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THE EFFECTIVENESS OF DYNAMIC MATHEMATICAL SOFTWARE IN THE INSTRUCTION OF THE UNIT CIRCLE

Edward Simons

THE EFFECTIVENESS OF DYNAMIC MATHEMATICAL SOFTWARE IN THE INSTRUCTION OF THE UNIT CIRCLE

A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Teaching: Mathematics

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by

Edward E. Simons Jr

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ABSTRACT

This study is attempting to test the effectiveness of dynamic computer models such as GeoGebra and Desmos on high school students' ability to understand key concepts with regards to the introduction of unit circle and the graphing of the sine and cosine functions.

Algebra two high school students of varying ages were chosen and randomly placed into two groups. Both groups were given the same pre-assessment and an identical lesson. The two groups' only difference occurred with the individual student practice portion of the lesson where one group did 'traditional' paper and pencil practice for graphing and solving while the other group used only computer models as their individual practice. Both groups were then reassessed by giving the same assessment again. Their levels of improvement were compared using standard statistical analysis and a mean comparison test. The results showed a statistically significant improvement in the student group that used the dynamic models versus the group that did not use the computer. The sample size was large enough to generate a confidence value of over 99% (99.3%) so we were able to reject the null hypothesis that there was

no difference between the group results and accept the hypothesis that the student group that used the computer models improved by a statistically significant amount. The non computer group improved by 7.7 percent while the computer aided group improved by over 49 percent. This represented an 88 percent increase in the scores of the computer group when compared with the control group. I was able to definitively conclude that the dynamic software did have a significant and positive effect on the students' learning of the unit circle.

It is hoped that this information will be used to help inform more effective instruction for high school and college students as they learn this topic. It also provides a strong argument for an increased emphasis on educating teachers to become more fluent in the use of dynamic models and software as both a demonstration tool and as an interactive tool for their students in a variety of math levels. These results may also have wider applications to many other math topics and math instruction in general.

ACKNOWLEDGEMENTS

There are many people that have contributed to my knowledge and to the material for this paper. First, I would like to thank the student subjects, their parents, and their teachers for allowing me into their classroom to conduct my research and answer my questions. Second, I would like to thank my academic advisors who have contributed so much to my thesis. These are Dr. Susan Addington, Dr. Jeremy Aikin, and my principal advisor Dr. Madeleine Jetter. I would especially like to thank Dr. Madeleine Jetter for her enormous patience, dedication and constant support. Without her input and tireless dedication this paper would not have been possible. Lastly, I would like to thank my beautiful wife Barbara for constantly pushing me to excel and for her patience while I worked.

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CHAPTER ONE

INTRODUCTION

History of Dynamic Software

The history of dynamic software goes back to the mid 80's with the introduction of MatLab in 1984. This program was almost exclusively used at the university level by doctoral students for research. It took nearly a decade for dynamic math programs to be created for the use of more basic math use such as geometry explorations. The first commercial Macintosh version of Geometer's Sketchpad was released in 1991. (2014, The Sketchpad Story The first Windows version was released in 1993. It wasn't until the release of the third version in 1995 that widespread use of the program began and the program's true potential was realized. This version was designed to run on Windows 95. This program was one of the earliest user-friendly dynamic geometry programs but it was only available at a cost of $50. The first free programs like GeoGebra were first introduced in 2001 and started to become more widely disseminated by 2006 (Szabo, A. 2011, May 16). If we use these dates as a reference to what has been available to most K-12 teachers, then free dynamic software has only been easily accessible for approximately fifteen years. Using myself as an example of how rapidly new ideas get introduced into the K12 curriculum, I was introduced to Geometer's Sketchpad in my university classes in 2005 which was within a few years of it's introduction. There were undoubtedly others who where introduced earlier but remember that these individuals where almost exclusively math majors. It is much less likely that any teachers below the high school level had any idea of the existence of, or exposure to this program. Also, the majority of professors that were instructing the educational courses did not demonstrate dynamic software let alone have any instructions on how to integrate it into a math curriculum. It is easy to see how slowly the use of dynamic software is being integrated into Mathematics curriculums due to this "educational inertia".

There is currently no organized or mandated usage of dynamic software in most current high school mathematics curriculums. However, it is mentioned in the Common Core State Standards (CCSS-M as one possible method to use for students to understand the effects of different geometric transformations. I could predict that some sort of mandated inclusion of dynamic lessons will occur eventually. But until then, it was only being done through the initiative of the Math teachers that have chosen to include it as part of their normal classroom lessons.

Significance

A study where the use of dynamic software is implemented into the curriculum by the teacher as a demonstrational tool would fit into the current body of research by highlighting more effective methods for precalculus instruction. This could be used by any math instructor regardless of their proficiency with the use of dynamic software. I believe that dynamic lessons and demonstrations could be incorporated into any math lesson at almost any level of instruction. Arguments could be made for their effectiveness at various levels but thousands of preconstructed models are already available on most of the free dynamic math sites. In a recent search in GeoGebra, I found literally hundreds of preconstructed demonstrations for "dividing fractions". Some results were very similar and not all were strictly covering division but this gives some idea of the amount of free resources available from this one site. These have been made almost exclusively by other teachers who are looking to demonstrate some math concept in a conceptually different way. It would also help to identify any particular concepts that may tend to more understandable if dynamic software is used as part of the instruction.

And lastly, it is important to quantify what (if any) level of increase in student understanding is being achieved by the use of these dynamic models. If significant benefits are found in levels of student understanding, it would provide convincing evidence that such methods should be incorporated into every precalculus curriculum and perhaps into other levels of mathematical instruction.

Background

As the use of computers continues to infiltrate into ever more aspects of our lives, I have observed that the use of technology is still conspicuously absent in the majority of high school math classrooms (at least in Southern California. There is the occasional video or perhaps an interactive link in a textbook that very few teachers bother to explore (I know I rarely did!. Most teachers do use computers for power points or perhaps google classroom, but that is different from using the computer as an interactive tool to directly learn and explore a mathematical concept. This idea of using the computer as an instructional tool goes well beyond a simple powerpoint presentation or google search for information.

It is my belief that all teachers need to incorporate more dynamic lessons in their math instruction. This leads to the problem of changing the general attitude of how math teachers approach their lessons. There are many facets to the problem of changing this behavior among teachers and students. One is the lack of motivation of teachers or perceived benefit for teachers to use this new technology in the classroom. If you ask a teacher if technology is beneficial to learning, teachers will almost always answer yes. The problems start occurring when the instruction starts and time constraints prevent teachers from ‘adding’ material to their established lessons. What teachers need to be aware of is that dynamic software replaces more traditional techniques. If used properly, it can save time for teachers because students can more rapidly answer questions with the use of pre established models. According to Alsina, C., & Nelsen, R. (2006), handmade graphs on paper or chalkboard are a tedious procedure at best with arguable benefit versus the time expenditure. There is some measured benefit from these traditional methods but we now know there are more effective tools available. With today’s calculators and computers we are able to quickly graph sophisticated functions and manipulate them easily.

It is also very instructive to see the graph react as a direct result of changing the input such as using a slider that can be given any range of inputs that will immediately show the results in the graph. This is much faster than anyone could see the same results by paper and pencil graphing. I have personally seen the confidence level of students increase in a single lesson when they are asked to graph a function using Desmos or Geogebra. And this confidence carries over to later instances where these same students are tasked with graphing using the traditional methods.

Student Difficulties in Working with Trigonometric Functions

Some of the difficulties that arise for students from the introduction of graphs of trigonometric functions are that it is different in several ways from what the students have been exposed to in previous levels of math instruction. Trigonometric functions are periodic, and it does not use any previously learned algebraic symbols. Students should have been exposed to curves that are from quadratic, cubic, exponential, and logarithmic functions but nothing that is a repeating pattern. Also, most of the key inflection points of trig functions reside at increments of pi/2 which also presents unique difficulties with many high school students attempting to graph trig functions with paper and pencil. They have almost exclusively been taught to choose small whole numbers as the best start to constructing graph. I have found that it is often the case that even when students can graph the sine and cosine functions correctly, they still have difficulty in matching key values in the graph to the key values they are usually required to learn when introduced to the unit circle. And finally, how often do students actually use graphs or know when to use them? According to Byers, P. E. (2010) ,"The idea of using an investigative approach in college should be explored. In particular, investigations using technology to capture the *dynamic* features of sinusoidal waveforms and the unit circle are warranted" (pg 182). It seems logical and essential to also introduce these investigations at the high school level as well.

Training Teachers to use Dynamic Software

Teachers tend to look upon new training as passing fads that fall in and out of favor. The longer a teacher has been teaching the less receptive they are to new ideas; (Makela, 1998 and for good reason. In their careers, veteran teachers have tried dozens of varying methods that are presented by administrators or instructors who can be somewhat less than well versed in the demands of a high school mathematics teacher. Veteran math teachers have usually settled on a few techniques they have found that work best for them and produce good results in their classrooms. Those same veteran teachers are emotionally and physically invested in those chosen methods and convincing them of trying a new method let alone learning an entire mathematical software packages such as Desmos or GeoGebra can be problematic. Precalculus and trigonometry topics in particular are hundreds of years old and most teachers of today are teaching the subject in a similar manner to when Euler's notation and influence was incorporated into regular classroom instruction.

Current Research

Geniuses like Euler could probably visualize the infinite variations that occur to create a moving picture in their minds. The rest of us mere mortals are now lucky enough to have a tool available to actually see a moving picture of the possible variations in functions like sine, cosine and tangent. But like any tool, one needs to know the correct application for that tool, and which version of the tool works the best for any given task. 'Dynamic geometry software' is available in many forms. On a recent google search, there were 36 different programs listed for two-dimensional geometry constructions, and eleven for three dimensional construction.

The NCTM (National Council for the Teaching of Mathematics) indicated that the use of varied representations are "essential elements in supporting students' understanding of mathematical concepts and relationships" Moyer, Niezgoda, & Stanley, (2005). I would go further and argue that there are some elements that are more effective at supporting student understanding than others. Much of the current pedagogical research relates to graphing calculators and their benefits to student learning. There is also a large body of research in 'computer saturated' mathematics instruction. It is widely recognized that a varied approach (differentiated instruction) to demonstrating math concepts allows students to choose the method(s) that fit their understanding and learning. Consistent use of dynamic software to demonstrate math topics should be used with the goal of getting the students to start *thinking* dynamically when they look at a static picture in a book or computer screen.

One attempt to do this was in a new method to teach trigonometry using a method called MNO (Burke and Olley, 2008) M is the map of the terrain to be covered, in this case the mathematical structure of the topic, connections and potential problems that students might encounter. N is the narrative, the sequencing of the lessons and activities. O is 'orientation', the activities which will serve as strategies to facilitate students' engagement with the topic. The study involved two lessons that involved competing strategies to teach trigonometric ratios. The first used a technique called the ratio method or SohCahToa that most math teachers are quite familiar with. If this is not familiar, SohCahToa is an acronym that helps students remember the structure of the three basic representations of the trigonometric ratios which are, Sine $\Theta = \frac{opposite}{hypotenuse}$ or

Soh, Cosine Θ = $\frac{adjacent}{hypotenuse}$ or Cah, and Tangent Θ = $\frac{opposite}{adjacent}$ or Toa. The other lesson used two dynamic models of the unit circle. Geometer's Sketchpad (GSP) was used to demonstrate the way the ratios are derived from a special right triangle in the unit circle and was given to the test group. The focus of the lesson was to compare the different strategies and the effects of GSP on both of was not directly on whether GSP improved learning but rather, which of the competing methods of learning trig. ratios was more effective with GSP as a part of both lessons. Previous studies had shown the ratio method to be most effective but the comparison was made without the use of dynamic models.

The Common Core State Standards recommend the use of dynamic geometry software, particularly in the learning of transformations. It is unusual that the standard is mentioning dynamic geometry as an example of what

students could do but not requiring or assessing student knowledge of dynamic software. This is most likely due to the influence of Common Core and the emphasis that this places on students to delve deeper and to learn ‘how’ the answers are derived rather than just memorizing a procedure. Teachers should teach these standards using dynamic tools. (C-GO, ‘Represent transformations in the plane using, e.g., transparencies and *geometry software*; describe transformations as functions that take points in the plane as inputs and give other points as outputs.’).

The demand for technology integrated into instruction is definitely increasing. In a European study supported by a software company (Alexander, et al, 2018) it reveals that universities are under increasing pressure to offer incoming students access to state-of-the-art technology because of the increased fees they are being asked to pay. According to the study, more than half of the students surveyed say having access to computers and the latest software is one of the most important factors when choosing an institution – more so than having well-qualified and accessible lecturers. About half of the students cited increased fees to explain their reasoning, and 89% of those starting university in 2012 “feel entitled to a better university experience”. It is natural to assume that US students feel similarly to our European counterparts.

I chose this topic because I have used Geometer’s Sketchpad, GeoGebra, and Desmos software regularly in my geometry classes and with precalculus. I was first introduced to dynamic software in my undergraduate math classes at CSUSB and I immediately saw the benefits of it. Being able to dynamically manipulate advanced geometry problems helped me tremendously in doing proofs, and finding solutions. Learning the use of the software has helped me enormously in my own math learning and I would love to be able to pass that on to others. I use it frequently when I teach Algebra I, II and Geometry but I use it the most when I teach Precalculus. I have had mostly positive feedback from students when I ask if they feel that computer demonstrations help them to better understand concepts rather than me at the front of the room using the whiteboard. So there is anecdotal evidence from students and teachers that using dynamic software increases understanding.

I have had the occasional student run with the use of Dynamic software as I did. These students seem to thrive but whether it is directly attributable to the software or not is debatable.

I have always had the sense that it helps but I have never done any quantitative comparison of the benefits (to students of Sketchpad or GeoGebra. I would like to know if this is really having an impact on student learning rather than wasting valuable instruction time. I would also like to know what types of mathematical concepts are best demonstrated dynamically rather than with static methods. This is what seems to be missing in the research that I have found.

I am not unaware of the drawbacks to dynamic software. The amount of time that is necessary to get students to log in and the initial lack of student familiarity with the program can lead to some serious delays in actual student learning. But an early investment of time in teaching some of the basic skills and controls that are necessary to learn programs like Desmos or GeoGebra can pay dividends later. Students still need to do basic compass and straightedge constructions so that they can see the basic principles involved. Moss (1997) states that computers can make it too easy just as calculators can make knowing multiplication less essential. Students still need to know the basics before using the more sophisticated tools. Moss (1997) found that students' multiplication skills deteriorate in proportion to their dependence on calculators. Similarly, some of the basic construction techniques such as segment bisectors, or copying segments can be lost by relying on dynamic software too much because there are computer tools that aid in these constructions.

In most of my searching, I found a few studies that related directly to precalculus instruction and one was a very comprehensive study by Kan Kan Chan and Siu Wai Leung who did research into the effectiveness of using dynamic software.

There is a fair amount of research into the benefits of dynamic software in K-12 math classes although much of the information is sporadic. One study that if found was a meta-analysis of other studies that examined the effectiveness of dynamic geometry software (DGS based upon the random effect model and used standardized mean differences of examinations to measure outcomes. DGS-based instruction was found to have a statistically significant positive influence on students' mathematical achievement in all levels of education, Chan & Leung (2014).

One paper that did measure this directly was in a study by Baharvand, Mohsen (2001) where students used the software themselves in doing the constructions and investigating properties. The results were markedly in support of the dynamic software having a positive effect. In another article by Keith Weber of Rutgers University, he states ’students need to relate diagrams of triangles to numerical relationships and to manipulate the symbols involved in those relationships’. This is precisely what is done when a dynamic model is shown.

The question that follows is how much the students gain in understanding and what level of knowledge is obtained from this. I also found a very thorough treatment of using Geometer’s Sketchpad with geometry students and the introductory phase of trigonometry and the learning of the basic trigonometric ratios. Steer, de Vila and Eaton (2009). The conclusions from this study were that 8th year students (8th grade) were able to successfully use the software to construct triangles that represent the standard trigonometric ratios. They did not show any significant increase in understanding when assessments were compared to the control classrooms. I immediately wondered if any long term data was available but was unable to find anything in my searches. This in itself is an indication that there are serious gaps in the research of the benefits of doing constructions. I believe that a study of this kind can add to the total already existing body of research be of benefit to instructors who are reasonably proficient in the use of Geometer's Sketchpad and GeoGebra. By increasing the individual segments of instruction that can be shown to receive benefit from dynamic software, the greater the emphasis will be to include these dynamic models in everyday instruction.

CHAPTER TWO

OVERVIEW

The purpose of this project will be to examine the effectiveness of using dynamic software as a demonstration tool and as an interactive tool for students who are being introduced to the unit circle, and graphing sine and cosine. The instructor of the experimental group will have students use dynamic software for a minimum of 15 minutes to investigate the relationship between the unit circle and the graph of sine and cosine. Due to time constraints, the students will use two different dynamic models that will be limited to a one day lesson. The primary focus of the study will be on students who are in the early stages of learning trigonometry and more specifically, the unit circle. The groups that were used for the study were mostly Algebra 2 students of between 14 to 17 years of age and near the end of the second semester. All of these students should have been introduced to trigonometric ratios the previous year when they took geometry but there was no mention of the unit circle and little or no graphing of trigonometric functions.

There are many applications in the various math curricula that lend themselves to interactive models, transformational geometry, student manipulated graphs of the sine and cosine functions, demonstrations of many of the trigonometric identities, the list goes on and on. I plan on trying to record any measurable benefits or drawbacks to the employment of several of these dynamic models in a normally paced Algebra 2 class.

Goals

The question that I would like to answer in my investigation is:

a) How Is student understanding of the unit circle, graph and its relationship with the trigonometric functions affected when a teacher uses dynamic demonstrations using a computer instead of static demonstrations?

b) If there is a measurable benefit of dynamic software, what portion of the lesson benefitted the most from it?

c) If there is no measurable benefit, I would like to explore what factors and conditions could be improved to try and provide some benefit.

CHAPTER THREE

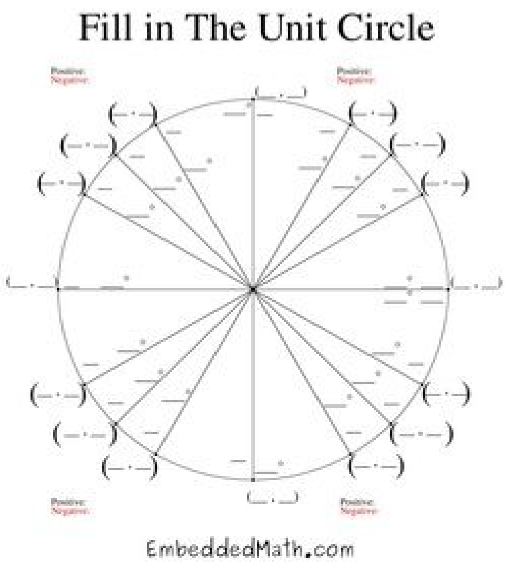
METHODOLOGY

This study was constructed in a pretest-posttest control grouped half-experimental pattern. Two groups of students were studied, one group (let's call them group A) did not receive any interaction with dynamic models and was taught a 'typical' lesson. The second group (group B) were taught the same material with the inclusion of dynamic software (see appendix A). The treatment group (B) used the dynamic models to replace the more typical paper and pencil type of practice. Both groups are high school students currently enrolled in Precalculus and Algebra two sections and were of varying ages and grade levels. Each class that was used was divided in half for their respective groups (A and B) by simply having every other student in each row move to one side of the class while the remaining group moved to the other side of the classroom. This should provide enough randomization to minimize any weighting of one group over the other in math ability. Three of the four classroom groups were taught by me while the fourth classroom was taught by their normal classroom teacher but using the same lesson plan. Each class received exactly the same instruction. Where the classes diverged were during the student practice portion of the lesson. Group A were asked to complete the included practice section of the lesson plan which consisted of the students using paper and pencil and trying to answer several questions that used the topic that was taught. The treatment group was given the link to the dynamic models and asked to explore several different topics. One example of a topic was having the students use a slider that controlled the period and record various answers for specific values of x (sin x).

Both groups were assessed at similar times in the curriculum. This occurred near the end of the second semester. Most of the Algebra 2 students should have had some exposure to the unit circle since it is now being taught in most standard geometry curriculums. The date that the lesson was taught was intentionally introduced to coincide with the trigonometry portion of the curriculum in Algebra 2. A single pre-assessment was given to both groups to determine their level of understanding. Prior student knowledge of trig functions and the unit circle were limited to some basic lessons in the Geometry class. This would have occurred two years prior for the students that are taking precalculus. Precalculus students also received some additional instructions on the unit circle in their Algebra 2 class which would have been one year before precalculus.

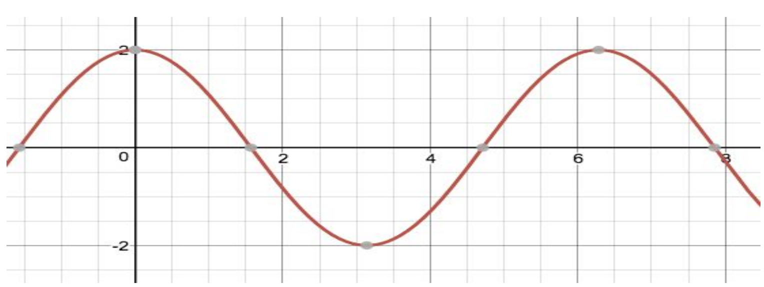
Assessment (pre and post)

1.



2. Explain how you derived the *x* and *y* coordinates for each location of the unit circle.

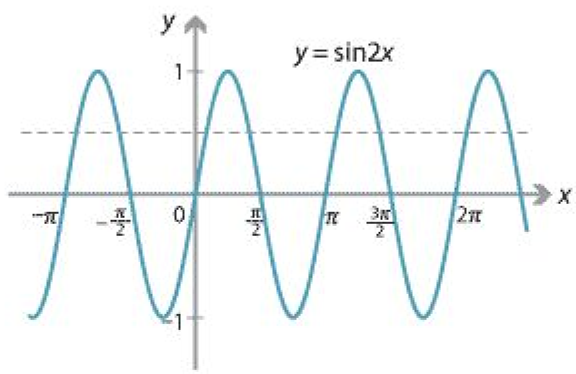
3. Write the function that represents the graph shown below.



4. How is the graph of the equation y = sin(x - π) different from the graph of y = sinx? What is the cause of this change?

5. Below is a graph of the function y = sin(2x). Explain (mathematically) how the graph of this function is similar and different from the graph of y = sinx. The word bank below is provided as a collection of terms that could be used in your explanation.

(Word Bank: Period, Amplitude, Shift, Midline, Key Values, Maximum, Minimum)



6. Define the intervals in which the sine function is increasing and decreasing in the interval (0 to 2π).

7. Explain why the equation $sin^2x + cos^2x = 1$ is true for all values of x between 0° and 360°

8. Say which is greater, sin 83 degrees or sin 175 degrees and how do you know.

9. Explain why the value of cosine can never be 2.

The purpose of having the students fill in the unit circle values in Question1 is to assess their current level of knowledge and/or memorization of the unit circle values. Question 2, 'Explain how you derived the *x* and *y* coordinates for each location of the unit circle', is designed to determine if they have the unit circle values memorized or do they calculate them from special right triangles or any other methods. Question 3 is assessing several things, first whether students can distinguish between the graphs of sine from cosine. Second, it is assessing their knowledge of period and amplitude. Questions 4 and 5 are to assess how well students understand transformations of trig functions involving shifts and compressions. Questions 6 "Define the intervals in which the sine function is increasing and decreasing in the interval (0 to 2π)", begins the second portion of the assessment. This question is specifically targeted at assessing the second dynamic demonstration that the test group used. We can expect to have a significant percentage of students that will be unable to completely fill in the unit circle values and also to incorrectly graph the sine and cosine functions. These students will be useful in evaluating any improvement that may occur in their ability and knowledge due to the non-dynamic versus the dynamic lessons.

CHAPTER FOUR

PEDAGOGY

The lesson plan (see appendix A that was selected was part of a Massachusetts' high school PreCalculus program by Douglas A Ruby ("6 Unit Circle," 2002 that was approved by that district and in general use. All classes that were part of the study were able to get through about half the lesson plan. The stoppage due to time occurred during question 4 of the lesson plan (see appendix. Both groups progressed to where they started estimating angles on the unit circle but the teachers were forced to cut this short in order to leave time for the post-assessment. This balanced nicely with the test classes' two dynamic models. The control group followed the included paper and pencil practice that was in the lesson while the dynamic group used two models that were available on geogebra. The first demonstration that group B was introduced to is an interactive model of the unit circle called "unit circle - exact values". The students were asked to use the software to explore the values that were preset and see how they matched the given table of values. The hope was that they could more quickly see how all these points on the unit circle are related to each other by quadrants. In a traditional precalculus section, the book 'Precalculus' by Larson and Hostetler, section 4.2 is where the unit circle is introduced with static pictures of the standard values similar to what is used in my pre assessment (see question 1 of the pre assessment). In the following section, (4.3) students are reintroduced to the 45,45,90 and 30,60,90 triangles and given the standard values in table form (below)

ratio \ angle θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Fig. 1, Trigonometry Table

Group B students were shown static pictures of the unit circle and the normal values for 0, 30, 45, 60, and 90 degrees. They are also shown the triangle relationships that generate these values with an interactive model in Geogebra. This dynamic demonstration was called “unwrapping the unit circle”. It allows students to click and drag a point (c) and watch how the sine, cosine, and tangent values change as a result. It also allows the students to click on sine cosine or tangent and the computer will trace out the graph from 0 to 2 pi. The hope here was to give group B a stronger visual connection from the unit circle to the graph of the trig functions. Group A will be given a non interactive practice with no dynamic models. Students from group B will be given an opportunity to interact with the dynamic models.

The two groups of students will be given identical assessments (see Appendix B and the results will be compared with special attention being paid to any significant differences that can be attributed to the computer software. The methods of comparison of the student data will be with the use of the statistical analysis. This will allow me to analyze the effectiveness of the assessment and how well it is targeted to the student group(s as well as giving me a detailed analysis of the students' assessment results. At the conclusion of the daily lesson each group was given the same assessment. The recorded data was coded to protect the students' identities. I will be looking for correlations between the student explanations and their answers they provide on the assessment.

The second construction (see Figure 2 that was shown will be a similar model of the unit circle but with the graph of the sine (or cosine function being traced out.

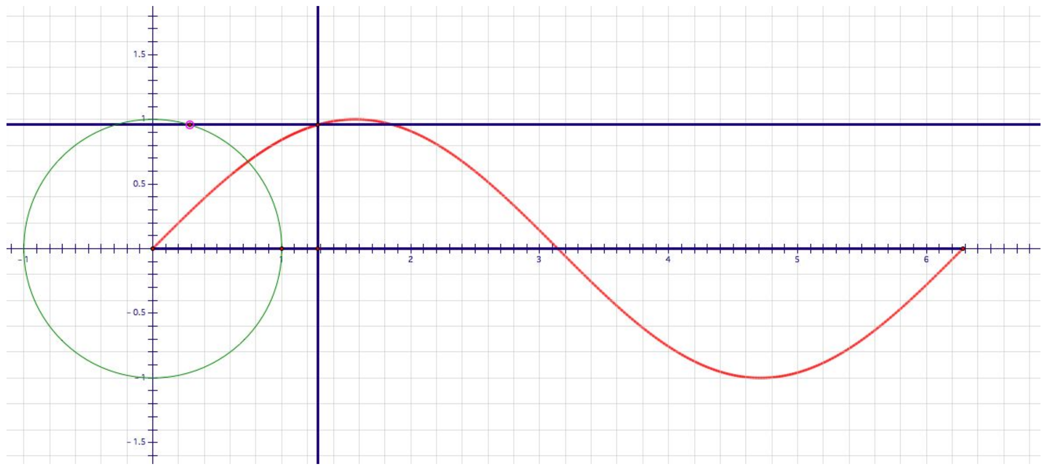


Fig. 2, Screenshot of the Dynamic model of a Sine and Cosine Generator

This type of activity is usually done by having students graph the key locations of sine and cosine at 0, 90, 180, 270, and 360 degrees. It may be beneficial to have students start the activity by doing the manual graph, and the interactive dynamic software following to help reinforce the connections between the two segments of the exercise. Particular emphasis will be placed upon how the position of the circle radius and its relationship to the position of the point that is graphing the trig function. My goal was for students to determine which values are the dependent and independent variables.

Upon the conclusion of the two main lessons the same assessment was given and comparisons made between the control and test groups. The pictures of the dynamic models below allow students to click and drag point B and get different values for sine and cosine. It is quickly apparent to the students how the trig functions are related to the unit circle that they are controlling. One negative aspect of this model is that the sine and cosine values are in decimal form. It could be more helpful if they were left in radical form so that they can be matched with special values like $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$ or $\frac{\sqrt{3}}{2}$.

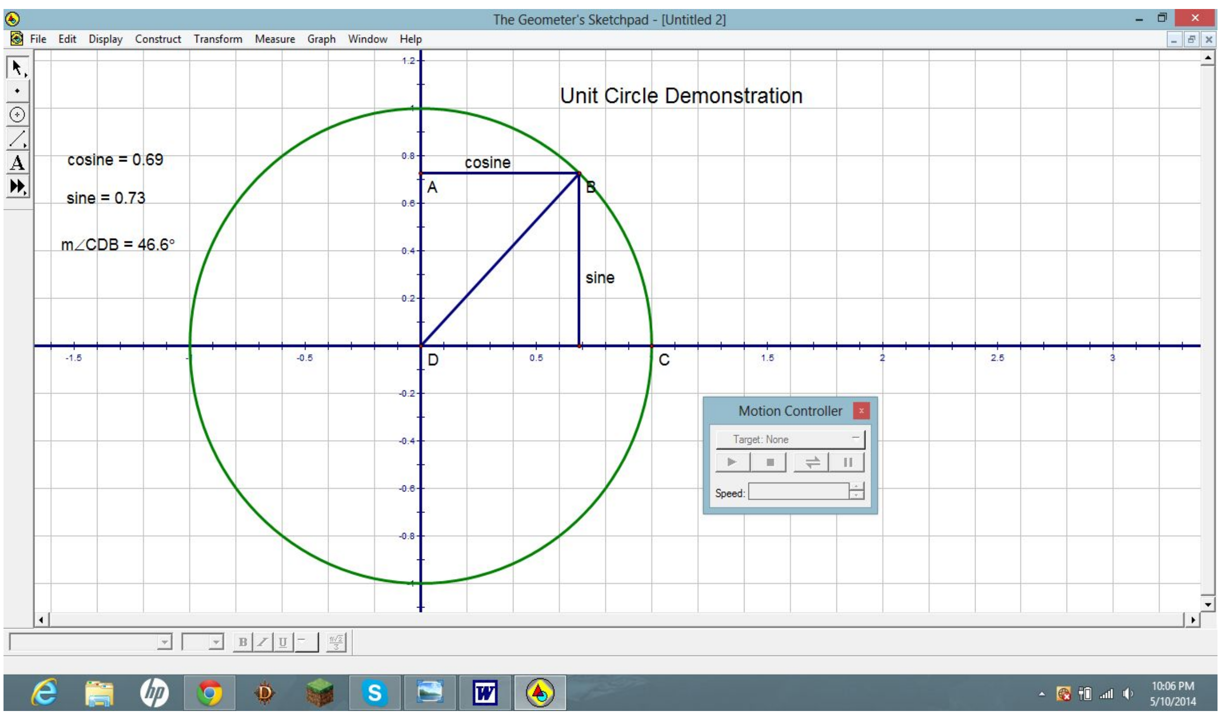


Fig. 3, Unit Circle Dynamic Model Generator for Sine and Cosine Values

The other model below allows students to see point B travel around the unit circle and trace out the graph. After this demonstration, students are asked to more closely examine the demonstration and relate what is generating the x and y values on the circle to the graph. Almost everyone quickly sees that the circumference is the x value and the y value transfer directly from the circle to the graph. The students are then encouraged to go back and look at the first dynamic model where they can see the link between the special right triangles and the sine and cosine function. This entire series of demonstrations takes about 10 minutes.

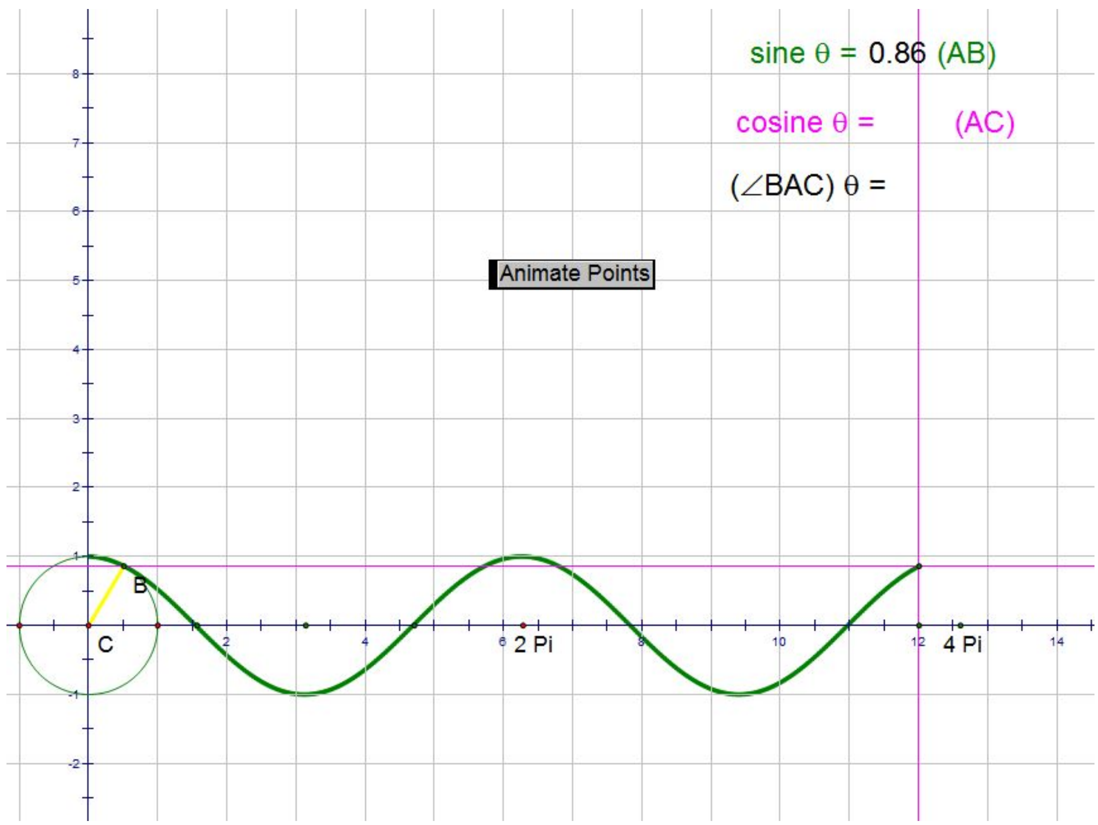


Fig. 4, Dynamic Model for Showing Variations in the Period of Sine and Cosine

CHAPTER FIVE

STATISTICAL DATA ANALYSIS

The body of data that was collected ended up coming from Algebra 2 students only (no precalculus students. A total of five different classrooms were used with four of them being taught by one teacher and the remaining one taught by me. Also recall that each classroom was randomly divided into Group A and Group B categories by the simple expedient of having every other student move to one side of the classroom and gathering the remaining students into the other side. I had a total of 125 students who were included in the study because a fair portion had to be excluded due to either missing the pre- or post-test or not turning in the consent form. These requirements placed a large burden of difficulty on the collection of data and will be addressed further in the final conclusions. The statistical significance of any differences was analyzed by using a test of the equality of two means. This was possible because the data was normally distributed. See graphed data below. An anomaly occurred with student's scoring 3 on the pretest with only 6 achieving that score out of a possible 128 students. Also a larger than expected portion of students scored a four. The two most commonly answered questions were questions one, with one point scored for over half the unit circle values filled in correctly and question two, where many students received one point for answering “from memorization”. There was a wide variety of different answers that contributed to the ‘4’ scores.

The question that showed the most improvement for both groups was question 1. Many more students were able to completely fill out the unit circle or to advance from previously not able to fill in any (zero points to more than half correct (1 point. The question that showed the most improvement for the computer group was question 3. More than half of that group were able to correctly identify the graph whereas less than 20% of the non-computer group improved on this question. More investigation on this would be necessary to accurately determine the exact reason for this but I can surmise that the one dynamic lesson that clearly shows the difference between the sine and cosine graph played a significant role in this.

The mean of all students’ pretest score was 3.008. The median score was 4 of the total pretest, with the difference between the median and the mean due to a large number of zero scores. The mean of the pretest scores for the non computer group was 2.74 and the mean of the pretest score for the computer group was 3.26.

	Mean/Median (pre test)	Mean/Median (post test)	Percent Mean improvement
Group A (non comp)	2.74 / 2	2.95 / 3	7.7%
Group B (comp)	3.26 / 4	4.88 / 4	49.7%

This difference in pretest means, although fairly large, is not significant because I am only trying to measure improvement from this base score (pretest. The mean of the post test for the non computer group was 2.95 out of a possible 16 being the perfect score, so that group improved marginally. One could argue that this was only a one day lesson so the students did not have time to absorb or assimilate the material. The mean of the computer group post test score was 4.88 out of a possible 16, (perfect score. The non computer group had an improvement in the mean score from the lesson of 0.21. While the treatment group that used the dynamic models mean increase by over 1.62. We can see from these numbers that there is a 1.41 point increase in the improvement of the overall score where the only variable was the use or absence of dynamic software. We can therefore note a difference in performance between student groups with regard to learning about the unit circle.

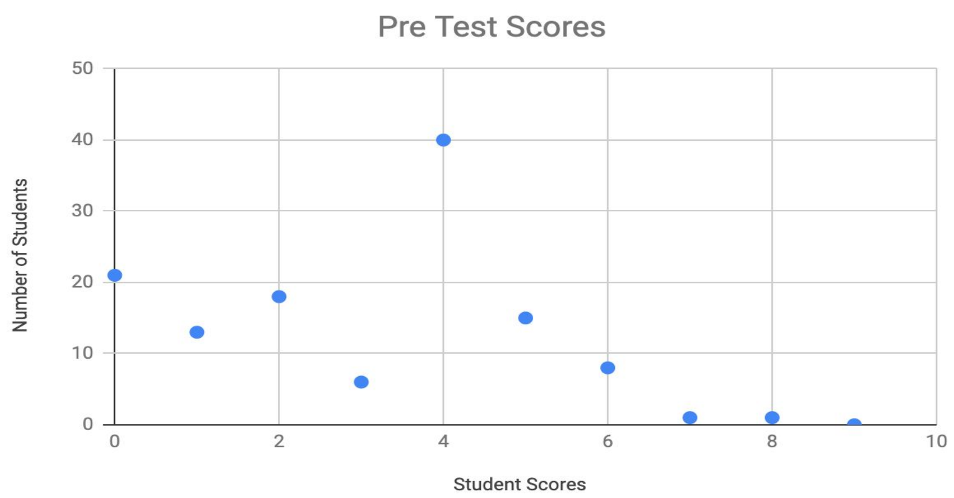


Fig. 5, Dot Plot of the Pre Test Scores

Mean Difference Test

Let's call the test scores for the comparison (non computer students X and the scores for the students that used the dynamic lesson Y. The mean value that I needed was found by subtracting the post test score minus the pretest score ($X_{post} - X_{pre} = \mu_x$). The same was done with the test group ($Y_{post} - Y_{pre} = \mu_y$). The means of each of these lists were then taken with the null hypothesis H_0: $\mu_x - \mu_y = 0$ against the alternative hypothesis H_1: $\mu_x - \mu_y < 0$. If we assume that *X* and *Y* are independent and normally distributed with a common variance, and if we assume respective random samples of groups X and Y, then we can use this method to find a test based on the statistic

$$T = \frac{\overline{X} - \overline{Y}}{\sqrt{\{[(n-1)S_x^2 + (m-1)S_Y^2]/(n+m-2)\}(1/n + 1/m)}}$$

T has a t distribution with r = n + m - 2 degrees of freedom when H_0 is true and the variances are equal. So, the hypothesis H_0 will be rejected in favor of H_1 if the observed value of T is less than $-t_{\propto}(n + m - 2)$. Calculating this gives -3.12(61-64) = **15.6,** so 3.12 < 15.6 and we can reject the null hypothesis.

From my data set, $\mu_x = 0.213$; (the mean of the post test, no computer minus pretest, no computer), $\mu_y = 1.609$; (the mean of the post test with computer minus pretest with computer). The variances, respectively are S_x^2 = 1.237 and S_Y^2 = 1.832. Using this information to determine if the differences in means is significant with respect to the variances, I used this information to get my T value.

$$\frac{0.90625 - 0.213}{\sqrt{\{[(61-1)1.237 + (64-1)1.832]/(61+64-2)\}(1/61 + 1/64)}} = 3.12$$

CHAPTER SIX

CONCLUSIONS

The T- value of 3.12 represents a confidence interval of over 99% that we can reject the null hypothesis and accept the hypothesis that there is a significant improvement in the test student population. As you can see from the graphs below, the non computer group showed a marginal positive shift while the computer group clearly showed a significant shift from lower pretest scores to higher post test scores. I was actually surprised by the magnitude of the difference between the two groups. I believe more research would be useful and beneficial but if this is shown to be a typical result of using dynamic models to demonstrate math concepts then a much greater emphasis should be placed upon math teacher instruction in this area.

I set the following graphs up in two different ways. The first set of graphs compares each groups' pre and post test so that the amount of improvement can be examined. The second set of graphs makes the comparison between groups so readers can compare the two groups' performance to each other based upon the pretest scores and the post test scores separately (see below).

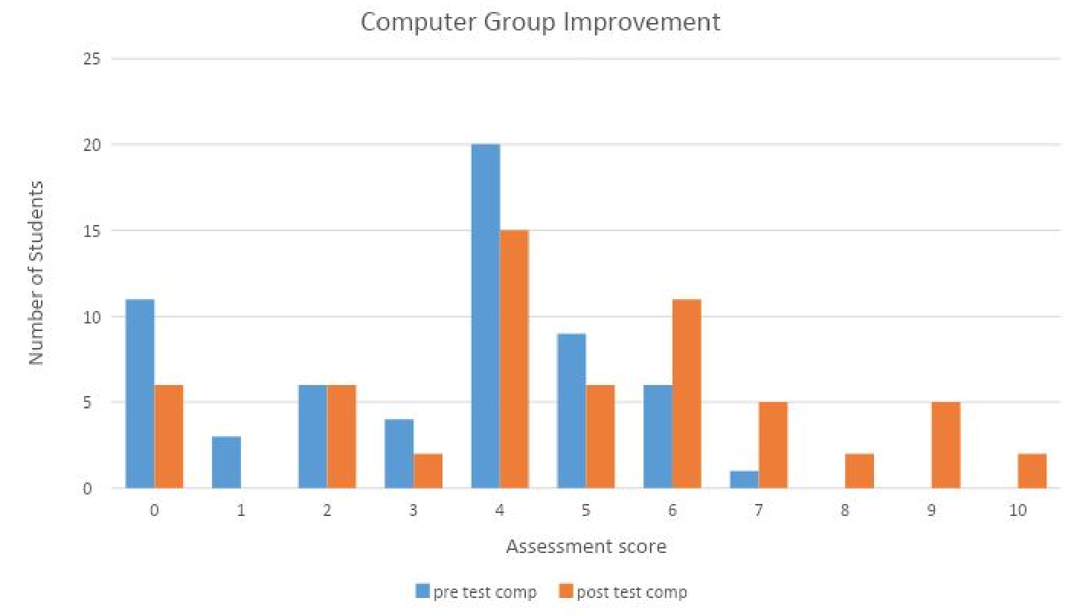


Fig. 6, Bar Graph Comparison of Computer Groups' Pre and Post Test Scores

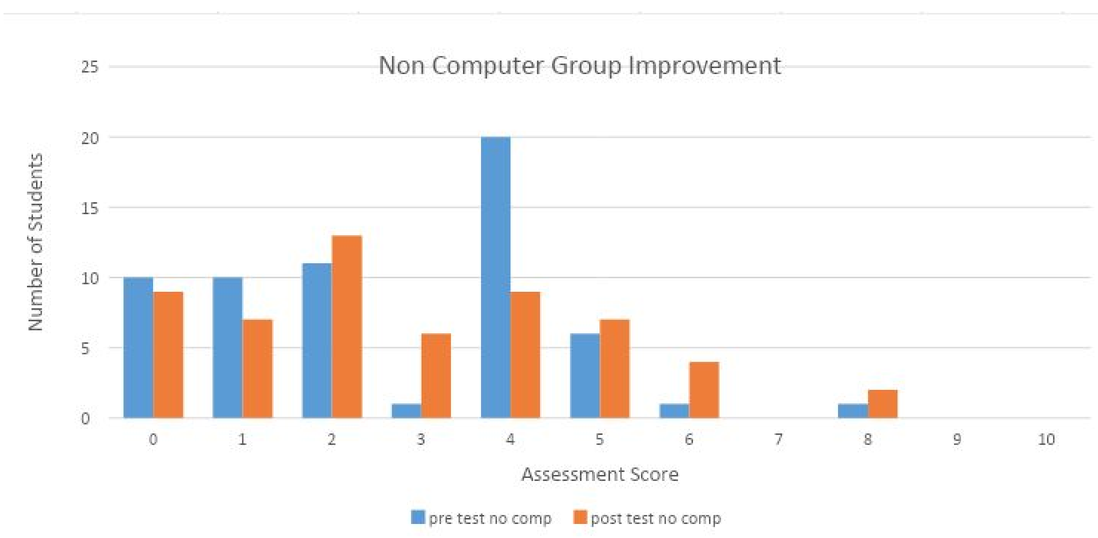


Fig. 7, Bar Graph Comparison of Non Computer Groups' Pre and Post Test

Scores

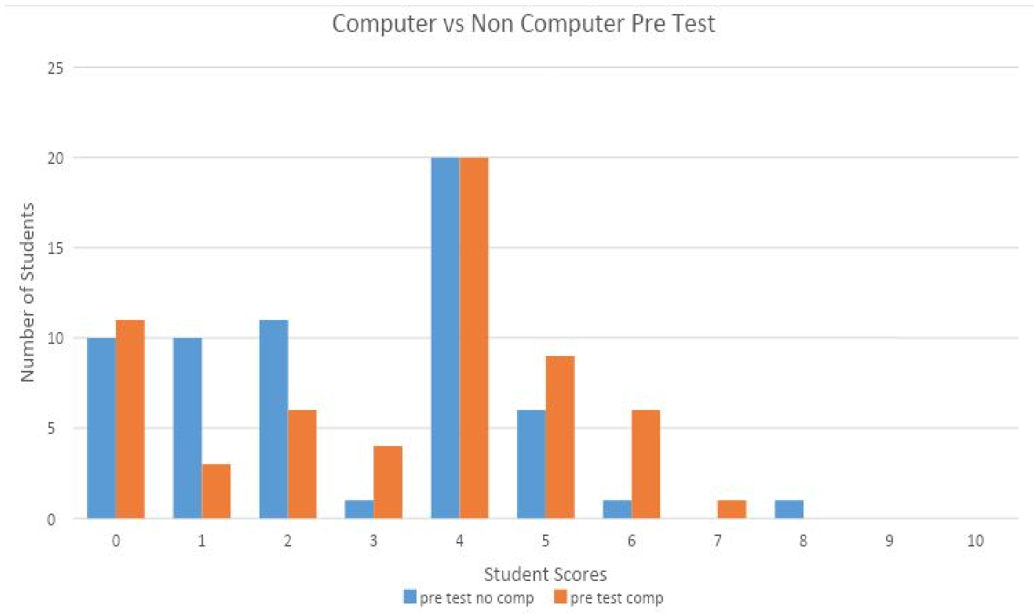


Fig. 8, Bar Graph Comparison of Pretest Scores for the Computer vs. Non Computer Groups

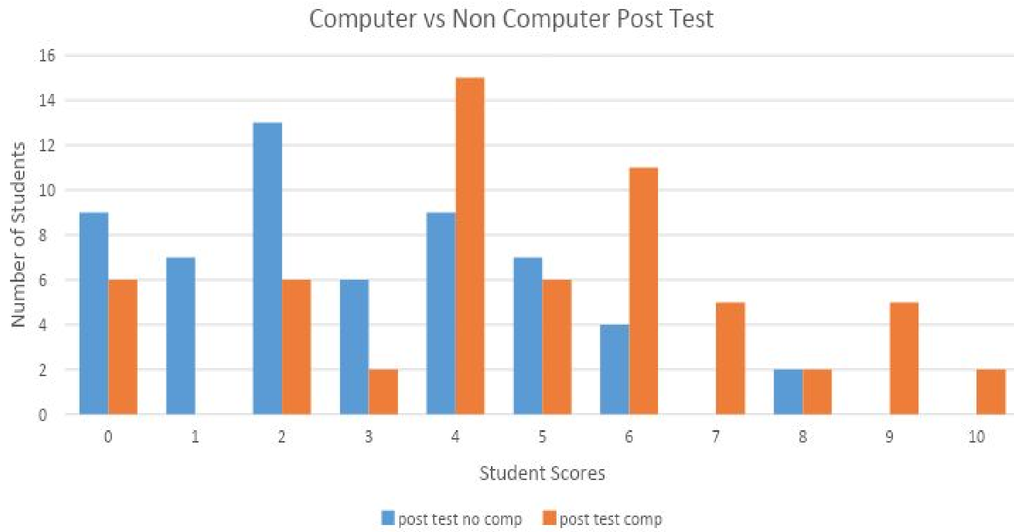


Fig. 9, Bar Graph Comparison of Post Test Scores for the Computer vs. Non Computer Groups

Several interesting facts are apparent when examining the graphs. The most noteworthy was the similarities in the pretest graph scores which is as expected. When this chart is compared to the graphs of the two groups' post test scores the large differences are immediately apparent. The means of the pretest non computer group and the computer group scores were 2.73 and 3.26

respectively. The post test score averages in the same order were 2.95 and 4.87. This can be seen when comparing the post test scores where the shift from lower scores to the higher end (right can be clearly seen. One fact that I found difficult to understand was the high number of scores of 5 in the pretest and post test. My hypothesis on the reason for this would have to do with the quiz scoring rubric and the ease with which a student with marginal understanding my get some questions correct while it would require a deeper understanding to get closer to a score of 10. (Recall that a perfect score by the rubric that I created would be 16)

From the pretest to the post test, the non computer group had zero scores that were 10 and 9 respectively. The computer group had 9 and 6 from the pre to post tests. Here again, we can see a significant drop in the number of zero scores which showed more improvement of the computer group.

CHAPTER SEVEN

FUTURE RECOMMENDATIONS AND RESEARCH

A longer term study would be more illuminating on any benefits of dynamic software on student learning. It would also be informative to track those students all through their high school years. I strongly suspect that a fair percentage may run with the use of dynamic software and start to explore on their own and use it as a personal tool to answer homework questions and to study.

I would also like to see research at the elementary levels that incorporate dynamic software. Such topics as fractions and basic multiplication and division could benefit or at least provide insight into how beneficial dynamic models are to all levels of math instruction.

This study provides strong evidence that increased usage of dynamic software can greatly benefit student learning of the unit circle. It would be logical to assume that dynamic software could provide benefits to other areas as well. One specific suggestion would be in examining how the changing slope and y-intercept in a linear equation affects the graph. This type of lesson lends itself to computers. Other examples would be in showing how increasing dx in integration provides greater accuracy for integral estimates using simpson's rule or the trapezoid rule. I have also seen very illuminating dynamic demonstrations of calculus topics such a Riemann Sums. Using a slider for dx, it is readily apparent that the sum approaches the value as dx approaches zero. The number of possible uses are endless. The greater the body of evidence the more widespread the usage should become which would provide benefits to all subsequent math lessons.

APPENDIX A

LESSON PLAN

Unit Circle Lesson Plan

By: Douglas A. Ruby **Date: 10/10/2002**
Class: Pre-Calculus II **Grades: 11/12**

INSTRUCTIONAL OBJECTIVES:

At the end of this lesson, the student will be able to:

1. Given a real number that is an integral multiple of halves, thirds, fourths, or sixths of π, find the point on the unit circle determined by it.
2. Given a point on the unit circle that is an integral multiple of fourths, sixths, eighths, or twelfths of the distance around the circle, find the real numbers between -2π and 2π that determine that point.
3. Given any arbitrary point on the unit circle with (x, y) coordinates satisfying the equation for the circle $x^2 + y^2 = 1$, identify the 6 trigonometric functions for the angle in standard position described by a ray drawn from the origin to that point.
4. Given any arbitrary point on the unit circle with (x, y) coordinates satisfying the equation for the circle $x^2 + y^2 = 1$, find the angle in standard position described by a ray drawn from the origin to that point.

Relevant Massachusetts Curriculum Framework

PC.M.1 – Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, particularly problems involving angular velocity and acceleration.
PC.P.3 - Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

MENTAL MATH – (5 Minutes)

Question 1: What are the values of the following Trigonometric Functions:

a) cos $-\pi/4$ - *Solution:* $\sqrt{2}/2$

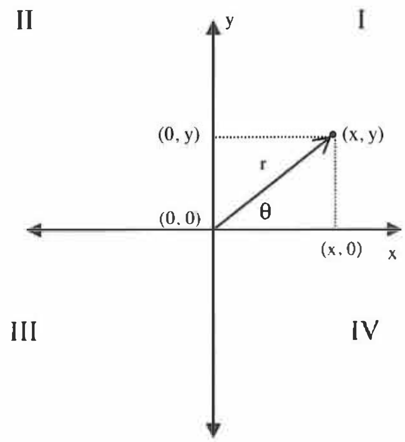
b) cos $9\pi/4$ - *Solution:* $\sqrt{2}/2$

c) tan 12π - *Solution:* 0

Question 2: What does the mnemonic **All S**tudents **T**ake **C**alculus stand for?

All Students Take Calculus means that for the four quadrants (in oprder) All functions, Sine, Tangent, and Cosine and their reciprocals are positive.

CLASS ACTIVITIES – (Note: 45 Minute Lesson Plan)



1 Review

If you will remember to a few days ago when we discussed *Angles in Standard Position*, we used the diagram to the left.

We defined an angle θ in standard position and the six trigonometric functions related to θ. The trig functions are related to the triangle formed by using the x, and y coordinates of the ray *r* drawn from the origin to the point *(x, y)*.

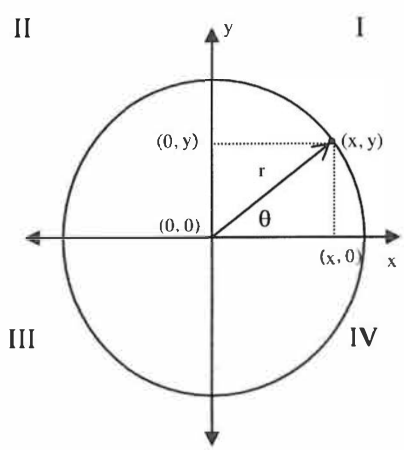
Therefore, the six trig functions are:

$$\sin\theta = \frac{side\_opposite\_\theta}{hypotenuse} = \frac{y}{r} \qquad \csc\theta = \frac{hypotenuse}{side\_opposite\_\theta} = \frac{r}{y}$$

$$\cos\theta = \frac{side\_adjacent\_\theta}{hypotenuse} = \frac{x}{r} \qquad \sec\theta = \frac{hypotenuse}{side\_adjacent\_\theta} = \frac{r}{x}$$

$$\tan\theta = \frac{side\_opposite\_\theta}{side\_adjacent\_\theta} = \frac{y}{x} \qquad \cot\theta = \frac{side\_adjacent\_\theta}{side\_opposite\_\theta} = \frac{x}{y}$$

Notice, that we can modify this diagram, by drawing a circle, whose center is the origin that intersects the ray *r* at point *(x, y)*. This would look like:



Notice also, that the equation of this circle is:

$$x^2 + y^2 = r^2$$

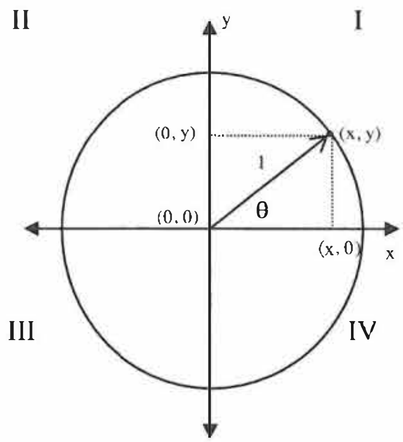
so that the length of the ray *r* is also the radius *r* of the circle. So for any point (x, y) anywhere on this circle, the same six trigonometric identities discussed above, still holds with respect to *x, y,* and *r*.

2. Introduction to the Unit Circle

Now suppose that instead of an arbitrary circle with a radius of r, we had what is called the *unit circle*. The unit circle has a radius r of 1 and is defined by the equation:

$$x^2 + y^2 = 1^2 = 1$$

So in our above looks like the diagram below.



For the unit circle, the six trigonometric functions are:

$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

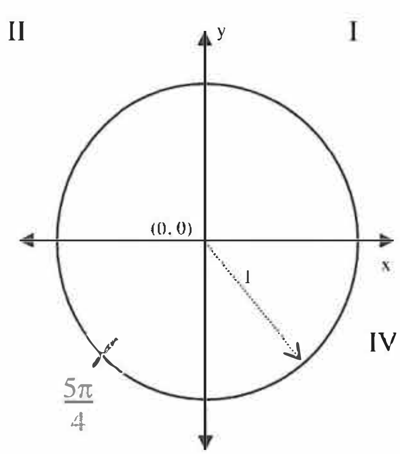
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Notice, that the cos θ and sin θ are just the x and y coordinates of *any* point on the circle! Further, since tan θ = y/x, it becomes clear that:

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\vartheta} \text{ and } \cot\theta = \frac{x}{y} = \frac{\cos\vartheta}{\sin\vartheta}$$

Thus, with this simple diagram, we have completed establishing the fundamental relationship between a triangle, a circle, the six trigonometric functions, and any (x, y) point in any four quadrants of the Cartesian plane that satisfies the equation $x^2 + y^2 = 1$ for the unit circle.

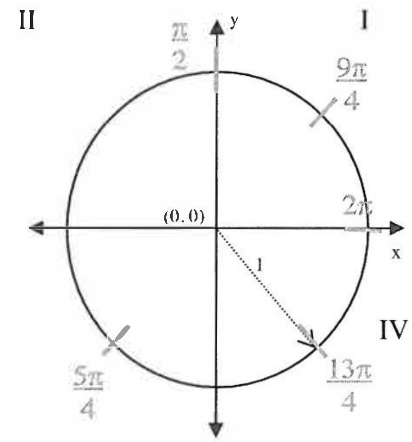
3. Finding Points on the Unit Circle.



Let's apply this knowledge. Example: Let's use the unit circle and our knowledge from our prior lesson on *Radians and Degrees* to draw out the points on the unit circle that match certain angles. For example: If we want to see where the angle θ = $\frac{5\pi}{4}$ is on the unit circle, we first draw the circle, then we mark the point on the circle corresponding to θ without actually drawing the ray marking the angle in standard position. This would look like: the drawing to the left: Notice, that since the radius $r = 1$, the point we marked for θ = $\frac{5\pi}{4}$ is also 5π/4 of the way around the circle, relative to the point (1,0) on the initial side of the angle in standard position. (*Note: Show this on the board!*)

Now you try the same with a Unit circle. Draw a unit circle. Starting at the point (1,0), mark points determined by the following real numbers:

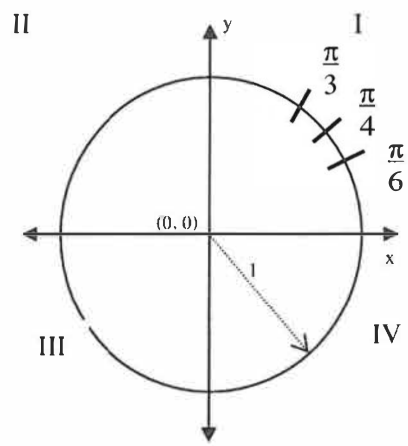
a) $\frac{\pi}{2}$ b) $\frac{5\pi}{4}$ c) 2π d) $\frac{9\pi}{4}$ e) $\frac{13\pi}{4}$



That is terrific. You will have three problems like this in your homework.

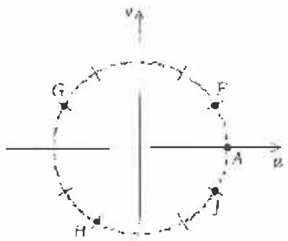
4. Estimating Angles on the Unit Circle

Now lets move on to estimating the real numbers between -2π and 2π that represent points on a circle relative to the point (1,0).



Let's assume that the first marks below represent the points that are π/6, π/4, and π/3, of the way around the circle from the initial point (1,0). Then, as you can see, we can represent any of the points on the circumference of the unit circle as real numbers that have the same value as the radian measure that we discussed in our *Radians and Degrees* lessons.

Let's look at the diagram below and estimate the real numbers between -2π and 2π for the indicated points:



Try Now: How far will a point move on the unit circle, counterclockwise, in going from point A to:

a) point F? $\pi/6$
b) point H? $5\pi/6$
c) point G? $4\pi/3$
d) point J? $11\pi/6$

Great! Now, lets move on.

5. Using the (x, y) coordinates on the unit circle to find the six trig functions.

We already found that the definitions to for the six standard trigonometric functions are highly related to the (x, y) coordinates of any point on the unit circle as shown below:

$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\vartheta} \qquad \cot\theta = \frac{x}{y} = \frac{\cos\vartheta}{\sin\vartheta}$$

Further, we already know that the values of the sine and cosine for the angles in standard positions of 30°, 45°, and 60° (π/6, π/4, and π/3) for all four quadrants. Finally, let's think about what happens when we go π/2 of the way (90°) around the unit circle. What are the (x, y) values for a point on the unit circle at θ = π/2? *(0, 1)* So what are the values of the sine, cosine, and tangent? *(sin θ = 1, cos θ = 0, tan θ = undefined)* And what are the values of the reciprocal functions cosecant, secant and cotangent? *(csc θ = 1, sec θ = undefined , cot θ = 0)* Great!

Now, suppose I go 5π/6 (150°) of the way around the unit circle. What is the (x, y) coordinates for that point? *(-sqrt(3)/2, ½)* So what are the values of the sine, cosine, and tangent? *(sin θ = ½, cos θ = -sqrt3/2, tan θ = -1/sqrt(3))* And what are the values of the reciprocal functions cosecant, secant and cotangent? *(csc θ = 2, sec θ = -2/sqrt(3) , cot θ = -sqrt(3))* Great!

So as you can see, we can figure out the (x, y) values for the angles in the standard positions of fourths (π/2), sixths (π/3), eighths (π/4), or twelfths (π/6) of the way around the unit circle.

6. Arbitrary Points on the Unit Circle

What if the point is not one of the standard points that is a multiple of fourths ($\pi/2$), sixths ($\pi/3$), eighths ($\pi/4$), or twelfths ($\pi/6$) of the way around the unit circle? Suppose I said that the (x, y) coordinate of the point on the unit circle is (.65, .7599342). Use your calculators to validate that it satisfies the equation for the unit circle. Then give me the six trigonometric functions for that point.

$x^2 + y^2 = (.65)^2 + (.7599342)^2 = .9999999883$ - *close enough!*

$\sin \theta = .7599342$	$\csc \theta = 1.31590$
$\cos \theta = .65$	$\sec \theta = 1.53846$
$\tan \theta = 1.16913$	$\cot \theta = .855337$

So we can find the six trigonometric functions. Let's quickly review **All** **S**tudents **T**ake **C**alculus.

Remember, that in Quadrant I, x and y are positive. In quadrant II, x is negative and y is positive. In quadrant III, x and y are both negative. In quadrant 4, x is positive and y is negative. Therefore, we can see that **All** **S**tudents **T**ake **C**alculus means that for the four quadrants (in oprder) **All** functions, **S**ine, **T**angent, and **C**osine and their reciprocals are positive. The other functions (in quadrants II, III, and IV) will be negative. Be sure to check your "sign" of your sine carefully! (pun intended).

7. Using the (x, y) coordinates on the unit circle to find the angle

The problem with knowing any arbitrary (x, y) coordinate on the unit circle and the six trig functions including their "sign" (i.e. whether they are positive or negative) is that we still don't know the angle, do we? To solve the problem of what is the angle described by an arbitrary point on the unit circle, we need to use something called *inverse trigonometric functions*.

Later on, we will cover the inverse trigonometric functions in more detail. But for the purposes of tonight's homework and for quizzes and tests, lets just use the functions on our calculators. There are six inverse trigonometric functions. For example, the inverse sine function is called the *in x* function. It is also written as the $\sin^{-1} x$ function. The argument x of the $\sin^{-1} x$ function is the value of $\sin \theta$. This means that *arcsin x* is an angleθ whose *sin* is x. Therefore, we can write the following equation:

$$\theta = \sin^{-1} x$$

Thus, in the example above, we know that the $\sin \theta = .7599342$. By using the $\sin^{-1}$ function on our calculators, we can find that:

$$\theta = \sin^{-1} .7599342 = .8632 \text{ radians} = 49.458^\circ$$

You should see the $\sin^{-1}$ function on your calculators. For the following examples, find the angle of rotation in radians and degrees for each of the (x, y) points. In some cases, you will not need your calculators!

a. (0 ,1)	b. (.25, .96825)	c. $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	d. (.141067, .99)
0 rad, 0^o	*1.318 rad. 75.5234^o*	*$\pi/6$ rad. 30^o*	*1.4293 rad. 81.89^o*

8. Homework discussion (5 Minutes)

HOMEWORK (Materials):

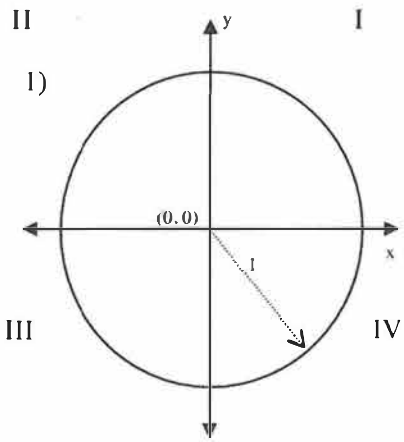
Source:

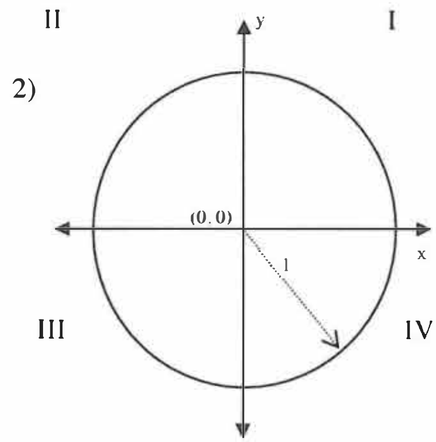
- Bittinger and Beecher, *Algebra and Trigonometry*, Section 6.4, pp. 391-404

For each of the exercises 1-3, sketch a unit circle, and mark the points determined by the given real numbers:

1. a) $\frac{\pi}{4}$ b) $\frac{3\pi}{2}$ c) $\frac{3\pi}{4}$ d) π e) $\frac{11\pi}{4}$

2. a) $\frac{\pi}{3}$ b) $\frac{5\pi}{6}$ c) $\frac{11\pi}{6}$ d) $\frac{13\pi}{6}$ e) $\frac{23\pi}{6}$





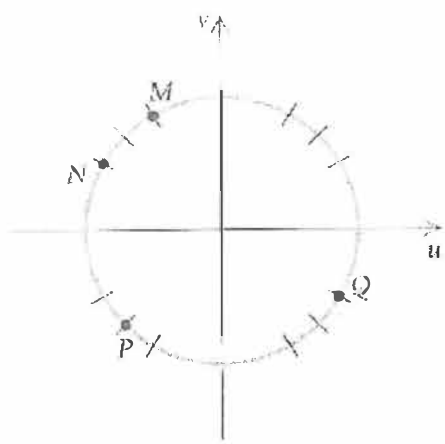
3. Find the real numbers M, N, P, and Q between -2π and 2π that determine each of the points on the unit circle.

$M=2\pi/3$

$N=5\pi/6$

$P=5\pi/4$

$P=11\pi/6$



Unit Circle – Mr. Ruby

What are the (x, y) coordinates on the unit circle for the following angles?

4. $\pi/6 = (\sqrt{3}/2, ½)$
5. $-\pi/6 = (\sqrt{3}/2, -½)$
6. $5\pi/4 = (-1/\sqrt{2}, -1/\sqrt{2})$
7. $-5\pi/4 = (-1/\sqrt{2}, 1/\sqrt{2})$

What are the values of the six trigonometric functions on the unit circle for the following angles?

8. $\pi/6$

$\sin\theta = ½$	$\csc\theta = 2$
$\cos\theta = \sqrt{3}/2$	$\sec\theta = 2/\sqrt{3}$
$\tan\theta = 1/\sqrt{3}$	$\cot\theta = \sqrt{3}$

9. $-\pi/6$

$\sin\theta = -½$	$\csc\theta = -2$
$\cos\theta = \sqrt{3}/2$	$\sec\theta = 2/\sqrt{3}$
$\tan\theta = -1/\sqrt{3}$	$\cot\theta = -\sqrt{3}$

10. $5\pi/4$

$\sin\theta = -1/\sqrt{2}$	$\csc\theta = -\sqrt{2}$
$\cos\theta = -1/\sqrt{2}$	$\sec\theta = -\sqrt{2}$
$\tan\theta = 1$	$\cot\theta = 1$

11. $-5\pi/4$

$\sin\theta = 1/\sqrt{2}$	$\csc\theta = \sqrt{2}$
$\cos\theta = -1/\sqrt{2}$	$\sec\theta = -\sqrt{2}$
$\tan\theta = -1$	$\cot\theta = -1$

12. ,423 radians

$\sin\theta = .4105$	$\csc\theta = 2.436$
$\cos\theta = .9119$	$\sec\theta = 1.097$
$\tan\theta = .4502$	$\cot\theta = 2.221$

For the following point, provide the six trigonometric values and the angle (in radians and degrees) outlined on the unit circle:

13. (.823, .56804)

$\sin\theta$ = *.56804*	$\csc\theta$ = *1.7604*
$\cos\theta$ = *.823*	$\sec\theta$ = *1.215*
$\tan\theta$ = *.6902*	$\cot\theta$ = *1.4489*

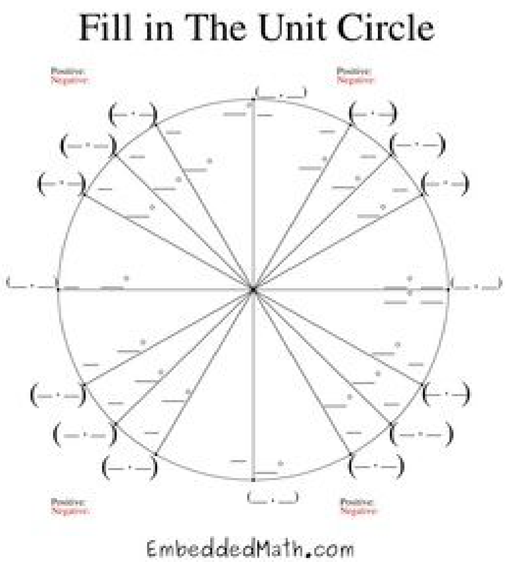
θ = *.6041223 radians = 34.614°*

APPENDIX B

PRE AND POST TEST

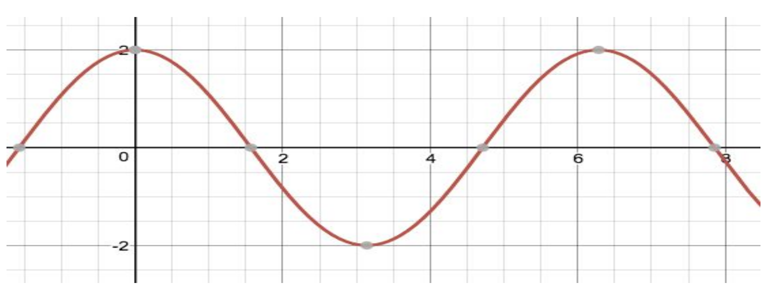
Assessment

1.



2. Explain how you derived the *x* and *y* coordinates for each location of the unit circle.

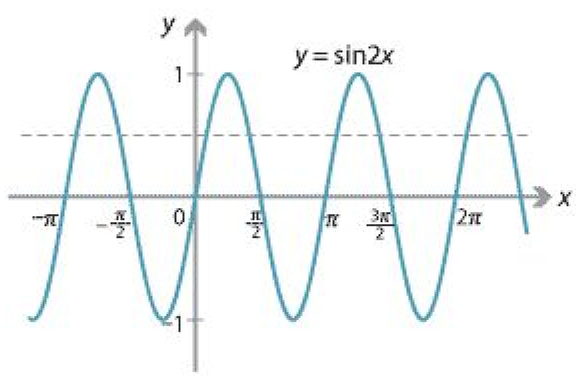
3. Write the function that represents the graph shown below.



4. How is the graph of the equation y = sin(x - π) different from the graph of y = sinx? What is the cause of this change?

5. Below is a graph of the function y = sin(2x). Explain (mathematically) how the graph of this function is similar and different from the graph of y = sinx. The word bank below is provided as a collection of terms that could be used in your explanation.

(Word Bank: Period, Amplitude, Shift, Midline, Key Values, Maximum, Minimum)



6. Define the intervals in which the sine function is increasing and decreasing in the interval (0 to 2π).

7. Explain why the equation $sin^2x + cos^2x = 1$ is true for all values of x between 0° and 360°

8. Say which is greater, sin 83 degrees or sin 175 degrees and how do you know.

9. Explain why the value of cosine can never be 2.

APPENDIX C

ASSESSMENT SCORING RUBRIC

ASSESSMENT SCORING RUBRIC

Question 1: Less than half the unit circle filled in correctly - 0 points

Between half correct and totally correct - 1 point

Completely filled in (correctly) - 2 points

Question 2: No explanation or a nonsensical answer - 0 points

Any type of mention of special right triangles or “from memorization” but not a complete demonstration of their understanding or an explanation of their reasoning. If a student answer “from memorization, they needed to have scored at least on point on question 1 - 1 point

An explanation that was clearly understood and demonstrated a level of deeper knowledge of the origin of the values. - 2 points

Question 3: No answer or any non Cosine answer with no shift. - 0 points

Any Cosine (or shifted Sine) answer with incorrect amplitude. - 1 Point

A Cosine (or shifted Sine) answer with a correct amplitude - 2 points

Question 4: An incorrect explanation of the difference and incorrect cause - 0 points

A correct explanation of the difference but incorrect cause or incorrect explanation with correct cause - 1 point

Correct explanation and cause - 2 points

Question 5: No correctly explained differences or similarities and key words used incorrectly - 0 points

Either one correct similarity or difference with at least one correct use of a wordbank word - 1 point

A correct similarity and a correct difference where both are explained by words in the wordbank - 2 points

Question 6: No correct explanation of any intervals - 0 points

Partially correct explanation with at least one interval correct - 1 points

All intervals correct with an explanation that is relevant - 2 points

Question 7: No correct explanation - 0 points

A partially correct explanation that demonstrates either some understanding of the max and min values of sine and cosine or with the Pythagorean theorem. - 1 point

An explanation that is clearly understandable and demonstrates knowledge of both the Pythagorean theorem and the values of sine and cosine- 2 points

Question 8: Incorrect with no or incorrect explanation - 0 points

Correct with no or incorrect explanation - 1 points

Correct with a correct explanation - 2 points

Question 9: No explanation or one that is nonsensical. - 0 points

A partially correct but incomplete explanation with no mention of the Pythagorean identity in question 7. - 1 point

A complete explanation that clearly shows depth of knowledge of sine and cosine and how they would evaluation in the Pythagorean identity or understanding of the graphed values never exceeding 1 or demonstration of the unit circle radius not exceeding 1. - 2 points

APPENDIX D

RAW DATA, STUDENT TEST SCORES

The left column includes every pretest score. The middle and right columns are the post test scores and they are separated by whichever group the students were in (control and test groups)

RAW DATA, STUDENT TEST SCORES

pre test score (all students)	post test (no computer)	post test (computer group)
3		3
0	2	
2		5
1	3	
3		5
8	6	
3	3	
2	2	
0		2
6		9
0		2
4	5	
5	7	
4	4	
0		2
3		7
4		5
2	2	
2	2	
6		7
1		4
1	1	
2	2	
0		0
pre test score (all students)	post test (no computer)	post test (computer group)
4	4	
5		6
5	6	
5		5
2		3

4	4	
0	0	
6		7
2	2	
4		6
1	1	
0		0
4	5	
4	5	
5		9
4		6
4		4
0	0	
6		10
6	8	
1	1	
4		9
5	2	
4	5	
4		4
1	2	
2		2
1		4
8	8	
4		8
5		6
0		2
pre test score (all students)	post test (no computer)	post test (computer group)
5		6
2	3	
5	3	
4	4	

4	4	
2		4
4		6
0	0	
0		0
2		5
1	0	
2		2
4	5	
3		6
0	1	
0		4
0	0	
4		6
7		8
4	4	
1	0	
3		4
4	4	
4	6	
0	0	
5		4
4	2	
2	2	
4		6
4		4
0	2	
1		4
pre test score (all students)	post test (no computer)	post test (computer group)
4	3	
0		0
4	6	

2	3	
4		4
1	2	
4		4
3		4
1	1	
2	2	
0		0
4	4	
4		6
5	6	
5		5
5		3
4	4	
0	0	
6		7
2		5
4		10
2	1	
0	0	
4	5	
4		5
5		9
4		6
4		4
0		0
6		7
2	3	
1	1	
pre test score (all students)	post test (no computer)	post test (computer group)
4		9
5	2	

4	5	
6		9
4		4
	2.12466648	2.628405223
	standard dev	standard dev

APPENDIX E

IRB APPROVAL LETTER

IRB APPROVAL LETTER

July 30, 2018

CSUSB INSTITUTIONAL REVIEW BOARD

Protocol Renewal

FY2017-134

Status: Approved

Mr. Ed Simons and Prof. Madeleine Jetter

Department of Mathematics

California State University, San Bernardino

5500 University Parkway

San Bernardino, California 92407

Dear Mr. Simons and Prof. Jetter:

Your protocol renewal to use human subjects, titled "The benefits of computer models on student understanding of the unit circle" has been reviewed and approved by the Chair of the Institutional Review Board (IRB). A change in your informed consent requires resubmission of your protocol's informed consent as amended for use in your study.

Your renewal is approved from September 09, 2018 to September 08,

2019. Please note the Cayuse IRB system will notify you when your protocol comes up for renewal at 90, 60, and 30 days before the protocol expires. If you are no longer conducting the study you can submit a study closure through the Cayuse IRB system.

You are required to notify the IRB if any substantive changes are made in your research prospectus/protocol, if any unanticipated adverse events are experienced by subjects during your continued research, and when your project has ended. If your project lasts longer than one year, you (the investigator/researcher) are required to notify the IRB by email or correspondence of *Notice of Project Ending* or *Request for Continuation* at the end of each year. Failure to notify the IRB of the above may result in disciplinary action. You are required to keep copies of the informed consent forms and data for at least three years.

If you have any questions regarding the IRB decision, please contact Michael Gillespie, Research Compliance Officer. Mr. Gillespie can be reached by phone at (909) 537-7588, by fax at (909) 537-7028, or by email at mgillesp@csusb.edu. Please include your application identification number (above) in all correspondence.

Best of luck with your research.

Sincerely,

Donna Garcia

Donna Garcia, Ph.D., IRB Chair

CSUSB Institutional Review Board

DG/MG

Dynamic lessons used

Nick Kochis, Unit Circle-Exact Values, (n.d.)

https://www.geogebra.org/m/G7xgNRxm

Nick Kochis, Unwrapping the Unit Circle, (n.d.)

https://www.geogebra.org/m/uW34CJcm

BIBLIOGRAPHY

Alsina, C., & Nelsen, R. (2006). Moving Frames. In *Math Made Visual: Creating images for understanding mathematics* (pp. 69-72). Mathematical Association of America. Retrieved from http://www.jstor.org.libproxy.lib.csusb.edu/stable/10.4169/j.ctt5hh9ks.17

Baharvand, Mohsen, (Dissertation, 2001). *A comparison of the effectiveness of computer-assisted instruction versus traditional approach to teaching geometry*

Burke, J. & Olley C. (2008) MNO – *Seeing the wood for the trees*. Presentation at BSRLM, King's College London November 2008

Burr, M. J. (2010). *Increasing math success with differentiated instruction*. Walden University.

Byers, P. E. (2010). *Transition to college mathematics: An investigation of trigonometric representations as a source of student difficulties* (Order No. NR64873). Available from ProQuest Dissertations & Theses A&I; ProQuest Dissertations & Theses Global: The Humanities and Social Sciences Collection. (748252848). Retrieved from http://libproxy.lib.csusb.edu/login?url=https://search-proquest-com.libproxy.lib.csusb.edu/docview/748252848?accountid=10359

Chan, Kan Kan. (2014). Dynamic geometry software improves mathematical achievement: Systematic review and meta-analysis. *Journal of educational computing research, 51*(3), 311-326.

Alexander, B., Adams Becker, S., Cummins, M., and Hall Giesinger, C. (2017). *Digital Literacy in Higher Education, Part II*: An NMC Horizon Project Strategic Brief. Volume 3.4, August 2017. Austin, Texas: The New Media Consortium.

Makela, V. (1998). *Effectiveness of teachers' professional development as assessed by teachers, teaching supervisors and students, proquest dissertations and theses.*

Moss, L. J. (2000). *The use of dynamic geometry software as a cognitive tool* (pp. 1-184). The University of Texas at Austin.

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of mathematical representations. *Technology-supported mathematics learning environments*, *67*, 17-34.

Ruby, D.A. 6 Unit Circle. (2002). Retrieved 2018, from https://www.scribd.com/document/125844970/6-Unit-Circle

Steer, J., de Vila, M. A., & Eaton, J. (2009). Trigonometry with Year 8: Part 1. *Mathematics teaching*, *214*, 42-44.

Szabo, A. (2011, May 16). GeoGebra History. Retrieved from https://prezi.com/dxtarvwzqwz_/geogebra-history/

Weber, Keith, (2005, Vol. 17). Mathematical Education Research Journal, . *Students' understanding of trigonometric functions*.

The Sketchpad Story. (n.d.). Retrieved from http://www.dynamicgeometry.com/General_Resources/The_Sketchpad_Story.html