FINITE TIME SLIDING MODE CONTROL FOR PIEZOELECTRIC ACTUATORS

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FINITE TIME SLIDING MODE CONTROL FOR PIEZOELECTRIC ACTUATORS

ABSTRACT

The piezoelectric actuator (PEA) is a device which performs very small displacements within the range of less than or equal to 100 µm. However, the PEAs suffer from the inherent non-linearity because of the hysteresis, creep and vibrations effects. These effects may cause a displacement error as high as 15% of the total range. This high error shows the necessity of a controller. Due to Sliding Mode Control (SMC) simple design steps, high robustness and low sensitivity to disturbances, it has been implemented to overcome and minimize the error. This research project has successfully accomplished the following three objectives. The first is to overcome the robustness and accuracy issue by applying the SMC method which is robust and accurate, using the mathematical model of the PEA which includes all the uncertainties and disturbances. The second is to apply the Terminal Sliding Mode Control (TSMC) concept to overcome the global infinite time stability to reach convergence or zero error in finite time. The third is to reach chattering free non-singular system. Chattering and singularity phenomena reduce the service lifetime of the PEA and create unwanted noise in the control input single. To reduce the chattering, sigmoid (sig) function has been used in the sliding function. This method is called continues terminal sliding mode. To overcome the singularity problem, the sliding function and the derivative of the sliding function does not result in terms with negative (fractional) powers which mean there will be no singularity. The chattering free non-singular finite time system has been proved mathematically and in the simulation.

Keywords: PEA, SMC, NTSMC, finite time stability.

TIME FINITE KAWALAN MOD GELONGSOR UNTUK PENGGERAK PIEZOELECTRIC

ABSTRAK

Penggerak Piezoelektrik adalah peranti yang melakukan anjakan yang sangat kecil dalam jarak yang kurang daripada atau sama dengan 100 µm. Walau bagaimanapun, penggerak piezoelektrik menderita linieriti yang tidak wujud kerana kesan histeresis, creep dan getaran. Kesan ini boleh menyebabkan ralat anjakan setinggi 15% daripada jumlah keseluruhan. Kesilapan yang tinggi ini menunjukkan keperluan peranti kawalan. Disebabkan oleh Sliding Mode Control (SMC) langkah-langkah reka bentuk yang mudah, ketahanan yang tinggi dan kepekaan yang rendah terhadap gangguan, ia telah dilaksanakan untuk mengatasi dan meminimumkan kesilapan. Projek penyelidikan ini berjaya mencapai tiga objektif berikut. Yang pertama adalah untuk mengatasi masalah keteguhan dan ketepatan dengan menggunakan SMC yang merupakan kaedah kawalan yang teguh dan tepat dengan menggunakan model matematik PEA yang merangkumi semua ketidakpastian dan gangguan. Yang kedua adalah menggunakan konsep Termonal Sliding Mode Control (TSMC) untuk mengatasi kestabilan masa tak terhingga global untuk mencapai penumpuan atau ralat sifar dalam time finite. Yang ketiga adalah untuk mencapai sistem bukan bersuara bebas chattering. Fenomena chattering dan singularity mengurangkan hayat perkhidmatan peranti dan mewujudkan hingar dalam input kawalan. Untuk mengurangkan chattering, fungsi sigmoid (sig) telah digunakan dalam fungsi gelongsor. Kaedah ini dipanggil continues terminal sliding mode. Dan untuk mengatasi masalah singulariti, fungsi gelongsor dan derivatif fungsi gelongsor tidak menghasilkan kuasa negatif (fraksional) yang bermaksud tidak akan ada singulariti. Sistem bukan bersendirian bebas chattering telah dibuktikan secara matematik dan dalam keputusan simulasi. Pengawal adalah Non-singular Terminal Sliding Mode Control (NTSMC) mencapai kestabilan dalam time finite.

Kata kunci: PEA, SMC, NTSMC, kestabilan time finite.

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Thank you,

Ahmed Al-Haddad

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LIST OF SYMBOLS AND ABBREVIATIONS

C_A	Total Piezoelectric Stack Actuators (PSA) Capacitances, Electrically Parallel with The Transformer
C_A	Linear Capacitance
F_A	The Stage Moving Part Transduced Force from The Electrical Side
R_0	The Equivalent Internal Resistance of the v_A
T_{em}	Electromechanical Transducer with The Transformer Ratio, The Piezoelectric Effect
a_1, a_0, and b_0	The Model Coefficients
b_s	The Stage Damping Coefficient
k_{amp}	Fixed Gain of the v_A
k_s	The Stage Mechanism Stiffness
m_s	The Stage Moving Part Equivalent Mass
$\dot{q}$	The Resulting Current Flowing Through the Circuit
q_c	The Linear Capacitance Stored Charge
q_p	The Mechanical Side Transduced Charge Due to The Piezoelectric Effect
v_h	Hysteresis Effect Voltage
v_A	Transducer Related Voltage
μm	Micrometer
DC	Direct Current
H	Hysteresis Effect
Hz	Hertz
LSS	SMC Uses A Linear Function for The Sliding Surface

MAE	Mean Absolute Error
NTSMC	Non-Singular Terminal Sliding Mode Control
$p(t)$	Lumped Disturbance of The Nominal System.
PEA	Piezoelectric Actuator
PEAs	Piezoelectric Actuators
PID	Proportional Integral Differential
q	The Total Charge in the PSA
RMSE	Root Mean Squared Error
SMC	Sliding Mode Control
TSMC	Terminal Sliding Mode Control
VSCS	Variable Structure Control System
$x(t)$	The End-Effector Output Displacement of The Stage
sgn	Signum Function
sig	Sigmoid Function
$v(t)$	The Control Input Voltage

LIST OF APPENDICES

CHAPTER 1: INTRODUCTION

This chapter is an overview of the micro/nano positioning technology. The main control issues in the piezoelectric actuator (PEA) device will be presented. Also, the aim and the objectives of this research project will be stated.

1.1 Background

Nanopositioning or micropositioning is defined as an accurate control in a very small area measured by nano/ micrometre scale at dimensions of 100 μm or less than that. This control approach is becoming very significant in the fields of nanotechnology applications (G.-Y. Gu et al., 2016). Examples of these applications are Nanomanufacturing equipment (Gozen & Ozdoganlar, 2012), data storage devices (Eleftheriou, 2012), atomic force microscopes (Yong, Moheimani, Kenton, & Leang, 2012), and micro/nano manipulators (Tang & Li, 2015; Tian, Shirinzadeh, Zhang, & Alici, 2009). Unlike the earlier technology actuators, piezoelectric actuators (PEAs) have the advantages of backlash and friction free, fast response, large motivating force, and high positioning resolution (Devasia, Eleftheriou, & Moheimani, 2007). In addition, PEAs are able to handle small displacements within a scale from sub-nanometer to a 100 micrometres (Adriaens, De Koning, & Banning, 2000; Aphale, Fleming, & Moheimani, 2007). Due to these advantages, they have been extensively utilized for nanoscale positioning control with applications in many control areas, such as biomedical technology, nanofabrication and optical inspection and nano storage of information. PEAs also used in nanopositioning application such as intelligent structure (Crawley & De Luis, 1987), scanning probe microscopy (Binnig & Smith, 1986), Nanomanipulation (Onal, Ozcan, & Sitti, 2011), atomic force microscopy (AFM) (Vogl, Ma, & Sitti, 2006), hard disk drives (Chan & Liao, 2006) and many other systems (Devasia et al., 2007; Sitti, 2001)

However, one of the key difficulties is the essential non-linearity in PEAs due to the hysteresis effect. The definition of the hysteresis effect is a mixture of the currently applied voltage and the past applied that results in output displacement error (Croft, Shed, & Devasia, 2001). As shown in Figure 1-1, this effect can cause a major error up to 10–15% of the total motion range (Adriaens et al., 2000). To reduce this error, a controller must be designed to correct the nonlinearity of the piezoelectric actuators.

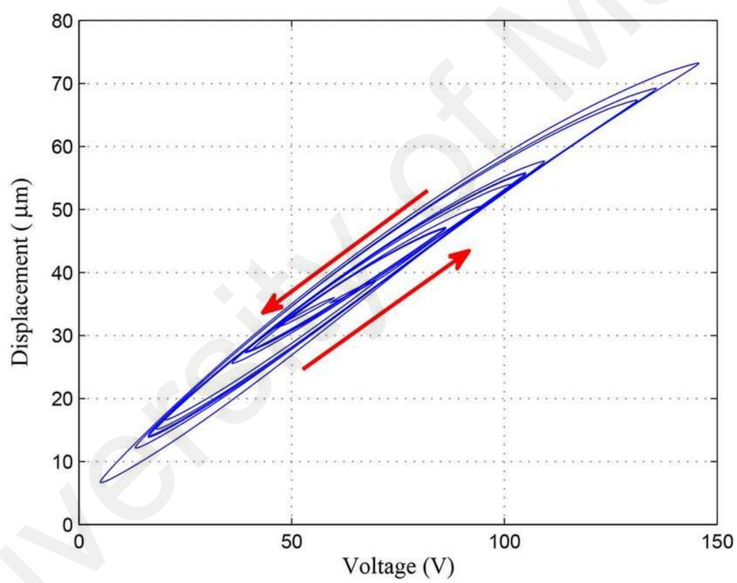


Figure 1-1: Hysteresis effect. Voltage applied and the displacement relationship in piezoelectric actuators (G.-Y. Gu et al., 2016)

1.2 Aims of the Project

To reduce the non-linearity that is caused by the hysteresis, a robust and accurate controller must be designed to overwhelm this issue. The controller must overcome the following challenges:

- Robustness and accuracy: preciseness and robustness against external disturbances, uncertainties, and parameter variations are the key features that any control efforts should achieve, especially in a very small environment like the PEA output.
- Global infinite time stability: the controller should speed up the convergence rate together with high precision. In other words, the global infinite time stability should be changed to finite time stability.
- Chattering problem: due to the discontinuous control function, this unwanted phenomenon reduces the service lifetime of the device and generates undesirable and destructive noise in the control input.

1.3 Objectives

1. To design a controller for piezoelectric actuators by implementing the Sliding Mode Control (SMC) technique to overcome the lack of robustness and accuracy.
2. To apply the Terminal Sliding Mode Control (TSMC) concept to reach stability in finite time.
3. To reach chatter free non-singular system.

1.4 Structure of The Thesis

Chapter two the literature review, this chapter is divided into four main sections. Firstly, it describes the piezoelectric actuator. Secondly, it shows the cause of the nonlinearity in the system and the control challenges. Thirdly, it overviews the SMC methods which are the Sliding Mode Control (SMC), the Terminal Sliding Mode Control (TSMC) and the Non-Singular Terminal Sliding Mode Control (NTSMC). Fourthly, it reviews some of the studies which apply the SMC methods to control the piezoelectric actuator. **Chapter three** the methodology, the mathematical modelling will be explained, and the control theory of the sliding mode control will be presented. The mathematical solution of the sliding mode control will be explained, and the finite time stability will be proved. **Chapter four** discusses and analyses the simulation results of two simulated experiments. Fourteen different step signals will be applied, and a complex signal will be given as a trajectory. Finally, **chapter five** is the conclusion.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter is divided into four main parts. The first part describes the piezoelectric actuator device by explaining the main components of the device and the principle of operation. The second part focuses on the cause of the nonlinearity in the system and the control challenges that should be solved by the controller. The third part gives an overview of the control method Sliding Mode Control (SMC). Also, it explains the differences between SMC, TSMC and NTSMC by listing the advantages and disadvantages of each controller respectively. The fourth part will be reviewing twelve other researchers works which apply the SMC methods to control the piezoelectric actuator.

2.2 Part 1: Piezoelectric Actuator Stages Device

2.2.1 Explanation of the structure

Figure 2-1 shows the main parts of the PEA stages device which is shown as a block

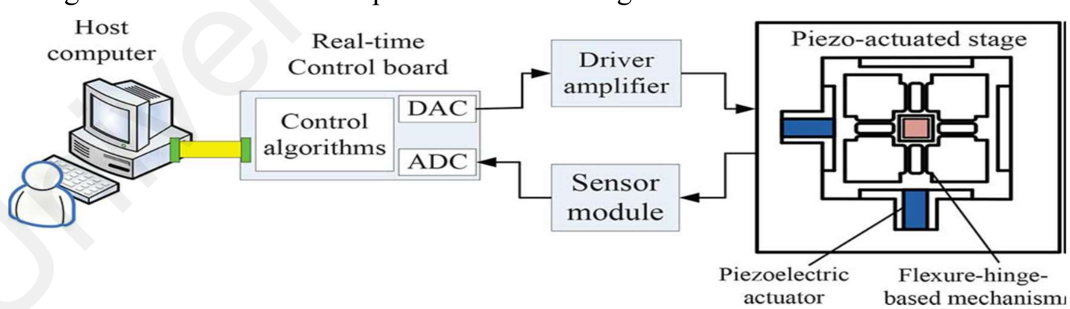


Figure 2-1: Block diagram of piezoelectric actuator stages device. Analog to Digital Converter (ADC) and Digital to Analog Converter (DAC), (G.-Y. Gu et al., 2016)

diagram. There are five main parts in the PEA stages device as it explained by (Devasia et al., 2007; Fleming & Leang, 2008; C.-X. Li, Gu, Yang, & Zhu, 2013), which are the flexure-hinge guided mechanism, the piezoelectric actuator, the driver amplifier, the sensor module and the control algorithms.

2.2.1.1 The flexure-hinge-guided mechanism

The main role of this part is to deliver movement. The elastic deformations principle used has the benefits of a monolithic structure which eliminate any sliding parts, thus the nonlinear effects can be avoided. The nonlinear effects that may occur if this part would not be used are backlash and friction.

2.2.1.2 The piezoelectric actuator

It is the heart of this process which produces the force that moves the mechanism. It has a relatively fast response, high resolution and high output force.

2.2.1.3 The driver amplifier

The signal usually needs to be amplified due to the low voltage control signals. This driver amplifies the PEA's control signal which can be voltage or charge control methods.

2.2.1.4 The sensor module

It is used to know the displacement measurement of PEA in the real-time. It has two parts, the sensor and signal conditioner which send the signal to the control system. The signal conditioner is utilized to change the sensing signal to a low voltage.

2.2.1.5 The control algorithms

In real time, this algorithm is made and applied by the control board. According to this program, the PEA moves.

2.2.2 Principle of operation

The piezoelectric actuator (PEA) is the main part of this device. PEAs perform displacement range from sub-nanometres to a hundred micrometre, due to that it gives piezo-actuated stages the advantage of wide displacement range. Piezoelectric materials are the key element in piezoelectric actuator, which are defined as intelligent material-based devices that have the ability to directly change an electrical signal to a physical movement (Devasia et al., 2007). At the beginning of this technology, piezoelectric crystals were used, such as Rochelle salt, quartz and tourmaline. However, piezoelectric ceramic materials discovery has strengthened the technology due to their powerful piezoelectric effect and high Curie temperature. Barium titanate and lead zirconate titanate (PZT) are examples of these materials that make this breakthrough possible. In addition, Figure 2-2 shows that the PEA can-shaped as tubes, rectangular patches or thin disks (Niezrecki, Brei, Balakrishnan, & Moskalik, 2001). The flexibility in sizing and formation has given the piezoelectric actuators an advantage compared to the conventional actuators.

The principle of piezoelectricity is the key concept of PEAs. It is an electromechanical connection between both the mechanical properties and the electrical properties of piezoelectric materials (Devasia et al., 2007). A measurable electric charge is produced when a mechanical pressure is applied to the piezoelectric materials, a phenomenon is known as the direct piezoelectric effect. And vice versa, when an electrical charge is applied, mechanical displacement occurs. This phenomenon is called the inverse piezoelectric effect.

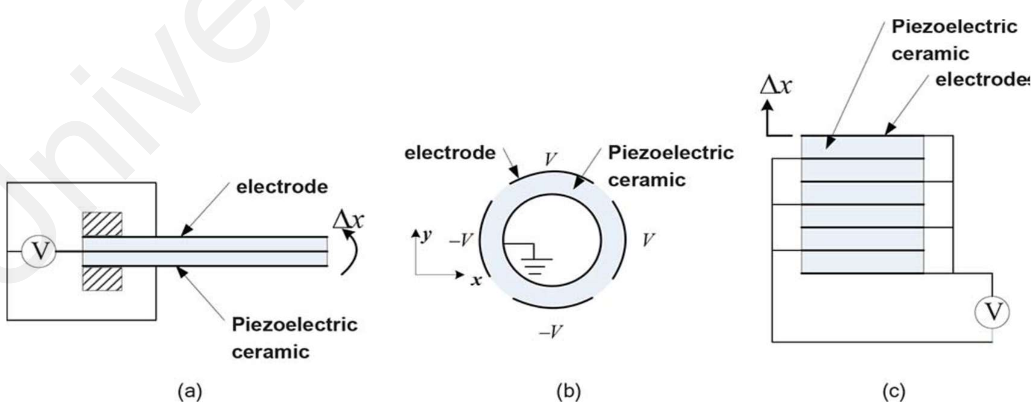


Figure 2-2: The three types of PEAs (a) thin disks (b)tube (c) rectangular patches (G.-Y. Gu et al., 2016)

2.3 Part 2: Modelling Control Challenges

The inherent nonlinearities of piezo-actuated stages have increased the difficulties of the system modelling. The three main problems which cause the nonlinearity are vibration, hysteresis and creep. It is known that the modern control theory is based on the system dynamic modelling, due to that the three nonlinearities in the piezo-actuated stages should be considered in the mathematical modelling. By considering these effects, it is expected that the designed controller can control the position accurately and reduce the error caused by these effects. In the following points, the three main nonlinearities occur in PEA will be discussed.

2.3.1 Creep:

Creep is associated with piezoelectric actuator output displacement drift phenomenon when it is exposed to an applied voltage (Jung & Gweon, 2000). It is an outcome of the crystalline domains slower rearrangement in an applied electric field. Throughout slow-speed operations, the creep effect becomes obvious over long time. When a positive voltage is given as an input, Figure 2-3 illustrates the piezoelectric actuator creep nonlinearity in an open loop system.

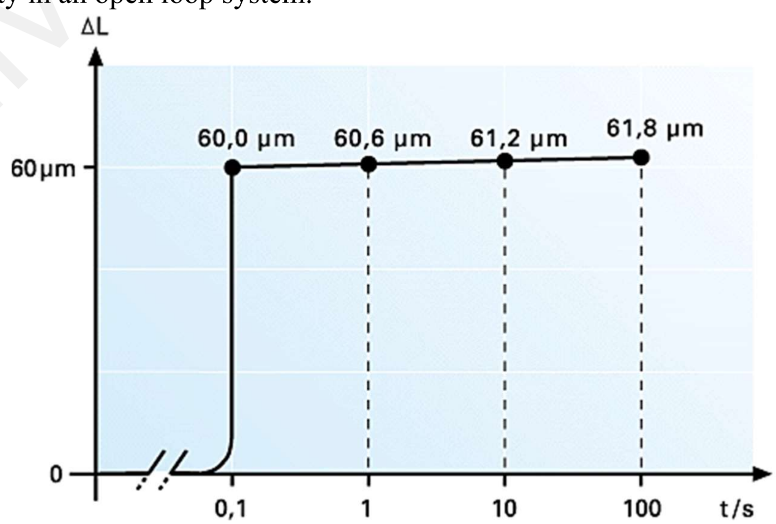


Figure 2-3: Creep in PEA during slow-speed operation (Noor & Ahmad, 2017)

In the graph, the positioning error increases as the time increase after the system have reached the desired value. However, to reverse this effect, a negative step voltage can be applied. Typically, this phenomenon can be compensated using proportional integral differential PID algorithms for instants, or any other closed-loop control law (G. Gu & Zhu, 2010).

In high-speed scanning applications, it has been reported that the creep effect can be neglected (Yong et al., 2012). However, the creep is a challenging issue in piezoelectric actuators slow-speed and open-loop devices, which many studies and effort have been made to overcome and to reduce the nonlinearity effect (Changhai & Lining, 2005; Jung, Shim, & Gweon, 2000; Rakotondrabe, Clévy, & Lutz, 2010).

2.3.2 Hysteresis:

Hysteresis is defined as a nonlinear nonsmoothed phenomenon that occurs between the applied voltage and the output displacement of the piezoelectric actuator (Visintin, 2013). The nonlinearity causes the amplitude- dependent behaviour due to multivalued and nonlocal memoryless of this nonlinearity, Figure 2-4 shows the nonlinear hysteresis effect.

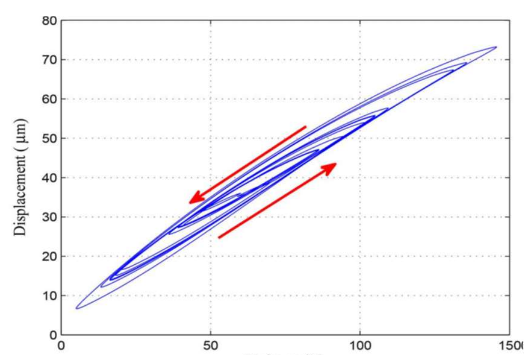


Figure 2-4: Hysteresis effect (G.-Y. Gu et al., 2016)

Consequently, the previous input voltage is dominant which influences the total displacement of the PEAs. Furthermore, as stated in the literature, hysteresis can be affected by the frequency of the input signals (Leang & Devasia, 2007). This shows that the hysteresis is a rate-dependent on the input signals frequency i.e. as the frequency increases the hysteresis nonlinearity increases as well, which shown in Figure 2-5.

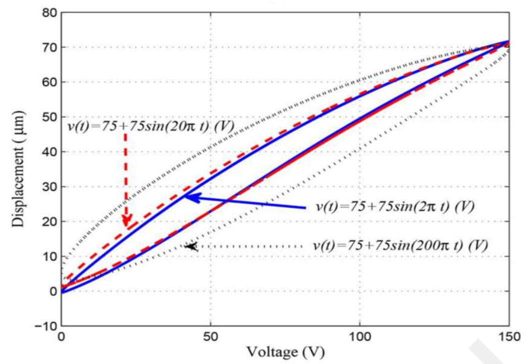


Figure 2-5: Hysteresis effect with different frequencies (G.-Y. Gu et al., 2016)

The hysteresis displacement error is as high as 15% of the total range. Moreover, as the frequency of the input signal increases, the error also may be increased (Cheng, Liu, Hou, Yu, & Tan, 2015) which magnifies the importance of modelling this effect to overcome this control problem.

2.3.3 Vibration:

In the piezo-actuated stages, vibration is the electromechanical dynamics behaviour (G.-Y. Gu & Zhu, 2013). The strong peak of the lightly damped resonances occurs in the piezo-actuated stages frequency response; because of low structural damping ratio and high stiffness characteristics; is shown in Figure 2-6.

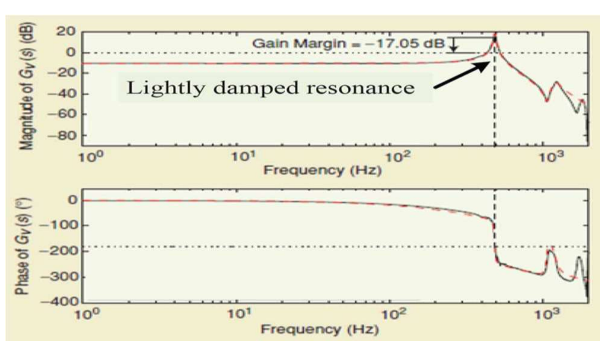
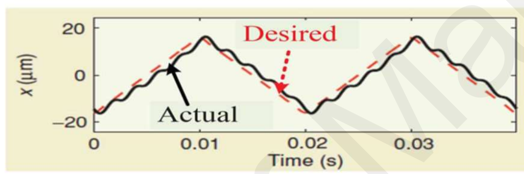


Figure 2-6: The frequency response (G.-Y. Gu et al., 2016)

Due to these two factors, a) the rapid phase drop related with the hysteresis nonlinearity (G.-Y. Gu, Zhu, Su, & Ding, 2013) and b) small structural damping ratio (G. M. Clayton, Tien, Leang, Zou, & Devasia, 2009), low-gain margin problem of the vibrational dynamics happened. Consequently, the oscillation or vibration of the motion excited easily with high-frequency input signals components as shown in Figure 2-7 as a consequence, the high-speed performance is limited (Yong et al., 2012). The range of input signals frequencies in the applications is 1/100 to 1/10 of the stages lowest resonant modes.



(b)

Figure 2-7: Vibration effect (G.-Y. Gu, Zhu, Su, Ding, & Fatikow, 2016)

2.4 Part 3: Control Methods

2.4.1 Background

Some conventional methods have been used to solve the nonlinearity problem by using controllers depending on the charging amplifies (G. Clayton, Tien, Fleming, Moheimani, & Devasia, 2008; Devasia et al., 2007; Ma, Huang, Liu, & Feng, 2011; Minase, Lu, Cazzolato, & Grainger, 2010) or limiting the amplitude of the voltage applied as an input (Leang, Zou, & Devasia, 2009). However, these two methods are not practical due to the high complexity and hard implementation. Therefore, other advanced control methods are being utilized by the researchers to overwhelm hysteresis, especially in PEAs applications. In general, there are two advanced control methods that are widely described in the literature.

The first method is by implementing the inverse-based feedforward compensation methods. In this method, the hysteresis must be modelled first. Many control schemes have been established to provide the positioning accuracy condition of PEAs nanopositioning systems. Unfortunately, the modelling methods have errors and uncertainties that cannot be avoided. The hysteresis modelling may take a long time to process which produces the error continuously (Q. Xu, 2017). Typical examples which have been done are Prandt-Ishlinskii model (Mokaberi & Requicha, 2008; Pesotski, Janocha, & Kuhnen, 2010), Preisach model (Jang, Chen, & Lee, 2009; Liu, Tan, Chen, Teo, & Lee, 2013), Bouc-Wen model (Huang & Lin, 2004), and Maxwell resistive capacitor (MRC) model (Yeh, Ruo-Feng, & Shin-Wen, 2008).

The second method is the feedback control methods which the hysteresis is not to be modelled. In this method, the system nonlinearities are considered as disturbances which can be blocked by the feedback controller. There are many examples of this method, such as SMC (sliding mode control) (Dong, Salapaka, & Ferreira, 2008; Liaw et al., 2007; Liaw, Shirinzadeh, & Smith, 2008; Shen, Jywe, Liu, Jian, & Yang, 2008; S. Yin, Ding, Abandan Sari, & Hao, 2013), robust control (Dong et al., 2008; S. Yin et al., 2013), repetitive control (C.-Y. Lin & Chen, 2011), and PID control (H. Xu, Ono, & Esashi, 2006).

In addition, these two methods can be combined by using both feedforward and feedback control design which are considered to be the third method as described by (C.-J. Lin & Yang, 2006; Song, Zhao, Zhou, & De Abreu-García, 2005).

2.4.2 Sliding Mode Control (SMC)

In this section, the basic concept of the sliding mode control will be explained, and the main advantages and disadvantages will be listed.

2.4.2.1 The development of SMC methods

Sliding mode control (SMC) is a control technique to control non-linear dynamic systems. SMC modifies the system dynamically by utilizing a discontinuous control signal to force the system to slide on a defined stable switching function (see Figure 2-8).

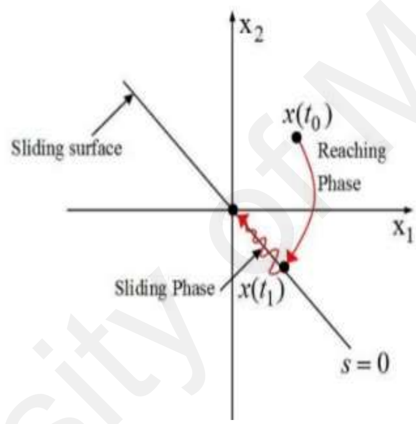


Figure 2-8: SMC the states sliding action to equilibrium point (Saxena, Tandon, Saxena, Rana, & Kumar, 2017)

The most significant advantages of SMC are the simple design steps, high robustness and low sensitivity. Due to that, SMC is a robust control to external disturbances, has low sensitivity to modelling errors and parameters changes. From the advantages above, therefore, SMC has been applied as a controller in many applications, such as motion control, position control, machine control, press control instrumentation and robotics (Sabanovic, 2011; Utkin, 2013).

The basic type of the **SMC** uses a linear function for the sliding surface (LSS). However, this controller sliding surface is inherently nonlinear control. The controller design steps are designing the sliding surface function which must be stable and selecting the control law which drives the system to the stable surface. The sliding surface function is based on the system required specifications which is needed to be performed. The control law is selected according to the dynamic and the sliding surface. It is important to know that the LSS controller is asymptotically stable which mean that the system reaches stability in infinite time (Q. Xu, 2017). In addition to that, the LSS controller has a chattering problem which reduces the lifetime of the controller (Q. Xu, 2017).

Many application requires a finite time stability, due to that, the terminal sliding-mode controller (TSMC) has been established to overcome the LSS limitations (X. Yu & Man, 1996). The main advantage of TSMC as compared to LSS is highly steady-state tracking precision and fast finite-time convergence (see Finger 2-9).

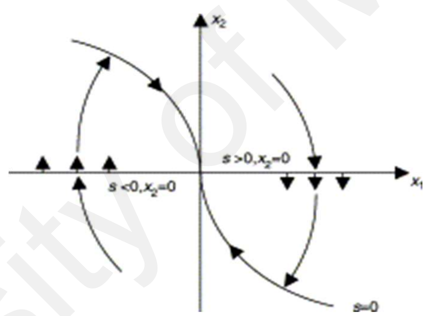


Figure 2-9: TSMC the sliding function is not linear (Feng, Yu, & Man, 2002)

Nevertheless, singularity considered as one of the TSMC controller problems. The singularity as defined in the state space is an area for maintaining the ideal TSM motion the controller needs will be infinitely high (see Figure 2-10) (J. Yin, Khoo, Man, & Yu, 2011).

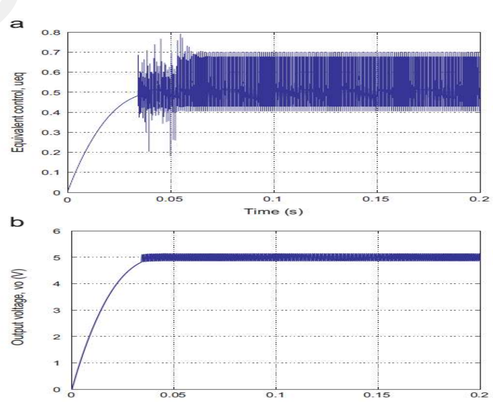


Figure 2-10: Terminal Sliding Mode Control output, (a) the control law effort and (b) the output voltage in DC-DC buck converter with the singularity problem effect clearly shown as noise on the waveform (Komurcugil, 2013)

The non-singular terminal sliding mode control (**NTSMC**) proposed in (Feng et al., 2002) solve the singularity problem simply and completely (see Figure 2-11). The limitation of this method is that it can only work on a second-order system and some high order system. To overcome this limitation, the back-stepping based non-singular TSMC control (Jianqing & Zibin, 2009) and derivative and integral TSMC control (Chiu, 2012) are proposed.

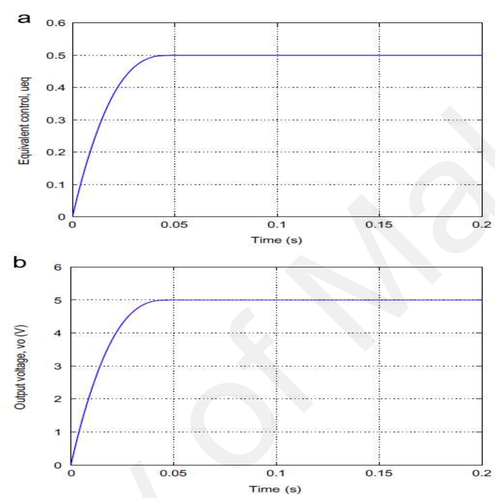


Figure 2-11: Non-singular terminal sliding mode control output waveforms, (a) the control law effort and (b) the output voltage in DC-DC buck converter with the singularity problem effect has been solved (Komurcugil, 2013)

2.5 Part 4: Different Studies of SMC Methods for Piezoelectric Actuator Control

In this section, SMC, TSMC and NTSMC methods which have been applied to control the piezoelectric actuators will be reviewed. The key concept of each method will be presented. The results, advantages and the tracking/position error graph will be shown. This section is organized according to the control method and the publishing date.

2.5.1 Sliding mode control SMC

2.5.1.1 The conventional sliding mode control

(Abidi & Sabanovic, 2007) this study has applied the sliding mode method to control the Piezoelectric actuator stages. The study has tried to solve the most important problems of the PEAs which are the hysteresis and the chattering. They clarify that the SMC has the following advantages, high-accuracy piezo actuator position, Fast response and it is not affected by disturbances and uncertainties. The method which has been used is the sliding-mode controller and the disturbance rejection method which increase the accuracy and the resistance to disturbances of the closed-loop system. To prove the success of the method, a comparison between the SMC and PID control has been done. As a result, the final displacement error decreased hence the method is proved to be more reliable than the PID controller. Figure 2-10 views the tracking presentation of the controller.

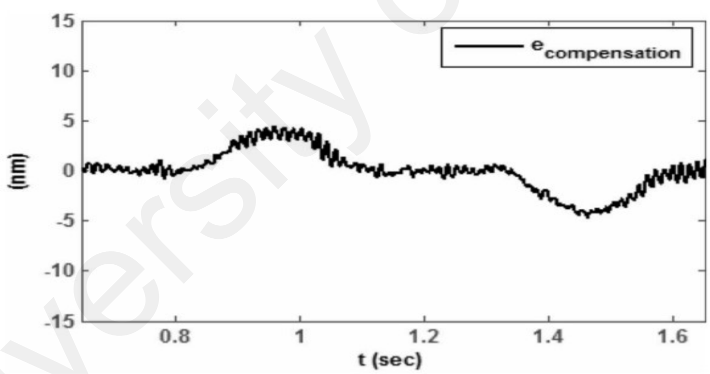


Figure 2-12: The tracking error of (Abidi & Sabanovic, 2007)

(Liaw et al., 2007), one of the unique features of this method is the controller function without any form of feed-forward compensation. It can eliminate external disturbances, non-linearities, and parametric uncertainties. The controlling idea depends on the requirement of a target output and the design of an improved sliding mode scheme, to reach the output by achieving the position tracking error of the system to be almost zero. Theoretically and after the mathematical analysis, the control method has been proven to be stable. The experimental study illustrates the tracking ability of the system and the control method effectiveness. The control system involves the estimated parameters and its bounds in addition to the bound of external disturbances and the nonlinear effects. Figure 2-11 shows the tracking performance of the controller.

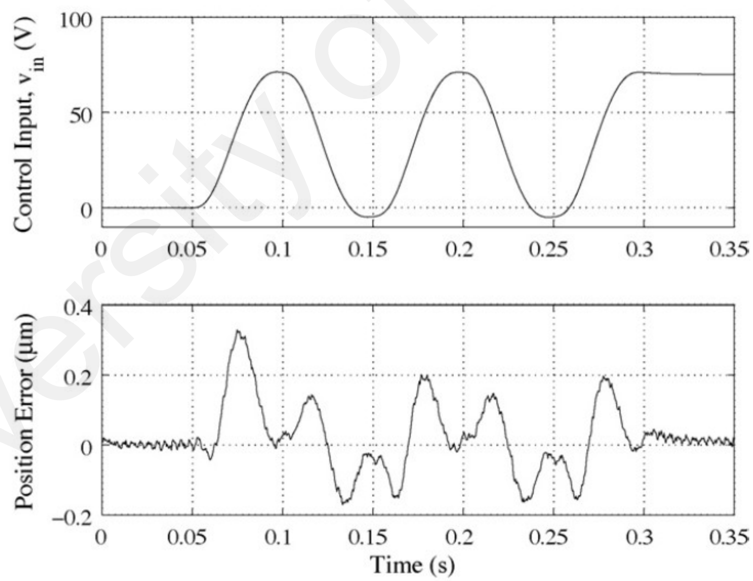


Figure 2-13: The control input and the tracking error of (Liaw, Shirinzadeh, & Smith, 2007)

(Shieh & Huang, 2007), one of the earlier methods has been used to overcome the SMC limitation is the filtering-type sliding-surface control (FTSSC). The main advantage of FTSSC is to reduce the charting problem, compared with the SMC. This improvement will increase the lifetime of the equipment and produce a better trajectory tracking of the piezoelectric positioning stage. To represent the dynamic of the system, the mass-spring mechanical system has been used. This representation contains the linear movement function of the system and the non-linear elements produced due to the hysteresis function. These two functions, in the spring mechanical system, describe the motion dynamics of the piezoelectric positioning stage. To prove the equations, the hysteresis response from both the suggested equations and the practical piezoelectric positioning stage have been compared. The system state space model developed according to the suggested equations with taking into castration that, the voltage applied to the stage is defined as an output of a new control variable. The result of the state space model shows a chattering reduction improvement, in addition to the SMC other advantages. Via filtering-type sliding-surface control, the following advantages are obtained a. tracking response with high performance, b. robustness and c. improvement in the reduction of the chattering problem. The main disadvantages are the complexity of the design and the positioning is not accurate enough. Figure 2-12 displays the tracking act of the controller.

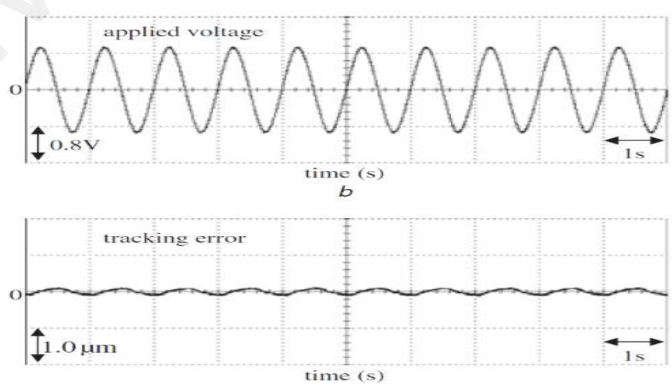


Figure 2-14: The control input and the tracking error of (Shieh & Huang, 2007)

(R. Xu et al., 2018), The major contributions of this study are described as follows; first, to find the unknown parameters of the Bouc-Wen model, the bat-inspired algorithm is implemented. Therefore, the model can define both the piezo-actuated stages hysteresis loops 'major and minor'. Second, to estimate the mathematical modelling uncertainty, the perturbation estimation method is employed also the unknown external disturbances can be estimated using the same method. Third, a novel reaching law has been designed for this controller to overcome the hysteresis nonlinearity effect. The experiment result proves the ability of this law to suppress the hysteresis nonlinearity. Forth, by the Lyapunov stability theory, the stability of the suggested control technique is examined. Figure 2-13 views the tracking presentation of the controller.

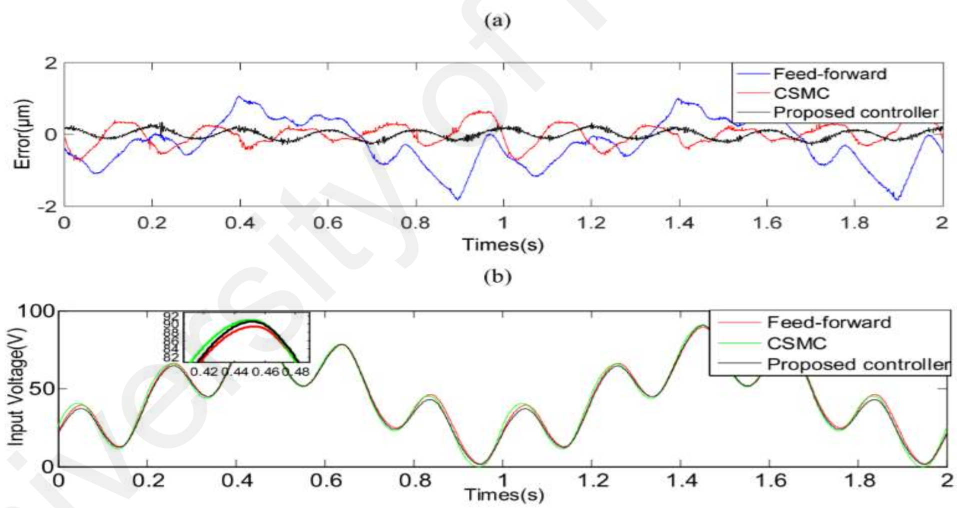


Figure 2-15: The tracking error and the control input of (R. Xu, Zhang, Guo, & Zhou, 2018)

2.5.1.2 The discrete sliding mode controller

(Khan et al., 2006), The main contribution of this study is, as the piezoelectric stage can be modelled like a nominal linear lumped parameters second order electromechanical system with voltage, as the input, position and as the output. Then, the robust controller should control this dynamic successfully. The PEA hysteresis is the main reason for nonlinear disturbance effect in the system. The main goal of this papers is designing a robust discrete sliding mode controller and disturbance rejection method to realize high position control accuracy in the Nano-scale, by removing hysteresis. To achieve the disturbance rejection, the sliding mode observer is applied. This observer method reflects the total disturbance caused by the hysteresis and external force applied to the system. Therefore, the observer able to estimate the lumped disturbance performing on the system. Figure 2-14 shows the tracking performance of the controller.

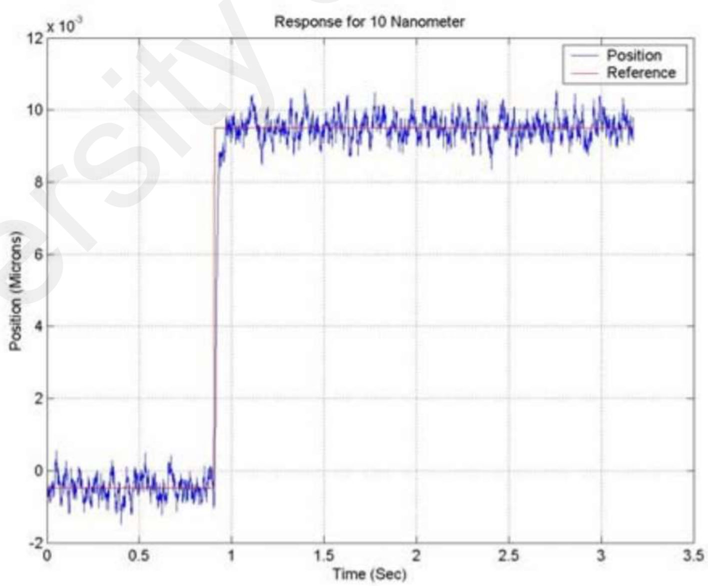


Figure 2-16: The step response of (Khan, Elitas, Kunt, & Sabanovic, 2006)

(Q. Xu, 2014), Chattering problem is one of the challenges during the design phase. Consequently, finding a solution to this problem is essential. The high order SMC is proved to overcome this problem. High-order SMC basic objective is to ensure that the system states arrive at the sliding surface. The sliding surface definition is $s = \dot{s} = \ddot{s} = \cdots = s^{(n-1)} = 0$, in this equation s is the sliding surface and n is the order of sliding mode control order. Therefore, the order of the sliding surface allows the derivative of sliding function s and its n-1 to be zero. The discontinuous control of the system which produces the chattering problem is transferred to the higher derivatives of the sliding function. In other words, the reduction of the magnitude of oscillations eases the chattering phenomenon. It is better for a sampled-data system to use discrete time SMC than normal SMC method. For that, the discrete time SMC has been selected to implement the design. Consequently, discrete-time high-order SMC is developed to be robust control technique for digital implementation. By using the stability theory, the system can be proved to be stable. In terms of motion tracking precision, the study has been determined that the 2nd-order DSMC is higher-ranking than the PID controller. The main advantage of this method is the clear reduction of the chattering problem and the position error, but the stability is infinite time stability. Figure 2-15 displays the tracking act of the controller.

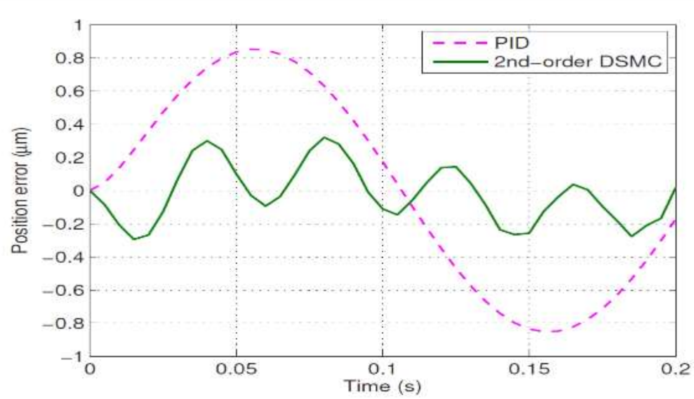


Figure 2-17: The tracking error of (Q. Xu, 2014)

2.5.1.3 The adaptive sliding mode controller

(Chen & Hisayama, 2008), this method has been used the Prandtl–Ishlinskii hysteresis model to control the positioning stage of piezoelectric actuators, with the adaptive concept. The main advantage of this technique is that only the controller formula parameters need to be estimated using the adaptive method. There is no requirement to identify or measure the real value of the system parameters. The control law or the input which has been proposed to control this system ensures the global stability of the PEA stages system controller. It has been chosen the design parameters to control the position error to be as small as possible. The main advantage is that the parameters are not required to be measured or identified which reduces the complexity of the design. Nonetheless, this method has infinite time stability of the system, difficult to obtain robust stability results and has low performance in high frequency. Figure 2-16 views the tracking presentation of the controller.

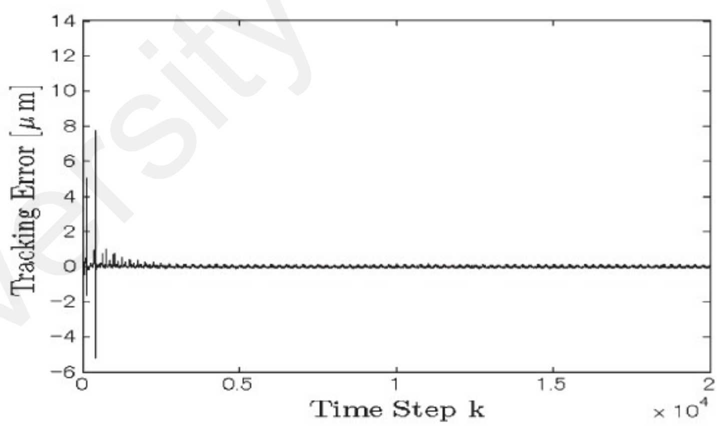


Figure 2-18: The tracking error of (Chen & Hisayama, 2008)

(Q. Xu, 2017), The major aim of this design is to achieve chattering-free SMC by using adaptive sliding mode control with uncertainty and disturbance estimation control strategy (ASMC-UDE). There is no need to use any reference model in this method. The hysteresis and other uncertainties in the system are considered to the nominal plan model as a lumped disturbance. These disturbances are suppressed by using the ASMC-UDE method. In this method, the reference input of the controller is the desired position trajectory. In other methods, the input reference is the voltage signal. This new reference is more practical for nanopositioning applications. The main properties of the ASMC-UDE strategy are a) chattering free, b) it does not require hysteresis modelling, c) can be used in any other system with the disturbance either matched or unmatched, d) reference model free. In this paper, Continuous control action has been developed, in place of the signum function had been utilized in SMC discontinuous control term. For this reason, ASMC-UDE is chattering-free inherently. The control gain tuned online by using the adaptive rules, that result to archive tracking error bound which has been predefined. Moreover, turn to the perturbation estimation technique, the estimation of the hysteresis term and the procedure of hysteresis modelling have been eliminated. Using Lyapunov analysis, the system has been proved to be stable. Figure 2-17 is a presentation of the tracking act of the controller.

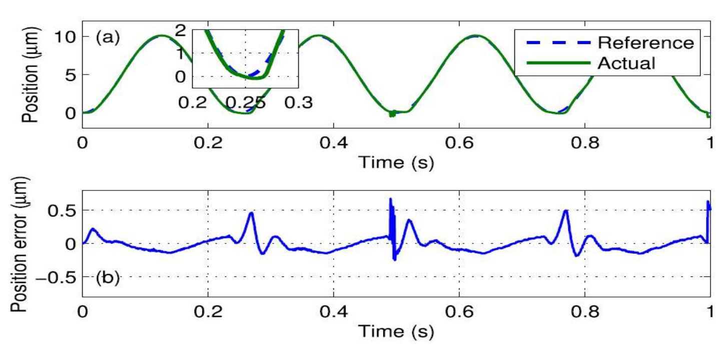


Figure 2-19: The tracking and the error of (Q. Xu, 2017)

2.5.2 Terminal sliding mode control

(J. Li & Yang, 2014), Reaching stability in finite time increases the system performance and precision, due to that the researchers are trying to devolve methods to convert the systems dynamics to equilibrium in finite time. Consequently, in a nonlinear system, the TSMC has been adopted because of the advantages listed earlier. Nevertheless, the tracking performance and oscillation consider as limitations due to the chattering. The chattering is caused by the discontinuous control and the high frequency that may excite dynamic modelling limit. The boundary layer technique could reduce the chattering, but this method is cancelling all the SMC features such as insensitivity to uncertainties and disturbances and finite time stability. The continuous TSMC avoid all this limitation, and to improve the robustness, sliding mode disturbance observer has been adopted. Figure 2-18 shows the tracking act of the controller.

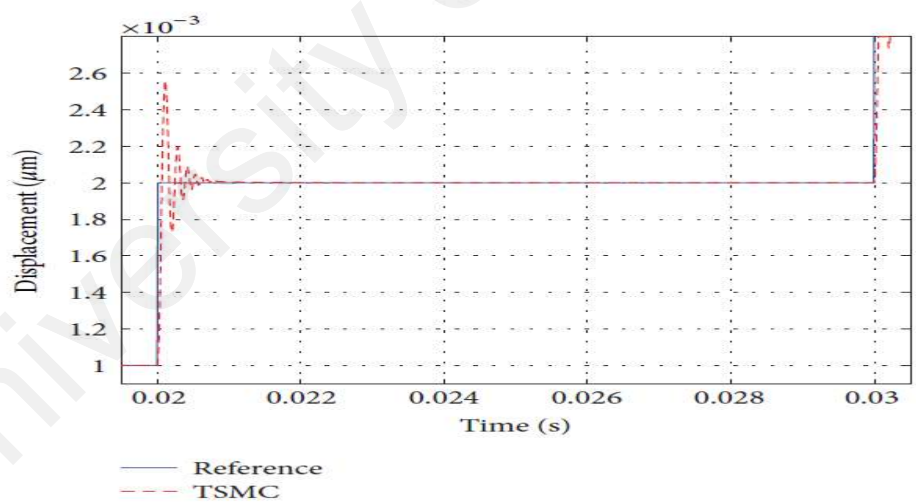


Figure 2-20: The step signal response of (J. Li & Yang, 2014)

(Q. Xu, 2015), the main novelty of this work is the second-order discrete-time TSMC (2-DTSMC) approach, and the implementation of this method experimentally for accurate motion control of a high-order plant model of the piezoelectric nanopositioning system. The 2-DTSMC system has been designed with using output feedback only, which is more practical for fewer sensors nanopositioning schemes, unlike the case of state feedback-based control. Furthermore, it does not depend on a hysteresis model and the state observer, which make the control design easier in both the procedure and facilitates application. The stability and usability of the control method have been proved by using a commercial nanopositioning platform as an experiment. The results of this experiment show that 2-DTSMC approach; in terms of motion tracking accuracy; is much better than the discrete-time sliding-mode control (DSMC), second-order DSMC, and DTSMC algorithms. Figure 2-19 shows the tracking performance of the controller, the high and low-frequency input tracking and the corresponding error exposed.

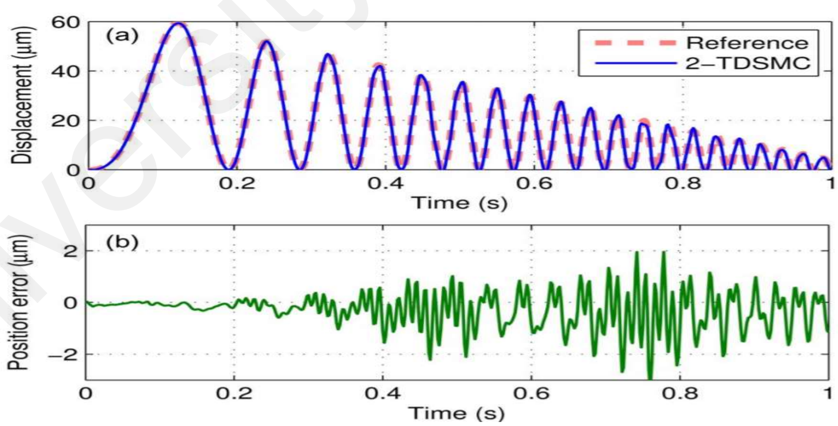


Figure 2-21: Tracking response and the tracking error of (Q. Xu, 2015)

2.5.3 The non-singular terminal sliding mode control

(Al-Ghanimi et al., 2015), This control method has used the fast non-singular terminal sliding mode (FNTSM) for Piezoelectric Actuators PEAs. Compared to the conventional sliding mode (CSM), this method is chatter free. One of the features is zero error convergence which means in the presence of disturbance and uncertainties, the system converts to the equilibrium point in finite time. FNTSM control has been designed according to the parametric uncertainties bounded information. To measure the feedback velocity, the model-free velocity estimator has been used. The system uncertain parameters bounded information are the controller design bases. The fast-tracking and accuracy of the controller; even with the disturbances and uncertainty of the system parameters; are the main advantages of this method. The experiments have proved that this method is more stable than the conventional sliding mode CSM and $H\infty$ controllers. Figure 2-20 views the tracking act of the controller.

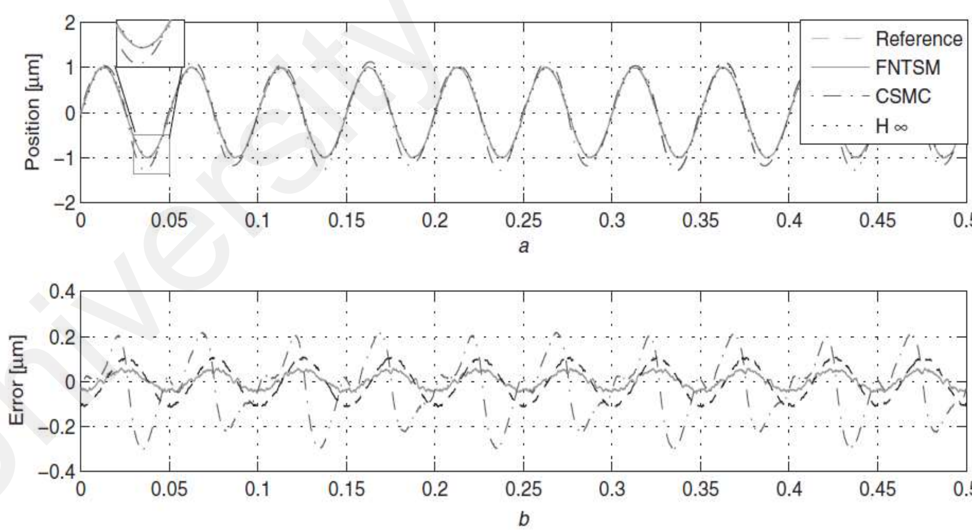


Figure 2-22: The tacking and tracking error of (Al-Ghanimi, Zheng, & Man, 2015)

(Al-Ghanimi et al., 2017), this control technique "which is systematic control method" has used the fast-non-singular terminal sliding mode (FNTSM) based on piezoelectric actuator online perturbation estimation technique. FNTSM is chatter free and finite time stability system even with the disturbance and uncertainties. Due to that, the convergence to a zero-error can be achieved. FNTSM control based on perturbation estimation (FNTSMPE) is the key contribution of this study. To estimate the states of the feedback system, a model-free robust precise differentiator has been implemented, from just quantifiable location signal. The robust performance and high-precision have been realized perfectly compared to the ordinary FNTSM control. Figure 2-21 displays the tracking presentation of the controller.

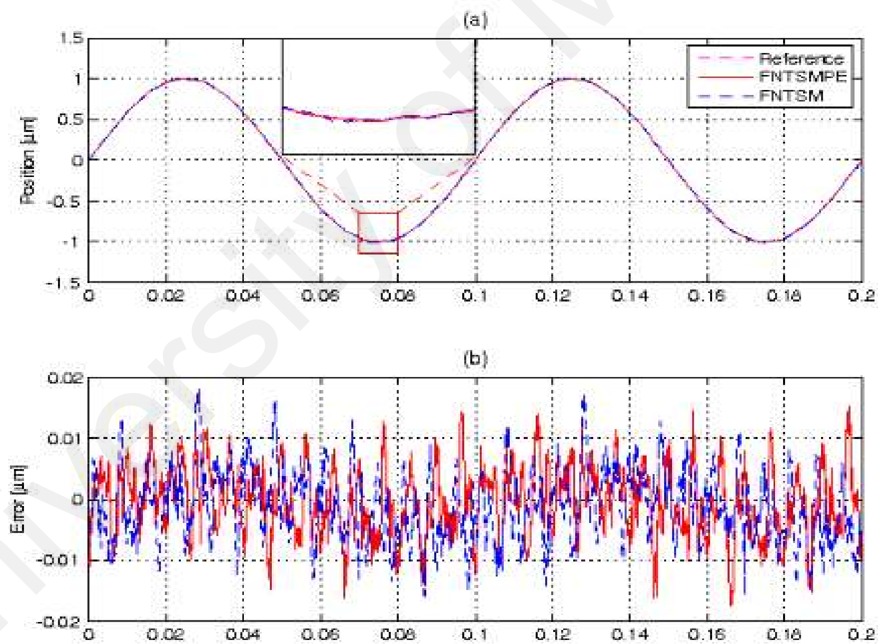


Figure 2-23: The tracking and tracking error of (Al-Ghanimi, Zheng, & Man, 2017)

2.6 Summary

Nanopositioning piezoelectric actuators (PEAs) is one of the most important technologies in nanotechnology applications. The materials which are used in the PEAs manufacturing are the piezoelectric ceramic materials such as Barium titanate and lead zirconate titanate (PZT) which has the advantage of the flexibility in sizing and formation. However, the hysteresis effect causes displacement error as high as 15% of the total range. In general, three main approaches of advanced control have been used which are feedforward compensation methods, feedback control methods and feedforward/feedback control design. SMC which is a feedback control method, has simple design steps, high robustness and low sensitivity to disturbances. The design steps are a) designing the sliding surface, and b) designing the control law.

There are three main types of SMC. The first is SMC, which uses a linear function sliding surface, due to that the system reaches stability in infinite time. In addition, it suffers from the chattering problem which reduces the lifetime of the controller. The second is TSMC, as many applications required a finite time stability, consequently, the TSMC controller has been developed to overcome sliding mode control limitations. Nonetheless, the singularity problem of the TSMC has raised the need of developing another type of SMC. The third is the NTSMC which has been proposed to solve the singularity issue simply and completely. The table 2-1 below summarizes the studies that have been reviewed in this chapter.

Table 2-1: Summary of the studies reviewed

Reference	Method	F. T	E%	C. F	H. M	Remarks
Abidi & Sabanovic, 2007	Sliding-Mode Controller and The Disturbance Rejection Method	No	0.5	No	No	One signal of 1micro used to show the tracking with very low frequency
Liaw et al., 2007	Enhanced Sliding Mode Motion Tracking Control Methodology	No	0.58	No	No	A sinusoidal signal with low frequency
Shieh & Huang, 2007	Filtering-Type Sliding-Surface Control	No	0.25	No	Yes	Sinusoidal signal 1Hz
R. Xu, Zhang, Guo, & Zhou, 2018	Sliding Mode Tracking Control with Perturbation Estimation for Hysteresis Nonlinearity	No	0.94	No	Yes	A complex harmonic signal has been given to examining the trajectory tracking
Khan, Elitas, Kunt, & Sabanovic, 2006	Discrete Sliding Mode Controller and Disturbance Rejection Method to Eliminate Hysteresis	No	Low At 1μm	No	Yes	Different step signals applied 1000 to 2 nm the error increase as the displacement decreases up to 25% at 5nm and 60% at 2nm
Q. Xu, 2014	Second-Order DSMC Control	No	1.5	No	No	Sinusoidal signal 5Hz
Chen & Hisayama, 2008	Adaptive Sliding Mode Control	No	High	No	Yes	Step input and the complex harmonic signal has used the error lower with time
Q. Xu, 2017	Adaptive Sliding Mode Control with Uncertainty and Disturbance Estimation	No	0.5	Yes	No	The sinusoidal signal of different frequencies has been applied and a triangle signal too
J. Li & Yang, 2014	Terminal Sliding Mode (TSM) Combined with A Sliding Mode Disturbance Observer (SMDO)	Yes	0.75	No	Yes	The sinusoidal signal of different frequencies applied and step signals too
Q. Xu, 2015	Second-Order Discrete-Time Terminal Sliding-Mode Control	Yes	0.25	Yes	No	A signal with varying amplitude and frequency applied
Al-Ghanimi, Zheng, & Man, 2015	Fast Non-Singular Terminal Sliding Mode FNSTSM	Yes	2.5	Yes	No	The sinusoidal signal of different frequencies has been applied and a triangle signal too
Al-Ghanimi, Zheng, & Man, 2017	FNSTSM with Perturbation Estimation	Yes	0.5	Yes	Yes	The sinusoidal signal of different frequencies has been applied and a triangle signal too

F.T (Finite Time) **H. M** (Hysteresis Modelling)

E% (The Error Percentage) **C. F** (Chattering Free System)

CHAPTER 3: METHODOLOGY

3.1 Introduction

Figure 3-1 shows the flowchart of the methodology to solve the control problem. In the beginning, the piezoelectric actuators control problem should be defined. According to the literature review, the PEAs suffer from the nonlinearity caused by creep, hysteresis and vibration or external disturbances. After studying the literature, the SMC has been selected to overcome these problems.

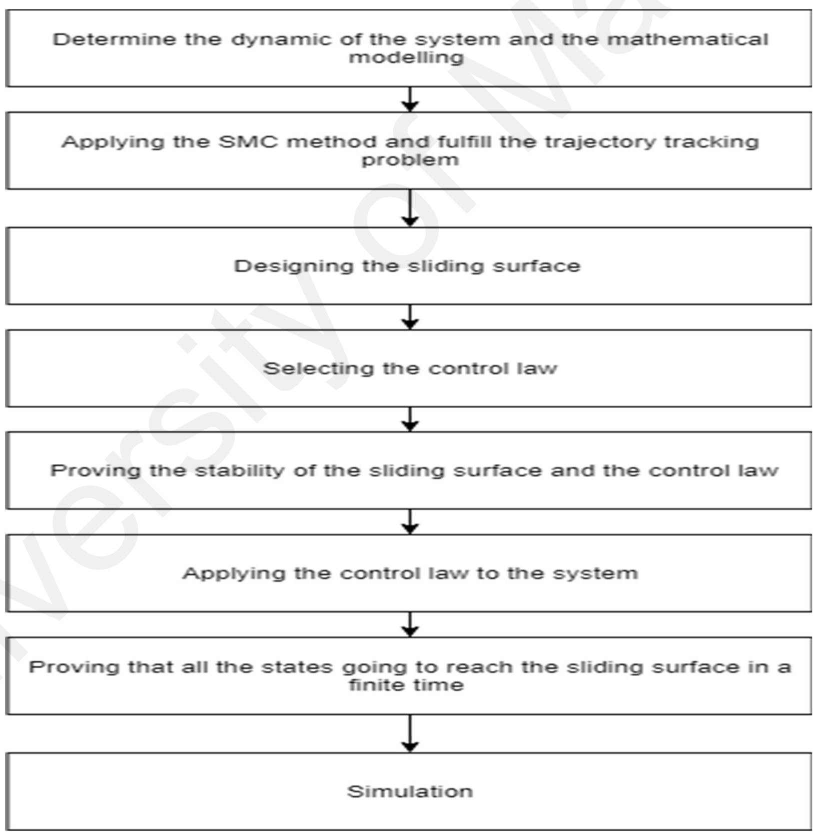


Figure 3-1: Methodology flowchart

To apply the sliding mode control and to prove that the control method overcomes the control problems, the mathematical modelling of the system should be constructed first. The mathematical model should be as easy and accurate as possible and contains all the nonlinearities of the system. The next step is to apply the control method to the mathematical model.

The main steps of designing the SMC controller are to choose the sliding surface and prove it stability using Hurwitz polynomial $p^2 + c_2 p + c_1 = 0$ then the construction of the control law or the control input using generalized Lyapunov stability theory.

To find the flowchart topics on the following mathematical solution, it is shown in the **<u>bold underlined</u>** font.

In this chapter, the mathematical modelling will be explained, and the mathematical control theory of the sliding mode control will be presented. The mathematical solution of the SMC will be explained, and the finite time system stability will be proved.

3.2 Mathematical Modelling

In (G.-Y. Gu et al., 2013) and (G.-Y. Gu, Zhu, Su, Ding, & Fatikow, 2015), **the mathematical modelling of the piezo-actuated stage has been developed.** The voltage-charge hysteresis, nonlinear electric behaviour and frequency response of the stage are included. Bearing in mind the voltage amplifier (v_A) drive the piezo-actuated stage in this general model as demonstrate in Figure 3-2.

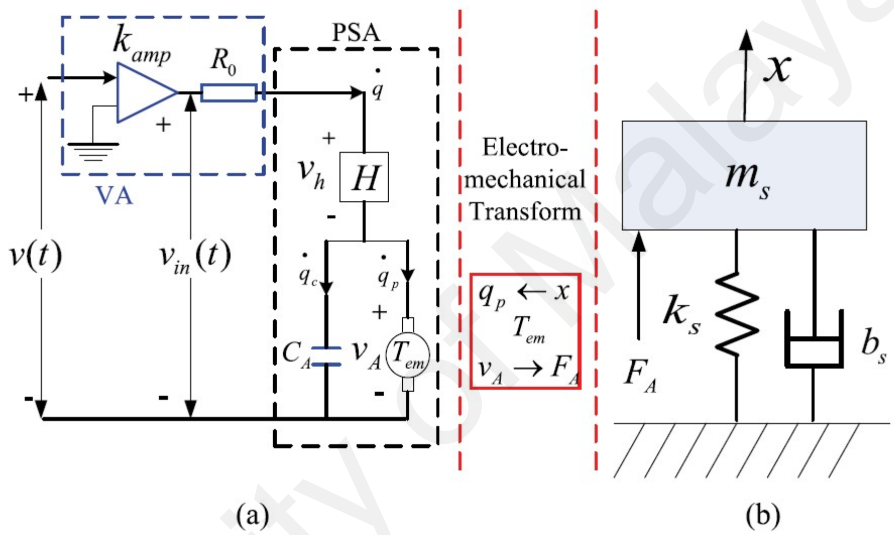


Figure 3-2: The mathematical modelling diagram (a) electrical (b) mechanical. piezoelectric stack actuators (PSA)

The PEA stage is described by these set of equations:

$$R_0\dot{q}(t) + v_h(t) + v_A(t) = k_{\text{amp}}v(t) \quad (1)$$

$$v_h(t) = H(q) \quad (2)$$

$$q(t) = q_c(t) + q_p(t) \quad (3)$$

$$v_A(t) = q_c(t)/C_A \quad (4)$$

$$q_p(t) = T_{\text{em}}x(t) \quad (5)$$

$$F_A = T_{\text{em}}\, v_A(t) \quad (6)$$

$$m_s\ddot{x}(t) + b_s\dot{x}(t) + k_sx(t) = F_A \quad (7)$$

Where:

$v(t)$	the control input voltage
k_{amp}	fixed gain of the v_A
R_0	the equivalent internal resistance of the v_A
T_{em}	electromechanical transducer with the transformer ratio, the piezoelectric effect
v_A	transducer related voltage
H	hysteresis effect
v_h	hysteresis effect voltage
C_A	total piezoelectric stack actuators (PSA) capacitances, electrically parallel with the transformer
q	the total charge in the PSA
$\dot{q}$	the resulting current flowing through the circuit
C_A	linear capacitance
q_c	the linear capacitance stored charge
q_p	the mechanical side transduced charge due to the piezoelectric effect
F_A	the stage moving part transduced force from the electrical side
m_s	the stage moving part equivalent mass
b_s	the stage damping coefficient
k_s	the stage mechanism stiffness
$x(t)$	the end-effector output displacement of the stage

From (1) – (7), the following equations can be found:

$$m_s\ddot{x}(t) + b_s\dot{x}(t) + (k_s + T_{\text{em}}^2 C_A)x(t) = T_{\text{em}} C_A q(t) \tag{8}$$

$$R_0 C_A \dot{q}(t) + q(t) - T_{\text{em}}x(t) = CA(k_{\text{amp}}v(t) - v_h(t)) \tag{9}$$

In practice, the equivalent internal resistance R_0= 0 (Peng & Chen, 2014). Thus, the electromechanical dynamic model (8) and (9) is reduced to

$$m_s\ddot{x}(t) + b_s\dot{x}(t) + k_s x(t) = f(t) + d(t) \tag{10}$$

where $f(t) = T_{\text{em}}k_{\text{amp}}v(t)$ and $d(t)$ represents the bounded disturbances with the nonlinear hysteresis effect $v_h(t)$ and external perturbation.

Remark: This model neglect the creep nonlinearity because it can be simply fixed with using any feedback controller PID controller as an example (Peng & Chen, 2014).

To simplify the control design, the **PEA stages system dynamics model** (10) is rewritten into the form (Q. Xu, 2017):

$$\ddot{x}(t) + a_1\dot{x}(t) + a_0 x(t) = b_0 u(t) + p(t) \tag{11}$$

where a_1, a_0, and $b_0$are the model coefficients, and *p(t)* represents the lumped disturbance of the nominal system. It is noteworthy that *p(t)* can be either matched (act on the control inputs example actuator faults) or unmatched disturbance (act on different channels from the control input example modelling uncertainties) (Ali & Langlois, 2013).

3.3 Mathematical Solution

3.3.1 Definitions and the lemma

Definition 1: in (1), the sign function is defined and the function $sig^a(x)$ is defined as $sig^a(x) = |x|^a sgn(x)$.

$$sgn(x) = \begin{cases} 1 & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -1 & ; \quad x < 0 \end{cases} \tag{1}$$

Definition 2: absolute function mathematical relation and the sign function is

$$|x| = xsgn(x).$$

Lemma: For any nonlinear system $\dot{x} = f(x), f(0) = 0,\ x \in D \subseteq \Re^n, x(0) = x_0$ by selecting constant coefficients r_1 to r_5 according to : $r_1 > 0, r_2 > 0, r_3 > 1, r_4 = 1 - \frac{1}{2r_3}, r_5 = 1 + \frac{1}{2r_3}$ in addition to the continuous Lyapunov function and radially unbounded function $V(x): \Re^n \to \Re^+ \cup \{0\}$ with the aim of $\dot{V}(x) \leq -r_1 V^{r_4}(x) - r_2 V^{r_5}(x)$. At that point, the equilibrium point $x = 0$ has a globally finite time stability. The system settling time can be found as $T_r \leq \pi r_3 (\sqrt{r_1 r_2})^{-1}$ (Sergey Parsegov, Andrey Polyakov, & Pavel Shcherbakov, 2013)

3.3.2 Piezoelectric actuator stage modelling representation and trajectory tracking

The mathematical model of the PEA stage system is represented as follow

$$\ddot{x}(t) + a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t) + p(t)$$

$$\begin{cases} x_1(t) = output\ displasment \\ \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -a_1 \dot{x_1}(t) - a_0 x_1(t) + b_0 u(t) + p(t) \end{cases} \tag{2}$$

Where a_1, a_0 and b_0 are the model coefficients, $u(t)$ is the control input, and *p(t)* is the lumped disturbance of the nominal system, a model of uncertainties and external disturbances which assumed to be time variant, and its and derivative upper bound are $(\begin{cases} ||p(t)|| \leq \eta_1 \\ ||\dot{p}(t)|| \leq \eta_2 \end{cases})$. Where $\eta_1$and η_2 are constants.

The **trajectory tracking errors can be stated** as

$\begin{cases} e_1 = x_1 - x_{1_d} \\ e_2 = x_2 - x_{2_d} \end{cases},$

By taking the derivative and considering (2):

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = -a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d}) + b_0 u(t)\ +\ p(t) - \dot{x}_{2_d} \end{cases} \tag{3}$$

To reach to the desired trajectory tracking the error model should reach zero ($e_i \rightarrow 0 \ \Longrightarrow\ x_i \rightarrow x_{i_d}$).

3.3.3 Design the TSMC

The first step is to decide which sliding surface should be used. For this design, **the following sliding surface will be defined as:**

$$s = \dot{e}_2(t) + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2) \tag{4}$$

Where,

c_1	+ constant
c_2	+ constant
α_1	+ number
α_2	+ number

and that $\begin{cases} \alpha_1 = N \\ \alpha_2 = \frac{N}{2-N} \end{cases}$ where N is a + number and less than one. (J. Li & Yang, 2014; Wang, Jiang, Chen, & Wu, 2018; S. Yu, Yu, Shirinzadeh, & Man, 2005)

Second, the **control input is selected:**

$$\begin{cases} u = \frac{1}{b_0}\left(u_r + u_{eq}\right) \\ \dot{u}_r = -k_1 sig^{\beta_1}(s) - k_2 sig^{\beta_2}(s) \ \ -\eta_2 sgn(s) \\ u_{eq} = -c_1 sig^{\alpha_1}(e_1) - c_2 sig^{\alpha_2}(e_2) \ + \dot{x}_{2_d} - (-a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d})) \end{cases} \quad (5)$$

Where

k_1	+ constant	u	Control law
k_2	+ constant	$\dot{u}_r$	Reaching law derivative
β_1	+ number	u_{eq}	Equivalent law
β_2	+ number		

Note that, $\rho_3 > 1$, $\beta_1 = 1 - \frac{1}{\rho_3}$, $\beta_2 = 1 + \frac{1}{\rho_3}$, and $\left\|\dot{p}(t)\right\| \leq \eta_2$

Theorem: from the tracking errors of the system defined in (3) and the sliding surface in (4), and the control input selected in (5) then the system is finite time stable. Therefore, the system state variables reach stability and this stability is a finite time which means the system error reaches zero in finite time. Accordingly, the output displacement of the PEA system will follow the desired trajectory tracking.

Proof: the proof should confirm the stability of the system at a finite time. And to do so, the: A) the sliding surface, and B) the control input, must be proved to be stable at a finite time.

Third, to **prove the sliding surface finite time stability**, $c_1, and\ c_2$ must be chosen to be Hurwitz polynomial to the following equation $c_0 p^2 + c_2 p + c_1 = 0$ i.e. all the coefficients must be positive or all the coefficients be negative. ■A

(SE Parsegov, AE Polyakov, & PS Shcherbakov, 2013; Zhang & Panda, 1999)

Forth, to **prove the control input finite time stability**, $\dot{e}_2(t)$ is substituted in the sliding surface in this way the control input is applied to the system.

$$s = \dot{e}_2(t) + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$$

$s = -a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d}) + b_0 u(t)\ +\ p(t) - \dot{x}_{2_d} + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$ substitute the value of u:

$s = -a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d}) + b_0 \frac{1}{b_0}\left(u_r + u_{eq}\right)\ +\ p(t) - \dot{x}_{2_d} + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$ substitute the value of u_{eq}:

$$s = -a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d}) + b_0 \frac{1}{b_0}\left(u_r + (-c_1 sig^{\alpha_1}(e_1) - c_2 sig^{\alpha_2}(e_2)\ + \dot{x}_{2_d} - (-a_1(e_2 + x_{2_d}) - \ a_0\ (e_1 + x_{1_d})))\right)\ +\ p(t) - \dot{x}_{2_d} + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$$

Please note that u_{eq} selected to have these vales to reach to the result bellow in (6).

$$s = u_r + p(t)\ \rightarrow \dot{s} = \dot{u}_r + \dot{p}(t) \quad (6)$$

The Lyapunov candidate function $V(x) = \frac{1}{2}s^2$ has the Lyapunov function conditions mentioned above in the Lemma, and so:

$\dot{V}(x) = s\dot{s}$, substitute (6):

$\dot{V}(x) = s\left(\dot{u}_r + \dot{p}(t)\right)$, substitute ($\dot{u}_r$):

$$\dot{V}(x) = s\left(-k_1 sig^{\beta_1}(s) - k_2 sig^{\beta_2}(s)\ - \eta_2 sgn(s) + \dot{p}(t)\right) \quad (7)$$

from to definitions 1 and 2 at section 3.3.1, (7) can be written as:

$$\dot{V}(x) = -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} - \eta_2|s| + s\dot{p}(t) \tag{8}$$

Please note that $\dot{u}_r$ selected to have these vales to reach to the result above in (8).

It is known that, if the absolute value of a term is taken, it can be written as bellow

$s\dot{p}(t) \leq |s\dot{p}(t)|$

So, $\dot{V}(x) \leq -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} - \eta_2|s| + |s\dot{p}(t)|$

$\dot{V}(x) \leq -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} - \eta_2|s| + |s|\big||\dot{p}(t)|\big|$

$\dot{V}(x) \leq -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} + |s|(-\eta_2 + \big||\dot{p}(t)|\big|)$

$\big||\dot{p}(t)|\big| \leq \eta_2$ so (8) can be written as:

$$\dot{V}(x) \leq -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} \tag{9}$$

Note that, this team $|s|(-\eta_2 + \big||\dot{p}(t)|\big|)$ is negative or 0 which mean it does not change the fact that $\dot{V}(x) \leq -value$ in (9), due to that it can be cancelled.

The Lyapunov candidate function can be written as $|s| = \sqrt{2}(V(x))^{\frac{1}{2}}$ and by substitution in (9)

$$\dot{V}(x) \leq -k_1\sqrt{2}(V(x))^{\frac{\beta_1+1}{2}} - k_2\sqrt{2}(V(x))^{\frac{\beta_2+1}{2}}$$

From the lemma $\dot{V}(x) \leq -r_1V^{r_4}(x) - r_2V^{r_5}(x)$ and by the control law the desired $\dot{V}(x)$ has been reached which proves the finite time stability of the control input. ■B

From ■A and ■B, the system (3) is stable at a finite time and **<u>all the states reach stability (on the sliding surface) in a finite time.</u>** The settling time is $T_r \leq$

$\pi\rho_3\left(\sqrt{\rho_1\rho_2}\right)^{-1}$ where $\rho_1 = k_1\left(\sqrt{2}\right)^{\beta_1+1}, \rho_2 = k_2\left(\sqrt{2}\right)^{\beta_2+1}, \rho_3 > 1, \beta_1 = 1 - \frac{1}{\rho_3}, \beta_2 = 1 + \frac{1}{\rho_3}$ ■

Note that s and the derivative $\dot{s}$ does not result in terms with negative (fractional) powers $\dot{s} = -k_1 sig^{\beta_1}(s) - k_2 sig^{\beta_2}(s) - \eta_2 sgn(s) + \dot{p}(t)$, and it is easily seen that the control $u = \frac{1}{b}\left(u_r + u_{eq}\right)$ and $\dot{V}(x) = -k_1|s|^{\beta_1+1} - k_2|s|^{\beta_2+1} - \eta_2|s| + s\dot{p}(t)$ does not contain any terms with negative (fractional) power, which mean there will be no singularity (X. Yu, Zhihong, Feng, & Guan, 2002).

Note that the chattering happens in the SMC due to the discontinuance switching of sliding mode, and to solve this problem and reduce the chattering, the sigmoid (sig) function has been used in $s = \dot{e}_2(t) + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$. This method is called continuous TSMC (J. Li & Yang, 2014; Wang et al., 2018; S. Yu et al., 2005) .

3.4 The Simulation Setup

The mathematical model of the PEA stage system is as follow

$$\ddot{x}(t) + a_1\dot{x}(t) + a_0 x(t) = b_0 u(t) + p(t)$$

$$\begin{cases} x_1(t) = output\ displasment \\ \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -a_1\dot{x_1}(t) - a_0 x_1(t) + b_0 u(t) + p(t) \end{cases}$$

tracking of $x_1(t)$ the output displacement, so trajectory tracking errors are defined as

$$\begin{cases} e_1 = x_1 - x_{1_d} \\ e_2 = x_2 - x_{2_d} \end{cases},$$

and by derivative:

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = -a_1(e_2 + x_{2_d}) - a_0\,(e_1 + x_{1_d}) + b_0 u(t) + p(t) - \dot{x}_{2_d} \end{cases}$$

The definition of a sliding surface for this design is

$$s = \dot{e}_2(t) + c_1 sig^{\alpha_1}(e_1) + c_2 sig^{\alpha_2}(e_2)$$

and the control input is

$$\begin{cases} u = \frac{1}{b}(u_r + u_{eq}) \\ \dot{u}_r = -k_1 sig^{\beta_1}(s) - k_2 sig^{\beta_2}(s) - \eta_2 sgn(s) \\ u_{eq} = -c_1 sig^{\alpha_1}(e_1) - c_2 sig^{\alpha_2}(e_2) + \dot{x}_{2_d} - (-a_1(e_2 + x_{2_d}) - a_0\,(e_1 + x_{1_d})) \end{cases}$$

The software has been used in this project is MATLAB and Simulink to simulate the system. The control coefficients have been selected as shown in Table 3-1:

Table 3-1: The control coefficients of the simulation

Parameter	Value
a_1	183.7
a_0	1.076×10^7
b_0	5.876×10^7
$p(t)$	$0.2\cos(x_1) + 0.1\sin(x_2)$
c_1	6
c_2	6
α_1	7/6
α_2	7/5
k_1	1
k_2	1
η_2	0.2
β_1	7/8
β_2	9/8
$x_1(0)$	5 0 (for step signals experiment)
$\dot{x}_1(0)$	-2 0 (for step signals experiment)

The Simulink diagram which designed for this simulation shown in Figure 3-3.

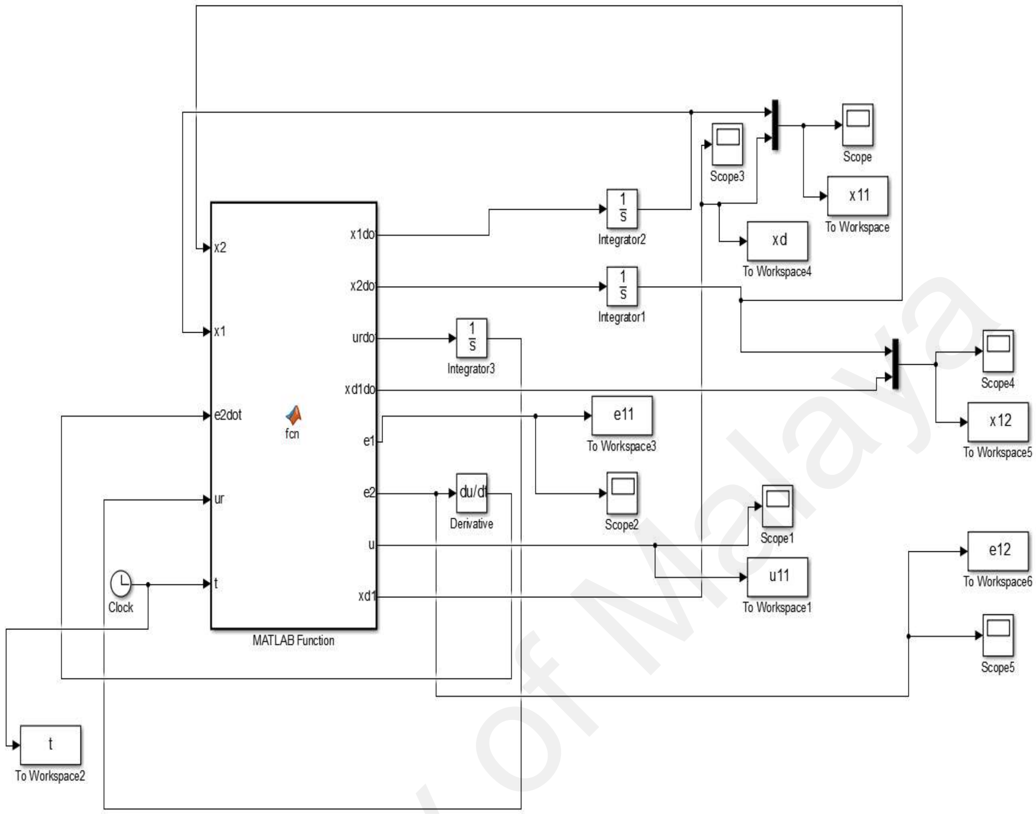


Figure 3-3: The simulation layout

Where:

t	The time
e1	e_1 trajectory tracking error 1
e11	e_1 trajectory tracking error 1 (showing the error 1)
e12	e_2 trajectory tracking error 2 (showing the error 2)
e2	e_2 trajectory tracking error 2
e2dot	$\dot{e}_2$ the derivative of trajectory tracking error 2
u	u is the control input
u11	u is the control input (showing the control input)
ur	u_r reaching law
urdo	$\dot{u}_r$ the derivative of reaching law
x1	x_1 the displacement
x11	x_1 the displacement (showing the real displacement and the desired displacement)
x12	x_2 the derivative of x_1 (showing the real speed and the desired speed)
x1do	$\dot{x}_1$ the derivative of x_1
x2	x_2 the derivative of x_1
x2do	$\dot{x}_2$ the derivative of x_2
xd	x_{1_d} the desired displacement (showing the desired displacement)
xd1	x_{1_d} the desired displacement
xd1do	$\dot{x}_{1d}$ the derivative of x_{1d}

CHAPTER 4: RESULTS & DISCUSSION

4.1 Introduction

In chapter three, the mathematical solution which proves the system finite time stability has been shown. This chapter discusses and analyses the simulation results. First, fourteen different step signals will be applied to the system and the system response will be investigated. Second, a complex signal will be given as a trajectory and the system trajectory tracking will be studied.

4.2 Step Signals

In this simulation, 14 different step signals applied to the system to examine the transient response capability. The step signals range from 1 to 100 micrometres. The initial value of the system is 0 micrometre. The graph (Figure 4-1) shows the system step response, and the table 4-1 illustrates the overshoot, the settling time and the rise time for all the 14 signals and the graphs (Figure 4-2 and Figure 4-3) show the date. According to the graphs and the table, the controller results are smooth and chatter free. The convergence (reaching to 0 error) using this controller is reached at a finite time.

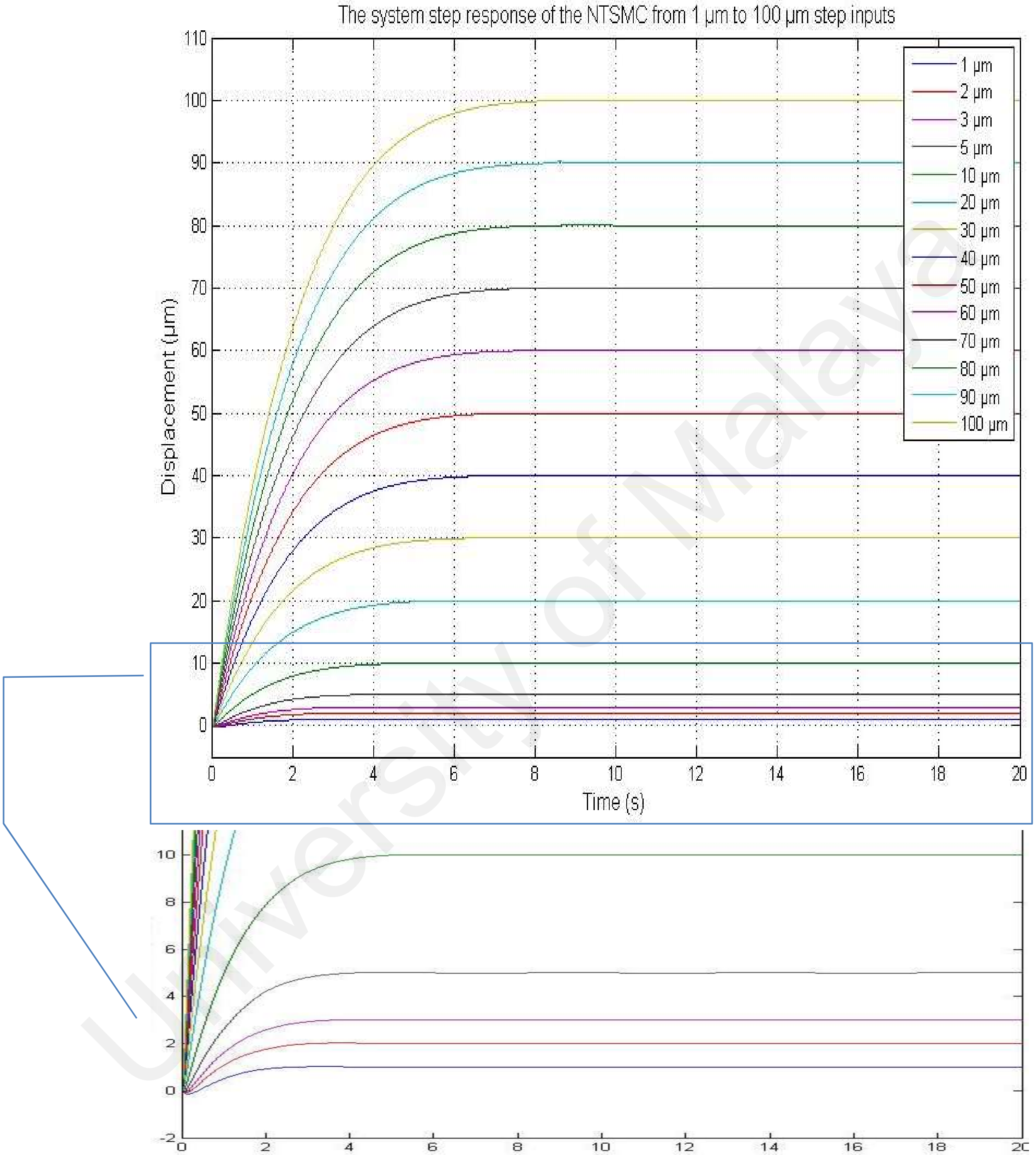


Figure 4-1: The step response of the system from 1 µm to 100 µm

Table 4-1: 1 to 100 µm step input signals results

Displacement (µm)	Settling Time (S)	Overshoot (%)	Rise Time (S)
1	3.75	2.8	1.52
2	2.71	0.975	1.77
3	2.89	0.79	1.95
5	3.26	0.422	2.16
10	3.91	0.227	2.5
20	4.45	0.09	2.85
30	4.85	0.063	3.08
40	5.11	0.054	3.24
50	5.33	0.028	3.38
60	5.53	0.03	3.49
70	5.68	0.022	3.59
80	5.75	0.025	3.68
90	5.91	0.021	3.75
100	6.06	0.02	3.82

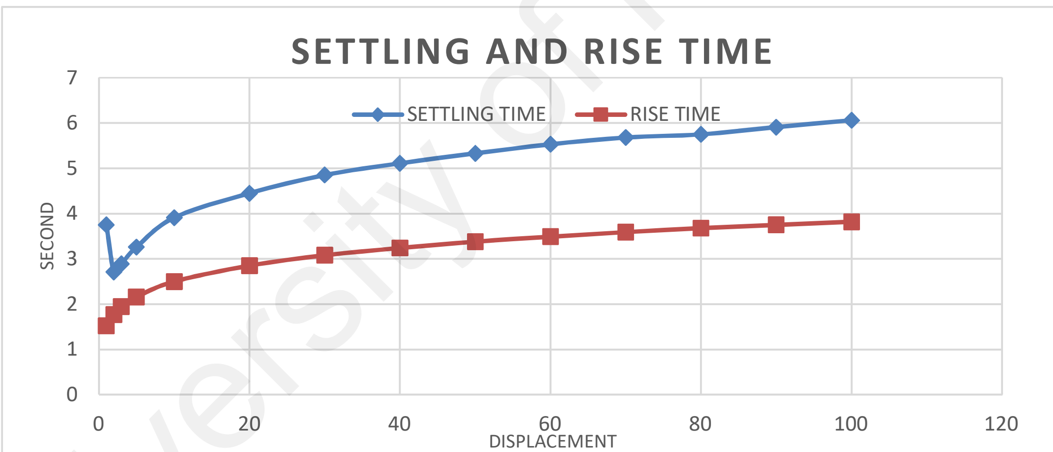


Figure 4-2: Settling and rise time results

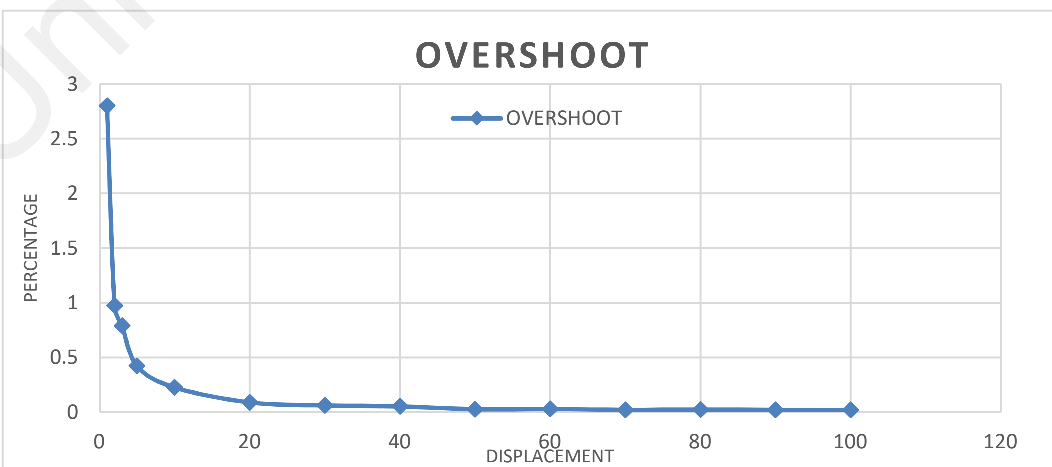


Figure 4-3: Overshoot result

4.2.1 Discussion

When 1 μm step signal has been applied to the controller, the overshoot of the system response was the maximum at 2.8% but the rising time was the minimum at 1.52s. As the step signal increases, the overshoot decreases but the rising time growths. For example, at 100 μm step signal, the overshoot of the system response was the minimum at 0.02% but the rising time was the maximum at 3.82s. These results show a direct proportion between the step signal value and the rising / settling time, in contrast, it demonstrates inverse proportion between the step signal value and the overshoot percentage. The controller has been responded to these signals and reaches stability in finite time with very low overshoot and without chattering or steady-state error

4.3 Complex Signal Tracking

In this simulation, a complex wave of 38 μm peak-to-peak (p-p) has been set as a desired trajectory for the system to examine the system trajectory tracking and the tracking errors. The wave equation is $12\sin(2\pi 0.5t) + 6\sin(10\pi 0.5t) + \sin(9\pi t)$. The initial value of the system is 5 micro meters. The graphs (Figure 4-4 to Figure 4-7) show the results and the table 4-2 illustrates the maximum error, Root Mean Squared Error and Mean Absolute Error. According to the graphs and the table, the controller results are smooth and chattering free. The convergence using this controller is reached in finite time.

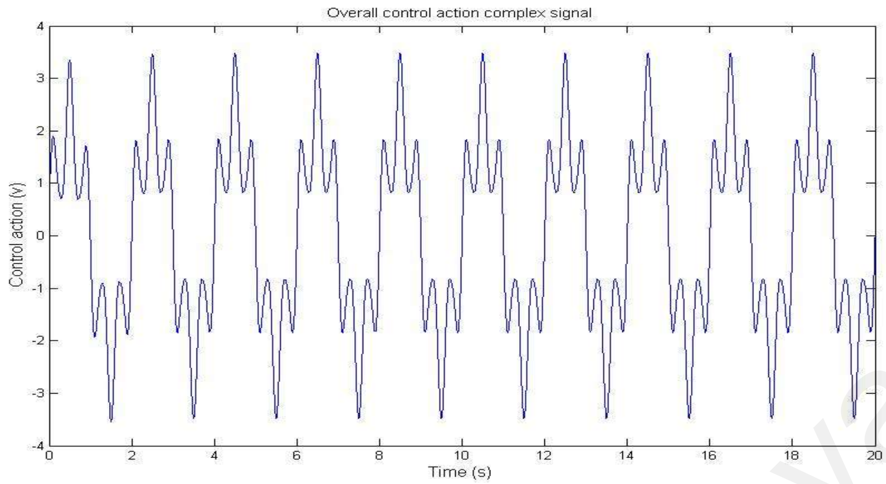


Figure 4-2: The control input

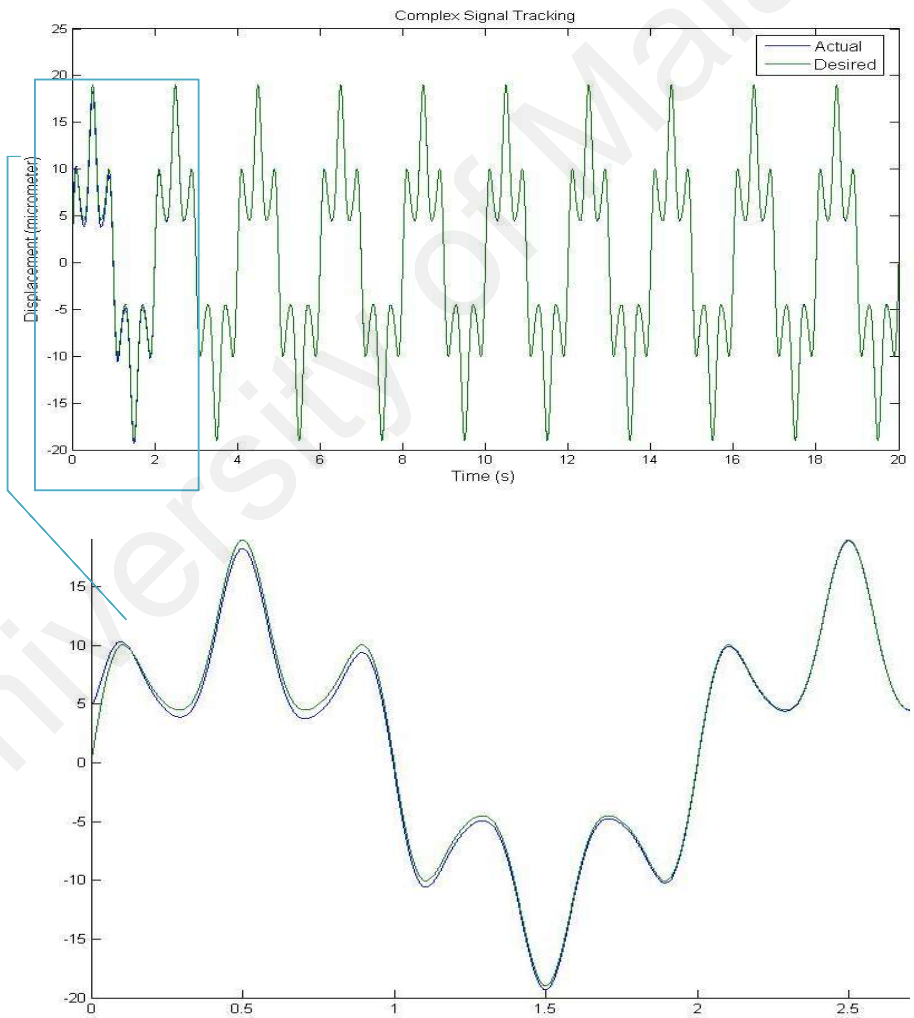


Figure 4-3: The trajectory tracking

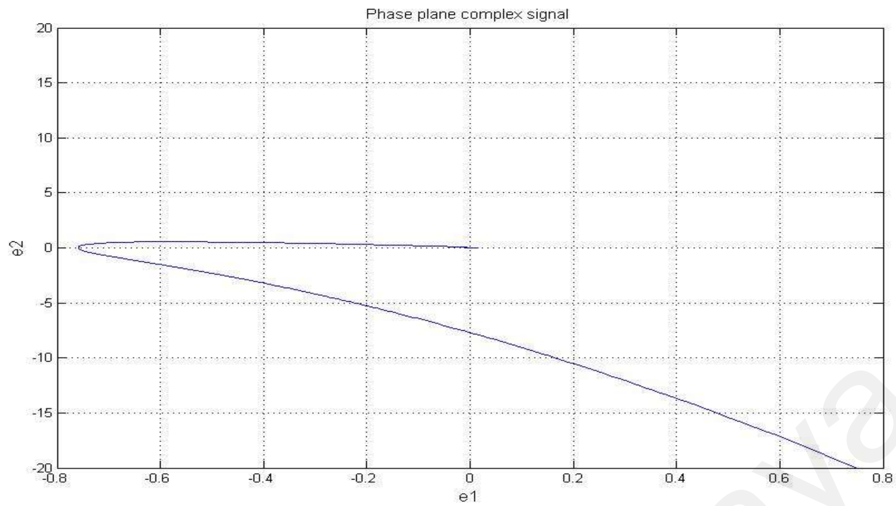


Figure 4-6: The phase plane

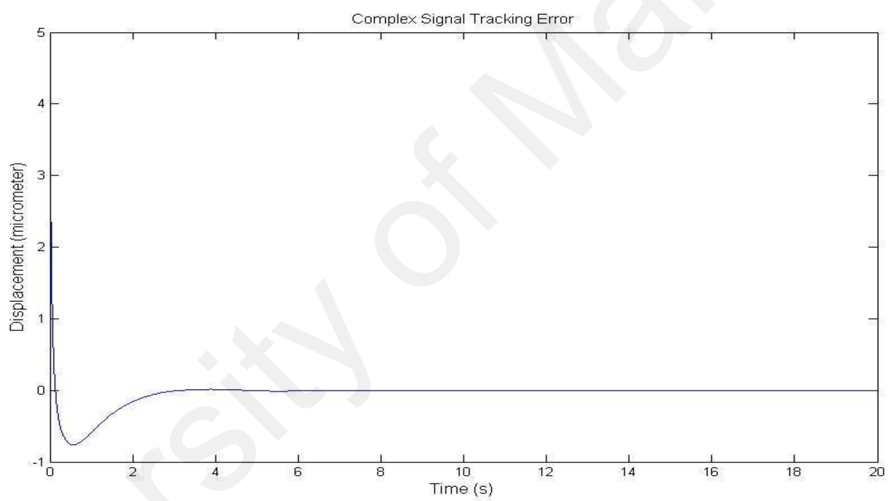


Figure 4-7: The trajectory tracking error

Table 4-2: Summary of the complex signal tracking errors

Performance	The response of TSMC
Max. error (μm)	0.7604 2.001%
RMSE (μm) Root Mean Squared Error	0.2208 0.581%
MAE (μm) Mean Absolute Error	0.0624 0.164%

4.3.1 Discussion

To increase the challenge on the controller and to prove the trajectory tracking control furthermore, a complex wave $12\sin(2\pi 0.5t) + 6\sin(10\pi 0.5t) + \sin(9\pi t)$ has been applied to the system as a desired trajectory. The results show a very good response of the system. The maximum error, RMSE and MAE are 2%, 0.58 and 0.16% respectively. The complex trajectory tracking is precise and chatter free. Additionally, according to the phase plane graph, the error has been reached to zero in finite time. The system is stable in finite time.

4.4 Summary

Considering the simulation, the terminal sliding mode controller has shown good results when it is applied to this system. Two experiments have been simulated using Simulink to evaluate the controller performance.

First, the positioning control performance has been examined by giving the controller 14 different step signals. When 1 μm step signal has been applied to the controller, the overshoot of the system response was the maximum at 2.8% but the rising time was the minimum at 1.52s. And as the step signal increases, the overshoot decreases but the rising time growths. For example, at 100 μm step signal, the overshoot of the system response was the minimum at 0.02% but the rising time was the maximum at 3.82s. These results show a direct proportion between the step signal and the rising and settling time, in contrast, it demonstrates inverse proportion between the step signal and the overshoot percentage. The controller has been responded to these signals and stability has been reached in finite time with very low overshoot and without chattering or steady state error.

Second, to increase the challenge on the controller and to prove the trajectory tracking control, a complex wave $12\sin(2\pi 0.5t) + 6\sin(10\pi 0.5t) + \sin(9\pi t)$ has been applied to the system as a desired trajectory. The results show a very good trajectory tracking of the system. The maximum error, RMSE and MAE are 2%, 0.58 and 0.16% respectively. The complex trajectory tracking control is precise and chatter free. Additionally, according to the phase plane graph, the zero error has been reached in finite time. The system is stable in finite time.

The error has reached as low as 2×10^{-4}% with time. The tracking is accurate even with the defined equation $0.2\cos(x_1) + 0.1\sin(x_2)$ as the lumped disturbance. Even if the disturbance is increased to $20\cos(x_1) + 10\sin(x_2)$, there is no significant change in the RMSE.

CHAPTER 5: CONCLUSION

5.1 Project Summary

The piezoelectric actuator is a device that performs very small displacement. The mean part of this device is the piezoelectric materials which are intelligent materials that can expand once an electrical charge is applied. The piezoelectric actuator suffers from the inherent non-linearity because of the hysteresis, creep and vibrations effects. The hysteresis is the core and the most significant nonlinearity which needs to be overcoming by the controller, as it causes a displacement error as high as 15% of the total range. One strategy to solve the nonlinearity problem is to use Sliding Mode Control (SMC) method due to the simple design steps, high robustness and low sensitivity to disturbance.

The first objective was designing a controller for piezoelectric actuators using SMC method to overcome the robustness and accuracy issue. SMC is a robust control method which means it is designed to function accurately even with noise or disturbance. The disturbances are unavoidable because the mathematical modelling and the actual plan always has some discrepancies. By using the mathematical modelling which includes all the disturbances, the uncertainties and disturbances can be eliminated. As a result, the system using the proposed controller is robust and accurate, which has been tested and verified using the step signal and the complex signal simulation.

The second objective was applying the TSMC concept to overcome the global infinite time stability and reach to stability in a finite time. To clarify this point, a finite time stability means that all the system states reach stability in finite time i.e. reach convergence or zero error in a finite time. The main advantage of the finite time stability is that it reaches stability and zero error faster than the global infinite time stability. The mathematical solution shows the steps to design the controller and the proof of the finite time stability according to the lemma and the Lyapunov stability theorem. As a result, the system finite time stability has been proved mathematically and in the simulation.

The third objective was reaching to chattering free non-singular system. Chattering happens in the sliding mode control due to the discontinuance switching of the sliding mode. To solve this problem and reduce the chattering, the continues terminal sliding mode method has been used by applying the sigmoid (sig) function in sliding surface function. It is also important to note that the derivative of the sliding surface along the system dynamics does not result in terms with negative (fractional) powers, which mean there will be no singularity. The chattering free non-singular design increases the service life of the product and increase the accuracy of the system.

In the end, the controller is NTSMC reach stability in finite time. The project objectives have been accomplished, the non-linearity of this system has been minimized and the simulation results demonstrations a smooth precise operation of the system.

5.2 Future Work

To reduce the convergence rate (the time needed to reach to zero error), many other SMC methods can be implemented.

For example, Terminal Sliding Mode (TSM) together with a Sliding Mode Disturbance Observer (SMDO), to improve the robustness of the TSMC. The SMDO is adopted to estimate the bounded disturbances and uncertainties at a finite time.

Also, Adaptive Sliding Mode Control with Uncertainty and Disturbance Estimation. An adaptive law of the control gain is developed to adjust the control gain in accordance with the tracking result. The adaptive rules allow predefining the magnitude of tracking error bound in advance, which is attractive for practical applications.

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APPENDIX A: THE STEP SIGNAL PROGRAM

```
function [x1dot,x2dot,urdot,xd1dot,e1,e2,u,xd1]  = fcn(x2,x1,e2dot,ur,t)

%% system configurations
A1=183.7;
a0=1.076*(10^7);
b0=5.876*(10^7);
f=-A1*x2-a0*x1;
g=b0;
k12=1;
c2=6;
c1=6;
d=0.2*cos(x1)+0.1*sin(x2);
etta2=0.2;

% N;
%%alpha1=N=alpha1s/alpha1m;
%%alpha2=N/(2-N)=alpha2s/alpha2m;

alpha1s=7;
alpha1m=6;
alpha2s=7;
alpha2m=5;

%% p3=8
%%beta1=1-(1/p3);
%%beta2=1+(1/p3);

beta1s=7;
beta1m=8;
beta2s=9;
beta2m=8;

%% the desired trajectory
xd1=10;
xd1dot=0;
xd1dotdot=0;


% tracking
e1=x1-xd1;
e2=x2-xd1dot;

%% sliding surface
s=e2dot+c1*((sign(e1))*(nthroot(abs(e1),alpha1m))^alpha1s)+c2*((sign(e2))*(nth
root(abs(e2),alpha2m))^alpha2s);

%% control input

urdot=(-
k12*(((sign(s))*(nthroot(abs(s),beta1m))^beta1s)+((sign(s))*(nthroot(abs(s),be
ta2m))^beta2s)))-etta2*sign(s);

ueq=-c1*((sign(e1))*(nthroot(abs(e1),alpha1m))^alpha1s)-
c2*((sign(e2))*(nthroot(abs(e2),alpha2m))^alpha2s)-(f)+xd1dotdot;

u=(1/g)*(ur+ueq);

%% the system
x1dot=x2;
x2dot=f+g*u+d;
```

APPENDIX B: THE COMPLEX SIGNAL PROGRAM

```
function [x1dot,x2dot,urdot,xd1dot,e1,e2,u,xd1]  = fcn(x2,x1,e2dot,ur,t)

%% system configurations
A1=183.7;
a0=1.076*(10^7);
b0=5.876*(10^7);
f=-A1*x2-a0*x1;
g=b0;
k12=1;
c2=6;
c1=6;
d=0.2*cos(x1)+0.1*sin(x2);
etta2=0.2;

%% N;
%%alpha1=N=alpha1s/alpha1m;
%%alpha2=N/(2-N)=alpha2s/alpha2m;

alpha1s=7;
alpha1m=6;
alpha2s=7;
alpha2m=5;

%% p3=8
%%beta1=1-(1/p3);
%%beta2=1+(1/p3);

beta1s=7;
beta1m=8;
beta2s=9;
beta2m=8;

%% the desired trajectory

xd1=12*sin(2*pi*0.5*t)+6*sin(10*pi*0.5*t)+sin(9*pi*t);
xd1dot=0.5*(24*pi*cos(2*0.5*pi*t)+60*pi*cos(10*0.5*pi*t))+9*pi*cos(9*pi*t);
xd1dotdot=0.25*(-48*pi*pi*sin(2*pi*0.5*t)-600*pi*pi*sin(10*pi*0.5*t))-
81*pi*pi*sin(9*pi*t);


% tracking
e1=x1-xd1;
e2=x2-xd1dot;

%% sliding surface
s=e2dot+c1*((sign(e1))*(nthroot(abs(e1),alpha1m))^alpha1s)+c2*((sign(e2))*(nth
root(abs(e2),alpha2m))^alpha2s);

%% control input

urdot=(-
k12*(((sign(s))*(nthroot(abs(s),beta1m))^beta1s)+((sign(s))*(nthroot(abs(s),be
ta2m))^beta2s)))-etta2*sign(s);

ueq=-c1*((sign(e1))*(nthroot(abs(e1),alpha1m))^alpha1s)-
c2*((sign(e2))*(nthroot(abs(e2),alpha2m))^alpha2s)-(f)+xd1dotdot;

u=(1/g)*(ur+ueq);

%% the system
x1dot=x2;
x2dot=f+g*u+d;
```