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Demonstrating open superstring amplitude relationship with Yang-Mills theory based on dual model limit

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Abstract

This paper demonstrates the relationship between superstrings, Yang-Mills fields, and dual models. We discovered a novel and profound connection between these theories and their respective prescriptions for the calculation of scattering amplitudes, which directly demonstrates the duality between Field Theory and String Theory in a novel manner.

1. Introduction

Dualities are important limits that connect String Theory and Field Theory. These relationships can be exploited in various scenarios and in various ways; therefore, they may be meaningful or have a foundation in physics. In this regard, this paper discusses various important aspects that clarify the meaning of such relationships. The search for the closed expressions of the N-boson scattering amplitude at tree level has been an extremely active area of research over the last few years. Technical procedures have been developed to efficiently compute the actual effective Lagrangian terms for the bosonic and fermionic terms in superstring theory. These advances are based on the existence of diverse relationships amongst superstring and Yang-Mills scattering amplitudes, such as Kleiss-Kuijf and BCJ relationships [1, 2]. The importance of these relationships lies in the fact that it is possible to write the N-point scattering amplitude at tree level in terms of (N-3)! sub-amplitudes [3]. Important progress has been made with the S-matrix method toward determining the terms of an effective Lagrangian for open superstrings at low energies. This method allows the computation of the $\alpha'-$expansion terms (Regge trajectory) of the effective Lagrangian from the N-point scattering amplitude of superstring theory [4]. Significant advances in the calculation of N-point Yang-Mills scattering amplitudes at tree level has helped the efficient implementation of a practical and compact algorithm for investigating the scattering amplitudes. Such advances are based on symmetry being revealed as cyclic and reflection properties, and on considering that the various kinetic structures present in the amplitude have vector space properties. The relationship of these considerations with the calculation of N-point scattering amplitudes at tree level in superstring theory can, in principle, be constructed and extended to the case of fermions through a supersymmetric transformation. The influence of these symmetries on calculations involving loops is also important and may lead to an efficient procedure for carrying out these calculations. These advances in perturbative string theory are related to the connection between the Born-Infeld Lagrangian and the Yang-Mills and S-Matrix method for calculating the terms corresponding to the α'^N order. These advances are also related to methods developed in the context of pure spinor formalism for the calculation of scattering amplitudes [5]. The S-matrix method allows the determination of the bosonic part of the effective Lagrangian of an open superstring at low energies. However, this method directly influences the determination of the theoretical scattering amplitudes, and allows us to obtain the scattering amplitude of the open superstring N-points at tree level in a closed form, which is developed using the pure spinor method and its variations through expansion to the corresponding terms of the coupling constant. This method is constructed in terms of kinematic and super-symmetric relationships. Another important aspect of the method is the initial treatment of the external gauge boson interactions and the subsequent incorporation of their interactions into the corresponding fermionic particles. Over the last 15 years, Various studies and reviews have particularly focused on the fact that the non-locality of string theory (existence of a set of massive states) implies that effective action at low energies in massless modes constitutes an infinite series in all orders of α'^N. First, this would apply to the effective tree-level Langrangian for gauge vectors in open-loop type I theory. Particularly, the effective Lagrangian in open string theory can be represented as a series expansion of the $F^{\mu\nu}$ and its covariant derivatives [6–12]. The S-matrix method has been used as an efficient approach toward calculating the scattering amplitudes to $3 \leqslant N \leqslant 7$ points for an open superstring at tree level. These results are consistent with the investigation results obtained for the scattering amplitudes of an open superstring in the formalism of pure spinors. Accordingly, this study investigated the calculation of the scattering amplitudes of an open superstring at tree level and at loop level, in relation to the low-energy effective Lagrangian of an open superstring. This investigation is also related to the possible relationships between symmetry and group theory, which can lead to to the computation of the higher orders of the coupling constant and their corresponding diagrams for the scattering amplitudes of superstrings.

An interesting relationship exists between the open superstring low energy effective lagrangian (OSLEEL) [13, 14] and Yang-Mills fields. This relationship manifests itself in the context of superstring amplitudes, because an important parameter of the theory, namely, the α' expansion of tree level open and closed superstrings, may be related to the coupling constant g and the mass m of external scalar particles. Essentially, if we consider the analysis of these parameters, that is, the case of the limit wherein α' and g go to zero, and m and $\lambda = g/\sqrt{\alpha'}$ are fixed, the relationship between the OSLEEL and Yang-Mills theory can be investigated.

The rest of this paper is structured as follows: section 2 discusses the relationships between the S-matrix in the Veneziano model and Field Theory based on $\lambda\phi^3$ theory. In section 3 we investigate the origin of these relationships. In section 4, the same viewpoint is extended to the superstring scattering amplitudes and Yang-Mills fields at tree level.

2. Dual models, S-matrix and field theory

The Feynman graphs (FG) of ϕ^3 theory can be generalized as dual diagrams based on the dual resonance model (DRM) [15–18]. The low-energy limit on the amplitude and N-point tree level amplitude can be related to the Born term of ϕ^3 theory. For the Veneziano amplitude, this can be done in an explicit manner. Moreover, this equivalence is constructed at tree level and can be extended to several loop orders. We can start this analysis from the well-known S-matrix approach, as follows [19]:

$$(p_1 \dots p_j|S|p_{j+1} \dots p_N) = i(2\pi)^4\delta^4(\Delta p)\prod_{i=1}^{N}\frac{T_N(p_1 \dots p_N)}{\sqrt{N!(2\pi)^3 2p_i^0}}, \tag{1}$$

where $T_N(p_1 \dots p_N)$ is the full N-point Veneziano amplitude and

$$\Delta p = \left(\sum_{i=1}^{j} p_i - \sum_{k=j+1}^{N} p_k\right).$$

Let us consider the full four-point Veneziano amplitude T_4, which can be calculated as follows:

$$T_4 = F_4(s, t) + F_4(t, u) + F_4(u, s), \tag{2}$$

with

$$F_4(s, t) = 1/2g^2 B(-\alpha'(s - m^2); -\alpha'(t - m^2)), \tag{3}$$

where B is the Beta function, α' is the slope, g is the dimensionless coupling constant, and m is the mass of the external scalar particles. By considering the following relationships:

$$\alpha_s = \alpha'(s - m^2), \tag{4}$$

and

$$\alpha_t = \alpha'(t - m^2), \tag{5}$$

the Beta function can be expanded as follows:

$$B(-\alpha_s, -\alpha_t) = \frac{\Gamma(-\alpha_s)\Gamma(-\alpha_t)}{\Gamma(-\alpha_s - \alpha_t)} \approx -\frac{\alpha_s + \alpha_t}{\alpha_s\alpha_t} \approx \frac{1}{\alpha'}\left(\frac{1}{m^2 - s} + \frac{1}{m^2 - t}\right). \tag{6}$$

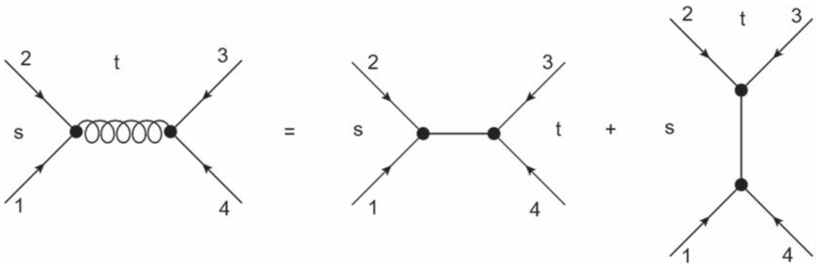

Figure 1. Diagram of four-point function in channels s and t.

From (3), by considering the expansion of the Beta function (6), $F_4(s, t)$ can be calculated as follows:

$$F_4(s,\ t) \underset{\alpha'\to 0}{=} 1/2\lambda^2\left(\frac{1}{m^2 - s} + \frac{1}{m^2 - t}\right) \tag{7}$$

and the full amplitude T_4 can be obtained from (2), as follows:

$$T_4 \underset{\alpha'\to 0}{=} \lambda^2\left(\frac{1}{m^2 - s} + \frac{1}{m^2 - t} + \frac{1}{m^2 - u}\right). \tag{8}$$

Interestingly, in this limit, the Veneziano amplitude recovers the field theory as follows:

$$\mathcal{L} = 1/2[\partial_\mu\partial^\mu - m^2\phi^2] + 1/6\lambda\phi^3. \tag{9}$$

In terms of dual diagrams or Feynman-like diagrams (FLD), this result demonstrates the reduction of the four-point function shown in figure 1.

The result (7) can also be obtained by explicitly calculating the Beta function as follows:

$$F_4(s,\ t) = 1/2\lambda^2\alpha' \int_0^1 dxx^{-\alpha'(s-m^2)-1}(1 - x)^{-\alpha'(t-m^2)-1}, \tag{10}$$

where the integral is calculated only when it diverges from the limit $\alpha' \to 0$ at points $x = 0$ and $x = 1$; the calculation result is obtained as follows:

$$\begin{aligned} F_4(s,\ t) &\approx 1/2\lambda^2\alpha'\left[\int_0^{\epsilon} dxx^{-\alpha'(s-m^2)-1} + \int_{1-\epsilon}^{1} (1 - x)^{-\alpha'(t-m^2)-1}\right] \\ &\approx 1/2\lambda^2\left(\frac{1}{m^2 - s} + \frac{1}{m^2 - t}\right). \end{aligned} \tag{11}$$

This approach can be extended to the N-point case as follows:

$$T_N = \sum_{\mathcal{P}} F_N(\mathcal{P}_{p_1},\ldots,\mathcal{P}_{p_N}), \tag{12}$$

where $\mathcal{P}$ represents any non-cyclic and non-anticyclic permutation of the external momenta $p_1, \ldots, p_N$, and

$$F_N(p_1 \ldots p_N) = \frac{g^{N-2}}{2^{N-3}}(\alpha')^{1/2(N-4)} B_N(p_1 \ldots p_N), \tag{13}$$

where B_N is the generalized Beta function written in terms of the $(N - 3)$ dimensional integral. In the context of Field Theory, it can be considered that the following relationship holds:

$$T_N = \sum_i F_i \tag{14}$$

However, F_i are not the equivalent Feynman graphs of the N-point amplitude, which can be obtained from common Feynman rules for propagators, as follows:

$$\frac{-i(2\pi)^4}{m^2 - p^2 - i\epsilon} \tag{15}$$

and for each vertex

$$i\lambda(2\pi)^4\left(\sum_i p_i\right). \tag{16}$$

This can be easily seen in (7), which is symmetric in channels s and t.

Thus far, we have considered an interesting equivalence between the dual resonance model and the ϕ^3 Feynman diagrams in the zero-slope limit $\alpha' \to 0$. Now, we will investigate this equivalence with regard to an open superstring low energy effective Lagrangian scenario.

3. Beta function

The amplitude of Veneziano (8) is a Beta function with contributions to channels s, t, and u. This amplitude reproduces the $\lambda\phi^3$ theory, as discussed in section 2. We consider that the agreement of the amplitudes obtained according to two different conceptual foundations within the context of Field Theory is not unjustified. Moreover, we argue that this relationship is based on the physical meaning and functional structure of the propagator, as follows:

$$\Delta(y-x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(y-x)}}{k^2 + m^2 - i\epsilon} \tag{17}$$

where k denotes the particle's four-momentum.

The amplitudes constructed based on Lagrangian Field Theory seem to be related to dual models for the same scattering processes, possibly because the Beta function exhibits duality. The amplitudes obtained from the Lagrangians naturally contain all terms appearing when the Beta function is expanded to powers of α', considering that the limits of $\alpha' \to 0$ and g^2/α' are kept constant. Dual models [20] can describe the scattering of particles with variable mass and spin that are exchanged in the t and s channels; the amplitude can be expressed as follows:

$$A(s, t) = -\sum_J \frac{g_J^2(-s)^J}{t - m^2} \tag{18}$$

where g_J is the coupling constant and J is the particle spin. The duality reflects $s \to t$ and $t \to s$ in (18).

4. Superstrings and Yang-Mills fields

The relationship between the Yang-Mills fields and dual models in the zero slope limit reveals an interesting connection to the string scattering amplitude, as discussed in section 2. Let us consider an open string massless boson amplitude at tree level, as follows:

$$\mathcal{A}^{(M)} = i(2\pi)^{10}\delta(k_1 + \ldots + k_M) \sum_{j_1,\cdots,j_M} tr(\lambda^{a_{j_1}} \ldots \lambda^{a_{j_M}}) A(j_1 \ldots j_M), \tag{19}$$

where M is the number of bosons and the sum Σ in the indices j_i is obtained over the non-cyclic permutations of group $1, \ldots, M$; λ^{a_j} are matrices in the adjoint representation of the Lie group; $A(j_i)$ is the sub-amplitude corresponding to the M-point amplitude of the open superstring. In the Ramond-Neveu-Schwarz (RNS) formalism, the sub-amplitude can be expressed as follows:

$$\begin{aligned} A(1, 2, \ldots, M) = {} & 2\frac{g^{M-2}}{(2\alpha')^{7M/4+2}}(x_{M-1} - x_1)(x_M - x_1) \\ & \times \int dx_2 \ldots dx_{M-2} \int d\theta_1 \ldots d\theta_{M-2} \prod_{i>j}^{M} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i . k_j} \\ & \times \int d\phi_1 \ldots d\phi_M e^{f_M(\eta, k, \theta, \phi)}, \end{aligned} \tag{20}$$

where

$$f_M(\eta, k, \theta, \phi) = \sum_{ij}^{M} \frac{(\theta_i - \theta_j)\phi_i(\eta_i . k_j)(2\alpha')^{11/4} - 1/2\phi_i\phi_j(\eta_i\eta_j)(2\alpha')^{9/2}}{x_i - x_j - \theta_i\theta_j}. \tag{21}$$

where θ_i and ϕ_i are Grassmann variables, the x_i variables are real, and $x_1 \ldots < x_M$; k_i and η_i are the i-th string momentum and polarization, respectively. The known symmetries in (20), namely, cyclicity

$$A(1, 2, \ldots, M-1, M) = A(M, 1, \ldots, M-2, M-1) \tag{22}$$

on-shell gauge invariance

$$A(1, 2, \ldots, M)|_{\eta_i = k_i} = 0,\ i = 1, \ldots M \tag{23}$$

and world-sheet parity

$$A(1,\, 2,\ldots M-1,\, M) = (-1)^{M} A(M-1,\, M-2,\ldots,1,\, M) \tag{24}$$

contain the principal ingredients for establishing a connection between the dual model prescription and tree level string scattering amplitude. Notably, each $(M-3)$ can be written as follows [21]:

$$\int_0^1 \frac{dx_i}{x_i} x_i^{L_0-1} = \int_0^1 dx_i x_i^{-\alpha(s_i)-1} x_i^{-\sum n a_n^+ a_n} \tag{25}$$

where $s_i = (p_1 + \ldots + p_{i+1})$, i $= 1, \ldots$, N-3. Here, L_0 can be defined as follows:

$$L_0 - 1 = \alpha'[M^2 - p^2], \tag{26}$$

where

$$M^2 = (1/\alpha')\sum_{n=1}^{\infty} n a_n^+ a_n - (1/\alpha'). \tag{27}$$

Interestingly, the string propagator can be expressed as follows:

$$D(p^2) = -\frac{i\alpha'}{(2\pi)^4}\frac{1}{L_0 - 1 - i\epsilon}, \tag{28}$$

This is analogous to the Feynman propagator, which is expressed as follows:

$$\frac{-i}{(2\pi)^4}\frac{1}{m^2 - p^2 - i\epsilon}. \tag{29}$$

By taking the vacuum expectation of the zero modes, we can obtain the following relationship:

$$<0,\, p_1|V(p_2)x_1 V(p_3)\ldots x_{N-3}V(p_{N-1})|0,\, p_N> = \prod^{N} (1 - x_{ij})^{-p_i p_j}, \tag{30}$$

where $x_{ij} = x_{i-1}x_i \ldots x_{j-2}$ is the product of the variables between i and j. Therefore, we can obtain the following expression for the N-point function:

$$B_N = g^{N-2}\int_0^1 \prod_{i=1}^{N-3} dx_i x_i^{-\alpha(s_i)-1} \prod_{1\leqslant i<j\leqslant N} (1 - x_{ij})^{-p_i \cdot p_j}. \tag{31}$$

Interestingly, we can reduce, for example, the expression (31) to $N = 4$ and obtain the Veneziano formula, as follows:

$$\begin{aligned} B_4 &= g^2 \int_0^1 dx x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1} = g^2 B(-\alpha(s);\, -\alpha(t)) \\ &= g^2 \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}, \end{aligned} \tag{32}$$

where $s = (p_1 + p_2)^2$ and $t = (p_2 + p_3)^2$.

For the total amplitude (12) to satisfy various physical conditions such as isospin and strangeness, which are important quantum numbers characterizing the hadron charge and can represent these physical properties through the generators λ of the group $U(N)$ consisting of $n \times n$ matrices [22], we should express it in a more general form, as follows:

$$T(1,\, 2,\, \ldots,\, N) = \sum tr(\lambda_1 \lambda_2 \ldots \lambda_M) B(1,\, 2,\, \ldots,\, M). \tag{33}$$

where the summation is made over the $(N-1)!$ inequivalent cyclic permutations of the external lines.

Now, we will demonstrate that it is possible to obtain the amplitude of an open string massless boson at tree level (19) from the properties of B_N. We can observe the factorability of B_N from the fact that we can cyclically combine consecutive lines in (13). Therefore B_N can be expressed by considering, for example, the pole in channel s as follows:

$$B(1,\, \ldots,\, N) \sim \frac{1}{m^2 - s}\sum_A B(1,\, 2,\, \ldots,\, P,\, A) B(A,\, P+1,\, \ldots,\, N) \tag{34}$$

where $s = -(k_1 + k_2 + \ldots + k_p)^2$. We can expand (34) to explicitly express the possible reversals in the N lines in a general form, as follows:

$$
\begin{aligned}
&Tr(\lambda_1 \ldots \lambda_P \lambda_{P+1} \ldots \lambda_M) B(1, \ldots, P, X) B(A, P+1, \ldots, M) \\
&+ Tr(\lambda_P \ldots \lambda_1 \lambda_{P+1} \ldots \lambda_N) B(P, \ldots, 1, X) B(A, P+1, \ldots, M) \\
&+ Tr(\lambda_1 \ldots \lambda_P \lambda_M \ldots \lambda_{P+1}) B(1, \ldots, P, X) B(A, M, \ldots, P+1) \\
&+ Tr(\lambda_P \ldots \lambda_1 \lambda_M \ldots \lambda_{P+1}) B(P, \ldots, 1, X) B(A, M, \ldots, P+1).
\end{aligned} \tag{35}
$$

Another important feature in the relationship that we want to establish between the open superstring amplitude and Yang-Mills theory as a limit of the dual model is the fact that, in string theory, the wave function is invariant under $\sigma \to \pi - \sigma$. Therefore, the oscillators change from $\alpha_n^i \to (-1)^n \alpha_n^i$, which affects the cyclicity of B_N and results in the following relationship:

$$B(P,\ldots,1, A) = (-1)^{P+1} B(1,\ldots,P, A). \tag{36}$$

Thus, we can write (35) as follows:

$$
\begin{aligned}
&Tr(\lambda_1 \ldots \lambda_P - (-1)^P \lambda_P \ldots \lambda_1) Tr(\lambda_{P+1} \ldots \lambda_M - (-1)^{M-P} \lambda_M \ldots \lambda_{P+1}) \\
&\times B(1,\ldots,P, A) B(A, M+1,\ldots,N).
\end{aligned} \tag{37}
$$

Hence, the duality and the fact that B_N in equation (31) is invariant under the cyclic permutations of momenta $p_1, \ldots, p_N$ gives the entire amplitude at tree level, which can be written by considering all of the previously mentioned properties, as follows:

$$T_N = \sum_{non-cyclic} B_N(\mathcal{P}p_1, \mathcal{P}p_2,\ldots,\mathcal{P}p_N), \tag{38}$$

where the summation is made over the non-cyclic permutations of the external legs. The main duality properties with regard to the graphs can be found in [23] and [24]. In this study, we used the notation of $\mathcal{P}p_{(i=1,\ldots,N)}$ to designate the external legs of the FLD.

It should be noted that the FLD correspond to different Feynman graphs, which can nevertheless be appropriately related [25].

The properties of (38) described in (37) establish a physical application for the scattering process in the context of the relationship that we want to establish. Considering each of the external mesons, we associate a combination of the $\lambda_i[i = 0, 1, \ldots 8]$ matrices of $SU(3)$, and each vertex of the operator contributions is multiplied by $(\lambda)_{\alpha\beta}$. The contraction $\delta_{\alpha\beta}$ is associated with each quark line. In this sense, the expression (31) is multiplied by the following expression:

$$cTr[\lambda_1, \lambda_2, \ldots, \lambda_N] \tag{39}$$

where c is a normalization constant, and (38) can be written as follows:

$$T_N = \sum_a Tr(\lambda_1 \lambda_2 \ldots \lambda_N) B_N(\mathcal{P}p_1, \mathcal{P}p_2,\ldots,\mathcal{P}p_N). \tag{40}$$

When the product of the λ matrices is a λ matrix, it uses the property of B_N factorization and has the following properties:

$$Tr(\lambda_i \lambda_j) = 2\delta_{ij} \tag{41}$$

and

$$Tr(A\lambda^a) Tr(B\lambda^a) = Tr(AB) \tag{42}$$

In the case wherein the group $SU(3)$ is represented by eight matrices ($\lambda_{i=1,\ldots,8}$), for example, it can be written using the properties of (39), as follows:

$$\frac{1}{2} Tr(\lambda_1 \lambda_2 \ldots \lambda_N) = \sum_{k=0}^{8} Tr(\lambda_1 \ldots \lambda_a \lambda_k) \times Tr(\lambda_k \lambda_{a+1} \ldots \lambda_N). \tag{43}$$

Therefore, it is obvious that the following relationship holds:

$$F_N = aTr[\lambda_1,\ldots,\lambda_N] B_N \tag{44}$$

Thus, a certain connection is established with the (19) part of the kinematics factors. The equation (44) completes the construction of the relationship that we set out to establish between the amplitude of an open superstring and the Yang-Mills amplitudes as a dual limit.

5. Conclusions

This study demonstrated the relationship between $\lambda\phi^3$ theory and dual models at tree level, and showed that this relationship can be extended to the case of an open superstring at low energies. The investigated relationship between the dual models and the Yang-Mills Lagrangians for various scattering amplitudes at tree level [15] is justified in terms of the propagator in equation (17), as explained in section 3. Moreover, this relationship can be extended to the scattering amplitude of the open superstring.

The relationship between the Yang-Mills fields and the dual models at the zero-slope limit is a strong scenario wherein duality plays a key role. The finding whereby, in this limit, the FLD recover and are reduced to the FG of ϕ^3 theory, when considering the expansion of the Beta function, is a relevant factor for the construction of scattering amplitudes at tree level according to the elementary concept of duality based on the properties of the Beta function. In this sense, the most basic level of the relationship between ϕ^3 theory and the expansion of the Beta function at the zero-slope limit appears as the most elementary level of what is considered duality. Moreover, the description of the effective Lagrangian for the scattering amplitudes of the open superstring at low energies establishes a very deep and fruitful relationship between the Yang-Mills fields and the treatment of the scattering amplitude at tree level given by string theory.

There seems to be an extensive relationship between Yang-Mills, string theory, and duality. These relationships may hold because the description of the scattering amplitude of the *N*-points at tree level from the FLD, *S*-matrix, and vertex operators, can be defined based on the relationships considered in the scattering calculation. Duality simply achieves this relationship in the integrals that result in the Beta function.

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