Essays on Risk Management of Financial Institutions:

Systematic Risk, Cross-sectional Pricing of Risk Factors, Parameter Errors affecting Risk Measures, and Credit Decisions under Parameter Uncertainty

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Referent: Professor Dr. Daniel Rösch
Korreferent: Professor Dr. Maik Dierkes
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Abstract

The events of the recent global financial crisis have highlighted several shortcomings in today's financial risk management and have motivated new research. This cumulative thesis presents four contributions in these current areas of research:

During the global financial crisis many financial institutions dealing with structured credit derivatives were exposed to severe unexpected losses. This indicates that systematic influences are decisively underestimated particularly with regard to structured products like securitized tranches of collateralized debt obligations (CDO). Therefore this cumulative thesis addresses these systematic effects in an analytical study. The provided simple model allows a closed-form comparison of both, bonds and tranches, with respect to their exposure to systematic risk. It is demonstrated that this exposure to systematic risk of tranches may be many times higher than the exposure of bonds, even if both products share the same through-the-cycle (TTC) rating grade, e.g., an 'Aaa' rating, measured by either default probability or expected loss. Particularly in economic downturns, default rates of tranches may be multiples of bonds. These results help to understand the observed high default rates of tranches during the financial crisis and show that classical TTC ratings are insufficient metrics for measuring risks of structured products.

During the financial turmoil the credit spreads for corporate bonds as wells as for credit derivatives such as Credit Default Swaps (CDS) or securitized tranches of CDO have largely increased. Thus, this cumulative thesis analyzes the pricing of systematic risk factors in credit default swap contracts in a two-stage empirical framework. In a first step contract-specific sensitivities (betas) to several systematic risk factors are estimated by time-series regressions using quoted CDS spreads of 339 U.S. entities from 2004 to 2011. In a second step it is shown that these contract-specific sensitivities are cross-sectionally priced in CDS spreads after controlling for individual risk factors. Particularly the credit market climate, the cross-market correlation, and the market volatility explain CDS spread changes and their corresponding sensitivities are priced in the cross-section. The basic risk factors explain about 83% (90%) of the CDS spreads prior to (during) the crisis.

In risk management, risk is often quantified by the application of risk measures, which are the outcome of a specific model and chosen model parameters. Since in practical application the 'true' model parameters are unknown, (credit) risk measures are prone to parameter errors. Therefore, this cumulative thesis investigates the popular risk measures Value-at-Risk (VaR) and conditional Value-at-Risk (cVaR) in a credit model with respect to errors in the model parameters. It is shown: 1) The cVaR can be more prone to estimation errors than the VaR. 2) The sensitivity to parameter errors is higher for lower probabilities of default, implying that highly rated risk buckets are more affected by parameters errors. 3) A higher confidence level, often considered to be safer, increases the impact of parameter errors. Again, this effect is more pronounced for highly rated risk buckets. These results have straightforward implications for practical applications: a preference for safety (i.e. preferring a lower probability of default and higher rating grades, a higher confidence level, and choosing cVaR over VaR due to possible tail risks) may increase vulnerability to parameter errors.

Additionally unknown 'true' model parameters lead not only to parameter errors, but also to parameter uncertainty in a Knightian sense, which can be noticeable particularly for credit risks and results in inadequate risk assessments and decisions. To date there is no clear-cut way of considering this kind of uncertainty in practical bank management. Therefore this cumulative thesis develops a simple method for assessing parameter uncertainty and accounting for it in two applications of credit risk models, namely (1) credit decision making as well as (2) computing uncertainty adjusted risk measures and capital buffers. An empirical application of this approach finds that (i) a credit decision can be substantially reversed (e.g. switched from a low-risk portfolio to a high-risk portfolio) once a decision maker's aversion against uncertainty is taken into account, and (ii) uncertainty adjusted capital buffers will yield in an add-on compared to the situation where only risk is considered.

The findings and proposals presented in this cumulative thesis are relevant to several interest groups, such as other researchers in the field of credit risk or derivatives, investors dealing with swap contracts, risk managers in financial institutions, and regulatory authorities or policy makers. Eventually, the presented results may help to take risk management to the next level in order to maintain financial stability and help to avoid mistakes that were made in the past.

Keywords: Credit Risk, Systematic Risk, Parameter Uncertainty

Zusammenfassung

Die jüngste globale Finanzkrise hat diverse Defizite im aktuellen Risikomanagement von Finanzinstitutionen aufgezeigt und hierdurch neue Forschungsfragen motiviert. In vier Beiträgen widmet sich diese kumulative Dissertation vier ausgewählten Teilfragen:

Während der globalen Finanzkrise haben Finanzinstitutionen, die mit strukturierten Derivaten handelten, hohe unerwartete Verluste erlitten. Diese Beobachtung ist ein Indiz dafür, dass systematische Risiken strukturierte Produkte wie Collateralized Debt Obligations (CDO) stärker beeinflussen als (unstrukturierte) Produkte wie Unternehmensanleihen. Daher wird in dieser kumulativen Dissertation ein analytisches Modell entwickelt, mit dem die Einflüsse von systematischen Risiken sowohl auf unstrukturierten Unternehmensanleihen sowie verbrieften Tranchen von CDOs untersucht werden können. Mit diesem Modell wird gezeigt, dass Tranchen aufgrund des 'Poolens' und 'Tranchierens' viel sensitiver auf makroökonomischen Veränderungen reagieren müssen, als dies Unternehmensanleihen mit identischem Ausgangsrating tun. Weiter wird gezeigt, dass gerade wirtschaftliche Abschwungsphasen die Ausfallraten von Tranchen im Vergleich zu gleich gerateten Unternehmensanleihen um ein Vielfaches erhöhen. Die Ergebnisse helfen die spezifischen Risikocharakteristiken von strukturierten Produkten besser zu verstehen und verdeutlichen, dass Ratingansätze, die einem 'Through-the-Cycle'-Ansatz folgen, diesen besonderen Eigenschaften von strukturierten Kreditderivaten nicht gerecht werden können.

Weiter sind während der jüngsten Finanzkrise die Prämienquotierungen (Spreads) von Unternehmensanleihen und Kreditderivaten wie Credit Default Swaps (CDS) oder strukturierten Produkten wie CDOs stark angestiegen. Daher untersucht diese kumulative Dissertation den Bepreisungseinfluss systematischer Risikofaktoren auf CDS Spreads der Jahre 2004 bis 2010 von 339 U.S. amerikanischen Unternehmen. In einem zweistufigen Regressionsverfahren werden zunächst die firmenspezifischen Sensitivitäten zu verschiedenen systematischen Risikofaktoren ermittelt. Anschließend wird gezeigt, dass diese firmenspezifischen Sensitivitäten im Querschnitt in den CDS Prämien eingepreist werden, auch wenn auf idiosynkratische Risikofaktoren wie Kreditratings, Liquidität und Verschuldungsgrad kontrolliert wird. Insbesondere das allgemeine Kreditmarktumfeld, die Kreuzmarktkorrelation sowie die Mark-

tvolatilität erklären die Änderungsraten von CDS Spreads und die zugehörigen firmenspezifischen Sensitivitäten beeinflussen die Risikoprämien der untersuchten Swap Kontrakte im Querschnitt. Im Zeitraum vor Beginn der Finanzkrise erklären die vorgeschlagenen Risikofaktoren bis zu 83% der CDS Spreads, während im Krisenzeitraum bis zu 90% der CDS Spreads durch die vorgeschlagenen Risikofaktoren erklärt werden können.

Im Risikomanagement werden Risiken häufig durch die Anwendung von Risikomaßen quantifiziert, wofür sowohl ein Modell als auch die zugehörigen Parameter spezifiziert werden müssen. In der realen Anwendung sind die 'wahren' Modelparameter jedoch nicht bekannt und müssen geschätzt werden. Dies kann zu Fehlspezifikationen führen, weshalb angewendete Risikomaße stets einem Parameterfehler ausgesetzt sind. Die Auswirkung dieser Fehlspezifikation auf die am häufigsten verwendeten Risikomaße im Kreditrisikomanagement, dem Value-at-Risk (VaR) sowie conditional Value-at-Risk (cVaR), werden in dieser kumulativen Dissertation untersucht. Es wird gezeigt: 1) Der cVaR ist anfälliger gegenüber Parameterfehler als der VaR. 2) Die Anfälligkeit beider Risikomaße gegenüber Fehlspezifikationen von Parametern steigt mit geringer Ausfallwahrscheinlichkeit der untersuchten kreditrisikobehafteten Portfolien. Dies bedeutet, dass gerade bonitätsstarke Portfolien einem erhöhten Parameterrisiko ausgesetzt sind. 3) Ein höheres angewendetes Konfidenzniveau, welches häufig mit einer höheren Sicherheit assoziiert wird, erhöht für beide untersuchte Risikomaße die Parametersensitivität und somit die Anfälligkeit gegenüber Parameterfehler. Diese Ergebnisse haben direkte Implikationen für die praktische Anwendung. Die oft angestrebte und auch von regulatorischer Aufsicht vorgegebene Präferenz zu einer erhöhten Sicherheit, ausgedrückt durch eine geringere Ausfallwahrscheinlichkeit des kreditrisikobehafteten Portfolios, ein höheres Konfidenzniveaus für die angewendeten Risikomaße, sowie die Wahl von Risikomaße, die extreme Verluste (tail risks) berücksichtigen, führen zu einer erhöhten Anfälligkeit gegenüber Parameterfehlern.

Weiter werden in den meisten Anwendungen von Kreditrisikomodellen die zugehörigen Modellparameter als bekannte Größen vorausgesetzt. In praktischen Anwendungen jedoch sind die wahren Parameter stets unbekannt und müssen durch Schätzungen ersetzt werden. Dies führt zu Parameterunsicherheit im Knightschen Sinne. Die Effekte hieraus können gerade im Kreditrisiko zu einer falschen Bewertung der tatsächlichen Risikohöhe und somit zu Fehlentscheidungen führen. Aktuell gibt es kein trennscharfes Verfahren

im bankspezifischen Risikomanagement, welches eine klare Abgrenzung von Risiko und Unsicherheit ermöglicht. Ein möglicher Ansatz zur Kreditentscheidung und Berechnung von Kreditrisikomaßen unter Unsicherheit wird in dieser kumulativen Dissertation vorgestellt. Eine empirische Anwendung dieses Verfahrens zeigt, dass Kreditentscheidungen substantiell verschieden sein können, wenn Parameterunsicherheit berücksichtigt wird. Beispielsweise können unter Unsicherheit Portfolien mit geringerer Bonität einem Portfolio mit höherer Bonität vorgezogen werden.

Die Arbeitsinhalte und -ergebnisse richten sich an Wissenschaftler im Kreditrisiko- oder Derivatebereich, an Investoren, die mit strukturierten Verbriefungen oder CDS-Kontrakten handeln, Risikomanager in Finanzinstitutionen, sowie regulatorische Aufsichtsinstanzen. Möglicherweise können mit den vorgestellten Verfahren Defizite im aktuellen Risikomanagement abgebaut werden und dazu beitragen die Finanzstabilität der Finanzmärkte sicherzustellen.

Schlagwörter: Kreditrisiko, Systematische Risiken, Parameterunsicherheit

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Chapter 1

Introduction

1.1 New Challenges for Financial Institutions' Risk Management

Financial institutions (FI) are companies (e.g., commercial or investment banks, insurances companies, mutual funds, investment companies) that focus on dealing with financial transactions, such as loans, deposits and investments. As financial intermediaries they perform the essential function of channeling funds from those with a surplus to those with a shortages of funds (Saunders and Cornett, 2008). Therefore, they have great importance for the overall economy and additionally they represent a large part of it. For example, at the end of 2013, the U.S. financial business held total assets of $82.89 trillion, in contrast to the $49.65 trillion of total assets held in the U.S. nonfinancial business.[1] The financial business differs from the nonfinancial business not only in its business model, but particularly in its investment and financing structure. For example, at the end of 2013 the U.S. nonfinancial business held approximately 40% financial assets (e.g., bonds, stocks, or loans) and 60% nonfinancial assets (e.g. real estate, equipment, machinery or production raw materials). In contrast, the U.S. financial business held 98% in financial assets and solely 2% in nonfinancial assets. Additionally the U.S. financial business was mainly financed by liabilities (92.7%), while the U.S. nonfinancial business was primarily financed by net worth (56.5%). From this particular investment and financing structure derive several *risks*, and therefore managing these *risks* appropriately became

[1] Reported numbers are publicly available in the Federal Reserve Statistical Release, Z.1. Financial Accounts of the Unites States, Third Quarter 2014.

one of the most important tasks of financial institutions' daily business.

Knight (1921) provides an early definition of the term 'risk' by a formal distinction from the term 'uncertainty'.

> The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome (...) is known (...), while in the case of uncertainty this is not true (Knight, 1921, p. 232).

In other words, Knight defined risk and uncertainty as random variation according to a known or unknown law, respectively. Therefore, these definitions focus on the degree of knowledge about the random variable, whereas the type of desired or undesired outcome is not specified. The differentiation helps to disclose the different natures of effects and allows to distinguish between them, and has led to a wide range of specific research, which has been thoroughly reviewed by Gilboa and Marinacci (2013). However, these definitions and the distinction between the terms risk and uncertainty have neither in the past nor today been part of everyday usage. To the contrary, these terms are either used synonymously in common language, or the term risk is used to refer to both uncertainty[2] as well as exposure to undesired consequences (Holton, 2004). Therefore, Knight's definitions are often criticized for not defining risk according to general custom and language. Thus, definitions of risk in the literature are more frequently related to uncertainty and exposure to undesired consequences. For example, "risk (...) involves both uncertainty and some kind of loss or damage that might be received" (Kaplan and Garrick, 1981, p. 12). Other authors define risk in terms of possible changes in values between two dates (Markowitz, 1952; Artzner et al., 1999). Holton (2004) states that it is impossible to define risk itself, but that we can operationally define some aspects of perceived risks. Therefore, he concludes that "it is meaningless to ask if a risk metric captures risk. Instead, ask if it is useful" (Holton, 2004, p. 204).

In the 1950s, decision-makers in FI and academic researchers in the field of finance started to analyze how risk can be practically managed (Crockford,

[2] Here the term uncertainty is used in common usage as a state of not knowing whether a proposition is true or false (see Holton, 2004).

1982). Initially, risk measures were first applied to quantify risk.[3] For example, Markowitz (1952) laid the foundation for modern portfolio theory and used variance[4] as a risk measure, recapturing the understanding of risk as changes in values. The development of financial products such as futures, options and swaps started in the 1970s, and with their introduction risk managers were provided with early tools to actively handle risk. Since then, risk management has become more and more important for finance-related practitioners and researchers.[5] In order to operationally define risks that FI confront, risk categories have been introduced. Commonly applied categories are: 'credit risk', 'market risk', 'liquidity risk', 'operational risk' and 'systemic risk' (Hull, 2009; Duffie and Singleton, 2012). Credit risk in particular has been, and remains, one of the most important risk categories. Large parts of banks' assets are prone to credit risk[6], and during the last decades the general amount of such credit-risky assets has tremendously increased in absolute as well as in relative terms (e.g. compared with the gross domestic product (GDP)). For example, "from 1978 to 2007, the amount of debt held by the [U.S.] financial sector soared from $3 trillion to $36 trillion, more than doubling as a share of [U.S.] gross domestic product" (Financial Crisis Inquiry Commission, 2011, p. xvii).

The increasing amount of credit-risky assets and the fact that traditionally, credit risk could only be managed during credit origination have led to the development of credit derivatives and more complex securitization structures such as collateralized debt obligations (CDO) (compare Schönbucher, 2003). A credit derivative allows to actively transfer credit risk, making credit risk manageable even after its origination. Credit derivatives increase the liquidity in the credit market and thus lower credit risk premiums (compare Duffie, 2008). Securitization structures can reduce borrowing costs, manage regulatory capital requirements, and provide new investment possibilities (Tavakoli, 2004). Therefore, the market of credit derivatives has experienced a tremen-

[3] Actually a risk measure can only be applied if a distinct random variable is assumed. Therefore, a risk measure can only be applied to risk according to the definition by Knight (1921).

[4] Variance is defined as the mean quadratic deviation of the random variable from its mean.

[5] For a historical review of risk management, see e.g. Crockford (1982), McNeil et al. (2010), or Dionne (2013).

[6] The ratio of loans and leases outstanding to total asset from U.S. commercial banks varies during the last 40 years between 53% and 63%. Data are publicly available from the Federal Reserve Bank of St. Louis (http://research.stlouisfed.org).

dous growth since their development.[7] Many practitioners and researchers judge the development of credit derivatives, particularly the development of CDOs, as one of the most important financial innovations in recent history (see Hull and White, 2008; Longstaff, 2010).

Since the late 1980s, risk management for FI is determined by regulatory requirements, which set minimum risk management standards. Although such requirements seek to enhance the financial stability, they can also provide incentives leading to contrary results.[8] Since 1998, all FIs with material international banking business of the ten most industrialized countries (G10) were affected by the first Basel Capital Accord (generally known as Basel I). This scheme formulates minimum standards for the regulation and supervision of banks, particularly with respect to credit risk. All affected banks were required to maintain a minimum capital ratio of capital to risk-weighted assets of 8% (see BCBS, 1988). This requirement was achieved by the G10 countries' banks in September 1993 (see, e.g., BCBS, 2001). However, under this regulatory scheme, banks had additional incentives to originate CDO securities, because this reduced the amount of regulatory capital they must hold. This procedure is called regulatory capital arbitrage and "exploits the large divergences that can arise between a portfolio's true economic risk and the [Basel I] Accord's measure of risk" (Jackson et al., 1999, p. 22). To address this shortcoming and other issues, such as the lack of consideration of market and operational risk, the Basel Committee 1999 issued a proposal for a new capital framework to replace Basel I (see BCBS, 1999). This led to the release of the 'Revised Capital Framework' in 2004, known as Basel II, which became effective January 2007 (see BCBS, 2004, 2014*a*). Even during its development, Basel II was already sharply criticized due to its heavy reliance on credit rating agencies (see Danielsson, 2002), using arguably inappropriate risk measures and still enabling capital arbitrage (see Danielsson et al., 2001). Unlike Basel I, Basel II never came entirely into effect, as the banks and authorities were distracted by the global financial crisis (GFC), confirming several postulated shortcomings of the regulatory framework. Therefore, during the financial turmoil, the Basel Committee already amended the regulation framework of Basel II in July 2009

[7] A comprehensive analysis of the developments in structured finance markets can be found in Löhr (2013*a*).

[8] "It is possible that at times the cumulative imbalance between products and infrastructure development [e.g. regulatory practices] could become large enough to jeopardize the very functioning of the financial system" (Merton, 1995, p. 471).

to enhance measurements of risks related to securitizations and trading book exposures (market risk); this amended framework is today known as Basel 2.5 (BCBS, 2009). In December, 2010 Basel III was released, which focuses on higher levels of capital requirements, and for the first time, incorporates 'liquidity risk' in the Basel Accord (BCBS, 2010, 2013). The requirements are constantly developed further (see BCBS, 2014*b*) and the Basel Committee is still working on the incorporation of new findings and lessons learned from the GFC. "Basel 4 is already on the regulatory horizon, even if the implementation of Basel 3 is only planned for 2019" (Embrechts et al., 2013, p. 2).

The GFC began in 2007 and, according to Melvin and Taylor (2009), involved the greatest financial dislocations since the Great Depression. The world trade flows declined by about 12% in 2009, which is in excess of the estimated loss of 5.4% in world GDP during the same period (see Chor and Manova, 2012). In retrospective, the reasons and development of the GFC have been analyzed, and several shortcomings have been identified.[9] For example Coffee (2009) and Partnoy (2010) argue that reliance on credit ratings[10] and trust in credit rating agencies (CRA) were among the main underlying causes of the crisis. Other authors identified the exaggerated use of structured financial instruments, such as CDOs, as import drivers of the crisis (see, e.g., Longstaff, 2010; Griffin and Tang, 2012). Some findings - such as the importance of liquidity risk, a risk category that had not been considered explicitly before - have already led to adaptations in risk management and regulatory frameworks (see Basel III).

Until now, however, research into risk has collectively not led to an exhaustive covering of the issue. To the contrary, the GFC motivated new research in risk management, and highlighted previously identified short-comings in today's risk management. This cumulative thesis contributes to the following four areas of research.

Firstly, the financial turmoil reveals large differences between risk characteristics of bonds and structured products. Even if both products are equally rated, which should be equivalent to holding the same creditworthiness, securitized tranches had higher impairment rates during the GFC. Particularly, high-

[9] Financial Crisis Inquiry Commission (2011) and Baily and Taylor (2014) provide a good overview of recent awareness of reasons, which has led to the GFC.

[10] A rating expresses an opinion about the creditworthiness of an obligor. Such ratings are determined in-house of FI or are bought by an obligor from credit-rating agencies (CRA) such as Standard & Poor's (S&P), Moody's and Fitch.

rated securitized tranches of CDO experienced far higher impairment rates in comparison with equally rated bonds. For example, the 5-year cumulative impairment rates for 'A'-rated CDO tranches increased from 5% in 2005 to 57% in 2009, while the impairment rates of equally rated bonds only changed from 0.56% to 0.81% in the same time period (compare Moody's, 2006*a*,*b*, 2010*a*,*b*). Several explications have been put forward to explain this mismatch between rating grades and real physical defaults. For example, rating agencies have been accused of issuing inflated high-rating grades. The rating agencies admit that parameter assumptions were not appropriate, but maintain the statement that these parameters were extrapolated from historical data (Griffin and Tang, 2011). Others argue that rating grades were inflated by ratings shopping - i.e. issuers acquiring ratings from several CRA and then choosing the most favorable (see, e.g., Skreta and Veldkamp, 2009). In addition to work focusing on the analysis on CRA, other research has shown that CDO structures were subject to higher systematic risk exposures due to their securitization (see, e.g., Krahnen and Wilde, 2008; Coval et al., 2009*a*; Eckner, 2009; Hamerle and Plank, 2009), and that ratings do not appropriately reflect this kind of risk for securitizations (see, e.g., Rösch and Scheule, 2009, 2010).

Secondly, credit spreads largely increased for corporate bonds as well as for credit derivatives such as CDS or securitized tranches during the financial turmoil. Again, particularly credit spreads on highly rated debt claims increased much more in relative terms than those of lower rated credit assets. This mismatch between credit ratings and market-priced default risk is referred to as a 'credit spread puzzle' in the corporate debt markets (see, e.g., Amato and Remolona, 2003; Chen, 2010). This phenomenon reveals that market participants claim premiums compensating risks that are not included in the product's physical default risk indicated by its rating grade. Several empirical studies analyze common risk factors as pricing drivers of credit spreads with respect to corporate bond markets (see, e.g., Collin-Dufresne et al., 2001; Chen, 2010; Iannotta and Pennacchi, 2011; Giesecke et al., 2011; Friewald et al., 2012; De Jong and Driessen, 2012). As suggested by Collin-Dufresne et al. (2001), Chen (2010) and Iannotta and Pennacchi (2011) for corporate debt, other authors also identified systematic risk factors that drive CDS spreads (see, e.g., Amato, 2005; Blanco et al., 2005; Ericsson et al., 2009; Gala et al., 2010; Berndt and Obreja, 2010; Arora et al., 2012; Wang et al., 2013). Most of these studies analyze dependencies of credit spreads or credit spread changes by focusing on time-series regressions. An exception is the analysis by Friewald et al. (2012), who use Fama-Macbeth cross-sectional regressions to show that liquidity is explicitly priced in bond markets.

These two research areas and the related financial literature highlight the importance of systematic risk with respect to credit derivatives in general, and for securitizations in particular. However, the recent literature does not provide a comprehensive analysis of sensitivity to systematic risk with respect to securitization. Additionally, the current literature has primarily shown the interference between systematic risk and credit spreads, but has not addressed how these sensitivities are cross-sectionally priced in the swap markets.

Thirdly the GFC highlighted the importance of choosing appropriate risk measures. Before and during the GFC the Value-at-risk (VaR) was the most popular risk measure in practical applications (see, e.g., Duffie and Pan, 1997) and Basel I as well as Basel II rely exclusively on this risk measure. However, the concept of VaR has then already been widely criticized. The VaR is neither convex, not sub-additive, and thus not a coherent risk measure in the sense of Artzner et al. (1999). Additionally, the VaR is merely a percentile of a probability distribution function, and therefore does not take into account any tail information beyond the VaR. Therefore, in the late 1990s, the conditional Value-at-Risk (cVaR), also referred to as Expected Shortfall (ES), has become the favored risk measure in academic resarch. Because it is coherent (see, e.g., Acerbi and Tasche, 2002; Frey and McNeil, 2002; Tasche, 2002) and holds other theoretical advantages such as convexity, it is easily optimized (see, e.g., Rockafellar and Uryasev, 2000). Therefore, Basel II was initially sharply criticized for the choice of the VaR as the risk measure.

> VaR is a misleading risk measure when the returns are not normally distributed, as is the case with credit, market and (...) operational risk. Moreover, it does not measure the distribution or extent of risk in the tail. (Danielsson et al., 2001, p. 4)

In 2012, the Basel Committee acknowledged this criticism (BCBS, 2012) when it recommended replacing the 99% VaR with the 97.5% cVaR in internal market risk models; the committee has also used the 97.5% cVaR to calibrate capital requirements under the revised market risk standardized approach. However, the committee still proposes a 99.9% VaR for the incremental capital charge for default risk in order to maintain consistency with the banking book treatment (see BCBS, 2012, 2013). There is ongoing academic debate about which risk measures are appropriate, and whether the cVaR is superior to the VaR. Current debates focus on diversification, aggregation, economic interpretation, extreme behavior, robustness and backtesting of VaR and cVaR.[11] However, the current literature lacks an analysis of how parameter errors, or the sensitivity to parameter changes, can affect these two risk measures particularly with respect to credit risk. Given that common risk models are highly sensitive to changes in parameters, this analysis became necessary.

Risk management always relies to some extent on the assessment of historical data (compare, e.g., Lo, 2001), an approach that is even advised by regulatory authorities (see, e.g., BCBS, 2013). However, the quality of historical data varies in many aspects, such as extent and consistency, and is particularly prone to extreme outliers that are not often observed, but still possible. Massive losses occurring after stable years or massive variation not observed in historical data are examples of this. Key words used here are 'fat tails' or 'black swan' (see, e.g., Taleb, 2009) and the recent GFC was possibly such an extreme outlier. Therefore, even if it was possible to determine true specific risk models - which is obviously not possible, inducing model uncertainty - all risk model parameters have to be estimated based on observable data using statistical and econometric techniques. These approaches are always prone to estimation errors; as a result, the values inserted for the parameters may not match the true underlying and unknown parameters. Particularly in cases of 'fat tails', these estimation errors cannot be described without further assumptions about known probability laws. Therefore, estimation errors lead to parameter uncertainty in the Knightian sense and, consequently, any risk management is prone to Knigthian uncertainty. This topic constitutes the fourth research area of this cumulative thesis.

One well-established approach in the literature to deal with such estimation errors or parameter uncertainty is called robust optimization. In these approaches, it is assumed that some or all model parameters are not given with certainty, but lie in a given 'uncertainty set'. Given these uncertainty sets, which are formulated as an additional optimization restriction, the worst possible parameter constellation is determined. Then, the solution of the robust optimization problems seeks to provide the 'best' possible decision given the worst-case parameter scenario.

[11] For an overview of recent literature see Embrechts et al. (2013) and Emmer et al. (2013).

So far, a consideration of parameter uncertainty or parameter errors is not explicitly given in regulatory requirements. The current requirements demand only that banks be conservative in their estimates (see, e.g., BCBS, 2013), without closer description of the general form or implementation of conservative estimation approaches.

In the 21st century, a plethora of contributions with respect to portfolio selection has been created.[12] However, current literature provides no robust optimization-based framework to deal with parameter uncertainty with respect to credit risk. Other frameworks that deal with parameter uncertainty with respect to credit risk often follow a Bayesian approach (compare Gössl, 2005; Dwyer, 2006; McNeil and Wendin, 2007; Kiefer, 2009; Tarashev, 2010; Chang et al., 2011). These approaches treat the unknown parameters as random variables and may model parameter-uncertainty aversion. But all approaches have in common that they cannot separately quantify or distinguish the effect of parameter uncertainty. Therefore, the current literature provides credit risk managers neither with a method to separately consider credit risk from parameter uncertainty, nor with an aid to quantify the effects of parameter uncertainty.

To conclude, this cumulative thesis contributes to each of these four research areas and analyzes

- sensitivities to systematic risk of structured products in comparison with bonds,
- the pricing of systematic risk factors in credit default swap contracts in a two-stage empirical framework,
- the popular risk measures VaR and cVaR in credit models with respect to errors in the model parameters,
- an economically based robust optimization framework for credit decisions under parameter uncertainty in the sense of Knight (1921).

Thus, the findings and proposals presented in this thesis are relevant to several interest groups, such as other researchers in the field of credit risk or derivatives, investors dealing with swap contracts, risk managers in financial

[12] For an overview see Fabozzi et al. (2010).

institutions and regulatory authorities or policy makers. Eventually, the presented results may help to take risk management to the next level in order to maintain financial stability and help to avoid mistakes that were made in the past.

1.2 Outline and Contributions

This cumulative thesis consists of one chapter for each of the four research areas previously mentioned. Each chapter provides a specific introduction, main contribution and conclusion.

Chapter 2 provides an analytical framework that allows a detailed comparison of risk characteristics of bonds and structured products, particularly CDO tranches, with respect to systematic risk. For this, we decompose the systematic risk factor within the commonly known and often applied Asymptotic Single Risk Factor (ASRF) model into both a super-systematic factor and sectoral risk components (compare Pykthin and Dev, 2002; Gordy and Howells, 2006). With this adjustment, we can define the probability of default (PD) as well as the expected loss (EL) for both products unconditional to systematic risk and conditional to a single realization of the super-systematic factor. By the unconditional PD and EL, both product classes can be equalized with respect to their creditworthiness, and as a result both products hold the same rating grade. Then, these equally rated products are analyzed with respect to their exposure - represented by the conditional PDs (CPD) and conditional EL (CEL) - to single realizations of the systematic risk factor. Additionally, this model allows an analysis of the effects of risk diversification and concentration in securitization.

We find that the CPD and CEL of tranches are much more sensitive to realizations of the systematic risk factor than those of corporate bonds. Particularly for systematic risk factor realizations representing an economic downturn, the CPD and CEL of tranches are many times higher than the CPD and CEL of an equally rated bond. We show that the risk characteristics CPD and CEL of securitized tranches depend on their level of subordination. Particularly for tranches of higher creditworthiness, the sensitivity to single realizations of the systematic risk factors increases disproportionately in comparison with same-rated bonds. With a MC approach, we demonstrate the effects of pooling and tranching with respect to risk diversification and concentration. We argue that the diversification of idiosyncratic and partial sectorial risk may lead to the concentration of systematic risk exposures. This may explain the unexpectedly high impairment rates during the GFC. Finally, we propose that classic ratings are insufficient metrics for measuring risk of structured securities, as they are especially prone to systematic risk compared with unstructured bonds.

Chapter 3 analyzes the pricing of systematic risk factors in credit default swap contracts in a two-stage empirical framework in line with Fama and MacBeth (1973). In the first pass, contract-specific sensitivities to several systematic risk factors are estimated by time-series regressions. Similarly to other authors (see, e.g., Amato, 2005; Blanco et al., 2005; Ericsson et al., 2009; Gala et al., 2010; Berndt and Obreja, 2010; Arora et al., 2012; Wang et al., 2013), we apply common state variables, such as 'changes in the spot rate', 'changes in the slope of the yield curve', 'changes in the market volatility', 'changes in the credit market climate', and 'changes in the cross-market correlation'[13] to reflect systematic risk. These common state variables cannot be observed directly, therefore we approximate them by proxies. The resulting contract-specific sensitivities are assigned to the second-pass cross-section regressions to analyze how and in what magnitude these sensitivities are cross-sectionally priced. This second-pass regression controls for individual risk factors, such as contract-specific PD (represented by an average rating provided by Markit), swap liquidity (represented by the contract's trading depth) and other individual risk factors such as firm leverage, market capitalization as well as assigned sectors. The empirical study refers to a comprehensive dataset of single-name CDS spreads provided by Markit. It comprises 339 U.S. entities from January 6th, 2004 to December 27th, 2010 and contains 124,413 weekly spreads. The data sample is divided into two time intervals. The first interval represents a moderate economic condition prior to the GFC, while the second covers the period of financial distress during the GFC.

The analysis shows that the contract-specific PD, represented by the credit rating, determines the general CDS spread level, but also illustrates that these spreads depend on the macroeconomic climate. Therefore, the spread levels vary over time and the premium based on the contract's rating does not sufficiently compensate for systematic risk. The first-pass regression shows that the 'credit market climate', the 'cross-market correlation' and the 'market volatil-

[13] We consider the average of quarterly cross-correlations referring to returns on numerous exchange, equity and credit markets.

ity' explain significant CDS spread changes. The second-pass regressions shows that in addition to a firm's physical default risk and other idiosyncratic risk premiums, particularly the sensitivities to the 'credit market climate', the 'cross-market correlation' and the 'market volatility' are cross-sectionally priced in the CDS spreads. We find that our basic risk factors explain about 83% of the CDS spreads prior to the crisis and about 90% during the crisis. This study presents a framework to identify contract-specific sensitivities to systematic risk and to quantify the factor-specific risk premium in the cross-section of CDS spreads.

Chapter 4 analyzes the sensitivity of the risk measures VaR and cVaR within the ASRF credit model. Here, 'sensitivity' is the effect of changes on the risk measures that occur when the model parameters are affected by parameter errors (e.g. estimation errors). For each risk measure, we derive the partial derivatives with respect to all model parameters and introduce specific key numbers. These key numbers are analyzed for several parameter settings that can be found for real credit portfolios from speculative to investment grade. Particularly, we analyze the risk measures' sensitivity for various confidence levels α and portfolio PD. The theoretical results are confirmed and illustrated within an empirical case study using publicly available default data from Moody's (2013) annual default reports.

We find that, aside from its theoretical advantages, the cVAR can be more prone to estimation errors than the VaR. This sensitivity is especially higher for lower PD, implying that highly-rated risk buckets are more affected by parameter errors. It is shown that a higher confidence level, often considered safer, increases the impact of parameter errors for both risk measures and that particularly the cVaR becomes even more sensitive. Again, both effects are more pronounced for highly rated risk buckets. It is demonstrated that this effect can be considerably reduced if a $\alpha = 99.5\%$ (in line with Solvency II) is applied, instead of $\alpha = 99.9\%$ (in line with the Basel Accord). However, it is shown that we may then be even more in favor of using the VaR rather than the cVaR as a risk measure. Therefore, especially for high-rated risk buckets and a high confidence level α, the VaR appears to be superior to the cVaR in credit risk.

Chapter 5 follows the definition of risk and uncertainty according to Knight (1921) and presents an economic framework for the quantification of parameter uncertainty in credit risk models. In the robust optimization-based framework, the degree of the decision-maker's uncertainty aversion or affinity can be quantified and decoupled from risk aversion. Thus, the framework provides a clear-cut distinction to risk. This framework is applied within a comprehensive Monte-Carlo simulation as well as on a publicly available default dataset from Moody's (2013) annual default reports.

With this framework, we firstly transfer the economic rationale of max-min optimization due to Gilboa and Schmeidler (1989) into the credit area. Secondly, as an extension to robust optimization, we introduce a new uncertainty set that covers the possibility of parameter uncertainty based on available data more accurately in comparison with other existing uncertainty sets. Thirdly, the economic rationale of an ambiguity-adverse decision-maker leads to a definition of risk measures under uncertainty. Based on the Monte-Carlo simulation and real default data, we demonstrate that uncertainty aversion requires premiums on risk measures and that portfolios with lower average PD and, thus, lower risk, can be affected by parameter uncertainty more strongly than portfolios with higher average PD (and thus higher risk). The implication is that even under a moderate degree of uncertainty aversion, a financial decision-maker might prefer a low-rated, high-risk portfolio over a high-rated low-risk portfolio if uncertainty aversion is taken into account.

Chapter 6 provides conclusions, and highlights practical implications and potential further research topics.

Chapter 2

An Analytical Approach for Systematic Risk Sensitivity of Structured Finance Products

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Chapter 3

Valuation of Systematic Risk in the Cross-Section of Credit Default Swap Spreads

The content of this chapter refers to the working paper 'Valuation of Systematic Risk in the Cross-Section of Credit Default Swap Spreads' by Claußen, A., Löhr, S., Rösch, D., and Scheule, H., 2014.

3.1 Introduction

During the Global Financial Crisis (GFC) the spreads of Credit Default Swaps (CDS) heavily increased across most CDS dealings on corporate debt claims, which was triggered by the high numbers of corporate defaults on bonds and loans.[14] While 31 Moody's-rated corporate issuers defaulted in 2006 on a total of 10.4 USD billion of loans and bonds, the number of defaulted issuers increased to 261 in 2009 on a total of 328.9 USD billion (Moody's, 2010*a*). In fact, the CDS spreads on high-rated debt claims, e.g., 'AAA'-rated bonds, increased much more rapidly than those on lower-rated credit assets, which

[14] Similar to insurance contracts, CDS – as credit derivatives – are linked to credit-risky assets such as corporate bonds, loans etc. In their role as protection seller, CDS investors periodically receive premium payments for covering losses in the underlying credit assets. These losses may be due to default events such as interest shortfalls or principal impairments, see Arora et al. (2012). Thus, in the absence of arbitrage, the fair CDS spread (risk premium) theoretically compensates for the default risk of the underlying credit asset.

may indicate a mismatch between credit ratings and the related default risk.[15]

On the corporate debt market this phenomenon takes part in the so-called credit spread puzzle (compare Amato and Remolona, 2003; Chen, 2010). Apart from addressing corporate default risk (Giesecke et al., 2011), several empirical studies recently looked beyond theoretical contingent claims and accounted for other pricing factors such as liquidity (Tang and Yan, 2010; Bongaerts et al., 2011; De Jong and Driessen, 2012; Friewald et al., 2012; Dick-Nielsen et al., 2012). As suggested by Collin-Dufresne et al. (2001); Chen (2010) and Iannotta and Pennacchi (2011) for corporate debt, other authors also identified systematic risk factors driving CDS spreads (e.g., Amato, 2005; Blanco et al., 2005; Gala et al., 2010; Arora et al., 2012; Wang et al., 2013; Berndt and Obreja, 2010).

Most of the recent studies analyze time-series properties of credit spreads or credit spread changes by focusing on time-series regressions. An exception are Friewald et al. (2012) who use Fama-Macbeth cross-sectional regressions to show that liquidity is priced in bond markets after controlling for other factors such as credit ratings. In summary, the current literature on both bond and CDS markets focuses on the identification of credit spread drivers and aims to answer the question of how these determinants are priced.

Our paper contributes to credit spread determinants in several ways. Firstly, we explicitly address systematic risk exposures of CDS contracts and identify at least three systematic risk factors beyond Merton's (1974) structural theory as important drivers for CDS spread changes. Thus, we suggest the *Credit Market Climate*, the *Market Volatility* and the *Cross-market Correlation* as common determinants of CDS spread changes.

Secondly, based on our CDS database from 2004 to 2010 containing weekly spread data of 339 U.S. firms we show that credit ratings do not sufficiently cover the overall credit risk priced in CDS spreads. We find that systematic risk is generally priced beyond the ratings of U.S. firms located in numerous economic sectors, e.g., financial, industrial and consumer goods.

Thirdly, we extend the current literature by applying a two-pass regression approach to CDS markets (similar to Fama and MacBeth, 1973) and thus we show that systematic risk exposures are cross-sectionally priced in swap mar-

[15] Subrahmanyam et al. (2014) find empirical evidence that the bankruptcy risk of underlying reference firms increases after the inception of CDS trading.

kets.[16] In the first pass, we identify common determinants of credit spread changes and provide contract-specific sensitivities (betas) to common risk factors by time-series regressions. In the second pass, we examine by cross-section regressions how these betas are cross-sectionally priced in CDS spreads after controlling for i) several individual factors such as credit ratings, contract liquidity and firm leverage and ii) sectoral influences. Thus, we calculate premiums for these systematic risk betas, similar to the CAPM's beta premium.[17] We find that these determinants of CDS spread changes are priced across several economic sectors, particularly in times of financial distress. Specifically, common risks related to the *Credit Market Climate*, the *Market Volatility* and the *Cross-market Correlation* are rewarded in the cross-section of CDS spread after controlling for other important pricing elements such as credit ratings and liquidity. The results of the cross-section regressions show that our set of variables – composed of systematic and non-systematic risk measures – allows us to explain about 80% of the observed CDS spreads in normal market environments and 90% during economic downturns. Furthermore, the OLS regression results are robust with respect to the inclusion of the Fama-French factors and other firm-specific factors such as the firm's leverage ratio and market capitalization. Our findings suggest that systematic risk is a decisive pricing factor, after controling for individual risk factors and sectoral influences.

Our empirical findings are important for at least three fields. Firstly, the contributions are relevant for asset pricing as they identify variables which determine spreads of swap contracts referring to credit risky assets. While previous literature analyzes the price impact of credit ratings (e.g., Ederington and Goh, 1993, 1998), we explicitly address the price impact of systematic risk in CDS spreads beyond ratings. Extending the current literature related to CDS and corporate debt, our findings are relevant for the valuation of CDS, and may provide further insight into the pricing of corporate bonds.

Secondly, the results are important for the regulation of financial markets. As discussed by Iannotta and Pennacchi (2011), there is a mismatch between regulatory capital for banks derived from credit ratings and credit spreads, as the latter might account for systematic risk, while credit ratings do not

[16] The initial two-pass regression approach was proposed by Fama and MacBeth (1973) to evaluate the cross-section of stock returns.

[17] According to the Capital Asset Pricing Model (CAPM), market participants can fully diversify idiosyncratic risks, but not market (systematic) risk which is therefore compensated by a risk (beta) premium (compare Sharpe, 1964).

appropriately reflect systematic risk. Current regulatory capital requirements for banks primarily focus on credit ratings, and therefore banks – or financial investors in general – are subject to misaligned incentives if systematic risk is priced: within a specific rating grade, banks may choose those investments with the highest systematic risk exposures due to the higher risk premiums linked to these products. This might be a threat to financial institutions and the whole financial system. By providing empirical evidence for the pricing of systematic risk on CDS markets beyond ratings, our paper also contributes to this discussion.

Thirdly, our findings might be important for pricing structured finance securities such as Collateralized Debt Obligations (CDOs). Since, for example, synthetic CDOs such as single-tranche CDOs (STCDOs) take on credit exposures through including CDS contracts, this work may also provide first insights into the valuation of such structured products, which are particularly exposed to systematic risk (see Coval et al., 2009*a*).[18]

The remainder of the paper is organized as follows. In Section 3.2, we provide the theoretical framework for our empirical analysis by introducing systematic and rather firm-specific spread determinants. Further, we describe the database and briefly discuss the proxies used. In Section 3.3, we firstly introduce the regression models within the two-pass approach and secondly provide the methodology to test whether corporate ratings appropriately reflect systematic risk. Thirdly, we provide our results and check the robustness of our findings by expanding our model framework to i) the Fama-French factors, ii) further firm-specific factors and iii) a principal component analysis. Section 3.4 concludes.

3.2 Determinants of Credit Default Swap Spreads

3.2.1 Theoretical Spread Determinants

Black and Scholes (1973) and Merton (1974) introduced an intuitive option-pricing framework for valuing corporate equity and debt. This structural framework by Merton (1974) provides an attractive approach to credit risk.

[18] Popular STCDOs are tranches of credit indices such as the North American CDX and iTraxx Europe index families. Each credit index represents a basket of the 125 most liquid CDS contracts on corporate names which exhibit an investment grade rating.

In structural models the default event is usually triggered when the firm's assets fall below a critical threshold.[19] The value of a firm's asset follows a simple random walk (firm value process) and the default threshold is a function of the amount of debt outstanding.

The values of debt claims are determined under the risk-neutral measure by computing the present value of their expected future cash flows discounted at the risk-free rate. Since a credit default swap extracts and transfers the default risk of corporate debt, CDS investors – in their role as protection seller – periodically receive a premium payment (premium leg) for covering losses in underlying debt claims (protection leg). In the absence of arbitrage and in the presence of risk-neutral valuation, the present value (PV) of the premium leg equals the PV of the protection leg. Hence, depending on the underlying debt claim future expected cash flows – namely the protection and premium payments – of the related CDS are analogously discounted to determine the fair CDS spread.[20]

Motivated by the structural framework, we uniquely define the CDS spread $S_{\vartheta,t}$ of contract ϑ at time t through 1) the price of underlying debt claims, 2) its related contractual cash flows, 3) the time-specific risk-free rate r_t, 4) common state variables $\mathcal{Y}_t$, which cross-sectionally affect all credit spreads simultaneously and 5) individual state variables $\mathcal{V}_{\vartheta,t}$, which are firm-specific. Thus, we define credit spreads similarly to Collin-Dufresne et al. (2001) extended by the common state variables $\mathcal{Y}_t$. This leads to

$$S_{\vartheta,t} := S_{\vartheta,t}\left(C_{\vartheta,t}(F_{\vartheta,t}), r_t, \mathcal{Y}_t, \mathcal{V}_{\vartheta,t}\right) \tag{3.1}$$

with contractual payments $C_{\vartheta,t}$ depending on the firm value $F_{\vartheta,t}$.[21] We suppose that credit spread changes are determined given the current values of the time-specific variables $\mathcal{Y}_t$ and $\mathcal{V}_{\vartheta,t}$ respectively. Also referring to the structural framework, we may predict i) determinants of CDS spread changes, and ii) whether changes in these variables should be positively or negatively correlated with changes in the CDS spreads.

Similar to other authors, we propose some common state variables reflecting

[19] Structural models were further investigated by Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Briys and De Varenne (1997), Gordy (2000), Collin-Dufresne and Goldstein (2001) and Gordy (2003).

[20] For more detailed information compare Amato (2005).

[21] See Collin-Dufresne et al. (2001) for more detailed information.

systematic risk:[22]

1. *Changes in the Spot Rate.* In theory, the static effect of a higher spot rate is to increase the risk-neutral drift of the firm value process (Longstaff and Schwartz, 1995; Duffee, 1998). The higher drift reduces the firm's probability of default and thus the price of related derivatives offering protection against default losses. We therefore expect that CDS spreads are negatively correlated with the risk-less interest rate.

2. *Changes in the Slope of the Yield Curve.* Independent from the structural framework, some authors argue that the interest term-structure is upon other factors mainly driven by i) the interest level and ii) the slope characteristics (Blanco et al., 2005).

   Often, the slope of the yield curve is seen as an indicator of economic wealth: while a positive slope indicates a prosperous economy, a negative one reflects expectations of an economic downturn. Hence, the CDS spread may decrease if an increasing slope of the interest curve indicates higher expected short rates, as also argued by Collin-Dufresne et al. (2001) for credit spreads.[23] By contrast, a decreasing term-structure may indicate an economic downturn leading to higher losses given default since recoveries are assumed to be negatively correlated with the macroeconomy (Frye, 2000; Altman, 2008; Bade et al., 2011). In this way, the liquidation risk for corporate debt may be higher leading to widening CDS spreads.

3. *Changes in the Market Volatility.* Since debt claims exhibit characteristics similar to a short position in a put option, it follows from the option-pricing framework that option prices increase with increasing volatility. Intuitively, with an increase of volatility, the firm's default probability increases and thus the related CDS spread increases due to the higher default risk.

4. *Changes in the Credit Market Climate.* The *Credit Market Climate* may reflect the market view of the overall credit risk. If the global economy

[22] Since systematic risk affects all market participants simultaneously, we aim to approximate this kind of risk by common risk variables. Note that state variables are generally not necessary in Merton's structural approach.

[23] Note that rising future short-term rates may lead to lower default probabilities and thus to lower CDS spreads.

is turning down in line with decreasing recoveries, the weakening market conditions should increase the firms' default risk as well as related losses. Thus, the increased credit risk on credit markets may lead to an increase of the overall credit spread level. The *Credit Market Climate* can be seen as a common market factor similar to the market index in the CAPM. It should be strongly affected by economic conditions. Therefore, we expect a cross-sectional increase of default risk due to weakening economic conditions leading to increased CDS spread levels. Hence, the CDS spreads should be positively correlated with the *Credit Market Climate*.

5. *Changes in the Cross-market Correlation.* Foresi and Wu (2005) argue that downside movements in any equity index are likely to be highly correlated with those in other markets as a result of global contagion. Expanding this argument to credit markets, we expect higher CDS spreads if cross-market correlations increase, because the prospects for risk diversification on global markets decrease. In turn, we expect lower CDS spreads if the dependencies across various markets – such as credit, equity, and exchange markets – decrease.

Lastly, non-systematic and thus rather individual spread determinants are proposed and discussed individually.

1. *Physical Default Probability.* Within the structural framework, the difference between the physical probability of default (PD) and the risk-neutral PD indicates the risk aversion of market participants. Under the risk-neutral measure, the drift parameter μ of the asset value process is changed to the risk-less rate r from which it follows that the risk-neutral PD is composed of the physical PD plus a correction term accounting for the risk aversion. By controlling for the physical PD, we quantify the premium for pure default risk apart from other major determinants. In line with intuition and ceteris paribus, the higher (lower) the firm's physical PD, the higher (lower) the CDS spread should be.

2. *Swap Liquidity.* Analogously to other authors who show that liquidity is priced in credit spreads of corporate bonds, we assume that CDS investors also claim a premium compensating for liquidity risk. Transferring these empirical findings to CDS markets, the contract's liquidity is expected to determine the CDS spread. Intuitively, CDS spreads should

rise if the contracts' liquidity, for example, measured by their trading volume, decreases and vice versa. Eventually, we expect a negative relationship between *Swap Liquidity* and swap spread.

3.2.2 Empirical Data

Our empirical study refers to a comprehensive data set of single-name CDS spreads provided by Markit. Overall, we analyze dollar-denoted CDS spreads of 339 U.S. American entities from January 6^{th}, 2004 to December 27^{th}, 2010.[24] By splitting the entire period into two different subsamples, we account for different market conditions before the GFC and in times of market turbulence during the GFC. Firstly, we define the period from January 6^{th}, 2004 to June 18^{th}, 2007 as time prior to the GFC (Pre-GFC). Secondly, we define the period from June 19^{th}, 2007 to December 27^{th}, 2010 as time of financial distress during the GFC.[25]

Table 3.1 summarizes the sample periods for the time-series regressions (TSR) and for the cross-sectional regressions (CSR).[26] The amount of related CDS spread observations and the number of considered entities are also denoted.

Table 3.1: Sample Period of Time-series and Cross-section Regressions

		Multiple Time-series and Cross-section Regressions		
Sample		Entire period	Pre-GFC	GFC
Maturity	From:	6^{th} of Jan 04	6^{th} of Jan 04	19^{th} of Jun 07
	Until:	27^{th} of Dec 10	18^{th} of Jun 07	27^{th} of Dec 10
	Amount:	339	339	339
Entities	Obs. per entity:	367	180	187
	Sum of obs.:	124,413	61,020	63,393

Notes: The table summarizes the sample maturities as well as the amount of CDS spread observations (obs.) covered by each sample. The period of the Pre-GFC reflects the time interval prior to the financial crisis and the GFC describes the time period during the crisis. Based on each sample, multiple time-series regressions as well as cross-sectional regressions are conducted.

[24] The contracts' document clause is MR (modified restructuring). The seniority is SNRFOR (senior unsecured debt). For more information compare Markit (2008). We select contracts which have at least 47 weekly spread notations per year.

[25] On June 18^{th}, 2007 it was reported for the first time that Merrill Lynch seized collateral from a Bear Stearns hedge fund invested heavily in subprime loans, which may have caused strong spread increases on credit markets over the following days.

[26] The corresponding regression models are introduced in the next section.

Overall, we investigate 124,413 weekly CDS spreads from 339 different issuers in the entire period, in which the number of CDS spreads per entity is 367. The Pre-GFC sample contains 180 weekly spreads per entity, which leads to 61,020 weekly observations in total. In the GFC sample, we examine 63,393 weekly CDS spreads with 187 observations per entity.

The U.S. companies are divided over ten economic sectors, e.g., financials (16.81%), industrials (14.16%) and consumer goods (13.57%). Table 3.2 summarizes the number of firms located in each sector and provides the sector-specific average spreads by sample.

Table 3.2: Investigated Economic Sectors

			Mean Spread		
U.S. Sector	Count	Count in %	Entire	Pre-GFC	GFC
Basic Materials	22	6.49	0.0184	0.0113	0.0253
Consumer Goods	46	13.57	0.0216	0.0113	0.0316
Consumer Services	58	17.11	0.0320	0.0162	0.0471
Financials	57	16.81	0.0220	0.0042	0.0389
Health Care	16	4.72	0.0137	0.0074	0.0198
Industrials	48	14.16	0.0123	0.0077	0.0168
Oil & Gas	29	8.55	0.0128	0.0082	0.0174
Technology	14	4.13	0.0156	0.0109	0.0202
Telecommunications	12	3.54	0.0291	0.0230	0.0349
Utilities	37	10.91	0.0119	0.0073	0.0163
Overall	339	100	0.0189	0.0107	0.0268

Notes: The table reports the amount of U.S. entities located in ten economic sectors and denotes the sector-specific mean CDS spreads by sample (Entire, Pre-GFC and GFC).

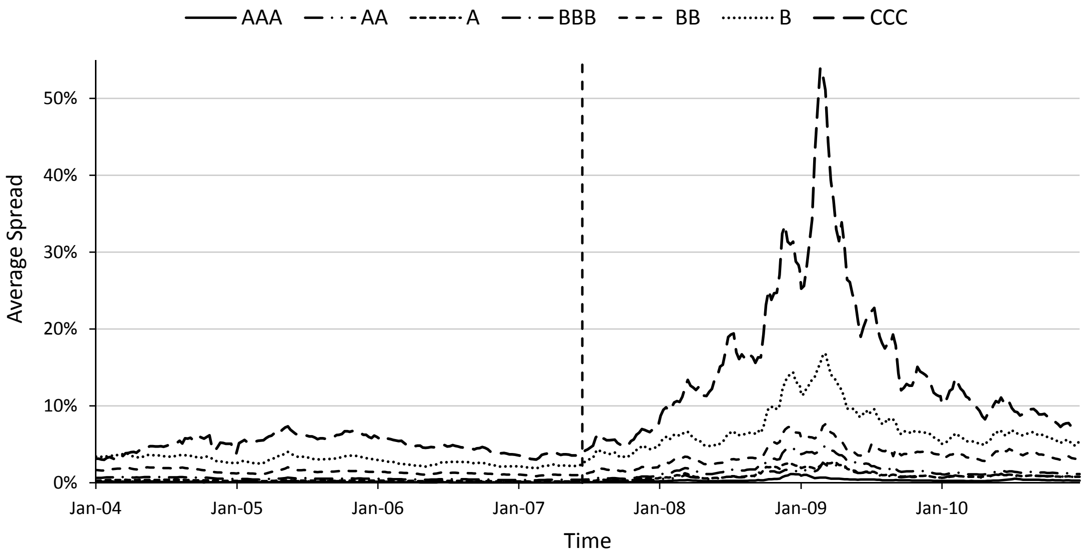
Since we investigate a wide range of U.S. firms, we may obtain a broad insight into the cross-sectional determinants of CDS spreads. The sector-specific average spreads vary by sample and across sectors. In order to account for sector-specific influences, we implement sector dummies in our CSR model.

Furthermore, all underlying contracts of the CDS are rated on a rating scale from 'AAA' to 'CCC'.[27] In Figure 3.1, we plot the time series of average CDS spreads per rating grade from January 6^{th}, 2004 to December 27^{th}, 2010 (x-axis). The y-axis denotes the average CDS spreads.

The average spread level generally varies depending on the rating grades: the average CDS spread of 'AAA'-rated underlyings (black line) is below all other grade-specific average spreads throughout, as theoretically assumed

[27] The rating scale contains average ratings referring to Moody's and S&P ratings. For more details compare www.markit.com.

Figure 3.1: Average Spreads by Rating



Notes: This figure shows time series of average CDS spreads for various rating grades, e.g., 'AAA', 'AA', 'A', from January 6^{th}, 2004 to December 27^{th} 2010. The spread function of 'AAA'-rated contracts (black line) is below all other spread functions since highest creditworthiness is linked to the lowest risk premium. In turn, the 'CCC'-based CDS spread function (dashed line) is located above all others. The entire sample is divided into the period prior to the financial crisis (Pre-GFC) and the GFC by the dashed vertical line.

above. By contrast, 'CCC'-rated contracts (dashed line) exhibit the highest average CDS spreads since they reflect the highest default risk. All grade-specific functions show that average spreads are rapidly increasing across all rating grades during the turmoil of the GFC.

Next, we choose the following proxies for the identified systematic state variables.

1. *Spot Rate.* The spot rate (SP) is approximated by changes in government bonds, as also suggested by other authors in the recent literature (compare Blanco et al., 2005; Avramov et al., 2007).[28] We use 5-year Treasury note rates provided by the U.S. Department of the Treasury.[29]

2. *Slope of the Yield Curve.* Analogously to Collin-Dufresne et al. (2001),

[28] However, due to several reasons, e.g., taxation treatment, scarcity premiums and benchmark status issues, it is often criticized that government bonds are no ideal proxy for the unobservable risk-free rate. In this concern, 5-year swap rates for dollars and euros are often proposed as a better proxy. For an insightful discussion see Blanco et al. (2005). We also incorporate corresponding swap rates for robustness.

[29] Other maturities such as 1-year, 2-years and 10-years are also investigated, but not reported since they lead to similar results.

among others, we define the slope of the term structure (STS) as the difference between the long-term and the short-term Treasury note rate. To capture slope effects, we use changes in spread differences on U.S. Treasury notes with 2-year and 10-year maturity. The slope may be interpreted as an indicator of the economic health and expectations of future short rates. Respective Treasury note rates are also provided by the U.S. Department of the Treasury.

3. *Market Volatility.* As benchmark for the *Market Volatility*, we utilize the VIX index provided by the Chicago Board Options Exchange. The VIX measures market expectation of near-term volatility conveyed by stock index option prices.[30] By using a wider range of strike prices rather than just at-the-money series, the VIX index additionally incorporates information from the volatility 'skew'. Thus, the VIX reflects investors' consensus view of future expected stock market volatility: since out-of-the money put options as well as in-the-money call options are considered for short maturities, and may be seen as an indicator for negative jumps in the S&P 500 index causing investors' fear. According to Collin-Dufresne et al. (2001), an increasing probability and magnitude of large negative jumps in the firm value should increase credit spreads, and thus CDS spreads (Blanco et al., 2005).

4. *Climate of Credit Markets.* As S&P 500 index returns are suggested to approximate the overall state of the economy (see Collin-Dufresne et al., 2001; Blanco et al., 2005), we analogously assume the index spread changes of the 5-year (5Y) CDX NA IG credit index (CDX) as proxy for the credit market conditions. The CDX is one of the most popular CDS indices covering a cross-sectoral basket of the 125 most liquid North American (NA) investment grade (IG) single-name CDS.[31] Index spreads of the CDX are provided by Markit.

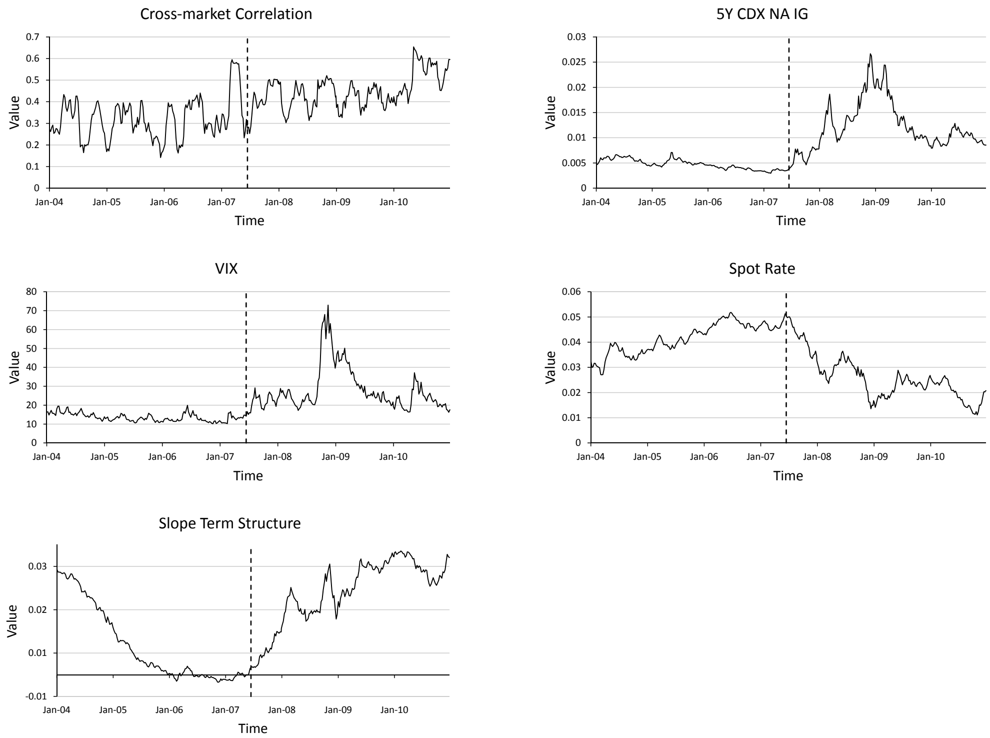
[30] The VIX uses a weighted average of options with a constant maturity of 30 days to expiration. The options refer to the S&P 500 index.

[31] The CDX index spread reflects an average credit spread on a basket of 125 CDS dealings. For each series, the composition of the underlying basket is fixed until maturity (almost six months). Depending on, e.g., default and liquidity criteria, the constituents of the basket may vary across series. Note that our database contains CDS spreads related to 205 CDS dealings, which are denoted at least once in several CDX series from January 2004 to December 2010. For a detailed description of the numerous CDX indices and series refer to www.markit.com.

5. *Cross-market Correlation*. We consider the average of quarterly cross-correlations referring to returns on numerous i) exchange, ii) equity and iii) credit markets. In this context, we suggest some indices to calculate the applied *Cross-market Correlation* (CMC), e.g., S&P 500, DAX 30, 5Y CDX NA IG, Dow Jones Industrial Average, Nikkei 225.

Figure 3.2 shows the times series of the systematic state variables from January 6^{th}, 2004 to December 27^{th}, 2010 (x-axes). The y-axes denote the states of the respective proxies. The dashed vertical lines divide the entire sample period into the samples Pre-GFC and GFC.

Figure 3.2: Time Series of Systematic State Variables



Notes: This figure shows time series of systematic state variables from January 6^{th}, 2004 to December 27^{th}, 2010. The following proxies for the state variables are plotted: the *Cross-Market Correlation* refers to the average cross-correlation across several market indices (upper-left). The time series of the 5Y CDX IG index spread represents the *Credit Market Climate* (upper-right). The *Market Volatility* is indicated by the time series of the VIX index (mid-left). The *Spot Rate* is approximated by the 5Y T-bill rate (mid-right). For the *Slope of The Term Structure*, we present time series of the difference between the 10Y and the 2Y T-bill rate (lower-left). The dashed vertical lines divide the entire sample into the two sub-samples (Pre-GFC and GFC).

Time series of the *Cross-market Correlation* (upper-left chart) fluctuated within the entire period in a moderate range between 0.13 (min) and 0.63 (max) with mean 0.36 and standard deviation (STD) 0.09.

As intuitively expected, the index spread of the CDX (upper-right chart) was moving sideways with relatively low volatility before the GFC. Indeed, during the GFC the volatility of the CDX strongly increased as well as its spread level. While its mean was denoted at almost 47 basis points (bp), its STD was at 9.8 bp prior to the crisis. In contrast to the Pre-GFC, the mean of the CDX was three times higher (136 bp) during the GFC, while its STD was six times higher (59 bp). The maximum spread was observed at the end of 2008 denoting at 280 bp, the minimum spread of 29 bp in January 2007, a few months before the GFC began.

The VIX index (mid-left chart) moved sideways from January 2004 until June 2007 with moderate volatility (index mean 13.6 and STD 2.2), increased clearly in the beginning of the GFC and reached its historical peak at around 80.9 in December 2008. Similarly to the other systematic risk factors, the mean of the VIX was clearly higher in times of crisis (2.3 times higher) than in moderate economic conditions and also its related STD (6.1 times higher). In the beginning of 2009, the VIX index clearly turned back on the index level reached in January 2008.

The *Spot Rate* in terms of the 5-year Treasury note rate (mid-right chart) was about 3% in January 2004, moved to around 5% in June 2007 and then decreased rapidly to 1.5% in 2009 due to the market turbulences on the credit markets.

A decreasing *Slope of the Term-structure* (lower-left chart), which we observed before the global financial crisis began in June 2007, indicates expectations of an economic downward movement (compare Bank for International Settlements, 2009). Increasing slope values as observed during the turmoil on financial markets, in turn, may have predicted an economic up-turn in the aftermath.

Eventually, the time series show that each systematic risk factor clearly behaves differently before the GFC than during the financial turmoil, as it is indicated by the factors' period-specific means and standard deviations. Basically motivated by the chronology of the GFC, the determination of our subsamples is also confirmed by both CDS spread descriptives and time-series analysis of the systematic state variables.

The correlation matrix in Table 3.3 refers to changes (Δ) in the systematic

Table 3.3: Sample-specific Correlation Matrix of Systematic Risk Factors

				Pre-GFC		
		ΔCMC	ΔCDX	ΔVIX	ΔSR	ΔSTS
	ΔCMC		0.0571	0.264	-0.0254	0.0726
	ΔCDX	0.0441		0.4116	-0.1067	-0.0734
GFC	ΔVIX	0.0673	0.6382		-0.1656	0.0219
	ΔSR	-0.0534	-0.4530	-0.3158		0.1949
	ΔSTS	-0.1589	0.0522	0.1152	0.1021	

Notes: The table shows the cross-correlations related to changes (Δ) in five systematic risk variables, namely the *Cross-market Correlation* (CMC), the CDX index, the VIX index, the *Spot Rate* (SR), and the *Slope of the Term Structure* (STS). While the upper triangle of the matrix refers to the cross-correlations of the Pre-GFC, the lower triangle shows the correlations of the GFC.

state variables identified above and reflects the linear dependency structure across these changes. The upper triangle of the matrix refers to correlations in the Pre-GFC and the lower triangle shows cross-correlations in the crisis.

According to Table 3.3, the proxy for the *Cross-market Correlation* and the proxies for the interest risk – *Slope of the Term Structure* and *Spot Rate* – exhibit the lowest overall Δ-dependencies on the other systematic risk factors in both samples. Table 3.3 also shows that the dependencies generally increase during the GFC. Nevertheless, most cross-correlations denote at low levels (about 0.10). We observe the highest correlation between the VIX and the CDX with 0.41 before the GFC and 0.64 during the financial crisis.

In the following, proxies for individual risk are provided.

1. *Physical Probability of Default.* Since a credit rating generally reflects an opinion of the obligor's creditworthiness, the highest-rated obligors ('AAA'-rated) are assumed to exhibit the lowest probability of default (PD), while lowest-rated ones ('C'-rated) exhibit the highest PD. Credit-rating agencies (CRA), for example, link their classical rating grades (ordinal scaled) to historical default rates of corporate bonds (Moody's, 2010*a*).[32] Hence, we use average ratings provided by Markit as proxy for the firm's physical default risk, similarly to Friewald et al. (2012).[33]

   Following Abid and Naifar (2006), we assume that the absolute CDS spread level is determined by the related obligor rating. The worse the

[32] Referring to the three major rating agencies – Moody's, S&P and Fitch – the rating grades are monotonically increasing with the obligor's creditworthiness (compare Moody's, 2012).
[33] Recall that Markit's average ratings are based on available Moody's and S&P ratings, see www.markit.com

rating of the obligor the higher the CDS spread level and vice versa. For simplicity, we apply a shortened rating scale which summarizes the available rating metrics. In our cross-sectional regressions, we account for five different rating classes RC_1 to RC_5, where the latter indicates lowest creditworthiness and RC_1 highest.[34]

CRAs such as Moody's and S&P provide ratings that are rather stable through business cycles (through-the-cycle ratings), see Moody's (1999) and S&P (2008). Thus, macroeconomic point-in-time information is rather neglected in such a through-the-cycle approach (Moody's, 1999).[35] Since CRAs mainly address firm-specific risks in their rating metrics rather than states of the global economy (common risk) (S&P, 2008), we consider credit ratings primarily as proxy for individual risk.

2. *Swap Liquidity.* As proposed by Gala et al. (2010) and Arora et al. (2012), we incorporate the contract's number of trades (trading depth) to proxy its liquidity. The data were provided by Markit and denoted as *Swap Liquidity* (LIQ).

Overall, each single-risk proxy i) reveals for itself significant explanatory power in respective univariate regressions, ii) significantly contributes to the explanation of the endogenous variable in our CSR and iii) has the power to innovate. The latter condition is especially important in terms of multi-collinearity: all of our systematic risk factors provide for themselves additional explanatory power.[36] In other words, the explanatory power of each proxy is i) not completely covered by the ensemble of other regressors, independent of the introduction order, and ii) its explanatory power is not the product of the entire ensemble.

[34] Due to the number of available ratings, RC_1 includes rating grades 'AAA', 'AA' and 'A' , RC_2 reflects 'BBB' ratings, RC_3 accounts for rating grade 'BB', RC_4 refers to 'B' ratings and RC_5 to 'CCC' ratings. The data set contains few 'AAA'- and 'AA'-rated entities.

[35] More detailed information on CRAs and their rating systems can be found in Krahnen and Weber (2001) or in Löffler (2004, 2013).

[36] The order of regressors does not matter in basic OLS regressions, but within OLS regressions based on normalized regressors which were additionally conducted for robustness. Within a normalized framework, the regressors are corrected for observable co-variances. In the end, the regressors' covariance matrix is a diagonal matrix with variances equal to one. The regressors' means are also standardized and equal null. The results accounting for multi-collinearity are not separately reported since they solely confirm the presented findings.

3.3 Empirical Evidence for Pricing Systematic Risk in CDS Spreads

3.3.1 Models in the Two-pass Regression Approach

In the first step of our two-pass regression procedure (similar to Fama and MacBeth, 1973), we estimate the CDS spread sensitivities (betas) to the proposed systematic state variables by multiple time-series regressions (TSR). For each CDS referring to entity $\vartheta \in \{1, ..., 339\}$ with CDS spread $S_{\vartheta,t}$ at time t we estimate the following time-series regression model, which was methodologically proposed by Collin-Dufresne et al. (2001) for credit spreads and also applied by Ericsson et al. (2009) and by Friewald et al. (2012).

$$\begin{aligned}\Delta S_{\vartheta,t} = \alpha_\vartheta + \beta_\vartheta^{CMC} \cdot \Delta CMC_t + \beta_\vartheta^{CDX} \cdot \Delta CDX_t + \beta_\vartheta^{VIX} \cdot \Delta VIX_t \\ + \beta_\vartheta^{SR} \cdot \Delta SR_t + \beta_\vartheta^{STS} \cdot \Delta STS_t + \varepsilon_{\vartheta,t}.\end{aligned} \tag{3.2}$$

$\Delta S_{\vartheta,t}$ denotes the spread change of the contract related to firm ϑ at time t.[37] α_ϑ describes the intercept, $\beta_\vartheta^{(\cdot)}$ denotes the coefficients of included regressors, Δ refers generally to changes in the state variables and $\varepsilon_{\vartheta,t}$ is the residual.[38]

In the second step, we examine the cross-section of CDS spreads by cross-section regressions, similarly to Friewald et al. (2012) who apply this type of regression to corporate bond spreads. Thus, our TSR beta estimates are used as regressors in the cross-sectional regression (CSR), along with additional variables such as the proposed individual risk factors. In the basic model setup, we consider the firm's ratings and the contract's liquidity. In addition, we account for firm-specific sectoral influences by sector dummies. In Section 3.3.4, we add further firm-specific risk factors, e.g., the firm's *Leverage Ratio* and *Market Capitalization*, as well as further systematic risk betas related to the Fama-French factors, in order to check the robustness of our findings.

After calculating the entities' average CDS spreads $\overline{S}_\vartheta$ by sample, we esti-

[37] In our TSR regressions, we regress weekly CDS spread changes by weekly changes in the common risk variables. Corresponding regressions are also conducted on a daily database leading to similar results. Due to the noise in high-frequency data, we focus on results related to the weekly database.

[38] Linkage of shortened declarations according to Section 3.2.2: *Cross-market Correlation* (CMC), *Credit Market Climate* (CDX), *Market Volatility* (VIX), *Spot Rate* (SR) and *Slope of the Term Structure* (STS).

mate the following cross-section regression model for each sample

$$\begin{aligned}\overline{S}_\vartheta = \alpha &+ \gamma^{CMC} \cdot \hat{\beta}_\vartheta^{CMC} + \gamma^{CDX} \cdot \hat{\beta}_\vartheta^{CDX} + \gamma^{VIX} \cdot \hat{\beta}_\vartheta^{VIX} + \gamma^{SR} \cdot \hat{\beta}_\vartheta^{SR} \\ &+ \gamma^{STS} \cdot \hat{\beta}_\vartheta^{STS} + \gamma^{LIQ} \cdot LIQ_\vartheta + \boldsymbol{\gamma}^{RC} \cdot \boldsymbol{RC}_\vartheta + \boldsymbol{\gamma}^{SI} \cdot \boldsymbol{SI}_\vartheta + \varepsilon_\vartheta^{CS}, \quad (3.3)\end{aligned}$$

where $\varepsilon_\vartheta^{CS}$ denotes the cross-sectional residual. $\hat{\beta}_\vartheta^{(\cdot)}$ denotes the parameter estimates of TSR regressors.[39] LIQ denotes the swap's average liquidity, **RC** and **SI** represent the firm-specific *Rating Classes* and *Sector Indicators* respectively, which are included as dummy variables.[40] α denotes the intercept and $\gamma^{(\cdot)}$ are the cross-sectional slope parameters. $\boldsymbol{\gamma}^{RC}$ and $\boldsymbol{\gamma}^{SI}$ represent vectors of estimators referring to the sector-specific and rating-specific dummy variables.

Table 3.4 gives a brief regressor overview and shows the predicted signs of coefficients related to the TSR and the CSR in line with the theoretical expectations presented in Section 3.2.1.

Table 3.4: Overview of Common Risk Factors and Predicted Signs

Variable	Description	Predicted Sign β (TSR)	γ (CSR)
	Systematic Risk Factors in Time-series Regressions		
ΔCMC	Change in the Cross-market Correlation	+	+
ΔCDX	Change in CDX index spread	+	+
ΔVIX	Change in implied volatility of S&P 500	+	+
ΔSR	Change in yield on 5-year Treasury yield	−	−
ΔSTS	Change in 10-year minus 2-year Treasury yield	−	−
	Non-systematic Risk Factors in the Cross-section Regression		
LIQ_ϑ	Liquidity of CDS Contract		−
RC_ϑ	Rating Dummy for Class 1 to 5		+
SI_ϑ	Sector Indicator for Sector 1 to 10		

Notes: The table shows included regressors of both the multiple time-series regressions (TSR) and the cross-section regressions (CSR). The predicted signs for the respective regression coefficients of the TSR (β) and CSR (γ) are also denoted.

[39] In the first pass of our regression approach, we use CDX index spread changes to regress contract-specific CDS spread changes in order to obtain the contract-specific regressor sensitivities (compare (3.2) and (3.3)). In this context, we assume that the possible endogeneity is marginal due to three reasons: i) more than one third of our examined CDS dealings are no constituents of CDX series, ii) for CDX constituents, the related index spread is only fractionally and temporary influenced by the contract-specific CDS spread, and iii) the main results related to our second-pass regressions are solely indirectly influenced by CDX spread changes, since we focus on the cross-section examination of sensitivities to CDX spread changes. We obtain robust regression results for a model which does not control for CDX.

[40] RC_1 (SI_1) represents the reference rating class (sector) included in the intercept.

For example, the estimates of the TSR which refer to changes in the CDX credit index should be positive since in theory an increase of the CDX index spread should commonly widen the CDS spreads. Alternatively, for the *Swap Liquidity LIQ*, we expect a negative relationship to the CDS spread. Hence, an increase of the liquidity should lead to a decrease of the CDS spread and vice versa.

3.3.2 Systematic Risk After Controlling for Credit Ratings

Firstly, we examine whether the firms' ratings have cross-sectional explanatory power with respect to the CDS spreads of 339 entities. Table 3.5 reports the regression results of rating-based CSRs.

Table 3.5: Cross-section Regressions by Rating Dummies

	Entire Period	Pre-GFC	GFC
Intercept	0.0065***	0.0025*	0.0103***
	(0.0017)	(0.0013)	(0.0028)
BBB-rated	0.0039*	0.0027*	0.005⁻
	(0.0021)	(0.0016)	(0.0034)
BB-rated	0.0217***	0.0116***	0.03***
	(0.0027)	(0.0021)	(0.0043)
B-rated	0.0461***	0.0252***	0.0676***
	(0.0031)	(0.0023)	(0.0049)
CCC-rated	0.0741***	0.0493***	0.1111***
	(0.0048)	(0.004)	(0.0077)
R^2	59.29%	45.29%	55.39%
No. Entities	339	339	339

Notes: The table summarizes the rating-based results of cross-section regressions. The parameters are statistically significant at the 1%-level (***), the 5%-level (**) and the 10%-level (*). R^2 denotes the coefficient of determination. 'No. Entities' reflects the number of entities considered in the cross-section regressions.

The intercept includes the reference rating class RC_1 referring to 'AAA', 'AA' and 'A' ratings. Thus, the intercept represents the basic spread level of swap contracts related to high-rated obligors. Furthermore, Table 3.5 shows that the worse the firm's rating the higher is the general risk premium for that CDS contract, which is in line with our expectations. The risk premiums seem to be higher in the financial crisis than prior to the GFC, which holds across all rating classes. The results show that firm ratings represent relevant information for pricing swap contracts cross-sectionally. A comparison of R^2

based on a comparable number of observations (see Table 3.3) indicates that ratings may explain more of the spread variation in times of financial distress than in moderate economic conditions (45.29% vs. 55.39%). These results may also indicate that market participants, who were involved in pricing swap contracts, relied more intensively on ratings during the GFC than prior to the crisis.

A simple preliminary test to examine whether CDS spreads are reflecting systematic risk after controlling for the risks reflected by CRA ratings, is to compare the rating-based mean CDS spreads of contracts having different sensitivities to systematic risk.[41] Similarly to Iannotta and Pennacchi (2011), we conduct univariate TSRs for each systematic regressor by rating class (R_1: 'AAA'-'A', R_2: 'BBB', R_3: 'BB', R_4 = 'B', and R_5: 'C'). For each sample period, the sensitivity to systematic risk is measured by the regressor's beta. CDS contracts exhibiting systematic risk sensitivities above or equal to the sample median are defined as contracts with high systematic risk exposures and thus attributed to Portfolio 1. Contracts with sensitivities below the sample median are attributed to Portfolio 2 (low systematic risk exposures). Afterwards, the portfolios' mean spreads are calculated by rating class and tested for equality via t-test.

Table 3.6 reports the median betas (β), the portfolio-specific mean spreads and the t-test results for each systematic risk factor, rating class and sample.

The median betas are monotonically increasing with rating classes. Hence, contracts related to the worse credit rating exhibit the highest betas to systematic risk. This may be due to the increase in the rating-specific spread level. Thus, swap contracts of poorly-rated firms may not necessarily exhibit the highest sensitivities to systematic risk. Although the sensitivities to systematic risk are not comparable across rating classes, the contracts' systematic risk sensitivities vary widely around the median beta within each rating class. This indicates that systematic risk exposures are underestimated in part by CRAs. This finding holds for all regressors and both samples.

Most of the portfolio-specific mean spreads significantly differ from each other in each sample and across all regressors. In most cases, we observe

[41] Iannotta and Pennacchi (2011) provide a similar test for credit spreads on corporate bonds.

Table 3.6: Systematic Risk Indication by Rating Class

		Pre-GFC			GFC		
			Mean CDS Spread			Mean CDS Spread	
	Rating Class	Median Beta	Portfolio 1 (above median)	Portfolio 2 (below median)	Beta Median	Portfolio 1 (above median)	Portfolio 2 (below median)
CMC	1	<0.0001	0.0024***	0.0025	0.0009	0.0128***	0.0079
	2	0.0001	0.0059***	0.0044	0.0014	0.0182***	0.0125
	3	0.0002	0.014	0.0141	0.0058	0.0518***	0.0293
	4	0.0014	0.0383***	0.0170	0.0187	0.1002***	0.0557
	5	0.0037	0.078***	0.0255	0.0228	0.1315***	0.1112
CDX	1	0.2074	0.0029***	0.0020	0.3926	0.0154***	0.0055
	2	0.4223	0.0063***	0.0041	0.5809	0.0206***	0.0102
	3	11,635	0.0194***	0.0090	11,506	0.0606***	0.0208
	4	16,865	0.0385***	0.0168	23,389	0.0984***	0.0574
	5	23,129	0.0781***	0.0254	46,139	0.1571***	0.0856
VIX	1	<0.0001	0.0029***	0.0020	0.0001	0.0154***	0.0055
	2	<0.0001	0.0062***	0.0042	0.0001	0.0201***	0.0107
	3	0.0001	0.0189***	0.0094	0.0002	0.0561***	0.0251
	4	0.0001	0.0384***	0.0168	0.0005	0.0977***	0.0582
	5	0.0002	0.0609***	0.0426	0.0011	0.1416***	0.1011
SR	1	-0.0144	0.0024***	0.0025	-0.1502	0.0054***	0.0150
	2	-0.0235	0.0043***	0.0060	-0.2388	0.0096***	0.0206
	3	-0.0632	0.0089***	0.0191	-0.5638	0.0186***	0.0614
	4	-0.0817	0.0219***	0.0333	-0.9909	0.0514***	0.1045
	5	-0.1624	0.0472***	0.0563	-15,973	0.1209	0.1219
STS	1	-0.0076	0.0026***	0.0023	0.0025	0.0117***	0.0090
	2	-0.0075	0.0056***	0.0047	-0.0210	0.0141***	0.0164
	3	-0.0046	0.0156***	0.0126	0.0177	0.0413*	0.0394
	4	0.0501	0.0322***	0.0231	-0.0470	0.0648***	0.0911
	5	-0.0400	0.0472***	0.0563	-0.2102	0.1416***	0.1011

Notes: This table reports mean CDS spreads per rating class depending on the sensitivity of CDS spread changes to five systematic risk factors: *Cross-market Correlation* (CMC), CDX index, VIX index, *Spot Rate* (SR) and *Slope of the Term Structure* (STS). Univariate regressions are conducted on CDS contracts in order to evaluate the median sensitivity (*Median Beta*) to the systematic risk proxies in each rating class. Afterwards portfolios are established in dependence on estimated betas. Portfolio 1 contains all CDS with betas above the median, while Portfolio 2 includes those with betas below the median. The results are reported for both samples, the Pre-GFC and the GFC. ***, **, and * indicate the statistical significance (1%-, 5%-, and 10%-level) of the t-test for equality of mean CDS spreads for contracts with beta estimates below and above the median.

higher average spreads for portfolios composed of high risk contracts.[42] From these empirical findings we conclude that CDS with higher systematic risk exposures are in general higher priced and that this systematic risk is not sufficiently reflected by credit ratings.

[42] The order of sensitivities is descending for each rating class. Thus, we observe the highest systematic risk concentration in *Portfolio 2*, if the regressor's sensitivity to systematic risk is expected to be negative, as is the case in terms of the *Spot Rate*. Again, the higher interest risk sensitivity leads on average to higher CDS spreads.

3.3.3 Empirical Results of Time-series and Cross-section Regressions

Figure 3.3 shows boxplots summarizing the estimation results of multiple time-series regressions across 339 entities by regressor and sample. All of the state variables in regression (3.2) have some ability to explain changes in the CDS spreads. Further, the signs of the estimated coefficients mostly correspond with our ex-ante expectations.

With respect to the GFC (right boxplot in each chart), the regression results show that signs of estimates agree on average with our expectations, except for the betas of the *Spot Rate* proxy. These beta estimates are expected to be negative, which is on average fulfilled prior to the GFC.[43] Hence, in the pre-crisis the SR corresponds to expectations and thus an increase in the SP tends on average to a decrease of CDS spreads across all firms. In times of financial distress, the beta estimates of the *Cross-market Correlation* are on average positive and thus CDS spreads tend to increase with increasing market correlation.

Coefficients of the slope proxy (SMT) are mainly negative during the *GFC*. As suggested in theory, positive expectations of the economy lead to a decrease in CDS spreads across most of the firms. We find further that the betas of the CDX index spread changes are positive throughout all samples. As expected, there is a positive relationship between CDX spread changes and CDS spread changes.

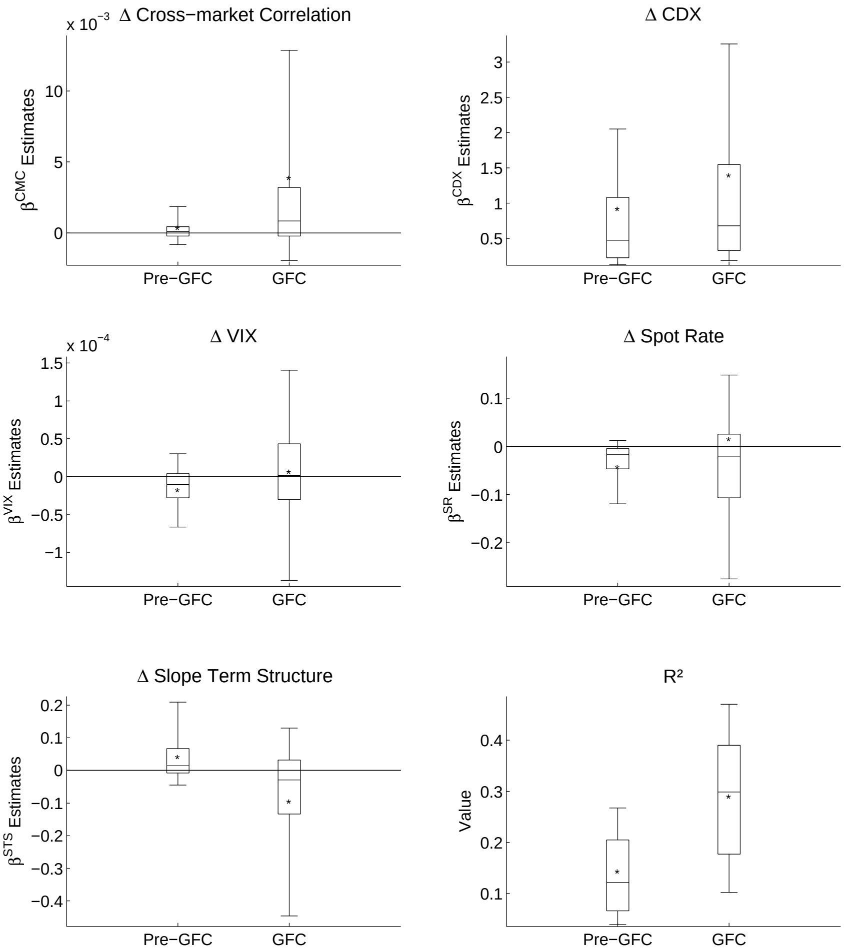
Regarding the Pre-GFC (left boxplot in each chart), the signs of betas correspond on average to theory except in case of the VIX and the STS. In terms of the VIX (STS), the respective beta estimates are on average negative (positive) before the crisis and thus contrary to our rationale.[44] Analogously to the empirical findings of Longstaff and Schwartz (1995), Duffee (1998) and Blanco et al. (2005) for credit spread changes, we find that an increase in the risk-free rate (SR) lowers the CDS spread for at least 75% of the firms prior to the crisis.

Similarly to other empirical studies (Collin-Dufresne et al., 2001; Ericsson et al., 2009; Friewald et al., 2012), the coefficient of determination R^2 ranges on average between 14.37% and 29.08%, as shown in the lower-right chart of

[43] While the median beta is negative, the mean beta is positive due to a few outliers.

[44] Note that there are still entities whose beta estimates meet our rationale, but not on average.

Figure 3.3: Estimation Results of Time-series Regressions



Notes: This figure provides boxplots referring to estimates of time-series regressions for the Pre-GFC and the GFC. Additionally, the lower-right chart shows boxplots related to the coefficients of determination (R^2). In each boxplot, the upper whisker$^+$ refers to the 90 percentile, while the lower whisker$^-$ refers to the 10 percentile. Asterisks denote the means.

Figure 3.3. We find that the explanatory powers of our applied systematic risk factors depend on the sample period. Our systematic state variables explain CDS spread changes much better in times of market turbulence than in moderate market conditions. This finding may justify the selection of our proxies for systematic state variables.

By contrast, most recent studies (e.g., Ericsson et al., 2009) primarily consider firm-specific risk factors in their TSR, but do not provide a cross-sectional spread examination, except Friewald et al. (2012). In our two-pass approach such individual risk factors are methodically omitted in the TSR (pass one), but explicitly considered in the CSR (pass two).[45] Nevertheless, we achieve comparable explanatory power in our first pass by focusing on systematic risk factors.

In the second step, we run the cross-sectional regressions (3.3) according to our two-pass regression methodology to identify significant cross-sectional pricing factors and their specified weights or spread premiums (γ) in pricing CDS contracts.

Table 3.7 shows the gamma estimates of the CSR for the two samples (Pre-GFC and GFC). While the left column in each sample reports results without sector dummies, the right column shows results under consideration of sector dummies which account for sectoral influences. Standard deviations are reported in parentheses and significance levels are marked with asterisks. In contrast to the additional individual variables in the CSR, which are deterministic, the betas of our systematic state variables are statistically estimated. Thus, they are generally stochastic and hence possibly misspecified. To account for related parameter estimation errors, we also report corrected standard deviations and corrected significances for the gamma estimates, as suggested by Shanken (1992).[46]

While the TSR estimates indicate the firm-specific sensitivity to the systematic risk factors, the CSR estimates may be interpreted as average pricing weights for the systematic risk factors across all CDS spreads. We find that mostly the CSR estimates significantly differ from null. Thus, TSR estimates are either positively ($\gamma > 0$) or negatively ($\gamma < 0$) priced.

[45] We are explicitly targeting at the product's sensitivity to systematic risk based on weekly data points. To avoid distortions due to time-constant firm-specific risk factors such as the firm ratings or corporate debt, we omit these factors in the first pass.

[46] The Shanken corrections are separated by a slash. For a thorough description of the applied correction procedures compare Shanken (1992); Shanken and Zhou (2007).

Table 3.7: Cross-section Estimates

	Pre-GFC		GFC	
Intercept	0.0098***/***	0.0095***/***	0.0165***/***	0.0164***/***
	(0.0014/0.0016)	(0.0017/0.0016)	(0.0033/0.0021)	(0.0036/0.002)
CMC	2.4531***/***	2.5994***/***	0.3227***/**	0.3335***/**
	(0.2492/0.8104)	(0.2455/0.8938)	(0.0673/0.1546)	(0.0678/0.159)
CDX	0.0074***/***	0.0075***/***	0.0127***/***	0.0126***/***
	(0.0004/0.0015)	(0.0004/0.0015)	(0.0006/0.0014)	(0.0006/0.0015)
VIX	41.1653***/***	41.1929***/**	33.0365***/***	32.8229***/***
	(5.7257/15.8017)	(5.6241/16.4716)	(3.6158/8.0606)	(3.674/8.5242)
SP	0.0072*/−	0.0077**/−	-0.0144***/***	-0.0144***/***
	(0.0039/0.0104)	(0.0039/0.0108)	(0.0018/0.003)	(0.0019/0.003)
STS	-0.0002−/−	0.0002−/−	-0.009***/***	-0.0093***/***
	(0.0013/0.0061)	(0.0013/0.0062)	(0.0012/0.0022)	(0.0012/0.0023)
LIQ	-0.0008***/***	-0.0008***/***	-0.0021***/***	-0.0019***/***
	(0.0001/0.0001)	(0.0001/0.0001)	(0.0004/0.0003)	(0.0004/0.0003)
‘BBB’-rated	-0.0001−/−	-0.0002−/−	0.0027−/***	0.0029*/***
	(0.001/0.0003)	(0.001/0.0004)	(0.0017/0.0005)	(0.0017/0.0005)
‘BB’-rated	0.0017−/*	0.0012−/−	0.0122***/***	0.0129***/***
	(0.0013/0.001)	(0.0013/0.0011)	(0.0022/0.001)	(0.0023/0.0012)
‘B’-rated	0.0077***/***	0.0066***/***	0.0258***/***	0.0258***/***
	(0.0016/0.0014)	(0.0016/0.0015)	(0.0028/0.0023)	(0.003/0.0024)
‘CCC’-rated	0.0125***/***	0.0108***/***	0.0445***/***	0.0457***/***
	(0.0028/0.0032)	(0.0028/0.0034)	(0.0043/0.0051)	(0.0045/0.0056)
Sector Dummies	No	Yes	No	Yes
R^2	81.70%	83.18%	89.89%	90.18%
No. Entities	339		339	

Notes: This table shows the estimation results referring to the cross-section regressions (CSR) of Equation (3.3) under consideration of both systematic and individual risk factors. Systematic risk factors are the *Cross-market Correlation* (CMC), the CDX index, the VIX index, the *Spot Rate* (SR) and the *Slope of the Term Structure* (STS). Non-systematic or individual risk factors are represented by the *Swap Liquidity* (*LIQ*) and the firm's rating. Sector dummies account for the sector in which the firm is operating. The results are provided for each subsample based on weekly CDS spread data. The parameters are statistically significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). Values in parenthesis describe the parameters' standard deviation (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination.

For example: given a positive beta (β), we observe with respect to a positive gamma ($\gamma > 0$) that the CDS spread increases if the firm's sensitivity to that common risk factor increases.

In times of financial distress (GFC), all systematic risk sensitivities (TSR-betas) exhibit significant explanatory power to the cross-section of CDS spreads, regardless of whether the standard deviations of gamma estimates are Shanken corrected. This means that the contracts' sensitivities to the systematic risk proxies are significantly priced in CDS contracts across all economic sectors. Thereby, the signs of all gamma coefficients correspond to our expectations.

In the pre-crisis, we observe a slight mismatch between our expectation and empirical findings with respect to the interest risk proxies. Prior to the GFC, the gamma estimates indicate that a higher sensitivity to the *Spot Rate* leads to a spread increase, but the corrected t-statistic shows that these estimates are not significantly priced. The *Slope of the Term Structure* also lacks statistical significance in the pre-crisis, but becomes statistically significant in the crisis. Therefore, we infer that market participants view the STS as an indicator of economic wealth, which is particularly priced in economic downturns, but less relevant in moderate economic conditions.

The gammas of the CDX, the CMC and the VIX reach high statistical significance in both samples, irrespective of whether we control for firm-specific risks, sector dummies and Shanken-corrected t-statistics. Thus, market participants seem to demand a positive risk premium depending on the *Cross Market Correlation*, the *Credit Market Climate* and the *Market Volatility*, independent from the sample period.

As found in previous literature, liquidity is also an economically and statistically significant pricing determinant, which is contract-specific. The estimates of the *Swap Liquidity* proxy are statistically significant across all samples. Market participants appear to claim a risk premium for the market liquidity of the CDS. Thus, in the cross-section an increase of the *Swap Liquidity* leads to a decrease of swap spreads and vice versa.

Regarding the rating classes, our empirical results confirm our expectations and show that the CDS spreads monotonically increase with decreasing firm rating. Again, we observe a strong increase of basic spread levels across all rating classes during the GFC. This general increase in CDS spreads may be due to extremely high default rates of investment grade bonds in this period, which may have caused many rating downgrades of these financial instruments as well. Hence, we suspect that the firms' rating information significantly determines the CDS spread levels across both samples, correction methods and swap contracts. As expected, we conclude that a high-rated firm may benefit from its higher creditworthiness by receiving a reduction in its CDS spread (lower spread level).

Moreover, we find that our empirical results hold across all economic sectors examined, since the inclusion of sector dummies does not materially affect our estimation results. Thus, we conclude that the introduced risk factors have economy-wide impacts on the pricing of swap contracts, after controlling for sectoral influences.

While the entire ensemble of risk factors accounts for almost 90.18% of the spread variation during the GFC, the model's R^2 is clearly lower prior to the crisis (83.18%). Thus, we find that the explanatory power of the regressor ensemble depends on the sample period and that the regressors best fit CDS spreads in the crisis. This finding also indicates that systematic risk betas of CDS contracts are particularly priced in economic downturns in alignment with increasing statistical significances of our systematic risk proxies in the GFC.

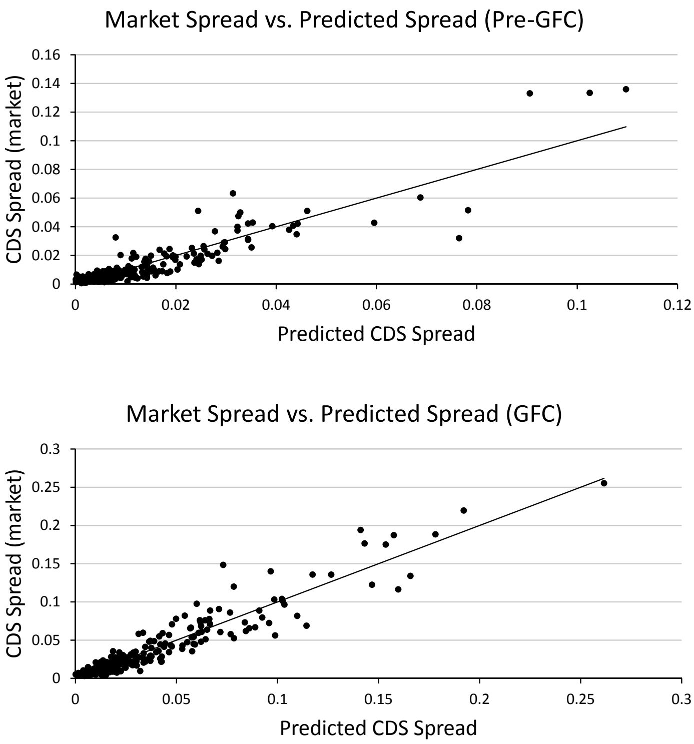
In Figure 3.4, we compare predicted CDS spreads with observed market spreads by sample in order to indicate the accuracy of our CSR model. The x-axes of the two charts denote the predicted CDS spreads and the y-axes denote the market CDS spreads.

Referring to regression (3.3), both scatter plots visualize the quality of our proposed CSR model. Since the spread predictions in the lower chart ($R^2 = 90.18\%$) are less scattered than in the upper one ($R^2 = 83.18\%$), we suggest that our CSR model – which explicitly addresses systematic risk – reaches the highest model accuracy in times of global financial distress.

3.3.4 Robustness

In the following, we extend our basic regression approach in several ways to show the robustness of our empirical findings. Firstly, we test whether our results hold, if the three Fama-French (FF) factors are included in our basic models. Thereby, we also examine if the FF factors provide additional explanatory power in the cross-section of swap spreads beyond the basic risk components (compare Fama and French, 1993). Secondly, we examine euro/dollar

Figure 3.4: CDS Spread Comparison (Market Spread vs. Model Spread)



Notes: This figure shows the comparison of market CDS spreads (y-axes) and model spreads (x-axes). The spread predictions are based on the estimation results related to the basic CSR model in Equation (3.3). While the upper chart refers to the period prior to the GFC (Pre-GFC), the lower chart shows the results for the GFC.

swap rates as alternate proxies for the *Spot Rate* and the *Slope of the Term Structure*. Thirdly, we conduct a principal component analysis (PCA) referring to the residuals of the multiple time-series regressions. The PCA may help to identify further potential candidates for systematic risk. Related to the PCA, we conduct new cross-sectional regressions in which we include the eigenvector of the first major component. By this, we test if this unknown systematic risk factor is cross-sectionally priced in the CDS spreads. Fourthly, we add the firm's *Leverage Ratio* and *Market Capitalization* as two additional firm-specific risk factors to the basic regression model. Lastly, we provide the regression results of the entire model in which all model extensions are simultaneously considered.[47]

Table 3.8 shows CSR results referring to the first three model extensions. Estimation results are presented for both sample periods (Pre-GFC and GFC) under consideration of sector and rating dummies. Respective standard deviations are reported in parentheses. Additionally, corrections for the standard deviations and significances are provided as suggested by Shanken (1992), which are separated by a slash.

In the first case (left column), the three Fama-French benchmark returns are included in the TSR. Afterwards, the estimated betas are added to the basic CSR model in Equation (3.3). The *Fama-French Excess Return* (FFR) describes the excess[48] return on the market, *Small Minus Big* (SMB) represents the performance of small stocks relative to big stocks, and *High Minus Low* (HML) denotes the performance of value stocks relative to growth stocks (compare Fama and French, 1993). Commonly, the Fama-French factors are used by investors seeking portfolio benchmark returns and by academics to explain the cross-section of stock returns.

We find that there is a negative relationship between the FFR and CDS spreads which is also statistically significant. This empirical result also follows economic intuition since positive excess returns may indicate a prosperous global economy with lower default risk in general. Thus, the CDS spreads should increase if the excess returns decrease and vice versa. In contrast to

[47] Not reported are robustness checks related to i) various window sizes of the cross-sectional regressions, e.g., rolling or fixed, and ii) other alternate proxies for, e.g., the *Slope of the Term Structure*, *Spot Rate* and *Cross-market Correlation*. These analyses lead to robust regression results.

[48] The excess return is defined as the difference between the return of the market portfolio (R_m) and the risk-less rate (r) (compare Fama and French, 1993).

the FFR, we observe shifts in the signs of estimators with respect to SMB and HML across samples. These sign changes make further interpretations somewhat difficult.

Table 3.8: Case-specific Cross-section Estimates

Case	Fama-French Factors		Alternate Interest Rates		Principal Component	
Sample	Pre-GFC	GFC	Pre-GFC	GFC	Pre-GFC	GFC
Intercept	0.0085***/***	0.0155***/***	0.0098***/***	0.0155***/***	0.0098***/***	0.0221***/***
	(0.0014/0.001)	(0.0036/0.0021)	(0.0017/0.0015)	(0.0037/0.002)	(0.0019/0.0017)	(0.0042/0.0029)
CMC	1.3524***/***	0.3823***/***	2.3556***/***	0.3015***/**	2.6067***/***	0.3237***/**
	(0.222/0.2979)	(0.0768/0.132)	(0.256/0.7025)	(0.0687/0.1521)	(0.2465/0.9009)	(0.0673/0.1578)
CDX	0.0062***/***	0.0114***/***	0.0073***/***	0.0121***/***	0.0075***/***	0.0126***/***
	(0.0004/0.0008)	(0.0007/0.0012)	(0.0004/0.0013)	(0.0006/0.0015)	(0.0004/0.0016)	(0.0006/0.0015)
VIX	32.3192***/***	27.8042***/***	49.522***/***	33.5933***/***	41.4919***/**	32.8809***/***
	(4.8401/9.7092)	(3.9545/6.1847)	(5.445/17.3408)	(3.6465/8.393)	(5.6819/16.6877)	(3.6428/8.5889)
SR (T-bills)	-0.0059*/−	-0.0152***/***			0.0076*/−	-0.0148***/***
	(0.0033/0.0051)	(0.0019/0.0033)			(0.0039/0.0108)	(0.0019/0.003)
STS (T-bills)	0.0047***/−	-0.0081***/***			0.0002−/−	-0.0096***/***
	(0.0013/0.0035)	(0.0012/0.0025)			(0.0013/0.0062)	(0.0012/0.0024)
SR (Swap Rate)			0.0025−/−	-0.0165***/***		
			(0.0042/0.0084)	(0.0018/0.0029)		
STS (Swap Rate)			-0.001−/−	-0.0093***/***		
			(0.0013/0.0058)	(0.0012/0.002)		
FFR	-0.065***/**	-0.075***/***				
	(0.0106/0.0265)	(0.0083/0.0257)				
SMB	0.0213**/*	-0.0128**/−				
	(0.0085/0.0124)	(0.0056/0.0105)				
HML	0.0151**/−	-0.0262***/***				
	(0.0066/0.0131)	(0.0033/0.0072)				
LIQ	-0.0007***/***	-0.0017***/***	-0.0008***/***	-0.0019***/***	-0.0008***/***	-0.002***/***
	(0.0001/0.0001)	(0.0004/0.0003)	(0.0001/0.0001)	(0.0004/0.0003)	(0.0001/0.0002)	(0.0004/0.0003)
PC 1					-0.0075−/−	-0.0863**/**
					(0.0189/0.0164)	(0.0338/0.04)
R^2	89.04%	90.57%	83.22%	90.08%	83.19%	90.38%

Notes: This table shows the estimation results referring to the cross-section regressions under consideration of three different cases: in the first case, three Fama-French Factors are added as regressor to the basic model of Equation (3.2). Thus, the explanatory power of betas related to the Fama-French excess return (FFR), *Small Minus Big* (SMB) and *High Minus Low* are tested in the presence of the basic risk factors (*Cross-market Correlation* (CMC), CDX index, VIX index, *Spot Rate* (SR), *Slope of the Term Structure* (STS) and *Swap Liquidity* (LIQ)). In the second case, runtime equivalent swap rates are considered in the basic CSR model instead of Treasury notes. In the third case, the basic model is expanded to the first component (PC 1) of the principal component analysis. In all three cases, dummy variables account for both the firm's rating and economic sector and the number of entities is 339. The results are provided for each sample (Pre-GFC and GFC). The parameters are statistically significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). Values in parenthesis describe the parameters' standard deviation (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination.

On the one hand, the R^2 increases from 83.19% to 89.04% through the consideration of the Fama-French factors in the pre-crisis. On the other hand, the R^2 remains on the same level with respect to the GFC (90.18% vs. 90.57%). Thus, we conclude that the Fama-French factors may increase the explanatory power of the basic model in times of moderate economic movements, but that the additional pricing information is strongly limited in times of an economic downturn.

In the second case (middle column), we examine the 5-year euro/dollar swap rate as alternate proxy for the *Spot Rate*, since some authors in the recent literature suggest swap rates as interest rate proxies rather than Treasury notes. Furthermore, the *Slope of the Term Structure* is now approximated by the difference of the 10-year swap rate and the 2-year swap rate. The new ensemble of systematic risk factors achieves similarly high R^2, whereas the gamma coefficients are roughly similar to those of the basic model. Since the coefficients of determination vary by less than 0.1% in each sample, we suggest that alternate interest rate proxies provide similar pricing information.

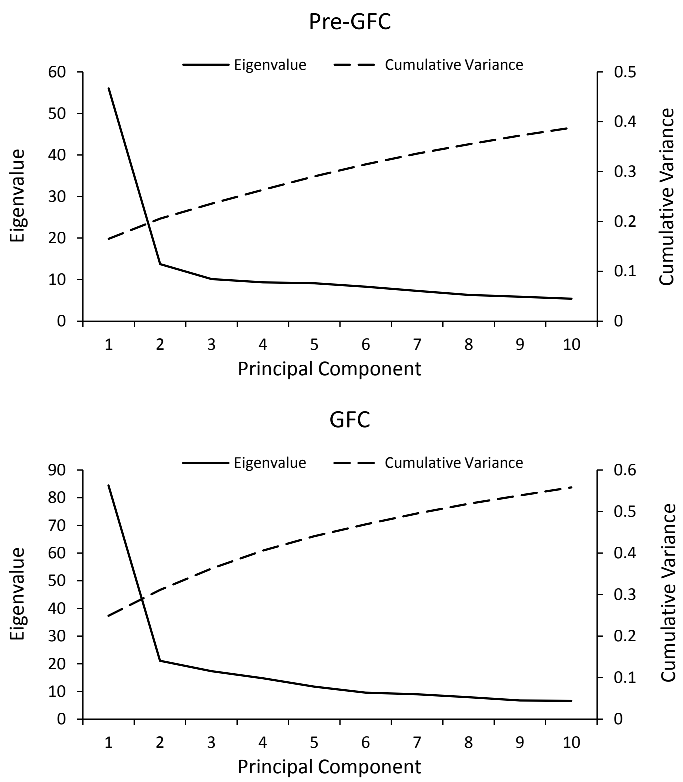
In the third case (right column), we conduct a principal component analysis (PCA) on the residuals of the multiple TSR to identify potential candidates for systematic risk omitted in this empirical study so far. By this, we examine if the TSR residuals are jointly driven by unknown systematic risk factors and we specify these principal components, similar to Collin-Dufresne et al. (2001). To test whether the specified principal components are priced by market participants in our CDS spreads cross-sectionally, we run subsequent second-pass regressions (CSR) in which we additionally include the eigenvector of the first major component.

Results of the PCA are plotted in Figure 3.5. While the upper chart shows the results of the PCA related to the Pre-GFC, the lower chart contains the PCA results for the GFC. The primary y-axes show the eigenvalues, the secondary y-axes denote the cumulative variance of identified components that are denoted on the x-axes.

Both charts demonstrate that the PCA leads to similar results in each sample. According to the scree test, the residuals of the time-series regressions are mainly driven by one major risk component that accounts for almost 17% of the cumulated variance prior to the GFC and for almost 25% during the GFC.

The right column of Table 3.8 summarizes the estimation results of the CSR after adding the first principal component. The results show that the influ-

Figure 3.5: Principal Component Analysis of Time-series Residuals



Notes: This figure shows the results of the principal component analysis (PCA) referring to the residuals of the time-series regressions. The PCA is provided for both subsamples. The upper chart refers to the Pre-GFC, the lower chart to the GFC. In each chart, the x-axis denotes the principal components and the primary y-axis reports the corresponding eigenvalues. The secondary y-axes show the cumulative variance of the principal components.

ence of the first component ($PC1$) is negative in both samples. Moreover, the principal component is significantly priced during the GFC, but not pre-crisis. Since the component is unknown, economic interpretations are somewhat difficult. But the coefficient indicates that there may be a source for systematic risk that is negatively correlated with CDS spreads. Thus, the swap spreads increase when the component's value decreases and vice versa.

Overall, the PCA indicates that there are some systematic drivers responsible for the shared variance of TSR residuals, but these drivers are not priced without cross-sectional restriction. Therefore, the use of the PCA is strongly limited. From the small pricing impact of the PCA component in combination with the relatively high explanatory power of our basic model framework, one may conclude that our valuation framework already considers the most important systematic as well as individual spread drivers and thus provides valuable insight into the pricing of swap contracts.

Table 3.9 reports the empirical results related to the CSR based on the last two model extensions. The estimation results refer to the Pre-GFC and the GFC. Standard deviations are reported in parentheses. The shanken-corrected standard deviations and significances are separated by a slash, see Shanken (1992).

Results reported in the left column refer to the basic model under consideration of two additional firm-specific factors: the firm's *Market Capitalization* (MC) and *Leverage Ratio* (LR). Independent from Merton's structural framework, we suppose that the firm's size somehow indicates the robustness of the firm against, e.g., economic downturns (compare Blume et al., 1998; Yan and Tang, 2007). We suggest that firms characterized by a large and well-diversified asset portfolio exhibit both a higher resistance to external shocks and a greater power to innovate (compare Porter, 1987; Hitt et al., 1996). Thus, we expect a positive risk premium for firms with lower market value for capital. Finally, we measure the firm size by the natural logarithm of the capital (compare Blume et al., 1998) and additionally calculate the book-to-market equity ratio based on a COMPUSTAT database.[49]

According to the structural theory, the default threshold is a function of outstanding debt claims. The higher the leverage, the higher the probability that the asset value processes pass the critical threshold. Hence, the default proba-

[49] Since the book-to-market equity ratio reaches no significance in our model framework, we focus solely on the firm's market capitalization.

Table 3.9: Cross-section Estimates Including Additional Risk Factors

Case	Firm-specific Risk Factors		Entire Model	
Sample	Pre-Crisis	GFC	Pre-Crisis	GFC
Intercept	$0.0029^{-/-}$	$0.0125^{-/*}$	$0.0026^{-/-}$	$0.0184^{**/**}$
	(0.0037/0.0024)	(0.0079/0.0071)	(0.0032/0.0016)	(0.0071/0.0089)
CMC	$2.8977^{***/***}$	$0.5783^{***/*}$	$1.8085^{***/***}$	$0.7574^{***/**}$
	(0.2552/1.0955)	(0.0775/0.3461)	(0.2538/0.3952)	(0.0817/0.3358)
CDX	$0.0056^{***/***}$	$0.0138^{***/***}$	$0.0052^{***/***}$	$0.0092^{***/***}$
	(0.0004/0.0017)	(0.0007/0.0032)	(0.0004/0.0012)	(0.0008/0.0024)
VIX	$48.5248^{***/**}$	$23.6418^{***/-}$	$50.5089^{***/***}$	$12.4364^{***/-}$
	(6.4512/19.9699)	(4.6039/14.6251)	(5.6951/15.5018)	(4.6586/9.3532)
SR (T-bills)	$0.0172^{***/-}$	$-0.0052^{-/-}$	$-0.0072^{*/-}$	$-0.0126^{***/*}$
	(0.0035/0.0207)	(0.0032/0.0057)	(0.0042/0.0083)	(0.0029/0.0069)
STS (T-bills)	$-0.0006^{-/-}$	$-0.0105^{***/*}$	$0.0058^{***/-}$	$-0.0045^{***/-}$
	(0.0016/0.0093)	(0.0015/0.0055)	(0.0016/0.0058)	(0.0014/0.0075)
FFR			$-0.1065^{***/**}$	$-0.0518^{***/-}$
			(0.0142/0.0522)	(0.0084/0.041)
SMB			$0.0035^{-/-}$	$-0.0299^{***/-}$
			(0.0141/0.022)	(0.0069/0.0285)
HML			$0.0206^{**/-}$	$-0.0402^{***/***}$
			(0.0083/0.0223)	(0.0037/0.0134)
LIQ	$-0.0008^{***/***}$	$-0.0017^{***/***}$	$-0.0008^{***/***}$	$-0.0007^{*/-}$
	(0.0001/0.0002)	(0.0005/0.0007)	(0.0001/0.0002)	(0.0004/0.0006)
MC	$0.0003^{-/-}$	$0^{-/-}$	$0.0002^{-/*}$	$-0.0002^{-/-}$
	(0.0003/0.0002)	(0.0006/0.0007)	(0.0002/0.0001)	(0.0005/0.0007)
LR	$0.0055^{**/***}$	$0.0034^{-/-}$	$0.0064^{***/***}$	$-0.0013^{-/-}$
	(0.0027/0.002)	(0.0054/0.0048)	(0.0023/0.0015)	(0.0045/0.0063)
PC 1			$0.0223^{*/-}$	$-0.0768^{***/*}$
			(0.013/0.0151)	(0.0274/0.0419)
Dummies (Rating & Sector)	Yes	Yes	Yes	Yes
R^2	91.29%	91.63%	93.87%	94.43%
No. Entities	225	225	225	225

Notes: This table shows the estimation results referring to the cross-section regressions under consideration of two different cases: in the first case, two more firm-specific risk factors – the firm's *Market Capitalization* and *Leverage Ratio* – are added to the basic CSR model of Equation (3.3). The *Entire Model* (case two) contains the basic risk factors (*Cross-market Correlation* (CMC), CDX index, VIX index, *Spot Rate* (SR), *Slope of the Term Structure* (STS) and *Swap Liquidity* (LIQ)), the three Fama-French factors (Fama-French excess Return (FFR), *Small Minus Big* (SMB) and *High Minus Low* (HML), additional firm-specific risk factors (*Market Capitalization* and *Leverage Ratio*) and the first component (PC 1) of the principal component analysis as further systematic risk factor. Dummy variables are included to account for both the firm's rating and economic sector. The results are provided for both samples (Pre-GFC and GFC) based on weekly CDS spread data of 225 entities. The parameters are statistically significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). The values in parenthesis describe the parameters' standard deviations (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination.

bility increases with increasing leverage. Therefore, we may expect a positive relationship between the leverage ratio and the observed CDS spread. Among others, Welch (2004) found that stock returns capture changes in leverage appropriately. Approximated by stock returns, Avramov et al. (2007) identified leverage as a main driver for credit spread changes. We approximate leverage by the following leverage ratio

$$\frac{\text{Book Value of Debt}}{\text{Market Value of Equity} + \text{Book Value of Debt}}$$

to proxy the firm's health according to Collin-Dufresne et al. (2001). Respective data is provided by COMPUSTAT. As this analysis requires additional data from COMPUSTAT the number of entities is reduced to 225.

According to the left column of Table 3.9, our main results hold with respect to the inclusion of these two firm-specific variables.[50] We find that the MC does not provide significant explanatory power – neither prior to the GFC or during. By contrast, the firm's LR constitutes a significant pricing determinant in moderate economic conditions which also corresponds to economic expectations: across all economic sectors an increase in the firm's leverage leads to an increase in the swap's risk premium. Overall, the inclusion of these firm-specific risk factors leads to an increase of the R^2 from 83.18% to 91.29% in the Pre-GFC, and causes relatively small benefits in times of the GFC, where the R^2 increases from 90.18% in the basic model to 91.63% in the extended model.

To check whether the effect sizes related to each model extension are complementary or not, we estimate the last model case in which the basic two-pass approach is simultaneously extended to the Fama-French factors, the *Leverage Ratio*, the *Market Capitalitzation* and the first principal component.[51] The respective regression results are reported in the right column of Table 3.9.

With respect to the models R^2, the entire ensemble of risk factors accounts for almost 94% of the CDS spread variation in both samples, which is highest compared to the R^2 of all other regression models.[52] We find that the main results are confirmed in the *Entire Model*: again, the OLS regression results show that all estimates of the systematic risk variables reach statistical signifi-

[50] Slight differences may be due to the lower amount of entities in this model setup.
[51] Here, Treasury notes constitute the reference interest rates.
[52] Note that the models' R^2 are not directly comparable with each other due to different numbers of entities in the data sets.

cance, independent from the sample period.[53] Thus, all systematic risk proxies are significantly priced in the cross-section of CDS spreads. Apart from the *Slope of the Term Structure*, all of these variables additionally meet economic expectations. Despite the fact that time-series characteristics of systematic risk variables vary by sample in terms of both their means and standard deviations (compare Figure 3.2), the quality of the *Entire Model* is almost identical in both samples. Since the *Entire Model* exhibits high explanatory power independent from the sample, this regression model is robust for subsampling.

Each case-specific model extension confirms the results of the basic approach. Thus, we identify the *Credit Market Climate* (CDX), the *Cross Market Correlations* (CMC) and the *Market Volatility* (VIX) as the most important systematic risk factors in the cross-sectional pricing process of swap contracts. The corresponding risk sensitivities (betas) are positively priced across all samples and model cases. This result indicates a positive correlation between these risk proxies and the cross-section of credit spreads. We find that CDS spreads significantly rise if one of these risk factors increases and vice versa which is in line with economic expectations. The applied model extensions may help to increase the model's explanatory power particularly with respect to moderate economic conditions. Additionally, we confirm liquidity as a further decisive determinant in pricing swap contracts. Corresponding to expectations, the contract's liquidity reveals a negative relationship to the CDS spread in both samples and we observe significant negative gamma estimates for all models. Hence, the results show that the contract's sensitivity to liquidity risk is compensated by a respective premium widening the spread if the liquidity of the contract decreases. Referring to the rating classes, the estimates are statistically significant in most cases. The empirical results show that market participants claim a higher risk premium for investing in low-rated swap contracts reflecting a lower creditworthiness of the rated obligor. This risk premium increases monotonically with rating classes and is paid in the cross-section of CDS spreads. All these findings hold, regardless of whether we account for the economic sector in which the firm is operating.

We conclude that systematic risk generally affects spreads of swap contracts relying on debt assets. We demonstrate that specific systematic risk variables

[53] The Shanken-corrected t-value of the VIX is not statistically significant in this model setup. Such distortions may generally be due to i) the lower amount of entities, ii) the higher number of regressors or iii) effects of multi-collinearity.

such as the *Credit Market Climate*, the *Cross-market Correlation* and the *Market Volatility* may play a major role in pricing credit default swaps. We find that the systematic risk exposures of CDS contracts vary by rating class and even within each rating class. We further show that these systematic risk exposures are priced beyond ratings. The explanatory power of our systematic risk determinants may generally vary by regressor and by sample, and we find that the influence of most systematic risk factors increases in economic downturns.

Overall, we argue in this empirical study from both an economic and a statistical perspective in order to demonstrate the relevance of provided systematic risk factors for pricing CDS contracts. The results hold while controlling for major firm-specific risk factors, other systematic risk proxies and sectoral influences.

3.4 Conclusion

The recent Global Financial Crisis has shown that macroeconomic shocks, e.g., caused by the U.S. housing crisis, may have a strong impact on global financial markets, particularly on the credit markets. Indeed, many credit market participants suffered from unexpectedly high default rates on corporate bonds or related financial instruments such as credit default swaps or collateralized debt obligations.

We find that most betas of our systematic state variables are significantly priced in each sample, given firm-specific risk variables and sector dummies. Our basic ensemble of risk factors explains about 83% of the cross-section of CDS spreads before the crisis and about 90% during the crisis. Thus, systematic risk seems to be priced in economic downturns. Moreover, we identify the firm's rating, *Leverage Ratio* and the contract-specific *Swap Liquidity* as the most important individual risk factors in pricing swap contracts. While the firm's rating is mostly significantly priced and its gamma estimates correspond to economic expectations, those of other risk factors, such as the *Market Capitalization* are not. Results related to the firm's *Leverage Ratio* are plausible from an economic point of view in both samples and this proxy is also significantly priced prior to the GFC.

Related to our systematic risk factors, we find that the sensitivity to the *Credit Market Climate* – approximated by the 5-year CDX NA IG credit index spread – significantly influences the cross-section of CDS spreads. From an economic perspective, we observe a positive sensitivity of CDS spread changes to changes in the CDX which leads to a positive risk premium in the contracts' cross-section. If the credit climate worsens, the CDS spreads increase significantly and vice versa. Hence, our empirical findings show that investors on CDS markets are monetarily compensated for this kind of common risk.

Furthermore, we find that the suggested *Cross-market Correlation* also significantly explains CDS spreads. To approximate the prospects of risk diversification across, e.g., stock, credit and exchange markets, we calculate the average cross-correlation related to specified markets. Both beta and gamma estimates also satisfy economic expectations: the higher (lower) the cross-market correlation the higher (lower) is the related systematic risk since market participants are more (less) constrained in their diversification efforts. Thus, we observe increasing CDS spreads in line with an increasing *Cross-market Correlation* (positive pricing effect) due to a positive sensitivity of CDS spreads to cross-correlation movements.

With the VIX index – indicating the *Market Volatility* – we identify another important determinant for the valuation of systematic risk in CDS spreads. Positive beta as well as gamma estimates, which are also statistically significant, confirm our theoretical expectations and suggest that market participants are positively rewarded for the market risk expressed through the volatility on stock markets. We find that if the volatility on stock markets is high (low) swap investors may receive a high (low) risk premium included in the CDS spread.

In order to check the robustness of our empirical findings, we provide further checks: to account for parameter estimation risk related to our two-pass regression approach, we provide corrected t-statistics for the gamma estimates, as proposed by Shanken (1992). Moreover, we extend our analysis to the Fama-French Factors (Fama and French, 1993). In both samples, the model accuracy increases in terms of the coefficient of determination (R^2), but this effect is particularly observable in moderate economic conditions. The inclusion of swap rates instead of Treasury notes in order to approximate the interest rate risk in terms of the *Spot Rate* and the *Slope of the Term Structure* leads to R^2, which are similarly as high as in the basic model. Eventually, the R^2 do not differ more than 0.1% in total. Therefore, we conclude that swap rates provide comparable pricing information to Treasury notes. Through a principal component analysis we identify at least one major component responsible for the shared variance of TSR residuals. We find that this principal component is significantly priced in CDS spreads across all entities during the GFC, but not prior to the crisis. Eventually, the results related to each model extension show that our main empirical findings hold, irrespective of the presence of these additional risk factors or proxy alternatives.

Apart from our findings, further research is suggested in other systematic risk variables such as market recovery risk, or counter-party risk since both factors may represent other relevant determinants of CDS spreads omitted in this study (compare Brigo and Chourdakis, 2009; Arora et al., 2012). Thereby, both risk variables may be evaluated either referring to credit markets in general (systematic) or explicitly as swap-specific risk factors.

In summary, our empirical study provides a valuable insight into the valuation of systematic risk in CDS spreads. We suggest that at least three of our systematic risk factors reflect decisive determinants in pricing credit default swaps in line with economic expectations. These systematic determinants may also play a decisive role in the valuation of synthetic CDOs since this type of asset securitization consists of CDS contracts. Thus, our empirical study shows the impact of systematic risk on the valuation of swap contracts, and the potential for further research in the valuation of structured securities.

Chapter 4

What Wags the Tail? How Parameter Errors Affect Risk Measures in Credit Models

The content of this chapter refers to the working paper 'What Wags the Tail? How Parameter Errors Affect Risk Measures in Credit Models' by Claußen, A., and Rösch, D., 2014.

4.1 Introduction

In the last decades, risk management has become one of the most important subjects of interest for financial institutions. *Risk*, according to Knight (1921), can be defined as random variation that follows a known probability law. Thus, by definition, risk is described by a probability distribution. When this probability distribution is complex, risk cannot be summarized by one or a few key numbers such as its moments. In practice, however, often only one key number is reported for the ranking of risk (e.g. rating) or the calculation of regulatory capital. To enable this approach *risk measures* have been introduced, which map risk into a single number. An example of one of the earlier risk measures is the variance (or volatility), which has for instance been used in modern portfolio theory as described by Markowitz (1952). Because the variance is a symmetric measure, it considers negative as well as positive deviations from expected values. While positive deviations are often not perceived as risk, the Value-at-Risk (VaR) has become a popular downside risk measure during the nineties, and is still one of the most frequently used measures in practice.

However, the VaR has also been widely criticized. The VaR is neither convex nor sub-additive in the general distribution case, and is therefore not a coherent risk measure in the sense of Artzner et al. (1999). Moreover, the VaR may exhibit multiple local extremes for discrete distributions (e.g. Mausser and Rosen, 1998) and is therefore hard to be optimized in these cases. Finally, the VaR is merely a percentile of a probability distribution, and therefore does not take into account any tail information beyond VaR. In contrast, the conditional Value-at-Risk (cVaR), also referred to as Expected Shortfall, has the property of coherence (e.g. Acerbi and Tasche, 2002; Frey and McNeil, 2002; Tasche, 2002), and is convex and easily optimized as shown by Rockafellar and Uryasev (2000). Moreover, cVaR considers tail-risk by definition. This risk measure has therefore become the favored risk measure in academia, and is the second most popular risk measure in practice today. Indeed, the Basel Committee on Banking Supervision recommends to replace the 99% VaR with the 97.5% cVaR in internal market risk models and has also used the 97.5% cVaR to calibrate capital requirements under the revised market risk standardized approach. However, the committee still proposes a 99.9% VaR for the incremental capital charge for default risk to maintain consistency with the banking book treatment (BCBS, 2012, 2013).

There is ongoing discussion about which risk measures are appropriate, and whether the cVaR is superior to the VaR. Current debates focus on diversification, aggregation, economic interpretation, extreme behavior, robustness and backtesting of VaR and cVaR. For an excellent overview of recent literature see Embrechts et al. (2013) and Emmer et al. (2013).

VaR, even though it is not coherent, is more easily backtested and generally more robust; in contrast, the cVaR is less robust, harder to backtest (not elicitable) and requires a larger sample size than the VaR to provide the same level of accuracy (Yamai and Yoshiba, 2005; Cont et al., 2010; Gneiting, 2011). Embrechts et al. (2014) have recently shown that the cVaR fulfills a new notation of robustness (*aggregation-robustness*) that is not fulfilled by the VaR. Emmer et al. (2013) present an alternative method for backtesting cVaRs and conclude that despite the caveats that apply to the estimation and backtesting of cVaR as well as the requirement of larger sample size, it can be considered as a good risk measure and seems to be the best for use in practice.

In this paper we analyze the specific sensitivity behavior of the VaR and cVaR within the popular Asymptotic Single Risk Factor credit model that underlies the Basel Accord (which is used by banks to determine their regulatory capital for default risk in the banking book under the Internal Rating Based (IRB) Approach). We understand *sensitivity* as the effect of errors in the model parameters on these two risk measures. This is also known as *quantile sensitivities*. Hong (2009) presents a way of estimating quantile sensitivities by a conditional-expectation form, especially when the derivatives cannot be derived analytically. Further he gives a good literature review over quantile estimation and gradient estimation. An analysis for the VaR and cVaR has been studied by Hong and Liu (2009) and Fu et al. (2009). More recently Hong et al. (2014) developed faster estimators for select portfolio credit risk model. In our paper we derive quantile sensitivities analytically. We will show that the cVaR - despite its theoretical advantages - can be more sensitive than the VaR, and the behavior depends in particular on the default probabilities of the loans in the portfolio as well as on the chosen confidence level α.

Our paper is related to work by Yamai and Yoshiba (2002), who analyze credit risk via Monte Carlo simulation and find that using the cVaR requires a larger sample size than VaR to achieve the same level of accuracy. Our study differs in several aspects and includes more detailed analyses.

As measures for vulnerability to parameter errors, we analytically calculate and analyze the partial derivatives of the VaR and cVaR with respect to the model parameters. We then formulate and analytically solve a robust optimization problem that incorporates these estimation errors, and analyze the result as a relative add-on for estimation error. This add-on can be interpreted as the factor by which capital (if computed via the risk measure) should be multiplied to satisfy a decision-maker who is adverse to (parameter) uncertainty.

We support our theoretically derived results with an empirical study using default data from Moody's rating agency, covering a 43-year period of defaults from 1970 to 2012. We provide the following three main contributions to the discussion about the superiority of the cVaR over the VaR:

First, we show that the credit model cVaR - even though it has theoretical advantages - can be more vulnerable to parameter errors than the VaR. Because our credit model is used in the Basel IRB approach, a shift from VaR to cVaR, as proposed for risks in the trading book (BCBS, 2013), would lead to higher impacts of parameter errors in the banking sector.

Second we find, that with a lower probability of loan default (PD) (or a better rating grade) the (relative) effects of parameter errors increase for both the VaR and the cVaR. Especially, for Investment Grade rated (IG-rated) risk buckets, the cVaR reacts stronger than the VaR.

Third, increasing the confidence level α leads to a higher (relative) impact of estimation errors, particularly for IG-rated risk buckets.

Thus, trying to be safer in a common sense (low PD, high confidence level α, choice of tail-considering risk measure cVaR) increases the effects of estimation errors.

4.2 Credit Model, Parameter Errors and Risk Measure Sensitivities

4.2.1 The Credit Model

Our analysis focuses on the Asymptotic Single Risk Factor (ASRF) credit model, which has become a standard credit portfolio model in the banking industry. It underlies the Basel Accord by which banks determine their regulatory capital under the IRB Approach; the ASRF is also used by banks and researchers as a ‘quick and dirty’ approach to calculate and measure general economic capital and credit portfolio risk. The foundation and derivation of the model is given by Vasicek (1987) and Gordy (2000, 2003). It is appealing because of its simplicity, its analytical tractability, its economic intuition, and its potential to model skewed loss distributions. As shown in Gordy (2000), the model can also easily be mapped onto other popular industry credit models. A large number of more complex models have been proposed, but considering them all is beyond the scope of our current study. However, even though our results are derived from a relatively simple model, we conjecture that they will prove to be robust even when compared with more complex models that include additional parameters.

The ASRF credit model assumes an infinitely fine-grained homogeneous portfolio of loans or bonds, with the risk being driven by a single common, systematic risk factor; idiosyncratic risk disappears owing to full diversification.

The distribution of credit portfolio loss $L(\cdot)$ in a given period is modeled by

$$L(Y,\rho,\pi) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho}\cdot Y}{\sqrt{1-\rho}}\right), \qquad Y \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{4.1}$$

where $\pi \in (0,1)$ is an unconditional probability of loan default (PD), $\rho \in (0,1)$ is the asset (return) correlation, Y is a standard normally distributed common systematic risk factor, and Φ is the standard normal CDF (with Φ^{-1} denoting its inverse).[54]

The CDF and PDF for (4.1) are derived in Vasicek (1991) and are given by the so-called 'Vasicek-distribution' with density

$$v(\ell) = \sqrt{\frac{1-\rho}{\rho}}\cdot \exp\left(-\frac{\left(\sqrt{1-\rho}\cdot\Phi^{-1}(\ell) - \Phi^{-1}(\pi)\right)^2}{2\cdot\rho} + \frac{\left(\Phi^{-1}(\ell)\right)^2}{2}\right)$$

for realizing loss $\ell \in (0,1)$ and CDF

$$V(\ell) = \mathbb{P}(L^{LHP}(Y,[\pi,\rho]) \le \ell) = \Phi\left(\frac{\sqrt{1-\rho}\cdot\Phi^{-1}(\ell) - \Phi^{-1}(\pi)}{\sqrt{\rho}}\right).$$

Depending on the parameter constellation the loss distribution is unimodal for $\rho < 0.5$ with the mode at

$$\Phi\left(\frac{\sqrt{1-\rho}}{1-2\rho}\cdot\Phi^{-1}(\pi)\right),$$

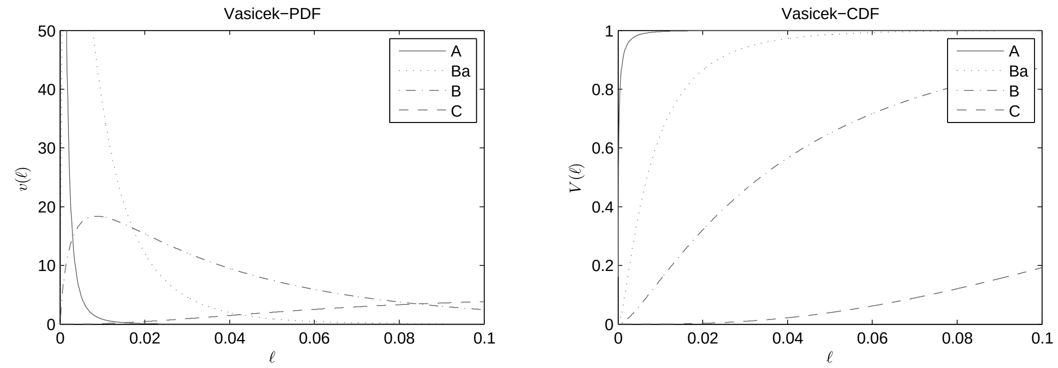
and large positive skewness, monotone for $\rho = 0.5$ and U-shaped for $\rho > 0.5$.

As an example for different loss profiles described by this specific distribution, we use point estimates from our empirical analysis given in Section 4.3.2. The solid line in Figure 4.1 is the shape of its PDF and CDF of 'A' rated risk bucket with estimates $[\hat{\rho}_A, \hat{\pi}_A] = [22.11\%, 0.05\%]$. The dotted line represents a 'Ba' portfolio with $[\hat{\rho}_{Ba}, \hat{\pi}_{Ba}] = [12.15\%, 1.05\%]$ and the dash-dot (respectively dashed) line illustrates the PDF and CDF of a 'B' (respectively 'C') risk bucket with estimates $[\hat{\rho}_B, \hat{\pi}_B] = [18.79\%, 4.95\%]$ (respectively $[\hat{\rho}_C, \hat{\pi}_C] = [14.26\%, 19.53\%]$).

All estimates of asset (return) correlations in Figure 4.1 are smaller than

[54] To simplify the notation, we skip the parameter of recovery rate RR by setting $RR = 0$. Therefore, if default occurs, the lender suffers a loss of 100%. If one wanted to consider a nonzero recovery rate, the risk measures derived below could be modified with the factor $(1 - RR)$.

Figure 4.1: Shape of Vasicek-distribution



Notes: This figure shows the shape of the PDF and CDF of the Vasicek-distribution. Point parameter estimates are resulting from a maximum-likelihood estimation of historical default data from Moody's from 1973-2012: $[\hat{\rho}_A, \hat{\pi}_A] = [22.11\%, 0.05\%]$, $[\hat{\rho}_{Ba}, \hat{\pi}_{Ba}] = [12.15\%, 1.05\%]$, $[\hat{\rho}_B, \hat{\pi}_B] = [18.79\%, 4.95\%]$ and $[\hat{\rho}_C, \hat{\pi}_C] = [14.26\%, 19.53\%]$.

50% and therefore all loss distributions are unimodal with positive skewness. For the 'Ba' risk bucket, for instance, the expected loss is 4.95%, and the mode equals 0.8629%. Due to the positive skewness, although small losses are more likely in absolute terms, a wide range of larger losses than the expected loss is also likely. This is captured by the respective $\text{VaR}_{99.9\%} = 9.43\%$ or $\text{cVaR}_{99.9\%} = 11.39\%$.

In this credit model the VaR and cVaR can be expressed analytically for confidence level $\alpha \in (0, 1)$:

$$VaR(\rho, \pi, \alpha) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right), \tag{4.2a}$$

$$cVaR(\rho, \pi, \alpha) = \frac{1}{1-\alpha}\Phi_2\left(\Phi^{-1}(\pi), \Phi^{-1}(1-\alpha), \sqrt{\rho}\right), \tag{4.2b}$$

where $\Phi_2(x_1, x_2, \varrho)$ is the cumulative standard bivariate normal distribution function given by

$$\Phi_2(x_1, x_2, \varrho) = \frac{1}{2\kappa\sqrt{1-\varrho^2}} \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} \exp\left(-\frac{x_1^2 - 2\varrho x_1 x_2 + x_2^2}{2(1-\varrho^2)}\right) dx_2 dx_1$$

and κ is the irrational ratio of a circle's circumference to its diameter,[55] and ϱ is the correlation between two standardized normally distributed random

[55] We introduce this notation because we use the variable π for the PD.

variables $X_i \sim N(0,1)$, $i \in 1,2$.

Generally speaking, these numbers return the level of loss which is not exceeded with a given (high) confidence probability (VaR), or the expected loss given that the VaR is exceeded (cVaR). Although the risk measures are defined $\forall \alpha \in (0,1)$ we focus our analysis on the case $0.5 < \alpha < 1$ as is common in theory and practice. Then it is obvious that for $\pi \to 0$ ($\pi \to 1$) the VaR and the cVaR converge to zero (one). If $\rho \to 0$, the VaR and cVaR converge to π. For the VaR, Höse and Huschens (2008) show that for $\rho \to 1$, the VaR converges to 0 if $\pi < 1-\alpha$, 0.5 if $\pi = 1-\alpha$ and 1 if $\pi > 1-\alpha$. Using the results from Meyer (2013), it follows that the cVaR approaches the upper Fréchet-Hoeffding bound for $\rho \to 1$ and goes to

$$\frac{\min(\pi, 1-\alpha)}{1-\alpha}.$$

Obviously, both risk measures are nonlinear functions in the parameters ρ, π and α. E.g. a fixed confidence level of 99.9%, $\rho = 20\%$ and $\boldsymbol{\pi} = [0.1\%, 1\%, 5\%]$ yields $VaR(20\%, \boldsymbol{\pi}, 99.9\%) = [2.81\%, 14.55\%, 38.44\%]$. If ρ is changed to 25% (e.g. due to an alternative estimation procedure) the VaRs increase to $VaR(25\%, \boldsymbol{\pi}, 99.9\%) = [3.72\%, 18.35\%, 45.42\%]$ (which are multiples of [1.32, 1.26, 1.18]). Thus, the relative change of the VaR increases with declining PD.

In any empirical and practical application of risk models, some values for the unknown parameters have to be specified and inserted. These values can be derived in different ways or combinations thereof; a simple way to make the models 'go live' in practice, is to make assumptions about the parameter values, which can be inferred from past experiences with the models or expert judgments and opinions. For this, no statistical technique and database is required. Another method is to calibrate the models using current or past market data from traded securities, such as bonds or credit derivatives. For the correlation, one could, for example, use spreads of tranches of collateralized debt obligations, which are actually financial contracts that trade assumptions about credit correlations. A weakness of this calibration method is that it requires assumptions about the market (such as absence of arbitrage), and allows calibration of risk-neutral measures only (rather than physical or real-world measures which are required for the VaR and cVaR). Finally, one can make use of databases with historical data, such as default or loss rates from loans or bonds, and estimate the real-world parameters using statistical-econometric techniques. For this, usually a rich history of data is required, which may not be readily available if a bank wants to infer the parameters for its own portfolio with sparse data only. All of these approaches make assumptions, and the chosen parameter values may not match the true underlying and unknown parameters. In other words, empirical approaches for making credit risk models 'go live' are prone to parameter errors. In some instances, such as when parameter estimates are derived from historical data, the potential degree of error can be derived by computing confidence intervals using statistical theory. We will provide an example for this in Section 4.3. Generally, however, parameters are measured with error, and it is hard to quantify the extent of these errors. Thus, in the next section we analyze how the outcome of the risk model (i.e. the VaR or the cVaR) is affected by parameter errors when no specific assumptions are made about their potential magnitudes.

4.2.2 Sensitivities of Risk Measures

As our first main result, we show that given an α-quantile, the cVaR can be more sensitive to errors in ρ and π than the VaR. This is particularly likely for specific parameter constellations $[\rho, \pi]$ that are usually associated with IG-rated risk buckets. To derive these results, we compute the partial derivatives of $VaR(\rho, \pi, \alpha)$ and $cVaR(\rho, \pi, \alpha)$ with respect to $\rho, \pi \in (0, 1)$. The partial derivatives with respect to $\alpha \in (0, 1)$ are also computed and will be used in Section 4.2.3.

The partial derivative of the $VaR(\rho, \pi, \alpha)$ with respect to ρ follows by using the chain and quotient rule,

$$\begin{aligned}\frac{\partial}{\partial \rho} VaR(\rho, \pi, \alpha) = & \frac{\sqrt{\rho} \cdot \Phi^{-1}(\pi) - \Phi^{-1}(1-\alpha)}{2\sqrt{\rho(1-\rho)^3}} \\ & \cdot \phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right)\end{aligned} \tag{4.3}$$

and is also given in Höse and Huschens (2008). For its partial derivative with respect to π, we apply again the chain rule leading to

$$\begin{aligned}\frac{\partial}{\partial \pi} VaR(\rho, \pi, \alpha) = & \frac{1}{\phi(\Phi^{-1}(\pi)) \cdot \sqrt{1-\rho}} \\ & \cdot \phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) > 0,\end{aligned} \tag{4.4}$$

where $\frac{\partial}{\partial x}\Phi^{-1}(x) = \phi(\Phi^{-1}(x))^{-1}$ follows by the inverse function theorem. Using this theorem as well as the quotient rule leads to the partial derivative of the VaR with respect to α:

$$\begin{aligned}\frac{\partial}{\partial \alpha} VaR(\rho,\pi,\alpha) = &\frac{\sqrt{\rho}}{\sqrt{1-\rho}\cdot\phi(\Phi^{-1}(1-\alpha))}\\ &\cdot \phi\left(\frac{\Phi^{-1}(\pi)-\sqrt{\rho}\cdot\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) > 0.\end{aligned} \tag{4.5}$$

The partial derivative of the $cVaR(\rho,\pi,\alpha)$ with respect to ρ is given by[56]

$$\begin{aligned}\frac{\partial}{\partial \rho} cVaR(\rho,\pi,\alpha) = &\frac{\phi\left(\Phi^{-1}(1-\alpha)\right)}{(1-\alpha)\cdot 2\sqrt{\rho}\cdot\sqrt{1-\rho}}\\ &\cdot \phi\left(\frac{\Phi^{-1}(\pi)-\sqrt{\rho}\cdot\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) > 0\end{aligned} \tag{4.6}$$

by using the general result in Plackett (1954) for the partial derivative of the bivariate normal distribution with respect to ρ

$$\frac{\partial}{\partial \rho}\Phi(x_1,x_2,\rho) = \frac{1}{2\kappa\sqrt{1-\rho^2}}\exp\left(-\frac{x_1^2-2\rho x_1 x_2 + x_2^2}{2\sqrt{1-\rho^2}}\right). \tag{4.7}$$

For the partial derivative of the cVaR with respect to π and α, we use the partial derivative of $\Phi_2(x_1,x_2,\varrho)$, with respect to x_1 or x_2 which can e.g. be found in Meyer (2013) leading to

$$\frac{\partial}{\partial \pi} cVaR(\rho,\pi,\alpha) = \frac{1}{1-\alpha}\cdot\Phi\left(\frac{\Phi^{-1}(1-\alpha)-\sqrt{\rho}\cdot\Phi^{-1}(\pi)}{\sqrt{1-\rho}}\right) > 0 \tag{4.8}$$

and

$$\frac{\partial}{\partial \alpha} cVaR(\rho,\pi,\alpha) = \frac{1}{1-\alpha}\cdot\big(cVaR(\rho,\pi,\alpha) - VaR(\rho,\pi,\alpha)\big) > 0. \tag{4.9}$$

From these partial derivatives we can infer some important properties of VaR and cVaR. We can approximate the effect of parameter changes (e.g. due to estimation errors) and, in particular, we can demonstrate the higher sensitivity of the cVaR compared with the VaR for changes in ρ.

The partial derivative with respect to ρ of the VaR for the above example

56 After the application of (4.7), we have added $\rho\cdot\Phi^{-1}(1-\alpha)^2 - \rho\cdot\Phi^{-1}(1-\alpha)^2 = 0$ in the numerator for further simplifications.

($\alpha = 99.9\%$, $\rho = 20\%$ and $\boldsymbol{\pi} = [0.1\%, 1\%, 5\%]$) equals according to (4.3) $\frac{\partial}{\partial\rho}VaR(20\%, \boldsymbol{\pi}, 99.9\%) = [\ 17.19\%,\ 73.18\%,\ 140.57\%]$, which is in relation to the VaR a multiple of

$$\frac{\frac{\partial}{\partial\rho}VaR(20\%, \boldsymbol{\pi}, 99.9\%)}{VaR(20\%, \boldsymbol{\pi}, 99.9\%)} = [6.12, 5.03, 3.66].$$

So, particularly for a low PD, the relative impact of parameter changes on the VaR is high. An approximation of the effect of parameter changes on the partial derivative can be computed via a first-order Taylor Series Expansion:

$$\begin{aligned} VaR(25\%, \boldsymbol{\pi}, 99.9\%) &\approx VaR(20\%, \boldsymbol{\pi}, 99.9\%) + 5\% \cdot \frac{\partial}{\partial\rho}VaR(20\%, \boldsymbol{\pi}, 99.9\%) \\ &\approx [3.67\%, 18.21\%, 45.47\%]. \end{aligned}$$

For $\pi_1 = 0.1\%$ and $\pi_2 = 1\%$ the approximation undervalues the true VaR, while for $\pi_3 = 5\%$ it overvalues the VaR. However, in the presented cases, the relative errors are therefore rather small (-1.43%, -0.76% and 0.122%, respectively).

Next, note that the cVaR is monotonously increasing in all three parameters ρ, π and α. In contrast, the VaR is only monotonously increasing in π and α, but not in ρ. Höse and Huschens (2008) argue that it is a common misunderstanding of the ASRF credit model that a higher asset correlation always increases the economic capital (for which the VaR is often used).[57]

Höse and Huschens (2008) show that if $0.5 < \alpha < 1$ and $0 < \pi < 1 - \alpha$

$$\frac{\partial}{\partial\rho}VaR(\rho, \pi, \alpha) \begin{cases} > 0 & \text{if} \quad 0 < \rho < \rho_{\max} \\ = 0 & \text{if} \quad \rho = \rho_{\max} \\ < 0 & \text{if} \quad \rho_{\max} < \rho < 1, \end{cases} \tag{4.10}$$

where $0 < \rho_{\max} = \left(\frac{\Phi^{-1}(1-\alpha)}{\Phi^{-1}(\pi)}\right)^2 < 1$ is the 'worst-case' correlation that maximizes the VaR. As seen from (4.10), if $0 < \pi < 1 - \alpha$ and ρ is larger than $\rho_{\max} < 1$, the VaR *decreases* for any further *increase* of ρ. Thus, particularly in the cases where the VaR decreases by increasing ρ, the cVaR is more (positively) sensitive because it is monotonously increasing in $\rho \in (0, 1)$.

[57] See e.g. Basel II Correlation Values - An Empirical Analysis of EL, UL and the IRB Model, Fitch Ratings 2008, "Adjusting the correlation values is a policy lever for regulators to achieve desired capital outcomes. For example, by increasing correlation assumptions, regulators are able to increase overall Basel II capital requirements."

Next, we generalize this for a smaller ρ. For this, we compute the ratio of $\frac{\partial}{\partial\rho}cVaR(\rho,\pi,\alpha)$ and $\frac{\partial}{\partial\rho}VaR(\rho,\pi,\alpha)$. After canceling down

$$\phi\left(\frac{\Phi^{-1}(\pi)-\sqrt{\rho}\cdot\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right)>0$$

and making some minor simplifications, we define

$$q_\rho(\rho,\pi,\alpha):=\frac{\frac{\partial}{\partial\rho}cVaR(\rho,\pi,\alpha)}{\frac{\partial}{\partial\rho}VaR(\rho,\pi,\alpha)}=\frac{\frac{1-\rho}{1-\alpha}\cdot\phi(\Phi^{-1}(1-\alpha))}{\sqrt{\rho}\cdot\Phi^{-1}(\pi)-\Phi^{-1}(1-\alpha)}. \tag{4.11}$$

If this ratio is larger (smaller) than one the cVaR is more (less) sensitive to changes in ρ than the VaR. Further, it gives the magnitude of the sensitivity of the cVaR versus the VaR.

Because (4.11) is larger than one for $\rho\to 0$ and the numerator in Equation (4.11) is a linear function in ρ with a negative slope while the denominator is a square root function in ρ with a negative slope, there must be an intersection point $\rho_{int}>0$ that results in the same value for the numerator and the denominator. It can be derived by

$$\begin{aligned}\frac{\partial}{\partial\rho}cVaR(\rho_{int},\pi,\alpha)&=\frac{\partial}{\partial\rho}VaR(\rho_{int},\pi,\alpha)\\ \frac{1-\rho_{int}}{1-\alpha}\cdot\phi(\Phi^{-1}(1-\alpha))&=\sqrt{\rho_{int}}\cdot\Phi^{-1}(\pi)-\Phi^{-1}(1-\alpha)\\ (1-\rho_{int})\cdot a&=\sqrt{\rho_{int}}\cdot b-c,\end{aligned}$$

where $a=\frac{\phi(\Phi^{-1}(1-\alpha))}{1-\alpha}$, $b=\Phi^{-1}(\pi)$ and $c=\Phi^{-1}(1-\alpha)$ leading to

$$\rho_{int}=\frac{\left(\sqrt{b^2+4\cdot(c+a)\cdot a}-b\right)^2}{4\cdot a^2}.$$

Therefore the cVaR is more sensitive than the VaR for all $\rho\in(0,\rho_{int})$.

We could not derive an analogous result for the sensitivities in π, therefore we analyze the following ratio of sensitivities numerically:

$$q_\pi(\rho,\pi,\alpha):=\frac{\frac{\partial}{\partial\pi}cVaR(\rho,\pi,\alpha)}{\frac{\partial}{\partial\pi}VaR(\rho,\pi,\alpha)}=\frac{\frac{1}{1-\alpha}\cdot\Phi\left(\frac{\Phi^{-1}(1-\alpha)-\sqrt{\rho}\cdot\Phi^{-1}(\pi)}{\sqrt{1-\rho}}\right)}{\frac{1}{\phi(\Phi^{-1}(\pi))\cdot\sqrt{1-\rho}}\cdot\phi\left(\frac{\Phi^{-1}(\pi)-\sqrt{\rho}\cdot\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right)}. \tag{4.12}$$

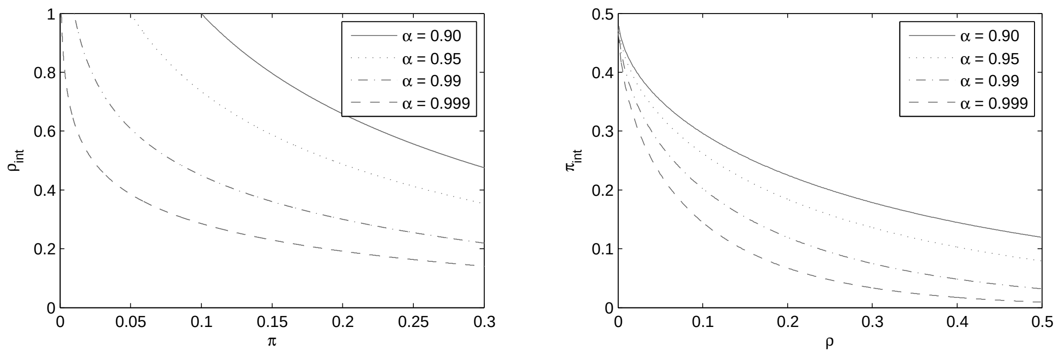
This ratio $q_\pi(\rho, \pi, \alpha)$ has the analogous interpretation as (4.11) but for changes in π. The numerical analysis reveals that the limiting value of $q_\pi(\rho, \pi, \alpha)$ for $\pi \to 0$ is larger than 1 for all reasonable parameter settings. The intersection point $0 < \pi_{int} < 0.5$ for the two partial derivatives in π can also be computed numerically by solving

$$\frac{\partial}{\partial \pi} cVaR(\rho, \pi_{int}, \alpha) = \frac{\partial}{\partial \pi} VaR(\rho, \pi_{int}, \alpha).$$

Both partial derivatives are monotonically decreasing in π (as confirmed numerically); therefore, we conclude the existence of only one intersection point. Thus, given ρ and α, the cVaR is more sensitive than the VaR for $0 < \pi < \pi_{int} < 0.5$.

Figure 4.2 illustrates these findigs. The left-hand panel shows ρ_{int} plotted for parameters $\pi \in (0, 0.3)$ and $\alpha \in \{0.90, 0.95, 0.99, 0.999\}$. For all parameter settings below the respective line, the cVaR is more sensitive to changes in ρ than the VaR. Similarly, the right-hand panel shows π_{int} for $\rho \in (0, 0.5)$ and $\alpha \in (0.90, 0.95, 0.99, 0.999)$. Again, for all parameter settings below the graph, the cVaR is more sensitive than the VaR in the sense of changing π.

Figure 4.2: ρ_{int} and π_{int} for $\pi \in (0, 0.3)$ and $\rho \in (0, 0.5)$



Notes: This figure shows ρ_{int} for given $\pi \in (0, 0.3)$ and π_{int} given $\rho \in (0, 0.5)$ for $\alpha \in \{0.90, 0.95, 0.99, 0.999\}$. For all parameter settings below the plotted line the cVaR reacts more sensitively in changes in the related parameter than the VaR.

To get an indication about potentially realistic magnitudes of the parameters, we compare these values with the supervisory asset correlation of the Basel risk weight formula, which is specified as a function of π yielding correlations between 0.12 and 0.24. The Basel formula is given by

$$\begin{aligned} 0.12 < \rho_{Basel} = 0.12 \cdot & \left(\frac{1 - \exp(-50 \cdot \pi)}{1 - \exp(-50)} \right) \\ & + 0.24 \cdot \left(1 - \frac{1 - \exp(-50 \cdot \pi)}{1 - \exp(-50)} \right) < 0.24. \end{aligned}$$

This shows that the cVaR is more sensitive than the VaR for most parameter settings, particularly for low probabilities of default. Realistic parameter setting (where this relation is reversed) are only found for a high π. However, in all cases that correspond to highly rated (IG-rated) bonds (e.g $\pi < 0.01$), which comprise the largest part in a typical portfolio, the cVaR has higher sensitivity than the VaR for reasonable values of ρ, e.g. those in line with the Basel formula.

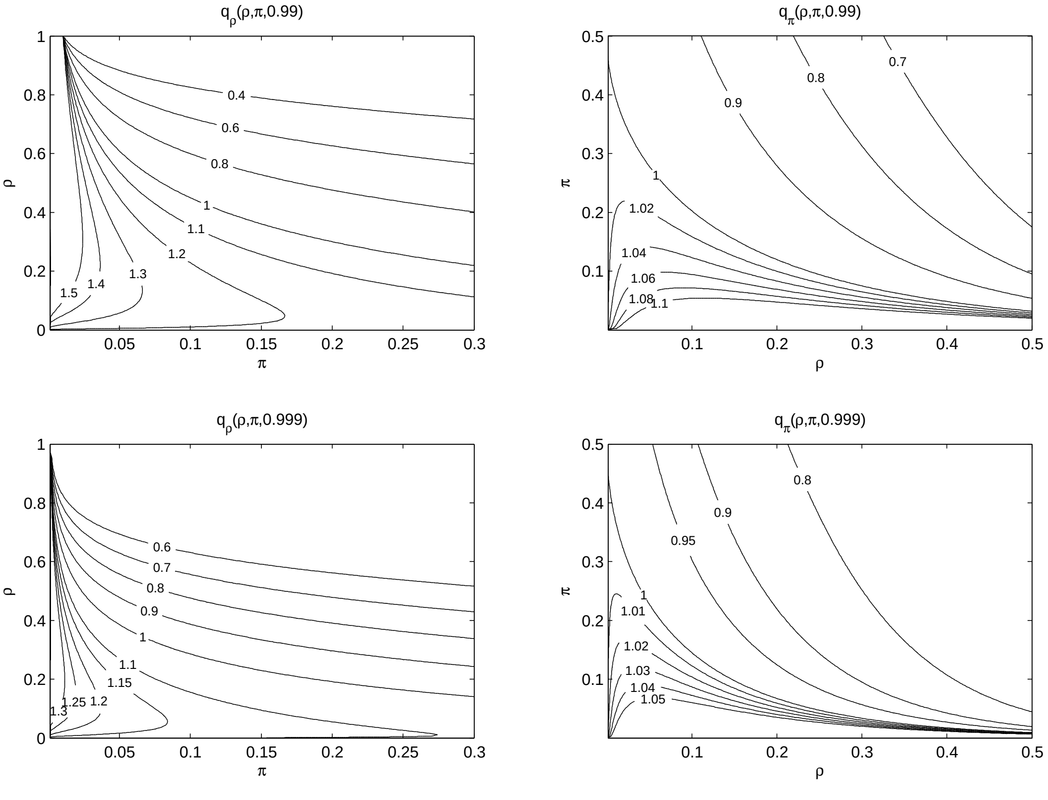
Concerning π, the cVaR is also more sensitive than the VaR for all parameter constellations that are typical for IG-rated risk buckets. Only for relatively high probabilities of default, e.g. $\pi = 0.10$, we see realistic parameter settings where the VaR is more sensitive to changes in π than the cVaR.

Importantly, Figure 4.2 also shows that for a lower α, the cVaR is more sensitive than the VaR for an even wider range of parameter settings (these results will be analyzed in more detail in Section 4.2.3). Beacuse this result holds for changes in ρ and π, we conjecture a higher sensitivity of the cVaR with respect to α as compared the VaR.

Figure 4.3 shows level curves (contour plots) of $q_\rho(\rho, \pi, \alpha)$ from (4.11) and $q_\pi(\rho, \pi, \alpha)$ from (4.12). The two left-hand panels show level curves of $q_\rho(\rho, \pi, \alpha)$ for $\alpha = 0.99$ (upper chart) and $\alpha = 0.999$ (lower chart). The contour lines in the graphs give the factor by which the cVaR is more sensitive in changes in ρ than the VaR in absolute terms. For example for $\alpha = 0.999$ the parameter setting $\rho = 0.20$ and $\pi = 0.05$ approximately hits the level curve 1.15 (exactly $q_\rho(0.20, 0.05, 0.999) = 1.144$), meaning that at this point, the sensitivity to changes in ρ of the cVaR is higher than the sensitivity of the VaR by factor 1.15 (exactly 1.144). For a lower PD, this ratio increases. For example, for $\alpha = 0.999$, $\rho = 0.20$ and $\pi = 0.01$, the cVaR-to-VaR-sensitivity ratio is at a factor $q_\rho(0.20, 0.05, 0.999) = 1.314$. Thus, for a higher rating - or a lower PD - the cVaR reacts much more sensitively to changes in ρ than the VaR.

In contrast, for large probabilities of default π and asset correlation ρ, we find parameter settings for which the cVaR is actually less sensitive in ρ than

Figure 4.3: Selected Level Curves of $q_\rho(\rho, \pi, \alpha)$ and $q_\pi(\rho, \pi, \alpha)$ with $\alpha \in \{0.99, 0.999\}$



Notes: This figure shows selected level curves of $q_\rho(\rho, \pi, \alpha)$ from (4.11) for $\pi \in (0, 0.3)$ and $\rho \in (0, 1)$ as well as $q_\pi(\rho, \pi, \alpha)$ from (4.12) for $\rho \in (0, 0.5)$ and $\pi \in (0, 0.3)$ for $\alpha \in \{0.99, 0.999\}$.

the VaR. For example, for $\rho = 0.40$, $\pi = 0.15$ and $\alpha = 0.999$, the sensitivity in ρ of the cVaR is less sensitive than the VaR by a factor of $q_\rho(0.40, 0.15, 0.999) = 0.8298$. However, these settings are not typical for IG-rated risk buckets, which are the major constituents of typical bank portfolios.

By comparing the upper left chart plotting $q_\rho(\rho, \pi, 0.99)$ with the lower chart plotting $q_\rho(\rho, \pi, 0.999)$, we see that with decreasing α, the strength of the sensitivity behavior of the cVaR increases compared with the sensitivity behavior of the VaR. This conforms to our previous result. For the two examples given above for $\alpha = 0.99$ the factors increase to $q_\rho(0.20, 0.05, 0.99) = 1.3404$ and $q_\rho(0.20, 0.05, 0.99) = 1.658$ respectively.

The two right-hand graphs show level curves of $q_\pi(\rho, \pi, \alpha)$ for $\alpha = 0.99$ (upper right-hand graph) and $\alpha = 0.999$ (lower right-hand panel). Overall, the effect of higher sensitivity of the cVaR with respect to π is less high than the sensitivity with respect to ρ. For example, for the parameter settings from above $q_\pi(0.20, 0.05, 0.999) = 1.023$ and $q_\pi(0.20, 0.01, 0.999) = 1.1433$, whereas $q_\pi(0.20, 0.05, 0.99) = 1.0942$ and $q_\pi(0.20, 0.01, 0.99) = 1.2640$.

These results have practical implications for financial institutions that use the cVaR instead of the VaR as a risk measure. Even though the cVaR has theoretical advantages, it is apparently more prone to parameter errors, and the true risk measure might therefore deviate more from the stated risk measure with the cVaR than with the VaR. This effect will be more pronounced for typical bank portfolio constituents (IG-rated securities).

The practical implications of our first contribution can be further investigated by analyzing relative add-ons

$$VaR_+(\hat{\rho}, \hat{\pi}, \alpha, \epsilon) = \frac{\max\limits_{(\rho,\pi)\in\Theta_\epsilon} VaR(\rho, \pi, \alpha) - VaR(\hat{\rho}, \hat{\pi}, \alpha)}{VaR(\hat{\rho}, \hat{\pi}, \alpha)} \tag{4.13}$$

and

$$cVaR_+(\hat{\rho}, \hat{\pi}, \alpha, \epsilon) = \frac{\max\limits_{(\rho,\pi)\in\Theta_\epsilon} cVaR(\rho, \pi, \alpha) - cVaR(\hat{\rho}, \hat{\pi}, \alpha)}{cVaR(\hat{\rho}, \hat{\pi}, \alpha)}, \tag{4.14}$$

where $\hat{\rho}$ and $\hat{\pi}$ are parameter values from expert judgments or parameter estimates (e.g. by maximum-likelihood estimation) and $\epsilon > 0$ defines an *uncer-*

tainty box

$$\Theta_\epsilon = \big((1-\epsilon)\cdot\hat{\rho}, (1+\epsilon)\cdot\hat{\rho}\big) \times \big((1-\epsilon)\cdot\hat{\pi}, (1+\epsilon)\cdot\hat{\pi}\big) \subseteq (0,1)^2 \qquad (4.15)$$

around these estimated parameters. These relative add-ons compute the relative deviation of the maximum risk measure from the estimated risk measure for a given uncertainty around the specified parameter values. Calculating these relative add-ons has two main advantages.

First, the maximization of the VaR or the cVaR subject to an uncertainty box takes into account that the unknown true parameters might be in an area around the parameter estimates. This agrees with the robust optimization literature that incorporates estimation uncertainties directly into the optimization algorithm under a deterministic worst-case approach. For a overview of developments, see e.g Zhu and Fukushima (2009), Huang et al. (2010) and Fabozzi et al. (2010) or more recently Zymler et al. (2013). Other restrictions, such as ellipsoidal or polyhedral uncertainty, could be used; however, by using a box-uncertainty, we can solve those restricted optimization problems analytically.

Second, the degree of uncertainty aversion is quantified by $\epsilon > 0$. A larger ϵ leads to a greater uncertainty box, resulting in a higher argument of the maximization problem. Then, the difference of the maximized risk measure and the estimated risk measure can be interpreted as an uncertainty premium. However, since the cVaR is by definition always larger than the VaR, the resulting uncertainty premium measured by the cVaR is consequentially larger in absolute terms than the premium measured by the VaR. To compare the different levels of sensitivity of the different risk measure, we correct for this by dividing the risk measures for given values evaluated at the estimates $[\hat{\rho}, \hat{\pi}]$. In economic terms, this relative add-on can be understood as the factor by which an uncertainty averse decision-makers (as quantified by $\epsilon > 0$) should multiply capital.

Given that the VaR is monotonously increasing in π, and given the results from Höse and Huschens (2008) the maximization problem in (4.13) for the VaR can be solved as

$$VaR_+(\hat{\rho}, \hat{\pi}, \alpha, \epsilon) = \frac{cVaR((\rho_+, (1+\epsilon)\cdot\hat{\pi}), \alpha) - cVaR(\hat{\rho}, \hat{\pi}, \alpha)}{cVaR(\hat{\rho}, \hat{\pi}, \alpha)}, \qquad (4.16)$$

where

$$\rho_+ = \begin{cases} (1+\epsilon)\cdot\hat{\rho}, & \text{if} \quad (1+\epsilon)\cdot\hat{\pi} \geq 1-\alpha \\ \rho_{\max}, & \text{if} \quad (1+\epsilon)\cdot\hat{\pi} < 1-\alpha \text{ and } (1-\epsilon)\cdot\hat{\rho} \leq \rho_{\max} \leq (1+\epsilon)\cdot\hat{\rho} \\ (1-\epsilon)\cdot\hat{\rho}, & \text{if} \quad (1+\epsilon)\cdot\hat{\pi} < 1-\alpha \text{ and } \rho_{\max} < (1-\epsilon)\cdot\hat{\rho} \\ (1+\epsilon)\cdot\hat{\rho}, & \text{if} \quad (1+\epsilon)\cdot\hat{\pi} < 1-\alpha \text{ and } \rho_{\max} > (1+\epsilon)\cdot\hat{\rho} \end{cases} \tag{4.17}$$

and $0 < \rho_{\max} = \left(\frac{\Phi^{-1}(1-\alpha)}{\Phi^{-1}(\pi)}\right)^2 < 1$ is the worst-case correlation from (4.10).

For the cVaR, we can use its monotonicity w.r.t. ρ and π from (4.6) and (4.8), and the solution to the maximization problem becomes

$$cVaR_+(\hat{\rho},\hat{\pi},\alpha,\epsilon) = \frac{cVaR((1+\epsilon)\cdot\hat{\rho},(1+\epsilon)\cdot\hat{\pi}),\alpha) - cVaR(\hat{\rho},\hat{\pi},\alpha)}{cVaR(\hat{\rho},\hat{\pi},\alpha)}. \tag{4.18}$$

To compare cVaR and VaR, we define the ratio of these two maximization results

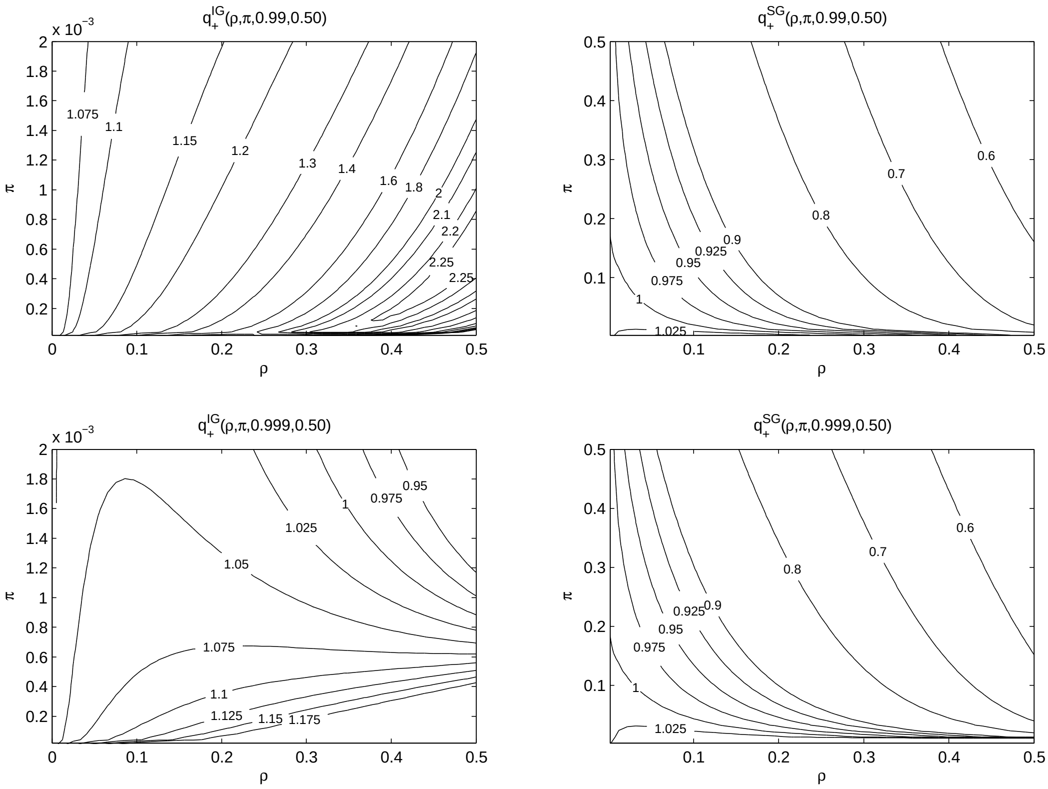
$$q_+(\hat{\rho},\hat{\pi},\alpha,\epsilon) = \frac{cVaR_+(\hat{\rho},\hat{\pi},\alpha,\epsilon)}{VaR_+(\hat{\rho},\hat{\pi},\alpha,\epsilon)}. \tag{4.19}$$

If this ratio is larger than one, the relative add-on of the cVaR is larger than the relative add-on of the VaR by that factor. In other words the cVaR reacts more sensitively to parameter estimation error than the VaR in those cases.

Selected contour plots of $q_+(\rho,\pi,\alpha,\epsilon)$ are shown in Figure 4.4, where ϵ is set to 0.5 in all charts. This choice of ϵ reflects a box size of approximately half the standard deviation of estimates in real-world application, as will be described in Section 4.3. The two charts on the left show $[\rho,\pi] \in (0,0.5)\times(0,0.002)$. This domain represents parameter settings compatible with IG-rated risk buckets and is denoted by $q_+^{IG}(\cdot)$. In the two charts on the right $[\rho,\pi] \in (0,0.5)\times(0.001,0.5)$, which represent parameter settings associated with speculative grade rated (SG-rated) risk buckets and is denoted by $q_+^{SG}(\cdot)$.

The graphs on the left show that for parameter settings associated with IG-rated risk buckets the parameter error aversion add-on for the cVaR is larger than for the VaR. The two charts on the right show that for SG-rated risk buckets this is not the case. Here, the VaR reacts more sensitively to parameter errors than the cVaR, as almost all level curves are smaller than one. Additionally, we see that with a lower α for IG-rated risk buckets, the cVaR reacts much more to parameter uncertainty than the VaR.

Figure 4.4: Selected Level Curves of $q_+(\rho, \pi, \alpha, 0.5)$ for $[\rho, \pi] \in (0, 0.5) \times (0, 0.5)$



Notes: This figure shows selected level curves of $q_+(\rho, \pi, \alpha, \epsilon)$ from (4.19) for $\epsilon = 0.5$. For the two left-hand graphs, the domain for IG-rated risk buckets is defined as $[\rho, \pi] \in (0, 0.5) \times (0, 0.002)$, while for the two right hand graphs a domain for SG-rated risk buckets is introduced as $[\rho, \pi] \in (0, 0.5) \times (0.001, 0.5)$.

4.2.3 Sensitivities and Confidence Levels

We now provide a more detailed analysis of the confidence level α. In the usual interpretation of α a higher value is associated with a lower probability of adverse outcomes exceeding the α-quantile; similarly, if capital buffers are linked to the risk measure at an α confidence level, a higher value implies c.p. higher capital and a lower likelihood of default. A higher α is therefore c.p. usually associated with more safety. We will now analyze the problems associated with these perceptions. For the third main result of our contribution we show that both the VaR and the cVaR (particularly for IG-rated risk buckets) are more sensitive to errors in ρ and π with increasing α-quantiles. In other words, while a high confidence level apparently delivers a high degree of safety, the vulnerability to parameter errors increases with the perceived confidence or safety.

For this analysis, we first define and analyze the ratios

$$\begin{aligned} q_\alpha^{\text{VaR}}(\rho,\pi,\alpha) &= \frac{\frac{\partial}{\partial\alpha} VaR(\rho,\pi,\alpha)}{VaR(\rho,\pi,\alpha)}, \text{ and} \\ q_\alpha^{\text{cVaR}}(\rho,\pi,\alpha) &= \frac{\frac{\partial}{\partial\alpha} cVaR(\rho,\pi,\alpha)}{cVaR(\rho,\pi,\alpha)} \end{aligned} \tag{4.20}$$

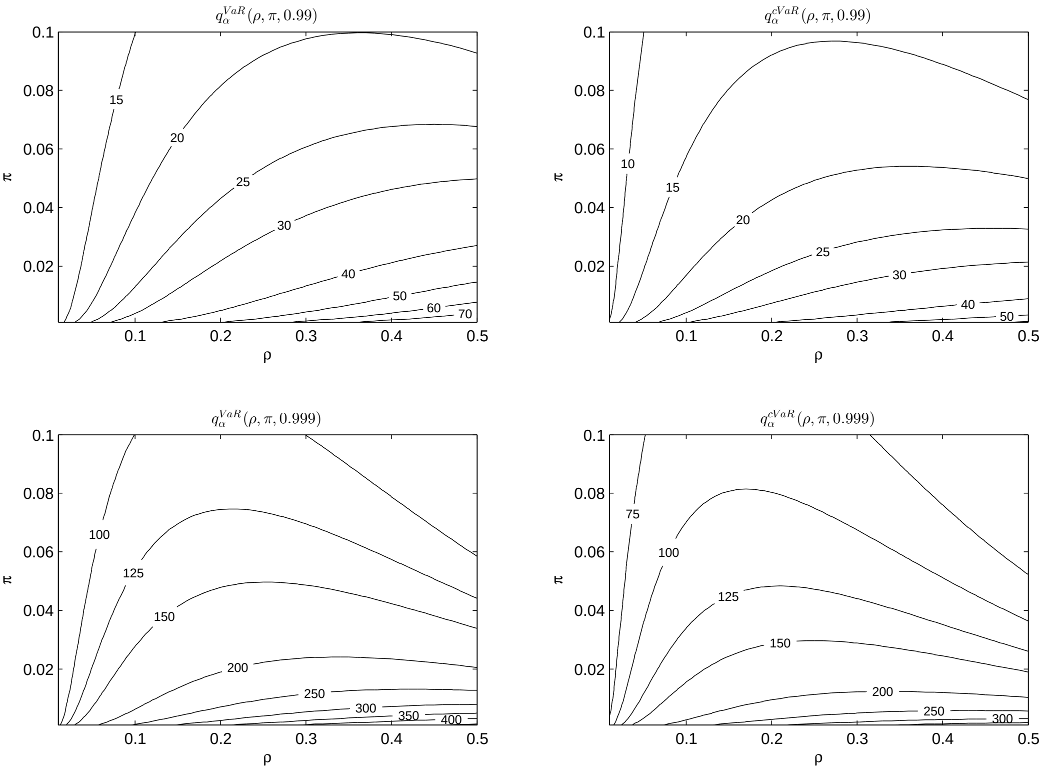
which measure the sensitivity of the VaR (cVaR) in changes in α accounting for the actual level of the risk measures. Selected level curves (contour plots) are shown in Figure 4.5, where the ratios are plotted for the VaR in the left-hand charts and the cVar in the right-hand charts as functions of ρ and π for $\alpha = 0.99$ (upper charts) and $\alpha = 0.999$ (lower charts).

Comparing the upper charts with the lower charts shows that increasing α from 99% to 99.9% raises the sensitivity of the measures by more than 500% even when controlling for the actual level. For example, we find $q_\alpha^{VaR}(20\%, 1\%, 99\%) = 35.38$, while $q_\alpha^{VaR}(20\%, 1\%, 99.9\%) = 233.14$. This equates to a multiple of 658.96%.

Further, the VaR reacts less sensitively in this regard. This is in line with the above results from Figures 4.2 and 4.3 where the cVaR becomes more sensitive in changes in ρ and π than the VaR by decreasing α. For this, see exemplary $q_\alpha^{cVaR}(20\%, 1\%, 99\%) = 28.42$ and $q_\alpha^{cVaR}(20\%, 1\%, 99.9\%) = 197.92$ which represents a multiple of 696.41%.

We analyze this effect in more detail for different parameter settings and

Figure 4.5: Selected Level Curves of $q_\alpha^{VaR}(\rho, \pi, \alpha)$ and $q_\alpha^{cVaR}(\rho, \pi, \alpha)$ with $\alpha \in \{0.99, 0.999\}$



Notes: This figure shows selected level curves off $q_\alpha^{\text{cVaR}}(\rho, \pi, \alpha)$ and $q_\alpha^{\text{VaR}}(\rho, \pi, \alpha)$ from (4.20). For all four graphs the domain is defined as $[\rho, \pi] \in (0, 0.5) \times (0, 0.1)$.

IG-rated and SG-rated risk buckets. Suppose that the real-world parameters $\theta \in \{\rho, \pi\}$ are estimated using statistical-econometric techniques. For ease of exposition, the estimator $\hat{\theta}$ shall be unbiased ($\mathbb{E}(\hat{\theta}) = \theta$) for each parameter and normally distributed with a known standard deviation $\sigma(\hat{\theta}) = \sqrt{\mathbb{E}\left((\hat{\theta} - \theta)^2\right)}$:

$$\hat{\theta} \sim N\left(\theta, \sigma(\hat{\theta})^2\right).$$

By inserting the estimators $\hat{\rho} \sim N(\rho, \sigma(\hat{\rho})^2)$ and $\hat{\pi} \sim N(\pi, \sigma(\hat{\pi})^2)$ in the functions of the VaR (4.2a) and cVaR (4.2b), these risk measures also become random variables and are given by

$$\mathcal{R}(\hat{\rho}, \pi, \alpha), \qquad \mathcal{R}(\rho, \hat{\pi}, \alpha) \qquad \text{and} \qquad \mathcal{R}(\hat{\rho}, \hat{\pi}, \alpha),$$

where $\mathcal{R}(\cdot) \in \{VaR(\cdot), cVaR(\cdot)\}$. While $\mathcal{R}(\hat{\rho}, \pi, \alpha)$ (respectively $\mathcal{R}(\rho, \hat{\pi}, \alpha)$) takes into account the estimation error in ρ (respectively π), the measure $\mathcal{R}(\hat{\rho}, \hat{\pi}, \alpha)$ includes the estimation errors in ρ and π simultaneously. To quantify the estimation error effect on the VaR in comparison with the cVaR, we calculate the β-quantiles of their distributions and divide them by its *true* value evaluated at $[\rho, \pi]$.

$$q_{\beta,\rho}^{\mathcal{R}(\cdot)} = \frac{Q_\beta\big(\mathcal{R}(\hat{\rho}, \pi, \alpha)\big)}{\mathcal{R}(\rho, \pi, \alpha)} \tag{4.21a}$$

$$q_{\beta,\pi}^{\mathcal{R}(\cdot)} = \frac{Q_\beta\big(\mathcal{R}(\rho, \hat{\pi}, \alpha)\big)}{\mathcal{R}(\rho, \pi, \alpha)} \tag{4.21b}$$

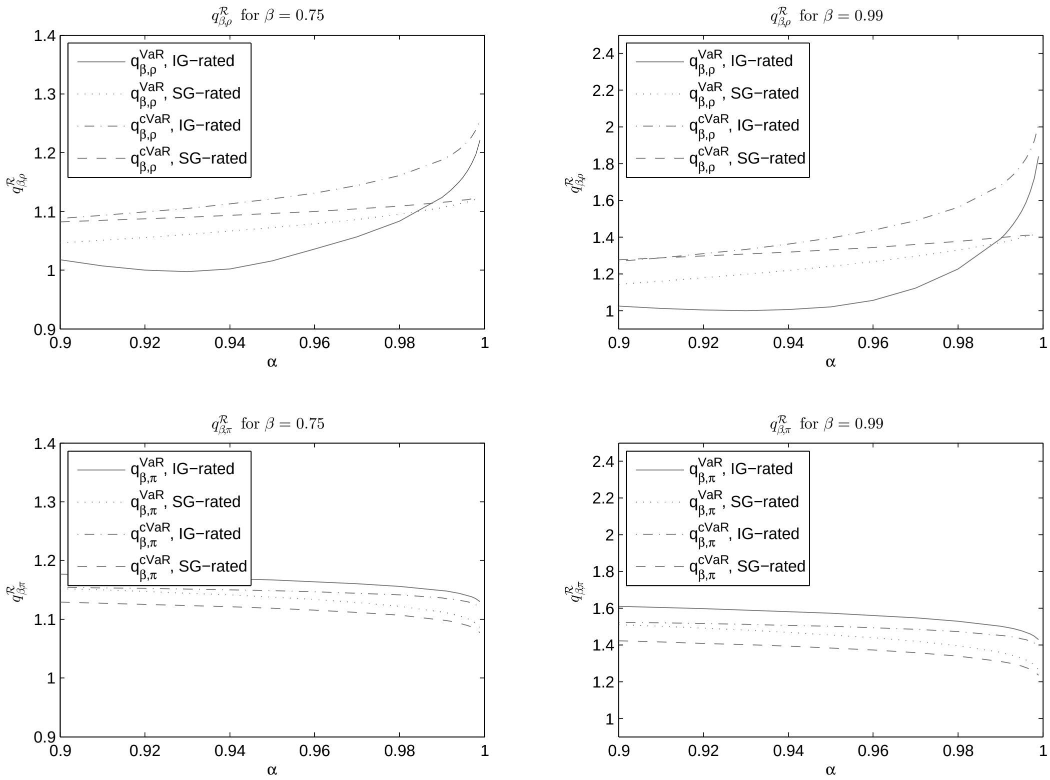
$$q_{\beta}^{\mathcal{R}(\cdot)} = \frac{Q_\beta\big(\mathcal{R}(\hat{\rho}, \hat{\pi}, \alpha)\big)}{\mathcal{R}(\rho, \pi, \alpha)} \tag{4.21c}$$

Thus, these measures describe the factors by which the random VaR or cVaR might be larger (or smaller) than the true risk measure for a given quantile level β. Therefore, these factors are influenced by the estimation error of the underlying parameters, represented by their standard deviation $\sigma(\hat{\theta})$, which depends on the quality of given data, the length of the data series and the estimation technique, as well as on the chosen β-quantile. A higher $\beta-$quantile in general leads to a higher factor.

Since the cVaR is monotonously increasing in ρ and π, we can calculate $q_{\beta,\rho}^{cVaR(\cdot)}$ and $q_{\beta,\pi}^{cVaR(\cdot)}$ analytically. The same we can do for $q_{\beta,\pi}^{VaR(\cdot)}$. For the other cases we provide results from a Monte Carlo simulation.

To demonstrate how this factor varies by changes in α, we vary α in $(0.9, 0.999)$ for two parameter settings and present the results in Figure 4.6. The parameter settings are based on the results from Section 4.3, which represent an IG-rated and SG-rated risk bucket ($\rho^{IG} = \rho^{SG} = 20\%, \pi^{IG} = 0.05\%, \pi^{IG} = 5\%$). For adequate comparison, we standardize the estimation error by setting $\sigma(\vartheta) = 0.25 \cdot \vartheta$, where $\vartheta \in \{\rho^{IG}, \rho^{SG}, \pi^{IG}, \pi^{SG}\}$[58].

Figure 4.6: $q^{\mathcal{R}(\cdot)}_{\beta,\rho}$ and $q^{\mathcal{R}(\cdot)}_{\beta,\pi}$ for $\beta \in \{0.75, 0.9\}$



Notes: This figure shows the measure $q^{\mathcal{R}(\cdot)}_{\beta,\rho}$ and $q^{\mathcal{R}(\cdot)}_{\beta,\pi}$ for $\beta \in \{0.75, 0.9\}$ and $\alpha \in (0.9, 0.999)$. Therefore it gives insight in possible risk overestimation given the same estimation error based on the choice of the risk measure VaR or cVaR.

The two graphs on the left-hand side show the evolution of the measures $q^{\mathcal{R}(\cdot)}_{\beta,\rho}$ and $q^{\mathcal{R}(\cdot)}_{\beta,\pi}$ for $\beta = 0.75$ and $\alpha \in (0.9, 0.999)$. The upper and lower graphs show the possible risk overestimation by parameter errors in ρ and π, respectively. The data indicate that for IG-rated risk buckets in particular the relative risk overestimation increases with a higher α. For $\alpha > 0.995$, this effect rises quasi

[58] Estimation errors for IG-rated risk buckets are higher than for SG-rated risk buckets due to the lack of observed losses. The (relative) estimation errors are often higher for ρ than for π.

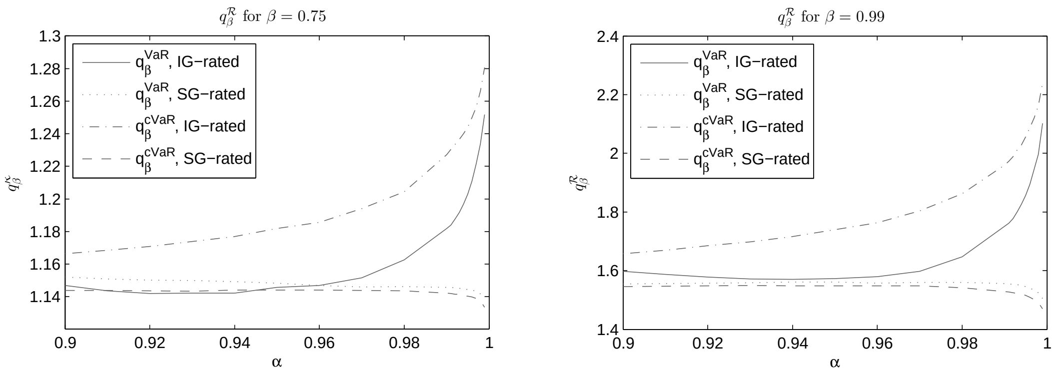
exponentially. Thus, for an IG-rated risk bucket under parameter estimation error in ρ, a considerably large risk overestimation is possible. Further, this effect is even more pronounced for the cVaR. In comparison with SG-rated risk buckets, we see only a small increase of this overestimation for both risk measures. Actually, the effect is almost constant; we therefore conclude that for SG-rated risk buckets under estimation error in ρ the choice of α does not matter much.

If we analyze this effect for estimation errors only in π, we see that for the IG-rated and SG-rated risk buckets the possible risk overestimation stays almost constant. For a large $\alpha > 0.995$, we see a slightly decreasing effect. Therefore we conclude that if there is only *uncertainty* in π, the choice of α is of minor importance in comparison with the *uncertainty* in ρ.

If we compare the left-hand graphs with the right hand-graphs, we see that with an increasing β from 0.75 to 0.99, the lines are almost scaled, as would be expected.

While Figure 4.6 only shows an analysis of a single estimation error, Figure 4.7 shows the results from an analysis considering both estimation error simultaneously by $q_\beta^{\mathcal{R}(\cdot)}$ for the same parameter sets. Note that these plots are based on Monte Carlo simulations and are therefore not always smooth.

Figure 4.7: $q_\beta^{\mathcal{R}(\cdot)}$ for $\beta \in \{0.75, 0.9\}$ and $\alpha \in (0.9, 0.999)$



Notes: This figure shows the measure $q_\beta^{\mathcal{R}(\cdot)}$ for $\beta \in \{0.75, 0.9\}$ and $\alpha \in (0.9, 0.999)$. This provides insights into possible risk overestimation given the same estimation error based on the choice of the risk measure VaR or cVaR.

The left-hand graph shows a possible risk overestimation for IG-rated risk buckets, which increases with a higher α. Again, especially for a high $\alpha > 0.995$, this effect increases almost exponentially. For SG-rated risk buckets, the choice of α does not lead to considerable higher or slower risk overestimation.

Again, the cVaR reacts more sensitively than the VaR to changes in parameters, and a comparison of the left- and right-hand graphs shows that the reported lines are almost scaled when β is increased from 0.75 to 0.99.

We therefore conclude that under parameter errors in ρ and π (especially for IG-rated risk buckets) the choice of the α-quantile for both risk measures VaR and cVaR is more important than previously acknowledged in the literature. Both risk measures react more sensitively to changes in ρ and π with higher α.

Therefore this gives risk to the conjecture that a smaller α-quantile could be wise in some practical situations in risk management, particularly considering the result that this effect increases exponentially for $\alpha > 0.995$.

If using a lower α one should recall the result of Section 4.2.2. There we have seen that with a lower α the cVaR becomes more sensitive to changes in the parameters ρ and π in comparison to the VaR. Thus, for lower α the VaR is superior to the cVaR concerning vulnerability to errors, and the VaR could be favored in practical applications, along with lower α.

4.2.4 Calibrating VaR and cVaR to the same Level of Capital

So far we have compared the behavior of the two risk measures under the same confidence level α. Because of the following relation,

$$cVaR_\alpha(\cdot) = VaR_\alpha(\cdot) + \mathbb{E}\big(X - VaR_\alpha(\cdot) \mid X > VaR_\alpha(\cdot)\big), \tag{4.22}$$

where $\mathbb{E}\big(X - VaR_\alpha(\cdot) \mid X > VaR_\alpha(\cdot)\big) > 0$, the cVaR exceeds the VaR in absolute terms in general. Therefore in some practical applications the α-quantile for the cVaR might be lowered in order to reduce this difference. For example the Basel Committee proposes a 97.5% cVaR instead of a 99% VaR for the internal models-based approach and used this adapted cVaR to determine the risk weights for the revised standardized approach to handle risk in the trading book. The Committee argues, that this confidence level for the cVaR provides a broadly similar level of risk capital, while providing a number of benefits, such as a more stable model output and less sensitivity to extreme outlier observations (BCBS, 2013). This adaptation is in general questionable.

Per definition the α cVaR provides the average outcome (e.g. loss) for an extreme event, which is likely to occur at a probability rate of $1-\alpha$. Therefore if the cVaR is considered as a risk measure and capital is allocated accordingly, in more than α per cent of cases the undesired events can be absorbed. If the confidence level of the cVaR is adapted in such a way, that the cVaR equals the original α VaR, then only α per cent of the cases can be absorbed and the advantage of tail-consideration is lost. In this case the choice of the adapted cVaR over the original VaR may only provide the above mentioned technical advantages, such as coherence, convexity, etc. However, these advantages are still accompanied by a higher sensitivity to parameter errors as will be shown below.

For a deeper analysis we define $\alpha^c \in (0,1)$ implicitly by the following equation

$$VaR(\rho, \pi, \alpha) = cVaR(\rho, \pi, \alpha^c), \tag{4.23}$$

which gives the cVaR confidence level $\alpha^c < \alpha$ such that the cVaR equals the VaR with confidence level α.

For example let $\rho = 0.20$, $\pi = 0.05$ and $\alpha = 0.99$, then

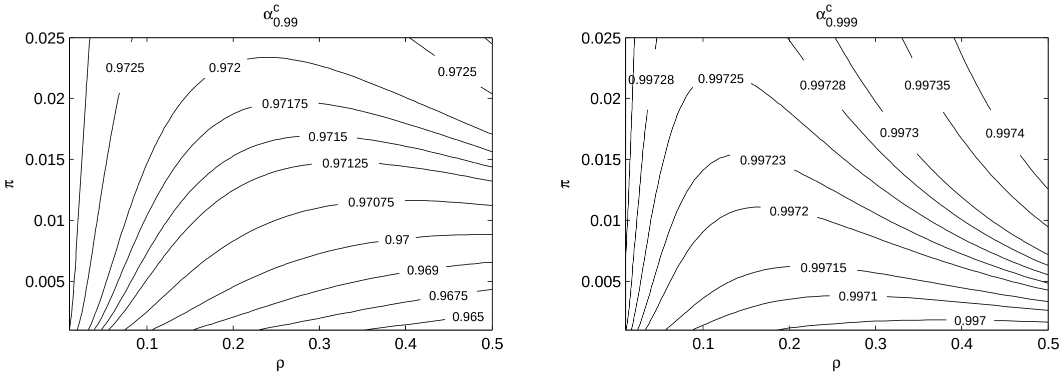
$$VaR(0.20, 0.05, 0.99) = 0.2496 = cVaR(0.20, 0.05, 0.973),$$

leading to $\alpha^c = 0.973$. Figure 4.8 provides this α^c for $\alpha \in \{0.99, 0.999\}$ and typical parameter settings for IG- and 'Ba'-rated risk buckets; $\rho \in (0, 0.5)$, $\pi \in (0, 0.025)$. As a result we find a nonlinear dependency between the parameter settings and given α-quantile. For the given parameter range the minimum of α^c for $\alpha = 0.99$ (for $\alpha = 0.999$) is approximately 0.9608 (0.9969) and the maximum is 0.9735 (0.9975). If one used the Basel proposal of a 97.5% cVaR instead of a 99% VaR for credit risk, one would find quite larger risk levels and sensitivities in the given parameter constellation.[59]

If we compute the implied α^c for the cVaR according to Equation (4.23), both risk measures lead to the same risk capital and we can again compare the

[59] If the considered portfolio has a PD larger than 20% it is even possible, that a 97.5% cVaR is actually smaller than a 99% VaR.

Figure 4.8: Implied α^c for $\rho \in (0, 0.5)$, $\pi \in (0, 0.025)$ and $\alpha \in \{0.9, 0.999\}$



Notes: This figure shows the implied α^c for which the cVaR equals the VaR for a confidence level α for $\rho \in (0, 0.5)$, $\pi \in (0, 0.025)$ and $\alpha \in \{0.9, 0.999\}$.

sensitivity to changes in the model parameter by the following ratios:

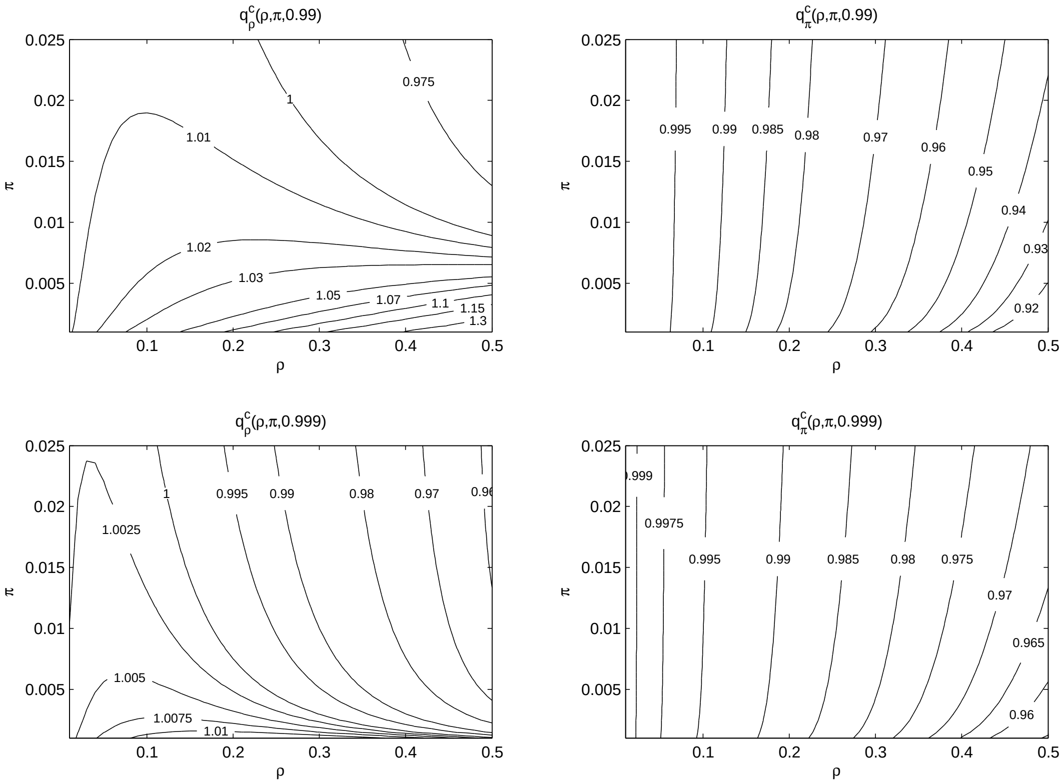
$$q^c_\rho(\rho, \pi, \alpha) := \frac{\frac{\partial}{\partial \rho} cVaR(\rho, \pi, \alpha^c)}{\frac{\partial}{\partial \rho} VaR(\rho, \pi, \alpha)}, \text{and} \tag{4.24}$$
$$q^c_\pi(\rho, \pi, \alpha) := \frac{\frac{\partial}{\partial \pi} cVaR(\rho, \pi, \alpha^c)}{\frac{\partial}{\partial \pi} VaR(\rho, \pi, \alpha)}.$$

Therefore, if $q^c_\theta > 1$ the sensitivity of the cVaR to changes in the parameter $\theta \in \{\rho, \pi\}$ is larger for the cVaR than for the VaR, although both risk measures denote the same level of risk capital in absolute terms.

Examples of these ratios are given in Figure 4.9 for the parameter settings which are typical for IG- and 'Ba'-rated risk buckets and $\alpha \in \{0.99, 0.999\}$ in line with Figure 4.8.

The two graphs on the left-hand side show for $\alpha = 0.99$ (upper chart) and $\alpha = 0.999$ (lower chart) the evolution of the ratio $q^c_\rho(\rho, \pi, \alpha)$, while the right-hand side shows the graphs for the ratio $q^c_\pi(\rho, \pi, \alpha)$. We find that especially the behavior with respect to ρ changes in a notable way for $\alpha = 0.99$, while it is rather stable and closer to one for $\alpha = 0.999$. For example for $\alpha = 0.99$ ($\alpha = 0.999$), $\rho^{IG} = 20\%$ and $\pi^{IG} = 0.05\%$, we find that the 96.72% (99.69%) cVaR is about 10% (1.9%) more sensitive to changes in ρ than the 99% (99.9%) VaR. As a result we conclude that especially for a smaller α - although the initial level of risk capital is the same - the cVaR is more sensitive to parameter errors in ρ than the VaR, which is in line with our former results. The graphs on the right-hand side confirm our previous result, that i) the sensitivity with respect to changes in π for both risk measures do not differ in a notable way and ii) that this behavior is almost independent from the choice of α.

Figure 4.9: q^c_ρ and q^c_π for $\rho \in (0, 0.5)$, $\pi \in (0, 0.025)$ and $\alpha \in \{0.9, 0.999\}$



Notes: This figure shows the measure q^c_ρ and q^c_π for $\rho \in (0, 0.5)$, $\pi \in (0, 0.025)$ and $\alpha \in \{0.9, 0.999\}$. This parameter range for ρ and π corresponds all parameter constellations which are plausible for IG- and 'Ba'-rated risk buckets. If the measure is larger one the cVaR reacts more sensitive to parameter changes than the VaR.

From this theoretical analysis, we conclude that the cVaR reacts more sensitively than the VaR to changes in ρ; even when it is adapted in such a way, that it reports the same level of risk capital as the α VaR. This higher sensitivity increases with lower PD and lower original α-quantile.

Hence, in credit risk a shift from VaR to cVaR can boost the relative effects of uncertainty, even if the tail-considering risk measure is calibrated to meet the level of risk capital of the VaR. This effect is increased, if the original α-quantile is lowered.

4.3 Empirical Results

4.3.1 Data

In this section we provide a case study to assess the effects of parameter errors in practical applications of financial institutions. We use an econometric approach to model parameter error, this is more comprehensible than an expert judgment approach, yields estimates for the real-world parameters rather than being risk-neutral, and delivers straightforward uncertainty bounds for the parameters using results from classic estimation theory. We use publicly available default data information from Moody's (2013) annual default report. The report lists the number of companies (ranked by rating grade) at the beginning of any given year, and reports what proportion of them had defaulted by the end of the year. This information is given for the seven rating grades 'Aaa', 'Aa', 'A', 'Baa', 'Ba', 'B' and 'Caa-C', and the data cover a 43-year period from 1970 to 2012. The first four rating grades are summarized as Investment Grades ('IG'), while the last three rating grates are Speculative Grades ('SG'). An example use of a subset of these data, restricted to SG default informations, has been reported by Tasche (2011) for the years 1990 to 2010.

The number of rated companies in this dataset increased from 1,032 in 1970 to 4,823 in 2012; and on average, there are 2,688 rated companies included per year. Because there were only very few defaults in the first two highest rating grates (no defaults in 'Aaa' and, only 6 in 'Aa') we exclude these two rating categories; this is consistent with the approach taken by McNeil and Wendin (2007) and Chang et al. (2011) who used a similar dataset from Standard & Poors. However, we did include these grades for our analysis of IG-rated risk buckets. Descriptive statistics of the default rates for each grade in this study are listed in Table 4.1.

Table 4.1 shows that the average of the historical one-year default rates of Moody's rated companies increases with rating grade, while the ratio of mean and standard deviation decreases. Therefore, the rating methodology seems appropriate, even though estimation errors were larger for higher rating grades.

Table 4.1: Basic Statistics of Historical One-year Default Rates (%) in Moody's (2013), 1970 - 2012

	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
Mean	0.04	0.17	1.07	5.03	22.09	0.08	3.86
Std	0.10	0.28	1.19	4.30	20.28	0.13	3.04
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.41
Median	0.00	0.00	0.81	4.70	18.26	0.00	3.13
Max	0.51	1.06	4.95	19.44	100	0.51	13.29

Notes: This table shows descriptive statistics of the historical one-year default rates for the rating categories 'A', 'Baa', 'Ba', 'B' and 'C'. The Investment Rating Grade 'IG' includes the four highest rating grades 'Aaa', 'Aa', 'A' and 'Baa', while the Speculative Rating Grade 'SG' contains the lowest rating grades 'Ba', 'B' and 'Caa-C'. We use: Std, standard deviation; Min, minimum; Max, maximum.

4.3.2 Parameter Estimation

As pointed out by Gordy and Heitfield (2010), the literature on parameter estimation for portfolio credit risk models has grown enormously over the last years. There are methods of moment estimators (see Gordy, 2000; Nagpal and Bahar, 2001; Frey and McNeil, 2003), and early applications included work by Hamerle et al. (2003*b*) and Löffler (2003). For examples of maximum-likelihood estimation of these model, see Frey and McNeil (2003), Hamerle et al. (2003*a*); Hamerle and Rösch (2005) and Düllmann et al. (2008). For a use of Bayesian MCMC estimators see McNeil and Wendin (2007). Apart from these kinds of point estimations for the parameters, the literature also focuses on the interval estimation of default probabilities, especially for low default portfolios, see Tasche (2011).

In this study we use the maximum-likelihood estimation method as described by Frey and McNeil (2003). If we observe a time series of defaults $\mathbf{d} = (d_1, ..., d_T)$ for a portfolio or bucket containing $\mathbf{n} = (n_1, ..., n_T)$ borrowers who are assumed to be homogeneous, we can derive the likelihood function

$$l(\rho, \pi, \mathbf{n}, \mathbf{d}) = \prod_{t=1}^{T} \int_{-\infty}^{\infty} b\left(d_t, n_t, \pi(y)\right) d\Phi(y). \tag{4.25}$$

where $b(.)$ denotes the probability function of the binomial distribution with n_t trials and probability $\pi(y)$. The conditional probability of default is given by

$$\pi(y) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho}\cdot y}{\sqrt{1-\rho}}\right). \tag{4.26}$$

The likelihood function (4.25) is logarithmized and numerically optimized with respect to the parameter ρ and π in the unit square. There are several algorithms solving this optimization problem, and the integral is evaluated by a global adaptive quadrature.

If a likelihood function $f(\aleph, \boldsymbol{\theta})$ for given data $\aleph$ and parameters $\boldsymbol{\theta}$ fulfills the regularity conditions,[60] the maximum-likelihood estimator $\hat{\boldsymbol{\theta}}$ is 1) consistent, 2) asymptotically normal, 3) asymptotically efficient, 4) achieves the so-called Cramer Rao Lower Bound and 5) is asymptotically $\hat{\boldsymbol{\theta}} \sim N(\boldsymbol{\theta}, \mathcal{I}(\boldsymbol{\theta})^{-1})$, where $\mathcal{I}(\cdot)$ is the information matrix

$$\mathcal{I}(\boldsymbol{\theta}) = \mathbb{E}_{\boldsymbol{\theta}}\left(-\frac{\partial^2 \log(f(\aleph|\boldsymbol{\theta}))}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}\right).$$

Therefore, all presented standard deviations are the result of inverting the numerically calculated negative Hessian matrix of logarithmized (4.25) at the estimates. Since these outcomes are an estimation of the *true* standard deviation, we denote them with $\hat{\sigma}(\hat{\theta})$ for $\theta \in \{\rho, \pi\}$.

As described by Gordy and Heitfield (2002, 2010), the panel dataset $\mathbf{d} = (d_1, ..., d_T)$ and $\mathbf{n} = (n_1, ..., n_T)$ may cover default data on large numbers of rated obligors $n_t \gg 0$, but in time-series dimension $t \in \{1, ..., T\}$ the available data contain only a few decades, or even just a few years. For example, the data by Moody's (2013) which are used here cover a 43-year period from 1970 to 2012 for more than 4,000 obligors. As shown by Gagliardini and Gourieroux (2005), large n_t is not sufficient for consistency of the parameter estimation. Instead, a large T is needed in the time-series dimension.

As already described by Gordy and Heitfield (2010), the standard errors show that even if the asymptotics are reliable and the estimators are unbiased, parameter estimates are prone to estimation error; this has also been analyzed by Löffler (2003) and Tarashev (2010). Moreover, when very few defaults are observed within a bucket, estimating the parameters becomes difficult or impossible. Therefore, when hardly any default occur, we might consider

[60] Conditions such as differentiability of the log of the likelihood function, compact parameter space, independent and identically distributed densities.

upper confidence bounds for the PD.

4.3.3 Empirical Evidence for Sensitivity Effects

Table 4.2 shows the estimation results for each rating grade and various time horizons (from 2003-2012 covering 10 years of data, 1993-2012 covering 20 years, and 1973-2012 covering 40 years). The first row shows the maximum-likelihood estimates $[\hat{\rho}, \hat{\pi}]$ for ρ and π, the second row shows their estimated standard errors $\hat{\sigma}(\hat{\rho})$ and $\hat{\sigma}(\hat{\pi})$ and the third row reports the ratio of standard errors and parameter estimates $\hat{\sigma}(\hat{\theta})/\hat{\theta}$, $\theta \in \{\rho, \pi\}$. Depending on the time period used for estimation, we obtain different parameter estimates. The range of parameter estimates is for $\hat{\rho} \in (8.62\%, 24.06\%)$ and $\hat{\pi} \in (0.05\%, 19.53\%)$. While $\hat{\pi}$ increases monotonously with rating grade, we find a decreasing relation for $\hat{\rho}$, with some exceptions. The estimated standard errors decrease with increasing sample size for most instances, as would be expected. Some exceptions are found among the lower rating grades and SG. The fourth and fifth row of each panel show the estimates for the VaR and cVaR given a confidence level of 99.9% and using the respective parameter estimates.[61]

Obviously, in all cases in Table 4.2 the cVaR is larger than the VaR, which is to be expected according to (4.22). For deteriorating rating grades, we obtain c.p. higher risk measures, indicating a monotonous risk-grading as we would expect. However, comparing the short with the longer panel, ratings 'IG', 'A', and 'Baa' exhibit the highest risk measures when only the short panel is used, while for the worse ratings 'B', 'Caa-C' and 'SG' the short panel yields the smallest risk measures. This is in line with commonly known observations from the last financial crisis during which high-rated investments faced in particular unexpectedly large losses.

For all three time horizons, we see that (with minor exceptions) the *relative* estimation error decreases with decreasing rating grade. In other words, especially for the high-rated risk buckets, financial institutions are faced with even larger (relative) estimation errors. Furthermore - and in line with expectations - the estimation errors decline with longer time horizon.

In the previous section, we have already seen that the sensitivity to parameter errors increases with rating quality. Now, we see empirically that better

[61] The reported VaR and cVaR are relatively high, since we set the recovery rate (RR) to zero. This could be corrected by using the linear factor (1-RR). The mean of all recovery rates reported in Moody's (2013) for the last 31 years is 41.67%.

Table 4.2: MLE Results for Historical One-year Default Rates from Moody's (2013) for Different Time Horizons (in per cent)

	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
2003 - 2012							
$[\hat{\rho}, \hat{\pi}]$	[24.06, 0.11]	[19.93, 0.19]	[16.4, 0.56]	[20.67, 1.61]	[10.57, 13.52]	[19.5, 0.13]	[9.38, 3.75]
$(\hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}))$	(21.09, 0.11)	(13.19, 0.13)	(9.82, 0.28)	(8.95, 0.72)	(4.46, 2.37)	(11.26, 0.09)	(3.91, 0.86)
$\{\hat{\sigma}(\hat{\rho})/\hat{\rho}, \hat{\sigma}(\hat{\pi})/\hat{\pi}\}$	{87.63, 94.31}	{66.21, 69.23}	{59.91, 49.99}	{43.28, 45.05}	{42.15, 17.52}	{57.73, 69.62}	{41.68, 22.93}
$VaR_{99.9}$	3.86	4.45	7.95	20.40	45.89	3.24	19.06
$cVaR_{99.9}$	5.56	6.07	10.08	24.81	49.67	4.50	21.66
1993 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
$[\hat{\rho}, \hat{\pi}]$	[21.26, 0.08]	[16.74, 0.2]	[9.56, 0.66]	[19.76, 3.29]	[10.53, 15.73]	[18.11, 0.11]	[8.62, 4.21]
$(\hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}))$	(15.44, 0.05)	(8.27, 0.08)	(4.73, 0.17)	(5.84, 0.87)	(3.33, 1.91)	(8.05, 0.05)	(2.58, 0.64)
$\{\hat{\sigma}(\hat{\rho})/\hat{\rho}, \hat{\sigma}(\hat{\pi})/\hat{\pi}\}$	{72.62, 65.48}	{49.42, 42.93}	{49.43, 25.41}	{29.55, 26.28}	{31.62, 12.12}	{44.44, 47.90}	{29.93, 15.12}
$VaR_{99.9}$	2.46	3.81	5.48	30.15	49.89	2.68	19.57
$cVaR_{99.9}$	3.56	5.08	6.62	35.2	53.65	3.71	22.07
1973 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
$[\hat{\rho}, \hat{\pi}]$	[22.11, 0.05]	[13.74, 0.18]	[12.15, 1.05]	[18.79, 4.95]	[14.26, 19.53]	[15.65, 0.09]	[9, 3.92]
$(\hat{\sigma}(\rho), \hat{\sigma}(\pi))$	(13.93, 0.03)	(5.89, 0.05)	(3.65, 0.20)	(4.07, 0.84)	(4.06, 2.20)	(5.91, 0.03)	(2.1, 0.44)
$\{\hat{\sigma}(\hat{\rho})/\hat{\rho}, \hat{\sigma}(\hat{\pi})/\hat{\pi}\}$	{63.00, 54.34}	{42.89, 29.38}	{30.05, 18.81}	{21.66, 17.03}	{28.51, 11.27}	{37.73, 32.93}	{23.32, 11.33}
$VaR_{99.9}$	1.83	2.88	9.43	36.54	63.04	1.91	19.13
$cVaR_{99.9}$	2.73	3.78	11.39	41.69	67.12	2.61	21.67

Notes: This table states the MLE results based on different time horizons for the rating categories 'A', 'Baa', 'Ba', 'B' and 'C'. The Investment Rating Grade 'IG' combines the four highest rating grades 'Aaa', 'Aa', 'A' and 'Baa', while the Speculative Rating Grade 'SG' summarizes the lowest rating grades 'Ba', 'B', 'Caa-C'. $[\hat{\rho}, \hat{\pi}]$ are the MLE-Estimates for the unknown parameter ρ and π, and $\hat{\sigma}(\hat{\theta})$, $\theta \in \{\rho, \pi\}$ is the estimated standard error. $VaR_{99.9}$ and $cVaR_{99.9}$ denotes the VaR, respectively cVaR at the estimation $[\hat{\rho}, \hat{\pi}]$.

risk buckets exhibit higher uncertainty around the parameter estimates. Both effects in combination will boost the vulnerability of risk measures for higher rated securities.

For additional insight, the next tables provide the sensitivity measures from Section 4.2.2 computed from the available real-world data. Table 4.3 shows the ratios $q_\rho(\rho, \pi, \alpha)$ from (4.11) and $q_\pi(\rho, \pi, \alpha)$ from (4.12) using the parameter estimates from the various time horizons and $\alpha \in (0.99, 0.999)$.

The interpretation of the numbers is as follows. If $q_\rho(\rho, \pi, \alpha) > 1$, the cVaR is (by that factor) more sensitive than the VaR to changes in ρ, respectively for $q_\pi(\rho, \pi, \alpha)$ for changes in π. In most instances, cVar is more sensitive than VaR. For example using the longest time horizon for A rated risk buckets, the cVaR is more sensitive to changes in ρ than the VaR by a factor of $q_\rho(22.11\%, 0.05\%, 99.9\%) = 1.7036$ or $q_\rho(22.11\%, 0.05\%, 99.0\%) = 2.6765$.

More economically this means: If the 99.9% cVaR (based on the 40 year time period) is used for an A-rated risk bucket for the allocation of risk capital, any estimation error in ρ reacts by factor 1.7036 more intensively than if the

Table 4.3: Application of $q_\rho(\rho, \pi, \alpha)$ from (4.11) and $q_\pi(\rho, \pi, \alpha)$ from (4.12) on Parameter Estimates from Table 4.2

	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
Data from 2003 - 2012							
$q_\rho(\hat{\boldsymbol{\theta}}, 0.99)$	2.4464	2.0698	1.7164	1.5633	1.2111	2.1628	1.356
$q_\pi(\hat{\boldsymbol{\theta}}, 0.99)$	1.5643	1.4468	1.3026	1.2146	1.0308	1.4849	1.1229
$q_\rho(\hat{\boldsymbol{\theta}}, 0.999)$	1.6069	1.5021	1.3651	1.2621	1.1022	1.5437	1.1989
$q_\pi(\hat{\boldsymbol{\theta}}, 0.999)$	1.3217	1.2668	1.1836	1.1065	1.0013	1.2937	1.0751
Data from 1993 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
$q_\rho(\hat{\boldsymbol{\theta}}, 0.99)$	2.4259	1.9344	1.5448	1.4175	1.1923	2.1329	1.3385
$q_\pi(\hat{\boldsymbol{\theta}}, 0.99)$	1.5714	1.4019	1.2309	1.1386	1.0186	1.4789	1.1136
$q_\rho(\hat{\boldsymbol{\theta}}, 0.999)$	1.6276	1.467	1.3102	1.1889	1.0899	1.5428	1.191
$q_\pi(\hat{\boldsymbol{\theta}}, 0.999)$	1.3407	1.2505	1.1552	1.0559	0.9919	1.296	1.07
Data from 1973 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'	IG	SG
$q_\rho(\hat{\boldsymbol{\theta}}, 0.99)$	2.6765	1.8417	1.539	1.3432	1.1414	2.0638	1.3487
$q_\pi(\hat{\boldsymbol{\theta}}, 0.99)$	1.6461	1.3698	1.2221	1.0983	0.9802	1.4604	1.1191
$q_\rho(\hat{\boldsymbol{\theta}}, 0.999)$	1.7036	1.4434	1.2944	1.1512	1.0437	1.5326	1.1959
$q_\pi(\hat{\boldsymbol{\theta}}, 0.999)$	1.3829	1.2402	1.1408	1.0297	0.9531	1.2944	1.0732

Notes: This table denotes $q_\rho(\rho, \pi, \alpha)$ from (4.11) and $q_\pi(\rho, \pi, \alpha)$ from (4.12) for the derived parameter estimates in Table 4.2 depending on time horizon and $\alpha \in (0.99, 0.999)$. $\hat{\boldsymbol{\theta}} = [\hat{\rho}, \hat{\pi}]$ where $\hat{\rho}$ and $\hat{\pi}$ are the result of the maximum-likelihood estimation.

VaR were used.

This underlines our previous theoretical results that for highly rated risk buckets, the cVaR generally reacts more sensitively to errors in the underlying parameters than the VaR, this effect increases for a decreasing α.

Because the ratio reported in Table 4.3 is only based on the partial derivatives of the two risk measures, it does not correct for the actual level of the VaR and cVaR. Therefore Table 4.4 shows the ratio between the relative VaR add-on (4.13) and the cVaR add-on from (4.14) as well as these individual add-ons. Because we have performed a maximum-likelihood estimation with estimates for the standard errors $\hat{\sigma}(\hat{\theta})$, $\theta \in \{\rho, \pi\}$ - compare Table 4.2 - we modify the uncertainty box from (4.15) for calculating the add-ons as

$$\begin{aligned}\Theta_k =& \big((1 - k \cdot \hat{\sigma}(\hat{\rho})) \cdot \hat{\rho}, (1 + k \cdot \hat{\sigma}(\hat{\rho})) \cdot \hat{\rho}\big) \times \\ & \big((1 - k \cdot \hat{\sigma}(\hat{\pi})) \cdot \hat{\pi}, (1 + k \cdot \hat{\sigma}(\hat{\pi})) \cdot \hat{\pi}\big) \subseteq (0,1)^2\end{aligned} \tag{4.27}$$

and by solving (4.13) and (4.14) with this new restriction. This uncertainty box scales with $k > 0$ and the estimated standard errors $\hat{\sigma}(\hat{\theta})$, $\theta \in \{\rho, \pi\}$. Thus, the box accouts for the size of the estimation errors for the parameter ρ and π. Then, we define the resulting ratio between these two new add-ons by

$$\hat{q}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k) = \frac{\widehat{cVaR}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k)}{\widehat{VaR}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k)}, \tag{4.28}$$

where

$$\widehat{\mathcal{R}}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k) = \frac{\max\limits_{(\rho,\pi)\in\Theta_k} \mathcal{R}(\rho, \pi, \alpha) - \mathcal{R}(\hat{\rho}, \hat{\pi}, \alpha)}{\mathcal{R}(\hat{\rho}, \hat{\pi}, \alpha)} \tag{4.29}$$

for $\mathcal{R} \in \{VaR, cVaR\}$. Again, these optimization problems can be solved analytically, analogously to (4.16) and (4.18).

This ratio can be interpreted as follows. If $\hat{q}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k) > 1$, the relative cVaR add-on from the defined uncertainty box Θ_k is larger than the relative VaR add-on by this factor. Since these add-ons are normalized, a ratio larger than one means that the cVaR actually reacts more sensitively to estimation errors than the VaR. In addition, the add-ons $\widehat{\mathcal{R}}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k)$ can be interpreted as a premium of an ambiguity averse investor in the sense of Gilboa and Schmeidler (1989) for the chosen $k > 0$. While $k = 0$ can be understood to be an ambiguity neutral investor, a larger k represents a higher ambiguity aversion. The results are reported in Table 4.4 for the three time horizons defined above and the rating grades ‘A’, ‘Baa’, ‘Ba’, ‘B’, ‘Caa-C’.[62]

Table 4.4 provides two dimensions of analysis; the first dimension is the relative risk measures add-on, and the second is the corresponding ratio of these add-ons.

Generally, a higher k obviously leads to higher relative add-ons for both risk measures. This is plausible, since with a higher k the uncertainty box from (4.27) becomes larger, representing a higher ambiguity aversion of the investor. This result holds for all time horizons and rating grades. Similarly, we find a higher relative cVaR and VaR add-on for shorter time horizons for all rating grades. Again, this result matches expectations, because the estimation errors $\hat{\sigma}(\hat{\rho})$ and $\hat{\sigma}(\hat{\pi})$ increase with shorter time series - compare Table 4.2 - resulting

[62] The results for IG- and SG-rated risk bucket are not reported in Table 4.4 due to space limitations. Generally, a IG-rated risk bucket has intermediate properties between ‘A’- and ‘Baa’-rated risk buckets, while a SG-rated risk bucket performs almost like a ‘Ba’-rated one.

Table 4.4: Application of $\hat{q}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k)$ from (4.28) and $\widehat{\mathcal{R}}_+(\cdot)$ from (4.29) on Parameter Estimates from Table 4.2

	'A'	'Baa'	'Ba'	'B'	'Caa-C'
2003 - 2012					
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 0.25)$	1.1685 (47.47% / 40.63%)	1.1158 (32.84% / 29.43%)	1.081 (24.09% / 22.29%)	1.0203 (17.45% / 17.11%)	1.0051 (7.18% / 7.14%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 1)$	1.2879 (248.74% / 193.14%)	1.1629 (158.78% / 136.55%)	1.0955 (109.65% / 100.09%)	1.01 (74.74% / 74%)	0.9918 (27.95% / 28.18%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 0.25)$	1.0444 (53.7% / 51.42%)	1.0347 (36.25% / 35.03%)	1.0234 (26.02% / 25.43%)	0.9798 (17.34% / 17.7%)	0.9819 (7.06% / 7.19%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 1)$	1.0393 (294.26% / 283.12%)	1.0324 (179.17% / 173.54%)	1.0154 (118.61% / 116.8%)	0.9582 (71.92% / 75.06%)	0.9678 (26.66% / 27.55%)
1993 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 0.25)$	1.1912 (35.76% / 30.02%)	1.1161 (21.6% / 19.35%)	1.0959 (14.7% / 13.41%)	1.0025 (9.89% / 9.86%)	1.0021 (4.98% / 4.97%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 1)$	1.3022 (178.05% / 136.73%)	1.1485 (98.72% / 85.96%)	1.1093 (63.59% / 57.33%)	0.9912 (40.56% / 40.92%)	0.9917 (19.4% / 19.56%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 0.25)$	1.0648 (41.51% / 38.99%)	1.0432 (24.11% / 23.11%)	1.045 (16.48% / 15.77%)	0.9694 (9.57% / 9.87%)	0.9796 (4.86% / 4.97%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 1)$	1.0813 (218.82% / 202.36%)	1.0475 (112.55% / 107.44%)	1.0478 (72.1% / 68.81%)	0.9534 (38.17% / 40.03%)	0.9687 (18.52% / 19.12%)
1973 - 2012	'A'	'Baa'	'Ba'	'B'	'Caa-C'
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 0.25)$	1.2304 (30.74% / 24.98%)	1.1263 (16.5% / 14.64%)	1.0732 (9.51% / 8.86%)	0.9948 (6.34% / 6.37%)	0.9701 (4.18% / 4.31%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.99, 1)$	1.3622 (149.53% / 109.77%)	1.1542 (73.32% / 63.52%)	1.0784 (40.01% / 37.10%)	0.9856 (25.57% / 25.94%)	0.9553 (16.01% / 16.76%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 0.25)$	1.0792 (36.4% / 33.73%)	1.0545 (18.76% / 17.79%)	1.0279 (10.33% / 10.04%)	0.9653 (6.06% / 6.28%)	0.9486 (3.81% / 4.01%)
$\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), 0.999, 1)$	1.1057 (189.01% / 170.94%)	1.0623 (85.24% / 80.24%)	1.0265 (43.51% / 42.39%)	0.9534 (23.91% / 25.08%)	0.9338 (14.08% / 15.07%)

Notes: This table denotes $\hat{q}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi}), \alpha, k)$ from (4.28) and $\widehat{\mathcal{R}}_+(\hat{\rho}, \hat{\pi}, \hat{\sigma}(\rho), \hat{\sigma}(\pi), \alpha, k)$ from (4.29) for the parameter estimates from Table 4.2 depending on time horizon, $\alpha \in (0.99, 0.999)$ and $k \in (0.25, 1)$. $\hat{\boldsymbol{\theta}} = [\hat{\rho}, \hat{\pi}]$ where $\hat{\rho}$ and $\hat{\pi}$ are the result of the maximum-likelihood procedure, while $\hat{\sigma}(\hat{\boldsymbol{\theta}}) = [\hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi})]$ and $\hat{\sigma}(\hat{\rho})$ and $\hat{\sigma}(\hat{\pi})$ are according estimated standard errors. Numbers in round brackets are the relative cVaR and VaR add-ons $\left(\widehat{cVaR}_+(\cdot) \ / \ \widehat{VaR}_+(\cdot)\right)$.

in a larger uncertainty box (4.27) and therefore a higher add-on.

In Table 4.2, we have already seen higher relative estimation errors for higher-rated risk buckets. Therefore, the higher-rated risk buckets experience a higher add-on for both risk measures. For example, for the 10 years time series, $\alpha = 0.999$ and $k = 1$ the risk measures for an A-rated grade are almost four times higher (relative add-ons equal almost 300%), while the risk measures for a 'Ba'-rated risk bucket are only doubled. We find this effect for all time series, and therefore propose that higher-rated risk buckets are more influenced by estimation errors.

Next, we observe the ratio of these add-ons $\hat{q}_+(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), \alpha, k)$. Notably, for the IG-rated risk buckets 'A', 'Baa' and 'Ba', this ratio is always larger than one. Therefore, the cVaR reacts more sensitively to parameter estimation error than the VaR in all IG-cases. This is in line with our theoretical analysis from Section 4.2.2. Again, under the same estimation error, represented by $\hat{\sigma}(\hat{\rho})$ and $\hat{\sigma}(\hat{\pi})$, and the same degree of ambiguity aversion, represented by $k \in \{0.25, 1\}$, the cVaR experiences a larger relative add-on. Therefore, a financial institution using the cVaR will be exposed to a higher degree of parameter error than an institution using the VaR.

This effect is more pronounced for high-rated risk buckets than for low rated ones. Thus, particularly in the case of IG-rated risk buckets, the VaR appears to be superior to the cVaR in its reaction to parameter estimation errors.

Another result of the theoretical analysis from Section 4.2.3 is confirmed by our analysis of real-world data in Table 4.2. We find that for the 'A'-, 'Baa'- and 'Ba'-rated risk buckets, the relative add-ons increase for a higher α. Therefore, with a higher α, both risk measures become more sensitive to estimation error. It should be noted that this effect is not observed for SG-rated risk buckets, in line with our results shown in Figure 4.6 and 4.7. Therefore, to reduce the vulnerability to estimation errors, it seems reasonable to choose a smaller α. Hower, it should be remembered that the cVaR reacts even more to the parameter estimation error compared with the VaR, as represented by higher $\hat{q}_{+}(\hat{\boldsymbol{\theta}}, \hat{\sigma}(\hat{\boldsymbol{\theta}}), \alpha, k)$.

4.4 Conclusion

In this analysis, we considered the effects of estimation errors on the VaR and cVaR in the ASFR model. For this purpose, we calculated analytically the partial derivatives of these risk measures with respects to the model parameters, and introduced specific key numbers for the analysis. In addition we considered the effect of the confidence level α on the effect-strength of parameter estimation errors.

The main findings are as follows. First, despite theoretical advantages of the cVaR over the VaR, the cVaR reacts more 'aggressively' to parameter errors than the VaR in practical empirical applications. Therefore, the VaR may be favored over the cVaR. Second, higher-rated risk buckets are more affected by estimation errors than lower-rated ones. Third, a higher confidence level α (being apparently safer) may boost the effect from estimation errors for IG-rated risk buckets. As a consequence, one might consider a lower α-quantile, where already a reduction from $\alpha = 0.999$ (in line with the Basel Accord) to $\alpha = 0.995$ (in line with solvency II) reduces the effects of uncertainty. Additionally, if we reduce α in such a way, we may be even more in favor of using the VaR rather than the cVaR as a risk measure; the relative cVaR parameter error add-on becomes higher in relation to the VaR add-on. Although the SG-rated risk buckets are more risky than IG-rated ones in terms of point measures such as probabilities of default or ratings, these risk buckets can be handled with higher certainty. Due to larger numbers of defaults in time-series data, an estimation procedure should lead to lower relative estimation errors. The overall risk measure add-ons from estimation errors are considerably lower than those of the IG-rated ones. Moreover, riskier buckets do not react as strongly to changes in the α-quantile.

To conclude, especially for high-rated risk buckets (IG-rated risk buckets) and a high confidence level α, the VaR appears to be superior to the cVaR in practice. In contrast for low-rated risk buckets (SG-rated risk buckets), neither the choice of α nor selection of risk measures has a decisive influence. Thus, the common approach to be more safe (low PD equivalent high rating, a high confidence level α, and choosing a tail-considering risk measure such as cVaR, all of which are advised by the Basel Accord) actually increases the effect of estimation errors in credit risk.

As result we support the current proposal of the Basel Accord to treat the IDR in the trading book according to the IRB approach. Any further consideration of a replacement of the 99.9% VaR by a cVaR for the determination of credit default risk capital has to take into account the general higher uncertainty effects of the cVaR especially for IG-rated risk buckets and lower confidence level α.

Chapter 5

Credit Risk Measures and Credit Decisions under Uncertainty

The content of this chapter refers to the working paper 'Credit Risk Measures and Credit Decisions under Uncertainty' by Claußen, A., and Rösch, D., 2014.

5.1 Introduction

An essential task of financial institutions is risk taking, such as credit risk, liquidity risk, interest rate risk, operational risk, regulatory risk, and reputation risk, but it is challenging to manage these risks appropriately. Therefore, risk management has become one of the most important subjects for financial institutions and academic researchers in the area of banking. More precisely, however, financial institutions not only face *risk*, but also *uncertainty*. Risk and uncertainty are usually referred to as defined by Knight (1921), who considers these factors as random variation according to probability law that is either known (risk) or unknown (uncertainty). This differentiation has led to a wide body of research, thoroughly reviewed by Gilboa and Marinacci (2013). Ellsberg (1961) shows that generally uncertainty matters in decision making because decision-makers are not neutral towards uncertainty. (Epstein, 1999, p. 579) states that

> the distinction between them is roughly that risk refers to situations where the perceived likelihoods of events of interest can be

> represented by probabilities, whereas uncertainty refers to situations where the information available to the decision-maker is too imprecise to be summarized by a probability measure. Thus the terms ‘vagueness’ or ‘ambiguity’ can serve as close substitutes.

In this article we focus on uncertainty (synonymous with ambiguity) with respect to credit risk, because it is one of the most significant risk classes for banks. Without consideration of uncertainty, credit risk modeling relies on a stochastic framework with implicit assumptions about models, stochastic variables and parameters. Using tools of standard statistics the potential future loss of a credit portfolio is usually modeled as a random variable. *Risk measures* (e.g. Value-at-Risk or Expected Shortfall to a predefined confidence level) are then derived from its probability distribution in order to aggregate the credit risk into one representative key figure. The bank can then use aggregated quantification of risk for processes such as making an investment decision or determining and allocating economic capital. If all assumptions about the model, the stochastic variables and the parameters hold with certainty (i.e. the model is correct), a financial decision-maker would face only *risk* as the outcome of the stochastic variables in the *true model*.

It is not possible to determine the *true model* for a real portfolio, but even if it was, all included parameters would have to be known with certainty. Because model parameters cannot be observed directly, they are usually estimated based on observable data using statistical and econometric techniques. This induces parameter estimation errors. As a result, the values inserted for the parameters will almost certainly not match the true underlying and unknown parameters. The errors can often not be described by a known probability law unless further assumptions are made. Therefore, the bank faces the challenge of using models affected by *parameter uncertainty*.

In this paper, we present an economic framework for the quantification of parameter uncertainty in credit risk models and thus provide a clear-cut distinction between uncertainty and risk. We show how a decision-maker derives his decision in an environment that includes both risk and uncertainty. The degree of a decision-maker’s aversion or affinity to uncertainty can be quantified and decoupled from the degree of risk aversion. We illustrate the approach and its practical implications by using a publicly available dataset. We show that portfolios with particularly high ratings (a lower average probability of default and, thus, low risk) can be affected by parameter uncertainty more strongly than portfolios with lower ratings (and, thus, higher risk). Even a decision-maker who has a moderate degree of aversion to uncertainty might then prefer a low-rated, high-risk portfolio to a high-rated, low-risk portfolio once uncertainty aversion is taken into account.

Our approach relates to four streams in the literature. The first considers modeling and estimation of risk in credit models, see Gordy (2000), Gordy (2003), Löffler (2003), Hamerle et al. (2003*b*), Duffie et al. (2007), Feng et al. (2008), Tarashev and Zhu (2008), Heitfield (2008), Duffie et al. (2009), Gordy and Heitfield (2010), Rösch and Scheule (2014). These papers either consider only risk given several known parameters, or provide insights into the magnitudes of estimation errors but do not suggest how to deal with these errors in a decision-theoretic environment. The second stream extends classical modeling and estimation approaches to Bayesian statistics, which considers parameters as random variables via a given prior. Important references in the credit risk field are (Gössl, 2005; Dwyer, 2006; McNeil and Wendin, 2007; Kiefer, 2009; Tarashev, 2010; Chang et al., 2011). A related, third stream is based on robust optimization, one of the most popular topics in the field of optimization and control. This approach seeks to minimize the negative impact of future random events when model parameters are unknown, and owes particular credit to Ben-Tal and Nemirovski (1998, 1999) and El Ghaoui and Lebret (1997); El Ghaoui et al. (1998). During the last decade, a number of contributions have been published about (market risk) portfolio selection (thoroughly reviewed and discussed by Fabozzi et al., 2010). A shared feature of Bayesian statistics and robust optimization is that they are both capable to deal with parameter errors from a mathematical technical rather than from an economic perspective. It is also possible to derive uncertainty-adjusted risk measures within these approaches, but there is a lack of economic rationale as to which prior to choose in the Bayesian framework or what size the uncertainty set should be in robust optimization. This is addressed in the fourth stream, which is based on work by Gilboa and Schmeidler (1989). These authors derive an optimization approach for an ambiguity-averse investor within a decision theoretic framework and thus explicitly consider an investor's attitude to risk *and* uncertainty simultaneously. This economic framework has been applied to mean-variance optimization within modern portfolio theory for stocks, see Goldfarb and Iyengar (2003); Tütüncü and Koenig (2004); Garlappi et al. (2007); Zhu et al. (2009), who all analyze best worst-case performance of an ambiguity-averse investor's optimal portfolio selection under given uncertainty sets.

Given these streams of literature, our paper provides the following contributions. Firstly, we transfer the economic rationale of max-min optimization as defined by Gilboa and Schmeidler (1989) to the credit area, and show how parameter uncertainty can be considered and quantified separately from risk in a credit model in line with economic theory, Bayesian statistics and robust optimization. The approach is more flexible and economically more intuitive than Bayesian statistics, as uncertainty aversion can be considered directly. Indeed, we show that the Bayes based frameworks are a special case of our approach and imply a given, fixed degree of uncertainty attitude. Secondly, as a further extension to robust optimization, we introduce a new uncertainty set that covers the possibility of parameter uncertainty based on available data most accurately by abandoning the assumption of normality which is valid only asymptotically (if at all). Thirdly, our approach delivers a definition of risk measures, such as Value-at-Risk or conditional Value-at-Risk (Expected Shortfall), under uncertainty. We show how uncertainty can be considered directly when risk measures are derived in practical applications. This provides additional insights into the current discussions of adequate risk measures, which focus on diversification, aggregation, economic interpretation and robustness.[63] Two important practical results are that i) uncertainty aversion requires premia on risk measures and that ii) decision makers can view a credit-risky asset or a portfolio with a lower risk as inferior to another asset or portfolio with higher risk if uncertainty aversion is adequately taken into account.

Our results have implications for risk management, particularly for the determination of economic capital and the risk-bearing capability of financial institutions, by providing a definition of uncertainty-adjusted risk measures and a decision-theoretic approach for credit portfolio selection. The results could also impact on banking supervision, since our framework could easily be adopted fo regulatory purposes, for example by considering parameter errors in supervisory rules or by providing safety buffers against uncertainty.

The remainder of the paper is structured as follows: Section 2 presents the basic model and related literature, and introduces the novel data-driven restriction. Section 3 analyses the framework and its implementation and compares it with other (Bayesian-based) approaches. Section 4 applies the

[63] For an overview of relevant literature see Embrechts et al. (2013) and Emmer et al. (2013).

concept to empirical data. Section 5 concludes and discusses the potential impact of our results on practical applications.

5.2 Credit Decision under Risk and Uncertainty

In line with Knight (1921), we define *risk* as random variation according to a known probability law, and *uncertainty* as random variation according to an unknown law. In order to define such laws, we use $(\Omega, \mathcal{F}, \mathbb{P})$ as the ordinary probability space and $L^0(\Omega, \mathcal{F}, \mathbb{P})$ as a set of $\mathcal{F}$-measurable almost surely finite random variables in that space. Then, financial risks in general, and credit risk in particular, can be represented by a *convex cone* $\mathcal{M} \subseteq L^0(\Omega, \mathcal{F}, \mathbb{P})$.[64] This cone is defined so that any random variable $L \in \mathcal{M}$ shall represent a loss of a credit portfolio over a given time horizon and is modeled as a function $L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})$ of

i. a random vector $\mathbf{Y} = (Y_1, Y_2, ..., Y_{n_\mathbf{y}})$, where each of $n_\mathbf{y} \in \mathbb{N}_0$ random variables $Y_i \in \mathcal{M}$ describes a possible future random variation according to a known law,

ii. a vector of $n_{\boldsymbol{\theta}} \in \mathbb{N}_0$ model parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_{n_{\boldsymbol{\theta}}}) \in \boldsymbol{\Theta} \subseteq \mathbb{R}^{n_{\boldsymbol{\theta}}}$, connecting the different random variables to the specific loss function and

iii. a vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, ...\gamma_{n_\gamma}) \in \boldsymbol{\Gamma} \subseteq \mathbb{R}^{n_\gamma}$, $n_{\boldsymbol{\gamma}} \in \mathbb{N}_0$, representing all possible decision alternatives.

According to Knight (1921), the *risk* of loss $L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})$ comes exclusively from the random outcome of $\mathbf{Y}$, because the parameters are deterministic. By using a *risk measure*, this risk is quantified. Alternatively, in economic terms according to Frey and McNeil (2002), we can interpret a risk measure applied to $L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})$ as the amount of capital that should be added as a buffer to a portfolio so that it becomes acceptable to an external or internal risk controller.

In general, a risk measure is a function that maps random variables to the real numbers, so any $\mathcal{R} : \mathcal{M} \to \mathbb{R} \cup \{\infty\}$ is a risk measure. In the literature

[64] $\mathcal{M}$ is a convex cone if $L_1 \in \mathcal{M}$ and $L_2 \in \mathcal{M}$ implies that $L_1 + L_2 \in \mathcal{M}$ and $\lambda L_1 \in \mathcal{M}\ \forall\ \lambda > 0$.

risk measures are often introduced axiomatically with specific conditions that should be met in order for the risk measure to be considered *good*. Examples include deviation measures (Pedersen and Satchell, 1998; Rockafellar et al., 2002), and the popular coherent risk measures in Artzner et al. (1999). Here, we do not choose such specific risk measures, but instead follow the broad principle 'Risk as a primitive' according to Brachinger and Weber (1997); we only claim the existence of a meaningful *risk ordering* in a binary relation $A \succsim B$. A is at least as risky as B.[65] In this setting, we can understand and use the expectation operator $\mathbb{E}(\cdot)$ or the standard deviation $\sigma(\cdot)$ as such a risk measure. In addition, we use the Value-at-Risk (VaR) and conditional Value-at-Risk (cVaR), also known as the Expected Shortfall. The VaR of a portfolio at the confidence level $\alpha \in (0,1)$ is given by

$$VaR_\alpha(L) = \inf\{l \in \mathbb{R} \mid \mathbb{P}(L > l) \leq 1 - \alpha\},$$

and the cVaR at confidence level $\alpha \in (0,1)$ is defined as

$$cVaR_\alpha(L) = \mathbb{E}(L|L \geq VaR_\alpha).$$

Generally the VaR returns the level of loss that is not exceeded with a given (high) confidence probability, while the cVaR gives the expected loss given the VaR is exceeded.

We use the expectation $\mathbb{E}(\cdot)$ and standard deviation $\sigma(\cdot)$ operators, because they are the first two moments of a random variable and therefore give a first indication about the modeled loss. Then we consider the VaR, as it is still the most commonly used risk measure in practice, and is used for applications such as allocating economic capital. However, the VaR has also been widely criticized. The VaR is neither convex nor sub-additive in the general distribution case, and is therefore not a coherent risk measure in the sense of the definition by Artzner et al. (1999). Moreover, the VaR may exhibit multiple local extremes for discrete distributions (e.g. Mausser and Rosen, 1998) and is therefore hard to optimize in these cases. Finally, the VaR is merely a percentile of a probability distribution, and therefore does not take into account any tail information beyond the VaR. In contrast, the cVaR has the property of coherence (see e.g. Acerbi and Tasche, 2002; Frey and McNeil, 2002; Tasche,

[65] This risk ordering is similar to the widely used monotonicity property.

2002), and is convex and easily optimized as shown by Rockafellar and Uryasev (2000). In addition, it considers tail-risk by definition. It has therefore become the favored risk measure in academic research and is the second most popular risk measure used in practice today. The discussion about appropriate risk measures, and whether the cVaR is superior to the VaR, is still very much alive. Current debates focus on issues related to diversification, aggregation, economic interpretation, extreme behavior and robustness of the VaR and cVaR; Embrechts et al. (2013) and Emmer et al. (2013) provide an excellent overview. Against this background, we use the cVaR as the fourth risk measure in our study.

After specifying the model $L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})$, a risk-averse investor (or other decision-maker) makes a decision represented by $\boldsymbol{\gamma}$ that minimizes the risk quantified by a risk measure, given some restriction (e.g. a minimum expected return, no shortsales, normalized asset values). Therefore, the investor faces the following optimization problem,

$$\min_{\boldsymbol{\gamma} \in \boldsymbol{\Gamma}} \mathcal{R}\left(L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})\right) \qquad \text{s.t.} \qquad r\left(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma}\right) \leq \mathbf{c}, \tag{5.1}$$

where $\mathcal{R}(\cdot)$ is a chosen risk measure and $r\left(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma}\right) \leq \mathbf{c}$ contains the restrictions.

Markowitz (1952) has introduced this optimization problem (5.1) for the allocation of wealth across assets, and laid the foundation for modern portfolio theory, known as the mean-variance framework. The variance is a symmetric measure, therefore negative as well as positive deviations are considered. Because positive deviations are often not perceived as a risk, the VaR became a popular downside risk measure in the last two decades. However, due to its undesirable mathematical characteristics, such as a lack of sub-additivity and convexity, Rockafellar and Uryasev (2000) have presented an approach for minimizing the cVaR rather than the VaR. In their initial article, the constraints covered long-only positions, non-negative portfolio weights, a fully invested portfolio constraint and a minimum expected return constraint. Krokhmal et al. (2002) extended the variety of constraints by introducing linear transaction costs, constraints on the portfolio shares (such as minimum and maximum portfolio shares of single assets) and liquidity constraints as bounds on the position changes. An application to portfolios with credit risk is presented by Andersson et al. (2001).

All portfolio selection problems have in common that the model parameters $\boldsymbol{\theta}$ (in classical portfolio selection model: expected returns, variances and covariances of returns; in credit risk: probability of default and default correlation) are unknown and are usually estimated from given data $\aleph$ (e.g. realized stock returns or credit default data). Thus these parameters are estimated with errors.[66] For stocks, it is well known that a small perturbation of inputs may lead to a large change in the optimal portfolio (see e.g. Best and Grauer, 1991; Broadie, 1993; Chopra and Ziemba, 1993; Michaud, 1989). This lack of robustness of parameters usually entails extreme positions in the assets of the optimal portfolio and delivers a poor out-of-sample performance, see e.g. Black and Litterman (1992). For credit risk, the existence and importance of parameter errors have only been acknowledged in some of the aforementioned publications.

Two ways of dealing with such estimation errors have emerged: Statistical approaches and optimization approaches. One popular statistical method for addressing parameter uncertainty follows a Bayesian approach, in which the unknown parameters are treated as random variables (for credit risk, see Gössl, 2005; Dwyer, 2006; McNeil and Wendin, 2007; Kiefer, 2009; Tarashev, 2010; Chang et al., 2011). The Bayesian decision maker combines prior beliefs about the parameters with evidence from observable data to construct a predictive (posterior) distribution of the parameters. Combing prior distribution and likelihood specifically addresses the uncertainty about the parameters. We will analyze the Bayesian approach and its relation to our method in more detail later.

As an optimization-based approach, *Robust Optimization* incorporates parameter uncertainty directly into the optimization algorithm and can be summarized as a deterministic worst-case approach. This method owes particular credit to Ben-Tal and Nemirovski (1998, 1999); El Ghaoui and Lebret (1997) and El Ghaoui et al. (1998). For an overview of current developments in portfolio selection, see Huang et al. (2010) and Fabozzi et al. (2010). All robust optimization problems have in common that the model parameters are not specified exactly, and it is assumed that these parameters belong to a given *uncertainty set*. These sets are described by additional *hard* uncertainty re-

[66] Note that parameter uncertainty arises even if the investor does not have to make an investment decision; for example, she may already have an active portfolio and needs to compute its risk to allocate economic capital accordingly.

strictions $u(\cdot)$, which must be satisfied (Ben-Tal and Nemirovski, 1999). Their structure and scale is specified by the modeller, typically based on statistical estimates (see Gregory et al., 2011). The structure represents the geometry or shape of the uncertainty set $u(\cdot)$, while the scale represents the magnitude of the structure. Common uncertainty restrictions $u(\cdot)$ are a finite set of scenarios (first introduced by Soyster, 1973) and box or ellipsoidal restrictions (e.g. Ben-Tal and Nemirovski, 1999; Goldfarb and Iyengar, 2003; Tütüncü and Koenig, 2004; Garlappi et al., 2007; Zhu and Fukushima, 2009; Boyle et al., 2012). Nearly all robust portfolio models construct uncertainty sets as ellipsoids, and while almost all literature about robust portfolio optimization considers the structure of $u(\cdot)$, there is only little research addressing the scale of it (Gregory et al., 2011).

The robust optimization framework incorporates parameter uncertainty, but without a decisive definition of scale, the decision maker's attitude towards uncertainty cannot be inherently included in the model. To address this issue, $u(\cdot)$ is often designed as a confidence interval. Early work in robust portfolio optimization that designs such uncertainty structures was done by Goldfarb and Iyengar (2003). Their uncertainty sets correspond to confidence regions around the least-squares estimate of the market parameters. Tütüncü and Koenig (2004) estimate $u(\cdot)$ using a bootstrapped sample as percentiles. Garlappi et al. (2007) introduced joint constraints instead of individual on expected returns constraints, and established the link between robust portfolio optimization and the work of Gilboa and Schmeidler (1989), who laid the economic decision-theoretic foundation for modeling ambiguity aversion. In these models, a non-neutral decision-maker behaves as if she maximizes, in every period, the expected utility given a worst-case belief that is chosen from a set of conditional probabilities (e.g. Epstein and Schneider, 2008). More recent work treating $u(\cdot)$ as a confidence interval based on box and ellipsoidal structures are from Zhu and Fukushima (2009) and Boyle et al. (2012).

We adapt the economics-based robust portfolio selection framework from Gilboa and Schmeidler (1989) and Garlappi et al. (2007) by extending equation (5.1) and adding an *additional maximization* to the set of possible parameters Θ, reflecting the investor's non-neutrality to parameter uncertainty. This additional maximization is restricted by an *additional uncertainty constraint* $u\left(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \aleph\right) \leq \boldsymbol{\epsilon}$, which can be interpreted as a specific (confidence) interval. If this interval is not empty, it explicitly acknowledges the possibility of estimation errors. We also add a constant $\psi \in \{-1, 1\}$. If $\psi = 1$, the investor is uncertainty averse, while for $\psi = -1$ she is ambiguity affine. These adaptations lead to the following min-max-problem.

Definition 1: *Optimization Problem under Risk and Uncertainty:*

$$\min_{\boldsymbol{\gamma}\in\boldsymbol{\Gamma}} \max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \psi \cdot \mathcal{R}\left(L(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma})\right) \tag{5.2}$$
$$\text{s. t. } r\left(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma}\right) \leq \mathbf{c} \quad \text{and} \quad u\left(\mathbf{Y}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\aleph}\right) \leq \boldsymbol{\epsilon}$$

For the derivation of the parameter estimates of the objective function and the restrictions, we use the maximum-likelihood approach.[67] If the likelihood function $\mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph})$ fulfills some regularity conditions,[68] the maximum-likelihood estimator $\hat{\boldsymbol{\theta}}$ is 1) consistent, 2) asymptotically normal, 3) asymptotically efficient, 4) achieves the Cramer Rao Lower Bound and 5) is asymptotically $\hat{\boldsymbol{\theta}} \sim N(\boldsymbol{\theta}, \mathcal{I}(\boldsymbol{\theta})^{-1})$, where $\mathcal{I}(\cdot)$ is the information matrix

$$\mathcal{I}(\boldsymbol{\theta}) = \mathbb{E}_{\boldsymbol{\theta}}\left(-\frac{\partial^2 log(\mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph}))}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}\right).$$

Based on such estimation results we define the *box uncertainty* for each parameter θ_j by

$$u_j(\theta_j, \hat{\theta}_j, \sigma(\hat{\theta}_j)) = \frac{(\theta_j - \hat{\theta}_j)^2}{(\sigma(\hat{\theta}_j))^2} \leq \epsilon_j, \tag{5.3}$$

where $\hat{\theta}_j$ is the estimate of θ_j and $\sigma(\hat{\theta}_j)$ its standard deviation. Then the approach delivers with a given probability an $n_{\boldsymbol{\theta}}$-dimensional box-region $\times_{j=1}^{n_{\boldsymbol{\theta}}}[\hat{\theta}_j - \sqrt{\epsilon_j} \cdot \sigma(\hat{\theta}_j), \hat{\theta}_j + \sqrt{\epsilon_j} \cdot \sigma(\hat{\theta}_j)]$ around the estimates $\hat{\boldsymbol{\theta}}$ that may include the unknown parameters. With higher estimation errors, induced by larger $\sigma(\hat{\theta}_j)$, the box increases.

If the parameters are estimated jointly, we define the *ellipsoid uncertainty* by

$$u(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}, \Sigma) = (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T \cdot \Sigma^{(-1)}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \leq \epsilon, \tag{5.4}$$

[67] Alternatively, a maximum-posteriori approach can be chosen. This allows the inclusion of prior beliefs or other information into the uncertainty framework.

[68] E.g. differentiability of the log likelihood function, compact parameter space and independent and identically distributed densities $f(\boldsymbol{\aleph}_i \mid \boldsymbol{\theta})$ (e.g. Wald, 1949).

where Σ is the covariance matrix of the estimates and $\Sigma^{(-1)}$ its inverse. This inequality describes an ellipsoid around the estimates and considers the standard deviations as well as the covariances of estimates.

The structure of the uncertainty restrictions (5.3) and (5.4) is defined by the left hand side of (5.3) and (5.4), while the size is determined by the choice of ϵ_j and ϵ. If the estimators are normally distributed and their standard deviations are known, (5.3) is standard normally distributed for each parameter θ_j and (5.4) is χ^2_n distributed with n degrees of freedom.[69] Under the assumptions of the asymptotic convergence of the maximum-likelihood estimator we define

$$\epsilon_j = \Phi^{-1}\left(1 - \frac{1-\beta}{2}\right)^2 \qquad \text{and} \qquad \epsilon = \chi^{-2}(\beta, dim(\boldsymbol{\theta})), \tag{5.5}$$

given an uncertainty aversion parameter $\beta \in (0, 1)$, similar to Garlappi et al. (2007); Zhu et al. (2009); Boyle et al. (2012). Now, the size of the uncertainty set, and therefore the uncertainty aversion, is measured by the choice of β, whereby a higher β implies a higher uncertainty aversion. The box and ellipsoid restrictions can be interpreted as one- and multi-dimensional confidence intervals, respectively.

From these definitions, we see that only a small part of available information of the likelihood is considered. The box uncertainty consists only of the modus (i.e. the ML estimates) and the standard deviation of the estimates, while the ellipsoid uncertainty also uses the correlation of the estimates. Therefore these uncertainty sets can be understood as an approximation of the likelihood function at its modus. This approximation performs well if the likelihood function equals a normal distribution function which is asymptotically correct.

In practice however, depending on the model, time length and data quality, the likelihood function does not necessarily converge towards the normal distribution. As a result, it can have several shapes, such as that of a bended ellipsoid, and it can even have multiple local maxima.[70] Therefore, we propose a new *data-driven uncertainty* restriction that takes into account all available information for a specific model. All of this information is embedded within

[69] In practice, standard deviations or the covariance matrix are not known and also have to be estimated. Consequently the Student-t-distribution and the Hotelings T-squared distribution can be used to determine ϵ_j and ϵ, respectively.

[70] These special shapes are covered neither by the Studend-t-distribution nor the Hotelings T-squared distribution.

the likelihood, and therefore we propose

$$\mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph}) \geq \epsilon(\beta) \tag{5.6}$$

as a new uncertainty structure, where its size is defined with respect to the choice of β by

$$\int\limits_{\{\boldsymbol{\theta} \ | \ \mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph}) \geq \epsilon(\beta)\}} \mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph}) \mathrm{d}\boldsymbol{\theta} = \beta \cdot \int\limits_{\Theta} \mathcal{L}(\boldsymbol{\theta}|\boldsymbol{\aleph}) \mathrm{d}\boldsymbol{\theta}. \tag{5.7}$$

Then the restriction is essentially a level curve of the likelihood, and by the choice of $\epsilon(\beta)$ according to (5.7) it can be considered as an exact β-confidence interval around the ML estimates. Therefore, it uses the specific shape of the likelihood.

Theoretically, the data-driven restriction has the best structure and the best consideration of uncertainty effects given a specific model according to the likelihood principle. Assume, for example, a model and observed data leading to a likelihood function with the shape of a pin. The modus of it shall be in the middle of its head. Then, the first two restrictions would cover the head of the pin relatively well, while the tip is not considered sufficiently. This becomes important if the maximum of the risk measure is at the end of the tip. In this case, the first two restrictions would far underestimate the possible effects of uncertainty.

5.3 Implementation

5.3.1 Credit Model, Estimation, and Estimation Error

We apply our uncertainty framework to the Asymptotic Single Risk Factor (ASRF) credit model that underlies the Basel Accord, which banks use to determine their regulatory minimum capital under the IRB Approach. The ASRF is also used by banks and researchers as a 'quick and dirty' approach to calculate and measure economic capital and credit portfolio risk. The foundation and derivation of the model is given by Vasicek (1987) and Gordy (2000, 2003). It is appealing because of its simplicity, its analytical tractability, its economic intuition, and its potential to model skewed loss distributions. As shown in Gordy (2000), the model can also easily be mapped onto other popular industry credit models. A large number of extensions have been proposed, but considering all of them here is beyond the scope of our study. More complex models usually require more instead of fewer parameters, and could therefore be dealt with in a way that is similar to the method presented here.

The ASRF credit model assumes an infinitely fine-grained homogeneous portfolio of loans or bonds, with the risk being driven by a single common, systematic risk factor; idiosyncratic risk disappears owing to full diversification. The distribution of credit portfolio loss $L(\cdot)$ in a given period is modeled by

$$L^{LHP}(Y, [\rho, \pi]) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot Y}{\sqrt{1-\rho}}\right), \qquad Y \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{5.8}$$

where $\pi \in (0,1)$ is an unconditional probability of loan default (PD), $\rho \in (0,1)$ is the asset (return) correlation, Y is a standard normally distributed common systematic risk factor, and Φ is the standard normal CDF (with Φ^{-1} denoting its inverse).[71]

In our study, we analyze the four risk measures expected loss, standard deviation, VaR, and cVaR, given as

$$\mathbb{E}\left(L^{LHP}(Y, [\rho, \pi])\right) = \pi, \tag{5.9a}$$

$$\sigma\left(L^{LHP}(Y, [\rho, \pi])\right) = \sqrt{\Phi_2(\Phi^{-1}(\pi), \Phi^{-1}(\pi), \rho) - \pi^2}, \tag{5.9b}$$

$$VaR_\alpha\left(L^{LHP}(Y, [\rho, \pi])\right) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot \Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right), \tag{5.9c}$$

$$cVaR_\alpha\left(L^{LHP}(Y, [\rho, \pi])\right) = \frac{1}{1-\alpha}\Phi_2\left(\Phi^{-1}(\pi), \Phi^{-1}(1-\alpha), \sqrt{\rho}\right), \tag{5.9d}$$

where $\Phi_2(x_1, x_2, \varrho)$ is the standard bivariate normal distribution function. Gordy (2003) shows that under the ASRF assumptions, the contribution of the VaR of a single entity is portfolio invariant, i.e. it does not depend on the characteristics of the portfolio in which it is held. Therefore, its risk can be measured by the VaR on a stand-alone basis.

As noted by Gordy and Heitfield (2010), the literature on parameter estimation for portfolio credit risk models has grown enormously over the last years. The early generation comprises non-parametric methods and meth-

[71] To simplify the notation, we skip the parameter of recovery rate RR by setting $RR = 0$. Therefore, if default occurs, the lender suffers a loss of 100%. If one wanted to consider a nonzero recovery rate, the risk measures derived below could be modified with the factor $(1 - RR)$.

ods of moment estimators (see Gordy, 2000), Nagpal and Bahar (2001). For Maximum-Likelihood estimators, see Gordy and Heitfield (2002), Frey and McNeil (2003), Hamerle et al. (2003*a*,*b*); Hamerle and Rösch (2005) and Düllmann et al. (2008). For a Bayesian MCMC approach, see McNeil and Wendin (2007). Apart from these kinds of point estimations for the parameters, the researchers have also developed interval estimation of default probabilities, especially for low default portfolios, see Tasche (2011).

In this study, we use the maximum-likelihood estimation method as described by Frey and McNeil (2003). If we observe a time series of defaults $\mathbf{d} = (d_1, ..., d_T)$ for a portfolio containing $\mathbf{n} = (n_1, ..., n_T)$ borrowers who are assumed to be homogeneous, we can derive the binomial-normal mixture likelihood function

$$\mathcal{L}^{LHP}(\rho, \pi, \mathbf{n}, \mathbf{d}) = \prod_{t=1}^{T} \int_{-\infty}^{\infty} b\left(d_t, n_t, \pi(y)\right) d\Phi(y). \tag{5.10}$$

where $b(.)$ denotes the probability function of the binomial distribution with n_t trials and a probability $\pi(y)$. This conditional probability of default is given by

$$\pi(y) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot y}{\sqrt{1-\rho}}\right). \tag{5.11}$$

The likelihood function (5.10) is logarithmized and numerically optimized with respect to the parameter ρ and π in the unit square. There are several algorithms solving this optimization problem, and the integral is evaluated by a global adaptive quadrature. In addition, one can numerically derive the second partial derivative at the estimates, and compute the inverse of the information matrix as an approximation for the standard errors.

It should be noted that the risk measures in (5.9) hold for the ASRF credit model in which all idiosyncratic risks are fully diversified away by infinite homogeneous loans or bonds. However, the estimates $\hat{\rho}$ and $\hat{\pi}$ are based on a binomial-normal mixture model with a finite number of loans. While there is also an ML estimator for the ASRF model (see Düllmann et al., 2008) we refer to the likelihood of the binomial-normal mixture model for the estimation. This is because in our empirical study, we use default data with a finite number of observed loans or bonds (occasionally with zero defaults in one or more years) that cannot be modeled in the ASRF. Alternatively, we could derive the risk measures for the specific numbers of observed loans or bonds based on the binomial-mixture model, thereby imposing longer computational times. However, since we consider more than 300 entities in almost all cases, the idiosyncratic risks are nearly diversified and the results do not change substantially.[72]

As described by Gordy and Heitfield (2002, 2010), the panel dataset $\mathbf{d} = (d_1, ..., d_T)$ and $\mathbf{n} = (n_1, ..., n_T)$ may cover default data on large numbers of rated obligors $n_t \gg 0$, but in time-series dimension $t \in \{1, ..., T\}$ the available data include only a few decades, or even just a few years. As shown by Gagliardini and Gourieroux (2005), a large n_t is not sufficient for consistency of the parameter estimation. Instead, a large T is needed in the time-series dimension. As already described by Gordy and Heitfield (2010), the standard errors show that even if the asymptotics are reliable and the estimators are unbiased, parameter estimates are prone to estimation error.

5.3.2 Simplified Credit Decision under Uncertainty

Using the ASRF credit model, we now simplify and solve the general min-max problem from (5.2). We assume a decision-maker who wants to invest into exactly one out of m portfolios with different risks. The investor cannot change the loans within the portfolios, is indifferent to other portfolio characteristics apart from risk, and has access to historical default data for each risk bucket. If the decision maker is uncertainty-averse (i.e. $\psi = 1$) her problem is

Definition 2: *Simplified Optimization Problem*

$$\begin{aligned} &\min_{\boldsymbol{\gamma} \in \{0,1\}^m} \max_{[\rho_i, \pi_i] \in (0,1)^2} \sum_{i=1}^{m} \gamma_i \cdot \mathcal{R}\left(L^{LHP}(Y, [\rho_i, \pi_i])\right) \\ &\text{s. t.} \sum_{i=1}^{m} \gamma_i = 1, \quad u\left(Y, [\rho_i, \pi_i], [\mathbf{n}_i, \mathbf{d}_i]\right) \leq \boldsymbol{\epsilon_i}, \ \forall i \in \{1, 2, ..., m\}. \end{aligned} \tag{5.12}$$

To solve this min-max problem (5.12) the m distinct inner maximization problems have to be solved; it is easy to see that the decision-maker therefore chooses the portfolio with the *lowest* worst-case risk.

Each inner maximization depends on two parameters $\boldsymbol{\theta}_i = [\rho_i, \pi_i] \in (0, 1)^2$,

[72] We applied and compared both methods and found qualitatively homogeneous results (data not shown).

therefore the integral (5.7) for the data-driven restriction can easily and quickly be evaluated numerically. For the sake of simplicity and easier reading, we skip the portfolio index i and obtain

Definition 3: *Risk Measure under Uncertainty*

$$\max_{[\rho,\pi]\in(0,1)^2} \mathcal{R}\left(L^{LHP}(Y,[\rho,\pi])\right) \quad \text{s. t.} \quad u\left(Y,[\rho,\pi],[\mathbf{n},\mathbf{d}]\right) \le \boldsymbol{\epsilon}. \tag{5.13}$$

Depending on the choice of risk measure $\mathcal{R}(\cdot)$ and uncertainty restriction $u\left(Y,[\rho,\pi],[\mathbf{n},\mathbf{d}]\right) \le \boldsymbol{\epsilon}$, each problem can be easily solved.

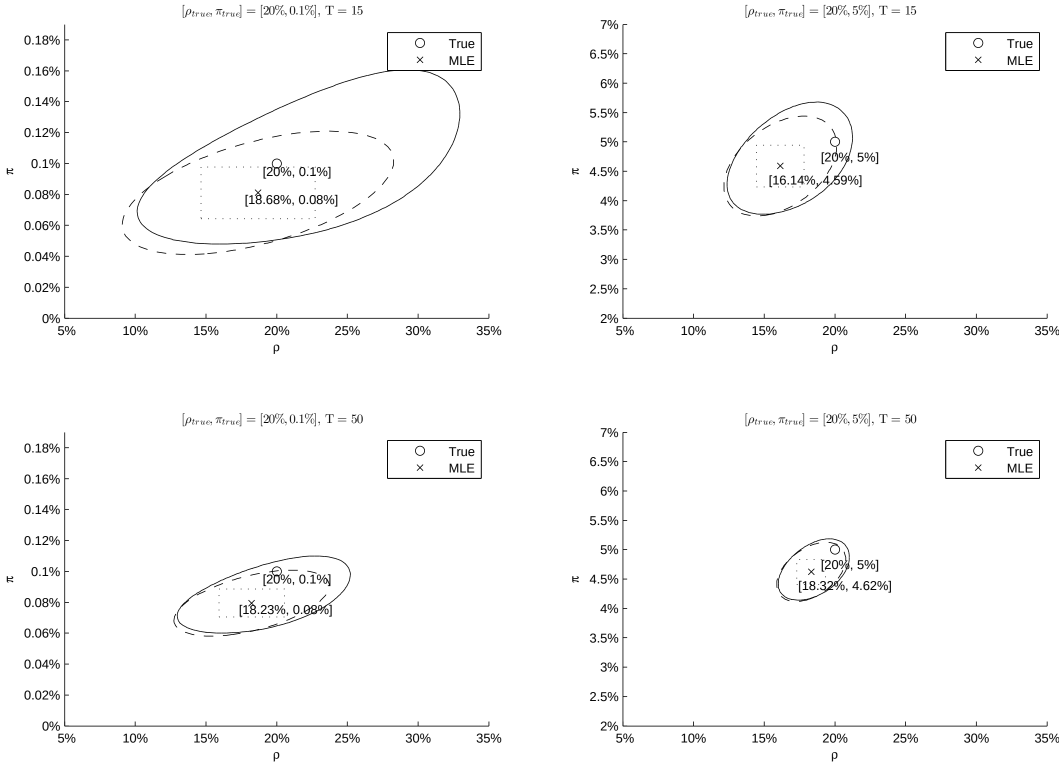
Thus, the min-max problem is reduced to the simpler inner-maximization problem. We analyze three restrictions and four risk measures more closely, resulting in twelve different optimization problems. Each problem is solved separately, leading to different arguments of the maximization. It is intuitive that the arguments of the maximization are generally distinct for each risk measure.[73]

For illustration purposes, we plot the three restrictions in Figure 5.1 for two parameter settings $[\rho_{true,1} = \rho_{true,2} = 20\%, \pi_{true,1} = 0.1\%, \pi_{true,2} = 5\%]$ and time lengths of $T_1 = 15$ years and $T_2 = 50$ years, respectively. For each of these four cases, we simulate a random default history based on the known true parameters and a given time length for 1,000 homogeneous entities. Under the assumption of an uncertainty aversion of moderate size ($\beta = 25\%$) and the results of a maximum-likelihood estimation, we derive the box restriction (R1), the ellipsoid restriction (R2) and the data-driven restriction (R3). R1 is represented as a dotted box, R2 is the dash ellipsoid and R3 is the full-line curved ellipsoid. For a clear visual interpretation, we have chosen a set of simulated default data, for which both estimates are smaller than the true parameters. These estimates are denoted as '$\times$' in Figure 5.1.

In all cases, the box restriction covers the smallest area around the estimates, followed by the ellipsoid restriction. Both restrictions are symmetrically centered around the estimates by definition, which is not the case for the data-driven restriction. R3 covers not only the largest area in all cases, but also considers far higher potential parameters for ρ and π due to its curved shape. Since all risk measures (with exception of VaR for $\pi < 1-\alpha$) are monotonously increasing in both model parameters, we expect a maximization subject to R3

[73] This might be counter-intuitive to the results from Rockafellar and Uryasev (2000), but here we optimize with respect to model parameters and not decision vectors.

Figure 5.1: Shape of Uncertainty Restrictions for Different Parameter Settings and Time Length



Notes: This figure shows the shape of the three restrictions depending on simulated loss data from the parameter setting $[\rho_{true,1} = \rho_{true,2} = 20\%, \pi_{true,1} = 0.1\%, \pi_{true,2} = 5\%]$ and time length $T_1 = 15$ and $T_2 = 50$. In all cases, 1,000 homogeneous entities are considered and a β-quantile of 25% is chosen. The restrictions 1, 2 or 3 are represented by the dotted box, dashed ellipsoid or full line ellipsoid, respectively.

leading to the largest values. Therefore, we recall the former pin-example and state that R1 and R2 can underestimate possible effects of parameter uncertainty.[74] Figure 5.1 illustrates the asymptotic convergence of the likelihood function (represented by R3) by a comparison of the upper (T_1=15) with the lower (T_2=50) graphs, in which the time series is increased. With longer time series, the shape of R3 converges to the shape of R2 due to the asymptotic properties.[75] Note also that R3 is more strongly curved for lower PD (left graphs) and therefore less well approximated by the other restrictions. Since it is curved into the upper right direction, and given the monotonous increases

[74] If a risk measure was maximized by the smallest allowed parameter constellation, R2 would overestimate the effects of uncertainty in comparison to R3.

[75] Note that even under consideration of a Hotelings T-squared distribution for T=15, the area of the ellipsoid uncertainty would increase by less than 10% and that the stretched curve of the data-driven restriction would not be covered.

of risk measures in the domain under consideration, we expect higher effects of parameter uncertainty and a larger underestimation from these effects by the restrictions R1 and R2, particularly for lower PD.

5.3.3 Risk Measures under Uncertainty

By introducing the simplification of the investment decision, the inner maximization problem (5.13) provides a definition of (credit) risk measures under uncertainty. If, for example, a decision-maker has to allocate economic capital for a given credit risky portfolio and is *uncertain* about the model parameters, he can insert his specific uncertainty aversion and solve the inner maximization problem (5.13). Alternatively, for regulatory purposes, a minimum coefficient for uncertainty aversion could be prescribed by regulatory authorities.

For an analysis of the effects of uncertainty on credit risk measures, we perform a Monte Carlo simulation study and generate a stylized portfolio of 1,000 borrowers for various scenarios.[76] We analyze three scenarios in which a uniform low-risk PD of 0.1%, a medium-risk PD of 1% and a high-risk PD of 5% is assumed for each borrower. In all cases, we set the asset correlation to 20%. These parameter settings approximate Moody's Investment Grade, 'Ba' and 'B' rated risk buckets, as we will see in more detail in Section 5.4.2. The time series is set to lengths $T = 15$, $T = 30$ or $T = 50$ years.

We randomly draw time series of default data and compute the parameter estimates and estimation errors. Then, we solve each inner maximization problem (5.13). For all cases, we set the α-quantile equal to 99.9%, in line with Basel IRB Approach, and vary the β-quantile according to 5%, 15% and 50%. We compute the difference to the true risk measures (computed by using the known parameters) in absolute and relative terms. For each setting, this procedure is repeated 1,000 times and the median of the results is reported in Table 5.1 ($T = 30$) and Table 5.2 ($T = 15$ and $T = 50$).

The first column in Table 5.1 reports the true risk measures, computed using the parameters, and the second column reports the absolute and relative difference (in parentheses) for the results of the maximum-likelihood estimates. For example, for the first parameter constellation ($\rho_{true} = 20\%$ and $\pi_{true} =$

[76] For a robustness check of idiosyncratic risk effects, portfolios with 100, 500 and 10,000 entities were analyzed in addition. This does not alter the general results, but due to less or more diversified idiosyncratic risk, the uncertainty add-ons became larger or smaller, respectively. Results are available upon request.

0.1%), the true 99.9% VaR is $VaR^{true}_{99.9\%} = 2.81\%$. The median 99.9% VaR of the 1,000 maximum-likelihood estimates is $VaR^{MLE}_{99.9\%} = 2.45\%$, yielding the reported absolute difference of $VaR^{MLE}_{99.9\%} - VaR^{true}_{99.9\%} = -0.36\%$. This gives the reported relative difference of -12.65%. Therefore in 50% of the Monte-Carlo replications the maximum-likelihood results underestimate the true risk by at least -12.65%. The remaining columns report the (relative) differences based on the uncertainty framework.

Table 5.1: Uncertainty Effects on Risk Measures (in per cent)

	$\rho_{\text{true}} = 20.00, \pi_{\text{true}} = 0.1$									
Riskmeasure			$\beta = 5$			$\beta = 15$			$\beta = 50$	
(true)	MLE	R1	R2	R3	R1	R2	R3	R1	R2	R3
$\mathbb{E}(\cdot) = 0.10$	-0.003	-0.0005	0.01	0.02	0.01	0.02	0.03	0.03	0.05	0.10
	(-3.42)	(-0.47)	(11.33)	(15.62)	(5.36)	(21.61)	(34.68)	(26.03)	(48.98)	(97.77)
$\sigma(\cdot) = 0.24$	-0.03	-0.02	0.03	0.05	0.01	0.07	0.12	0.11	0.19	0.39
	(-10.55)	(-6.26)	(11.46)	(19.27)	(3.70)	(30.66)	(48.07)	(46.89)	(80.06)	(162.2)
$\text{VaR}_{99.9} = 2.81$	-0.36	-0.21	0.30	0.48	0.08	0.85	1.45	1.35	2.53	5.07
	(-12.65)	(-7.51)	(10.82)	(17.17)	(2.82)	(30.33)	(51.6)	(48.16)	(90.19)	(180.71)
$\text{cVaR}_{99.9} = 3.96$	-0.58	-0.37	0.41	0.71	0.07	1.30	2.09	2.00	3.74	7.35
	(-14.75)	(-9.22)	(10.43)	(18.01)	(1.78)	(32.73)	(52.73)	(50.50)	(94.56)	(185.71)
	$\rho_{\text{true}} = 20.00, \pi_{\text{true}} = 1.00$									
Riskmeasure			$\beta = 5$			$\beta = 15$			$\beta = 50$	
(true)	MLE	R1	R2	R3	R1	R2	R3	R1	R2	R3
$\mathbb{E}(\cdot) = 1.00$	-0.03	-0.01	0.05	0.07	0.02	0.12	0.16	0.15	0.29	0.43
	(-3.07)	(-1.34)	(5.4)	(6.76)	(1.94)	(12.31)	(15.82)	(15.08)	(28.71)	(42.71)
$\sigma(\cdot) = 1.55$	-0.08	-0.05	0.09	0.12	0.03	0.23	0.30	0.34	0.59	0.87
	(-5.49)	(-3.02)	(5.86)	(7.56)	(2.17)	(14.76)	(19.23)	(22.08)	(38.04)	(56.33)
$\text{VaR}_{99.9} = 14.55$	-0.91	-0.54	0.90	1.15	0.26	2.34	3.05	3.49	6.07	8.70
	(-6.25)	(-3.69)	(6.16)	(7.93)	(1.8)	(16.06)	(20.95)	(24.01)	(41.72)	(59.78)
$\text{cVaR}_{99.9} = 18.14$	-1.18	-0.69	1.10	1.46	0.31	2.94	3.74	4.32	7.46	10.54
	(-6.50)	(-3.81)	(6.07)	(8.03)	(1.72)	(16.22)	(20.6)	(23.83)	(41.09)	(58.11)
	$\rho_{\text{true}} = 20.00, \pi_{\text{true}} = 5.00$									
Riskmeasure			$\beta = 5$			$\beta = 15$			$\beta = 50$	
(true)	MLE	R1	R2	R3	R1	R2	R3	R1	R2	R3
$\mathbb{E}(\cdot) = 5.00$	-0.03	0.03	0.28	0.31	0.16	0.52	0.60	0.63	1.07	1.31
	(-0.59)	(0.70)	(5.67)	(6.24)	(3.23)	(10.37)	(11.94)	(12.52)	(21.33)	(26.19)
$\sigma(\cdot) = 5.24$	-0.18	-0.09	0.22	0.25	0.09	0.51	0.62	0.79	1.27	1.63
	(-3.38)	(-1.66)	(4.12)	(4.82)	(1.81)	(9.80)	(11.74)	(15.01)	(24.22)	(31.16)
$\text{VaR}_{99.9} = 38.44$	-1.24	-0.63	1.41	1.66	0.59	3.45	4.17	5.31	8.58	11.01
	(-3.21)	(-1.64)	(3.66)	(4.31)	(1.53)	(8.97)	(10.84)	(13.81)	(22.31)	(28.64)
$\text{cVaR}_{99.9} = 43.85$	-1.57	-0.84	1.45	1.75	0.51	3.70	4.48	5.68	9.41	11.96
	(-3.59)	(-1.92)	(3.30)	(3.99)	(1.16)	(8.43)	(10.22)	(12.96)	(21.47)	(27.28)

Notes: This table shows the difference to the true risk measures in per cent. The numbers in parentheses give the relative difference to the true value in per cent. MLE represents the results at the maximum-likelihood estimate, while R1, R2 and R3 are the results from our uncertainty approach using restriction 1, 2 and 3. The time length of simulated default data is set to $T = 30$. All numbers are the median of a Monte Carlo simulation study with 1,000 repetitions.

Higher β results in higher absolute and relative differences for all risk measures and parameter settings. This is because all three uncertainty areas become wider with higher β and all risk measures are monotonously increasing in π and ρ in the domain under consideration. The largest differences result for R3, while R1 leads to the smallest differences. A lower PD results in a relatively higher difference, because with lower PD the quality of estimation decreases. This leads to larger ambiguity sets around the estimates, as seen in Figure 5.1. Due to the curving of R3 into the upper-right direction, particularly for low-default portfolios, the third restriction leads to much higher differences. Even though the relationship $\mathbb{E}(\cdot) < \sigma(\cdot) < VaR^{99.9} < cVaR^{99.9}$ holds for (absolute) differences, we cannot conduct any clear ordering for the relative differences. The cVaR exhibits the highest relative difference for low-default risk buckets, while this is not the case for the high-default portfolios. This indicates that the cVaR can be more sensitive to parameter uncertainty than the VaR, especially for lower PD (or better rating grade).

Table 5.2 shows the effects of different time lengths. We restrict the reported results to the analysis of the VaR and the three parameter settings for the time-series lengths of $T = 15$ and $T = 50$. The first column reports the true parameters $[\rho_{true}, \pi_{true}]$, the average parameter estimates $[\hat{\rho}, \hat{\pi}]$ and average standard errors $[\sigma(\hat{\rho}), \sigma(\hat{\pi})]$. The second column reports the true 99.9% VaR for each case. The third column shows the absolute (first row) and the relative differences (second row, numbers in parentheses) to the true risk measures for the maximum-likelihood estimates. The number of cases out of 1,000 replications in which the true VaR is underestimated is given in square brackets. The following columns show these results based on the uncertainty framework for the three different restrictions and the three level of uncertainty aversion $\beta \in \{5\%, 15\%, 50\%\}$.

As expected, smaller time series result in higher absolute and relative differences, because the estimation quality decreases. All three uncertainty sets become wider, and the differences increase. The risk bucket with the lowest PD and the shortest sample size exhibits the largest (relative) differences. With a smaller sample size, the outcome of different uncertainty sets differ much more. For example, for $T = 50$ and $\beta = 15\%$, the relative difference of the VaR for the risk buckets with the lowest PD is 24.71% for R2 and 36.18% for R3, compared with 22.27% vs 71.36% when $T = 15$. So, with a lower PD, higher uncertainty aversion and smaller sample size, the approximations of the box and ellipsoid uncertainty set of the data-driven restriction become worse

Table 5.2: Uncertainty Effects on the VaR under Different Restriction and Time Length (in per cent)

$[\rho_{true}, \pi_{true}]$ $[\hat{\rho}, \hat{\pi}]$ $[\sigma(\hat{\rho}), \sigma(\hat{\pi})]$	$\text{VaR}_{99.9}^{true}$	T = 15									
			$\beta = 5$			$\beta = 15$			$\beta = 50$		
		MLE	R1	R2	R3	R1	R2	R3	R1	R2	R3
[20.00, 0.100]	2.81	-0.84	-0.68	-0.04	0.32	-0.35	0.63	2.00	1.32	3.04	8.15
[14.43, 0.097]		(-29.96)	(-24.14)	(-1.32)	(11.27)	(-12.29)	(22.27)	(71.36)	(46.96)	(108.14)	(290.13)
[10.39, 0.055]		[611]	[585]	[504]	[462]	[536]	[445]	[362]	[395]	[307]	[156]
			{8.95}	{43.74}	{64.22}	{28.25}	{83.71}	{144.87}	{118.26}	{203.87}	{475.45}
[20.00, 1.00]	14.55	-1.83	-1.31	0.48	1.10	-0.31	2.50	4.08	4.11	7.86	14.29
[18.54**, 1.04**]		(-12.58)	(-9.03)	(3.32)	(7.55)	(-2.12)	(17.17)	(28.01)	(28.26)	(54.02)	(98.18)
[7.25, 0.43]		[582]	[562]	[473]	[446]	[516]	[395]	[329]	[331]	[240]	[140]
			{3.94}	{18.14}	{22.61}	{12.05}	{33.03}	{44.46}	{45.07}	{71.65}	{115.56}
[20.00, 5.00]	38.44	-2.49	-1.62	1.28	1.94	0.10	4.25	5.76	6.69	11.34	16.57
[18.73***, 5.15***]		(-6.46)	(-4.21)	(3.32)	(5.05)	(0.25)	(11.05)	(14.98)	(17.4)	(29.51)	(43.11)
[6.39, 1.40]		[582]	[557]	[468]	[450]	[496]	[401]	[351]	[328]	[238]	[160]
			{2.29}	{10}	{11.69}	{6.91}	{17.78}	{22.01}	{24.66}	{36.56}	{51.05}
$[\rho_{true}, \pi_{true}]$ $[\hat{\rho}, \hat{\pi}]$ $[\sigma(\hat{\rho}), \sigma(\hat{\pi})]$	$\text{VaR}_{99.9}^{true}$	T = 50									
			$\beta = 5$			$\beta = 15$			$\beta = 50$		
		MLE	R1	R2	R3	R1	R2	R3	R1	R2	R3
[20.00, 0.10]	2.81	-0.30	-0.19	0.24	0.36	0.04	0.69	1.02	1.07	1.95	3.26
[18.47*, 0.10**]		(-10.77)	(-6.73)	(8.41)	(12.95)	(1.55)	(24.71)	(36.18)	(37.98)	(69.45)	(116.03)
[7.21, 0.04]		[559]	[542]	[451]	[416]	[489]	[359]	[317]	[302]	[220]	[141]
			{4.55}	{21.41}	{25.94}	{14.06}	{39.68}	{51.94}	{54.72}	{89.99}	{144.06}
[20.00, 1.00]	14.55	-0.69	-0.38	0.72	0.88	0.22	1.86	2.20	2.73	4.73	6.17
[19.32***, 1.01***]		(-4.71)	(-2.64)	(4.97)	(6.02)	(1.54)	(12.78)	(15.12)	(18.77)	(32.54)	(42.38)
[4.36, 0.24]		[575]	[544]	[435]	[421]	[476]	[355]	[325]	[287]	[182]	[132]
			{2.18}	{9.98}	{10.97}	{6.62}	{18}	{20.55}	{24.31}	{38.39}	{48.19}
[20.00, 5.00]	38.44	-0.83	-0.36	1.24	1.38	0.56	2.87	3.25	4.25	6.71	8.00
[19.56***, 5.01***]		(-2.15)	(-0.94)	(3.23)	(3.58)	(1.46)	(7.48)	(8.46)	(11.07)	(17.46)	(20.81)
[3.84, 0.80]		[561]	[530]	[443]	[436]	[472]	[343]	[324]	[284]	[199]	[152]
			{1.27}	{5.57}	{5.88}	{3.84}	{9.92}	{10.99}	{13.7}	{20.48}	{24.02}

Notes: This table shows the difference to the true VaR in per cent. The numbers in parenthesis give the relative difference to the true VaR. MLE represents the results at the maximum-likelihood estimates, while R1, R2 and R3 are the results from our uncertainty approach using restrictions 1, 2 and 3. The time-length of simulated default data is set to T=15 and T=50. All numbers are the median of a Monte Carlo simulation study with 1,000 repetitions. The parameters estimates $[\hat{\rho}, \hat{\pi}]$ are statistically significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). Square brackets denotes the absolute number of cases out of 1,000 repetitions, in which the true VaR is underestimated. Number in curly braces give the relative add-on to the VaR at the estimates.

and may lead to an uncertainty effect underestimation.

The maximum-likelihood results underestimate the true VaR (see numbers in square brackets) in more than 50% of the cases each, in particular for short time series and low PD, as described comprehensively by Gordy and Heitfield (2010). This holds for the VaR and the cVaR. Therefore, in practical applications, the odds of an underestimation of the true risk (i.e. economic capital) are high due to parameter uncertainty or simply due to *bad luck in observed data*; odds increase particularly for high rated risk buckets. Using $\beta > 0$, these odds can be reduced. For example, for the highest PD and the shortest time series, the number of underestimations can be reduced from 582 out of 1,000 repetitions to 450 ($\beta = 5\%$) or 351 ($\beta = 15\%$). This reduction is achieved by an add-on or uncertainty premium to the VaR, the median of which is given in Table 5.2 in each fourth row in curly brackets in relative terms. Therefore, about 130 underestimations are avoided in the above example by a relative increase of the VaR of 11.69%.

By increasing β, which results in a higher relative add-on (uncertainty premium), the number of underestimations can be further reduced. However, what is an appropriate choice of β? To answer this question at least heuristically, we compute the ratio of uncertainty premium and the underestimation reduction. For example for $\beta = 15\%$, $\pi_{true} = 1\%$ and R3, the number of underestimations is reduced by about 250 and the premium is 44.46% ($T = 15$). This gives a ratio of 0.18%. Each underestimation that can be avoided costs about 0.18% in relation to the original VaR. For the longer time series, a ratio of 0.08% results. With lower PD c.p. the premium per effective reduction increases. Thus, especially for high-rated risk buckets, protection against uncertainty effects requires higher uncertainty premia. Moreover, the premium per effective reduction ratio increases with higher β. Therefore, any additional marginal protection against uncertainty requires an even higher uncertainty premium.

In summary we conclude the following. Firstly, high-rated (low-risk) buckets are more prone to parameter uncertainty than lower-rated (riskier) buckets. Secondly, if the decision-maker faces portfolios with low PD and short sample sizes, the possible uncertainty effects are best covered by the data-driven restriction. Thirdly, with higher uncertainty aversion, these effects are boosted and the restriction R3 becomes even more important. Fourthly, although there might be no *best* β, an intermediate size of about $\beta = 15\%$ appears to provide a reasonable trade-off between an underestimation reduction to 1/3 and the magnitude of uncertainty premium. Fifthly, especially for risk buckets with low PD (low risk) the VaR appears to be less sensitive to parameter uncertainty than the cVaR.

Regarding the restrictions, we conclude that R1 should be used only as a first proxy. Although the optimization problem can be solved analytically, the (relative) differences underestimate the possible parameter uncertainties in comparison with the other restrictions. R2 is suitable for higher PD risk buckets, longer time series and smaller β as an appropriate restriction, since it approximated R3 reasonably well in these cases. R3 includes all data information and implied uncertainty. Therefore, this seems to be the most appropriate restriction.

5.3.4 Comparison with Bayesian Approaches

Next, we show that our framework includes other existing *uncertainty approaches* in credit risk using Bayesian statistics, where the (unknown) parameters $\boldsymbol{\theta}$ are treated as random variables. A Bayesian decision-maker combines prior beliefs about the parameters with evidence from observable data $\boldsymbol{\aleph}$ to construct a predictive distribution of the parameters:

$$p(\boldsymbol{\theta}|\boldsymbol{\aleph}) = \frac{p(\boldsymbol{\aleph} \mid \boldsymbol{\theta}) \cdot p(\boldsymbol{\theta} \mid \boldsymbol{\xi})}{\int p(\boldsymbol{\aleph} \mid \boldsymbol{\theta}) d\boldsymbol{\theta}}, \tag{5.14}$$

where $\boldsymbol{\xi}$ is a vector of hyper-parameters describing the prior distribution $p(\boldsymbol{\theta} \mid \boldsymbol{\xi})$, $p(\boldsymbol{\aleph} \mid \boldsymbol{\theta})$ is the sampling distribution (i.e. likelihood) and $\int p(\boldsymbol{\aleph} \mid \boldsymbol{\theta}) d\boldsymbol{\theta}$ is the marginal likelihood. Bayesian approaches focus on the specification of reasonable priors and the derivation of the posterior distribution for $\boldsymbol{\theta}$, including technical sampling methods (e.g. Gössl, 2005; Dwyer, 2006; McNeil and Wendin, 2007; Kiefer, 2009; Tarashev, 2010; Chang et al., 2011). The random vector of 'candidate' values $\boldsymbol{\theta}_c$ is sampled from the posterior $p(\boldsymbol{\theta}|\boldsymbol{\aleph})$ and inserted into the model-specific loss function $L([\mathbf{Y},\ \boldsymbol{\theta}_c])$. Therefore, it becomes a joint probability distribution function of model-specific random variables, represented by $\mathbf{Y}$, and additional random variables represented by $\boldsymbol{\theta}_c$. Finally a risk measure $\mathcal{R}(\cdot)$ is usually computed, using the entire joint probability distribution $\mathcal{R}\big(L(\mathbf{Y},\ \boldsymbol{\theta}_c)\big)$, see Gössl (2005); Dwyer (2006); Chang et al. (2011). Either, the expectation $\mathbb{E}_{p(\boldsymbol{\theta}|\boldsymbol{\aleph})}\big(\mathcal{R}\left(L(\mathbf{Y}, \boldsymbol{\theta}_c)\right)\big)$ of the risk measure over the posterior is evaluated (Garlappi et al., 2007), or the risk measure of the whole joint probability distribution is computed; alternatively, if being very conservative, an extreme quantile of the distribution of risk measures sampled from $\boldsymbol{Y}$ and $p(\boldsymbol{\theta}|\boldsymbol{\aleph})$ can also be used (Tarashev, 2010).

To simplify matters, we analyze uncertainty only in ρ in the following section. We assume that the posterior distribution $p(\rho|\boldsymbol{\aleph})$ for the candidate value ρ_c is already derived as a beta distribution $\beta(\rho_c \mid \boldsymbol{\aleph})$ with a standard deviation of 5% and a modus of 20%. This modus shall equal the maximum-likelihood estimate for the unknown correlation ρ. The true PD is assumed to be known either as 0.1% or 1%. As a risk measure, we consider the VaR for confidence levels $\alpha \in \{95\%, 99\%, 99.9\%\}$. Then, we sample

10^6 candidate values for ρ_c from $\beta(\rho_c \mid \aleph)$, and for each candidate we draw a standard normal distributed y. As a result we obtain the outcome of the three different Bayesian-based approaches: The VaR of the joint distribution $A1 : VaR_\alpha\big(L^{LHP}(Y, \rho_c, \pi_{true})\big)$, the expectation of the VaR over the posterior beta-distribution $A2 : \mathbb{E}_{\beta(\rho_c|\aleph)}\big(VaR_\alpha\big(L^{LHP}(Y, \rho_c, \pi_{true})\big)\big)$ and the extreme α-quantile of the VaR distribution $A3 : L^{LHP}(\Phi^{-1}(1-\alpha), \beta^{-1}(\alpha), \pi_{true})$, where $\beta^{-1}(\cdot)$ is the inverse of the posterior distribution $\beta(\rho_c \mid \aleph)$.

Table 5.3 shows the add-ons to the risk measure using the maximum-likelihood estimates and the relative add-ons in parentheses. For each number, we also compute the *implied* β for the box uncertainty restriction, which is given in square brackets. This can be interpreted as the value for β that leads to the same add-on in our uncertainty framework and the corresponding Bayesian-based approach. For example, for PD of $\pi_{true} = 0.1\%$ and a 99.9% confidence level, the VaR using the ML estimate of $\rho = 20\%$ is $VaR^{MLE}_{99.9\%} = 2.81\%$. The first approach ($A1$) results in a VaR of 3.14%. This corresponds to the reported add-on of $VaR_{99.9\%}\big(L^{LHP}(Y, \rho_c, 0.1\%)\big) - VaR^{MLE}_{99.9\%} = 0.33\%$, and a relative add-on of 11.71%. To get this add-on from our uncertainty framework based on the box uncertainty, a β of 26.62% is necessary.

Table 5.3: VaR Add-on for Bayesian Approaches and Implied β (in per cent)

		$\pi_{\text{true}} = 0.1$			$\pi_{\text{true}} = 1$	
	$\alpha = 95$	$\alpha = 99$	$\alpha = 99.9$	$\alpha = 95$	$\alpha = 99$	$\alpha = 99.9$
VaR^{MLE}_α	0.42	1.10	2.81	3.77	7.53	14.55
A1	-0.01 (-1.48) [-19.36]	0.02 (2.02) [7.56]	0.33 (11.71) [26.62]	-0.01 (-0.22) [-0.60]	0.26 (3.45) [15.91]	1.65 (11.31) [34.36]
A2	-0.00 (-0.84) [-10.64]	0.03 (3.07) [12.61]	0.21 (7.59) [19.31]	0.04 (1.04) [8.09]	0.27 (3.62) [15.15]	0.82 (5.61) [17.52]
A3	0.02 (4.83) [84.07]	0.54 (48.96) [99.3]	3.93 (139.97) [99.96]	0.64 (16.88) [94.6]	4.02 (53.42) [99.4]	15.54 (106.81) [99.98]

Notes: This table shows the (relative) add-on to the VaR at the MLE in per cent for different Bayesian-based approaches (in parentheses). Numbers in square bracket describe the implied β based on our maximization framework using the box uncertainty. Given this β, our uncertainty framework results in the same differences as the respective Bayesian-based approach.

From the implied β we see that i) all other approaches comply with our uncertainty framework, given a specific β and ii) the implied uncertainty aversion changes with the choice of approach, risk bucket and confidence level α. The implied uncertainty aversion for the third (conservative) approach ranges from 84.07% to 99.98% and is the result of the presumed comonotonicity of the model-specific random variable Y and the posterior distribution $\beta(\rho_c \mid \aleph)$. For the first (second) approach the uncertainty aversion ranges between -19.36% (-10.64%) and 34.36% (19.31%), with a negative β representing uncertainty affinity. Generally the absolute and relative differences become larger with higher α. Therefore, α measures not only the degree of *risk* aversion, but also the degree of implied *uncertainty* aversion in the Bayesian approach. Because of this, risk and uncertainty are not separately quantified or distinguished. Moreover, for small α, a negative add-on is possible. This implies a negative uncertainty premium and a decision-maker who is uncertainty affine.

We conclude that the Bayesian approaches are compatible with our framework and cover uncertainty effects, but do not explicitly separate the effects of risk and uncertainty for given prior and posterior, this implies a specific degree of uncertainty aversion, and may even lead to the possibly undesired result of implied uncertainty affinity.

5.4 Empirical Example

5.4.1 Data

We use publicly available default data from Moody's (2013) annual default report. The report lists the number of companies (ranked by rating grade) at the beginning of any given year, and reports what proportion of them had defaulted by the end of the year. This information is given for the seven rating grades 'Aaa', 'Aa', 'A', 'Baa', 'Ba', 'B' and 'Caa-C'. The first four rating grades are summarized as Investment Grades ('IG'), while the last three rating grates are Speculative Grades ('SG').

In our study, we consider a 32-year period from 1981 to 2012. The number of rated companies in this dataset increases from 1,247 in 1981 to 4,823 in 2012. On average, there are 3,226 rated companies per year. The two highest rating grades are excluded due to an insufficient number of defaults (no defaults in 'Aaa', and only 6 in 'Aa') when the data are analyzed per rating grade, in line with McNeil and Wendin (2007) and Chang et al. (2011). We did include these grades for our analysis of IG-rated risk buckets. Descriptive statistics of the default rates for each grade are shown in Table 5.4. The mean and standard deviation of the historical one-year default rates of Moody's rated companies increase with rating grade, while the ratio of mean and standard deviation decreases.

Table 5.4: Basic Statistics of Historical One-year Defaults (%) Rate in Moody's (2013), 1981-2012

	A	Baa	Ba	B	Caa-C	IG	SG	Total
Mean	0.05	0.19	1.14	4.99	22.17	0.09	4.50	1.72
Std	0.11	0.30	1.20	3.94	19.64	0.14	3.04	1.30
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.16
Median	0.00	0.00	0.82	4.53	19.02	0.02	3.44	1.28
Max	0.51	1.06	4.95	16.02	100.00	0.51	13.29	5.97

Notes: This table shows descriptive statistics of the historical one-year default rates for the rating categories 'A', 'Baa', 'Ba', 'B' and 'Caa-C'. The Investment Rating Grade 'IG' includes the four highest rating grades 'Aaa', 'Aa', 'A' and 'Baa', while the Speculative Rating Grade 'SG' contains the lowest rating grades 'Ba', 'B' and 'Caa-C'. We use: std, standard deviation; min, minimum; max, maximum.

5.4.2 Risk Measures under Uncertainty

We apply the inner maximization problem (5.13) to the default data. For the specification of the box and the ellipsoid restriction, the parameter estimates and standard errors are derived via the maximum-likelihood approach and given in Table 5.5. The first column shows the maximum-likelihood estimates $[\hat{\rho}, \hat{\pi}]$ and their statistical significance. The second column shows their estimated standard errors $[\hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi})]$. The range of parameter estimates is $\hat{\rho} \in (6.81\%, 21.09\%)$ and $\hat{\pi} \in (0.06\%, 19.45\%)$. While $\hat{\pi}$ increases monotonously with worsening rating grade, we find a roughly decreasing trend for $\hat{\rho}$, with some exceptions. Except for 'A' and 'Baa' all estimates are highly significant. The next four columns show the estimates for the risk measures $\mathbb{E}(\cdot)^{MLE}, \sigma(\cdot)^{MLE}, VaR_{99.9}^{MLE}$ and $cVaR_{99.9}^{MLE}$ using a recovery rate of zero for simplicity.

For deteriorating rating grades we obtain c.p. higher risk measures. This indicates a monotonous risk-grading as we would expect. Additionally, in all cases, the $cVaR_{99.9\%}^{MLE}$ is larger than the $VaR_{99.9\%}^{MLE}$, which is to be expected according to

$$cVaR_\alpha(\cdot) = VaR_\alpha(\cdot) + \mathbb{E}\big(X - VaR_\alpha(\cdot) \mid X > VaR_\alpha(\cdot)\big),$$

where $\mathbb{E}\big(X - VaR_\alpha(\cdot) \mid X > VaR_\alpha(\cdot)\big) > 0$.

Table 5.5: MLE Results for Historical One-year Default Rates from Moody's (2013), 1981-2012

	$[\hat{\rho}, \hat{\pi}]$	$[\hat{\sigma}(\hat{\rho}), \hat{\sigma}(\hat{\pi})]$	$\mathbb{E}(\cdot)^{MLE}$	$\sigma(\cdot)^{MLE}$	$\mathrm{VaR}_{99.9}^{MLE}$	$\mathrm{cVaR}_{99.9}^{MLE}$
A	[21.09, 0.06*]	[13.85, 0.03]	0.06	0.16	1.95	2.86
Baa	[14.19**, 0.20***]	[6.35, 0.06]	0.20	0.32	3.22	4.21
Ba	[13.65***, 1.16***]	[4.33, 0.25]	1.16	1.33	11.24	13.61
B	[20.61***, 5.18***]	[4.66, 1.00]	5.18	5.48	40.06	45.60
Caa-C	[14.46***, 19.45***]	[4.19, 2.28]	19.45	10.75	63.28	67.37
IG	[16.11**, 0.10***]	[6.35, 0.04]	0.10	0.20	2.14	2.94
SG	[7.87***, 4.50***]	[1.95, 0.51]	4.50	2.81	19.4	21.77
Total	[6.81***, 1.71***]	[1.71, 0.21]	1.71	1.19	8.73	10.03

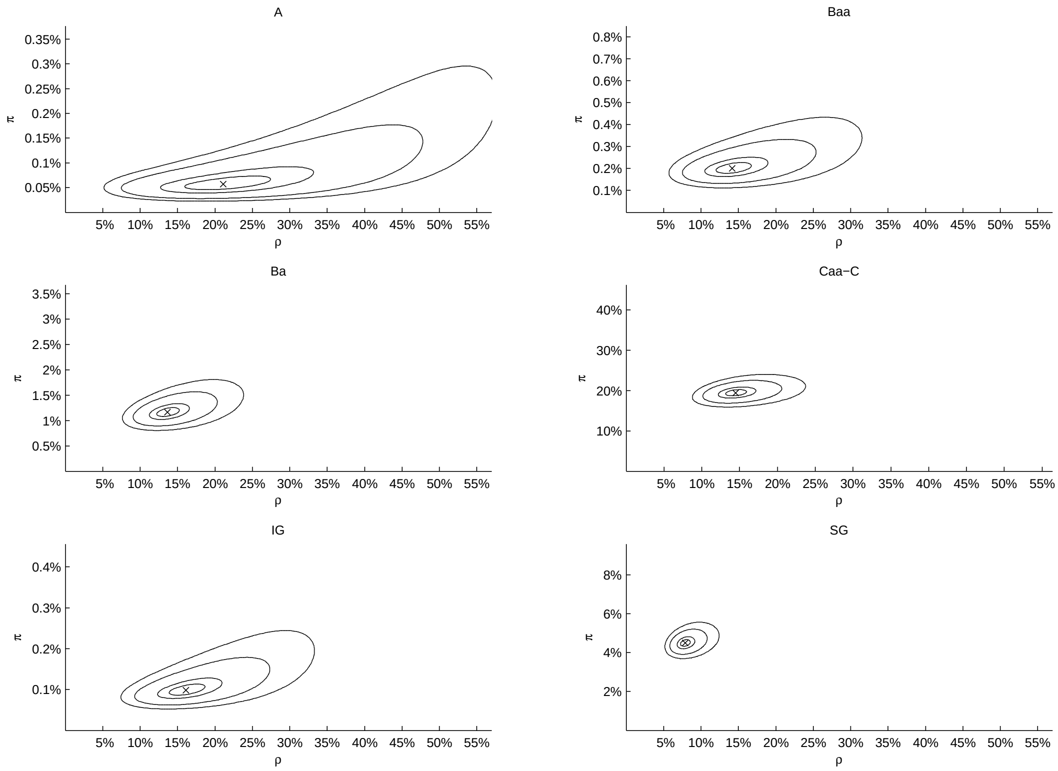
Notes: This table shows the MLE results for Moody's based on a yearly time horizon from 1981 to 2012 for the rating categories 'A', 'Baa', 'Ba', 'B' and 'Caa-C'. $[\hat{\rho}, \hat{\pi}]$ are the MLE-estimates and $\sigma(\theta), \theta \in \{\rho, \pi\}$ are their standard errors. The parameters are statistically significant at the 1%-level (***), the 5%-level (**), or the 10%-level (*).

The relative estimation error decreases with decreasing rating grade (with minor exceptions). Particularly, high-rated entities may be prone to parameter uncertainty. This is supported by contour plots of the likelihood function $\mathcal{L}^{LHP}(\rho, \pi|\mathbf{n}, \mathbf{d})$ for the rating grades 'A', 'Baa', 'Ba', 'Caa-C', 'IG' and 'SG' given in Figure 5.2. The '$\times$' denotes the maximum-likelihood estimates and the chosen contour plots equal the data-driven restriction for $\beta \in \{5\%, 15\%, 50\%, 75\%\}$. Therefore the inner and outer lines show the 5% and 75% confidence region, respectively, for the unknown model parameters.

As in our simulation study, we obtain strongly curved wide shapes for high rated risk buckets, while the low rated risk buckets produce nearly ellipsoidal narrow shapes. Particularly for the higher-rated low-risk buckets, the contours can obviously not be approximated by the other two restrictions. Additionally, a wider shape for the restriction may lead to larger uncertainty add-ons c.p.

Table 5.6 shows the results for the inner maximization problem (5.13) for the four risk measures and three uncertainty restrictions, using 'A' and 'Caa-C' for illustration. As the true risk measures are unknown, the first column reports the risk measures using the maximum-likelihood estimates. According to our simulation study, these may have a more than 50% chance of being an underestimation of the 'true' unknown risk measures. The next columns show the absolute and relative add-on to each risk measure than can be interpreted as a (relative) uncertainty premium. For example, for the 'A'-rated risk buckets, the maximum-likelihood approach estimates a 99.9% VaR

Figure 5.2: $\mathcal{L}^{LHP}(\rho, \pi|\mathbf{n}, \mathbf{d})$ for Selected Moody's Rating Grades (1981-2012)



Notes: This figure shows the likelihoods $\mathcal{L}^{LHP}(\rho, \pi|\mathbf{n}, \mathbf{d})$ for the one year default rate from Moody's for the rating categories 'A', 'Baa', 'Ba', 'Caa-C', 'IG' and 'SG' for a time horizon from 1981 to 2012. The contour plots show the data-driven restriction for $\beta \in \{5\%, 15\%, 50\%, 75\%\}$.

of $VaR^{MLE}_{99.9\%} = 1.95\%$. Under restriction R3 and a uncertainty aversion of $\beta = 15\%$, which might reduce the odds of an underestimation to 1/3, we get a 99.9% VaR of $VaR^{\beta=15\%,R3}_{99.9\%} = 5.34\%$. This corresponds to the reported add-on of $VaR^{\beta=15\%,R3}_{99.9\%} - VaR^{MLE}_{99.9\%} = 3.39\%$, yielding a relative add-on of 173.73% (in parenthesis).

As in the simulation, a higher β increases the add-on. Similarly, the data-driven restriction R3 leads to the highest (relative) add-ons, while R1 leads to the smallest add-on. Again, there is no clear ranking of risk measures with respect to their sensitivity to uncertainty effects. For the low-default portfolio, the $cVaR(\cdot)$ is affected most strongly in terms of relative add-on, followed by $VaR(\cdot)$, $\sigma(\cdot)$ and $\mathbb{E}(\cdot)$. For the high-default portfolio it is $\sigma(\cdot)$, followed $VaR(\cdot)$ and $cVaR(\cdot)$. The risk bucket with lowest PD (grade 'A') exhibits a considerably larger relative add-on (or uncertainty premium). For example, for $\beta = 0.15$, a relative premium of 173.73% of the VaR is necessary

Table 5.6: Uncertainty Effects Based on Risk Measure and Restriction for Moody's Rated Risk Buckets, 1981-2012 (in per cent)

Risk measure	$\beta = 5$			$\beta = 15$			$\beta = 50$		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
A									
$\mathbb{E}(\cdot)_{MLE} = 0.06$	0.00	0.01	0.02	0.01	0.02	0.04	0.02	0.04	0.07
	(3.49)	(17.91)	(32.66)	(10.58)	(31.89)	(71.11)	(37.72)	(65.85)	(122.05)
$\sigma(\cdot)_{MLE} = 0.16$	0.01	0.06	0.11	0.04	0.11	0.25	0.15	0.26	1.03
	(7.32)	(35.50)	(65.75)	(22.98)	(67.62)	(152.94)	(94.70)	(163.21)	(637.21)
$\text{VaR}_{99.9}^{MLE} = 1.95$	0.16	0.78	1.44	0.50	1.50	3.39	2.09	3.67	14.79
	(8.14)	(40.07)	(74.03)	(25.66)	(76.85)	(173.73)	(107.39)	(188.30)	(758.50)
$\text{cVaR}_{99.9}^{MLE} = 2.86$	0.25	1.25	2.29	0.79	2.41	5.39	3.34	6.01	22.42
	(8.69)	(43.58)	(80.24)	(27.52)	(84.30)	(188.43)	(116.98)	(210.29)	(784.41)
Caa-C									
Risk measure	$\beta = 5$			$\beta = 15$			$\beta = 50$		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
$\mathbb{E}(\cdot)_{MLE} = 19.45$	0.14	0.73	0.77	0.43	1.30	1.43	1.54	2.69	3.08
	(0.74)	(3.75)	(3.95)	(2.22)	(6.68)	(7.33)	(7.91)	(13.80)	(15.85)
$\sigma(\cdot)_{MLE} = 10.75$	0.15	0.63	0.68	0.44	1.12	1.27	1.57	2.29	2.79
	(1.37)	(5.90)	(6.33)	(4.13)	(10.46)	(11.84)	(14.61)	(21.35)	(26.00)
$\text{VaR}_{99.9}^{MLE} = 63.28$	0.66	2.80	3.00	1.98	4.90	5.52	6.77	9.66	11.59
	(1.05)	(4.43)	(4.74)	(3.12)	(7.74)	(8.72)	(10.69)	(15.27)	(18.31)
$\text{cVaR}_{99.9}^{MLE} = 67.37$	0.67	2.84	3.04	2.00	4.95	5.57	6.76	9.64	11.52
	(1.00)	(4.22)	(4.52)	(2.96)	(7.34)	(8.27)	(10.03)	(14.31)	(17.09)

Notes: This table shows the add-on to the MLE risk measures in per cent. The numbers in parentheses return the relative add-on based on the MLE value in per cent. MLE represents the results from the standard maximum-likelihood approach, while R1, R2 and R3 are the results from our approach using restriction 1, 2 and 3.

for the 'A'-rated risk bucket in order to yield a specific underestimation odds reduction, compared with a relative add-on of only 8.72% for the 'Caa-C'-rated risk bucket given approximately the same odds reduction. This result underlines our previous finding that high-rated risk buckets may be more prone to parameter uncertainty than lower-rated ones.

In Table 5.7, we present the (relative) add-ons for all rating grades, the data-driven restriction, the risk measure VaR and cVaR and $\beta \in \{5\%, 15\%, 50\%\}$. We find that the relative add-on increases with higher rating grades at an increasing rate. In this data sample, moving one rating grade higher roughly doubles the impact of parameter uncertainty; indeed, when moving from 'Baa' to 'A' it is more than tripled. For example, using a 5% β-quantile, the relative VaR add-on is 13.76% for 'Ba', 27.43% for 'Baa', and 74.03% for 'A'. A risk manager holding an 'A'-rated portfolio should consider an approximate surplus of 75% of the VaR at MLE estimates as an uncertainty premium, even under small ambiguity aversion ($\beta = 5\%$). According to the simulations, this premium may reduce the chance of an underestimation of the *true* VaR to about 45%. If the risk manager is more uncertainty-averse (e.g. $\beta = 15\%$, with odds of underestimation of about 1/3), he has to increase the VaR by about 174%; with an aversion of 50% (and odds of underestimation of about 15%) he should increase the VaR by more than 750%. In contrast, for a 'B'-rated risk bucket with a high uncertainty aversion of $\beta = 50\%$, reducing the odds of underestimation to about 15%, he only needs an uncertainty premium of 33.08%.

Table 5.7: Uncertainty Effects Based on VaR and cVaR for all Moody's Rating Categories under the Data-driven Restriction, 1981-2012 (in per cent)

Rating	$\text{VaR}^{MLE}_{99.9\%}$	$\beta = 5$	$\beta = 15$	$\beta = 50$	$\text{cVaR}^{MLE}_{99.9\%}$	$\beta = 5$	$\beta = 15$	$\beta = 50$
A	1.95	1.44	3.39	14.79	2.86	2.29	5.39	22.42
		(74.03)	(173.73)	(758.5)		(80.24)	(188.43)	(784.41)
Baa	3.22	0.88	1.79	5.18	4.21	1.22	2.49	7.19
		(27.43)	(55.68)	(161.01)		(29.08)	(59.13)	(170.78)
Ba	11.24	1.55	2.99	7.40	13.61	1.92	3.72	9.14
		(13.76)	(26.64)	(65.87)		(14.13)	(27.32)	(67.12)
B	40.06	3.38	5.94	13.25	45.60	3.68	6.44	14.13
		(8.44)	(14.84)	(33.08)		(8.08)	(14.12)	(30.99)
Caa-C	63.28	3.00	5.52	11.59	67.37	3.04	5.57	11.52
		(4.74)	(8.72)	(18.31)		(4.52)	(8.27)	(17.09)
IG	2.14	0.63	1.3	3.96	2.94	0.92	1.89	5.76
		(29.45)	(60.71)	(184.89)		(31.2)	(64.45)	(196.18)
SG	19.40	1.26	2.43	5.5	21.77	1.45	2.8	6.33
		(6.5)	(12.5)	(28.36)		(6.68)	(12.84)	(29.07)
Total	8.73	0.72	1.29	3.09	10.03	0.86	1.55	3.71
		(8.2)	(14.78)	(35.44)		(8.55)	(15.42)	(37.00)

Notes: This table shows the (relative) add-ons to the MLE VaR and MLE cVaR under the data-driven restriction in per cent.

5.4.3 Credit Decision under Uncertainty in Practical Application

Finally, we show how the consideration of uncertainty effects can impact investment decisions. According to our simplified credit-decision model (5.12), a decision-maker prefers the portfolio that has the smallest risk measures given his uncertainty aversion β. We focus on the data-driven restriction and the VaR. To solve the min-max problems (5.12), we use the results of the inner maximization problems (5.13) from Table 5.7.

According to Table 5.7, the VaR of an 'A'-rated risk bucket is lower than that of a 'Baa'-rated risk bucket, if it is evaluated using the ML estimates (with no uncertainty aversion). However, since the higher-rated risk bucket is more prone to parameter uncertainty, high uncertainty aversion yields a VaR that is smaller for 'Baa' than for 'A' (e.g. $VaR_{99.9\%}^{\beta=50\%,Moody's,A} = 6.27\% > VaR_{99.9\%}^{\beta=50\%,Moody's,Baa} = 5.86\%$). Actually, an investor with an uncertainty aversion higher than a 'break-even' $\beta = 24.38\%$ would prefer the 'Baa'-rated risk buckets over the 'A'-rated one. Thus, a credit decision can be reversed if uncertainty is taken into account.[77]

These results are derived from a time horizon with a length $T = 32$. In practical applications, financial institutions often do not have such a long data history. Therefore, we also analyze uncertainty effects based on a rolling window approach, whereby a time horizon with length $T = 15$ is assumed in each step. The rolling window starts from 1991 to 2005, and ends with 1998 to 2012. For each step of the rolling window, we solve the corresponding inner maximization problem for a 99.9% VaR under a 15% uncertainty aversion based on the data-driven restriction.[78] The results are presented in Figure 5.3, in which the VaR using the ML estimates is represented by the solid line, and the risk measure under the consideration of uncertainty is shown by the dashed line.

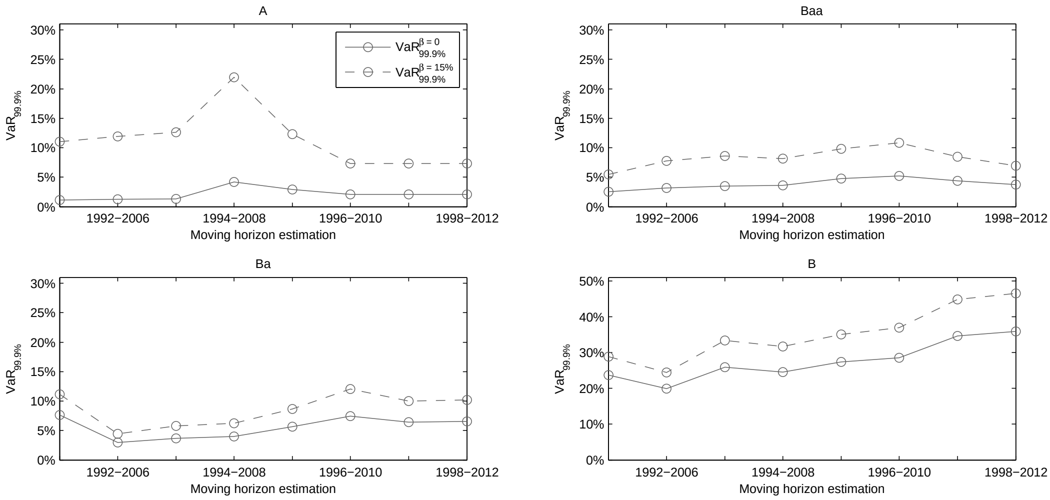
Again, the relative add-on increases with higher rating grade, as the relative difference between the solid and the dashed line is much larger for higher-rated risk buckets. We also computed the average of the relative add-ons of the eight time horizons for each rating grade (data not shown). For 'A', this average is 513.81%, for 'Baa' it is 115.58%, for 'Ba' it amounts to 54.44%, for 'B' it is 27.34%, and for 'Caa-C' it is only 12.48%. Based on the Monte Carlo study, we presume that each of these relative add-ons reduces the general odds of an underestimation of the true risk measure to about 1/3. Therefore, these results highlight the generally higher uncertainty premium for high rated risk buckets.

For this data set, a decision-maker with a 15% uncertainty aversion would again prefer a lower-rated risk bucket over a higher-rated one, especially when comparing 'A' with 'Baa'. For example, based on the most recent information in our data set from 1998 to 2012, an ambiguity-neutral investor would prefer

[77] This holds true even if an additional restriction on a minimum expected return is imposed. If the investor chooses the low-risk portfolio in the setting without uncertainty, she will still switch to the high-risk portfolio after considering uncertainty, because normally the high-risk portfolio will offer an expected return that is at least as high as that of the low-risk portfolio.

[78] Other risk measures, different β-quantiles and other settings of the rolling window approach were also analyzed, yielding qualitatively similar results.

Figure 5.3: Uncertainty Effects on the VaR for Selected Rating Categories under a Rolling Window Approach



Notes: This figure compares the 99.9%-VaR for selected rating grades based on the maximum-likelihood approach with the results from the uncertainty framework under $\beta = 15\%$. The results are presented for a rolling window, and each estimation is based on $T = 15$ starting with 1991-2005 and ending with 1998-2012.

'A' with a VaR of 2.08% over 'Baa' with a VaR of 3.75%. With an uncertainty aversion of 15%, however, the investor would compute a VaR of 7.30% for the higher-rated bucket, while he would obtain a VaR of 6.91% for the lower-rated one. This switch in investment behavior between different investors can be found almost for any time horizon, and also for a shift from 'Baa' to a 'Ba'.

5.5 Conclusion

We present a framework to show how parameter uncertainty (equivalent parameter ambiguity) can be quantified and distinguished from risk in credit models. We derive a general min-max problem of a credit decision-maker, and introduce a novel data-driven restriction to model the possibility of parameter uncertainty. This allows us the derivation and definition of risk measures under uncertainty.

We show that the odds of underestimations of risk measures can be reduced when the agent is uncertainty-averse. We also show that uncertainty aversion comes at the cost of a premium, which is added to the risk measure. We demonstrate that alternative Bayesian methods dealing with parameter uncertainty are covered by our framework. Actually, these approaches are special cases with an implied degree of uncertainty attitude.

Our empirical analysis of historic default data shows that portfolios with particularly high ratings (low risk) are more affected by parameter uncertainty than portfolios with lower ratings (high risk). We find that even under a moderate degree of uncertainty aversion, a decision-maker might therefore prefer a lower-rated risk bucket over a high-rated one, in contrast to a decision maker who ignores parameter uncertainty.

The approach could offer financial institutions a generally applicable method to address parameter uncertainty in credit portfolio decisions, and enables them to compute uncertainty-adjusted risk measures such as the VaR. Our method could also provide policy makers and regulatory authorities with a tool for imposing regulatory rules that explicitly prescribe a capital buffer for parameter uncertainty.

Our approach has the potential to be developed into a more complex method, future work could include an investigation of more general objective functions and further restrictions, such as minimum expected returns or constraints on portfolio weights. Moreover, our current paper only addresses parameter uncertainty, but future applications could extend the setting to uncertainty with respect to the model. Even though parameter uncertainty seems to be much more relevant than model uncertainty to credit risk (Hamerle and Rösch, 2005), considering these two factors together could nevertheless result in a more generally applicable approach, because both of these factors are interrelated. Finally, another important research topic would of course be a thorough analysis of the degree of uncertainty aversion in practical applications.

Chapter 6

Conclusion, Practical Implications and Further Research Topics

Each final section of the Chapters 2, 3, 4, and 5 summarizes the main findings of this cumulative thesis in detail and in context of the addressees. Within this chapter, we focus on the practical implications and aim to identify possible further research topics. At the end, we provide a brief overall conclusion.

In Chapter 2, we provided a model that allows a closed-form comparison of both bonds and tranches with respect to their exposure to systemic risk. We demonstrated that due to pooling and tranching, idiosyncratic risks are diversified, but that the exposure to systematic risks is concentrated. Therefore, we find that tranches' conditional probabilities of default and conditional expected losses are much more sensitive to realizations of a systematic risk factor than those of equally rated corporate bonds. Our findings show that tranches are generally much more prone to systematic risk than bonds, and that the effect size increases with tranches' seniorities.

With the European regulation on Credit Rating Agencies (Regulation (EC) No 1060/2009) CRAs have to distinguish structured finance instruments from other instrument, in their rating categories. For example S&P uses for this a (sf) suffix, but have not changed the definition of the rating or represented opinion about the issue's or issuer's creditworthiness (S&P, 2012). However, our findings suggest a basis for the development of rating metrics that reflect more appropriately the product-specific exposures to systematic risk. These new metrics may incorporate higher moments of loss distributions with respect to realization of modeled systematic risk factors, and may therefore include sensitivities. For higher transparency, the one-dimensional rating metrics of the CRAs may then be extended to multi-dimensional measures, as proposed in Jenkinson (2008).

Our analysis focuses on (single) structured securitization, but the framework can easily be extended to multiple structured securitizations such as CDO squared. Although the derivation of analytical results might be challenging, a Monte Carlo approach is easily implemented. With such an approach, the specific sensitivity of multiple structured securitizations can be analyzed, accounting for heterogeneous borrowers and sectors; this approach goes beyond simplifying assumptions of the ASRF credit model. It is to be expected that due to several layers of pooling and tranching, CDO squared tranches are even more prone to systematic risk. This result would emphasize that a single (sf) suffix from the CRA is not sufficient to cover the specific risk characteristics of structured instruments.

In Chapter 3 we analyzed systematic risk exposures of CDS contracts referring to numerous U.S. firms located in several branches. With a two-pass regression framework, we identified at least three systematic risk factors - 'credit market climate', 'cross-market correlation' and the 'market volatility' - as important drivers for CDS spread changes. Additionally, we showed that the sensitivities to these systematic risk factors are cross-sectionally priced in CDS spreads after controlling for individual risk factors such as credit ratings, liquidity, firm leverage and sectorial influences.

The applied empirical study allows for several extensions. Firstly, other representations of systematic risk, such as market recovery risk or individual counter-party risk (compare Brigo and Chourdakis, 2009; Arora et al., 2012) and the pricing of their sensitivities may be analyzed. Secondly, the analysis may be extended to the examination of sector-specific risk factors. Both of these extensions may provide additional pieces for the solution of the 'credit spread puzzle'. Since structured products are more sensitive to systematic risk (compare Chapter 2), a similar analysis with respect to structured securities could enable a dismantling of the valuation of structured securities. For this, as argued by Loehr (2014), the application of a dynamic panel regression approach may help to identify common determinants of tranche spreads. Löhr (2013*b*)

In Chapter 4, we analyzed the most commonly used risk measures VaR and cVaR in the ASRF credit model with respect to errors in the model parameters. We found that the cVaR can be more prone to estimation errors than the VaR, and that this sensitivity increases with a lower probability of default of the underlying portfolio, or with a higher confidence level α.

Therefore, we support the current proposal of the Basel Accord to use a 99.9% VaR for the incremental capital charge for default risk (BCBS, 2012, 2013), while the Basel Committee recommends to replace the 99% VaR with the 97.5% cVaR in internal market risk models and has used the 97.5% cVaR to calibrate capital requirements under the revised market risk standardized approach. Any further consideration of a replacement of the 99.9% VaR by a cVaR for the determination of credit default risk capital has to take into account the higher sensitivity to parameter errors of the cVaR.

The analysis with respect to credit risky portfolios can be extended to securitizations. Firstly, it can be analyzed whether risk measures of securitizations are comparatively more prone to parameter errors than those of portfolios. Secondly, with the deduced key numbers from Chapter 4, it can be studied whether the cVaR can be more prone to estimation errors than the VaR for structured products. Both studies may have direct implications for the formulation of capital requirements for structured products.

In Chapter 5 we presented an economic framework to show how parameter uncertainty can be qualified and distinguished from risk in credit models. As a direct implication of this framework, we define risk measures under uncertainty and additionally provide a novel data-driven restriction to model the possibility of parameter uncertainty as accurately as possible. With this framework, we show that portfolios with particularly low PD (high rating grades) are more affected by parameter uncertainty than portfolios with higher PD (low rating grades). As a result, even under a moderate degree of uncertainty aversion, a decision-maker might prefer a lower-rated risk bucket over a high-rated one.

This framework enables decision-makers of financial institutions to address parameter uncertainty in credit portfolio decisions, and allows them to compute uncertainty-adjusted risk measures. It provides politicians and regulators with a first tool for imposing regulatory rules that explicitly prescribe a capital buffer for parameter uncertainty.

Our framework lays the general foundations for how parameter uncertainty can be considered in credit risk. Therefore, future work can address several extensions. A thorough analysis of the degree of uncertainty aversion for practical application may refine our heuristic proposal. Then, the robust optimization problem restricted to credit models can be extended by more general objective functions and further restrictions, such as minimum expected returns or constraints on portfolio weights. The general framework can easily be extended to credit decisions with respect to structured financial products such as CDO-tranches. Within the ASRF model, the most commonly used risk measures VaR and cVaR of such tranches can be described analytically. Therefore, the resulting optimization problems should be from the same magnitude of difficulty. We propose that structured instruments are much more prone to parameter uncertainty, and that this higher sensitivity can be measured with this framework. Again, our framework may then be a good starting point for the development of supervisory rules to buffer against uncertainty.

In conclusion, this cumulative thesis contributes to four current research areas with respect to risk management. It firstly addresses the exposure to systematic risk of structured securities. It is shown that CDO tranches are much more sensitive to systematic risk than equally-rated corporate bonds. The findings facilitate an understanding of the natural behavior of securitizations, and highlight the importance of the development of new rating metrics covering their generic properties. Secondly, in this thesis we identify three important systematic risk factors - 'credit market climate', 'cross-market correlation', and 'market volatility' -, which are significantly priced in the cross-section of CDS spreads. Thirdly, we show that the cVaR may be more prone to parameter errors than the VaR within a commonly used credit risk model. The sensitivities become larger for lower PD (higher rating) and higher confidence level α. Thus, the commonly used approach to be safer (low PD, high confidence level α, and choosing a tail-considering risk measure such as cVaR), actually increases the effect of parameter errors in credit risk. And fourthly, we introduce a robust optimization-based framework to incorporate parameter uncertainty into the credit decision process. The framework helps to separate the effect of parameter uncertainty from the original credit risk, and enables a separate quantification of each effect. Overall, the findings presented in this thesis may help to advance the risk management in financial institutions, may motivate better and more transparent rating metrics, and may lead to new appropriate regulatory requirements.

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Lebenslauf des Verfassers

Wissenschaftlicher Werdegang

06/2015	**Gottfried Wilhelm Leibniz Universität Hannover** Abschluss der Promotion zum Dr. rer. pol. mit der Note 'summa cum laude'.
seit 03/2010	**Gottfried Wilhelm Leibniz Universität Hannover** Wissenschaftlicher Mitarbeiter und Doktorand am Institut für Banken und Finanzierung der Wirtschaftswissenschaftlichen Fakultät.
10/2004 – 02/2010	**Gottfried Wilhelm Leibniz Universität Hannover** Diplomstudium der Mathematik mit Schwerpunkt numerischer Optimierung und Nebenfach Betriebswirtschaftslehre, Abschluss mit Auszeichnung (Note 1.0).

Publikationen

2014	'An Analytical Approach for Systematic Risk Sensitivity of Structured Finance Products', in *Review of Derivatives Research* 17(1), pp. 1-37 mit S. Löhr und D. Rösch
2011	'Credit Ratings und Kapital für Verbriefungstransaktionen', in *Risiko Manager* (9), mit S. Löhr, K. Lützenkirchen und D. Rösch.