Market Beta and Factor Risk Premia in Financial Markets

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Fabian Hollstein

Abstract

This thesis investigates the properties of assets' market betas and the pricing of aggregate uncertainty in financial markets. Chapter 1 introduces the main concepts and delivers an overview of the subsequent chapters.

Chapter 2 conducts a comprehensive comparison of market beta estimation techniques. We study the performance of several historical, time-series model, and option-implied estimators for realized market beta. Thereby, we find the hybrid methodology, combining historical return data and option-implied information, to consistently outperform all other approaches. In addition, all other approaches, including fully implied and GARCH-based methods for dynamic conditional beta, are dominated by a simple beta estimate based on historical (co-) variances and a Kalman filter based approach. Our conclusions remain unchanged after performing several robustness checks.

Based on the findings in Chapter 2, in particular that the historical daily estimator for beta performs notably well, Chapter 3 studies the value of high-frequency data for beta estimation. Using intra-day high-frequency return data, we comprehensively analyze the performance of beta estimation based on such data. We find that, overall, the value of high-frequency data is limited. From a statistical viewpoint both the historical high-frequency approach and the hybrid approach using historical returns and options

prices work more or less equally well, while a combination of both approaches can improve performance. On the other hand, if we are interested in the economic implications of beta estimation a positive risk-return relationship cannot be detected using the high-frequency estimator while the hybrid approach appears to contain superior information. Our results extend to the estimation of downside beta.

Motivated by the empirical failure of market beta to fully account for the variations observable in the cross-section of stock returns, Chapter 4 studies whether further risk factors, in particular aggregate economic uncertainty, are priced in financial markets. In line with the predictions of a stylized theoretical model with stochastic volatility, we find that time-varying aggregate economic uncertainty commands an economically substantial and statistically significant negative risk premium. Aggregate uncertainty, marked-off from risk, is proxied with market volatility-of-volatility measured by the VVIX index. A two-standard deviation increase in aggregate uncertainty factor loadings is associated with a decrease in average annual returns ranging from 6.3 % to 18.7 %. This phenomenon can neither be explained by aggregate volatility, jump risk, and several other canonical, liquidity, and returns distributions characteristics, nor by a crisis effect.

Finally, Chapter 5 concludes and outlines possible future directions for research.

Keywords: Market beta estimation, high-frequency data, aggregate economic uncertainty

Zusammenfassung

Diese Arbeit beschäftigt sich mit den Eigenschaften von und Schätzmethoden für Marktbetafaktoren verschiedener Aktien. Des Weiteren wird untersucht, inwiefern gesamtwirtschaftliche ökonomische Unsicherheit in Kapitalmärkten gepreist ist. Kapitel 1 stellt die Hauptkonzepte vor und liefert einen Überblick über die nachfolgenden Kapitel.

Kapitel 2 präsentiert eine umfangreiche Analyse von verschiedenen Möglichkeiten, um Beta zu schätzen. Wir testen verschiedene historische, Zeitreihenmodell-basierte und optionsimplizite Verfahren für die Beta-Schätzung und evaluieren diese Verfahren mit dem nachfolgend realisierten Beta. Unsere Resultate deuten darauf hin, dass das hybride Verfahren von Buss & Vilkov (2012), das historische und options-implizite Daten kombiniert, am besten funktioniert. Außerdem sind alle weiteren Verfahren, unter anderem komplett options-implizite und GARCH-Modellbasierte Verfahren, einer einfachen historischen Schätzmethode sowie einem Kalman Filter basierten Verfahren unterlegen. Diese Schlussfolgerungen werden durch etliche Robustheitsanalysen bestätigt.

Auf Grundlage der Ergebnisse des 2. Kapitels, insbesondere motiviert durch den Fakt, dass das simple historische Schätzverfahren auf Basis von täglichen Renditen sehr gut funktioniert, untersuchen wir in Kapitel 3 den Wert von Intra-Day-Hochfrequenzdaten für die Beta-Schätzung. Unsere

Ergebnisse zeigen auf, dass Hochfrequenzdaten nur von beschränktem Wert für die Schätzung von Beta sind. Aus einer statistischen Perspektive betrachtet liefern Hochfrequenzschätzer und das hybride Buss & Vilkov (2012) Verfahren ungefähr gleich gute Resultate. Dagegen kann eine simple Kombination beider Verfahren die Schätzgenauigkeit für das nachfolgend realisierte Beta weiter verbessern. Andererseits ergibt sich aus der ökonomischen Perspektive, d.h. der Frage, ob Unterschiede in den Schätzungen für Beta auch Unterschiede in nachfolgenden Renditen abbilden, ein konträres Bild. Mit historischen Schätzern, die tägliche oder Hochfrequenzdaten benutzen, lässt sich kein signifikanter Trade-off zwischen Rendite und Risiko feststellen, während das hybride Verfahren unter diesem Gesichtspunkt deutlich besser funktioniert. Die hier beschriebenen Resultate gelten gleichermaßen für die Schätzung von Downside Beta.

Da Marktbeta alleine, wie in zahlreichen empirischen Studien gezeigt, die Variationen im Querschnitt der Aktienrenditen nicht komplett erklären kann, untersucht Kapitel 4 welche weiteren Risikofaktoren in Kapitalmärkten, speziell im Aktienmarkt, gepreist sind. Dabei wird der Schwerpunkt vor allem auf die gesamtwirtschaftliche ökonomische Unsicherheit gelegt. Als Ausgangspunkt zeigen wir mit einem einfachen theoretischen Modell mit stochastischer Volatilität, dass gesamtwirtschaftliche Unsicherheit in Kapitalmärkten potentiell gepreist sein kann. Unsere empirischen Ergebnisse indizieren, dass zeitvarianter ökonomischer Unsicherheit eine ökonomisch bedeutsame und statistisch stark signifikante negative Risikoprämie anhaftet. Wir benutzen die Volatilität der Volatilität des Marktes gemessen durch den VVIX Index der Chicago Board Options Exchange, um den Grad an gesamtwirtschaftlicher ökonomischer Unsicherheit, abgegrenzt von Risiko, der zur jeweiligen Zeit im Markt vorhanden ist, zu bestimmen. Unsere Ergebnisse zeigen, dass eine um zwei Standardabweichungen höhere Faktorsensitivität gegenüber der gesamtwirtschaftlichen Unsicherheit mit einer im Durchschnitt um 6.3 % bis 18.7 % reduzierten annualisierten Rendite einzelner Aktien einhergeht. Diese Ergebnisse können weder durch andere Risikofaktoren, wie z.B. gesamtwirtschaftliches Risiko, dem Risiko extremer Sprünge in Renditen, sowie vieler weiterer bekannter Faktoren, noch durch einen distinguierten Kriseneffekt erklärt werden.

Abschließend präsentiert Kapitel 5 Schlussfolgerungen und liefert Anregungen für mögliche zukünftige Forschungsthemen.

Schlagwörter: Marktbeta-Schätzung, Hochfrequenzdaten, gesamtwirtschaftliche ökonomische Unsicherheit

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Chapter 1

Introduction

The development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966) provides an important cornerstone in modern financial economics. If the assumptions of the CAPM are fulfilled, the model predicts that all assets can be priced by only one risk factor, i.e., the market risk premium. Asset's equilibrium rates of return then depend on their sensitivity to changes in the market risk premium, i.e., their beta factors. These factors, however, are not observable and hence need to be estimated. Studying the properties of market beta on the one hand is important to test the model predictions, and, on the other hand, to better understand the dynamic developments occurring on financial markets.

Chapter 2 makes use of the recent developments made in estimating beta in various different ways. Beta can be estimated simply from historical return data (Fama & MacBeth, 1973; Baker, Bradley, & Wurgler, 2010), based on historical return data specifying a time-series model (Pagan, 1980; Engle, 2014), or using data implied from options markets either combined with historical return data (French, Groth, & Kolari, 1983; Buss & Vilkov, 2012) or solely (Chang, Christoffersen, Jacobs, & Vainberg, 2012). Faff, Hillier, & Hillier (2000) compare historical and time-series models and several of the authors mentioned previously compare the approach they propose to subsets of existing models. Overall, however, these various ways of estimating beta, thus far, have not been comprehensively examined and compared.

Chapter 2 of this thesis, to the best of our knowledge, provides the first comprehensive and thorough empirical study on the performance of a wide range of market beta estimation techniques, including several historical, time-series model, and option-implied estimation approaches. Furthermore, a novel hybrid estimator for beta that corrects option implied volatility for the volatility risk premium is proposed. We study the information content of different approaches to estimate subsequent realized beta (based on daily return data) in univariate and encompassing regressions (Mincer & Zarnowitz, 1969) and determining the estimation accuracy using the root mean squared error (RMSE) criterion.

Our empirical evidence suggests that the hybrid approach proposed by Buss & Vilkov (BV) (2012), which combines option implied with historical return information, turns out to outperform all other methods. The simple historical benchmark model as well as an approach based on the Kalman filter and a random walk (RW) are shown to work comparatively well, while GARCH-based models of dynamic conditional beta and fully option implied approaches produce serious errors. We further show that the BV approach works so well mainly because, in combining historical and option implied information, it ensures that the estimates are adjusted to be unbiased in their value-weighted cross-sectional averages.

Motivated by the fact that historical beta, based on daily return data, works notably well, as well as by recent advances in financial economics using intra-day high-frequency data, Chapter 3 studies the value of intra-day high-frequency data for beta estimation. Several authors show that forecasts for the moments and co-moments of the return distribution can be obtained with greater precision once employing intra-day high-frequency data (e.g., Andersen & Bollerslev, 1998; Bollerslev & Zhang, 2003; Amaya, Christoffersen, Jacobs, & Vasquez, 2015). Bollerslev & Zhang (2003), Barndorff-Nielsen & Shephard (2004), and Andersen, Bollerslev, Diebold, & Wu (2005, 2006) derive the estimator for realized beta and show that it delivers a consistent estimate for the true underlying integrated beta. The use of high-frequency data may enable the researcher to make use of this consistency without having to rely on the traditionally imposed long historical windows which entail restrictions on the stability of the underlying processes and economic conditions. If these restrictions are not satisfied estimates can attain a lot of noise.

Consequently, Chapter 3 investigates the usefulness of high-frequency data for estimating beta. To the best of our knowledge, Chapter 3 delivers the first comprehensive and thorough empirical study on the statistical and economic performance of option-implied and historical market beta estimation techniques, including high-frequency return data. We use intra-day data to obtain both a more precise statistical evaluation of different ex ante estimates as well as presumably more precise estimates of ex ante historical or hybrid beta. Additionally, we provide evidence on optimal combinations of estimators and add an economic evaluation criterion relating ex ante estimates for beta with the cross-section of subsequent excess returns using Black, Jensen, & Scholes (1972) regressions. Finally, we provide evidence on the estimation of downside beta. While conditional risk premia have recently attracted much attention (Ang, Chen, & Xing, 2006a; Lettau, Maggiori, & Weber, 2014), so far only little work has been done with regard to the concrete estimation of downside beta.

The results of Chapter 3 indicate that the value of intra-day high-frequency data for beta estimation is only limited. While on the one hand, regarding the statistical evaluation especially over short horizons, high-frequency estimates are shown to be quite precise, on the other hand, the estimator fails to create economic value. These results are independent of the sampling frequency for high-frequency beta (using intervals of 5 up to 130 minutes). From a statistical viewpoint the hybrid estimator of Buss & Vilkov (2012) works more or less equally well and a simple combination of the high-frequency and hybrid estimators outperforms the individual models quite consistently. Regarding economic value, the hybrid model clearly outperforms historical daily and high-frequency models being the only approach that is able to detect a significantly positive cross-sectional relation between beta and subsequent excess returns.

The intertemporal CAPM (ICAPM) by Merton (1973) and the Arbitrage Pricing Theory (APT) by Ross (1976) provide important extensions of the classical CAPM. These models predict that equilibrium rates of return can also be influenced by further factors, especially in the intertemporal CAPM setting variables ought to be priced in financial markets that predict changes in the future investment opportunity set. For example, Fama & French (1993) motivate the introduction of a risk factor for size and book-to-market with the ICAPM, implying that changes in these factors are related to changes in expected future market returns or volatility. In another very important study, Ang, Hodrick, Xing, & Zhang (2006b) show that there exists a substantial risk premium on aggregate volatility in financial markets.

Connected to the classical distinction between risk and uncertainty pioneered by Knight (1921), defining risk as measurable uncertainty that can be captured using numerical probabilities while anything that cannot be described by numerical probabilities ought to be defined as uncertainty, Chapter 4 studies whether time-varying economic uncertainty is priced in financial markets. Following a large stream in the literature that measures risk using first-order beliefs, i.e. return volatility, and what Knight called "unmeasurable uncertainty" with second-order beliefs, i.e. the variation in the probability distribution of the payoffs (e.g., Segal, 1987; Nau, 2003; Seo, 2009; Baltussen, Van Bekkum, & Van Der Grient, 2015), we measure aggregate uncertainty using the VVIX index.[1]

Building on a simple stylized theoretical model, based on the standard ICAPM with recursive preferences and consumption uncertainty, we show that under these fairly common assumptions aggregate uncertainty is potentially priced in the cross-section of asset returns. For the empirical methodology we follow Ang et al. (2006a) and Cremers, Halling, & Weinbaum (2015) studying the contemporaneous relationship of asset's factor loadings on innovations in aggregate uncertainty and realized returns. The main contribution of Chapter 4 is that, to the best of our knowledge, we are the first to examine whether aggregate uncertainty, captured by the natural non-parametric VVIX measure, is priced in financial markets, particularly in the cross-section of stock returns.

Our results suggest that time-varying aggregate economic uncertainty commands an economically substantial and statistically significant negative risk premium. Using single and double portfolio sorts with a battery of control variables, we find that stocks with high sensitivities to innovations in aggregate uncertainty underperform those with low sensitivities by about 11.7 % per year. This finding cannot be explained by any of the single control variables. Using cross-sectional Fama & MacBeth (1973) regressions, we find that a two-standard deviation increase in aggregate uncertainty factor loadings is associated with a significant decrease in average annual returns that ranges from 6.3 % to 18.7 %, depending on the cross-sectional model specification.

This thesis proceeds as follows. Chapter 2 provides a comprehensive

[1] The VVIX index represents the 30-day forward-looking option-implied volatility of the volatility index (VIX), i.e. the volatility-of-volatility.

empirical study on the estimation of beta. Chapter 3 studies the value of high-frequency data for beta estimation. Chapter 4 examines whether aggregate uncertainty is priced in the cross-section of equity returns. Finally, Chapter 5 summarizes the main findings of this thesis and suggests several lines for future research.

For reasons of improved readability, especially of the separate parts constituting the complete thesis, each chapter is self-contained. This means, variables and acronyms are redefined in each chapter. Whenever possible, notations are consistent throughout the thesis in order to facilitate the reading.

Chapter 2

Estimating Beta*

2.1 Introduction

Ever since the development of the capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966) and the arbitrage pricing theory (APT) by Ross (1976), the concept of beta (i.e., the covariation of an asset with the relevant risk factors) plays a crucial role in financial economics. For many applications such as asset pricing, portfolio choice or risk management, market beta is the single most important parameter of interest. However, beta factors are not directly observable and hence they need to be estimated.

The main contribution of this chapter is that we are, to the best of our knowledge, the first to provide a comprehensive and thorough empirical study on the performance of a wide range of market beta estimation techniques, including several historical, time-series model, and option-implied estimation approaches. Additionally, we propose a new estimator for beta that corrects option-implied volatilities for the volatility

*This chapter is based on the Article "Estimating Beta" authored by Fabian Hollstein and Marcel Prokopczuk, Journal of Financial and Quantitative Analysis, forthcoming.

risk premium.

Our main results can be summarized as follows. The approach proposed by Buss & Vilkov (BV) (2012), combining option-implied with historical return information, turns out to outperform all other methods in estimating realized beta (based on daily return data). We determine outperformance in two dimensions (i) informational efficiency and (ii) estimation accuracy. The BV approach is both shown to be informationally more efficient in encompassing regressions compared to all other approaches and it yields the lowest out-of-sample estimation errors, employing the root mean squared error (RMSE) criterion. The simple historical benchmark model as well as an approach based on the Kalman filter and a random walk (RW) are shown to work comparatively well, while GARCH-based models of dynamic conditional beta and fully option-implied approaches produce serious errors. We further show that the BV approach works so well mainly because, in combining historical and option-implied information, it ensures that the estimates are adjusted to be unbiased in their value-weighted cross-sectional averages.

The most basic approach to estimate beta is to simply estimate covariances and variances from a time-series of historical return data. However, this approach faces the problem that beta coefficients exhibit significant time variation (e.g., Blume, 1975; Ferson & Harvey, 1991, 1993). To adress this concern, several approaches (e.g., GARCH-based) have been developed to capture this variability. More recently, it has been suggested that one could incorporate information from the options market, where all information available to investors should be contained in today's prices, thereby overcoming the inertia inherently generated by historical estimates, even when applying a rolling-window approach.

Regarding volatility estimation, which is closely related to beta estimation, numerous studies have been performed. Examining the performance of option-implied versus the historical volatility estimation approach the results of early studies differ (e.g., Canina & Figlewski, 1993; Christensen & Prabhala, 1998; Fleming, 1998). More recently, there seems to be a consensus that implied volatility (IV) estimates are to be favored.

Jiang & Tian (2005) show that model-free IV outperforms at-the-money (ATM) IV and historical volatility. Frijns, Tallau, & Tourani-Rad (2010) and Taylor, Yadav, & Zhang (2010)[1] show a superior performance of IV compared to different time-series models. Prokopczuk & Wese Simen (2014a) show that adjusting for the volatility risk premium improves the performance of IV. Thus, there exists ample evidence on the performance of volatility estimators.[2]

Surprisingly, however, the estimation of beta has received considerably less attention in the literature. Faff et al. (2000) find a superior performance of time-series models (especially of those using the Kalman filter) over historical estimators for beta in an in-sample analysis, while not presenting any out-of-sample evidence.

The relative underrepresentation of research studying beta estimation in the extant literature might, to some extent, be caused by the fact that beta requires information on correlations, which is not as easily obtained from options as is information on volatilities. Only very recently have several authors developed option-implied approaches to estimate beta.

Chang et al. (2012) develop such an option-implied approach and show that it often outperforms the historical beta in a cross-sectional analysis. Baule, Korn, & Saßning (2015) compare various different fully implied beta estimators. They obtain the best performance using betas based on implied variances. However, Chang et al. (2012) do not compare their approach

[1]For the one month horizon estimator.

[2]Other papers on volatility estimation include Jorion (1995), Guo (1996), Poon & Granger (2003), Szakmary, Ors, Kyoung Kim, & Davidson III (2003), Martens & Zein (2004), Agnolucci (2009) or Charoenwong, Jenwittayaroje, & Low (2009).

for implied betas directly to other existing approaches and Baule et al. (2015) only compare the performance of fully implied estimators among one another and to the simple historical estimator relying on a small Dow Jones Industrial Average (DJIA) 30 sample.

Buss & Vilkov (2012) propose another implied approach imposing a correction on historical correlations and compare it to historical, hybrid, and the Chang et al. (2012) implied beta estimator. However, they do not examine the performance of time-series models and other fully implied beta estimation techniques. Furthermore, they limit their attention to a comparatively long horizon of one year. Very recently, Engle (2014) and Bali, Engle, & Tang (2015) show that dynamic conditional beta does well in a cross-sectional analysis.

The remainder of this chapter is organized as follows. Section 2.2 describes our data set and methodology, providing an overview of the approaches considered. In Section 2.3 we present our empirical results. Section 2.4 checks the robustness of our results and Section 2.5 finally presents our conclusions. In the appendix to this chapter, which can be found in Section A, we present the results of additional analyses.

2.2 Data and Methodology

2.2.1 Data

We base our study on the S&P 500 market index and its constituents for the sample period between January 01, 1996 and December 31, 2012.[3] Additionally, we perform a robustness analysis on a sample based on the

[3]The starting date of our study is thereby determined by the start of the OptionMetrics database in January 1996.

DJIA.[4] We obtain daily and monthly price data as well as data on dividend payments and shares outstanding from the Center for Research in Security Prices (CRSP) for the period from January 01, 1994 until December 31, 2012.[5,6] To be able to compute historical and time-series model estimates right from the start of our study period and to perform a portfolio sorting using non-overlapping data, this data starts two years before the main sample period.

Options data are from the IvyDB OptionMetrics Volatility Surface that directly provides implied volatilities for standardized delta levels and maturities.[7] We use options with approximately six months to maturity since we want to obtain six-month estimates for beta. As a robustness check we also repeat the analysis with options of approximately one, three, and twelve months to maturity. We select out-of-the-money (OTM) options, namely puts with deltas larger than -0.5 and calls with deltas smaller than 0.5. Thereby we obtain options data for 438 stocks in 1996 growing to 493 stocks in 2010 out of the 500 contained in the S&P 500 at each respective date. On average, options data on 472 stocks is available. Data on the risk-free rate is collected from the IvyDB zero curve file.

2.2.2 Option-Implied Moments

Several of the beta estimation approaches are based on option-implied moments. Therefore we follow Bakshi, Kapadia, & Madan (BKM) (2003),

[4]The sample period for the DJIA dataset begins on January 01, 1998 as options on the DJIA are traded no earlier than October 1997 at the Chicago Board of Options Exchange (CBOE). We do not start before the beginning of the new year to avoid spurious findings caused by potentially small initial trading volumes in the new market.

[5]The data for monthly estimators, that can be found in the appendix to this chapter, starts on January 01, 1986.

[6]Data on the DJIA is not available through CRSP, therefore we obtain price data from the Bloomberg database.

[7]IvyDB uses a kernel smoothing algorithm and only reports standardized options "if there exists enough option price data on that date to accurately interpolate the required values". For more details refer to the IvyDB technical document.

who make use of the property that any payoff can be spanned using a continuum of OTM puts and calls (Bakshi & Madan, 2000) and Jiang & Tian (2005) to compute model-free option-implied volatility, skewness and kurtosis.[8] For that, we first compute ex-dividend stock prices. Secondly, for any given stock and trading day, we interpolate implied volatilities using a cubic spline across moneyness levels (K/S, strike-to-spot), equally spaced between 0.3 percent and 300 percent, to obtain a grid of 1,000 implied volatilities (Chang et al., 2012). Implied volatilities outside the range of available strike prices are extrapolated using the value for the smallest, resp. largest, available moneyness level (as in Jiang & Tian, 2005 and Chang et al., 2012). The volatilities are used to compute Black–Scholes option prices for calls, $C(.)$, if K/S>1 and puts, $P(.)$, if K/S<1. These are used to obtain the prices of the volatility (QUAD), the CUBIC, and the quartic (QUART)

[8]Note that Jiang & Tian (2005) compute implied volatility only. The procedure for skewness and kurtosis, though, is equivalent.

contract (Jiang & Tian, 2005):

$$\begin{aligned}
\text{QUAD} &= \int_S^\infty \frac{2\left(1-\ln\left[\frac{K}{S}\right]\right)}{K^2} C(\tau,K)dK \qquad (2.1)\\
&+ \int_0^S \frac{2\left(1+\ln\left[\frac{S}{K}\right]\right)}{K^2} P(\tau,K)dK,\\
\text{CUBIC} &= \int_S^\infty \frac{6\ln\left[\frac{K}{S}\right]-3\left(\ln\left[\frac{K}{S}\right]\right)^2}{K^2} C(\tau,K)dK \qquad (2.2)\\
&+ \int_0^S \frac{6\ln\left[\frac{S}{K}\right]+3\left(\ln\left[\frac{S}{K}\right]\right)^2}{K^2} P(\tau,K)dK,\\
\text{QUART} &= \int_S^\infty \frac{12\left(\ln\left[\frac{K}{S}\right]\right)^2-4\left(\ln\left[\frac{K}{S}\right]\right)^3}{K^2} C(\tau,K)dK \qquad (2.3)\\
&+ \int_0^S \frac{12\left(\ln\left[\frac{S}{K}\right]\right)^2+4\left(\ln\left[\frac{S}{K}\right]\right)^3}{K^2} P(\tau,K)dK.
\end{aligned}$$

The integrals are approximated, following Dennis & Mayhew (2002), using a trapezoidal rule. The option-implied moments can be computed as:

$$\begin{aligned}
\mu^{\mathbb{Q}} &= e^{r_t^f(T-t)} - 1 - \frac{e^{r_t^f(T-t)}}{2}\text{QUAD} - \frac{e^{r_t^f(T-t)}}{6}\text{CUBIC} \qquad (2.4)\\
&-\frac{e^{r_t^f(T-t)}}{24}\text{QUART},\\
(\sigma^{\mathbb{Q}})^2 &= e^{r_t^f(T-t)}\text{QUAD} - (\mu^{\mathbb{Q}})^2, \qquad (2.5)\\
\text{skew}^{\mathbb{Q}} &= \frac{e^{r_t^f(T-t)}\text{CUBIC} - 3\mu^{\mathbb{Q}}e^{r_t^f(T-t)}\text{QUAD} + 2(\mu^{\mathbb{Q}})^3}{[e^{r_t^f(T-t)}\text{QUAD} - (\mu^{\mathbb{Q}})^2]^{3/2}}, \qquad (2.6)\\
\text{kurt}^{\mathbb{Q}} &= \frac{e^{r_t^f(T-t)}\text{QUART} - 4\mu^{\mathbb{Q}}e^{r_t^f(T-t)}\text{CUBIC} + 6(\mu^{\mathbb{Q}})^2e^{r_t^f(T-t)}\text{QUAD} - 3(\mu^{\mathbb{Q}})^4}{[e^{r_t^f(T-t)}\text{QUAD} - (\mu^{\mathbb{Q}})^2]^2}, \qquad (2.7)
\end{aligned}$$

where r_t^f denotes the risk-free rate and $T-t$ the time to maturity of the contract. $(\sigma^{\mathbb{Q}})^2$, $\text{skew}^{\mathbb{Q}}$, and $\text{kurt}^{\mathbb{Q}}$ are the option-implied variance, skewness, and kurtosis, respectively. In the following, we use the respective values obtained to compute beta estimates that require option-implied moments.

2.2.3 Beta Estimation

Realized Beta Following Andersen et al. (2006) we use daily log-returns to compute realized beta (RB):[9]

$$\beta_{j,t}^{\mathrm{R}} = \frac{\sum_{\tau=1}^{N} r_{j,\tau} r_{M,\tau}}{\sum_{\tau=1}^{N} r_{M,\tau}^2}, \tag{2.8}$$

where $r_{j,\tau}$ and $r_{M,\tau}$ refer to the (excess) return of asset j and the market (excess) return at time τ, respectively. N is the number of observations during the time period under investigation.

Andersen et al. (2006) show that under only weak regularity conditions is this a consistent measure for the true underlying integrated beta. While Hansen & Lunde (2006) strongly advise using realized volatility when evaluating volatility models, we follow that spirit using ex post realized beta to evaluate all the respective ex ante estimates obtained using the different beta estimation methods.

Historical Beta Closely related to the above approach, we compute historical estimates (HIST) in the usual way, following Fama & MacBeth (FM) (1973) and many others, regressing an asset's (excess) return on the market (excess) return:

$$\beta_{j,t} = \frac{\mathrm{cov}(r_j, r_M)}{\mathrm{var}(r_M)}. \tag{2.9}$$

We utilize beta estimated using one year of daily returns as, e.g., in Baker et al. (2010).[10]

Dynamic Conditional Beta We estimate both dynamic conditional beta with GARCH models for the (co-) volatilities and AR-type

[9] We refer to past realized beta as a possible ex ante beta estimation technique as HIST_6.

[10] We also test the standard FM beta computed using five years of monthly return data. The results on that (and other montly beta estimators) can be found in the appendix to this chapter.

models that impose certain factor dynamics directly on the beta series. We refer to both types as time-series models.

We consider dynamic conditional beta (Engle, 2014 and Bali et al., 2015) using a Dynamic Conditional Correlation GARCH model (DCC) as proposed by Engle (2002) and Cappiello, Engle, & Sheppard (2006), incorporating both the empirically well-established leverage effect by allowing for an asymmetric effect of positive and negative return innovations, as well as an asymmetric reaction of correlations on innovations in variances.[11] First, univariate volatility models are estimated as GJR GARCH (as proposed by Glosten, Jagannathan, & Runkle, 1993):

$$\begin{aligned} r_t &= \mu + a_t && (2.10) \\ a_t &\sim N(0, \sigma_t^2) && (2.11) \\ h_t^2 &= \omega + (\alpha + \gamma \mathrm{I}_{t-1}[r_{t-1} < \mu]) a_{t-1}^2 + \beta h_{t-1}^2, && (2.12) \end{aligned}$$

where r_t is the daily (monthly) asset return, μ is the mean return, and a_t represents the return innovations. $\mathrm{I}_{t-1}[r_{t-1} < \mu]$ is an indicator function taking the value of one if r_{t-1} is lower than μ and zero otherwise.

The return innovation series is assumed to be conditionally (on the time $t-1$ information set, $\Im_{t-1}$) normally distributed with mean zero and conditional covariance matrix H_t, which can be decomposed as shown below in equation (2.13). Once the univariate models are estimated, standardized residuals $\epsilon_{i,t} = a_{i,t}/\sqrt{h_{i,t}}$ (with $h_{i,t}$ being the respective variance element in H_t) can be used to estimate the correlation parameters (see Cappiello et al.

[11] See Black (1976), Christie (1982), and many others.

(2006)):

$$
\begin{aligned}
H_t &= D_t P_t D_t, && (2.13)\\
P_t &= Q_t^{*-1} Q_t Q_t^{*-1}, && (2.14)\\
Q_t &= (\bar{P} - A'\bar{P}A - B'\bar{P}B - G'\bar{N}G) + A'\epsilon_{t-1}\epsilon_{t-1}'A && (2.15)\\
&\quad + G'n_{t-1}n_{t-1}'G + B'Q_{t-1}B.
\end{aligned}
$$

D_t is a diagonal matrix containing the standard deviations of the individual assets. A, B, and G are $k \times k$ parameter matrices, $n_t = I[\epsilon_t < 0] \circ \epsilon_t$ is a $k \times 1$ indicator function where $\circ$ denotes the Hadamard product (element-wise multiplication). Q_t^* is a diagonal matrix containing the square roots of the respective diagonal elements of Q_t, ensuring that P_t is a valid correlation matrix.

We use the model in the bivariate case (i.e., $k = 2$) for each estimation including an asset-return series and that of the market index at a rolling estimation window of one year for daily returns, thereby computing an estimate for the respective beta in each month of our sample period.[12] We choose a rolling window instead of an expanding window to allow for structural changes to be incorporated more quickly. The estimation of all time-series models is conducted by maximum likelihood. Using the parameter estimates, we iteratively estimate the covariances and betas for all days until the end of the forecast horizon. The time t estimate is then obtained as the average beta over the forecast horizon.

For robustness, we also test the Constant Conditional Correlation Model (CCC) of Bollerslev (1990), neither imposing a dynamic structure on correlations (only on the volatilities) nor an asymmetric effect of return innovations, thereby leaving more degrees of freedom for the estimation process.

[12] For monthly estimators, we use the monthly returns of the past 60 months instead of the past one year of daily returns. The results on monthly estimators can be found in the appendix to this chapter.

Kalman Filter Models We also include approaches directly imposing a factor structure on beta and using the Kalman filter (see, e.g., Pagan, 1980 and Black, Fraser, & Power, 1992). As underlying dynamics, we consider a random walk (equation (2.16), RW), a random walk with drift (equation (2.17), $\mathrm{RW_D}$), an AR(1) (equation (2.18), AR), and an ARMA(1,1) (equation (2.19), ARMA) model. In all four cases, the standard CAPM security market line is taken as the measurement equation and the transition equation describes the chosen model for the dynamic evolution of beta in state-space form:

$$\beta_{j,t}^{\mathrm{RW}} = \beta_{j,t-1} + \epsilon_{j,t}, \tag{2.16}$$

$$\beta_{j,t}^{\mathrm{RW}_D} = \phi_0 + \beta_{j,t-1} + \epsilon_{j,t}, \tag{2.17}$$

$$\beta_{j,t}^{\mathrm{AR}(1)} = \phi_1 \beta_{j,t-1} + \epsilon_{j,t}, \tag{2.18}$$

$$\beta_{j,t}^{\mathrm{ARMA}(1,1)} = \phi_1 \beta_{j,t-1} + \epsilon_{j,t} + \theta_1 \epsilon_{j,t-1}. \tag{2.19}$$

We estimate the models analogous to those for dynamic conditional beta, and also use one year of daily returns.

A drawback when using the time-series models (both GARCH and Kalman) is that stability of the model structure has to be assumed. Ghysels (1998) shows that if the factor structure hypothesized is inherently misspecified, the errors made may even increase, compared to a static factor model, which might be the major concern regarding this class of estimators. Nevertheless, a superior performance of the time-series model estimators is certainly possible, if the true dynamics is approximated sufficiently well.

Option-Implied and Hybrid Betas Siegel (1995) points out that an implicit beta could be obtained directly through the use of exchange options, an option to exchange the shares of a firm for the shares of a market index. Unfortunately, however, these exchange options are currently not traded. Thus, one has to rely on some identifying assumption in order to obtain an implicit beta and thereby make use of the inherently forward-looking information that can be obtained from option prices. It should be taken into consideration, though, that the implied approaches yield estimates (at least partially) under the risk-neutral probability measure which is likely to differ from the actual physical probability measure if these sources of risk are priced in the market.[13]

We consider several possibilities for option-implied betas, making use of the model-free implied moments discussed above. These include the hybrid approach of French et al. (FGK) (1983), that directly combines historical correlations and option-implied volatilities, and that of Buss & Vilkov (2012), who use the property that the implied variance of the market index has to be the same as the implied variance of the value-weighted portfolio of all market constituents (first relation) and combine that with a technical condition for implied correlations to translate from physical ($\rho^{\mathbb{P}}_{ij,t}$) to risk-neutral correlations ($\rho^{\mathbb{Q}}_{ij,t}$), namely $\rho^{\mathbb{Q}}_{ij,t} = \rho^{\mathbb{P}}_{ij,t} - \alpha_t(1 - \rho^{\mathbb{P}}_{ij,t})$.[14] Combining the two relations and solving for α_t, implied correlations can be computed. Thus, a beta estimate under the risk-neutral probability measure, $\mathbb{Q}$, is obtained by:

$$\beta^{\mathbb{Q}}_{j,t} = \frac{\sigma^{\mathbb{Q}}_{j,t} \sum_{i=1}^{N}(\omega_{i,t}\sigma^{\mathbb{Q}}_{i,t}\rho^{\mathbb{Q}}_{ji,t})}{(\sigma^{\mathbb{Q}}_{M,t})^2}, \tag{2.20}$$

where $\sigma^{\mathbb{Q}}_{j,t}$ and $\sigma^{\mathbb{Q}}_{M,t}$ denote the implied volatilities obtained in equation (2.5) for individual stocks and the market index, respectively.[15] $\omega_{i,t}$ denotes the weight of the N individual assets in the market index at a certain point in time. One main disadvantage of this approach is the fact that it requires

[13]See, e.g., Carr & Wu (2009) and Driessen, Maenhout, & Vilkov (2009) for literature on the the price of volatility and correlation risk, respectively.

[14]Making sure both that the matrix is a correlation matrix (all correlations not exceeding one and the matrix being positive definite) and that it matches with empirical observations, namely that implied correlations are higher than empirical ones and that the correlation risk premium is higher for lowly correlated stocks. For more details, refer to Buss & Vilkov (2012).

[15]Hereafter, to avoid the notation getting to messy, we suppress the superscript Q for risk-neutral moments.

information on all the constituents of the index considered. The estimates are likely to be biased if implied volatilities are not available for all stocks of which the market index consists.

Additionally, we investigate the fully implied approach by Chang et al. (CCJV) (2012) that solely relies on options data. Their estimator is given by

$$\beta_{j,t}^{\mathrm{CCJV}} = \left(\frac{\mathrm{skew}_{j,t}}{\mathrm{skew}_{M,t}}\right)^{1/3}\left(\frac{\sigma_{j,t}}{\sigma_{M,t}}\right), \tag{2.21}$$

using the identifying assumption that the skewness of the idiosyncratic shock equals zero with $\mathrm{skew}_{j,t}$ and $\sigma_{j,t}$ denoting the implied skewness and volatility of individual stocks and with $j = M$ those of the market index, respectively.[16] As pointed out by Chang et al. (2012), the first part of equation (2.21) can be regarded as a correlation proxy.

Further fully implied estimators for beta also rely on certain assumptions on the return moments. A first approach makes use of the restriction made by Skintzi & Refenes (SR) (2005), who impose the assumption that the return correlation is identical for all stocks in the cross-section. This yields the estimator

$$\beta_{j,t}^{\mathrm{SR}} = \frac{\omega_{j,t}\sigma_{j,t} + \sum_{i\neq j}\omega_{i,t}\rho_t\sigma_{j,t}\sigma_{i,t}}{\sigma_{M,t}^2}. \tag{2.22}$$

Kempf, Korn, & Saßning (2015) propose two further possibilities that are related to the above approaches. The first approach assumes that the proportion of idiosyncratic variance is equal for all stocks in the cross-section (KKS1), resulting in

$$\beta_{j,t}^{\mathrm{KKS1}} = \frac{\sigma_{j,t}}{\sum_{i=1}^{N}\omega_{i,t}\sigma_{i,t}}. \tag{2.23}$$

[16] Note that equation (2.21) only yields an estimate if the individual stock's skewness is negative given the usually observed negative skewness of the market. This is an obvious shortcoming, especially for practical purposes.

Secondly, they propose a beta estimator imposing the restriction that the proportion of idiosyncratic kurtosis is identical for all stocks in the cross-section (KKS2). This yields:

$$\beta_{j,t}^{\text{KKS2}} = \frac{\text{kurt}_{US,j,t}^{1/4}}{\sum_{i=1}^{N} \omega_{i,t} \text{kurt}_{US,i,t}^{1/4}}, \tag{2.24}$$

where $\text{kurt}_{US,j,t}$ is the unscaled kurtosis as obtained in Section II.B, equation (2.7).[17]

Risk Premium Adjustment The option-implied approaches estimate beta under the risk-neutral probability measure. However, in most situations we are interested in beta under the physical probability measure. We therefore propose a new hybrid estimator for beta that employs forward-looking information from option prices and, at the same time, corrects for volatility risk premia. To obtain this estimator, we follow the procedure in Prokopczuk & Wese Simen (2014a) to implement an adjustment for the volatility risk premium. For that, we compute average variance risk premia for a period of just under two years:[18]

$$\text{ARVRP}_{j,t}^2 = \frac{1}{504 - \tau} \sum_{i=t-504}^{t-\tau} \frac{\sigma_{j,i,i+\tau}^2}{\text{RV}_{j,i,i+\tau}^2}. \tag{2.25}$$

$ARVRP_{j,t}^2$ denotes the average relative variance risk premium from $t - 504$ to $t - \tau$, $\sigma_{j,i,i+\tau}^2$ is the model-free implied variance of asset j at time i for the period until $i + \tau$ as obtained in equation (2.5), $RV_{j,i,i+\tau}^2$ is the realized variance over the time period ranging from i to $i + \tau$, and τ denotes the estimation horizon. To compute the risk premium adjustment we require at least one hundred non-missing observations of both $\sigma_{j,i,i+\tau}^2$ and the corresponding $RV_{j,i,i+\tau}^2$. We then obtain the risk premium adjusted

[17]Note that in equation (2.7) the kurtosis is scaled. To obtain the unscaled fourth moment one has to multiply equation (2.7) by the squared implied variance. The implied variance is provided in equation (2.5).

[18]For the risk premium adjustment for evaluation horizons of three months and less, we compute average variance risk premia for a period of just under one year.

implied volatility $RMFIV_{j,t,T}$ for each point in time t as:

$$\text{RMFIV}_{j,t} = \frac{\sigma_{j,t}}{\text{ARVRP}_{j,t}}. \tag{2.26}$$

The risk premium adjusted beta (RPadj), is then computed by using the historical correlation $\rho_{j,t}$ and risk premium adjusted implied volatilities for individual assets and the market index:

$$\beta_{j,t}^{\text{RPadj}} = \rho_{j,t} * \frac{\text{RMFIV}_{j,t}}{\text{RMFIV}_{M,t}}. \tag{2.27}$$

2.3 Empirical Results

2.3.1 Summary Statistics and Correlation Analysis

Panel A of Table 2.1 reports summary statistics on the different beta estimation techniques. It can be seen that the value-weighted average beta over all stocks in the S&P 500 (Mean_{vw}), a quantity which theoretically has to be equal to one, is substantially different from that value in some cases, suggesting that approaches which experience such deviations likely yield biased estimates. While the value-weighted average beta of RW is very close to one, those of RW_D, AR, ARMA, and the GARCH DCC and CCC are far off with values of 1.06, 0.98, 1.04, 0.93, and 1.04, respectively. Looking at approaches employing information from the options market, we find that the value-weighted averages of especially the hybrid FGK and the fully implied CCJV, as well as SR and RPadj also are clearly different from one, with values of 0.84, 1.15, 1.04, and 1.02, respectively. By construction, the quantity is exactly equal to one for BV, KKS1, and KKS2, while it is relatively close for HIST and HIST_6. The time-series model betas RW, RW_D, ARMA, and DCC are shown to vary strongly, with minimum values smaller than -8 and maximum values greater than 16, potentially inducing large errors for these extreme values. Furthermore, by construction, the fully

Table 2.1: Summary Statistics and Sample Correlations

This table provides summary statistics on the different beta estimation techniques in Panel A. All methods utilize (if necessary) daily return data and estimate beta for six months. The sample period spans from January 1996 (beginning with estimates for February 1996) until December 2012. Nobs denotes the number of monthly estimates, Mean and Mean_{vw} are the equal- and value-weighted averages of the estimates over the entire sample period, respectively. Std. dev., Median, Min, and Max present further summary statistics on the overall standard deviation, median, minimum, and maximum estimate, respectively. Panel B presents the sample correlation coefficients among the different beta estimation techniques on the basis of individual estimates.

Panel A. Summary Statistics

	Nobs	Mean	Mean_{vw}	Std. dev.	Median	Min	Max
RW	98,179	1.0036	1.0026	0.5720	0.9313	-29.076	19.833
RW_D	98,179	1.0713	1.0618	0.7650	0.9807	-36.212	24.796
AR	98,179	0.9662	0.9753	0.7770	0.8349	-2.0388	20.986
ARMA	98,179	1.0547	1.0369	1.0053	0.8724	-8.6868	48.673
DCC	98,176	0.9511	0.9304	0.8068	0.8648	-13.997	16.734
CCC	98,179	1.0731	1.0430	0.7009	0.9348	-0.6028	17.905
HIST	98,243	1.0022	1.0033	0.4680	0.9360	-0.6675	4.6485
HIST_6	98,630	1.0021	1.0015	0.5036	0.9344	-0.9818	7.7906
FGK	94,889	0.8473	0.8436	0.3927	0.7998	-0.8230	5.7060
RPadj	90,190	1.0321	1.0206	0.5208	0.9498	-0.8554	5.5901
BV	95,043	1.0427	1.0000	0.3756	0.9870	-0.4646	6.9280
CCJV	89,530	1.2211	1.1505	0.4706	1.1579	0.0220	6.2420
SR	95,755	1.1077	1.0391	0.3693	1.0297	0.1215	6.2270
KKS1	95,755	1.1074	1.0000	0.3733	1.0260	0.1237	6.2000
KKS2	95,755	1.1038	1.0000	0.3635	1.0258	0.1341	6.3583

Panel B. Sample Correlation Coefficients

RW	RW_D	AR	ARMA	DCC	CCC	HIST	HIST_6	FGK	RPadj	BV	CCJV	SR	KKS1	KKS2	
*	0.91	0.76	0.68	0.54	0.64	0.82	0.85	0.79	0.80	0.77	0.52	0.61	0.61	0.61	RW
	*	0.79	0.70	0.51	0.60	0.76	0.86	0.74	0.75	0.72	0.49	0.58	0.57	0.57	RW_D
		*	0.67	0.44	0.52	0.68	0.76	0.66	0.68	0.64	0.43	0.49	0.48	0.48	AR
			*	0.41	0.48	0.59	0.68	0.57	0.60	0.57	0.40	0.48	0.47	0.47	ARMA
				*	0.71	0.59	0.57	0.56	0.55	0.53	0.39	0.45	0.44	0.44	DCC
					*	0.71	0.68	0.66	0.66	0.64	0.48	0.56	0.56	0.56	CCC
						*	0.93	0.90	0.92	0.88	0.58	0.69	0.68	0.68	HIST
							*	0.87	0.88	0.84	0.57	0.67	0.66	0.66	HIST_6
								*	0.95	0.89	0.67	0.71	0.70	0.70	FGK
									*	0.90	0.64	0.74	0.73	0.73	RPadj
										*	0.70	0.88	0.88	0.87	BV
											*	0.77	0.77	0.77	CCJV
												*	1.00	0.99	SR
													*	1.00	KKS1
														*	KKS2

implied CCJV, SR, KKS1, and KKS2 cannot adopt negative values, casting some doubt on their performance.

Panel B of Table 2.1 presents the sample correlation coefficients among betas obtained with different estimation techniques on the basis of their estimates for individual assets. We note very high correlations greater than 0.9 among the fully implied estimates (namely KKS1, KKS2, and SR), FGK and the risk premium adjusted estimates (RPadj), HIST and HIST_6, as well as among HIST and RPadj. When comparing the remaining estimators, in many cases the correlations are only moderate or quite low. The smallest correlation among the estimates of the remaining approaches is observed between DCC and CCJV, amounting to only 0.39. This shows that the estimated values vary substantially across the different approaches, providing evidence for the need to study their performance further.

2.3.2 Information Content

A common way to evaluate the performance of ex ante estimates are Mincer & Zarnowitz (1969) regressions. We therefore regress the six-month (ex post) realized beta on the different (ex ante) beta estimates in the following way:

$$\beta^{\mathrm{R}}_{t,T} = a + b\zeta_{t,T} + \epsilon_t. \tag{2.28}$$

$\beta^{R}_{t,T}$ denotes the realized beta in the period ranging from t to T and $\zeta_{t,T}$ stands for one beta estimate in univariate regressions or a vector of several beta estimates in encompassing regressions. With the approach in equation (2.28) we can test for the informational efficiency and unbiasedness of the respective estimates.[19] As Hansen & Lunde (2006) show, using logarithmically transformed variables for the regressions, while making the

[19]While the value-weighted average betas we examine in Section III.A indicate that some approaches are biased on average, with the portfolio approach we employ here, we can test for unbiasedness on a rather individual level.

regression procedure less sensitive to outliers (Pagan & Schwert, 1990), often leads to inconsistent rankings of the estimation models if an unbiased but imperfect proxy for the true evaluation variable is used. They further show that level Mincer–Zarnowitz regressions are robust to (mean zero) errors in the proxy. Consequently, we stick to levels instead of logs to obtain results that are more robust.

Unbiasedness is tested in univariate regressions by performing a Wald test, imposing the joint hypothesis of a being equal to zero and b being equal to one. For an unbiased model we should not be able to reject the underlying hypothesis. Informational efficiency can be tested in encompassing regressions by constraining the slope parameters of alternative estimators to zero, thereby determining if the respective approaches contain information beyond that of a baseline model. If, in encompassing regressions, one estimator is to be more informative it must have a significant slope estimate and the explanatory power must rise compared to the restricted model. Additionally, we test the joint hypothesis of one slope parameter being equal to one and the second slope parameter being equal to zero. The underlying hypothesis of this test states that one approach fully subsumes all information contained in the other approach it is tested with.

To conduct our analysis we follow the approach suggested by Fama & MacBeth (1973). At the end of each month, we form five value-weighted portfolios out of the individual stocks in our sample. We sort the stocks according to their estimate for historical beta (of equation (2.9)) obtained in an estimation period (sorting period) strictly before the estimation period of the historical beta serving as one beta estimate in an ascending order and compute estimates as well as realizations for beta for each of these portfolios.[20] This approach ensures that we obtain a certain range in the estimated values and delivers results that are comparable. At the same time, this avoids a bias in our analyses related to a potential errors-in-the-variables problem. To keep the analysis comparable, we can only include those estimates in our sample where all approaches yield an estimate.[21]

To keep the presentation manageable, we select at least one approach from each model family to perform our main analysis. We select Historical (HIST), the Kalman filter random walk (RW), DCC GARCH (DCC), the hybrid FGK and BV, the fully implied CCJV and KKS1, and the risk premium adjusted (RPadj) approach, and consider the remaining methods in the robustness analysis in Section IV. In all analyses, we evaluate the approaches using realized beta during the subsequent six months. Table 2.2 presents the regression results for daily estimation approaches.[22]

Panel A of Table 2.2 presents the results of the univariate regressions for each estimation approach and each of the five portfolios. It can be seen that in most cases the intercept estimate is significantly (at 5 %)[23] different from zero and the estimate for the slope coefficient is significantly different from one.[24] Only the approaches HIST, RW, KKS1, and BV yield non-significant

[20]For example, using daily data and estimating beta at the end of January 1996, evaluating it in the period February – July 1996, the estimation of historical beta uses return data from February 1995 until the end of January 1996. The portfolio sorting is carried out according to the estimate for historical beta using return data between February 1994 and the end of January 1995. If historical return data is not available, the quantity is set to one. The procedure for monthly analysis is performed accordingly, starting the first sorting period in February 1986.

[21]The major cause of reduction results from the impossibility of computing the CCJV beta in some cases, as pointed out in Section II.C and the general unavailability of sufficient options and return data (see Section III.A).

[22]A further analysis on the approaches using monthly return data can be found in the appendix to this chapter.

[23]Further mentions of (non-) significance will always refer to the five percent significance level.

[24]Note that for univariate regressions the t-statistics of the slope coefficients test the hypothesis of those being equal to one and not, as is usually done, equal to zero. In the multivariate regressions, the t-statistics refer to the usual hypothesis that the parameters are equal to zero.

Table 2.2: Univariate and Encompassing Estimates for Realized Beta – Daily Data

This table presents the results from regressions of ex post realized beta over the horizon of six months on competing ex ante estimates for each of the five portfolios, respectively. Each row corresponds to one regression. Each month, we sort our estimates in an ascending order according to the historical (Equation (2.9)) beta obtained in a sorting period strictly before the estimation period of the historical beta. The sorting and estimation periods are without overlap and have equal length. The stocks are then allocated to quintile portfolios. Portfolio betas and returns are value-weighted. a denotes the regression intercept, b_{HIST}, b_{RW}, b_{DCC}, b_{FGK}, b_{CCJV}, b_{KKS1}, b_{BV}, and b_{RPadj} refer to the slope coefficients for the historical, random walk, DCC, FGK, CCJV, KKS1, BV, and risk premium adjusted estimate for beta, respectively. For each regression coefficient we report Newey & West (1987) t-statistics ($t(.)$) using 6 lags, where we test a against zero and b against one in the univariate regressions (Panel A), while we test both against zero in the multivariate regressions (Panel B). R^2_{adj} denotes the adjusted R^2 of the regressions. The columns $Wald_1$ and $Wald_2$ refer to the Wald test statistic and p gives the corresponding p-values. In the univariate regressions in Panel A, for the Wald test (reported in $Wald_1$) we test the joint hypothesis of the intercept being equal to zero and the slope coefficient being equal to one. In the multivariate regressions in Panel B, the joint underlying hypothesis is that the first slope coefficient is equal to one and the second slope coefficient is equal to zero and vice versa for $Wald_1$ and $Wald_2$, respectively. t-statistics and p-values in **bold** font indicate significance at the 5 % level. The sample period is January 1996 until December 2012 with 198 (188 when RPadj is included) monthly observations.

Panel A. Univariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
HIST	1	0.23	(1.50)	0.67	(-1.49)															0.32	14.37	**(0.00)**
	2	0.19	(1.53)	0.80	(-1.39)															0.57	17.57	**(0.00)**
	3	0.14	(0.85)	0.85	(-0.90)															0.55	3.93	**(0.02)**
	4	0.01	(0.06)	0.97	(-0.24)															0.71	7.58	**(0.00)**
	5	0.69	**(3.58)**	0.49	**(-3.74)**															0.21	34.13	**(0.00)**
RW	1	0.19	(1.67)			0.73	(-1.66)													0.43	12.65	**(0.00)**
	2	0.21	**(2.35)**			0.77	**(-2.33)**													0.63	20.10	**(0.00)**
	3	0.21	(1.79)			0.78	(-1.93)													0.57	11.03	**(0.00)**
	4	0.21	**(2.61)**			0.80	**(-2.83)**													0.67	14.34	**(0.00)**
	5	0.54	**(3.81)**			0.60	**(-3.94)**													0.37	32.13	**(0.00)**
DCC	1	0.45	**(7.28)**					0.33	**(-7.28)**											0.18	113.20	**(0.00)**
	2	0.56	**(7.30)**					0.38	**(-6.84)**											0.34	185.88	**(0.00)**
	3	0.74	**(9.40)**					0.22	**(-9.91)**											0.16	263.39	**(0.00)**
	4	0.90	**(11.31)**					0.19	**(-10.73)**											0.13	295.27	**(0.00)**
	5	1.12	**(12.61)**					0.21	**(-14.07)**											0.10	159.85	**(0.00)**

Table 2.2: Univariate and Encompassing Estimates for Realized Beta – Daily Data (continued)

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
FGK	1	0.38	**(3.22)**							0.49	**(-2.66)**									0.14	58.54	**(0.00)**
	2	0.40	**(4.29)**							0.63	**(-3.08)**									0.40	142.91	**(0.00)**
	3	0.46	**(4.60)**							0.60	**(-3.66)**									0.44	207.42	**(0.00)**
	4	0.50	**(7.53)**							0.64	**(-5.04)**									0.58	378.24	**(0.00)**
	5	0.58	**(4.87)**							0.72	**(-2.49)**									0.46	208.34	**(0.00)**
CCJV	1	0.36	**(3.67)**									0.30	**(-6.92)**							0.09	351.04	**(0.00)**
	2	0.60	**(8.61)**									0.24	**(-11.66)**							0.07	229.19	**(0.00)**
	3	0.70	**(9.08)**									0.21	**(-11.98)**							0.08	259.32	**(0.00)**
	4	0.69	**(12.42)**									0.33	**(-13.31)**							0.29	232.77	**(0.00)**
	5	0.81	**(8.85)**									0.42	**(-7.35)**							0.30	82.12	**(0.00)**
KKS1	1	-0.61	(-1.64)											1.56	(1.23)					0.25	128.08	**(0.00)**
	2	-1.22	**(-3.86)**											2.27	**(3.65)**					0.37	44.23	**(0.00)**
	3	-0.77	**(-1.99)**											1.78	**(1.99)**					0.49	25.23	**(0.00)**
	4	-0.34	**(-2.64)**											1.38	**(3.21)**					0.70	65.79	**(0.00)**
	5	0.19	(0.61)											0.97	(-0.14)					0.23	46.67	**(0.00)**
BV	1	-0.16	(-0.84)													1.13	(0.49)			0.50	37.11	**(0.00)**
	2	-0.32	**(-2.50)**													1.34	**(2.42)**			0.75	26.82	**(0.00)**
	3	-0.30	**(-2.63)**													1.30	**(2.57)**			0.80	27.26	**(0.00)**
	4	-0.33	**(-3.58)**													1.33	**(3.90)**			0.80	36.75	**(0.00)**
	5	-0.05	(-0.19)													1.10	(0.49)			0.42	21.69	**(0.00)**
RPadj	1	0.40	**(4.45)**															0.38	**(-4.61)**	0.11	32.35	**(0.00)**
	2	0.43	**(5.37)**															0.50	**(-5.15)**	0.30	42.15	**(0.00)**
	3	0.52	**(5.88)**															0.43	**(-6.35)**	0.22	47.04	**(0.00)**
	4	0.55	**(4.33)**															0.49	**(-4.25)**	0.32	54.87	**(0.00)**
	5	0.46	**(4.06)**															0.65	**(-4.24)**	0.57	49.63	**(0.00)**

Panel B. Multivariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
HIST + RW	1	0.19	(1.64)	-0.06	(-0.27)	0.78	**(3.60)**													0.43	32.90	**(0.00)**	10.20	**(0.00)**
	2	0.19	(1.95)	0.16	(1.10)	0.63	**(4.24)**													0.63	24.98	**(0.00)**	16.44	**(0.00)**
	3	0.14	(1.06)	0.38	**(2.08)**	0.47	**(2.47)**													0.59	13.15	**(0.00)**	16.15	**(0.00)**
	4	0.04	(0.31)	0.65	**(3.40)**	0.30	**(2.58)**													0.72	5.86	**(0.00)**	34.75	**(0.00)**
	5	0.52	**(4.01)**	0.04	(0.37)	0.57	**(3.72)**													0.37	59.62	**(0.00)**	26.02	**(0.00)**
HIST + DCC	1	0.23	(1.45)	0.68	**(2.54)**			-0.01	(-0.09)											0.31	11.24	**(0.00)**	125.00	**(0.00)**
	2	0.20	(1.59)	0.78	**(5.34)**			0.02	(0.30)											0.57	8.16	**(0.00)**	268.73	**(0.00)**
	3	0.13	(0.80)	0.88	**(4.78)**			-0.03	(-0.62)											0.55	4.31	**(0.01)**	539.63	**(0.00)**
	4	-0.02	(-0.15)	1.07	**(9.25)**			-0.08	(-1.82)											0.72	5.50	**(0.00)**	1093.13	**(0.00)**
	5	0.70	**(3.56)**	0.46	**(2.56)**			0.03	(0.38)											0.21	27.93	**(0.00)**	193.53	**(0.00)**
HIST + FGK	1	0.25	(1.62)	0.86	**(3.92)**					-0.26	(-1.53)									0.33	13.59	**(0.00)**	51.15	**(0.00)**
	2	0.19	(1.53)	0.78	**(3.93)**					0.02	(0.21)									0.57	8.11	**(0.00)**	72.45	**(0.00)**
	3	0.17	(0.99)	0.65	**(2.97)**					0.20	**(2.08)**									0.57	8.42	**(0.00)**	76.34	**(0.00)**
	4	0.07	(0.58)	0.74	**(5.12)**					0.21	**(2.78)**									0.73	9.26	**(0.00)**	125.49	**(0.00)**
	5	0.61	**(5.41)**	-0.05	(-0.41)					0.75	**(4.32)**									0.46	85.67	**(0.00)**	13.22	**(0.00)**
HIST + CCJV	1	0.00	(0.03)	0.64	**(3.08)**							0.26	**(3.01)**							0.38	23.50	**(0.00)**	126.26	**(0.00)**
	2	0.06	(0.54)	0.78	**(5.75)**							0.15	**(2.98)**							0.60	15.68	**(0.00)**	321.11	**(0.00)**
	3	-0.05	(-0.34)	0.83	**(5.67)**							0.19	**(3.73)**							0.62	21.10	**(0.00)**	421.97	**(0.00)**
	4	-0.04	(-0.40)	0.87	**(8.45)**							0.14	**(3.26)**							0.75	16.57	**(0.00)**	633.55	**(0.00)**
	5	0.60	**(5.35)**	0.24	**(2.31)**							0.32	**(3.28)**							0.33	51.34	**(0.00)**	90.18	**(0.00)**

Table 2.2: Univariate and Encompassing Estimates for Realized Beta – Daily Data (continued 2)

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
HIST + KKS1	1	-0.45	(-1.44)	0.50	**(2.68)**									0.97	**(2.72)**					0.39	25.54	**(0.00)**	28.66	**(0.00)**
	2	-0.15	(-0.35)	0.71	**(2.92)**									0.46	(0.70)					0.58	9.89	**(0.00)**	77.09	**(0.00)**
	3	-0.43	(-1.40)	0.57	**(4.28)**									0.87	**(2.62)**					0.61	18.94	**(0.00)**	54.13	**(0.00)**
	4	-0.24	(-1.80)	0.55	**(3.23)**									0.70	**(3.03)**					0.75	16.80	**(0.00)**	41.74	**(0.00)**
	5	0.18	(0.59)	0.29	(1.54)									0.64	(1.65)					0.27	38.56	**(0.00)**	6.08	**(0.00)**
HIST + BV	1	-0.17	(-0.83)	-0.04	(-0.19)											1.17	**(2.84)**			0.50	51.16	**(0.00)**	1.33	(0.27)
	2	-0.37	**(-2.65)**	-0.16	(-1.08)											1.56	**(5.56)**			0.76	87.05	**(0.00)**	21.20	**(0.00)**
	3	-0.32	**(-3.03)**	-0.16	(-1.20)											1.48	**(6.67)**			0.80	131.87	**(0.00)**	23.80	**(0.00)**
	4	-0.31	**(-3.41)**	0.12	(0.74)											1.19	**(6.19)**			0.80	47.02	**(0.00)**	25.40	**(0.00)**
	5	-0.08	(-0.28)	-0.14	(-1.10)											1.28	**(3.90)**			0.42	74.66	**(0.00)**	1.74	(0.18)
HIST + Rpadj	1	0.27	**(2.23)**	1.51	**(3.27)**													-0.91	**(-2.22)**	0.42	34.61	**(0.00)**	96.30	**(0.00)**
	2	0.20	(1.79)	1.13	**(3.32)**													-0.33	(-1.23)	0.61	21.32	**(0.00)**	150.32	**(0.00)**
	3	0.15	(0.98)	1.12	**(3.50)**													-0.27	(-1.21)	0.59	12.75	**(0.00)**	170.66	**(0.00)**
	4	0.00	(-0.00)	0.94	**(5.41)**													0.04	(0.34)	0.70	1.74	(0.18)	233.18	**(0.00)**
	5	0.52	**(5.26)**	-0.08	(-0.71)													0.69	**(5.55)**	0.57	133.89	**(0.00)**	37.86	**(0.00)**
RW + BV	1	-0.11	(-0.48)			0.27	(1.41)									0.82	**(1.97)**			0.52	30.16	**(0.00)**	5.26	**(0.00)**
	2	-0.28	(-1.64)			0.09	(0.70)									1.21	**(4.08)**			0.75	72.44	**(0.00)**	20.00	**(0.00)**
	3	-0.33	**(-2.67)**			-0.09	(-0.96)									1.42	**(7.07)**			0.80	134.95	**(0.00)**	22.18	**(0.00)**
	4	-0.30	**(-2.86)**			0.08	(1.07)									1.22	**(8.10)**			0.80	87.04	**(0.00)**	25.15	**(0.00)**
	5	0.05	(0.19)			0.25	**(2.09)**									0.76	**(2.84)**			0.44	41.63	**(0.00)**	4.52	**(0.01)**
DCC + BV	1	-0.15	(-0.74)					0.05	(0.57)							1.07	**(3.47)**			0.50	208.56	**(0.00)**	1.94	(0.15)
	2	-0.30	**(-2.21)**					0.02	(0.42)							1.31	**(7.56)**			0.75	532.63	**(0.00)**	19.43	**(0.00)**
	3	-0.30	**(-2.48)**					0.02	(0.53)							1.28	**(9.45)**			0.80	1311.92	**(0.00)**	21.61	**(0.00)**
	4	-0.35	**(-3.95)**					-0.04	(-1.16)							1.39	**(15.99)**			0.81	1602.58	**(0.00)**	27.57	**(0.00)**
	5	-0.09	(-0.34)					-0.07	(-0.95)							1.20	**(4.78)**			0.42	301.63	**(0.00)**	1.66	(0.19)
FGK + BV	1	-0.16	(-0.84)							0.01	(0.08)					1.12	**(3.90)**			0.50	101.26	**(0.00)**	1.25	(0.29)
	2	-0.33	**(-2.35)**							-0.03	(-0.29)					1.37	**(6.55)**			0.75	194.72	**(0.00)**	19.30	**(0.00)**
	3	-0.31	**(-2.32)**							-0.02	(-0.22)					1.33	**(6.49)**			0.80	270.92	**(0.00)**	21.22	**(0.00)**
	4	-0.36	**(-3.28)**							-0.04	(-0.34)					1.38	**(8.06)**			0.80	202.66	**(0.00)**	24.70	**(0.00)**
	5	0.37	(1.35)							0.54	**(2.47)**					0.31	(0.89)			0.46	14.22	**(0.00)**	9.05	**(0.00)**
FGK + RPadj	1	0.37	**(3.60)**							0.53	(1.03)							-0.03	(-0.07)	0.15	24.60	**(0.00)**	37.49	**(0.00)**
	2	0.38	**(5.57)**							0.70	(1.89)							-0.05	(-0.15)	0.44	33.77	**(0.00)**	74.89	**(0.00)**
	3	0.48	**(6.81)**							0.80	**(3.13)**							-0.21	(-1.02)	0.50	49.57	**(0.00)**	123.95	**(0.00)**
	4	0.48	**(5.23)**							0.73	**(3.97)**							-0.06	(-0.29)	0.58	46.90	**(0.00)**	138.12	**(0.00)**
	5	0.45	**(3.92)**							0.08	(0.30)							0.59	**(2.69)**	0.57	43.59	**(0.00)**	37.15	**(0.00)**
CCJV + BV	1	-0.49	**(-2.75)**									0.34	**(4.03)**			1.15	**(4.88)**			0.61	261.08	**(0.00)**	31.41	**(0.00)**
	2	-0.45	**(-3.58)**									0.15	**(3.43)**			1.31	**(9.48)**			0.78	665.00	**(0.00)**	35.11	**(0.00)**
	3	-0.38	**(-3.23)**									0.10	**(2.27)**			1.27	**(10.65)**			0.82	985.15	**(0.00)**	33.22	**(0.00)**
	4	-0.32	**(-3.46)**									0.04	(0.94)			1.27	**(13.67)**			0.80	836.43	**(0.00)**	26.43	**(0.00)**
	5	-0.01	(-0.03)									0.04	(0.34)			1.03	**(3.77)**			0.42	117.32	**(0.00)**	0.70	(0.50)
KKS1 + BV	1	-0.25	(-1.04)											0.16	(0.45)	1.07	**(3.45)**			0.50	55.23	**(0.00)**	1.52	(0.22)
	2	-0.26	(-1.15)											-0.08	(-0.19)	1.36	**(5.95)**			0.75	197.56	**(0.00)**	19.23	**(0.00)**
	3	-0.06	(-0.38)											-0.50	(-1.75)	1.55	**(7.91)**			0.81	211.12	**(0.00)**	27.83	**(0.00)**
	4	-0.33	**(-3.48)**											0.01	(0.03)	1.32	**(6.00)**			0.80	77.14	**(0.00)**	24.40	**(0.00)**
	5	0.02	(0.07)											-0.17	(-0.29)	1.21	**(2.40)**			0.42	32.64	**(0.00)**	1.05	(0.35)
BV + Rpadj	1	-0.15	(-0.86)													1.24	**(3.95)**	-0.15	(-0.95)	0.50	5.74	**(0.00)**	127.01	**(0.00)**
	2	-0.34	**(-2.92)**													1.45	**(7.65)**	-0.10	(-1.00)	0.76	26.68	**(0.00)**	302.79	**(0.00)**
	3	-0.31	**(-3.80)**													1.44	**(11.47)**	-0.13	(-1.84)	0.81	29.86	**(0.00)**	486.00	**(0.00)**
	4	-0.37	**(-3.91)**													1.37	**(12.78)**	-0.01	(-0.13)	0.80	28.66	**(0.00)**	399.84	**(0.00)**
	5	0.40	(1.68)													0.09	(0.26)	0.61	**(3.26)**	0.57	37.18	**(0.00)**	37.04	**(0.00)**

values for some portfolios. The joint hypothesis of a being equal to zero and b being equal to one is rejected in any case, suggesting that the approaches yield biased estimates. HIST and RW obtain the lowest values for the Wald test, but still the null hypothesis of unbiasedness is strongly rejected. For each portfolio except the last, BV yields the highest adjusted R^2 followed by RW and HIST, indicating that those three approaches exhibit the highest explanatory power. RPadj has the highest explanatory power for portfolio five. Noteworthy are also the particularly poor performances of DCC and CCJV, yielding very high values for the Wald statistic and obtaining values for the adjusted R^2 that are below 0.35 for all portfolios.

Looking at the results of the encompassing regressions in Panel B of Table 2.2 we find that DCC obtains a rather poor performance with HIST and BV being informationally more efficient for all portfolios, meaning that in bivariate regressions the coefficients for the latter are significantly different from zero whereas those of DCC are not. The BV approach yields a significant slope parameter in every case and is informationally more efficient compared to most other beta estimation approaches. Whenever the slope parameters of other methods competing with BV are significantly different from zero, they are economically not very large. Only FGK and RPadj do yield statistically significant and economically large estimates for one portfolio in a joint encompassing regression together with BV. The explanatory power increases in every case when adding the BV beta to all other models. The hypothesis that one approach subsumes all information contained in another approach (indicated by the tests $Wald_1$ and $Wald_2$) is rejected in most cases. There are some cases, though, where the hypothesis that BV subsumes all information contained in, e.g., HIST, DCC, or KKS1 cannot be rejected. Combining HIST and the fully implied KKS1, the latter is shown to contain some additional information, making a significant contribution in three out of five cases and, at least slightly, increasing the explanatory power. Comparing FGK and its risk premium adjusted counterpart, RPadj, the results are not clear. FGK is favored for two portfolios and RPadj is favored for one, so it remains unclear if the risk premium adjustment yields an improvement.[25] In addition, looking at the univariate regressions, further analysis shows that the intercept estimates for FGK and RPadj do not differ significantly. This also conflicts with the possible view that an uniform bias may consistently be removed.

In contrast, comparing our risk premium adjusted beta estimator with the BV approach, the latter performs clearly better. While our approach corrects implied volatilities for the well-established variance risk premium and therefore obtains an estimate under the physical probability measure, the BV approach corrects for the risk premium at the level of correlations in the opposite direction and obtains an estimate under the risk-neutral probability measure. Given that realizations under the real world probability measure are of interest, this is somewhat surprising. A potential explanation is offered by Chang et al. (2012), who show that for certain parameter constellations the bias caused by the use of risk-neutral moments can be quite small. In other words, in the case of beta, there are biased moments in both the numerator and the denominator and the two effects may cancel out.

Overall, our results on estimators based on daily return data suggest that Buss & Vilkov (2012) approach yields informationally most efficient though not entirely unbiased estimates. Furthermore, the random walk approach, the simple historical estimator, and KKS1 are shown to possess some informational efficiency, when comparing them to the remaining approaches.

[25]However, one has to keep in mind that the regression may not be too informative; given the very high correlations between the two, serious problems related to multicollinearity arise.

2.3.3 Estimation Accuracy

Turning the focus on out-of-sample estimation accuracy, we employ the loss function most commonly applied in the literature, namely root mean squared errors (RMSE) to evaluate the performance of the different beta estimation techniques:

$$\text{RMSE} \;=\; \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\beta^{\text{R}}_{t,T} - \zeta_{t,T})^2}. \tag{2.29}$$

Here, n is the number of estimation windows, $\beta^{R}_{t,T}$ again denotes the realized beta over a period from t until T, and $\zeta_{t,T}$ is the respective beta estimate. Patton (2011) shows that only the mean squared errors (MSE) criterion, as opposed to other commonly used loss functions like mean average errors (MAE), mean average percentage errors (MAPE), and mean squared percentage errors (MSPE), is robust to the presence of (mean zero) noise in the evaluation proxy, so we choose this loss function.[26]

Table 2.3 summarizes the estimation errors using daily return data. We observe that BV yields the smallest average RMSE over the five portfolios (as indicated by *italic* font), followed by RW and HIST. The fully implied CCJV and the GARCH DCC achieve the worst and second-worst performance, respectively. Adjusting for the volatility risk premium clearly reduces the average estimation error (comparing RPadj to FGK). Overall, both FGK and RPadj, as well as KKS1, can be found in the mid-range regarding the estimation accuracy.

To further examine the results, we analyze whether the differences we observe are statistically significant. The remainder of Table 2.3 presents the average differences in root mean squared errors in the upper triangular

[26]We present the results on other loss functions, including MAE, MAPE, and MSPE in the appendix to this chapter.

Table 2.3: Estimation Errors: Six-Month Horizon – Daily Data

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. The first row reports the average root mean squared errors (RMSE) of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in brackets indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
avg.	0.1381	0.1301	0.2676	0.2203	0.2783	0.1704	*0.1164*	0.1718
HIST		0.0079	-0.1295	-0.0822	-0.1402	-0.0323	0.0217	-0.0337
		(0.0)	**(-1.0)**	(-0.8)	(-0.8)	(-0.4)	(0.2)	(-0.6)
RW	0.0002		-0.1374	-0.0901	-0.1482	-0.0403	0.0138	-0.0416
	(-0.4)		**(-1.0)**	**(-1.0)**	(-0.8)	(-0.2)	(0.2)	(-0.6)
DCC	0.0639	0.0637		0.0473	-0.0108	0.0972	0.1512	0.0958
	(1.0)	**(1.0)**		(0.0)	(0.0)	(0.6)	**(1.0)**	(0.8)
FGK	0.1001	0.0999	0.0362		-0.0581	0.0499	0.1039	0.0485
	(1.0)	**(1.0)**	(0.2)		(-0.6)	(0.6)	**(1.0)**	(0.2)
CCJV	0.1143	0.1141	0.0504	0.0142		0.1079	0.1620	0.1066
	(1.0)	**(1.0)**	(0.6)	(0.4)		(0.8)	**(1.0)**	**(1.0)**
KKS1	0.0113	0.0110	-0.0526	-0.0888	-0.1031		0.0541	-0.0013
	(0.4)	(0.6)	(-0.8)	(-0.8)	**(-1.0)**		(0.8)	(-0.2)
BV	-0.0111	-0.0113	-0.0750	-0.1112	-0.1254	-0.0224		-0.0554
	(-0.6)	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.6)
RPadj	0.0256	0.0254	-0.0383	-0.0745	-0.0888	0.0143	0.0367	
	(0.8)	(0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	(0.2)	**(1.0)**	

matrix and the respective median differences in the lower triangular matrix. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is statistically significant (e.g., 0.4 indicates that the differences for two out of five portfolios are significant). If the differences are significant for all five portfolios, the figure is printed in **b**old font. Significance is tested by the modified Diebold–Mariano (Harvey, Leybourne, & Newbold, 1997) and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

We find that BV always obtains lower average root mean and median squared errors than the other methods. These differences are statistically significant for all portfolios compared to DCC, FGK, and CCJV, and at least three portfolios compared to KKS1 and RPadj, whereas when comparing to HIST and RW, the RMSE of BV is significantly lower for one portfolio and the root median SE is significantly lower for three portfolios. HIST and RW yield significantly lower estimation errors than all other methods, except BV and KKS1, for at least three out of the five portfolios and than KKS1 for at least one portfolio. Overall, the evidence indicates that the BV approach obtains the best out-of-sample accuracy, followed by RW and HIST.

2.4 Robustness

2.4.1 More Portfolios

We test whether the results obtained so far are robust to building more portfolios. Thus, we build 10, 25, and 50 portfolios and in the limit we also consider the case of individual stocks. Table 2.4 reports the results, which are quite similar to our previous findings. We observe that the average errors in general increase with the number of portfolios. Independently of the number of portfolios, BV always obtains the lowest average RMSE, yielding the lowest error for a minimum of 60 percent when building portfolios. In the case of estimates for individual stocks, BV also obtains the lowest average RMSE, though not much can be stated as each approach has its share where it yields the lowest errors, indicating that all approaches work well for some stocks. Overall, the BV, HIST, and RW approaches also perform best when increasing the number of portfolios.

2.4.2 Different Horizons

To further examine the robustness of our results we perform the evaluation using different time horizons, namely one, three, and twelve months. We estimate the values for option-implied methods using options with approximately one (three, twelve) months to maturity and adjust the horizon for time-series models to the respective time frame, evaluating all the methods using realized beta over the subsequent one, three, and twelve months, respectively.

Panel A of Table 2.5 reports the estimation errors of our main methods and their significance for the one-month evaluation period. We find that using this evaluation horizon yields the same result with BV, HIST, and RW being the approaches with the best out-of-sample estimation accuracy. BV obtains the lowest average RMSE. Comparing the mean and median differences of the estimation errors, only in relation to KKS1, HIST and RW do not yield a significantly lower error at least for eighty percent of the portfolios, while BV always does. For some portfolios, BV yields significantly lower median errors compared to HIST and RW. The results for three and twelve months in Panels B and C are qualitatively equal. BV always obtains the lowest average RMSE, with significantly lower errors in many cases

Table 2.4: Estimation Errors: More Portfolios

This table reports the root mean squared (RMSE) errors of the competing estimators, using daily return data, for realized beta over the horizon of six months, for different counts of portfolios. Each month, we form N portfolios with N amounting to 5, 10, 25, 50, and in the limit we also consider the case of solely individual assets (in this case we compute the values of the loss functions for each asset in every month of our sample period individually and average over all errors). The stocks are allocated into N portfolios in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). The numbers in parentheses denote the count (as proportions) of portfolio series for which a certain approach yields the lowest error among those presented in the table. For each loss function, the lowest average errors among all approaches are indicated by *italic* font.

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
5 Portfolios								
avg. RMSE	0.1381	0.1301	0.2676	0.2203	0.2783	0.1704	*0.1164*	0.1718
	(0.00)	(0.20)	(0.00)	(0.00)	(0.00)	(0.00)	(0.60)	(0.20)
10 Portfolios								
avg. RMSE	0.1518	0.1474	0.2868	0.2273	0.2901	0.1854	*0.1304*	0.1837
	(0.00)	(0.20)	(0.00)	(0.00)	(0.00)	(0.00)	(0.80)	(0.00)
25 Portfolios								
avg. RMSE	0.1735	0.1787	0.3260	0.2397	0.3099	0.2075	*0.1540*	0.1998
	(0.00)	(0.12)	(0.00)	(0.00)	(0.00)	(0.00)	(0.84)	(0.04)
50 Portfolios								
avg. RMSE	0.1975	0.2131	0.3683	0.2558	0.3323	0.2340	*0.1808*	0.2198
	(0.10)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.80)	(0.08)
individual assets								
avg. RMSE	0.5374	0.5866	0.8309	0.5592	0.6616	0.5826	*0.5363*	0.5379
	(0.09)	(0.12)	(0.13)	(0.17)	(0.15)	(0.13)	(0.08)	(0.11)

Table 2.5: Estimation Errors: Different Horizons – Daily Data

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over horizons of one (Panel A), three (Panel B), and twelve (Panel C) months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. In each Panel, the first row reports the average root mean squared errors (RMSE) of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the panels report the difference in estimation errors. The upper triangular matrices report the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrices report the average root median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

<u>*Panel A. One Month*</u>

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
avg.	0.1637	0.1515	0.2231	0.2193	0.4285	0.2051	*0.1483*	0.1977
HIST		0.0122	-0.0594	-0.0556	-0.2648	-0.0414	0.0153	-0.0340
		(0.0)	(-0.8)	(-0.8)	(-0.8)	(-0.2)	(0.0)	(-0.8)
RW	-0.0022		-0.0717	-0.0678	-0.2770	-0.0536	0.0031	-0.0462
	(-0.2)		**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.4)	(0.0)	(-0.8)
DCC	0.0375	0.0397		0.0038	-0.2053	0.0181	0.0748	0.0254
	(1.0)	**(1.0)**		(0.0)	(-0.8)	(0.4)	**(1.0)**	(0.0)
FGK	0.0525	0.0547	0.0150		-0.2092	0.0142	0.0709	0.0216
	(1.0)	**(1.0)**	(0.0)		(-0.8)	(0.4)	**(1.0)**	(0.0)
CCJV	0.1550	0.1572	0.1175	0.1025		0.2234	0.2801	0.2308
	(1.0)	**(1.0)**	**(1.0)**	(0.8)		(0.8)	**(1.0)**	(0.8)
KKS1	0.0189	0.0211	-0.0186	-0.0336	-0.1361		0.0567	0.0074
	(0.4)	(0.6)	(-0.4)	(-0.4)	**(-1.0)**		(0.6)	(0.0)
BV	-0.0041	-0.0020	-0.0417	-0.0567	-0.1592	-0.0231		-0.0493
	(-0.4)	(-0.2)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.8)
RPadj	0.0340	0.0362	-0.0035	-0.0185	-0.1210	0.0151	0.0382	
	(0.8)	(0.8)	(0.0)	(-0.6)	**(-1.0)**	(0.4)	(0.8)	

Table 2.5: Estimation Errors: Different Horizons – Daily Data (continued)

Panel B. Three Months

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
avg.	0.1380	0.1281	0.2259	0.2057	0.3335	0.1771	*0.1191*	0.1791
HIST		0.0100	-0.0879	-0.0676	-0.1955	-0.0391	0.0189	-0.0411
		(0.0)	**(-1.0)**	(-0.8)	(-0.8)	(-0.2)	(0.0)	(-0.8)
RW	-0.0031		-0.0979	-0.0776	-0.2054	-0.0491	0.0089	-0.0511
	(-0.4)		**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.2)	(0.0)	(-0.8)
DCC	0.0494	0.0525		0.0203	-0.1075	0.0488	0.1068	0.0468
	(1.0)	**(1.0)**		(0.0)	(-0.8)	(0.4)	**(1.0)**	(0.4)
FGK	0.0784	0.0815	0.0289		-0.1278	0.0286	0.0866	0.0265
	(1.0)	**(1.0)**	(0.2)		(-0.8)	(0.4)	**(1.0)**	(0.0)
CCJV	0.1313	0.1344	0.0818	0.0529		0.1564	0.2144	0.1544
	(1.0)	**(1.0)**	(0.8)	(0.6)		(0.8)	**(1.0)**	(0.8)
KKS1	0.0151	0.0182	-0.0343	-0.0633	-0.1162		0.0580	-0.0020
	(0.6)	(0.6)	(-0.6)	(-0.6)	(-0.8)		(0.8)	(-0.2)
BV	-0.0087	-0.0056	-0.0581	-0.0871	-0.1400	-0.0238		-0.0600
	(-0.6)	(-0.2)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.6)
RPadj	0.0305	0.0336	-0.0189	-0.0479	-0.1008	0.0154	0.0392	
	(0.8)	**(1.0)**	(-0.6)	(-0.8)	**(-1.0)**	(0.4)	**(1.0)**	

Panel C. Twelve Months

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
avg.	0.1488	0.1423	0.3374	0.2307	0.2674	0.1677	*0.1227*	0.1895
HIST		0.0065	-0.1886	-0.0820	-0.1187	-0.0190	0.0261	-0.0407
		(0.0)	**(-1.0)**	(-0.8)	(-0.8)	(0.0)	(0.2)	(-0.4)
RW	-0.0019		-0.1951	-0.0884	-0.1251	-0.0254	0.0196	-0.0471
	(0.0)		**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.2)	(0.2)	(-0.6)
DCC	0.0714	0.0734		0.1067	0.0700	0.1697	0.2147	0.1479
	(1.0)	**(1.0)**		(0.0)	(0.2)	(0.8)	**(1.0)**	**(1.0)**
FGK	0.1003	0.1023	0.0289		-0.0367	0.0630	0.1080	0.0413
	(1.0)	**(1.0)**	(-0.2)		(-0.2)	(0.8)	**(1.0)**	(0.0)
CCJV	0.1188	0.1208	0.0474	0.0185		0.0997	0.1447	0.0780
	(1.0)	**(1.0)**	(0.2)	(0.0)		(0.8)	**(1.0)**	(0.8)
KKS1	0.0054	0.0073	-0.0660	-0.0950	-0.1135		0.0450	-0.0217
	(0.4)	(0.4)	(-0.8)	(-0.8)	**(-1.0)**		(0.8)	(-0.2)
BV	-0.0163	-0.0144	-0.0877	-0.1167	-0.1352	-0.0217		-0.0667
	(-0.6)	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.4)
RPadj	0.0214	0.0234	-0.0500	-0.0789	-0.0974	0.0161	0.0378	
	(0.8)	**(1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(0.4)	(0.6)	

compared to the other approaches.

Summing up, when changing the evaluation period to one, three, or twelve months, BV, RW and, to a slightly lesser extent, HIST are still the best approaches.

2.4.3 Further Models for Implied Beta

We examine further possible beta estimators utilizing information from option prices. Looking at Panel A of Table 2.6, we find KKS2, based on implied kurtosis, to obtain quite similar results as KKS1, based on implied volatilities. Both yield a little higher estimation errors than SR, which is also based on implied volatilities, while all three yield substantially lower estimation errors compared to CCJV. However, HIST, RW, and BV still yield even lower errors. To summarize, even the simple historical benchmark is to be preferred over all the fully implied methods taken into consideration. The assumptions that have to be made on (co-) moments for the fully implied estimators therefore seem to be invalid. BV, RW, and HIST, utilizing correlations from historical return data, consistently outperform these models.

2.4.4 Option Liquidity

As option-implied approaches strongly rely on precise estimates for the option-implied moments, the rather poor performance could be caused by poor quality of the options data, resulting in imprecise moment estimates. To check for that, we repeat our analysis for all stocks contained in the DJIA 30.[27] The DJIA includes 30 of the largest U.S. companies that both commonly have more options traded (in terms of strike prices) and exhibit

[27] Since some methods require information on all members of an index, it is not possible to just select a subset of stocks from the S&P 500. Thus, we focus on an index that has significantly fewer members than the S&P 500.

Table 2.6: Estimation Errors: Six-Month Horizon – Daily Data (Further Implied and DJIA)

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five (Panel A), respectively two (Panel B), portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. In each panel, the first row reports the average root mean squared errors (RMSE) of the estimation models over the five (two) portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the panels report the difference in estimation errors. The upper triangular matrices report the differences in root mean squared estimation errors, averaged over the five (two) portfolios. Similarly, the lower triangular matrices report the average root median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

Panel A. Further Implied Estimators

	HIST	RW	CCJV	SR	KKS1	KKS2	FGK	RPadj	BV
avg.	0.1381	0.1301	0.2783	0.1674	0.1704	0.1720	0.2203	0.1718	*0.1164*
HIST		0.0079	-0.1402	-0.0293	-0.0323	-0.0339	-0.0822	-0.0337	0.0217
		(0.0)	(-0.8)	(-0.2)	(-0.4)	(-0.4)	(-0.8)	(-0.6)	(0.2)
RW	0.0002		-0.1482	-0.0372	-0.0403	-0.0419	-0.0901	-0.0416	0.0138
	(-0.4)		(-0.8)	(-0.2)	(-0.2)	(-0.2)	**(-1.0)**	(-0.6)	(0.2)
CCJV	0.1143	0.1141		0.1110	0.1079	0.1063	0.0581	0.1066	0.1620
	(1.0)	**(1.0)**		(0.8)	(0.8)	(0.8)	(0.6)	**(1.0)**	**(1.0)**
SR	0.0153	0.0150	-0.0991		-0.0030	-0.0046	-0.0529	-0.0044	0.0510
	(0.6)	(0.4)	**(-1.0)**		(0.4)	(0.4)	(-0.6)	(0.0)	(0.8)
KKS1	0.0113	0.0110	-0.1031	-0.0040		-0.0016	-0.0499	-0.0013	0.0541
	(0.4)	(0.6)	**(-1.0)**	(-0.2)		(-0.4)	(-0.6)	(-0.2)	(0.8)
KKS2	0.0135	0.0133	-0.1008	-0.0018	0.0022		-0.0482	0.0003	0.0557
	(0.4)	(0.6)	**(-1.0)**	(-0.2)	(0.4)		(-0.6)	(-0.2)	(0.8)
FGK	0.1001	0.0999	-0.0142	0.0849	0.0888	0.0866		0.0485	0.1039
	(1.0)	**(1.0)**	(-0.4)	(0.6)	(0.8)	(0.8)		(0.2)	**(1.0)**
RPadj	0.0256	0.0254	-0.0888	0.0103	0.0143	0.0121	-0.0745		0.0554
	(0.8)	(0.8)	**(-1.0)**	(0.2)	(0.2)	(0.2)	**(-1.0)**		(0.6)
BV	-0.0111	-0.0113	-0.1254	-0.0263	-0.0224	-0.0246	-0.1112	-0.0367	
	(-0.6)	(-0.6)	**(-1.0)**	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	

Table 2.6: Estimation Errors: Six-Month Horizon – Daily Data (Further Implied and DJIA) (continued)

Panel B. DJIA

	HIST	RW	DCC	CCJV	SR	KKS1	KKS2	FGK	RPadj	BV
avg.	0.1569	*0.1558*	0.2356	0.2627	0.2434	0.1798	0.1730	0.2067	0.1965	0.1560
HIST		0.0011	-0.0787	-0.1058	-0.0865	-0.0229	-0.0161	-0.0498	-0.0396	0.0009
		(0.0)	**(-1.0)**	**(-1.0)**	(-0.5)	(-0.5)	(-0.5)	**(-1.0)**	(-0.5)	(0.0)
RW	-0.0005		-0.0798	-0.1069	-0.0876	-0.0240	-0.0173	-0.0509	-0.0407	-0.0002
	(0.0)		**(-1.0)**	(-0.5)	(-0.5)	(-0.5)	(-0.5)	**(-1.0)**	(-0.5)	(0.0)
DCC	0.0439	0.0444		-0.0271	-0.0078	0.0558	0.0626	0.0289	0.0391	0.0796
	(1.0)	**(1.0)**		(-0.5)	(0.0)	(0.5)	(0.5)	(0.0)	(0.0)	(0.5)
CCJV	0.0821	0.0826	0.0382		0.0193	0.0829	0.0897	0.0560	0.0662	0.1067
	(1.0)	**(1.0)**	(0.5)		(0.0)	**(1.0)**	**(1.0)**	(0.5)	(0.5)	**(1.0)**
SR	0.0784	0.0789	0.0344	-0.0037		0.0636	0.0704	0.0367	0.0469	0.0874
	(0.5)	(0.5)	(0.0)	(-0.5)		(0.5)	(0.5)	(0.0)	(0.5)	(0.5)
KKS1	0.0207	0.0212	-0.0232	-0.0614	-0.0577		0.0068	-0.0269	-0.0167	0.0238
	(0.5)	(0.5)	(-0.5)	**(-1.0)**	(-0.5)		**(1.0)**	(-0.5)	(0.0)	(0.5)
KKS2	0.0105	0.0110	-0.0334	-0.0716	-0.0679	-0.0102		-0.0337	-0.0235	0.0170
	(0.5)	(0.5)	(-0.5)	**(-1.0)**	(-0.5)	**(-1.0)**		(-0.5)	(-0.5)	(0.5)
FGK	0.0650	0.0655	0.0211	-0.0171	-0.0134	0.0443	0.0545		0.0102	0.0507
	(1.0)	**(1.0)**	(0.0)	(0.0)	(0.0)	(0.5)	(0.5)		(0.0)	(0.5)
RPadj	0.0318	0.0323	-0.0122	-0.0503	-0.0466	0.0111	0.0213	-0.0332		0.0405
	(1.0)	**(1.0)**	(-0.5)	(-0.5)	(0.0)	(0.5)	(0.5)	(-0.5)		**(1.0)**
BV	0.0093	0.0098	-0.0347	-0.0728	-0.0691	-0.0114	-0.0012	-0.0557	-0.0225	
	(0.5)	(0.5)	**(-1.0)**	**(-1.0)**	(-0.5)	**(-1.0)**	(-0.5)	**(-1.0)**	**(-1.0)**	

a much higher liquidity compared to smaller stocks in the S&P 500.[28] While Chakravarty, Gulen, & Mayhew (2004) find that option market price discovery is related to trading volume, considering only very liquid options may yield more precise estimates for the implied approaches.

The results are presented in Panel B of Table 2.6. We find that the fully implied approaches CCJV, SR, KKS1, and KKS2 still obtain larger errors in comparison to HIST, RW, and BV.[29] The results further show that HIST, RW, and BV obtain significantly smaller mean and median errors for at least one of the two portfolios. Thus, even when restricting the sample to the

[28]Note that although the total turnover on S&P 500 index options is substantially higher than that on DJIA index options, on average the daily total contract volume on DJIA index options of about 23,000 should be sufficiently high to obtain accurate moment estimates, even at the six-month horizon.

[29]The results for the one-month horizon, where both the options on the index an these on the individual assets should be more frequently traded, are qualitatively equal.

DJIA, the fully implied models are inferior to a simple historical estimate. The adjustment for the volatility risk premium (RPadj), that also may be better fitted for better-quality options data, yields lower errors compared to FGK, whereas the differences are not statistically significant in most cases. Finally, under the presumably better options data, the average RMSE of HIST, RW, and BV are approximately equal, with RW yielding the lowest average RMSE. The RMSE of HIST, RW, and BV are significantly lower than those of the remaining approaches for at least one of the two portfolios.

2.4.5 Further Time-Series Models

We investigate further models imposing a time-varying structure on beta, namely a random walk with drift, a first-order autoregressive (AR(1)) model, an autoregressive moving average ARMA(1,1) model, and the CCC Model of Bollerslev (1990), as well as realized beta over the past six months (HIST_6).[30] Lewellen & Nagel (2006) argue that the results of short-term regressions provide conditional parameters without the use of conditioning variables as long as the parameters are relatively stable within that short period. Consequently, HIST_6 might deliver better conditional estimates than the simple historical estimator using one year of historical return data.

In Table 2.7 we find RW_D, AR, ARMA, and especially HIST_6 to perform quite well, while CCC is clearly outperformed.[31] Furthermore, it can be seen that the less complex structure as in CCC yields a slight improvement, when comparing the results to those of DCC. HIST_6 obtains a smaller average RMSE compared to the historical estimate over one year, so short-term conditional estimates may be better fitted. The average RMSE

[30]Note that adding a drift in models (2.18) and (2.19) for AR and ARMA does especially affect ARMA adversely for long horizons. Consequently, we only report the results of the models without drift.

[31]Note that the values change slightly compared to those in Table 2.3, as we are able to retain more estimates since CCJV is not included in the analysis.

Table 2.7: Estimation Errors: Six-Month Horizon – Daily Data (Further Time-Series Models)

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. The first row reports the average root mean squared errors (RMSE) of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

	HIST	$HIST_6$	RW	RW_D	AR	ARMA	DCC	CCC	BV
avg.	0.1355	0.1283	0.1269	0.1870	0.1865	0.1899	0.2638	0.2572	*0.1152*
HIST		0.0072	0.0086	-0.0515	-0.0510	-0.0543	-0.1283	-0.1217	0.0203
		(0.0)	(0.0)	(-0.4)	(-0.8)	(-0.8)	**(-1.0)**	(-0.8)	(0.2)
$HIST_6$	-0.0041		0.0014	-0.0587	-0.0582	-0.0616	-0.1356	-0.1289	0.0131
	(-0.2)		(0.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.8)	(0.2)
RW	-0.0022	0.0019		-0.0601	-0.0596	-0.0630	-0.1370	-0.1303	0.0117
	(-0.2)	(0.0)		(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.8)	(0.2)
RW_D	0.0394	0.0435	0.0416		0.0005	-0.0029	-0.0769	-0.0702	0.0718
	(0.8)	**(1.0)**	**(1.0)**		(0.0)	(0.0)	(-0.8)	(0.0)	(0.8)
AR	0.0345	0.0385	0.0366	-0.0049		-0.0034	-0.0774	-0.0707	0.0713
	(1.0)	**(1.0)**	**(1.0)**	(-0.2)		(0.2)	(-0.8)	(0.0)	(0.8)
ARMA	0.0369	0.0410	0.0391	-0.0025	0.0024		-0.0740	-0.0673	0.0746
	(1.0)	**(1.0)**	**(1.0)**	(-0.2)	(-0.2)		(-0.8)	(-0.2)	(0.8)
DCC	0.0638	0.0679	0.0660	0.0244	0.0293	0.0269		0.0067	0.1486
	(1.0)	**(1.0)**	**(1.0)**	(0.8)	(0.8)	(0.8)		(0.0)	**(1.0)**
CCC	0.0338	0.0379	0.0360	-0.0055	-0.0006	-0.0031	-0.0299		0.1420
	(1.0)	**(1.0)**	**(1.0)**	(0.0)	(0.0)	(0.2)	(-0.8)		(0.8)
BV	-0.0137	-0.0096	-0.0115	-0.0531	-0.0482	-0.0506	-0.0775	-0.0475	
	(-0.8)	(-0.6)	(-0.6)	(-0.8)	**(-1.0)**	(-0.8)	**(-1.0)**	(-0.8)	

of RW_D, AR, and ARMA are only moderate, but clearly higher than those of BV, which overall again yields the lowest average RMSE. Regarding significance, we find that HIST, $HIST_6$, RW, and BV yield significantly lower errors than AR, ARMA, DCC and CCC for at least four portfolios. Only in rare cases are there significant differences among the formerly mentioned, but in these cases, they are mostly in favor of BV. Overall we find, beside HIST and RW, $HIST_6$ to be a valuable alternative to the BV approach.[32]

2.4.6 Bias Removal

As we discuss in Section III.A, some estimators are heavily biased with their cross-sectional value-weighted average estimate for beta, a quantity that theoretically has to be equal to one if a full market index is used, being substantially different from that value. A possible improvement could be to try and remove the bias implied by these deviations.[33]

A first simple method we try is to standardize the estimators in a way that their cross-sectional value-weighted average exactly equals one. For that, for each approach, we simply divide each estimate by the cross-sectional value-weighted mean beta of that approach at that time. We apply the technique on all estimators but those that fulfill the condition already by construction (e.g., KKS1 and BV). The results are shown in Panel A of Table 2.8. Indeed, the simple bias removal seems to be working, in particular for the two hybrid estimators FGK and RPadj, reducing their RMSE almost to the level of BV with a significant difference for no portfolio, comparing BV to FGK.[34] Consequently, the main benefit

[32]The results for the one-month horizon, where the conditional estimates of the time-series approaches are likely much more precise, are qualitatively equal. Overall, BV delivers lower RMSE compared to all time-series approaches.

[33]We thank an anonymous referee for suggesting this.

[34]As can be seen in the appendix to this chapter, further analysis shows that BV still is informationally more efficient compared to the bias-removed FGK and RPadj.

of the BV approach seems to be not the adjustment of correlations to the risk-neutral probability measure but rather the ensurement that the estimates combining option-implied and historical return information are approximatively unbiased in their cross-sectional average. For CCJV and DCC, the bias removal yields a substantial improvement, however these estimators are still inferior, while the improvement is quite small for HIST and RW.

We also employ more refined bias-removal techniques in the spirit of Mincer & Zarnowitz (1969) using regression techniques as in equation (2.28).[35] In a first approach we form portfolios as in Section III.B, obtain estimates for each approach and then perform the univariate regression for each approach separately, pooling all 60 (12 months times 5 portfolios) unadjusted ex ante estimates for each approach i as well as the corresponding ex post realized portfolio beta estimates during the twelve months $t-17$ up to $t-6$ (as realized beta with a six-month window is only available up to $t-6$ at time t):

$$\beta^{\mathrm{R}} \quad = \quad a_{i,t} + b_{i,t}\beta_i^{\mathrm{UNADJ}} + \epsilon_j. \tag{2.30}$$

β_i^{UNADJ} is the vector of pooled initial portfolio beta estimates of one approach, while β^{R} denotes the corresponding pooled realized beta vector. Subsequently, after obtaining the regression coefficients $\hat{a}_{i,t}$ and $\hat{b}_{i,t}$, we manipulate the current estimates, inserting them into the equation

$$\beta_{i,j,t}^{\mathrm{ADJ}} \quad = \quad \hat{a}_{i,t} + \hat{b}_{i,t}\beta_{i,j,t}^{\mathrm{UNADJ}}, \tag{2.31}$$

where $\beta_{i,j,t}^{\mathrm{ADJ}}$ is the adjusted estimate of approach i and asset j at time t.

A second approach could be to try to remove the bias in the same spirit combining it with the estimate for historical beta (HIST). For that,

[35] We consider further possibilities to try and remove bias in the appendix to this chapter.

we perform a bivariate regression of portfolio realized beta on each approach and HIST over the twelve months $t-17$ up to $t-6$. The final adjustment is then performed as follows:

$$\beta_{i,j,t}^{\mathrm{ADJ}} = \hat{a}_{i,t} + \hat{b}_{i,t}^{\beta}\beta_{i,j,t}^{\mathrm{UNADJ}} + \hat{b}_{i,t}^{\mathrm{HIST}} HIST_{j,t}. \quad (2.32)$$

$HIST_{j,t}$ is the estimate for historical beta at time t and $\hat{b}_{i,t}^{\beta}$ and $\hat{b}_{i,t}^{\mathrm{HIST}}$ are the regression coefficients on the considered approach and HIST, respectively.

The results on these approaches are presented in Panels B and C of Table 2.8.[36] The results on the adjustment of equation (2.31) indicate an even slightly higher average RMSE for BV while all other approaches also yield (in many cases significantly) higher RMSE than the initial BV. The adjustment of equation (2.32), combining the estimates with HIST, yields an improvement for DCC and the implied estimators FGK, CCJV, and RPadj, indicating that not all information on historical returns is incorporated in these estimators. For all remaining approaches, including BV, the combination with HIST yields a higher RMSE compared to the simpler bias removal using equation (2.31).

Consequently, a simple bias removal is shown to be valuable in particular for hybrid estimators. Furthermore, our results suggest that a regression-based bias-removal cannot further improve the performance of BV, HIST, and RW.

2.5 Conclusion

This chapter examines the performance of a wide range of approaches to estimating an asset's market beta. Specifically, we investigate several constant and time-varying models relying on historical return data and

[36] Note that the results for the uncorrected $\mathrm{BV}^{\mathrm{UC}}$ differ from those in previous tables as the bias correction first needs 17 months of data before it starts, delaying the start of the evaluation period.

Table 2.8: Bias Removal

This table reports the out-of-sample estimation errors of competing bias-removed estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. Panel A presents the results on a simple bias removal, while Panels B and C present the results on bias removals using regression techniques. In each panel, the first row reports the average root mean squared errors (RMSE) of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the tables report the difference in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences. BV^{UC} refers to the non-corrected BV estimates.

Panel A. Simple Bias Removal

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
avg.	0.1304	0.1263	0.1820	0.1200	0.1633	0.1704	*0.1164*	0.1272
HIST		0.0041	-0.0516	0.0104	-0.0329	-0.0400	0.0141	0.0032
		(-0.2)	(-0.8)	(0.4)	(-0.4)	(-0.4)	(0.2)	(0.0)
RW	-0.0029		-0.0556	0.0063	-0.0370	-0.0441	0.0100	-0.0009
	(0.0)		**(-1.0)**	(0.2)	(-0.4)	(-0.4)	(0.2)	(0.2)
DCC	0.0169	0.0199		0.0619	0.0187	0.0115	0.0656	0.0548
	(1.0)	**(1.0)**		**(1.0)**	(0.0)	(0.0)	(0.8)	(0.8)
FGK	-0.0135	-0.0106	-0.0305		-0.0433	-0.0504	0.0037	-0.0072
	(-0.6)	(-0.4)	**(-1.0)**		(-0.6)	(-0.4)	(0.0)	(-0.4)
CCJV	0.0150	0.0180	-0.0019	0.0285		-0.0071	0.0470	0.0361
	(0.6)	(0.6)	(-0.4)	**(1.0)**		(0.0)	(0.8)	(0.4)
KKS1	0.0092	0.0122	-0.0077	0.0227	-0.0058		0.0541	0.0432
	(0.4)	(0.4)	(-0.4)	(0.6)	(-0.2)		(0.8)	(0.4)
BV	-0.0132	-0.0102	-0.0301	0.0004	-0.0282	-0.0224		-0.0108
	(-0.6)	(-0.6)	(-0.8)	(0.0)	**(-1.0)**	**(-1.0)**		(0.0)
RPadj	-0.0093	-0.0064	-0.0263	0.0042	-0.0243	-0.0185	0.0038	
	(-0.2)	(-0.4)	**(-1.0)**	(0.0)	(-0.8)	(-0.6)	(0.6)	

Table 2.8: Bias Removal (continued)

Panel B. Regression Technique

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj	BV^{UC}
avg.	0.1941	0.1630	0.2595	0.1976	0.2461	0.1601	0.1286	0.2066	*0.1203*
HIST		0.0311	-0.0654	-0.0035	-0.0520	0.0340	0.0655	-0.0125	0.0738
		(0.0)	(-0.6)	(-0.4)	(-0.4)	(0.0)	(0.4)	(-0.4)	(0.4)
RW	-0.0142		-0.0965	-0.0346	-0.0831	0.0029	0.0345	-0.0436	0.0427
	(-0.6)		**(-1.0)**	(-0.8)	(-0.4)	(0.0)	(0.2)	(-0.6)	(0.2)
DCC	0.0516	0.0657		0.0619	0.0134	0.0994	0.1310	0.0529	0.1392
	(0.8)	**(1.0)**		(0.6)	(0.0)	(0.8)	**(1.0)**	(0.4)	**(1.0)**
FGK	0.0207	0.0349	-0.0309		-0.0486	0.0374	0.0690	-0.0091	0.0773
	(0.6)	(0.8)	(-0.6)		(-0.2)	(0.8)	(0.8)	(0.0)	(0.8)
CCJV	0.0320	0.0462	-0.0195	0.0113		0.0860	0.1176	0.0395	0.1258
	(0.4)	**(1.0)**	(-0.4)	(0.4)		(0.6)	(0.6)	(0.2)	(0.6)
KKS1	-0.0148	-0.0006	-0.0663	-0.0354	-0.0468		0.0316	-0.0465	0.0398
	(-0.6)	(-0.2)	**(-1.0)**	(-0.6)	**(-1.0)**		(0.0)	(-0.8)	(0.4)
BV	-0.0199	-0.0057	-0.0715	-0.0406	-0.0519	-0.0051		-0.0781	0.0082
	(-1.0)	(-0.4)	**(-1.0)**	(-0.8)	**(-1.0)**	(-0.2)		(-0.8)	(0.0)
RPadj	0.0201	0.0342	-0.0315	-0.0006	-0.0120	0.0348	0.0399		0.0863
	(0.6)	(0.8)	(-0.6)	(0.0)	(-0.6)	(0.8)	**(1.0)**		(0.8)
BV^{UC}	-0.0317	-0.0175	-0.0833	-0.0524	-0.0637	-0.0169	-0.0118	-0.0517	
	(-1.0)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.4)	**(-1.0)**	

Panel C: Regression Technique Combining with HIST

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj	BV^{UC}
avg.	0.1941	0.1879	0.1998	0.1898	0.1765	0.1716	0.1367	0.1894	*0.1203*
HIST		0.0062	-0.0057	0.0043	0.0176	0.0224	0.0574	0.0047	0.0738
		(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.4)	(0.0)	(0.4)
RW	-0.0061		-0.0119	-0.0019	0.0114	0.0162	0.0512	-0.0015	0.0676
	(-0.2)		(0.0)	(0.0)	(0.0)	(0.2)	(0.2)	(0.0)	(0.6)
DCC	0.0044	0.0104		0.0099	0.0232	0.0281	0.0630	0.0104	0.0794
	(0.2)	(0.4)		(0.0)	(0.0)	(0.4)	(0.6)	(0.0)	(0.6)
FGK	-0.0011	0.0050	-0.0054		0.0133	0.0182	0.0531	0.0004	0.0695
	(0.0)	(0.2)	(-0.2)		(0.0)	(0.6)	(0.4)	(0.0)	(0.6)
CCJV	0.0016	0.0077	-0.0027	0.0027		0.0049	0.0398	-0.0129	0.0562
	(0.0)	(0.0)	(-0.4)	(-0.2)		(0.0)	(0.4)	(0.0)	(0.8)
KKS1	-0.0116	-0.0055	-0.0160	-0.0105	-0.0133		0.0349	-0.0178	0.0513
	(-0.8)	(-0.4)	(-0.8)	(-0.8)	(-0.2)		(0.0)	(-0.4)	(0.2)
BV	-0.0202	-0.0141	-0.0246	-0.0192	-0.0219	-0.0086		-0.0527	0.0164
	(-1.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.2)	(0.0)
RPadj	-0.0019	0.0042	-0.0062	-0.0008	-0.0035	0.0098	0.0184		0.0691
	(0.0)	(0.4)	(-0.2)	(0.0)	(-0.2)	(0.8)	**(1.0)**		(0.6)
BV^{UC}	-0.0317	-0.0256	-0.0360	-0.0306	-0.0333	-0.0201	-0.0115	-0.0298	
	(-1.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.8)	(-0.2)	**(-1.0)**	

additionally several methods including or solely relying on option-implied information.

In summary, estimators using historical information only perform well if they do not make too strong structural assumptions, like the simple historical beta and the Kalman filter approach with a random walk parametrization. In contrast, models that make strong assumptions on the volatility and correlation processes (like the GARCH-based DCC) are shown to produce very large errors.

Including information from option prices is shown to be valuable to some extent. Fully implied methods, having the big advantage of employing the forward-looking information from options markets, nonetheless adhere the major shortcoming that they cannot attain negative values. Consequently, even the models that are on average unbiased by construction (KKS1 and KKS2) produce substantial errors. Avoiding strong and seemingly invalid identifying assumptions, the hybrid approaches, combining historical return data with forward-looking information from the options market, are shown to produce the lowest errors. In particular, the hybrid approach of Buss & Vilkov (2012) consistently performs best regarding informational efficiency as well es estimation accuracy. These results are shown to be robust both to building more portfolios and different estimation horizons. Furthermore, we find that the main benefit of BV, compared to other hybrid approaches, is that it ensures that the estimates are adjusted to be unbiased in their value-weighted cross-sectional averages.

Overall, although the BV approach appears the method of choice, one major shortcoming of this method (and other hybrid approaches that try a simple bias correction) has to be borne in mind. The methodology requires information on a full market index. Consequently it cannot be employed for assets that are not included in an index, nor if there is insufficient option-implied information for *all* assets in the index. Therefore, whenever the BV approach is not applicable, our results indicate that one should rely either on RW or a simple estimate based on historical returns, since both quite consistently outperform all other approaches.

A Appendix

A.1 Information Content – Monthly Data

Monthly Estimators In this section, we repeat the analyses on the information content of Section 2.3.2 using monthly return data. The results are reported in Table A.1. Except for KKS1 and BV, the univariate regressions in Panel A of the Table show all approaches to be biased for all portfolios, having an intercept significantly different from zero, a slope parameter significantly different from one, and consequently a strongly significant Wald test. For BV, the Wald test yields significant values in only three out of the five cases, while KKS1 is shown to be biased for four of the five portfolios.[37] Throughout all approaches, except KKS1, that does not rely on return data at all, the adjusted R^2 is substantially smaller than that in the regressions using daily estimates, with most values being close to zero. The highest adjusted R^2 is obtained for the BV approach, being the only one, except KKS1, that has substantial explanatory power over all portfolios.

In the encompassing regressions in Panel B of Table A.1, the general performance is poor and not much can be stated about methods being informationally more efficient or subsuming one another, except that the BV approach turns out to be informationally more efficient compared to all other approaches. For some portfolios, BV even subsumes all information incorporated in these approaches, while it is also shown to be informationally more efficient than KKS1, which does not rely on return data at all. Again, the adjusted R^2 substantially increases when adding BV as an additional explanatory variable in every case.

[37] Note that the overall results change as the stocks are sorted differently using monthly historical beta obtained in the sorting period.

Table A.1: Univariate and Encompassing Estimates for Realized Beta – Monthly Data

This table presents the results from regressions of ex post realized beta over the horizon of six months on competing monthly ex ante estimates for each of the five portfolios, respectively. Each row corresponds to one regression. Each month, we sort our estimates in an ascending order according to the historical (equation (9)) beta obtained in a sorting period strictly before the estimation period of the historical beta. The sorting and estimation periods are without overlap and have equal length. The stocks are then allocated to quintile portfolios. Portfolio betas and returns are value-weighted. a denotes the regression intercept, b_{HIST}, b_{RW}, b_{DCC}, b_{FGK}, b_{CCJV}, b_{KKS1}, b_{BV}, and b_{RPadj} refer to the slope coefficients for the historical, random walk, DCC, FGK, CCJV, KKS1, BV, and risk premium adjusted estimate for beta, respectively. For each regression coefficient we report Newey & West (1987) t-statistics ($t(.)$) using 6 lags, where we test a against zero and b against one in the univariate regressions (Panel A), while we test both against zero in multivariate regressions (Panel B). R^2_{adj} denotes the adjusted R^2 of the regressions. The columns $Wald_1$ and $Wald_2$ refer to the Wald test statistic and p gives the corresponding p-values. In the univariate regressions in Panel A, for the Wald test (reported in $Wald_1$) we test the joint hypothesis of the intercept being equal to zero and the slope coefficient being equal to one. In multivariate regressions in Panel B, the joint underlying hypothesis is that the first slope coefficient is equal to one and the second slope coefficient is equal to zero and vice versa for $Wald_1$ and $Wald_2$, respectively. t-statistics and p-values in **bold** font indicate significance at the 5 % level. The sample period is January 1996 until December 2012 with 198 (188 when RPadj is included) monthly observations.

Panel A. Univariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
HIST	1	0.96	**(7.15)**	-0.24	**(-6.59)**															0.03	166.45	**(0.00)**
	2	0.70	**(13.19)**	0.28	**(-11.46)**															0.29	360.43	**(0.00)**
	3	0.69	**(6.80)**	0.32	**(-6.69)**															0.17	94.84	**(0.00)**
	4	0.71	**(16.71)**	0.33	**(-15.23)**															0.32	197.35	**(0.00)**
	5	1.06	**(9.26)**	0.10	**(-10.05)**															0.00	111.86	**(0.00)**
RW	1	0.85	**(5.78)**			-0.08	**(-5.66)**													0.00	66.45	**(0.00)**
	2	0.69	**(12.83)**			0.27	**(-11.83)**													0.29	332.39	**(0.00)**
	3	0.85	**(10.03)**			0.16	**(-10.25)**													0.04	128.90	**(0.00)**
	4	0.89	**(13.56)**			0.16	**(-13.71)**													0.13	434.42	**(0.00)**
	5	0.90	**(8.13)**			0.23	**(-7.93)**													0.04	75.82	**(0.00)**
DCC	1	0.69	**(11.48)**					0.16	**(-11.22)**											0.04	234.97	**(0.00)**
	2	0.82	**(17.21)**					0.14	**(-16.23)**											0.11	599.91	**(0.00)**
	3	0.90	**(19.36)**					0.12	**(-17.39)**											0.07	482.00	**(0.00)**
	4	1.01	**(12.57)**					0.05	**(-13.18)**											0.01	565.31	**(0.00)**
	5	1.28	**(11.09)**					-0.08	**(-12.96)**											0.01	250.60	**(0.00)**

Table A.1: Univariate and Encompassing Estimates for Realized Beta – Monthly Data (continued)

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
FGK	1	0.92	**(6.51)**							-0.23	**(-4.60)**									0.02	478.03	**(0.00)**
	2	0.77	**(13.16)**							0.26	**(-7.99)**									0.14	1274.35	**(0.00)**
	3	0.86	**(7.81)**							0.21	**(-4.96)**									0.07	915.11	**(0.00)**
	4	0.86	**(9.42)**							0.26	**(-5.96)**									0.14	1252.45	**(0.00)**
	5	0.86	**(3.63)**							0.37	**(-2.24)**									0.09	400.07	**(0.00)**
CCJV	1	0.50	**(6.30)**									0.29	**(-9.49)**							0.20	489.70	**(0.00)**
	2	0.93	**(12.03)**									0.00	**(-14.73)**							-0.01	762.84	**(0.00)**
	3	0.83	**(13.43)**									0.16	**(-14.91)**							0.13	647.51	**(0.00)**
	4	0.93	**(13.67)**									0.12	**(-13.66)**							0.06	517.53	**(0.00)**
	5	1.09	**(6.43)**									0.08	**(-6.29)**							0.01	172.05	**(0.00)**
KKS1	1	-0.14	(-0.46)											1.07	(0.21)					0.18	43.42	**(0.00)**
	2	-0.06	(-0.37)											1.03	(0.19)					0.36	16.29	**(0.00)**
	3	0.07	(0.32)											0.94	(-0.28)					0.26	1.07	(0.34)
	4	0.07	(1.06)											0.98	(-0.26)					0.65	108.81	**(0.00)**
	5	0.32	(0.84)											0.78	(-0.62)					0.09	25.82	**(0.00)**
BV	1	-0.13	(-0.51)													1.12	(0.41)			0.27	5.13	**(0.01)**
	2	0.03	(0.38)													0.96	(-0.46)			0.50	1.41	(0.25)
	3	-0.04	(-0.19)													1.04	(0.22)			0.34	0.92	(0.40)
	4	0.01	(0.15)													1.01	(0.16)			0.65	14.32	**(0.00)**
	5	-0.58	(-0.80)													1.53	(0.83)			0.29	10.05	**(0.00)**
RPadj	1	0.95	**(9.63)**															-0.25	**(-7.93)**	0.08	347.48	**(0.00)**
	2	0.84	**(19.14)**															0.11	**(-15.70)**	0.09	876.96	**(0.00)**
	3	0.94	**(10.14)**															0.08	**(-7.97)**	0.02	525.27	**(0.00)**
	4	1.01	**(12.91)**															0.07	**(-11.12)**	0.02	583.86	**(0.00)**
	5	0.72	**(4.21)**															0.42	**(-3.46)**	0.34	131.35	**(0.00)**

Panel B. Multivariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
HIST + RW	1	0.93	**(5.31)**	-0.26	(-1.36)	0.06	(0.30)													0.03	106.21	**(0.00)**	57.50	**(0.00)**
	2	0.68	**(12.23)**	0.14	(1.20)	0.15	(1.34)													0.30	287.83	**(0.00)**	308.31	**(0.00)**
	3	0.73	**(7.78)**	0.47	**(3.75)**	-0.19	(-1.73)													0.20	101.20	**(0.00)**	172.57	**(0.00)**
	4	0.70	**(16.81)**	0.44	**(8.54)**	-0.10	(-1.55)													0.34	200.15	**(0.00)**	581.89	**(0.00)**
	5	0.98	**(8.90)**	-0.20	(-1.21)	0.38	**(2.16)**													0.05	67.32	**(0.00)**	52.07	**(0.00)**
HIST + DCC	1	0.88	**(6.76)**	-0.34	(-1.75)			0.23	**(2.69)**											0.11	124.33	**(0.00)**	139.59	**(0.00)**
	2	0.69	**(12.80)**	0.26	**(3.76)**			0.02	(0.38)											0.29	277.63	**(0.00)**	586.20	**(0.00)**
	3	0.68	**(6.86)**	0.33	**(3.08)**			0.00	(-0.10)											0.17	94.36	**(0.00)**	481.68	**(0.00)**
	4	0.73	**(17.28)**	0.40	**(5.52)**			-0.09	(-1.40)											0.36	209.05	**(0.00)**	874.35	**(0.00)**
	5	1.09	**(9.78)**	0.19	(1.71)			-0.13	(-1.22)											0.03	63.93	**(0.00)**	249.66	**(0.00)**
HIST + FGK	1	0.98	**(6.70)**	-0.21	(-0.74)					-0.07	(-0.19)									0.03	106.29	**(0.00)**	66.46	**(0.00)**
	2	0.71	**(11.72)**	0.32	**(5.26)**					-0.08	(-0.75)									0.29	279.90	**(0.00)**	184.41	**(0.00)**
	3	0.69	**(6.32)**	0.34	**(3.34)**					-0.03	(-0.17)									0.17	94.53	**(0.00)**	129.74	**(0.00)**
	4	0.71	**(11.28)**	0.32	**(5.98)**					0.01	(0.10)									0.32	191.77	**(0.00)**	185.97	**(0.00)**
	5	0.91	**(4.58)**	-0.05	(-0.32)					0.39	(1.22)									0.08	74.17	**(0.00)**	28.60	**(0.00)**
HIST + CCJV	1	0.51	**(3.53)**	0.00	(-0.02)							0.29	**(4.60)**							0.20	148.58	**(0.00)**	153.31	**(0.00)**
	2	0.60	**(8.01)**	0.29	**(4.30)**							0.07	(1.84)							0.30	287.30	**(0.00)**	613.00	**(0.00)**
	3	0.46	**(4.99)**	0.34	**(4.68)**							0.17	**(4.77)**							0.33	139.76	**(0.00)**	569.66	**(0.00)**
	4	0.49	**(5.28)**	0.36	**(8.12)**							0.16	**(2.73)**							0.44	255.28	**(0.00)**	731.89	**(0.00)**
	5	0.95	**(4.42)**	0.10	(1.14)							0.08	(0.58)							0.01	61.56	**(0.00)**	154.70	**(0.00)**

Table A.1: Univariate and Encompassing Estimates for Realized Beta – Monthly Data (continued 2)

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
HIST + KKS1	1	-0.01	(-0.04)	-0.11	(-0.75)									1.01	**(3.22)**					0.18	144.71	**(0.00)**	1.05	(0.35)
	2	0.11	(0.60)	0.12	(1.46)									0.75	**(3.36)**					0.38	333.46	**(0.00)**	4.07	**(0.00)**
	3	-0.02	(-0.11)	0.24	**(3.05)**									0.79	**(4.12)**					0.35	147.03	**(0.00)**	14.01	**(0.00)**
	4	0.09	(1.12)	0.07	(1.66)									0.89	**(9.26)**					0.66	476.49	**(0.00)**	2.59	(0.08)
	5	0.00	(-0.01)	0.18	**(2.26)**									0.86	**(2.55)**					0.11	78.51	**(0.00)**	3.36	**(0.04)**
HIST + BV	1	0.04	(0.19)	-0.26	(-1.80)											1.14	**(4.11)**			0.31	189.65	**(0.00)**	6.97	**(0.00)**
	2	-0.01	(-0.07)	-0.04	(-0.39)											1.04	**(4.34)**			0.50	436.33	**(0.00)**	0.63	(0.53)
	3	0.00	(-0.02)	0.10	(1.07)											0.91	**(4.14)**			0.35	145.73	**(0.00)**	2.00	(0.14)
	4	-0.02	(-0.30)	-0.05	(-0.76)											1.10	**(8.65)**			0.65	470.53	**(0.00)**	1.02	(0.36)
	5	-0.54	(-0.78)	-0.11	(-0.82)											1.62	**(2.37)**			0.29	125.20	**(0.00)**	6.05	**(0.00)**
HIST + Rpadj	1	1.03	**(6.96)**	-0.15	(-0.63)													-0.21	(-1.07)	0.09	116.88	**(0.00)**	222.80	**(0.00)**
	2	0.71	**(14.65)**	0.34	**(4.61)**													-0.08	(-1.19)	0.25	299.41	**(0.00)**	762.55	**(0.00)**
	3	0.65	**(6.59)**	0.45	**(4.08)**													-0.10	(-0.88)	0.21	101.69	**(0.00)**	479.41	**(0.00)**
	4	0.73	**(14.95)**	0.41	**(7.05)**													-0.11	(-1.58)	0.38	219.57	**(0.00)**	732.49	**(0.00)**
	5	0.80	**(4.48)**	-0.07	(-0.44)													0.44	**(2.45)**	0.34	138.86	**(0.00)**	90.22	**(0.00)**
RW + BV	1	-0.02	(-0.07)			-0.19	(-1.10)									1.16	**(3.90)**			0.28	111.93	**(0.00)**	2.64	(0.07)
	2	-0.09	(-0.51)			-0.10	(-0.91)									1.18	**(4.22)**			0.51	475.01	**(0.00)**	2.43	(0.09)
	3	-0.04	(-0.21)			-0.05	(-0.70)									1.10	**(5.36)**			0.34	228.79	**(0.00)**	0.71	(0.49)
	4	-0.05	(-0.81)			-0.09	**(-2.37)**									1.16	**(13.74)**			0.68	1285.41	**(0.00)**	7.59	**(0.00)**
	5	-0.58	(-0.81)			0.00	(-0.04)									1.53	**(2.28)**			0.28	102.16	**(0.00)**	4.80	**(0.01)**
DCC + BV	1	-0.23	(-0.95)					0.16	**(2.52)**							1.12	**(3.80)**			0.31	208.44	**(0.00)**	6.65	**(0.00)**
	2	0.04	(0.47)					0.02	(0.56)							0.93	**(8.42)**			0.50	875.86	**(0.00)**	0.68	(0.51)
	3	-0.07	(-0.33)					-0.02	(-0.61)							1.09	**(5.29)**			0.34	624.76	**(0.00)**	0.38	(0.68)
	4	0.02	(0.24)					-0.02	(-0.81)							1.02	**(17.42)**			0.65	1692.89	**(0.00)**	0.46	(0.63)
	5	-0.53	(-0.81)					-0.14	(-1.52)							1.63	**(2.58)**			0.33	405.54	**(0.00)**	11.27	**(0.00)**
FGK + BV	1	-0.15	(-0.64)							0.03	(0.20)					1.14	**(3.95)**			0.27	119.18	**(0.00)**	0.49	(0.61)
	2	0.04	(0.46)							0.03	(0.36)					0.92	**(7.23)**			0.50	301.03	**(0.00)**	0.58	(0.56)
	3	-0.04	(-0.19)							0.00	(0.02)					1.04	**(4.87)**			0.33	184.81	**(0.00)**	0.09	(0.91)
	4	-0.02	(-0.30)							-0.11	(-1.27)					1.12	**(9.38)**			0.67	478.53	**(0.00)**	4.46	**(0.01)**
	5	-0.61	(-0.85)							0.20	(0.91)					1.40	**(2.61)**			0.31	70.06	**(0.00)**	8.56	**(0.00)**
FGK + RPadj	1	0.84	**(6.34)**							0.47	(0.98)							-0.47	(-1.59)	0.10	78.69	**(0.00)**	227.91	**(0.00)**
	2	0.78	**(15.25)**							0.26	(1.24)							-0.02	(-0.15)	0.10	153.20	**(0.00)**	625.12	**(0.00)**
	3	0.78	**(8.12)**							0.66	**(2.80)**							-0.28	(-1.56)	0.12	113.82	**(0.00)**	420.96	**(0.00)**
	4	0.79	**(10.33)**							0.83	**(4.47)**							-0.40	**(-2.87)**	0.31	182.36	**(0.00)**	648.99	**(0.00)**
	5	0.95	**(6.83)**							-0.59	**(-2.33)**							0.70	**(2.97)**	0.40	95.44	**(0.00)**	109.56	**(0.00)**
CCJV + BV	1	-0.25	(-0.85)									0.24	**(2.73)**			0.98	**(2.96)**			0.40	238.48	**(0.00)**	22.43	**(0.00)**
	2	-0.13	(-1.07)									0.10	**(2.35)**			1.02	**(9.91)**			0.53	957.44	**(0.00)**	7.53	**(0.00)**
	3	0.00	(0.01)									0.05	(1.27)			0.94	**(4.96)**			0.35	585.67	**(0.00)**	1.89	(0.15)
	4	-0.02	(-0.29)									0.05	(1.47)			0.98	**(15.43)**			0.66	1267.05	**(0.00)**	3.17	**(0.04)**
	5	-0.62	(-0.81)									0.05	(0.45)			1.51	**(2.48)**			0.29	253.35	**(0.00)**	5.38	**(0.01)**
KKS1 + BV	1	-0.27	(-0.89)											0.36	(0.84)	0.92	**(2.09)**			0.28	13.82	**(0.00)**	2.04	(0.13)
	2	0.17	(1.00)											-0.45	(-1.06)	1.28	**(4.18)**			0.51	30.80	**(0.00)**	2.58	(0.08)
	3	-0.05	(-0.24)											0.10	(0.38)	0.96	**(4.07)**			0.33	11.42	**(0.00)**	0.21	(0.81)
	4	-0.01	(-0.11)											0.51	**(2.41)**	0.54	**(2.79)**			0.68	9.60	**(0.00)**	9.03	**(0.00)**
	5	-0.50	(-0.78)											-0.32	(-0.58)	1.77	(1.85)			0.29	30.09	**(0.00)**	6.03	**(0.00)**
BV + Rpadj	1	-0.06	(-0.22)													1.09	**(3.62)**	-0.06	(-0.62)	0.29	4.78	**(0.01)**	310.14	**(0.00)**
	2	0.04	(0.35)													0.96	**(6.56)**	-0.02	(-0.36)	0.44	3.88	**(0.02)**	1051.38	**(0.00)**
	3	-0.08	(-0.38)													1.09	**(5.42)**	-0.01	(-0.07)	0.35	5.00	**(0.01)**	609.45	**(0.00)**
	4	0.00	(0.07)													1.08	**(13.20)**	-0.07	(-1.63)	0.69	17.40	**(0.00)**	1582.54	**(0.00)**
	5	-0.30	(-0.51)													1.00	**(2.07)**	0.30	**(2.08)**	0.43	31.07	**(0.00)**	117.38	**(0.00)**

Daily versus Monthly Estimators Table A.2 presents the results of directly comparing estimators relying on daily versus monthly data. It can be seen that daily estimators are mostly informationally more efficient than their monthly counterparts for HIST, RW, and BV when evaluating the estimates using realized beta computed from six months of daily returns. Thus, especially when estimating beta for short horizons, relying on daily data is favorable. Naturally, our study design inherently favors estimators based on daily data by evaluating the estimations using realized beta, which is itself based on daily data. Furthermore, the time period, in addition to only the sampling frequency (daily versus monthly), differs between daily and monthly estimates (one year versus five years). Consequently, part of the difference in informational efficiency could also be induced by that. Thus, caution has to be applied when aiming to generalize these findings.

In summary (including the results of the main part), estimators using daily instead of monthly return data yield a better performance, and in both cases the Buss & Vilkov (2012) approach is most favorable regarding informational efficiency.

A.2 Estimation Accuracy – Additional Loss Functions

We examine three additional loss functions, commonly applied in the literature, namely mean absolute errors (MAE), mean absolute percentage errors (MAPE), and mean squared percentage errors (MSPE) to evaluate the performance of the different beta estimation techniques:

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} \mid \beta^R_{t,T} - \zeta_{t,T} \mid, \tag{A.1}$$

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n} \mid \frac{\beta^R_{t,T} - \zeta_{t,T}}{\beta^R_{t,T}} \mid, \tag{A.2}$$

$$\text{MSPE} = \frac{1}{n}\sum_{t=1}^{n} \left(\frac{\beta^R_{t,T} - \zeta_{t,T}}{\beta^R_{t,T}}\right)^2. \tag{A.3}$$

Table A.2: Encompassing Estimates for Realized Beta – Daily and Monthly Data

This table presents the results from regressions of ex post realized beta over the horizon of six months on competing ex ante estimates for each of the five portfolios, respectively. Each row corresponds to one regression. Each month, we sort our estimates in an ascending order according to the historical (equation (9)) beta obtained in a sorting period strictly before the estimation period of the historical beta. The sorting and estimation periods are without overlap and have equal length. The stocks are then allocated to quintile portfolios. Portfolio betas and returns are value-weighted. a denotes the regression intercept, b_{HIST}, b_{RW}, b_{DCC}, b_{FGK}, b_{CCJV}, b_{BV} and b_{RPadj} refer to the slope coefficients for the historical, random walk, DCC, FGK, CCJV, BV and risk premium adjusted estimate for beta, respectively. For each regression coefficient we report Newey & West (1987) t-statistics ($t(.)$) using 6 lags, where we test a and b against zero in multivariate regressions. R^2_{adj} denotes the adjusted R^2 of the regressions. The columns $Wald_1$ and $Wald_2$ refer to the Wald test statistic and p gives the corresponding p-values. In multivariate regressions, the joint underlying hypothesis is that the first slope coefficient is equal to one and the second slope coefficient is equal to zero and vice versa for $Wald_1$ and $Wald_2$, respectively. t-statistics and p-values in **bold** font indicate significance at the 5 % level. The sample period is January 1996 until December 2012 with 198 monthly observations.

		a	$t(a)$	b_{HIST_d}	$t(b)$	b_{HIST_m}	$t(b)$	b_{RW_d}	$t(b)$	b_{RW_m}	$t(b)$	b_{BV_d}	$t(b)$	b_{BV_m}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
$HIST_d + HIST_m$	1	0.24	(1.59)	0.73	**(3.16)**	-0.10	(-0.96)									0.34	11.27	**(0.00)**	167.03	**(0.00)**
	2	0.20	(1.57)	0.87	**(5.29)**	-0.08	(-0.81)									0.60	6.37	**(0.00)**	246.71	**(0.00)**
	3	0.13	(1.00)	0.89	**(4.33)**	-0.03	(-0.27)									0.63	3.95	**(0.02)**	154.66	**(0.00)**
	4	-0.05	(-0.53)	0.83	**(5.47)**	0.19	(1.49)									0.76	4.86	**(0.01)**	87.40	**(0.00)**
	5	0.67	**(3.30)**	0.45	**(2.85)**	0.05	(0.40)									0.21	33.92	**(0.00)**	151.58	**(0.00)**
$RW_d + RW_m$	1	0.20	(1.88)					0.82	**(5.13)**	-0.12	(-0.96)					0.45	11.36	**(0.00)**	147.55	**(0.00)**
	2	0.19	**(2.21)**					0.85	**(6.61)**	-0.06	(-0.78)					0.65	11.27	**(0.00)**	213.45	**(0.00)**
	3	0.18	(1.64)					0.79	**(5.10)**	0.01	(0.12)					0.64	10.88	**(0.00)**	189.17	**(0.00)**
	4	0.19	**(2.15)**					0.81	**(9.78)**	0.01	(0.10)					0.71	11.91	**(0.00)**	236.56	**(0.00)**
	5	0.55	**(3.78)**					0.60	**(5.61)**	-0.01	(-0.08)					0.37	28.22	**(0.00)**	228.85	**(0.00)**
$BV_d + BV_m$	1	-0.28	(-1.55)									0.72	**(2.09)**	0.56	(1.55)	0.53	5.90	**(0.00)**	14.45	**(0.00)**
	2	-0.38	**(-2.91)**									1.25	**(4.04)**	0.16	(0.43)	0.75	21.18	**(0.00)**	81.94	**(0.00)**
	3	-0.29	**(-2.75)**									1.26	**(5.34)**	0.02	(0.10)	0.83	22.99	**(0.00)**	88.69	**(0.00)**
	4	-0.37	**(-3.46)**									1.24	**(3.31)**	0.13	(0.30)	0.83	31.14	**(0.00)**	63.57	**(0.00)**
	5	-0.09	(-0.49)									0.82	**(2.45)**	0.31	(0.94)	0.44	2.27	(0.11)	14.89	**(0.00)**

Here, n is the number of estimation windows, $\beta_{t,T}^{R}$ again denotes the realized beta over a period from t until T, and $\zeta_{t,T}$ is the respective beta estimate.[38] Patton (2011) shows that only MSE, as opposed to the loss functions employed here (MAE, MAPE, and MSPE), is robust to the presence of noise in the evaluation proxy. Thus, further care has to be applied when interpreting the results presented here.

Daily Data Table A.3 summarizes the estimation errors using daily return data in more detail. Starting with MAE in Panel A, we observe that BV yields the smallest estimation error (as indicated by *italic* font) for four and RW yields the smallest estimation error for one portfolio(s). On average, BV obtains the lowest error, followed by RW and HIST. Considering RMSE in Panel B, the results are quite similar. Regarding MAPE and MSPE in Panels C and D, the results rather favor RW, but for four and three portfolios BV still yields the smallest MAPE and MSPE, respectively. Performing best in the portfolio with lowest historical betas during the sorting period, RW yields the smallest average MAPE and MSPE. For all loss functions the fully implied CCJV and the GARCH DCC achieve the worst and second-worst performance, respectively.

To further examine the results, we analyze whether the differences we observe in Table A.3 are statistically significant. Table A.4 presents the mean differences in absolute errors (AE), squared errors (SE), absolute percentage errors (APE), and squared percentage errors (SPE) in the upper triangular matrices and the respective median differences in the lower triangular matrices.

Looking at Panel A in Table A.4 we find that BV always obtains lower average mean and median absolute errors than the other methods. These

[38] Note that the percentage loss functions exhibit very high values when realized beta gets close to zero which, unlike in many other situations such as volatility estimation, is certainly possible in the case of beta. Thus, MAPE and MSPE must be interpreted with care.

Table A.3: Estimation Errors: Six-Month Horizon – Daily Data

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. Panels A and B report the mean absolute errors (MAE) and the root mean squared errors (RMSE) of the estimation models for each portfolio, respectively. Panel C reports the mean absolute percentage errors (MAPE) and panel D reports the mean squared percentage errors (MSPE). avg. denotes the respective errors averaged over all five portfolios. For each portfolio and the average, the lowest errors among all approaches are indicated by *italic* font.

Panel A. Mean Absolute Errors (MAE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1114	*0.1024*	0.1732	0.1592	0.2952	0.1762	0.1104	0.1408
2	0.0873	0.0794	0.1687	0.1771	0.2289	0.1028	*0.0693*	0.1229
3	0.0679	0.0699	0.1792	0.1636	0.2040	0.0720	*0.0512*	0.1134
4	0.0621	0.0667	0.1752	0.1757	0.1781	0.0790	*0.0549*	0.1033
5	0.1780	0.1632	0.2684	0.2599	0.2167	0.1743	*0.1407*	0.1536
avg.	0.1013	0.0963	0.1929	0.1871	0.2246	0.1209	*0.0853*	0.1268

Panel B. Root Mean Squared Errors (RMSE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1517	*0.1375*	0.2283	0.2006	0.3498	0.2252	0.1423	0.1883
2	0.1177	0.1110	0.2293	0.2019	0.2929	0.1592	*0.0933*	0.1696
3	0.0971	0.0984	0.2511	0.1879	0.2609	0.1141	*0.0724*	0.1580
4	0.0788	0.0864	0.2636	0.2019	0.2188	0.0999	*0.0735*	0.1475
5	0.2452	0.2174	0.3656	0.3091	0.2693	0.2537	0.2003	*0.1953*
avg.	0.1381	0.1301	0.2676	0.2203	0.2783	0.1704	*0.1164*	0.1718

Table A.3: Estimation Errors: Six-Month Horizon – Daily Data (continued)

Panel C. Mean Average Percentage Errors (MAPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	1.0532	*0.6647*	1.1013	1.0683	2.5807	1.8380	0.9794	1.5415
2	0.1275	0.1171	0.2281	0.2281	0.3479	0.1839	*0.1102*	0.1925
3	0.0892	0.0886	0.2084	0.1850	0.2547	0.1020	*0.0666*	0.1478
4	0.0598	0.0640	0.1680	0.1632	0.1694	0.0715	*0.0507*	0.1006
5	0.1231	0.1141	0.1862	0.1801	0.1589	0.1118	*0.0917*	0.1104
avg.	0.2905	*0.2097*	0.3784	0.3649	0.7023	0.4614	0.2597	0.4186

Panel D: Mean Squared Percentage Errors (MSPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	63.482	*16.589*	49.609	60.094	391.58	187.67	45.473	140.73
2	0.0537	*0.0473*	0.1181	0.0812	0.3632	0.1838	0.0509	0.1549
3	0.0278	0.0239	0.0919	0.0492	0.1556	0.0489	*0.0153*	0.0754
4	0.0066	0.0078	0.0685	0.0357	0.0456	0.0084	*0.0051*	0.0250
5	0.0276	0.0221	0.0614	0.0433	0.0405	0.0212	*0.0140*	0.0200
avg.	12.719	*3.3379*	9.9898	12.061	78.436	37.587	9.1116	28.202

differences are statistically significant for all portfolios compared to DCC, FGK, CCJV, KKS1, and RPadj, whereas when comparing to HIST and RW, the MAE is significantly lower for two and one portfolio(s) and the median AE is significantly lower for four and three portfolios, respectively. HIST and RW outperform all other methods (except KKS1 and BV) for at least four out of the five portfolios. Examining the other loss functions SE, APE, and SPE the picture is quite similar, except that RW obtains the smallest average (mean) errors in both percentage loss functions, but even so the (net) significance is in favor of BV, which also has the smallest average median errors over all loss functions including the percentage loss functions. Nevertheless, the evidence indicates that overall the BV approach obtains the best out-of-sample accuracy, followed by RW and HIST.

Monthly Data Looking at the estimators using monthly return

Table A.4: Differences of Estimation Errors: Six-Month Horizon – Daily Data

This table reports the differences in the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months. In panel A–C, the upper triangular matrix reports the mean differences of absolute (AE), as well as differences in mean absolute percentage (APE), and squared percentage (SPE) estimation errors, respectively, averaged over the five portfolios. Similarly, the lower triangular matrices report the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

Panel A. Absolute Errors (AE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		0.0050	-0.0916	-0.0858	-0.1232	-0.0195	0.0161	-0.0255
		(0.0)	**(-1.0)**	**(-1.0)**	(-0.8)	(-0.2)	(0.4)	(-0.8)
RW	0.0002		-0.0966	-0.0908	-0.1283	-0.0246	0.0110	-0.0305
	(-0.4)		**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.2)	(0.2)	(-0.8)
DCC	0.0639	0.0637		0.0058	-0.0316	0.0721	0.1076	0.0661
	(1.0)	**(1.0)**		(0.0)	(-0.4)	(0.8)	**(1.0)**	**(1.0)**
FGK	0.1001	0.0999	0.0362		-0.0374	0.0662	0.1018	0.0603
	(1.0)	**(1.0)**	(0.2)		(-0.2)	(0.8)	**(1.0)**	(0.8)
CCJV	0.1143	0.1141	0.0504	0.0142		0.1037	0.1393	0.0978
	(1.0)	**(1.0)**	(0.6)	(0.4)		(0.8)	**(1.0)**	**(1.0)**
KKS1	0.0113	0.0110	-0.0526	-0.0888	-0.1031		0.0356	-0.0059
	(0.4)	(0.6)	(-0.8)	(-0.8)	**(-1.0)**		**(1.0)**	(-0.2)
BV	-0.0111	-0.0113	-0.0750	-0.1112	-0.1254	-0.0224		-0.0415
	(-0.8)	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.6)
RPadj	0.0256	0.0254	-0.0383	-0.0745	-0.0888	0.0143	0.0367	
	(0.8)	(0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	(0.2)	**(1.0)**	

Table A.4: Differences of Estimation Errors: Six-Month Horizon – Daily Data (continued)

Panel B. Absolute Percentage Errors (APE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		0.0808	-0.0879	-0.0744	-0.4118	-0.1709	0.0308	-0.1280
		(0.0)	(-0.8)	(-0.8)	(-0.6)	(0.0)	(0.4)	(-0.6)
RW	-0.0015		-0.1687	-0.1552	-0.4926	-0.2517	-0.0500	-0.2089
	(-0.4)		(-0.8)	(-0.8)	(-0.8)	(0.0)	(0.6)	(-0.6)
DCC	0.0643	0.0658		0.0135	-0.3239	-0.0830	0.1187	-0.0402
	(1.0)	**(1.0)**		(0.0)	(-0.2)	(0.6)	**(1.0)**	(0.4)
FGK	0.1016	0.1032	0.0373		-0.3374	-0.0965	0.1052	-0.0536
	(1.0)	**(1.0)**	(0.2)		(0.0)	(0.6)	(0.8)	(0.4)
CCJV	0.1278	0.1294	0.0635	0.0262		0.2409	0.4426	0.2837
	(1.0)	**(1.0)**	(0.8)	(0.0)		(0.8)	(0.8)	(0.8)
KKS1	0.0187	0.0202	-0.0456	-0.0830	-0.1092		0.2017	0.0429
	(0.4)	(0.6)	(-0.8)	(-0.6)	**(-1.0)**		(0.8)	(-0.2)
BV	-0.0083	-0.0068	-0.0726	-0.1099	-0.1361	-0.0270		-0.1588
	(-0.8)	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(-0.6)
RPadj	0.0308	0.0323	-0.0336	-0.0709	-0.0971	0.0121	0.0390	
	(0.8)	(0.8)	**(-1.0)**	(-0.8)	**(-1.0)**	(0.2)	**(1.0)**	

Panel C. Squared Percentage Errors (SPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		9.3816	2.7297	0.6589	-65.7168	-24.8677	3.6079	-15.4823
		(0.0)	(-0.8)	(-0.6)	(-0.6)	(0.0)	(0.0)	(0.0)
RW	-0.0003		-6.6519	-8.7227	-75.0984	-34.2492	-5.7737	-24.8639
	(-0.4)		(-0.8)	(-0.8)	(-0.6)	(0.0)	(0.2)	(0.0)
DCC	0.0142	0.0145		-2.0708	-68.4465	-27.5973	0.8782	-18.2120
	(1.0)	**(1.0)**		(0.2)	(0.0)	(0.4)	(0.8)	(0.4)
FGK	0.0255	0.0258	0.0113		-66.3757	-25.5265	2.9490	-16.1412
	(1.0)	**(1.0)**	(0.4)		(0.0)	(0.4)	(0.6)	(0.2)
CCJV	0.0435	0.0438	0.0293	0.0180		40.8492	69.3247	50.2345
	(1.0)	**(1.0)**	(0.6)	(0.0)		(0.8)	(0.8)	(0.6)
KKS1	0.0064	0.0067	-0.0079	-0.0192	-0.0371		28.4755	9.3853
	(0.4)	(0.4)	(-0.8)	(-0.6)	**(-1.0)**		(0.4)	(0.0)
BV	-0.0008	-0.0005	-0.0150	-0.0263	-0.0442	-0.0071		-19.0902
	(-0.8)	(-0.6)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**		(0.0)
RPadj	0.0062	0.0065	-0.0081	-0.0194	-0.0373	-0.0002	0.0069	
	(0.8)	(0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	(0.2)	**(1.0)**	

Table A.5: Estimation Errors: Six-Month Horizon – Monthly Data

This table reports the out-of-sample estimation errors of competing estimators, using monthly return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. Panels A and B report the mean absolute errors (MAE) and the root mean squared errors (RMSE) of the estimation models for each portfolio, respectively. Panel C reports the mean absolute percentage errors (MAPE) and panel D reports the mean squared percentage errors (MSPE). avg. denotes the respective errors averaged over all five portfolios. For each portfolio and the average, the lowest errors among all approaches are indicated by *italic* font.

Panel A. Mean Absolute Errors (MAE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1504	0.1209	0.1818	0.2784	0.2308	0.1104	*0.0911*	0.2296
2	0.1324	0.1287	0.1799	0.2976	0.2280	0.0593	*0.0478*	0.2213
3	0.1052	0.1229	0.1762	0.2899	0.2176	0.0608	*0.0571*	0.2094
4	0.1005	0.1449	0.1850	0.2852	0.1726	0.0619	*0.0437*	0.1982
5	0.1812	0.1701	0.2156	0.2982	0.2087	0.1036	*0.0822*	0.1772
avg.	0.1339	0.1375	0.1877	0.2899	0.2115	0.0792	*0.0644*	0.2071

Panel B. Root Mean Squared Errors (RMSE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.2067	0.1661	0.2311	0.3078	0.2802	0.1392	*0.1122*	0.2717
2	0.1650	0.1593	0.2279	0.3139	0.2686	0.0784	*0.0645*	0.2652
3	0.1285	0.1504	0.2366	0.3134	0.2596	0.0872	*0.0826*	0.2618
4	0.1256	0.1911	0.2275	0.3034	0.2135	0.0756	*0.0556*	0.2367
5	0.2330	0.2083	0.2997	0.3436	0.2637	0.1713	*0.1412*	0.2041
avg.	0.1718	0.1751	0.2446	0.3164	0.2571	0.1103	*0.0912*	0.2479

Table A.5: Estimation Errors: Six-Month Horizon – Monthly Data (continued)

Panel C. Mean Average Percentage Errors (MAPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1867	0.1568	0.2283	0.3413	0.3085	0.1617	*0.1261*	0.3009
2	0.1465	0.1401	0.1941	0.3194	0.2527	0.0668	*0.0527*	0.2441
3	0.1053	0.1217	0.1739	0.2847	0.2180	0.0592	*0.0554*	0.2116
4	0.0950	0.1377	0.1707	0.2666	0.1660	0.0569	*0.0410*	0.1881
5	0.1518	0.1423	0.1743	0.2418	0.1755	0.0780	*0.0629*	0.1489
avg.	0.1371	0.1397	0.1883	0.2908	0.2241	0.0845	*0.0676*	0.2187

Panel D. Mean Squared Percentage Errors (MSPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.0625	0.0504	0.0822	0.1346	0.1610	0.0575	*0.0301*	0.1529
2	0.0350	0.0316	0.0593	0.1127	0.0929	0.0089	*0.0056*	0.0897
3	0.0166	0.0220	0.0538	0.0927	0.0683	0.0066	*0.0057*	0.0730
4	0.0147	0.0340	0.0419	0.0801	0.0436	0.0047	*0.0028*	0.0526
5	0.0375	0.0291	0.0540	0.0723	0.0496	0.0120	*0.0085*	0.0289
avg.	0.0333	0.0334	0.0582	0.0985	0.0831	0.0179	*0.0105*	0.0794

data in Tables A.5 and A.6, the picture is even clearer. We find that BV, computed with the correlations over the past five years of monthly returns, significantly outperforms all other approaches based on monthly return data.[39] Moreover, BV based on monthly return data seems not to produce larger outliers compared to the other methods, since the results from mean and median loss functions are quite similar. While the fully implied KKS1, that does not rely on return data at all, is the second-best estimator, the historical estimate (HIST), based on five years of monthly returns, significantly outperforms all other approaches at least partially

[39] Note that the even lower average errors compared to the BV approach using daily return data result from the slightly different sorting approach using five years of monthly returns, yielding substantially less dispersion in the respective beta estimates for the portfolios and thereby reducing the probability of large errors (and even more strongly reducing the probability of high percentage errors, as realized beta only rarely comes close to zero). When sorting the daily estimates in the same way all, loss functions yield lower errors when using daily return data.

Table A.6: Differences of Estimation Errors: Six-Month Horizon – Monthly Data

This table reports the differences in the out-of-sample estimation errors of competing estimators, using monthly return data, for realized beta over the horizon of six months. In panel A–D, the upper triangular matrix reports the mean differences of absolute (AE), root mean squared (SE), as well as differences in mean absolute percentage (APE), and squared percentage (SPE) estimation errors, respectively, averaged over the five portfolios. Similarly, the lower triangular matrices report the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

Panel A. Absolute Errors (AE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		-0.0036	-0.0537	-0.1559	-0.0776	0.0547	0.0696	-0.0732
		(0.0)	(-0.6)	**(-1.0)**	(-0.8)	(0.8)	**(1.0)**	(-0.8)
RW	0.0074		-0.0502	-0.1524	-0.0740	0.0583	0.0731	-0.0696
	(0.2)		(-0.6)	**(-1.0)**	(-0.6)	(0.8)	(0.8)	(-0.8)
DCC	0.0373	0.0300		-0.1022	-0.0238	0.1085	0.1233	-0.0195
	(1.0)	(0.6)		**(-1.0)**	(0.0)	**(1.0)**	**(1.0)**	(-0.4)
FGK	0.1695	0.1621	0.1321		0.0783	0.2107	0.2255	0.0827
	(1.0)	**(1.0)**	**(1.0)**		(0.8)	**(1.0)**	**(1.0)**	**(1.0)**
CCJV	0.0801	0.0727	0.0427	-0.0894		0.1323	0.1472	0.0044
	(0.8)	(0.8)	(0.6)	**(-1.0)**		**(1.0)**	**(1.0)**	(0.0)
KKS1	-0.0497	-0.0571	-0.0870	-0.2192	-0.1298		0.0148	-0.1279
	(-0.8)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**		(0.8)	**(-1.0)**
BV	-0.0590	-0.0664	-0.0963	-0.2285	-0.1391	-0.0093		-0.1428
	(-1.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.8)		**(-1.0)**
RPadj	0.0871	0.0797	0.0498	-0.0823	0.0071	0.1368	0.1461	
	(0.8)	(0.8)	(0.6)	**(-1.0)**	(0.0)	**(1.0)**	**(1.0)**	

Table A.6: Differences of Estimation Errors: Six-Month Horizon – Monthly Data (continued)

Panel B. Squared Errors (SE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		-0.0033	-0.0728	-0.1446	-0.0853	0.0614	0.0806	-0.0761
		(-0.4)	(-0.6)	**(-1.0)**	(-0.6)	(0.8)	**(1.0)**	(-0.8)
RW	0.0074		-0.0695	-0.1414	-0.0821	0.0647	0.0838	-0.0728
	(0.4)		(-0.8)	**(-1.0)**	(-0.6)	(0.6)	**(1.0)**	(-0.6)
DCC	0.0373	0.0299		-0.0719	-0.0125	0.1342	0.1534	-0.0033
	(1.0)	(0.6)		(-0.8)	(0.0)	**(1.0)**	**(1.0)**	(0.0)
FGK	0.1695	0.1621	0.1321		0.0593	0.2061	0.2252	0.0685
	(1.0)	**(1.0)**	**(1.0)**		(0.4)	**(1.0)**	**(1.0)**	(0.6)
CCJV	0.0801	0.0727	0.0427	-0.0894		0.1468	0.1659	0.0092
	(0.8)	**(1.0)**	(0.6)	**(-1.0)**		**(1.0)**	**(1.0)**	(0.2)
KKS1	-0.0497	-0.0571	-0.0870	-0.2192	-0.1298		0.0191	-0.1376
	(-0.8)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**		(0.6)	(-0.8)
BV	-0.0590	-0.0664	-0.0963	-0.2285	-0.1391	-0.0093		-0.1567
	(-1.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.8)		(-0.8)
RPadj	0.0871	0.0797	0.0498	-0.0823	0.0071	0.1368	0.1461	
	(0.8)	(0.8)	(0.4)	**(-1.0)**	(-0.2)	**(1.0)**	**(1.0)**	

Panel C. Absolute Percentage Errors (APE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		-0.0026	-0.0512	-0.1537	-0.0871	0.0526	0.0694	-0.0817
		(-0.2)	(-0.6)	**(-1.0)**	(-0.8)	(0.8)	**(1.0)**	(-0.8)
RW	0.0019		-0.0486	-0.1511	-0.0844	0.0552	0.0721	-0.0790
	(0.2)		(-0.6)	**(-1.0)**	(-0.6)	(0.8)	(0.8)	(-0.8)
DCC	0.0368	0.0349		-0.1025	-0.0359	0.1038	0.1206	-0.0305
	(0.8)	(0.6)		**(-1.0)**	(0.0)	(0.8)	**(1.0)**	(-0.2)
FGK	0.1697	0.1678	0.1329		0.0666	0.2063	0.2231	0.0720
	(1.0)	**(1.0)**	**(1.0)**		(0.8)	**(1.0)**	**(1.0)**	(0.8)
CCJV	0.0776	0.0757	0.0408	-0.0921		0.1396	0.1565	0.0054
	(0.8)	(0.8)	(0.6)	**(-1.0)**		**(1.0)**	**(1.0)**	(0.0)
KKS1	-0.0530	-0.0549	-0.0898	-0.2227	-0.1306		0.0169	-0.1342
	(-0.8)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**		(0.8)	**(-1.0)**
BV	-0.0616	-0.0635	-0.0984	-0.2312	-0.1392	-0.0086		-0.1511
	(-1.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.4)		**(-1.0)**
RPadj	0.0877	0.0858	0.0509	-0.0820	0.0101	0.1407	0.1493	
	(0.8)	(0.8)	(0.6)	**(-1.0)**	(0.0)	**(1.0)**	**(1.0)**	

Table A.6: Differences of Estimation Errors: Six-Month Horizon – Monthly Data (continued 2)

Panel D. Squared Percentage Errors (SPE)

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
HIST		-0.0002	-0.0250	-0.0652	-0.0498	0.0153	0.0227	-0.0461
		(-0.2)	(-0.4)	**(-1.0)**	(-0.8)	(0.8)	**(1.0)**	(-0.6)
RW	0.0000		-0.0248	-0.0650	-0.0497	0.0155	0.0229	-0.0460
	(0.4)		(-0.6)	**(-1.0)**	(-0.8)	(0.8)	(0.8)	(-0.8)
DCC	0.0097	0.0097		-0.0402	-0.0248	0.0403	0.0477	-0.0212
	(0.8)	(0.6)		(-0.8)	(0.0)	(0.8)	**(1.0)**	(0.2)
FGK	0.0682	0.0682	0.0585		0.0154	0.0805	0.0879	0.0191
	(1.0)	**(1.0)**	**(1.0)**		(0.2)	**(1.0)**	**(1.0)**	(0.4)
CCJV	0.0243	0.0243	0.0146	-0.0439		0.0651	0.0725	0.0037
	(0.8)	**(1.0)**	(0.6)	**(-1.0)**		**(1.0)**	**(1.0)**	(0.2)
KKS1	-0.0093	-0.0094	-0.0190	-0.0776	-0.0336		0.0074	-0.0615
	(-0.8)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**		(0.6)	**(-1.0)**
BV	-0.0100	-0.0101	-0.0197	-0.0783	-0.0343	-0.0007		-0.0689
	(-1.0)	(-0.8)	**(-1.0)**	**(-1.0)**	**(-1.0)**	(-0.4)		**(-1.0)**
RPadj	0.0296	0.0296	0.0199	-0.0386	0.0053	0.0389	0.0396	
	(0.8)	(0.8)	(0.4)	**(-1.0)**	(-0.2)	**(1.0)**	**(1.0)**	

relying on historical return data, except monthly RW, in at least two of the five portfolios each, while RW is also frequently outperformed for some portfolios.

A.3 Detailed Results on Section 2.3

In this section, we present the results of Section 2.3, namely longer horizons, further models for implied beta, option liquidity, and further time-series models, in more detail. In Table A.7, we report the RMSE of each of the individual portfolios, instead of only the averages. The discussion of the results can be found in Section 2.3.

A.4 Bias Removal

In this section, we provide further insight on the information content of estimators on which we have performed the simple bias removal, scaling the

Table A.7: Portfolio Root Mean Squared Errors (Section 2.3)

This table reports the out-of-sample estimation errors of competing estimators, using daily return data, for realized beta over the horizon of six months for each portfolio in Panels D to F and over the horizon indicated in the panel headlines for Panels A to C. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. We report the root mean squared errors (RMSE) of the estimation models for each portfolio. avg. denotes the errors averaged over all five portfolios. For each portfolio and the average, the lowest errors among all approaches are indicated by *italic* font.

Panel A: One-Month Horizon

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1659	*0.1520*	0.2047	0.2088	0.4956	0.2540	0.1613	0.1883
2	0.1352	0.1293	0.1869	0.1892	0.4499	0.1810	*0.1231*	0.1634
3	0.1202	0.1112	0.1998	0.1907	0.4135	0.1437	*0.1036*	0.1815
4	0.1180	0.1098	0.1971	0.1932	0.3700	0.1381	*0.1086*	0.1826
5	0.2793	0.2550	0.3272	0.3145	0.4134	0.3085	*0.2452*	0.2726
avg.	0.1637	0.1515	0.2231	0.2193	0.4285	0.2051	*0.1483*	0.1977

Panel B: Three-Month Horizon

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1417	*0.1254*	0.1979	0.1885	0.4076	0.2300	0.1344	0.1651
2	0.1176	0.1073	0.1937	0.1867	0.3563	0.1709	*0.1026*	0.1581
3	0.0978	0.0953	0.2057	0.1766	0.3313	0.1241	*0.0800*	0.1676
4	0.0909	0.0916	0.2060	0.1872	0.2787	0.1060	*0.0813*	0.1753
5	0.2421	0.2207	0.3265	0.2894	0.2935	0.2545	*0.1973*	0.2296
avg.	0.1380	0.1281	0.2259	0.2057	0.3335	0.1771	*0.1191*	0.1791

Panel C: Twelve-Month Horizon

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj
1	0.1638	0.1532	0.2761	0.2111	0.3387	0.2190	*0.1492*	0.2038
2	0.1318	0.1269	0.2954	0.2159	0.2643	0.1519	*0.0995*	0.1842
3	0.1102	0.1068	0.3394	0.1995	0.2325	0.1117	*0.0804*	0.1623
4	0.0835	0.0920	0.3559	0.2078	0.2073	0.1045	*0.0807*	0.1505
5	0.2545	0.2327	0.4202	0.3194	0.2942	0.2516	*0.2038*	0.2465
avg.	0.1488	0.1423	0.3374	0.2307	0.2674	0.1677	*0.1227*	0.1895

Table A.7 Portfolio Root Mean Squared Errors (Section 2.3) (continued)

Panel D. Further Implied

	HIST	RW	CCJV	SR	KKS1	KKS2	FGK	RPadj	BV
1	0.1517	*0.1375*	0.3498	0.2383	0.2252	0.2279	0.2006	0.1883	0.1423
2	0.1177	0.1110	0.2929	0.1660	0.1592	0.1598	0.2019	0.1696	*0.0933*
3	0.0971	0.0984	0.2609	0.1234	0.1141	0.1142	0.1879	0.1580	*0.0724*
4	0.0788	0.0864	0.2188	0.0900	0.0999	0.1007	0.2019	0.1475	*0.0735*
5	0.2452	0.2174	0.2693	0.2193	0.2537	0.2575	0.3091	*0.1953*	0.2003
avg.	0.1381	0.1301	0.2783	0.1674	0.1704	0.1720	0.2203	0.1718	*0.1164*

Panel E. DJIA

	HIST	RW	DCC	CCJV	SR	KKS1	KKS2	FGK	RPadj	BV
1	0.1375	*0.1274*	0.2003	0.3062	0.2985	0.1889	0.1829	0.1771	0.1829	0.1517
2	0.1763	0.1842	0.2710	0.2192	0.1884	0.1708	0.1632	0.2363	0.2102	*0.1604*
avg.	0.1569	*0.1558*	0.2356	0.2627	0.2434	0.1798	0.1730	0.2067	0.1965	0.1560

Panel F. Further Time-Series Models

	HIST	HIST_6	RW	RW_D	AR	ARMA	DCC	CCC	BV
1	0.1499	*0.1347*	0.1359	0.1642	0.1879	0.1702	0.2275	0.2047	0.1441
2	0.1144	0.1080	0.1061	0.1593	0.1699	0.1440	0.2266	0.2067	*0.0953*
3	0.0921	0.0963	0.0950	0.1515	0.1468	0.1237	0.2448	0.2247	*0.0699*
4	0.0782	0.0800	0.0844	0.1455	0.1400	0.1610	0.2633	0.2549	*0.0722*
5	0.2430	0.2225	0.2130	0.3144	0.2878	0.3502	0.3570	0.3949	*0.1947*
avg.	0.1355	0.1283	0.1269	0.1870	0.1865	0.1899	0.2638	0.2572	*0.1152*

estimates so that the value-weighted cross-sectional average beta of each estimation techniques equals one (Section 2.4.6). We perform univariate and encompassing regressions as in equation (2.28). The results are presented in Table A.8.

For HIST and RW the bias (as can be seen in Table 2.1) is very small and the removal does not change much. Considering the Wald test, for only one portfolio the null hypothesis of unbiasedness cannot be rejected for HIST. DCC and CCJV are still biased, but their explanatory power

Table A.8: Univariate and Encompassing Estimates for Realized Beta – Bias Removal

This table presents the results from regressions of ex post realized beta on competing ex ante estimates for each of the five portfolios, respectively. Each row corresponds to one regression. Each month, we sort our estimates in an ascending order according to the historical (equation (9)) beta obtained in a sorting period strictly before the estimation period of the historical beta. The sorting and estimation periods are without overlap and have equal length. The stocks are then allocated to quintile portfolios. Portfolio betas and returns are value-weighted. a denotes the regression intercept, b_{HIST}, b_{RW}, b_{DCC}, b_{FGK}, b_{CCJV}, b_{KKS1}, b_{BV}, and b_{RPadj} refer to the slope coefficients for the historical, random walk, DCC, FGK, CCJV, KKS1, BV, and risk premium adjusted estimate for beta, respectively. For each regression coefficient we report Newey & West (1987) t-statistics ($t(.)$) using 6 lags, where we test a against zero and b against one in the univariate regressions (Panel A), while we test both against zero in the multivariate regressions (Panel B). R^2_{adj} denotes the adjusted R^2 of the regressions. The columns $Wald_1$ and $Wald_2$ refer to the Wald test statistic and p gives the corresponding p-values. In the univariate regressions in Panel A, for the Wald test (reported in $Wald_1$) we test the joint hypothesis of the intercept being equal to zero and the slope coefficient being equal to one. In the multivariate regressions in Panel B, the joint underlying hypothesis is that the first slope coefficient is equal to one and the second slope coefficient is equal to zero and vice versa for $Wald_1$ and $Wald_2$, respectively. t-statistics and p-values in **bold** font indicate significance at the 5 % level. The sample period is January 1996 until December 2012 with 198 (188 when RPadj is included) monthly observations.

Panel A. Univariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
HIST	1	0.20	(1.30)	0.72	(-1.22)															0.35	13.44	**(0.00)**
	2	0.16	(1.31)	0.86	(-1.05)															0.63	21.67	**(0.00)**
	3	0.04	(0.28)	0.96	(-0.24)															0.66	1.62	(0.20)
	4	-0.03	(-0.40)	1.02	(0.25)															0.80	3.43	**(0.03)**
	5	0.71	**(5.23)**	0.49	**(-5.66)**															0.24	39.38	**(0.00)**
RW	1	0.17	(1.51)			0.76	(-1.42)													0.45	12.12	**(0.00)**
	2	0.20	**(2.27)**			0.79	**(-2.12)**													0.65	22.75	**(0.00)**
	3	0.17	(1.53)			0.83	(-1.56)													0.64	8.35	**(0.00)**
	4	0.20	**(2.96)**			0.82	**(-3.03)**													0.71	12.72	**(0.00)**
	5	0.57	**(5.13)**			0.58	**(-5.36)**													0.37	33.34	**(0.00)**

Table A.8: Univariate and Encompassing Estimates for Realized Beta – Bias Removal (continued)

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p
DCC	1	0.39	**(4.01)**					0.42	**(-4.05)**											0.23	67.85	**(0.00)**
	2	0.40	**(4.19)**					0.56	**(-4.10)**											0.53	87.22	**(0.00)**
	3	0.37	**(2.69)**					0.61	**(-2.83)**											0.45	35.89	**(0.00)**
	4	0.53	**(3.54)**					0.50	**(-3.89)**											0.39	63.78	**(0.00)**
	5	0.80	**(9.54)**					0.41	**(-11.99)**											0.36	135.22	**(0.00)**
FGK	1	0.19	(1.19)							0.70	(-1.31)									0.35	9.67	**(0.00)**
	2	0.13	(1.14)							0.86	(-1.10)									0.65	7.55	**(0.00)**
	3	0.04	(0.28)							0.95	(-0.32)									0.73	0.73	(0.48)
	4	-0.11	(-1.22)							1.10	(1.25)									0.76	2.90	(0.06)
	5	0.49	**(3.44)**							0.67	**(-3.12)**									0.31	13.76	**(0.00)**
CCJV	1	-0.89	**(-3.16)**									1.90	**(2.67)**							0.45	189.18	**(0.00)**
	2	-1.05	**(-4.72)**									2.08	**(4.63)**							0.65	93.58	**(0.00)**
	3	-0.85	**(-3.16)**									1.86	**(3.15)**							0.66	51.58	**(0.00)**
	4	-0.56	**(-4.98)**									1.59	**(5.55)**							0.73	81.18	**(0.00)**
	5	-0.32	(-1.31)									1.39	**(2.07)**							0.34	82.04	**(0.00)**
KKS1	1	-0.61	(-1.64)											1.56	(1.23)					0.25	128.08	**(0.00)**
	2	-1.22	**(-3.86)**											2.27	**(3.65)**					0.37	44.23	**(0.00)**
	3	-0.77	**(-1.99)**											1.78	**(1.99)**					0.49	25.23	**(0.00)**
	4	-0.34	**(-2.64)**											1.38	**(3.21)**					0.70	65.79	**(0.00)**
	5	0.19	(0.61)											0.97	(-0.14)					0.23	46.67	**(0.00)**
BV	1	-0.16	(-0.84)													1.13	(0.49)			0.50	37.11	**(0.00)**
	2	-0.32	**(-2.50)**													1.34	**(2.42)**			0.75	26.82	**(0.00)**
	3	-0.30	**(-2.63)**													1.30	**(2.57)**			0.80	27.26	**(0.00)**
	4	-0.33	**(-3.58)**													1.33	**(3.90)**			0.80	36.75	**(0.00)**
	5	-0.05	(-0.19)													1.10	(0.49)			0.42	21.69	**(0.00)**
RPadj	1	0.21	(1.35)															0.71	(-1.28)	0.35	13.61	**(0.00)**
	2	0.18	(1.50)															0.82	(-1.40)	0.62	14.97	**(0.00)**
	3	0.09	(0.67)															0.90	(-0.69)	0.68	2.65	(0.07)
	4	-0.03	(-0.40)															1.02	(0.32)	0.79	1.43	(0.24)
	5	0.61	**(4.46)**															0.56	**(-4.75)**	0.34	31.93	**(0.00)**

Panel B. Multivariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	b_{RW}	$t(b)$	b_{DCC}	$t(b)$	b_{FGK}	$t(b)$	b_{CCJV}	$t(b)$	b_{KKS1}	$t(b)$	b_{BV}	$t(b)$	b_{RPadj}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
FGK + BV	1	-0.16	(-0.78)							0.01	(0.03)					1.12	**(2.62)**			0.50	42.00	**(0.00)**	1.24	(0.29)
	2	-0.34	(-1.96)							-0.07	(-0.34)					1.44	**(3.77)**			0.75	45.89	**(0.00)**	19.38	**(0.00)**
	3	-0.32	(-1.90)							-0.05	(-0.17)					1.36	**(3.18)**			0.80	34.58	**(0.00)**	21.14	**(0.00)**
	4	-0.30	**(-3.18)**							0.22	(0.93)					1.08	**(3.84)**			0.80	23.36	**(0.00)**	25.91	**(0.00)**
	5	-0.14	(-0.48)							-0.26	(-0.86)					1.44	**(2.95)**			0.42	31.95	**(0.00)**	1.83	(0.16)
BV + RPadj	1	-0.15	(-0.67)													1.08	**(2.22)**	0.03	(0.11)	0.49	3.55	**(0.03)**	35.82	**(0.00)**
	2	-0.37	**(-2.27)**													1.52	**(4.33)**	-0.13	(-0.66)	0.76	24.97	**(0.00)**	64.97	**(0.00)**
	3	-0.36	**(-2.61)**													1.54	**(4.83)**	-0.18	(-0.95)	0.80	24.32	**(0.00)**	60.85	**(0.00)**
	4	-0.28	**(-2.65)**													0.95	**(3.80)**	0.32	(1.64)	0.81	30.87	**(0.00)**	9.21	**(0.00)**
	5	-0.16	(-0.35)													1.33	**(1.80)**	-0.13	(-0.34)	0.41	2.37	(0.10)	46.17	**(0.00)**

increases substantially compared to the non-bias-removed estimates. The biggest impact of the bias removal is obtained on FGK and RPadj, leaving both unbiased for two out of five portfolios after setting their value-weighted cross-sectional average to one. Nevertheless, in encompassing regressions together with BV, BV is still shown to be informationally more efficient.

Table A.9 presents the results of further possibilities to try and remove the bias in the estimates. In particular, we perform the regressions as described in Section 2.4.6 on the level of individual estimates. This might be more precise than the portfolio approach considered in the main part. For

Table A.9: Bias Removal – Further Possibilities

This table reports the out-of-sample estimation errors of competing bias-removed estimators, using daily return data, for realized beta over the horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in an ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas and returns as value-weighted averages. Panel A presents the results on a simple bias removal, while Panels B and C present the results on bias removals using a regression technique. In each panel, the first row reports the average root mean squared errors (RMSE) of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the tables report the difference in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences. BV^{UC} refers to the non-corrected BV estimates.

Table A.9: Bias Removal – Further Possibilities (continued)

Panel A. Regression Technique – Individual

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj	BV^{UC}
avg.	0.2034	0.2066	0.1715	0.1791	0.1570	0.1609	0.1699	0.1754	*0.1216*
HIST		-0.0033	0.0318	0.0243	0.0464	0.0425	0.0335	0.0280	0.0818
		(0.0)	(0.2)	(0.0)	(0.4)	(0.6)	(0.2)	(0.2)	(0.8)
RW	0.0039		0.0351	0.0276	0.0497	0.0458	0.0367	0.0312	0.0851
	(0.0)		(0.2)	(0.0)	(0.4)	(0.6)	(0.2)	(0.2)	(0.6)
DCC	-0.0118	-0.0157		-0.0075	0.0146	0.0106	0.0016	-0.0039	0.0499
	(-0.6)	(-0.8)		(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(0.6)
FGK	-0.0063	-0.0101	0.0055		0.0221	0.0182	0.0092	0.0037	0.0575
	(-0.6)	(0.0)	(0.0)		(0.2)	(0.0)	(0.0)	(0.0)	(0.8)
CCJV	-0.0127	-0.0166	-0.0009	-0.0064		-0.0039	-0.0129	-0.0184	0.0354
	(-0.6)	(-0.8)	(-0.2)	(-0.4)		(0.0)	(0.0)	(-0.2)	(0.4)
KKS1	-0.0083	-0.0122	0.0035	-0.0021	0.0043		-0.0090	-0.0145	0.0393
	(-0.4)	(-0.8)	(-0.2)	(-0.4)	(0.0)		(0.0)	(0.0)	(0.6)
BV	-0.0046	-0.0085	0.0072	0.0017	0.0081	0.0038		-0.0055	0.0483
	(-0.6)	(-0.4)	(0.2)	(-0.2)	(0.4)	(0.0)		(0.0)	(0.6)
RPadj	-0.0049	-0.0088	0.0069	0.0013	0.0077	0.0034	-0.0004		0.0538
	(-0.6)	(-0.4)	(0.2)	(0.0)	(0.4)	(0.4)	(0.2)		(0.6)
BV^{UC}	-0.0360	-0.0399	-0.0242	-0.0298	-0.0234	-0.0277	-0.0315	-0.0311	
	(-0.8)	(-0.8)	(-0.8)	(-0.8)	(-0.6)	(-0.8)	(-0.8)	(-0.8)	

Panel B. Regression Technique Combining with HIST – Individual

	HIST	RW	DCC	FGK	CCJV	KKS1	BV	RPadj	BV^{UC}
avg.	0.2034	0.2225	0.2045	0.1950	0.1935	0.1926	0.1808	0.1942	*0.1216*
HIST		-0.0191	-0.0011	0.0083	0.0098	0.0108	0.0225	0.0091	0.0818
		(-0.2)	(0.0)	(0.2)	(0.0)	(0.0)	(0.2)	(0.0)	(0.8)
RW	0.0063		0.0179	0.0274	0.0289	0.0299	0.0416	0.0282	0.1009
	(0.4)		(0.2)	(0.2)	(0.4)	(0.6)	(0.4)	(0.2)	(0.6)
DCC	-0.0025	-0.0088		0.0095	0.0110	0.0119	0.0237	0.0103	0.0829
	(0.0)	(-0.4)		(0.2)	(0.0)	(0.0)	(0.2)	(0.0)	(0.8)
FGK	0.0047	-0.0016	0.0072		0.0015	0.0025	0.0142	0.0008	0.0735
	(0.2)	(-0.2)	(-0.4)		(0.0)	(-0.2)	(0.0)	(0.0)	(0.8)
CCJV	-0.0029	-0.0092	-0.0005	-0.0076		0.0009	0.0127	-0.0007	0.0719
	(-0.4)	(-0.8)	(0.2)	(-0.4)		(0.0)	(0.0)	(0.0)	(0.6)
KKS1	-0.0021	-0.0084	0.0004	-0.0068	0.0008		0.0117	-0.0016	0.0710
	(-0.2)	(-0.2)	(-0.2)	(-0.4)	(0.0)		(0.0)	(0.2)	(0.8)
BV	0.0009	-0.0054	0.0034	-0.0038	0.0039	0.0030		-0.0134	0.0593
	(0.0)	(-0.4)	(-0.2)	(-0.4)	(0.2)	(-0.2)		(0.0)	(0.8)
RPadj	0.0032	-0.0031	0.0057	-0.0015	0.0061	0.0053	0.0023		0.0726
	(0.2)	(-0.4)	(0.0)	(0.0)	(0.2)	(0.0)	(0.2)		(0.8)
BV^{UC}	-0.0360	-0.0423	-0.0336	-0.0407	-0.0331	-0.0339	-0.0370	-0.0392	
	(-0.8)	(-0.8)	(-0.6)	(-0.8)	(-0.8)	(-0.8)	(-0.8)	(-0.6)	

each firm, we first regress the six-month ex post realized beta on the ex ante estimates obtained by each approach using the estimates and realizations available at time t during the period $t-17$ up to $t-6$ (as realized beta with a six-month window is only available up to $t-6$ at time t), namely 12 monthly observations. After obtaining the regression coefficients $\hat{a}$ and $\hat{b}_{\beta}$, we manipulate the current estimates using the following equation:

$$\beta_{j,t}^{\text{ADJ}} \quad = \quad \hat{a} + \hat{b}\beta_{j,t}^{\text{UNADJ}}, \tag{A.4}$$

where $\beta_{j,t}^{\text{ADJ}}$ and $\beta_{j,t}^{\text{UNADJ}}$ are the adjusted and unadjusted estimates, respectively. In a second approach, analog to the main part, we combine the estimates with HIST.

In Panel A of Table A.9, we present the results for the individual regression approach. It can be seen that all approaches produce a larger average RMSE compared to the uncorrected BV^{UC}. Combining the estimates with HIST, shown in Panel B, also does not yield an improvement.

Chapter 3

The Value of High-Frequency Data for Beta Estimation*

3.1 Introduction

Despite being regularly challenged in empirical studies, the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966), it appears, has "survived" during the last fifty years and there is some indication that it is still the most widely used model for applications in financial economics, especially in practice (Graham & Harvey, 2001). The major reasons for that may lie in the model's simplicity and intuitive appeal, predicting that the equilibrium rates of return are solely determined by one factor which captures an asset's exposure to systematic risk, i.e., its beta.

Using high-frequency data has proven useful in many fields of financial economics. This holds especially for the estimation of volatility. There is a vast amount of evidence suggesting that using high-frequency data

*This chapter is based on the Working Paper "The Value of High-Frequency Data for Beta Estimation" authored by Fabian Hollstein, Marcel Prokopczuk, and Chardin Wese Simen, 2015.

significantly improves volatility forecasts as well as value-at-risk calculations (e.g., Andersen & Bollerslev, 1998; Andersen, Bollerslev, Diebold, & Labys, 2003; Chen & Ghysels, 2011). There also is evidence that higher moments, i.e. skewness and kurtosis, can be measured more precisely using high-frequency data. Amaya et al. (2015) find relations of high-frequency realized skewness and kurtosis with subsequent returns. Furthermore, Bollerslev & Zhang (2003) show that, using high-frequency data, the factor loadings of the twenty-five Fama–French portfolios can be measured with increased accuracy. Additionally, the use of high-frequency data is beneficial not only from a statistical but also from an economic perspective. For instance, Fleming, Kirby, & Ostdiek (2003) establish the economic value of high-frequency data in an asset allocation setting.

Recent advances in the estimation of beta using high-frequency return data and data implied from the options market suggest that changes in beta can be captured more easily compared to classically employed long historical windows and partially give rise to a possible (empirical) revival of the conditional version of the classical CAPM. Andersen et al. (2006) show that, under weak regulatory conditions, realized beta delivers a consistent measure of the true underlying integrated beta. This finding theoretically motivates the use of finer grids for sampling return data. To the best of our knowledge, though, we are the first to provide a comprehensive and thorough empirical study on the statistical and economic performance of option-implied and historical market beta estimation techniques, including high-frequency return data.

Chapter 2 shows that, among many competing approaches, the Buss & Vilkov (BV) (2012) hybrid estimator, using options prices and daily return data, adheres a superior performance for estimating beta compared to several historical, time-series, and option-implied approaches relying on daily and monthly return data. However, unlike in Chapter 2, in this chapter we make use of the potentially superior high-frequency return data.

We make a number of additional contributions examining several aspects of beta estimation. First, we examine the statistical properties, i.e. the informational efficiency and estimation accuracy of ex ante estimators for ex post beta using high-frequency realized beta for both the statistical examination of beta estimates and as an additional historical estimator for beta. Additionally, we provide evidence on optimal combinations and bias-corrections of different estimators to obtain more precise estimates.

We also impose an economic evaluation criterion for the analysis. While it appears appealing to be able to predict future beta, the most important property an estimate for beta ought to have is its significance in explaining securities' returns. Cross-sectionally, higher beta firms should have higher expected returns. Consequently, we employ a cross-sectional test to evaluate the empirical validity of the risk–return trade-off for the historical daily, high-frequency, and hybrid estimators with various specifications and over various different time horizons. To the best of our knowledge, we are the first to compare the cross-sectional implications for the risk–return trade-off of historical daily, high-frequency, and hybrid estimators.

Furthermore, we provide evidence on the estimation of downside beta. Downside risk and conditional risk premia have recently attracted much attention. The use of high-frequency data for downside beta is potentially very important since the general difficulty in estimating it is the lack of regular return observations below a certain threshold. However, the estimation of downside beta, thus far, has received only little attention. We fill this gap providing an empirical analysis on the estimation accuracy and the cross-sectional pricing of downside beta employing historical daily, high-frequency, and hybrid estimation methods.

Our main results can be summarized as follows. The use of high-frequency data for beta estimation does not in general appear to create value. While from a statistical viewpoint the use of high-frequency return data can be beneficial, the economic value of using high-frequency return data appears to be limited.

Regarding informational efficiency and estimation accuracy, high-frequency historical and hybrid estimators work more or less equally well. Especially over short time horizons of up to three months, the historical high-frequency estimator is shown to provide proper conditional estimates superior to those of the hybrid and daily historical models, whereas over longer time horizons the hybrid BV yields slightly better estimation accuracy.

For the most important aspect of beta estimation, economic value, the hybrid BV estimator family turns out to clearly outperform historical daily and high-frequency models. On average, a significantly positive relation of beta and subsequent excess returns can be detected, albeit not of the magnitude predicted by the CAPM. For the historical models with daily returns, only a weak relation is found, while for the high-frequency estimators a positive risk–return trade-off cannot be uncovered at all.

Additionally, we show that the main results of Chapter 2 hold using high-frequency realized beta to evaluate competing estimators. The approach proposed by Buss & Vilkov (2012), as well as high-frequency estimators, turns out to be informationally more efficient compared to the historical estimator and produces lower, though not always significantly lower, estimation errors.

We also show that, once employing high-frequency returns, the actual sampling frequency for historical realized beta estimators is of second-order importance examining sampling intervals of five minutes and more. Differences in estimation errors among the high-frequency estimators are typically small and insignificant, while estimation errors generally rise slightly with decreasing sampling frequency. High-frequency approaches are typically informationally more efficient and more accurate compared to historical estimators relying on daily return data. Using high-frequency correlations for BV, on the other hand, does not in general improve the estimation accuracy.

Regarding adjustments and combinations of estimators, we find that a simple combination of the high-frequency and hybrid BV approaches quite consistently outperforms the individual approaches from a statistical evaluation standpoint while its economic value, in turn, is also inferior compared to that of only using BV.

Lastly, our results extend to the estimation of downside beta. The estimation accuracy of high-frequency and hybrid estimators is more or less equally good, while only the hybrid BV estimator has power in explaining the cross-section of subsequent stock returns.

We show that our results are robust to various alternative specifications. Building more portfolios, using alternative sampling frequencies, or different time horizons from one month up to two years for the evaluation of beta estimates, the results are qualitatively equal. We further show that our results on the economic value of beta estimation are robust to different evaluation frequencies also ranging from one month to two years.

Turning the focus on the classical methodology employed for beta estimation, simply using long historical windows of monthly return data has the major drawback is that beta coefficients are shown to exhibit significant time variation (e.g., Blume, 1975; Ferson & Harvey, 1991, 1993). To obtain conditional estimates of beta, Lewellen & Nagel (2006) suggest relying on short historical windows. On the other hand, to obtain a reliable estimate, one needs a large sample of observations, which implies a trade-off between precision and the need for truly conditional estimates. Recent developments for estimating beta using high-frequency data may serve to reconcile these two arguments. Bollerslev & Zhang (2003), Barndorff-Nielsen

& Shephard (2004), and Andersen et al. (2005, 2006) derive the estimator for realized beta and examine its properties. The use of high-frequency data can deliver the observations needed to make use of the results of Andersen et al. (2006) that realized beta yields a consistent estimate of the true underlying integrated beta without relying on very long historical windows, which imposes assumptions on the stability of the underlying processes and economic conditions which, in reality, may fluctuate heavily over time.

A part of the analysis related to the above argument is the question of optimal sampling frequency. While realized beta is a consistent measure for the true underlying integrated beta, on the one hand, it should be optimal to use time-frames as small as possible to obtain as many observations as feasible. On the other hand, though, due to microstructure noise and infrequent trading, this strategy will fail at some point. Already in the 1970s, when daily returns started becoming available for empirical research, authors argued that covariances were severely underestimated when assets are infrequently and non-synchronously traded (Scholes & Williams, 1977; Epps, 1979). Naturally, when using intra-day data, the problem of infrequent trading becomes even more severe. For example, Bollerslev, Li, & Todorov (2015) use intervals as long as 75 minutes to account for such concerns. Since we concentrate on the S&P 500, i.e., the largest companies in the U.S. that are presumably very liquid and frequently traded, it is likely that reliable estimates can also be obtained using higher sampling frequencies. Consequently, we analyze the effects of different sampling schemes on estimation accuracy.

The major alternative to using historical return data only is to additionally incorporate the inherently forward-looking information incorporated in option prices. Buss & Vilkov (2012) and Chang et al. (2012) show that beta estimators employing information from the options market perform well in predicting the cross-section of stock returns. However, Buss & Vilkov (2012) evaluate beta only based on one-month subsequent returns and none of the two studies compares with the potentially superior high-frequency historical estimates for beta.

Presenting a comprehensive study comparing market beta estimation techniques, Chapter 2 shows that the hybrid methodology of BV is informationally more efficient and has smaller estimation errors compared to all other approaches examined (including GARCH-based and fully implied methods) relying on daily return data. However, in Chapter 2 we also find that the simple historical estimator based on daily returns works comparatively well, not yielding estimation errors that are significantly higher than those of BV for all the specifications examined. In particular, the simple historical estimator is shown to be clearly superior to any estimator that uses option-implied data only or GARCH-specifications. Therefore, it appears worthwhile investigating whether the hybrid approach of BV is still favorable when using beta estimated with high-frequency return data as a competing estimator.[1]

Finally, we connect to the literature on downside beta. Ang et al. (2006a) use the disappointment aversion model of Gul (1991) to demonstrate that assets with higher betas, conditional on low realizations of the market return, can be regarded as particularly risky. Lettau et al. (2014) show that the downside risk CAPM can price the cross-section of returns of equities, and many other asset classes. However, we use ex ante estimates for downside beta instead of examining only a contemporaneous relationship and further study the properties of various different methods to estimate downside beta.

The remainder of this chapter is organized as follows. Section 3.2

[1] One argument for why high-frequency data might not yield better estimates is provided by Gilbert, Hrdlicka, Kalodimos, & Siegel (2014). They argue that for opaque firms, the market needs longer to understand the implications of news on systematic risk.

describes our data set and methodology. In Section 3.3 we present our empirical results. Section 3.4 checks the robustness of our results. Finally, Section 3.5 concludes. The appendix to this chapter contains details on the estimation of option-implied moments.

3.2 Data and Methodology

3.2.1 Data

We base our study on the S&P 500 market index and its constituents for the sample period between January 01, 1996 and December 31, 2014.[2] Additionally, we perform a robustness analysis on a sample based on the Dow Jones Industrial Average (DJIA).[3]

We obtain daily and monthly price data as well as data on dividend payments and shares outstanding from the Center for Research in Security Prices (CRSP) for the period from January 01, 1994 until December 31, 2014. To be able to compute historical estimates right from the start of our study period and to perform a portfolio sorting using non-overlapping data, this data starts two years before the main sample period. High-frequency return data is gathered from the Thomson Reuters Tick History (TRTH) database. We sample the data at five-minute intervals. Additionally, we examine different sampling frequencies of up to 130 minutes. To ensure the reliability of the high-frequency data, we perform the appropriate standard data cleaning operations as outlined in Barndorff-Nielsen, Hansen, Lunde, & Shephard (2009).

[2] The starting date of our study is thereby determined by the start of the OptionMetrics and Tick History databases in January 1996.

[3] The sample period for the DJIA dataset begins on January 01, 1998 as options on the DJIA are traded no earlier than October 1997 at the Chicago Board of Options Exchange (CBOE). We do not start before the beginning of the new year to avoid spurious findings caused by potentially small initial trading volumes in the new market.

Options data are from the IvyDB OptionMetrics Volatility Surface that directly provides implied volatilities for standardized delta levels and maturities.[4] For the main analysis, we use options with approximately six months to maturity since we want to obtain six-month estimates for beta. As a robustness check, we also repeat the analysis with options of approximately one, three, twelve, eighteen, and twenty-four months to maturity. We select out-of-the-money (OTM) options, namely puts with deltas larger than -0.5 and calls with deltas smaller than 0.5. We use the formulas provided by Bakshi et al. (2003) to compute model-free implied moments. A more detailed outline of the procedure is presented in Section B.1 in the appendix to this chapter.

We thereby obtain options and high-frequency return data for 438 and 447 stocks in 1996 growing to 493 and 488 stocks at the respective peaks, both in 2010, out of the 500 contained in the S&P 500 at each respective date, respectively.[5] On average, options data on 472 stocks and sufficient high-frequency return data on 478 stocks is available. Data on the risk-free rate is collected from the IvyDB zero curve file.

3.2.2 Beta Estimation

Realized Beta Following Andersen et al. (2006) we use high-frequency log-returns to compute realized beta:

$$\beta_{j,t}^{\mathrm{R}} = \frac{\sum_{\tau=1}^{N} r_{j,\tau} r_{M,\tau}}{\sum_{\tau=1}^{N} r_{M,\tau}^2}, \tag{3.1}$$

[4]IvyDB uses a kernel smoothing algorithm and only reports standardized options "if there exists enough option price data on that date to accurately interpolate the required values". For more details refer to the IvyDB technical document.

[5]Note that options data was only available until the end of August 2014 when we started this study. The first estimate for high-frequency beta is made at the end of June 1996 since we need six months to obtain the estimates and the TRTH database starts no earlier than January 01, 1996.

where $r_{j,\tau}$ and $r_{M,\tau}$ refer to the return of asset j and the market return at time τ, respectively. N is the number of observations during the time period under investigation. While Hansen & Lunde (2006) strongly advise using realized volatility when evaluating volatility models, we follow that spirit using ex post realized beta to evaluate all the respective ex ante estimates obtained using the different beta estimation methods. As an additional estimator, we denote ex ante realized beta by $\mathrm{HF}_{freq,\tau\text{ mon}}$ with $freq$ being the sampling frequency of the returns and τ mon indicating the length of the estimation period. Whenever τ mon is missing, the length of the estimation period matches that of the evaluation period.

Historical Beta Closely related to the above approach, we compute historical estimates (HIST) in the usual way, following Fama & MacBeth (1973) and many others, regressing an asset's excess return on the market excess return:

$$\beta_{j,t}^{\mathrm{HIST}} = \frac{\mathrm{cov}(r_j, r_M)}{\mathrm{var}(r_M)}. \tag{3.2}$$

The main historical estimator utilizes one year of daily returns as do, e.g., Baker et al. (2010).

Hybrid Beta We consider the approach of Buss & Vilkov (2012), who combine model-free implied volatilities and historical correlations to estimate beta. The authors use the property that the implied variance of the market index has to be the same as the implied variance of the value-weighted portfolio of all market constituents (first relation) and combine that with a technical condition for implied correlations to translate from physical ($\rho_{ij,t}^{\mathbb{P}}$) to risk-neutral correlations ($\rho_{ij,t}^{\mathbb{Q}}$), namely $\rho_{ij,t}^{\mathbb{Q}} = \rho_{ij,t}^{\mathbb{P}} - \alpha_t(1 - \rho_{ij,t}^{\mathbb{P}})$.[6] Combining these two relations and solving for α_t, implied correlations

[6] Making sure both that the matrix is a correlation matrix (all correlations not exceeding one and the matrix being positive definite) and that it matches with empirical observations, namely that implied correlations are higher than empirical ones and that the correlation risk premium is higher for lowly correlated stocks. For more details, refer to Buss & Vilkov (2012).

can be computed. Thus, a beta estimate under the risk-neutral probability measure is obtained by:

$$\beta_{j,t}^{\text{BV}} = \frac{\sigma_{j,t}^{\mathbb{Q}} \sum_{i=1}^{N} (\omega_{i,t} \sigma_{i,t}^{\mathbb{Q}} \rho_{ji,t}^{\mathbb{Q}})}{(\sigma_{M,t}^{\mathbb{Q}})^2}, \tag{3.3}$$

where $\sigma_{j,t}^{\mathbb{Q}}$ and $\sigma_{M,t}^{\mathbb{Q}}$ denote the implied volatilities for individual stocks and the market index, respectively. $\omega_{i,t}$ denotes the weight of the N individual assets in the market index at a certain point in time. We utilize the BV approach in the usual style using daily returns over one year to compute correlations. We also use different time horizons to estimate correlations matching the evaluation horizon (denoted by $\text{BV}_{\tau \text{ mon}}$) and high-frequency correlations (BV_{freq}) to obtain alternative specifications for BV, where the variables are as previously defined. The implied volatilities needed for the approach are extracted from options whose expiration matches the evaluation horizon, i.e. six months for the main analysis.

3.3 Empirical Results

3.3.1 Summary Statistics and Correlation Analysis

Panel A of Table 3.1 reports summary statistics on the different beta estimation techniques. It can be seen that the value-weighted average beta over all stocks in the S&P 500 (Mean_{vw}) is very close or exactly equal to one for all approaches. Thereby, it can be seen that the problem of infrequent trading is not severe, since even for the five-minute interval the value-weighted average beta is only slightly below one. The equally-weighted average is lower for the high-frequency models compared to the hybrid models. Consequently, it appears that smaller firms tend to have lower betas for high-frequency estimators compared to the BV methods. Furthermore, it

Table 3.1: Summary Statistics and Sample Correlations

This table provides summary statistics on the different beta estimation techniques (Panel A) and sample correlation coefficients among the different beta estimation techniques on the basis of pooled individual estimates over the entire sample period (Panel B). The sample period spans from January 1996 (beginning with estimates for February 1996) until December 2014. Nobs denotes the number of monthly estimates, Mean and Mean_{vw} are the equal- and value-weighted averages of the estimates over the entire sample period, respectively. Std. dev., Median, Min, and Max present further summary statistics on the overall standard deviation, median, minimum, and maximum of all individual estimates, respectively.

Panel A. Summary Statistics

	Nobs	Mean	Mean_{vw}	Std. dev.	Median	Min	Max
HIST	110,277	1.0073	1.0032	0.4555	0.9461	-0.6675	4.6485
$\text{HIST}_{6\text{ mon}}$	110,692	1.0054	1.0012	0.4892	0.9447	-0.9818	7.7906
HF_{5}	106,376	0.9369	0.9874	0.4378	0.8679	-1.5517	4.1109
$\text{HF}_{5,1\text{ mon}}$	108,685	0.9387	0.9872	0.4888	0.8608	-8.9032	5.5420
HF_{15}	106,375	0.9567	0.9956	0.4496	0.8837	-1.2501	4.3931
HF_{30}	106,375	0.9662	0.9974	0.4574	0.8938	-2.0210	4.4675
HF_{75}	106,375	0.9820	0.9991	0.4748	0.9086	-2.7018	5.4052
HF_{130}	106,375	0.9814	0.9936	0.4759	0.9104	-2.4115	5.5074
BV	105,811	1.0463	1.0000	0.3680	0.9960	-0.4827	6.6757
$\text{BV}_{6\text{ mon}}$	106,233	1.0470	1.0000	0.3776	0.9959	-1.0182	6.7820
BV_{5}	103,542	1.0518	1.0000	0.3606	0.9869	0.0398	6.4297
BV_{15}	103,542	1.0503	1.0000	0.3684	0.9855	-0.2166	6.5612
BV_{30}	103,543	1.0521	1.0000	0.3705	0.9875	-0.3033	6.5797
BV_{75}	103,544	1.0548	1.0000	0.3763	0.9908	-0.3070	6.6407
BV_{130}	103,544	1.0548	1.0000	0.3763	0.9908	-0.3070	6.6407

can be noted that methods relying on historical returns have higher standard deviations compared to the hybrid models (about 0.45 vs. around 0.37).

Panel B of Table 3.1 presents the sample correlation coefficients among betas obtained with different estimation techniques on the basis of their pooled estimates for individual assets during the entire sample period. Generally, we note very high correlations around 0.9 and higher. The lowest correlations arise between BV or $\text{BV}_{6\text{ mon}}$ and $\text{HF}_{5,1\text{ mon}}$ and BV_{5}

Table 3.1: Summary Statistics and Sample Correlations (continued)

Panel B. Correlation Coefficients of Different Estimates

HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	HF_{15}	HF_{30}	HF_{75}	HF_{130}	BV	$BV_{6\ mon}$	BV_5	BV_{15}	BV_{30}	BV_{75}	BV_{130}	
*	0.92	0.87	0.78	0.88	0.89	0.90	0.91	0.88	0.86	0.78	0.80	0.80	0.81	0.81	HIST
	*	0.86	0.78	0.88	0.89	0.91	0.92	0.84	0.88	0.76	0.77	0.78	0.79	0.79	$HIST_{6\ mon}$
		*	0.90	0.99	0.98	0.95	0.94	0.81	0.81	0.82	0.83	0.82	0.82	0.81	HF_5
			*	0.89	0.88	0.86	0.85	0.76	0.76	0.77	0.77	0.77	0.77	0.76	$HF_{5,1\ mon}$
				*	0.99	0.97	0.96	0.83	0.83	0.83	0.84	0.84	0.84	0.83	HF_{15}
					*	0.98	0.97	0.84	0.85	0.83	0.84	0.85	0.84	0.84	HF_{30}
						*	0.98	0.85	0.86	0.83	0.84	0.84	0.85	0.84	HF_{75}
							*	0.85	0.86	0.82	0.83	0.84	0.84	0.85	HF_{130}
								*	0.97	0.94	0.94	0.95	0.95	0.95	BV
									*	0.93	0.94	0.94	0.95	0.95	$BV_{6\ mon}$
										*	1.00	0.99	0.99	0.99	BV_5
											*	1.00	0.99	0.99	BV_{15}
												*	1.00	0.99	BV_{30}
													*	1.00	BV_{75}
														*	BV_{130}

and $HIST_{6\ mon}$, respectively, amounting to 0.76. The implications of these high correlations are twofold. First, it may be hard to detect significant differences between the approaches since, to a large extent, they appear to carry similar information. Secondly, we have to take care of multicollinearity issues possibly inflating the standard errors in encompassing regressions performed in the next section.

3.3.2 Information Content

A common way to evaluate the performance of ex ante estimates is to use Mincer & Zarnowitz (1969) regressions. We therefore regress the six-month (ex post) realized beta on the different (ex ante) beta estimates in the following way:

$$\beta_{t,T}^{\mathrm{R}} = a + b\zeta_{t,T} + \epsilon_t. \quad (3.4)$$

$\beta_{t,T}^{R}$ denotes the realized beta in the period ranging from t to T and $\zeta_{t,T}$ stands for one beta estimate in univariate regressions or a vector of several beta estimates in encompassing regressions. With the approach in equation (3.4) we can test for the informational efficiency and unbiasedness of the respective estimates. As Hansen & Lunde (2006) show, using logarithmically transformed variables for the regressions, while making the regression procedure less sensitive to outliers (Pagan & Schwert, 1990), often leads to inconsistent rankings of the estimation models if an unbiased but imperfect proxy for the true evaluation variable is used. They further show that Mincer–Zarnowitz regressions in levels are robust to (mean zero) errors in the evaluation proxy. Consequently, we stick to levels instead of logs to obtain results that are more robust.

Unbiasedness is tested in univariate regressions by performing a Wald test, imposing the joint hypothesis of a being equal to zero and b being equal to one. For an unbiased model we should not be able to reject the underlying hypothesis. Informational efficiency can be tested in encompassing regressions by constraining the slope parameters of alternative estimators to zero, thereby determining if the respective approaches contain information beyond that of a baseline model. If, in encompassing regressions, an estimator is to be more informative it must have a significant slope estimate and the explanatory power must rise compared to the restricted model. Additionally, we test the joint hypothesis of one slope parameter being equal to one and the second slope parameter being equal to zero. The underlying hypothesis of this test states that one approach fully subsumes all information contained in the other approach it is tested with.

To conduct our analysis, we follow the approach suggested by Fama & MacBeth (1973). At the end of each month, we build five value-weighted portfolios out of the individual stocks in our sample. To sort the stocks, we use as an instrument the stocks' estimate for (daily) historical beta obtained in an estimation period (sorting period) strictly before the estimation period of the historical beta serving as one beta estimate. We sort the stocks in an ascending order and compute estimates as well as realizations for beta for each of these portfolios.[7] This approach ensures that we obtain a certain range in the estimated values and delivers results that are comparable without particularly loading on the measurement errors of one of the approaches. To keep the analysis comparable, we can only include those estimates in our sample where all approaches yield an estimate.[8]

To keep the presentation manageable, we select at least one approach from each model family to perform our main analysis. We select historical and six-month historical ($\text{HIST}_{6\text{ mon}}$) using daily return data, five-minute high frequency over six (HF_5) and one month(s) ($\text{HF}_{5,1\text{ mon}}$), BV and $\text{BV}_{6\text{ mon}}$, as well as the high-frequency hybrid BV_5 relying on five-minute return data, and consider the methods estimated with further sampling frequencies in the robustness analysis in Section IV. In all analyses, we evaluate the approaches using high-frequency realized beta during the subsequent six months.

Table 3.2 presents the regression results for the main estimation approaches employing high-frequency five-minute realized beta to evaluate the ex ante estimates. Panel A of Table 3.2 presents the results of the univariate regressions for each of the five portfolios. It can be seen that in many cases the intercept estimate is significantly (at 5 %)[9] different from zero and the estimate for the slope coefficient is significantly different from

[7]For example, using daily data and estimating beta at the end of January 1996, evaluating it in the period February – July 1996, the estimation of historical beta uses return data from February 1995 until the end of January 1996. The portfolio sorting is carried out according to the estimate for historical beta using return data between February 1994 and the end of January 1995. If historical return data for the sorting period is not available, the sorting beta is set to one.

[8]Note that, while analyzing the value of high-frequency data for beta estimation, sorting on past low-frequency beta might be regarded as non-optimal. However, the sorting is only designed to ensure that the resulting portfolios have a certain spread in their beta estimates. Each of the portfolios is examined separately.

[9]Further mentions of (non-)significance in this section will always refer to the 5 % significance level.

one.[10]

For all approaches there are only some portfolios yielding non-significant values for the intercept and slope coefficients. The joint hypothesis of a being equal to zero and b being equal to one, however, is rejected in any case, suggesting that all approaches yield biased estimates.

For three portfolios, BV yields the highest adjusted R^2, while the short-term $\text{HF}_{5,1\text{ mon}}$ and $\text{BV}_{6\text{ mon}}$ have the highest adjusted R^2 for one portfolio each. The estimates of these three approaches and HF_5 exhibit the highest explanatory power, which is substantially higher than that of the traditional historical daily estimators, e.g., for BV, the adjusted R^2 is higher by 12 up to 32 percentage points compared to HIST. For the high-frequency approaches the picture looks similar.

Turning the focus to the results of the encompassing regressions in Panel B of Table 3.2 we find the high-frequency estimators and BV to be informationally more efficient than HIST. The adjusted R^2 rises when adding these models to HIST and the slope coefficient on BV, HF_5, and $\text{HF}_{5,1\text{ mon}}$ is significant as opposed to that on HIST, which generally yields a non-significant slope coefficient when combined with these models. The relation of HIST and BV_5 is not entirely clear. Comparing the high-frequency estimators using six months of return data (HF_5) to that employing only the returns during the preceding month ($\text{HF}_{5,1\text{ mon}}$), the shorter-term estimator appears to be favored in the extreme portfolios 1 and 5 where only the latter has a significant slope estimate. However, for the remaining portfolios none of the two approaches is informationally more efficient than the other, with significant slope estimates for both.

The picture is also not entirely clear when placing HF_5 or $\text{HF}_{5,1\text{ mon}}$

[10] Note that for univariate regressions the t-statistics of the slope coefficients test the hypothesis of those being equal to one and not, as is usually done, equal to zero. In the multivariate regressions, the t-statistics refer to the usual hypothesis that the parameters are equal to zero.

Table 3.2: Univariate and Encompassing Estimates for Realized Beta – Five-Minute Data

This table presents the results from regressions of ex post realized beta over the time horizon of six months on competing ex ante estimates for each of the five portfolios, respectively. Each row corresponds to one regression. Each month, we sort our estimates in ascending order according to the historical beta obtained in a sorting period strictly before the estimation period of the historical beta. The sorting and estimation periods are without overlap and have equal length. The stocks are then allocated to quintile portfolios. Portfolio betas are determined as value-weighted averages. a denotes the regression intercept and $b(.)$ denote the slope coefficients for the different models to estimate beta. For each regression coefficient we report the corresponding Newey & West (1987) t-statistics ($t(b)$) using six lags, where we test a against zero and b against one in the univariate regressions (Panel A), while we test both against zero in the multivariate regressions (Panel B). R^2_{adj} denotes the adjusted R^2 of the regressions. The columns $Wald_1$ and $Wald_2$ refer to the Wald test statistic and p indicates the corresponding p-values. In the univariate regressions in Panel A, for the Wald test (reported in $Wald_1$) we test the joint hypothesis of the intercept being equal to zero and the slope coefficient being equal to one. In the multivariate regressions in Panel B, the joint underlying hypothesis is that the first slope coefficient is equal to one and the second slope coefficient is equal to zero and vice versa for $Wald_1$ and $Wald_2$, respectively. t-statistics and p-values in **bold** font indicate significance at the 5 % level. The sample period is January 1996 until December 2014 with 222 monthly observations.

Panel A. Univariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	$b_{HIST_{6\ mon}}$	$t(b)$	b_{HF_5}	$t(b)$	$b_{HF_{5,1\ mon}}$	$t(b)$	b_{BV}	$t(b)$	$b_{BV_{6\ mon}}$	$t(b)$	b_{BV_5}	$t(b)$	R^2_{adj}	$Wald_1$	p
HIST	1	0.37	**(3.47)**	0.47	**(-3.36)**													0.31	81.79	**(0.00)**
	2	0.19	(1.66)	0.80	(-1.58)													0.67	21.64	**(0.00)**
	3	0.04	(0.31)	0.94	(-0.46)													0.75	7.04	**(0.00)**
	4	0.03	(0.23)	0.93	(-0.62)													0.68	33.43	**(0.00)**
	5	0.80	**(3.87)**	0.39	**(-4.14)**													0.24	122.70	**(0.00)**
$HIST_{6\ mon}$	1	0.34	**(4.40)**			0.52	**(-4.38)**											0.45	92.00	**(0.00)**
	2	0.22	**(2.55)**			0.75	**(-2.63)**											0.68	29.54	**(0.00)**
	3	0.14	(1.38)			0.83	(-1.62)											0.71	15.71	**(0.00)**
	4	0.10	(1.03)			0.88	(-1.40)											0.72	22.02	**(0.00)**
	5	0.72	**(3.87)**			0.45	**(-4.05)**											0.34	109.49	**(0.00)**
HF_5	1	0.17	**(1.99)**					0.75	**(-2.03)**									0.52	14.06	**(0.00)**
	2	0.07	(0.72)					0.94	(-0.58)									0.78	8.15	**(0.00)**
	3	0.13	(1.50)					0.87	(-1.59)									0.80	10.81	**(0.00)**
	4	0.11	(1.17)					0.90	(-1.23)									0.80	5.89	**(0.00)**
	5	0.50	**(2.79)**					0.62	**(-2.86)**									0.39	32.60	**(0.00)**

Table 3.2: Univariate and Encompassing Estimates for Realized Beta – Five-Minute Data (continued)

		a	$t(a)$	b_{HIST}	$t(b)$	$b_{HIST_{6\ \mathrm{mon}}}$	$t(b)$	b_{HF_5}	$t(b)$	$b_{HF_{5,1\ \mathrm{mon}}}$	$t(b)$	b_{BV}	$t(b)$	$b_{BV_{6\ \mathrm{mon}}}$	$t(b)$	b_{BV_5}	$t(b)$	R^2_{adj}	$Wald_1$	p
$\mathrm{HF}_{5,1\ \mathrm{mon}}$	1	0.18	**(3.40)**							0.74	**(-3.50)**							0.64	25.52	**(0.00)**
	2	0.10	(1.59)							0.90	(-1.49)							0.79	9.05	**(0.00)**
	3	0.16	**(2.81)**							0.83	**(-2.95)**							0.82	20.61	**(0.00)**
	4	0.17	**(2.82)**							0.84	**(-2.90)**							0.80	16.19	**(0.00)**
	5	0.45	**(4.31)**							0.66	**(-4.38)**							0.52	39.46	**(0.00)**
BV	1	0.11	(0.87)									0.77	(-1.38)					0.43	49.72	**(0.00)**
	2	-0.26	**(-2.86)**									1.26	**(2.60)**					0.81	49.42	**(0.00)**
	3	-0.34	**(-5.14)**									1.32	**(4.74)**					0.86	85.52	**(0.00)**
	4	-0.36	**(-3.12)**									1.33	**(3.07)**					0.81	28.79	**(0.00)**
	5	0.09	(0.58)									0.96	(-0.31)					0.56	14.17	**(0.00)**
$\mathrm{BV}_{6\ \mathrm{mon}}$	1	0.15	(1.33)											0.71	(-1.94)			0.43	55.59	**(0.00)**
	2	-0.20	(-1.90)											1.18	(1.65)			0.75	31.06	**(0.00)**
	3	-0.26	**(-3.24)**											1.24	**(2.95)**			0.83	56.29	**(0.00)**
	4	-0.31	**(-2.95)**											1.28	**(2.93)**			0.80	21.35	**(0.00)**
	5	0.14	(1.01)											0.93	(-0.68)			0.58	15.10	**(0.00)**
BV_5	1	-0.14	(-0.71)													1.05	(0.19)	0.36	116.69	**(0.00)**
	2	-0.54	**(-3.06)**													1.54	**(2.91)**	0.61	56.08	**(0.00)**
	3	-0.52	**(-3.29)**													1.50	**(3.20)**	0.76	67.05	**(0.00)**
	4	-0.34	**(-2.94)**													1.33	**(3.04)**	0.79	25.96	**(0.00)**
	5	0.18	(1.23)													0.91	(-0.80)	0.47	24.89	**(0.00)**

Panel B. Multivariate Regressions

		a	$t(a)$	b_{HIST}	$t(b)$	$b_{HIST_{6\ \mathrm{mon}}}$	$t(b)$	b_{HF_5}	$t(b)$	$b_{HF_{5,1\ \mathrm{mon}}}$	$t(b)$	b_{BV}	$t(b)$	$b_{BV_{6\ \mathrm{mon}}}$	$t(b)$	b_{BV_5}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
HIST + HF_5	1	0.16	**(2.06)**	-0.13	(-0.89)			0.89	**(5.77)**									0.52	136.29	**(0.00)**	14.34	**(0.00)**
	2	0.06	(0.68)	-0.25	(-1.37)			1.20	**(5.52)**									0.79	82.67	**(0.00)**	4.97	**(0.01)**
	3	0.11	(1.10)	0.13	(0.46)			0.76	**(2.91)**									0.80	33.44	**(0.00)**	11.39	**(0.00)**
	4	0.12	(1.24)	-0.07	(-0.47)			0.95	**(6.72)**									0.80	65.40	**(0.00)**	5.90	**(0.00)**
	5	0.50	**(2.73)**	-0.08	(-0.55)			0.70	**(2.89)**									0.39	131.80	**(0.00)**	26.46	**(0.00)**
HIST + $\mathrm{HF}_{5,1\ \mathrm{mon}}$	1	0.18	**(3.15)**	-0.01	(-0.15)					0.75	**(9.30)**							0.64	211.33	**(0.00)**	23.01	**(0.00)**
	2	0.09	(1.22)	0.16	(1.55)					0.75	**(6.37)**							0.80	90.09	**(0.00)**	8.93	**(0.00)**
	3	0.09	(1.19)	0.28	(1.78)					0.62	**(4.35)**							0.83	53.27	**(0.00)**	28.44	**(0.00)**
	4	0.09	(1.13)	0.24	**(2.41)**					0.67	**(7.47)**							0.81	78.91	**(0.00)**	24.08	**(0.00)**
	5	0.44	**(4.07)**	0.02	(0.23)					0.65	**(6.79)**							0.52	195.58	**(0.00)**	31.98	**(0.00)**
HIST + BV	1	0.12	(0.87)	0.05	(0.30)							0.71	**(2.54)**					0.43	97.05	**(0.00)**	7.99	**(0.00)**
	2	-0.27	**(-2.83)**	-0.03	(-0.28)							1.30	**(7.08)**					0.81	104.22	**(0.00)**	19.59	**(0.00)**
	3	-0.31	**(-3.84)**	0.16	(1.17)							1.13	**(6.85)**					0.87	102.67	**(0.00)**	44.19	**(0.00)**
	4	-0.39	**(-3.32)**	-0.15	(-0.84)							1.51	**(6.51)**					0.81	75.39	**(0.00)**	28.83	**(0.00)**
	5	0.05	(0.26)	-0.22	**(-2.76)**							1.24	**(6.08)**					0.59	244.39	**(0.00)**	7.75	**(0.00)**
HIST + BV_5	1	-0.02	(-0.12)	0.23	(1.41)											0.71	**(2.43)**	0.39	83.72	**(0.00)**	6.38	**(0.00)**
	2	-0.08	(-0.46)	0.58	**(2.93)**											0.50	(1.75)	0.68	18.66	**(0.00)**	48.43	**(0.00)**
	3	-0.32	(-1.83)	0.47	**(2.58)**											0.84	**(2.93)**	0.79	24.05	**(0.00)**	61.04	**(0.00)**
	4	-0.32	**(-2.71)**	0.13	(0.83)											1.17	**(5.88)**	0.79	60.93	**(0.00)**	26.17	**(0.00)**
	5	0.17	(1.15)	-0.05	(-0.56)											0.97	**(6.22)**	0.47	166.35	**(0.00)**	1.38	(0.25)

Table 3.2: Univariate and Encompassing Estimates for Realized Beta – Five-Minute Data (continued 2)

		a	$t(a)$	b_{HIST}	$t(b)$	$b_{HIST_{6\ \mathrm{mon}}}$	$t(b)$	b_{HF_5}	$t(b)$	$b_{HF_{5,1\ \mathrm{mon}}}$	$t(b)$	b_{BV}	$t(b)$	$b_{BV_{6\ \mathrm{mon}}}$	$t(b)$	b_{BV_5}	$t(b)$	R^2_{adj}	$Wald_1$	p	$Wald_2$	p
$\mathrm{HF}_5 + \mathrm{HF}_{5,1\ \mathrm{mon}}$	1	0.17	**(2.77)**					0.08	(0.63)	0.68	**(6.09)**							0.64	52.26	**(0.00)**	23.42	**(0.00)**
	2	0.06	(0.79)					0.44	**(2.85)**	0.50	**(4.07)**							0.81	16.70	**(0.00)**	16.26	**(0.00)**
	3	0.13	(1.94)					0.35	**(2.16)**	0.51	**(3.11)**							0.82	26.21	**(0.00)**	26.53	**(0.00)**
	4	0.12	(1.51)					0.44	**(3.45)**	0.45	**(3.86)**							0.82	20.21	**(0.00)**	29.96	**(0.00)**
	5	0.38	**(3.22)**					0.17	(1.62)	0.54	**(5.61)**							0.53	66.65	**(0.00)**	35.72	**(0.00)**
$\mathrm{HF}_5 + \mathrm{BV}$	1	0.08	(0.63)					0.54	**(3.33)**			0.32	(1.26)					0.55	21.91	**(0.00)**	38.65	**(0.00)**
	2	-0.17	(-1.50)					0.34	(1.87)			0.83	**(3.40)**					0.82	25.56	**(0.00)**	27.58	**(0.00)**
	3	-0.24	**(-2.40)**					0.26	(1.94)			0.97	**(4.82)**					0.87	79.50	**(0.00)**	52.54	**(0.00)**
	4	-0.19	(-1.71)					0.43	**(3.84)**			0.74	**(4.49)**					0.83	29.49	**(0.00)**	48.21	**(0.00)**
	5	0.09	(0.58)					0.00	(-0.01)			0.96	**(5.05)**					0.56	76.49	**(0.00)**	0.22	(0.80)
$\mathrm{HF}_5 + \mathrm{BV}_5$	1	0.06	(0.35)					0.64	**(4.57)**							0.24	(0.97)	0.53	14.70	**(0.00)**	38.07	**(0.00)**
	2	0.12	(0.76)					0.99	**(4.74)**							-0.11	(-0.33)	0.78	2.02	(0.13)	121.43	**(0.00)**
	3	-0.08	(-0.52)					0.64	**(4.91)**							0.43	(1.89)	0.81	15.30	**(0.00)**	79.03	**(0.00)**
	4	-0.14	(-1.23)					0.50	**(3.58)**							0.64	**(3.40)**	0.82	18.68	**(0.00)**	45.18	**(0.00)**
	5	0.19	(1.35)					0.17	(0.70)							0.72	**(2.52)**	0.48	47.67	**(0.00)**	2.73	(0.07)
$\mathrm{HF}_{5,1\ \mathrm{mon}} + \mathrm{BV}$	1	0.11	(1.09)							0.63	**(6.54)**	0.19	(0.94)					0.65	27.65	**(0.00)**	78.79	**(0.00)**
	2	-0.15	(-1.42)							0.40	**(4.08)**	0.75	**(4.38)**					0.83	34.18	**(0.00)**	36.85	**(0.00)**
	3	-0.21	**(-2.24)**							0.28	**(2.64)**	0.93	**(5.15)**					0.88	82.98	**(0.00)**	55.12	**(0.00)**
	4	-0.16	(-1.73)							0.42	**(5.69)**	0.72	**(5.79)**					0.84	51.15	**(0.00)**	60.05	**(0.00)**
	5	0.15	(1.07)							0.27	**(2.50)**	0.63	**(3.84)**					0.58	52.27	**(0.00)**	6.24	**(0.00)**
$\mathrm{BV} + \mathrm{BV}_{6\ \mathrm{mon}}$	1	0.12	(0.92)									0.43	(0.83)	0.33	(0.75)			0.44	8.89	**(0.00)**	14.93	**(0.00)**
	2	-0.26	**(-3.19)**									1.68	**(7.08)**	-0.43	(-1.93)			0.81	22.64	**(0.00)**	50.67	**(0.00)**
	3	-0.34	**(-5.46)**									1.62	**(4.78)**	-0.29	(-0.94)			0.87	40.93	**(0.00)**	50.75	**(0.00)**
	4	-0.36	**(-2.92)**									1.03	**(2.06)**	0.29	(0.70)			0.81	27.94	**(0.00)**	27.35	**(0.00)**
	5	0.12	(0.83)									0.17	(0.26)	0.77	(1.19)			0.58	4.83	**(0.01)**	1.17	(0.31)
$\mathrm{BV} + \mathrm{BV}_5$	1	0.04	(0.28)									0.65	**(2.04)**			0.20	(0.54)	0.43	8.40	**(0.00)**	14.74	**(0.00)**
	2	-0.10	(-0.91)									1.59	**(11.48)**			-0.51	**(-2.49)**	0.82	26.98	**(0.00)**	169.51	**(0.00)**
	3	-0.33	**(-4.01)**									1.33	**(7.43)**			-0.02	(-0.09)	0.86	39.69	**(0.00)**	153.85	**(0.00)**
	4	-0.38	**(-3.30)**									0.86	**(3.25)**			0.49	**(2.04)**	0.81	32.82	**(0.00)**	41.89	**(0.00)**
	5	0.13	(0.83)									1.35	**(2.29)**			-0.42	(-0.74)	0.57	2.52	(0.08)	25.96	**(0.00)**

in a joint regression with BV. In these cases, both models regularly adhere significant slope coefficients, meaning that the estimators partially contain complementary information. Based on this insight, a combination of both approaches could potentially be useful.[11]

Additionally, BV_5 is shown to be quite clearly inferior compared to BV, with a significant slope parameter for only two portfolios. One of the two significant slope parameters is even negative, implying a negative relation between beta estimates and realization when the information in BV is already given.[12] The superiority of BV over the high-frequency BV_5 might on the one hand seem surprising since the use of high-frequency historical data has been shown to improve the informational efficiency of estimates for beta over the historical approach based on a daily frequency. On the other hand, however, it has to be noted that BV makes use of the full correlation matrix of all index constituents, for which the problem of potential non-synchronous and infrequent trading becomes much more severe than when just estimating the covariance between an asset's return and that of the market, as is done for the historical high-frequency estimators.

The hypothesis that one approach subsumes all the information contained in another approach (indicated by the tests $Wald_1$ and $Wald_2$) is rejected in most cases, meaning that all approaches, to some extent, contain some information that others have not incorporated and none of the models is fully perfectly specified.

Overall, using high-frequency realized beta, we first confirm the results of Chapter 2 that BV is superior in terms of informational efficiency compared to the historical estimator using daily return data. Secondly,

[11] We examine the issue of combinations in Section 3.3.5.

[12] The negative slope estimate on BV_5 is most probably caused by the near-multicollinear relation to BV, which is constructed very similarly, with a correlation amounting to 0.94. The general conclusion that BV is superior, however, remains valid.

we show that the high-frequency estimators perform quite well. They are informationally more efficient compared to the historical estimator using daily return data. In a joint regression with BV, both models appear to add valuable information, while none of the two models is distinctly favored.[13]

3.3.3 Estimation Accuracy

Turning the focus on out-of-sample estimation accuracy, we employ the loss function most commonly applied in the literature, namely the root mean squared error (RMSE) criterion, to evaluate the performance of the different beta estimation techniques:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\beta^{\mathrm{R}}_{t,T} - \zeta_{t,T})^2}. \tag{3.5}$$

Here, n is the number of estimation windows, $\beta^{R}_{t,T}$ again denotes the realized beta over a period from t until T, and $\zeta_{t,T}$ is the respective beta estimate. Patton (2011) shows that only the mean squared error (MSE) criterion, as opposed to other commonly employed loss functions, is robust to the presence of (mean zero) noise in the evaluation proxy, so we choose this loss function. We test for significance in RMSE using the modified Diebold–Mariano test proposed by Harvey et al. (1997) and for significance in root median squared error (RMedSE) with the non-parametric Wilcoxon signed rank test. For the Wilcoxon signed rank test, we perform the significance test only on the first series of non-overlapping differences in errors to account for possible serial correlation (Diebold & Lopez, 1996).

[13]For the sake of brevity, we do not report further results using different frequencies to estimate ex post realized beta (e.g., 15, 30, 75, 130 minutes, or daily returns). These results actually are qualitatively equal and available upon request.

Since for this approach we have to discard observations, the statistical power of the test is reduced.[14] These results should thus be interpreted cautiously.

Table 3.3 summarizes the estimation errors using five-minute return data. It can be seen that the short-window high-frequency $\text{HF}_{5,1\text{ mon}}$ yields the smallest average RMSE over the five portfolios (as indicated by *italic* font), followed by HF_5 and BV. The differences in RMSE, however, are hardly ever significant. On the other hand, the differences in RMedSE are significant in some instances.

Comparing the approaches relying on historical return data, the median errors are significantly smaller when using high-frequency returns compared to daily returns in at least two out of the five portfolios, while the RMSEs are only significantly smaller for one and zero portfolio(s). The mean and median errors are not significantly different comparing BV to HIST, delivering only a very weak indication for a superior estimation accuracy. On the other hand, significant differences between HF_5, $\text{HF}_{5,1\text{ mon}}$, and BV also cannot be established, neither in RMSEs nor in RMedSEs. The estimation errors for BV_5 are slightly higher compared to the best models; however, the differences are significant in few instances only.

Overall, the evidence indicates that the approaches HF_5, $\text{HF}_{5,1\text{ mon}}$, and BV obtain the best out-of-sample accuracy, while differences are mostly not significant, neither between the models mentioned nor compared to other models.[15]

[14] While, strictly speaking the Wilcoxon signed rank test incorporates the joint null hypothesis of zero median in the loss differentials as well as a symmetric distribution, we stick to this test instead of an alternative only testing on zero median, like the simple sign test, since the Wilcoxon signed rank test turns out more powerful in many applications (Conover, 1999).

[15] Examining the estimated spread in the beta vs. the realized spread, e.g., of the 5-1 portfolio, the results are qualitatively similar. BV and high-frequency models adhere more or less equal RMSE for the spread while that of the historical daily models is significantly higher in most cases.

Table 3.3: Estimation Errors

This table reports the out-of-sample estimation errors of competing estimators for five-minute realized beta over the time horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. The first row reports the average root mean squared error of the estimation models over the five portfolios. The lowest error among all approaches is indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

	HIST	$\text{HIST}_{6\text{ mon}}$	HF_5	$\text{HF}_{5,1\text{ mon}}$	BV	$\text{BV}_{6\text{ mon}}$	BV_5
avg.	0.1247	0.1177	0.0907	*0.0865*	0.0915	0.0932	0.1127
HIST		0.0070	0.0340	0.0382	0.0332	0.0315	0.0120
		(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)
$\text{HIST}_{6\text{ mon}}$	-0.0052		0.0271	0.0312	0.0262	0.0245	0.0050
	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_5	-0.0160	-0.0108		0.0041	-0.0009	-0.0026	-0.0221
	(-0.4)	(-0.2)		(0.0)	(0.0)	(0.0)	(-0.2)
$\text{HF}_{5,1\text{ mon}}$	-0.0164	-0.0112	-0.0004		-0.0050	-0.0067	-0.0262
	(-0.6)	(-0.2)	(0.0)		(0.0)	(-0.2)	(-0.2)
BV	-0.0113	-0.0061	0.0047	0.0051		-0.0017	-0.0212
	(0.0)	(0.0)	(0.0)	(0.2)		(0.0)	(0.0)
$\text{BV}_{6\text{ mon}}$	-0.0105	-0.0053	0.0055	0.0059	0.0008		-0.0195
	(-0.4)	(-0.2)	(0.0)	(0.2)	(0.0)		(0.0)
BV_5	-0.0068	-0.0016	0.0092	0.0096	0.0045	0.0037	
	(0.2)	(0.0)	(0.2)	(0.2)	(0.4)	(0.2)	

3.3.4 Estimation Errors Through Time

A potentially interesting topic lies in the development of estimation errors over time. Figure 3.1 depicts the cumulative average absolute errors (AEs) of $\text{HF}_{5,1\text{ mon}}$ and BV compared to those of HIST over time.[16] Business cycle contractions as reported by the National Bureau of Economic Research (NBER) are indicated by shaded areas. It can be seen that during the late 1990s the estimation errors are highest for the hybrid BV model. Later on, when liquidity in options markets increases, BV performs considerably better.[17] Especially around crisis periods, BV yields a superior estimation accuracy compared to HIST and, to a lesser extent, $\text{HF}_{5,1\text{ mon}}$.[18] This is most likely caused by the fact that during changes in economic conditions, the option-implied moments can adjust much faster and more frequently compared to historical covariances. Overall, $\text{HF}_{5,1\text{ mon}}$ delivers the lowest cumulative average AEs. Furthermore, since it only needs one month of historical high-frequency data, differences in crisis times compared to BV are only moderate. From the year 2001 on, BV yields an estimation accuracy even slightly better than $\text{HF}_{5,1\text{ mon}}$.

3.3.5 Bias-Removal and Combinations

As it is shown in Section 3.3.2, all the estimation techniques are biased and partially carry complementary information. None of the models

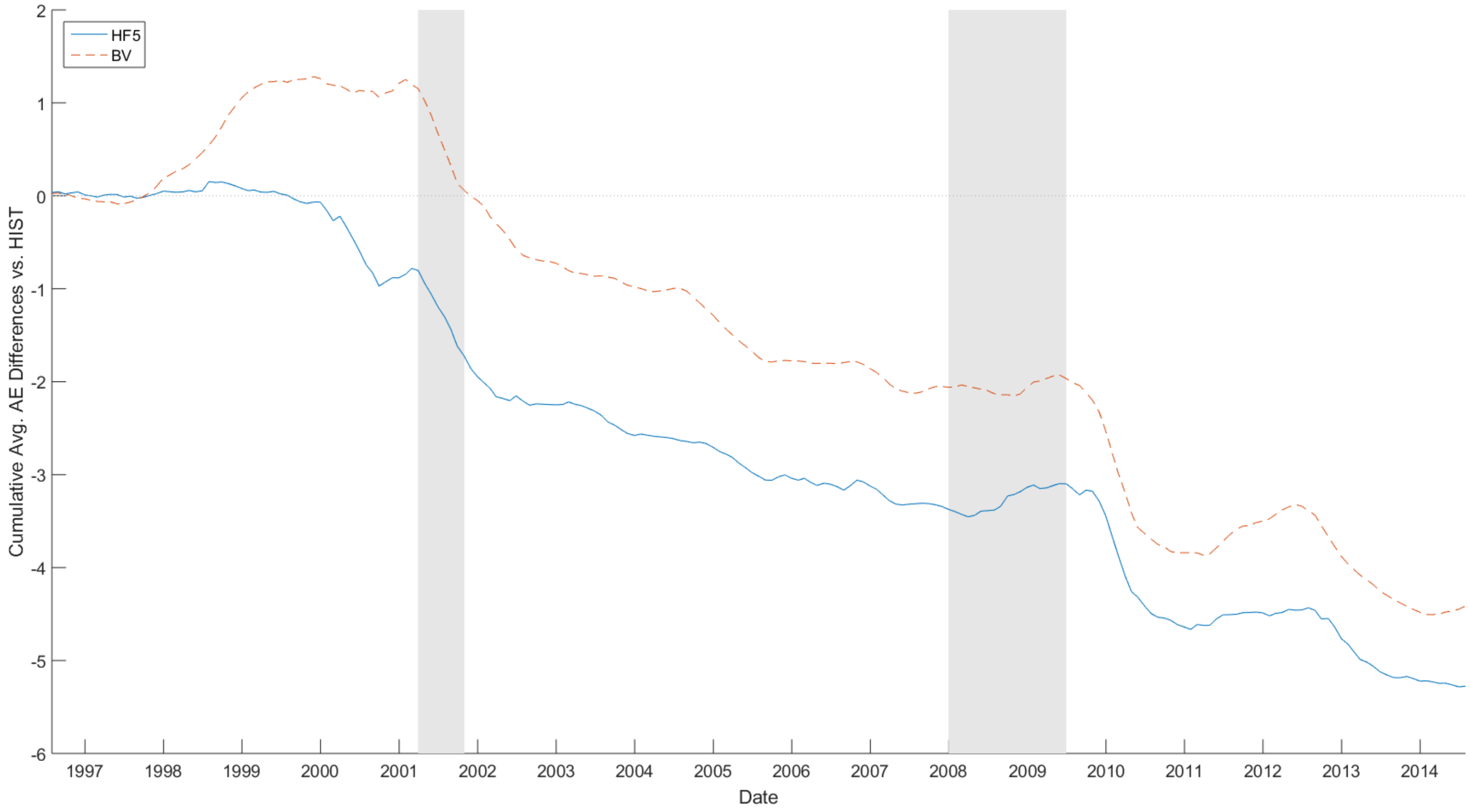
[16] We plot the differences of the cumulative errors of one model minus those of HIST. This means that a downward trend indicates the superior estimation accuracy of the model examined compared to HIST and vice versa.

[17] E.g., the average daily total contract volume in S&P 500 index options increases from about 94,200 during the late 1990s to roughly 473,800 for the remainder of our sample period.

[18] Note that we are dealing with estimation horizons of up to one year and an evaluation horizon of six months. Models relying on one year of historical return data are therefore influenced by the crisis period until eleven months after the end of the contraction period. Furthermore, all evaluation periods starting five months and less before the beginning of a contraction period at least partially contain crisis times.

Figure 3.1: Cumulative Error Differences Through Time

This figure plots the cumulative average absolute (or, equivalently, root squared) error (AE) differences of $\mathrm{HF}_{5,1\ \mathrm{mon}}$ (*solid*) and BV (*dashed*) compared to HIST over our sample period 1996–2014. The shaded areas indicate time periods marked as business cycle contractions by the NBER.



fully subsumes all information contained in another, and in bivariate encompassing regressions it often occurs that both models yield a significant slope coefficient. These findings suggest that by removing the bias or combining estimates it might be possible to further increase the estimation accuracy. Bates & Granger (1969) note that the combination of estimation techniques may prove worthwhile, especially when the estimates combined are based on different sets of information. To investigate this, we try three basic approaches. The first is just a simple combination of estimators. While simple ad hoc combinations are easy to implement, the procedure might not provide the optimal result. On the other hand, Clemen (1989) and Timmermann (2006) provide evidence that, offering diversification gains, first of all, combinations of multiple individual estimates can substantially increase the estimation accuracy. Secondly, they show that such simple combinations often work reasonably well or even better compared to more complex approaches of combining estimates. However, with two further approaches we try to find the (ex ante) optimal correction on individual methods, trying to remove the bias that is inherent in any of the estimation techniques (see Section 3.3.2), and optimal combinations of estimators. The approach may be considered as an ex ante optimal AR(1) model.

Specifically, we employ bias-removal and combining techniques in the spirit of Mincer & Zarnowitz (1969) using regression techniques as in equation (3.4). We build portfolios as in Section 3.3.2, obtain estimates for each approach and then perform the uni- or multivariate regressions, pooling all unadjusted ex ante estimates for each approach as well as the corresponding ex post realized portfolio beta estimates up to $t - k$ (since realized beta with a k-month window is only available up to $t - k$ at time t)

in separate vectors.[19] Since the portfolio characteristics are relatively stable, we employ an expanding window instead of a rolling window to make use of a maximum length of history in order to estimate the parameters with greater precision.[20] The regression equation takes the following form:

$$\beta^{\mathrm{R}} \quad = \quad a_t + b_t^{(1)} \beta^{(1,\mathrm{unadj})} + b_t^{(2)} \beta^{(2,\mathrm{unadj})} + \epsilon. \tag{3.6}$$

$\beta^{(1,\mathrm{unadj})}$ is the vector of pooled initial portfolio beta estimates of one approach and, optionally, $\beta^{(2,\mathrm{unadj})}$ indicates a possible further approach to be included, while β^{R} denotes the corresponding pooled realized beta vector. At every point in time the estimation moves forward, five additional observations are added to each of these vectors. Subsequently, after obtaining the time-t regression coefficients, we manipulate the current estimates, inserting them into (3.7):

$$\beta_t^{\mathrm{ADJ}} \quad = \quad \hat{a}_t + \hat{b}_t^{(1)} \beta_t^{(1,\mathrm{unadj})} + \hat{b}_t^{(2)} \beta_t^{(2,\mathrm{unadj})}. \tag{3.7}$$

β_t^{ADJ} is the vector of adjusted estimates at time t and $\hat{a}_t$, $\hat{b}_t^{(1)}$, and $\hat{b}_t^{(2)}$ are the respective estimated regression coefficients.[21]

The results are presented in Table 3.4. We reexamine the models HIST, $\mathrm{HF}_{5,1\ \mathrm{mon}}$, and BV. Additionally, we study simple combinations of estimators where, e.g., BV_HF[25] implies that BV and $\mathrm{HF}_{5,1\ \mathrm{mon}}$ are combined placing a weight of 25 % in the model formerly mentioned, and BV_HF_HIST[33] refers to a combined estimator placing a weight of one third to each, BV, $\mathrm{HF}_{5,1\ \mathrm{mon}}$, and HIST. Furthermore, we have uni- and multivariate model combinations obtained as described above. These

[19]We start the procedure after having twelve months of estimates and realizations (i.e., sixty observations for both the dependent and independent variable) to perform the bias-removal.

[20]We also try a rolling window approach. The results, however, suggest that the expanding window approach is indeed superior.

[21]Note that now $\beta_t^{(1,\mathrm{unadj})}$ and $\beta_t^{(2,\mathrm{unadj})}$ are assigned a t-subscript, since we only use the vector of current estimates instead of the pooled vector of all previous estimates.

Table 3.4: Bias-Removal and Combinations

This table reports the out-of-sample estimation errors of competing bias-removed and combined estimators for realized beta over the time horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. The first row reports the average root mean squared error of the estimation models over the five portfolios. The lowest error among all approaches is indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

Table 3.4: Bias-Removal and Combinations (continued)

	HIST	$HF_{5,1\ mon}$	BV	BV_HF^{25}	BV_HF^{50}	BV_HF^{75}	$BV\_HF\_HIST^{33}$	$HIST^{bias}$	BV^{bias}	HF^{bias}	BV_HF^{bias}	BV_HIST^{bias}	HF_HIST^{bias}	$BV\_HF\_HIST^{bias}$	$HF_{1\ mon}\_HF^{bias}$
avg.	0.1278	0.0883	0.0930	0.0789	0.0765	0.0817	0.0849	0.1290	0.0938	0.0916	0.0882	0.1061	0.1047	0.0980	0.0973
HIST		0.0398	0.0344	0.0492	0.0514	0.0460	0.0431	0.0078	0.0408	0.0412	0.0481	0.0333	0.0381	0.0445	0.0410
		(0.0)	(0.0)	(0.2)	(0.2)	(0.2)	(0.2)	(0.0)	(0.2)	(0.0)	(0.2)	(0.4)	(0.2)	(0.2)	(0.0)
$HF_{5,1\ mon}$	-0.0207		-0.0055	0.0093	0.0115	0.0061	0.0032	-0.0321	0.0009	0.0013	0.0083	-0.0066	-0.0018	0.0046	0.0012
	(-0.6)		(0.0)	(0.8)	(0.2)	(0.0)	(0.0)	(0.0)	(0.2)	(0.0)	(0.6)	(0.0)	(0.0)	(0.0)	(0.0)
BV	-0.0150	0.0057		0.0148	0.0170	0.0116	0.0087	-0.0266	0.0064	0.0068	0.0137	-0.0011	0.0037	0.0101	0.0066
	(-0.4)	(0.2)		(0.2)	(0.6)	(0.8)	(0.2)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
BV_HF_{25}	-0.0242	-0.0035	-0.0092		0.0022	-0.0032	-0.0061	-0.0414	-0.0084	-0.0080	-0.0011	-0.0159	-0.0111	-0.0047	-0.0082
	(-0.6)	(-0.2)	(-0.2)		(0.0)	(0.0)	(0.0)	(-0.2)	(-0.2)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
BV_HF^{50}	-0.0270	-0.0063	-0.0120	-0.0028		-0.0054	-0.0083	-0.0436	-0.0106	-0.0102	-0.0033	-0.0181	-0.0133	-0.0069	-0.0104
	(-0.6)	(-0.2)	(-0.4)	(0.0)		(-0.2)	(0.0)	(-0.2)	(0.0)	(-0.2)	(0.0)	(0.0)	(-0.4)	(0.0)	(-0.2)
BV_HF_{75}	-0.0226	-0.0019	-0.0076	0.0016	0.0044		-0.0029	-0.0382	-0.0052	-0.0048	0.0021	-0.0127	-0.0079	-0.0015	-0.0050
	(-0.4)	(0.0)	(-0.6)	(0.2)	(0.2)		(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)
$BV\_HF\_HIST_{33}$	-0.0207	0.0001	-0.0056	0.0036	0.0064	0.0019		-0.0353	-0.0023	-0.0019	0.0051	-0.0098	-0.0050	0.0014	-0.0020
	(-1.0)	(0.0)	(-0.4)	(0.2)	(0.2)	(0.0)		(-0.2)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$HIST^{bias}$	-0.0081	0.0126	0.0069	0.0161	0.0189	0.0145	0.0126		0.0330	0.0334	0.0403	0.0255	0.0303	0.0367	0.0332
	(0.0)	(0.4)	(0.2)	(0.6)	(0.8)	(0.4)	(0.8)		(0.2)	(0.0)	(0.2)	(0.2)	(0.2)	(0.2)	(0.0)
BV^{bias}	-0.0176	0.0032	-0.0025	0.0067	0.0095	0.0051	0.0031	-0.0095		0.0004	0.0073	-0.0075	-0.0027	0.0037	0.0002
	(-0.4)	(0.2)	(-0.2)	(0.2)	(0.2)	(0.0)	(0.2)	(-0.4)		(0.0)	(0.2)	(0.0)	(0.0)	(0.2)	(0.0)
HF^{bias}	-0.0211	-0.0003	-0.0060	0.0032	0.0060	0.0015	-0.0004	-0.0130	-0.0035		0.0069	-0.0079	-0.0031	0.0033	-0.0002
	(-0.6)	(0.0)	(-0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(-0.4)	(-0.2)		(0.6)	(0.0)	(0.0)	(0.0)	(0.0)
BV_HF^{bias}	-0.0235	-0.0028	-0.0085	0.0007	0.0035	-0.0009	-0.0029	-0.0155	-0.0060	-0.0025		-0.0148	-0.0100	-0.0036	-0.0071
	(-0.6)	(-0.2)	(-0.4)	(0.2)	(0.0)	(0.0)	(-0.4)	(-0.8)	(-0.2)	(-0.2)		(0.0)	(-0.4)	(0.0)	(0.0)
BV_HIST^{bias}	-0.0193	0.0014	-0.0043	0.0049	0.0077	0.0033	0.0013	-0.0113	-0.0018	0.0017	0.0042		0.0048	0.0112	0.0077
	(-0.4)	(0.2)	(0.0)	(0.2)	(0.4)	(0.4)	(0.2)	(-0.2)	(0.2)	(0.2)	(0.4)		(0.0)	(0.0)	(0.0)
HF_HIST^{bias}	-0.0226	-0.0019	-0.0076	0.0016	0.0044	0.0000	-0.0019	-0.0145	-0.0050	-0.0015	0.0010	-0.0033		0.0064	0.0029
	(-0.6)	(0.0)	(-0.2)	(0.4)	(0.2)	(-0.2)	(0.0)	(-0.8)	(-0.2)	(0.0)	(0.4)	(-0.2)		(0.6)	(0.0)
$BV\_HF\_HIST^{bias}$	-0.0245	-0.0038	-0.0095	-0.0003	0.0025	-0.0019	-0.0039	-0.0165	-0.0070	-0.0035	-0.0010	-0.0052	-0.0020		-0.0035
	(-0.6)	(0.0)	(-0.2)	(0.0)	(0.2)	(0.0)	(-0.2)	(-0.8)	(-0.2)	(0.0)	(0.0)	(-0.4)	(-0.2)		(0.0)
$HF_{1\ mon}\_HF^{bias}$	-0.0240	-0.0033	-0.0090	0.0002	0.0030	-0.0014	-0.0034	-0.0159	-0.0065	-0.0030	-0.0005	-0.0047	-0.0014	0.0005	
	(-0.6)	(0.0)	(-0.2)	(0.0)	(0.2)	(0.0)	(0.0)	(-0.8)	(-0.2)	(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)	

estimators are indicated with the superscript "bias". It can be seen that the simple ad hoc combination of BV and $\text{HF}_{5,1\text{ mon}}$ obtains the lowest average estimation errors. These are significantly lower compared to the initial models of BV, $\text{HF}_{5,1\text{ mon}}$, and HIST for at least one out of the five portfolios in RMSE and RMedSE, respectively. Looking at the individual bias-corrected models, the average RMSE can be slightly reduced compared to the initial models. However, these reductions are mostly insignificant. Looking at ex ante optimal combinations of estimators, it also appears that the estimation accuracy can be slightly, but insignificantly, improved for most possible combinations in relation to their constituents. The ex ante optimal combination of BV and $\text{HF}_{5,1\text{ mon}}$ delivers significantly more precise estimates compared to HIST and $\text{HF}_{5,1\text{ mon}}$ for at least one portfolio in RMSE and four portfolios in RMedSE. However, the simple fifty-fifty combination of both models yields an even lower average RMSE. Although this difference is not significant, given the simplicity of the approach it can be regarded as clearly favorable.[22]

Overall, it appears to be valuable to use a combination of the high-frequency and BV estimators. An ex ante optimal combination delivers proper estimation accuracy. However, a simple combination of the two estimators performs even slightly better.

3.3.6 Beta and Subsequent Returns

In this section, we examine the economic value of beta predictability, namely the relation of current estimates for beta and subsequent returns. A beta estimation methodology is favorable if a higher estimate is associated with a

[22]We also consider the Bayesian shrinkage approach proposed by Diebold & Pauly (1990) with the empirical Bayes estimator and the prior of equal weights and zero intercept for all combinations examined. The results differ only slightly from those of just the regression approach, while the simple ad hoc combination of BV and $\text{HF}_{5,1\text{ mon}}$ with equal weights still yields lower average estimation errors.

higher expected return during the following period. If the CAPM is a valid asset pricing model, there should be a positive and monotonic relationship between firms' betas and their expected returns.[23] Motivated by these theoretical insights, we set out to investigate whether the beta forecasts are consistent with these predictions.

To perform the analysis, for each approach and at the end of each month, we sort the stocks into N portfolios according to their current beta estimate. We compute portfolio betas and excess returns over the subsequent six months as value-weighted averages. This approach has the advantage that on the one hand it maximizes the spread in expected beta and on the other hand keeps the portfolio properties stable, while betas of individual assets may vary more strongly over time.

For the first part of the analysis, we sort the stocks into five portfolios and examine their average excess returns, testing whether the pattern in returns is monotonically increasing with beta. The results are shown in Panel A of Table 3.5. This table delivers a few insights that appear noteworthy. First, we can detect, respectively, that the risk–return relation of the historical estimators employing daily data (HIST, $\text{HIST}_{6\text{ mon}}$) are rather flat or the increments in average portfolio returns appear erratic. The overall pattern is even slightly negative for the high-frequency estimators, while for the BV estimators the risk–return relation is increasing on average. To thoroughly test for a monotonic pattern between portfolio betas and returns, we employ the monotonicity test of Patton & Timmermann (2010). To detect a significantly positive beta-return relation, one needs to be able to reject the null hypothesis of a monotonically decreasing relation, while not being able to reject the hypothesis of a monotonically increasing pattern. Overall, we cannot detect a significantly monotonically relationship for any

[23]Furthermore, if the CAPM is not valid, the market portfolio is still a risk factor in many models, e.g., Fama & French (1993) or Hou, Xue, & Zhang (2015).

of the approaches. The lowest p-values of the test for a monotonically decreasing pattern are obtained for BV and $BV_{6\ \mathrm{mon}}$. However, they are still above 10 %. After all, it might be hard to detect significant monotonicity since, when doing the empirical examination, the theoretically always positive relation between beta and expected returns flips around when applying it to beta vs. realized returns in times of negative realized market excess returns (Pettengill, Sundaram, & Mathur, 1995). While average returns ought still be higher for the high-beta portfolio, the monotonically decreasing pattern during these periods most likely prevents us from being able to reject the overall hypothesis of a monotonically decreasing pattern.

To further explore the risk–return relation, we examine cross-sectional regressions in the spirit of Black et al. (1972). Specifically, we build ten, twenty-five, and fifty portfolios following the procedure outlined above. For each portfolio and methodology, we compute the averages of the ex ante portfolio betas and ex post excess returns. Finally, we regress the vector of average realized portfolio excess returns on the vector of average betas. For an approach to work well, the intercept estimate should be close to and indistinguishable from zero. Furthermore, the slope coefficient ought to be significantly different from zero to indicate a positive relation between risk and returns. To validate one of the basic CAPM predictions, the slope coefficient, in magnitude, should also be close to 7.1 % which is the average annualized market excess return during the period under investigation, using the S&P 500 total return index as proxy for the market portfolio.

The empirical results can be found in Panel B of Table 3.5. It turns out that none of the models fully matches the predictions made by the CAPM. For all approaches and independent of the number of portfolios formed, the intercept estimate is significantly different from zero. The intercept parameters for the historical approaches relying on daily return data are around 5 %. For ten portfolios, none of the slope parameters

Table 3.5: Return Statistics and Cross-Sectional Regressions

This table reports return statistics and cross-sectional regressions in the manner of Black et al. (1972). For each methodology, we build five (Panel A), ten, twenty-five, and fifty (Panel B) portfolios into which the stocks are allocated in ascending order according to their current beta estimates. We determine portfolio betas and excess returns over the subsequent six months as value-weighted averages. Excess returns are annualized. Panel A presents return statistics. The lines exp. beta denote the average expected portfolio beta, while av. ret indicates the average subsequent portfolio excess return of each portfolio. $p(decr.)$ and $p(incr.)$ denote the p-values of the monotonicity test of Patton & Timmermann (2010), with the null hypothesis of monotonically decreasing and monotonically increasing relation of beta and returns, respectively. In panel B, we regress the portfolios' average excess return over rolling six-month windows on the average beta estimates for the respective portfolios. Const. and Slope denote the regression intercept and slope, while p-value indicates the respective p-value using Ordinary Least Squares (OLS) standard errors. The rows adj R^2 present the adjusted R^2 of the regressions. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Return Statistics

		1	2	3	4	5	5 *minus* 1	$p(decr.)$	$p(incr.)$
HIST	exp. beta	0.52	0.76	0.94	1.17	1.61	1.09		
	av. ret	0.0658	0.0722	0.0662	0.0721	0.0687	0.0029	(0.353)	(0.331)
$HIST_{6\ mon}$	exp. beta	0.48	0.74	0.94	1.18	1.65	1.17		
	av. ret	0.0614	0.0714	0.0684	0.0800	0.0681	0.0067	(0.302)	(0.667)
HF_5	exp. beta	0.53	0.71	0.85	1.05	1.51	0.98		
	av. ret	0.0812	0.0739	0.0692	0.0724	0.0682	-0.0129	(0.390)	(0.260)
$HF_{5,1\ mon}$	exp. beta	0.48	0.69	0.86	1.07	1.58	1.10		
	av. ret	0.0751	0.0777	0.0682	0.0784	0.0731	-0.0019	(0.637)	(0.626)
BV	exp. beta	0.64	0.84	0.99	1.17	1.52	0.87		
	av. ret	0.0680	0.0657	0.0842	0.0838	0.0922	0.0241	(0.241)	(0.847)
$BV_{6\ mon}$	exp. beta	0.63	0.84	0.99	1.18	1.53	0.89		
	av. ret	0.0609	0.0688	0.0764	0.0870	0.0891	0.0282	(0.134)	(0.561)
BV_5	exp. beta	0.68	0.85	0.98	1.15	1.51	0.83		
	av. ret	0.0687	0.0680	0.0870	0.1007	0.0905	0.0218	(0.278)	(0.824)
BV_HF^{50}	exp. beta	0.59	0.78	0.92	1.11	1.51	0.93		
	av. ret	0.0684	0.0668	0.0713	0.0829	0.0767	0.0083	(0.240)	(0.653)

Table 3.5: Return Statistics and Cross-Sectional Regressions (continued)

Panel B. Cross-Sectional Regressions

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{6\ mon}$	BV_5	BV_HF^{50}
10 Portfolios								
Const.	0.0655***	0.0632***	0.0836***	0.0805***	0.0416***	0.0432***	0.0453***	0.0612***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.005)	(0.000)
Slope	0.0063	0.0086	-0.0110	-0.0067	0.0374***	0.0340***	0.0382***	0.0138
p-value	(0.452)	(0.169)	(0.119)	(0.143)	(0.002)	(0.001)	(0.008)	(0.130)
adj R^2	0.07	0.22	0.28	0.25	0.72	0.75	0.61	0.26
25 Portfolios								
Const.	0.0645***	0.0659***	0.0876***	0.0779***	0.0544***	0.0502***	0.0541***	0.0596***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0094	0.0086	-0.0145***	-0.0015	0.0263***	0.0291***	0.0306***	0.0176***
p-value	(0.192)	(0.184)	(0.009)	(0.722)	(0.004)	(0.000)	(0.000)	(0.003)
adj R^2	0.07	0.08	0.26	0.01	0.30	0.52	0.45	0.32
50 Portfolios								
Const.	0.0661***	0.0684***	0.0850***	0.0799***	0.0565***	0.0530***	0.0593***	0.0639***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0109*	0.0091	-0.0079*	-0.0005	0.0273***	0.0292***	0.0271***	0.0166***
p-value	(0.088)	(0.114)	(0.082)	(0.895)	(0.001)	(0.000)	(0.000)	(0.001)
adj R^2	0.06	0.05	0.06	0.00	0.22	0.38	0.30	0.20

for the historical estimators is significant. However, with the number of portfolios, significance increases. For fifty portfolios, the slope coefficients for the one-year historical daily approach is significant at 10 %. For the high-frequency historical estimators, the intercept estimate is about 8 % which, together with the usually insignificant or even slightly significantly negative slope estimates, implies a flat relation of beta and returns. The result that high-frequency beta is not priced in the cross-section of stock returns is consistent with recent evidence in Bollerslev et al. (2015) who show that continuous and total beta, as opposed to discontinous beta, do not adhere a significant price of risk. Turning the focus on the BV models, the intercept estimate is typically smaller than that for the historical estimators with values around 5 %. However, it is still highly significant in all cases, already contradicting one of the CAPM implications. The slope estimates for the BV models lie between 2.6 % and 3.8 % in magnitude, which is too small compared to the average market excess return, contradicting the second prediction of the CAPM. On the other hand, the slope coefficients are highly significant, at least implying a positive risk–return relation, maybe not with the magnitude desired but substantial nevertheless. As is expected, given the results outlined before, the combination of the high-frequency estimator and BV implies a positive, though weaker compared to solely using BV, relation of beta and subsequent excess returns.

The particularly bad performance of the high-frequency estimators in detecting a relationship between risk and returns at all may appear surprising at the first glance, since looking at the statistical examination, it performs notably well. Overall, however, it seems that BV contains information about the cross-section of future returns beyond that of any of the historical approaches. The major reason for that is probably that BV uses option-implied volatilities over a time period matching the evaluation horizon. This means that the procedure uses inherently forward-looking information, whereas all historical approaches have to rely on the assumption that beta, and hence the risk–return trade-off, is stable throughout both the estimation and evaluation horizons. Furthermore, the evaluation approach presented in this section strongly loads on measurement errors of the individual approaches by sorting according to the respective current estimates. This means that high measurement error stocks are likely to be clustered in the extreme portfolios, preventing the errors from being fully diversified in these portfolios. Consequently, it appears that measurement errors are stronger in the historical, and particularly high-frequency, estimators for some stocks. Using the MSE decomposition suggested by Mincer & Zarnowitz (1969) delivers some support for this conjecture. While the average RMSE, using the initial specification with five portfolios sorted by a common instrument, are approximately equal for BV and the high-frequency models, the bias component for BV is substantially higher (about 15 % of total MSE compared to roughly 2 %).[24] The inefficiency part is approximately equal around 10 % for each of the approaches, while the random error part, in turn, is substantially higher for the high-frequency models (around 88 % vs. 75 % of the total MSE).

3.3.7 Downside Beta

In this section, we examine the estimation accuracy of different models for downside beta. This is important since several recent studies show that downside beta is an important factor for pricing the cross-section of returns for stocks and other asset classes (e.g., Ang et al., 2006a; Lettau et al., 2014). For the estimation of downside beta, however, one has typically to rely on long estimation windows in order to get sufficient data points that provide a reliable estimate. Therefore, the availability of high-frequency data is potentially crucial, since, for any potential specification, more data points are available and hence a more precise estimation ought to be possible. To do the analysis, we have to adjust our main models to estimate downside beta. Subsequently, we examine the statistical and economic properties of these estimators.

Since the estimation changes slightly for conditional beta factors, we quickly outline how we adjust our main models. Realized downside beta is estimated as follows:

$$\beta_{j,t}^{\mathrm{R},-} = \frac{\sum_{\tau=1}^{N} r_{j,\tau} r_{M,\tau} I[r_{M,\tau} < \theta]}{\sum_{\tau=1}^{N} r_{M,\tau}^2 I[r_{M,\tau} < \theta]}. \tag{3.8}$$

$I[r_{M,\tau} < \theta]$ is an indicator function returning the value one if the condition in brackets is fulfilled and zero otherwise. θ is an exogenously defined threshold.

[24] The average total RMSE of BV is 0.099 compared to 0.105 for HF_5 and 0.132 for $\mathrm{HF}_{5,1\ \mathrm{mon}}$, while nothing can be stated about significance since each of the approaches is sorted differently and therefore the estimates of beta are to be evaluated by different estimates for realized beta.

We define the threshold as zero.[25] We estimate historical beta with

$$\beta_{j,t}^{\text{HIST},-} = \frac{\text{cov}(r_j, r_M | r_M < \theta)}{\text{var}(r_M | r_M < \theta)}. \tag{3.9}$$

The hybrid downside beta is obtained as

$$\beta_{j,t}^{\text{BV},-} = \frac{\sigma_{j,t}^{\mathbb{Q},-} \sum_{i=1}^{N} (\omega_{i,t} \sigma_{i,t}^{\mathbb{Q},-} \rho_{ji,t}^{\mathbb{Q},-})}{(\sigma_{M,t}^{\mathbb{Q},-})^2}, \tag{3.10}$$

with $\sigma_{j,t}^{\mathbb{Q},-}$ and $\sigma_{M,t}^{\mathbb{Q},-}$ being the option-implied downside volatilities while $\rho_{ji,t}^{\mathbb{Q},-}$ is the risk-neutral downside correlation applying the transformation of physical to risk-neutral correlations suggested by Buss & Vilkov (2012).

For the hybrid approach to be obtainable using the types of options currently available we need the assumption that the relative variance risk premium conditional on an asset's return being below a certain threshold is the same as that conditional on the market return being below the corresponding threshold. This implies that the following relation holds:

$$\begin{aligned} \frac{\text{var}^{\mathbb{P}}(r_j | r_j < \theta)}{\text{var}^{\mathbb{P}}(r_j | r_M < \theta)} &= \frac{\text{var}^{\mathbb{Q}}(r_j | r_j < \theta)/RVRP_j}{\text{var}^{\mathbb{Q}}(r_j | r_M < \theta)/RVRP_j} \\ &= \frac{\text{var}^{\mathbb{Q}}(r_j | r_j < \theta)}{\text{var}^{\mathbb{Q}}(r_j | r_M < \theta)} = \nu, \end{aligned} \tag{3.11}$$

where $\mathrm{RVRP_j} = \text{var}^{\mathbb{Q}}(r_j)/\text{var}^{\mathbb{P}}(r_j)$ denotes the relative variance risk premium of asset j. Further assuming that the quantity ν obtained with the conditional variances from the historical relation under $\mathbb{P}$ is stable over short horizons, we can convert the implied variance conditional on the asset's return being below θ to the conditional variance, given the market return is below the corresponding threshold. We obtain ν using the past one year of daily returns and, subsequently, transform $\text{var}^{\mathbb{Q}}(\mathrm{r_j}|\mathrm{r_j} < \theta)$ to $\text{var}^{\mathbb{Q}}(\mathrm{r_j}|\mathrm{r_M} < \theta)$ dividing it by ν.[26]

[25] For the sake of brevity we do not report the results on other downside thresholds as, e.g. the average market return during the sample period (Ang et al., 2006a), or the average market return during the sample period minus one sample standard deviation (Lettau et al., 2014). The results using these thresholds are qualitatively similar.

[26] We further describe the procedure of obtaining the lower partial moments in Section B.1 in the appendix to this chapter.

Table 3.6: Estimation Errors: Downside Beta

This table reports the out-of-sample estimation errors of competing estimators for five-minute realized downside beta over the time horizon of six months for each portfolio. We define the downside threshold as $\theta = 0$. We build five quintile portfolios into which the stocks are allocated in ascending order according to their historical downside beta in the sorting period (taking place directly before the estimation period for historical downside beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. The first row reports the average root mean squared errors of the estimation models over the five portfolios. The lowest error among all approaches is indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average root median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{6\ mon}$	BV_5	BV_HF^{50}
avg.	0.1143	0.1140	0.0864	0.0932	0.0848	0.0862	0.0872	*0.0697*
HIST		0.0003	0.0279	0.0210	0.0294	0.0280	0.0270	0.0445
		(0.0)	(0.0)	(0.0)	(0.2)	(0.0)	(0.0)	(0.8)
$HIST_{6\ mon}$	-0.0003		0.0276	0.0207	0.0291	0.0277	0.0268	0.0443
	(0.0)		(0.4)	(0.2)	(0.2)	(0.2)	(0.0)	(0.8)
HF_5	-0.0153	-0.0149		-0.0068	0.0016	0.0002	-0.0008	0.0167
	(-0.2)	(-0.6)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$HF_{5,1\ mon}$	-0.0139	-0.0136	0.0013		0.0084	0.0070	0.0060	0.0235
	(-0.2)	(-0.2)	(0.0)		(0.0)	(0.0)	(0.0)	(1.0)
BV	-0.0178	-0.0174	-0.0025	-0.0038		-0.0014	-0.0024	0.0151
	(-0.2)	(0.0)	(0.0)	(0.0)		(0.0)	(0.0)	(0.4)
$BV_{6\ mon}$	-0.0156	-0.0152	-0.0003	-0.0016	0.0022		-0.0010	0.0165
	(-0.2)	(0.0)	(0.0)	(0.2)	(0.0)		(0.0)	(0.4)
BV_5	-0.0157	-0.0154	-0.0004	-0.0018	0.0020	-0.0002		0.0175
	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(0.2)
BV_HF^{50}	-0.0247	-0.0244	-0.0095	-0.0108	-0.0070	-0.0092	-0.0090	
	(-0.6)	(-0.6)	(0.0)	(-0.6)	(-0.4)	(-0.4)	(-0.4)	

Table 3.7: Cross-Sectional Regressions: Downside Beta

This table reports cross-sectional regressions in the manner of Black et al. (1972). We define the downside threshold as $\theta = 0$. For each methodology, we build ten, twenty-five, and fifty portfolios into which the stocks are allocated in ascending order according to their current estimates. We determine portfolio betas and excess returns over the subsequent six months as value-weighted averages. Excess returns are annualized. We regress the portfolios' average excess return over rolling six-month windows on the average beta estimates for the respective portfolios. Const. and Slope denote the regression intercept and slope, while p-value indicates the respective p-value using OLS standard errors. The rows adj R^2 present the adjusted R^2 of the regressions. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{6\ mon}$	BV_5	BV_HF^{50}
10 Portfolios								
Const.	0.0865***	0.0762***	0.0876***	0.0756***	0.0634***	0.0516***	0.0653***	0.0630***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	-0.0089	-0.0022	-0.0146*	-0.0019	0.0184***	0.0284***	0.0193**	0.0132**
p-value	(0.196)	(0.812)	(0.068)	(0.762)	(0.005)	(0.000)	(0.018)	(0.030)
adj R^2	0.20	0.01	0.36	0.01	0.64	0.87	0.52	0.47
25 Portfolios								
Const.	0.0839***	0.0728***	0.0874***	0.0759***	0.0612***	0.0557***	0.0672***	0.0637***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	-0.0043	0.0052	-0.0129**	0.0003	0.0227***	0.0271***	0.0192***	0.0149**
p-value	(0.531)	(0.457)	(0.017)	(0.942)	(0.000)	(0.000)	(0.003)	(0.017)
adj R^2	0.02	0.02	0.22	0.00	0.59	0.71	0.33	0.22
50 Portfolios								
Const.	0.0840***	0.0752***	0.0865***	0.0767***	0.0636***	0.0574***	0.0713***	0.0653***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	-0.0028	0.0053	-0.0083	0.0027	0.0223***	0.0282***	0.0167***	0.0174***
p-value	(0.629)	(0.314)	(0.109)	(0.498)	(0.000)	(0.000)	(0.002)	(0.001)
adj R^2	0.00	0.02	0.05	0.01	0.42	0.47	0.19	0.22

Table 3.6 presents the results for estimation accuracy of our main models. Regarding the individual models, BV yields the lowest average RMSE while that of HF_5 is only slightly higher and the historical models using daily return data yield the highest estimation errors. However, the differences among the individual models are insignificant most of the time. Regarding the cross-sectional relation of downside beta and subsequent returns, the results are presented in Table 3.7. A significantly positive relation of downside beta and subsequent returns can be detected neither for the historical daily nor for the high-frequency models. For the high-frequency estimators the relation is even significantly negative in two instances. For the hybrid models, independently of the specification, we detect a significantly positive relation of downside beta and subsequent returns. The slope estimates are at around 2–3 % smaller in magnitude compared to those for "total" beta presented in Table 3.5. Overall, the results on estimation and cross-sectional pricing of partial downside beta are qualitatively similar to those of "total" beta.

3.4 Robustness

3.4.1 Long-Run vs. Short-Run Estimation Accuracy

To examine the robustness of our results we perform the evaluation using different time horizons, namely one, three, twelve, eighteen, and twenty-four months. We estimate the values for option-implied methods using options with the appropriate time to maturity (e.g., one month for the one-month time horizon, etc.) and adjust the estimation horizon for high-frequency models to the respective time-frame, evaluating all the methods using realized beta over the subsequent one, three, twelve, eighteen, and twenty-four months, respectively.

Table 3.8: Estimation Errors: Different Time Horizons

This table reports the out-of-sample estimation errors of competing estimators, for realized beta over time horizons of one (Panel A), three (Panel B), twelve (Panel C), eighteen (Panel D), and twenty-four (Panel E) months for each portfolio. We build five quintile portfolios into which the stocks are allocated in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. In each Panel, the first row reports the average root mean squared errors of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the panels report the difference in estimation errors. The upper triangular matrices report the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrices report the average root median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrices, respectively. The sign indicates the direction of the significant differences.

<u>*Panel A. One Month*</u>

	HIST	$HIST_{1\ mon}$	HF_5	BV	$BV_{1\ mon}$	BV_5	BV_HF^{50}
avg.	0.1192	0.1255	0.0798	0.0938	0.1014	0.1026	*0.0752*
HIST		-0.0063	0.0394	0.0254	0.0178	0.0166	0.0440
		(-0.2)	(1.0)	(0.6)	(0.4)	(0.4)	**(1.0)**
$HIST_{1\ mon}$	0.0076		0.0457	0.0318	0.0241	0.0230	0.0504
	(0.4)		**(1.0)**	(0.8)	(0.6)	(0.4)	**(1.0)**
HF_5	-0.0169	-0.0244		-0.0140	-0.0216	-0.0228	0.0046
	(-0.8)	**(-1.0)**		(-0.4)	(-0.6)	(-0.6)	(0.6)
BV	-0.0053	-0.0129	0.0115		-0.0076	-0.0088	0.0186
	(-0.4)	(-1.0)	(0.6)		(-0.2)	(-0.4)	**(1.0)**
$BV_{1\ mon}$	-0.0018	-0.0094	0.0150	0.0035		-0.0012	0.0263
	(-0.4)	(-0.6)	(0.8)	(0.2)		(0.0)	**(1.0)**
BV_5	0.0010	-0.0066	0.0178	0.0063	0.0028		0.0274
	(0.0)	(-0.6)	(0.8)	(0.6)	(0.2)		**(1.0)**
BV_HF^{50}	-0.0197	-0.0272	-0.0028	-0.0143	-0.0178	-0.0206	
	(-1.0)	**(-1.0)**	(-0.2)	**(-1.0)**	**(-1.0)**	**(-1.0)**	

Table 3.8: Estimation Errors: Different Time Horizons (continued)

Panel B. Three Months

	HIST	$\text{HIST}_{3\ \text{mon}}$	HF_5	$\text{HF}_{5,1\ \text{mon}}$	BV	$\text{BV}_{3\ \text{mon}}$	BV_5	BV_HF^{50}
avg.	0.1174	0.1070	0.0750	0.0796	0.0866	0.0897	0.1032	*0.0700*
HIST		0.0105	0.0424	0.0379	0.0308	0.0277	0.0142	0.0474
		(0.0)	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)	**(1.0)**
$\text{HIST}_{3\ \text{mon}}$	-0.0043		0.0320	0.0274	0.0203	0.0172	0.0038	0.0370
	(0.0)		(1.0)	(0.6)	(0.2)	(0.2)	(0.2)	**(1.0)**
HF_5	-0.0184	-0.0142		-0.0046	-0.0117	-0.0147	-0.0282	0.0050
	(-0.8)	(-0.8)		(-0.4)	(-0.2)	(-0.6)	(-0.6)	(0.2)
$\text{HF}_{5,1\ \text{mon}}$	-0.0169	-0.0126	0.0015		-0.0071	-0.0102	-0.0236	0.0096
	(-0.6)	(-0.6)	(0.2)		(-0.2)	(-0.4)	(-0.4)	(0.6)
BV	-0.0089	-0.0046	0.0096	0.0081		-0.0031	-0.0165	0.0167
	(-0.4)	(-0.4)	(0.2)	(0.2)		(0.0)	(-0.4)	(0.8)
$\text{BV}_{3\ \text{mon}}$	-0.0088	-0.0046	0.0096	0.0081	0.0000		-0.0134	0.0198
	(-0.4)	(-0.4)	(0.4)	(0.2)	(0.0)		(-0.2)	(0.8)
BV_5	-0.0027	0.0016	0.0158	0.0143	0.0062	0.0062		0.0332
	(-0.2)	(-0.4)	(0.4)	(0.2)	(0.4)	(0.4)		(0.8)
BV_HF^{50}	-0.0226	-0.0183	-0.0042	-0.0057	-0.0137	-0.0138	-0.0200	
	(-1.0)	**(-1.0)**	(-0.2)	(-0.6)	(-0.8)	(-0.8)	(-0.6)	

Panel C. Twelve Months

	HIST	HF_5	$\text{HF}_{5,1\ \text{mon}}$	BV	BV_5	BV_HF^{50}
avg.	0.1405	0.1216	0.1070	0.1045	0.1290	*0.0910*
HIST		0.0189	0.0335	0.0360	0.0115	0.0495
		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_5	-0.0080		0.0146	0.0171	-0.0074	0.0306
	(-0.2)		(0.0)	(0.0)	(0.0)	(0.0)
$\text{HF}_{5,1\ \text{mon}}$	-0.0137	-0.0057		0.0025	-0.0220	0.0160
	(0.0)	(0.0)		(0.0)	(0.0)	(0.2)
BV	-0.0167	-0.0086	-0.0030		-0.0245	0.0135
	(0.0)	(0.0)	(0.0)		(0.0)	(0.0)
BV_5	-0.0143	-0.0062	-0.0006	0.0024		0.0380
	(0.0)	(0.0)	(0.0)	(0.2)		(0.0)
BV_HF^{50}	-0.0228	-0.0147	-0.0091	-0.0061	-0.0085	
	(-0.2)	(-0.4)	(0.0)	(0.0)	(-0.2)	

Table 3.8: Estimation Errors: Different Time Horizons (continued 2)

Panel D. Eighteen Months

	HIST	HIST$_{18\text{ mon}}$	HF$_5$	HF$_{5,1\text{ mon}}$	BV	BV$_{18\text{ mon}}$	BV$_5$	BV_HF^{50}
avg.	0.1559	0.1620	0.1385	0.1247	0.1154	0.1121	0.1375	*0.1038*
HIST		-0.0061	0.0173	0.0311	0.0404	0.0437	0.0184	0.0521
		(0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.2)
HIST$_{18\text{ mon}}$	0.0064		0.0235	0.0373	0.0466	0.0499	0.0245	0.0582
	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF$_5$	-0.0014	-0.0078		0.0138	0.0231	0.0264	0.0010	0.0347
	(0.0)	(0.0)		(0.0)	(0.2)	(0.2)	(0.0)	(0.2)
HF$_{5,1\text{ mon}}$	-0.0058	-0.0122	-0.0044		0.0093	0.0126	-0.0128	0.0209
	(0.0)	(0.0)	(0.0)		(0.2)	(0.2)	(0.0)	(0.2)
BV	-0.0168	-0.0232	-0.0154	-0.0110		0.0033	-0.0221	0.0116
	(0.0)	(0.0)	(0.0)	(0.0)		(0.0)	(0.0)	(0.0)
BV$_{18\text{ mon}}$	-0.0169	-0.0233	-0.0155	-0.0111	-0.0001		-0.0254	0.0083
	(0.0)	(0.0)	(0.0)	(0.0)	(0.2)		(0.0)	(0.0)
BV$_5$	-0.0167	-0.0231	-0.0154	-0.0110	0.0001	0.0002		0.0337
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)		(0.0)
BV_HF^{50}	-0.0170	-0.0234	-0.0156	-0.0112	-0.0002	-0.0001	-0.0003	
	(0.0)	(-0.2)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)	

Panel E. Twenty-Four Months

	HIST	HIST$_{24\text{ mon}}$	HF$_5$	HF$_{5,1\text{ mon}}$	BV	BV$_{24\text{ mon}}$	BV$_5$	BV_HF^{50}
avg.	0.1649	0.1722	0.1458	0.1372	0.1215	0.1201	0.1421	*0.1109*
HIST		-0.0074	0.0191	0.0276	0.0433	0.0448	0.0227	0.0540
		(0.2)	(0.0)	(0.0)	(0.2)	(0.2)	(0.0)	(0.2)
HIST$_{24\text{ mon}}$	0.0254		0.0264	0.0350	0.0507	0.0521	0.0301	0.0614
	(0.0)		(0.0)	(-0.2)	(0.2)	(0.2)	(0.0)	(0.2)
HF$_5$	0.0179	-0.0075		0.0086	0.0243	0.0257	0.0037	0.0349
	(0.0)	(0.0)		(0.0)	(0.2)	(0.2)	(0.0)	(0.2)
HF$_{5,1\text{ mon}}$	0.0010	-0.0243	-0.0169		0.0157	0.0171	-0.0049	0.0264
	(0.0)	(0.0)	(0.0)		(0.2)	(0.2)	(0.0)	(0.2)
BV	-0.0173	-0.0426	-0.0352	-0.0183		0.0014	-0.0206	0.0107
	(0.0)	(0.0)	(-0.2)	(0.0)		(0.0)	(0.0)	(-0.2)
BV$_{24\text{ mon}}$	-0.0188	-0.0442	-0.0367	-0.0199	-0.0016		-0.0220	0.0093
	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)		(0.0)	(-0.2)
BV$_5$	-0.0163	-0.0416	-0.0342	-0.0173	0.0010	0.0025		0.0313
	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)	(0.0)		(0.0)
BV_HF^{50}	-0.0134	-0.0388	-0.0313	-0.0145	0.0038	0.0054	0.0029	
	(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	

Panel A of Table 3.8 reports the estimation errors of our main methods and their significance for the one-month evaluation period. We find that using this evaluation horizon the five-minute high-frequency estimator yields the lowest estimation errors among the individual models. These are significantly lower compared to those of the historical models relying on daily return data for at least four of the five portfolios. Compared to BV, the RMSE is significantly lower for two portfolios. Consequently, the relation is not entirely clear, but there is strong indication that for the one-month time horizon one should rely on HF_5. The superiority of the one-month high-frequency estimator indicates that beta is quite stable over the short term and the argument of Lewellen & Nagel (2006), that these short-term estimates deliver proper conditional forecasts over short time horizons, appears to hold. The combined estimator BV_HF^{50}, however, yields estimation errors that are significantly lower for all portfolios compared to all other approaches except HF_5. Compared to HF_5, the RMSE is significantly lower for three and the RMedSE for one portfolio, so the two models can be ranked with some but not full confidence.

For the three-month time horizon, shown in Panel B of Table 3.8, the results are quite similar. The high-frequency estimator, now using a three-month estimation horizon, appears to deliver the best estimation accuracy among the individual models. While the one-month high-frequency estimator yields results that are only slightly worse, the differences in RMSE compared to BV are mostly not significant for the two approaches. Still, the combination of $HF_{5,1\ mon}$ and BV overall yields the best estimation accuracy, which is significantly better than that of all models but the high-frequency estimators.

When looking at longer time horizons, namely twelve, eighteen, and twenty-four months in Panels C – E of Table 3.8, the ranking of the high-frequency estimators and the BV models topples slightly towards the direction of BV. Either the usual twelve-month BV or the BV model using daily returns over the time horizon matching the evaluation horizon obtain the lowest average RMSE of all individual models. Naturally, over longer horizons, the inherently forward-looking information employed in BV yields better conditional estimates even compared to short historical windows. The differences, however, are mostly insignificant. Furthermore, independent of the evaluation horizon, BV_HF^{50} yields the best estimation accuracy, but the differences are at most weakly significant, especially compared to the BV models.

Summing up, especially over short time horizons up to three months, the high-frequency estimator works quite well concerning the statistical evaluation methods, while for the twelve-month and longer time horizon(s) the results on the estimation accuracy of approaches examined become indistinguishable. Over all time horizons, the combination of the high-frequency and BV estimators delivers the best estimation accuracy.

3.4.2 Further Models

In this section, we examine further possibilities for the estimation of beta. In particular, we consider further high-frequency estimators using return frequencies of 15, 30, 75, and 130 minutes and further BV models employing high-frequency correlations estimated on the basis of 15-, 30-, 75-, and 130-minute returns.

The results are presented in Table 3.9. It can be seen that the average RMSE is higher for lower-frequency HF estimators compared to the five-minute approach, while these differences are mostly not significant. Turning the focus on the optimal sampling frequency for BV, the average estimation errors are slightly lower for lower sampling frequencies, with BV based on daily returns yielding the lowest average RMSE. However, once more, in

Table 3.9: Estimation Errors: Further Models

This table reports the out-of-sample estimation errors of competing estimators for five-minute realized beta over the time horizon of six months for each portfolio. We build five quintile portfolios into which the stocks are allocated in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. The first row reports the average root mean squared errors of the estimation models over the five portfolios. The lowest errors among all approaches are indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the five portfolios. Similarly, the lower triangular matrix reports the average median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.4 indicates that the differences for two out of five portfolios are statistically significant). If the differences are significant for all five portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

Table 3.9: Estimation Errors: Further Models (continued)

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	HF_{15}	HF_{30}	HF_{75}	HF_{130}	BV	$BV_{6\ mon}$	BV_5	BV_{15}	BV_{30}	BV_{75}	BV_{130}
avg.	0.1247	0.1177	0.0907	*0.0865*	0.0927	0.0961	0.1009	0.1045	0.0915	0.0932	0.1127	0.1082	0.1076	0.1046	0.1074
HIST		0.0070	0.0340	0.0382	0.0320	0.0286	0.0238	0.0201	0.0332	0.0315	0.0120	0.0165	0.0171	0.0201	0.0173
		(0.0)	(0.2)	(0.0)	(0.2)	(0.2)	(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$HIST_{6\ mon}$	-0.0052		0.0271	0.0312	0.0251	0.0216	0.0168	0.0132	0.0262	0.0245	0.0050	0.0095	0.0101	0.0131	0.0104
	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_5	-0.0160	-0.0108		0.0041	-0.0020	-0.0055	-0.0102	-0.0139	-0.0009	-0.0026	-0.0221	-0.0176	-0.0170	-0.0140	-0.0167
	(-0.4)	(-0.2)		(0.0)	(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)
$HF_{5,1\ mon}$	-0.0164	-0.0112	-0.0004		-0.0061	-0.0096	-0.0144	-0.0180	-0.0050	-0.0067	-0.0262	-0.0217	-0.0211	-0.0181	-0.0208
	(-0.6)	(-0.2)	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)	(-0.2)	(-0.2)	(-0.2)	(-0.2)	(-0.2)
HF_{15}	-0.0118	-0.0066	0.0042	0.0046		-0.0035	-0.0082	-0.0119	0.0012	-0.0006	-0.0201	-0.0156	-0.0150	-0.0120	-0.0147
	(-0.4)	(-0.2)	(0.0)	(0.0)		(0.0)	(-0.4)	(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_{30}	-0.0095	-0.0042	0.0066	0.0070	0.0023		-0.0048	-0.0084	0.0046	0.0029	-0.0166	-0.0121	-0.0115	-0.0085	-0.0112
	(-0.2)	(0.0)	(0.0)	(0.2)	(0.0)		(-0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_{75}	-0.0077	-0.0025	0.0083	0.0087	0.0040	0.0017		-0.0036	0.0094	0.0077	-0.0118	-0.0073	-0.0067	-0.0037	-0.0065
	(-0.2)	(-0.2)	(0.0)	(0.2)	(0.2)	(0.2)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_{130}	-0.0074	-0.0022	0.0086	0.0090	0.0044	0.0021	0.0003		0.0130	0.0113	-0.0082	-0.0037	-0.0031	-0.0001	-0.0028
	(-0.4)	(0.0)	(0.2)	(0.4)	(0.2)	(0.0)	(0.2)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
BV	-0.0113	-0.0061	0.0047	0.0051	0.0005	-0.0019	-0.0036	-0.0039		-0.0017	-0.0212	-0.0167	-0.0161	-0.0131	-0.0159
	(0.0)	(0.0)	(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$BV_{6\ mon}$	-0.0105	-0.0053	0.0055	0.0059	0.0013	-0.0011	-0.0028	-0.0031	0.0008		-0.0195	-0.0150	-0.0144	-0.0114	-0.0141
	(-0.4)	(-0.2)	(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
BV_5	-0.0068	-0.0016	0.0092	0.0096	0.0049	0.0026	0.0009	0.0006	0.0045	0.0037		0.0045	0.0051	0.0081	0.0054
	(0.2)	(0.0)	(0.2)	(0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(0.4)	(0.2)		(0.0)	(0.0)	(0.0)	(0.0)
BV_{15}	-0.0088	-0.0036	0.0072	0.0076	0.0030	0.0006	-0.0011	-0.0014	0.0025	0.0017	-0.0020		0.0006	0.0036	0.0009
	(-0.2)	(0.0)	(0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(0.0)	(0.2)	(0.2)	(-0.4)		(0.0)	(0.0)	(0.0)
BV_{30}	-0.0086	-0.0034	0.0074	0.0078	0.0032	0.0009	-0.0009	-0.0012	0.0027	0.0019	-0.0018	0.0002		0.0030	0.0003
	(-0.2)	(0.0)	(0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(0.0)	(0.2)	(0.2)	(-0.6)	(-0.2)		(0.0)	(0.0)
BV_{75}	-0.0088	-0.0036	0.0072	0.0076	0.0030	0.0007	-0.0010	-0.0014	0.0025	0.0017	-0.0019	0.0000	-0.0002		-0.0027
	(-0.2)	(-0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(-0.4)	(-0.2)	(0.0)		(0.0)
BV_{130}	-0.0099	-0.0047	0.0061	0.0065	0.0019	-0.0004	-0.0022	-0.0025	0.0014	0.0006	-0.0031	-0.0011	-0.0013	-0.0011	
	(-0.2)	(-0.2)	(0.2)	(0.2)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(-0.2)	(-0.2)	(-0.2)	(-0.2)	(0.0)	

general the differences are insignificant in the majority of the cases.

3.4.3 More Portfolios and Different Sampling Frequencies

We test whether the results obtained so far are robust to building more portfolios and different sampling frequencies for realized beta used to evaluate the ex ante estimates. Thus, for the first part of the analysis, we build ten, twenty-five, and fifty portfolios and in the limit we also consider the case of individual stocks. Panel A of Table 3.10 reports the results, which are quite similar to our previous findings. We observe that the average errors in general increase with the number of portfolios where the diversification of idiosyncratic measurement errors works less well. Regarding the individual models, independently of the number of portfolios, one of the high-frequency estimators obtains the lowest average RMSE. For five and ten portfolios, the short-term $HF_{5,1\ \mathrm{mon}}$ has the lowest estimation errors, while for a larger number of portfolios the six-month HF_5 delivers the most precise estimates on average. The estimation errors of the BV approaches are generally a bit higher than those of the high-frequency approaches, but smaller than those of HIST. Looking at the combined estimate, BV_HF^{50}, it turns out to yield the best estimation accuracy as long as portfolios are formed. It thereby has the lowest estimation errors for at least 76 % of the portfolios. Only for individual assets is the average RMSE for HF_5 slightly smaller.

The results for different sampling frequencies for realized beta used to evaluate the ex ante estimates are presented in Panel B of Table 3.10. The results are consistent with our previous findings. Among the individual models $HF_{5,1\ \mathrm{mon}}$ yields the lowest average RMSE independent of the sampling frequency in realized beta. BV and HF_5 also yield relatively low average RMSEs. Further analysis reveals that, as hitherto, in most cases the

Table 3.10: Estimation Errors: More Portfolios and Different Sampling Frequencies

This table reports the root mean squared errors of the competing estimators for realized beta over the time horizon of six months, for different counts of portfolios (Panel A) and different frequencies for the evaluation proxy (Panel B). Each month, we form N portfolios with N amounting to 5, 10, 25, and 50, and in the limit we also consider the case of solely individual assets (in this case we compute the values of the loss functions for each asset in every month of our sample period individually and average over all errors). The stocks are allocated into N portfolios in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). The numbers in parentheses denote the count (as proportions) of portfolio series for which a certain approach yields the lowest error among those presented in the table. For each specification, the lowest average errors among all approaches are indicated by *italic* font.

Panel A. More Portfolios

	HIST	$\text{HIST}_{6\text{ mon}}$	HF_5	$\text{HF}_{5,1\text{ mon}}$	BV	$\text{BV}_{6\text{ mon}}$	BV_5	BV_HF^{50}
5 Portfolios								
avg. RMSE	0.1247	0.1177	0.0907	0.0865	0.0915	0.0932	0.1127	*0.0749*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(1.00)
10 Portfolios								
avg. RMSE	0.1358	0.1286	0.1001	0.0999	0.1040	0.1062	0.1245	*0.0854*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(1.00)
25 Portfolios								
avg. RMSE	0.1538	0.1503	0.1148	0.1200	0.1270	0.1294	0.1438	*0.1029*
	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)	(0.00)	(0.00)	(0.96)
50 Portfolios								
avg. RMSE	0.1730	0.1734	0.1289	0.1411	0.1508	0.1539	0.1654	*0.1207*
	(0.00)	(0.00)	(0.22)	(0.00)	(0.02)	(0.00)	(0.00)	(0.76)
Individual Assets								
avg. RMSE	0.3008	0.3247	*0.2252*	0.2687	0.2984	0.3066	0.3069	0.2325
	(0.11)	(0.10)	(0.20)	(0.21)	(0.09)	(0.08)	(0.11)	(0.11)

Table 3.10: Estimation Errors: More Portfolios and Different Sampling Frequencies (continued)

Panel B. Different Sampling Frequencies

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{6\ mon}$	BV_5	BV_HF^{50}
5 min								
avg. RMSE	0.1247	0.1177	0.0907	0.0865	0.0915	0.0932	0.1127	*0.0749*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(1.00)
15 min								
avg. RMSE	0.1234	0.1168	0.0938	0.0897	0.0929	0.0949	0.1164	*0.0774*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(1.00)
30 min								
avg. RMSE	0.1248	0.1188	0.0979	0.0928	0.0974	0.0994	0.1221	*0.0819*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(1.00)
75 min								
avg. RMSE	0.1282	0.1232	0.1053	0.1000	0.1046	0.1070	0.1306	*0.0903*
	(0.00)	(0.00)	(0.00)	(0.20)	(0.00)	(0.00)	(0.00)	(0.80)
130 min								
avg. RMSE	0.1285	0.1222	0.1055	0.0998	0.1074	0.1087	0.1333	*0.0917*
	(0.00)	(0.00)	(0.00)	(0.20)	(0.00)	(0.00)	(0.00)	(0.80)
daily								
avg. RMSE	0.1337	0.1250	0.1156	0.1103	0.1125	0.1127	0.1384	*0.1005*
	(0.00)	(0.00)	(0.00)	(0.20)	(0.00)	(0.00)	(0.00)	(0.80)

differences in RMSE are not statistically significant among the individual models. On the other hand, the combination of BV and $HF_{5,1\ mon}$ yields the lowest overall average RMSE independent of the sampling frequency for ex post realized beta. These differences are in general significant for some portfolios in RMSE and RMedSE.

3.4.4 Option and Stock Liquidity

Since both option-implied and high-frequency approaches strongly rely on precise and up-to-date measures of option and stock prices, it appears worthwhile to examine a highly liquid subset of our total sample where these conditions are most probably are. To do that, we repeat our main analysis for all stocks contained in the DJIA 30. The DJIA includes 30 of the largest U.S. companies that commonly have more trading activity in both stocks and in options compared to other securities.

The results are presented in Table 3.11. Regarding the individual models, the six-month high-frequency estimator yields the lowest average RMSE, followed by $HF_{5,1\ \mathrm{mon}}$ and the BV models. So, also given the relatively large magnitude in the differences of 0.013 comparing HF_5 and BV, on first glance the presumably better quality data appears to be more important for the high-frequency estimators compared to the hybrid BV estimators. Differences in RMSE, however, are mostly not significant. Considering median estimation errors, the differences are significant more often. However, still no fully clear statement on which model is to be preferred can be made since the estimation errors for the BV models are significantly higher for at most one of the two portfolios only. Additionally, turning the focus on the combination of BV and $HF_{5,1\ \mathrm{mon}}$, this simple model once more yields the lowest average estimation errors, which turn out to be significantly lower mainly in RMedSE compared to all models except the short-term historical and the high-frequency estimators, while in RMSE the differences are only significant compared to the BV models.[27]

Consequently, it turns out that the results appear not to be influenced strongly by the liquidity of the underlying securities and their derivatives used to extract information on beta.

3.4.5 Cross-Sectional Robustness

The analysis in Section 3.3.6 suggests that only the BV models and, to some extent the historical models using daily returns, are able to detect a positive cross-sectional relation of beta and subsequent excess returns. To examine whether this finding is specific to the models examined and the six-month evaluation horizon we perform a robustness test using additional models

[27] At 10 %, however, the RMSE for BV_HF^{50} is significantly smaller than those of HIST for both portfolios.

Table 3.11: Estimation Errors: DJIA

This table reports the out-of-sample estimation errors of competing estimators for realized beta over the time horizon of six months for each portfolio. We build two portfolios into which the stocks are allocated in ascending order according to their historical beta in the sorting period (taking place directly before the estimation period for historical beta without overlap and with equal length). We determine portfolio betas as value-weighted averages. The first row reports the average root mean squared errors of the estimation models over the two portfolios. The lowest error among all approaches is indicated by *italic* font. The remainder of the table reports the differences in estimation errors. The upper triangular matrix reports the differences in root mean squared estimation errors, averaged over the two portfolios. Similarly, the lower triangular matrix reports the average root median differences of estimation errors. We compute the difference between the errors of the model *[name in row]* and those of the model *[name in column]*. The absolute numbers in parentheses indicate the share of portfolios for which the difference is significant at 5 % (e.g., 0.5 indicates that the differences for one out of two portfolios are statistically significant). If the differences are significant for all portfolios, the figure is printed in **bold** font. Significance is tested by the modified Diebold–Mariano and the Wilcoxon signed rank tests for the upper and lower triangular matrix, respectively. The sign indicates the direction of the significant differences.

	HIST	$HIST_{6\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{6\ mon}$	BV_5	BV_HF^{50}
avg.	0.1003	0.1060	0.0783	0.0860	0.0916	0.0913	0.0951	*0.0738*
HIST		-0.0057	0.0221	0.0143	0.0087	0.0090	0.0052	0.0265
		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$HIST_{6\ mon}$	-0.0077		0.0277	0.0200	0.0144	0.0147	0.0109	0.0321
	(0.0)		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
HF_5	-0.0150	-0.0072		-0.0077	-0.0133	-0.0130	-0.0168	0.0044
	(-0.5)	(0.0)		(-0.5)	(0.0)	(0.0)	(0.0)	(0.0)
$HF_{5,1\ mon}$	-0.0176	-0.0099	-0.0027		-0.0056	-0.0053	-0.0091	0.0121
	(-0.5)	(0.0)	(0.0)		(0.0)	(0.0)	(0.0)	(0.5)
BV	-0.0014	0.0063	0.0136	0.0162		0.0003	-0.0035	0.0178
	(0.0)	(0.0)	(0.0)	(0.5)		(0.0)	(0.0)	**(1.0)**
$BV_{6\ mon}$	-0.0011	0.0066	0.0138	0.0165	0.0003		-0.0038	0.0175
	(0.0)	(0.0)	(0.0)	(0.5)	(0.0)		(0.0)	**(1.0)**
BV_5	-0.0017	0.0061	0.0133	0.0160	-0.0002	-0.0005		0.0213
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(0.5)
BV_HF^{50}	-0.0221	-0.0144	-0.0072	-0.0045	-0.0207	-0.0210	-0.0205	
	(-1.0)	(0.0)	(0.0)	(0.0)	**(-1.0)**	**(-1.0)**	**(-1.0)**	

and various alternative estimation and, accordingly, evaluation horizons. For each approach and time horizon we perform Black et al. (1972) cross-sectional regressions of average portfolio excess returns on average portfolio betas using different counts of portfolios. The results on additional models are presented in Panel A of Table 3.12. It can be seen that the results are consistent with our previous findings. For the high-frequency models, the relation of beta and returns is flat, while for the BV high-frequency models, there is a significantly positive relation of beta and subsequent excess returns, while the slope is too small in magnitude to match the main CAPM predictions. This holds independently of the sampling frequency used for the estimators. For the high-frequency BV models using 15- up to 130-minute sampling frequencies, the estimates for the slope coefficients are of similar magnitude compared to the main BV models. Further results on the main models over different time horizons, using twenty-five portfolios, are presented in Panel B of Table 3.12. These are qualitatively equal to the results for the six-month time horizon. For the historical models using daily data, we find a weakly positive relation of beta and subsequent returns which is most pronounced for the twelve–month time horizon. For other time-frames, the relation is regularly insignificant. For the high-frequency estimators, over short time horizons the beta-return relation is flat, while it is even significantly negative over long time horizons. For the BV models, the slope estimate is significantly positive for almost any specification and time horizon. Just as for the six-month time horizon the slope coefficient is sizable but not quite as large as the average market excess return, while the intercept estimate is significant in almost every case.

Table 3.12: Cross-Sectional Regressions: Robustness

This table reports cross-sectional regressions in the style of Black et al. (1972). For each methodology, we build N portfolios into which the stocks are allocated in ascending order according to their current estimates. We determine portfolio betas and excess returns over the subsequent six months as value-weighted averages. We regress the portfolios' annualized average excess return over rolling windows on the average beta estimates for the respective portfolios. In Panel A, we examine the results using additional models and in Panel B, we present the results for different time horizons building fifty portfolios, each. Const. and Slope denote the regression intercept and slope, while p-value indicates the respective p-value using OLS standard errors. The rows adj R^2 present the adjusted R^2 of the regressions. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %. If one model has already shown up for a certain time-frame, this is indicated by a "–"-sign in the first row. We abstain from repeatedly reporting these models.

Panel A. Further Models

	HF_{15}	HF_{30}	HF_{75}	HF_{130}	BV_{15}	BV_{30}	BV_{75}	BV_{130}
10 Portfolios								
Const.	0.0721***	0.0712***	0.0765***	0.0699***	0.0442***	0.0454***	0.0492***	0.0492***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.006)	(0.003)	(0.002)
Slope	0.0013	0.0031	-0.0034	0.0032	0.0392***	0.0374**	0.0330**	0.0315**
p-value	(0.839)	(0.570)	(0.456)	(0.540)	(0.005)	(0.010)	(0.015)	(0.015)
adj R^2	0.01	0.04	0.07	0.05	0.66	0.58	0.54	0.55
25 Portfolios								
Const.	0.0793***	0.0755***	0.0792***	0.0722***	0.0529***	0.0531***	0.0536***	0.0576***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	-0.0050	-0.0007	-0.0033	0.0032	0.0320***	0.0309***	0.0307***	0.0260***
p-value	(0.412)	(0.896)	(0.518)	(0.560)	(0.000)	(0.000)	(0.000)	(0.005)
adj R^2	0.03	0.00	0.02	0.01	0.48	0.42	0.43	0.29
50 Portfolios								
Const.	0.0809***	0.0791***	0.0802***	0.0744***	0.0617***	0.0594***	0.0622***	0.0642***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	-0.0039	-0.0019	-0.0014	0.0036	0.0250***	0.0260***	0.0241***	0.0210***
p-value	(0.464)	(0.689)	(0.747)	(0.420)	(0.000)	(0.000)	(0.001)	(0.002)
adj R^2	0.01	0.00	0.00	0.01	0.25	0.28	0.22	0.19

Table 3.12: Cross-Sectional Regressions: Robustness (continued)

Panel B. Different Time Horizons

	HIST	$HIST_{\tau\ mon}$	HF_5	$HF_{5,1\ mon}$	BV	$BV_{\tau\ mon}$	BV_5	BV_HF^{50}
One Month								
Const.	0.0675***	0.0760***	0.0754***	–	0.0545***	0.0667***	0.0498***	0.0584***
p-value	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0029	-0.0002	-0.0032		0.0240**	0.0118	0.0328***	0.0148
p-value	(0.777)	(0.984)	(0.732)		(0.040)	(0.242)	(0.001)	(0.305)
adj R^2	0.00	0.00	0.00		0.08	0.03	0.21	0.02
Three Months								
Const.	0.0653***	0.0629***	0.0749***	0.0791***	0.0605***	0.0533***	0.0566***	0.0657***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0085	0.0162**	0.0002	-0.0038	0.0223***	0.0281***	0.0274***	0.0105
p-value	(0.244)	(0.011)	(0.980)	(0.361)	(0.009)	(0.000)	(0.000)	(0.122)
adj R^2	0.03	0.13	0.00	0.02	0.13	0.24	0.28	0.05
Twelve Months								
Const.	0.0594***	–	0.0825***	0.0736***	0.0458***	–	0.0439***	0.0596***
p-value	(0.000)		(0.000)	(0.000)	(0.000)		(0.000)	(0.000)
Slope	0.0113**		-0.0108***	-0.0015	0.0316***		0.0355***	0.0136***
p-value	(0.017)		(0.010)	(0.576)	(0.000)		(0.000)	(0.001)
adj R^2	0.11		0.13	0.01	0.46		0.37	0.20
Eighteen Months								
Const.	0.0619***	0.0610***	0.0803***	0.0731***	0.0378***	0.0421***	0.0368***	0.0591***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0053	0.0055	-0.0127***	-0.0059**	0.0330***	0.0301***	0.0352***	0.0094**
p-value	(0.136)	(0.179)	(0.003)	(0.016)	(0.000)	(0.000)	(0.000)	(0.019)
adj R^2	0.05	0.04	0.17	0.12	0.30	0.29	0.29	0.11
Twenty-Four Months								
Const.	0.0644***	0.0672***	0.0919***	0.0775***	0.0483***	0.0481***	0.0425***	0.0674***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Slope	0.0004	-0.0038	-0.0268***	-0.0129***	0.0177**	0.0197**	0.0244***	-0.0023
p-value	(0.907)	(0.501)	(0.000)	(0.000)	(0.029)	(0.012)	(0.006)	(0.534)
adj R^2	0.00	0.01	0.30	0.29	0.10	0.12	0.15	0.01

3.5 Conclusion

This study analyzes whether intra-day high-frequency data adds value for beta estimation. We find that historical beta estimated with high-frequency returns delivers relatively precise estimates for ex post realized beta. Especially over short time horizons, high-frequency estimators appear to deliver accurate conditional estimates. Regarding informational efficiency and especially for longer time horizons the hybrid beta of Buss & Vilkov

(2012) employing information from the options market performs equally well or slightly better compared to the high-frequency estimator. When aiming to estimate ex post realized beta with high precision, we further show that it appears worthwhile to impose a simple method to combine both the historical high-frequency and the hybrid estimate for beta. This approach consistently delivers the lowest estimation errors that are significantly lower compared to any of the individual models, especially over short time horizons.

On the other hand, when evaluating beta estimates by their economic value, i.e., the cross-sectional predictability of subsequent excess returns, the hybrid BV approach is clearly favorable. While the approach cannot fully reconcile empirical observations with the CAPM predictions, at least it predicts a highly significant risk–return trade-off using beta to proxy for risk. Historical models using high-frequency or daily return data mostly imply a flat relation of beta and subsequent returns.

We further show that these results also hold for downside beta and employing various robustness tests.

B Appendix

B.1 Model-Free Option-Implied Volatility

The hybrid beta estimation approach is based on option-implied moments. Therefore we follow Bakshi et al. (2003), who make use of the property that any payoff can be spanned using a continuum of OTM puts and calls (Bakshi & Madan, 2000) and Jiang & Tian (2005) to compute model-free option-implied volatility. For that, we first compute ex-dividend stock prices. Secondly, for any given stock and trading day, we interpolate implied volatilities using a cubic spline across moneyness levels (K/S, strike-to-spot), equally spaced between 0.3 % and 300 %, to obtain a grid of 1,000 implied volatilities (Chang et al., 2012). Implied volatilities outside the range of available strike prices are extrapolated using the value for the smallest, resp. largest, available moneyness level (as in Jiang & Tian, 2005 and Chang et al., 2012). The volatilities are used to compute Black–Scholes option prices for calls, $C(.)$, if K/S>1 and puts, $P(.)$, if K/S<1. These are used to obtain the prices of the volatility (QUAD), the CUBIC, and the quartic (QUART) contract (Jiang & Tian, 2005):

$$\begin{aligned}
\text{QUAD} &= \int_S^\infty \frac{2\left(1-\ln\left[\frac{K}{S}\right]\right)}{K^2} C(\tau,K)dK \qquad (\text{B.1})\\
&+ \int_0^S \frac{2\left(1+\ln\left[\frac{S}{K}\right]\right)}{K^2} P(\tau,K)dK,\\
\text{CUBIC} &= \int_S^\infty \frac{6\ln\left[\frac{K}{S}\right]-3\left(\ln\left[\frac{K}{S}\right]\right)^2}{K^2} C(\tau,K)dK \qquad (\text{B.2})\\
&+ \int_0^S \frac{6\ln\left[\frac{S}{K}\right]+3\left(\ln\left[\frac{S}{K}\right]\right)^2}{K^2} P(\tau,K)dK,\\
\text{QUART} &= \int_S^\infty \frac{12\left(\ln\left[\frac{K}{S}\right]\right)^2-4\left(\ln\left[\frac{K}{S}\right]\right)^3}{K^2} C(\tau,K)dK \qquad (\text{B.3})\\
&+ \int_0^S \frac{12\left(\ln\left[\frac{S}{K}\right]\right)^2+4\left(\ln\left[\frac{S}{K}\right]\right)^3}{K^2} P(\tau,K)dK.
\end{aligned}$$

The integrals are approximated, following Dennis & Mayhew (2002), using a trapezoidal rule. The option-implied moments can be computed as:

$$\begin{aligned}
\mu^{\mathbb{Q}} &= e^{r_t^f(T-t)} - 1 - \frac{e^{r_t^f(T-t)}}{2}\text{QUAD} - \frac{e^{r_t^f(T-t)}}{6}\text{CUBIC} \qquad (\text{B.4})\\
&- \frac{e^{r_t^f(T-t)}}{24}\text{QUART},\\
(\sigma^{\mathbb{Q}})^2 &= e^{r_t^f(T-t)}\text{QUAD} - (\mu^{\mathbb{Q}})^2, \qquad (\text{B.5})
\end{aligned}$$

where r_t^f denotes the risk-free rate and $T-t$ the time to maturity of the contract. $(\sigma^{\mathbb{Q}})^2$ is the option-implied variance.

To obtain conditional, respectively partial implied moments, we build on Andersen & Bondarenko (2013) and Andersen, Bondarenko, & Gonzalez-Perez (2015) who develop the concept of corridor implied volatility which can be used to split model-free implied volatility into different parts for different intervals of the underlying asset price. Specifically, we follow the steps outlined above to obtain the grid of OTM option prices. Following on from that we use the alternative approach to obtain model-free implied volatility pioneered by Britten-Jones & Neuberger (2000) as:

$$(\sigma^{\mathbb{Q}})^2 = 2\int_0^{\infty} \frac{M(\tau, K)}{K^2} dK, \tag{B.6}$$

with $M(\tau, K)$ being the minimum price of put and call with strike K. We impose the threshold Se^{θ} with θ being equal to zero:

$$(\sigma^{\mathbb{Q},-})^2 = 2\int_0^{Se^{\theta}} \frac{M(\tau, K)}{K^2} dK. \tag{B.7}$$

We employ the discrete approximation of the integral using a trapezoidal rule as described above.

Chapter 4

Aggregate Uncertainty Affects Stock Returns*

4.1 Introduction

An essential distinction between risk and uncertainty has been emphasized ever since the seminal work of Knight (1921). He defines risk as measurable uncertainty that can be represented by numerical probabilities while there is also unmeasurable uncertainty which cannot be captured as easily. In another very important contribution, Ellsberg (1961) shows that there is a strong effect of uncertainty on investors' decisions while controlling for risk. Keynes (1936, p. 154) describes the impact of uncertainty on prices as "the outcome of the mass psychology of a large number of ignorant individuals [which] is liable to change violently as the result of a sudden fluctuation of opinion due to factors which do not really make much difference to the prospective yield". Particularly, when considering the serious market distortions caused by the recent financial crisis (and the crises before), one

*This chapter is based on the Working Paper "Aggregate Uncertainty Affects Stock Returns" authored by Fabian Hollstein and Marcel Prokopczuk, 2015.

sees that this "mass psychology" cannot be ignored for applications in asset pricing. Consequently, taking into account uncertainty (ambiguity[1]) appears to be a good starting point.

A simple stylized theoretical model, based on the standard Intertemporal Capital Asset Pricing Model (ICAPM) with recursive preferences and consumption uncertainty predicts that, beside the commonly employed risk-return trade-off, there exists also an uncertainty-return trade-off. To this end, aggregate uncertainty surrounding the consumption growth process can be regarded as a state variable. The intuition behind this is that time-varying aggregate uncertainty induces changes in the investment opportunity set, as higher uncertainty may lower the expectation of future market returns or increase expectations of future volatility of stock returns and hence worsen the expected risk-return trade-off. Furthermore, apart from a rational model motivation, uncertainty may also be priced through the channels of individuals with preferences different to standard expected utility preferences. Ambiguity-averse investors are likely to demand compensation for holding stocks with high exposure to such aggregate uncertainty. In a recent experimental study, Füllbrunn, Rau, & Weitzel (2014) show that, under certain conditions, ambiguity aversion can be reflected in capital markets.

Our main contribution is that, to the best of our knowledge, we are the first to examine the pricing of aggregate uncertainty proxied by the Chicago Board Options Exchange (CBOE) Volatility of the Volatility Index (VVIX), a natural non-parametric measure of stock market volatility-of-volatility, in the cross-section of expected stock returns. We further show that the uncertainty-return trade-off, predicted by a simple theoretical model

[1]Previous studies use the terms "uncertainty" and "ambiguity" synonymously. We mostly stick to the term "uncertainty" here, while referring to widespread terms like "ambiguity aversion" when talking about attitudes toward uncertainty.

without clear reference to the sign of the trade-off, is priced negatively. While risk is commonly represented by first-order beliefs, i.e. return volatility, we follow a plethora of literature which models what Knight called "unmeasurable uncertainty" with second-order beliefs, i.e. the variation in the probability distribution of the payoffs (e.g., Segal, 1987; Nau, 2003; Seo, 2009; Baltussen et al., 2015).

In the empirical methodology, we follow Ang et al. (2006a) and Cremers et al. (2015). Specifically, we estimate factor loadings on innovations in aggregate uncertainty on the level of individual stocks using daily returns. We sort stocks into portfolios according to their contemporaneous factor loadings and examine the portfolio returns over the same period. This approach clearly meets the first requirement for a factor risk based explanation, namely that there have to be contemporaneous patterns between factor loadings and average returns. The second requirement, that risk exposures are robust to controlling for stock characteristics and other factor loadings, is addressed by performing double sorts and Fama & MacBeth (1973) regressions with respect to a battery of control variables.

Our main result is that aggregate uncertainty is a significantly priced factor in the cross-section of stock returns. We find that stocks with high sensitivities to innovations in aggregate uncertainty have low average returns, while stocks with low sensitivities to innovations in aggregate uncertainty have significantly higher average returns. Sorting stocks into quintile portfolios, the hedge portfolio buying stocks with high and selling stocks with low sensitivities to innovations in aggregate uncertainty experiences an annual value-weighted return and 4-factor alpha of approximately -11.68 % and -13.88 %, respectively.

As increasing uncertainty is likely to induce a deterioration in the investment opportunity set, risk-averse investors are likely to be inclined to hedge against that by buying stocks with high sensitivity toward aggregate uncertainty, i.e., stocks that do well when aggregate uncertainty rises. Furthermore, another potential explanation for our findings is that ambiguity-averse investors want to hedge against changes in aggregate uncertainty. Consequently, both rational investors who feature risk-aversion and investors that exhibit non-expected utility with ambiguity aversion potentially demand stocks they anticipate will do well if aggregate uncertainty rises, lowering the average returns of these stocks.

Using double sorts, we find that our results cannot be explained by beta, size, book-to-market, aggregate volatility, as well as liquidity, returns distributions characteristics, and various other control variables. In accordance to the results of the portfolio sorts, using Fama & MacBeth (1973) regressions, we find that innovations in aggregate uncertainty command an economically substantial and statistically significant negative price of risk, with a two-standard deviation increase in aggregate uncertainty factor loadings being associated with a significant decrease in average annual returns that ranges from 6.3 % to 18.7 %. We show that our results are robust to the incorporation of various control variables. We perform several additional checks to further examine the robustness of our results. Jointly estimating various factor sensitivities in multivariate regressions, the effect of aggregate uncertainty remains significant. We also find the effect to persist when controlling for effects of the recent financial crisis or when using realized measures of aggregate uncertainty.

The remainder of this chapter is organized as follows. Section 4.2 presents an overview of the related literature. Section 4.3 presents a simple model, suggesting a trade-off between aggregate uncertainty and returns. Section 4.4 describes our dataset and methodology. Section 4.5 presents our empirical results with portfolio sorts and cross-sectional regressions. Section 4.6 checks the robustness of our results. Finally, Section 4.7 concludes. Detailed variable definitions are provided in Chapter C.1 in the appendix at the end of this chapter. Sections C.2 and C.3 of the appendix provide further robustness analyses.

4.2 Literature Review

We add to a large body of research in asset pricing, a cornerstone being the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966). Several authors (e.g., Banz, 1981; Fama & French, 1992), though, show that market beta, used alone, fails to explain the cross-sectional variation in asset returns. Addressing this concern, the ICAPM by Merton (1973) provides an important extension of the classical CAPM. It is shown that, once investors act to maximize their expected utility of lifetime consumption instead of only one period, as in the basic CAPM, current asset demands are affected by the possibility of uncertain changes in future investment opportunities. Consequently, if there is a state variable related to changes in the investment opportunity set, the assets' sensitivities to this state variable should be priced in the cross-section of returns. Campbell (1993, 1996) provides important extensions to the ICAPM framework, imposing a loglinear approximation to the budget constraint instead of assuming decision intervals as infinitely small, with which long-run effects can be better studied. Very recently, Campbell, Giglio, Polk, & Turley (2014) extend the Campbell (1993) framework allowing for stochastic volatiltiy.

Fama & French (1993), among many others, motivate their findings of a size and book-to-market risk factor with the ICAPM. Subsequently, Ang et al. (2006b) and Adrian & Rosenberg (2008) show that market volatility is a priced risk factor in the cross-section of stock returns, carrying a significantly negative risk premium.[2] Examining market skewness and kurtosis, Chang, Christoffersen, & Jacobs (2013) find that market skewness is a priced factor, while kurtosis is not. Furthermore, adding market skewness and kurtosis decreases the significance of the market volatility factor. Cremers et al. (2015) separate the effects of jump and volatility risk and show that both carry a significantly negative risk premium in the cross-section of stock returns. Han & Zhou (2012) show that the market variance risk premium (VRP) carries a significantly negative risk premium as well. We shed further light on the pricing of these factors studying their relation to aggregate uncertainty.

This chapter is also related to the literature dealing with uncertainty. Barsky & De Long (1993) argue that there exists substantial uncertainty about the structure of the aggregate dividend process in the U.S.. Several papers introduce a setup in which learning about uncertain probabilities is required (e.g., Pastor & Veronesi, 2003; Leippold, Trojani, & Vanini, 2008; Ozoguz, 2009; Cremers & Yan, 2012) while others point to the impossibility of observing probabilities at all (e.g., Hansen, Sargent, & Tallarini, 1999; Bossaerts, Ghirardato, Guarnaschelli, & Zame, 2010).

Epstein & Wang (1994) show that when introducing uncertainty indeterminate equilibria can result which can cause sizable volatility. Cao, Wang, & Zhang (2005) demonstrate that the presence of uncertainty can lead to limited market participation, and Zhang (2006) reports that information uncertainty can impose stock price continuation. Anderson, Ghysels, & Juergens (2009) examine the effect of risk and uncertainty on expected returns, measuring aggregate uncertainty with the disagreement among professional forecasters' expectations. They find empirical evidence for an uncertainty-return trade-off. As opposed to Anderson et al. (2009),

[2]Other papers on volatility and the cross-section of returns are Coval & Shumway (2001), Goyal & Santa-Clara (2003), Bali & Cakici (2008), and Fu (2009).

we use a market-based measure of aggregate uncertainty which delivers day-by-day observations instead of a quarterly measure based on the forecasts of relatively few agents.

We further build upon the results of Bloom (2009) who provides a structural framework to analyze the impact of uncertainty shocks. In this framework, higher uncertainty causes firms to make use of their "real options", postponing hiring and investment decisions when uncertainty rises. Consequently, a sharp rise in uncertainty potentially generates recessions. While Bloom (2009) measures uncertainty with simple volatility and concentrates on simultaneous effects of a change in uncertainty on all firms, we extend this point of view by using a more sophisticated measure for uncertainty, separating the effects of risk and uncertainty and, more importantly, by allowing for different exposures to aggregate uncertainty of different firms. While a rise in aggregate uncertainty has only little impact on some firms, others are affected much more heavily, making these firms more "risky".

In their formulation of a structural model with recursive preference and consumption uncertainty, Bali & Zhou (2015) also show that there exists both a risk-return as well as an uncertainty-return trade-off. For their empirical analysis, they substitute consumption volatility-of-volatility with the market variance risk premium and find a positive coefficient on the uncertainty-return trade-off. Building upon a similar model as Bali & Zhou (2015), we choose to use a more direct and intuitive measure of consumption volatility-of-volatility, and hence aggregate economic uncertainty, namely market volatility-of-volatility.

Baltussen et al. (2015) use the smooth ambiguity model of Klibanoff, Marinacci, & Mukerji (2005) to show that second-order beliefs (represented by volatility-of-volatility) can, on the one hand, be interpreted as a proxy for uncertainty and, on the other hand, potentially play an important role in investor's utility functions. They further show that individual stock's idiosyncratic volatility-of-volatility carries a significantly negative risk premium. As opposed to this study, Baltussen et al. (2015) primarily use idiosyncratic volatility-of-volatility and cannot detect a significant effect using past sensitivities from aggregate factor specifications with high-minus-low idiosyncratic volatility-of-volatility portfolios or the volatility-of-volatility from at-the-money (ATM) S&P 500 options. We, in turn, examine model-free aggregate market volatility-of-volatility, represented by the VVIX, as a state variable concentrating on systematic instead of idiosyncratic effects. Barnea & Hogan (2012) show that there is a negative variance risk premium in VIX options, meaning that investors, on average, are willing to accept a negative payoff in order to insure against increasing aggregate uncertainty. This provides further evidence that investors also take account of aggregate uncertainty. We shed further light on this.

This chapter is also connected to that of Bollerslev, Tauchen, & Zhou (2009), who extend the long-run risks model of Bansal & Yaron (2004) incorporating time-varying volatility-of-volatility. They show that, within the model, volatility-of-volatility has an effect on the equity premium. An empirical result of Bollerslev et al. (2009) is that the market variance risk premium significantly explains the time series variation in the equity risk premium. Drechsler & Yaron (2011) present another general equilibrium model, which introduces infrequent jumps in the persistent component of consumption and dividend growth. In these model surroundings, they show that the variance risk premium is linked to fluctuating volatility, having a large predictive power for stock market returns. Barndorff-Nielsen & Veraart (2012) propose a probabilistic model that allows for stochastic volatility-of-volatility providing further arguments for a relation between volatility-of-volatility and the variance risk premium. While these papers concentrate mostly on the variance risk premium and its implications for the equity risk premium we employ a much broader dataset using the cross-section of equity returns and concentrate on the effect of volatility-of-volatility.

Another paper closely related to ours is Huang & Shaliastovich (2014), who show that there is a volatility-of-volatility risk premium in the cross-section of S&P 500 and VIX options. However, we study the pricing of aggregate uncertainty in the cross-section of equity returns. Chen, Chung, & Lin (2014) develop a general equilibrium model in which, beside market beta and variance risk, the variance of the market variance affects asset prices. They measure volatility-of-volatility using high-frequency index option data and empirically find volatility-of-volatility to carry a significantly negative risk premium. Our study differs from theirs both theoretically in the model framework and empirically in that we directly use the VVIX index provided by the CBOE, instead of a high frequency intraday realized variance measure of the VIX index. Consequently, we use a forward-looking volatility measure of forward-looking volatility instead of past variation in forward-looking volatility. Several papers show that using implied instead of historical volatility estimates significantly improves the estimation accuracy (e.g., Jiang & Tian, 2005; Prokopczuk & Wese Simen, 2014a). While there is no direct evidence on second order volatility estimation accuracy, it is intuitively appealing to use the estimation technique that is shown to work best for both the first and in succession the second order estimation of current volatility.

4.3 Model Formulation

We build on the results of Campbell et al. (2014) and Bali & Zhou (2015) imposing a stylized intertemporal asset pricing model with stochastic volatility to motivate the existence of both a risk-return and an uncertainty-return trade-off.

The representative agent is assumed to have Epstein & Zin (1989) preferences with the value function V_t as

$$V_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{4.1}$$

where C_t is the consumption at time t, and the preference factors of the representative agent are denoted by δ, the subjective discount factor, and γ, the coefficient of relative risk-aversion. As is commonly done, for convenience we define $\theta = (1-\gamma) / (1-1/\psi)$, with ψ being the elasticity of intertemporal substitution. As shown by Epstein & Zin (1991), the corresponding stochastic discount factor (SDF) can be expressed as

$$M_{t+1} = \left(\delta \left(\frac{C_t}{C_{t+1}} \right)^{1/\psi} \right)^{\theta} \left(\frac{W_t - C_t}{W_{t+1}} \right)^{1-\theta}, \tag{4.2}$$

with W_t being the market value of the agent's consumption stream. The logarithm of the SDF is then

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1}, \tag{4.3}$$

with $r_{t+1} = \ln (W_{t+1} / (W_t - C_t))$ being the log return on wealth and $g_{t+1} = \Delta c_{t+1}$ being the log consumption growth. We follow Bollerslev et al. (2009) and Bali & Zhou (2015) assuming the following joint dynamics for consumption growth and consumption growth volatility

$$g_{t+1} = \mu_g + \sigma_{g,t} z_{g,t+1} \tag{4.4}$$

$$\sigma_{g,t+1}^2 = a_\sigma + \rho_\sigma \sigma_{g,t}^2 + \sqrt{q_t} z_{\sigma,t+1} \tag{4.5}$$

$$q_{t+1} = a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}. \tag{4.6}$$

μ_g is the constant mean growth rate, $\sigma_{g,t}^2$ denotes the conditional variance of consumption growth, q_t represents the volatility uncertainty process, while $z_{g,t+1}$, $z_{\sigma,t+1}$, and $z_{q,t+1}$ describe independent i.i.d. $N(0,1)$ processes. The parameters satisfy $a_\sigma > 0$, $a_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$, and $\varphi_q > 0$.

Let ω_t denote the logarithm of the price-dividend ratio, or price-consumption or wealth-consumption ratio, of the asset that pays the consumption endowment. To find the equilibrium, one can conjecture a solution for ω_t as an affine function of the state variables $\sigma_{g,t}^2$ and q_t,

$$\omega_t = A_0 + A_\sigma \sigma_{g,t}^2 + A_q q_t. \tag{4.7}$$

Using the standard Campbell & Shiller (1988) approximation $r_{t+1} = \kappa_0 + \kappa_1 \omega_{t+1} - \omega_t + g_{t+1}$, a solution for the coefficients A_0, $A_\sigma < 0$, and $A_q < 0$ can be obtained. Substituting the Campbell & Shiller (1988) approximation into Equation (4.3) one obtains a pricing kernel without reference to consumption growth (Bali & Zhou, 2015; Campbell et al., 2014):

$$m_{t+1} = \theta \ln \delta + \frac{\theta}{\psi}\kappa_0 - \frac{\theta}{\psi}\omega_t + \frac{\theta}{\psi}\kappa_1 \omega_{t+1} - \gamma r_{t+1}. \tag{4.8}$$

Assuming a conditional joint lognormal distribution with time-varying volatility for the asset returns, the risk premium on any asset is given by

$$E_t(r_{j,t+1}) - r_{f,t} + \frac{1}{2}Var_t(r_{j,t+1}) = -Cov_t\left(m_{t+1}, r_{j,t+1}\right). \tag{4.9}$$

Inserting the pricing kernel without reference to consumption growth in Equation (4.8) into Equation (4.9), one can obtain an ICAPM pricing relation of the following form:

$$E_t(r_{j,t+1}) - r_{f,t} + \frac{1}{2}Var_t(r_{j,t+1}) = \gamma Cov_t\left(r_{t+1}, r_{j,t+1}\right) - \frac{\theta}{\psi}\kappa_1 Cov_t\left(\omega_{t+1}, r_{j,t+1}\right). \tag{4.10}$$

Following Bollerslev et al. (2009) and Bali & Zhou (2015), we substitute out the consumption growth volatility with $Var_t(r_{t+1}) = \sigma_{g,t}^2 + \kappa_1^2\left(A_\sigma^2 + A_q^2 \varphi_q^2\right)$ when inserting Equation (4.7) into Equation (4.10). This yields:

$$
\begin{aligned}
E_t(r_{j,t+1}) - r_{f,t} + \frac{1}{2}Var_t(r_{j,t+1}) + \frac{\theta}{\psi}\kappa_1 A_\sigma Cov_t\left(Var_{t+1}(r_{t+2}), r_{j,t+1}\right) & \\
= \quad & \gamma Cov_t\left(r_{t+1}, r_{j,t+1}\right) \\
& + \frac{\theta}{\psi}\kappa_1\left[A_\sigma \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right) - A_q\right] Cov_t\left(q_{t+1}, r_{j,t+1}\right) \\
= \quad & \gamma Var_t(r_{t+1})\frac{Cov_t\left(r_{t+1}, r_{j,t+1}\right)}{Var_t(r_{t+1})} \\
& + \frac{\theta}{\psi}\kappa_1\left[A_\sigma \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right) - A_q\right] Var_t(q_{t+1})\frac{Cov_t\left(q_{t+1}, r_{j,t+1}\right)}{Var_t(q_{t+1})} \\
\equiv \quad & Y \cdot \beta_{j,t}^{\mathrm{M}} + Z \cdot \beta_{j,t}^{\mathrm{V}}.
\end{aligned} \tag{4.11}
$$

Apart from the variance term $Var_t(r_{j,t+1})$, and a trade-off of returns with future variance indicated by $Cov_t\left(Var_{t+1}(r_{t+2}), r_{j,t+1}\right)$, there is the usual risk-return trade-off $Y \equiv \gamma Var_t(r_{t+1})$, and an uncertainty-return trade-off $Z \equiv \frac{\theta}{\psi}\kappa_1\left[A_\sigma \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right) - A_q\right] Var_t(q_{t+1})$ can be detected from this formulation. At this stage, we deviate from Bali & Zhou (2015) who proxy consumption volatility-of-volatility q_t with the variance risk premium. Instead, we directly use the asset market volatility-of-volatility to proxy for this economic uncertainty.

In the following sections, we empirically examine whether the theoretical prediction of an uncertainty-return trade-off derived above holds. Furthermore, the model does not make a clear prediction on the sign of the uncertainty-return trade-off, as opposed to the risk-return trade-off which is clearly signed by the coefficient of relative risk-aversion.

4.4 Data and Methodology

4.4.1 Data

We base our study on all stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotations (NASDAQ) that are classified as ordinary common shares (Center for Research in Security Prices (CRSP) share codes 10 or 11), excluding closed-end funds and REITS (SIC codes 6720–6730 or 6798), for the sample period between January 01, 2007 and December 31, 2014. We obtain data on the VIX and the VVIX from the CBOE. The VIX is constructed so that it represents the model-free 30-day implied volatility of the S&P 500 index. On February 24, 2006, the CBOE began trading options written on the VIX and recently, with the time series beginning in 2007, the CBOE started reporting the VVIX, which represents the model-free 30-day implied volatility of the VIX.[3] The beginning of the reporting of the VVIX on January 01, 2007 restricts the beginning of our sample period to that date.[4] For a robustness check, we also obtain five-minute intraday high-frequency data on the VIX from the Thompson Reuters Tick History (TRTH) database.

We obtain daily and monthly price data as well as data on dividend payments, trading volumes, firm age, and shares outstanding from the CRSP. Following Amihud (2002) and Zhang (2006), we exclude "penny stocks" with prices below \$ 5. Additionally, we require a market

[3]For reliable implied moments, option liquidity is an important issue. While the trading volume of VVIX options in 2006 was quite low, with several thousand contracts per day, it increased to more than one million contracts per day in 2013. For more information, refer to the CBOE homepage.

[4]In principle, we could compute the implied volatility of the VIX prior to that date using the results of Bakshi et al. (2003). We refrain from that to avoid spurious findings caused by potentially small initial trading volumes in the newly created VIX options market.

capitalization of at least 225 million dollars (D'Avolio, 2002; Baltussen et al., 2015). These two thresholds serve to eliminate the most illiquid stocks that exhibit potential microstructure problems and may bias the results (Fama & French, 2008). Furthermore they ensure that only stocks with relatively low short-sale constraints are selected (D'Avolio, 2002). We adjust for delisting returns following Shumway (1997) and Shumway & Warther (1999).

Balance sheet and income statement data is obtained from the Compustat database. Options data are from the IvyDB OptionMetrics database.[5] Data on the Fama & French (1993) and momentum factors as well as the risk-free (Treasury Bill) rate are collected from Kenneth French's data library. Data on the Pastor & Stambaugh (2003) liquidity factor is obtained from Robert Stambaugh's homepage.

Chapter C.1 of the appendix contains a more detailed description of all variables used in this chapter.

4.4.2 Empirical Framework

Our goal is to test whether the main model prediction holds and stocks with different sensitivities to innovations in aggregate uncertainty have different average returns. For that, we follow a large body in the asset pricing literature, examining the contemporaneous relation between realized factor loadings and realized returns (e.g., Black et al., 1972; Fama & MacBeth, 1973; and Fama & French, 1993; among many others). Ang et al. (2006a) argue that while pre-formation factor loadings reflect both actual variation in factor loadings and measurement error, post-formation factor loadings are almost exclusively affected by stock return covariations with risk factors. Additionally, they point out that if risk exposures, and

[5] Options data has only been available up to 31 August, 2014 when we started this project. So, all tests that include options data (e.g., Idio. vol-of-vol and dSkew) are performed for the sample period January 01, 2007 until August 31, 2014.

hence factor loadings, are highly time-varying, pre-formation factor loadings might be poor predictors of ex post risk exposures leaving the analysis with low power to detect relations between factor loadings and realized returns. Addressing these concerns, our research design follows Ang et al. (2006a) and Cremers et al. (2015) by estimating factor loadings for individual stocks using daily returns over rolling annual periods from the regression:

$$r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^{\mathrm{M}}_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^{\mathrm{V}}_{j,t} dVVIX_{\tau} + \epsilon_{j,\tau}. \tag{4.12}$$

$r_{j,\tau}$ is the daily return of asset j on day τ, $r_{M,\tau}$ is the return of the market on that day, and $r_{f,\tau}$ is the risk-free rate. $dVVIX_{\tau}$ is the daily innovation in the VVIX index.

Beside the fact that further factors like those from Fama & French (1993) are not predicted to be priced in our simple model, Ang et al. (2006b) argue that including additional factors in the regression in Equation (4.12) may add a lot of noise. We control for further factors when performing the time series and cross-sectional asset pricing tests. As a robustness check, to account for possible model misspecification we also consider multivariate joint factor loading estimations controlling for several aggregate risk factors previously documented in the literature:

$$r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^{\mathrm{M}}_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^{\mathrm{V}}_{j,t} dVVIX_{\tau} + \beta^{\zeta}_{j,t}\zeta_{\tau} + \epsilon_{j,\tau}. \tag{4.13}$$

ζ_{τ} contains one or more market factors, such as the Fama & French (1993) and Carhart (1997) factors, the daily change in the volatility index (dVIX) as described by Ang et al. (2006b), the innovations in market skewness and kurtosis (dSkew, dKurt) as shown by Chang et al. (2013), the Cremers et al. (2015) Straddle Vol and Jump factors, or innovations in the market variance risk premium (Han & Zhou, 2012; dVRP). dVIX is of particular interest as the model predicts a trade-off of returns and future variance, for which current implied market variance might, to some extent, be a proper proxy for tomorrow's market variance. For the regressions in Equations (4.12) and (4.13), we use daily returns over rolling annual periods to estimate the sensitivities. For each period and stock, we require at least one hundred non-missing return observations in order to estimate the factor sensitivities.[6]

Turning the focus on the measurement of innovations in an economic variable, there generally exists a trade-off between a possible errors-in-variables problem using simple first differences, if that fails to completely filter out the expected movement, versus the danger of misspecifying a more complex equation for the expected movement in a variable (Chen, Roll, & Ross, 1986). We choose to measure the innovations in the VVIX index using the daily first differences in the variable because it is highly serially correlated with a first-order autocorrelation of 0.94 during our sample period. Therefore, the current value of VVIX appears to be a relatively good proxy for the tomorrow's expectation making the first difference quite adequately capture its innovation. For robustness, we also consider measuring innovations in aggregate uncertainty by fitting an ARMA(1,1) model on the complete time series of the VVIX index. This approach results in a measure of innovations of $dVVIX_\tau = VVIX_\tau - 0.9989VVIX_{\tau-1} + 0.1063dVVIX_{\tau-1}$. The results of both approaches are qualitatively equal, which is further discussed in the next section.

[6]For the factor loading estimation regressions, potential low explanatory power might be a concern. In the basic specification, we find the model in Equation (4.12) to exhibit an average R-squared of 0.28 with median 0.23. Thus, it can be concluded that the factor loading regressions possess substantial explanatory power. In these regressions, the coefficient $\beta^{\mathrm{M}}_{j,t}$ is significant (at 10 %) in 83 % of the cases while the coefficient $\beta^{\mathrm{V}}_{j,t}$ is significantly different from zero in about 18 % of the cases.

Table 4.1: Sample Correlations of Different Aggregate Factors

This table presents the sample correlation coefficients of the aggregate factors dVVIX, dVIX, Straddle vol, dSkew, dKurt, Jump, dVRP, MKT, SMB, HML, Momentum, dPol, $\text{dVVIX}_{\text{ARMA}}$, and $\text{dVoVIX}_{\text{ARMA}}$. Detailed variable definitions are provided in the appendix.

dVVIX	dVIX	Straddle vol	dSkew	dKurt	Jump	dVRP	MKT	SMB	HML	Momentum	dPol	$\text{dVVIX}_{\text{ARMA}}$	$\text{dVoVIX}_{\text{ARMA}}$	
*	0.64	-0.07	-0.05	0.00	0.49	0.37	-0.54	-0.09	-0.16	0.02	-0.04	0.99	0.28	dVVIX
	*	-0.07	0.00	-0.01	0.45	0.78	-0.84	-0.06	-0.31	0.25	-0.05	0.64	0.21	dVIX
		*	-0.04	0.03	-0.67	0.01	-0.01	-0.03	-0.03	0.08	-0.05	-0.07	-0.15	Straddle vol
			*	-0.83	0.02	0.00	-0.01	0.03	0.00	-0.01	0.00	-0.05	0.02	dSkew
				*	-0.04	0.01	0.00	0.04	-0.03	0.04	-0.03	-0.01	-0.04	dKurt
					*	0.21	-0.29	-0.05	-0.09	-0.02	0.02	0.48	0.29	Jump
						*	-0.64	0.03	-0.24	0.24	-0.06	0.36	0.13	dVRP
							*	0.15	0.44	-0.43	0.04	-0.53	-0.14	MKT
								*	-0.03	0.00	-0.02	-0.09	-0.04	SMB
									*	-0.58	0.05	-0.16	-0.03	HML
										*	-0.04	0.02	-0.09	Momentum
											*	-0.04	0.05	dPol
												*	0.30	$\text{dVVIX}_{\text{ARMA}}$
													*	$\text{dVoVIX}_{\text{ARMA}}$

4.5 Empirical Results

4.5.1 Descriptive Statistics

In addition to various firm characteristics, we consider the impact of several aggregate state variables that have previously been examined in the literature. In Table 4.1 we report the sample correlations between daily innovations in aggregate uncertainty (dVVIX), innovations in aggregate volatility (dVIX; Ang et al., 2006b), the Fama & French (1993) and Carhart (1997) factors, and also the factors on market skewness and kurtosis of Chang et al. (2013), stochastic volatility and jump risk of Cremers et al. (2015), and innovations in the market variance risk premium (Han & Zhou, 2012).

First, we note that whether innovations are measured as simple first differences (dVVIX) or as innovations in an ARMA(1,1) model ($\text{dVVIX}_{\text{ARMA}}$) in fact does not make a big difference since the correlation between the two measures is almost perfect with 99 %.

For a factor risk explanation in the sense of the ICAPM, a state variable must be associated with future deterioration in the investment opportunity set, so there should be some correlation of factor realizations with the (future) market excess return. While it is hard to tell which horizon to choose for future impacts, we can examine current correlations. There, we find a very high negative correlation of -0.84 of the contemporaneous market return with the first difference in the VIX and quite high correlations of -0.64 with dVRP and of around 0.4 and -0.4 with HML and Momentum, respectively.[7] The correlation of dVVIX with MKT is also substantial with -0.54. Overall, this simple correlation analysis delivers some support for the view of aggregate uncertainty being a state variable in the sense of the ICAPM. The linear relation between dVVIX and dVRP, Jump, and dVIX, to which it might be related by construction, is also quite substantial though not perfect with values of 0.37, 0.49, and 0.64, respectively. Correlations of dVVIX with other factors are negligible. Consequently, the factor representing aggregate uncertainty appears to be distinct from other factors documented previously.

Further summary statistics are provided in Table 4.2. In Panel A it can be seen that mean and median innovations in the VVIX are close to zero. Measuring innovations in the VVIX using the first difference is shown to result in a factor with very low autocorrelation (-0.13), whereas using residuals from the fitted ARMA-model reduces the first-order autocorrelation to practically zero. The remaining factors

[7] The very high correlation between dVIX and MKT in our sample period between January 01, 2007 and December 31, 2014 might indicate problems of multicollinearity. Consequently, our results have to be interpreted with care when dVIX is included.

Table 4.2: Summary Statistics

Panel A of this table presents summary statistics on the aggregate factors dVVIX, dVIX, Straddle vol, dSkew, dKurt, Jump, dVRP, MKT, SMB, HML, Momentum, dPol, $\mathrm{dVVIX}_{\mathrm{ARMA}}$, and $\mathrm{dVoVIX}_{\mathrm{ARMA}}$. Detailed variable definitions are provided in the appendix. Panel B provides yearly summary statistics on the VVIX and Panel C shows yearly summary statistics on the individual stocks' sensitivities to aggregate uncertainty, β^{V}, with the sensitivity estimation starting in the year denoted first in the first column. Mean, Median, and Std. dev. refer to the sample average, median, and standard deviation of the factors, respectively. P10 and P90 refer to the 10 % and 90 % percentiles, respectively. Autocorr(1) presents the first order autocorrelation.

Panel A. Market Factors

Variable	Mean	Median	Std. dev.	Autocorr(1)	P10	P90
dVVIX	0.00010	-0.00350	0.0452	-0.1117	-0.0454	0.0499
dVIX	0.00004	-0.00110	0.0205	-0.1553	-0.0178	0.0185
Straddle vol	-0.00004	0.00105	0.0149	-0.0062	-0.0158	0.0153
dSkew	-0.00944	-0.00680	0.1749	-0.0542	-0.2042	0.1729
dKurt	0.01942	-0.01618	0.4852	-0.0512	-0.4282	0.4820
Jump	-0.00165	-0.00986	0.0524	-0.0930	-0.0426	0.0408
dVRP	0.00000	0.00001	0.0096	-0.0398	-0.0040	0.0045
MKT	0.00035	0.00100	0.0140	-0.0962	-0.0143	0.0136
SMB	0.00006	0.00015	0.0059	-0.0605	-0.0066	0.0064
HML	-0.00004	-0.00010	0.0062	-0.0097	-0.0059	0.0058
Momentum	0.00002	0.00060	0.0110	0.1262	-0.0099	0.0100
dPol	0.00339	-0.00364	0.0532	0.1642	-0.0490	0.0639
$\mathrm{dVVIX}_{\mathrm{ARMA}}$	0.00100	-0.00276	0.0449	-0.0029	-0.0451	0.0521
$\mathrm{dVoVIX}_{\mathrm{ARMA}}$	0.00074	-0.00108	0.0161	0.1191	-0.0144	0.0168

Panel B. VVIX Summary Statistics

Year	Mean	Median	Std. dev.	P10	P90
2007	0.8768	0.8618	0.1331	0.7194	1.0469
2008	0.8185	0.7741	0.1560	0.6763	1.1088
2009	0.7978	0.7913	0.0863	0.6954	0.9225
2010	0.8836	0.8622	0.1307	0.7538	1.0346
2011	0.9294	0.9146	0.1021	0.8182	1.0491
2012	0.9484	0.9384	0.0838	0.8416	1.0740
2013	0.8052	0.7960	0.0897	0.6967	0.9203
2014	0.8301	0.7991	0.1433	0.6771	0.9990
total	0.8638	0.8456	0.1310	0.7069	1.0393

Table 4.2: Summary Statistics (continued)

Panel C. β^{V} Summary Statistics

Year	Mean	Median	Std. dev.	P10	P90
2007 – 2008	0.0171	0.0133	0.0661	-0.0530	0.0951
2008 – 2009	0.0563	0.0454	0.1051	-0.0523	0.1812
2009 – 2010	0.0147	0.0114	0.0511	-0.0410	0.0751
2010 – 2011	0.0162	0.0136	0.0423	-0.0305	0.0663
2011 – 2012	0.0122	0.0094	0.0470	-0.0377	0.0674
2012 – 2013	0.0082	0.0056	0.0460	-0.0405	0.0618
2013 – 2014	0.0151	0.0116	0.0424	-0.0293	0.0643
total	0.0202	0.0134	0.0824	-0.0403	0.0886

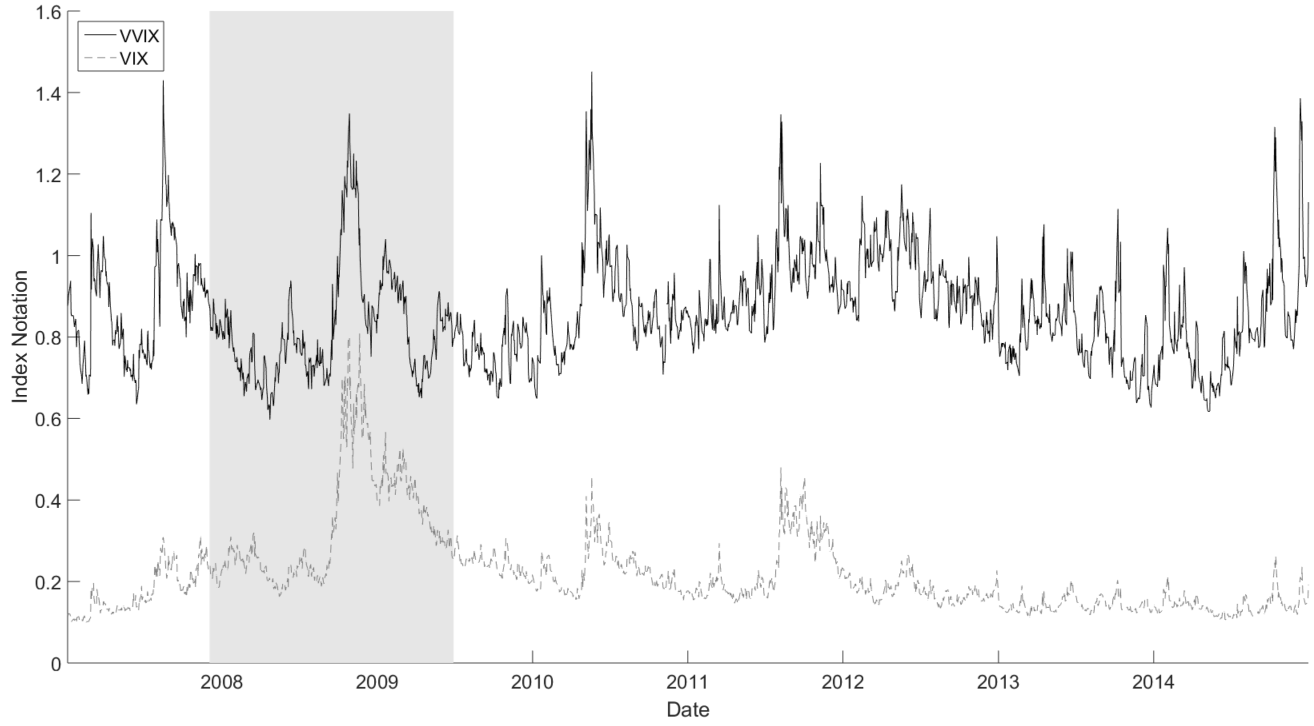
are mostly constructed as returns with means close to zero and negligible autocorrelations.

Panels B and C of Table 4.2 present yearly summary statistics on the VVIX and $\beta^{\mathrm{V}}_{j,t}$ factors of individual stocks, respectively. It is quite interesting to observe that the yearly average level of the VVIX is smallest in the crisis year 2009. In the years 2011 and 2012 it was substantially higher with values above 0.9, while there is a sharp decrease in 2013, almost returning to the 2009 level. The mean and median sensitivities of individual stocks to innovations in aggregate uncertainty, $\beta^{\mathrm{V}}_{j,t}$, presented in Panel C, are mostly close to 0.01 while the highest average value and standard deviation among the estimates is observed during the rolling annual estimation periods starting in 2008.

The time series of VIX and VVIX are plotted in Figure 4.1. The average level of the VVIX (0.86) is substantially higher than that of the VIX (0.22). The VVIX exhibits pronounced spikes that correspond with certain crisis events like the Bear Sterns Hedge Funds Collapse (August 2007), the Lehman Brothers bankruptcy (September 2008), the Freddie Mac and Fannie Mae crisis (May 2010), or the near collapse of the Russian rouble (December 2014). Consequently, this stylized evidence provides further

Figure 4.1: Time Series of VIX and VVIX

This figure plots the time series of the VIX (*dashed-grey*), and the VVIX (*solid-black*) during the sample period January 01, 2007 until December 31, 2014. The shaded area indicates the time period marked as business cycle contraction by the NBER.



insights for the VVIX being a proper proxy for economic uncertainty, as such events, beside increasing risk, impose large shocks on probability distribution surrounding the aggregate consumption growth process, toward which the VVIX is shown to be sensitive.

4.5.2 Single Portfolio Sorts and Characteristics

At the beginning of each month, we sort the stocks in ascending order with respect to their sensitivities to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the following year. We form quintile portfolios, so that quintile 1 contains the stocks with the lowest exposure to aggregate uncertainty while quintile 5 contains those with the highest uncertainty factor loadings. The hedge portfolio (5 minus 1) buys the quintile of stocks with the highest exposure and simultaneously sells the stocks in the quintile with the lowest exposure to aggregate uncertainty. The portfolio sorting approach maximizes the spread in the exposure to aggregate uncertainty and, thus, differences in average returns can be quite accurately attributed to differences in the sorting variable. Fama & French (2008) raise concerns that by building value-weighted portfolios the hedge portfolio can be dominated by few big stocks, whereas for equally weighted portfolios the hedge portfolio can be dominated by micro caps. To address these issues, we analyze both value-weighted and equally weighted portfolios.[8] When value-weighting, within each quintile, we weight the stocks by their relative market value at the beginning of the estimation period for $\beta^{\mathrm{V}}_{j,t}$. When weighting equally, all stocks adhere the same weight. While our research design involves successive twelve-month periods employing partly overlapping information, it introduces moving average effects. To account for that, in all analyses,

[8]The results on equally weighted portfolios can be found in the appendix to this chapter.

Table 4.3: Portfolios Sorted by Exposure to Aggregate Uncertainty – Value-Weighted

At the beginning of each month, we form value-weighted quintile portfolios based on the stock's sensitivities to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the following year. To obtain the sensitivities, we regress daily excess stock returns on dVVIX, controlling for MKT as in Equation (4.12). Stocks with the lowest $\beta^{\mathrm{V}}_{j,t}$ are sorted into portfolio 1, those with the highest into portfolio 5. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. We reform the portfolios after one month. The row labeled Mean return is based on monthly simple returns. CAPM alpha, FF-3 alpha, 4-factor alpha, and 5-factor alpha refer to the alphas of the CAPM, the Fama & French (1993) 3-factor, Carhart (1997) 4-factor, and the 5-factor (including liquidity) models, respectively. The segment NYSE only restricts the sample of stocks to those that are traded at the NYSE at the beginning of the estimation period. The segment Factor loadings denotes the average annual factor loadings, where β^{M}, β^{V}, and β^{dVIX} refer to the factor loadings on the market factor, dVVIX, and dVIX. The segment Stock characteristics presents average (value-weighted) portfolio characteristics with Mkt. share denoting the average market share of the portfolios. The remaining variable definitions are provided in the Appendix. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

we adjust the standard errors following Newey & West (1987) using twelve lags.[9]

Table 4.3 reports various summary statistics for the quintile portfolios sorted by contemporaneous aggregate uncertainty. We find the average annual raw return to adhere a strictly monotonically decreasing pattern, from 12.8 % in quintile 1 to 1.1 % in quintile 5. The difference in raw returns of −11.7 % between quintiles 5 and 1 is statistically significant at 1 %. Looking at the line labeled CAPM alpha, which reports the results when controlling for systematic risk, we find an even stronger effect for the

[9]While theoretically only eleven lags are required, we follow Ang et al. (2006a) and Cremers et al. (2015) including an additional lag for robustness.

Table 4.3: Portfolios Sorted by Exposure to Aggregate Uncertainty – Value-Weighted (continued)

Rank	1	2	3	4	5	5 minus 1
Mean return	0.1281***	0.1001**	0.0812**	0.0624	0.0113	-0.1168***
	(0.009)	(0.011)	(0.048)	(0.217)	(0.858)	(0.000)
CAPM alpha	0.0298**	0.0188***	-0.0046	-0.0411***	-0.1145***	-0.1443***
	(0.010)	(0.007)	(0.353)	(0.002)	(0.000)	(0.000)
FF-3 alpha	0.0252*	0.0209***	-0.0031	-0.0344**	-0.1144***	-0.1396***
	(0.100)	(0.000)	(0.566)	(0.042)	(0.000)	(0.000)
4-factor alpha	0.0262*	0.0192***	-0.0057	-0.0327**	-0.1126***	-0.1388***
	(0.090)	(0.000)	(0.134)	(0.049)	(0.000)	(0.000)
5-factor alpha	0.0457***	0.0210***	-0.0067*	-0.0562***	-0.1357***	-0.1814***
	(0.000)	(0.000)	(0.097)	(0.000)	(0.000)	(0.000)
NYSE only						
4-factor alpha	0.0123	0.0168**	-0.0021	-0.0399***	-0.1119***	-0.1242***
	(0.424)	(0.013)	(0.727)	(0.000)	(0.000)	(0.000)
Factor loadings						
β^{M}	0.8734***	0.8868***	1.0431***	1.2425***	1.5987***	0.7253***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
β^{V}	-0.0440***	-0.0072***	0.015***	0.0407***	0.0881***	0.1321***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
β^{dVIX}	-0.0843***	-0.0218***	0.0285***	0.0930***	0.1998***	0.2841***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Return characteristics						
Std. deviation	0.2315	0.1853	0.1971	0.2453	0.3111	0.1632
Skewness	-0.9567	-1.0053	-0.8500	-0.5373	-0.0324	0.0085
Kurtosis	3.6807	3.4233	3.2257	2.8214	3.0424	2.4601
Stock characteristics						
Mkt. share	0.2500	0.2990	0.2269	0.1457	0.0784	-0.1716
Size ($*10^{-6}$)	81.541	86.145	70.863	63.085	41.550	-39.991
Book-to-market	0.5197	0.4877	0.5008	0.5427	0.5389	0.0192
Bid-ask spread	0.0007	0.0006	0.0006	0.0008	0.0011	0.0004
Amihud illiquidity ($*10^{6}$)	4.2099	1.0524	1.9819	3.6976	9.4581	5.2482
Age	39.683	43.020	36.717	30.224	24.820	-14.863
Leverage	0.5495	0.5432	0.5521	0.5634	0.5926	0.0432
MAX	0.0653	0.0543	0.0603	0.0725	0.1030	0.0377
Volatility	0.0227	0.0210	0.0228	0.0250	0.0290	0.0063

5 minus 1 portfolio of -14.4 % which is also significant at 1 %. Controlling for the Fama & French (1993) (FF-3) and Carhart (1997) (4-factor) factors leads to alphas of -14.0 and -13.9 % per year, both being significant at 1 %. Further including the Pastor & Stambaugh (2003) factor in addition to the factors previously mentioned (5-factor) yields an alpha for the 5 minus 1 portfolio of -18.1 % per year which is also significant at 1 %. Consequently, accounting for systematic risk factors, the stocks in portfolio 5 in particular are expected to earn substantially higher returns than realized. For each factor model specification, the alphas of the portfolio of stocks with the highest sensitivities to innovations in aggregate uncertainty is significantly negative at 1 %. Especially, adding the liquidity factor strongly decreases the alpha of the 5 minus 1 portfolio. Restricting the sample to stocks that are traded on the NYSE only slightly reduces the underperformance of stocks with high exposure to aggregate uncertainty while the 4-factor alpha is -12.4 % and still significant at 1 %.

Both the market betas and the sensitivities to dVIX differ strongly across the five portfolios. The market beta of the hedge portfolio amounts to 0.72, being significantly different from zero at 1 %. This would, following the logic of the CAPM, predict a substantially positive excess return, whereas the return that is realized is significantly negative. While the exposure to aggregate uncertainty has to be monotonically increasing by construction, the sensitivity to dVIX also is monotonically increasing from portfolio 1 to 5, with a factor sensitivity of 0.28 for the hedge portfolio, which is statistically significant at 1 %. Age is often argued to be a good proxy for uncertainty (Zhang, 2006), so the observation that firms with high sensitivities to aggregate uncertainty are, on average, substantially younger is completely in line with this. The portfolios differ also, among other things, in average Size, Amihud illiquidity, and $Kurtosis$ and so there may be other factors that can potentially explain the effect of aggregate uncertainty we find in univariate sorts.[10]

4.5.3 Double Sorts

To address the concerns regarding the difference in factor loadings and characteristics among the different portfolios, we examine the performance of the portfolios sorted by sensitivities to innovations in aggregate uncertainty, controlling for different other factors and characteristics that have been previously shown to explain the cross-section of stock returns.

For that, at the beginning of each month, we first sort the stocks in ascending order with respect to the characteristic we want to control for. We form quintile portfolios. Afterwards, within each quintile, we sort stocks based on their uncertainty-sensitivity into another five quintile portfolios, which results in a total of 25 portfolios. The five portfolios sorted on the exposure to aggregate uncertainty are then obtained by averaging over the respective quintiles within each quintile of the control characteristic. This means that the uncertainty-sensitivity quintile 1 is the average of uncertainty-sensitivity quintiles 1 across all quintiles sorted on the control characteristic, and so on. Thus, we obtain quintile portfolios on the exposure to aggregate uncertainty controlling for another characteristic without making assumptions on the parametric form of the relationships. Again, we obtain the 5 minus 1 hedge portfolio buying the final portfolio 5 and selling portfolio 1. First, we consider value-weighted portfolios, where within each of the 25 intermediate step portfolios we weight the stocks by their relative market value at the beginning of the estimation period for the exposures to aggregate uncertainty. We then also consider equally weighted portfolios.

The results for value-weighted double sorts are presented in Table

[10]The results on equally weighted portfolios, presented in the appendix to this chapter, are qualitatively similar. Returns and alphas of the 5 minus 1 hedge portfolio are all negative and statistically significant at 1 %.

Table 4.4: Double Sorts – Value-Weighted

This table reports Carhart (1997) 4-factor alphas for double-sorted portfolios. At the beginning of each month, we first sort stocks into quintiles based on the characteristics denoted in the first column. Then, within each quintile, we sort stocks based on their uncertainty-sensitivity ($\beta^{\mathrm{V}}_{j,t}$) into another five quintile portfolios. The five portfolios sorted on $\beta^{\mathrm{V}}_{j,t}$ are then obtained by averaging over the respective quintiles within each quintile of the other characteristic, thus we obtain $\beta^{\mathrm{V}}_{j,t}$ quintile portfolios controlling for another characteristic. We reform the portfolios after one month. This procedure is performed for each of the characteristics. We report the main control variables for value-weighted returns. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Rank	1	2	3	4	5	5 minus 1
Beta	0.0096	-0.0081	-0.0118	-0.0318***	-0.0679***	-0.0776***
	(0.586)	(0.192)	(0.182)	(0.004)	(0.000)	(0.002)
Size	0.0041	0.0118	0.0036	-0.028***	-0.1005***	-0.1047***
	(0.751)	(0.126)	(0.231)	(0.000)	(0.000)	(0.001)
Book-to-market	0.0191	0.0114***	-0.0095**	-0.0299***	-0.1036***	-0.1227***
	(0.202)	(0.005)	(0.019)	(0.007)	(0.000)	(0.001)
dVIX	-0.0161**	-0.0066	-0.0097	-0.0189**	-0.0749***	-0.0588**
	(0.035)	(0.196)	(0.110)	(0.014)	(0.000)	(0.019)
Bid-ask spread	-0.0516***	-0.0234***	-0.0541***	-0.0816***	-0.1332***	-0.0817***
	(0.000)	(0.006)	(0.000)	(0.000)	(0.000)	(0.007)
Momentum	0.0182	0.0200***	0.0013	-0.0352***	-0.1127***	-0.1309***
	(0.197)	(0.000)	(0.804)	(0.004)	(0.000)	(0.000)
Short-term reversal	0.0239	0.0146***	-0.0008	-0.0361***	-0.1168***	-0.1406***
	(0.100)	(0.004)	(0.900)	(0.001)	(0.000)	(0.000)
Age	0.0240	0.0197**	-0.0040	-0.0359***	-0.1124***	-0.1364***
	(0.127)	(0.012)	(0.269)	(0.007)	(0.000)	(0.001)
Leverage	0.0286**	0.0133***	-0.0104**	-0.0214*	-0.1010***	-0.1296***
	(0.039)	(0.000)	(0.035)	(0.096)	(0.000)	(0.001)

4.4. We report Carhart (1997) 4-factor alphas and robust Newey & West (1987) p-values in brackets. We find the effect of high uncertainty-sensitivity underperforming low uncertainty-sensitivity stocks to strongly persist. Controlling for the firm characteristics Size, Book-to-market,[11] Momentum, Short-term reversal, Age, and Leverage also leads to an economically large effect on the hedge portfolio between -14 and -10 % per year, which is statistically significant at 1 % in all cases. When controlling for Beta, the dVIX-sensitivity, and Bid-ask spread, the effect weakens, with 4-factor alphas of the hedge portfolio of -7.6, -5.9, and -8.2 % per year. Since the alphas decrease in absolute terms, this means that part of the effect of aggregate uncertainty can be assigned to those control variables. However, the 4-factor alphas are still significantly different from zero at least at 5 %.[12]

Consequently, controlling for various canonical characteristics does not affect our main result that the uncertainty-return trade-off is priced with a negative sign.

4.5.4 Regression Tests

The portfolio sorts present strong evidence that sensitivities to innovations in aggregate uncertainty are related to returns. The double sorts indicate that the effect cannot be explained by any other factor or firm characteristic individually. Following on from that, in this section, we estimate Fama & MacBeth (1973) regressions that simultaneously control for different variables and test if the stock's sensitivity to innovations in aggregate uncertainty contains information about stock returns beyond that of various other firm characteristics. Lo & MacKinlay (1990) and Lewellen, Nagel,

[11] As Fama & French (1997, 2008) show that SMB and HML loadings vary over time, they suggest sticking to current Size and Book-to-market factors as more current proxies. We follow their advice.

[12] The results on equally weighted double sorts, shown in the appendix to this chapter, are qualitatively equal.

& Shanken (2010) argue against the use of portfolios in cross-sectional regressions, since the particular method, by which the portfolios are formed can severely influence the results. Furthermore, Ang, Liu, & Schwarz (2010) show that creating portfolios ignores important information on individual factor loadings and leads to higher asymptotic standard errors of risk premium estimates. Consequently, we utilize this additional information and, at the same time, avoid the specification of breakpoints, performing the analysis on individual stocks rather than stock portfolios.

Each month, we perform cross-sectional regressions of stock excess returns over the following year on stocks' sensitivities to innovations in aggregate uncertainty and one or more control variables, adhered over the same period. We winsorize all regressors at the 1st and 99th percentile to restrict the effect of outliers (Fama & French, 2008; Baltussen et al., 2015). For the regressions, we use OLS (equally weighted) or WLS (value-weighted) with a diagonal weighting matrix, where the inverse of the firm's market value at the end of the previous month is along the diagonal, with the following regression equation:[13]

$$r_{j,t} - r_{f,t} = \alpha_t + \lambda_t^{\mathrm{M}} \beta_{j,t}^{\mathrm{M}} + \lambda_t^{\mathrm{V}} \beta_{j,t}^{\mathrm{V}} + \lambda_t^{\zeta} \beta_{j,t}^{\zeta} + \epsilon_{j,t}. \tag{4.14}$$

$r_{j,t}$ is the annual return of stock j and $r_{f,t}$ is the risk-free rate during that period. $\beta_{j,t}^{\mathrm{M}}$ and $\beta_{j,t}^{\mathrm{V}}$ are the stock's market beta and sensitivity to innovations in aggregate uncertainty over the evaluation period, respectively. The term $\beta_{j,t}^{\zeta}$ denotes a vector collecting further variables hypothesized to explain returns. λ_t^{V} and λ_t^{ζ} are the risk premia associated with the respective variables, while $\epsilon_{j,t}$ is the prediction error.

In the next step, we perform tests on the time series averages $\bar{\alpha}$, $\bar{\lambda}^{\mathrm{M}}$, $\bar{\lambda}^{\mathrm{V}}$, and $\bar{\lambda}^{\zeta}$ of the estimated monthly intercept and slope coefficients, $\hat{\alpha}_t$, $\hat{\lambda}_t^{\mathrm{M}}$, $\hat{\lambda}_t^{\mathrm{V}}$, and $\hat{\lambda}_t^{\zeta}$. We account for potential autocorrelation, heteroskedasticity, and

[13]The results for value-weighted regressions can be found in the chapter appendix.

errors-in-variables concerns, computing robust Newey & West (1987) (again using twelve lags) and Shanken (1992) adjusted p-values based on the time series of coefficient estimates.

Table 4.5 reports the results of the basic Fama & MacBeth (1973) regressions. We report the results of a regression of excess returns on $\beta^{\mathrm{V}}_{j,t}$, $\beta^{\mathrm{M}}_{j,t}$, and various other canonical characteristics. In the basic regression specification suggested by our theoretical model (ii), the yearly market price of aggregate uncertainty (coefficient on $\beta^{\mathrm{V}}_{j,t}$) is -0.9920 with a p-value smaller than 0.001 which corresponds to a t-statistic of -3.71, clearly clearing the hurdle defined by Harvey, Liu, & Zhu (2015), who suggest accounting for potential data mining and publication bias by defining the critical t-ratio as 3.0 instead of 2.0 for newly discovered risk factors. Consequently, a two-standard deviation increase across stocks in their uncertainty-sensitivity is associated with a 16.34 % decrease in average annual returns.[14]

Naturally, adding further explanatory variables partly reduces both the magnitude and significance of the risk premium estimates on aggregate uncertainty, but adding ln(Size), Book-to-market, Bid-ask spread,[15] Momentum, and Short-term reversal in models (iii) to (iv) and (vi) to (viii) does not change much. The coefficient on uncertainty-sensitivity remains economically large and highly significant at 1 %. Adding dVIX in model (v) reduces the significance in the risk premium on aggregate uncertainty, but the p-value is still below 5 %. Consequently, both measures dVVIX and dVIX appear to carry at least partially similar information.[16]

[14]This number is obtained as follows. We need the sample mean of the cross-sectional standard deviation among the sensitivities to innovations in aggregate uncertainty from Table 4.2 which amounts to 0.0824. Plugging in yields $-0.9920 * (2 * 0.0824) = -0.1634$.

[15]Amihud (2002) argues that the Bid-ask spread is a more precise measure of (il-)liquidity than the one he develops.

[16]Note, though, that the correlation between factor loadings on dVVIX and dVIX amounts to only 47 % (compared to the factor correlation of 64 %), which makes it very unlikely that severe problems of multicollinearity are caused.

Table 4.5: Fama–MacBeth Regressions

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{V}_{j,t}$) over the same time, and a series of stock characteristics, all also measured over the following year. Detailed variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Factor	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)
Constant	0.1054	0.0932***	0.0463	0.1082***	0.0863**	0.1193***	0.0990***	0.0924***	0.0856	0.4391***	0.4522***	0.4299***	0.4453***	0.4544***
	(0.174)	(0.006)	(0.570)	(0.001)	(0.010)	(0.000)	(0.002)	(0.007)	(0.312)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
dVVIX	-0.7847***	-0.9920***	-0.9746***	-1.0465***	-0.5151**	-0.9692***	-0.9639***	-0.9767***	-1.0333***	-0.5782**	-0.5568**	-0.6216**	-0.5843**	-0.5779**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.050)	(0.000)	(0.000)	(0.000)	(0.000)	(0.029)	(0.029)	(0.018)	(0.041)	(0.035)
Beta		0.0097	0.0117	0.0181	0.0211	0.0109	-0.0093	0.0049	0.0199	0.0276	0.0067	0.0268	0.0283	0.0080
		(0.890)	(0.868)	(0.795)	(0.764)	(0.877)	(0.888)	(0.944)	(0.777)	(0.686)	(0.913)	(0.692)	(0.680)	(0.897)
ln(Size)			0.0031						0.0014	-0.0201***	-0.0208***	-0.0197***	-0.0216***	-0.0222***
			(0.437)						(0.743)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Book-to-market				-0.0416**					-0.0396**	-0.0342**	-0.0401***	-0.0354**	-0.0362**	-0.0415***
				(0.012)					(0.017)	(0.016)	(0.003)	(0.018)	(0.010)	(0.003)
dVIX					-0.2057***					-0.2061***	-0.2048***	-0.1926***	-0.2065***	-0.1953***
					(0.000)					(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid-ask spread						-19.266***				-40.369***	-37.854***	-39.861***	-42.364***	-40.436***
						(0.000)				(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Momentum							-0.0402				-0.0562			-0.0507
							(0.426)				(0.273)			(0.331)
Short-term reversal								-0.0132			-0.0445			-0.0455
								(0.830)			(0.495)			(0.485)
ln(Age)												0.0015		0.0020
												(0.791)		(0.712)
Leverage													0.0366	0.0259
													(0.154)	(0.338)
adj. R^2	0.0187	0.0762	0.0796	0.0835	0.0891	0.0868	0.0881	0.0823	0.0874	0.1188	0.1322	0.1212	0.1224	0.1385

Thus, particularly when dVIX is included in the regression, the coefficient denoting the market price of aggregate uncertainty is substantially smaller, amounting to about -0.5 compared to about -1 when aggregate volatility is not included as an explanatory variable. In Section 4.6.2 we deliver further investigations on the relation of aggregate uncertainty and aggregate volatility.

Adding several canonical characteristics jointly leaves the market price of aggregate uncertainty negative, with p-values below 5 %. Many of the risk premium estimates on the other canonical characteristics are not significantly different from zero, which is consistent with recent evidence that beta is not priced in the cross-section of stock returns (Frazzini & Pedersen, 2014) but partly conflicts with the view that prominent return anomalies have attenuated recently (Chordia, Subrahmanyam, & Tong, 2014). We use model (x) as base specification when adding further variables. It includes widely accepted characteristics that, except for Beta, empirically are shown to be connected to average returns during our sample period. Models (xii) to (xiv) show that adding ln(Age) and Leverage does not have a big impact on the market price of aggregate uncertainty.[17]

4.6 Robustness

4.6.1 Further Control Variables

In this section, we include further control variables to perform double sorts and regression tests. We control for various returns distributions characteristics (e.g., Co-Skewness, Downside Beta, or Idio. vol-of-vol), liquidity related characteristics (like Amihud illiquidity or Turnover), and

[17]The results on value-weighted regressions, shown in the appendix to this chapter, are qualitatively equal.

market factors (e.g., dSkew, Straddle vol, or Jump).

Table 4.6: Double Sorts (Further Control Variables) – Value-Weighted

This table reports Carhart (1997) 4-factor alphas for double-sorted portfolios. At the beginning of each month, we first sort stocks into quintiles based on the characteristics denoted in the first column. Then, within each quintile, we sort stocks based on their uncertainty-sensitivity ($\beta^{\mathrm{V}}_{j,t}$) into another five quintile portfolios. Portfolio returns are value-weighted. The five portfolios sorted on $\beta^{\mathrm{V}}_{j,t}$ are then obtained by averaging over the respective quintiles within each quintile of the other characteristic, thus we obtain $\beta^{\mathrm{V}}_{j,t}$ quintile portfolios controlling for another characteristic. We reform the portfolios after one month. This procedure is performed for each of the characteristics. We categorize control variables into groups of returns distributions characteristics (Panel A), liquidity-related characteristics (Panel B), and market factors (Panel C). The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Returns Distributions Characteristics

Rank	1	2	3	4	5	5 minus 1
Idio. volatility	-0.0086	-0.0345***	-0.0599***	-0.0728***	-0.0974***	-0.0888**
	(0.578)	(0.000)	(0.000)	(0.000)	(0.000)	(0.019)
Co-Skewness	0.0212	0.0193***	-0.0031	-0.0333**	-0.1136***	-0.1348***
	(0.170)	(0.001)	(0.438)	(0.011)	(0.000)	(0.001)
Co-Kurtosis	0.0280**	0.0258***	0.0067	-0.0322**	-0.1103***	-0.1383***
	(0.034)	(0.000)	(0.369)	(0.018)	(0.000)	(0.000)
Downside beta	0.0292**	0.0057	-0.0032	-0.0351**	-0.0874***	-0.1166***
	(0.015)	(0.586)	(0.590)	(0.011)	(0.000)	(0.000)
MAX	0.0357**	0.0159**	-0.0218**	-0.0501***	-0.0888***	-0.1244***
	(0.016)	(0.024)	(0.014)	(0.000)	(0.000)	(0.000)
Idio. vol-of-vol	0.0438***	0.0214***	0.0193***	0.0015	-0.0654**	-0.1093***
	(0.003)	(0.000)	(0.001)	(0.901)	(0.011)	(0.005)
Volatility	0.0030	-0.0033	-0.0054	-0.0444***	-0.0922***	-0.0952***
	(0.819)	(0.747)	(0.570)	(0.000)	(0.000)	(0.009)
Skewness	0.0200	0.0252***	-0.0012	-0.0309**	-0.1046***	-0.1247***
	(0.146)	(0.000)	(0.798)	(0.018)	(0.000)	(0.001)
Kurtosis	0.0207	0.0238***	-0.0088*	-0.0368***	-0.0958***	-0.1164***
	(0.110)	(0.000)	(0.079)	(0.009)	(0.000)	(0.002)

Table 4.6: Double Sorts (Further Control Variables) - Value-Weighted (continued)

Panel B. Liquidity- Related Characteristics

Rank	1	2	3	4	5	5 minus 1
PS liquidity	-0.0043 (0.885)	-0.0082 (0.700)	-0.0311* (0.066)	-0.0566*** (0.000)	-0.1362*** (0.000)	-0.1319*** (0.000)
Amihud illiquidity	-0.0485*** (0.000)	-0.0312*** (0.000)	-0.0319*** (0.000)	-0.0608*** (0.000)	-0.1329*** (0.000)	-0.0844*** (0.001)
Volume	-0.0302*** (0.005)	-0.0155* (0.087)	-0.0205*** (0.000)	-0.0472*** (0.000)	-0.1234*** (0.000)	-0.0932*** (0.000)
Turnover	0.0249* (0.070)	0.0077* (0.072)	-0.0023 (0.619)	-0.0338*** (0.001)	-0.095*** (0.000)	-0.1199*** (0.000)

Panel C. Market Factors

Rank	1	2	3	4	5	5 minus 1
dSkew	0.0094 (0.627)	-0.0014 (0.907)	-0.0135* (0.096)	-0.0483*** (0.000)	-0.1157*** (0.000)	-0.1251*** (0.000)
dKurt	0.0136 (0.489)	-0.0041 (0.720)	-0.0162* (0.065)	-0.0470*** (0.000)	-0.1211*** (0.000)	-0.1347*** (0.000)
Straddle vol	-0.0027 (0.881)	-0.0110 (0.244)	-0.0144** (0.034)	-0.0404*** (0.000)	-0.1064*** (0.000)	-0.1037*** (0.001)
Jump	-0.0003 (0.985)	-0.0055 (0.574)	-0.0155** (0.030)	-0.0457*** (0.000)	-0.0965*** (0.000)	-0.0962*** (0.000)
dVRP	-0.0071 (0.522)	0.0032 (0.764)	-0.0054 (0.396)	-0.0331*** (0.000)	-0.0947*** (0.000)	-0.0876*** (0.007)
dPol	0.0010 (0.972)	-0.0145 (0.493)	-0.0290* (0.076)	-0.0567*** (0.000)	-0.1221*** (0.000)	-0.1232*** (0.000)
Forec. uncertainty	0.0118 (0.354)	0.0061 (0.253)	-0.0098** (0.018)	-0.0365*** (0.001)	-0.1092*** (0.000)	-0.1210*** (0.002)

The results on double sorts are presented in Table 4.6. The return-uncertainty trade-off persists, independently of which variable we control for and also independently of the return weighting scheme. For most control variables, the 4-factor alpha of the hedge portfolio amounts to about 12 % (10 %) for value-weighted (equally weighted) returns. However, for some controls, e.g. Idio. Volatility or Amihud illiquidity the 4-factor alphas decrease. Thus, it appears that part of the effect of aggregate uncertainty can be explained by these control variables. Nevertheless, the 4-factor alpha is highly statistically significant in any case. Controlling for alternative variables that are potentially related to uncertainty, policy uncertainty (dPol), and forecaster uncertainty, the results are still significant.

The results of regression tests can be found in Table 4.7. In Panel A, we report the results controlling for various returns distributions characteristics. We find idiosyncratic volatility, MAX, and Skewness to carry a significant price of risk when adding them to our base model. On the other hand, neither do Co-Skewness, Downside Beta, Idio. vol-of-vol carry a significant price of risk.

In all cases, the market price of aggregate uncertainty is statistically significant, leastwise at 10 %. In Panel B, we control for various liquidity-related characteristics like the Amihud illiquidity measure or Turnover, but adding these variables does not change the previous results. The coefficient on uncertainty-sensitivity is always significant at 10 %. In Panel C, we control for different market factors that are estimated separately. Adding dSkew, dKurt, Straddle vol, Jump, dVRP, and dPol does not change our basic results. While the aggregate market factors partially carry significant risk premia, this does not affect the estimates on the market price of aggregate uncertainty that are statistically significant in every specification. Including Forec. uncertainty, a variable supposedly to some extent capturing similar information as aggregate uncertainty, the p-value of the market price of aggregate uncertainty turns out to be slightly above 10 %. However, the coefficient on Forec. uncertainty is not significant either, which along with the high adjusted R^2 this delivers some indication a multicollinear relation.[18]

[18]Since Forec. uncertainty is measured on a quarterly basis while dVVIX yields daily notations, estimation of correlations has severe limitations.

Table 4.7: Fama–MacBeth Regressions (Further Control Variables)

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{V}_{j,t}$) over the same time, and a series of stock characteristics, all also measured over the following year. Detailed variable definitions are provided in the appendix. Panel A examines returns distribution characteristics and Panels B and C show liquidity-related characteristics and market factors, respectively. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Returns Distributions Characteristics

	(xv)	(xvi)	(xvii)	(xviii)	(xix)	(xx)	(xxi)	(xxii)	(xxiii)	(xxiv)
Constant	0.6075***	0.4359***	0.4298***	0.4419***	0.2402**	1.1328***	0.5142***	0.4503***	0.4619***	0.5289***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.024)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
dVVIX	-0.6551***	-0.5731**	-0.5734**	-0.6345**	-0.3854*	-0.7815**	-0.6418**	-0.6047**	-0.6155**	-0.6461**
	(0.006)	(0.030)	(0.032)	(0.043)	(0.074)	(0.020)	(0.011)	(0.021)	(0.020)	(0.011)
Beta	0.0744	0.0263	0.0253	0.0627	-0.0377	0.0686	0.0175	0.0098	0.0119	0.0194
	(0.190)	(0.699)	(0.709)	(0.388)	(0.429)	(0.316)	(0.784)	(0.884)	(0.858)	(0.765)
ln(Size)	-0.0289***	-0.0198***	-0.0194***	-0.0204***	-0.0094*	-0.0633***	-0.023***	-0.02***	-0.0207***	-0.0227***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.081)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Book-to-market	-0.0414***	-0.0348**	-0.0341**	-0.0331**	-0.0190	-0.0203	-0.0463**	-0.04**	-0.0413**	-0.0439**
	(0.007)	(0.016)	(0.019)	(0.022)	(0.185)	(0.306)	(0.036)	(0.033)	(0.030)	(0.039)
dVIX	-0.2070***	-0.2080***	-0.2023***	-0.2170***	-0.1941***	-0.2331***	-0.1927***	-0.1929***	-0.1931***	-0.1965***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid-ask spread	-34.827***	-39.751***	-40.077***	-39.844***	-44.502***	-199.268***	-32.459***	-32.908***	-32.418***	-33.220***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Idio. volatility	-4.5888***									
	(0.000)									
Co-Skewness		-0.0013								
		(0.280)								
Co-Kurtosis			0.0000							
			(0.182)							
Downside beta				-0.0329						
				(0.299)						
MAX					1.7544***					
					(0.002)					
Idio. vol-of-vol						0.1420				
						(0.474)				
Volatility							-0.7012			-1.4161*
							(0.362)			(0.098)
Skewness								0.0083***		0.0248***
								(0.000)		(0.001)
Kurtosis									0.0000	-0.0007**
									(0.946)	(0.025)
adj. R^2	0.1406	0.1224	0.1201	0.1248	0.1438	0.1658	0.1341	0.1287	0.1283	0.1398

Table 4.7: Fama–MacBeth Regressions (Further Control Variables) (continued)

Panel B. Liquidity-Related Characteristics

	(xxv)	(xxvi)	(xxvii)	(xxviii)
Constant	0.1712**	0.1862**	-0.3321***	0.1025
	(0.021)	(0.018)	(0.000)	(0.152)
dVVIX	-0.5547**	-0.5755**	-0.5129*	-0.5679**
	(0.033)	(0.030)	(0.070)	(0.027)
Beta	0.0033	0.0291	0.0317	0.0336
	(0.964)	(0.676)	(0.650)	(0.620)
ln(Size)	-0.0038	-0.0056		-0.0002
	(0.214)	(0.137)		(0.956)
Book-to-market	-0.0378**	-0.0369**	-0.0264*	-0.0388**
	(0.015)	(0.012)	(0.069)	(0.011)
dVIX	-0.2198***	-0.2101***	-0.1759***	-0.2061***
	(0.000)	(0.000)	(0.000)	(0.000)
PS liquidity	0.0200			
	(0.116)			
Amihud illiquidity		-0.9346***		
		(0.000)		
ln(Volume)			0.0253***	
			(0.000)	
Turnover				-0.0027
				(0.466)
adj. R^2	0.1182	0.1052	0.1099	0.1029

4.6.2 Multivariate Estimation

In this section, we examine the robustness of our results to jointly estimating the sensitivities to innovations in aggregate uncertainty with those to other factors, as presented in Equation (4.13). Table 4.8 reports the results of Fama & MacBeth (1973) regressions when the sensitivities to the different factors are obtained in a joint multivariate sensitivity estimation regression. Incorporating the Fama & French (1993) factors (xxxix) leaves the effect of aggregate uncertainty strongly significant at 1 %. Adding the other market factors like dVIX, dSkew, dKurt, Straddle vol, Jump, or dVRP (models (xl) to (xlvii)) does not change much. The price of risk on HML regularly is significant at 5 % while that on MKT and SMB never is. The coefficient on uncertainty-sensitivity is statistically significant at 1 % in any case. The coefficient on dVIX is substantially less significant when estimating the

Table 4.7: Fama–MacBeth Regressions (Further Control Variables) (continued 2)

Panel C. Market Factors

	(xxix)	(xxx)	(xxxi)	(xxxii)	(xxxiii)	(xxxiv)	(xxxv)	(xxxvi)	(xxxvii)	(xxviii)
Constant	0.4631***	0.4698***	0.4726***	0.4420***	0.4393***	0.4315***	0.4626***	0.4331***	0.5082***	0.4908***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
dVVIX	-0.6784**	-0.6309**	-0.6116**	-1.0625***	-1.0538***	-1.1378***	-0.6947***	-0.6114**	-0.6076**	-0.5122
	(0.012)	(0.020)	(0.020)	(0.000)	(0.001)	(0.000)	(0.010)	(0.015)	(0.018)	(0.104)
Beta	0.0258	0.0253	0.0214	0.0247	0.0236	0.0256	0.0279	0.0283	0.0046	0.0121
	(0.719)	(0.726)	(0.762)	(0.727)	(0.737)	(0.712)	(0.690)	(0.676)	(0.950)	(0.863)
ln(Size)	-0.0216***	-0.0220***	-0.0221***	-0.0201***	-0.0198***	-0.0195***	-0.0215***	-0.0198***	-0.0236***	-0.0221***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.006)
Book-to-market	-0.0330**	-0.0318**	-0.0313**	-0.0370**	-0.0374**	-0.0372**	-0.0360**	-0.0338**	-0.0357**	-0.0223
	(0.015)	(0.017)	(0.016)	(0.018)	(0.016)	(0.019)	(0.016)	(0.016)	(0.020)	(0.208)
dVIX	-0.2065***	-0.2047***	-0.2140***				-0.2239***	-0.1125	-0.2014***	-0.2482***
	(0.000)	(0.000)	(0.000)				(0.000)	(0.119)	(0.000)	(0.000)
Bid-ask spread	-41.090***	-41.045***	-40.044***	-41.138***	-41.431***	-40.713***	-41.097***	-39.689***	-43.454***	-53.003***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
dSkew	-2.0165*		-2.6088							
	(0.057)		(0.362)							
dKurt		5.8227*	-1.1536							
		(0.078)	(0.892)							
Straddle vol				0.1384***		0.1578**				
				(0.002)		(0.028)				
Jump					-0.3319	0.0611	0.0782			
					(0.153)	(0.853)	(0.716)			
dVRP								-0.0527**		
								(0.026)		
dPol									0.1696	
									(0.357)	
Forec. uncertainty										0.0000
										(0.357)
adj. R^2	0.1285	0.1282	0.1371	0.1150	0.1166	0.1215	0.1255	0.1241	0.1329	0.2411

sensitivities jointly with dVVIX compared to the analysis in which both are estimated separately. Consequently, although both appear to partly carry similar information, our measure for aggregate uncertainty seems to incorporate even more, or more precise, information for stock returns. The prices of risk on Jump and dVRP are significant, whereas the remaining factors are only partly significant.

4.6.3 Crisis Effects

Our sample period contains the recent financial crisis that has imposed large fluctuations on asset markets. Völkert (2015) shows that there was a considerable change in the shape of the risk-neutral distribution of the VIX during the crisis. Consequently, our uncertainty explanation could potentially be imposed by a crisis effect. We check for that by introducing a crisis dummy taking the value one at every month where half of the following year or more falls in the period from September 2008 until August 2009 (August 2010), or into months indicated as business cycle contractions by the National Bureau of Economic Analysis (NBER). Table 4.9 presents the results. Defining the crisis period as September 2008 until August 2009 in Panel A, we find the crisis dummy of the 5 minus 1 hedge portfolio to take a value of -13.7 % per annum for value-weighted returns. The negative sign of the crisis dummy indicates that the uncertainty-return trade-off is even more pronounced in times of crises. Yet, the crisis dummy is not statistically significant. Nevertheless, although the 4-factor alphas of the hedge portfolio is smaller in magnitude, with a value of -11.6 % per annum compared to the specification without a crisis dummy in Table 4.3, it is still significant at 1 %. Using a longer crisis period in Panel B does not affect the results for the 4-factor alphas, but the crisis dummy of the hedge portfolio shrinks heavily. Directly using the NBER definition for recessions (Panel C), the

Table 4.8: Fama–MacBeth Regressions – Multivariate Estimation

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the same time and/or a series of $\beta^{\zeta}_{j,t}$ estimates obtained simultaneously in a joint regression as in Equation (4.13), using daily data over the following year. MKT is the return on the market portfolio. Further detailed variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

	(xxxix)	(xl)	(xli)	(xlii)	(xliii)	(xliv)	(xlv)	(xlvi)	(xlvii)
Constant	0.0755**	0.0894***	0.0781**	0.0959***	0.0840**	0.0849**	0.0726**	0.0908***	0.0757**
	(0.038)	(0.008)	(0.026)	(0.003)	(0.012)	(0.016)	(0.042)	(0.006)	(0.027)
dVVIX	-0.9167***	-0.9510***	-0.8549***	-0.8845***	-0.8022***	-0.9880***	-0.8936***	-0.9540***	-0.8879***
	(0.003)	(0.000)	(0.003)	(0.001)	(0.004)	(0.000)	(0.002)	(0.000)	(0.003)
MKT	0.0454	0.0182	0.0412	0.0094	0.0338	0.0176	0.0456	0.0164	0.0437
	(0.520)	(0.794)	(0.547)	(0.890)	(0.612)	(0.791)	(0.499)	(0.813)	(0.527)
SMB	-0.0136		-0.0101		-0.0079		-0.0099		-0.0114
	(0.380)		(0.509)		(0.616)		(0.490)		(0.462)
HML	-0.0575**		-0.0581**		-0.0563**		-0.0565**		-0.0597***
	(0.010)		(0.010)		(0.012)		(0.014)		(0.007)
dVIX		-0.2644**	-0.2630**	-0.2404**	-0.2411**				
		(0.012)	(0.017)	(0.015)	(0.021)				
dSkew				-1.6169	-0.9444				
				(0.109)	(0.283)				
dKurt				6.1918*	4.0815				
				(0.055)	(0.139)				
Straddle vol						0.1318**	0.0890		
						(0.026)	(0.113)		
Jump						-0.7197***	-0.6277**		
						(0.010)	(0.039)		
dVRP								-0.0645**	-0.0698**
								(0.043)	(0.033)
adj. R^2	0.1127	0.0887	0.1193	0.1044	0.1287	0.0875	0.1202	0.0877	0.1199

Table 4.9: Crisis Effects

At the beginning of each month, we form quintile portfolios based on the stock's sensitivities to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the following year. To obtain the sensitivities, we regress daily stock returns on dVVIX, controlling for MKT as in Equation (4.12). Stocks with the lowest $\beta^{\mathrm{V}}_{j,t}$ are sorted into portfolio 1, those with the highest into portfolio 5. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. 4-factor alpha refers to the Carhart (1997) 4-factor alpha, while Crisis dummy is a dummy variable taking the value one if at least half of the following year falls in the period indicated by the respective panel headlines. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Crisis Period September 2008–August 2009

Rank	1	2	3	4	5	5 minus 1
4-factor alpha	0.0163	0.0193***	-0.0067	-0.0211	-0.0993***	-0.1156***
	(0.340)	(0.000)	(0.438)	(0.172)	(0.000)	(0.003)
Crisis dummy	0.0581	-0.0009	0.0061	-0.0686*	-0.0786	-0.1367
	(0.122)	(0.966)	(0.865)	(0.061)	(0.294)	(0.153)

Panel B. Crisis Period September 2008–August 2010

Rank	1	2	3	4	5	5 minus 1
4-factor alpha	0.0140	0.0165***	0.0045	-0.0211	-0.1144***	-0.1283***
	(0.425)	(0.002)	(0.204)	(0.219)	(0.000)	(0.003)
Crisis dummy	0.0516***	0.0115	-0.0430***	-0.0490	0.0075	-0.0441
	(0.007)	(0.126)	(0.000)	(0.155)	(0.826)	(0.346)

Panel C. Crisis During Recessions Indicated by the NBER

Rank	1	2	3	4	5	5 minus 1
4-factor alpha	0.0435***	0.0277***	-0.0157**	-0.0580***	-0.1218***	-0.1653***
	(0.009)	(0.000)	(0.034)	(0.002)	(0.000)	(0.001)
Crisis dummy	-0.0552	-0.0271	0.0319*	0.0802*	0.0291	0.0842
	(0.133)	(0.114)	(0.074)	(0.066)	(0.729)	(0.470)

4-factor alpha of the hedge portfolio remains highly significant while the crisis dummy again does not yield a significant coefficient. To summarize, there is only weak indication for a crisis effect. However, controlling for crises, the result that stocks with high sensitivity to innovations in aggregate uncertainty strongly underperform stocks with low exposure to aggregate uncertainty strongly persists.

4.6.4 Predicting Future Exposure to Aggregate Uncertainty

In the previous sections, we demonstrate a strongly negative relation between the sensitivities to innovations in aggregate uncertainty and stock returns. In this section, we examine the cross-sectional relation of several ex ante firm characteristics and factor sensitivities and the ex post sensitivities to innovations in aggregate uncertainty. Cross-sectional predictors of the stock's uncertainty-sensitivities are measured during the twelve months directly prior to the twelve-month estimation period for sensitivities to innovations in aggregate uncertainty. The results are reported in Table 4.10. We find past uncertainty-sensitivities to significantly predict future relative uncertainty-sensitivities. The coefficient, however, is only 0.1272, which is far from one, and the explanatory power is very small with 1.5 %. Consequently, we test further variables that may predict the stock's future exposure to aggregate uncertainty. Univariately, in models (ii) to (xx), we find past Beta, Book-to-market, dVIX, ln(Age), Leverage, MAX, and Jump to significantly predict future relative exposure to aggregate uncertainty at 1 %, though the explanatory power is negligible for all of these models. In multivariate regressions of models (xxi) and (xxii), we find past dVVIX, Beta, dVIX, Momentum, and Volatility to most strongly predict future relative uncertainty-sensitivities.

Table 4.10: Determinants of Relative Sensitivities to Aggregate Uncertainty

This table presents average coefficient estimates from monthly equally weighted Fama & MacBeth (1973) regressions. Each month, we regress the sensitivity to innovations in aggregate uncertainty during the following year on a constant and a series of stock characteristics, all measured over the past year. Detailed variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Factor	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)
Constant	0.0190***	-0.0116***	-0.0238	0.0115***	0.0045***	0.0084***	0.0055**	0.0011	0.0037*	0.0052	0.0061***
	(0.001)	(0.005)	(0.111)	(0.000)	(0.004)	(0.005)	(0.020)	(0.573)	(0.085)	(0.182)	(0.006)
dVVIX	0.1272***										
	(0.000)										
Beta		0.0192***									
		(0.002)									
ln(Size)			0.0023*								
			(0.059)								
Book-to-market				-0.0016***							
				(0.002)							
dVIX					0.0325***						
					(0.000)						
Bid-ask spread						-0.4705*					
						(0.079)					
Momentum							-0.0051**				
							(0.048)				
Age								0.0022***			
								(0.004)			
Leverage									0.0099***		
									(0.009)		
Idio. volatility										0.0172	
										(0.901)	
Co-Skewness											0.0003***
											(0.003)
Downside beta											
MAX											
Idio. vol-of-vol											
dSkew											
dKurt											
Straddle vol											
Jump											
dVRP											
Volatility											
adj. R^2	0.0153	0.0173	0.0055	0.0043	0.0094	0.0076	0.0017	0.0010	0.0019	0.0086	0.0033

Table 4.10: Determinants of Relative Sensitivities to Aggregate Uncertainty (continued)

Factor	(xii)	(xiii)	(xiv)	(xv)	(xvi)	(xvii)	(xviii)	(xix)	(xx)	(xxi)	(xxii)
Constant	-0.0064***	0.0028	0.0135***	0.0049***	0.0052***	0.0054***	0.0053***	0.0044***	0.0053	0.0463*	0.0540*
	(0.001)	(0.286)	(0.000)	(0.003)	(0.004)	(0.005)	(0.004)	(0.003)	(0.465)	(0.073)	(0.089)
dVVIX										0.0789***	0.0820***
										(0.000)	(0.000)
Beta										0.0179***	0.0238***
										(0.000)	(0.000)
ln(Size)										-0.0036**	-0.0037**
										(0.011)	(0.024)
Book-to-market										0.0039	0.0044
										(0.281)	(0.188)
dVIX										0.0515***	0.0622***
										(0.007)	(0.004)
Bid-ask spread										4.2810***	3.0667*
										(0.006)	(0.099)
Momentum										-0.0052**	-0.0067***
										(0.028)	(0.001)
Age										-0.0006	-0.0017**
										(0.197)	(0.017)
Leverage										0.0050	0.0061
										(0.219)	(0.157)
Idio. volatility											0.6587
											(0.101)
Co-Skewness										0.0001	0.0002
										(0.542)	(0.116)
Downside beta	0.0129***									-0.0027	-0.0047
	(0.000)									(0.377)	(0.113)
MAX		0.0221									-0.1234
		(0.441)									(0.163)
Idio. vol-of-vol			-0.0259*							-0.0143	-0.0145
			(0.074)							(0.170)	(0.327)
dSkew				-0.0317							0.3211
				(0.799)							(0.254)
dKurt					0.1469						1.7025**
					(0.490)						(0.039)
Straddle vol						-0.0320*				-0.0220*	-0.0201
						(0.052)				(0.054)	(0.229)
Jump							0.1379***			-0.0857***	-0.0639
							(0.004)			(0.010)	(0.168)
dVRP								0.0105**		-0.0053	-0.0044
								(0.011)		(0.460)	(0.559)
Volatility									0.0853		-0.3048***
									(0.624)		(0.001)
adj. R^2	0.0090	0.0073	0.0071	0.0031	0.0016	0.0063	0.0069	0.0073	0.0133	0.1020	0.1414

Since we have seen that past uncertainty-sensitivities significantly predict relative future uncertainty-sensitivities, we examine the relation of average returns and past sensitivities to innovations in aggregate uncertainty. Table 4.11 reports the results. Sorting the stocks by past exposure to aggregate uncertainty is seen to produce a smaller, though also significant, spread in average returns. For value-weighted returns, the annual 4-factor alpha amounts to -6.2 %, which is statistically significant at 1 %. Consequently, the sensitivities to aggregate uncertainty appear to be somewhat stable over time. However the positive spread in ex post exposures to aggregate uncertainty, when sorting on ex ante uncertainty-sensitivities, is substantially smaller than that produced sorting by contemporaneous factor loadings (see Table 4.3).

Ang et al. (2006a) point out that pre-formation factor loadings, beside actual variation in exposures toward that factor, additionally reflect measurement error effects. This measurement error increases during phases of high return volatility. Consequently, since markets are highly volatile during our sample period, high variation in the exposures to aggregate uncertainty may be obtained.[19] These theoretical arguments suggest that pre-formation factor loadings cannot fully capture ex post factor loadings in an adequate fashion. Consequently, smaller spreads in ex post exposures to innovations in aggregate uncertainty are obtained.

4.6.5 Realized Measure of Aggregate Uncertainty

The results of this study, so far, could be criticized on the grounds of a relatively short sample period that may induce spurious findings. To test for that we use, taking account of the literature on volatility estimation, an inferior measure of aggregate uncertainty, which is only

[19]While the average value of the VIX is 19.44, % during the period 1990 until 2006 it is considerably higher with 21.84 % during our sample period 2007 until 2014.

Table 4.11: Pre-Formation Factor Loadings

At the beginning of each month, we form value-weighted quintile portfolios based on the stock's sensitivities to aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the past year. To obtain the sensitivities, we regress daily excess stock returns on dVVIX, controlling for MKT as in Equation (4.12). Stocks with the lowest $\beta^{\mathrm{V}}_{j,t}$ are sorted into portfolio 1, those with the highest into portfolio 5. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. We reform the portfolios after one month. We report Carhart (1997) 4-factor alphas for portfolios based on returns over the following year. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %. The columns ex post β^{V} report the average annual ex post sensitivities to aggregate uncertainty of the portfolios.

Rank	1	2	3	4	5	5 minus 1
Mean return	0.1270**	0.1203**	0.1287**	0.1223*	0.1266*	-0.0004
	(0.018)	(0.024)	(0.025)	(0.067)	(0.088)	(0.990)
CAPM alpha	0.0126	0.0089	-0.0008	-0.0268**	-0.0482**	-0.0608*
	(0.208)	(0.455)	(0.912)	(0.018)	(0.044)	(0.065)
3-factor alpha	0.0145	0.0165**	-0.0055	-0.0405***	-0.0422***	-0.0567***
	(0.152)	(0.017)	(0.485)	(0.006)	(0.000)	(0.001)
4-factor alpha	0.0164*	0.0188***	-0.0063	-0.0424***	-0.0457***	-0.0621***
	(0.066)	(0.007)	(0.377)	(0.002)	(0.000)	(0.000)
ex post β^{V}	-0.0011	0.0014	0.0047	0.0087	0.0189	0.0200

partially forward-looking. Specifically, we use five-minute high-frequency data on the VIX from the TRTH database and estimate intraday realized volatility for each day. The high-frequency data is available for the period from January 01, 1996 until December 31, 2014. Thus, the sample period is extended substantially. We perform standard data cleaning operations following Rösch, Subrahmanyam, & Van Dijk (2014). Following on from that, we obtain daily realized volatilities of the VIX and compute the innovations ($dVoVIX_t$) from a fitted ARMA model with $dVoVIX_t =$

$VoVIX_t - 0.9941 VoVIX_{t-1} + 0.6803 dVoVIX_{t-1}$.[20] We estimate factor sensitivities using Equation (4.12), replacing dVVIX by dVoVIX.

We present the results in Table 4.12. For value-weighted portfolios, the return differential between stocks with high and those with low sensitivities to aggregate uncertainty is substantially smaller than when using the VVIX in Table 4.3 with about −6.3 %. This estimate is significant at 10 %. The uncertainty-return trade-off persists when controlling for systematic risk or employing the 3-factor model while a significant alpha cannot be obtained with the 4-factor and 5-factor models. Even though the portfolio of stocks with high sensitivities to innovations in aggregate uncertainty has significantly negative alphas, these cannot be detected in the hedge portfolio as the alpha of the portfolio with the lowest sensitivities is also negative. Consequently, when accounting for momentum and liquidity, a significant effect cannot be found. Furthermore, when restricting the sample to stocks that are traded on the NYSE or when restricting the sample to the horizon in which data on the VVIX is available (2007 until 2014), the effect is just about significant at 10 %. This is fully in line with our motivation of dVVIX being a superior measure for aggregate uncertainty compared to dVoVIX.

4.7 Conclusion

Using a simple stylized theoretical model we show that, beside the well-established risk-return trade-off, investors also face an uncertainty-return trade-off. In our empirical study, we verify this prediction, finding a clear and very robust negative risk premium on aggregate uncertainty.

We use both uni- and bivariate portfolio sorts and show that the

[20] To measure innovations of the realized volatility of the VIX, it is indeed necessary to fit an ARMA model, since simple first differences still adhere a substantial autocorrelation of −0.38. Consequently, this should be considered when estimating tomorrow's expected volatility.

Table 4.12: Realized Measure of Aggregate Uncertainty

At the beginning of each month, we form value-weighted (Panel A) and equally weighted (Panel B) quintile portfolios based on the stock's sensitivities to innovations in aggregate uncertainty ($\beta_{j,t}^{\mathrm{V}}$), measured as the realized volatility of the VIX, over the following year. The extended sample period spans from January 1996 to December 2014. To obtain the sensitivities, we regress daily excess stock returns on dRVIX, controlling for MKT as in Equation (4.12). Stocks with the lowest $\beta_{j,t}^{\mathrm{V}}$ are sorted into portfolio 1, those with the highest into portfolio 5. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta_{j,t}^{\mathrm{V}}$ and simultaneously selling the stocks in the quintile with the lowest $\beta_{j,t}^{\mathrm{V}}$. We reform the portfolios after one month. The row labeled Mean return is based on monthly simple returns. CAPM alpha, FF-3 alpha, 4-factor alpha, and 5-factor alpha refer to the alphas of the CAPM, the Fama & French (1993) 3-factor, Carhart (1997) 4-factor, and the 5-factor (including liquidity) models, respectively. The segment NYSE only restricts the sample of stocks to those that are traded at the NYSE at the beginning of the estimation period. The segment Horizon 07-14 presents the results when restricting the sample period to the time frame between 2007 and 2014. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Rank	1	2	3	4	5	5 minus 1
Mean return	0.1076**	0.1197***	0.1133***	0.0922**	0.0450	-0.0626*
	(0.013)	(0.000)	(0.001)	(0.016)	(0.407)	(0.054)
CAPM alpha	0.0015	0.0308***	0.0195**	-0.0083	-0.0834***	-0.0849**
	(0.943)	(0.001)	(0.018)	(0.297)	(0.000)	(0.015)
FF-3 alpha	-0.0157	0.0257***	0.0138**	-0.0102	-0.0694***	-0.0537*
	(0.426)	(0.001)	(0.039)	(0.207)	(0.000)	(0.077)
4-factor alpha	-0.0252	0.0264***	0.0118*	-0.0074	-0.0682***	-0.0431
	(0.125)	(0.000)	(0.065)	(0.283)	(0.000)	(0.131)
5-factor alpha	-0.0368***	0.0239***	0.0141**	-0.0072	-0.0559***	-0.0191
	(0.009)	(0.000)	(0.031)	(0.323)	(0.001)	(0.409)
NYSE only						
4-factor alpha	-0.0217	0.0171**	0.0143**	-0.0077	-0.0597***	-0.0380*
	(0.141)	(0.018)	(0.047)	(0.248)	(0.000)	(0.093)
Horizon 07-14						
4-factor alpha	-0.0398**	0.0210**	0.0230***	-0.0110	-0.0727***	-0.0329*
	(0.029)	(0.016)	(0.000)	(0.305)	(0.000)	(0.100)

quintile portfolio of stocks with the highest sensitivity toward innovations in aggregate uncertainty underperforms the quintile of stocks with the lowest exposure to aggregate uncertainty by about 14 % per annum in terms of 4-factor alphas. Using regression tests, we estimate the cross-sectional market price of aggregate uncertainty to be both economically substantial and statistically highly significant. The estimated risk premium on aggregate uncertainty cannot be explained by known risk factors. Our results are also consistent with any multifactor model, in which aggregate uncertainty is priced with a negative sign if investors relate a positive change in aggregate uncertainty to future unfavorable shifts in the investment opportunity set.

C Appendix

C.1 Variable Definitions

Main Control Variables

- **Age** (Zhang, 2006, "Age") is the number of years up to time t since a firm first appeared in the CRSP database. In regressions, we take the natural logarithm to remove the extreme skewness in this variable.

- **Aggregate volatility** (Ang et al., 2006b, "dVIX"), is denoted by the coefficient $\beta_{j,t}^{dVIX}$ in the regression $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta_{j,t}^{M}(r_{M,\tau} - r_{f,\tau}) + \beta_{j,t}^{dVIX} dVIX_{\tau} + \epsilon_{j,\tau}$ using daily returns over the examination period (Ang et al., 2006b), where $dVIX$ is defined as the first difference in the VIX from the Chicago Board Options Exchange (CBOE).

- **Beta** is the coefficient $\beta_{j,t}^{M}$ obtained by the regression in Equation (4.12).

- **Bid-ask spread** ("Bid-ask spread") is the stock's average daily bid-ask spread over the examination period.

- **Book-to-market** (Fama & French, 1992, "Book-to-market") is the weighted average of book equity divided by market equity over the examination period. The basic quantity is updated every 12 months for the beginning of the year. Book equity is defined as stockholder's equity, plus balance sheet deferred taxes and investment tax credit, plus post-retirement benefit liabilities, minus the book value of preferred stock.

- **Leverage** (Bhandari, 1988, "Leverage") is defined as the weighted average of one minus book equity (see "Book-to-market") divided by total assets (Compustat: AT). The basic quantity is updated every 12 months for the beginning of the year.

- **Momentum** (Jegadeesh & Titman, 1993, "Momentum") is the cumulative stock return over the period from $t-12$ until $t-1$.

- **Short-term reversal** (Jegadeesh, 1990, "Short-term reversal") is the preceding month's stock return (from $t-1$ to t).

- **Size** (Banz, 1981, "Size") is the average of firm's market capitalization over the examination period. Market Capitalization is computed as the product of the price times the number of shares outstanding. In regressions, we take the natural logarithm to remove the extreme skewness in this variable.

Further Controls

- **Amihud illiquidity** (Amihud, 2002, "Amhiud illiquidity") is the absolute value of the stock's return divided by the daily dollar volume, averaged over the examination period. Specifically, it is $Illiq_t = \frac{1}{n}\sum_{\tau=1}^{n}\frac{|r_{j,\tau}|}{Volume\$_\tau}$, with the daily dollar volume ($Volume\$_\tau$, in thousand dollars) being calculated as last trade price times shares traded on day τ, while the summation is taken over all n trading days during the examination period.

- **Co-Skewness** (Harvey & Siddique, 2000, "Co-Skewness") is the coefficient $\beta^C_{j,t}$ on the squared market excess return in the regression $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^M_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^C_{j,t}(r_{M,\tau} - r_{f,\tau})^2 + \epsilon_{j,\tau}$, including the market excess return and the squared market excess return, estimated using daily returns over the examination period.

- **Demand for lottery** (Bali, Cakici, & Whitelaw, 2011, "MAX") is the average of the five highest daily returns during the examination period.

- **Downside beta** (Ang et al., 2006a, "Downside beta") is the coefficient $\beta^D_{j,t}$ in the regression $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^D_{j,t}(r_{M,\tau} - r_{f,\tau}) + \epsilon_{j,\tau}$, using daily returns over the examination period only when the market return is below the average daily market return over that year.

- **Forecaster uncertainty** (Anderson et al., 2009, "Forc. uncertainty") is the coefficient β^F_j in the regression $r_{j,\tau} - r_{f,\tau} = \alpha_j + \beta^F_j dunc_\tau + \beta^M_j(r_{M,\tau} - r_{f,\tau}) + \epsilon_{j,\tau}$, where $dunc_\tau$ is the quarterly innovation in the weighted variance of predictions on the market return. The regression is performed once for each security using quarterly returns over the whole sample period. We construct the forecasts following Anderson et al. (2009) using the Survey of Professional Forecasters.

- **Idiosyncratic volatility** (Ang et al., 2006b, "Idio. volatility") is the standard deviation of the residuals $\epsilon_{j,\tau}$ in the Fama & French (1993) 3-factor model $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^L_{j,t} L_\tau + \beta^M_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^S_{j,t} SMB_\tau + \beta^H_{j,t} HML_\tau + \epsilon_{j,\tau}$, using daily returns over the examination period. SMB_τ and HML_τ denote the returns on the Fama & French (1993) factors.

- **Idiosyncratic volatility-of-volatility** (Baltussen et al., 2015, "Idio. vol-of-vol") is the volatility of the at-the-money Black & Scholes (1973) option implied volatility (IV) over the examination period divided by the average IV over that period $VoV_{j,t} = \frac{\sigma(IV_{j,\tau})}{\mu(IV_{j,\tau})}$. We use the data cleaning procedure as described by Baltussen et al. (2015) and require at least one hundred non-missing IV observations in order to compute the quantity.

- **Kurtosis** ("Kurtosis") is the stock's scaled fourth moment, computed using daily returns over the examination period.

- **NYSE only** ("NYSE only") is a dummy variable that takes the value of one if the stock is traded at the NYSE at time t and zero otherwise.

- **Pastor–Stambough liquidity** (Pastor & Stambaugh, 2003, "PS liquidity") is the coefficient $\beta^L_{j,t}$ in the following regression $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^L_{j,t} L_\tau + \beta^M_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^S_{j,t} SMB_\tau + \beta^H_{j,t} HML_\tau + \epsilon_{j,\tau}$, where L_τ is the liquidity factor provided by Lubos Pastor and $r_{M,\tau} - r_{f,\tau} = MKT_\tau$, SMB_τ, and HML_τ are the Fama–French factors provided by Kenneth R. French. We run the regression using the monthly returns during the examination period.

- **Skewness** (Xu, 2007, "Skewness") is the stock's scaled third moment computed using daily returns over the examination period.

- **Stochastic volatility** (Cremers et al., 2015, "Straddle vol"), **market skewness and kurtosis** (Chang et al., 2013, "dSkew", "dKurt"), **aggregate jump risk** (Cremers et al., 2015, "Jump"), **market variance risk premium** (Han & Zhou, 2012, "dVRP"), and **policy uncertainty** (Brogaard & Detzel, 2015, "dPol") are the coefficients $\beta^F_{j,t}$ in the regression $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta^M_{j,t}(r_{M,\tau} - r_{f,\tau}) + \beta^F_{j,t} F_\tau + \epsilon_{j,\tau}$, using daily returns over the examination period (Ang et al., 2006b), where F is one of the following:
  - **Innovations in implied market skewness and kurtosis** (Chang et al., 2013), which are defined as the difference of daily implied skewness (kurtosis) computed from S&P 500 index options using the formulas of Bakshi et al. (2003) and its expectation, which is obtained fitting an ARMA(1,1) model on the complete time series of skewness (kurtosis) estimates.

The resulting measure of innovations in market skewness then is $dSkew_\tau = Skew_\tau - 0.9956Skew_{\tau-1} + 0.5707dSkew_{\tau-1}$, that of innovations in market kurtosis is $dKurt_\tau = Kurt_\tau - 0.9981Kurt_{\tau-1} + 0.6231dKurt_{\tau-1}$.

- **Innovations in policy uncertainty** (Brogaard & Detzel, 2015) are obtained by fitting an ARMA(1,1) model on the Baker, Bloom, & Davis (2013) policy uncertainty index using trading days only. The resulting measure of innovations in policy uncertainty is $dPol_\tau = Pol_\tau - 0.9962Pol_{\tau-1} + 0.8394dPol_{\tau-1}$. We obtain data on the policy uncertainty index from the authors' webpage.

- **Innovations in the market variance risk premium** (Han & Zhou, 2012), where the market variance risk premium is defined as the difference between the risk-neutral expected variance (VIX^2) and the physical expected variance of the S&P 500 index over a 30-day horizon using daily return data. First, we compute the expected variance ($EV_{\tilde{\tau}}$) under the physical measure by regressing the annualized realized variance ($RV_{\tilde{\tau}+30}$) on the lagged implied ($VIX^2_{\tilde{\tau}}$) and the lagged annualized historical ($RV_{\tilde{\tau}}$) realized variance, using an expanding window of daily data that is available at time τ, starting with data from January 01, 1996 ($\tilde{\tau}$ refers to those dates) $RV_{\tilde{\tau}+30} = \alpha_\tau + \beta_\tau VIX^2_{\tilde{\tau}} + \gamma_\tau RV_{\tilde{\tau}} + \epsilon_{\tilde{\tau}+30}$ in a first step and then computing $EV_\tau = \hat{\alpha_\tau} + \hat{\beta_\tau} VIX^2_{\tilde{\tau}} + \hat{\gamma_\tau} RV_\tau$. The market variance risk premium (VRP_τ) is obtained as $VRP_\tau = VIX^2_\tau - EV_\tau$. $dVRP$ is obtained as the first difference in VRP.

- **Market-neutral straddle returns** (Cremers et al., 2015), that are computed by first constructing ATM zero beta straddles.

Afterwards the $StraddleVol$ factor is the return of a gamma neutral and vega positive portfolio of the two straddles maturing in the next month and the month after next, while the $Jump$ factor is the return of a gamma positive and vega neutral portfolio using the same straddles. To construct the factors, Black & Scholes (1973) option sensitivities are used.

- **Turnover** (Datar, Y Naik, & Radcliffe, 1998, "Turnover") is the number of shares traded in one month divided by the total shares outstanding, averaged over all months in the examination period.
- **Volatility** (Zhang, 2006, "Volatility") is the stock's standard deviation computed using daily returns over the examination period.
- **Volume** (Gervais, Kaniel, & Mingelgrin, 2001, "Volume") is the stock's average daily dollar trading volume over the examination period. In regressions, we take the natural logarithm to remove the extreme skewness in this variable.

C.2 Equally Weighted Sorts

Single Portfolio Sorts and Characteristics

The results on equally weighted portfolios, presented in Table C.1, are qualitatively similar to those of the value-weighted portfolios, shown in Table 4.3. Returns and alphas of the 5 minus 1 hedge portfolio are at about -11 %, all negative and statistically significant at 1 % for each of the models we test.

Table C.1: Portfolios Sorted by Exposure to Aggregate Uncertainty – Equally Weighted

At the beginning of each month, we form equally weighted quintile portfolios based on the stock's sensitivities to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the following year. To obtain the sensitivities, we regress daily excess stock returns on dVVIX, controlling for MKT. Stocks with the lowest $\beta^{\mathrm{V}}_{j,t}$ are sorted into portfolio 1, those with the highest into portfolio 5. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. We reform the portfolios after one month. The row labeled Mean return is based on monthly simple returns. CAPM alpha, FF-3 alpha, and 4-factor alpha refer to the alphas of the CAPM, the Fama & French (1993) 3-factor, Carhart (1997) 4-factor, and the 5-factor (including liquidity) models, respectively. The segment NYSE only restricts the sample of stocks to those that are traded at the NYSE at the beginning of the estimation period. The segment Factor loadings denotes the average annual factor loadings, where β^{M}, β^{V}, and β^{dVIX} refer to the factor loadings on the market factor, dVVIX, and dVIX. The segment Stock characteristics presents average (equally weighted) portfolio characteristics with Mkt. share denoting the average market share of the portfolios. The remaining variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Table C.1: Portfolios Sorted by Exposure to Aggregate Uncertainty – Equally Weighted (continued)

Rank	1	2	3	4	5	5 minus 1
Mean return	0.1366**	0.1246***	0.1100**	0.0879*	0.0284	-0.1081***
	(0.016)	(0.009)	(0.020)	(0.082)	(0.623)	(0.000)
CAPM alpha	0.0230	0.0278***	0.0137	-0.0152	-0.0873***	-0.1103***
	(0.152)	(0.008)	(0.294)	(0.313)	(0.000)	(0.000)
FF-3 alpha	0.0026	0.0161**	0.0020	-0.0245***	-0.1023***	-0.1048***
	(0.821)	(0.018)	(0.600)	(0.000)	(0.000)	(0.000)
4-factor alpha	0.0039	0.0149**	0.0011	-0.0260***	-0.106***	-0.1099***
	(0.735)	(0.026)	(0.753)	(0.000)	(0.000)	(0.000)
5-factor alpha	0.0205***	0.0226***	0.0042	-0.0297***	-0.1163***	-0.1368***
	(0.009)	(0.000)	(0.105)	(0.000)	(0.000)	(0.000)
NYSE only						
4-factor alpha	0.0140	0.0203**	0.0053	-0.0077**	-0.0791***	-0.0932***
	(0.417)	(0.021)	(0.273)	(0.037)	(0.000)	(0.000)
Factor loadings						
β^{M}	1.0236***	1.0549***	1.1604***	1.3134***	1.6208***	0.5972***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
β^{V}	-0.0511***	-0.0071***	0.0160***	0.0420***	0.0987***	0.1498***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
β^{dVIX}	-0.0739***	0.0042	0.0553***	0.1180***	0.2449***	0.3187***
	(0.000)	(0.502)	(0.000)	(0.000)	(0.000)	(0.000)
Return characteristics						
Std. deviation	0.2705	0.2247	0.2245	0.2425	0.2800	0.0903
Skewness	-0.4030	-0.4694	-0.3409	-0.2567	-0.2104	0.3806
Kurtosis	2.8357	2.6719	2.6955	2.7900	2.5149	2.9773
Stock characteristics						
Mkt. share	0.2500	0.2990	0.2269	0.1457	0.0784	-0.1716
Size ($*10^{-6}$)	8.1557	9.5966	7.1994	4.6868	2.5425	-5.6132
Book-to-market	0.5827	0.5916	0.5949	0.6120	0.6288	0.0460
Bid-ask spread	0.0018	0.0013	0.0014	0.0015	0.0019	0.0002
Amihud illiquidity ($*10^{6}$)	83.487	21.095	27.479	36.389	51.683	-31.804
Age	21.367	24.765	22.933	20.422	17.109	-4.2581
Leverage	0.5133	0.5494	0.5241	0.5127	0.5206	0.0073
MAX	0.0909	0.0739	0.0760	0.0844	0.1105	0.0196
Volatility	0.0299	0.0267	0.0276	0.0295	0.0329	0.0028

Double Sorts

The results on equally weighted double sorts, controlling for the canonical characteristics, can be found in Table C.2. Controlling for Beta, dVIX, and Bid-ask spread, the alphas of the hedge portfolio turn out smaller compared to single sorts, while it is still at about -11 % for the remaining control variables. In case of controlling for dVIX, the 4-factor alpha is not significant with a p-value slightly above 10 %.

Imposing further control variables in Table C.3, the results are not affected. The uncertainty-return trade-off can clearly be detected in every case.

C.3 Value-Weighted Regression Tests

Fama–MacBeth Regressions – Value-Weighted

Table C.4 reports the results of value-weighted Fama & MacBeth (1973) regressions. We report the results of a regression of excess returns on $\beta^{\mathrm{V}}_{j,t}$, $\beta^{\mathrm{M}}_{j,t}$, and various other canonical characteristics. In the basic regression specification suggested by our theoretical model (ii), the yearly price of aggregate uncertainty risk (coefficient on $\beta^{\mathrm{V}}_{j,t}$) is -1.0449 with a p-value smaller than 0.002, which corresponds to a t-statistic of -3.28, also clearly clearing the hurdle defined by Harvey et al. (2015). Consequently, a two-standard deviation increase across stocks in their uncertainty-sensitivity is associated with a 17.22 % decrease in average annual returns.

Adding ln(Size), Book-to-market, Bid-ask spread, Momentum, and Short-term reversal in models (iii) to (iv) and (vi) to (viii) does not change much. The coefficient on uncertainty-sensitivity remains economically large and highly significant at 1 %. Adding dVIX in model (v) strongly reduces the significance in the risk premium on aggregate uncertainty, but the p-value

Table C.2: Double Sorts – Equally Weighted

This table reports Carhart (1997) 4-factor alphas for double-sorted portfolios. At the beginning of each month, we first sort stocks into quintiles based on the characteristics denoted in the first column. Then, within each quintile, we sort stocks based on their uncertainty-sensitivity ($\beta^{\mathrm{V}}_{j,t}$) into another five quintile portfolios. The five portfolios sorted on $\beta^{\mathrm{V}}_{j,t}$ are then obtained by averaging over the respective quintiles within each quintile of the other characteristic, thus we obtain $\beta^{\mathrm{V}}_{j,t}$ quintile portfolios controlling for another characteristic. We reform the portfolios after one month. This procedure is performed for each of the characteristics. We report the main control variables for equally weighted returns. The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{\mathrm{V}}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{\mathrm{V}}_{j,t}$. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Rank	1	2	3	4	5	5 minus 1
Beta	-0.0107	-0.0010	-0.0035	-0.0233***	-0.0733***	-0.0626***
	(0.371)	(0.844)	(0.309)	(0.000)	(0.000)	(0.001)
Size	0.0044	0.0114	0.0016	-0.0275***	-0.1020***	-0.1064***
	(0.723)	(0.139)	(0.635)	(0.000)	(0.000)	(0.000)
Book-to-market	0.0086	0.0133**	0.0014	-0.0234***	-0.0981***	-0.1067***
	(0.459)	(0.017)	(0.710)	(0.000)	(0.000)	(0.000)
dVIX	-0.0310***	0.0018	-0.0030	-0.0163***	-0.0636***	-0.0326
	(0.001)	(0.709)	(0.453)	(0.000)	(0.000)	(0.107)
Bid-ask spread	0.0021	-0.0007	-0.0092**	-0.0280***	-0.0760***	-0.0780***
	(0.870)	(0.918)	(0.029)	(0.000)	(0.000)	(0.008)
Momentum	0.0101	0.0164**	0.0023	-0.0255***	-0.1024***	-0.1125***
	(0.423)	(0.016)	(0.478)	(0.000)	(0.000)	(0.000)
Short-term reversal	0.0054	0.0136**	0.0031	-0.0298***	-0.1028***	-0.1082***
	(0.642)	(0.035)	(0.381)	(0.000)	(0.000)	(0.000)
Age	0.0089	0.0149**	0.0001	-0.0244***	-0.1010***	-0.1099***
	(0.454)	(0.036)	(0.971)	(0.000)	(0.000)	(0.000)
Leverage	0.0083	0.0129*	-0.0043	-0.0241***	-0.0982***	-0.1064***
	(0.427)	(0.055)	(0.248)	(0.000)	(0.000)	(0.000)

Table C.3: Double Sorts (Further Control Variables) – Equally Weighted

This table reports Carhart (1997) 4-factor alphas for double-sorted portfolios. At the beginning of each month, we first sort stocks into quintiles based on the characteristics denoted in the first column. Then, within each quintile, we sort stocks based on their uncertainty-sensitivity ($\beta^{V}_{j,t}$) into another five quintile portfolios. Portfolio returns are equally weighted. The five portfolios sorted on $\beta^{V}_{j,t}$ are then obtained by averaging over the respective quintiles within each quintile of the other characteristic, thus we obtain $\beta^{V}_{j,t}$ quintile portfolios controlling for another characteristic. We reform the portfolios after one month. This procedure is performed for each of the characteristics. We categorize control variables into groups of returns distributions characteristics (Panel A), liquidity-related characteristics (Panel B), and market factors (Panel C). The column labeled 5 minus 1 refers to the hedge portfolio buying the quintile of stocks with the highest $\beta^{V}_{j,t}$ and simultaneously selling the stocks in the quintile with the lowest $\beta^{V}_{j,t}$. Robust Newey & West (1987) p-values using 12 lags are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Returns Distributions Characteristics

Rank	1	2	3	4	5	5 minus 1
Idio. Volatility	0.0096	-0.0023	-0.0158***	-0.0387***	-0.065***	-0.0746***
	(0.407)	(0.750)	(0.001)	(0.000)	(0.000)	(0.001)
Co-Skewness	0.0085	0.0136*	0.0031	-0.0256***	-0.1039***	-0.1125***
	(0.467)	(0.057)	(0.380)	(0.000)	(0.000)	(0.000)
Co-Kurtosis	0.0095	0.0146**	0.0026	-0.0253***	-0.1057***	-0.1152***
	(0.431)	(0.013)	(0.484)	(0.000)	(0.000)	(0.000)
Downside Beta	0.0043	0.0102*	-0.0004	-0.0291***	-0.0968***	-0.1011***
	(0.712)	(0.061)	(0.928)	(0.000)	(0.000)	(0.000)
MAX	0.0030	0.0114*	-0.0047	-0.0334***	-0.0881***	-0.0911***
	(0.777)	(0.088)	(0.236)	(0.000)	(0.000)	(0.000)
Idio. Vol-of-Vol	0.0558***	0.0379***	0.0243***	0.0098	-0.0502*	-0.1060***
	(0.003)	(0.000)	(0.001)	(0.312)	(0.083)	(0.008)
Volatility	0.0221*	0.0108	0.0086**	-0.0266***	-0.0754***	-0.0975***
	(0.056)	(0.130)	(0.013)	(0.000)	(0.000)	(0.000)
Skewness	0.0248**	0.0189***	0.0049	-0.017***	-0.0921***	-0.1169***
	(0.032)	(0.001)	(0.109)	(0.000)	(0.000)	(0.000)
Kurtosis	0.0240**	0.0178***	0.0060**	-0.0184***	-0.0898***	-0.1138***
	(0.034)	(0.003)	(0.035)	(0.000)	(0.000)	(0.000)

Table C.3: Double Sorts (Further Control Variables) – Equally Weighted (continued)

Panel B. Liquidity-Related Characteristics

Rank	1	2	3	4	5	5 minus 1
PS liquidity	-0.0121	-0.0051	-0.0160	-0.0442***	-0.1181***	-0.1060***
	(0.619)	(0.793)	(0.319)	(0.000)	(0.000)	(0.000)
Amihud illiquidity	-0.0485***	-0.0312***	-0.0319***	-0.0608***	-0.1329***	-0.0844***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Volume	-0.0020	0.0054	-0.0022	-0.0219***	-0.0914***	-0.0894***
	(0.871)	(0.512)	(0.557)	(0.000)	(0.000)	(0.004)
Turnover	0.0036	0.0114*	0.0018	-0.0297***	-0.0989***	-0.1025***
	(0.772)	(0.075)	(0.592)	(0.000)	(0.000)	(0.000)

Panel C. Market Factors

Rank	1	2	3	4	5	5 minus 1
dSkew	-0.0015	0.0059	-0.0052	-0.0328***	-0.1051***	-0.1037***
	(0.919)	(0.592)	(0.504)	(0.000)	(0.000)	(0.000)
dKurt	-0.0007	0.0043	-0.0075	-0.0297***	-0.1051***	-0.1045***
	(0.964)	(0.690)	(0.336)	(0.000)	(0.000)	(0.000)
Straddle vol	-0.0117	0.0042	-0.0082	-0.0261***	-0.0969***	-0.0853***
	(0.388)	(0.685)	(0.285)	(0.000)	(0.000)	(0.000)
Jump	-0.0148	0.0051	-0.0058	-0.0302***	-0.0929***	-0.0781***
	(0.207)	(0.619)	(0.463)	(0.000)	(0.000)	(0.000)
dVRP	-0.0183*	0.0060	-0.0027	-0.0222***	-0.0748***	-0.0564**
	(0.087)	(0.340)	(0.426)	(0.000)	(0.000)	(0.014)
dPol	-0.0154	-0.0068	-0.0172	-0.0399***	-0.1161***	-0.1007***
	(0.514)	(0.732)	(0.281)	(0.002)	(0.000)	(0.000)
Forec. uncertainty	0.0023	0.0119*	-0.0010	-0.0280***	-0.0968***	-0.0991***
	(0.831)	(0.071)	(0.804)	(0.000)	(0.000)	(0.000)

is only slightly above 10 %. Again, particularly when dVIX is included in the regression, the coefficient denoting the price of aggregate uncertainty risk is substantially smaller amounting to about -0.5 compared to about 1 when aggregate volatility is not included as an explanatory variable. Adding several canonical characteristics jointly leaves the price of aggregate uncertainty risk negative with p-values close to 10 %. Models (xii) to (xiv) show that adding ln(Age) and Leverage does not have a big impact on the price of aggregate uncertainty risk.

Table C.4: Fama–MacBeth Regressions – Value-Weighted

This table presents average coefficient estimates from monthly value-weighted Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{V}_{j,t}$) over the same time, and a series of stock characteristics, all also measured over the following year. Detailed variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Factor	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)
Constant	0.0865	0.0893**	0.2567**	0.2567**	0.0808**	0.0976**	0.0943**	0.0889**	0.2851**	0.4673***	0.4512***	0.4514***	0.4388***	0.4287***
	(0.198)	(0.028)	(0.024)	(0.024)	(0.049)	(0.016)	(0.016)	(0.027)	(0.012)	(0.001)	(0.001)	(0.002)	(0.005)	(0.004)
dVVIX	-0.8848***	-1.0449***	-1.0672***	-1.0672***	-0.5034	-1.0299***	-0.9913***	-1.0150***	-1.0829***	-0.5683*	-0.5098	-0.5716*	-0.6455*	-0.5948
	(0.005)	(0.002)	(0.002)	(0.002)	(0.106)	(0.002)	(0.003)	(0.002)	(0.002)	(0.079)	(0.118)	(0.073)	(0.097)	(0.120)
Beta		0.0058	-0.0041	-0.0041	0.0157	0.0107	-0.0106	0.0008	0.0069	0.0187	0.0015	0.0161	0.0238	0.0047
		(0.941)	(0.956)	(0.956)	(0.836)	(0.890)	(0.887)	(0.992)	(0.925)	(0.792)	(0.982)	(0.819)	(0.733)	(0.942)
ln(Size)			-0.0096						-0.0108*	-0.0212***	-0.0201**	-0.0185**	-0.0188**	-0.0165**
			(0.139)						(0.094)	(0.008)	(0.012)	(0.016)	(0.026)	(0.036)
Book-to-market				-0.0096					-0.0375*	-0.0233	-0.0276*	-0.0225	-0.0302*	-0.033**
				(0.139)					(0.058)	(0.188)	(0.079)	(0.208)	(0.069)	(0.033)
dVIX					-0.2459***					-0.2358***	-0.2447***	-0.2308***	-0.2135***	-0.2166***
					(0.001)					(0.001)	(0.000)	(0.002)	(0.010)	(0.007)
Bid-ask spread						-21.423***				-53.722***	-51.349***	-53.289***	-53.656***	-53.108***
						(0.000)				(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Momentum							-0.0421				-0.0618			-0.0645
							(0.353)				(0.170)			(0.157)
Short-term reversal								0.0025			-0.0321			-0.0369
								(0.976)			(0.674)			(0.626)
ln(Age)												-0.0080		-0.0062
												(0.103)		(0.211)
Leverage													-0.0207	-0.0242
													(0.578)	(0.514)
adj. R^2	0.0421	0.1605	0.1710	0.1710	0.1801	0.1658	0.1789	0.1726	0.1860	0.2144	0.2370	0.2197	0.2261	0.2510

Table C.5: Fama–MacBeth Regressions (Further Control Variables) – Value-Weighted

This table presents average coefficient estimates from monthly value-weighted Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{V}_{j,t}$) over the same time, and a series of stock characteristics, all also measured over the following year. Detailed variable definitions are provided in the appendix. Panel A examines returns distribution characteristics and Panels B and C show liquidity-related characteristics and market factors, respectively. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

Panel A. Returns Distributions Characteristics

	(xv)	(xvi)	(xvii)	(xviii)	(xix)	(xx)	(xxi)	(xxii)	(xxiii)	(xxiv)
Constant	0.6332***	0.4565***	0.4452***	0.4764***	0.2926**	0.6303***	0.4689***	0.4480***	0.4631***	0.4640***
	(0.000)	(0.002)	(0.001)	(0.001)	(0.032)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
dVVIX	-0.6331**	-0.5608*	-0.6279*	-0.7308*	-0.3697	-0.5224	-0.5328	-0.4887	-0.4732	-0.5374
	(0.042)	(0.082)	(0.060)	(0.088)	(0.167)	(0.128)	(0.124)	(0.123)	(0.130)	(0.122)
Beta	0.0663	0.0180	0.0212	0.0771	-0.0407	0.0504	0.0158	0.0060	0.0085	0.0145
	(0.279)	(0.799)	(0.764)	(0.423)	(0.478)	(0.438)	(0.841)	(0.935)	(0.904)	(0.859)
ln(Size)	-0.0297***	-0.0206**	-0.0200***	-0.0218***	-0.0129*	-0.0280***	-0.0207***	-0.0195**	-0.0204***	-0.0203***
	(0.000)	(0.011)	(0.007)	(0.005)	(0.087)	(0.007)	(0.007)	(0.011)	(0.009)	(0.007)
Book-to-market	-0.0327*	-0.0228	-0.0213	-0.0251	-0.0128	-0.0184	-0.0283	-0.0251	-0.0262	-0.0272
	(0.053)	(0.191)	(0.196)	(0.161)	(0.493)	(0.333)	(0.150)	(0.163)	(0.119)	(0.119)
dVIX	-0.2368***	-0.2399***	-0.2244***	-0.2379***	-0.2418***	-0.2369***	-0.2323***	-0.2280***	-0.2307***	-0.2350***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.000)	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)
Bid-ask spread	-42.456***	-52.088***	-53.460***	-51.830***	-66.713***	-127.035***	-45.305***	-46.011***	-46.510***	-44.886***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Idio. Volatility	-5.0290***									
	(0.000)									
Co-Skewness		0.0005								
		(0.702)								
Co-Kurtosis			-0.0001							
			(0.339)							
Downside Beta				-0.0575						
				(0.283)						
MAX					2.0682***					
					(0.002)					
Idio. Vol-of-Vol						-0.1460				
						(0.349)				
Volatility							-0.3940			-0.4028
							(0.778)			(0.784)
Skewness								0.0025		0.0039
								(0.804)		(0.720)
Kurtosis									-0.0002	-0.0001
									(0.583)	(0.803)
adj. R^2	0.2337	0.2206	0.2216	0.2279	0.2406	0.2418	0.2407	0.2360	0.2321	0.2525

Table C.5: Fama–MacBeth Regressions (Further Control Variables) – Value-Weighted (continued)

Panel B. Liquidity-Related Characteristics

	(xxv)	(xxvi)	(xxvii)	(xxviii)
Constant	0.3509***	0.3094***	0.0703	0.3054***
	(0.000)	(0.006)	(0.483)	(0.006)
dVVIX	-0.5345*	-0.5540*	-0.5520*	-0.5571*
	(0.054)	(0.079)	(0.084)	(0.078)
Beta	-0.0002	0.0164	0.0255	0.0299
	(0.998)	(0.817)	(0.728)	(0.658)
ln(Size)	-0.0154***	-0.0128**		-0.0125**
	(0.006)	(0.044)		(0.042)
Book-to-market	-0.0302	-0.0331*	-0.0312*	-0.0368**
	(0.101)	(0.072)	(0.088)	(0.044)
dVIX	-0.2886***	-0.2451***	-0.2388***	-0.2425***
	(0.000)	(0.001)	(0.002)	(0.001)
PS liquidity	0.0098			
	(0.491)			
Amihud illiquidity		-1.1730***		
		(0.000)		
ln(Volume)			0.0009	
			(0.861)	
Turnover				-0.0065
				(0.209)
adj. R^2	0.2397	0.2056	0.2014	0.2141

Multivariate Estimation

Table C.6 reports the results of value-weighted Fama & MacBeth (1973) regressions when the sensitivities to the different factors are obtained in a joint multivariate sensitivity estimation regression. The results are similar to those of the usual regression tests without a weighting scheme. Incorporating the Fama & French (1993) factors (xxxvii) leaves the effect strongly significant at 1 %. Adding the other market factors like dVIX, dSkew, dKurt, Straddle vol, Jump, or dVRP (models (xl) to (xlvii)) does not change much. The price of risk on HML regularly is significant at 10 % while that on MKT and SMB never is. The coefficient on uncertainty-sensitivity is statistically significant at 1 % in any case. The coefficient on dVIX is substantially less significant when estimating the sensitivities jointly with dVVIX compared to the analysis in which both are estimated separately.

Table C.5: Fama–MacBeth Regressions (Further Control Variables) – Value-Weighted (continued 2)

Panel C. Market Factors

	(xxix)	(xxx)	(xxxi)	(xxxii)	(xxxiii)	(xxxiv)	(xxxv)	(xxxvi)	(xxxvii)	(xxviii)
Constant	0.4798***	0.4798***	0.4818***	0.4926***	0.4923***	0.4740***	0.5037***	0.4560***	0.5594***	0.4727***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.000)	(0.002)
dVVIX	-0.6836**	-0.6803*	-0.6837*	-1.1308***	-1.1250***	-1.1572***	-0.7351**	-0.5663*	-0.4797	-0.5907*
	(0.050)	(0.058)	(0.065)	(0.001)	(0.004)	(0.004)	(0.040)	(0.071)	(0.108)	(0.075)
Beta	0.0135	0.0159	0.0064	0.0186	0.0210	0.0226	0.0230	0.0207	-0.0048	0.0192
	(0.856)	(0.833)	(0.930)	(0.803)	(0.776)	(0.755)	(0.753)	(0.769)	(0.949)	(0.789)
ln(Size)	-0.0218***	-0.0219***	-0.0214***	-0.0227***	-0.0229***	-0.0219***	-0.0238***	-0.0207***	-0.0263***	-0.0215**
	(0.003)	(0.002)	(0.001)	(0.004)	(0.005)	(0.006)	(0.003)	(0.007)	(0.001)	(0.011)
Book-to-market	-0.0220	-0.0228	-0.0232	-0.0277	-0.0265	-0.0268	-0.0237	-0.0231	-0.0202	-0.0232
	(0.191)	(0.145)	(0.141)	(0.163)	(0.178)	(0.177)	(0.207)	(0.159)	(0.309)	(0.177)
dVIX	-0.2311***	-0.2260***	-0.2237***				-0.2725***	-0.1745	-0.2773***	-0.2206***
	(0.002)	(0.004)	(0.004)				(0.001)	(0.127)	(0.000)	(0.002)
Bid-ask spread	-53.020***	-53.294***	-51.543***	-55.651***	-55.573***	-54.651***	-54.120***	-52.400***	-55.2134***	-52.9124***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
dSkew	-1.6520		-2.0666							
	(0.155)		(0.484)							
dKurt		5.1874	-1.1943							
		(0.136)	(0.889)							
Straddle vol				0.0873		0.0656				
				(0.181)		(0.312)				
Jump					-0.2278	-0.0632	0.2603			
					(0.482)	(0.876)	(0.357)			
dVRP								-0.0582		
								(0.166)		
dPol									0.483*	
									(0.056)	
Forec. uncertainty										0.0000
										(0.676)
adj. R^2	0.2321	0.2310	0.2430	0.2119	0.2138	0.2198	0.2298	0.2253	0.2458	0.2272

The prices of risk on Jump and dVRP are significant, whereas the remaining factors are only partly significant.

Table C.6: Fama–MacBeth Regressions – Multivariate Estimation (Value-Weighted)

This table presents average coefficient estimates from monthly value-weighted Fama & MacBeth (1973) regressions. Each month, we regress excess stock returns during the following year on a constant, the sensitivity to innovations in aggregate uncertainty ($\beta^{\mathrm{V}}_{j,t}$) over the same time and/or a series of $\beta^{\zeta}_{j,t}$ estimates obtained simultaneously in a joint regression as in Equation (12) in the main paper, using daily data over the following year. Detailed variable definitions are provided in the appendix. Robust Newey & West (1987) p-values using 12 lags, that also incorporate the Shanken (1992) errors-in-variables correction, are reported in parentheses. The stars indicate significance with one star (*) denoting significance at 10 %, two (**) at 5 %, and three (***) stars at 1 %.

	(xxxix)	(xl)	(xli)	(xlii)	(xliii)	(xliv)	(xlv)	(xlvi)	(xlvii)
Constant	0.0550	0.0894***	0.0566	0.0959***	0.0681*	0.0849**	0.0504	0.0764**	0.0583
	(0.189)	(0.008)	(0.167)	(0.003)	(0.089)	(0.016)	(0.213)	(0.037)	(0.149)
dVVIX	-0.9871***	-0.9510***	-0.9196***	-0.8845***	-0.8912***	-0.9880***	-0.9822***	-0.9436***	-0.9303***
	(0.005)	(0.000)	(0.007)	(0.001)	(0.009)	(0.000)	(0.006)	(0.001)	(0.006)
MKT	0.0418	0.0182	0.0413	0.0094	0.0296	0.0176	0.0465	0.0244	0.0388
	(0.586)	(0.794)	(0.583)	(0.890)	(0.686)	(0.791)	(0.522)	(0.728)	(0.608)
SMB	0.0074		0.0097		0.0081		0.0106		0.0088
	(0.812)		(0.746)		(0.773)		(0.722)		(0.766)
HML	-0.0408*		-0.0400*		-0.0432**		-0.0397*		-0.0412*
	(0.060)		(0.064)		(0.034)		(0.063)		(0.059)
dVIX		-0.2644**	-0.3102**	-0.2404**	-0.2826**				
		(0.012)	(0.013)	(0.015)	(0.018)				
dSkew				-1.6169	-0.1274				
				(0.109)	(0.902)				
dKurt				6.1918*	2.1572				
				(0.055)	(0.511)				
Straddle vol						0.1318**	0.0697		
						(0.026)	(0.120)		
Jump						-0.7197***	-0.6956***		
						(0.010)	(0.005)		
dVRP								-0.0541**	-0.0890**
								(0.018)	(0.024)
adj. R^2	0.2173	0.1798	0.2299	0.2046	0.2427	0.1770	0.2272	0.1808	0.2299

Chapter 5

Conclusion and Further Research

5.1 Summary and Conclusion

This thesis investigates the properties of asset's market beta and the pricing of aggregate uncertainty in financial markets. Chapter 2 comprehensively studies the statistical properties of different methods to estimate an asset's market beta. We find that the hybrid methodology proposed by Buss & Vilkov (2012) performs best both in terms of informational efficiency and estimation accuracy. Furthermore, we find that the simple historical benchmark model as well as a Kalman filter based approach with a random walk parametrization perform well in terms of both evaluation criteria. On the other hand, fully option implied models or GARCH-based time-series approaches are shown to produce large pricing errors.

Chapter 3 studies the value of intra-day high-frequency data for beta estimation. We employ high-frequency return data both to obtain a presumably more precise statistical examination of ex ante estimates as well as for an additional historical estimator. Additionally, we present evidence on optimal combinations of estimators and impose an economical evaluation criterion in the analysis. We find that the results of Chapter 2 hold using high-frequency data. Furthermore, we find that the value of intra-day high-frequency data for beta estimation is limited. From the statistical evaluation viewpoint, especially over short time horizons, the historical high-frequency estimator is shown to yield precise estimates, whereas from an economic perspective the approach cannot uncover a positive risk-return trade-off. Regarding the economic evaluation, the hybrid approach proposed by Buss & Vilkov (2012) performs clearly best, detecting a positive risk-return trade-off, albeit not of the magnitude predicted by the CAPM. Using the statistical examination, the BV approach performs more or less equally well compared to the high-frequency estimator.

Chapter 4 uses a simple stylized theoretical model to introduce the possibility of the existence of an uncertainty-return trade-off in financial markets in addition to the well-established risk-return trade-off. For the empirical analysis we use uni- and bivariate portfolio sorts as well as cross-sectional Fama & MacBeth (1973) regressions and find aggregate uncertainty, measured by the VVIX, to be significantly priced with a negative sign. Specifically, we find that the quintile portfolio of stocks with the highest sensitivity toward innovations in aggregate uncertainty underperforms the quintile of stocks with the lowest exposure to aggregate uncertainty by about 14 % per annum in terms of 4-factor alphas. In cross-sectional regressions, a two-standard deviation increase in aggregate uncertainty factor loadings is associated with a significant decrease in average annual returns that ranges from 6.3 % to 18.7 %. These findings cannot be explained by known risk factors or a crisis effect.

The findings presented in this thesis have important implications for both academics and market participants in practice. First of all, beta is important for many applications in asset pricing, portfolio choice or risk management. Financial managers should estimate beta, if applicable, using the hybrid method of Buss & Vilkov (2012). This ensures estimates that perform well from a statistical viewpoint, i.e. are accurate and informationally efficient, and, more importantly, serve to detect a positive risk-return trade-off in the cross-section of stock returns. The BV approach, however, is not applicable for all stocks. It requires options data for all constituents of a market index and the index itself. This means that for many assets, the hybrid BV methodology cannot provide an estimate. In these cases, the asset manager should stick to a simple historical estimate instead of a GARCH or fully option implied alternative. If the asset manager has intra-day high-frequency data at hand, this is of limited value. Using high-frequency data, he obtains more precise estimates compared to the historical daily estimator, but if the BV approach is feasible he should stick to that since it is superior from an economic perspective. The findings presented here also have several implications for the academic literature. The conclusions drawn above hold in academic applications alike, meaning that one should stick to the BV approach while using high-frequency data for beta estimation has only little value.

Lastly, showing that aggregate uncertainty is priced in the stock market provides another important contribution that may help us understanding financial markets better. Market participants can decide whether they want to hedge against increases in aggregate uncertainty or expose their portfolios to that factor earning the substantial risk premium attached to it. Showing that once aggregate uncertainty is imposed many of the previously documented anomalies or risk factors are not priced, this thesis also contributes to the literature trying to separate "real" risk factors from those spuriously detected.

5.2 Suggestions for Further Research

Especially related to the measurement of beta using option-implied data as well as intra-day high-frequency return data, several potentially interesting topics for future research arise. Using such data, beta can be obtained on a day-by-day basis. Building on that there are several fields of potential future research.

First, given that using the hybrid BV methodology is available for various time horizons using options data with suitable maturity, we can study the term structure of beta. The main reason for term-structure effects in implied beta may be caused by the fact that certain economic shocks do not affect systematic risk at the short end, but potentially have large implications at the long end (or vice versa). We can study the implications of a positive or negative term structure for firms and their future returns.

Secondly, we can test the risk-return trade-off predicted by the CAPM at the very short end. The CAPM is a one-period model. The exact length of this period, however, remains unspecified. Consequently, we can test the model on a daily or weekly basis. Bali et al. (2015) show that dynamic conditional beta, based on a GARCH-specification, is doing well in a cross-sectional analysis. However, the results of their study might depend on the specific model chosen. Using realized or implied betas, we are able to obtain model-free or semi-parametric daily and weekly conditional estimates for beta. Employing these estimates of beta, we can test the conditional CAPM at the very short end following the approaches of Lewellen & Nagel (2006) and Ang & Kristensen (2012).

Furthermore, using day-by-day estimates, we can study time-variations and test for jumps in beta. We can study how heavily the systematic risk of individual assets is fluctuating. We further could examine what firm characteristics (e.g., size, book-to-market, industry, age) can explain time-variations in beta. Moreover, using the non-parametric test of Lee & Mykland (2008) or a related test and our daily beta estimates, we could identify jumps in betas of the stocks in the S&P 500. Once we have detected such movements, we can study their causes. To do this, we can relate them to scheduled and unscheduled news following the approach in Prokopczuk & Wese Simen (2014b).

Lastly, using the hybrid beta estimates for different time horizons, combined with a term structure of risk-free interest rates and an appropriate estimate for the term-structure of expected returns for the market portfolio we could examine the term-structure of asset's expected returns.

Finally, the evidence presented in Chapter 4 may help to develop a factor model related to fundamentals. The model by Fama & French (1993) is often criticized on the ground that the size and book-to-market lack a clear theoretical foundation. Recent developments (e.g., Hou et al., 2015; Fama & French, 2015) in factor models can also be criticized on that grounds. Consequently, we could study closer the relation of aggregate uncertainty and further macroeconomic fundamentals with the factors in existing models and, finally, maybe develop a new factor model which based on economic variables, incorporating aggregate uncertainty.

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