A Game-theoretic Approach to Fairness for a Distributed Reservation-based Medium Access

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ABSTRACT

The increasing number of mobile devices and the requested ubiquitous connectivity along with the growth in use of multimedia and file transfer applications pose an enormous challenge to existing transmission technologies. For networks with changing topologies due to mobile devices, a distributed medium access is particularly interesting. Furthermore, multimedia and file transfer applications require high data rates with guaranteed and predictable access to provide quality of service. When considering such high data rate networks, ECMA-368 provides distributed medium access with guaranteed resource allocation for small-sized networks. Besides guaranteed access, the fairness a system offers is an important quality of service property. For this reason, we present a fairness analysis of a distributed reservation-based medium access control protocol such as has been specified in ECMA-368.

Many real-world situations can be abstracted by models with strategically interacting decision-makers. For those decision-making processes, game theory provides mathematical tools to predict the outcome of such an interaction. Originating in economics, game theory has been applied to different research fields such as politics, biology or telecommunications. In this thesis, we employ game theory to analyse the strategic interaction of network nodes in a distributed reservation-based protocol. We show that the unfair slot allocation, which we identify in the fairness analysis, is the rational outcome in the original protocol. An introduced algorithm that relaxes the reservation, however, is proven to drive the game to a fair slot allocation, if players are rational.

In this thesis, we provide an analysis of ECMA-368 that covers throughput, delay of the transmitted packets and fairness. ECMA-368 applies a distributed beaconing system to organise medium access and network management. Due to the fixed order in the beacon phase, reservations can only be made in a first-come, first-served manner. We show that the individual throughput and delay depend highly on the position of a node's beacon but is independent of the network size. Therefore, the

earlier a node transmits her beacon in the beacon phase, the more privileged she is. Thus, the more channel time she can allocate and hence, the better she perceives the fairness. Relaxing the fixed order in the beacon phase to employ round-robin or random beaconing is shown to achieve long-term fairness. Due to the selfishness of the network nodes, however, both methods lack short-term fairness.

We model the distributed reservation-based protocol as a multi-stage game and show that the identified unfair slot allocation is the rational Nash Equilibrium. To achieve short-term fairness, we introduce a relaxed reservation method that provides discriminated players with a means to enhance their resource share. For the static 2-player game, we provide the strategies that correspond to the fair Bayesian Nash Equilibrium. The direct determination of this equilibrium, however, is complex. Therefore, we consider the repeated game, in which players learn from their opponents' behaviour and adapt their actions to optimize their utility. With an update algorithm that follows Bayes' rule, we show that the game converges to the fair Perfect Bayesian Nash Equilibrium, if players are symmetric in their estimates about their opponents' behaviour. If players are rational, they choose the same smallest initial estimates and hence, reach the fair slot allocation. If we exclude boundary effects, we further prove that for all other initial estimates the game converges to nearly fair slot allocations.

Simulations extend the analysis to larger networks and indicate that the results of the 2-player game also hold for the N-player game. Thus, equal initial estimates drive the game to the fair equilibrium. The convergence time grows linearly with the network size. It also increases with the impact that future utilities have in a player's decision and decreases with the parameter of the relaxed reservation method.

Keywords: Fairness, distributed medium access, Bayesian, game theory

ZUSAMMENFASSUNG

Die steigende Zahl mobiler Geräte, ihre allgegenwärtige Konnektivität, sowie der Zuwachs von Multimedia- und Dateiübertragungen stellen eine enorme Herausforderung für bestehende Übertragungstechnologien dar. Für Netzwerke mit wechselnder Topologie durch mobile Geräte ist ein verteilter Medienzugriff besonders interessant. Darüber hinaus erfordern Multimedia- und Dateiübertragungen eine hohe Übertragungsrate mit garantiertem und vorhersehbaren Zugriff, um Dienstgüte zu gewährleisten. Bei der Betrachtung solcher hochdatenratigen Netzwerke bietet der ECMA-368 einen verteilten Medienzugriff mit garantierter Ressourcenzuteilung für kleine Netzwerke. Neben dem gesicherten Zugriff ist die Fairness eines Systems ein wichtiges Dienstgütekriterium. Aus diesem Grund behandelt diese Arbeit die Fairness eines verteilten reservierungsbasierten Protokolls zur Steuerung des Medienzugriffs, wie es der ECMA-368 spezifiziert.

Viele reale Situationen können durch Modelle mit strategisch interagierenden Entscheidungsträgern abstrahiert werden. Für diese Entscheidungsprozesse bietet die Spieltheorie mathematische Werkzeuge, um das Ergebnis einer solchen Interaktion vorherzusagen. Seinen Ursprung hat die Spieltheorie in der Ökonomie. Mittlerweile jedoch findet sie in unterschiedlichen Forschungsfeldern wie der Politik, Biologie oder Telekommunikation Anwendung. In der vorliegenden Arbeit wird Spieltheorie verwendet, um die strategische Interaktion von Netzwerkknoten in einem verteilten reservierungsbasierten Protokoll zu analysieren. Es wird gezeigt, dass die unfaire Zeitschlitzzuteilung, die das Ergebnis der vorangegangenen Fairness-Analyse ist, das rationale Resultat des ursprüngliches Protokolls ist. Ein neu eingeführter Algorithmus zur Lockerung der Reservierung bei rationalen Spielern jedoch erreicht eine gerechte Aufteilung der Ressourcen.

In dieser Arbeit wird eine Analyse des ECMA-368 erbracht, die den Durchsatz, die Verzögerung der übertragenen Pakete und die resultierende Fairness untersucht. Der ECMA-368 verwendet ein verteiltes Beaconing-System, um den Medienzugriff und das Netzwerkmanagement zu organisieren. Aufgrund der festen Reihenfolge in der Beacon-Phase können Reservierungen nur im first-come, first-served Verfahren durchgeführt werden. Es wird gezeigt, dass der individuelle Durchsatz und die individuelle Verzögerung stark von der Position des Beacons eines Knotens, nicht jedoch von der Netzwerkgröße abhängt. Je früher ein Knoten sein Beacon in der Beacon-Phase senden kann, desto privilegierter ist er. Damit hat dieser Knoten mehr Sendezeit zur Verfügung und die Fairness ist zu seinen Gunsten verschoben. Wird die starre Reihenfolge der Beacon-Phase für eine Round-Robin oder eine zufällige Reihenfolge aufgelöst, so stellt sich auf lange Sicht Fairness ein. Aufgrund des Eigennutzes der Netzwerkknoten fehlt es beiden Methoden jedoch an der Gewährleistung von Fairness, wenn ein kurzer Zeitraum betrachtet wird.

In dieser Arbeit wird das verteilte reservierungsbasierte Protokoll als ein mehrstufiges Spiel modelliert und gezeigt, dass die identifizierte unfaire Zeitschlitzzuteilung das rationale Nash Gleichgewicht ist. Um Fairness auf kurze Sicht zu erreichen, wird die Reservierung gelockert. Auf diese Weise erhalten benachteiligte Spieler eine Möglichkeit, ihren Resourcenanteil zu erhöhen. Für das statische Spiel mit zwei Spielern werden die Strategien im fairen Bayesian Nash Gleichgewicht bestimmt. Die direkte Bestimmung dieses Gleichgewichts ist eine komplexe Aufgabe. Aus diesem Grund wird das wiederholte Spiel betrachtet, in dem die Spieler vom Verhalten ihrer Gegner lernen und ihre eigenen Züge anpassen, um ihren Nutzen zu optimieren. Rationale Spieler wählen für ihre anfängliche Bewertung den gleichen kleinstmöglichen Wert und erreichen so eine faire Zeitschlitzzuteilung. Bei Ausschluss von Randeffekten wird weiterhin gezeigt, dass alle anderen anfänglichen Bewertungen das Spiel zu annähernd fairen Zeitschlitzzuweisungen führen.

Simulationen erweitern die Analyse auf größere Netzwerke und deuten darauf hin, dass die Ergebnisse des Spiels mit zwei Spielern auch für Spiele mit N Spielern gelten. Demnach führen gleiche anfängliche Bewertungen das Spiel zum fairen Gleichgewicht. Die Konvergenzzeit wächst linear mit der Netzwerkgröße. Ferner nimmt sie mit dem Einfluss zu, den zukünftiger Nutzen auf die Entscheidung der Spieler hat. Der Parameter des gelockerten Reservierungsverfahrens vermindert die Zeit bis zur Konvergenz.

Schlagwörter: Fairness, verteilter Medienzugriff, Bayesian, Spieltheorie

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Part I

DISSERTATION

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INTRODUCTION

Social protocols have been necessary to organise the exchange of information ever since people have needed to communicate with each other. Those protocols did not only contain the vocabulary and prescribe the syntactics of the language. They also included and still include written and unwritten rules on how to communicate. Some things we should keep in mind when it comes to the rules involving communication are: "who is first to talk", "who is next to talk", "what happens when a new person joins in the discussion", "how long is someone allowed to talk", "what happens if some people accidentally or intentionally talk at the same time" or "is there anyone who decides individually every time how to communicate".

Consider a classical classroom situation for example. There is a teacher and several students. The teacher will do most of the talking. The students are required to listen while the teacher is talking and to raise their hands if they have a question or know the answer to one of the teacher's questions. If students raise their hands, it is the teacher who chooses the one to speak. Therefore, in this example, it is the teacher alone who decides every time which student is allowed to speak. She can also request someone to stop talking and handles circumstances in which students accidentally or intentionally talk at the same time. Another example to consider is a meeting. There might be one person in the group that acts as a moderator and controls the communication. However, the group could also agree on taking turns with everyone allowed to talk for a certain amount of time.

When people started to use electronic devices that communicate with their users and also among themselves, there was again a need for protocols to handle this information exchange. As for communication among humans, those protocols have to deal with the order and duration of communication. We distinguish between centralized and distributed protocols.

In centralized protocols, there is one device that decides which device is allowed to communicate when. We can compare this with the teacher-student situation, when it is the teacher who decides to allow her students to talk or to talk herself. The teacher has an outstanding position, while the students can be considered equals in this communication relationship. For electronic devices, the protocol used with the Universal Serial Bus (USB) [64] behaves in a similar way. Here, the host such as a laptop acts as a master, while the attached devices such as USB flash drives, digital cameras or printers are considered slaves. Another example is a Wireless Local Area Network (WLAN) [36] that is operated in infrastructure mode. Here, an access point is in charge of coordinating the client devices.

The more mobile devices are available, the more flexible does the topology of a network become. Frequent topology changes and many devices with similar capabilities, however, make it difficult to clearly distinguish between master and slave devices. While the teacher-student situation unambiguously was a situation of this kind, the meeting situation, in contrast, needs to be considered. If one of the people moderates the debate, this is clearly centralized communication. However, if during the discussion none of the participants takes on this role, the communication itself needs to be distributively organised. To discuss the electronic analogue, further assume that in this group of people everyone carries a mobile device and they find that in order to communicate they need to connect the devices so that they can exchange data. Once agreed on a protocol, e.g. people switch on their WLAN interfaces in ad-hoc mode, those devices can connect distributively with each other and form a network of devices with equal communication rights.

Hence, with the increasing popularity of mobile devices, wireless ad-hoc solutions of distributed communication attract more and more attention. Additionally, the capabilities of mobile devices increase from generation to generation. More devices support real-time multimedia applications and the transfer of large files. Thus, the connectivity of mobile devices has to cope with the quality of service (QoS) requirements of such applications. For real-time applications, one important concern is the predictability of communication.

Distributed communication can further be classified into competitive and contention-free communication. Consider again the meeting example. If the communication was competitive, people that intend to contribute to the discussion would try to make themselves heard and simply start talking. The question that arises is who will be listened to in the case that some persons talk at the same time. Is it the person that speaks the loudest or more insistent on making a point? Further, one asks what impact such behaviour would have on subsequent encounters. Some may avoid another collision whereas others may speak even louder in order to be heard.

If we consider contention-free communication, people could have decided in advance to take turns. Thus, whenever it is a person's turn, she may talk for a predefined amount of time if she has anything to contribute, otherwise it is the next person's turn. Obviously, the predictability of when to talk and for how long is much larger in the contention-free communication than in the competitive communication. However, this example also shows that contention-free communication does not per se result in absolute predictability. The people in the meeting do not know exactly when they will be able to talk again. But they do know the maximum time they have to wait if every person in the room contributes to the discussion. So, even though it depends on the particular design of the actual protocol, distributed contention-free communication suggests to best meet the QoS requirements of real-time applications and their connectivity between mobile devices [5]. This is the reason why, in this work, we consider distributed contention-free communication.

With the growth in use of multimedia and file transfer applications the required data rates for communication increases. If the effectively provided data rates are not able to cope with the demand, a means to distribute the available rate among the requested transmissions is necessary, which immediately raises the question of fairness. So when considering distributed contention-free communication, we particularly analyse the issue of fairness. Recall the rules concerning social protocols raised at the beginning of this chapter. They already reflected this very important aspect of quality of service. Intuitively, a situation is fair, if all participants are treated equally, i.e., all people receive the same share of resources. However, the concept of fairness can also be defined differently. People could be assigned a weight that depends, for instance, on the amount of information they have to share or its importance. The larger the weight of a person, the larger her share of time to talk. This is referred to as weighted fairness.

Both the terms distributed communication as well as contention-free communication can be analysed from different view points. In this work, we consider distributed medium access control (MAC) protocols that provide contention-free communication. Again remember the meeting situation. The social protocols discussed in this example arranged the order in which people were allowed to speak and raised the question of the length of time they were admitted to talk. Those are two of the main aspects that are dealt with by medium access control protocols. As the name suggests, they are concerned with the access to the medium, thus, when and how a device is allowed to access the communication medium such as a cable or the air in the case of wireless transmissions. There are different approaches to provide distributed contention-free medium access. In the following, we provide some examples and shed light on the various levels of contention-free access and their implication on fairness.

1.1 APPROACHES TO DISTRIBUTED RESERVATION MAC PROTOCOLS

One of the oldest network protocols is ALOHA [2]. It was proposed in the early 1970s to provide for wireless transmissions between the University of Hawaii's main campus near Honolulu and its colleges, which were scattered along the islands of Hawaii. With more than one device requiring access to the medium, there was a need for a medium access control protocol.

In ALOHA, data is split into equally sized packets. Once a packet is ready for transmission, the device sends it and waits for a response. If no reply arrives in time, the packet is assumed to be lost due to a collision with another packet, thus, it needs to be retransmitted. An enhancement of ALOHA is slotted ALOHA [3] that requires synchronization between devices. Time is divided into slots whose length is such that exactly one packet can be transmitted in one time slot. Devices are then required to start the transmission of their packets at the beginning of a time slot, so a new packet transmission cannot start during an ongoing one. Thereby, the probability of packet collisions is reduced. In order to further improve satellite communication that was performed with slotted ALOHA, Crowther et al. proposed Reservation-ALHOA [22]. Initially, the access to the radio channel is the same as with slotted ALOHA. Thus, devices contend for time slots if there is data for transmission. However, unlike slotted ALOHA, once a device successfully contended for a time slot, she has it periodically reserved until either she explicitly frees the slot or has finished her transmission and the slot becomes idle.

While both the original and the slotted ALOHA are clearly protocols with competitive access, Reservation-ALOHA reduces collisions to the time when devices compete for a reservation. Reservation-ALOHA adapts to the nature of the input traffic. Therefore, the throughput, i.e., the rate of successful packet delivery, varies from slotted ALOHA's throughput to that of fixed assigned time slots. However, the issue of fairness, i.e., how to prevent users from starving if they did not successfully contend for a slot, remains. One remedy is to exclude some slots from reservation and including a fairness algorithm into the contention phase [45]. If we consider cellular wireless networks, another option to obtain a reservation is to use a resource partitioning pattern. The available frequency range is divided into partitions and distributed onto the base stations. These can reserve slots within their partition without contention. Every time slot the partitions are shifted to the next base station which respects the existing reservation but can access the remaining slots if it does not interfere with the reserved ones [9].

Usually, wireless LAN that is standardized in the family of IEEE 802.11 [36] is not accounted for as a contention-free protocol, but subsumed under contention-based protocols. However, if we have a closer look, it does contain contention-free aspects. Basically, devices that have packets ready for transmission sense whether the channel is idle. If there is no ongoing transmission, they start transmitting their packets. Hence, roughly speaking, if a user started a transmission and no other did so at the same instant, the transmitting user has the medium reserved for her transmission because any other user will sense that the channel is busy and, thus, refrain from transmitting. However, there are users that cannot sense a transmission because they are too far away, but would interfere with it. To prevent this from happening, the request to send (RTS)/clear to send (CTS) handshake has been established. A user with packets for transmission initially sends an RTS when the channel is idle and the receiver replies with a CTS. Therefore, any user in transmission range of both the transmitter and the receiver is aware of the upcoming transmission and will refrain from interfering with it. So the RTS/CTS can be seen as a per-packet reservation procedure. Fairness is well analysed in IEEE 802.11. If IEEE 802.11 is operated in infrastructure mode it is shown to be short-term fair for two hosts [13, 19]. This is important for low latency applications and upper layers [13, 43]. Measurements, however, indicate that the short-term fairness of IEEE 802.11 degrades with increasing number of hosts in the network [14, 19].

There are several extensions of the IEEE 802.11. The amendment IEEE 802.11e defines procedures to support applications with quality of service requirements in local area networks. With the Enhanced Distributed Channel Access (EDCA), IEEE 802.11e provides distributed service differentiation that enables high priority traffic to be processed faster. However, there also exist hybrid MAC scheduling schemes with distributed resource reservation for IEEE 802.11e. In those schemes information regarding the reservation parameters such as service start time, mean data rate, frame size or delay bounds need to be distributed either by implicit or explicit signalling [69]. Alternatively, EDCA can be extended with features of a centralized protocol to provide distributed resource reservation, admission control and scheduling. Users distribute their own admission requests to all other users that simultaneously decide about it. All users are assumed to have the same level of information, thus, they reach the same admission decision [33]. The efficiency of the resource distribution further improves, if users actively release reserved resources once they have finished their transmissions [68].

So there are several distributed protocols that partially provide resource reservation. While IEEE 802.11 contends on a per-packet basis, the transmission slot in Reservation-ALOHA is reserved until implicitly or explicitly released. However, both Reservation-ALOHA and IEEE 802.11 contend for transmission time and only once they are successful, is the transmission phase reserved.

Another option, with even less contention, has been proposed in ECMA-368 [1]. As with slotted ALOHA, time is slotted and divided into superframes. As opposed to slotted ALOHA, however, superframes are further divided into a beacon phase and a phase for data transmission. Generally, each user in the network has to transmit a beacon in the beacon phase. Besides other information users employ their beacons to publish reservations for the upcoming data transfer phase. Thus, contention only occurs, when users join the network and compete for a beacon slot using Reservation-ALOHA. Once they have gained a beacon slot, it is reserved until they release it by leaving the network. In contrast to Reservation-ALOHA though, the beacon slot comes not only with one transmission slot but offers the opportunity to reserve several time slots in the transmission phase.

This last example of a distributed reservation-based protocol has significant potential for real-time applications that require guaranteed and predictable access to the medium. The reason for this lies in the absence of contention in the reservation process while still providing a completely distributed system. However, the issue of fairness is yet to be considered. In this work, we fill this gap and present a thorough analysis of throughput, delay and fairness of the distributed reservation protocol in ECMA-368. We show that in high-load scenarios the influence of the reservation rules of ECMA-368 are negligible. Then, we introduce a distributed algorithm that drives the users of the network to a fair slot allocation.

The examples provided in this section can be abstracted by models where several decision-makers interact with each other. Recall the meeting example. If people distributively decide when to talk, they strategically interact with each other. This means that the decision of one person to talk directly influences the decisions of the other persons and vice versa. For those decision-making processes, game theory provides mathematical tools to analyse the behaviour of the decision-makers. Hence, it offers methods to predict the outcome of such an interaction. In this work, we apply game theory to analyse the distributed reservation method of ECMA-368. Furthermore, we provide a profound analysis of the game including the introduced algorithm and determine its equilibria as well as show its convergence.

1.2 THESIS CONTRIBUTIONS

In this thesis, we provide a fairness analysis of a distributed reservation-based protocol and present an algorithm to overcome the identified unfairness. We apply game theory to model the interaction between the decision-makers and show that the introduced algorithm guides the game to a fair equilibrium.

Firstly, we analyse ECMA-368 and determine its capacities and fundamental limitations regarding throughput, delay of the transmitted packets and fairness. For this analysis, we implement a Java based tool that determines all possible reservation patterns and weights them with their likelihood given a Poisson model for the frame arrival process. Due to the fixed beacon order, reservations can only be made in a first-come, first-served manner. We show that the individual throughput and delay highly depends on the position of a user's beacon and evaluate the impact on the perceived fairness.

In the next step, we evaluate modifications of the beacon phase in order to enhance the fairness. One alternative to the fixed order of beacons is the randomization of the beacon slots. If users have to randomly choose a beacon slot every z superframes, long-term fairness can be achieved at the cost of additional beacon collisions every time users choose a new beacon slot. Users can only reserve and transmit if they have successfully transmitted a beacon beforehand. Therefore, we show that in contrast to the fixed beacon order the maximum achievable throughput is reduced. Instead of randomly changing the beacon order, we also alter the order in a round-robin fashion. Though this achieves maximum throughput, we argue why it also only provides long-term fairness.

To achieve short-term fairness, we introduce a relaxed reservation method that provides discriminated users with a means to enhance their throughput. In preparation of a game-theoretic analysis that determines the implications of this relaxed reservation, we model the distributed reservation protocol with fixed beacon order as a multi-stage game. For the static game, we identify the Nash Equilibria, subgame-perfect equilibria and determine the Pareto- and socially-optimal as well as fair equilibria.

We then modify the game model to account for the relaxed reservation method and determine the Bayesian Nash Equilibria of this game for 2 players. The Bayesian Nash Equilibrium is the equivalent of a Nash Equilibrium for a game with imperfect information. The imperfection in this game regards information that players have to estimate about each other in order to execute the relaxed reservation method.

Since the immediate determination of the Bayesian Nash Equilibrium of a game is a tedious task, we model the dynamic game for 2 players, i.e., the static game is repeated several times. In every period of the game, players learn from their opponent's behaviour and adapt their own actions accordingly. We show that with a proper learning rule the game converges to a Perfect Bayesian Nash Equilibrium, if certain boundary effects are excluded. Further we show that the Perfect Bayesian Nash Equilibrium is fair if and only if both players start with equal initial estimates, and nearly fair for most of the remaining cases.

For the evaluation of larger networks, we set up a simulation environment. We employ the network simulator OMNeT++ [65] as a framework and implement the behaviour of the distributed reservation protocol of ECMA-368 enhanced by the relaxed reservation method and the required belief update rule. Simulations show that the results also apply for games with more than two players. We further observe that the convergence time increases linearly with the number of players in the game. It also grows with the discount factor, but decreases with the parameter of the relaxed reservation method.

1.3 THESIS OUTLINE

Chapter 2 provides a brief introduction into ECMA-368 and presents related work that evaluates this protocol regarding throughput, channel utilization and fairness as well as the consequences of the beaconing approach. We further render an introduction into game theory and review related work that applies game-theoretic methods in the field of communication networks and especially resource allocation.

Chapter 3 presents the research problem that is addressed in this thesis. It further highlights the contributions and explains their relevance.

In Chapter 4, we analyse the ECMA-368 regarding throughput, delay of packet transmission and the corresponding fairness. We particularly focus on the fairness perceived by individual users and show that the order of the beacons heavily impact this perceived fairness. We then evaluate the influence of random beaconing as well as round-robin beaconing on the fairness.

Chapter 5 provides a game-theoretic analysis of the distributed reservation protocol with fixed beacon order and determines the Nash Equilibria, subgame-perfect equilibria and identifies the Pareto- and socially-optimal as well as fair equilibria. We present a relaxed reservation method to overcome the unfairness identified in Chapter 4 and model this as a static multi-stage game with imperfect information. For the 2-player game we determine the Bayesian Nash Equilibria.

Chapter 6 extends the static game to a 2-player dynamic game. In this dynamic game players observe their opponent's behaviour and adapt their own actions to maximize their benefits. We show under which conditions the game converges to a Perfect Bayesian Nash Equilibrium. Furthermore, we evaluate the Perfect Bayesian Nash Equilibria regarding their fairness. Simulations extend the game to networks with more than two users and different parameter sets of the introduced algorithm.

Finally, Chapter 7 concludes this work and raises possible future extensions.

2

FUNDAMENTALS AND RELATED WORK

This chapter introduces ECMA-368 [1] as the distributed reservation protocol that constitutes the basis of this work. We present related work that analyses ECMA-368 regarding throughput, channel utilization, fairness and distributed beaconing. An introduction into game theory explains elements and methods relevant for this thesis. Furthermore, we review related work that employs game-theoretic methods in the field of communication networks and particularly focus on resource allocation.

2.1 FUNDAMENTALS OF ECMA-368

In 2002, the Federal Communications Commission (FCC) of the USA granted in an amendment of the Part 15 rules a 7500 MHz spectrum (3.1 GHz-10.6 GHz) for the unlicensed use of ultra wideband (UWB) devices for communication and measurement [27]. UWB commonly denotes a signal that covers a very large bandwidth [12]. The FCC defined that signals below 2.5 GHz are considered UWB signals, if for their fractional bandwidth f_f it holds that $f_f \geq 0.2$ [27]. The fractional bandwidth f_f refers to the energy bandwidth over the center frequency of the signal:

$$f_f = \frac{f_H - f_L}{\frac{f_H + f_L}{2}}, \tag{1}$$

where f_H and f_L are the upper and lower frequencies of the -10 dBm emission point. Signals above the threshold of 2.5 GHz, however, are referred to as UWB signals, if their bandwidth exceeds 500 MHz. The FCC further defined a spectrum mask that poses strict rules on the spectral density as well as the maximum peak power. Despite this restrictive regulation of the transmission power, the opening of such a large portion of spectrum attracted several major chip manufacturers [12].

Parallel to the opening of the spectrum for UWB devices, the IEEE 802.15.3a Task Group was formed in 2001 to investigate solutions for the development of high-speed and low power wireless personal area networks (WPAN). Both main proposals considered in the IEEE 802.15.3a Task Group included a physical layer based on UWB. An overview of the medium access control protocols and proposals for UWB is given in [10, 56]. In 2006, the task group disbanded and one of the proposals was standardized as ECMA-368 [1] by ECMA International. The ECMA-368 standardizes both the physical and the MAC layer of a device. On the physical layer it applies a multi-band orthogonal frequency division multiplexing (MB-OFDM) approach. The frequency band from 3.1 GHz-10.6 GHz is divided into 14 bands, each 528 MHz in length. Thus, each of those bands provides sufficient bandwidth for a UWB signal. The standard offers data rates from 53.3 Mbit/s up to 480 Mbit/s, hence, it is applicable for high-speed applications.

In this work, we consider the access to the medium. Therefore, the next paragraphs present the main principles for medium access in ECMA-368. The ECMA-368 specifies a fully distributed communication network without central coordinator. Instead, it uses a distributed beaconing scheme to coordinate medium access and support dynamic network organisation. Time is assumed to be slotted and a superframe structure is applied as depicted in Figure 1. Each superframe starts with a beacon phase, followed by a data transfer phase. Generally, each node that has joined the network is required to transmit a beacon during the beacon phase to inform the other nodes about her own status and her particular view of the network.

To join an existing network, a node first identifies its beacon phase. Once she has found an empty and feasible beacon slot, she attempts to transmit her beacon using Reservation-ALOHA. Beacons are not sent to a particular recipient, so a node does not receive an explicit acknowledgement that her beacon has been successfully transmitted. However, each node's beacon contains a map of beacon slots and their owners. So when a node receives a beacon which includes her own beacon slot mapped to herself, she knows that she has successfully attained this beacon slot. Once the network is established, the assignment of a beacon slot to a node remains, even if she is inactive. So the order of the nodes in the beacon phase is fixed.

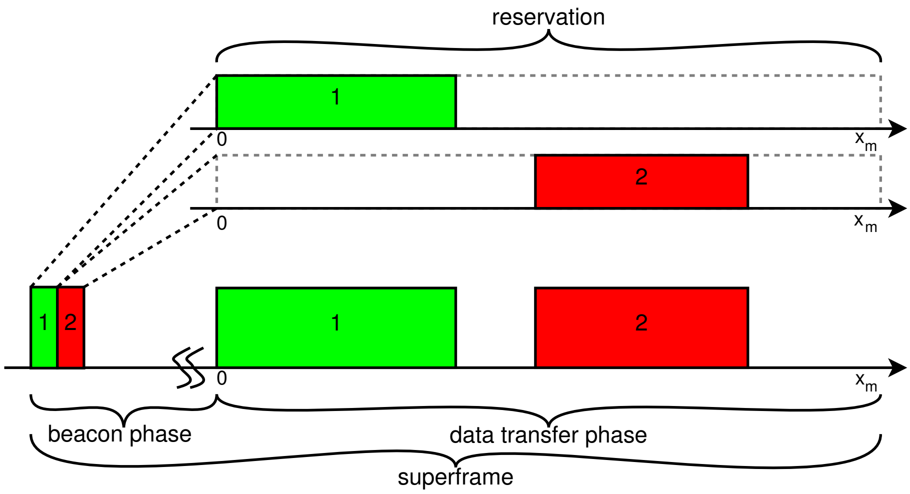

Figure 1: Superframe structure of ECMA-368. The length of a superframe is fixed to 256 slots. A beacon phase variable in length is followed by a data transfer phase that consists of the remaining x_m slots. During their beacons nodes can make reservations for the upcoming data transfer phase.

The standard provides both contention-free and contention-based medium access. Generally, every time slot in the data transfer phase that has not been reserved by one of the nodes in the network is open for contention-based access. Nodes that need guaranteed access to the medium can use the distributed reservation protocol (DRP) to reserve channel time. The transmitter negotiates the time, i.e., which slots in the data transfer phase of the superframe to use for transmission, with her receiver and both publish the chosen time slots in their beacons. Recall that the order of the beacons in the beacon phase is fixed once established. This implies that the access to the medium is granted in a first-come, first-served manner, since previously published reservations constrain subsequent ones.

ECMA-368 imposes several rules and policies regarding the location of the reserved slots in the data transfer phase as well as the length of a reservation. For a better understanding of those rules, the superframe structure is often described as a 16x16 matrix [8] as depicted in Figure 2. Each superframe is divided into 256 slots, each 256 µs in length. These slots are grouped into 16 zones, each represented by one column. Zones are further distinguished by their isozone affiliation as indicated in Figure 2. The first zone includes the beacon phase and does not belong to any of the four isozones.

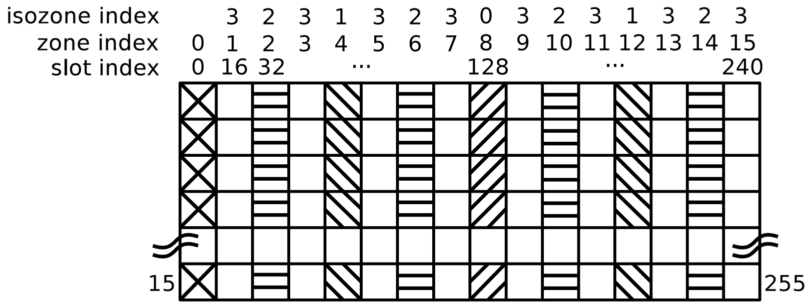

Figure 2: A superframe consists of 256 slots that are equally grouped into 16 zones. Depending on their position in the superframe the zones are further grouped into 4 isozones. The first zone includes the beacon phase and is not part of an isozone.

When nodes publish their reservations, they have the option to label it as a safe reservation. If they marked their reservations as safe, they do not have to release channel time when requested by other nodes. A general policy, however, states that nodes shall not safely reserve more than 112 slots within a superframe and at most one safe reservation block per zone. A block refers to the consecutively reserved slots within a single zone. In this work, we consider safe reservations only, since they guarantee channel access. Thus, all rules presented in the subsequent paragraphs are concerned with safe reservations.

In the worst case, the limit of 112 safe slots per node causes the channel to be saturated while supporting only two nodes. To prevent this from happening, the standard further requires nodes to comply with a policy that concerns the maximum number of reserved slots per zone. This policy relates the maximum number of consecutive slots in a zone to the index of the first reserved slot according to Table 1. Slots in a zone are numbered from 0 up to 15. So if a node requests to reserve the first slot in a zone, i.e., slot index 0, the maximum number of slots that she is allowed to safely reserve in this zone is 8 slots. If a node decides to start her reservation with slot index 6, however, she may only reserve 4 slots in this zone.

The standard not only provides policies how to choose a reservation block within a particular zone, but also which zone to choose for a reservation. So when choosing a zone, a node is required to minimize the isozone index. By this, the standard aims to equally spread the allocations in the superframe. So if a node intends to reserve a block of 8 slots and the first 8 slots in zone 8 are taken already, she tries either zone 4 or zone 12.

index of 1^{st} slot	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
max #slots	8	7	6	5	4	4	4	4	4	4	4	4	4	3	2	1

Table 1: Maximum number of slots that are allowed to be reserved in a zone depending on the slot index of the first reserved slot. Thus, if a reservation starts in slot index 8, for instance, it may contain at most 4 slots.

The standard provides two alternatives of the distributed reservation protocol. Nodes that apply hard reservation can exclusively use the reserved time slots for their own transmission. If a node does not require the reserved slots anymore, she has to explicitly release the reservation. The alternative soft reservation, however, does not require an explicit release of resources. When nodes other than the reservation owner sense the channel idle, they are allowed to access it using the contention-based protocol that is also applied in the slots that have not been reserved at all. This protocol is referred to as prioritized channel access (PCA) and provides differentiated access to the channel such as EDCA of IEEE 802.11e (cf. to Chapter 1).

In this section, we presented an overview of the structure of the medium access control protocol specified in ECMA-368. We further introduced the distributed reservation protocol and described the main policies regarding reservation patterns in the data transfer phase. In the following section, we discuss related work that analyses the performance of the medium access in ECMA-368, the implication of different reservation patterns and the distributed beaconing algorithm.

2.2 RELATED WORK ON ECMA-368

In this section, we discuss related work on ECMA-368. It includes papers that analyse the performance of ECMA-368 as well as works that determine the influence of the reservation pattern on the performance. Finally, we consider evaluations of the distributed beaconing algorithm.

In 2005, ECMA-368 had not been standardized yet. Instead, it was under consideration in the IEEE 802.15.3a Task Group. At that time, Hiertz et al. published a throughput analysis of this standard proposal [35]. In their analysis, they assumed an ideal channel and determined the saturation throughput for both the contention-free distributed reservation protocol as well as the contention-based prioritized channel access. Further, they considered all three acknowledgement strategies that are provided by ECMA-368. With an immediate acknowledgement, the receiver has to transmit an acknowledgement for each individual frame, while no acknowledgement does not require a confirmation from the receiver. A compromise between those two acknowledgement strategies is the delayed acknowledgement. Here, the receiver awaits a certain amount of frames, before she transmits an acknowledgement for all received frames. In their analysis, the authors in [35] show that the distributed reservation protocol always outperforms the contention-based access protocol due to the absence of collisions. Furthermore, they confirm that the saturation throughput is highest when no acknowledgement is required and lowest for immediate acknowledgement. The delayed acknowledgement strategy achieves a throughput in between those two. Frame aggregation further increases the maximum achievable throughput, since it bundles several higher layer frames into one MAC layer frame and thus reduces the overhead.

The results for an ideal channel given by [35] provide an upper bound for the saturation throughput. A more realistic approach includes the impact of a non-ideal channel. The standard proposals for IEEE 802.15.3a included both line-of-sight and non-line-of-sight channel models [30]. Depending on the underlying channel model the expected bit error rate at the receiver varies and hence the saturation throughput. In [70], the authors extend an EDCA model by including the effects of the bit error rate, the transmission limit given in ECMA-368 and the ECMA-368 specific timings to provide a saturation throughput analysis of the prioritized channel access. For an ideal channel it can be observed that the saturation throughput increases with the frame length due to the reduction of overhead. In the case of a non-ideal channel, however, the opposite trend becomes visible. The frame error rate increases with the length of a frame, thus, the larger the frame the smaller the throughput becomes. Those two effects combined imply that there is a frame length that maximizes the throughput. This optimal frame length can be determined for each access category provided in ECMA-368 [70].

A non-ideal channel not only affects the saturation throughput of the contention-based access protocol but also the distributed reservation protocol. Assume an indoor scenario, where the channel suffers from shadowing. Depending on the chosen transmission mode that specifies the applied modulation and coding scheme, there is an optimal frame length that maximizes the achievable throughput. With these results throughput optimization can be performed that adjusts both the frame length and the transmission mode given the current channel quality [71]. The maximum throughput, however, only slightly varies with the applied reservation method [50]. Recall that nodes choose between hard and soft reservation. In the case of hard reservation, a node has to explicitly release the resources, if she does not require them anymore. In the case of soft reservation, though, every other node is able to access reserved slots, if the reservation owner's queue becomes idle.

The imagined deployment of UWB were small-sized networks with high data rate requirements such as entertainment set ups in home environments. With several of such systems in place, we are faced with dense network topologies [57]. ECMA-368 provides several rules to coordinate the coexistence of several networks [1]. If those networks operate in the same frequency range, though, this coexistence comes at the cost of throughput loss. There are different levels of network coexistence that can be measured by their connectivity. Full connectivity refers to a topology, in which all nodes are in the same beacon group. The beacon group of a node refers to the set of nodes that transmit beacons with the same beacon phase start time than that of the tagged node [1]. Decreasing the connectivity implies that some of those nodes become members of the extended beacon group. This further includes the beacon groups of all nodes in the tagged node's beacon group. Thus, they become 2-hop neighbours of the tagged node. The extreme case is that networks are totally separated so much so that there is no interference between them.

In [57], the authors model the decrease of connectivity between networks by varying the attenuation factor of a wall that physically separates the networks. For totally separated networks, a network's throughput is maximal, while in the topology with full connectivity all networks share the maximum throughput. In between those extreme cases, the network throughput depends on the level of interference induced by the surrounding networks. An interference-aware reservation method that considers link feedback and senses the channel to optimize the choice of time slots and the transmission mode increases the achievable throughput for all levels of connectivity [57]. The throughput can further be raised if instead of a random slot allocation, slots are grouped. By this the overhead due to required guard times can be reduced. The level of connectivity does not only affect the network throughput if the reservation-based protocol of ECMA-368 is considered. It has an even larger effect on the performance when the contention-based protocol is applied [58]. For fully meshed networks, the achievable throughput of the contention-based access is smaller than that of the contention-free access due to its probabilistic nature. If we consider the channel reuse that can be achieved by interference awareness, the reservation-based protocol benefits more than the contention-based protocol.

Another effect that lowers the system throughput are exposed nodes. Recall that an exposed node defers from transmitting because a node in her neighbourhood already transmits. The receiver of her neighbour's transmission, however, is not in her own transmission range and thus, would not be disturbed by the exposed node's transmission. So, if the exposed node transmits, the capacity of the system increases. The beacons that are exchanged in ECMA-368, already contain information about a node's neighbourhood. In [59], the authors propose to use this information to identify the 1- and 2-hop neighbours and mitigate the influence of exposed nodes. Since the number of exposed nodes increases with the network size, the impact of this method grows with the network size. In the distributed reservation protocol, its application results in a system capacity gain of up to 30 %.

So far, we have not particularly considered the impact of the reservation pattern on the throughput. The ECMA-368 provides precise rules how to choose the time slots in a superframe. If the required amount of slots is equally distributed among the zones in the superframe such that the delay requirements of the application are met, an upper bound for the utilization of the system is given [23]. With this baseline in mind, the level of degradation that occurs if the reservation rules imposed by ECMA-368 are followed can be determined. The authors in [23] apply an isozone-fit reservation strategy that tries to keep the superframe well-structured and symmetric.

If the attempted isozone cannot cope with the required number of slots, the request is dropped to the subsequent isozone. The throughput degradation caused by the high blocking probability of the isozone-fit algorithm compared with the baseline can be reduced, if requests do not have to be entirely dropped to the next isozone but split between isozones.

If nodes remove their reservations, the remaining reservations in a superframe become fragmented, thus a compaction algorithm that reassembles the remaining reservations leads to a better utilization of the superframe. The combination of those improvements of the isozone-fit strategy reaches a throughput close to the baseline [23]. Recall that ECMA-368 poses restrictions on how many slots a node is allowed to reserve in a zone. The higher the starting slot index is, the smaller the amount of slots (cf. Table 1). Thus, heterogeneous traffic achieves a larger utilization than homogeneous traffic, because flows that require only a small amount of slots fill the gaps between large flows [23]. Another possibility to increase the system throughput when considering the reservation pattern, is the introduction of priorities. In [47], the authors use the beacons to additionally distribute flow information, in particular the size of a flow. Nodes then distributively determine a new beacon order according to the nodes' flow sizes from largest to smallest. By this rearrangement large flows obtain a higher priority than small flows.

Besides the throughput or utilization of the system, timing related aspects such as the waiting or service time are important performance measures. The delay in an ideal channel provides a lower bound, since no retransmissions have to be considered. Due to its deterministic behaviour, the delay of the distributed reservation protocol is bounded irrespective of the traffic load. Thus, if the reservation of a flow is admitted, quality of service can be guaranteed. In the contention-based protocol, however, the delay performance is good for light traffic but suffers from large delays in heavy traffic [35].

In non-ideal channels, the influence of the signal-to-noise ratio (SNR) on the throughput and thus on the delay has to be considered. Assume reservations to be non-uniformly distributed and the channel to be subject to indoor people shadowing. For a Poisson model of the frame arrival rate it holds that the more variable the reservation pattern, the larger the mean waiting time. This effect increases with the traffic load [49, 50]. However, the average waiting time in the queue decreases with an increasing signal-to-noise ratio. This can be attributed to the fact that better channel quality requires less retransmissions. While hard reservation highly gains with the SNR, soft reservation experiences only small improvements [49, 50]. If soft and hard reservation are directly compared, the prior suffers from a larger waiting time than the latter. This is again due to the Poisson model which denotes that the transmission queue can become empty, although there are reserved slots left.

Soft reservation then implies that those remaining slots can be occupied by other nodes. Thus, it occurs that in the case that new frames arrive at the tagged node, she cannot transmit them in her current reservation, because the remaining slots have been lost to another transmission. Hence, the newly arrived frames suffer from an additional delay, since they have to wait until the next reservation block. This difference vanishes the larger the traffic load is. The reason is that high traffic load connotes that the probability of the transmission queue to become empty is small, thus, under heavy load soft reservation degrades to hard reservation [49, 50].

The decision whether to choose the distributed reservation or the contention-based protocol for transmission, is not trivial. In [60], the authors consider video streaming and propose to reserve below the peak rate and transmit the remaining frames via the contention-based protocol. When deciding which frames to transmit contention-based and which contention-free, they consider two approaches. If there is one queue for each type, frames first fill the reservation queue and remaining frames are located in the second queue. Thus, only frames in the second queue have to contend for channel access. Recall that under heavy load, soft reservation degrades to hard reservation. Thus, if the second queue is not empty, soft and hard reservation perform equally. If the second queue is empty, soft reservation allows other nodes to transmit more frames during the contention phase.

The alternative option is to maintain a common queue for both access methods. This, however, could lead to situations in which the queue is empty at the time the reservation is present or a large number of frames requires access via the contention-based protocol. The latter induces a high collision probability and thus increases the service time. With less contention in the case of two separate queues, the service time is minimal [60].

In the analysis of the reservation-based medium access in ECMA-368, there is little work that explicitly considers fairness. In [6, 7], the authors analyse the impact of the reservation process on throughput fairness using the Gini index, which is a means for statistical dispersion and measures the inequality among values of a frequency distribution [32]. For a mix of isochronous and asynchronous traffic, the flexible use of reservation and contention-based access provides the highest fairness index when compared to fixed assignment of traffic type and access method [6, 7].

The authors in [6, 7] analysed fairness, but did not attempt to induce fairness. The goal to reach fairness is set in [40]. In a cooperative approach, nodes use their beacons to distribute flow information, in particular the lower and upper bounds of the service rate to guarantee quality of service of their flows. From these values and the current allocation of resources among the flows, nodes distributively determine a satisfaction level, which is also included in the beacons. To achieve fair resource allocation, the satisfaction level is required to be the same for all nodes, hence the resource allocation has to be adjusted accordingly. New nodes are admitted, if the new global satisfaction level does not fall below a certain threshold [40].

All papers presented that did not only analyse but implemented strategies to attain some predefined goal have in common that they assume nodes to be maximizing a global aim. Thus, nodes with a minor priority for instance voluntarily release slots to provide higher prioritised flows with more channel time [40, 47]. In this work, however, we assume nodes to solely maximize their own utility. The fairness problem that we demonstrate to be inherent in this protocol is solved by introducing an algorithm that drives the nodes to a fair slot allocation.

While [6, 7] used the Gini index to quantify the accomplished fairness, another widely used fairness index is defined by Jain et al. in [38]:

$$f(x) = \frac{[\sum_{i=1}^{N} x_i]^2}{N \sum_{i=1}^{N} x_i^2}, \quad x_i \geq 0, \tag{2}$$

with x_i the resource allocation of node i and N the number of nodes in the network. The index is defined in the interval $[0, 1]$ and measures the equality of the slot allocation x. If all nodes receive the same resource allocation, i.e., $x_i = x_j$ for all $i \neq j \in \mathbb{N}$, the fairness index $f(x) = 1$ and the system is 100% fair. If we consider a single node and her perception of fairness, we have to determine the fair allocation mark. It is defined as follows:

$$x_f = \frac{\sum_{i=1}^{N} x_i^2}{\sum_{i=1}^{N} x_i} \tag{3}$$

For the i^{th} node the algorithm is x_i / x_f fair. The overall fairness is then given as the average of the fairness perceived by the nodes in the network.

The assumption that all rendered works have in common is the existence of an established network. To provide for such a network, though, the beaconing algorithm and especially the collision resolution methods are integral parts of ECMA-368. In [48], the authors determine the theoretical limit of the node density that the beaconing algorithm can still cope with. They derive an approximation for the beacon phase length depending on the node density.

The ECMA-368 is an example of a self-coordinated network, which is challenged by topology changes resulting from node mobility and nodes being switched on or off [66]. Assume a short range network, so a beacon slot cannot be used by multiple nodes, as it would induce beacon collisions. The beaconing algorithm designed in ECMA-368 requires nodes joining in the network to randomly choose a slot in a fixed sized extended window of the current beacon phase. The more new nodes join in the network, the more the extended window slides to the end of the beacon phase. However, the maximum length of the beacon phase is fixed. Thus, there are two problems. The first problem arises because joining nodes are required to choose a slot between the highest taken slot and the maximum beacon slot. In case a new node chooses the maximum beacon slot, no other new node is able to join in the network, because no free and feasible slot remains. Thus, the second new node fails to join in the network. The second problem arises, if the second last slot is occupied and two or more nodes randomly choose the same last slot, because the one before is occupied. Here, a deadlock occurs. In [66], the authors evaluate the failure probability and convergence time of nodes joining a network and propose a more flexible joining scheme to optimize these parameters.

In this section, we presented related work on ECMA-368. Papers that analyse the performance of ECMA-368 cover aspects such as throughput and delay. Little work considers the aspect of fairness such as done in this work. Furthermore, we reviewed papers that evaluate the impact of the reservation algorithm on channel utilization and resource allocation. Those papers assume nodes to maximize a global aim and distributively determine the optimal slot allocation. In this work, however, we assume nodes to maximize their personal utility. To still achieve the global fairness goal, we introduce an algorithm that provides discriminated nodes with a means to enhance their channel share. By this, the network reaches a fair slot allocation. Finally, we presented papers that cover aspects of the beaconing algorithm. In our work, we identify the fixed beacon order as the reason for the unfairness inherent in the protocol. Neither the randomization of the beacon slots nor applying a round robin scheme in the beacon phase provides for short-term fairness. Instead, we keep the fixed beacon order and relax the reservation to provide discriminated nodes with a means to increase their share and thus, achieve fairness.

2.3 FUNDAMENTALS OF NON-COOPERATIVE GAME THEORY

Many real-world situations can be abstracted by models where several decision-makers interact with each other. For those decision-making processes, game theory provides mathematical tools to analyse possible and likely behaviour of the decision-makers. Hence, it offers methods to predict the outcome of such an interaction. In order for game theory to be applicable, though, there has to exist a strategic interaction between the players in the game [52]. Thus, a player's decision has to depend on the other players' past, present and future actions. The outcome of the decision process of all players induces a certain welfare level at each single decision-maker. This welfare level reflects a preference relation of the possible outcomes of the process and can be different for each individual decision-maker.

Game theory is used in several fields to analyse strategic interaction, such as real games, economy, politics or telecommunication. An example of a real game that can be analysed with game theory is the game chess. Here, a player's move heavily depends on what her opponent did last and what the player expects her to do in the sequel of the game. Many findings in game theory are due to analyses of economic problems. The market entry game, for instance, studies the following situation: Someone deliberates opening a new store in a certain area. However, in this area there already exists an equivalent established store. Whether or not the new store will open depends on the established store's past and expected future behaviour towards the opening of a new store. In politics, the behaviour of politicians towards each other can be investigated with the help of game theory. Which specific stances politicians take up depends on their opponents' expected actions and hence yields in a situation of strategic interaction. Finally, in telecommunication there are several topics that are applicable for game theory. During spectrum auctions bidders decide about their bids not only taking into account their own limit but also the other bidders' expected actions. The decision about a route in a network is also marked by conflicting demands that are predestined to be analysed with game theory.

There exist two methodologies in game theory: non-cooperative and cooperative game theory. Non-cooperative game theory is well established. Much research has been done in this area and several books provide a thorough introduction to this field, such as [24, 26, 31, 52, 54, 55]. Non-cooperative game theory considers the individual players with their possible actions and strategies. Each player decides about her actions depending on her individual expected payoff. On the other hand, there is cooperative game theory. The literature corpus is much smaller, still there are several books that introduce this topic in a well-defined manner, such as [17, 54, 55, 67]. Cooperative game theory considers coalitions of players, rather than focussing on the individual players. The outcome in a cooperative game is not the combination of actions of the players but the coalition that is formed and its corresponding payoff. Cooperative game theory is often referred to as payoff-driven [67]. Since the specific actions of the players is not relevant to analyse a cooperative game, it is even applicable for situations in which those actions are unknown [55].

In this work, we consider how the fixed beacon slot order affects the fairness of the resource allocation. We introduce an algorithm that relaxes the reservation and thus, induces strategic interaction between the nodes in the network, which makes it applicable for a game-theoretic analysis. In this analysis, we show that the algorithm drives the game to an outcome with a fair allocation. Players are assumed to maximize their own utility, thus, the tools of non-cooperative game theory are more applicable to analyse this scenario than the tools of cooperative game theory.

In the following sections, we introduce non-cooperative game theory and explain the game structure itself as well as solution concepts available for different game types. Non-cooperative game theory provides a mathematical framework to analyse strategic interaction between individual players. Hence, when establishing a game, one has to answer the following questions [51]: Who are the players? What actions are available to them? What are the players' objectives? Does the game have an equilibrium? If yes, is it unique? Is there a dynamic process for players to update their strategies according to the course of the game? If yes, what is it and does it converge to some equilibrium?

In order to be able to answer those questions, this section formally explains the structure of non-cooperative games and introduces relevant terms when describing those games. The underlying assumptions are discussed and classification options of games are presented. Additionally, we illustrate different solution concepts and give metrics to evaluate available solutions.

2.3.1 *Definitions and terminology*

In the subsequent section, we use bold letters for sets and corresponding non-bold letters representing their respective cardinalities. A non-cooperative game Γ has at least the following three elements: a finite set of players $\mathbf{N}$, an action space $\mathbf{A}_i$ and a utility or payoff function u_i for each player i. The players in the games considered in this work are the nodes in the network. The action space $\mathbf{A}_i$ contains all feasible actions player i can carry out. A particular action of player i is denoted by a_i. Hence,

$a = (a_i)_{i \in \mathbf{N}}$ refers to the action profile of all players and $\mathbf{A} = \times_{i \in \mathbf{N}} \mathbf{A}_i$ is the set of all action profiles. The actions of all players except player i are usually denoted by a_{-i}.

The action profile that is effectively played is called the outcome of the game. Each player i is assumed to rank the different outcomes according to her own and personal preference. This preference relation for player i is represented by her utility or payoff function u_i. The value of this function for a certain outcome $u_i(a)$ is called player i's payoff of outcome of a. Which action player i decides to play in a game depends on her strategy s_i. A strategy is a complete plan of actions for every possible situation in the game. Here, complete means that a strategy also covers situations that never arise.

There are some basic assumptions that often underlie game theory. One is that players are rational and act strategically. The rationality assumption is a model of the players' individual behaviour. It presumes that players know all of their own possible alternative actions, i.e., player i is aware of her entire action space $\mathbf{A}_i$. Besides her own action space, there is other information that can be available to a player. This information includes the other players' payoffs, for instance. If they have no knowledge in this vein, the rationality assumption presumes that they have expectations about such unknowns. Another important feature of the rationality assumption is that each player has a proper preference relation between any two possible outcomes. This relation has to be unambiguous and is reflected in the utility function. Players are assumed to be payoff maximizers, which does not imply that players have to be selfish. The payoff a player receives from a certain outcome is not necessarily monetary. It represents a player's rating of a potential outcome and can also be altruistic depending on the player's personal type.

A player's strategy is defined as a complete plan of actions. With players being payoff maximizers, they are supposed to find the specific strategy that maximizes their expected payoff in all situations. The assumption that they act rationally and strategically requires that they are always able to find this optimal strategy and never make mistakes. This assumption is not very realistic, since real life games are usually complex with many possible actions or players or many external unknowns. Hence, players are often unable or not willing to calculate every possible payoff and

		player 2	
		L	R
player 1	T	2 , 0	0 , 4
	B	0 , 1	1 , 3

Figure 3: Example for a strategic game. Player 1 has the options T(op) and B(ottom), player 2 the options L(eft) and R(ight). The maximum payoffs a player can achieve given her opponent's actions are encircled. For instance, given player 1 plays T, the maximum player 2 can get is a payoff of 4, if she plays R.

strategy. Therefore, they might not find the optimal strategy. Despite this lack of realism, rationality is a very common assumption.

Another prevailing assumption is common knowledge about certain aspects of the game, e.g. the structure of the game, payoffs and possible actions. Assume, for instance, that all players know the structure of the game. Common knowledge requires that every player knows that all other players know the structure of the game. Further it means that all players know that all players know that all players know the structure of the game and ad infinitum. In many game-theoretic analyses, perfect recall is assumed. With perfect recall players are able to remember all previous moves. For complex or long games, this is also a less realistic assumption, since players are likely to forget at least some of what happened before.

After the explanation of the underlying assumptions and the ingredients of a game, the following paragraphs describe classification options. So games can be divided into strategic and extensive games. Figure 3 gives an example for a game in strategic form. This game has two players. Player 1's available actions are T(op) and B(ottom), player 2's action space contains L(eft) and R(ight). The first number in a cell is player 1's payoff for the corresponding strategy profile, the second one is player 2's payoff. If player 2 plays L(eft), the best response for player 1 is to play T(op), since a payoff of 2 is larger than 0, which she would get with B(ottom). If player 2 plays R(ight), the action B(ottom) is best. Hence, the optimal strategy for player 1 is to play T(op), if player 2 plays L(eft), and B(ottom), if she plays R(ight). The payoffs for the optimal strategies are encircled in Figure 3.

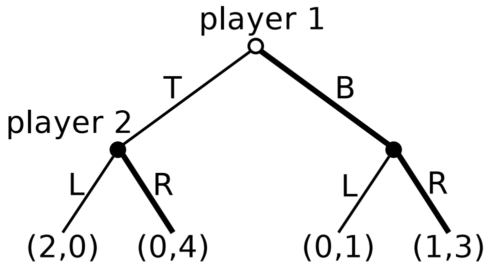

Figure 4: Example for an extensive game. Player 1 has the options T(op) and B(ottom), subsequently, player 2 chooses between L(eft) and R(ight). Best responses are indicated by a thick branch.

Games in strategic form represent simultaneous play. Simultaneity is not restricted to its temporal meaning. Situations in which the chosen action is not revealed to the opponent before she makes her own decision can also be represented by a game in strategic form. When a player makes her decision, she has to take into account her belief about what her opponent will do and vice versa. Thus, a strategic game results in a circular situation with players guessing about each other's moves.

Extensive games, on the other hand, usually illustrate sequential play. They are depicted as trees such as the example in Figure 4. Decision trees contain decision nodes, at which one of the players decides about her next move and branches that prescribe possible courses of the game. An extensive game ends in terminal nodes. The game depicted here has again two players with the alternative actions T(op)/B(ottom) and L(eft)/R(ight), respectively. First to choose is player 1. Once she has decided to play either T(op) or B(ottom), player 2 chooses between L(eft) and R(ight). The payoffs are specified at the terminal nodes.

In strategic games, players thought in a circle taking into account their beliefs about the other players' actions. In extensive games, players are confronted with a look-ahead situation. Players that move first have to consider what the subsequent players will do in response, i.e., what the future consequences will be. So we can illustrate the order of the players, their options and what they know at each moment in the game about the past actions. If players are assumed to observe all previous moves, i.e., they always know at which point in the decision tree they reside, this game is denoted as one with perfect information. In case any information about a player's move is hidden from any other player, beliefs have to be formed, which is then considered to be a game with imperfect information. In contrast, incomplete information relates to an asymmetric distribution of information about the rules of the game. Here, some players have private information, e.g. about their types or personal payoffs. The type of a player refers to her characteristics, e.g. an aggressive or a compliant player. Those games of incomplete information, however, can be reformulated as games with imperfect information according to Harsanyi [34].

The family of such games are called Bayesian extensive games with observable actions. In those games, players know every move their opponents have ever made. However, they cannot observe the initial move that is made by nature. This initial move determines which type the players are. Players do not reveal this information, thus, each player has private information about the payoffs she receives. Formally, this game type is written as [55]:

Definition 2.1 The tuple $\langle \Gamma, (\Theta_i), (p_i), (u_i) \rangle$ denotes a Bayesian extensive game with observable action, where

- Γ is an N-player extensive game with perfect information and simultaneous moves

and for each player $i \in \mathbf{N}$

- Θ_i is a finite set of possible types θ_i of player i,
- $p_i(\theta_i)$ is a probability measure on Θ_i that represents the probability that player i is selected to be of type θ_i. The measures p_i are stochastically independent and positive for all $\theta_i \in \Theta_i$ and
- $u_i(\theta, h)$ is the payoff player i receives, if she is of type θ and the history of the game Γ is given as h.

The history h contains all actions prior to the current stage. Let the action profile at stage k be denoted by a^k. It describes all players' actions at stage k. The history of the game at stage k is then given by h^k and contains all actions prior to stage k, so $h^k \equiv (a^1, a^2, \ldots, a^{k-1})$, with $h^1 \equiv \varnothing$ the history at the start of the game.

A third option to classify games is their number of interactions. We distinguish between static and dynamic games. In static games, the game is played once, while in dynamic games, players repeatedly interact with each other. Thus, in dynamic games, an ongoing relationship between players can be established. This relationship can lead to a climate in which it is possible that cooperation, punishments and rewards or the development of learning behaviour arise.

In dynamic games, we further differentiate games by how much their players value future payoffs. Myopic players only consider the payoff in the current stage. In their decision process, expected future payoffs do not have any value for them. On the contrary, long-sighted players take into account expected future payoffs when they decide about their current action. Generally, future payoffs are discounted by a discount factor δ in order to reflect how much influence future payoffs have.

In this section, we introduced the main ingredients of a game and presented different means to classify games. The next section covers solution concepts for different game types.

2.3.2 *Solution concepts*

When analysing a game, the goal is to study the stability of the possible outcomes in order to determine the equilibria of the game. When an equilibrium has been identified, it has to be shown whether or not it is unique. In the next section, we further elaborate the evaluation of those equilibria.

The most common stability measures are Nash Equilibria. In a Nash Equilibrium each player's strategy is a mutual best response to the strategies that are played by her opponents. Assume S^* to be the Nash Equilibrium of the game. Therefore, player i has no incentive to deviate from strategy s_i^*, since there is no other strategy s_i that increases her utility. Formally, it can be written as:

Definition 2.2 (Nash Equilibrium) A strategy profile S^* is a Nash Equilibrium [31] if, for all players i,

$$u_i(s_i^*, S_{-i}^*) \geq u_i(s_i, S_{-i}^*) \quad \text{for all } s_i \in \mathbf{S}_i.$$

Recall the example in Figure 3. The circles indicated the players' best responses to their opponents' actions. In the action profile (B, R), both players play a best response to each other, hence this profile marks a Nash Equilibrium.

In sequential plays with perfect information, backward induction is a common way to identify credible Nash Equilibria. The player that moves first, guesses what her opponent will do afterwards before deciding about her own move. Consider the example in Figure 4 again. Here, player 1 identifies player 2's optimal moves before determining her best response to them. In the case player 1 played T, player 2 will play R. If she played B, player 2 will respond with R. Knowing that, player 1 decides between the action profile (T, R) and (B, R). Hence, she chooses to play B, because the utility she gains from (B, R) is larger than the one she gains from playing (T, R).

For a game with imperfect information, backward induction is not applicable because players cannot forecast precisely their opponents' behaviour. In these games, the concept of a subgame-perfect equilibrium has been established to determine credible equilibria. To define a subgame-perfect equilibrium, we first provide the definition of a proper subgame.

Definition 2.3 (Proper Subgame) A proper subgame G [31] of an extensive-form game T consists of a single node and all its successors in game T, with the property that if nodes $x' \in G$ and $x'' \in h(x')$ then $x'' \in G$. The information sets and payoffs are inherited from the original game.

So a subgame is a subset of a game in extensive form. A subgame-perfect equilibrium then generates a Nash Equilibrium in each of those subgames if they are individually analysed.

Definition 2.4 (Subgame-perfect (Nash) equilibrium) A strategy profile S is a subgame-perfect equilibrium (SPE) [25] of a finite extensive-form game if it induces a Nash Equilibrium in each proper subgame of the original game.

With the concept of a subgame-perfect equilibrium, we are able to determine credible behaviour in games with imperfect information. A Nash Equilibrium that is not subgame-perfect is not a reasonable equilibrium, since it poses an incredible threat. For Bayesian extensive games with observable action the equivalent of a Nash Equilibrium is called a Bayesian Nash Equilibrium and is defined as:

Definition 2.5 (Bayesian Nash Equilibrium) A Bayesian Nash Equilibrium (BNE) in a game of incomplete information with a finite number of types θ_i for each player i, prior distribution p, and pure-strategy spaces S_i is a Nash Equilibrium of the "expanded game" in which each player i's space of pure strategies is the set $S_i^{\Theta_i}$ of maps from Θ_i to S_i [31].

Thus, a Bayesian Nash Equilibrium is a consistency check and requires players to play their best responses given their beliefs about the distribution of types of the other players. In dynamic games, we make use of the solution concept Perfect Bayesian Nash Equilibrium that combines the ideas of subgame perfection, Bayesian Nash Equilibrium and Bayesian inference. So strategies have to generate a Bayesian Nash Equilibrium in every "continuation game" [31]. Such an equilibrium consists of two elements. First is the behavioural strategy $\sigma_i(\theta_i)$, i.e., the strategy of player i given that she is of type θ_i. Second is the probability measure on Θ_i denoted as $\mu_i(h)$. It refers to the common belief of the players except player i about player i's type after the history h of the game. The equilibrium is formally defined as:

Definition 2.6 (Perfect Bayesian Nash Equilibrium) Consider a Bayesian extensive game with observable actions be given by the tuple $\langle \Gamma, (\Theta_i), (p_i), (u_i) \rangle$. A pair $((\sigma_i), (\mu_i))$ is a Perfect Bayesian Nash Equilibrium (PBE) of the game [55] if the following conditions are met:

- sequential rationality: the strategy $\sigma_i(\theta_i)$ of each θ_i of each player i has to induce an optimal outcome for θ_i in the subsequent play for any information set of the game
- correct initial beliefs: $\mu_i(\varnothing) = p_i$ for each $i \in \mathbf{N}$
- action-determined beliefs: players' belief about player i is influenced only by player i's action

- Bayesian updating: if player i's action at h is consistent with $\mu_i(h)$, the new belief is derived by Bayes' rule from player i's action, i.e.,

$$\mu_i(h,a)(\theta_i') = \frac{\sigma_i(\theta_i')(h)(a_i) \cdot \mu_i(h)(\theta_i')}{\sum_{\theta_i \in \Theta_i} \sigma_i(\theta_i)(h)(a_i) \cdot \mu_i(h)(\theta_i)},$$

  until the player's behaviour contradicts her strategy, which leads to a new conjecture about her type.

In this section, we provided the solution concepts for both simultaneous and extensive-form games. Furthermore, we introduced the concept of a Perfect Bayesian Nash Equilibrium for dynamic games with imperfect information. In the next section, we cover the evaluation of the equilibria that have been identified.

2.3.3 *Means to evaluate an equilibrium*

When evaluating an identified equilibrium, the goal is to study its properties. The properties that we focus on in this thesis are Pareto-optimality, social-optimality and different measures of fairness. To evaluate the goodness of a Nash Equilibrium, we use the notion of Pareto-optimality as defined in [52].

Definition 2.7 (Pareto-optimality) A Nash Equilibrium S^* is Pareto-optimal, if there is no other strategy profile S such that $u_i(S) \geq u_i(S^*)$ for all players i and $u_i(S) > u_i(S^*)$ for some player i.

This means that in a Pareto-optimal Nash Equilibrium there is no strategy profile S that increases one player's payoff without decreasing another player's payoff. Besides Pareto-optimality, a desirable Nash Equilibrium is also socially-optimal [26].

Definition 2.8 (Social-optimality) A Nash Equilibrium S^* is socially-optimal, if there is no other strategy profile S such that $\sum_{i \in \mathbf{N}} u_i(S) > \sum_{i \in \mathbf{N}} u_i(S^*)$.

In a socially-optimal Nash Equilibrium, society's welfare which is defined as the sum of the utilities of all players in the game cannot be increased. Therefore, any socially-optimal Nash Equilibrium has to be Pareto-optimal but not vice-versa. A third way to evaluate a Nash Equilibrium regards its fairness. There are several definitions of fairness. Bertsekas and Gallager describe the term max-min fairness [16]. If we apply it on the utilities of players of a game, it can be defined as:

Definition 2.9 (Max-min Fairness) A feasible distribution of utilities u is max-min fair, if and only if an increase of player i's utility within the domain of feasible utilities must be at the cost of a decrease of some player j's already smaller utility. Formally, for any other feasible u', if $u'_i > u_i$ then there must exist some player j such that $u_j \leq u_i$ and $u'_j < u_j$. According to [18], if a solution exists, it is unique.

While max-min fairness favours smaller utilities, the concept of proportional fairness [39] characterizes a resource allocation, which is inversely proportional to the players' requests.

Definition 2.10 (Proportional fairness) A feasible distribution of utilities u is proportionally fair, if and only if for any other feasible utility u' the aggregate of proportional changes is zero or negative, i.e.,

$$\sum_{i \in \mathbf{N}} \frac{u'_i - u_i}{u_i} \leq 0, \quad \text{for all } u'. \tag{4}$$

The fairness definitions so far only provided a binary decision about the fairness, i.e., an allocation is either considered as fair or not. Remember that Jain et al. [38] provided a fairness index that is continuous in the interval $[0,1]$. If we map the index introduced in (2) to utilities, we obtain:

Definition 2.11 (Jain's fairness index) The fairness index defined by Jain et al. [38] of the utility profile u is given as:

$$f(u) = \frac{[\sum_{i=1}^{N} u_i]^2}{N \sum_{i=1}^{N} u_i^2}, \quad \text{with } u_i \geq 0, \quad f(u) \in [0,1], \tag{5}$$

with u_i player i's utility and N the number of nodes in the network. Larger values of $f(u)$ indicate better fairness.

In this section, we gave a brief introduction into non-cooperative game theory. We described the main elements of a game and gave a classification of games. Moreover, we explained solution concepts for the different game types and provided means to evaluate the equilibria of a game.

2.4 RELATED WORK ON NON-COOPERATIVE GAME THEORY

In this section, we review papers that apply game-theoretic tools on communication network problems. The works discussed determine the equilibria of the games, consider games with imperfect information and further deliver an insight into various methods on how to design a game in order to guarantee that it converges to a particular equilibrium. We also include a description of the chain store paradox, which is closely linked to the game model applied in this thesis.

Game theory is a means to analyse strategic interaction between decision-makers. It provides mathematical tools to analyse the possible and likely behaviour of those decision-makers. An important result of a game-theoretic analysis is the determination of the equilibrium outcomes. In [28, 29], for instance, a multi-radio multi-channel problem is modelled as a game with a finite number of selfish players that individually attempt to maximize their total bitrate. In [20, 37], the authors present mathematical analyses of random access games and determine their Nash Equilibria. Besides the determination of the Nash Equilibria of a game, it is also important to evaluate their properties. In [28, 29], the authors show which Nash Equilibria are max-min fair according to Definition 2.9 and which are also coalition-proof according to the definition in [15].

Very often games offer several Nash Equilibria. When considering their properties, however, those Nash Equilibria are usually not the same, i.e., some might be more preferable than others. Therefore, there is a need to introduce algorithms as rules of the games to drive the game to the desired Nash Equilibrium. In [20, 28, 29], the authors present algorithms so that the game converges to a certain Nash Equilibrium and analyse the convergence time as well as the influence of the number of players. The random access game in [37] further includes an analysis of the game asymptotic behaviour as the number of players approaches infinity.

Many findings in game theory are due to analyses of economic situations. In [61], Selten presents an extensive form game that reveals an inconsistency between game theory and plausible human behaviour. He explains this paradox with the limited rationality of human behaviour, while game theory usually assumes rationality. In his work, Selten discusses two versions of a market game. The first version is closely connected to the game model of this thesis and thus explained in the following. There is one incumbent that holds stores in m markets. In each of those m markets, there is one possible entrant. In the first period of the game, the possible entrant in market 1 decides whether or not to enter. If she enters, it is the incumbent's turn and she has to decide whether or not to fight the entrant. The utility functions are set in such a way that both players lose, if there is a fight. However, if there is no fight, the entrant gains and the incumbent is neutral. In the second period of the game, the possible entrant in market 2 decides about her entrance, followed by the incumbent's reaction assuming the same utility functions as in market 1. The same procedure applies for all subsequent markets.

To analyse this game, we apply backward induction, thus, we start the analysis in the last period. In this last market, the incumbent maximizes her utility, if she does not fight. Knowing this, the possible entrant in the last market, enters. In the second-last market, there is no reason for the incumbent to fight entry, because she loses in the current market and it does not deter entry in the last market. Hence, the incumbent does best, if she does not fight in the second-last market. Thus, the possible entrant in the second-last market enters. If we follow induction theory, all potential entrants should and will enter and the incumbent should and will never fight. Intuitively, however, an incumbent would attempt to deter entry by fighting. This constitutes the paradox [61].

In the game that Selten presents, markets are totally independent of each other. In real life, however, the lack of rationality in humans leads to a linkage of markets. In [44, 53], the authors connect the behaviour in the originally independent markets by means of imperfect information. They show that the cost of predation in the short-run, i.e., losing due to fighting, is worthwhile considering the reputation effect and hence the expected gain in subsequent periods. Thus, predation becomes rational, if it deters entry. Fundamental in applying this is asymmetric information. Players in this game with imperfect information maintain a belief about their opponents' types. This belief reflects a player's estimate whether or not her opponent behaves rationally. If an entrant plays against a rational incumbent, she gains if she enters. If she plays against an irrational incumbent, however, she loses, if she enters. Thus, her decision highly depends on her estimate about the incumbent's type.

Recall that in static games with imperfect information, the equivalent to a Nash Equilibrium is a Bayesian Nash Equilibrium. The equilibrium in the corresponding dynamic game is referred to as a Perfect Bayesian Nash Equilibrium. In [21, 46], the authors model the other players' channel gains as the Bayesian components of an imperfect random access game and determine the equilibria both for the static and the corresponding dynamic games. In such an equilibrium, beliefs have to be consistent with the outcome of the game and players have to play a best response given their equilibrium beliefs. The direct determination of these equilibrium beliefs is a tedious task. In dynamic games, however, in which the static game is repeatedly played, players have the opportunity to gather information about their opponents, deduce conjectures about their behaviour and update their beliefs accordingly. An example provides [63], where the authors introduce a bio-inspired learning algorithm to encourage cooperation in a random access game.

The current section provided a brief introduction to Selten's chain store paradox [61] and the extensions of his game that provide for the imperfection of human behaviour [44, 53]. In this thesis, we similarly model the considered protocol with a relaxed reservation algorithm as a Bayesian game. As in [44], we consider two-sighted uncertainty, i.e., neither the incumbent nor the entrants are aware of their opponent's type. We consider, however, a single entrant repeatedly meeting the same entrant in different markets. The additional papers that have been presented in this section, give an impression of how game theory has been applied to analyse communication networks.

3

PROBLEM STATEMENT

In the previous chapter, we reviewed related work on the analysis of ECMA-368. We have observed that there is little research that explicitly considers fairness. In this chapter, we describe and summarize the research questions that are addressed in this work, which cover the aspect of fairness in a distributed reservation protocol.

The literature review revealed the lack of fairness analyses of the distributed reservation protocol specified in ECMA-368. The analyses in [6, 7] determined the fairness depending on traffic type and access method. However, the authors did not consider the fairness perceived by the individual nodes. Fairness, however, should not only include the overall system fairness, but extend to an analysis of the individual nodes' gain in the system. Thus, we pose the following questions:

> *Is there a fairness issue inherent in the distributed reservation protocol specified in ECMA-368? How is this fairness issue reflected in the fairness perceived by the individual nodes? Which parameters of the distributed reservation protocol influence the fairness? How does the fairness depend on the size of the network?*

The ECMA-368 divides time into superframes of fixed length. Each superframe starts with a beacon phase, followed by a phase for data transmission. When nodes join a network, they first have to identify the beacon phase and attempt to place a beacon in an available beacon slot. Once they have attained a beacon slot, they have successfully joined the network and keep this beacon slot for the entire time that they are affiliated with the network. It emerges that the fixed order in the beacon phase discriminates nodes that transmit in later beacon slots. In [40], the authors aim at achieving fairness in the distributed reservation protocol of ECMA-368. Nodes are requested to publish flow information in their beacons. Based on this information, nodes then distributively determine a global satisfaction level and resources are allocated such that all nodes in the network achieve this level. Hence, some nodes voluntarily refrain from slots, so that other nodes also achieve the satisfaction level. However, nodes are not necessarily altruistic, thus we require a method that does not rely on good behaviour of nodes. Consider, for instance, a network that is operated with nodes designed by different vendors. If a node is constructed to be selfish and thus, gains a larger share of resources, it might be more attractive for potential customers. So vendors could have an economic incentive to design selfish nodes rather than altruistic ones. Thus, the algorithm that redistributes resources has to be robust against such selfish behaviour.

Further, we differentiate fairness regarding the time span it corresponds to. Long-term fairness is achieved, if the system converges to a fair allocation of resources after a long time. Short-term fairness is an even stronger property, which requires the system to be fair for smaller time intervals. Especially for flows that require low latency such as real-time applications, short-time fairness is an essential quality of service property [13]. So we pose the following questions:

> *How does a round-robin beaconing scheme perform regarding fairness? How does a randomization of the beacon phase influence the fairness? Can short-term fairness be achieved for any of those flexible beaconing schemes? Alternatively, does a relaxation of the reservation method achieve short-term fairness? For this relaxed reservation method, what are the parameters that influence the fairness? What are the parameters that influence the convergence time?*

The distributed reservation protocol, especially when considering a relaxation of the reservation method, constitutes a situation with strategic interaction. Thus, game theory provides tools to determine the outcomes, i.e., the resource allocation in the equilibrium, which can then be evaluated regarding its fairness.

4

FAIRNESS ANALYSIS OF ECMA-368 AND BEACONING ALTERNATIVES

In this chapter, we study the distributed reservation-based medium access specified in ECMA-368 [1] in order to identify the capacity and capabilities of the standard as well as its fundamental limitations. Recall that the ECMA-368 MAC architecture is fully distributed, so there is no central coordinator. Instead, it uses a distributed beaconing system to coordinate the medium access. Those beacons are used in order to perform device discovery, support dynamic network organisation and mobility. During the beacon phase stations announce time slots they intend to use during the data transfer phase. Since the beacon order in the beacon phase is fixed, this announcement is organised in a first-come, first-served manner. Contention only occurs when nodes join the network and compete for a beacon slot using Reservation-ALOHA. The first-come, first-served reservation mechanism in the beacon phase implies that nodes have to cope with the channel time that remains after prior nodes have placed their reservations.

We show in this chapter that this may end up in a situation where nodes are not able to reserve as much channel time as they require. In the worst case they may not be able to reserve any time at all. So we highlight the influence of the first-come, first-served reservation method on the arising unfairness, in which nodes may even starve. First, we present the protocol model that we found our analysis on. It includes the assumptions we make as well as an illustration of the different set of rules regarding a node's reservation. By evaluating different set of rules we identify the influence of the particular rules that ECMA-368 poses on the reservation process. Furthermore, we introduce the system model of the numerical analysis. For this model, we explain how we map a node's requirement of a certain data rate on a reservation. Since we assume a Poisson model for the frame arrival process, the domain of definition is infinite. Thus, we provide a cardinality analysis to reduce the probability space. Finally, we present the numerical analysis that determines throughput, delay and fairness of ECMA-368. The results of this fairness analysis have been previously published in [11]. We conclude this chapter considering beaconing alternatives such as randomizing the beacon slot order and the introduction of a round-robin mechanism in the beacon phase.

4.1 PROTOCOL MODEL

In this section, we describe the assumptions we make regarding the offered traffic as well as the channel. We also explain the policy sets that are evaluated in this work.

In our analysis, we assume an error-free channel. Hence, no frames are lost due to channel conditions. If we relaxed this assumption, we would also have to account for retransmitted frames. This would worsen the unfairness, since privileged nodes would have to reserve even more channel time in order to account for their possible retransmissions. This would leave even less channel time for subsequent nodes. We further assume a Poisson model for the frame arrival process. The analysis, however, is easily stretched to any other distribution by remodelling the according probabilities. The problem of unfairness that we illustrate in this chapter, though, will remain.

Recall that in Section 2.1, we stated that we focus on safe reservation. We presume that nodes prefer predictable and guaranteed channel access, so safe reservations that do not have to be released on request are justified. We assume nodes to only reserve for already arrived frames, thus, we do not have to include an algorithm that predicts the traffic. This further induces that no waste of channel time occurs. However, this assumption induces an additional delay. Frames that become ready for transmission during a reservation block have to wait for the next reservation block because there will not be enough time in the current block.

Furthermore, for analytical convenience, we presume that the reservation negotiation process has already taken place. By this, we focus on the reservation and its impact on the slot allocation. The first node in the beacon phase publishes her reservation in her beacon and subsequent nodes place their reservations for the remaining time of the superframe. Finally, we focus on a single superframe, so we assume that frames, which cannot be transmitted in the current superframe are dropped at its end. Remember that ECMA-368 has been designed for high-rate scenarios. If we consider real-time applications, information quickly loses its importance, if it is delayed. The assumptions explained cause reservations in a superframe to be independent of previous superframes. Thus, we do not have to consider queueing effects but are only influenced by the frame arrival rate. Due to limited channel time a gap might arise between the number of slots a node requires to reserve and the amount it is able to reserve.

As described in Section 2.1, ECMA-368 imposes several policies regarding the reservation of channel time in the data transfer phase. Since the consideration of all policies is complex to implement in real systems, we analyse different sets of policies to evaluate their impact on fairness. By this, we determine situations in which a policy subset sufficiently approximates the true result. We classify policies into those that relate to the amount of slots, which we refer to as rules, and those that relate to the location of slots in the data transfer phase, which we call strategies.

Reservation rules per node:

- *Basic*: A node is limited to 112 slots per superframe and 8 slots per zone.
- *Full*: A node is limited to 112 slots per superframe. The maximum number of slots per zone depends on the index of the first reserved slot according to Table 1 in Section 2.1.

Reservation strategies per node:

- First-Fit (FF): Nodes fill the superframe from the beginning, i.e., they choose zones with indices as low as possible.
- Min-Fit (MF): Nodes choose zones with the largest number of empty slots and apply FF on those. By this, they minimize the number of zones they require.
- Policy-Fit (PF): Nodes choose zones in the order of the isozones and apply FF on those.

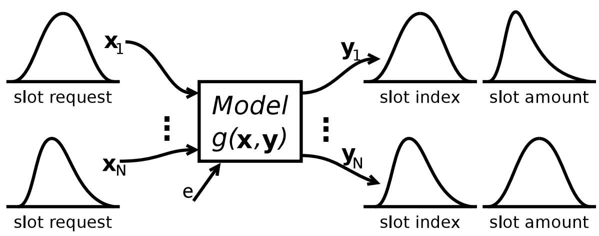

Figure 5: System model that relates the distribution of the number of required slots to the actually reserved slots. The output considers both the distribution of the amount of slots and the reserved slot indices for a network with N nodes.

In this section, we presented the protocol model that we analyse in the remaining sections. We justified the assumptions and introduced the different policy sets that we consider in our fairness analysis. In the next section, we address the system model, i.e., we explain how we do the analysis.

4.2 SYSTEM MODEL

In this section, we describe our system model. First, we specify the notation of the input and output signals of our system, which are the distributions of the required reservations and the actually generated reservations, respectively. Then we illustrate how each input signal is generated from the frame arrival rate, which is assumed to follow a Poisson distribution.

The allocation process is depicted in Figure 5. In the following, we use bold letters for random variables and bold capital letters for vectors of random variables. The i^{th} input signal $\mathbf{x}_i$ is a random variable that represents the number of slots that node i requires to support her application. The order of nodes in this model is given by their order in the beacon phase, i.e., without loss of generality node i transmits in the i^{th} beacon slot. Any restrictions imposed by the environment, e.g. the amount of slots that is reserved for the beacon phase, are comprised in the input signal e. The output signal $\mathbf{y}_i$ is a random variable that contains a vector of the reserved slots given by their indices as well as the sum of the reserved slots of node i, with $\mathbf{y}_i = g(\mathbf{x}_i, \mathbf{y}_1, \ldots, \mathbf{y}_{i-1})$. Note that node i's slot allocation $\mathbf{y}_i$ does not only depend on her own request $\mathbf{x}_i$, but also on all reservations $\mathbf{y}_k$ that have been previously announced, for $1 \leq k < i$.

Consider x_i to be a sample of the random variable $\mathbf{x}_i$. Then every ordered input N-tuple $X = (x_1, x_2, \dots, x_N)$ is a possible realization of requests of all nodes. So for each of these ordered input N-tuples X, we calculate the corresponding ordered output N-tuple $Y = (y_1, y_2, \dots, y_N)$ that considers the rules that are imposed by ECMA-368 as well as the limited channel time. We assume that the frame arrival process is given by a Poisson model. From this, we can determine the distribution of the required slots. Knowing the probability of each ordered input N-tuple X, we obtain the output signals $\mathbf{y}_i$, for $i \in [1, N]$, by weighting the output N-tuple Y with the probability of the corresponding input N-tuples.

In the subsequent paragraphs, we relate the mean number of slots that a node requires to support her application, i.e., λ_{slots}, to the frame arrival rate λ of her application. For our analysis, we exclude the possibility to transmit across slot borders. Thus, if a frame cannot be transmitted within the remaining time of a time slot, we assume it to be entirely transmitted in the next time slot. This denotes a worst-case situation. The number of frames m_{slot} that can be transmitted in a single time slot is given by (6):

$$m_{\text{slot}} = \left\lfloor \frac{T_{\text{slot}}}{\frac{l}{R} + T} \right\rfloor = \left\lfloor \frac{T_{\text{slot}} \cdot R}{l + T \cdot R} \right\rfloor, \tag{6}$$

with T_{slot} the slot length, R the bit rate used for the transmission, l the frame length and T any additional time that is necessary to transmit a frame, e.g. time required for acknowledgements.

Assume that a node's application has generated x_{SF} frames in the current superframe. Then, she requires exactly k slots to support her application, if it holds that $x_{\text{SF}} \in [(k-1)m_{\text{slot}} + 1, km_{\text{slot}}]$. To determine the probability distribution of the required slots, let $q_{x_{\text{SF}}}$ be the probability of x_{SF} generated frames in a superframe. Then the probability that a node requests k slots is given by (7):

$$P_{\lambda_{\text{SF}}}(X = k \text{ slots}) = \begin{cases} q_0, & \text{for } k = 0 \\ \displaystyle\sum_{x_{\text{SF}}=(k-1)m_{\text{slot}}+1}^{km_{\text{slot}}} q_{x_{\text{SF}}}, & \text{for } k > 0 \end{cases} \tag{7}$$

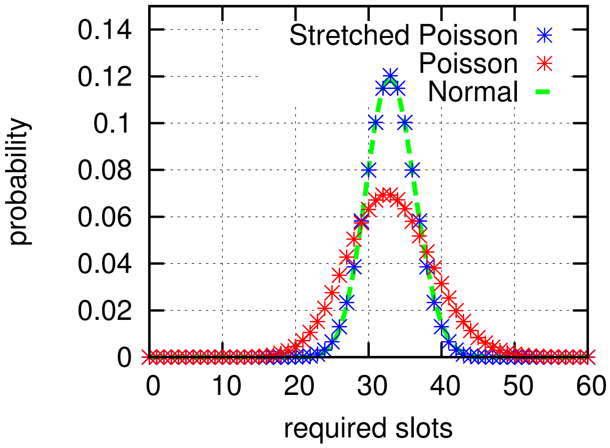

Figure 6: Probability distribution for $\lambda = 1500 \frac{\text{frames}}{s}$. With $m_{\text{slots}} = 3$, the corresponding average number of required slots is given by $\lambda_{\text{slots}} = 33 \frac{\text{slots}}{\text{SF}}$. The figure depicts a general Poisson distribution with $\lambda = \lambda_{\text{slots}}$ and the actual distribution given by (7) denoted as stretched Poisson. For large values of λ_{slots}, the latter can be approximated by a normal distribution.

A frame arrival rate λ implies that on average $\lambda_{\text{SF}} = \lambda T_{\text{SF}}$ frames arrive during a superframe of length T_{SF}. Hence, the average number of slots required is given by $\lambda_{\text{slots}} = \left\lceil \frac{\lambda_{\text{SF}}}{m_{\text{slot}}} \right\rceil = \left\lceil \frac{\lambda T_{\text{SF}}}{m_{\text{slot}}} \right\rceil$. Assume for instance that a node's application generates frames at an average rate of $\lambda = 1500 \frac{\text{frames}}{s}$. To determine the corresponding λ_{slots}, we previously calculate the number of frames that can be transmitted in a single slot m_{slots}. Recall that the slot length in ECMA-368 is given as $T_{\text{slot}} = 256$ µs. In an unacknowledged transmission, the standard further defines $T = 1.875$ µs. If we assume a transmission rate of $R = 53.3$ Mbit/s and a frame length of $l = 500$ byte, then m_{slot} is given as:

$$m_{\text{slot}} = \left\lfloor \frac{256 \cdot 10^{-6} \cdot 53.3 \cdot 10^{6}}{500 \cdot 8 + 1.875 \cdot 10^{-6} \cdot 53.3 \cdot 10^{6}} \right\rfloor = 3. \tag{8}$$

So with $T_{\text{SF}} = 65536$ µs, the node requires an average number of slots given by $\lambda_{\text{slots}} = 33 \frac{\text{slots}}{\text{SF}}$ to support her application. Figure 6 depicts a general Poisson distribution with $\lambda = \lambda_{\text{slots}}$ as well as the stretched Poisson distribution determined by (7). We observe that $P_{\lambda_{\text{SF}}}$ can be approximated for large values by a normal distribution with $\mu = \lambda_{\text{slots}}$ and $\sigma^2 = \frac{\lambda_{\text{slots}}}{m_{\text{slot}}}$.

In this section, we introduced the system model including the notation. We further explained the relation between the frame arrival rate and the number of slots a node requires in order to cope with this frame arrival rate. Since the support of the Poisson distribution is $[0, \infty)$, in the next section, we introduce a means to reduce the permutation space.

4.3 CARDINALITY OF NUMERICAL ANALYSIS

In the previous section, we assumed that the frame arrival rate λ follows a Poisson distribution, which is defined in the interval $[0, \infty)$. Thus, the corresponding stretched Poisson distribution of the required number of slots with parameter λ_{slots} is also defined in the interval $[0, \infty)$. So generally, each node i could require any positive number of slots $x_i \in [0, \infty)$. In this section, we reduce the cardinality of our analysis. For this purpose, we truncate the probability distribution such that a target probability of $(1 - \varepsilon)$ is achieved for a minimal interval $\Delta x_i = [x_{i,\text{low}}, x_{i,\text{up}}]$, with $i \in [1, N]$. With $\varepsilon \ll 1$, we therefore restrict our analysis to the N-tuples within this interval at the cost of introducing a known error.

Our scenario consists of N nodes. So for each of those N nodes we have to find the smallest interval $\Delta x_i = [x_{i,\text{low}}, x_{i,\text{up}}]$ such that the accumulated probability of the corresponding input N-tuples achieves the target probability of $(1 - \varepsilon)$. Let p_{k_i} be the probability that the value of the random variable $\mathbf{x}_i$ is equal to k, for $k = 0, 1, \ldots$. The corresponding cumulative distribution function is given by $F(x_i)$. So the probability that $\mathbf{x}_i$ is in Δx_i is given by:

$$\underset{\Delta x_i = [x_{i,\text{low}}, x_{i,\text{up}}]}{F(\Delta x_i)} = P\{x_{i,\text{low}} \leq \mathbf{x}_i \leq x_{i,\text{up}}\} = \sum_{k = x_{i,\text{low}}}^{x_{i,\text{up}}} p_{k_i} \tag{9}$$

Recall that the probability p_{k_i} that node i requires k slots depends on the frame arrival rate of node i's application. We assume that nodes' applications are not intertwined, thus, the probabilities p_{k_i}, for all $i \in [1, N]$, are independent of each other. The probability that $x_i \in \Delta x_i$, for all $i \in [1, N]$, is then given by:

$$\underset{\Delta x_i = [x_{i,\text{low}}, x_{i,\text{up}}],\ \forall i \in [1,N]}{F(\Delta x_1, \ldots, \Delta x_N)} = \sum_{k_N = x_{N,\text{low}}}^{x_{N,\text{up}}} \cdots \sum_{k_1 = x_{1,\text{low}}}^{x_{1,\text{up}}} p_{k_1} p_{k_2} \cdots p_{k_N} \tag{10}$$

We assume that nodes are symmetric, thus, all nodes' applications generate frames at the same arrival rate $\lambda_i = \lambda$, for all $i \in [1, N]$. Subsequently, the probability of a node to require k slots is the same for all nodes. So we can set $p_{k_i} = p_k$ and hence, $\Delta x_i = \Delta x = [x_{\text{low}}, x_{\text{up}}]$, $\forall\ i \in [1, N]$. Thus, we rewrite (10) to:

$$\underset{\Delta x = [x_{\text{low}}, x_{\text{up}}]}{F(\Delta x)} = \left[\sum_{\Delta x} p_k \right]^N \tag{11}$$

In this section, we intend to reduce the cardinality of the analysis by considering only those realizations that cover the target probability $(1 - \varepsilon)$. We find that we have to determine the lower and upper bounds x_{low} and x_{up}, respectively, that comply with (11), while minimising Δx:

$$\begin{aligned} &\min\ \Delta x = x_{\text{up}} - x_{\text{low}} \\ &\text{subject to} \\ &\left[\sum_{\Delta x} p_k \right]^N \geqslant 1 - \varepsilon \end{aligned} \tag{12}$$

In the subsequent paragraphs, we briefly describe the algorithm to solve the minimization problem of (12). We have shown in Section 4.2 that the probability distribution of the required slots follows a stretched Poisson distribution with parameter λ_{slots}, if the application generates frames at an arrival rate λ, whereas a Poisson model is assumed for the frame arrival rate process. From this, we know that the probability distribution of the required number of slots, i.e., the stretched Poisson distribution in Figure 6, has a single maximum for $x = \lambda_{\text{slots}}$. This property causes the subsequently proposed algorithm to be optimal.

To solve the minimization problem, we start with the maximum and set the initial bounds to $x_{\text{low}} = x_{\text{up}} = \lambda_{\text{slots}}$. As long as the target probability has not been achieved yet, we either increment x_{up} or decrement x_{low} by one depending on which corresponding probability is higher. Note that once x_{low} is equal to zero, any further step increments x_{up}. The procedure is summarized in Algorithm 1.

In this section, we introduced a means to reduce the cardinality of our analysis. We determined the smallest interval of the required number of slots of a node that

Algorithm 1 Algorithm to determine the minimum interval that complies with (12). Starting from the maximum of the probability function, probability p increased by stepwise enlarging the interval until the target probability is accomplished.

Input: probability to require $k = p_{\lambda_{\text{slots}}}$ slots
Input: number of nodes N
Input: target probability $(1 - \varepsilon)$
probability $p = p_{\lambda_{\text{slots}}}$
lower bound $x_{\text{low}} = \lambda_{\text{slots}}$
upper bound $x_{\text{up}} = \lambda_{\text{slots}}$
// if target probability is not achieved yet
while $p^N < 1 - \varepsilon$ **do**
  if $x_{\text{low}} > 0$ **and** $p_{x_{\text{low}}-1} \geq p_{x_{\text{up}}+1}$ **then**
    $x_{\text{low}} \leftarrow x_{\text{low}} - 1$
    $p \leftarrow p + p_{x_{\text{low}}}$
  else
    $x_{\text{up}} \leftarrow x_{\text{up}} + 1$
    $p \leftarrow p + p_{x_{\text{up}}}$
  end if
end while

covers a target probability of $(1 - \varepsilon)$ introducing a known error. In the next section, we discuss the numerical results obtained by an exhaustive examination of all cases.

4.4 Numerical results and discussion

For small values of the number of nodes N, we can solve the described problem with an exhaustive examination of all cases at reasonable expense. According to Knuth [41], we refer to this analysis technique as *generating* all combinatorial objects, here input N-tuples X, and *visiting* each object. This emphasizes that we analyse only one generated object at a time.

Large values of N, however, require different approaches such as Monte-Carlo simulations, because the cardinality of X increases exponentially with N. We restrict our analysis to networks with $N \leq 5$, since they are sufficient to provide an insight into the problem. For our analysis, we developed a multi-threaded distributed Java program that implements the different sets of reservation rules and strategies. It generates all possible combinations of slot requirements X and determines the corresponding output N-tuples Y considering the reservation rules and strategies. To determine the output signals $\mathbf{y}$, the output N-tuples Y are weighted with their probability to occur. Tests have shown that for $N = 5$ and an error $\varepsilon = 10^{-3}$ it takes about 3.5 days to perform the calculations on a single processor. Since the calculations for the input N-tuples are independent of each other, we can massively parallelise them.

In our scenario, we assume a Poisson model for the frame arrival process with parameter λ. Recall that in Section 2.1 we stated that we restrict our analysis to safe reservations, since only they provide guaranteed and predictable channel access. Safe reservations denote that a node's reservation is limited to 112 slots. Thus, any slot requirement larger than 112 slots is truncated to this limit. To evaluate whether the consequential aggregation of probability influences the results, we perform our analysis for frame arrival rates that correspond to an average slot requirement of $\lambda_{\text{slots}} \in [0, 120]$. The first zone of a superframe, i.e., the first 16 slots, is considered reserved for beaconing. To abstract from the chosen frame length and transmission data rate, we present the results in terms of slots instead of data rate. Applying the formulas in Section 4.2, though, specific results can easily be determined.

In the following paragraphs, we provide numerical results regarding the throughput and delay of each node and evaluate the corresponding fairness indices. The throughput is given by the average number of reserved slots. The delay is calculated from the distribution of the nodes' reserved slots. Jain's fairness index as introduced in Section 2.2 for the overall as well as the perceived fairness is applied for both the average number of reserved slots as well as the corresponding mean delay.

The graphs in Figure 7 show the average number of reserved slots against the average number of slots a node requires to support her application. The scenario is depicted for 5 nodes and different sets of reservation strategies and rules. The left graph of Figure 7 shows the *Basic* rule with the policy-fit reservation strategy. Remember that the *Basic* rule implies that nodes are only limited by the maximum number of slots per node and 8 slots per zone. With policy-fit, nodes have to consider the order of the isozones when choosing a zone for their reservation.

The right figure of Figure 7 illustrates the results for the *Full* rule. The outer graph depicts policy-fit, whereas the inner one shows a section of the min-fit reservation strategy. Note that the graphs for first-fit and policy-fit are identical, so the results

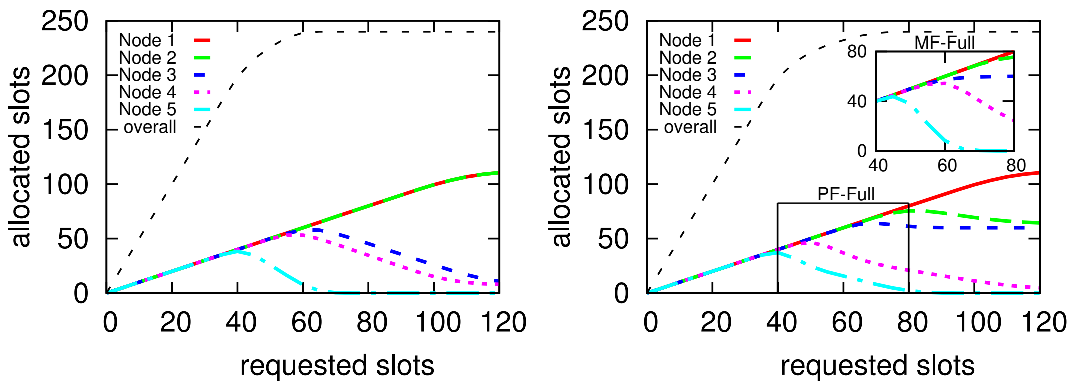

Figure 7: Reserved number of slots versus required number of slots with policy-fit-*Basic* (left), policy-fit-*Full* (right - outer figure), min-fit-*Full* (right - inner figure). With *Basic* (left), nodes 1 and 2 are bounded by the maximum number of slots, i.e., 112 slots. With *Full* (right), node 1 is bounded by 112 slots, the other nodes by the per-zone limitation. We further observe that node 4's maximum number of slots is higher for min-fit than for policy-fit (framed sections), which is due to a more efficient channel utilisation. Note that the graph for first-fit-*Full* is identical to policy-fit-*Full* and therefore omitted.

for policy-fit also hold for first-fit. *Full* refers to the rule that nodes have to account for the per-zone restrictions given by Table 1. With min-fit, nodes choose zones that offer the largest number of unoccupied slots. By this, nodes minimize the number of zones required to support their applications. The overall curve in both the left and the right figure is the sum of the nodes' reservations, i.e., the network throughput.

If we consider the first two nodes, we observe that for *Basic* both nodes are only limited by the maximum of 112 slots, which is the maximum number of slots per node. For *Full*, however, this applies only for the first node. The second node's number of reserved slots drops due to the per-zone limitation according to Table 1. In this case, node 3 can take advantage and increases her throughput if we compare it to the *Basic* rule in Figure 7 (left). Furthermore, we see that with policy-fit-*Full* in Figure 7 (right), node 4 and 5's throughput drops, even though the channel is not yet saturated. Node 5's decrease with the *Full* rule, however, is not as steep as in case of the *Basic* rule. This can also be traced back to the per-zone policy that limits node 4's throughput and therefore leaves more slots for node 5. If we compare the framed section of policy-fit-*Full* with the inner graph that represents min-fit-*Full*, we observe a relevant difference. With min-fit, node 4 gains more resources, specifically, it is able to reserve as much as in the *Basic* case in Figure 7 (left). The reason is

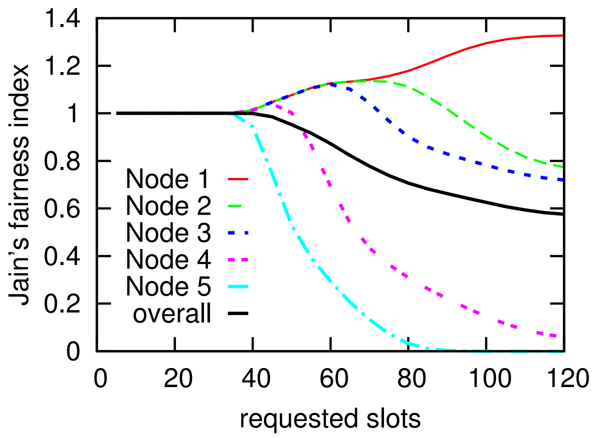

Figure 8: Jain's fairness of reserved slots with policy-fit-*Full* and 5 nodes. The overall fairness drops with increasing traffic load. The fairness perceived by the individual nodes (cf. (3) in Section 2.2) differs with the node index. Nodes with low index have an advantage over those with high index.

that with min-fit, nodes prior to node 4 choose different zones than with policy-fit, leading to a better channel utilisation. This implies less channel time for node 5, though, resulting in the steep decrease in the inner graph of Figure 7 (right).

Figure 8 shows Jain's fairness index for 5 nodes against the average number of required slots. It depicts both the overall fairness as well as the fairness perceived by the different nodes exemplary for policy-fit-*Full*. Note that the value for a node's perceived fairness can be larger than 1 as defined in (3), which marks the favouritism of this particular node. We observe that for low load the system achieves a fairness of 1, which denotes the maximum achievable fairness. By the time the channel is saturated, the overall fairness drops to about 0.6, since resources are not equally distributed anymore. Considering the perceived fairness, we notice that the first three nodes perceive the fairness higher than the average, while nodes 4 and 5's perceived fairness is below the average. The reason lies in the sharing mechanism. While the first three nodes achieve a share beyond the fair share, nodes 4 and 5 gain a very small share. Furthermore, we observe that the slope of the overall fairness curve becomes less steep when the fairness of the fifth node is zero, as from there, its share in the overall fairness does not change anymore.

We conclude for the throughput that both first-fit and policy-fit cause a drop in the throughput of the fifth node even though the channel is not saturated, whereas this does not happen for min-fit due to the more efficient channel utilisation. If we consider the fairness, all strategies achieve fairness for low load. High load, however, decreases the overall fairness for networks with 5 nodes by 0.4 and the perceived fairness curves diverge. We further conclude that the reservation strategy, i.e., which zones to choose for reservation, has no significant influence on the results. So we deduce that in high load scenarios any reservation strategy may be used.

Recall that we stated in the beginning of this section that the restriction to networks with $N \leq 5$ is sufficient to provide an insight into the fairness problem. From the throughput results we infer that a node's share of resources does not depend on the number of nodes in the network, but solely on her position in the beacon phase. Thus, an increase of the network size does not change the throughput results but only deteriorates the fairness results. So we further conclude that even though for high load the reservation rule, i.e., whether or not nodes' reservations in a zone depend on the slot indices, has a significant influence on the results, this influence quickly becomes negligible as the network size increases.

In the remaining paragraphs of this section, we discuss the influence of the different reservation strategies on the delay. Remember that in the beacon phase we reserve channel time for the frames that arrived in the previous superframe. So to focus on the impact of the reservation strategies, we define the delay of a frame as the slot index, in which the frame has been transmitted. Equivalently, the average delay of a node is the average of the slot indices that she used for her transmissions. To illustrate the allocation of slots among the different nodes, consider Figure 9. The graphs in this figure show the probability that a node reserves a particular slot index. All figures reflect a network with 5 nodes, the *Full* rule, and a mean slot requirement of $\lambda_{\text{slots}} = 45$ slots/SF as an example that well illustrates the different reservation strategies. The first zone is reserved for beaconing, all following zones display the same reservation pattern of 8-4-4 slots due to the *Full* rule.

The top figure illustrates the distribution of slot indices for the first-fit strategy. The zones in the beginning of the superframe are shared among the first 3 nodes. When the first node has fulfilled its request, node 4's probability to reserve increases. After that, node 5's probability to reserve becomes visible. It is only at the end of the superframe that she is able to reserve the largest part of a zone. Even though

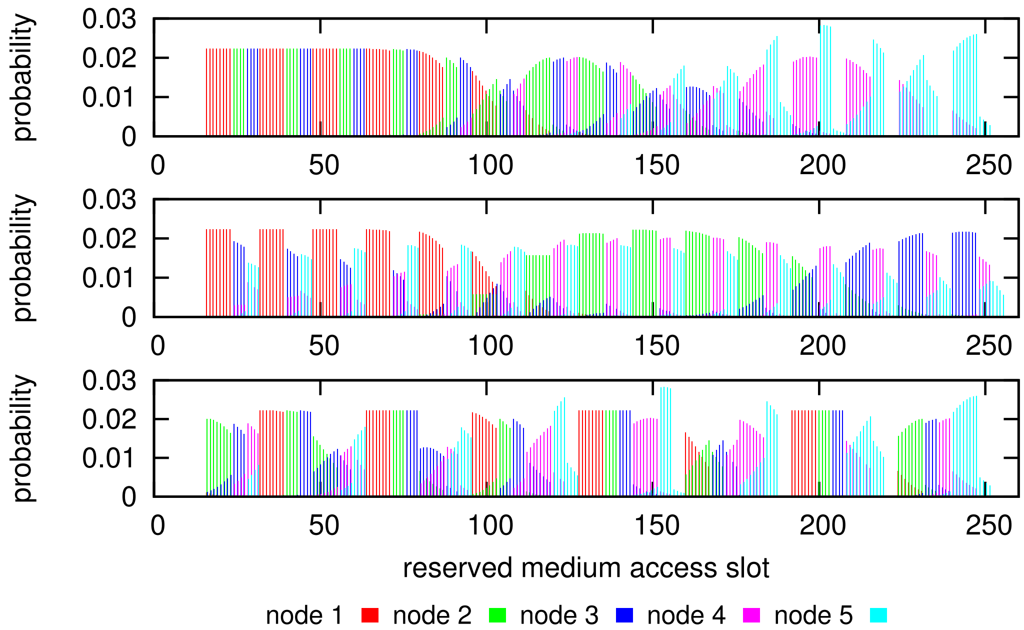

Figure 9: Distribution of slot indices with $\lambda_{\text{slots}} = 45$ slots/SF and different reservation strategies: first-fit-*Full* (top), min-fit-*Full* (centre) and policy-fit-*Full* (bottom). The graphs depict the probability that a node reserves a certain slot index. First-fit fills the superframe from the start, min-fit reserves in zones with the largest number of unoccupied slots, while policy-fit complies with the isozone order.

we notice empty slots in the range of indices 200 and higher, i.e., the channel is not yet saturated, node 5's throughput requirements cannot be entirely fulfilled as it has been seen in Figure 7 already. Here, the limitations of the per-zone restrictions become apparent in the figure.

In contrast, the min-fit strategy, which is depicted in the central figure, was able to fulfil all requests for $\lambda_{\text{slots}} = 45$ slots/SF. The first node reserves in a first-fit manner until she has reserved the number of slots that she requires to support her application. The second node, however, chooses zones that have the fewest number of reserved slots per zone. So she prefers not to reserve in zones that node 1 has already reserved in, but in the subsequent ones, i.e., in the middle of the superframe. Node 3's reservation starts towards the end of the superframe for the same reason. Since her requirements cannot be fulfilled by the remaining empty zones, she additionally chooses zones at the beginning of the superframe. Both nodes 4 and 5 behave comparably to the first-fit strategy, since on average the number of remaining slots for reservations is equal for each zone.

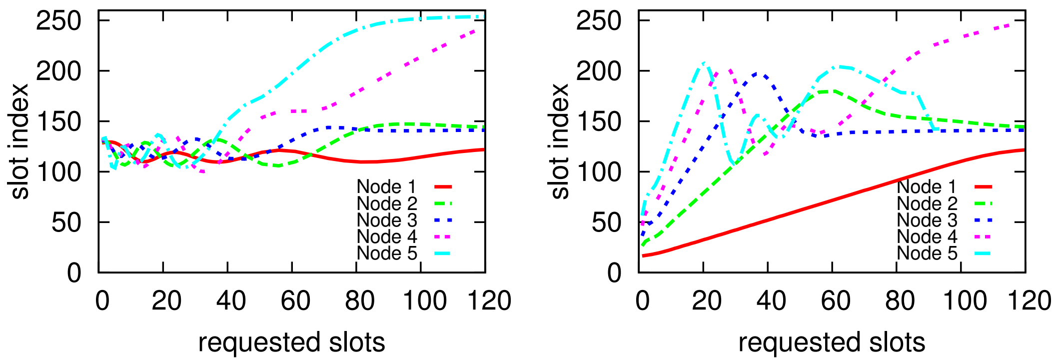

Figure 10: Mean slot index used by a node versus the required number of slots with policy-fit-*Full* (left) and min-fit-*Full* (right). For low load, policy-fit has a higher but more stable delay than min-fit. The higher the load, however, the more similar the curves for policy-fit and min-fit become. For clarity, graphs are shown with lines though functions are discrete.

Finally, the bottom figure shows the policy-fit strategy. Here, node 1's reservation probability is mainly found in the first three isozones. Nodes 2 and 3 also start their reservations in isozones 0 to 2, but still have a significant probability to reserve in isozone 3. Both nodes 4 and 5's probability is spread across the superframe with an emphasis on isozone 3. As for first-fit, we notice empty slots in the superframe.

The mean delay for policy-fit-*Full* and min-fit-*Full* and all considered traffic loads is illustrated in Figure 10. Recall that Figure 9 showed that policy-fit evenly spreads the reserved slot indices around the middle of the superframe. For low load, this results in a stable and similar delay for all nodes as observed in Figure 10 (left). Under saturated conditions, however, nodes with higher indices, e.g. nodes 4 and 5, are pushed towards the end of the superframe causing their delays to increase, while the delay of nodes with lower indices remains stable.

The min-fit strategy, in contrast, chooses zones with the largest number of empty slots. Except for the first node, this results in an oscillating behaviour as observed in Figure 10 (right). To explain the oscillating behaviour for the min-fit strategy, consider the individual nodes. The first node reserves in a first-fit manner, so her mean delay linearly increases with the required number of slots. The second node chooses zones that provide the largest number of available slots. For low load, the last zones of the superframe are not yet occupied, hence, she chooses those slots and her delay increases with the traffic load. For higher load, however, the empty zones

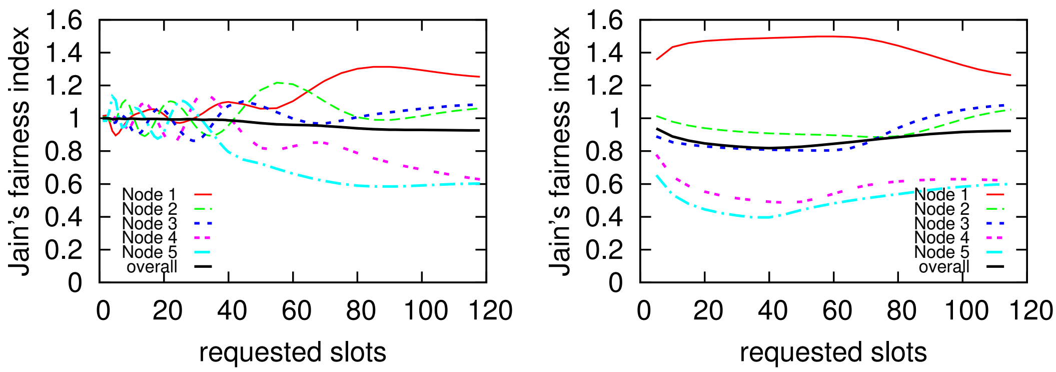

Figure 11: Jain's fairness considering the allocation of slot indices with policy-fit-*Full* (left) and first-fit-*Full* (right) and 5 nodes. For low load and policy-fit-*Full*, the system is fair. High load leads to a slight drop in the overall fairness. The perceived fairness, however, highly diverges. For first-fit-*Full* the fairness curves highly diverge for any load. For high load, the strategies diverge to the same results. Note that for clarity we draw continuous lines even though the results are discrete.

towards the end of the superframe do not suffice her needs, thus, she additionally reserves slots at the beginning of the superframe, which decreases her average delay. A further increase of the traffic load implies that even less empty zones at the end of the superframe are available and she has to reserve even more slots in the beginning of the superframe, so her average delay decreases even more. The same procedure applies for the subsequent nodes. When the maximum number of slots per node and zone is reached, the delays converge to their final values.

Figure 11 shows Jain's fairness index and the fairness perceived by the individual nodes for the delay. Here, we define x_i in (2) and (3) as the reciprocals of the slot indices. The figure depicts the scenario with policy-fit-*Full* (left) and first-fit-*Full* (right) for 5 nodes. For low load and policy-fit-*Full*, the stable and similar delay that has been shown in Figure 10 (left) results in a fairness index close to 1 for the overall and the perceived fairness indices. In high load scenarios, the perceived fairness for policy-fit diverges, since node 4 and 5's delay increases because nodes with higher indices are pushed towards higher slot indices or even out of the superframe. With first-fit-*Full* (right), however, the perceived fairness indices highly diverge for any load. For all strategies including min-fit-*Full*, the curves converge to similar values.

We conclude that policy-fit guarantees an almost constant mean delay for all nodes, since reservations are spread around the middle of the superframe. For low load though, this connotes that the mean delay is higher than for the other strategies.

All strategies achieve an overall delay fairness close to 1 in all traffic situations, but policy-fit additionally provides perceived fairness for low load. In high load situations, however, the perceived delay fairness curves for policy-fit diverge as well and coincide with the results for the other strategies. Thus, as with the fairness results for the throughput analysis, the reservation strategy has only marginal influence on the delay fairness in high-load scenarios. In the subsequent sections, we briefly show the impact of alternative beaconing systems on the fairness.

4.5 BEACONING ALTERNATIVES

The lack of fairness in ECMA-368 that we identified in this chapter so far is due to the fixed order in the beacon phase. This order develops during the set up of the network, when nodes use Reservation-ALOHA to join the network. This order only slightly changes when nodes leave or new nodes join the network. In order to achieve optimal resource allocation among competing nodes, medium access rules must be flexible enough to adapt to the specific scenario, being robust against the network size, the channel state and the applications' requirements.

In this section, we break open the fixed beacon order and make it more flexible, i.e., the privileged nodes in the beginning of the beacon phase change over time. When comparing the medium access of the stations over a long period of time, i.e., several superframes, we argue that this approach is capable of meeting a certain global fairness criterion, if nodes have to support similar traffic patterns and traffic requirements are time-invariant. However, we conjecture that the same fairness criterion is not met, if we focus on a short period of time, e.g. one superframe. This is due to the selfish behaviour of nodes that evens out over long time intervals. If a node is allowed to transmit her beacon in the first beacon slot, she reserves as much channel time as needed not considering that other nodes might require slots themselves. In this section, we consider random as well as round robin beaconing.

One way of a more flexible beacon order is random beaconing. If the beacon phase is randomized, nodes randomly choose a new beacon slot every z superframes. Here, z is a parameter that accounts for the number of nodes, the traffic patterns or the degree of unfairness in the original beaconing scheme. If we consider the long term that accounts for several factors of z superframes, all nodes are equally likely to occupy a beacon slot at the beginning, the middle and the end of the superframe. So if we consider the results of Section 4.4, all nodes equally access the medium and thus, both the long-term throughput and the mean delay are equal for all nodes.

However, random beaconing has a severe drawback. If nodes randomly choose new beacon slots every z superframes, there is a likelihood of beacon collisions. In ECMA-368, the length of the superframe is given and split into beacon and data transfer phase. So in order to have a large transfer phase, the beacon phase has to be as short as possible. The contraction algorithm provided in ECMA-368 leads to a beacon phase with hardly any unused beacon slot.

Assume there are m available beacon slots, of which $N \leq m$ are occupied by the nodes in the network. Thus, there are $a = m - N$ additional empty beacon slots. If all N nodes in the network randomly choose a new beacon slot at the same time, beacon collisions are likely to occur. A collision in the beacon slot, however, implies that the involved nodes cannot make a reservation for the upcoming superframe. Thus, nodes that are involved in a beacon collision, suffer from throughput reduction.

In the previous sections, we have shown that the severity of the fairness problem is largest in high-load scenarios. Therefore, in the rest of this work, we focus on greedy nodes. To estimate the throughput reduction, we further abstract from the reservation rules and strategies and suppose that there is no reservation limit for a node. This implies that the first node that successfully transmits a beacon in the beacon phase, reserves the entire data transfer phase x_m. Thus, with deterministic beaconing, we achieve a utilization in the data transfer phase of 100 %. To determine the throughput in case of random beaconing, we compute the probability P_s that there is at least one successfully transmitted beacon assuming that the nodes' draws are independent and identically distributed (iid). The probability P_s is given as $P_s = 1 - P_c$, with P_c the probability that all beacons collided. The achievable network throughput for random beaconing then becomes $G = (1 - P_c)x_m$.

Figure 12 shows the probability P_c that all beacons collide depending on the network size determined by enumeration. Generally, the probability decreases with

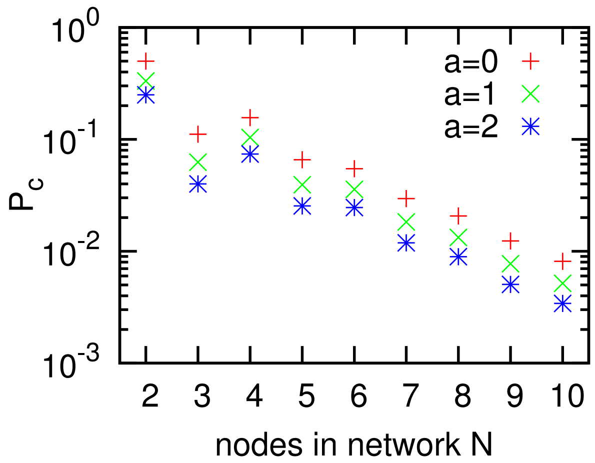

Figure 12: Probability that the data transfer phase of a superframe remains empty due to the collision of all beacons. Generally, the probability that all beacons collide decreases with increasing number of nodes in the network. When the number of available beacon slots is equal to the number of nodes, i.e., $a = 0$, the collision probability is maximal. The more additional beacon slots are present, the smaller is the collision probability.

increasing number of nodes in the network. We observe, however, that the graph is not monotonically decreasing. Consider for instance the increase of the collision probability from $N = 3$ to $N = 4$ in the case of no additional beacon slots $a = 0$. For $N = 3$ and $m = N$ available beacon slots, there are $3^3 = 27$ possible combinations how the nodes choose their beacon slots. If all nodes choose the same beacon slot, all beacons collide, i.e., $P_c = \frac{N}{N^N} = \frac{3}{27} = \frac{1}{9}$.

For $N = 4$ and $m = N$, there are $4^4 = 256$ possible combinations how the nodes choose their beacon slots. Here, however, there are two alternatives how all beacons collide. First, all beacons collide, if all nodes choose the same beacon slot. Second, all beacons collide, if two nodes pairwise choose the same beacon slot. Thus, the probability that all beacons collide becomes $P_c = \frac{N+\binom{N}{2}(N-1)!}{N^N} = \frac{4+6\cdot 6}{256} = \frac{5}{32} > \frac{1}{9}$. Hence, the increase in the probability that all beacons collide is caused by the set of additional combinations of chosen beacon slots.

Increasing the number of available beacon slots m so much so that $a > 0$ corresponds to a decrease in the probability that all beacons collide. Thus, the larger the network and the larger the number of spare beacon slots a, the more likely it is that at least one node successfully transmits a beacon and consequently transmits in the data transfer phase. Since nodes are equally likely to transmit the first successfully sent beacon due to the iid assumption, the individual throughput of each node i becomes $g_i = \frac{G}{N} = \frac{P_s x_m}{N} = \frac{(1-P_c)x_m}{N}$.

If we consider round-robin beaconing instead of random beaconing, we achieve long-term fairness without the cost of beacon collisions, so $G = x_m$. Round-robin beaconing implies that nodes move their beacon to the adjacent beacon slot every z superframes. Hence, a node returns to her initial beacon slot after $N \cdot z$ superframes. If we consider the period of $N \cdot z$ superframes, every node has had the same transmission opportunities. Thus, the long-term mean throughput and delay is equal for all nodes, i.e., $g_i = \frac{G}{N}$. Due to the illustrated selfish behaviour of nodes, however, neither round-robin nor random beaconing achieves short-term fairness.

We conclude that a change in the beacon order, either random or deterministic, achieves fairness, if we consider several superframes. In our analysis, we assumed greedy nodes, because we have previously shown that the unfairness increases with the traffic load. We further prescinded from the reservation rules and strategies and assumed that nodes are not limited in their reservation. Thus, the first node that can successfully transmit a beacon is able to reserve all available slots in the data transfer phase. With this abstraction, the original beacon order achieves a channel utilization of 100 % and so does round-robin beaconing. Random beaconing, however, suffers from throughput reduction due to beacon collisions. While both random and round-robin beaconing achieve long-term fairness, neither of them is able to provide short-term fairness.

4.6 SUMMARY

In this chapter, we identified the unfairness inherent in the distributed reservation-based medium access provided in ECMA-368 and showed that beaconing alternatives such as random or round-robin beaconing only provide long-term fairness. In particular, we evaluated various sets of reservation rules and strategies of ECMA-368 to identify the influence of different protocol aspects on fairness. For a Poisson model of the frame arrival process, we determined the rate of the required transmission slots and provided a numerical analysis.

We showed that in high-load scenarios the reservation strategy that determines the location of the reservation within the superframe has no significant impact on neither the throughput nor the delay and hence, not on the fairness. The reservation rules that determine the maximum amount a node is allowed to reserve influence the fairness, but this influence quickly becomes negligible with increasing network size. The reason for the identified unfairness lies in the fixed order of beacons in the beacon phase. The earlier a node transmits her beacon, the more privileged she is, which is considerably reflected is the nodes' values for the perceived fairness. In high-load scenarios for a network with 5 nodes, the first three nodes have perceived fairness values significantly larger than the average, while the fairness perceived by nodes 4 and 5 approaches zero. We further deduce that a node's resource share is independent of the number of nodes in the network, so increasing the network size aggravates the fairness issue.

As we showed that the fairness issue is due to the fixed beacon order in the beacon phase, in the last section of this chapter, we designed the beacon phase more flexible. Both the randomization of the beacon phase and the deterministic alteration in a round-robin manner achieved long-term fairness. While round-robin accomplished the same utilization as the original fixed beaconing, the randomization occurs at the cost of beacon collisions that induce a reduction of throughput.

While both alternative beacon orderings attain long-term fairness, neither of them is able to provide short-term fairness. In the next chapter, we pursue the realization of a fair resource allocation when accounting for a single superframe. Instead of changing the beacon order, we accept the given order of beacons and introduce an algorithm that relaxes the absoluteness of the reservation. Using non-cooperative game theory as a tool to analyse strategic interaction, we show that the introduced algorithm achieves short-term fairness.

5

STATIC GAME OF DISTRIBUTED RESERVATION PROTOCOL

So far, we have seen that a distributed reservation protocol with fixed beacon order guarantees channel access without collisions. The bandwidth, though, is not necessarily distributed in a fair manner among the nodes that are part of the network. In the previous chapter, we have further observed that applying a round-robin or random beaconing scheme in the beacon phase achieves long-term fairness but both lack short-term fairness. While round-robin is able to guarantee predictable channel access, this is naturally not the case with a random beaconing scheme.

In this chapter, we model the distributed reservation protocol with fixed beacon order as a multi-stage game, in which each single beacon slot and the data transfer phase as a whole is considered a stage in the game. For this game, we determine the Nash Equilibria and subgame-perfect equilibria and evaluate which of them are Pareto- and socially-optimal as well as fair equilibria. We keep the fixed beacon order and then deviate from the absoluteness of the reservation and determine the Bayesian Nash Equilibria of the game.

In the previous chapter, we have shown that large parts of the rules in ECMA-368 that regard the reservation have only marginal influence on the fairness in high load scenarios. For this reason, we abstract the ECMA-368 to a general distributed reservation-based protocol. The protocol model that we use is explained in the next section, followed by the corresponding game model.

5.1 PROTOCOL MODEL OF STATIC GAME

In this section, we describe the protocol model that we consider in our game-theoretic analysis. As before, we assume a time-slotted system with a superframe structure, whose length is given and fix. Each superframe starts with a beacon phase, which we also hold fix in length for convenience. The remaining x_m slots of the superframe are available for data transmission. The association procedure remains the same as in ECMA-368, hence, as soon as a node is associated with the network and has obtained a beacon slot, the order of nodes in the beacon phase is fixed. Without loss of generality, we refer to the node that occupies the i^{th} beacon slot as node i.

Initially, we assume the reservation process to be the same as the one described in Section 2.1. Recall that ECMA-368 specifies several rules and strategies regarding the location and amount of a node's reservation. In Chapter 4, we showed that in high load scenarios the strategies, which determine the reservation zone, have no influence on fairness. We further evaluated the impact of the reservation rules, which limit the amount of slots that a node is allowed to reserve. In high load scenarios, this limitation does not sufficiently provide fairness. Thus for the game-theoretic analysis, without loss of generality, we assume that nodes reserve in a first-fit manner and neglect any zone limitations. We further revoke the restriction on the maximum number of slots a single node may reserve. Hence, in the worst case scenario of heavy load the first node is able to reserve all x_m slots that are available for data transmission, so there are no slots left for the subsequent nodes.

5.2 GAME MODEL OF STATIC GAME

In this section, we present a multi-stage game to model the distributed reservation medium access control protocol with fixed beaconing. In particular, the multi-stage game consists of $N + 1$ stages. The first N stages model the beacon slots of the N players in the game. In those first N stages players play sequentially. After the beacon phase, we assume that all players simultaneously decide about their actual transmission after having observed all reservations. Thus, in the last stage, i.e., stage $N + 1$, they play simultaneously. Subsequently, we define the notation and present the utility functions that we apply. We use bold letters for sets and corresponding non-bold letters representing their respective cardinalities.

The game considered in this work is given by $\Gamma(\mathbf{N}, \mathbf{A}_i, u_i)$. Recall that $\mathbf{N}$ denotes the set of players. The actions player i can carry out are given by set $\mathbf{A}_i$, her utilities from the arising outcomes are given by u_i. Here, players of the game are the nodes associated with the network. We refer to node i as player i. The actions are the number of slots players choose to reserve and transmit in. The utility is a function of the achieved throughput and transmission cost. The first N stages of this game represent sequential play and refer to the reservations announced by players 1 through N in their respective beacons. Recall that we join player i with node i and, hence, with beacon slot i. This means that player i makes a move in stage i of the game by announcing her reservation. In the stages 1 until $i-1$ and $i+1$ until N, she does not move but observes her opponents' moves, i.e., she listens during the prior and remaining reservation stages. Stage $N+1$ is simultaneous play and refers to all players' transmission decision, i.e., all players concurrently decide which slots they transmit in. Note that player i's action in stage i limits her action space in stage $N+1$ because a player may only transmit in previously reserved slots.

The action a player chooses in the stages of the game is prescribed in her strategy. This strategy defines a complete plan of actions for each stage k of the game and every possible history h^k. So player i's strategy s_i is a sequence of $\{s_i^k\}_{k=1}^{N+1}$, whereas s_i^k maps $\mathbf{H}^k$ to $\mathbf{A}_i(\mathbf{H}^k)$, so $s_i^k(h^k) \in \mathbf{A}_i(h^k)$. A strategy profile then includes the strategies of all players. It is denoted by $S = (s_1, \ldots, s_N)$. S_{-i} terms the strategy profile for all players except player i. Player i's strategy s_i basically contains two elements for every possible history of the game. The first element is the number of slots she reserves in stage i subject to the reservations announced in the previous stages. The second element of her strategy concerns her transmission decision given all players' reservation decisions.

In non-cooperative games, players are assumed to be payoff-maximizers, i.e., they play a strategy that maximizes their utility. We define the utility as a function of the throughput reduced by the corresponding transmission costs. Hence, the utility is directly related to the transmission stage, but only indirectly to the reservation stages and depends on the number of successfully and unsuccessfully occupied slots. A slot is unsuccessfully used by a player, if a collision occurred. We assume a single-hop network with a single transmission channel. Thus, if two or more nodes transmit in the same time slot, it results in a collision. We restrict ourselves to an ideal channel, so frames cannot be lost due to channel conditions.

We assume a player's utility function to consist of two terms. On the one hand, players receive a gain from successfully transmitting, on the other hand, there are transmission costs no matter whether the transmission was successful or not. We denote the number of slots player i transmits in as x_i, which is composed of $x_i = r_i + w_i$, with r_i the number of slots player i successfully transmits in and w_i the number of slots in which a collision occurs. Thus, we denote player i's utility from playing strategy S as:

$$u_i(S) = \gamma_i(r_i) - \xi_i(r_i, w_i),$$

with $\gamma_i(r_i)$ the gain player i achieves from r_i successfully used slots. This gain is diminished by the costs $\xi_i(r_i, w_i)$. Note that the costs further include the costs for unsuccessful transmissions w_i. In order for a player to transmit at all, $u_i(S)$ has to be greater than zero, which is assumed to be the utility from not transmitting. We assume that the transmission costs per time slot, e.g. the required energy to transmit, do not change with the transmission amount. Thus, we assume linear costs, such as $\xi_i(r_i, w_i) = c_i(r_i + w_i)$, with $c_i \in \mathbb{R}_+^*$ a constant.

Defining the gain $\gamma_i(r_i)$ can be more differentiated. If we assume every transmitted bit to be equally important, the gain of successfully transmitting in r_i slots $\gamma_i(r_i)$ can be a linearly increasing function such as $\gamma_i(r_i) = p_i r_i$, with $p_i \in \mathbb{R}_+^*$ a constant. However, utility functions are commonly represented through continuously differentiable, monotonically increasing and strictly concave functions according to the law of diminishing returns [62]. Consider for instance an application that first transmits some basic information, e.g. the basic information in a picture, and the following frames transport enhancements of this information. So the more the application has already transmitted, the less valued the transmission of the next parts are. The corresponding gain can be represented by $\gamma_i(r_i) = p_i \ln(r_i + 1)$. Note that we move the ln-function into the origin to ensure that players receive zero rather than negative gain, if they are not transmitting.

With those two approaches, we derive two alternative utility functions. For the linear case, the utility function is given as $u_i(S) = p_i r_i - c_i(r_i + w_i)$. Without collisions this utility function becomes:

$$u_i(S) = (p_i - c_i)r_i. \tag{13}$$

For player i to transmit, her utility has to be positive, hence $p_i \geq c_i \geq 0$. With marginally decreasing utility functions, we derive $u_i(S) = p_i \ln(r_i + 1) - c_i(r_i + w_i)$. Without collisions we rewrite this utility function to:

$$u_i(S) = p_i \ln(r_i + 1) - c_i r_i. \tag{14}$$

To ensure that transmitting always results in a utility greater or equal to zero, it has to hold that $\frac{p_i \ln(r_i+1)}{r_i} \geq c_i \geq 0$.

In the subsequent sections, we consider the linear and the strictly concave utility functions to cover both the case of equal importance of information and the law of diminishing returns. We consider a single superframe, i.e., we play a static game.

5.3 NASH EQUILIBRIA OF THE STATIC GAME AND THEIR FAIRNESS

The analysis of games usually aims at identifying stable outcomes, the so-called Nash Equilibria. Furthermore, for sequential games, the term of a subgame-perfect equilibrium has been established to determine credible behaviour and thus, eliminates Nash Equilibria that pose incredible threats. First, we have to determine the Nash Equilibria for the proper subgames of a sequential game. In the case of perfect information, we then perform backward induction to determine whether it is reasonable for the players to actually arrive at those Nash Equilibria. Only those Nash Equilibria that are reasonable can be considered subgame-perfect.

In this section, we identify the Nash Equilibria and subgame-perfect equilibria of the described static game. Then we evaluate those Nash Equilibria with respect to Pareto- and social optimality as well as fairness. Pareto-optimality implies that no player can increase her utility without decreasing another player's utility, while in a socially-optimal Nash Equilibrium society's welfare is furthermore maximized. In order to determine the subgame-perfect equilibria of the game, we identify the proper subgames that are the entire game and the transmission subgame. With transmission subgame, we refer to stage $N+1$ of the game, in which all players simultaneously decide about their transmission. For this transmission subgame, we identify the following Nash Equilibria.

Lemma 5.1 *S^* is an NE in the transmission subgame, if each slot is used by at most one node and the number of used slots is the minimum of the number of available slots x_m and the aggregated number of slots required by the players. If we denote the number of slots required by player i as req_i, this is formally written as* $\min(\sum_{i \in \mathbf{N}} req_i, x_m)$.

In a Nash Equilibrium, there is no incentive for a player to unilaterally change her action. If every player is able to successfully transmit in as many slots as she requests, there is obviously no reason for any player to deviate and thus, it is stable. If players are not able to transmit as much as they request but all slots are taken by other players, they do not have an incentive to transmit more, since this would only induce costs but no additional gain. Formally, we write:

Proof We prove Lemma 5.1 by contradiction. Assume that more than one player transmits in a certain slot. Then each of those players would do better by unilaterally deviating from the current transmission pattern and refrain from transmitting in this multiple used slot in order to save the costs of the unsuccessful transmission. Hence, in an NE each slot can only be occupied by one player. Next, assume that the number of occupied slots is smaller than $\sum_{i \in \mathbf{N}} req_i$ and also smaller than x_m. Then each node with $x_i \leq req_i$ can do better by unilaterally deviating and additionally transmitting in an unoccupied slot. This concludes the proof. ∎

Subgame-perfect equilibria are those equilibria that induce Nash Equilibria in every subgame. The simultaneous transmission in stage $N+1$ is a proper subgame. If we extend this transmission subgame backwards to the reservation stages, we can characterize the subgame-perfect strategies. Thus, the subgame-perfect equilibria consist of the reservation and the transmission decision.

Lemma 5.2 *Let S^* be a pure strategy Nash Equilibrium in the transmission subgame. Then any feasible reservation vector $b = (b_1, b_2, \dots, b_n)$, with $b_i \in [s_i^*, x_m], \forall i \in \mathbf{N}$ forms an SPE $= [(b_1, s_1^*), \dots, (b_i, s_i^*), \dots, (b_n, s_n^*)]$.*

Proof If we consider the Nash Equilibria in the transmission subgame S^*, only reservations b_i that are in the interval $[s_i^*, x_m]$ are feasible, since player i shall not transmit more than she has reserved. In a static game, players cannot use the reservation phase in order to build up a reputation and manipulate the players' behaviour in the transmission subgame. Hence, any feasible reservation induces a Nash Equilibrium in the transmission subgame. ∎

In the following, we evaluate the identified Nash Equilibria regarding Pareto- and social-optimality as well as their fairness.

Lemma 5.3 *Any pure-strategy Nash Equilibrium in the game presented is Pareto-optimal.*

In a Pareto-optimal equilibrium, no player can increase her utility without decreasing another player's utility.

Proof We prove Lemma 5.3 by contradiction. Assume that there is an NE that is not Pareto-optimal. This implies that we can improve one player's utility without reducing another player's utility. There are two ways to increase a player's utility. First, we could allocate more slots to this particular player. According to Lemma 5.1, in an NE all slots are taken (or req_i is reached by each player i), so this is not possible. Second, we could reduce the collisions and therefore the costs the particular player experiences. According to Lemma 5.1 an NE does not contain colliding slots, so this is not possible either. Hence, we cannot increase a player's utility. This contradicts the assumption. Therefore, the Nash Equilibrium must be Pareto-optimal. ∎

Besides Pareto-optimality, another characteristic of a desirable Nash Equilibrium is social-optimality [26]. In a socially-optimal Nash Equilibrium, society's welfare as the sum of the utilities of all players in the game cannot be increased. Therefore, any socially-optimal Nash Equilibrium has to be Pareto-optimal but not vice-versa. In the game presented, socially-optimal Nash Equilibria are characterized as follows:

Lemma 5.4 *For symmetric players with linear utility functions all Pareto-optimal Nash Equilibria are socially-optimal. For symmetric players with monotonically increasing but marginally decreasing utility functions the Nash Equilibrium S^* is socially-optimal, if and only if $s_i^* = s_j^*, \forall\, i, j \in \mathbf{N}$.*

Generally, social-optimality does not consider individual players but focusses on the society as a whole. With linear utilities the aggregated utility is independent of the allocation of slots among the players. In the case of concave utility functions, though, the specific allocation is relevant. Consider two players, one player uses all slots and hence maximizes her utility, while the second player cannot transmit and has a utility of zero. With a concave utility function, player 1 yielding some slots to the second player reduces her utility, but increases player 2's utility by a larger amount. Hence, in sum society's welfare increased. The formal proof is given below.

Proof Social-optimality is given if $U = \sum_{i=1}^{N} u_i$ is maximized, with N the number of players. If we assume greedy players, any equilibrium that allocates all slots among the players without collisions, is Pareto-optimal. In a Nash Equilibrium there are no collisions, thus, $x_i = r_i$, so if we denote the available number of slots as x_m, then $x_N = x_m - \sum_{i=1}^{N-1} x_i$.

For linear utilities, $U = \sum_{i=1}^{N}(p_i - c_i)x_i$. To determine society's welfare we have to maximize U. Taking the first partial derivative for player i yields:

$$\begin{aligned}
\frac{\partial U}{\partial x_i} &= \frac{\partial}{\partial x_i}\left((p_1 - c_1)x_1 + (p_2 - c_2)x_2 + \ldots + (p_N - c_N)x_N\right) \\
&= \frac{\partial}{\partial x_i}((p_1 - c_1)x_1 + (p_2 - c_2)x_2 + \ldots + (p_N - c_N)(x_m - \sum_{j=1}^{N-1} x_j)) \\
&= (p_i - c_i) - (p_N - c_N).
\end{aligned} \tag{15}$$

Assuming symmetric players, thus $p_i = p_N$ and $c_i = c_N$, it holds that $\frac{\partial U}{\partial x_i} = 0$. Thus, for linear utility functions any distribution of slots among the players is considered socially-optimal. For marginally decreasing utility functions, U is given by $U = \sum_{i=1}^{N}(p_i \ln(x_i + 1) - c_i x_i)$. The first partial derivative for player i yields:

$$\begin{aligned}\frac{\partial U}{\partial x_i} &= \frac{\partial}{\partial x_i}(p_1 \ln(x_1+1) - c_1 x_1) + \ldots + p_N \ln(x_N+1) - c_N x_N) \\ &= \frac{\partial}{\partial x_i}(p_1 \ln(x_1+1) - c_1 x_1) + \ldots p_N \ln(x_m - \sum_{j=1}^{N-1} x_j + 1) - c_N(x_m - \sum_{j=1}^{N-1} x_j)) \\ &= \frac{p_i}{x_i+1} - c_i - \frac{p_N}{(x_m - \sum_{j=1}^{N-1} x_j) + 1} + c_N. \end{aligned} \tag{16}$$

If we again set the first derivative equal to zero and assume symmetric players, we obtain $x_i = x_m - \sum_{j=1}^{N-1} x_j$. Hence, in the socially-optimal case, all players receive the same share of resources, thus, $s_i^* = s_j^*, \forall i, j \in \mathbf{N}$. ∎

So far, we have evaluated the Nash Equilibria of the game regarding their optimality. In the following paragraphs, we identify the correlation of notions of fairness given in Section 2.3.3 with the determined Nash Equilibria for symmetric players.

Lemma 5.5 *For symmetric players the NE with $x_i = x^*, \forall\, i \in \mathbf{N}$ is max-min fair.*

Proof For symmetric players, $x_i = x^*, \forall\, i \in \mathbf{N}$ implies that $u_i = u_j, \forall\, i, j \in \mathbf{N}$. Therefore, player i can only increase her utility u_i by decreasing another player j's utility. Since player j's utility u_j is equal to her own utility u_i, it is max-min fair. According to [18], if a solution exists, it is unique. ∎

Lemma 5.6 *For symmetric players the NE with $x_i = x^*, \forall\, i \in \mathbf{N}$ is proportionally fair.*

Proof According to (4) in Definition 2.10, the allocation profile $x = (x_1, x_2, \ldots, x_N)$ is proportionally fair, if for any feasible allocation profile $x' = (x_1', x_2', \ldots, x_N')$ it holds that $\sum_{i\in\mathbf{N}} \frac{u_i' - u_i}{u_i} \leq 0$.

First, we consider players with linear utility functions, thus, $u_i = (p_i - c_i)x_i$. So we write $\sum_{i\in\mathbf{N}} \frac{(p_i-c_i)x_i' - (p_i-c_i)x_i}{(p_i-c_i)x_i} = \sum_{i\in\mathbf{N}} \frac{x_i' - x_i}{x_i} \leq 0$. In the proposed proportionally fair Nash Equilibrium x_{pf}, it holds that $x_i = x^*, \forall\, i \in \mathbf{N}$. Thus, $\sum_{i\in\mathbf{N}} \frac{x_i' - x_i}{x_i} = \frac{\sum_{i\in\mathbf{N}}(x_i' - x^*)}{x^*}$.

The allocation x' is a Nash Equilibrium, so $\sum_{i \in \mathbf{N}} x^* = \sum_{i \in \mathbf{N}} x'_i = x_m$. With this, $\frac{\sum_{i \in \mathbf{N}}(x'_i - x^*)}{x^*} = 0$, which proves that the Nash Equilibrium x_{pf} with $x_i = x^*, \forall\, i \in \mathbf{N}$ is proportionally fair. We further argue that this proportionally fair equilibrium is unique. Definition 2.10 states that (4) has to apply for any feasible x', thus, if there was another proportionally fair Nash Equilibrium x'', (4) would also have to hold for $x' = x_{\text{pf}}$, with $x_i = x^*, \forall\, i \in \mathbf{N}$. So, we write $\sum_{i \in \mathbf{N}} \frac{(x^* - x''_i)}{x''_i} \leq 0$. This, however, does not hold because the sum of the positive summands, i.e., if $x''_i < x^*$, is always larger than the negative summands, i.e., if $x''_i > x^*$. Thus x_{pf}, with $x_i = x^*, \forall\, i \in \mathbf{N}$, is the unique proportionally fair Nash Equilibrium for players with linear utilities.

Now, we consider players with concave utility functions $u_i = p_i \ln(x_i + 1) - c_i x_i$. According to [39], the proportionally fair solution for a logarithmic utility function is unique. In the equilibrium proposed, $x_i = x^*, \forall i \in \mathbf{N}$. For symmetric players, we further set $p_i = p$ and $c_i = c, \forall i \in \mathbf{N}$. So we have to show that:

$$\begin{aligned} \frac{\sum_{i \in \mathbf{N}} \left(p \ln(x'_i + 1) - c x'_i - p \ln(x^* + 1) + c x^*\right)}{p \ln(x^* + 1) - c x^*} &\leq 0 \\ \frac{\sum_{i \in \mathbf{N}} \left(p \ln(x'_i + 1) - c x'_i\right) - N(p \ln(x^* + 1) - c x^*)}{p \ln(x^* + 1) - c x^*} &\leq 0 \end{aligned} \tag{17}$$

Since, in Lemma 5.4 the Nash Equilibrium with $x_i = x^*, \forall i \in \mathbf{N}$ was shown to be the socially-optimal solution, $N(p \ln(x^* + 1) - c x^*) \geq \sum_{i \in \mathbf{N}} \left(p \ln(x'_i + 1) - c x'_i\right)$. Thus, the equilibrium with $x_i = x^*, \forall i \in \mathbf{N}$ is also the unique proportionally fair Nash Equilibrium for players with marginally decreasing utility functions. ∎

Finally, we find the Nash Equilibrium that maximizes Jain's fairness index.

Lemma 5.7 *For symmetric players, the Nash Equilibrium with $x_i = x^*, \forall\, i \in \mathbf{N}$ maximizes Jain's fairness index.*

Proof According to [38], the index is maximized if all players gain the same utility, so $u_i = u_j$, for all $i, j \in \mathbf{N}$. For symmetric players equal utilities $u_i(x_i) = u_j(x_j)$ are given, if $x_i = x_j, \forall\, i, j \in \mathbf{N}$. ∎

5.4 SYSTEM MODEL OF STATIC 2-PLAYER GAME WITH RELAXED RESERVATION

We have seen in the previous section that in the static game all Nash Equilibria are Pareto-optimal. In those stable allocations, it holds that no player can increase her utility without decreasing another player's utility. For linear utility functions, all Pareto-optimal Nash Equilibria are socially-optimal, hence, they optimize society's welfare. For monotonically increasing and marginally decreasing utility functions only the allocation given by $x_i = x^*, \forall\, i \in \mathbf{N}$ is socially-optimal. This allocation with equally shared resources is further max-min and proportionally fair and maximizes Jain's fairness index for both utility functions. So there is a unique allocation that complies with all properties.

Since we assumed that players are payoff-maximizers, there is only one Nash Equilibrium that occurs in this static game. It is characterized by player i's reservation $x_i = \min(req_i, x_m - \sum_{j<i} x_j)$, with $req_i \leq x_m$. Hence, the first node will use as many slots as possible, limited only by the maximum number of slots she requires and the maximum number of slots available for data transmission x_m, so $x_1 = \min(req_i, x_m)$. All subsequent nodes have to cope with the remaining slots. As we have seen before, this Pareto-optimal allocation is socially-optimal for linear but not for concave utility functions. Marginally decreasing utility functions, however, much better represent the application's view and requirements in a network. For this reason, the remainder of this work is dedicated to identifying a reservation mechanism that enables players to achieve the fair Nash Equilibrium in a game with concave utility functions.

The current section describes the modifications of the reservation procedure to overcome the unfairness for marginally decreasing utility functions that has been identified in the previous section. The variation presented provides discriminated players with a means to increase their share of resources and thus, improves the fairness in the network. So far, it has not been possible for players to reserve slots that have already been taken by another player. With the modification, however, players are allowed to doubly reserve another player's slots to a certain extent. By this means, the unfair slot allocation is altered to increase the fairness.

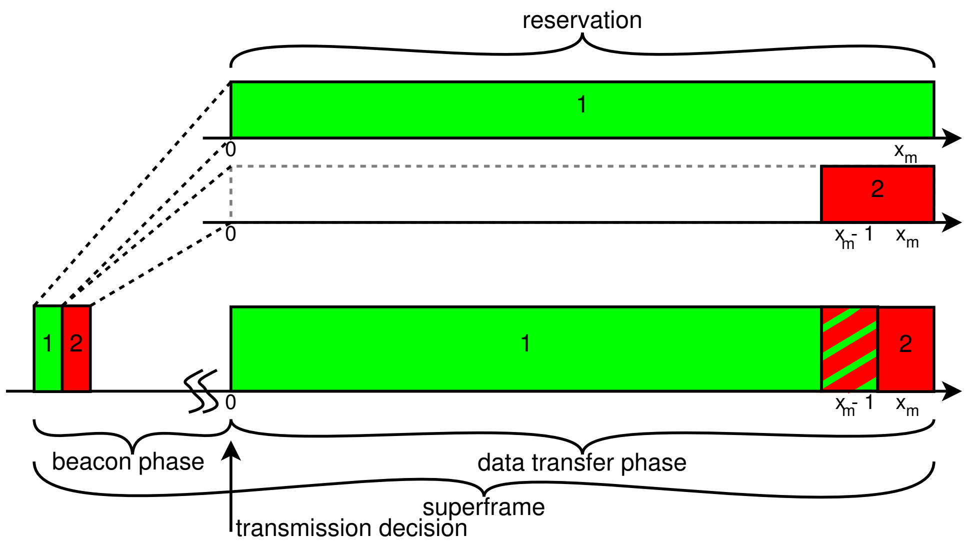

Figure 13: Superframe with beacon phase and data transfer phase. Order of nodes in the beacon phase is the same as in the data transfer phase. Node 1 reserves x_m slots, while node 2 doubly reserves the last two slots. In the data transfer phase, node 1 backs off from the last slot, but not from the second last. Hence, player 2 successfully transmits in one slot and a collision occurs in the second last slot of the data transfer phase.

Without loss of generality, we still demand the reserved slots of a node to be consecutive and require them as early as possible in the data transfer phase. Thus, the order of the nodes in the beacon phase is preserved in the data transfer phase. Figure 13 depicts a superframe with two active nodes. Here, node 1 reserves the entire data transfer phase as illustrated in the reservation element of beacon 1, thus leaving no exclusive transmission time for node 2.

The relaxed reservation mechanism that we propose in this section, overcomes this unequal allocation. It provides players with the opportunity to doubly reserve up to y_{max} slots of their directly previous node's reservation. Recall that we assumed consecutive slots. Thus, with $y_{max} = 2$ in Figure 13, for instance, node 2 can overlap node 1's last and second last slot. If she decides for it, the reservation element in beacon 2 contains the last and the second last slot of the data transfer phase.

Doubly reserving a slot during reservation is interpreted as a threat to generate a collision in those slots during the data transfer phase. Therefore, three scenarios are possible in the data transfer phase, if a slot has been doubly reserved by node 2 in the beacon phase:

1. Both nodes back off from the specific slot, thus, the slot is idle during data transmission.
2. One node backs off from the specific slot, thus, the other node successfully transmits in this slot.
3. Neither node backs off from the specific slot, thus, both nodes transmit in this slot and a collision occurs.

In Figure 13, two slots have been doubly reserved. At the beginning of the data transfer phase, both nodes make a transmission decision. In this example, node 1 backs off from the doubly reserved slot x_m, whereas node 2 does not back off at all. As a consequence, a collision occurs in slot $x_m - 1$, but no collision occurs in slot x_m. Hence, node 1 successfully transmits in $x_m - 2$ slots, while node 2 successfully transmits in one slot, i.e., in slot x_m of the data transfer phase. Thus, the slot allocation is slightly shifted towards better fairness.

In the proposed relaxed reservation mechanism, slot reservations are taken distributively by the nodes in the network according to the maximization of their individual utility function. A node's decision, however, strongly depends on the behaviour of the contending nodes, since the nodes' throughput and thus, their utility, is degraded by collisions. For this reason, we study the performance of the relaxed reservation mechanism using game-theoretic analysis, which is a proven remedy to modelling such strategic interactions.

5.5 GAME MODEL OF STATIC 2-PLAYER GAME WITH RELAXED RESERVATION

In this section, we introduce a 2-player Bayesian multi-stage game to model the relaxed reservation mechanism. The multi-stage game is comparable to the model presented in Section 5.2. In particular, the 2-player game consists of three stages; in the first two stages, players play sequentially, while in the last stage, players play simultaneously. However, it further includes a Bayesian component that reflects the imperfect information about the players' behaviour in matters of the double reservations presented.

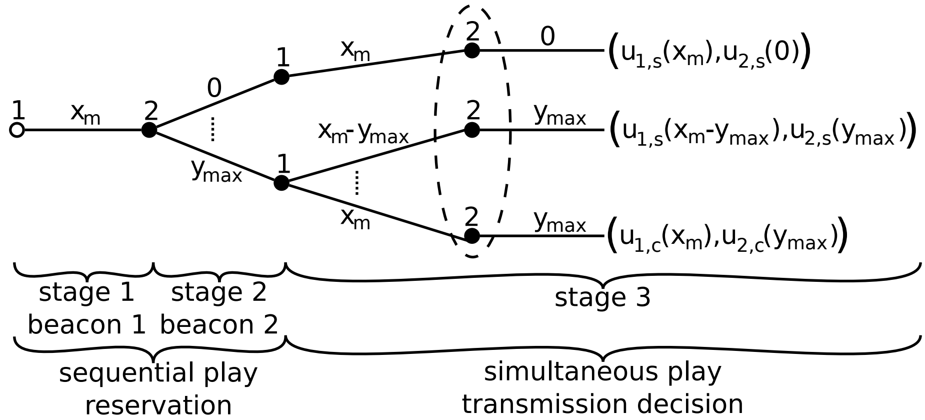

Figure 14: Decision tree of the 2-player multi-stage game. In stage 1, player 1 reserves x_m slots. In stage 2, player 2 announces her reservation and chooses between zero and y_{max} slots. Stage 3 is simultaneous play with both players deciding about whether or not to retreat from challenged slots depending on their utilities.

The players of the games are the nodes associated with the network, which are considered to be greedy. This means that they transmit in the entire data transfer phase, if they have the opportunity to do so. The players' strategies are the number of slots they reserve and transmit in, their utility functions include the expected throughput and transmission costs.

Figure 14 depicts the decision tree of the 2-player multi-stage game. In the sequential play at the beginning, players announce their reservations in their corresponding beacons. In stage 1, the greedy player 1 initially reserves the entire data phase, i.e., x_m slots. In opposition to the model in Section 5.2, though, player 2 is henceforth able to reserve up to y_{max} slots of player 1's reservation. Thus, in stage 2, player 2 doubly reserves an amount of slots in the interval $[0, y_{max}]$. If not stated otherwise, we assume $y_{max} = 1$. In stage 3, players make a transmission decision. If there are doubly reserved slots, both players concurrently decide how many of those doubly reserved slots they back off from.

Recall that in non-cooperative games, players are assumed to play a strategy that maximizes their utility. Player 1's strategy contains two elements. The first element is the number of slots she reserves in stage 1. While she initially reserves x_m slots, in the subsequent superframes, we assume that she reserves those slots that she used for transmission in the previous superframe. Recall Figure 13, player 1 backs off from her last slot, thus, her next reservation will not contain this slot anymore. This means that we assume that a player does not reclaim slots she has backed off from.

The second element of her strategy concerns her transmission decision. Essentially, a player transmits in a doubly reserved slot, if this increases her expected utility. Player 1 maintains an estimate of the probability that player 2 transmits in a doubly reserved slot. In superframe t this estimate is referred to as $\mu_{1,t}$. Given $\mu_{1,t}$, player 1 determines her expected utility in superframe t, i.e., $E[u_{1,t}(x_1)]$, from transmitting in x_1 slots including the doubly reserved slot. On the other hand, the deterministic utility that player 1 receives, if she does not transmit in the doubly reserved slot but successfully transmits in the remaining $x_1 - 1$ slots, is denoted as $u_{1,s}(x_1 - 1)$. Hence, if $E[u_{1,t}(x_1)] \geq u_{1,s}(x_1 - 1)$, she does not back off.

Player 2's strategy is inversely composed. The first element of her strategy concerns her reservation decision in stage 2. It contains the determination whether a doubly reserved slot yields a larger utility than no double reservation. Hence, player 2 maintains an estimate about player 1's transmission probability $\mu_{2,t}$. In superframe t, she determines her expected utility $E[u_{2,t}(x_2)]$ from transmitting in x_2 slots including a doubly reserved slot given her estimate $\mu_{2,t}$. If $E[u_{2,t}(x_2)]$ is larger than her utility from not transmitting in a double reserved slot, i.e., $E[u_{2,t}(x_2)] \geq u_{2,s}(x_2 - 1)$, she doubly reserves player 1's last slot.

The second element of player 2's strategy concerns her actual transmission. Since the basis for her reservation decision has not changed from stage 2 to stage 3, we assume that she follows through with her reservation. Hence, if in stage 2, player 2 doubly reserves a slot, in stage 3, she will decide to transmit in this slot. However, her opponent, player 1, is not aware of this deterministic behaviour.

For symmetric players the determination of their expected utilities in superframe t denoted by $E[u_{i,t}(x_i)]$ for $i \in [1, 2]$ from transmitting in a doubly reserved slot is analogous and given as:

$$E[u_{i,t}(x_i)] = \mu_{i,t} u_{i,c}(x_i) + (1 - \mu_{i,t}) u_{i,s}(x_i). \tag{18}$$

Here, $u_{i,c}(x_i)$ refers to player i's utility from transmitting in x_i slots, of which one slot collides and $u_{i,s}(x_i)$ denotes player i's utility from successfully transmitting in x_i slots. Hence, player 1 transmits in a doubly reserved slot and player 2 doubly reserves a slot, if and only if:

$$E[u_{i,t}(x_i)] \geq u_{i,s}(x_i - 1)$$
$$\mu_{i,t} u_{i,c}(x_i) + (1 - \mu_{i,t}) u_{i,s}(x_i) \geq u_{i,s}(x_i - 1) \tag{19}$$

Recall that in Section 5.2, we introduced two utility functions. Here, we focus on the strictly concave utility function (14). So the utility from successfully transmitting in x_i slots is given as $u_{i,s}(x_i) = p_i \ln(x_i + 1) - c_i x_i$. The utility from transmitting in x_i slots, of which one slot collided, is given as $u_{i,c}(x_i) = p_i \ln(x_i) - c_i x_i$. Hence, if we reformulate (19) and assume symmetric players, player i transmits in the doubly reserved slot in superframe t, if and only if:

$$\mu_{i,t} \leq 1 - \frac{c}{p \ln(\frac{x_i+1}{x_i})}. \tag{20}$$

In the next section, we identify the Bayesian Nash Equilibria of the static game.

5.6 BAYESIAN NASH EQUILIBRIA

For the static game, we identify the Bayesian Nash Equilibria, in which all players play their mutually best responses conditional to their beliefs [31, p. 215]. In those equilibria, players are indifferent about transmitting in a doubly reserved slot, i.e., risking a collision, and backing off from it. With the utility functions described in Section 5.2, the Bayesian Nash Equilibria are given as the equilibrium of (20):

$$\mu_i^*(x_i) = 1 - \frac{c}{p \ln(\frac{x_i+1}{x_i})}. \tag{21}$$

As with the Nash Equilibria and subgame-perfect equilibria, a Bayesian game can have several Bayesian Nash Equilibria. We define a Bayesian Nash Equilibrium to be desirable, if it has certain properties.

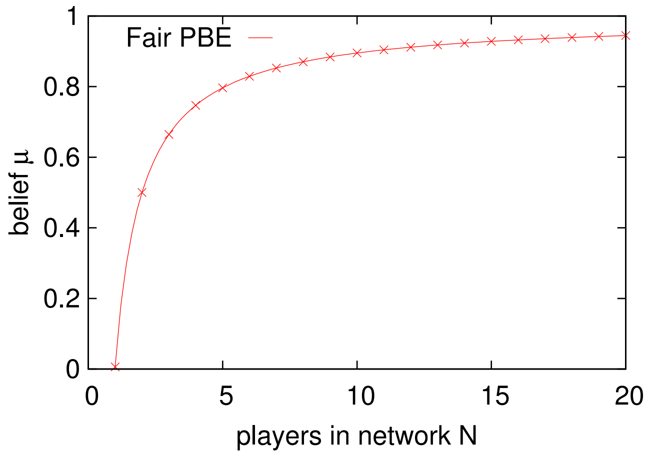

Figure 15: Depending on the number of players, the graph denotes the belief players have about their opponents' transmission probability in the fair Bayesian Nash Equilibrium according to (21) with $c = 1$ and $p = x_m + 1$. Note that in the fair Bayesian Nash Equilibrium $x_i = x_m/N$ for all $i \in \mathbf{N}$.

Definition 5.1 A Bayesian Nash Equilibrium is desirable, if it is Pareto-optimal, socially-optimal and fair.

In the desirable Bayesian Nash Equilibrium for marginally decreasing utility functions, it holds that $x_i = \frac{x_m}{N}, \forall\, i \in \mathbf{N}$. Figure 15 shows the belief in the desirable Bayesian Nash Equilibrium depending on the number of players for such a concave utility function. We assume greedy players, so player i's utility has to be maximal for $x_i = x_m$. If we calculate the maximum of $u_i(S) = p_i \ln(x_i + 1) - c_i x_i$ in the interval of feasible allocations $x_i \in [0, x_m]$ and set $x_i = x_m$, we obtain $p = c(x_m + 1)$. To ease calculation, we choose $c = 1$ and hence $p = c(x_m + 1) = x_m + 1$. We observe that the larger the number of nodes in the network, the larger the players' beliefs in the equilibrium. Thus, in order to play the Bayesian Nash Equilibrium of the game, players need to know the number of nodes in the network to determine their equilibrium beliefs given their share of resources.

5.7 SUMMARY

In this section, we have presented a game model of a distributed reservation protocol with fixed beacon order. Since we have shown in the previous chapter that for such a protocol a fairness problem arises when the traffic load is beyond saturation, we assumed players to be greedy.

We introduced two kinds of utility functions and identified the Nash Equilibria, which are all Pareto-optimal and the subgame-perfect equilibria of the game. For linear utility functions we showed that all Nash Equilibria are further socially-optimal. For monotonically increasing but marginally decreasing utility functions, only the Nash Equilibrium that is characterized by $x_i = x^*, \forall\, i \in \mathbf{N}$ is socially-optimal. This equilibrium with resources equally distributed among the players further complies with the definitions for max-min and proportional fairness and maximizes Jain's fairness index for both utility functions.

Assuming that greedy players are not altruistic this fair allocation does not occur by itself. Considering rational players, it is an unfair Nash Equilibrium that arises. Therefore, we presented a relaxed reservation mechanism to rearrange the slots to increase the fairness of the allocation. With this mechanism discriminated players are given a means to enhance their position. To implement this mechanism, players have to maintain a belief about their opponents' behaviour, so we finally determined the Bayesian Nash Equilibria of the static 2-player game. In the next chapter, we extend this game to a repeated game and determine the corresponding Perfect Bayesian Nash Equilibria.

6

DYNAMIC GAME OF DISTRIBUTED RESERVATION PROTOCOL

So far we have accounted for a one-shot game. In order to play the desired equilibrium of a game, though, all players have to correctly determine this equilibrium, which can be a tedious task. However, if we consider a repeated game, we are able to introduce algorithms that can be analysed in terms of their convergence towards the desired Nash Equilibrium.

In this chapter, we model a dynamic 2-player game that is a repeated version of the static 2-player game of the previous chapter. For the dynamic game, we find a function that relates a player's belief to her share of slots. This function marks the points at which a player is indifferent about transmitting and not transmitting in a challenged slot. Thus, this function represents the candidates of a Perfect Bayesian Nash Equilibrium. We further introduce a belief update algorithm. Recall that players maintain beliefs about their opponent's transmission behaviour. In a dynamic game, players observe their opponent's actions in every period of the game and update their beliefs. This belief in return affects the player's own action in the subsequent period. Having determined the candidates of Perfect Bayesian Nash Equilibria of the game, we evaluate which of those points are stable or quasi-stationary. For each set of initial beliefs and discount factor, we identify exactly one allocation of slots $x^* = (x_{1,t^*}, x_{2,t^*})$ that is stable or quasi-stationary. Besides their stability we evaluate those points in terms of fairness. For a stable point to be a Perfect Bayesian Nash Equilibrium of the dynamic game, the applied belief update algorithm has to drive the game to this stable point. Thus, we determine the initial conditions by which the game emerges to the stable point, hence, the Perfect Bayesian Nash Equilibrium. We show that for players with equal initial conditions, the game converges to the Perfect Bayesian Nash Equilibrium which is in fact fair. Simulations further suggest that in networks with more than two players the relaxed reservation algorithm also yields a fair allocation of resources if players have equal initial beliefs. Finally, simulations reveal that the convergence time scales linearly with network size. The convergence time decreases with the parameter of the relaxed reservation algorithm $y_{\max}$ and increases with the discount factor that reflects how much the players value the future.

6.1 GAME MODEL OF DYNAMIC 2-PLAYER GAME WITH RELAXED RESERVATION

In this section, we repeat the static game an infinite number of times. The t-th repetition corresponds to superframe t and is referred to as period t of the dynamic game. With the relaxed reservation procedure, we model this game inversely to Selten's chain store model (cf. Section 2.4). We derive the expected utilities for a player to transmit and not to transmit in a challenged slot. With this, we find a function for which a player is indifferent about transmitting and not transmitting. This function relates a player's belief about her opponent's behaviour to the amount of slots she transmits in and denotes candidates for Perfect Bayesian Nash Equilibria of the dynamic game.

In Selten's chain store model, the author analyses a situation with a single incumbent that is faced with several possible entrants that sequentially decide about their entry. So the incumbent faces a different player in each round. In our model, the incumbent player 1 faces the same possible entrant player 2 in every period. Selten showed in [61] that there is only one possible outcome for such a game: all entrants enter, since an incumbent is better off accommodating than fighting, when she faces entry. This result, though rational, does not seem very intuitive. In [53] and [44], the authors extend the chain store model to a game with incomplete information and model it as a game with imperfect information to reflect that in reality fighting can be a rational strategy for the incumbent to deter entry.

In the game presented in this chapter, the imperfectness regards the knowledge about the other player's payoffs. We consider two-sighted uncertainty. On the one hand, the entrant maintains an estimate about the probability that her entry, i.e., her double reservation, is met by a transmitting incumbent. On the other hand, the incumbent maintains an estimate about the probability that the entrant follows through with her threat of generating a collision.

Note the following: the potential entrant has made her final decision during the reservation, which is the entry phase in Selten's model. If she has decided to doubly reserve slots of her opponent's reservation, we assume that she will follow through with this and transmit in those slots. This knowledge, however, is not available to her opponent. The incumbent cannot deduce from a double reservation that her opponent will transmit in the corresponding slots. Thus player 1 considers her belief to be an estimate of whether or not player 2 follows through with a double reservation. In retrospective, however, this estimate rather reflects the probability that even though she has met a double reservation with transmitting and therefore a collision occurred, her opponent tries again in the next period.

In repeated games, players account for future utilities. When a player decides about her current action, she takes into consideration the expected effect of her behaviour on the other player's future behaviour and how this in return influences her own future utilities. Those future utilities are discounted with the common discount factor $\delta \in [0, 1)$. For our analysis of the dynamic game, we consider two types of players: long-sighted and myopic players. The more long-sighted players are, the more the future plays a role in their decision, hence, the larger δ is. Myopic players constitute the degenerated case of $\delta = 0$. They play a best response to the expected behaviour of their opponents. This decision process does not consider the impact on future reservations or transmissions. Thus, they solely take into account their current belief about their opponent's behaviour and their expected payoff.

In the static game presented in Chapter 5, a player transmits in a doubly reserved slot, if and only if her expected utility is larger or equal to the deterministic utility she receives if she does not transmit in the challenged slot (19). The left-hand side of (19), i.e., $E[u_{i,t}(x_i)]$, represents the expected utility she receives, if player i decides to transmit in a doubly reserved slot and hence, risks a collision. Recall that in the dynamic game, we repeat the static game an infinite number of times. Thus, to reformulate $E[u_{i,t}(x_i)]$ in (19) to account for future utilities, we have to make assumptions about future actions. For ease calculation of the expected utilities, we assume that if a collision occurs, the challenged player backs off from the collided slot in all subsequent periods and is not challenged again. Thus, in the case of a collision, player i with $i \in [1,2]$ receives a period-0 utility of $u_{i,c}(x_i)$. Every additional period τ adds the discounted utility $\delta^\tau u_{i,s}(x_i - 1)$. In total this sums up to $u_{i,c}(x_i) + u_{i,s}(x_i - 1)(\delta + \delta^2 + \ldots)$. However, if despite a doubly reserved slot no collision occurs, she receives a discounted utility of $\delta^\tau u_{i,s}(x_i)$ for every period $\tau \in [0, \infty)$ assuming that no further challenges occur. Thus, $E[u_{i,t}(x_i)]$ becomes:

$$\begin{aligned} E[u_{i,t}(x_i)] = \mu_{i,t} &\left(u_{i,c}(x_i) + u_{i,s}(x_i - 1)(\delta + \delta^2 + \ldots)\right) \\ &+ (1 - \mu_{i,t}) \left(u_{i,s}(x_i)(1 + \delta + \delta^2 + \ldots)\right) \end{aligned} \tag{22}$$

The right-hand side of (19) represents the deterministic utility player i receives, if she decides from the beginning to back off from the doubly reserved slot, thus no collision can occur. In this case, recall our assumption that she will not reserve this slot again. Assuming that no further challenges occur, her discounted utility from backing off is given as $u_{i,s}(x_i - 1)(1 + \delta + \delta^2 + \ldots)$. If we replace the geometric series with their closed forms, we obtain that player i transmits in a doubly reserved slot, if and only if:

$$\begin{aligned} E[u_{i,t}(x_i)] = \mu_{i,t} \left(u_{i,c}(x_i) + \frac{\delta}{1-\delta} u_{i,s}(x_i - 1)\right) + (1 - \mu_{i,t}) \left(\frac{1}{1-\delta} u_{i,s}(x_i)\right) \\ \geq \frac{1}{1-\delta} u_{i,s}(x_i - 1). \end{aligned} \tag{23}$$

With the utility function given by (14), we reformulate (23), so that player i transmits in a doubly reserved slot, if and only if for player i's belief in superframe t it holds that:

$$\mu_{i,t} \leq \frac{p \ln(\frac{x_i+1}{x_i}) - c}{p \ln(\frac{x_i+1}{x_i}) - \delta c}, \text{ for } i \in [1,2], \tag{24}$$

with p the prize a player gains for a successful transmission, c the transmission cost, δ the discount factor and x_i the number of slots player i transmits in.

As we have done for the previously defined games, we aim to identify the Nash Equilibria of the game. For repeated multi-stage games with imperfect information the equivalent of a Nash Equilibrium is denoted a Perfect Bayesian Nash Equilibrium as defined in Definition 2.6. According to [31, p. 326], a Perfect Bayesian Nash Equilibrium is "a set of strategies and beliefs such that, at any stage of the game, strategies are optimal given the beliefs, and the beliefs are obtained from equilibrium strategies and observed using Bayes' rule". So, if we take the equilibrium condition of (24), we identify candidates for the belief in the Perfect Bayesian Nash Equilibria:

$$\mu_i^*(x_{i,t}) = \frac{p\ln(\frac{x_{i,t}+1}{x_{i,t}}) - c}{p\ln(\frac{x_{i,t}+1}{x_{i,t}}) - \delta c}, \text{ for } i \in [1,2], \tag{25}$$

In the course of the game, players learn from their opponents' behaviour, regularly update their beliefs and adapt their reservations accordingly. In order to complete the requirements of a Perfect Bayesian Nash Equilibrium, Section 6.2 introduces a belief update algorithm that complies with Bayes' rule. It describes a decentralised belief update algorithm that iteratively adapts players' reservations to the outcome of previous superframes. Although each player greedily maximizes her individual utility, this algorithm introduces self-enforcing fairness in the network.

In the following section, we explain the belief update algorithm and how it relates to the stages of the game. Then we describe how the belief and the corresponding slot allocation evolves in the course of the game. In the current section, we have identified candidates for Perfect Bayesian Nash Equilibria, given by (25). In Section 6.3, we show which of these slot allocations is the stable one given the players' initial beliefs. In Section 6.4, we show for which discount factor δ and initial beliefs the stable allocation is Pareto- and socially-optimal as well as fair. Further, we generalize the result and determine the dependence of fairness on the discount factor and the initial beliefs. In Section 6.5, we show that with the belief update algorithm presented the game converges to a stable or quasi-stationary allocation. Finally, Section 6.6 gives simulation results for the N-player game and evaluates the impact of varying the maximum number of overlaps y_{max} and the discount factor δ on the convergence time.

6.2 BELIEF UPDATE ALGORITHM

In this section, we describe how the beliefs impact the players' reservation and transmission decisions for each superframe t. So given a player's belief, we illustrate how many slots this player reserves and transmits in. Furthermore, we explain the events of the game that affect the belief in return. At the end of a superframe, players have observed their opponent's behaviour in the last period of the game and update their beliefs accordingly. With this, we illustrate the temporal evolution of the beliefs and the corresponding slot allocations in the course of the game.

For the static game, we introduced the beliefs that players maintain in the course of the game. So in the 2-player game, player 1's belief $\mu_{1,t}$ is her a-priori estimate in superframe t that player 2 follows through, when she has challenged player 1's slots. Player 2's belief $\mu_{2,t}$ is her a-priori estimate in superframe t that player 1 transmits, when player 2 challenges her slots. The number of slots player i transmitted in during superframe $t-1$ is given as $x_{i,t-1}$. In the following, we consider superframe t and relate the elements of the algorithm to the three stages of the game, i.e., beacon slot 1, beacon slot 2 and the transmission decision:

- Stage 1: This stage corresponds to beacon slot 1, in which player 1 decides on her reservation, i.e., she reserves the uncollided slots of her last transmission $x_{1,t-1}$ and decides whether to reserve a collided slot again.
- Stage 2. This stage corresponds to beacon slot 2, in which player 2 decides on her reservation. Basis for this decision is the number of uncollided slots of player 2's transmission in superframe $t-1$ and her current belief $\mu_{2,t}$. Given this, player 2 doubly reserves the last slot of player 1's reservation, if this increases her expected utility $E[u_{2,t}(x_2)]$ according to (23).
- Stage 3: This stage corresponds to the players' transmission decisions. Now, players have full knowledge about any double reservations, which influences their transmission decisions.
  - Player 1: If there is a doubly reserved slot, she keeps this slot, if this increases her expected utility according to (23) given $\mu_{1,t}$.

– Player 2: Her transmission decision always equals her reservation decision, since the foundation for her decision has not changed since then.

So far, we have assumed that players have an a-priori belief. In the following, we address this belief and explain how it is determined. At the beginning of a superframe, players face uncertainty of whether or not they will see a collision in this next superframe. For Bayesian inference such as requested here, the distribution of the prior probability is often modelled by a beta distribution [4].

In general, for some parameter α and β the beta density for a random variable z is proportional to $z^{\alpha-1}(1-z)^{\beta-1}$. Its mean is given by $E[Z] = \frac{\alpha}{\alpha+\beta}$. In our game, we interpret α and β in terms of the number of superframes with collisions Φ and the number of superframes without collisions Ψ, respectively, and write $E[Z] = \mu$. So players keep counters for the number of superframes with and without collisions. At the end of superframe t, players know whether or not there has been a collision in the current superframe. If there was a collision, $\Phi_{i,t} = \Phi_{i,t-1} + 1$ and $\Psi_{i,t} = \Psi_{i,t-1}$, else $\Phi_{i,t} = \Phi_{i,t-1}$ and $\Psi_{i,t} = \Psi_{i,t-1} + 1$. Note that $\Phi_{i,0} = \phi_{i,0}$ and $\Psi_{i,0} = \psi_{i,0}$ are initial values for the parameters of the beta distribution to model the uncertainty before there are any observations.

For the reservation and transmission decisions in superframe t, the belief is determined by the events in the superframes up to $t-1$. So the prior belief for superframe t is given by:

$$E[Z_{i,t}] = \mu_{i,t} = \frac{\Phi_{i,t-1}}{\Phi_{i,t-1} + \Psi_{i,t-1}}. \tag{26}$$

At the end of superframe t, the posterior belief has to be determined, hence, the value for μ is updated. Since the beta distribution is self-conjugate[1], the posterior distribution is also a beta distribution [42], whereas the Φ is incremented in the case of a collision and the Ψ in the case there was no collision as explained above. Hence, player i's posterior belief in superframe t about player j's probability to transmit in a doubly reserved slot is given as the ratio of the number of superframes with

1 Prior and posterior are called conjugate distributions, if the posterior distribution is of the same family as the prior distribution.

collisions to the total number of superframes. This posterior belief of superframe t becomes the a-priori belief of superframe $t+1$ (27):

$$\mu_{i,t+1} = \frac{\Phi_{i,t}}{\Phi_{i,t} + \Psi_{i,t}}. \tag{27}$$

We split $\Phi_{i,t}$ into the initial value $\phi_{i,0}$ and the number of observed collisions in the course of the game $\phi_{i,t}$ and write $\Phi_{i,t} = \phi_{i,0} + \phi_{i,t}$. The same holds for $\Psi_{i,t} = \psi_{i,0} + \psi_{i,t}$. In the game presented, we assume perfect observation, i.e., both players interpret collisions correctly as collisions and successful transmissions as such, hence, there is no false detection rate. This implies that both players observe the same number of collisions, hence, we set $\phi_{1,t} = \phi_{2,t} = \phi_t$ and $\psi_{1,t} = \psi_{2,t} = \psi_t$. With this, player i's prior belief of superframe $t+1$ (27) becomes:

$$\mu_{i,t+1} = \frac{\phi_{i,0} + \phi_t}{\phi_{i,0} + \psi_{i,0} + \phi_t + \psi_t}. \tag{28}$$

So far, we have explained how the belief influences the reservation and transmission decisions in a single superframe and how those decisions in return affect the posterior belief. Next, we extend the analysis to the temporal evolution of the players' beliefs. The temporal evolution of the players' beliefs and the corresponding slot allocations are depicted in Figure 16. The x-axes represent the number of slots that the depicted player holds normalized by x_m. The y-axes give the players' current beliefs about their opponents' probability to transmit in a doubly reserved slot. The convex curve $\mu_i^*(x_{i,t})$ given by (25), here with $\delta = 0.5$, marks the points in which a player receives the same utility whether or not she transmits in a doubly reserved slot. A point located above $\mu_i^*(x_{i,t})$ refers to a situation, in which the player is not willing to doubly reserve or to transmit in a doubly reserved slot. If in Figure 16 (left) a point is located below $\mu_1^*(x_{1,t})$, player 1 will transmit if challenged by player 2. Analogously, player 2 will challenge player 1 if the current point in Figure 16 (right) is below $\mu_2^*(x_{2,t})$.

In the example in Figure 16, initial beliefs are set to $\mu_{i,0} = 0.5$. Since players are greedy, at the beginning of the game player 1 reserves all available slots x_m, thus her trajectory starts at (1,0.5). With no slots left for player 2, her belief is below $\mu_2^*(x_{2,t})$,

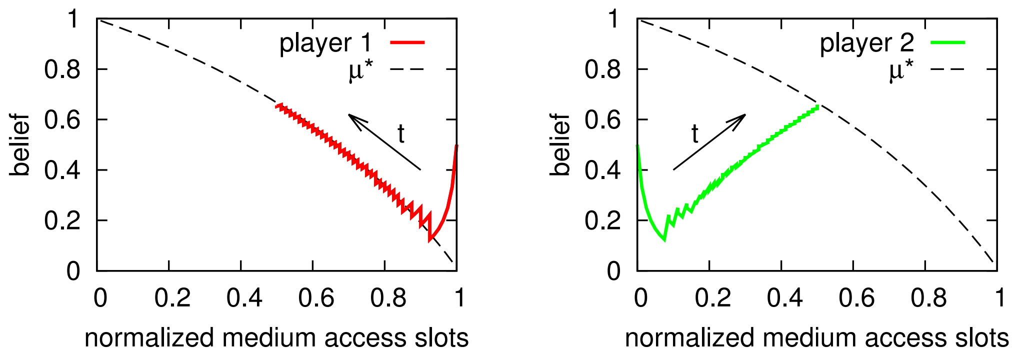

Figure 16: Temporal evolution of the players' beliefs and slot shares given the initial beliefs $\mu_{1,0} = \mu_{2,0} = 0.5$ and the discount factor $\delta = 0.5$. Player 1 (left) retreats from challenged slots as long as her trajectory is above $\mu_1^*(x_{1,t})$ and fights if below. Player 2 (right) challenges player 1's slots as long as her belief is below $\mu_2^*(x_{2,t})$. She gains slots when player 1 retreats.

thus she will doubly reserve player 1's last slot. Player 1 backs off until her trajectory first intersects with $\mu_1^*(x_{1,t})$. Note that if player 1 backs off, there is no collision, hence, the players' beliefs decrease. Simultaneously, every slot that player 1 loses because she is backing off, is gained by player 2. In this example, players started with equal initial beliefs, thus the trajectories are symmetric. At the time player 1's trajectory is below $\mu_1^*(x_{1,t})$, she starts to transmit in a doubly reserved slot, hence, collisions occur. This results in an increase of the beliefs with no change in the slot allocation, i.e., the vertical sections of the trajectories. If player 1's trajectory is above $\mu_1^*(x_{1,t})$ again, the process of backing off followed by collisions re-occurs. So as long as player 2 is challenging player 1, i.e., she is doubly reserving player 1's slots, player 1 either backs off and loses the slot or transmits and a collision occurs. Hence, as long as player 2 is challenging player 1, the number of slots player 2 transmits in is equal to the number of superframes without collisions plus the next challenged slot, i.e., $x_{2,t} = \psi_t + 1$, for $t \leq t^*$ with $t^* = \min\{t : \mu_{2,t+1} \geq \mu_2^*(x_{2,t})\}$.

Figure 16 suggests that the temporal evolution of the players' beliefs is deterministic. So every time players choose their initial beliefs to be $\mu_{1,0} = \mu_{2,0} = 0.5$, the trajectory looks as depicted in the graphs of Figure 16. However, when a player chooses her initial belief, she has no information about her opponent's choice. Thus, she can only estimate which of the possible trajectories actually arises in the game.

In Section 6.1, we have identified candidates for Perfect Bayesian Nash Equilibria given by (25) that are the convex curves in Figure 16. Here, we presented a belief update algorithm that follows Bayes' rule. In the next section, we merge those two findings and determine the actual Perfect Bayesian Nash Equilibrium of the game. It is characterized by a combination of beliefs and slot allocation that results in a stable slot distribution.

6.3 PERFECT BAYESIAN NASH EQUILIBRIUM

With (25) of Section 6.1, we have identified candidates for Perfect Bayesian Nash Equilibria of the dynamic game. In Section 6.2, we have further introduced a belief update algorithm that follows Bayes' rule. In this section, we select those candidates that are stable or quasi-stationary, once reached.

A slot allocation is considered stable, if it does not change over time. Thus, once the stable slot allocation is reached, the update of the players' beliefs must not induce the players to transmit in a different number of slots than the equilibrium allocation. A slot allocation is considered to be quasi-stationary, if it does not change for ν sequences of the game.

In this section, we identify two ranges for the players' initial beliefs. For the initial beliefs $\mu_{2,0} \leq \mu_{1,0}$, we show that the slot allocation (x_{1,t^*}, x_{2,t^*}) is stable and hence, marks the Perfect Bayesian Nash Equilibrium of the game. For $\mu_{2,0} > \mu_{1,0}$, the respective slot allocation is quasi-stationary, i.e., changes after ν sequences.

Theorem 6.1 *The slot allocation (x_{1,t^*}, x_{2,t^*}) in superframe t^*, with $t^* = \min\{t \geq t' : \mu_{2,t+1} \geq \mu_2^*(x_{2,t})\}$, where $t' = \min\{t : \mu_{1,t+1} < \mu_1^*(x_{1,t})\}$, and $t^* = \phi_{t^*} + \psi_{t^*}$:*

1. *is stable and thus, is the Perfect Bayesian Nash Equilibrium of the game, if it holds for the players' initial beliefs that $\mu_{2,0} \leq \mu_{1,0}$ and*

2. *is quasi-stationary, i.e., the slot allocation does not change for $\nu(1 + \Delta t)$ superframes, with finite ν that depends on c, p, δ, x_m, x_{2,t^*} and $\Delta t = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$, for the initial beliefs $\mu_{2,0} > \mu_{1,0}$.*

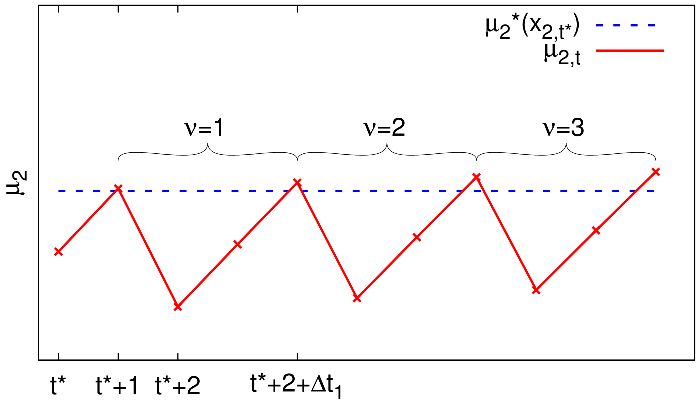

Figure 17: Player 2's belief $\mu_{2,t}$ over time beyond the equilibrium at $t = t^*$. If her belief is below $\mu_2^*(x_{2,t^*})$, she challenges and collisions occur, which results in an increase of her belief. Once her belief is above $\mu_2^*(x_{2,t^*})$, no collisions occur, thus, her belief decreases. Note that time is discrete and lines are only drawn for clarity.

In order to prove that the slot allocation (x_{1,t^*}, x_{2,t^*}) is stable, we have to show that for $t > t^*$ every double reservation results in a collisions because a collision does not change the slot allocation.

Recall Figure 16, in which the graphs depict the trajectories of the players' beliefs versus slot allocation for $t \leq t^*$. In contrast, Figure 17 shows player 2's trajectory of her belief $\mu_{2,t}$ versus time for $t \geq t^*$, i.e., once the equilibrium has been reached. By definition in Theorem 6.1, $t = t^* + 1$ is the first time player 2's belief μ_{2,t^*+1} is beyond $\mu_2^*(x_{2,t^*})$, given there was an earlier time $t' \leq t^*$ at which player 1's belief $\mu_{1,t'+1}$ was below $\mu_1^*(x_{1,t'})$. The circumstances at $t = t^*$ and $t = t^* + 1$ are depicted in the first two points of Figure 17. Every time a player's belief is larger than her corresponding μ^*, she does not meet a challenge. Thus, in superframe $t^* + 1$, player 2 does not challenge player 1's reservation. Without a challenge, there is no collision, thus, according to (28) the players' beliefs decrease. This decrease can be observed in Figure 17 from $t = t^* + 1$ to $t = t^* + 2$.

Once a player's belief is smaller than her corresponding μ^*, however, she does meet a challenge. Therefore, with $\mu_{2,t^*+2} < \mu_2^*(x_{2,t^*})$, player 2 challenges player 1's reservation and according to (28) the beliefs increase. Provided that for $t \geq t^*$ player 1 always transmits if challenged, this sequence of a decreasing belief followed by an increase of the belief is repeated from then on. We denote the number of sequences of this kind as ν.

The slot allocation (x_{1,t^*}, x_{2,t^*}) is stable, if it holds with every number of sequences ν, with $\nu \in \mathbb{N}$. The slot allocation is quasi-stationary, if it holds only with a finite number of sequences ν, with $\nu \in \mathbb{N}$. In the following, we evaluate in Lemmas 6.1-6.3 the first sequence, so for $\nu = 1$, and then extend the results in Lemma 6.4 to $\nu > 1$. Recall that we assume that $t = t^* + 1$ is the first superframe, in which player 2 does not challenge because it holds with her belief $\mu_{2,t^*+1} \geq \mu_2^*(x_{2,t^*})$, given that there is a time $t' \leq t^*$, for which player 1's belief is $\mu_{1,t'+1} < \mu_1^*(x_{1,t'})$.

1. In Lemma 6.1, we show that player 2 does not challenge for exactly one superframe. This means that at $t = t^* + 2$ it holds that $\mu_{2,t^*+2} < \mu_2^*(x_{2,t^*})$ and she challenges again.

2. In Lemma 6.2, we show that player 2 challenges player 1's reservation until $t = t^* + 1 + \Delta t_1$, with $\Delta t_1 = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$. This means that at $t = t^* + 1 + \Delta t_1$ player 2's belief is again larger than $\mu_2^*(x_{2,t^*})$ and the end of the first sequence has been reached.

3. In Lemma 6.3, we show the prerequisite of stability. Thus, we show that player 1 transmits in challenged slots and hence collisions occur.

Lemmas 6.1-6.3 consider the first sequence denoted by $\nu = 1$ in Figure 17. With these results, we determine the equilibrium slot allocation (x_{1,t^*}, x_{2,t^*}). In a final step, we extend the analysis to $\nu > 1$, with $\nu \in \mathbb{N}$. Thus, we demonstrate the stability or quasi-stationary of the determined slot allocation.

4. In Lemma 6.4, we show that for $\mu_{2,0} \leq \mu_{1,0}$ the determined slot allocation (x_{1,t^*}, x_{2,t^*}) is stable for every sequence $\nu \to \infty$. Furthermore, for $\mu_{2,0} > \mu_{1,0}$ we give the limit for the number of sequences ν that the slot allocation (x_{1,t^*}, x_{2,t^*}) is quasi-stationary.

In the following, we consider the first sequence $\nu = 1$ of Figure 17. At the end of superframe t^*, players have seen ϕ_{t^*} superframes with and ψ_{t^*} superframes without collisions. According to the definition in Theorem 6.1, the posterior belief of player 2 in superframe t^*, which is the a-priori belief of superframe t^*+1, is then given as:

$$\mu_{2,t^*+1} = \frac{\phi_{2,0} + \phi_{t^*}}{\phi_{2,0} + \psi_{2,0} + \phi_{t^*} + \psi_{t^*}} \geq \mu_2^*(x_{2,t^*}). \tag{29}$$

Therefore, in superframe t^*+1, player 2 does not challenge player 1's reservation, hence, no collision occurs. Thus, with the increase of the number of superframes without collisions ψ the players' beliefs decrease. The lack of challenging player 1's reservation implies that the slot distribution remains the same whereas the belief decreases. In Lemma 6.1, we show that player 2 does not challenge player 1's reservation for exactly one superframe. So at $t = t^*+2$, player 2's belief μ_{2,t^*+2} is again smaller than $\mu_2^*(x_{2,t^*})$ and she restarts challenging player 1.

Lemma 6.1 *Let* $t^* = \min\{t \geq t' : \mu_{2,t+1} \geq \mu_2^*(x_{2,t})\}$*, where* $t' = \min\{t : \mu_{1,t+1} < \mu_1^*(x_{1,t})\}$ *with* $\mu_2^*(x_{2,t}) < 1$*. Then at* $t = t^*+2$*, that is after one superframe of not challenging player 1's reservation, player 2 starts challenging again, because her belief is smaller than* $\mu_2^*(x_{2,t^*})$*.*

Proof For $t = t^*+1$ it holds that $\mu_{2,t} \geq \mu_2^*(x_{2,t^*})$, so we have to show that at $t = t^*+2$ for player 2 it holds that $\mu_{2,t} < \mu_2^*(x_{2,t^*})$. In superframe t^*+1, there has been no collision because player 2 does not challenge player 1's reservation. Thus, the number of superframes with and without collisions is given as $\phi_{t^*+1} = \phi_{t^*}$ and $\psi_{t^*+1} = \psi_{t^*}+1$, respectively. So, we have to show that the a-priori belief of superframe t^*+2 complies with:

$$\mu_{2,t^*+2} = \frac{\phi_{2,0} + \phi_{t^*}}{\phi_{2,0} + \psi_{2,0} + \phi_{t^*} + (\psi_{t^*}+1)} < \mu_2^*(x_{2,t^*}). \tag{30}$$

We solve $\mu_{2,t^*+1} \geq \mu_2^*(x_{2,t^*})$ given by (29) for ϕ_{t^*} and obtain that the number of superframes with collisions at $t = t^*$ has to comply with:

$$\phi_{t^*} \geq \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})}(\psi_{t^*} + \psi_{2,0}) - \phi_{2,0}. \tag{31}$$

We require the minimum ϕ_{t^*} for which (31) holds, so this can be written as:

$$\phi_{t^*} = \left\lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})}(\psi_{t^*}+\psi_{2,0}) - \phi_{2,0} \right\rceil . \tag{32}$$

If we insert (32) into (30) and re-arrange the terms, we obtain:

$$\mu_{2,t^*+2} = \frac{\psi_{t^*}+\psi_{2,0}}{\psi_{t^*}+\psi_{2,0}+(1-\mu_2^*(x_{2,t^*}))}\mu_2^*(x_{2,t^*}). \tag{33}$$

The discount factor δ, which is one of the parameters that determines $\mu_2^*(x_{2,t^*})$, is defined in the interval $[0,1)$. So, according to (25), the candidate for a Perfect Bayesian Nash Equilibrium $\mu_2^*(x_{2,t^*})$ is also in the interval $[0,1)$. Hence, with $(1-\mu_2^*(x_{2,t^*})) > 0$, the prefactor of $\mu_2^*(x_{2,t^*})$ on the right-hand side of (33) is smaller than one, hence, $\mu_{2,t^*+2} < \mu_2^*(x_{2,t^*})$. This connotes that player 2 starts to challenge player 1's reservation in superframe t^*+2, i.e., one superframe after the equilibrium has been reached. ∎

Lemma 6.1 showed that player 2 starts to challenge player 1's reservation in superframe t^*+2. If we refer to Figure 17 again, Lemma 6.1 demonstrates that at $t = t^*+2$ the graph of player 2's belief $\mu_{2,t}$ is below $\mu_2^*(x_{2,t^*})$. In the subsequent Lemma 6.2, we show that player 2 challenges player 1's reservation for Δt_1 superframes given that player 1 meets the challenge with transmitting. Thus, at $t = t^*+2+\Delta t_1$, player 2's belief rises above $\mu_2^*(x_{2,t^*})$ again.

Lemma 6.2 *Let $t^* = \min\{t \geq t' : \mu_{2,t+1} \geq \mu_2^*(x_{2,t})\}$, where $t' = \min\{t : \mu_{1,t+1} < \mu_1^*(x_{1,t})\}$ and $\mu_2^*(x_{2,t}) < 1$. Assuming that player 1 always transmits in a double reservation and thus only collisions occur, player 2 challenges for $\Delta t_1 = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$ superframes, i.e., until $t = t^*+2+\Delta t_1$.*

Proof We assume that player 1 always transmits in a doubly reserved slot. Consequently, every subsequently challenged superframe experiences a collision, thus according to (28), the players' beliefs increase and the slot allocation remains the same. We assume that it takes Δt_1 superframes with collisions until player 2's belief is larger than $\mu_2^*(x_{2,t^*})$ again and she terminates challenging player 1's reservation.

To prove Lemma 6.2, we have to show that $\Delta t_1 = \min\{\Delta t : \mu_{2,t} \geq \mu_2^*(x_{2,t^*})$, with $t = t^* + 2 + \Delta t\} = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$. So we write:

$$\Delta t_1 = \min\left\{\Delta t : \frac{\phi_{2,0} + (\phi_{t^*} + \Delta t)}{\phi_{2,0} + \psi_{2,0} + (\phi_{t^*} + \Delta t) + (\psi_{t^*} + 1)} \geq \mu_2^*(x_{2,t^*})\right\}. \tag{34}$$

If we insert ϕ_{t^*} given by (32) into (34), we obtain the number of superframes that player 2 challenges player 1 as:

$$\Delta t_1 = \left\lceil \frac{\mu_2^*(x_{2,t^*})}{1 - \mu_2^*(x_{2,t^*})} \right\rceil. \tag{35}$$

Thus, player 2 challenges player 1's reservation until $t = t^* + 2 + \Delta t_1$, where $\Delta t_1 = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$. In superframe $t = t^* + 2 + \Delta t$, player 2's belief is larger than $\mu_2^*(x_{2,t^*})$, thus, she does not challenge in this superframe. This implies that she challenges player 1's reservation in the interval $[t^* + 2, t^* + 1 + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil]$. ∎

For the previous lemmas, we assumed that player 1 transmits in every doubly reserved slot within the determined interval. In Lemma 6.3, we show that this assumption holds.

Lemma 6.3 *Given that player 2 challenges player 1's reservation for* $t \in [t^* + 2, t^* + 1 + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil]$, *player 1 transmits in every doubly reserved slot. Thus, every superframe in the specified interval experiences a collision.*

Proof Player 1 transmits in a doubly reserved slot in superframe t, if and only if $\mu_{1,t} < \mu_1^*(x_{1,t^*})$. To prove that player 1 transmits provided that player 2 challenges in the specified interval, we have to consider its bounds. If there are only collisions, the beliefs increase monotonically. Thus, if the condition $\mu_{1,t} < \mu_1^*(x_{1,t^*})$ holds for the bounds, it also holds within the interval.

The lower bound is given by superframe $t_{low} = t^* + 2$, thus, it has to hold that $\mu_{1,t_{low}} = \mu_{1,t^*+2} < \mu_1^*(x_{1,t^*})$:

$$\mu_{1,t_{low}} = \frac{\phi_{1,0} + \phi_{t^*}}{\phi_{1,0} + \psi_{1,0} + \phi_{t^*} + (\psi_{t^*} + 1)} < \mu_1^*(x_{1,t^*}). \tag{36}$$

The upper bound of the specified interval is given by $t_{up} = t^* + 1 + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$. Therefore, in superframe t_{up}, the number of superframes with and without collisions is given by $\phi_{t_{up}} = \phi_{t^*} + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil - 1$ and $\psi_{t_{up}} = \psi_{t^*} + 1$, respectively. Hence, $\mu_{1,t_{up}} < \mu_1^*(x_{1,t^*})$ becomes:

$$\mu_{1,t_{up}} = \frac{\phi_{1,0} + (\phi_{t^*} + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil - 1)}{\phi_{1,0} + \psi_{1,0} + (\phi_{t^*} + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil - 1) + (\psi_{t^*} + 1)} < \mu_1^*(x_{1,t^*}). \tag{37}$$

If $\mu_2^*(x_{2,t^*}) > 0$, it holds that $\lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil - 1 \geq 0$, thus $\mu_{1,t_{up}} \geq \mu_{1,t_{low}}$. Therefore, the inequality $\mu_{1,t_{low}} < \mu_1^*(x_{1,t^*})$ is true, if $\mu_{1,t_{up}} < \mu_1^*(x_{1,t^*})$. Thus, it is sufficient to determine the minimum ϕ_{t^*}, for which (37) holds given player 1's initial beliefs $\mu_{1,0}$ and the parameters of the game. Reformulating (37) yields:

$$\phi_{t^*} = \left\lceil (\psi_{t^*} + \psi_{1,0} + 1) \frac{\mu_1^*(x_{1,t^*})}{1 - \mu_1^*(x_{1,t^*})} - \left\lceil \frac{\mu_2^*(x_{2,t^*})}{1 - \mu_2^*(x_{2,t^*})} \right\rceil - \phi_{1,0} + 1 \right\rceil . \tag{38}$$

If the number of superframes with collisions ϕ_{t^*} complies with (38), player 1's belief $\mu_{1,t}$ is always smaller than $\mu_1^*(x_{1,t^*})$ for $t \in [t^* + 2, t^* + 1 + \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil]$. Hence, player 1 transmits in every challenged slot of the first sequence $\nu = 1$ in Figure 17. The consequence are collisions, which do not change the slot allocation (x_{1,t^*}, x_{2,t^*}), hence the allocation is stable in this first sequence. ∎

Recall Figure 17 again. In Lemma 6.1, we show that player 2 does not challenge player 1's reservation in superframe $t^* + 1$. As a consequence, the players' beliefs decrease. In Lemma 6.2, we demonstrate that player 2 challenges player 1's reservation for $\Delta t_1 = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$ superframes. Finally, Lemma 6.3 provides the proof that player 1 in fact transmits in each of those challenged superframes, such that the slot allocation remains the same in this first sequence $\nu = 1$.

In the subsequent paragraphs, we determine this slot allocation $(x_{1,t^*}, x_{2,t*})$. If we set (32) equal to (38) and neglect the ceiling functions, we can determine x_{2,t^*} for which both requirements regarding the number of superframes with collisions hold and obtain the following expression:

$$(\psi_{t^*} + \psi_{1,0} + 1)\frac{\mu_1^*(x_{1,t^*})}{1 - \mu_1^*(x_{1,t^*})} - (\psi_{t^*} + \psi_{2,0} + 1)\frac{\mu_2^*(x_{2,t^*})}{1 - \mu_2^*(x_{2,t^*})} = \phi_{1,0} - \phi_{2,0} \quad (39)$$

Consider for example a game with symmetric players that choose equal initial values for ϕ and ψ, so $\phi_{1,0} = \phi_{2,0}$ and $\psi_{1,0} = \psi_{2,0}$. Then (39) simplifies to:

$$\frac{\mu_1^*(x_{1,t^*})}{1 - \mu_1^*(x_{1,t^*})} = \frac{\mu_2^*(x_{2,t^*})}{1 - \mu_2^*(x_{2,t^*})}, \quad (40)$$

which holds for $x_{1,t^*} = x_{2,t^*}$. Thus, in the stable slot allocation for symmetric players with equal initial belief values the resources are equally distributed.

Generally, player 2 gains every slot player 1 backs off from. In superframe t^*, player 2 additionally challenges and transmits in one of player 1's slots, thus, $x_{2,t^*} = \psi_{t^*} + 1$. Player 1, however, transmits in the maximum number of slots x_m reduced by the number of slots she backed off from ψ_{t^*}. So in superframe t^*, player 1 transmits in $x_{1,t^*} = x_m - \psi_{t^*} = x_m - x_{2,t^*} + 1$ slots.

Recall that $\mu_{i,t+1}$ in (28) was defined as $\mu_{i,t+1} = \frac{\phi_{i,0}+\phi_t}{\phi_{i,0}+\psi_{i,0}+\phi_t+\psi_t}$. Thus, we have to provide initial values for the number of superframes with and without collisions, $\phi_{i,0}$ and $\psi_{i,0}$, respectively. If not specified otherwise, we set $\psi_{i,0} = 1$ and $\phi_{i,0} = \frac{\mu_{i,1}}{1-\mu_{i,1}}\psi_{i,0} = \frac{\mu_{i,1}}{1-\mu_{i,1}}$. Furthermore, we approximate $\ln(1 + \frac{1}{x}) \approx \frac{1}{x}$ for $\frac{1}{x} \ll 1$. Thus, with the initial values for $\phi_{i,0}$ and $\psi_{i,0}$ and the relation between x_{1,t^*} and x_{2,t^*}, we rewrite (39) to:

$$\frac{(x_{2,t^*} + 1)(2x_{2,t^*} - (x_m + 1))}{x_{2,t^*}(x_m + 1 - x_{2,t^*})} = \frac{c(1-\delta)}{p}\frac{\mu_{1,0} - \mu_{2,0}}{(1 - \mu_{1,0})(1 - \mu_{2,0})} := a. \quad (41)$$

The variable a is a shorthand term for the right-hand side of (41) and thus, is given externally by the cost c, the prize p, the discount factor δ and the players' initial beliefs $\mu_{1,0}$ and $\mu_{2,0}$. With this, we solve (41) for x_{2,t^*} and normalize by x_m:

$$\frac{x_{2,t^*}}{x_m} = \frac{(x_m+1)(1+a) - 2 + \sqrt{((x_m+1)(1+a) - 2)^2 + 4(x_m+1)(2+a)}}{2(2+a)x_m}. \quad (42)$$

Lemmas 6.1-6.3 show that the slot allocation (x_{1,t^*}, x_{2,t^*}) that can be determined by (42) is stable for the first sequence $\nu = 1$. Moreover, (42) illustrates how a player's share is correlated with her initial belief. The smaller player 2 chooses her initial belief, the larger her share given fixed initial belief of player 1.

In the following paragraphs, we extend the stability analysis to $\nu > 1$. Lemmas 6.1 and 6.2 are independent of ν. Thus, it always takes one superframe for $\mu_{2,t}$ to fall below $\mu_2^*(x_{2,t^*})$. Furthermore, it always takes $\Delta t = \Delta t_1 = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$ superframes for $\mu_{2,t}$ to rise above $\mu_2^*(x_{2,t^*})$ again. Thus, the course of the graph in Figure 17 is equal for each sequence provided that player 1 transmits in every challenged slot.

It remains to determine the upper bound ν^* for the number of sequences that player 1 does in fact transmit if challenged. In Lemma 6.3, we illustrated that it is sufficient to evaluate the end of a sequence. For sequence ν the end is reached at $t_\nu = t^* + \nu(\Delta t + 1) - 1$, with the number of superframes with and without collisions given by $\phi_{t_\nu} = \phi_{t^*} + \nu \cdot \Delta t - 1$ and $\psi_{t_\nu} = \psi_{t^*} + \nu$, respectively. So, we have to determine for which sequences ν^* it holds that:

$$\nu^* = \max \left\{ \nu : \mu_{1,t_\nu+1} = \frac{\phi_{1,0} + \phi_{t^*} + \nu \cdot \Delta t - 1}{\phi_{1,0} + \phi_{t^*} + \nu \cdot \Delta t - 1 + \psi_{1,0} + \psi_{t^*} + \nu} < \mu_1^*(x_{1,t^*}) \right\}. \quad (43)$$

With ϕ_{t^*} given by (32), $\Delta t = \lceil \frac{\mu_2^*(x_{2,t^*})}{1-\mu_2^*(x_{2,t^*})} \rceil$, $\psi_{1,0} = 1$, $x_{1,t^*} = x_m + 1 - x_{2,t^*}$ $\psi_{t^*} = x_{2,t^*} - 1$ and the approximation $\ln(1 + \frac{1}{x}) \approx \frac{1}{x}$ for $\frac{1}{x} \ll 1$, we rewrite (43) and obtain:

$$\nu^* = \max \left\{ \nu : x_{2,t^*}^2 (2p + \gamma) - x_{2,t^*}(x_m + 1)(p + \gamma) > \nu p (x_m + 1 - 2x_{2,t^*}) \right\}, \quad (44)$$

using γ as a shorthand term for $\gamma = c(1 - \delta)(\phi_{1,0} - \phi_{2,0} - 1)$. The slot allocation (x_{1,t^*}, x_{2,t^*}) is stable, if there is no upper bound for ν^* that solves (44). In contrast, we refer to the slot allocation (x_{1,t^*}, x_{2,t^*}) to be quasi-stationary, if there is an upper bound for ν^*. There are three cases that need to be considered:

1. Players are symmetric, thus, they have equal initial beliefs $\mu_{1,0} = \mu_{2,0}$. In (40), we have shown that in the corresponding slot allocation players share the resources equally, so $x_{1,t^*} = x_{2,t^*}$.

2. Player 1 chooses an initial belief larger than player 2's belief, so $\mu_{1,0} > \mu_{2,0}$. According to (42), this corresponds to $x_{1,t^*} < x_{2,t^*}$.

3. Player 1 chooses an initial belief smaller than player 2's belief, so $\mu_{1,0} < \mu_{2,0}$. According to (42), this corresponds to $x_{1,t^*} > x_{2,t^*}$.

In Lemma 6.4, we demonstrate that in the first and second case, i.e., $\mu_{1,0} \geq \mu_{2,0}$, the corresponding slot allocation $x_{1,t^*} \leq x_{2,t^*}$ is stable, so we show that (44) holds for any $\nu > 1$, with $\nu \in \mathbb{N}$. Furthermore, we show that in the third case of $\mu_{1,0} < \mu_{2,0}$ there is an upper bound ν^* for the number of sequences ν, thus the slot allocation $x_{1,t^*} > x_{2,t^*}$ is considered to be quasi-stationary.

Lemma 6.4 *With initial beliefs $\mu_{1,0} \geq \mu_{2,0}$, the corresponding slot allocation (x_{1,t^*}, x_{2,t^*}) is stable for any sequence $\nu \in \mathbb{N}$. If players' initial beliefs are $\mu_{1,0} < \mu_{2,0}$, the limit for the number of sequences ν^* that the slot allocation (x_{1,t^*}, x_{2,t^*}) is quasi-stationary is given by* $\nu^* < \frac{c(1-\delta)x_{2,t^*}^3 + 2(p - c(1-\delta)(x_m+1))x_{2,t^*}^2 + (x_m+1)(c(1-\delta)(x_m+1) - 3p)x_{2,t^*} + (x_m+1)^2 p}{p(x_m+1-x_{2,t^*})(x_m+1-2x_{2,t^*})}$.

Proof The first case to be considered is the game with symmetric players. As noted before, players with equal initial beliefs reach a fair slot allocation. If we insert the fair slot allocation into (44), we observe that the function is independent of the number of sequences ν. Hence (44) holds for any $\nu \in \mathbb{N}$ and is thus, stable.

In the second case, players are asymmetric in their beliefs with $\mu_{1,0} > \mu_{2,0}$, which corresponds to player 2 gaining a larger share of resources. Knowing that $x_{1,t^*} < x_{2,t^*}$ implies that the right-hand side of the inequality in (44) is negative, so we reformulate (44) and write:

$$\nu > \frac{x_{2,t^*}^2(2p+\gamma) - x_{2,t^*}(x_m+1)(p+\gamma)}{p(x_m+1-2x_{2,t^*})}, \tag{45}$$

using the shorthand term $\gamma = c(1-\delta)(\phi_{1,0} - \phi_{2,0} - 1)$. Recall that stability is given, if (44) holds for $\nu \in \mathbb{N}$. Hence, we have to show that the right-hand side of the inequality (45) is less than 1. By this, any $\nu \geq 1$ complies with (44). With $x_{1,t^*} < x_{2,t^*}$, the denominator of the right-hand side of (45) is negative, so we have to show that:

$$x_{2,t^*}^2(2p+\gamma) - x_{2,t^*}(x_m+1)(p+\gamma) > p(x_m+1-2x_{2,t^*}), \tag{46}$$

with $\gamma = c(1-\delta)(\phi_{1,0} - \phi_{2,0} - 1)$. Recall that the equilibrium slot allocation (x_{1,t^*}, x_{2,t^*}) is determined by players' initial beliefs, which in turn are formed using the initial values for $\phi_{1,0}$ and $\phi_{2,0}$. Thus, (46) is indeterminate because it contains both x_{2,t^*} and $\phi_{i,0}$, for $i \in [1,2]$. As a consequence, we make use of (41) to solve (46).

In (41), we substitute the initial beliefs by $\mu_{i,0} = \frac{\phi_{i,0}}{\phi_{i,0}+1}$, for $i \in [1,2]$, rearrange the equation and obtain:

$$\frac{p(x_{2,t^*}+1)(2x_{2,t^*} - (x_m+1))}{x_{2,t^*}(x_m+1-x_{2,t^*})} = c(1-\delta)(\phi_{1,0} - \phi_{2,0}). \tag{47}$$

With the help of (47) we solve (46). If we insert the left-hand side of (47) for $c(1-\delta)(\phi_{1,0} - \phi_{2,0})$ in the shorthand term γ of (46), we can reduce (46) to:

$$\frac{x_{2,t^*}((x_m+1) - x_{2,t^*})^2}{x_m+1-x_{2,t^*}} > 0. \tag{48}$$

In the stability analysis that we are evaluating in the current paragraphs, we consider the case that $x_{1,t^*} < x_{2,t^*}$. Thus, slot allocations for which $x_{1,t^*} < x_{2,t^*}$ and (48) hold, are stable. Hence, next we evaluate the left-hand side of (48) to identify those stable slot allocations.

Equation (48) has a zero-crossing at $x_{2,t^*} = 0$ and a double zero-crossing at $x_{2,t^*} = x_m + 1$, which is a removable discontinuity. Furthermore, there is a maximum turning point at $x_{2,t^*} = \frac{x_m+1}{2}$. Knowing the shape of the curve of the left-hand side of (48), we deduce that for $x_{1,t^*} < x_{2,t^*}$ it is larger than zero, so (48) holds. Thus, (46) and (45) hold. So for $x_{1,t^*} < x_{2,t^*}$ the stability requirement of (44) holds for any sequence $\nu > 0$, which proves that the slot allocation (x_{1,t^*}, x_{2,t^*}) is stable.

In the third case players are asymmetric in their beliefs with $\mu_{1,0} < \mu_{2,0}$, which implies that player 1 gains a larger share of resources than player 2. Knowing that $x_{1,t^*} > x_{2,t^*}$ implies that the right-hand side of the inequality in (44) is negative, so we reformulate (44) and write:

$$\nu < \frac{x_{2,t^*}^2(2p+\gamma) - x_{2,t^*}(x_m+1)(p+\gamma)}{p(x_m+1-2x_{2,t^*})}, \tag{49}$$

with the shorthand term $\gamma = c(1-\delta)(\phi_{1,0} - \phi_{2,0} - 1)$. If we insert the left-hand side of (47) for $c(1-\delta)(\phi_{1,0} - \phi_{2,0})$ in the shorthand term γ of (49), we are able to solve the indeterminate (49) and obtain the limit of the number of sequences $\nu^* < \frac{c(1-\delta)x_{2,t^*}^3 + 2(p - c(1-\delta)(x_m+1))x_{2,t^*}^2 + (x_m+1)(c(1-\delta)(x_m+1) - 3p)x_{2,t^*} + (x_m+1)^2 p}{p(x_m+1-x_{2,t^*})(x_m+1-2x_{2,t^*})}$. ∎

In this section, we have shown that for initial beliefs $\mu_{1,0} \geq \mu_{2,0}$, there exists a stable slot allocation (x_{1,t^*}, x_{2,t^*}), with $x_{1,t^*} \leq x_{2,t^*}$. Moreover, in the case that $\mu_{1,0} < \mu_{2,0}$, we have determined the number of sequences ν^* for which the corresponding slot allocation $x_{1,t^*} > x_{2,t^*}$ is quasi-stationary. In the next section, we evaluate the fairness of the identified slot allocations.

6.4 FAIRNESS OF PERFECT BAYESIAN NASH EQUILIBRIUM

In this section, we show how the fairness according to Jain [38] depends on the players' initial beliefs as well as the discount factor δ, with $\delta \in [0, 1)$. First, we illustrate that the Perfect Bayesian Nash Equilibrium of the game is Pareto- and socially-optimal and has a fairness of one, only if players have equal initial beliefs. Then we generalize to determine how the fairness index relates to the players' initial beliefs and the chosen discount factor δ.

In Section 6.1, we identified candidates for Perfect Bayesian Nash Equilibria. Those candidates were given by (25) that determines the belief $\mu_i^*(x_{i,t})$ at which player i is indifferent about transmitting and not transmitting given that she transmits in $x_{i,t}$ slots. In Section 6.3, we showed which of those candidates are reached in the game given the players' initial beliefs $\mu_{1,0}$ and $\mu_{2,0}$. In Theorem 6.2 we show that only if players choose equal initial beliefs the game reaches the stable slot allocation $x_{1,t^*} = x_{2,t^*}$ that maximizes Jain's fairness index and can thus, be considered both Pareto- and socially-optimal.

Theorem 6.2 *The Perfect Bayesian Nash Equilibrium of the game is Pareto- and socially-optimal and Jain's fairness index is maximized, iff both players have the same initial beliefs.*

Proof According to the Lemma 5.7 in Section 5.3, an equilibrium maximizes the fairness according to Jain if both players receive the same share of resources, i.e., $x_{1,t^*} = x_{2,t^*}$. This equilibrium is also Pareto- and socially-optimal (Lemmas 5.3,5.4). If we insert $x_{1,t^*} = x_{2,t^*}$ in (39), which is the first equation to determine the stable allocation of the game, and set $\psi_{1,0} = \psi_{2,0} = 1$ as explained before, we obtain:

$$0 = \phi_{1,0} - \phi_{2,0}. \tag{50}$$

Thus, in order to reach the fair allocation of $x_{1,t^*} = x_{2,t^*}$, the initial values $\phi_{1,0}$ and $\phi_{2,0}$ have to be equal. As a consequence, the players' initial beliefs have to be the same. Hence, we showed that the Perfect Bayesian Nash Equilibrium is fair, if and only if both players have the same initial beliefs. ∎

In the next paragraphs, we consider the Perfect Bayesian Nash Equilibria that occur if the players choose unequal initial beliefs $\mu_{1,0} \neq \mu_{2,0}$. So we generalize the result of Theorem 6.2 and, provided that there is a certain target fairness index, we study which combination of initial beliefs leads to it.

Recall that $u_i(x_i)$ denotes the utility player i gains from transmitting in x_i slots. The utility profile u then denotes the vector of utilities, i.e., $u = (u_1(x_1), u_2(x_2))$. For our fairness analysis, we consider a target fairness index $f(u)$. First we have to identify the corresponding equilibrium slot allocation. Secondly, we determine the initial beliefs $(\mu_{1,0}, \mu_{2,0})$ that result in the required slot allocation. Recall (5), which stated that Jain's fairness index is given as a function of the first two moments of the players' utilities:

$$f(u) = \frac{[\sum_{i=1}^{N} u_i]^2}{N \sum_{i=1}^{N} u_i^2}, \quad u_i \geq 0, \quad f(u) \in [0,1].$$

If we solve this for player 2 in the 2-player game, this yields:

$$u_2 = \frac{1 \mp 2\sqrt{f(u)(1-f(u))}}{2f(u)-1} u_1 \tag{51}$$

$$p \ln(x_2+1) - cx_2 = \frac{1 \mp 2\sqrt{f(u)(1-f(u))}}{2f(u)-1} \left(p \ln(x_1+1) - c(x_1)\right) \tag{52}$$

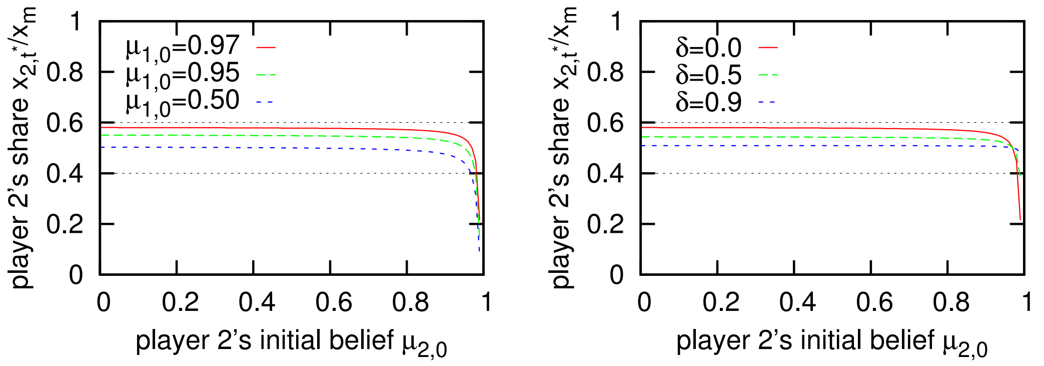

Figure 18: Player 2's share depending on her initial belief $\mu_{2,0}$ given player 1's initial belief and the common discount factor. For $\delta = 0$ (left), player 2's share increases with player 1's initial belief. For a fixed initial belief of player 1 $\mu_{1,0} = 0.97$ (right), we observe that player 2's share increases the smaller the discount factor. Note that if player 2's share is larger than the fair share of $x_{2,t^*}/x_m$, the equilibrium is stable. Shares smaller than the fair share are quasi-stationary.

Let x_2' be the solution to (52), with $x_1 = x_m - x_2 + 1$. Then we achieve a fairness index of $f(u)$ if the equilibrium slot distribution is given by $(x_{1,t^*}, x_{2,t^*}) = (x_m + 1 - x_2', x_2')$.

Equation (41) represents the relationship between the equilibrium slot allocation and the initial beliefs given the discount factor δ. So if we set $x_{2,t^*} = x_2'$ and solve (41) for $\mu_{2,0}$, we determine the initial belief $\mu_{2,0}$ depending on player 1's initial belief $\mu_{1,0}$ for which the target fairness index $f(u)$ is achieved and obtain:

$$\mu_{2,0} = \frac{q(1-\mu_{1,0}) - \mu_{1,0}}{q(1-\mu_{1,0}) - 1}, \text{with } q = \frac{p(x_2'+2)(2x_2' - (x_m+1))}{c(1-\delta)x_2'((x_m+1) - x_2')}. \tag{53}$$

Figure 18 plots player 2's share $x_{2,t^*}/x_m$ in the equilibrium allocation given by (42) against different initial beliefs $\mu_{2,0}$. In Figure 18 (left), the discount factor is set to $\delta = 0$, so players do not take into account future utilities, and the curves are depicted for different initial beliefs of player 1. In Figure 18 (right), player 1's initial belief is set to $\mu_{1,0} = 0.97$ and the parameter of the curves is the discount factor δ.

For our analysis, we consider a target fairness index of $f(u) = 0.999$. If we substitute this target fairness index in (52), player 2's share of resources has to be $x_{2,t^*}/x_m \in [0.4, 0.6]$. Thus, any share $x_{2,t^*}/x_m \in [0.4, 0.6]$ results in a fairness index $f(u) \geq 0.999$. The dotted lines at $x_{2,t^*}/x_m = 0.4$ and $x_{2,t^*}/x_m = 0.6$ in the graphs of Figure 18 denote those bounds on player 2's share.

Hence, we have to determine the combinations of initial beliefs $\mu_{1,0}$ and $\mu_{2,0}$ that correspond to $x_{2,t^*}/x_m \in [0.4, 0.6]$ to achieve or exceed the target fairness index of $f(u) = 0.999$. The intersection of the curves with the lower border $x_{2,t^*}/x_m = 0.4$ is given by (53). So as long as player 2's initial belief $\mu_{2,0}$ is smaller than this threshold belief, the target fairness is achieved or even exceeded.

In Figure 18 (left), we observe that the larger player 1's initial belief $\mu_{1,0}$, the larger is player 2's maximum initial belief $\mu_{2,0}$ that still achieves the target fairness. Recall that a fairness of one, which corresponds to $x_{2,t^*}/x_m = 0.5$, is only achieved when $\mu_{1,0} = \mu_{2,0}$. Figure 18 (left) also illustrates the previous findings that as long as player 2 chooses an initial belief $\mu_{2,0} < \mu_{1,0}$, she receives a share larger than the fair share. Analogously, she receives a share smaller than the fair share, if she selects an initial belief $\mu_{2,0} > \mu_{1,0}$. Recall, that for $\mu_{2,0} > \mu_{1,0}$, the equilibrium slot allocation is only quasi-stationary.

Next, we consider Figure 18 (right), which shows the dependence of player 2's share on the discount factor δ. As expected from Theorem 6.2, we observe that independent of the discount factor δ all curves intersect for $\mu_{2,0} = \mu_{1,0} = 0.97$. For $\mu_{2,0} \neq \mu_{1,0}$, we note that the impact of the initial beliefs diminishes for larger valuation of the future, i.e., increasing δ. So, the larger δ, the flatter is the corresponding curve and hence, the closer is player 2's share to the fair share.

We conclude from this section that players receive the fair share if and only if both players choose the same initial belief, no matter what their discount factor δ is. Further, if we assume that players intend to maximize their share, player 2 has an incentive to choose an initial belief as small as possible. If we further assume that player 1 anticipates this behaviour, she also has an incentive to choose an initial belief as small as possible to gain at least the fair share. Hence, the rational initial beliefs are $\mu_{1,0} = \mu_{2,0} = 0$. This combination of initial beliefs coincides with the fair and stable slot allocation for any discount factor of the players. In the next section, we analyse the convergence of the game. Thus, we answer the question under which conditions the game arrives at the identified equilibrium slot allocations.

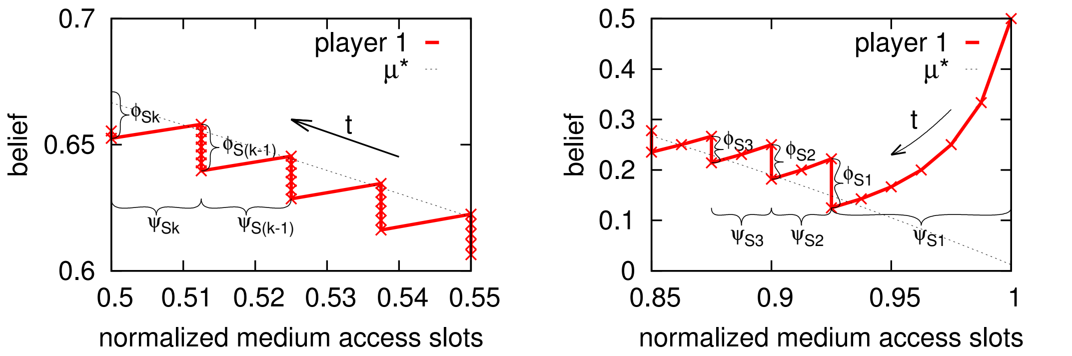

Figure 19: Detail of Figure 16 (left). The figure on the right shows the initial sections of player 1's trajectory, while the figure on the left shows the last sections before the equilibrium. Note the different ranges of the x- and y-axes. The ψ_{Sl} denote the superframes without collisions, while ϕ_{Sl} denote the superframes with collisions.

6.5 CONVERGENCE TO PERFECT BAYESIAN NASH EQUILIBRIUM

In Section 6.3, we identified the stable point of the game given the players' initial beliefs. However, so far we have not shown that the game actually arrives at this stable point. In this section, we discuss the convergence of the dynamic game.

Theorem 6.1 states that the slot allocation (x_{1,t^*}, x_{2,t^*}) is stable. So player i's belief in the equilibrium has to be $\mu_{i,t^*+1} = \frac{\phi_{i,0}+\phi_{t^*}}{\phi_{i,0}+\psi_{i,0}+\phi_{t^*}+\psi_{t^*}}$. Hence, the game must have undergone ϕ_{t^*} superframes with and ψ_{t^*} superframes without collisions. In this section, we show for which initial beliefs $\mu_{1,0}$ and $\mu_{2,0}$, we pass the point (x_{1,t^*}, x_{2,t^*}) in the course of the game with ϕ_{t^*} and ψ_{t^*}. Thereby, we prove that we reach the stable allocation and hence, that the game converges to the Perfect Bayesian Nash Equilibrium of the game.

The graphs Figure 19 are a detail of Figure 16 (left). They show the initial sections of the trajectory of player 1's belief versus slot allocation (right) and the last sections before the equilibrium is reached (left). Note the different ranges of the x- and y-axes. In the graphs, $\psi_{S1}, \psi_{S2}, \ldots$ denote those sections that relate to superframes without collisions. Section ψ_{S1} in Figure 19 (right), for instance, connotes that at $t = t_1$ the game has undergone $\psi_{t_1} = \psi_{S1}$ superframes without collisions, which results in player 1's belief μ_{1,t_1} to fall below $\mu_1^*(x_{1,t_1})$. Recall that prior to the equilibrium the number of slots that player 1 uses for transmission is directly related to the number of superframes without collisions, i.e., $x_{1,t_1} = x_m - \psi_{S1}$. At the end of section ψ_{S2}, which we assume to be reached at $t = t_2$, the game has undergone $\psi_{t_2} = \psi_{S1} + \psi_{S2}$ superframes without collisions. Player 1's share is then $x_{1,t_2} = x_m - \psi_{S1} - \psi_{S2}$. So at $t = t_k$ in Figure 19 (left), the game has undergone $\psi_{t_k} = \sum_{l=1}^{k} \psi_{Sl}$ superframes without collisions. So if $t_k = t^*$, we have discovered the k for which $\sum_{l=1}^{k} \psi_{Sl} = \psi_{t^*} = x_{2,t^*} - 1 = x_m - x_{1,t^*} + 1$.

Analogously, $\phi_{S1}, \phi_{S2}, \ldots$ denote those sections of the trajectory that relate to superframes with collisions. Note that ϕ_{Sl} is not equal to the difference in the beliefs, as suggested by the label of the y-axes. Rather, it is the number of superframes with collisions that correspond to this difference in belief. For instance, assume the belief at the lower edge is given by $\mu = \frac{\phi}{\phi+\psi}$, then the belief at the respective upper edge is given by $\mu = \frac{\phi+\phi_{Sl}}{\phi+\psi+\phi_{Sl}}$.

Assume that we have identified k such that $\sum_{l=1}^{k} \psi_{Sl} = \psi_{t^*}$ holds. Thus, we have reached the point (x_{1,t^*}, x_{2,t^*}). This slot allocation is stable, if the number of superframes with and without collisions are given by ϕ_{t^*} and ψ_{t^*}, respectively. By definition $\psi_{t^*} = x_{2,t^*} - 1$, so this condition holds. It remains to be shown that the number of superframes with collisions is equal to ϕ_{t^*}.

In Figure 19 (left), we observe that it is not a unique number of superframes with collisions that corresponds to the particular slot allocation (x_{1,t^*}, x_{2,t^*}), but an interval, i.e., section ϕ_{Sk} of the trajectory. To show that the game converges to the slot allocation (x_{1,t^*}, x_{2,t^*}), we have to show that the effective number of superframes with collisions ϕ_{t^*} is within this interval, i.e., falls into section ϕ_{Sk} of the trajectory.

The lower edge of section ϕ_{Sk} in Figure 19 (left) is given by $\phi_{\text{k,low}} = \sum_{l=1}^{k-1} \phi_{Sl}$, while the upper edge is given by $\phi_{\text{k,up}} = \sum_{l=1}^{k} \phi_{Sl}$, with known k, for which $\sum_{l=1}^{k} \psi_{Sl} = \psi_{t^*}$. Thus, we have to show that $\phi_{\text{k,low}} \leq \phi_{t^*} \leq \phi_{\text{k,up}}$ holds. Note that in Figure 19 (left) the trajectory of player 1's belief ends below $\phi_{\text{k,up}}$. The reason is that player 2 stopped challenging player 1, thus, the belief decreases. So in this example it holds that $\phi_{t^*} \in [\phi_{\text{k,low}}, \phi_{\text{k,up}}]$.

At the upper edge, player 1's belief, which we denote by $\mu_{1,t_{\text{k,up}}}$, has just risen above $\mu_1^*(x_{1,t_{\text{k,up}}})$. So we write:

$$\mu_{1,t_{\text{k,up}}} = \frac{\phi_{1,0} + \phi_{\text{k,up}}}{\phi_{1,0} + \phi_{\text{k,up}} + \psi_{1,0} + \sum_{l=1}^{k} \psi_{Sl}} \geq \mu_1^*(x_{1,t_{\text{k,up}}}), \tag{54}$$

with $x_{1,t_{\text{k,up}}} = x_{1,t^*} = x_m - \sum_{l=1}^{k} \psi_{Sl} = x_m - \psi_{t^*} = x_m - x_{2,t^*} + 1$ and $\psi_{1,0} = 1$. We reformulate (54) and use the approximation $\ln(1+\frac{1}{x}) \approx \frac{1}{x}$ for $\frac{1}{x} \ll 1$ to determine the minimum $\phi_{\text{k,up}}$ that solves (54):

$$\phi_{\text{k,up}} = \left\lceil \frac{x_{2,t^*}(p - cx_{1,t^*})}{cx_{1,t^*}(1-\delta)} - \phi_{1,0} \right\rceil . \tag{55}$$

Player 1's belief at the lower edge is denoted by $\mu_{1,t_{\text{k,low}}}$. Unfortunately, we cannot form this belief analogous to $\mu_{1,t_{\text{k,up}}}$. However, if we consider Figure 19 (left) again, we notice that player 1's belief at the lower edge $\mu_{1,t_{\text{k,low}}}$ only differs from her belief at the previous section's upper edge $\mu_{1,t_{\text{k-1,up}}}$ in the number of superframes without collisions. So, analogously, we form $\mu_{1,t_{\text{k-1,up}}}$ with $\phi_{\text{k-1,up}} = \phi_{\text{k,low}}$:

$$\mu_{1,t_{\text{k-1,up}}} = \frac{\phi_{1,0} + \phi_{\text{k,low}}}{\phi_{1,0} + \phi_{\text{k,low}} + \psi_{1,0} + \sum_{l=1}^{k-1} \psi_{Sl}} \geq \mu_1^*(x_{1,t_{\text{k-1,up}}}), \tag{56}$$

with $x_{1,t_{\text{k-1,up}}} = x_m - \sum_{l=1}^{k-1} \psi_{Sl} = x_m - \psi_{t^*} + \psi_{Sk} = x_m - x_{2,t^*} + 1 + \psi_{Sk}$ and $\psi_{1,0} = 1$. If we reformulate (56) according to (55) using the approximation $\ln(1+\frac{1}{x}) \approx \frac{1}{x}$ for $\frac{1}{x} \ll 1$, we obtain the lower bound $\phi_{\text{k,low}}$:

$$\phi_{\text{k,low}} = \left\lceil \frac{(x_{2,t^*} - \psi_{Sk})(p - c(x_{1,t^*} + \psi_{Sk}))}{c(x_{1,t^*} + \psi_{Sk})(1-\delta)} - \phi_{1,0} \right\rceil \tag{57}$$

So far, we have determined the interval $[\phi_{\text{k,low}}, \phi_{\text{k,up}}]$, for which the number of superframes with collisions corresponds to the slot allocation (x_{1,t^*}, x_{2,t^*}). In the subsequent paragraphs, we show on which condition ϕ_{t^*} as part of the stable allocation, is within these bounds. Hence, we present the condition for the game to converge to the Perfect Bayesian Nash Equilibrium.

The number of superframes with collisions in the equilibrium ϕ_{t^*} is given by (32). With $\psi_{2,0} = 1$, $\psi_{t^*} = x_{2,t^*} - 1$ and the approximation $\ln(1+\frac{1}{x}) \approx \frac{1}{x}$ for $\frac{1}{x} \ll 1$, we rewrite (32) to:

$$\phi_{t^*} = \left\lceil \frac{x_{2,t^*}(p - cx_{2,t^*})}{cx_{2,t^*}(1-\delta)} - \phi_{2,0} \right\rceil . \tag{58}$$

First, we consider the lower bound. Hence, we have to find for which x_{2,t^*} it holds that $\phi_{t^*} \geq \phi_{\text{k,low}}$. We neglect the ceiling function for both sides of the inequality. If the inequality holds, then it also holds if the ceiling function is applied on both sides. We reformulate the inequality $\phi_{t^*} \geq \phi_{\text{k,low}}$ and obtain:

$$c(1-\delta)(\phi_{1,0}-\phi_{2,0}) \geq \frac{(x_{2,t^*}-\psi_{Sk})(p-c(x_{1,t^*}+\psi_{Sk}))}{x_{1,t^*}+\psi_{Sk}} - p + cx_{2,t^*}. \tag{59}$$

If we insert the left-hand side of (47) for $c(1-\delta)(\phi_{1,0}-\phi_{2,0})$ in (59) to make the equation determinate, we obtain:

$$\frac{(x_{2,t^*}-\psi_{Sk})(p-c(x_{1,t^*}+\psi_{Sk}))}{x_{1,t^*}+\psi_{Sk}} - \frac{p(x_{2,t^*}+1)}{x_{1,t^*}} + \frac{p}{x_{2,t^*}} + cx_{2,t^*} \leq 0. \tag{60}$$

Recall that player i's utility (14) was given by $u_i(S) = p\ln(x_i+1) - cx_i$. For greedy players we require player i's utility to be maximized for $x_i = x_m$, so we set $p = c(x_m+1)$. For ease calculation, $c = 1$ and hence $p = c(x_m+1) = x_m+1$. Further, we substitute $x_{1,t^*} = x_m - x_{2,t^*} + 1$ such that (60) becomes:

$$\begin{aligned}\psi_{Sk}x_{2,t^*}^3 + x_{2,t^*}^2(2(x_m+1)(1-\psi_{Sk})-\psi_{Sk}^2) + x_{2,t^*}(x_m+1)(\psi_{Sk}^2-2\psi_{Sk}-3x_m-3)\\ + x_m^3 + x_m^2(\psi_{Sk}+3) + x_m(2\psi_{Sk}+3) + \psi_{Sk} + 1 \leq 0. \quad (61)\end{aligned}$$

If we find the x_{2,t^*} for which (61) holds, we have determined the slot allocation, for which the lower bound of superframes with collisions $\phi_{\text{k,low}}$ is met. Figure 20 is a sketch of the left-hand side of (61) for $\psi_{Sk} = 1$. We observe that the left-hand side of (61) is positive for $x_{2,t^*} = 0$ and negative for $x_{2,t^*} = \frac{x_m+1}{2}$. Thus, there exists a finite crossing in the interval $[0, \frac{x_m+1}{2}]$, which we refer to as $x_{2,t^*,z1}$. For $x_{2,t^*} = x_m$, the left-hand side of (61) is negative, while it is positive for $x_{2,t^*} = 2x_m$. Thus, there is a second crossing in $[x_m, 2x_m]$. Since the left-hand side of (61) is of grade 3, there can be no additional crossing in the interval $[0, x_m]$. Hence, the inequality (61) holds for $x_{2,t^*} \in [x_{2,t^*,z1}, x_m]$. Increasing ψ_{Sk} enlarges this interval, so we conclude that the lower bound for ϕ_{t^*} is met at least for $x_{2,t^*} \in [x_{2,t^*,z1}, x_m]$.

Recall that a stable slot allocation with $x_{1,t^*} \leq x_{2,t^*}$ is reached with players' initial beliefs $\mu_{1,0} \geq \mu_{2,0}$. For the crossing $x_{2,t^*,z1}$ it holds that $x_{2,t^*,z1} < \frac{x_m+1}{2}$, thus for

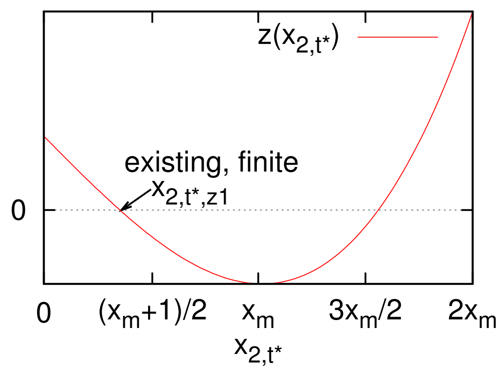

Figure 20: Sketch of (61) for $\psi_{Sk} = 1$. There is a zero in the interval $[0, \frac{x_m+1}{2}]$ and a second zero in $[x_m, 2x_m]$. So in the interval of interest $[0, x_m]$, the lower bound is met for $x_{2,t^*} \in [x_{2,t^*,z1}, x_m]$.

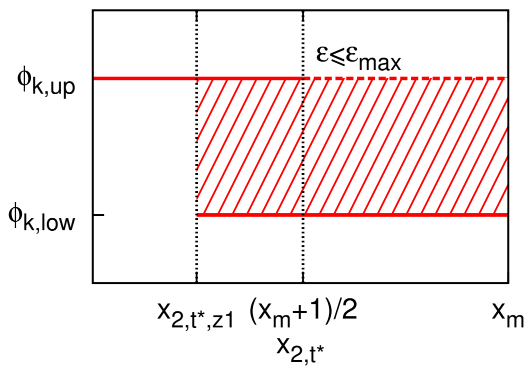

Figure 21: Summary of x_{2,t^*}, for which the game converges. The hatched area indicates that for $x_{2,t^*} \geq x_{2,t^*,z1}$ both the lower and upper bounds are met, thus, the game converges, if $\epsilon \leq \epsilon_{\max}$.

all $x_{1,t^*} \leq x_{2,t^*}$ the left-hand side of (61) holds. Thus, the lower bound is met for all $\mu_{1,0} \geq \mu_{2,0}$. Analogously, a quasi-stationary slot allocation with $x_{1,t^*} > x_{2,t^*}$ is reached with initial beliefs $\mu_{1,0} < \mu_{2,0}$. So moreover, the lower bound is met if $\mu_{1,0} < \mu_{2,0}$ with the corresponding $x_{2,t^*} \geq x_{2,t^*,z1}$.

In the following paragraphs we consider for which initial beliefs the upper bound $\phi_{\mathrm{k,up}}$ is met such that $\phi_{t^*} \leq \phi_{\mathrm{k,up}}$. Initially, we neglect the ceiling function for both sides of the inequality again, so we write:

$$\frac{x_{2,t^*}(p - cx_{2,t^*})}{cx_{2,t^*}(1-\delta)} - \phi_{2,0} \leq \frac{x_{2,t^*}(p - cx_{1,t^*})}{cx_{1,t^*}(1-\delta)} - \phi_{1,0}. \tag{62}$$

If we solve (62) by inserting the left-hand side of (47) for $c(1-\delta)(\phi_{1,0} - \phi_{2,0})$ to make the inequality determinate, we obtain that $x_{1,t^*} \geq x_{2,t^*}$. This means that for $x_{2,t^*} \leq \frac{x_m+1}{2}$ the upper bound is met. Hence, the game converges to the quasi-stationary points for which it holds that $x_{2,t^*} \geq x_{2,t^*,z1}$ and to the stable point $x_{1,t^*} = x_{2,t^*} = \frac{x_m+1}{2}$. According to (62), however, the game does not converge to the stable points with $x_{1,t^*} < x_{2,t^*}$. To further elaborate the convergence to those points, we write $\phi_{t^*} \leq \phi_{\mathrm{k,up}}$ considering the ceiling functions:

$$\left\lceil \frac{x_{2,t^*}(p - cx_{1,t^*})}{cx_{1,t^*}(1-\delta)} \right\rceil - \left\lceil \frac{x_{2,t^*}(p - cx_{2,t^*})}{cx_{2,t^*}(1-\delta)} \right\rceil \geq \phi_{1,0} - \phi_{2,0} \tag{63}$$

We denote the term remainder of a ceiling function as ϵ, such that $\epsilon = \lceil z \rceil - z$. Then, ϵ_1 and ϵ_2 are the remainder of the ceiling functions that result in $\phi_{\mathrm{k,up}}$ and ϕ_{t^*}, respectively. So, we reformulate the inequality given by (63):

$$\frac{x_{2,t^*}(p - cx_{1,t^*})}{cx_{1,t^*}(1-\delta)} + \epsilon_1 - \frac{x_{2,t^*}(p - cx_{2,t^*})}{cx_{2,t^*}(1-\delta)} - \epsilon_2 \geq \phi_{1,0} - \phi_{2,0}, \tag{64}$$

Solving (64) for $\epsilon = \epsilon_2 - \epsilon_1$ by inserting the left-hand side of (47) for the term $c(1-\delta)(\phi_{1,0} - \phi_{2,0})$ to make the inequality determinate, we obtain:

$$\epsilon \leq \frac{p(x_{1,t^*} - x_{2,t^*})}{c(1-\delta)x_{1,t^*}x_{2,t^*}} = \frac{p(x_m + 1 - 2x_{2,t^*})}{c(1-\delta)x_{2,t^*}(x_m + 1 - x_{2,t^*})} = \epsilon_{\max} \tag{65}$$

Thus, if the difference of the rounding remainder of ϕ_{t^*} and $\phi_{\mathrm{k,up}}$, i.e., $\epsilon = \epsilon_2 - \epsilon_1$, is smaller than the threshold given by $\epsilon_{\max}$, the game converges to the stable slot allocations with $x_{1,t^*} < x_{2,t^*}$.

In this section, we evaluated the convergence of the game presented. We showed that it converges, if the number of superframes with collisions in equilibrium ϕ_{t^*} meets both an upper and lower bound, $\phi_{\mathrm{k,up}}$ and $\phi_{\mathrm{k,low}}$, respectively. Figure 21 summarizes the slot allocations x_{2,t^*} for which those upper and lower bounds are met. We considered three different ranges for the Perfect Bayesian Nash Equilibria. In the first case, player 1's initial belief is smaller than player 2's initial belief, i.e., $\mu_{1,0} < \mu_{2,0}$, which results in a quasi-stationary slot allocation such that player 1 gains a larger share than player 2, $x_{1,t^*} > x_{2,t^*}$. For this range, the upper bound $\phi_{\mathrm{k,up}}$ is always met. The lower bound $\phi_{\mathrm{k,low}}$, however, is met only for $x_{2,t^*} > x_{2,t^*,z1}$. Thus, for $x_{2,t^*} \in [x_{2,t^*,z1}, \frac{x_m+1}{2})$ the game converges to a quasi-stationary slot allocation. The second case considered players symmetric in their beliefs, so $\mu_{1,0} = \mu_{2,0}$ and the corresponding slot allocation $x_{1,t^*} = x_{2,t^*} = \frac{x_m+1}{2}$ is stable. For this homogeneous case, we showed that the game converges since both the upper and lower bounds are met. The last case considered players asymmetric in their beliefs with $\mu_{1,0} > \mu_{2,0}$. For the corresponding stable slot allocations it holds that $x_{1,t^*} < x_{2,t^*}$. In our analysis, we considered the remainder of the ceiling functions for $\phi_{\mathrm{k,up}}$ and ϕ_{t^*}, such that $\epsilon = \epsilon_2 - \epsilon_1$. We showed that the game converges to the stable slot allocations, if for the difference of the remainders it holds that $\epsilon \leq \epsilon_{\max}$.

6.6 GAMES WITH n PLAYERS AND THE INFLUENCE OF SYSTEM PARAMETERS

In the previous sections, we studied the 2-player game and analysed its convergence and the equilibrium allocation of the belief update algorithm presented. In this section, we perform simulations to evaluate the influence of the number of players on each player's share of resources in the equilibrium. We find that the belief update algorithm results in an equivalent resource distribution for any network size, if a player's share is set in relation to the network size.

Further, we have a closer look at the convergence time and its dependence on the network size as well as on the maximum number of overlaps y_{max} and the players' discount factor δ. We show that the convergence time grows linearly with the network size, but decreases rapidly with an increase of the maximum number of overlaps y_{max}. The discount factor δ negatively influences the convergence time. So increasing δ induces the players to retreat less likely from challenged slots, thus, more time is needed to reach the equilibrium slot allocation.

For the simulations, we extend the network simulator OMNeT++ [65] by the relevant elements of the distributed reservation protocol considered in this work. In the simulations, the superframe consists of 16 slots for the beacon phase and an additional 80 slots for data transmission, so we set $x_m = 80$. As in the previous sections, the cost is assumed to be $c = 1$ and hence $p = c(x_m + 1) = x_m + 1 = 81$. By this, a player gains the maximum utility, when she transmits in the maximum number of slots x_m.

From Section 6.4 we know that players in the 2-player game have an incentive to choose an initial belief smaller than their opponent to maximize their share of resources. So the rational choice is to have $\mu_{1,0} = \mu_{2,0} = 0$. In the N-player game, we have to extend the notation and write $\mu_{i,j,0} = \mu_{j,i,0} = 0$ for all players $i \neq j \in \mathbf{N}$. Note that $\mu_{i,j,0}$ refers to player i's initial belief about player j. As illustrated in Figure 13, players can only be involved in collisions with players that occupy the slots adjacent to their owns. For this reason, players maintain a belief about both their predecessor and the subsequent player, thus, if we consider player i as an example, she maintains the beliefs $\mu_{i,j,t}$, with $j \in \{i-1, i+1\}$.

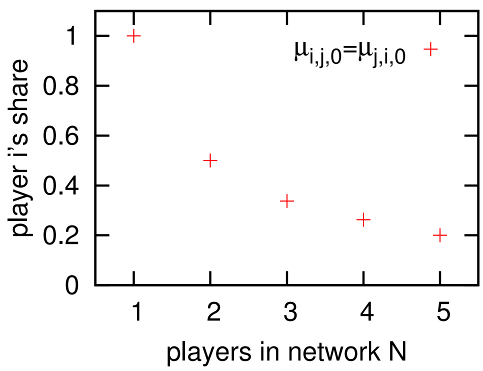

Figure 22: This figure depicts the share each player transmits in when the game has converged for different network sizes. Simulations have been performed with $\mu_{i,j,0} = \mu_{j,i,0}$, thus, all players receive the same share.

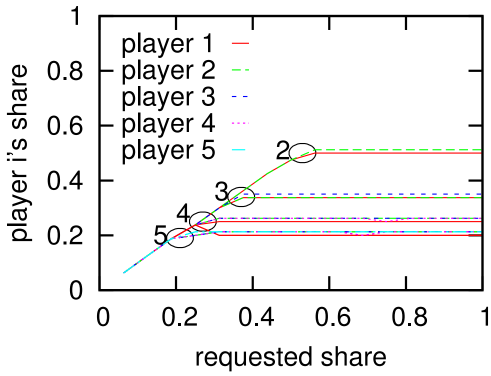

Figure 23: Slot allocation for discount factor $\delta = 0.5$, initial beliefs of $\mu_{i,j,0} = 0$ and $y_{max} = 1$. When saturation is reached, the algorithm settles to a fair equilibrium allocation as already indicated by Figure 22.

First, we consider the equilibrium allocation of the N-player game. Figure 22 depicts a player's share of resources in the equilibrium allocation for different network sizes. Here, the initial beliefs are set to $\mu_{i,j,0} = \mu_{j,i,0} = 0$ for all $i \neq j \in \mathbf{N}$, the maximum number of overlaps to $y_{max} = 1$ and the discount factor is set to $\delta = 0.5$. We observe that all players equally share available slots, so the finding that games with equal initial beliefs for all players reach a fair slot allocation, also holds for larger networks. Extensive simulations further suggest that if the players' initial beliefs are within the borders identified in the previous sections, the N-player game converges to nearly fair allocations which are omitted for reasons of clarity.

Figure 23 additionally depicts the slot allocation for different network sizes not only for greedy players but also for low-load scenarios. The simulations have been run with a discount factor set to $\delta = 0.5$, with initial beliefs of $\mu_{i,j,0} = 0$, for all $i \neq j \in \mathbf{N}$ and the maximum number of overlap $y_{max} = 1$. We observe that for low-load scenarios, i.e., when the sum of all players' requested shares does not exceed the available resources, all players can for obvious reasons transmit in the requested share. For high-load scenarios, the results of Figure 22 of players equally sharing resources can be observed again.

From the simulations presented so far, we conclude that the algorithm fairly distributes resources for both high- and low-load scenarios. However, there is a difference in the convergence time if we compare the scenario where the sum of the players' requests is just beyond the point of channel saturation and the scenario with greedy players. In the case of greedy players, initially there is one player that transmits in all available slots and all other players have to challenge. This basically implies that there is an immediate challenge to all players, which implies that all players' beliefs rise and players quickly retreat from their allocated slots.

Consider, however, a scenario where the sum of all players' requests is little more than the capacity of the channel. Assume, for instance, that all players' requests can be granted except the last player's request. While in the case of greedy players, all players except the first player challenged, here, in the beginning of the game, it is only the last player that challenges. Thus, for those players that are initially not involved in a collision, the number of superframes without collision increases. Recall that the belief given by (28) is the fraction of superframes with collisions over all superframes. By the time the first collision with a player occurs that is initially not involved in collisions the impact on the belief is very small and so is the belief itself. We conclude that the more time passes before the first collision occurs, the longer it takes for the slots to get redistributed and, hence, for the equilibrium slot allocation to be reached.

In the next paragraphs, we focus on the convergence time for greedy players and evaluate how it depends on the maximum number of overlaps y_{max} and the discount factor δ. If we fix those values, the convergence time varies with the choice of the initial beliefs $\mu_{i,j,0}$. In the case of equal initial beliefs, the convergence time is largest for $\mu_{i,j,0} = 0$ and decreases with increasing $\mu_{i,j,0}$. In the 2-player game, this decrease is explained by (32) and is transferable to the N-player game. In (32), the initial belief is subtracted to obtain the number of superframes with collisions in equilibrium ϕ_{t^*}, hence, a larger initial belief induces a smaller ϕ_{t^*}, thus, the convergence time decreases with the initial belief.

In Figure 24, we consider an initial belief of $\mu_{i,j,0} = 0$, which gives the upper bound of the convergence time. Figure 24 (left) plots the convergence time against different network sizes for different maximum number of overlaps y_{max}. In this simulations, the discount factor is set to $\delta = 0.5$. We observe that the larger the

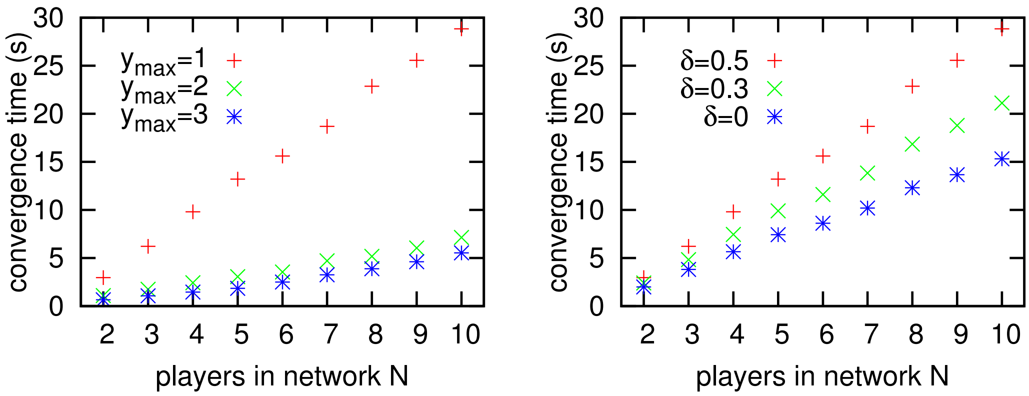

Figure 24: The convergence time grows linearly with the network size. In both figures, the initial beliefs are set to $\mu_{i,j,0} = 0$. The left figure shows that for $\delta = 0.5$ doubling the maximum number of overlaps from $y_{max} = 1$ to $y_{max} = 2$ leads to a significant drop in the convergence time. However, a further increase has only little impact. The right figure shows that for $y_{max} = 1$ the convergence time increases with δ.

network, the longer the algorithm needs to converge to the equilibrium allocation. The reason for this is that more slots need to be redistributed among the players if there are more players in the network. An increase of the maximum number of overlaps y_{max}, however, leads to a drop in the convergence time. The larger the number of overlaps, the more slots can be redistributed within a single superframe, thus, the overall time for the same redistribution is reduced. Note that in Figure 24 (left), we notice that the increase of the maximum number of overlaps from $y_{max} = 1$ to $y_{max} = 2$, results in a substantial decline of the convergence time, while the increase to $y_{max} = 3$ only slightly reduces the convergence time. This is due to the fact that the value for y_{max} is an upper bound for the effective amount of challenged slots per superframe. While at the beginning of the game, players make use of this upper bound, in the course of the game, the effective number of overlaps decreases.

Figure 24 (left) considered the impact of y_{max} on the convergence time. In contrast, Figure 24 (right) depicts the convergence time for different network sizes depending on the discount factor δ. We observe that the convergence time increases with the discount factor. Thus, the time to converge is minimized for $\delta = 0$. A large discount factor implies that players value the future. Hence, the larger the discount factor, the more do players consider future utilities when deciding whether or not to retreat from a challenged slot. Thus, the larger the discount factor, the less likely they are to retreat and it takes more time to redistribute the same amount of slots.

In this section, we evaluated games with different network sizes and traffic loads, varying the discount factor and the maximum number of overlaps. In all games, players started with an initial belief of $\mu_{i,j,0} = 0$, which marks the upper bound of the convergence time. We have seen that for any network size and traffic load, the game converges to the fair allocation of resources if players choose equal initial beliefs. Furthermore, we conclude that the convergence time increases with the network size and and the discount factor, while it decreases with the maximum number of overlaps.

6.7 SUMMARY

In this chapter, we extended the static game of Chapter 5 to a dynamic, i.e., repeated, game. In the dynamic game, players are able to learn from their opponents' behaviour. We showed that for almost all initial estimates about the opponents' behaviour the introduced belief update rule drives the game to an at least nearly fair resource allocation. The altered reservation method allows the resources to be distributed more evenly. This implies that some player's throughput will be increased at the cost of some other player's throughput. The network throughput, however, is at maximum decreased by the maximum number of overlaps $y_{\max}$, which are the challenged slots in a superframe. Corresponding to the rearrangement of resources, players' delays differ if we compare it with the original protocol. If we consider the average delay, though, it remains the same.

For the static game we introduced beliefs that players maintain in the course of the game. In the 2-player game, player 1's belief $\mu_{1,t}$ is her a-priori estimate in superframe t that player 2 follows through when she has challenged player 1's reservation. Player 2's belief $\mu_{2,t}$ is her a-priori estimate in superframe t that player 1 transmits when player 2 challenges her slots. Given those beliefs and the players' value of future utilities represented by the discount factor δ, we identified for which pairs of belief and slot allocation players are indifferent about challenging and not challenging, if player 2, or transmitting and not transmitting in a challenged slot, if player 1. Those pairs are potential candidates for Perfect Bayesian Nash Equilibria.

To determine the actual Perfect Bayesian Nash Equilibrium, however, we further require a belief update algorithm that complies with Bayes' rule. Thus, in this chapter, we presented a belief update algorithm that defines the rule how players adapt their beliefs given the other players' actions. The uncertainty in the belief is modelled by a beta-distribution which is known to follow Bayes' rule. So the belief is given as as the fraction of superframes in which a collision occurred. It is updated every superframe after observing the outcome of the superframe.

Having introduced the beliefs, the belief update algorithm and the candidates for Perfect Bayesian Nash Equilibria, we showed that for player 1's initial belief $\mu_{1,0}$ larger than player 2's initial belief $\mu_{2,0}$, i.e., $\mu_{1,0} > \mu_{2,0}$, the resulting slot allocation (x_{1,t^*}, x_{2,t^*}) is stable. In those stable allocations, player 1's number of slots x_{1,t^*} is smaller than player 2's number of slots x_{2,t^*}, so $x_{1,t^*} < x_{2,t^*}$. For equal initial beliefs $\mu_{1,0} = \mu_{2,0}$, the stable slot allocation is given by $x_{1,t^*} = x_{2,t^*}$. If player 1's initial belief is smaller than player 2's initial belief, i.e., $\mu_{1,0} < \mu_{2,0}$, the corresponding slot allocation (x_{1,t^*}, x_{2,t^*}) with $x_{1,t^*} > x_{2,t^*}$ is quasi-stationary for ν sequences.

Knowing the stable and quasi-stationary slot allocations, we evaluated their fairness. We showed that the fair slot allocation with $x_{1,t^*} = x_{2,t^*}$ is reached for $\mu_{1,0} = \mu_{2,0}$ only. For such equal initial beliefs, the allocation is independent of the discount factor δ and the parameters cost c and prize p. Given a target fairness index $f(u)$ and a discount factor δ, we further determined for which combination of initial beliefs this target fairness index is achieved.

If player 1's initial belief is larger than player 2's initial belief, player 2 gains towards player 1. So for player 2 it is rational to choose an initial belief $\mu_{2,0}$ as small as possible. Assuming that player 1 anticipates this, she also has an incentive to choose her initial belief $\mu_{1,0}$ as small as possible. Hence, if both players act rationally, they both choose $\mu_{1,0} = \mu_{2,0} = 0$, thus, the fair slot allocation arises.

Last in determining the actual Perfect Bayesian Nash Equilibria of the game, we showed for which initial beliefs or the respective slot allocations the game converges. We demonstrated that if player 2 transmits in the equilibrium point less than the fair share but beyond some threshold, the game converges to the quasi-stationary slot allocations. With equal initial beliefs, the game converges to the stable fair allocation with $x_{1,t^*} = x_{2,t^*}$. For all points, in which player 2 transmits in more than the fair share, the game converges, if some threshold $\epsilon_{\max}$ is not exceeded.

In this chapter, we identified the Perfect Bayesian Nash Equilibria depending on the players' initial beliefs and evaluated the fairness of those slot allocations. For our analysis, we assumed perfect observation, so players correctly interpret collisions and successful transmissions. Taking the previous findings into account, we can analyse the impact if we relaxed this assumption. So players would maintain different values for the number of superframes with and without collisions, ϕ_t and ψ_t, respectively, which in return would result in a different slot allocation in equilibrium. Consider for example that player 1's observation is perfect, while player 2 interprets some successful transmissions as collisions. Assume ϕ_{t^*} and ψ_{t^*} to be the values in equilibrium with perfect observation. Thus, for player 1 the number of superframes with and without collisions in the new equilibrium is unchanged and given as $\phi_{1,t^*} = \phi_{t^*}$ and $\psi_{1,t^*} = \psi_{t^*}$, respectively. For player 2, however, those are given as $\phi_{2,t^*} = \phi_{t^*} + \sigma$ and $\psi_{2,t^*} = \psi_{t^*} - \sigma$, with σ the number of incorrectly observed superframes.

Recall that player i's belief in superframe $t+1$ was given by (28). Player 1's equilibrium belief is equal to that in the scenario with perfect observation. Player 2's belief μ_{2,t^*+1}, however, is given by $\mu_{2,t^*+1} = \frac{\phi_{2,0}+(\phi_{t^*}+\sigma)}{\phi_{2,0}+(\phi_{t^*}+\sigma)+\psi_{2,0}+(\psi_{t^*}-\sigma)} = \frac{\phi_{2,0}+(\phi_{t^*}+\sigma)}{\phi_{2,0}+\phi_{t^*}+\psi_{2,0}+\psi_{t^*}}$. Note that instead of $\phi_{2,0} + (\phi_{t^*} + \sigma)$, we can also write $(\phi_{2,0} + \sigma) + \phi_{t^*}$ in the numerator and interpret σ to increase player 2's initial belief $\mu_{2,0}$. Thus, we compare the scenario in which player 2 does not perfectly observe the stages of the game with a game with perfect observation and the initial beliefs $\mu_{1,0}$ and $\mu_{2,0} + \sigma$. We have shown that most initial estimates result in a nearly fair slot allocation, so the impact of relaxing the assumption of perfect observation is likely to be negligible.

Finally, we performed simulations for larger networks and found that if players choose equal initial beliefs, the equilibrium allocation is fair for the N-player game. We further observed that the convergence time increases linearly with the network size as more slots have to be redistributed. It also increases with the discount factor δ but decreases with the maximum number of overlaps y_{max}.

In this chapter, we showed that the introduced algorithm that relaxes the reservation rules guides the dynamic game to a fair slot allocation. In Chapter 4, the distributed reservation-based protocol that uses a beacon phase with fixed beacon order to organise medium access such as the ECMA-368 was shown to result in an unfair slot allocation. In this chapter, we demonstrated that a slight alteration of the reservation rules that comes at the cost of a limited amount of collisions per superframe bounded by the maximum number of overlaps $y_{\max}$ induces fairness without requiring the nodes in the network to be altruistic and behave nicely.

7

CONCLUSIONS AND FUTURE WORK

In this thesis, we contributed a fairness analysis of a distributed reservation-based medium access control protocol and developed an algorithm that alters the reservation mechanism in order to overcome the identified unfairness. The analysed protocol and the introduced algorithm are modelled using game theory to capture the strategic interaction between the nodes in the network. Finally, we showed that the engineered algorithm drives the game to a fair equilibrium. The distributed reservation-based protocol considered in this work was designed for high data rate networks with small coverage such as multimedia home environments. Assuming that those networks consist of a limited number of nodes, the algorithm presented scales well. Especially, when we consider applications that are active for a longer time, the convergence time amortises. In the following, we summarize the conclusions of our work and convey possible future extensions.

We presented an analysis of the distributed reservation protocol specified in ECMA-368 and showed how throughput, delay and fairness depend on the reservation rules given in ECMA-368. For the analysis, we implemented a Java tool that determines all feasible reservation patterns and weights them with their probability to occur given a Poisson model of the frame arrival process. We showed that throughput and delay depend on the position of a node's beacon in the beacon phase. In high-load scenarios the impact of the reservation rules in ECMA-368 is insignificant or quickly lessens with growing network size. Hence, we revealed potential unfairness inherent in the protocol due to its first-come, first-served reservation method. The earlier a node's beacon in the beacon phase, the more privileged it is and thus, the better she perceives the fairness of the system. In contrast, nodes that transmit their beacons towards the end of the beacon phase have to cope with the remaining time and thus, have a degrading fairness for high-load scenarios.

We discover that the network size does not influence the amount of resources a node can allocate. Once the channel is saturated, any additional node does not transmit at all, while the previous nodes' share stays the same. Thus, we conclude that the fairness issue aggravates with increasing traffic. For this reason, the subsequent analyses considered greedy nodes. The fixed beacon order has been identified as the reason for the unfairness. However, we showed that both an alteration of the beacon phase to provide for round-robin beaconing and the randomization of the beacon phase only lead to long-term fairness and neither of them can provide short-term fairness.

Game theory is a proven remedy to modelling strategic interaction. So, we modelled the distributed reservation-based protocol with fixed beaconing as a static multi-stage game. For this game, we determined the Nash Equilibria and subgame-perfect equilibria. All Nash Equilibria are Pareto-optimal, i.e., there is no strategy profile that increases one player's payoff without decreasing another player's payoff. Considering linear utility functions, all Nash Equilibria are further socially-optimal. Thus, all Nash Equilibria maximize society's welfare, which is defined as the sum of all players' utilities. The Nash Equilibrium, in which resources are equally distributed, is furthermore max-min and proportionally fair and maximizes Jain's fairness index. This particular Nash Equilibrium is additionally unique in that it complies with all fairness criteria and is socially-optimal, if we choose the utility function to be monotonically increasing and strictly concave according to the law of diminishing returns. This utility function reflects the application's view point, if we consider the application to first transmit some basic information that is enhanced with the information carried in the subsequent frames.

Assuming that players are not altruistic, the fair Nash Equilibrium with equally shared resources does not arise. As a remedy, we introduced a relaxed reservation method that provides discriminated players with a means to enhance their share. This method requires players to maintain an estimate about their opponent's behaviour, so we determined the Bayesian Nash Equilibrium of the corresponding 2-player game. The direct attainment of a Bayesian Nash Equilibrium is complex, therefore, we repeatedly played the static game whereby we generated a dynamic game. In this dynamic game, players observe and learn from their opponent's behaviour. After every period of the game, i.e., every superframe, they update their estimates according to a predefined principle that complies with Bayes' rule. The updated estimate then directly influences their actions in the next superframe.

In this dynamic game, we showed that the game converges to a stable fair resource allocation, if both players choose the same initial estimates about each other's behaviour. The game also converges to a stable allocation, in which the second player gains a larger share than player 1, if her estimate is smaller than player 1's and some threshold is met. However, if the second player's estimate is larger than her opponent's estimate, the game converges to an allocation that is quasi-stationary for a defined time and in which the first player gains the larger share. While only equal estimates result in the fair Perfect Bayesian Nash Equilibrium, almost all remaining estimates induce nearly fair allocations.

If we assume player 2 to be rational, she chooses an initial estimate as small as possible as this maximizes her share of resources. Assuming that player 1 anticipates this behaviour, she also minimizes her initial estimate for the same reason. Thus, the rational set of initial estimates is that both are equal and zero. Hence, for rational players the fair equilibrium is reached. In this fair Perfect Bayesian Nash Equilibrium, the result is independent of the discount factor δ that connotes the value the future has for the players. For unequal initial estimates that lead to nearly fair allocations, though, a large discount factor increases the fairness. With a large discount factor, player 1 is less willing to pass on slots, so player 2 cannot gain much more than the fair share.

Simulations extended the analysis to larger networks and indicated that the results of the 2-player game can be extended to the N-player game. So equal initial estimates lead to a fair resource allocation among the nodes in the network. Finally, simulations showed that the convergence time increases linearly with the network size. It also grows with the discount factor, because nodes that value the future are less likely to retreat from challenged slots. The convergence time, however, decreases with the maximum overlap of the relaxed reservation method, since this induces that more slots can be redistributed at the same time.

In this work, we showed that a distributed reservation-based medium access control protocol that uses a beacon phase with fixed beacon order to organise medium access such as the ECMA-368 results in an unfair slot allocation. However, we demonstrated that a minimal alteration of the reservation rules that comes at the cost of a finite number of collisions per superframe yields fairness. In particular, the introduced algorithm makes fairness self-enforcing, so the nodes in the network are not required to be altruistic, but are assumed to maximize their own and personal utility functions. Hence, with the provided alteration of the reservation rules, the properties of predictable and guaranteed medium access in a distributed system without central coordinator can be enriched with the property of fairness.

In order to extend this work, the simulation results that regard the N-player game as well as the impact of the system parameters should be analytically verified to complete the analysis. Furthermore, alternative belief update algorithms could be evaluated to reduce the convergence time to make it more applicable for larger networks. Recall that we argued that the more superframes without collisions occur, the longer it takes before slots can be redistributed. It is inherent in the belief update algorithm that the system becomes attenuated in those situations. Therefore, future approaches for the belief update should deal with the occurrence of such situations.

Part II

APPENDIX

GLOSSARY OF ACRONYMS

BNE	Bayesian Nash Equilibrium
CTS	Clear to send
DRP	Distributed Reservation Protocol
EDCA	Enhanced Distributed Channel Access
FCC	Federal Communications Commission
iid	independent and identically distributed
MAC	Medium Access Control
MB-OFDM	Multi-band Orthogonal Frequency Division Multiplexing
NE	Nash Equilibrium
PBE	Perfect Bayesian Nash Equilibrium
PCA	Prioritized Channel Access
QoS	Quality of Service
RTS	Request to send
SNR	Signal-to-noise ratio
SPE	Subgame-perfect Equilibrium
USB	Universal Serial Bus
UWB	Ultra Wideband
WLAN	Wireless Local Area Network
WPAN	Wireless Personal Area Network

GLOSSARY OF SYMBOLS

$\mathbf{A}_i(h^k)$	player i's set of feasible actions given the history h^k
a^k_{-i}	actions chosen in stage k by all players except player i
a^k	action profile at stage k
a^k_i	action chosen by player i in stage k
c_i	player i's cost for transmitting in a single slot
f_f	fractional bandwidth
$f(u)$	fairness index given the utility profile u
h^k	history of the game at stage k
i, j	player indices i, j
l	frame length
m_{slot}	#frames that can be transmitted in a single time slot
N	#players in the network
p_i	prize player i gains when successfully transmitting in a single slot
period	periods of the game refer to superframes
R	transmission bit rate
r_i	#slots player i successfully transmits in
S^*	strategy profile in equilibrium
S_{-i}	strategy profile for all players except player i
s_i	strategy of player i
S	strategy profile
stage	stages of the game refer to beacon slots 1-N and the transmission phase
T_{SF}	superframe length
T_{slot}	slot length
$u_{i,c}(x_i)$	player i's utility from transmitting in x_i, of which the transmission in one slot is unsuccessful
$u_{i,s}(x_i)$	player i's utility from successfully transmitting in x_i slots
$u_{i,t}$	player i's utility in superframe t
u_i	player i's utility
u	utility profile
w_i	#slots player i unsuccessfully transmits in
$x_{i,t}$	#slots player i transmits in in superframe t
x_f	fair allocation mark

x_m	maximum #slots available for data transmission in a single superframe
y_{max}	maximum #overlaps a player is allowed to do per superframe
α, β	parameters of the beta-distribution
δ	discount factor, i.e., value of the future
ϵ_{max}	maximum value ϵ might have for the game to meet the upper bound for convergence
ϵ	difference of remainder of ceiling operation for ϕ_{t^*} and $\phi_{\mathrm{k,up}}$
$\gamma_i(r_i)$	player i's gain from successfully transmitting in r_i slots
Γ	non-cooperative game
$\xi(r_i, w_i)$	player i's costs from transmitting in $r_i + w_i$ slots
λ_{slots}	mean #slots required to support application with frame arrival rate λ
λ	frame arrival rate
$\mu_{i,t}$	a-priori belief of player i in superframe t
ν	#sequences of $(1 + \Delta t)$ superframes that the slot allocation x^* is quasi-stationary
μ_i^*	player i's belief in equilibrium
$\phi_{i,0}$	initial #superframes with collisions for player i at $t = 0$
$\phi_{i,t}$	#superframes with collisions player i has seen until superframe t
ϕ_{Sl}	l'th segment of #superframes with collisions
ϕ_{t^*}	#superframes with collisions until the equilibrium in superframe t^* is reached
ϕ_t	#superframes with collisions until superframe t
$\psi_{i,0}$	initial #superframes without collisions for player i at $t = 0$
$\psi_{i,t}$	#superframes without collisions player i has seen until superframe t
ψ_{Sl}	l'th segment of #superframes without collisions
ψ_{t^*}	#superframes without collisions until the equilibrium in superframe t^* is reached
ψ_t	#superframes without collisions until superframe t
$\sigma_i(\theta_i)$	behavioural strategy of player i given that she is of type θ_i
Θ_i	set of possible types of player i
θ_i	player i's type
$\blacksquare$	Halmos symbol to end a proof

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