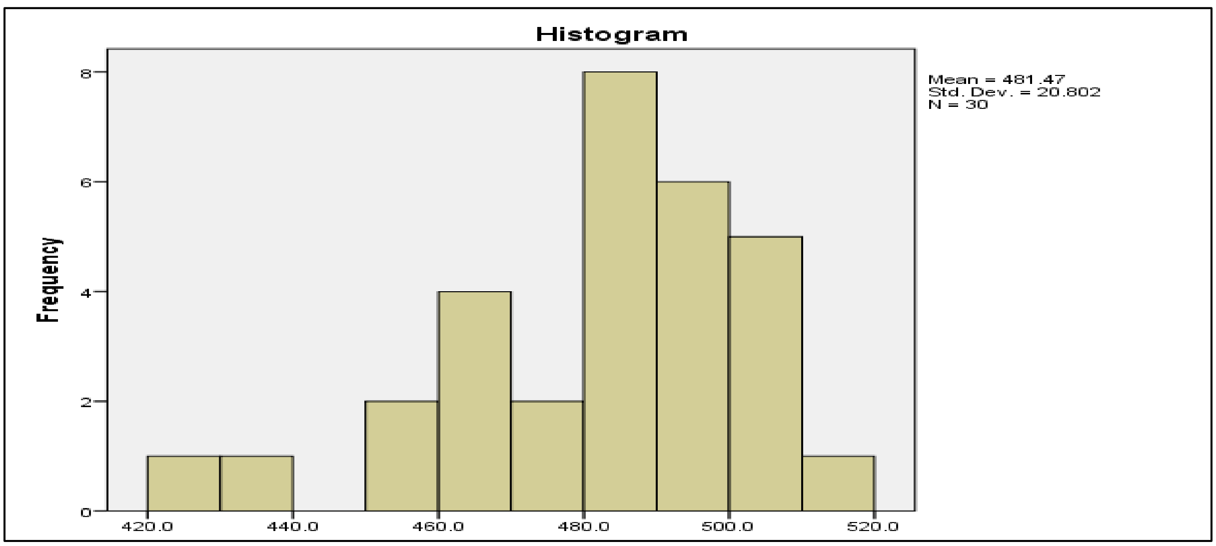
Table 17: Normality Test for the Mathematics Scores of the Students not Using Hands-on Equations

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Diagnostic Overall Scale Score-2	.148	30	.090	.924	30	.035

a. Lilliefors Significance Correction

The insight from the K-S normality test (ρ = 0.090 > 0.05) depicts that the assumption of normal distribution in the student mathematics score holds at α = 0.05. However, the Shapiro-Wilk normality test (ρ = 0.035 < 0.05) does not agree with the normality assumption. The visual analysis from the histogram (Figure 1) slightly agrees with the outcome of the K-S normality test.

Figure 2: Diagnostic Overall Scale Score

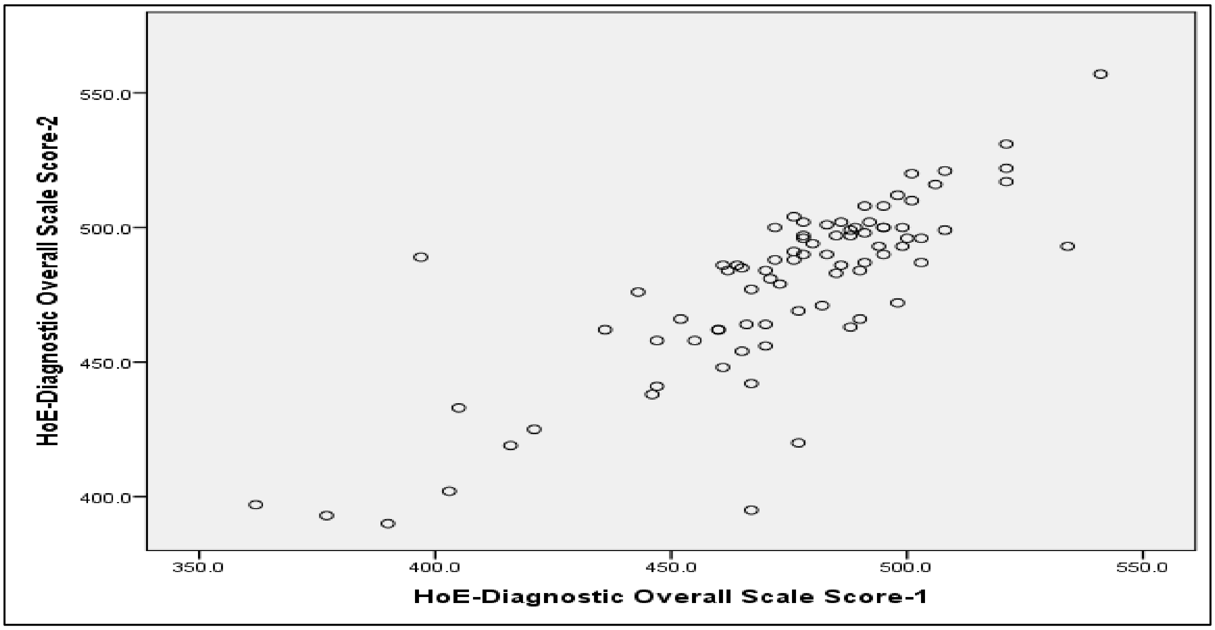


Diagnostic Overall Scale Score 2 Not using Hands-on Equations

Linearity Test:

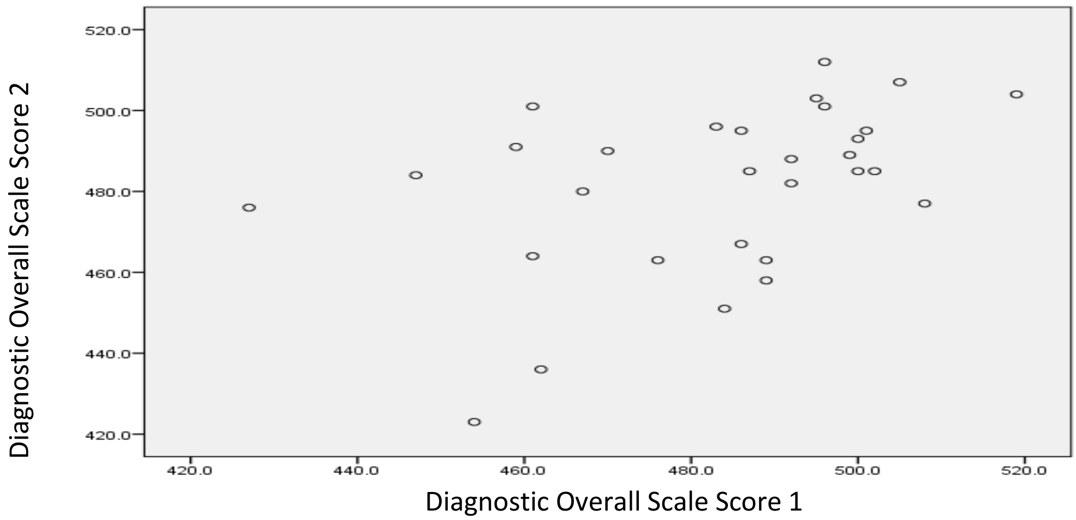
The linearity assumption is based on the premise that both the dependent factor variable and the explanatory factor variable are linearly related. Figures 2 and 3 present the scatterplots to assess the linearity in the student mathematics scores for the group that did receive instruction using Hands-on Equations and the group that was not subjected to the manipulative respectively.

Figure 3: Linearity Test for the Mathematics Scores of Students Using Hands-on Equations



The scatterplot visual (Figure 3) indicates that the mathematics scores for the students who did not receive instruction using Hands-on Equations was linearly related to the pretest scores. The stated conclusion is derived because of the curved-shaped appearance of the scatterplots. However, there seems to be a weak linear relationship between the student mathematics scores (Diagnostic Scale Score-2) and the pretest scores for the participants that did receive instruction using Hands-on Equations.

Figure 4: Linearity Test for the Mathematics Scores of Students not Using Hands-on Equations



Homogeneity (Constant Variance) Test:

The validity of the ANCOVA analysis is also based on the assumption that there is an unstable (constant) variance attributed to the residuals of the archetype model. A homogeneity test was conducted using the Levene's constant variance test. A summary of the Levene's constant variance test outcome for hypotheses 1, 2 and 3 is depicted in Table 17, 18 and 19 respectively.

Table 18: Homogeneity Test for Equality of Variance: Hypothesis 1

		Levene's Test for Equality of Variances	
		F	Sig. (p)
Diagnostic Overall Scale Score-2	Equal variances assumed	3. 164	0. 078
	Equal variances not assumed		

Table 19: Homogeneity Test for Equality of Variance: Hypothesis 2

		Levene's Test for Equality of Variances	
		F	Sig.
Diagnostic Overall Scale Score-2	Equal variances assumed	12. 062	0. 001
	Equal variances not assumed		

Table 20: Homogeneity Test for Equality of Variance: Hypothesis3

		Levene's Test for Equality of Variances	
		F	Sig.(p)
Diagnostic Overall Scale Score-2	Equal variances assumed	3.164	0. 078
	Equal variances not assumed		

The outcome of the homogeneity test reveals that only in model 1 and 3 did the constant variance assumption hold with respect to the student mathematics achievement scores because the F-values had a $\rho>0.05$. However, in model 2 ($F = 12.06$; $\rho = 0.001 < 0.05$), the constant variance assumption was not held at the 5% level.

Therefore, in summary, the assumptions of normality, linearity, and the constant variance (homogeneity) have been partially met in the data that was employed to conduct the ANCOVA analysis. This situation is likely to have a slight influence on the reliability of the ANCOVA analysis outcome.

Combined Results

The third null hypothesis is based on two independent observations from the "Special Education" and the "Exposure to Hands-on Equations" factor variables. Therefore, the results need to be combined in terms of weightage, as there is a contradiction between the results. While the first set of results (Score-1) does not show the intercept of Special Education and Intervention to play a significant role in defining students' mathematics performance, Score-2 showed the intercept to have a significant effect. The null hypothesis 1 (**$H_0$1**) was not accepted, which suggests that there is a significant difference in the mathematics achievement scores of students with and without special education needs for the participants that did receive instruction using Hands-on Equations. However, the initial hypothesis 2 defined by (**$H_0$2**) was not rejected, which means that there is no substantial difference in the student mathematics scores for the group that did receive instruction using Hands-on Equations and the group that was not subjected to the manipulative. Finally, the null hypothesis 3 (**$H_0$3**) was not accepted. There is a substantial variation in the mathematics scores of the students with and without a disability in Southeastern Georgia; for the group that did receive instruction using the Hands-on Equations and the group that did not experience instruction with the manipulative.

CHAPTER 5: DISCUSSION, CONCLUSION, AND RECOMMENDATIONS

Overview

This chapter will present a critical analysis of the study results in light of the literature review and previous empirical findings. The aim of this chapter is to compare and analyze the present findings with the previous finding, identify study limitations, and new insights resulting from the study.

Discussion

In the following section, the researcher will address the three research questions, analyzing if special education and Hands-on Equations have any role to play in the overall mathematics achievement of students. To present a critical analysis, results obtained from data surveys will be compared and analyzed with previous findings and assertions.

Challenges for Ninth-Grade Students with Learning Disabilities

One of the primary premises of the study is based on the argument that children with disabilities have a more difficult time grasping concepts of mathematics as compared to those without disabilities. It was based on the premise of the need for Hands-on Equations was proposed and evaluated for the ninth-grade students of two different schools; one utilizing the mentioned intervention and the other which does not practice the intervention. So, one of the primary tests conducted in the study was to see if mathematics scores for children enrolled in the schools differed among the ones receiving special education and the ones who were not receiving special education.

The test results clearly showed students who did not receive special education scored higher in their mathematics test as compared to the ones who received special education. A mean difference of -15.31 was reported for the first research question based on the Diagnostic

Scale Score-2 (refer to Table 9). This result confirms the assertion made by Yell et al. (2012) that students with disabilities find it increasingly difficult to meet the evaluation standard as they move to a higher grade-level. Referring to the Piaget's Cognitive Development theory, it is the fourth stage in which formal operational development takes place, where children learn how to make meaning of their learning experiences and relate it to their previous knowledge and experience. It is this stage where a critical understanding is expected of the children with disabilities to compete with their classmates without disabilities.

Significance of Hands-on Equations for Ninth-Grade Students

Addressing the problem statement of the study, the primary objective of the research was to get more certainty regarding the use and significance of Hands-on Equations to improve mathematics achievement of ninth-grade students. Many studies (Barber & Borenson, 2008; Brown, 2011; Jimenez, 2011; Liendenbach& Raymond, 1996; Skaggs, 2007), using students without disabilities as the test subjects, found a positive correlation between the use of Hands-on Equations and the overall mathematics achievement of the students. On the other hand, studies by Garderen, et al. (2012), Schulte & Stevens (2015), and Stevens & Schulte (2017) observed Hands-on Equations failed to achieve the desired output when the test population was students with learning disabilities. Based on this foundation, the present study was a step forward to analyze the impact of the use of Hands-on Equations on mathematics scores in general, considering both students with and without disabilities.

Contrary to the previous studies (Carbonneau et al., 2013; Gurbuz, 2010; Sherman &Bisanz, 2009), which found a significant positive association of Hands-on Equations with mathematics achievement, the current study did not find any significant correlation, either positive or negative. A surprising finding was the scores for the students with disabilities were slightly higher when they received no intervention ($\mu 1=484.66$) as compared to those who received the intervention ($\mu 2= 466.46$). However, since no statistical correlation was reported, this difference can be associated with mere chance, coincidence, or variation in the two sample sizes. Thus, in general, the present study fails to confirm all previous studies advocating the use of Hands-on Equations for improved mathematics achievement. The majority of the previous studies were conducted with fourth and fifth-grade students aged between 8-11 years. This age bracket falls under the third stage of ‘concrete operational development,’ according to Piaget’s theory of cognitive development (Piaget, 1965). The difference in results could be explained as the application of Hands-on Equations for ninth-grade students is yet to be properly developed or because supplementary tools are required to make a noticeable impact on the overall performance output.

As the mathematics concepts become more intricate with each level, it is important to question and understand how the chosen teaching tools correspond to the three learning stages involved in mathematics learning, as proposed by Bruner (1977). The use of manipulatives is one part of the overall learning cycle, and it needs to be backed by exercises of hypothesis testing, model building, and experimentation so students can recreate the problem in different scenarios (Kitta & Kapinga, 2015). It has been learned tools of narrative argumentations, symbolic representations, and audio-visual aids. All of these tools contribute to the overall understanding and conceptualizing of a complex or compound mathematics tool (Belenky & Nokes, 2009; Krummheuer, 2013). Hands-on Equations should then be fitted with these tools in the best possible way, considering the difficulty level of the target problems and the particular needs or knowledge-level of the target students.

Significance of Hands-On Equations for Ninth-Grade Students with Disabilities

The application and correlation of Hands-on Equations with mathematics achievement becomes dubious when the target group of students is the one with disabilities (Schulte & Stevens 2015; and Stevens & Schulte, 2017). Previous researchers have identified the added complexity when dealing with students who require special education, and therefore, building a significant correlation becomes more difficult (Steele, 2010). A two-factor test with the dependent variable of the Diagnostic Overall Scale Score seconded the arguments presented by Schulte & Stevens (2015) and Stevens & Schulte (2017). The observed population was divided into four groups; students with disabilities, students without disabilities, students who received instruction using Hands-on Equations, and the ones who did not. The intersection of these four sets results is included in this study (*refer to Table 14*). It can be seen as children with disabilities, who received instruction with Hands-on Equations scored slightly less ($\mu=466.46$) in the second test (Overall Scale Score-2) than those who did not receive the intervention ($\mu=484.66$. Again, with no statistically significant correlation detected, these differences in performance scores cannot be directly associated with the intervention).

One of the crucial findings from the literature was students with learning disabilities often suffer from other problems like difficulty memorizing, paying attention, and organizing (Bottge et al., 2010; Cortiella, 2011). While Hands-on Equations focuses on conceptualization and visualization, the other pertinent problems of children with difficulties often remain unaddressed. In addition, children with disabilities are aware they are different from their fellow students, and this awareness often hinders their self-confidence, self-esteem, and communication skills. Therefore, assuming utilization of Hands-on Equations will work for all or the majority of the students with learning disabilities would be an overstatement. The argument made by Geary

(2011) needs to be emphasized here: a student-oriented and individual need-based intervention is required to target the unique skills and challenges of the students with and without disabilities when teaching algebra, geometry, measurement, and other mathematics concepts.

In my opinion, the real benefits of Hands-on Equations can only be obtained when the teacher understands the grass root-level issues of students like having problems in recalling a concept, anxiety when solving a text exam, or lack of productive feedback on errors from the tutor. The suggestion of Wang (2013) regarding research-based teaching should also be taken into account in order to test and evaluate different techniques with different populations, rather than using a one-fit-for-all approach.

Implications

At the end of the discussion and analysis, it is a good time to summarize the findings and present some implications which can be applied in a real-world scenario. While the literature is consistent on the significant correlation between the use of Hands-on Equations and math achievement, the present study did not find any such correlation. This means the significance of the traditional ways of teachings through lectures and note-taking cannot be undermined. However, the overall mathematics achievement of the target population was average at best, and this means there is still some room for improvement in how mathematics and its different modules are taught in the classrooms.

The researcher agrees with the viewpoint of the involvement of abstract thinking in a number of mathematics modules, including algebra and geometry, as noted by a number of researchers (Connely-Fukawa, 2012; Donohue et al., 2013; Rainbolt & Gallian, 2010). For teachers, those abstract concepts might make complete sense, but it may sound very vague and irrelevant to students, especially the ones with learning disabilities. Thus, the tutors need to make efforts in relating those abstract concepts to everyday scenarios, allowing students to relate objects with them and make meaning out of the concepts. As noted in Bruner's theory of development, teachers should go from the inactive level to the iconic level and then to the symbolic level. This means before introducing algebraic notations or theorems; students should understand the process of getting there or the sense of why such a concept is needed to solve a problem.

Owing to Dienes' theory of learning mathematics, the use of manipulatives can help in the early stages of learning, where students are still getting familiar with the concepts, their representation, and symbolism. However, the final stage of formalization is when students should be able to use their inductive and deductive reasoning to explain, elaborate, and replicate a problem (Dienes, 1973). A general observation is students are given some problems to solve, and tutors try to give them multiple similar problems to strengthen their concepts. However, repetition is not the same as replication, which invites students to come up with similar problems to the ones given by the tutor. Thus, tutors need to encourage a critical thinking and problem-solving environment in the class, allowing students to think or act as tutors themselves. This activity needs to be backed by regular formative assessment and feedback, as guided by Strickland and Maccini (2010). Rather than a yearly or bi-annual evaluation, students, particularly the ones with learning disabilities, need regular feedback and assessment to embed a concept correctly into their minds.

Finally, the use of Hands-on Equations and other manipulatives for teaching mathematics concepts should be guided by evidence-based research for the target audience. As noted in the current research, Hands-on Equations may not guarantee improved mathematics achievement, particularly with children that have special learning needs. It is obvious a different plan cannot be prepared for individual students, so the best way could be for teachers to assess the needs of their class and blend those needs into a single or a specialized lesson plan. It is important to mention again the grassroots-level issue of lack of confidence or fear and anxiety should be addressed before tutors introduce any kind of learning interventions to their students. The studies by Brodesky& Gross (2009) and Burns & Hamm (2011) can be used as a guideline for breaking down complex mathematics concepts into simple, tangible, and concrete objects which can help in the process of visualizing and conceptualizing a problem. Finally, it is the tutor who should be the judge of the best and workable exercises to be brought into the classrooms for making the course more interactive and students' need-oriented.

Limitations

One of the study limitations was a high number of missing values, which resulted in a limited sample size when the univariate analysis was made. As the analysis is based on the interception of values, a single missing value would result in dismissing all values of that particular observation. However, there are only meager chances of this limitation to have altered the study results. Future studies should use a larger sample size to get more intercepted observations and then compare their findings with the current study. Moreover, it is believed such interventions often take time to have an impact, and therefore, using a longitudinal study would have produced results over a time-series which could then be compared to see any pattern of performance improvement. However, a longitudinal study was not possible for this study due to time, resources, and permission constraints.

Recommendations for Future Researchers

In the end, I would like to make some recommendations to future researchers who might be interested in conducting their research in a similar field of study.

- Researchers with enough time and resources can adopt a longitudinal research and sampling approach, where they collect data from the study participants during different intervals and monitor any improvement in their mathematics aptitude or performance scores.
- Researchers can also conduct module-specific research, focusing on Hands-on Equations implementation and efficiency for different mathematics modules, for example, algebra, geometry, and fractional measurements.
- Researchers who would like to take a more theoretical approach can try to expand or enrich the presented learning theories by targeting the use of intervention at different developmental stages during mathematics learning, either collecting and comparing data from different grade levels or at different module stages in the same grade level.
- Researchers should focus more on the primary grades for maximum impact of the Hands-on Equations learning system. Solving linear equations is a sixth grade standard. By the time a student reaches the ninth grade they are examining the relationships and reasoning with the equations. By focusing on the earlier grades such as primary, it will be a preview for the student and it will build the self-esteem for the struggling learner.
- In order for the Hands-on Equations program to be fully successful, the instructor must be fully vested in its success with proper training.

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APPENDIX A: GEORGIA STANDARDS OF EXCELLENCE

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APPENDIX B: POWER ANALYSIS USING G*POWER