Essays on Organizational Design and Teams

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Abstract

Since Adam Smith's *"The Wealth of Nations"* the division of labor and specialization of workforce have been viewed as an elementary organizational principle. Recent theoretical and empirical research has brought into consideration broader elements of organizational design, focusing on flexible workplace practices, the structure of incentives, and the behavioral aspects of workplace interactions. However, the theoretical literature mainly considers team building and organizational design as exogenous parameters. This dissertation endogenizes these choice variables, focusing on organizational design and teams from three divergent perspectives: that of agents, that of managers, and that of firms.

The first chapter develops a group work model with endogenous team size and analyzes how overconfidence influences optimal size and welfare of teams. The second chapter endogenizes a manager's optimal number of direct reports and shows how managers can exploit their organizational authority to shield themselves against replacement. The third chapter investigates a firm's optimal organizational strategy and offers a new informational and incentive-based explanation for the use of job rotation.

This dissertation enriches the debate on factors relevant to decisions about team size, contributes to the literature on incentives created by organizational change and, more generally, to the research on endogenous job design. Our results outline that overconfidence fosters team formation and can increase the welfare of agents in team settings. Team size and task allocation also play roles in maintaining managerial power through the strategy of "divide et impera". Questions of optimal task assignment and organizational design show how in the presence of implicit incentives job rotation may emerge endogenously.

Keywords: Team size, Overconfidence, Implicit incentives, Task delegation, Job rotation

Zusammenfassung

Seit Adam Smith und *"Der Wohlstand der Nationen"* gelten Arbeitsteilung und Arbeitsspezialisierung als ein elementares Organisationsprinzip von Unternehmen. In der neueren theoretischen und empirischen Forschung wird über weitergehende Aspekte einer optimalen organisatorischen Struktur diskutiert. Diese fokussieren sich beispielsweise auf flexible Arbeitsmethoden, die Gestaltung von Anreizen wie auch auf verhaltenspsychologische Aspekte. Dennoch betrachtet die theoretische Literatur Teambildung und Organisationsgestaltung als exogene Faktoren. Die vorliegende Dissertation endogenisiert diese Entscheidungsvariablen und untersucht Teams und die Frage des optimalen Organisationsdesigns aus drei verschiedenen Perspektiven: der von Agenten, der von Managern und der von Firmen.

Das erste Kapitel präsentiert ein Modell koordinierter Zusammenarbeit mit endogener Teamgröße und analysiert, wie sich Selbstüberschätzung auf die optimale Teamgröße und Wohlfahrt der Teammitglieder auswirkt. Das zweite Kapitel untersucht, wie Manager ihre organisatorische Weisungsbefugnis durch Aufgabendelegation zum Zwecke persönlicher Machterhaltung im Unternehmen nutzen können. Das dritte Kapitel befasst sich mit der Frage der optimalen Organisationsstruktur in Unternehmen und zeigt eine neue anreiz- und informationsbasierte Erklärungsgrundlage für die Implementierung von Arbeitsplatzrotation.

Die vorliegende Dissertation bereichert die kontroverse Debatte hinsichtlich der relevanten Entscheidungsfaktoren für die Wahl der optimalen Teamgröße, trägt zur Literatur über Anreizsteuerung durch organisatorische Veränderungen sowie allgemein zur Forschung über endogene Arbeits- und Organisationsgestaltung bei. Unsere Modellergebnisse belegen, dass Selbstüberschätzung Teambildung fördert und die Wohlfahrt von Agenten in Teams erhöhen kann. Aspekte der optimalen Teamgröße und Aufgabenallokation spielen auch in der Frage der Machtbefugnis und -erhaltung von Managern durch die Strategie von "divide et impera" eine Rolle. Die Problematik der optimalen Aufgabenallokation und Organisationsgestaltung zeigt, wie bei impliziten Anreizen Arbeitsplatzrotation endogen entstehen kann.

Schlagworte: Teamgröße, Selbstüberschätzung, Implizite Anreize, Aufgabendelegation, Arbeitsplatzrotation

Contents

Preface

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Introduction

Since Adam Smith's *"The Wealth of Nations"* the division of labor and specialization of workforce have been viewed as an elementary organizational principle. Recent theoretical and empirical research has brought into consideration broader elements of organizational design, focusing on flexible workplace practices, the structure of incentives, and the behavioral aspects of workplace interactions. However, the theoretical literature mainly considers team building and organizational design as exogenous parameters. This dissertation endogenizes these choice variables, focusing on organizational design and teams from three divergent perspectives: that of agents, that of managers, and that of firms. Specifically, this dissertation consists of three theoretical models that are a joint work with Hendrik Hakenes.

The first chapter develops a basic group work model with endogenous team size. In teams that are too small, complementarities cannot develop properly, whereas in teams that are too large, the free-rider problem becomes overwhelming. If team members are overconfident, effort levels increase, and the free-rider problem is partially resolved. Under certain conditions, optimal team size increases. However, even if coworkers' efforts are substitutes, overconfidence can drive team-building. Agents are then prevented from overinvesting in effort, leading to welfare improvements at the individual level.

The second chapter endogenizes a manager's optimal number of direct reports and shows how managers can exploit their organizational authority to shield themselves against replacement. Although the probability of hiring a star performer increases with the number of direct reports, each employee completes a smaller fraction of the overall task, such that learning about the employees' individual abilities occurs more slowly. We show that a manager maximizes the probability of retaining his job if he delegates a task to an infinite number of employees. Through the trade-off for the manager between decreasing his private costs of being replaced and increasing labor coordination costs, our model derives predictions of when managers tend to choose an excessively large number of direct reports, creating inefficiencies at the firm level.

The third chapter analyzes a firm's optimal organizational strategy and offers a new informational and incentive-related explanation for the use of job rotation. The longer an agent is employed in a job, the more the principal will have learned about his ability through the history of performance. With implicit incentives, influence perceptions and effort incentives decrease over time. Rotating agents to a different job deletes learning effects about ability, creating fresh impetus for effort. However, job rotation also reduces the time horizon, and thus reduces rents from working and also incentives. In this trade-off, we derive conditions for the desirability of job rotation and show how in the presence of career concerns job rotation may emerge endogenously. Finally, our model allows for more general comments on the optimal rotation frequency as well as the preferred organizational design of a firm.

This dissertation enriches the debate on factors relevant to decisions about team size, contributes to the literature on incentives created by organizational change and, more generally, to the research on endogenous job design. Our results outline that overconfidence fosters team formation and can increase the welfare of agents in team settings. Team size and task allocation also play roles in maintaining managerial power through the strategy of "divide et impera". Questions of optimal task assignment and organizational design show how in the presence of implicit incentives job rotation may emerge endogenously. Thus, going beyond traditional views of organizational structure, this dissertation takes a broader perspective, one that encompasses the behavioral effects of workplace interactions, informational and incentive-based considerations, to offer insights relevant to an expanded concept of optimal organizational structure.

1 Optimal Team Size and Overconfidence[1]

Abstract

How large should a team be? We develop a basic group work model with endogenous team size. In teams that are too small, complementarities cannot develop properly, whereas in teams that are too large, the free-rider problem becomes overwhelming. If team members are overconfident, effort levels increase, and the free-rider problem is partially resolved. Under certain conditions, optimal team size increases. However, even if coworkers' efforts are substitutes, overconfidence can drive team-building. Agents are then prevented from overinvesting in effort, leading to welfare improvements at the individual level.

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1.1 Introduction

Teams are an important organizational structure, both inside and outside of corporations. In professional life, teams of varying sizes are increasingly common throughout organizations. Some teams are small: start-up businesses are initially small, and in the professions, partnerships often have only two members. Some are extremely large: international accounting firms may comprise thousands of partners. So how large should a team optimally be? On the one hand, complementarities can be better exploited in larger teams, increasing productivity. On the other hand, a large team size can occasion productivity losses due to free-rider problems. The existing literature, starting with the seminal work of Marschak and Radner (1972), emphasizes that team size is an elementary structural variable determining team performance. However, this literature views team size as an exogenous parameter. In reality, the size of teams is often a choice variable influenced by various economic factors. Surprisingly, the question of the optimal size of a team has seldom been addressed by economic theory.

In this paper, we first construct a basic model with rational agents who can form teams to benefit from complementarities. Nevertheless, because effort levels are unobservable, there is a free-rider problem that increases with team size, and as a result, there is an optimal team size. We incorporate into our model the finding that people tend to be overconfident about their skills and, therefore, may not behave as predicted by standard economic theory. Behavioral characteristics are especially influential in the context of teams, as overconfidence affects individual comparisons and, therefore, influences the behavior of each team member. Intuitively, an agent who believes that his personal ability is greater than that of his coworkers might not be interested in teamwork. Consequently, with overconfidence, teams may potentially be too small. So, why do overconfident agents join teams at all? One obvious answer is that, although overconfidence is not individually optimal in teams, agents can benefit from the increased effort of their overconfident team members through complementarities. However, from the perspective of agents' welfare, overconfidence can drive team-building, even if individual contributions act as substitutes for each other, as it prevents agents from making costly overinvestments in effort. Therefore, the aim of our paper is to identify the factors relevant to decisions about team size and, thus, contribute to the debate on optimal team design.

Teams of varying size are prevalent in a wide range of contexts. They can occur as independent economic entities, consisting only of team members, or they may be components within firms. In contrast to the standard agency framework, where principal and agents are integrated into a hierarchical organizational structure, partnership members have joint ownership of assets and simultaneously act as agents in production. According to Levin and Tadelis (2005), partnerships emerge as a desirable organizational form when production is largely based on human capital, as profit sharing structures make agents more selective with regard to whom they accept as partners. Precisely because behavioral biases can affect agents' expectations regarding (relative) ability, overconfidence determines team formation and optimal team size in partnerships. Thus, overconfidence has been shown to be characteristic of entrepreneurs (Cooper, Woo, and Dunkelberg (1988), Camerer and Lovallo (1999), Koellinger, Minniti, and Schade (2007), Cassar (2010), Townsend, Busenitz, and Arthurs (2010)) and more prevalent among firm owners than managers within firms (Busenitz and Barney (1997), Forbes (2005)). Therefore, this study mainly pertains to interdependent self-managed teams of entrepreneurs, including professionals or business partnerships in law, architecture, or investment banking. However, our work can also be relevant to teams within corporations, including engineers who share bonuses on projects, sales teams with group bonuses, or also professional sport teams. More generally, our model also pertains to other types of cooperation, such as strategic alliances or networks, where teams collaborate on a cross-company basis. Prominent examples of the latter include airline alliances, networks in the automotive industry, and intra-firm partnerships in the computer and communication industry. Co-partnership is also adopted by public agencies or governmental institutions. In between, public-private partnerships of public authorities and private industry are, for example, widely used in the fields of R&D, infrastructure, and health care.

Although partners operate as firm owners, they are exposed to moral hazard. If effort is unobservable, profit sharing may lead to free-rider problems in teams. While a smaller partnership size can reduce free-riding, it may also entail synergy losses. Correspondingly, partnerships in medical practices usually consist of small teams. In comparison, in accounting or consulting, they often operate as medium-sized and large partnerships relying on international expertise that requires extensive cooperation among team members. In addition, academic authorship shows large variations in team size: in large-scale scientific experiments, as in physics or clinical trials, heavy collaboration is required, with teams often consisting of thousands of members. By contrast, in economics, teams of two or three authors are much more common. Therefore, we seek to shed light in the analysis below on why these differences in team size exist.

Literature. This paper is relevant to the literature on teams and team size as well as to research on overconfidence in groups. Beginning with the conception of Alchian and Demsetz (1972) that the existence of complementary activities may encourage team formation, Holmström (1982) considers the problem of free-riding when team members' individual contributions are not perfectly observable. Similarly, Kandel and Lazear (1992) focus on profit sharing in partnerships, finding that free-rider problems lead to declining equilibrium effort levels when team size increases. In contrast, profit sharing is a widely applied and favorably regarded reward system for providing effort incentives in teams (Weitzman and Kruse (1996), Prendergast (1999)). Consequently, complementarities may benefit firm-wide incentives in partnerships, even in the presence of free-riding (Adams (2006)).

Although the literature on teams has a lengthy history, it continues to view team size as an exogenous parameter. Only recently have contributions sought to endogenize the determination of optimal team size. Qian (1994) investigates the optimal span of control and wage levels for different hierarchical tiers of a firm. Similarly, Ziv (1993, 2000) analyzes the relationship between internal information structure, optimal size, and the hierarchical composition of a firm. Related to our work, Huddart and Liang (2005) study optimal task assignment and team size in partnerships in which agents exert effort in both production and monitoring. Although monitoring activities may increase incentives in production, they are also subject to free-riding. As a result, small teams are optimal if task assignment is symmetric. Otherwise, specialization of agents into either production or monitoring can contain shirking in both tasks and benefit large teams. Similarly, Liang, Rajan, and Ray (2008) analyze optimal team size, monitoring activity, and incentive contracts for managers and workers. In their principal-agent framework, the variance of performance measurement increases with team size, requiring higher levels of monitoring to sustain productivity. To derive the basic trade-offs with respect to the size and performance of groups, this paper abstracts from monitoring activities and from interdependencies between managers and a principal and focuses on the fundamental idea of group work as the interaction of coequal agents. In addition, our model accounts for behavioral effects that are fundamental to the question of optimal organizational design.

Recent economic literature has introduced the psychological finding that agents tend to overestimate their relative abilities (Taylor and Brown (1988), De Bondt and Thaler (1995)) and are overoptimistic about future outcomes (Weinstein (1980)). The core idea is built on the insight that a positive self-image and optimistic goal-setting correlate with increased motivation and greater effort (Felson and Reed (1986), Latham and Locke (1991), Benabou and Tirole (2002)). Specifically, even though informed agents should anticipate personal overconfidence in equilibrium, empirical evidence suggests that individuals evaluate themselves more optimistically than do others (Lewinsohn, Mischel, Chaplin, and Barton (1980)), believe that they are better than other people (Taylor and Brown (1994)), expect others to have overoptimistic self-perceptions (Krueger (1998)), and “agree to disagree” with the perspectives of other individuals (Hales (2009)). In addition, in the theoretical literature, heterogeneous prior beliefs are not only assumed to be essential to understanding economic problems (Morris (1995)) but may reflect overconfidence, as in Van den Steen (2004), where individuals disagree about the individual probabilities of success. Similarly, overconfidence is modeled as agreement to disagree about skill perceptions (Odean (1998), Fang and Moscarini (2005), Gervais and Goldstein (2007), Santos-Pinto (2008, 2010), Gervais, Heaton, and Odean (2011)) or also the likelihood of favorable outcomes (de la Rosa (2011)). While recognition of discrepant beliefs helps us abstract from screening and signaling devices in our model and, thereby, extract a pure overconfidence effect, Squintani (2006) shows the existence of an equilibrium in which overconfident players correctly anticipate each others’ strategies.

With regard to business settings, Hvide (2002) emphasizes that overconfidence increases an agent’s expectations of his outside opportunities in the job market and therefore can raise his bargaining power vis-à-vis the firm. Gervais and Odean (2001) present a multiperiod learning model in which overconfidence is endogenous. In their model, perceptions of ability increase with success and decrease with failure, so that self-evaluations become more accurate over time. Goel and Thakor (2008) analyze the interaction between managerial overconfidence and investment choices, effort incentives, promotion prospects, and compensation contracts. Related to overconfidence effects in teams, Bernardo and Welch (2001) show that overconfident entrepreneurs utilize their own information and are less likely to engage in herd behavior, which provides valuable information to their social group. However, groups in which many entrepreneurs rely on personal information make more mistakes and therefore suffer from attrition. As a result, moderate levels of overconfidence can be valuable if groups are sufficiently large to benefit from the information externality. Corgnet (2010) investigates the role of information in the formation of partnerships in the presence of overconfidence. Claims of excessive output shares induced by overconfidence negatively affect cooperation. Only with incomplete information about a partner's ability can overconfidence enable efficient team formation, as it reduces the informational rent of workers. Santos-Pinto (2010) analyzes overconfidence effects in tournaments. In his model, overconfidence can increase a firm's welfare, while it decreases the individual utilities of agents. Fang and Moscarini (2005) investigate the effect of overconfidence on optimal incentive contracts when a principal faces many agents. First, an independent wage policy allows the firm to tailor contracts to agents' expected abilities (sorting effect). Second, overconfident agents may react asymmetrically to good vs. bad outcomes (morale effect) such that, given overconfidence, a non-differentiation wage policy may be optimal. Similarly, Santos-Pinto (2008) considers a principal agent setting in which two agents have correct beliefs about personal ability but mistaken beliefs about coworker abilities. In this case, the principal can benefit from overconfidence if both agents are offered interdependent incentive schemes.

This paper mostly pertains to Gervais and Goldstein (2007), who develop a teamwork model with one rational and one overconfident agent. In the presence of complementarities, the enhanced effort of the biased agent increases the rational agent's effort incentives, resulting in increased profits for both workers. In our work, the advantageousness of teamwork in the presence of overconfidence is not necessarily based on the existence of complementarities between agents. As we endogenize team size, our model suggests that, even if team members' outputs are substitutes, team formation can benefit overconfident agents. Altogether, we are able to apply behavioral effects to a wider range of settings of team interaction, analyze the implications of varying group sizes on individual performance with overconfidence, and model the impact of overconfidence on the choice of optimal team size.

The remainder of this paper is organized as follows. Section 1.2 develops the basic model with rational agents, and section 1.3 provides the equilibrium analysis. Section 1.4 introduces overconfidence into the model, followed by a welfare analysis in section 1.5. Section 1.6 provides evidence regarding overconfidence and discusses the implications of the model. Section 1.7 contains our conclusions. All proofs are in the Appendix.

1.2 The Model

We consider an infinite pool of identical risk-neutral agents who can form teams for the purpose of production. The size of each team is endogenously chosen at an initial stage. Outcomes are independent between teams. Let $n \geq 1$ denote the set of agents on a team that generate a joint one-period output of

$$Y = \sum_{i=1}^{n} y_i + \kappa \sum_{i=1}^{n} \sum_{j \neq i}^{n} y_i \, y_j, \tag{1.1}$$

where the individual contributions are given by

$$y_i = a_i\, e_i.$$

Joint output Y is the sum of two components: the first component is the sum of individual outputs y_i of all team members. The choice of effort $e_i \geq 0$ of each agent is endogenous. The parameter $a_i > 0$ measures an agent's ability. Hence, for the same effort and cost of effort, workers of high ability contribute more than those of low ability, with skill and effort representing complements in production.[2] We assume that ability is equal for all agents, $a_i = a$, in situations where the production technology requires workers of a certain quality level. The second component is an interaction term measuring synergies obtained from group work, determined by the parameter κ. Thus, team output is additive in individual outputs and multiplicative in the combined outputs of two team members at a given time, additive over all pairs of agents in a team, yielding $n\,(n-1)/2$ combinations. Thus, if $\kappa > 0$, team members' outputs are complements in production, with the output of one agent raising the productivity of each coworker.[3] In contrast, if $\kappa < 0$, the productivity of an agent decreases through the interaction with other team members in which case agents' outputs are substitutes. In the absence of any externalities, $\kappa = 0$, joint output is the sum of the team members' individual contributions.[4] Each worker exerts effort at a private cost of

$$c(e_i) = c\,\frac{e_i^2}{2}. \tag{1.2}$$

We assume that effort decisions are non-contractible, as they are made simultaneously and are not observable by other agents. The joint output of a team, given in (1.1), is the only observable measure of inputs. Focusing on symmetric equilibria, joint output is equally divided among all team members. Hence, agents are rewarded not only on the basis of their own performance but also on the basis of the performance of fellow team members: this is the source of the free-rider problem. The extent of the free-rider problem is determined by the cost factor $c > 0$ of the effort cost function.

Agents are effort-averse and benefit from compensation. Team output is Y, as in (1.1); hence, the per-person expected end-of-period output is $E[Y]/n$. Agents thus

[2]Synergetic effects between ability and effort are outlined in the effort-performance literature (see, Bonner and Sprinkle (2002), for an overview) and commonly applied in teamwork models (see, for example, Gervais and Goldstein (2007), Liang, Rajan, and Ray (2008) or Santos-Pinto (2008)).

[3]To simplify our analysis, we assume constant returns to scale. However, our general results also apply to the case of decreasing returns to scale.

[4]In less collaborative teams, joint output mostly depends on the team members' individual contributions, and complementarities are small. As an example, in a team of call center agents, calls are served independently, and only a small amount of coordination is required. In contrast, a software development team involves extensive coordination among analysts, database administrators, support engineers, designers, architects, and programmers. In this case, joint output mainly depends on the combined outputs of all team members rather than on their individual contributions. Possibly, team members can also reduce each other's positive effects. An example is a team of emergency physicians. Increasing team size can increase coordination requirements and reduce the effectiveness of emergency assistance.

maximize their individual profit net of effort costs,

$$\max_{e_i} E[\Pi_i] = \frac{E[Y]}{n} - c(e_i) = \frac{1}{n}\left(\sum_{i=1}^{n} y_i + \kappa \sum_{i=1}^{n}\sum_{j\neq i}^{n} y_i\, y_j\right) - c\,\frac{e_i^2}{2}. \tag{1.3}$$

Because the incentive problems facing each agent and team are identical, we can limit our analysis to one exemplary team with $n \geq 1$ agents.[5] The timing of the model is as follows.

$t = 0$ Agents form production teams. The team size $n \geq 1$ is endogenously chosen by the agents.

$t = \frac{1}{2}$ All agents simultaneously exert effort e_i, based on the expected efforts of fellow team members.

$t = 1$ Joint output Y is realized and equally distributed among team members.

The formation of teams is similar to the formation of coalitions in cooperative game theory. Therefore, we solve for coalition-proof Nash equilibria (see Bernheim, Peleg, and Whinston (1987)). The idea is that a set of teams is an equilibrium if no team has an incentive to break apart in order for members to join other teams.

1.3 Equilibrium Analysis with Rational Agents

In order to determine the optimal team size of rational agents, we first determine the agents' equilibrium effort levels. More specifically, we examine a situation in which agents cannot observe their coworkers' efforts and in which they thus anticipate the equilibrium values. In equilibrium, each agent's effort is the best response to the expected effort of his fellow team members. Let us consider the decision problem from the perspective of a representative agent $i = 1$ collaborating with $n - 1 \geq 0$ identical coworkers. Then, replacing y_1 by $a\,e_1$ and y_j by $a\,e_j$ for $j \neq 1$ in the individual profit function and solving for agent 1's first-order condition, we have

$$e_1^* = \frac{a}{n\,c}\,\big((n-1)\,\kappa\, a\, e_j^* + 1\big)\,, \tag{1.4}$$

where "starred" variables denote equilibrium values. As all agents are of the same ability level, they anticipate that peers will exert the same effort level that they themselves exert. Then, in a symmetric equilibrium, all effort choices are identical, $e_1^* = e_j^* = e^*$. This yields the following lemma.

[5] We abstract from the problem of integrity, except in cases where n falls below 1. As aggregate output is the only contractible variable, note that for $n = 1$, output is completely assignable to the agent producing it. Consequently, there is no free-riding. It follows that if the algebraic term falls below 1, output would be more than assignable, which would result in a negative free-rider problem. Intuitively, this equilibrium is not feasible. The general intuition is that for $n < 1$, one would prefer smaller teams. In this case, we focus on the boundary solution of $n = 1$.

Lemma 1 *The equilibrium effort of rational agents is*

$$e^* = \frac{a}{n\,c - (n-1)\,\kappa\,a^2} \tag{1.5}$$

for $\kappa < c/a^2$ if $n > 1$. For $\kappa \geq c/a^2$ and $n > 1$, the equilibrium effort is infinitely large.

Lemma 1 outlines several characteristics of rational agents' equilibrium effort. Intuitively, for $n > 1$, the synergy term κ raises the productivity of each coworker in a team and therefore increases each agent's chosen effort level. In addition, as skill and effort are complements in production, optimal effort increases with ability level a. Consequently, if $\kappa > 0$, the positive impact of complementarities (and ability level) on output increases with team size. However, the larger the team is, the smaller the feedback effect of personal effort on the equally-divided team output. That is, the free-rider problem increases with team size. For $\kappa < c/a^2$, the equilibrium effort decreases with team size. This result holds even more strongly if agents' efforts are substitutes, $\kappa < 0$, as then both κ and c reduce effort incentives as team size increases.

If each agent chooses e^*, we can incorporate the equilibrium effort into the aggregate output function, where $y_1 = a\,e^*$, and $y_j = a\,e^*$ for all $j \neq 1$. This entails that rational agents' expected individual profit depends on team size:

$$E[\Pi] = \frac{a^2\,\left(2\,n\,c - c - (n-1)\,\kappa\,a^2\right)}{2\,\left(n\,c - (n-1)\,\kappa\,a^2\right)^2}. \tag{1.6}$$

Similarly to the equilibrium effort e^*, both ability level a and the interaction term κ increase expected individual profit, whereas c leads to an increased marginal cost of effort and lower profit. Again, ability level not only increases each team member's individual productivity, it also influences individual profit through complementarities.

How large should a team optimally be? Consider a team composed of n of agents. Under what conditions is n the coalition-proof Nash equilibrium team size? The team will include an additional (marginal) team member if each existing team member expects that he will be better off as a result, that is, if each agent's expected individual profit increases. On the one hand, profit sharing implies that the free-rider problem increases with team size, so that both individual contributions and the private costs of effort decrease. Furthermore, the loss in output outweighs savings associated with the decreased cost of effort; therefore, each team member's profit declines. On the other hand, if $\kappa > 0$, then the value of complementarities increases with team size such that each team member can benefit from the additional synergy created by the interaction between the additional agent and each current team member. Overall, the optimal team size increases, provided that the negative effect on output share is outweighed by the additional synergy gain. Thus, the coalition-proof Nash equilibrium team size is reached when the first-order condition, $\partial E[\Pi]/\partial n = 0$, holds, yielding

$$0 = \frac{a^2\,(2\,c - \kappa\,a^2)}{2\,\left(n^*\,c - (n^*-1)\,\kappa\,a^2\right)^2} - \frac{a^2\,(c - \kappa\,a^2)\,\left(2\,n^*\,c - c - (n^*-1)\,\kappa\,a^2\right)}{\left(n^*\,c - (n^*-1)\,\kappa\,a^2\right)^3}. \tag{1.7}$$

There are two marginal effects on an individual team member: the first term is positive and shows that each agent can benefit from lower effort costs if team size increases. Moreover, for $\kappa > 0$, each additional team member increases the value of complementarities. However, due to the free-rider problem, the effect of complementarities between each pair of agents decreases with each additional team member. Thus, for $\kappa < c/a^2$, the complementarity effect increases at a decreasing rate. The second term is negative and shows that the larger is the team size, the smaller is each team member's share in both individual contributions and joint complementarities. Moreover, the forgone output outweighs the reduction in the cost of effort. However, complementarities and ability level reduce the decline in marginal effort and, therefore, also reduce the loss in each agent's output share. In equilibrium, the positive effect (complementarities) exactly offsets the negative effect (loss in output share). This yields the following proposition.

Proposition 1 *In a coalition-proof Nash equilibrium, rational agents choose the team size*

$$n^* = \frac{2\,c^2 - 2\,c\,\kappa\,a^2 + \kappa^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,a^2 + \kappa^2\,a^4} \tag{1.8}$$

for $\kappa < c/a^2$. For $\kappa \geq c/a^2$, the optimal team is infinitely large. Furthermore, for $\kappa < 0$, the above term falls below 1, in which case the optimal solution is $n^* = 1$.

Proposition 1 shows that team formation, $n^* > 1$, is only beneficial if there is a positive complementarity effect that can compensate for the decrease in each agent's output share; that is, if $\kappa > 0$. In this case, the agents' optimal team size increases with the interaction term κ and the ability level a but decreases with the cost factor c. In contrast, if $\kappa \leq 0$, the optimal team size is always $n^* = 1$. Intuitively, if coworkers' efforts are substitutes, i.e., $\kappa < 0$, joint output decreases with team size. A second effect is that output is negatively affected by the free-rider problem, so that team-building remains suboptimal even in the absence of synergies, $\kappa = 0$. Altogether, efficient team formation can be ensured only if coworkers' individual outputs are complements in production.

1.4 The Model with Overconfidence

To analyze the effects of overconfidence on both agents' incentives to exert effort and the choice of optimal team size, we assume that agents are overconfident in their abilities. More specifically, each agent perceives his own ability to be $\hat{a} = b\,a$, although his actual ability is a. The parameter $b > 1$ measures the degree of overconfidence, assumed to be equal for all agents. Consequently, each agent chooses some effort level but, due to overconfidence, believes that output will be larger for a given effort level. More specifically, each agent believes that he is the only high-ability agent and that the other team members are overconfident. That is, each agent believes that he can generate a large output with only a small effort but that his coworkers believe that they are themselves highly productive. On the one hand, this causes him to believe that he is the most able member of the team, giving rise to an asymmetry in the composition of team members' expected ability levels. As an increase in team size would increase the fraction of low ability agents, the incentives to build the team might be reduced. On the other hand, in the presence of complementarities, each agent can benefit from overconfidence and the increased efforts of his coworkers, such that for our representative agents, it also makes sense to increase effort, and team-building may be beneficial.

In teams, overconfidence is relevant not only in absolute terms but also through individual comparisons. Accordingly, overconfidence is modeled as an asymmetric belief regarding ability that incorporates not only an overestimation of one's own abilities but also of one's abilities relative to coworkers. Thus, our modeling methodology employs an "agree to disagree" approach. That is, each agent is unaware of his own overconfidence; otherwise, he would not be biased. Aware that his teammates expect him to be overconfident, he agrees to disagree with them. From an ex ante perspective, a rational agent would benefit most from teamwork if his fellow team members were overconfident. In that case, he could benefit from the increased effort of his coworkers and free-ride on their overconfidence. If, additionally, the overconfident team members assume that the rational agent is overconfident, they would expect him to exert a greater effort in equilibrium and, therefore, would further increase their own efforts in order to exploit complementarities. Similarly, for an overconfident agent, it is advantageous to convince coworkers of one's high expectations of one's own ability in order to increase coworkers' efforts through the channel of complementarities. With symmetric beliefs, each agent correctly assesses his coworkers' overconfidence, recognizing that his team members are similarly aware of the biases of their coworkers.

More specifically, each agent perceives his personal ability to be $\hat{a} = b\,a$ but is aware of the overconfidence of fellow team members and, therefore, knows that their ability is a. Due to team members' overconfidence, each agent anticipates that his coworkers will overinvest in effort and choose an effort level that corresponds to the ability level $\hat{a}$, each agent believing that this effort level is optimal only for oneself and not for one's coworkers. Although agents are identical, their expectations are asymmetric, and thus, they may anticipate a different effort level from their peers. However, as all agents agree to disagree, they are aware of this asymmetry in ability expectations. Given that the degree of overconfidence b is equal for all agents, and the biases of team members are symmetrically known to all agents, each agent faces an identical maximization problem. Consequently, we can consider the decision problem from the perspective of a representative agent $i = 1$. Thus, we replace y_1 by $\hat{a}\,e_1 = b\,a\,e_1$ (because agent 1 believes he is more able than the others) and y_j by $a\,e_j$ for $j \neq 1$ (because agent 1 believes the others are not very able, although they believe that they are) in the expected individual profit function of (1.3). Solving for agent 1's first-order condition yields

$$e_1^B = \frac{b\,a}{n\,c}\,\left((n-1)\,\kappa\,a\,e_j^B + 1\right), \tag{1.9}$$

where the superscript B indicates that agents are biased. Now, agent 1 knows that the other agents also believe that he himself is overconfident and less able than he believes he is. He also knows that his coworkers all know that the other coworkers are overconfident. Thus, the maximization problem is symmetric: each agent believes that his personal ability is $b\,a$ and that the coworkers' abilities are a but that each coworker will choose an effort level that corresponds to ability level $b\,a$. Moreover, agent 1 knows that each coworker will attribute ability level a to each of his coworkers' abilities, including to agent 1. Consequently, agent 1 expects other agents to exert the same level of effort that he himself exerts. Thus, in equilibrium, each agent will choose the same effort and anticipate this effort level from each of the other agents, $e_j^B = e_1^B = e^B$. This yields the following lemma.

Lemma 2 *The equilibrium effort of overconfident agents is*

$$e^B = \frac{b\,a}{n\,c - (n-1)\,\kappa\,b\,a^2} \tag{1.10}$$

for $\kappa < c/\left(b\,a^2\right)$ if $n > 1$. For $\kappa \geq c/\left(b\,a^2\right)$ and $n > 1$, the equilibrium effort is infinitely large.

The main result here is that overconfidence increases the equilibrium effort for all team sizes, $e^B > e^*$, and thus mitigates the free-rider problem. The rationale is that biased agents overestimate their marginal product of effort and, therefore, increase their equilibrium effort relative to the rational model. Additionally, if $\kappa \neq 0$ and $n > 1$, positive or negative interaction effects arise from the increased efforts of team members, depending on whether coworkers' outputs are complements ($\kappa > 0$) or substitutes ($\kappa < 0$) in production. Specifically, if $\kappa > 0$, each agent will work more because he sees that his colleagues are overconfident and, consequently, will themselves work more; thus, coworkers' efforts are complements. However, for $\kappa < c/\left(b\,a^2\right)$, equilibrium effort decreases with team size. That is, as the free-rider problem increases, the effect of overconfidence on the equilibrium level of effort diminishes.

As there are no principals in our model, agents choose a team size that maximizes their expected individual profits. Ex post, the profit function of the standard model given in (1.3) holds, as the agents' true abilities do not change. Nevertheless, we must assess overconfident agents' decisions from the perspective of their ex ante expected profit, as the choice of the optimal team size relies on their biased expectations. Specifically, agent 1 believes the equilibrium effort e^B is optimal for him, and he also believes that the other agents are fooled into choosing a too high effort level because they believe they are of high ability. If everyone chooses e^B, then this effort level can be substituted into the aggregate profit function, where again $y_1 = \hat{a}\,e^B = b\,a\,e^B$, and $y_j = a\,e^B$ for all $j \neq 1$. Agent 1 believes he contributes more to aggregate output than the others, owing to his uniquely high abilities (although this is untrue, in fact). With symmetric beliefs, we obtain the overconfident agents' expected individual profits as a function of team size n,

$$E\left[\Pi^B\right] = \frac{b\,a^2\,\left((2\,n + b - 2)\,c - (n-1)\,\kappa\,b\,a^2\right)}{2\,\left(n\,c - (n-1)\,\kappa\,b\,a^2\right)^2}. \tag{1.11}$$

Based on each agent's overestimation of his or her personal productivity and the resulting increase in equilibrium effort, expected individual profitability increases in the presence of overconfidence for all team sizes, $E[\Pi^B] > E[\Pi]$. Thus, overconfidence influences expected individual profitability in four ways. First, expected profitability increases because agents (mistakenly) believe that their personal productivity is higher than their true productivity. Second, agents (correctly) expect increased profitability for $n > 1$ to result from an increase in effort induced by the overconfidence of all team members. Third, if $\kappa \neq 0$ and $n > 1$, expected individual profit (correctly) incorporates additional positive ($\kappa > 0$) or negative ($\kappa < 0$) interaction effects between all team members. Forth, due to personal bias, each agent (mistakenly) overvalues the extent of synergy effects. All four effects are interlinked and, thus, feed back on overconfident agents' optimal team size.

To illustrate the decision problem, consider a team consisting of $n = 1 + (n-1)$ agents. Agent 1 believes that he is especially skilled and that the other $n-1$ agents are less skilled, even though they believe they are highly skilled. Thus, agent 1 believes that an additional (marginal) team member will increase the share of less skilled but overconfident agents. Consequently, agent 1 believes that he is the only team member who chooses his effort level correctly and that, although the fraction of less skilled agents will increase with each additional team member, he will benefit from the overinvestment in effort of his overconfident coworkers (and consequently also from the overinvestment in effort of each additional overconfident coworker). That is, he believes that his coworkers' overconfidence causes incorrect decision making not only regarding optimal effort but also regarding optimal team size. However, all team members must agree upon each additional marginal team member. Consequently, when deciding the optimal team size, each agent has different beliefs about the source of aggregate profits. In particular, each agent believes that he contributes more to the joint output than the other team members. Nevertheless, as each coworker is symmetrically aware of each coworker's overconfidence (including the overconfidence of agent 1), all agents agree that their additional benefit from an additional (marginal) team member is $\partial E[\Pi^B]/\partial n$. In a coalition-proof Nash equilibrium, this derivative must vanish. We have a standard first-order condition:

$$0 = \frac{b\,a^2\,(2\,c - \kappa\,b\,a^2)}{2\,(n^B\,c - (n^B-1)\,\kappa\,b\,a^2)^2} - \frac{b\,a^2\,(c-\kappa\,b\,a^2)\,\big((2\,n^B + b - 2)\,c - (n^B-1)\,\kappa\,b\,a^2\big)}{(n^B\,c - (n^B-1)\,\kappa\,b\,a^2)^3}. \tag{1.12}$$

Similarly to the basic model with rational agents, (1.12) indicates the two marginal effects on each individual team member: the first term, which is positive, shows that each agent's cost in effort decreases with team size, while, for $\kappa > 0$, the value of complementarities increases with team size. As overconfidence raises each agent's equilibrium effort, the decline in the marginal cost of effort is larger relative to the rational model. For the same reason, the value of complementarities also increases, and, similarly to the rational model, the complementarity effect increases at a decreasing rate. The second term, which is negative, shows that, for $\kappa < c/(b\,a^2)$, each agent's output share decreases with team size, thus reducing each agent's profit. As complementarities, ability level, *and* degree of overconfidence increase equilibrium effort, this negative effect is moderated to a greater degree than in the rational model. However, as each agent overestimates his or her personal contribution to the joint output as well as to the complementarity effect, the expected marginal loss in profit exceeds the overconfident agent's real marginal profit loss. While the negative effect lowers incentives for team formation, the increased positive complementarity effect increases the benefits of teamwork. In equilibrium, these two marginal effects exactly offset each other, yielding the following proposition:

Proposition 2 *In a coalition-proof Nash equilibrium, overconfident agents choose a team size of*

$$n^B = \frac{-2\,c^2\,(b-2) + 2\,c\,\kappa\,(b-2)\,b\,a^2 + \kappa^2\,b^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4} \tag{1.13}$$

for $\kappa < c/(b\,a^2)$. For $\kappa \geq c/(b\,a^2)$, the optimal team is infinitely large. Furthermore, for

$$\kappa < \frac{2\,c\,(b-1)}{(2\,b-1)\,b\,a^2}, \tag{1.14}$$

the above term falls below 1, in which case the optimal solution is $n^B = 1$.

Proposition 2 shows that, although overconfident agents believe that their personal contribution to the joint output is the highest among team members, team formation can nevertheless be beneficial. This is the case if joint output mainly depends on team members' combined contributions rather than on their individual outputs, that is, if κ is positive and sufficiently large. Intuitively, as overconfidence increases agents' equilibrium efforts, it also increases the value of complementarities, potentially outweighing the negative effect of team size. In contrast, as $n \geq 1$, the optimal team size remains $n^B = 1$ if $\kappa \leq 0$. The rationale for this is that biased agents believe that, due to their superior abilities, their individual profits will exceed the profits of rational agents, irrespective of the actual size of the team. As the free-rider problem increases with team size, cooperation only appears beneficial if it involves additional advantages through complementarities.

Let us now compare the optimal team sizes of rational and overconfident agents. While in the absence of complementarities, $\kappa \leq 0$, both rational and biased agents choose to work alone, $n^* = n^B = 1$, this is not the case when $\kappa > 0$. Intuitively, as overconfident agents only build teams when complementarities are sufficiently large, they reduce their optimal team size relative to the rational model if κ is positive but below the threshold of (1.14). We obtain the following result.

Proposition 3 *Assume that*

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-1)\,b\,a^2} \tag{1.15}$$

and let $\kappa < c/(b\,a^2)$. Thus, overconfidence increases agents' optimal team size, $n^B > n^*$, if and only if

$$b > \frac{2\,\left(2\,c^3 - 4\,c^2\,\kappa\,a^2 + c\,\kappa^2\,a^4\right)}{\kappa\,a^2\,\left(4\,c^2 - 6\,c\,\kappa\,a^2 + \kappa^2\,a^4\right)} \tag{1.16}$$

for $\kappa < (3-\sqrt{5})\,c/a^2$. For $\kappa \geq (3-\sqrt{5})\,c/a^2$, the optimal team size increases for all values of the degree of overconfidence b.

Although increasing team size increases the expected proportion of agents with (only) ability a, overconfidence also increases agents' equilibrium effort, incentivizing team formation in the presence of complementarities. That is, if complementarities are sufficiently strong, overconfident agents increase their optimal team size for all values of the degree of overconfidence b and $n^B > n^*$. Otherwise, the optimal team size increases only if b exceeds the threshold (1.16). Thus, for large values of b, the complementarity effect is determined by the increase in effort induced by overconfidence rather than by the value of synergy parameter $\kappa > 0$. Stated differently, if the synergy term $\kappa > 0$ is small, a large overestimation of ability b, and thus a large increase in coworkers' equilibrium effort, is required to increase optimal team size, and vice versa. Thus, the larger is the initial ability a, the larger is the effect of overconfidence on biased agents' optimal team size. By contrast, the larger is the cost factor c, the larger is the free-rider problem and the smaller is the effect of overconfidence and thus also the increase in team size. If follows that, although overconfident agents view themselves as superior to all other coworkers, their optimal team size increases for both sufficiently high degrees of overconfidence and sufficiently large complementarities.

1.5 Overconfidence and Welfare

One can examine the effects of overconfidence on agents' welfare from two perspectives. While the *ex ante* approach considers welfare evaluations that are based on agents' expectations, the *ex post* approach incorporates realized rather than anticipated welfare (Starr (1973), Harris and Olewiler (1979), Sandmo (1983)). Although ex post optimality is not considered a standard measure in welfare economics (see, for example, Hammond (1981)), it has gained importance in measurements of welfare in overconfidence models. As overconfidence stems from ex ante disagreement about skill levels, expected outcomes, or the interpretation of performance signals, a measure of ex ante welfare fundamentally depends on the definition and extent of such biases. Moreover, it reflects the anticipated, rather than realized, utility of biased agents and therefore is not applicable to measuring, comparing, and evaluating the consequences of overconfidence. Accordingly, the welfare criterion of ex post optimality is assumed to be more appropriate if agents differ in their beliefs (Nielsen (2003), Kurz (2009)) and, in particular, if overconfidence effects prevail (Nielsen (2009)).[6]

The model with overconfidence considers overconfident agents' decision problem based on their ex ante expected welfare. However, in contrast to the model with rational agents, the ex ante expected profit of overconfident agents deviates from the individual profit that is realized at the end of the period. Therefore, to analyze overconfident agents' well-being at the end of the period, it is convenient to consider the ex post welfare perspective. This concept, moreover, allows us to compare the welfare of rational and overconfident teams and to assess the circumstances

[6]Consistent with this approach, the impact of overconfidence on agents' ex-post experienced utility is considered, for example, in the models of Gervais and Goldstein (2007), Sandroni and Squintani (2007), Santos-Pinto (2008), and Gervais, Heaton, and Odean (2011).

under which overconfidence may be beneficial or detrimental to agents. Our main purpose is to determine whether overconfidence may be desirable in teamwork, for example, through the mitigation of team-related free-rider problems, or whether the effects of wrong decision-making associated with overconfidence outweigh the associated benefits. While, with rational expectations, it appears natural to use agents' preferences to evaluate team formation and optimal team size, with overconfidence, possible policy implications must account for the incorrect decision making of biased agents. For example, if overconfidence generally results in welfare losses, public policy may be designed to encourage individuals to more accurately evaluate their skills. If there are (potential) team-related benefits to overconfidence, policy makers may wish to facilitate teamwork and encourage the formation of partnerships. For instance, governments may create start-up subsidy programs and support other forms of partnership formation.

More specifically, as all agents are overconfident, we must account for the fact that our representative agent $i = 1$ only has ability a. Then, for overconfident agents' equilibrium effort e^B, we replace y_1 by $a\, e^B$, and for all $j \neq 1$ in the initial profit function of (1.3), we replace y_j by $a\, e^B$. This yields the expected real individual profit of overconfident agents as a function of team size:

$$E\left[\Pi^W\right] = \frac{b\, a^2\ (2\, n\, c - b\ (c + (n-1)\, \kappa\, a^2))}{2\ (n\, c - (n-1)\, \kappa\, b\, a^2)^2}, \tag{1.17}$$

where the welfare perspective is denoted by the superscript W. Intuitively, as the overconfident agent's ex ante expected individual profit is based on an overestimation of one's personal outcome, it exceeds the ex post expected individual profit for all team sizes, $E[\Pi^B] > E[\Pi^W]$. Consequently, it is of interest to determine which team size maximizes (1.17) as the expected real individual profit, given overconfidence. Thus, from the perspective of each agent's individual welfare, we must account for the fact that the agent has mistaken beliefs about the source of aggregate profits. Specifically, as all agents are identical, each team member contributes equally to the joint output. Consequently, agent 1 has ability a instead of $\hat{a} = b\, a$. Again, we can separate the positive effect of complementarities from the negative effect of team size on each agent's fraction of the joint output. The first order condition, $\partial E[\Pi^W]/\partial n = 0$, yields

$$0 = \frac{b\, a^2 \left(2\, c - \kappa\, b\, a^2\right)}{2 \left(n^W c - \left(n^W - 1\right) \kappa\, b\, a^2\right)^2} - \frac{b\, a^2 \left(c - \kappa\, b\, a^2\right) \left(2\, n^W c - b \left(c + \left(n^W - 1\right) \kappa\, a^2\right)\right)}{\left(n^W c - \left(n^W - 1\right) \kappa\, b\, a^2\right)^3}. \tag{1.18}$$

Given that all agents are symmetrically informed of coworkers' overconfidence, the marginal positive team size effect corresponds to (1.12). However, with overconfidence, each agent believes that he contributes more to the joint output than others and consequently also to the complementarity effect. Thus, the marginal loss to each agent's output share, given by the second (negative) term, is less than each agent's expectations. Consequently, the overconfident agents' optimal team size might be too small. By equating the positive and (real) negative marginal effects, we obtain the welfare-optimal team size with overconfidence. Proposition 4 outlines the result.

Proposition 4 *Under the equilibrium effort choice e^B, overconfident agents' individual welfare is maximized by a team size of*

$$n^W = \frac{b\,(2\,c^2 - 2\,c\,\kappa\,b\,a^2 + \kappa^2\,b\,a^4)}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4} \tag{1.19}$$

for $\kappa < c/(b\,a^2)$. For $\kappa \geq c/(b\,a^2)$, the optimal team is infinitely large. Furthermore, for $b < 1.5$ and

$$\kappa < \frac{2\,c\,(b-1)}{(2\,b-3)\,b\,a^2} \tag{1.20}$$

the above term falls below 1, in which case the optimal solution is $n^W = 1$.

In the numerical example of Figure 1.1, the welfare-optimal team size n^W exceeds both the rational and the overconfident agents' optimal team sizes, n^* and n^B, respectively, for positive and, under certain conditions, negative values of κ. One major difference is that, while, for $\kappa \leq 0$, both rational and overconfident agents choose to work alone, $n^* = n^B = 1$, this does not necessarily hold for the welfare-optimal team size n^W. Therefore, it is of interest to compare the optimal team sizes of Propositions 1, 2, and 4. This also involves the question of the values of κ for which team formation is beneficial with respect to agents' welfare. Proposition 5 outlines the result.

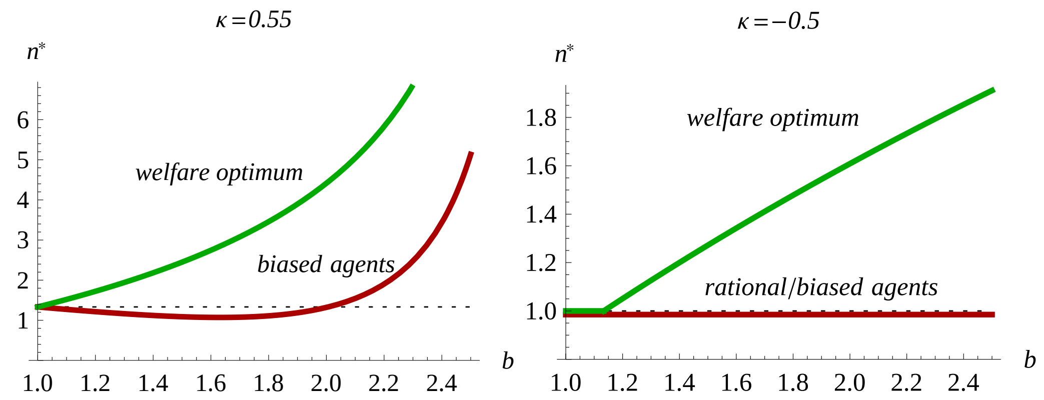

Figure 1.1: Optimal Team Size n^* as a Function of the overconfidence degree b

The graphs indicate the optimal team sizes of rational agents n^* (dashed line) and of overconfident agents n^B (red line) and the welfare-optimum, given overconfidence, n^W (green line). The parameters are $a = 1.5$ and $c = 3.5$.

Proposition 5 *Assume that $\kappa < c/(b\,a^2)$. Then the welfare-optimal team size n^W exceeds the overconfident agents' optimal team size n^B if and only if*

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-3)\,b\,a^2} \tag{1.21}$$

for $b < 1.5$. For $b \geq 1.5$, $n^W > n^B$ for all values of κ.

Proposition 5 shows that the welfare-optimal team size n^W exceeds overconfident agents' optimal team size n^B if the degree of overconfidence is high, $b \geq 1.5$, or if κ is not excessively negative and (1.21) holds. In this case, the welfare-optimal team size increases in the degree of overconfidence. Intuitively, even if overconfident agents increase their optimal team size, misinterpretation of individual outcomes makes each team member believe that he or she is more productive than his or her peers, so that the incentive to build the team is reduced. In fact, Figure 1.1 shows that, for sufficiently large values of complementarities, $\kappa > 0$, and the overconfidence level, b, biased agents increase their optimal team size relative to the rational model but choose a team size that is too small, $n^W > n^B > n^*$. Without complementarities, $\kappa \leq 0$, biased and rational agents always choose to work alone, whereas under certain conditions, the welfare-optimal team size exceeds this choice and team-building is welfare-optimal, $n^W > n^B = n^* = 1$.

The rationale behind this result is that, without teamwork, overconfidence leads to a costly overinvestment in effort. Intuitively, this overinvestment in effort increases in the degree of overconfidence. If team size increases, the free-rider problem emerges and reduces the level of overinvestment in effort. Additionally, with complementarities, $\kappa > 0$, agents can benefit from the increased effort of all team members. If coworkers' outputs are substitutes, $\kappa < 0$, team-building reduces effort and the costs of overinvestment in effort but, on the other hand, causes losses in profitability. While this first effect dominates for a high degree of overconfidence b, if there is extensive overinvestment in effort, the contrary holds if b is small and κ is highly negative. Accordingly, for $b < 1.5$, team-building remains beneficial, provided effort interaction is not highly negative, so that (1.21) holds. Note that, if $\kappa < 0$, the reduction in the cost of effort in teams arises through two channels. First, the free-rider problem emerges in teams and reduces equilibrium effort levels. Second, a negative κ further reduces effort and effort costs. These two effects are linked: the larger is the free-rider problem, the more it reduces the costs of effort in teams, and the smaller is the lost profitability associated with a negative κ. Hence, the larger are the marginal costs c, the more negative κ can be, with team-building remaining beneficial. Overall, our results suggest that team-building should be incentivized when overconfidence is present.

With the optimal team size results of Propositions 1, 2, and 4, we can derive the ex post expected equilibrium individual profits for rational and overconfident agents. This enables us to examine the conditions under which overconfidence increases agents' individual welfare relative to the rational equilibrium and thus determine whether overconfidence is desirable for teamwork. Substituting n for the optimal team size of rational agents n^* in (1.6), we obtain rational agents' expected equilibrium individual profit of

$$E\left[\Pi\left(n^*\right)\right] = \begin{cases} \frac{\left(\kappa\, a^3 - 2\, c\, a\right)^2}{8\, c^2\, (c - \kappa\, a^2)}, & \text{if } \kappa > 0, \\ \frac{a^2}{2\, c}, & \text{otherwise.} \end{cases} \tag{1.22}$$

Similarly, substituting n for the optimal team size of biased agents n^B in (1.17) yields overconfident agents' expected equilibrium individual profit of

$$E\left[\Pi^W\left(n^B\right)\right] = \begin{cases} \frac{b\left(\kappa\, b\, a^3-2\, c\, a\right)^2\left(3\,\kappa\,(b-1)\, b\, a^2-c\,(3\, b-4)\right)}{8\, c\,(c-\kappa\, b\, a^2)\,(c\,(b-2)-\kappa\,(b-1)\, b\, a^2)^2}, & \text{if } \kappa > \frac{2\, c\,(b-1)}{(2\, b-1)\, b\, a^2}, \\ -\frac{(b-2)\, b\, a^2}{2\, c}, & \text{otherwise.} \end{cases} \quad (1.23)$$

Note that (1.23) combines biased agents' optimal team size n^B with their expected real individual profit $E[\Pi^W]$. This allows us to examine the impact of the optimal team size of overconfident agents on their individual profit levels, as these will be experienced ex post. Finally, incorporating the welfare-optimal team size with overconfidence, n^W, into (1.17), we obtain the equilibrium individual profit of

$$E\left[\Pi^W\left(n^W\right)\right] = \begin{cases} \frac{\left(\kappa\, b\, a^3-2\, c\, a\right)^2}{8\, c\,(c-\kappa\,(b-1)\, a^2)\,(c-\kappa\, b\, a^2)}, & \text{if } b \geq 1.5 \text{ or if } \kappa > \frac{2\, c\,(b-1)}{(2\, b-3)\, b\, a^2}, \\ -\frac{(b-2)\, b\, a^2}{2\, c}, & \text{otherwise.} \end{cases} \quad (1.24)$$

The above equations show that κ plays a decisive role in the question of team-building, including with respect to the welfare implications of rational versus overconfident teams. Given the optimal team sizes of rational and overconfident agents, we can determine the values of κ under which overconfidence can increase individual welfare relative to the rational model. The result is summarized in Proposition 6.

Proposition 6 *(i) For $n = n^W$, individual welfare is higher (lower) or equal for overconfident agents than for rational agents only if $\kappa \geq 0$ ($\kappa < 0$).*
(ii) For $n = n^B$, there is a cut-off value $\underline{\kappa} > 0$ such that individual welfare is higher (lower) or equal for overconfident agents than for rational agents only if $\kappa \geq \underline{\kappa}$ ($\kappa < \underline{\kappa}$).

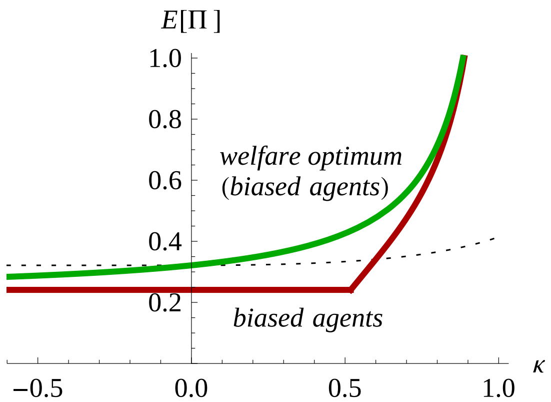

Figure 1.2: Effect of Overconfidence b and Output Interaction κ on Agents' Welfare

The graphs show the ex post expected individual profits $E[\Pi(n^*)]$ for the optimal team size of rational agents (dashed line), $E[\Pi^W(n^B)]$ for the optimal team size of overconfident agents (red line), and $E[\Pi^W(n^W)]$ for the welfare-optimal team size with overconfidence (green line). The parameters are $a = 1.5$, $c = 3.5$, as before, and $b = 1.5$.

As exemplified in Figure 1.2, for $\kappa < 0$, overconfidence always leads to a lower profit than in the rational equilibrium. Proposition 2 shows that overconfident agents refrain from teamwork in this case, $n^B = 1$. On the other hand, Proposition 5 shows that, under certain conditions, overconfident teams are beneficial even in the absence of effort complementarities, $n^W > 1$. It follows that, although the individual profits of biased agents cannot exceed individual profits in the rational equilibrium in this case, team-building can reduce the profitability losses of overconfident agents and increase their welfare relative to individual work. Thus, facilitating team-building in the presence of overconfidence can increase agents' welfare, especially if the relative degree of overconfidence is high or if κ is not excessively negative.

If $\kappa > 0$, the welfare-optimal team size n^W increases the welfare of overconfident agents compared with rational ones. Given biased agents' optimal team size n^B, individual welfare increases and approaches the welfare optimum only if complementarities are sufficiently strong, $\kappa > \underline{\kappa}$. This finding has the following economic implication: although the optimal team size of overconfident agents is based on an overestimation of individual profits, it can make overconfident agents better off compared with rational ones. That is, overconfidence and the misinterpretation of individual outcomes need not be detrimental to agents. Moreover, as our model allows for endogenously-determined team size, it shows that in the presence of complementarities, overconfidence can be welfare-improving, even given large degrees of overconfidence.

1.6 Implications and Discussion

A result of our model is that in partnerships, overconfidence regarding ability increases the effort provisions of agents, mitigates team-related free-rider problems, and increases biased agents' expected individual profits for all team sizes. Indeed, overconfidence effects appear to be typical of entrepreneurs and appear likely to explain excessive entry into new businesses. In a laboratory experiment, Camerer and Lovallo (1999) provide evidence that excessive business entry is based on an "inside view" of entrepreneurs that ignores past statistics in evaluating the current situation and, therefore, may lead to overoptimistic forecasts. By contrast, the "outside view" produces more accurate forecasts, as it relies on past situations of similar types and, therefore, allows one to draw conclusions regarding the likelihood of future success (Kahneman and Lovallo (1993)). Excessive entry is shown to be more likely when performance depends on skill and, thus, is likely to result from overconfidence of individuals in their personal abilities relative to those of other market participants, a phenomenon known as "reference group neglect". Specifically, Cooper, Woo, and Dunkelberg (1988) show that new business owners are overly optimistic about their future prospects and attribute a higher probability to their personal success than to that of other entrepreneurs of similar types. Overconfidence is thus also associated with heavy workloads, and decisions to start new businesses are perhaps based by larger perceived outcomes than agents expect to obtain inside firms. In effect, the empirical literature confirms that entrepreneurs of start-up firms exhibit higher levels of overconfidence than managers in large organizations (Busenitz and Barney (1997), Forbes (2005)).

Consequently, while new market entry of entrepreneurs is attributed to the "inside view" of overoptimism, the impact of overconfidence on subsequent performance is less clear-cut. Obviously, overoptimism about ability and future success may induce myopic decision making that ignores the past performance of other market entrants. Therefore, overconfidence and excessive business entry are expected to be linked to high failure rates of new businesses (Camerer and Lovallo (1999)). In our model, overconfident agents believe they are overly productive and, therefore, overestimate their expected end-of-period profits. From an ex post perspective, entrepreneurs should realize lower profits compared with their ex ante expectations. Moreover, the overestimation of personal contributions dampens team formation incentives, so that overconfident agents tend to choose team sizes that are too small. In particular, if complementarities (and thus optimal team size) are small, overconfident agents are worse off relative to rational ones, while the relationship is reversed if complementarities are sufficiently large. That is, the business entry of larger partnerships should be associated with lower failure rates than the business entry of smaller partnerships. In effect, numerous studies conducted in different countries and markets find high failure rates of entrepreneurs within their first years of activity and that business failures are especially prevalent among small startups (Dunne, Roberts, and Samuelson (1988, 1989), Wagner (1994), Geroski (1995), Mata, Portugal, and Guimaraes (1995)). Similarly, entrepreneurs experience lower initial earnings and lower earnings growth compared with employees inside firms (Hamilton (2000)). Consistent with our results, overconfidence may have ambiguous overall effects on entrepreneurial performance. While Koellinger, Minniti, and Schade (2007) and Hmieleski and Baron (2008, 2009) confirm a negative relationship, Prussia, Anderson, and Manz (1998) show that entrepreneurs' self-efficacy has a positive performance effect. Similarly, Chandler and Jansen (1992) find that "the most successful entrepreneurs are strongly convinced of their ability".

Our general results are consistent with the experimental evidence of Rulliere, Santos-Pinto, and Vialle (2011) confirming that in teams, overconfidence increases the effort provision of agents, mitigates moral hazard problems, and raises team output if complementarities exist. At the same time, we emphasize that, even in the presence of effort substitutes, overconfident agents can increase their welfare relative to individual work if they choose to form teams. At the aggregate level, our results propose that supporting the creation of partnerships can improve economic performance. In effect, entrepreneurship and new firm foundations are perceived as determining factors in economic performance (for recent contributions, see Wennekers and Thurik (1999), Audretsch and Keilbach (2004), Stel, Carree, and Thurik (2005), Audretsch, Bönte, and Keilbach (2008)). For example, R&D alliances and research networks are frequently promoted by governmental subsidy programs, including tax interventions and direct government participation through public private partnerships (see, for example, Hagedoorn (1993, 2002), Osborn and Hagedoorn (1997), Sakakibara (1997), Narula and Dunning (1998), Narula and Duysters (2004)). From a performance-based perspective, complementarities and synergy generation are seen as the main reasons for R&D cooperation, while partnership formation is acknowledged to advance performance (Hagedoorn and Schakenraad (1994), Mitchell and Singh (1996), Ahuja (2000), Baum, Calabrese, and Silverman (2000), Sampson (2007), Lin, Yang, and Arya (2009)). Similarly, related to our theoretical results, Becker and Dietz (2004) confirm that the likelihood of project innovation increases with the number of partners cooperating in an R&D project. Nevertheless, according to Hagedoorn, Link, and Vonortas (2000), public intervention should carefully consider possible adverse aspects of partnership formation and firm collaboration that, for example, can lead to decreased market competition and the creation of monopolies in existing and future markets.

1.7 Conclusion

By incorporating free-riding effects, synergies, and behavioral effects into a group work context, our paper focuses on three key elements that determine the optimal size of a team. As the free-rider problem increases with team size, rational agents only form teams if certain complementarities can be exploited. The introduction of overconfidence increases agents' equilibrium efforts and mitigates team-related free-rider problems. Nevertheless, because overconfident agents overestimate their individual profits, they increase optimal team size for a sufficiently high degree of overconfidence and sufficiently large complementarities. From agents' welfare perspective, overconfident agents tend to choose team sizes that are too small. Although overconfidence is only welfare-optimal if complementarities are present, it can drive team-building, even if coworkers' efforts are substitutes. In this case, overconfidence prevents agents from making costly overinvestments in effort and increases their welfare relative to individual work.

One main result is that the misinterpretation of individual profits somewhat diminishes the welfare of overconfident agents. Nevertheless, their optimal team size can be welfare-increasing relative to the rational model if complementarities are sufficiently strong. More generally, our results suggest that overconfident agents are more likely to self-select themselves into teamwork if the production technology requires coordination among team members. Considering the decision problem from the opposite perspective, our model proposes that teamwork should expand when overconfidence is present. With overconfidence, team size should be larger and, under certain conditions, exceed biased agents' optimal team size choice.

Based on these considerations, this analysis could be expanded further, under alternative assumptions concerning coworkers' abilities, expected levels of overconfidence, or also divergent information structures. Another direction for further research would be to analyze the interaction of overconfidence with other (costly) incentive systems, such as monitoring activities. By mitigating the free-rider problem in teams, monitoring can increase effort incentives and consequently also optimal team size of both rational and overconfident agents. However, even if costless, it can reduce the overconfident agents' welfare if it leads to an excessive effort overinvestment, especially if coworkers' efforts are substitutes. A further aspect would be to extend the model to a multi-period setting. This could yield novel insights with regard to the choice of optimal team size from a dynamic perspective. Related to this topic, Gervais and Odean (2001) show that traders' overconfidence increases with success and decreases with failure, such that over time, traders learn to have more accurate assessments of their ability. Likewise, in our model, the ex post individual profit of overconfident agents deviates from their ex ante expected value. Therefore, biases could diminish over time and shift optimal team size toward the rational equilibrium in a long-term consideration.

In light of the rising importance of team structures in modern organizational settings, our paper contributes to fundamental research on optimal team design. By incorporating overconfidence effects into our model, we relate the research on team size with that on overconfidence effects in teams. In addition to previous work on overconfidence in teams, we show that biased agents can benefit from cooperation without any effort interaction and even when individuals' efforts are substitutes, especially if the relative degree of overconfidence is high. In addition to accounting for existing empirical research, our results provide new testable predictions related to overconfidence effects in teams, contribute to the literature on endogenous organizational design, and speak to the efficient provision of incentives to agents.

1.8 Appendix

Proof of Lemma 1. In deciding his labor supply, each agent balances his end-of period profit, $E[\Pi_i]$, with his costs of effort, $c(e_i)$. As joint output is equally divided among team members, each agent's output Y/n depends on the contributions of his coworkers. For a team with $n \geq 1$ agents, consider the decision problem from the perspective of a representative agent $i = 1$ cooperating with $n-1 \geq 0$ identical coworkers. According to (1.3), agent 1's maximization problem is

$$\max_{e_1} E[\Pi_1] = \frac{E[Y]}{n} - c(e_1) = \frac{1}{n}\left(\sum_{i=1}^{n} y_i + \kappa \sum_{i=1}^{n}\sum_{j\neq i}^{n} y_i\, y_j\right) - c\,\frac{e_1^2}{2}. \tag{1.25}$$

As all coworkers are identical, the sum of individual outputs of all team members is given by $y_1 + (n-1)\, y_j$. Accordingly, agent 1 interacts with $n-1$ coworkers, while the $n-1$ coworkers additionally interact with $(n-2)/2$ coworkers. Hence, for a team consisting of $n = 1 + (n-1)$ agents, there are $n\,(n-1)/2$ interactions. Then, with individual output $y_1 = a_1\, e_1$ and coworkers' individual outputs $y_j = a_j\, e_j$ for $j \neq 1$, agent 1 solves

$$\begin{aligned}\max_{e_1} E[\Pi_1] &= \frac{y_1 + (n-1)\, y_j + (n-1)\,\kappa\, y_j\,(y_1 + (n-2)\, y_j/2)}{n} - c\,\frac{e_1^2}{2} \\ &= \frac{a_1\, e_1 + (n-1)\, a_j\; e_j + (n-1)\,\kappa\, a_j\, e_j\,(a_1\, e_1 + (n-2)\, a_j\, e_j/2)}{n} - c\,\frac{e_1^2}{2}.\end{aligned} \tag{1.26}$$

With equal abilities, $a_i = a_j = a$, the maximization problem is

$$\max_{e_1} E[\Pi_1] = \frac{a\;(e_1 + (n-1)\, e_j + (n-1)\,\kappa\, a\, e_j\,(e_1 + (n-2)\, e_j/2))}{n} - c\,\frac{e_1^2}{2}. \tag{1.27}$$

As coworkers' efforts e_j cannot be observed, agent 1's equilibrium effort, denoted e_1^*, is the best response to the conjectured efforts of his coworkers, e_j^*. The first-order condition, $\partial E[\Pi_1]/\partial e_1 = 0$, yields

$$\begin{aligned} 0 &= \frac{(n-1)\,\kappa\, a^2\, e_j^* + a}{n} - c\, e_1^*, \\ e_1^* &= \frac{a}{n\, c}\,\left((n-1)\,\kappa\, a\, e_j^* + 1\right), \end{aligned} \tag{1.28}$$

which corresponds to (1.4). As all agents are identical, $e_1^* = e_j^* = e^*$ holds, and we can rewrite (1.28) as

$$e^* = \frac{a}{n\,c - (n-1)\,\kappa\,a^2}. \tag{1.29}$$

Note that (1.29) applies for $\kappa < c/a^2$ if $n > 1$; otherwise, the equilibrium effort is infinitely large. ■

Proof of Proposition 1. Incorporating the equilibrium effort of Lemma 1 into (1.26), with $y_1 = y_j = a\,e^*$ for all $j \neq 1$, renders rational agents' individual profit $E[\Pi]$ as a function of team size n. Thus, corresponding to (1.6), each agent solves for the optimal team size n^*:

$$\max_n E[\Pi] = \frac{a^2\,(2\,n\,c - c - (n-1)\,\kappa\,a^2)}{2\,(n\,c - (n-1)\,\kappa\,a^2)^2}. \tag{1.30}$$

The first-order condition, $\partial E[\Pi]/\partial n = 0$, yields

$$\begin{aligned} 0 &= \kappa\,a^3 + 2\,c\,a\left(\frac{c}{n^*\,c - (n^*-1)\,\kappa\,a^2} - 1\right), \\ n^* &= \frac{2\,c^2 - 2\,c\,\kappa\,a^2 + \kappa^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,a^2 + \kappa^2\,a^4}. \end{aligned} \tag{1.31}$$

Note that (1.31) applies for $\kappa < c/a^2$; otherwise, the optimal team size is infinitely large. Additionally, as $n \geq 1$, the optimal team size is $n^* = 1$ for $\kappa < 0$; that is, in cases when the algebraic term in (1.31) is less than 1. ■

Proof of Lemma 2. With overconfidence, agent 1 believes that his personal ability is $\hat{a}_1 = b\,a_1 = b\,a$. Agent 1 is aware of his coworkers' overconfidence. Consequently, he knows that the abilities of his coworkers are $a_j = a$, even though each coworker believes that his personal ability is $\hat{a}_j = b\,a_j = b\,a$. Thus, the (expected) individual contribution is $y_1 = b\,a_1\,e_1 = b\,a\,e_1$, and the coworkers' individual contributions are $y_j = a_j\,e_j = a\,e_j$ for $j \neq 1$. Similarly to the proof of Lemma 1, we can rewrite agent 1's maximization problem of (1.26) as

$$\begin{aligned} \max_{e_1} E\left[\Pi_1^B\right] &= \frac{E\left[Y^B\right]}{n} - c(e_1) \\ &= \frac{y_1 + (n-1)\,y_j + (n-1)\,\kappa\,y_j\,(y_1 + (n-2)\,y_j/2)}{n} - c\,\frac{e_1^2}{2} \\ &= \frac{b\,a_1\,e_1 + (n-1)\,a_j\,e_j + (n-1)\,\kappa\,a_j\,e_j\,(b\,a_1\,e_1 + (n-2)\,a_j\,e_j/2)}{n} - c\,\frac{e_1^2}{2} \\ &= \frac{a\,(b\,e_1 + (n-1)\,e_j + (n-1)\,\kappa\,a\,e_j\,(b\,e_1 + (n-2)\,e_j/2))}{n} - c\,\frac{e_1^2}{2}. \end{aligned} \tag{1.32}$$

Again, agent 1's equilibrium effort, denoted e_1^B, is the best response to the conjectured efforts of his biased coworkers, e_j^B. The first-order condition, $\partial E[\Pi_1^B]/\partial e_1 = 0$, thus yields

$$0 = \frac{b\,a + (n-1)\,\kappa\,b\,a^2\,e_j^B}{n} - c\,e_1^B,$$

$$e_1^B = \frac{b\,a}{n\,c}\left(1 + (n-1)\,\kappa\,a\,e_j^B\right), \tag{1.33}$$

which corresponds to (1.9). Agent 1 knows that each coworker expects the personal contribution to be $y_j = b\,a\,e_j$, although the actual contribution is $y_j = a\,e_j$. Moreover, agent 1 knows that his coworkers believe that he has only ability a. He also knows that the coworkers are aware of all the other coworkers' overconfidence. Consequently, each agent believes that his own ability is $b\,a$ and that other agents' abilities are a. However, each coworker will choose an effort level that corresponds to the ability level $b\,a$. Moreover, agent 1 knows that each coworker will attribute ability level a to his coworkers, including agent 1. Thus, the "agree to disagree" approach implies that, in equilibrium, each agent will choose the same effort level but believes that this effort level is only optimal for oneself and not for the coworkers. Thus, for $e_1^B = e_j^B = e^B$, we can rearrange (1.33) to obtain

$$e^B = \frac{b\,a}{n\,c - (n-1)\,\kappa\,b\,a^2}. \tag{1.34}$$

Note that (1.34) applies for $\kappa < c/(b\,a^2)$ if $n > 1$; otherwise, the equilibrium effort is infinitely large. ■

Proof of Proposition 2. Incorporating the equilibrium effort of Lemma 2 into (1.32), with the (expected) individual contribution of $y_1 = b\,a\,e^B$, and the coworkers' contributions of $y_j = a\,e^B$ for $j \neq 1$, yields agent 1's expected individual profit, $E[\Pi_1^B]$, as a function of team size n. Note that each agent believes that his personal contribution to the joint output is the highest compared to one's coworkers. That is, agents disagree about the source of the joint output. However, all agents choose the same effort level, and the biases of the coworkers are symmetrically known to all team members. With symmetric beliefs, $E[\Pi_1^B] = E[\Pi_j^B] = E[\Pi^B]$ holds and agents agree upon each additional team member's contribution to this joint output, $\partial E[\Pi^B]/\partial n$. Each agent then solves

$$\max_n E\left[\Pi^B\right] = \frac{b\,a^2\,\left((2\,n + b - 2)\,c - (n-1)\,\kappa\,b\,a^2\right)}{2\,\left(n\,c - (n-1)\,\kappa\,b\,a^2\right)^2}. \tag{1.35}$$

The first-order condition, $\partial E[\Pi^B]/\partial n = 0$, yields

$$0 = \frac{b\,a\,\left(2\,\left(n^B + b - 2\right)\,c^2 - \left(3\,n^B + 2\,b - 4\right)\,c\,\kappa\,b\,a^2 + \left(n^B - 1\right)\,\kappa^2\,b^2\,a^4\right)}{\left(n^B - 1\right)\,\kappa\,b\,a^2 - n^B\,c},$$

$$n^B = \frac{-2\,c^2\,(b-2) + 2\,c\,\kappa\,(b-2)\,b\,a^2 + \kappa^2\,b^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4}. \tag{1.36}$$

Similarly, (1.36) applies for $\kappa < c/(b\,a^2)$; otherwise, the optimal team size is infinitely large. Additionally, as $n \geq 1$, the optimal team size is $n^B = 1$ for

$$\kappa < \frac{2\,c\,(b-1)}{(2\,b-1)\,b\,a^2}, \tag{1.37}$$

that is, if the algebraic term of (1.36) is less than 1. ■

Proof of Proposition 3. From a comparison of Propositions 1 and 2, it follows that, with overconfidence, $b > 1$, the agents' optimal team size increases, $n^* < n^B$, if

$$\frac{2\,c^2 - 2\,c\,\kappa\,a^2 + \kappa^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,a^2 + \kappa^2\,a^4} < \frac{-2\,c^2\,(b-2) + 2\,c\,\kappa\,(b-2)\,b\,a^2 + \kappa^2\,b^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4},$$

$$0 < \frac{2}{2\,c - \kappa\,a^2} - \frac{1}{c - \kappa\,a^2} + \frac{1}{c - \kappa\;b\,a^2} - \frac{2\,b}{2\,c - \kappa\,b\,a^2}. \quad (1.38)$$

Note that, as $n^* > 1$ only if $\kappa > 0$, and $n^B > 1$ only if κ is positive and sufficiently large, (1.38) can only be fulfilled if

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-1)\,b\,a^2}, \quad (1.39)$$

hence, if (1.15) holds and $\kappa < c/(b\,a^2)$. Equating (1.38) to zero and solving for b yields $n^* < n^B$ if

$$b > \frac{2\;(2\,c^3 - 4\,c^2\,\kappa\,a^2 + c\,\kappa^2\,a^4)}{\kappa\,a^2\;(4\,c^2 - 6\,c\,\kappa\,a^2 + \kappa^2\,a^4)} \quad (1.40)$$

for $\kappa \neq (3-\sqrt{5})\,c/a^2$. Otherwise, the algebraic term becomes infinitely large within the feasible parameter range. From this, it is easy to see that, for $c/(b\,a^2) > \kappa > 2\,c\,(b-1)/((2\,b-1)\,b\,a^2)$, (1.38) holds for all values of the degree of overconfidence b if $\kappa \geq (3-\sqrt{5})\,c/a^2$. Consequently, the threshold of (1.40) is applied in the range of $\kappa < (3-\sqrt{5})\,c/a^2$ for $c/(b\,a^2) > \kappa > 2\,c\,(b-1)/((2\,b-1)\,b\,a^2)$. ■

Proof of Proposition 4. As, given overconfidence, agents' true abilities do not change, $a_1 = a_j = a$, their individual profit function is consistent with the profit function of the rational model and is therefore given by (1.26) and (1.27). Conversely, when taking overconfident agents' expected individual profit of (1.32) as a basis, we must take into account that agent 1's ability is a rather than $\hat{a} = b\,a$. Consequently, we obtain

$$\begin{aligned} E\left[\Pi_1^W\right] &= \frac{E[Y]}{n} - c(e_1) \\ &= \frac{y_1 + (n-1)\,y_j + (n-1)\,\kappa\,y_j\,(y_1 + (n-2)\,y_j/2)}{n} - c\,\frac{e_1^2}{2} \\ &= \frac{a_1\,e_1 + (n-1)\,a_j\;e_j + (n-1)\,\kappa\,a_j\,e_j\,(a_1\,e_1 + (n-2)\,a_j\,e_j/2)}{n} - c\,\frac{e_1^2}{2} \\ &= \frac{a\;(e_1 + (n-1)\,e_j + (n-1)\,\kappa\,a\,e_j\,(e_1 + (n-2)\,e_j/2))}{n} - c\,\frac{e_1^2}{2}. \end{aligned} \quad (1.41)$$

Combining the overconfident agents' equilibrium effort of Lemma 2 in (1.41) with the contribution of $y_1 = a\,e^B$ and the coworkers' contributions of $y_j = a\,e^B$ for $j \neq 1$, we obtain the overconfident agents' real individual profit $E[\Pi^W]$ as a function of team size n. The new maximization problem is then

$$\max_n E\left[\Pi^W\right] = \frac{b\,a^2\;(2\,n\,c - b\;(c + (n-1)\,\kappa\,a^2))}{2\;(n\,c - (n-1)\,\kappa\,b\,a^2)^2}, \quad (1.42)$$

which corresponds to (1.17). The first-order condition, $\partial E[\Pi^W]/\partial n = 0$, yields the welfare-optimal team size with overconfidence,

$$0 = \frac{b\,a^2\,\left(-2\,\left(n^W - b\right)\,c^2 + \left(3\,n^W - 2\,b\right)\,c\,\kappa\,b\,a^2 - \left(n^W - 1\right)\,\kappa^2\,b^2\,a^4\right)}{2\,\left(n^W\,c - \left(n^W - 1\right)\,\kappa\,b\,a^2\right)^3},$$

$$n^W = \frac{b\,\left(2\,c^2 - 2\,c\,\kappa\,b\,a^2 + \kappa^2\,b\,a^4\right)}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4}. \tag{1.43}$$

In addition, (1.43) applies for $\kappa < c/(b\,a^2)$; otherwise, the optimal team size is infinitely large. Additionally, as $n \geq 1$, the optimal team size is $n^W = 1$ for $b < 1.5$ and

$$\kappa < \frac{2\,c\,(b-1)}{(2\,b-3)\,b\,a^2}, \tag{1.44}$$

that is, if the algebraic term of (1.43) is less than 1. ■

Proof of Proposition 5. A comparison of the optimal team sizes n^B and n^W of Propositions 2 and 4 yields

$$n^W - n^B = \frac{b\,\left(2\,c^2 - 2\,c\,\kappa\,b\,a^2 + \kappa^2\,b\,a^4\right)}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4} - \frac{-2\,c^2\,(b-2) + 2\,c\,\kappa\,(b-2)\,b\,a^2 + \kappa^2\,b^2\,a^4}{2\,c^2 - 3\,c\,\kappa\,b\,a^2 + \kappa^2\,b^2\,a^4}$$
$$= \frac{4\,c\,(b-1)}{2\,c - \kappa\,b\,a^2}. \tag{1.45}$$

As (1.45) is positive for $\kappa < c/(b\,a^2)$, we obtain $n^W > n^B$, provided that $n^W > 1$ holds. According to the proof of Proposition 4, $n^W > 1$ holds for all values of κ if $b \geq 1.5$. Otherwise, if $b < 1.5$, then $n^W > 1$, and consequently also $n^W > n^B$ if and only if

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-3)\,b\,a^2}, \tag{1.46}$$

hence, if (1.21) holds. ■

Proof of Proposition 6. As in the proposition, the proof consists of two parts.
(i) The first part of Proposition 6 compares the expected equilibrium profit of rational agents, $E[\Pi(n^*)]$, with the ex post expected individual profit of overconfident agents that is based on the welfare-optimal team size n^W, $E[\Pi^W(n^W)]$. In particular, according to (1.22) and (1.24), we obtain the following results:

First, for $0 < \kappa < c/(b\,a^2)$, team formation is optimal for both rational and overconfident agents such that $n^* > 1$ and $n^W > 1$ holds. This yields

$$E\left[\Pi^W\left(n^W\right)\right] - E\left[\Pi\left(n^*\right)\right] = \frac{\left(\kappa\,b\,a^3 - 2\,c\,a\right)^2}{8\,c\,\left(c - \kappa\,(b-1)\,a^2\right)\,\left(c - \kappa\,b\,a^2\right)} - \frac{\left(\kappa\,a^3 - 2\,c\,a\right)^2}{8\,c^2\,\left(c - \kappa\,a^2\right)} > 0. \tag{1.47}$$

From this, it is easy to see that for $0 < \kappa < c/(b\,a^2)$, (1.47) is strictly positive for all values of the degree of overconfidence $b > 1$, so that $E[\Pi^W(n^W)] > E[\Pi(n^*)]$ holds.

Second, for $\kappa \leq 0$, we obtain $n^* = 1$, while $n^W > 1$ if either $b \geq 1.5$ or if

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-3)\,b\,a^2}, \tag{1.48}$$

hence, if (1.21) holds. In this parameter range, the expected profit difference is

$$\begin{aligned} E\big[\Pi^W\big(n^W\big)\big] - E\big[\Pi\big(n^*\big)\big] &= \frac{\left(\kappa\,b\,a^3 - 2\,c\,a\right)^2}{8\,c\,\left(c - \kappa\,(b-1)\,a^2\right)\,\left(c - \kappa\,b\,a^2\right)} - \frac{a^2}{2\,c} \\ &= \frac{\kappa\,a^4\,\left(4\,c\,(b-1) + \kappa\,(4-3\,b)\,b\,a^2\right)}{8\,c\,\left(c - \kappa\,(b-1)\,a^2\right)\,\left(c - \kappa\,b\,a^2\right)} \leq 0. \end{aligned} \tag{1.49}$$

For $\kappa < 0$, (1.49) is negative within the feasible parameter range. The threshold value is $\kappa = 0$, where (1.49) equals zero, and consequently $E[\Pi^W(n^W)] = E[\Pi(n^*)]$.

Third, for $\kappa < 0$ and $n^W = 1$; hence, if (1.48) is not fulfilled, and $b < 1.5$, we obtain

$$E\big[\Pi^W\big(n^W\big)\big] - E\big[\Pi\big(n^*\big)\big] = -\frac{(b-2)\,b\,a^2}{2\,c} - \frac{a^2}{2\,c} = -\frac{(b-1)^2\,a^2}{2\,c} < 0. \tag{1.50}$$

In this parameter range, $E[\Pi^W(n^W)] < E[\Pi(n^*)]$. That is, for a team size of $n = n^W$, overconfidence increases (decreases) individual welfare only if $\kappa > 0$ ($\kappa < 0$).

(ii) The second part of Proposition 6 compares the expected equilibrium profit of rational agents, $E[\Pi(n^*)]$, with the ex post expected individual profit based on overconfident agents' optimal team size n^B, $E[\Pi^W(n^B)]$. Comparing (1.22) and (1.23) entails the following:

First, both $n^* > 1$ and $n^B > 1$ if

$$\kappa > \frac{2\,c\,(b-1)}{(2\,b-1)\,b\,a^2}, \tag{1.51}$$

hence if (1.15) holds, and $\kappa < c/(b\,a^2)$. In this parameter range, we obtain

$$\begin{aligned} &E\big[\Pi^W\big(n^B\big)\big] - E\big[\Pi\big(n^*\big)\big] \\ &\quad= \frac{b\,\left(\kappa\,b\,a^3 - 2\,c\,a\right)^2\,\left(3\,\kappa\,(b-1)\,b\,a^2 - c\,(3\,b-4)\right)}{8\,c\,\left(c - \kappa\,b\,a^2\right)\,\left(c\,(b-2) - \kappa\,(b-1)\,b\,a^2\right)^2} \\ &\quad\quad - \frac{\left(\kappa\,a^3 - 2\,c\,a\right)^2}{8\,c^2\,\left(c - \kappa\,a^2\right)} \\ &\quad= \frac{1}{8\,c^2}\left(\frac{\left(\kappa\,b\,a^3 - 2\,c\,a\right)^2}{\left(c - \kappa\,b\,a^2\right)}\,\frac{c\,b\,\left(3\,\kappa\,(b-1)\,b\,a^2 - c\,(3\,b-4)\right)}{\left(c\,(b-2) - \kappa\,(b-1)\,b\,a^2\right)^2}\right. \\ &\quad\quad \left. - \frac{\left(\kappa\,a^3 - 2\,c\,a\right)^2}{c - \kappa\,a^2}\right). \end{aligned} \tag{1.52}$$

From this, it is easy to see that as $b > 1$, the first bracketed fraction of (1.52) is positive and always greater than the third negative fraction. Depending on the parameter values, the second bracketed fraction can be positive or negative. However, κ reduces the denominator and increases the numerator of the second fraction within the given parameter range. Thus, the second fraction is strictly increasing in

κ. Moreover, for $\kappa \to c/(b\,a^2)$, the second fraction tends to $b > 1$. There is then a threshold value $\underline{\kappa} > 0$ with $E[\Pi^W(n^B)] = E[\Pi(n^*)]$. It follows that (1.52) is positive for $\kappa > \underline{\kappa}$, while it is negative for $\kappa < \underline{\kappa}$.

Second, if $0 < \kappa < c/(b\,a^2)$, but (1.51) is not fulfilled, overconfident agents choose to work alone, $n^B = 1$, while the optimal team size of rational agents is some $n^* > 1$. In this parameter range, we obtain

$$\begin{aligned} E\big[\Pi^W\big(n^B\big)\big] - E\big[\Pi\big(n^*\big)\big] &= -\frac{(b-2)\,b\,a^2}{2\,c} - \frac{\left(\kappa\,a^3 - 2\,c\,a\right)^2}{8\,c^2\,\left(c - \kappa\,a^2\right)} \\ &= \frac{a^2\,\left(\kappa\,a^2 - c\,\left(3 + 4\,(b-2)\,b + c/\left(c - \kappa\,a^2\right)\right)\right)}{8\,c^2} < 0. \end{aligned} \tag{1.53}$$

For $\kappa < c/(b\,a^2)$, (1.53) is strictly negative such that $E[\Pi^W(n^B)] < E[\Pi(n^*)]$ holds.

Third, for $\kappa \leq 0$, both rational and overconfident agents refrain from teamwork, $n^* = n^B = 1$. We obtain

$$E\big[\Pi^W\big(n^B\big)\big] - E\big[\Pi\big(n^*\big)\big] = -\frac{(b-2)\,b\,a^2}{2\,c} - \frac{a^2}{2\,c} = -\frac{(b-1)^2\,a^2}{2\,c} < 0, \tag{1.54}$$

which corresponds to (1.50) because $n^W = n^B = n^* = 1$. Consequently, for $n^W = n^B = 1$, overconfidence is always welfare-decreasing. Overall, for a team size of $n = n^B$, overconfidence increases (decreases) agents' individual welfare only if $\kappa > \underline{\kappa}$ ($\kappa < \underline{\kappa}$). ■

2 Divide et Impera: Curbing Agents' Duties to Remain in Office[1]

Abstract

By endogenizing a manger's optimal number of direct reports, we show how managers can exploit their organizational authority to shield themselves against replacement. Although the probability of hiring a star performer increases with the number of direct reports, each employee completes a smaller fraction of the overall task, such that learning about the employees' individual abilities occurs more slowly. We show that a manager maximizes the probability of retaining his job if he delegates a task to an infinite number of employees. Through the trade-off for the manager between decreasing his private costs of being replaced and increasing labor coordination costs, our model derives predictions of when managers tend to choose an excessively large number of direct reports, creating inefficiencies at the firm level.

[1] We thank Matthias Kräkel, Stefan Wielenberg, and Ian Jewitt for helpful comments. Participants in the 11th Brucchi Luchino Labor Economics Workshop in Trento, the 16th Colloquium on Personnel Economics in Tübingen, and the 75th Annual Conference of the German Academic Association for Business Research in Würzburg also provided helpful suggestions.

2.1 Introduction

In January 2010, Sara Mathew became CEO of Dun & Bradstreet Corporation, the world's leading provider of commercial information. This change was accompanied by an increase of direct subordinates from six agents under the direction of her predecessor to sixteen direct reports. One of the main reasons for this sudden more than doubling of the number of direct reports was that Sara Mathew "wanted to stay on top". After gaining experience and becoming more secure in her position, Sara Mathew feels more "comfortable" with only seven direct reports.[2]

The example of Sarah Mathew suggests that the number of subordinates seems to be an instrument for protecting a manager's position. By increasing the number of direct reports, a manager can reduce the influence and the visibility of each subordinate and thus strengthen the personal power inside the firm. This can be especially important for new managers, who are primarily confronted with a high level of job uncertainty. Sarah Mathew indicates a general trend in the "being the boss"-strategy of top executives. The findings of empirical analyses also indicate that managers have substantially expanded their number of direct reports over the past two decades, strengthening their power at the top hierarchy level (Rajan and Wulf (2006) and Guadalupe, Li, and Wulf (2012)).

In this paper, we present a novel conception of how top executives can exploit their personal working position to protect themselves from being replaced. More specifically, in his seminal work Mintzberg (1973) highlights that, by definition, a manager has formal authority over organizational decisions. In this work, we return to the roots of this definition and theoretically explore the possible consequences of allocating this authority to managers when they face career concerns. Career concerns are associated with biased actions on the part of the manager with a focus on the objective of not being replaced. We examine the nature of these incentives by endogenizing the question of a manager's optimal number of direct reports, if a principal assigns him responsibility over task delegation. Given our choice to focus on managerial career concerns and optimal organizational design, our model principally applies to diverse corporate structures and different hierarchy types, where a principal has to delegate part of his authority to a manager. Furthermore, since its famous implementation by Julius Caesar, the "divide et impera"-strategy has played an important role, not only in the field of economics but also for the protection of the personal power of top-ranking politicians.[3]

In the theoretical model, a principal hires a manager, who chooses the number of ex-ante identical employees to perform a task. While the manager has authority over

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[2]Description and citations based on Nielsen, G. R., and Wulf, J.: "How Many Direct Reports?", Harvard Business Review, April 2012.

[3]In the German political system, the number of Parliamentary State Secretaries is not limited by law and, hence, is selected by the respective minister. Although the scope of work does not change, the number of these secretaries has frequently increased, most recently since Angela Merkel took office in 2005. The example of the Federal Minister of Health, Daniel Bahr, who was appointed Parliamentary State Secretary by the former Federal Minister of Health, shows that Parliamentary State Secretaries can replace incumbent ministers and, therefore, represent a potential threat to their positions.

organizational decisions, the principal has the authority to replace the manager. Although the probability of hiring a star performer increases with the number of employees, each employee performs a smaller fraction of the overall task such that learning about employees' individual abilities occurs more slowly. The model shows that a manager minimizes the probability of his or her replacement if he delegates a task to an infinite number of employees.

Although increasing the number of direct reports can increase tenure and with it the on-the-job benefits of managers, task coordination becomes more costly. Thus, the manager's optimal number of direct reports results from a trade-off between decreasing his personal cost of being replaced and increasing labor coordination costs. Therefore, an increase in the number of employees leads to various sources of inefficiencies. This involves not only direct costs of task coordination, but can also be associated with indirect costs of possible suboptimal internal recruitment decisions. Consequently, managerial career concerns and the choice of the optimal number of direct reports are not only a private concern of the manager, they moreover represent a source of inefficiencies in the firm's global context.

We provide comparative static results on the formation and extent of these inefficiencies. The complex interactions between different exogenous variables and their impact on the manager's endogenous team size choice generate new insights in the provision of managerial incentives. One such insight is that, for a comparatively low labor cost factor, the optimal number of employees is inversely related to the manager's ability level. Consequently, an efficient organizational design assigns managers that face a low probability of replacement, such as high-ability managers, to high cost sectors and assigns complex tasks to them and vice versa.

Literature. This work relates to many other fields of the existing literature. First, it is relevant to the literature on managerial incentives arising from career concerns. Based on the seminal work of Fama (1980) and Holmström (1999), who outline the limitations on explicit payment structures, this literature primarily focuses on implicit incentives arising from career concerns and their implications for effort levels, investment decisions, or also compensation contracts (Holmström and Ricart i Costa (1986), MacLeod and Malcomson (1989), Scharfstein and Stein (1990), Gibbons and Murphy (1992), Dewatripont, Jewitt, and Tirole (1999a), Kräkel and Sliwka (2009)). This work mostly pertains to models that investigate the influence of career concerns on organizational decisions (Dewatripont, Jewitt, and Tirole (1999b), Bar-Isaac (2007)), and focuses on a novel consideration, the consequences of career concerns on a manager's choice of task delegation.

Second, this work refers to studies on the optimal allocation of authority, beginning with Fama and Jensen (1983) who analyze the factors facilitating separation of ownership and control. Similarly, based on the approach of Grossman and Hart (1986) and Hart and Moore (1990), Aghion and Tirole (1997) distinguish between formal authority as the right to decide and real authority as the effective control over decisions. Delegation of formal authority increases an agent's incentives for information acquisition, but also encourages the agent's participation. Related to our topic, formal authority is more likely to be delegated for decisions that are relatively more important to agents compared to principals, for example if the former can utilize extensive private benefits. Similarly, Baker, Gibbons, and Murphy (1999) investigate the role of information for the delegation of informal authority arguing that formal authority is only entitled to principals having the final say about decisions. Additionally, Stole and Zwiebel (1996) analyze intra-firm bargaining between principal and agents on a firm's organizational decisions. Our work is also tied to the general analysis on the optimal hierarchical composition of a firm (Williamson (1967), Calvo and Wellisz (1978), Rosen (1982), Qian (1994), Rajan and Zingales (2001)). Besides exploring technological issues for the optimal span of control or wage scales at different hierarchical tiers, this literature also offers incentive-based explanations. In our work, the choice of the optimal number of direct reports is not primarily motivated by technological reasons. Furthermore, we do not focus on managerial task delegation as a mechanism to provide incentives to agents, but moreover as a strategic instrument for managers to secure their personal working position.

Third, our model relates to the research on managerial turnover and entrenchment. In detail, Shleifer and Vishny (1989) analyze the problem of manager-specific investments that increase the cost of replacement of the manager. In Stulz (1988) managers strategically exert influence on their voting rights which impacts capital structure and firm value. Also Zwiebel (1996) and Fluck (1999) analyze the interaction between capital structure decisions and the extent of managerial entrenchment. Related to this topic, Hermalin and Weisbach (1998) focus on the establishment of personal loyalty that reduces the intensity by which entrenched managers are monitored, making it difficult to replace them. Further models analyze interrelations between severance pay and managerial entrenchment (Almazan and Suarez (2003)), or also the implications of entrenchment on risk-management (Kumar and Rabinovitch (2010)). While the literature on managerial entrenchment outlines how managers can use their decision autonomy to promote private benefits, the literature on on managerial turnover primary considers the advantages and consequences of managerial turnover with respect to efficiency (Höffler and Sliwka (2003), Sliwka (2007)). Our work primary focuses on organizational inefficiencies resulting from managerial entrenchment, based on learning about agents' abilities and the probability of managerial replacement.

Forth, there are models that directly explore managerial career concerns and inefficiencies in organizational design as they relate to employment, task assignment and delegation. Carmichael (1988) shows that tenure plays a key role in creating incentives to hire individuals who prove more suited to the position than themselves. Prendergast (1995) presents a theory of responsibility in organizations. He argues that if a manager collects skills by performing tasks, he then delegates too few tasks to his subordinates and thereby hoards responsibility to increase his future wages. Related to this topic, Sliwka (2001) shows that delegation reduces the power of middle managers, as subordinates become able to demonstrate their ability. He focuses on the resulting problem of managers becoming reluctant to delegate. In this paper, we explore how a manager's endogenous team size choice can be used as an instrument to protect his personal working position. We thereby offer a new understanding of how organizational authority can lead to managerial inefficiencies through incentives related to their career concerns.

The remainder of the paper is organized as follows. Section 2.2 develops the basic model, followed by an equilibrium analysis of the pure information problem in section 2.3. Section 2.4 incorporates coordination costs into the model. Section 2.5 provides a comparative static analysis. Finally, section 2.6 concludes. All proofs are available in the Appendix.

2.2 The Model

Consider a risk-averse firm (the principal) that employs a manager of a publicly known ability $A \geq 0$ to perform a one-period task. The task has the "length" $L > 0$, standing for the amount of labor required. In general, L is the aggregate task, consisting of a set of small subtasks that can be assigned to agents $i = 1, \ldots, n$, where the number n is endogenous. The manager can choose the number of agents at the beginning of the period.[4] Without loss of generality, let us assume that the task is an interval of subtasks, $[0; L]$, and subtasks are subintervals of length l_i, such that $L = \sum_{i=1}^{n} l_i$. Then, agent i performs the subtask l_i, with $\bigcup_{i=1}^{n} l_i = L$. Each agent has an unobservable ability a_i, independent and standard normally distributed, $a_i \sim \mathcal{N}(0, 1)$. Aggregate output Y is the sum of the outputs of single agents, $Y = \sum_{i=1}^{n} y_i$. Outputs of single agents are observable, they consist of the ability and an error term, $y_i = l_i\, a_i + \epsilon_i$, where the error terms are independent random variables with zero mean and a standard deviation that depends on the length of the task assigned to the agent.

An agent's output y_i can be used as a signal of his ability. The longer an agent works on his subtask, the more obvious his true abilities will become; the less noise there will be. We assume that ϵ_i has standard deviation $\sigma \cdot \sqrt{l_i}$. This assumption has the following micro-foundation. For integer l_i, one can say that the agent carries out l_i atomic work steps. If the output for each work step is $1 \cdot a_i + \epsilon$, and each error term ϵ is independently and identically distributed with $\epsilon \sim \mathcal{N}(0, \sigma)$, then the aggregate y_i is exactly like above, $y_i = l_i\, a_i + \epsilon_i$, where ϵ_i has variance $l_i\, \sigma^2$ and thus standard deviation $\sigma \cdot \sqrt{l_i}$.[5] We assume that output measures are observable to all parties. That is, manager and principal have the same level of information at every point in time.

Potentially, subtasks could be asymmetric. In the extreme case, some agents are then assigned to subtasks of zero size, which is equivalent to a reduction in team size n. Let us therefore concentrate on the symmetric case, where the manager's team size corresponds to the relevant number of agents. Then, agent i completes a share of $l_i = L/n$ of the overall task and produces an output of

$$y_i = l_i\, a_i + \epsilon_i = L/n \cdot a_i + \epsilon_i. \tag{2.1}$$

In the symmetric case, the measurement error ϵ_i decreases with task size L and increases with the number of agents n. All subtasks have an identical marginal

[4] We abstract from integer problems, except that $n \geq 1$.

[5] For non-integer l_i, the micro-foundation is similar, one only needs to start from smaller atomic tasks. For continuous l_i, one can model the noise as a Wiener process.

effect on the signal y_i. The resultant output is a noisy indicator of ability, and measurement becomes noisier if the number of agents grows. The intuition is that if the manager increases his team size, each agent completes a smaller fraction of the overall task and the variance of the measurement error increases. This weakens the precision of each signal and, in turn, the updating of each agent's ability.

The manager receives private benefits that are associated with the job.[6] However, the principal may replace the manager with an agent with expected ability $\hat{a}_i(y_i)$, conditional on observed output.[7] To incorporate the downside associated with the expectation of an agent's ability, we use an exponential utility function to value the principal's utility,

$$U(a) = -\mathrm{e}^{-\rho a}.$$

The parameter ρ represents a constant absolute risk aversion. The cut-off rule for managerial replacement derives from individual comparisons of the known utility provided by the manager and the ex-post expected utility provided by the agent. Consequently, the manager retains his job if $U(A) \geq E[U(\hat{a}_i(y_i))]$ holds for each individual comparison.[8] It follows that by choosing his optimal span of control, the manager seeks to retain his position.

The timing of the model is as follows:

1. *Principal*: employs a manager with a known ability of $A > 0$ to perform a task.

2. *Manager*: chooses the number of agents $n \geq 1$ with a priori unknown abilities $a_i \sim \mathcal{N}(0, 1)$ to complete the task.

3. *Agents*: complete subtasks. Individual outputs y_i are realized to update the agents' expected abilities.

4. The principal decides on managerial replacement:

   a) *The manager is retained* and enjoys private benefits if $U(A) \geq E[U(\hat{a}_i(y_i))]$ holds for all $i = 1, ..., n$.

   b) *The manager is replaced* by the agent with the highest expected utility $E[U(\hat{a}_i(y_i))]$, otherwise.

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[6]For the sake of simplicity, private benefits are normalized to zero in the model.

[7]Rosen (1982) argues that managerial ability is a crucial determinant of firm productivity. The basic rationale for focusing on ability as a decision criterion for managerial replacement refers to Meyer (1991), who argues that the output in the manager's position can be more related to ability than in the previous hierarchical levels. Additionally, the promotion of an intra-firm agent can have advantages over hiring an external worker. One aspect is that the market wage only reflects the general human capital of a worker, while specific human capital represents an additional costless benefit to a firm (Ortega (2003)). Although we do not explicitly model these relationships, their intuitions support our theoretical approach.

[8]Arya and Mittendorf (2011) show that without effort incentives, the choice of individual performance metrics produces the most informative signal of an agent's ability. As aggregation reduces the relative informativeness of each agent's signal, our results could also be applied to the case of aggregate performance measures.

2.3 The Pure Information Problem

To determine the manager's optimal span of control, we will initially analyze the problem set from the perspective of a representative agent. As, initially, all agents have identically unknown abilities and complete an equal share of the overall task, the results are valid for any agent. We will investigate the determinants of the probability of managerial replacement, given the signal of a single agent. Based on these results, we will extend our analysis by incorporating the signals of all agents and their impact on the manager's replacement probability. Therefore, this second analysis accounts both for the effects of 1) the signal of each agent and of 2) the number of signals produced.

According to (2.1), the distribution of each signal y_i results from the distributions of $a_i \sim \mathcal{N}(0,1)$ and $\epsilon_i \sim \mathcal{N}(0, \sigma^2\, l_i) = \mathcal{N}(0, \sigma^2\, L/n)$. Then, updating follows from

$$y_i = l_i\, a_i + \epsilon_i \sim \mathcal{N}\big(0, l_i^2 + \sigma^2\, l_i\big) = \mathcal{N}\big(0, L^2/n^2 + \sigma^2\, L/n\big). \tag{2.2}$$

The variance of each signal is the sum of the variances of the prior and the error term. It follows that the variance of a signal always exceeds the variance of an agent's ability, as it also incorporates measurement error, which increases with team size and decreases with task complexity. Consequently, the measurement variance decreases with the size of each subtask L/n. The output y_i determines the updating of an agent's ability according to the Bayes' theorem for normally distributed random variables. Accordingly, an agent's ex-post expected ability equals

$$\begin{aligned} \hat{a}_i(y_i) &= \frac{1}{1 + l_i/\sigma^2}\, 0 + \frac{l_i}{l_i + \sigma^2}\, y_i \\ &= \underbrace{\frac{n\,\sigma^2}{L + n\,\sigma^2}}_{=:q}\, 0 + \underbrace{\frac{L}{L + n\,\sigma^2}}_{=:1-q}\, y_i = \frac{L}{L + n\,\sigma^2}\, y_i. \end{aligned} \tag{2.3}$$

The posterior variance of an agent's ability results from the precision of both the prior and the conditional variances. Therefore, the *a posteriori* distribution of an agent's ability, given the signal y_i, is

$$\hat{a}_i | y_i \sim \mathcal{N}\Big(\frac{l_i}{l_i + \sigma^2}\, y_i, \frac{1}{1 + l_i/\sigma^2}\Big) = \mathcal{N}\Big(\frac{L}{L + n\,\sigma^2}\, y_i,\ \frac{n\,\sigma^2}{L + n\,\sigma^2}\Big). \tag{2.4}$$

The ex-post expected ability of an agent $\hat{a}_i(y_i)$ consists of a weighted average of the prior a_i (which equals zero) and the output y_i. The weight on the prior, denoted q, corresponds to the variance of the ex-post estimation of an agent's ability. Equivalently, the weight on the output measure is $1-q$. Both weights sum to 1, and hence, an increase in the measurement precision reduces the weight on the prior estimation of ability and vice versa. Equivalently, a smaller weight on the prior corresponds to a smaller variance in the ex-post estimation of ability and therefore to a larger weight on the signal y_i. Furthermore, learning decreases the variance of an agent's expected ability in comparison to the variance of the prior $\big(\frac{n\,\sigma^2}{L+n\,\sigma^2} < 1\big)$ and increases the precision of a_i. Updating increases with the number of observations, determined by task size L, and decreases with team size n and with the variance of the error term σ^2. The opposite holds for the variance of the a posteriori distribution of an agent's ability.

The cut-off rule for managerial replacement derives from individual comparisons of the known utility provided by the manager and the ex-post expected utility of an agent. Consequently, for a single signal y_i, the manager will retain his job if $U(A) \geq E[U(\hat{a}_i(y_i))]$ or, correspondingly, if

$$-e^{-\rho A} \geq \int_{-\infty}^{\infty} -e^{-\rho \tilde{a}} f(\tilde{a})\, d\tilde{a}. \tag{2.5}$$

Incorporating (2.4), we can rearrange (2.5) to

$$-e^{-\rho A} \geq -e^{-\frac{\rho\,(2\,L\,y_i - n\,\rho\,\sigma^2)}{2\,(L + n\,\sigma^2)}}, \tag{2.6}$$

or, equivalently, to

$$A \geq \frac{2\,L\,y_i - n\,\rho\,\sigma^2}{2\,(L + n\,\sigma^2)}. \tag{2.7}$$

Rearranging (2.7) yields the principal's optimal replacement rule. Lemma 3 outlines the result.

Lemma 3 *Define*

$$\bar{y} = A + \frac{n\,(2\,A + \rho)\,\sigma^2}{2\,L}, \tag{2.8}$$

and let $Y = \{y_1, .., y_n\}$*. Then, the principal replaces the manager only if the best signal* y_i *exceeds the critical* $\bar{y}$*, that is if* $\max\{y_1, .., y_n\} > \bar{y}$*.*

The critical signal $\bar{y}$ increases with the manager's ability A, the principal's risk aversion ρ, the variance of performance measurement σ^2, and the team size n and decreases with task size L. The higher the manager's ability A, the higher is the buffer range for the manager, $\mathrm{d}\bar{y}/\mathrm{d}A > 1$. In consideration of the principal's risk aversion ρ, a change in manager only occurs if the expectation of an agent's uncertain type significantly outweighs the manager's type. The higher the risk aversion of the principal, the greater the extent that the manager's ability has to be surpassed by an agent. Thus, risk aversion increases the benefits of the manager concerning his objective of not being replaced. The variance of the error term σ^2 increases the uncertainty of performance measurement and therefore decreases the updating of an agent's ability. The lower the weight on the signal y_i, the higher is the maximum possible $\bar{y}$. If the team size n increases, the precision of each signal and consequently the level of updating decrease and $\bar{y}$ rises. For an infinite number of agents, $\bar{y}$ also converges to infinity. In contrast, $\bar{y}$ decreases with the task size L. That is, L positively relates to the size of the subtask $l_i = L/n$ performed by each agent and therefore strengthens the task precision that, in turn, decreases $\bar{y}$. Thus, task complexity and team size have precisely opposite effects on $\bar{y}$. However, there is one essential difference: the larger the team size, the higher is the maximum $\bar{y}$, but simultaneously, the more numerous the signals and thereby the greater is the probability of hiring a star performer. Thus, by considering the probability of being replaced, the manager has to account for the impact of his team size choice on 1) the precision of a single signal and 2) the number of signals produced.

The distribution of an agent's signal is given by (2.2). Let $\Phi(y)$ be the distribution function of the standard normal distribution. Then, the distribution function of one signal y_i equals

$$F(y_i) = \Phi\left(y_i \sqrt{\frac{1}{1+\sigma^2\, n/L}}\right). \tag{2.9}$$

As the number of signals increases with the span of managerial control, we derive the distribution of the maximum of n signals, $\Phi(y)^n$. By incorporating (2.8), we can derive the probability that the maximum of the n signals will fall *below* the critical $\bar{y}$. We obtain the probability that the manager will be *retained* of

$$\begin{aligned} \Pr\{\max_{i\le n} y_i \le \bar{y}\} &= \bigvee_{i=1}^{n} \Pr(y_i \le \bar{y}) = \prod_{i=1}^{n} \Pr(y_i \le \bar{y}) = \Pr(y \le \bar{y})^n \\ &= \Phi\left(\left(A + \frac{n\,(2\,A+\rho)\,\sigma^2}{2\,L}\right)\sqrt{\frac{1}{1+\sigma^2\, n/L}}\right)^n. \end{aligned} \tag{2.10}$$

The complementary probability that the maximum of the n signals will *exceed* the critical $\bar{y}$, that is, the probability that the manager will be *replaced*, multiplied by the cost factor $k > 0$, determines the manager's replacement costs of $C_R(n) = k\,(1 - \Phi(y \le \bar{y})^n)$. Without loss of generality, we set $k = 1$. This yields

$$\begin{aligned} C_R(n) &= \Pr\{\max_i y_i > \bar{y}\} = 1 - \Pr(y \le \bar{y})^n \\ &= 1 - \Phi\left(\left(A + \frac{n\,(2\;A+\rho)\,\sigma^2}{2\,L}\right)\sqrt{\frac{1}{1+\sigma^2\, n/L}}\right)^n. \end{aligned} \tag{2.11}$$

As a next step, we can analyze the effect of team size on the manager's costs of the personal replacement $C_R(n)$. Proposition 7 outlines the result.

Proposition 7 *Managerial replacement costs $C_R(n)$ are maximized at some strictly positive number of agents $\hat{n}$. As $n \ge 1$, $C_R(n)$ is minimized when $n^* \to \infty$.*

As exemplified in figure 2.1, the replacement cost curve $C_R(n)$ is not monotonically decreasing with team size n. In the hypothetical case of $n = 0$, $C_R(n)$ is zero and increases (decreases) with n for $n < \hat{n}$ ($n > \hat{n}$). The slope of $C_R(n)$ results from the basic trade-off between the increasing probability of hiring a star performer and the decreasing probability of identifying one. On the one hand, if the manager increases the number of agents, the probability of employing an agent of superior ability increases, which in turn increases the probability that the manager will be replaced. On the other hand, if the manager divides the task among a large number of agents, each agent can only complete a small fraction of the overall task. This decreases the updating of each agent's ability and hence the probability of identifying a star performer. While this first effect dominates for small team sizes ($n < \hat{n}$), for large team sizes ($n > \hat{n}$), this relationship is reversed and replacement costs are negatively related to team size. For $n \to \infty$, $C_R(n)$ tends to zero. Consequently, as $n \ge 1$ by definition, the manager optimally hires as many agents as possible.

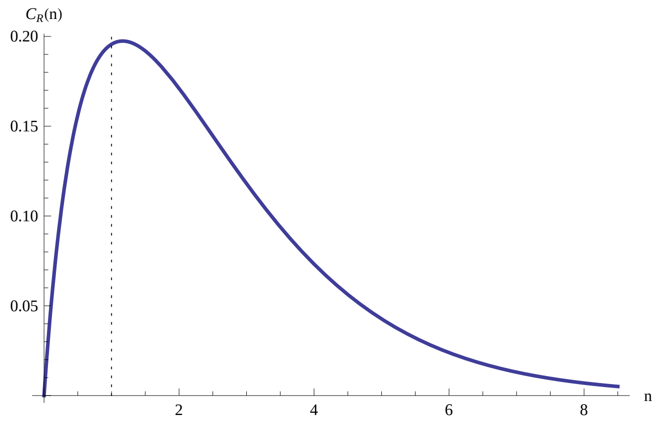

Figure 2.1: Managerial Replacement Costs $C_R(n)$ as a Function of Team Size n

The parameters are $L = 1$, $\rho = 5$, $\sigma = 1$ and $A = 0.1$. The vertical dotted line represents the minimal team size frontier of $n = 1$.

2.4 Introducing Coordination Costs

To obtain an interior optimum, the equilibrium of $n^* \to \infty$ must be expensive. Here, we focus on labor coordination costs $C_{LC}(n)$ and assume that the larger the team size n, the more coordination is required for integrating the individual contributions of team members.[9] To provide some intuition, suppose that each infinitesimal task requires an information input from each other task. If the task is performed by the same agent, this coordination becomes less costly.[10] Then, with the cost parameter $c > 0$, coordination costs can take the form of

$$C_{LC}(n) = \sum_{i=1}^{n} \sum_{j \neq i}^{n} c\, l_i\, l_j = \sum_{i=1}^{n} \sum_{j \neq i} c\, \frac{L}{n} \frac{L}{n} = c\, \frac{n-1}{n}\, L^2. \tag{2.12}$$

Coordination costs measure the divisibility of the task. If the cost factor c is high, the division of tasks between different agents results in complex combinations, while, for a small c, agents are easily exchangeable.[11] Therefore, coordination is required for two subtasks at a time, for all pairs of tasks, which yields $n\,(n-1)/2$ combinations.

[9] The consideration of labor coordination costs represents a standard approach in the team production literature. In their seminal work, Becker and Murphy (1992) emphasize that specialization and the division of labor fundamentally depend on coordination costs that increase with the number of agents. Therefore, coordination costs can be associated with different sources of inefficiencies, such as principal agent conflicts, problems of task coordination and monitoring or communication difficulties.

[10] Our results also apply in more complex cases, for example, if we assume that information is only required with some probability or only for higher stages of the overall task.

[11] For example, coordination costs can be low in companies, where divisions are geographically proximate and production can be coordinated easily. Additionally, the nature of the task itself can be subject to divergent marginal coordination costs. Considering the example of a call center, it is expected that marginal costs for coordinating different calls are comparatively low, while the development of a complex software system may require extensive coordination among team members.

Furthermore, the larger the size of the subtask l_i, the more input is required for subtask l_j and vice versa. That is, total coordination costs increase with the number of tasks n and with the subtask size $l_i = L/n$. More generally, if coordination costs subproportionally increase with subtask size, determined by the parameter α, with $0 < \alpha \leq 1$, we obtain

$$C_{LC}(n) = \sum_{i=1}^{n} \sum_{j \neq i}^{n} c\, l_i\, l_j^{\alpha} = \sum_{i=1}^{n} \sum_{j \neq i}^{n} c\, \frac{L}{n}\, \frac{L^{\alpha}}{n^{\alpha}} = c\, \frac{n-1}{n^{\alpha}}\, L^{1+\alpha}. \tag{2.13}$$

Consequently, α measures the extent of the task coordination problem. That is, for every pair of subtasks l_i and l_j, the smaller the α and the larger the subtask size l_j, the lower is the cost increase for coordinating the subtasks. In the symmetric case, $l_i = L/n$, marginal coordination costs therefore increase (decrease) with α for $L > n$ ($L \leq n$). Overall, according to (2.11) and (2.13), the manager chooses an optimal team size, denoted n^*, that minimizes his total costs as the sum of 1) the costs of the personal replacement $C_R(n)$ and of 2) the costs of labor coordination $C_{LC}(n)$. The manager solves

$$\begin{aligned} &\min_n (C_R(n) + C_{LC}(n)) \\ &= 1 - \Phi\left(\left(A + \frac{n\,(2\,A + \rho)\,\sigma^2}{2\,L}\right)\sqrt{\frac{1}{1 + \sigma^2\, n/L}}\right)^n + c\, \frac{n-1}{n^{\alpha}}\, L^{1+\alpha}. \end{aligned} \tag{2.14}$$

This objective function reveals the two costs, $C_R(n)$ and $C_{LC}(n)$, that the manager has to trade-off. On the one hand, the costs of labor coordination $C_{LC}(n)$ are strictly increasing with the number of agents n and therefore limit the manager's optimal span of control. On the other hand, as $n \geq 1$, the manager minimizes his replacement costs $C_R(n)$ by choosing an infinite number of agents. Moreover, as $C_R(n)$ is not monotonically decreasing with n, the manager has to account for both the effect of the increasing probability of hiring a star performer, which dominates for small team sizes ($n < \hat{n}$), and the effect of a decreasing visibility of each agent's ability, which dominates for large team sizes ($n > \hat{n}$). More specifically, it is important to determine the cases in which the manager optimally chooses $n^* = 1$ compared to the equilibrium of $n^* > 1$. As the single agent equilibrium represents a technically feasible option, the manager causes inefficiencies in terms of labor coordination costs if he chooses $n^* > 1$. Consequently, it is important to derive conditions in which the manager is incentivized to create these inefficiencies. Proposition 8 outlines the result.

Proposition 8 *There is a cost factor $\bar{c} > 0$ such that for $c \geq \bar{c}$, it is optimal to employ only one agent, $n^* = 1$. For $c < \bar{c}$, the optimal number of agents is some $n^* > 1$ that is characterized implicitly by*

$$\frac{\partial C_R(n)}{\partial n} = -\frac{\partial C_{LC}(n)}{\partial n} = c\, \frac{L^{1+\alpha}}{n^{1+\alpha}}\, (n\,(\alpha - 1) - \alpha). \tag{2.15}$$

Figure 2.2 depicts the interaction of replacement costs $C_R(n)$ and labor coordination costs $C_{LC}(n)$ for the two results of $n^* > 1$ and $n^* = 1$. Obviously, a local minimum of total costs can only be located in the range of $n > \hat{n}$, where $C_R(n)$ decreases

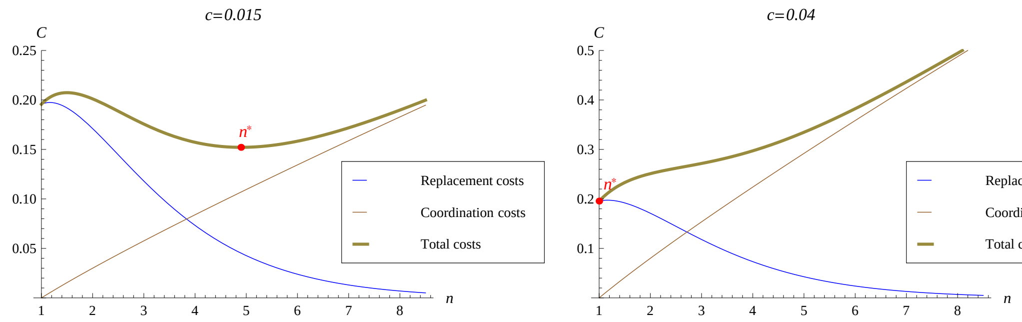

Figure 2.2: Managerial Replacement Costs $C_R(n)$ and Labor Coordination Costs $C_{LC}(n)$ as Functions of Team Size n

The parameters are $L = 1$, $\rho = 5$, $\sigma = 1$, $A = 0.1$, as before, and $\alpha = 0.5$.

with n. Here, the effect of a decreasing signal precision outweighs the effect of an increasing probability of hiring an agent of superior ability. In this range, the manager has to trade-off increasing labor coordination costs $C_{LC}(n)$ and decreasing replacement costs $C_R(n)$. If the labor cost factor c is comparatively small, marginal labor costs are low and the manager is incentivized to increase his span of control. The optimal team size is $n^* > 1$ (left figure). Conversely, a large cost factor can prevent the manager from increasing his span of control. This is the case if the labor cost increase from hiring an additional agent outweighs the associated decrease in replacement costs in the range of $n > \hat{n}$. Then, total costs are strictly increasing with team size, $n^* = 1$ (right figure). High marginal labor costs may also cause higher total costs at the minimum of the total cost curve than under the single agent equilibrium and $n^* = 1$. Overall, labor costs represent a key factor concerning the inefficiencies resulting from a manager's choice of his optimal span of control and, more generally, from his organizational authority. By assigning managers to different coordination cost sectors, the principal can indirectly influence the occurrence and the extent of these inefficiencies. Therefore, an efficient organizational design assigns managers that face a low probability of replacement, such as high-ability managers, to high coordination cost sectors, as these managers are less likely to choose a high span of control and thus are less likely to create extensive inefficiency costs.

2.5 Comparative Static Results

Implicit differentiation of (2.14) enables us to study how the manager adjusts his optimal team size n^* to changes in the exogenous parameters A, ρ, σ^2 or L. Proposition 9 outlines the results.

Proposition 9 *For c sufficiently small, the optimal number of agents n^* is decreasing with the manager's ability A, the principal's risk aversion ρ, the variance of performance measurement σ^2, or increasing with task size L.*

First, if the manager's ability A increases, replacement costs $C_R(n)$ decrease and the manager enjoys a larger buffer range. As labor coordination costs $C_{LC}(n)$ increase

Parameters	n^*
$A \nearrow$	$\searrow$
$\rho \nearrow$	$\searrow$
$\sigma^2 \nearrow$	$\searrow$
$L \nearrow$	$\nearrow$

Table 2.1: Effects of Variations of Parameters

with team size, the manager is incentivized to decrease the optimal number of agents n^*. Conversely, if A decreases, the manager has to increase his team size to maintain the former replacement probability. However, labor costs increase with team size, and the two cost functions have opposite effects. Overall, if the cost factor c is sufficiently small, the manager's optimal team size n^* negatively relates to A. While this economic intuition holds, no general expression for the cost factor c can be derived because of the implicit function governing the optimal team size (see the appendix).[12] However, if this equilibrium is ensured, then high-quality managers prefer smaller teams, as for them the probability of being replaced is lower compared to low quality managers.

Second, if the principal's risk aversion ρ concerning the workers' uncertain expected abilities increases, the manager enjoys a greater buffer range: the ex-post expected ability of an agent has to substantially surpass the manager's ability, and hence the manager's probability of retaining the job increases. Therefore, he can decrease the number of agents to reduce labor costs. Conversely, if ρ decreases, the manager is incentivized to increase his optimal team size if c is comparatively low. Therefore, risk aversion increases the job security of the manager, which encourages an increase in organizational efficiency.

Third, if the variance of performance measurement σ^2 decreases, the manager is incentivized to increase his optimal team size if marginal labor costs are not excessively high. Conversely, if σ^2 increases and performance measurement becomes less precise, the manager can benefit from the decreased informativeness of the agents' signals. The decreased probability of replacement adjusts the manager's optimal team size downwards, thereby reducing his organizational inefficiencies.

Fourth, if task complexity L rises, the size of each agent's subtask $l_i = L/n$ increases and with it the precision of the ex-post estimation of each agent's ability. This, in turn, increases the probability that the manager will be replaced, providing him incentives to increase his optimal team size. However, while a change in the manager's ability A, the principal's risk aversion ρ or the variance of performance measurement σ^2 directly influences replacement costs $C_R(n)$ and only indirectly influences labor

[12] This general result also holds for the comparative static results of the remaining exogenous parameters ρ, σ^2, and L. Furthermore, note that our comparative static results do not apply for the equilibrium of $n^* = 1$, that is, for $c \geq \bar{c}$. Then, using the example of an increase in managerial ability A, the manager cannot further reduce the optimal team size n^*, as $n \geq 1$ by definition. In contrast, if A decreases, the manager may not be incentivized to increase his team size if marginal labor costs c are high, such that $n^* = 1$.

costs $C_{LC}(n)$ through a change in n^*, the task size L, in addition, directly influences $C_{LC}(n)$. While, from the standpoint of the replacement costs, the optimal team size has to increase to maintain the former replacement probability, from the standpoint of the labor costs, an increase in n results in a twofold increase in labor costs: first, labor costs increase due to an increase in n and, second, due to an increase in L. This, in turn, implies that, when considering the four exogenous parameters A, ρ, σ^2 and L, the task size represents the most influential variable in restricting managers' organizational inefficiencies. Furthermore, as task size L and marginal labor costs c have a multiplicative effect on overall labor costs, labor costs and organizational inefficiencies resulting from a manager's "divide et impera"-strategy can be minimized by allocating managers facing a low probability of replacement, such as high-ability managers (high A), to high cost sectors (high c) and assigning complex tasks (high L) to them and vice versa. Related to our example of Sarah Mathew, our comparative static results account for the observation that new managers with a lower managerial ability are incentivized to increase their span of control, while, if managerial ability increases, replacement costs decrease and managers decrease their optimal team size and with it also the costs of labor coordination.

2.6 Conclusion

In this paper, we present a new source of inefficiencies resulting from managerial discretion over organizational decisions. Although the probability of hiring a star performer increases with the number of agents, the opposite holds for the probability of identifying one. Our model shows that a manager decreases the probability of being replaced by a subordinate, if he delegates a task to a sufficiently large number of agents. However, because labor costs increase with team size, the manager's optimal span of control results from the trade-off of decreasing replacement costs and increasing labor coordination costs.

Of particular note, our work derives the following implications: in settings with low marginal labor costs team sizes will be larger, especially if the manager faces a high probability of replacement. Moreover, for a sufficiently small labor cost factor, the optimal team size negatively relates to managerial ability, the principal's risk aversion concerning the agents' unknown abilities and the uncertainty of performance measurement. Conversely, the optimal team size increases with task complexity. Therefore, our work yields clear predictions on the optimal internal design of organizations to limit organizational inefficiencies resulting from a manager's "divide et impera"-strategy. Accordingly, an efficient organizational structure allocates managers facing a low probability of replacement, for example high-ability managers, to high cost sectors and assigns complex tasks to them, and vice versa.

The results of our work provide several hypotheses that may guide future research. First, related to the result that a larger span of control decreases the agents' probability of promotion, it would be of interest to reconsider the consequences of managerial actions by incorporating the agents' explicit and implicit incentives regarding production and wages. Second, analyzing the model from a dynamic perspective could generate new insights on the manager's optimal team size choice and the preferred allocation of agents to tasks. Especially with respect to further learning effects about agents' abilities, a manager may be incentivized to regroup agents among different jobs or tasks or even to exchange them completely. Third, a restriction on the span of control can be used as an instrument to exert pressure on the manager. The investigation of its effectiveness and comparing it to other instruments could yield interesting insights on the optimal provision of managerial incentives.

By endogenizing a manager's optimal team size choice, our paper helps to explain the recent empirical puzzle concerning the remarkable shift to greater spans of managerial control (Guadalupe, Li, and Wulf (2012)). The results of our work generate new insights on the optimal allocation of managers to organizations and derive testable predictions concerning the impact of environmental variables on the optimal span of managerial control. Overall, our work highlights how organizational authority can reinforce the power of managers through the strategy of "divide et impera".

2.7 Appendix

Proof of Lemma 3. In the general case of the normal distribution, with a prior distribution of $a \sim \mathcal{N}(\mu_a, \sigma_a^2)$, the posterior, given the observation y, is

$$\hat{a} \mid y \sim \mathcal{N}(\mu_{\hat{a}(y)}, \sigma^2_{\hat{a}(y)}),$$

with

$$\mu_{\hat{a}(y)} = \mu_a + (y - \mu_y)\, \frac{\sigma_a^2}{\sigma_a^2 + \sigma_y^2}.$$

The posterior variance of an agent's ability is the sum of the prior variance and the conditional variance on the basis of their inverses and equals

$$\sigma^2_{\hat{a}(y)} = \frac{1}{1/\sigma^2_{\hat{a}(y)}} = \frac{1}{1/\sigma_a^2 + 1/\sigma_y^2}.$$

In our case, with $a_i \sim \mathcal{N}(0,1)$ and $\epsilon_i \sim \mathcal{N}(0, \sigma^2\, l_i) = \mathcal{N}(0, \sigma^2\, L/n)$, and $y_i = l_i\, a_i + \epsilon$, the ex-post expected ability is

$$\hat{a}_i(y_i) = 0 + (y_i - 0)\, \frac{1}{1^2 + \frac{\sigma^2}{l_i}} = \frac{1}{1 + \frac{\sigma^2}{L/n}}\, y_i = \frac{L}{L + n\, \sigma^2}\, y_i, \tag{2.16}$$

which corresponds to the result of (2.3). The variance of the posterior is

$$\sigma^2_{\hat{a}(y)} = \frac{1}{1/1^2 + 1/\left(\frac{\sigma^2}{l_i}\right)} = \frac{1}{1 + 1/\left(\frac{\sigma^2}{L/n}\right)} = \frac{n\,\sigma^2}{L + n\,\sigma^2}. \tag{2.17}$$

It follows that the posterior of an agent's ability is distributed according to (2.4). Using this result, with an exponential utility function of the form $U(a) = -\mathrm{e}^{-\rho\, a}$, $U(A) \geq E[U(\hat{a}_i(y_i))]$ is fulfilled if (2.7) holds. Solving for y gives $\bar{y}$ of Lemma 3 as the upper threshold for an individual signal y_i for the manager to retain his position.

■

Proof of Proposition 7. The first derivative of $C_R(n)$ with respect to n equals

$$\frac{\partial C_R(n)}{\partial n} = -\,(1 - C_R(n))\Bigg(\log[(1-C_R(n))^{\frac{1}{n}}] + \frac{z(n)\,n\,\sigma^2\,(2\,A\,(L+n\,\sigma^2)+\rho\,(2\,L+n\,\sigma^2))}{L\sqrt{2\,\pi}\,(L+n\,\sigma^2)\,\sqrt{1+n\,\sigma^2/L}}\Bigg), \tag{2.18}$$

with

$$z(n) = \frac{e^{-\frac{(n\,\rho\,\sigma^2+2\,A\,(L+n\,\sigma^2))^2}{8\,L\,(L+n\,\sigma^2)}}}{\Phi\Big(\Big(A+\frac{n\,(2\,A+\rho)\,\sigma^2}{2\,L}\Big)\sqrt{\frac{1}{1+\sigma^2\,n/L}}\Big)} = \frac{e^{-\frac{(n\,\rho\,\sigma^2+2\,A\,(L+n\,\sigma^2))^2}{8\,L\,(L+n\,\sigma^2)}}}{\Big(1+erf[\frac{A+\frac{n\,2\,(A+\rho)\,\sigma^2}{2\,L}}{\sqrt{2+2\,n\,\sigma^2/L}}]\Big)} = \frac{e^{-\frac{(n\,\rho\,\sigma^2+2\,A\,(L+n\,\sigma^2))^2}{8\ L\,(L+n\,\sigma^2)}}}{2\,(1-C_R(n))^{\frac{1}{n}}}. \tag{2.19}$$

A range of $0 < z(n) \leq 1$ applies for (2.19). Furthermore, $z(n)$ increases with L and decreases with ρ, σ, n, and A. The denominator represents the probability that the critical $\bar{y}$ will not be met, given a single signal y_i. This probability decreases with L and increases with ρ, σ, n, and A. Accordingly, this relationship is reversed for the whole equation (2.19), as its numerator is positive.

To see that $C_R(n)$ is maximal at some strictly positive value of $\hat{n}$, note that for $n = 0$, $C_R(n) = 0$ and for $n > 0$, we obtain $1 > C_R(n) > 0$. That is, $\frac{\partial C_R(n)}{\partial n} > 0$ for small values of n ($n < \hat{n}$). Conversely, according to (2.18), $\frac{\partial C_R(n)}{\partial n} > 0$ if

$$0 > \frac{z(n)\,n\,\sigma^2\,(2\,A\,(L+n\,\sigma^2)+\rho\,(2\,L+n\,\sigma^2))}{L\sqrt{2\,\pi}\,(L+n\;\sigma^2)\,\sqrt{1+n\,\sigma^2/L}} + \log[(1-C_R(n))^{\frac{1}{n}}]. \tag{2.20}$$

The second term of the right-hand side of (2.20) becomes arbitrarily close to 0 for large values of n. Therefore, this inequality is not fulfilled if n is sufficiently large and $\frac{\partial C_R(n)}{\partial n} < 0$ holds for $n > \hat{n}$. Therefore, for $n \to \infty$, $C_R(n)$ tends to zero. ■

Proof of Proposition 8. According to (2.14), the manager minimizes $C_R(n) + C_{LC}(n)$. For $n = 1$, we obtain $C_{LC}(n) = 0$, and for $n > 1$, $\frac{\partial C_{LC}(n)}{\partial n} > 0$ holds. According to the proof of Proposition 7, for $n = 0$, we obtain $C_R(n) = 0$, and for $n > 0$, $C_R(n) > 0$ holds. The replacement cost curve is concave, with $\frac{\partial C_R(n)}{\partial n} > 0$ for $n < \hat{n}$ and $\frac{\partial C_R(n)}{\partial n} < 0$ for $n > \hat{n}$. For $n \to \infty$, we obtain $C_R(n) \to 0$. As $\frac{\partial C_{LC}(n)}{\partial n} > 0$, then, for $n \to \infty$, we obtain $C_{LC}(n) \to \infty$ and therefore also $C_R(n) + C_{LC}(n) \to \infty$.

For $n < \hat{n}$, both $\frac{\partial C_R(n)}{\partial n} > 0$ and $\frac{\partial C_{LC}(n)}{\partial n} > 0$ and therefore we obtain $\frac{\partial C_R(n)+C_{LC}(n)}{\partial n} > 0$. For $n > \hat{n}$, we obtain $\frac{\partial C_R(n)}{\partial n} < 0$ and $\frac{\partial C_{LC}(n)}{\partial n} > 0$. In this range, there can be a local minimum of total costs at some strictly positive value of n^*, where the positive slope of the labor cost curve equals the negative slope of the replacement cost curve, implicitly characterized by (2.15). The underlying implicit function $g(n, x) = \frac{\partial C_R(n)+C_{LC}(n)}{\partial n} = 0$ is derived in the proof of Proposition 9, equation (2.21).

Next, we will determine the cases in which the manager optimally chooses $n^* = 1$, so that we can derive conditions required for $n^* > 1$. If c is comparatively large, total costs at the local minimum can be higher than for the single-agent equilibrium, $C_R(n) + C_{LC}(n) \mid_{n=n_{min}} > C_R(n) + C_{LC}(n) \mid_{n=1}$, such that $n^* = 1$. Alternatively, if c is sufficiently large, then $\frac{\partial C_{LC}(n)}{\partial n} > \frac{\partial C_R(n)}{\partial n}$ for $n > \hat{n}$ and $n^* = 1$. Consequently, for sufficiently small values of the cost factor ($c < \bar{c}$), the manager optimally chooses $n^* > 1$, while, for comparatively large values of the cost factor ($c \geq \bar{c}$), the optimal team size is $n^* = 1$. ■

Proof of Proposition 9. We use the Implicit Function Theorem to prove the comparative static results. The first order condition for a minimum requires that $\frac{\partial(C_R(n)+C_{LC}(n))}{\partial n} = 0$. Differentiating (2.14) with respect to n and equating to zero yields our implicit function $g(n, x) = 0$ of

$$\begin{aligned} g(n,x) &= \frac{\partial(C_R(n) + C_{LC}(n))}{\partial n} = 0 \\ &= -c\,\frac{L^{1+\alpha}}{n^{1+\alpha}}\,(n\,(\alpha-1)-\alpha) - (1-C_R(n))\,\Bigg(\log[(1-C_R(n))^{\frac{1}{n}}] \\ &+ \frac{z(n)\,n\,\sigma^2\,(2\,A\,(L+n\,\sigma^2)+\rho\,(2\,L+n\,\sigma^2))}{L\,\sqrt{2\,\pi}\,(L+n\,\sigma^2)\,\sqrt{1+n\,\sigma^2/L}}\Bigg), \end{aligned} \tag{2.21}$$

with x standing for one of the exogenous parameters A, L, ρ or σ^2.

From the proofs of Propositions 7 and 8, it follows that $\frac{\partial C_{LC}(n)}{\partial n} > 0$ and $\frac{\partial C_R(n)}{\partial n} < 0$ for $n > \hat{n}$. The first order condition requires that $-\frac{\partial C_{LC}(n)}{\partial n} = \frac{\partial C_R(n)}{\partial n}$, or equivalently, that

$$c\,\frac{L^{1+\alpha}}{n^{1+\alpha}}\,(n\,(\alpha-1)-\alpha) = \frac{\partial C_R(n)}{\partial n}. \tag{2.22}$$

The comparative static results follow from $\frac{\partial n^*}{\partial x} = -\frac{\frac{\partial g(n,x)}{\partial x}}{\frac{\partial g(n,x)}{\partial n}}$. The second order condition for a minimum requires that the sign of the denominator is positive, $\frac{\partial g(n,x)}{\partial n} = \frac{\partial^2(C_R(n)+C_{LC}(n))}{\partial n^2} > 0$. Therefore, as $\frac{\partial g(n,x)}{\partial n} \neq 0$, we fulfill the conditions of the Implicit Function Theorem. Now it is sufficient to consider the sign of the numerator $-\frac{\partial g(n,x)}{\partial x}$ for each exogenous parameter. If the first derivative is negative, n^* is increasing with x or vice versa. The results are given in the following steps.

Part 1. Consider the case of $x = A$. The optimal team size n^* decreases with A if $\frac{\partial n^*}{\partial A} = -\frac{\frac{\partial g(n,A)}{\partial A}}{\frac{\partial g(n,A)}{\partial n}} < 0$ or, equivalently, if $\frac{\partial g(n,A)}{\partial A} > 0$. Differentiating (2.21) with respect to A, rearranging terms and using the result of (2.22) yields the condition

$$\begin{aligned} 0 < &-4\,L\,\Bigg(n\,\sigma^2 + 2\,(L+n\,\sigma^2)\,\Bigg(1 - \frac{c\,L^{1+\alpha}\,(n\,(\alpha-1)-\alpha)}{n^\alpha\,(1-C_R(n))}\Bigg)\Bigg) \\ &+ n\,\sigma^2\Bigg(2\,A + \rho + \frac{L\,\rho}{L+n\,\sigma^2}\Bigg)\,\Bigg(2\,A\,L + n\,(2\,A+\rho)\,\sigma^2 \\ &+ 2\,z(n)\,L\,\sqrt{\frac{2}{\pi}}\,\sqrt{1+\frac{n\,\sigma^2}{L}}\Bigg). \end{aligned} \tag{2.23}$$

As $z(n)$ is in the positive term of (2.23), we set $z(n) = 0$. As (2.23) has to be positive overall, this replacement corresponds to our worst-case condition for the fulfillment of the comparative static result. Furthermore, for the purpose of simplification, we prove our results given the special case of $L \leq 1$, such that $L \leq n$ holds and labor coordination costs decrease with α. Therefore, we can set $\alpha = 0$, as α is in the negative term of (2.23) and obtain our worst-case condition of

$$\begin{aligned} 0 < S_A(A) = &- 4\,L\left(n\,\sigma^2 + 2\,(L + n\,\sigma^2)\left(1 + \frac{c\,L\,n}{1 - C_R(n)}\right)\right) \\ &+ n\,\sigma^2\,(2\,A\,L + n\,(2\,A + \rho)\,\sigma^2)\left(2\,A + \rho + \frac{L\,\rho}{L + n\,\sigma^2}\right). \end{aligned} \tag{2.24}$$

It is convenient to define the right-hand side of (2.24) as $S_A(A)$. We complete our proof by deriving conditions for which $S_A(A = 0) > 0$ and $\frac{\partial S_A(A)}{\partial A} > 0$. Then, the right-hand side of (2.24) is positive for $A = 0$ and is increasing with A, and hence is positive for any value of A. First, $S_A(A = 0) > 0$ if

$$c < \frac{(1 - C_R(n))\,(-8\,L^3 - 20\,L^2\,n\,\sigma^2 + 2\,L\,n^2\,(\rho^2 - 6)\,\sigma^4 + n^3\,\rho^2\,\sigma^6)}{8\,L^2\,n\,(L + n\,\sigma^2)^2}. \tag{2.25}$$

Note that $(1 - C_R(n))$ corresponds to the result of (2.10) and represents the probability that the manager will retain his job. As $n \geq 1$, we obtain $0 < (1 - C_R(n)) < 1$. From this it is simple to show that, for any $0 < (1 - C_R(n)) < 1$, the first derivative of (2.25) with respect to n is positive and, therefore, (2.25) is increasing with n. That is, for any $0 < (1 - C_R(n)) < 1$, we can set $n = 1$ and obtain our worst-case condition for the cost parameter c of

$$c < \frac{(1 - C_R(n))\,(-8\,L^3 - 20\,L^2\,\sigma^2 + 2\,L\,(\rho^2 - 6)\,\sigma^4 + \rho^2\,\sigma^6)}{8\,L^2\,(L + \sigma^2)^2}. \tag{2.26}$$

From (2.26) follows that, for any $0 < (1 - C_R(n)) < 1$, $S_A(A = 0) > 0$ if c is sufficiently small. Finally, for any $0 < (1 - C_R(n)) < 1$, the first derivative of (2.24) with respect to A equals

$$\frac{\partial S_A(A)}{\partial A} = 4\,n\,(2\,A + \rho)\,\sigma^2\,(L + n\,\sigma^2) > 0. \tag{2.27}$$

As (2.27) is positive, n^* is decreasing with A for any $0 < (1 - C_R(n)) < 1$ if (2.27) holds.

Part 2. Consider the case of $x = \rho$. The optimal team size n^* decreases with ρ if $\frac{\partial n^*}{\partial \rho} = -\frac{\frac{\partial g(n,\rho)}{\partial \rho}}{\frac{\partial g(n,\rho)}{\partial n}} < 0$ or, equivalently, if $\frac{\partial g(n,\rho)}{\partial \rho} > 0$. Differentiating (2.21) with respect to ρ, rearranging terms and using the result of (2.22) yields the condition

$$\begin{aligned} 0 < &- 4\,L\,(4\,L + 3\,n\,\sigma^2) + \frac{8\,c\,L^{2+\alpha}\,(n\,(\alpha - 1) - \alpha)\,(L + n\,\sigma^2)}{n^\alpha\,(1 - C_R(n))} \\ &+ n\,\sigma^2\left(2\,A + \rho + \frac{L\,\rho}{L + n\,\sigma^2}\right)\Bigg(2\,A\,L + n\,(2\;A + \rho)\,\sigma^2 \\ &+ 2\,z(n)\,L\,\sqrt{\frac{2}{\pi}}\,\sqrt{1 + \frac{n\,\sigma^2}{L}}\Bigg). \end{aligned} \tag{2.28}$$

As $z(n)$ is in the positive term of (2.28), we set $z(n) = 0$. As (2.28) has to be positive overall, this replacement corresponds to our worst-case condition for the fulfillment of the comparative static result. For the same reason, in the special case of $L \leq 1$, we can set $\alpha = 0$, as α is in the negative term of (2.28). We obtain

$$\begin{aligned} 0 < S_\rho(A) = & -4\,L\,(4\,L + 3\,n\,\sigma^2) - \frac{8\,c\,L^2\,n\,(L + n\,\sigma^2)}{1 - C_R(n)} \\ & + n\,\sigma^2\,(2\,A\,L + n\,(2\,A + \rho)\,\sigma^2)\left(2\,A + \rho + \frac{L\,\rho}{L + n\,\sigma^2}\right). \end{aligned} \tag{2.29}$$

We define the right-hand side of (2.29) as $S_\rho(A)$. We complete our proof by deriving conditions for which $S_\rho(A = 0) > 0$ and $\frac{\partial S_\rho(A)}{\partial A} > 0$. Then, the right-hand side of (2.29) is positive for $A = 0$ and is increasing with A, and hence is positive for any value of A. First, $S_\rho(A = 0) > 0$ if

$$c < \frac{(1 - C_R(n))\,(-16\,L^3 - 28\,L^2\,n\,\sigma^2 + 2\,L\,n^2\,(\rho^2 - 6)\,\sigma^4 + n^3\,\rho^2\,\sigma^6)}{8\,L^2\,n\,(L + n\,\sigma^2)^2}. \tag{2.30}$$

For any $0 < (1 - C_R(n)) < 1$, the first derivative of (2.30) with respect to n is positive and (2.30) is increasing with n. That is, for any $0 < (1 - C_R(n)) < 1$, we can set $n = 1$ and obtain our worst-case condition for the cost parameter c of

$$c < \frac{(1 - C_R(n))\,(-16\,L^3 - 28\,L^2\,\sigma^2 + 2\,L\,(\rho^2 - 6)\,\sigma^4 + \rho^2\,\sigma^6)}{8\,L^2\,(L + \sigma^2)^2}. \tag{2.31}$$

From (2.31), it follows that, for any $0 < (1 - C_R(n)) < 1$, $S_\rho(A = 0) > 0$ if c is sufficiently small. Finally, for any $0 < (1 - C_R(n)) < 1$, the first derivative of (2.29) with respect to A equals

$$\frac{\partial S_\rho(A)}{\partial A} = 4\;n\;(2\;A + \rho)\;\sigma^2\;(L + n\;\sigma^2) > 0. \tag{2.32}$$

As (2.32) is positive, n^* is decreasing with ρ for any $0 < (1 - C_R(n)) < 1$ if (2.31) holds.

Part 3. Consider the case of $x = \sigma^2$. The optimal team size n^* decreases with σ^2 if $\frac{\partial n^*}{\partial \sigma^2} = -\frac{\frac{\partial g(n,\sigma^2)}{\partial \sigma^2}}{\frac{\partial g(n,\sigma^2)}{\partial n}} < 0$ or, equivalently, if $\frac{\partial g(n,\sigma^2)}{\partial \sigma^2} > 0$. Differentiating (2.21) with respect to σ^2, rearranging terms and using the result of (2.22) yields the condition

$$\begin{aligned} 0 < & -8 + \frac{9\,c\,L^{1+\alpha}\,(n\;(\alpha - 1) - \alpha)}{n^\alpha\,(1 - C_R(n))} + \frac{6\,n\,\sigma}{L + n\,\sigma} - \frac{4\,n\,(2\,A + \rho)\,\sigma}{2\,L\,(A + \rho) + n\,(2\,A + \rho)\,\sigma} \\ & + \frac{z(n)\,n\,\sqrt{\frac{2}{\pi}}\,\sigma\,\sqrt{1 + \frac{n\,\sigma}{L}}\,(2\,L\,(A + \rho) + n\,(2\,A + \rho)\,\sigma)}{(L + n\,\sigma)^2} \\ & + \frac{n\,\sigma\,\left(2\,A + \rho + \frac{L\,\rho}{L + n\,\sigma}\right)\left(2\,A + \frac{n\,\rho\,\sigma}{L + n\,\sigma}\right)}{2\,L}. \end{aligned} \tag{2.33}$$

As $z(n)$ is in the positive term of (2.33), we set $z(n) = 0$. As (2.33) has to be positive overall, this replacement corresponds to our worst-case condition for the fulfillment of the comparative static result. For the same reason, in the special case of $L \leq 1$, we can set $\alpha = 0$, as α is in the negative term of (2.33). We obtain

$$\begin{aligned} 0 < S_{\sigma^2}(A) = & -\frac{9\,c\,L\,n}{1-C_R(n)} + \frac{1}{2}\Bigg(-12 + \frac{n\,(2\,A+\rho)^2\,\sigma}{L} + \frac{L^2\,\rho^2}{(L+n\,\sigma)^2} \\ & - \frac{L\,(12+\rho^2)}{L+n\,\sigma} + \frac{16\,L\,(A+\rho)}{2\,L\,(A+\rho)+n\,(2\,A+\rho)\,\sigma}\Bigg). \end{aligned} \tag{2.34}$$

We define the right-hand side of (2.34) as $S_{\sigma^2}(A)$. We complete our proof by deriving conditions, for which $S_{\sigma^2}(A=0) > 0$ and $\frac{\partial S_{\sigma^2}(A)}{\partial A} > 0$. Then, the right-hand side of (2.34) is positive for $A = 0$ and is increasing with A, and hence is positive for any value of A. First, $S_{\sigma^2}(A=0) > 0$ if

$$\begin{aligned} c < & \frac{(1-C_R(n))}{18\,L^2\,n\,(L+n\,\sigma)^2\,(2\,L+n\,\sigma)}(-32\,L^4 - 64\,L^3\,n\,\sigma \\ & + 4\,L^2\,n^2\,(\rho^2-11)\,\sigma^2 + 4\,L\,n^3\,(\rho^2-3)\,\sigma^3 + n^4\,\rho^2\,\sigma^4). \end{aligned} \tag{2.35}$$

For any value of $0 < (1-C_R(n)) < 1$, the first derivative of (2.35) with respect to n is positive and (2.35) is increasing with n. That is, for any $0 < (1-C_R(n)) < 1$, we can set $n = 1$ and obtain our worst-case condition for the cost parameter c of

$$\begin{aligned} c < & \frac{(1-C_R(n))}{18\,L^2\,(L+\sigma)^2\,(2\,L+\sigma)}(-32\,L^4 - 64\,L^3\,\sigma \\ & + 4\,L^2\,(\rho^2-11)\,\sigma^2 + 4\,L\,(\rho^2-3)\,\sigma^3 + \rho^2\,\sigma^4). \end{aligned} \tag{2.36}$$

From (2.36), it follows that, for any $0 < (1-C_R(n)) \leq 1$, $S_{\sigma^2}(A=0) > 0$ if c is sufficiently small. Finally, for any $0 < (1-C_R(n)) < 1$, the first derivative of (2.34) with respect to A equals

$$\begin{aligned} \frac{\partial S_{\sigma^2}(A)}{\partial A} = & \frac{1}{2}\Bigg(\frac{4\,n\,(2\,A+\rho)\,\sigma}{L} - \frac{32\,L\,(A+\rho)\,(L+n\,\sigma)}{(2\,L\,(A+\rho)+n\,(2\,A+\rho)\,\sigma)^2} \\ & + \frac{16\,L}{2\,L\,(A+\rho)+n\,(2\,A+\rho)\,\sigma}\Bigg). \end{aligned} \tag{2.37}$$

From that it is simple to show that, for any $0 < (1-C_R(n)) < 1$, (2.37) is positive if (2.37) or, equivalently, if (2.36) holds. Therefore, for any $0 < (1-C_R(n)) < 1$, n^* is decreasing with σ^2 if (2.35) holds.

Part 4. Consider the case of $x = L$. The optimal team size n^* increases with L if $\frac{\partial n^*}{\partial L} = -\frac{\frac{\partial g(n,L)}{\partial \sigma^2}}{\frac{\partial g(n,L)}{\partial n}} > 0$, or, equivalently, if $\frac{\partial g(n,L)}{\partial L} < 0$. Differentiating (2.21) with respect to L, rearranging terms and using the result of (2.22) yields the condition

$$\begin{aligned} 0 > & 4\,L\left(2\,A+\rho+\frac{L\,\rho}{L+n\,\sigma^2}\right)\Bigg(-\frac{2\,c\,L^{1+\alpha}\,(n\,(\alpha-1)-\alpha)\,(L+n\,\sigma^2)}{n^{\alpha}\,(1-C_R(n))} \\ & + 3\,(2\,L+n\,\sigma^2)\Bigg) - n\,\sigma^2\left(2\,A+\rho+\frac{L\,\rho}{L+n\,\sigma^2}\right)^2 \\ & \left(2\,A\,L+n\,(2\,A+\rho)\,\sigma^2+2\,z(n)\,L\sqrt{\frac{2}{\pi}}\sqrt{1+\frac{n\,\sigma^2}{L}}\right) - 16\,L^2 \\ & \left(A+\rho+\frac{c\,L^{1+\alpha}\sqrt{2\,\pi}\,(n\,(\alpha-1)-\alpha)\,(1+\alpha)\,(L+n\,\sigma^2)\sqrt{1+\frac{n\,\sigma^2}{L}}}{n^{2+\alpha}\,z(n)\,(1-C_R(n))\,\sigma^2}\right). \end{aligned} \tag{2.38}$$

First, it is simple to see that, for a sufficiently small value of c, the last term of (2.38) is strictly negative and decreases with A. Therefore, we can exclusively focus on the first two terms of (2.38). Here, we set $z(n) = 0$, as $z(n)$ is in the negative part of the second term, which represents our worst-case condition. For the same reason, assuming our special case of $L \leq 1$, we set $\alpha = 0$, as α is in the positive part of the first term. This yields

$$\begin{aligned} 0 > S_L(A) =& 4\,L\left(6\,L + 3\,n\,\sigma^2 + \frac{2\,c\,L\,n\,(L + n\,\sigma^2)}{1 - C_R(n)}\right) \\ & - n\,\sigma^2\,(2\,A\,L + n\,(2\,A + \rho)\,\sigma^2)\left(2\,A + \rho + \frac{L\,\rho}{L + n\,\sigma^2}\right). \end{aligned} \tag{2.39}$$

We define the right-hand side of (2.39) as $S_L(A)$. We complete our proof by deriving conditions for which $S_L(A = 0) < 0$ and $\frac{\partial S_L(A)}{\partial A} < 0$. Then, the right-hand side of (2.39) is negative for $A = 0$ and is decreasing with A, and hence is negative for any value of A. First, $S_L(A = 0) < 0$ if

$$c < \frac{(1 - C_R(n))\,(2\,L + n\,\sigma^2)\,(-12\,L^2 - 12\,L\,n\,\sigma^2 + n^2\,\rho^2\,\sigma^4)}{8\,L^2\,n\,(L + n\,\sigma^2)^2}. \tag{2.40}$$

For any $0 < (1 - C_R(n)) \leq 1$, the first derivative of (2.40) with respect to n is positive and therefore (2.40) is increasing with n. That is, for any $0 < (1 - C_R(n)) < 1$, we can set $n = 1$ and obtain our worst-case condition for the cost parameter c of

$$c < \frac{(1 - C_R(n))\,(2\,L + \sigma^2)\,(-12\,L^2 - 12\,L\,\sigma^2 + \rho^2\,\sigma^4)}{8\,L^2\,(L + \sigma^2)^2}. \tag{2.41}$$

From (2.41), it follows that, for any $0 < (1 - C_R(n)) \leq 1$, $S_L(A = 0) < 0$ if c is sufficiently small. Finally, for any $0 < (1 - C_R(n)) \leq 1$, the first derivative of (2.39) with respect to A equals

$$\frac{\partial S_L(A)}{\partial A} = -4\,n\,(2\,A + \rho)\,\sigma^2\,(L + n\,\sigma^2) < 0. \tag{2.42}$$

As (2.42) is negative, n^* is increasing with L for any $0 < (1 - C_R(n)) \leq 1$ if c is sufficiently small. ■

3 On the Incentive Effect of Job Rotation[1]

Abstract

The longer an agent is employed in a job, the more the principal will have learned about his ability through the history of performance. With implicit incentives, influence perceptions and effort incentives decrease over time. Rotating agents to a different job deletes learning effects about ability, creating fresh impetus for effort. However, job rotation also reduces the time horizon, and thus reduces rents from working and also incentives. In this trade-off, we derive conditions for the desirability of job rotation and show how in the presence of career concerns job rotation may emerge endogenously. Finally, our model allows for more general comments on the optimal rotation frequency as well as the preferred organizational design of a firm.

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3.1 Introduction

Job rotation is a management strategy of periodically transferring employees to different jobs within an organization. Recent empirical research reports a rapid increase in flexible workplace practices, with job rotation as one of the three main characteristics of those practices.[2] For example, workers are rotated through various projects, between tasks in a single department, or even between business units in different regions or countries. Job rotation is not only a tool to fill temporary assignments, but also to rotate people at their regular job on a monthly, weekly, or actually daily basis. Traditional wisdom suggests that there are direct costs and benefits to job rotation that relate to technical aspects of specialization and learning. Obviously, even though rotating an engineer into a sales department can develop new skills, job-specific human capital will be lost and the acquisition of new knowledge is time-consuming. In this paper, we present a new explanation for the use of job rotation and argue that job rotation also has indirect costs and benefits through the effects of information and incentives that relate to a worker's career concerns. We highlight that job rotation is a tool for limiting the amount of performance information available to the market and therefore for modifying the effort incentives of workers.

By investigating the informational and incentive-related role of job rotation, the main contribution of this paper is to emphasize how erasing past performance information by job rotation can create new incentives. More specifically, related to the seminal work of Fama (1980) and Holmström (1999), firms often have to commit themselves to provide implicit incentives to their employees. The attempt to shape the personal skill perceptions serves as an incentive device. If an agent works in a job for a long time, the principal will have learned much about his abilities and thus the incentives to increase the principal's beliefs decrease over time. Job rotation results in an information loss about the agent's abilities and therefore increases his incentives to incur costly effort. However, anticipating rotation, an agent will cut back on his effort in the first place, due to a smaller pay-back period of his effort. These divergent effort interrelations create novel insights into the optimal provision of incentives to workers. One such insight is that, under certain conditions, the agent's average utility is non-monotonic in his rotation frequency.

To formally analyze the incentive role of job rotation, we formulate a principal agent setting with an infinite horizon. A risk-neutral firm owner and an agent with an ex-ante unknown ability engage in an employment relationship where the agent exerts effort to improve the market expectations of his skill and consequently his future compensation. Job rotation deletes past performance signals in drawing inference about the agent's skill and therefore relocates his incentives to incur costly effort. Ex-ante, when the agent's type is little known, effort incentives and influence

[2] Osterman (1994, 2000) documents establishment rates of about 35% for U.S. manufacturing firms with 50 or more employees. Pil and MacDuffie (1996) provide evidence on the adoption of job rotation for assembly plant workers around the world. Based on data in the U.S., Gittleman, Horrigan, and Joyce (1998) report adoption rates of about 40%. In a survey of Danish private sector firms, Eriksson and Ortega (2006) outline that rotation schemes were implemented for nearly 20% of hourly paid workers.

perceptions about ability are large. As time goes by, the agent's type is partially revealed and the incentives to exert effort decrease. Also, when the agent comes closer to the rotation date, he becomes lazy because his type is virtually revealed by then, and the information will be erased soon, so why bother to work? Hence, for long maturities, there is overworking in the beginning and underworking towards the rotation date. Furthermore, for a too large rotation frequency, the agent knows that he will not stay on the task for long, so he will not exert much effort in the first place. Consequently, a positive but finite rotation frequency is optimal.

We explore how job rotation influences the agent's effort incentives and average utility in a dynamic perspective. Specifically, if the agent's ability is virtually known ex ante, then firms place less reliance on output when forecasting the agent's type and the incentives to incur costly effort are small. A large duration until rotation can raise the agent's incentives to increase beliefs about his type and prevents him from underinvesting in effort. In contrast, the more dispersed the prior, the larger is the impact of the agent's effort on the firms' estimate of his ability. If time until rotation is long, then effort incentives can be inefficiently large, until more information is revealed. The agent can then only achieve the optimal workload by increasing the frequency of rotation. In light of such phenomena, this paper investigates the determinants for an agent's optimal assignment and characterizes the impact of this choice on learning about types and the incentives given to workers.

Altogether, our work contributes to the literature on incentives provided by organizational change and, more generally, to the research on endogenous job design. It allows us to shed light on the complex relationships between organizational assignment, learning effects about ability, the effort incentives given to workers, and their consequences on overall utility within an economic model. Our results offer a novel theoretical approach for the lack of empirical evidence for existing explanations to job rotation that relate to learning about abilities and the motivation of employees, emphasizing that job rotation is less common for incentivizing long-tenured employees with limited future prospects (Campion, Cheraskin, and Stevens (1994), Eriksson and Ortega (2006)).[3] In a more general context, our results may also explain the recent trends on workplace reorganizations, including the increased dissolution of long-term contracts, rising inter-firm job changes, limited employment durations, the use of flexible staffing, and the adoption of triangular employment relations with many employers and temporary or contingent contracts.[4]

Literature. Our paper relates to other fields within the existing literature. First, it refers to models on implicit incentives in the presence of career concerns, originating from the seminal work of Fama (1980) and Holmström (1999), who offer explanations for the limitations on explicit payment structures. This work is mostly

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[3] This employee motivation argument was elaborated in the context of so-called plateaued employees (see Stites-Doe (1996)).

[4] Several studies provide evidence on workplace restructuring and the implementation of flexible staffing arrangements, primary referring to the US labor market (see, for example, Kalleberg (2000), Gramm and Schnell (2001), Houseman (2001), or Kalleberg, Reynolds, and Marsden (2003)).

connected to models that study the influence of career concerns on organizational decisions (Dewatripont, Jewitt, and Tirole (1999b), Bar-Isaac (2007)), as well as the role of information on the strengths of implicit incentives (Dewatripont, Jewitt, and Tirole (1999a), Mukherjee (2008, 2010), Koch and Morgenstern (2010), Koch and Peyrache (2011)). Related to our approach, Arya and Mittendorf (2011) analyze the impact of organizational decisions as the choice between aggregated and disaggregated performance measures on the incentives given to workers. Although aggregation results in informational drawbacks, it increases the sensitivity of performance measurement to the updating of an agent's ability and with it the incentives to incur costly effort. In line with the basic result that improved information may reduce the strengths of implicit incentives, this paper newly investigates the role of a firm's organizational design as the choice between specialization and job rotation on learning about types and the incentives provided to workers.

Second, several models study the incentive effects of information disclosure and performance feedback when agents face career concerns and effort is history-dependent (Acemoglu, Kremer, and Mian (2008), Gershkov and Perry (2009), Aoyagi (2010), Casas-Arce (2010), Ederer (2010), Goltsman and Mukherjee (2011)). Most connected to our work, Kovrijnykh (2007) presents a model of career uncertainty and analyzes how reputational incentives interact with the possibility of career change. Martinez (2009) investigates how employment history and beliefs about future productivity affect motivation. Similarly, Hansen (2012) studies the effects of interim performance evaluations on agents' incentives to influence beliefs about future effort. While these models primary examine how an agent's effort incentives are influenced by the market expectations of the agent's effort, our work analyzes the relationship between performance information that is determined by an agent's assignment and the strengths of implicit incentives. More specifically, by investigating the effect of current effort on future expectations of an agent's ability, we show how limiting the amount of information available through job rotation impacts learning about an agent's type and therewith relocates the incentives to incur costly effort.

Third, there are parallels to the literature on optimal task assignment (Ricart i Costa (1988), Meyer (1991, 1994), Bernhardt (1995), Ortega (2003), Bar-Isaac and Hörner (2011)). These models concentrate on technological questions related to the optimal task assignment and its implications for learning about agents' abilities, wage levels, promotion decisions, and the associated incentives, including considerations regarding the external labor market. Our approach is also tied to the general analysis on optimal contract length that is based on implicit contracting. Here, our work most closely relates to Jovanovic (1979) who studies a model of optimal job matching where an agent learns his productivity through the observation of output. Turnover is created when the agent's job-specific productivity turns out to be low, while the agent remains in the current job if he forms a good match with the firm. Similarly, Cantor (1988) analyzes how a worker's effort incentives and recontracting costs relate to the length of a labor contract when agents face career concerns. The impetus behind this work is to investigate the information- and incentive-based role of job rotation. Therefore, an agent's task assignment and the optimal retention duration in a job are considered as strategic instruments to shift the effort incentives provided to agents, influenced by learning effects about their abilities and therefore by future compensation.

Forth, several models directly address the reasons why job rotation is useful. With regard to incentive-based explanations, Ickes and Samuelson (1987) argue that if higher efforts yield more demanding future remuneration schemes, agents are incentivized to cut back on their effort. Job rotation resolves this ratchet effect by disentangling the influence of current performance on future incentives. However, it results in a loss of specific human capital. Carmichael and MacLeod (1993) show that if workers are trained in more than one job, they will be motivated to reveal labor-saving technical change compared to single-skilled workers, as the former can be transferred to other jobs inside the firm and therefore are not threatened with dismissal. Cosgel and Miceli (1999) assume that job rotation reduces the boredom of monotonous jobs, but, on the other hand, suffers the loss of job-specific human capital. Eguchi (2005) outlines that job rotation can prevent agents from performing private activities within their regular work, as these become more profitable as tenure increases. Recent literature analyzes informational benefits of job rotation, either related to learning about the productivity of tasks (Arya and Mittendorf (2004)), or about the productivity of employees ((Meyer, 1994), Ortega (2001), Arya and Mittendorf (2006a,b), Prescott and Townsend (2006), Müller (2011)). Specifically, Arya and Mittendorf (2004) argue that job rotation helps to extract information about the productivity of tasks from employees, as this information can no longer be used against them in the case of rotation. In contrast, Ortega (2001) analyzes job rotation as an instrument to learn a few traits about many dimensions of a worker's ability instead of to learn a great deal about only a few dimensions. Arya and Mittendorf (2006a) focus on sorting benefits of job rotation, based on the result that only versatile employees optimally self-select themselves into rotation programs. Finally, Müller (2011) argues that job rotation leads to multiple performance evaluations of workers. This resolves confirmatory bias problems faced by supervisors, although job rotation sacrifices job-specific human capital. Our work relates to the incentive-based role of information with regard to job rotation and presents a complementary approach that is not linked to typical technological reasons for and against job rotation. Moreover, in contrast to the previous literature, job rotation represents an instrument for reducing the information about agents' abilities and thereby for systematically channeling their efforts towards increases in the benefits of a company's workforce.

From the standpoint of the literature on employee turnover, our paper most closely relates to Höffler and Sliwka (2003), who analyze the impact of managerial replacement on effort incentives provided to workers. In their paper, the dismissal of a manager results in a positive effort effect, based on an increased uncertainty about the subordinates' relative abilities. On the other hand, it reduces the quality of task allocation due to the loss of information. This work contributes to the growing literature on information and incentives provided by organizational change and highlights how in the presence of implicit contracts job rotation may emerge endogenously. Therefore, in addition to positive effort effects, our paper also focuses on negative effort incentives caused by the loss of past performance information. We consider a firm's optimal organizational assignment, clarify the advantages and drawbacks of job rotation programs, and present a new instrument for influencing learning effects about ability. Altogether, our paper focuses on the efficient provision of incentives to workers and creates novel insights into the longstanding debate on the optimal internal design of organizations.

The remainder of this paper is organized as follows. Section 3.2 develops the main model, and section 3.3 investigates the trade-offs of an agent's optimal rotation frequency. Section 3.4 provides evidence regarding job rotation and discusses the implications of the model. Section 3.5 contains our conclusions. All proofs are available in the Appendix.

3.2 The Model

We develop a dynamic principal agent setting with an infinite horizon. Consider a competitive labor market where, at some point in time, $t = 0$, one out of many identical firms (the principal) employs an agent for the purpose of production. At heart of this model we study the agent's optimal organizational assignment that is given by some T: after T periods the agent will be rotated to a different job inside the firm. To abstract from integer problems, we assume that the agent works continuously on a task. Aggregate output $Y(t)$ after time t, with $0 \leq t \leq T$, is an additively linear function of the agent's unobservable ability, a, his endogenous effort choice e, and an error term, ϵ, and is given by

$$Y(t) = a\,t + \int_0^t e\,\mathrm{d}t + \epsilon. \tag{3.1}$$

To avoid signaling issues associated with mixed strategies, we assume that all market participants share common prior beliefs about the agent's ability a that is normally distributed with zero mean and variance α^2. This implies that the agent himself does not know his type as, otherwise, a high type could go for different contracts than low types. By working on a task, principal and agent learn about the agent's ability over time through the observation of output. The agent's ability a and the error term ϵ are independently distributed, where ϵ follows a normal distribution with zero mean and a variance that depends on the time span t. We assume that ϵ has variance $\sigma^2\,t$. This assumption has the following micro-foundation. Suppose that aggregate working time can be divided into infinitesimal time intervals. Then, with independent increments, for a time span of length $\mathrm{d}t$, output is given by

$$\mathrm{d}Y(t) = a\,\mathrm{d}t + e\,\mathrm{d}t + \sigma\,\mathrm{d}W(t), \tag{3.2}$$

where $W(t)$ is a Wiener process generating noise. According to the basic properties of the standard Wiener process, $W(t)$ has zero mean and variance t. Then, output $Y(t)$ after time t is normally distributed with mean $a\,t + \int_0^t e\,\mathrm{d}t$ and with variance $\sigma^2\,t$. It follows that the larger the interval of observation, the smaller is the variance of the output measure relative to time t, and the more precise is then the updating of an agent's ability. Consequently, the longer the agent works on a task, the more obvious his true abilities will become; the less noise there will be.

Our main focus is to analyze the incentive effect of rotating the agent to a different job. Therefore, we investigate the informational role of job rotation that relates to the observability of the history of an agent's productivity. We assume that job rotation deletes previous performance information in drawing inference about the agent's skill and take complete deletion of information as the extreme case. This independence assumption guarantees that no information about past performance signals is transmitted to the new working department as well as to the outside labor market and involves drawing a new value of the agent's ability. Moreover, as information that is once inside the market cannot be unlearned at a later period, this assumption implies that the market cannot observe past performance signals, but rather that performance signals are private information of principal and agent. Consequently, we refer to an evaluation system where the final working department completes an agent's written character at the end of employment.[5]

To capture incentives of career reputation, we assume contract incompleteness in that output is observable, but not verifiable. That is, the principal cannot write a pay-for-performance contract based on output and is restricted to fixed wage payments, determined by a competitive labor market and conditioned on observed output. The desire to shape the principal's expectation of the agent's skill may therefore provide an impetus for effort. However, effort is costly with a quadratic cost structure of

$$c(e) = c\,\frac{e^2}{2}. \tag{3.3}$$

Then, in each period of length $\mathrm{d}t$, an effort $e \geq 0$ increases output by $e\,\mathrm{d}t$, but has a cost of $c(e)\,\mathrm{d}t = c\,e^2/2\,\mathrm{d}t$.

Competition for the worker is modeled as follows. We assume that the agent has all bargaining power at contract negotiation, reflecting a competitive labor market that consists of many homogeneous firms. In this case, the agent proposes the initial contract and is free to choose any period length T until rotation. Moreover, we assume that at each renegotiation stage $t \in [0, T]$, the firm faces sufficiently large delay costs if a consensus decision is not achieved. These delay costs reflect the firm's impatience in ex post renegotiation implying that the agent keeps all the bargaining power to propose subsequent contracts.[6] Thus, at each date $t \in [0, T]$, the agent makes a take-it-or-leave-it offer to the firm. The firm can either accept the offer or dismiss the agent. This condition requires that at each renegotiation stage, the minimum expected utility necessary to induce a firm to hire the agent must yield the firm at least zero expected utility. Stated differently, as the agent has bargaining power over pay, he ultimately reaps all the benefits from employment and becomes residual claimant of the aggregate surplus. Hence, at each date, the agent's wage moves up and down. The agent dislikes spending effort and benefits from compensation. Consequently, at each date t, the agent's utility is represented

[5]Similarly, Prescott and Townsend (2006) analyze a principal-agent model with multiple-stage production and assume that job rotation hinders agents in becoming informed about a project's interim performance measures. Also Höffler and Sliwka (2003) assume that after dismissing a manager, the successor receives no information about an agent's past performance signals, neither from the old manager, nor from the principal. Likewise, in a dynamic model of reputation, Bar-Isaac (2007) assumes that an agent's productive history is only observable at the location where it is produced, while the agent's reputation is lost if he moves to a different location.

[6]Aghion, Dewatripont, and Rey (1994) outline that the allocation of full bargaining power in ex post renegotiation to one contractual party represents a well-founded assumption, as it can be achieved by established contractual instruments, such as penalties, or default options.

by

$$U_t = w_t - c\,\frac{e_t^2}{2} = E[a_t + e_t + \epsilon_t] - c\,e_t^2/2 = E[a_t + e_t] - c\,\frac{e_t^2}{2}, \qquad (3.4)$$

as $E[\epsilon_t] = 0$ for all t.

As the agent himself learns his ability over time, his ex ante commitment to T might turn out to be suboptimal. In detail, to limit the visibility of his type, an agent who turns out to be of low ability might choose to rotate more frequently than in the initial steady state. Contrary, a high ability type might prefer to stay in the job for a longer time period such that his abilities will remain apparent. We refer to T as the choice of technology and assume that the agent's ex ante rotation decision is irreversible ex post. That is, once chosen, it is prohibitedly costly for the agent to change his rotation date at a future period $t < T$.[7] Given that all job separations are at the agent's initiative, this also applies for the case where the agent induces the firm to dismiss him purely for the purpose of recontracting his rotation frequency. This assumption strengthens the trade-off between information and incentives that relates to the agent's employment without affecting bargaining and the information structure with respect to the outside labor market.

The timing of the model is as follows.

$t = 0$ One of many identical firms employs an agent with an unknown ability a.

The agent commits to a binding duration until rotation T.

$t < T$ The agent works continuously on a task and exerts effort e.

At each date, firm and agent learn output $Y(t)$ to update the agent's expected ability.

The agent continuously renegotiates his wage w_t by making a take-it-or leave-it-offer to the firm.

$t = T$ Payoffs are realized.

The firm rotates the agent to a different job, and past performance signals are deleted.

3.3 Timing of Rotation

In order to investigate how incentives and output interact with the agent's assignment, we will analyze how the timing of rotation relates to learning about the agent's

[7] Similarly, Mukherjee (2008, 2010) analyzes a firm's optimal disclosure policy and assumes that a firm's ex ante choice of job design is irreversible ex post. Also in the model of Mukherjee and Vasconcelos (2011) a firm's task assignment decision is binding and cannot be revised at a future date. Likewise, Poutvaara, Takalo, and Wagener (2012) study a model of optimal contract duration where the possibility of premature contract renegotiation is excluded.

ability. From a dynamic perspective, as job rotation deletes past performance information in drawing inference about the agent's skill, it may have ambiguous effects on expected future wages, the incentives to incur costly effort, and with it also on the agent's utility. Specifically, the agent's incentives to exert costly effort stem from the desire to increase his productivity in order to improve the market expectation of his ability. That is, the agent maximizes his expected utility, given the market expectations of his effort. Being aware of the agent's utility function, the market anticipates the agent's effort level. Consequently, in equilibrium, the agent's optimal effort corresponds with the market expectation of his effort. As ability and effort additively increase the agent's productivity, learning is not affected by the market expectation of the agent's effort. That is, taking the market expectation of the agent's effort as given, we can only look at whether the agent has an incentive to unobservedly increase efforts. In detail, if the agent increases his efforts while the market believes he does not, his future wages increase because the market partly assigns the resulting productivity increase to an increase in his ability. However, even though the agent's efforts can't be observed, one essential equilibrium condition is that there are no asymmetries between the agent who knows his effort choice and the market in anticipating the agent's unobservable actions. Only this condition implies that in equilibrium, the market is not fooled such that learning effects and the expectations of the agent's ability are consistent for both the market and the agent. This, in turn, implies that if the agent increases his efforts (beyond the market expectations) to establish a favorable reputation of being a high type, the market will anticipate the agent's actions and attribute the increase in productivity to an increase in effort. However, the agent's effort is also chosen accurately to the effect that any effort reduction would be recognized as a signal of lower ability. Furthermore, as the agent is paid his expected productivity, effort is incentive-compatible. That is, the agent receives all the benefits from employment and profits from the resulting output increase in the end, but has to bear the effort costs.

To begin with, assume that the agent is at date $t < T$, and he has already produced $Y(t)$. Then, according to (3.1), the distribution of the output $Y(t)$ results from the distributions $a \sim \mathcal{N}\big(0, \alpha^2\big)$ and $\epsilon \sim \mathcal{N}\big(0, \sigma^2\, t\big)$. Consequently, updating follows from

$$Y(t) - \int_0^t \hat{e}\,\mathrm{d}t = a\,t + \epsilon \sim \mathcal{N}\big(0, \alpha^2\,t^2 + \sigma^2\,t\big), \tag{3.5}$$

where $\hat{e}$ denotes the market conjecture of the agent's effort. The variance of the output is the sum of the variances of the prior and of the error term. It follows that the variance of the output always exceeds the variance of the agent's ability as it also incorporates measurement error. However, the larger the interval of observation, the smaller is the variance of the output measure relative to the time span t and the updating of the agent's ability is then more precise. As the agent's equilibrium effort is anticipated, the market uses $Y(t) - \int_0^t \hat{e}\,\mathrm{d}t$ to forecast the agent's ability. According to the Bayes' theorem for normally distributed random variables, the agent's perceived type is then normally distributed with mean (expected type)

$$\hat{a} = E\left[a \mid Y(t) - \int_0^t \hat{e}\,\mathrm{d}t\right] = \frac{1}{t}\left(\underbrace{\frac{\sigma^2\,t}{\alpha^2\,t^2 + \sigma^2\,t}}_{:=q}\,0 + \underbrace{\frac{\alpha^2\,t^2}{\alpha^2\,t^2 + \sigma^2\,t}}_{:=1-q}\left(Y(t) - \int_0^t \hat{e}\,\mathrm{d}t\right)\right)$$
$$= \frac{\alpha^2}{\alpha^2\,t + \sigma^2}\left(Y(t) - \int_0^t \hat{e}\,\mathrm{d}t\right). \tag{3.6}$$

The posterior variance of the agent's ability results from the precisions of both the prior and of the conditional variances and equals

$$\hat{\alpha}^2 = \frac{1}{t^2}\left(\frac{1}{1/(\alpha^2\,t^2) + 1/(\sigma^2\,t)}\right) = \frac{\alpha^2\,\sigma^2}{\alpha^2\,t + \sigma^2}. \tag{3.7}$$

Then, the *a posteriori* distribution of the agent's ability, after observation of $Y(t)$, is given by

$$\hat{a} = E\left[a \mid Y(t) - \int_0^t \hat{e}\,\mathrm{d}t\right] \sim \mathcal{N}\left(\frac{\alpha^2}{\alpha^2\,t + \sigma^2}\left(Y(t) - \int_0^t \hat{e}\,\mathrm{d}t\right), \frac{\alpha^2\,\sigma^2}{\alpha^2\,t + \sigma^2}\right). \tag{3.8}$$

The agent's expected ability in period t consists of a weighted average of the prior $\hat{a}_0$ (which equals zero), and of the signal $Y(t) - \int_0^t \hat{e}\,\mathrm{d}t$. The sum of both weights is 1 in each case; thus, the weight on the prior, denoted q, increases if the new information is very noisy (large σ). Contrary, if the prior is very noisy (large α), more weight is put on the output measure $1-q$. Furthermore, learning reduces the variance of the agent's expected ability such that its estimation becomes more precise. Equivalently, the increase in $Y(t) - \int_0^t \hat{e}\,\mathrm{d}t$ is approximately proportional to $a\,t$ such that if tenure tends to infinity, $t \to \infty$, the agent's expected ability converges to the real ability, $\hat{a} \to a$, and its variance converges to zero, $\hat{\alpha}^2 \to 0$. That is, updating reduces the uncertainty about the agent's ability α, it consequently reduces further learning effects, and with it also the strengths of implicit incentives. Based on this basic result that increased information may reduce future incentives, the main focus of our work is to analyze how job rotation can disentangle future output from past learning effects and therefore relocate the agent's incentives to incur costly effort.

Taking the firms' expectations of the agent's effort as given, assume that at date t, the agent increases his effort by an infinitesimal $\mathrm{d}e$ for an infinitesimal time span $\mathrm{d}t$. That is, for the remaining $T-t$ periods until rotation, aggregate output is increased by $\mathrm{d}e\,\mathrm{d}t$. Hence, the agent expects that for all dates $t' \in [t, T]$, the firms' expected value of the agent's type is

$$\frac{\alpha^2}{\alpha^2\,t' + \sigma^2}\left(Y(t') + \mathrm{d}e\,\mathrm{d}t\right). \tag{3.9}$$

That is, it is increased by

$$\frac{\alpha^2}{\alpha^2\,t' + \sigma^2}\,\mathrm{d}e\,\mathrm{d}t. \tag{3.10}$$

As, at each date t, the agent is paid his expected marginal product, his aggregate wage from t to T increases by

$$\mathrm{d}e\,\mathrm{d}t \int_t^T \frac{\alpha^2}{\alpha^2\,t' + \sigma^2}\,\mathrm{d}t' = \mathrm{d}e\,\mathrm{d}t\,\log\left[\frac{\alpha^2\,T + \sigma^2}{\alpha^2\,t + \sigma^2}\right]. \tag{3.11}$$

The agent will increase his effort as long as the perceived benefits exceed the costs associated with this increase. The cost function is $c\,e^2/2$, hence the additional cost for this effort increase is $c\,\mathrm{d}e^2/2\,\mathrm{d}t$. The first order condition thus yields

$$c\,\mathrm{d}e^2/2\,\mathrm{d}t = \mathrm{d}e\,\mathrm{d}t\,\log\left[\frac{\alpha^2\,T+\sigma^2}{\alpha^2\,t+\sigma^2}\right]. \tag{3.12}$$

This gives the following lemma.

Lemma 4 *The agent's equilibrium effort is*

$$e^* = \log\left[\frac{\alpha^2\,T+\sigma^2}{\alpha^2\,t+\sigma^2}\right]/c. \tag{3.13}$$

Lemma 4 outlines a few basic characteristics of the agent's optimal effort e^*. Intuitively, as job rotation deletes past performance information in drawing inference about the agent's skill, effort is only worthwhile for $t < T$. Consequently, when the duration until rotation is extended, the wage derived from effort increases. Hence, the optimal effort increases with the payback period $T - t$. The effect of career concerns incentives implies that if skill is known, $\alpha = 0$, the optimal effort is zero, $e^* = 0$. That is, the larger the prior type uncertainty (large α) and the more precise the performance measurement (small σ), the larger are the incentives to invest in increasing visibility and the more effort the agent spends in equilibrium.

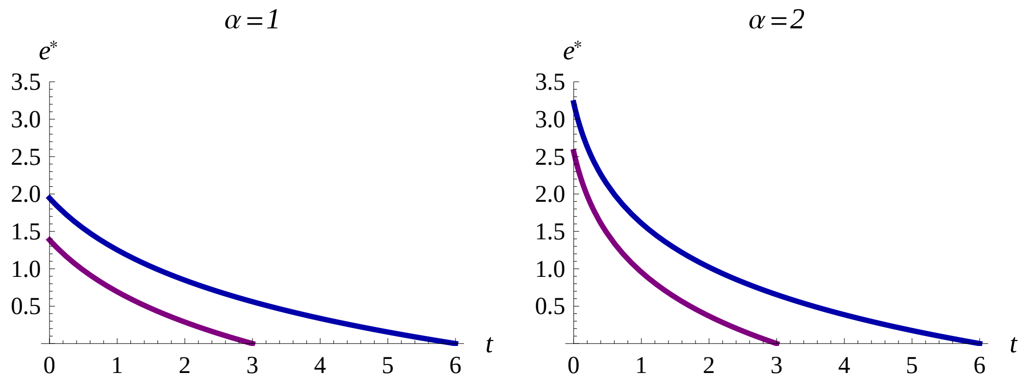

Figure 3.1: Optimal effort e^* as a function of time t

The parameters are $\sigma = 1$, $c = 1$, $\alpha = 1$ (left figure), and $\alpha = 2$ (right figure). We have $T = 6$ for the blue curve, and $T = 3$ for the purple curve.

Figure 3.1 shows the equilibrium effort e^* as a function of time t when varying the duration until rotation T and given different values of the prior type uncertainty α. Three things are visible. *First*, by comparing the left and right figures, the higher the uncertainty about the agent's type α, the more the agent has to prove, and the more effort he spends. *Second*, the longer the time until rotation $T - t$, the higher the uncertainty about the type, and the longer the agent can benefit from a wage increase from effort. Thus, for small t, effort is high. As time reaches $t = T$, the agent does not spend any effort at all. *Third*, the functions are convex in t. If the second effect were the only time effect prevalent, then the effort should be linear in

$T-t$. It is not. This means that the additional information that is aggregated over time plays an important role. In detail, firms not only place less weight on output when prior type uncertainty decreases, but they also are less likely to change their old estimate. Thus, at the beginning (for small t), the agent's type is little known, hence incentives to exert effort and influence perceptions about the type are large. As time goes by (for larger t), the agent's type is partially revealed, hence incentives to exert effort decrease.

Our results indicate that the strengths of career concerns decline as performance information accumulates such that learning effects about ability and the worker's future wage become less sensitive to output over time. *First*, for long maturities, an agent might overinvest in effort at the beginning, but underinvest in effort when coming closer to the rotation date, as all performance information will be erased soon. *Second*, for a too large rotation frequency, an agent might exert less effort in the first place as effort pays off for not very long. This yields the final question: how large is the optimal duration until rotation T^*? As the firm's profit is zero, we need to aggregate the agent's utility over time and calculate the agent's average. This is because the number of assignments increases with the agent's rotation frequency. At each date t, the agent is paid his expected productivity, hence his utility is

$$w_t - c\,\frac{e_t^2}{2} = E\left[a_t + e_t\right] - c\,\frac{e_t^2}{2}.$$

The average is then

$$\begin{aligned}\bar{U} &= \frac{1}{T}\int_0^T \left[E\left[a_t + e_t\right] - c\,\frac{e_t^2}{2}\right] \mathrm{d}t \\ &= \frac{1}{T}\int_0^T \left[E\left[a_t + \log\left[\frac{\alpha^2 T + \sigma^2}{\alpha^2 t + \sigma^2}\right]/c\right] - \log\left[\frac{\alpha^2 T + \sigma^2}{\alpha^2 t + \sigma^2}\right]^2/(2\,c)\right] \mathrm{d}t \\ &= \frac{\sigma^2}{2\,c\,\alpha^2\,T}\,\log\left[1 + \frac{\alpha^2\,T}{\sigma^2}\right]^2. \end{aligned} \tag{3.14}$$

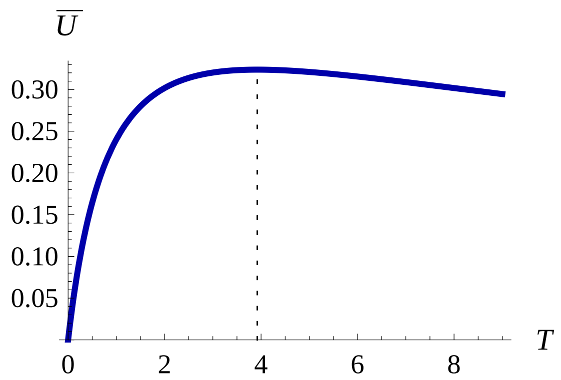

Figure 3.2: Agent's average utility $\bar{U}$ as a function of the duration until rotation T

The parameters are $\sigma = 1$, $c = 1$, and $\alpha = 1$, as before. The dashed line gives the optimal duration until rotation T^*.

This average depends on the cost factor c, of course, and on the fraction $\alpha^2\,T/\sigma^2$. Intuitively, if performance measurement is very noisy (large σ), learning effects are small and the effect of effort is low. A large duration until rotation T increases the payback period of the agent's effort and can incentivize him to spend more effort. Contrary, if the prior is highly dispersed (large α), learning effects and effort incentives are large. Cutting back the duration until rotation can prevent the agent from overworking such that smaller values of T are optimal. In other words, job rotation smooths effort incentives over time and therefore may increase the agent's utility due to convex effort costs. Figure 3.2 shows the agent's average utility over time, as dependent on the duration until rotation T. We are interested in the optimal duration T^* (in the picture, a little smaller than 4). The first order condition, $\partial \bar{U}/\partial T = 0$, yields

$$
\begin{aligned}
0 &= \left(1 + \frac{\alpha^2\, T^*}{\sigma^2}\right) \log\left[1 + \frac{\alpha^2\, T^*}{\sigma^2}\right] - 2\,\frac{\alpha^2\, T^*}{\sigma^2}, \\
T^* &= \frac{\sigma^2}{\alpha^2}\, T_1, \qquad (3.15)
\end{aligned}
$$

where T_1 is the solution to

$$
\begin{aligned}
0 &= (1 + T_1)\, \log\left[1 + T_1\right] - 2\, T_1, \\
T_1 &\approx 3.9215. \qquad (3.16)
\end{aligned}
$$

This proves the following proposition.

Proposition 10 *The optimal duration until rotation is* $T^* = 3.9215 \cdot \sigma^2/\alpha^2$.

The optimal rotation frequency is thus $1/T^* = 1/T_1 \cdot \alpha^2/\sigma^2 \approx 0.2550 \cdot \alpha^2/\sigma^2$. The better the agent's type is already known ex ante, in comparison to the variance of the noise ϵ, the smaller is α, and the less he should be rotated within the firm. Why? If the agent's type is virtually known (small α), then his incentives to work are small and suboptimal. Therefore, he needs to be incentivized to spend more effort, and this can be achieved by giving him a long horizon, so the incentives to increase beliefs about his type are higher. In fact, not only does he need to be incentivized, he also wants to be incentivized more because ultimately, he reaps all the benefits himself. If his type is virtually unknown (large α), he has an incentive to work like crazy especially in the beginning, until more information is revealed. Potentially, he might even overwork. If time until rotation is long, incentives to work can be inefficiently large. The agent can then only achieve the optimal workload by cutting down the duration until rotation, hence by increasing the rotation frequency.

Considering the perspective of the (unmodeled) periods beyond rotation, $t \geq T$, suggests that the optimal duration until rotation T^* may become subject to mixed strategies, as the agent will have superior information about his abilities compared to other market participants if human capital is not job-specific. Otherwise, the optimal duration until the following rotation will always remain the same as in the first equilibrium and therefore will maximize the agent's infinite horizon profits.

Our general results indicate that the optimal rotation frequency $1/T^*$ is small, when noise is primary induced by the uncertainty about external shocks σ rather than by the uncertainty about the agent's type α. That is, the optimal duration until rotation T^* increases when the noise of performance measurement increases, or the variance of ability decreases. For any such change, learning effects about ability decrease and the market is less likely to change the initial perception about the agent's type. Consequently, incentives to invest in visibility decrease such that the agent needs to be incentivized by a longer pay-back period of his effort; that is, a larger duration until rotation. In the extreme case, when prior type uncertainty tends to zero, $\alpha \to 0$, the optimal time until rotation tends to infinity, $T^* \to \infty$, and thus the optimal rotation frequency tends to zero, $1/T^* \to 0$. Consequently, a major result is that job rotation can only be beneficial in case when career uncertainty exists.

3.4 Implications and Discussion

With regard to empirical evidence, a study by Eriksson and Ortega (2006) reappraises three major explanations for the adoption of job rotation: employee learning, employer learning, and employee motivation. While our model rules out the possibility of typical technological arguments to job rotation, such as the acquisition of new knowledge through exposure to different tasks (employee learning hypothesis), it is consistent with stylized facts that relate to the employer learning and the employee motivation hypothesis. The employee motivation argument assumes that job rotation should incentivize long-tenured employees with limited advancement opportunities. Contrary, the employer learning argument focuses on job rotation as an instrument to learn different traits of a worker's ability. Abstracting from such multi-branched learning effects, our results propose a complementary explanation, one that connects learning effects about ability with incentive-related evidence for job rotation. In the argumentation of our model, a firm learns more about an agent's ability if prior type uncertainty is large and if performance measurement is relatively precise. In this case, agents should rotate more frequently. Consequently, of interest is, when the prior type uncertainty is likely to be large in comparison to the noise of performance measurement. *First*, at lower hierarchical levels, job design takes the form of standardized tasks and routine work is more common. That is, exogenous shocks are more likely to persist at higher hierarchical tiers, as projects become more complex and also may include or at least depend on the contributions of other agents. Here, performance measurement is less precise and thus learning effects about ability are small. *Second*, type uncertainty should decrease with an agent's work experience in the labor market. Consequently, the frequency of rotation should be smaller for senior workers and higher for new hires and employees at lower hierarchical levels of a firm. Consistent with this result, Campion, Cheraskin, and Stevens (1994) identify a negative relationship between self-selection into rotation programs and organizational tenure. With regard to different hierarchical occupations, employees with more routine work, like clerical workers, secretaries, and administrative assistants generally prefer higher rotation rates than executives. Similarly, Eriksson and Ortega (2006) emphasize that the implementation of job rotation negatively relates to employees' tenure. Moreover, promotion prospects and rotation rates are shown to be higher at new or fast-growing firms. Likewise, our model accounts for the finding that firms with flatter hierarchical structures and therefore limited prospects of promotion generally face lower rates of rotation. Therewith, our results offer a novel theoretical explanation for the lack of empirical evidence for existing motivational approaches to job rotation identifying that job rotation is less common for motivating plateaued employees without future prospects, but more plays a role for agents with shorter employee tenure, such as young professionals, who are primarily incentivized by career concerns objectives.

3.5 Conclusion

In this paper, we offer a new information- and incentive-based explanation for the use of job rotation. By considering the question of an agent's optimal task assignment, we highlight how in the presence of implicit incentives job rotation may emerge endogenously. More specifically, our model shows that learning about an agent's ability and the incentives to incur costly effort decrease over time. Rotating agents to a different job results in an information loss about past performance signals, creating new impetus to effort. However, job rotation also reduces the time horizon and thus reduces rents from working and incentives. In this trade-off, our model analyzes the impact of environmental variables on the desirability and the optimal extent of job rotation. As a result, the optimal duration until rotation increases as incentives to increase beliefs about the agent's type decline. Thus, the more dispersed the prior relative to the noise of performance measurement, the more often agents should be rotated inside the firm.

By analyzing a firm's optimal organizational strategy, the main contribution of our work is to characterize how implicit contracts impact optimal job design and how employees may benefit from job rotation programs. Therefore, we offer a new explanation for the lack of empirical evidence connecting employer learning arguments with existing motivational approaches to job rotation. In line with the findings that job rotation is more common among young employees and therefore is not appropriate for reducing the boredom of long-tenured workers (Campion, Cheraskin, and Stevens (1994), Eriksson and Ortega (2006)), we present a novel motivational explanation for the use of job rotation, confirming that job rotation can be advantageous in the presence of implicit incentives and hence is more effective in early careers of employees. More generally, abstracting from the traditional view that technical factors, such as the implementation of a new technology, can promote the implementation of job rotation (see, for example, Gittleman, Horrigan, and Joyce (1998)), we propose that information and incentives also play a pivotal role in the desirability of job rotation and the question of the optimal organizational strategy of firms. Our results are consistent with recent empirical evidence emphasizing the role of information with regard to job rotation and the incentives given to workers (Hertzberg, Liberti, and Paravisini (2010), Hentschel, Muehlheusser, and Sliwka (2012)).[8]

[8] Hertzberg, Liberti, and Paravisini (2010) find evidence that job transfers and the anticipation of rotation can remove loan officers' incentives to withhold bad news, as self-reporting has a smaller negative effect on their career than if bad news is uncovered by the successor. In relation to the literature on employee turnover, Hentschel, Muehlheusser, and Sliwka (2012) identify that managerial replacement can increase the subordinates' incentives to demonstrate their skills.

Our model implies that job rotation is not meant to be a panacea for the provision of incentives to workers; rather it is a means that should be used in moderation to establish a proper balance between effort incentives in the early and later periods of an agent's employment. This is because job rotation is related to both positive and negative incentives that are based on learning effects about ability. Based on these results, several theoretical extensions of this model can be considered. One aspect would be to extend the model framework by analyzing the interaction of job rotation with other forms of incentive-enhancing policies, such as monitoring activities. Specifically, monitoring could increase the precision of output measurement and therefore complement the incentive effect of rotation programs. Another consideration would be to investigate the implementation of job rotation in team settings. The entering of a new team agent would introduce uncertainty concerning the relative abilities of all team members and therefore can create effort incentives even for non-rotating employees, especially in the presence of aggregated or relative performance evaluations.

Taking a broader view of the results, our work contributes to the debate on the optimal internal design of organizations. Although the implementation of job rotation can also be affected by other factors, such as organizational requirements, our model offers a new understanding of how job rotation impacts learning effects about abilities and influences the behavior of agents through their career concerns incentives.

3.6 Appendix

Proof of Lemma 4. At date t, the agent can spend effort costs $c\,e^2/2$ such that for the remaining time period of $t' \in [t,T]$ output is increased by e. More specifically, at date t, an effort cost of $c\,\mathrm{d}e^2/2\,\mathrm{d}t$ increases output from t to T by $\mathrm{d}e\,\mathrm{d}t$. Thus, as the agent's wage proportionally increases with his expected ability, it is increased for all dates $t' \in [t,T]$ by

$$(1-q)\,\mathrm{d}e\,\mathrm{d}t = \frac{\alpha^2}{\alpha^2\,t' + \sigma^2}\,\mathrm{d}e\,\mathrm{d}t, \tag{3.17}$$

where $1-q$ denotes the weight on the output measure when updating the agent's ability. Consequently, according to (3.10), (3.11), and (3.12), the agent balances the resulting benefits with his costs of effort. Thus, the agent solves

$$\max_e \mathrm{d}e\,\mathrm{d}t \int_t^T \frac{\alpha^2}{\alpha^2\,t' + \sigma^2}\,\mathrm{d}t' - c\,\frac{\mathrm{d}e^2}{2}\,\mathrm{d}t = \mathrm{d}e\,\mathrm{d}t\,\log\left[\frac{\alpha^2\,T + \sigma^2}{\alpha^2\,t + \sigma^2}\right] - c\,\frac{\mathrm{d}e^2}{2}\,\mathrm{d}t. \tag{3.18}$$

The first-order condition yields the equilibrium effort of

$$e^* = \log\left[\frac{\alpha^2\,T + \sigma^2}{\alpha^2\,t + \sigma^2}\right]/c, \tag{3.19}$$

which corresponds to the result of (3.13). Note that as e^* is free of the market conjectures, the existence of a unique equilibrium is ensured. ■

Proof of Proposition 10. According to (3.4), at each date $t \in [0, T]$, the agent is paid his expected productivity, $w_t = E[a_t + e_t + \epsilon_t] = E[a_t + e_t]$. Then, applying the equilibrium effort e^* of Lemma 4, and given the distributions $a \sim \mathcal{N}\big(0, \alpha^2\big)$ and $\epsilon \sim \mathcal{N}\big(0, \sigma^2\, t\big)$, the agent's average utility is given by

$$\begin{aligned} \bar{U} &= \frac{1}{T} \int_0^T \left[E\left[a_t + \log\left[\frac{\alpha^2\, T + \sigma^2}{\alpha^2\, t + \sigma^2}\right] / c \right] - \log\left[\frac{\alpha^2\, T + \sigma^2}{\alpha^2\, t + \sigma^2}\right]^2 / (2\, c) \right] \mathrm{d}t \\ &= \frac{\sigma^2}{2\, c\, \alpha^2\, T} \log\left[1 + \frac{\alpha^2\, T}{\sigma^2}\right]^2, \end{aligned} \tag{3.20}$$

which corresponds to the result of (3.14). The agent solves for the optimal duration until rotation T^*,

$$\max_T \bar{U} = \frac{\sigma^2}{2\, c\, \alpha^2\, T} \log\left[1 + \frac{\alpha^2\, T}{\sigma^2}\right]^2. \tag{3.21}$$

The first order condition, $\partial \bar{U} / \partial T = 0$, yields

$$0 = \left(1 + \frac{\alpha^2\, T^*}{\sigma^2}\right) \log\left[1 + \frac{\alpha^2\, T^*}{\sigma^2}\right] - 2\, \frac{\alpha^2\, T^*}{\sigma^2}, \tag{3.22}$$

which corresponds to the result of (3.15). With $T^* = \sigma^2 / \alpha^2\, T_1$, we can rewrite (3.22) to

$$\begin{aligned} 0 &= (1 + T_1)\, \log\left[1 + T_1\right] - 2\, T_1, \\ T_1 &\approx 3.9215. \end{aligned} \tag{3.23}$$

That is, the optimal duration until rotation is $T^* = \sigma^2/\alpha^2 \cdot T_1 \approx 3.9215 \cdot \sigma^2/\alpha^2$. The optimal frequency of rotation then equals $1/T^* = 1/T_1 \cdot \alpha^2/\sigma^2 \approx 0.2550 \cdot \alpha^2/\sigma^2$. ■

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