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The Sequential Probability Ratio Test (SPRT) in Feature Extraction and Expert Systems in Nuclear Material Management

Thomas Jay Harrison
University of Tennessee, Knoxville

To the Graduate Council:

I am submitting herewith a thesis written by Thomas Jay Harrison entitled "The Sequential Probability Ratio Test (SPRT) in Feature Extraction and Expert Systems in Nuclear Material Management." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Nuclear Engineering.

J. Wesley Hines, Major Professor

We have read this thesis and recommend its acceptance:

Belle R. Upadhyaya, Laurence F. Miller

Accepted for the Council:
Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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THE SEQUENTIAL PROBABILITY RATIO TEST (SPRT) IN FEATURE EXTRACTION AND EXPERT SYSTEMS IN NUCLEAR MATERIAL MANAGEMENT

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Thomas Jay Harrison
May 2004

Dedication

This thesis is dedicated to my parents, Mary and Bob Campbell. Their love, support, and encouragement have made all the difference and have brought me to this point in my life.

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This thesis would not be possible without the following contributors:

- Joseph Bowling of the University of Tennessee, who has conducted research on this project from the expert system end. The results presented in this thesis would be at best marginally useful without his application.
- Chris Pickett and Rick Oberer of the Y-12 National Security Complex, who have provided the framework and overall direction for this research project. The research itself would not exist without their personal involvement and vision.
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Abstract

The Y-12 National Security Complex in Oak Ridge, TN, maintains the nation's stockpile of highly enriched uranium (HEU) for use in nuclear weapons. A proposed system for monitoring the HEU is the Continuous Automated Vault Inventory System (CAVIS), which uses radiation and mass detectors. Radionuclides decay stochastically (in a random manner that can be approximated by statistical analysis) and normal electronics and computer failures are inevitable. Therefore the system can and does experience spurious alarms arising from normal decay characteristics and system operation and not from material removal.

To reduce the spurious alarms and their associated costs, CAVIS operators desire a system to monitor the monitoring system. This system will alert operators and security personnel in the event of an actual alarm and assist operators in diagnosing and correcting false alarms. The system of choice for this task is an expert system, using a knowledge base to diagnose and propose remedies for system malfunctions.

The expert system requires information on which to base its decisions, and thus uses a feature extraction system to provide it the pertinent data. This feature extraction system uses the Sequential Probability Ratio Test (SPRT) to examine the radiation detector data and identify departures from the expected signal characteristics. The SPRT thus proves useful in the management of nuclear material. In addition to the SPRT, the feature extraction system uses several other analytical methods including statistical runs tests.

This thesis outlines and explains the development and use of the SPRT and the other methods for the feature extraction and the use of the feature extraction system. Although the CAVIS uses radiation and mass detectors, this research uses only the radiation detector information as its basis for monitoring and feature extraction. This research shows that radiation detector signals, when collected conscientiously (without changing the statistical characteristics of the measured attribute), do meet the requirement of normality necessary for the correct SPRT operation.

Further, this thesis applies the feature extraction system with simulated and real data as collected in a laboratory setting. These applications show that the feature extraction system is an excellent choice for use in a nuclear material management situation.

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1.0 Introduction and Organization

This chapter gives a brief introduction and outline of the thesis. It gives some background information as necessary and describes the general contributions made by this research. It then describes the content and purpose of each chapter.

1.1 Introduction

The Y-12 National Security Complex in Oak Ridge, TN, maintains the nation's stockpile of highly enriched uranium (HEU) for use in nuclear weapons. In general, HEU is any amount of uranium in which the fissile isotope ^{235}U has been enriched from its natural 0.7% by weight to over 20% by weight. In nuclear weapon terminology, HEU refers to uranium in which ^{235}U has been enriched to over 90%. Y-12 proposes to accomplish this stewardship mission through the Continuous Automated Vault Inventory System (CAVIS). The CAVIS continuously monitors the gamma activity and mass of the stockpiled material to maintain a complete record of its location. If the activity or mass values change, CAVIS alerts the operators and security personnel to a potentially serious incident.

However, this system has potential problems. Since radionuclides decay in a stochastic (specifically, a Poisson) process, this renders the system susceptible to false alarms in the gamma detector system. Moreover, electronics and computer failures contribute to the false alarm occurrence. This mixture of random events and certain eventual equipment degradation complicates the task of responsible stewardship.

The CAVIS operators thus require a system to monitor their monitoring system. This system is designed to alert the operators and personnel in the event of an actual alarm, and in the case of false alarms, will assist the operators in diagnosing and correcting any responsible system faults.

This monitoring system bases its diagnostic and corrective algorithms on human knowledge, organized and implemented as an expert system. The expert system will use rules and inferential logic in conjunction with sensor status information. However, the expert system will require more than just simple system status; it will require several features extracted from both current and past system status. This iterative and progressive feature extraction system architecture simplifies the expert system computation by performing all necessary computations.

1.2 Contributions

This research contributes to the field of nuclear material management by combining common statistical methods (runs tests and hypothesis testing) with artificial intelligence methods (feature extraction and expert systems) within the framework of nuclear material management. The fusion of the two fields provides a robust and flexible system for diagnosing and correcting hardware, software, and security issues in material storage.

The research may also be used in other fields such as manufacturing. The only requisite is that the random variables measured have essentially static parameters. However, this research may be modified to account for dynamic parameters.

1.3 Organization

After the introduction and organization (chapter 1), this thesis presents information on feature extraction (chapter 2) including the sequential probability ratio test and control charts. The literature survey for the SPRT states several characteristics of the SPRT, such as its superiority over other testing methods and its equivalence to Bayes solutions. It also includes a review of a paper on its current use in a related nuclear material monitoring situation and a review of other industrial applications. The feature extraction portion of the chapter discusses different statistical methods beyond hypothesis testing used to analyze data.

After the literature survey is a radioactive decay review (chapter 3). Within this section the basics of statistical distributions and radiation are explained so that several basic assumptions that form the basis for this research are validated. Following the radiation review the section revisits the SPRT more explicitly with a derivation and an explanation of the necessary terms and concepts. This section also includes detailed explanations of alternative methods for hypothesis testing.

Next the thesis moves to methodology (chapter 4). In this section the physical setup of the basic storage problem is demonstrated. Next it explains the particular values and methods for both the SPRT and the feature extraction are given and explained with pertinent parametric studies. The actual use of the SPRT on provided Y-12 data sets follows.

After methodology the results (chapter 5) appear. This section lists the outcomes of applying the SPRT and feature extraction to data sets, both real and fabricated. The section explains both the physical and statistical attributes of the real data.

Conclusions and future work (chapter 6) follow the results. In the conclusions the effectiveness of this research is addressed. Any problems in research application are also addressed. Any research not completed or analyzed before the completion of this thesis is described in future work. This also includes any improvements not incorporated into the current research.

After future work is the references (chapter 7) section. All computer code and materials not present in the main body of the thesis are present in the final section, the appendix (chapter 8).

In the next chapter the thesis deals with feature extraction. It will give a brief description of why feature extraction is attractive and then supply background and description of some feature extraction methods. These methods include the sequential probability ratio test (SPRT) and statistical controls charts.

2.0 Feature Extraction

Since this research requires the extraction of information from collected data samples, a review of feature extraction proves useful as well. The term "feature extraction" is rather nebulous; however, the concept is fairly straightforward. This chapter explains some feature extraction methods.

Feature extraction in this research refers to the gleaning of important and useful information from relatively unimportant and possibly obscuring data. The true state of a system may not be evident from an initial examination of the data collected; it may be necessary to sort through to find underlying trends and conditions.

For example, measuring an automobile's position at regular intervals gives only information about its position with respect to time. By doing analysis on this position information (that is, extracting features from the data) its velocity and acceleration may be determined. Conversely, by measuring the acceleration of a body at regular intervals gives only information about its acceleration with respect to time. By doing analysis of this acceleration data (that is, extracting features from the data) its velocity and position may be determined. This is the nature of feature extraction.

2.1 The SPRT

As will be shown in chapter 3, the radiation characteristic of stored HEU follows a Gaussian (normal) distribution. Therefore the common and effective techniques of analyzing normal distributions would be quite helpful. For reference when dealing with distributions, in this thesis the terms "Gaussian" and "normal" are used interchangeably.

When measuring any variable with any arbitrary probability density function, after collecting a sufficient number of sets of the data the means of those data sets will fall into a normal distribution. This characteristic of random variables is known as the Central Limit Theorem. Thus, the situation of measuring continuous normal random variables has extensive history in manufacturing and industry.

2.1.1 Sequential Analysis

In *Sequential Analysis*, Abraham Wald states that the problem of hypothesis testing is actually the problem of parameter estimation for a given random variable [Wald, 1947]. A given distribution may have any number of parameters (typical distributions such as the binomial, Poisson, and normal distributions have two [n, π], one [μ], and two parameters [μ, σ^2], respectively). A statement about the value of each parameter is called a simple hypothesis if it determines uniquely the value of all parameters, or a composite hypothesis if it is consistent with more than one value for some parameter. In other words, hypothesizing that the mean is M and the variance is V is a simple hypothesis while hypothesizing that the mean is between M_1 and M_2 is a composite hypothesis. In general these parameters have no *a priori* restrictions, that is, they may take any value (except restricted to positive values for variances and standard deviations).

The basic procedure for hypothesis testing requires collecting a number of observations of the random variable. The number of observations is known as the sample size, and these observations are assumed to be independent and identically distributed. The test procedure then applies the hypothesis test rule that if the sample is sufficient to reject the null hypothesis, the sample belongs in the critical region.

The critical region is the first of two regions into which the sample can be placed. The second region contains the sample if the sample is insufficient to reject the null hypothesis. Thus, hypothesis testing may be viewed as the determination of a critical region, and efficient hypothesis testing may be viewed as the selection of the optimal critical region.

In any hypothesis testing procedure, there exists a probability of error. These errors are either Type I (rejecting when true) or Type II (failure to reject when false) – also known as false alarm rate (α) or missed alarm rate (β), respectively. Depending on the situation, one error may be favored over another. The quantity α is the size of the critical region and the quantity $(1-\beta)$ is the power of the critical region. The critical region that has the highest power in the class of all regions of equal size is the most powerful region; that is, of a set of critical regions of equal size, the one with the greatest power is the preferred.

Therefore, hypothesis testing may be viewed as the minimalization of β for a given α. The practice of minimalizing β in order to create the most powerful critical region is the basis of the (then) current hypothesis testing procedure as formulated by Jerzy Neyman and Egon S. Pearson [Neyman, 1936]. This makes the optimal β a function of α, although Neyman and Pearson allow either α or β to be chosen arbitrarily. Neyman and Pearson show that the most powerful region for testing the null hypothesis H_0 against the alternative hypothesis H_1 is the region consisting of all samples which satisfy the inequality

$$\frac{\prod_{i=1}^{n} f_1(x_i)}{\prod_{i=1}^{n} f_0(x_i)} \geq k \tag{1}$$

where i is the index ranging from 1 to n, the number of samples, $f_0(x)$ and $f_1(x)$ are functions of the random variable x given the null hypothesis and the alternative hypothesis, respectively, and k is a constant chosen so that the region will have the required size α.

For example, assume the random variable of interest has a normal distribution with known variance $\sigma^2 = 1$ and two possible means, μ_0 associated with H_0 and μ_1 associated with H_1. For a normal distribution, the above values have the form of

$$\prod_{i=1}^{n} f_0(x_i) = \frac{1}{(2\pi\sigma^2)^{\frac{n}{2}}} \exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_0)^2\right] \tag{2}$$

$$\prod_{i=1}^{n} f_1(x_i) = \frac{1}{(2\pi\sigma^2)^{\frac{n}{2}}} \exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_1)^2\right] \tag{3}$$

Then the inequality takes the form of

$$\frac{\exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_1)^2\right]}{\exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_0)^2\right]} \geq k \tag{4}$$

which can be rearranged as

$$\exp\left[\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_0)^2 - \frac{1}{2}\sum_{i=1}^{n}(x_i - \mu_1)^2\right] \geq k \tag{5}$$

Wald continues to demonstrate that the SPRT is the most powerful test with a given sample size. Conversely, the SPRT also requires a smaller sample size to achieve a given α. This smaller sample size can be referred to as a sample size savings.

According to Wald, the SPRT gives a sample size savings of at least 47% over the other test procedure for α and β values between 0.01 and 0.05; although this research uses $\beta = 0.1$, there is no reason to expect the savings to decrease substantially.

2.1.2 Statistical Decision Functions

In *Statistical Decision Functions*, Wald shows that the class of all sequential probability ratio tests is a complete class of decision functions for deciding between hypotheses H_1 and H_2 [Wald, 1971]. Relating it to the Bayes solutions for decision functions proves this statement. Since the Bayes and Wald solutions are identical, the SPRT is equivalent to more classical methods for hypothesis testing and decision functions.

2.1.3 Selected Papers in Statistics and Probability

Wald and Wolfowitz provide a proof of the optimal characteristic of the SPRT in their paper "Optimum Character of the Sequential Probability Ratio Test" [Wald, 1957]. This proof shows the generalization that of all tests with the same power, the sequential probability ratio test requires on average the fewest observations. This result is imperative in its selection as the optimal test method and validates the statement that the SPRT provides a significant savings over other hypothesis testing methods.

2.1.4 SNM Portal Monitors

Canberra's SNM Portal Monitors have been in use as a means of diversion control at the exits of material access areas and facilities producing or storing fissile material or special nuclear material (SNM) for over 15 years [Davidson]. These portals monitors may also be used at borders to assist in searching for diverted material. Portal monitors developed jointly by Los Alamos National Laboratory and Canberra Nuclear monitor pedestrians and vehicles leaving a material access area with minimum delay. These portal monitors collect and analyze radiation data – neutron or gamma or both – emitted by weapons-grade or reactor-grade nuclear material.

These portal monitors are designed with three significant criteria. First, they should possess the maximum detection sensitivity for SNM. Second, they should have few nuisance alarms to minimize unnecessary detainment of persons or vehicles. Third, they should have special security features designed to prevent tampering or altering the operation of the system performance.

These monitors are specialized – they are designed for use in physical setups with certain characteristics. The specific characteristics first involve isotopic content. The portals are set up for either Pu or U – the two elements have significantly different emissions in both the neutron and gamma spectra. The isotopic enrichment of U is also important – high-enriched U has a higher gamma emission rate than low-enriched or natural U.

The next characteristic is the chemical composition. Metal fluorides have higher neutron emission rates than metal oxides, so the form of the nuclear material has a direct impact on the neutron detection limit.

The third characteristic is the particle size. Dense high-Z materials such as Pu and U attenuate gammas emitted at the center of the lump when the material is formed in large particles. Smaller particles allow more gamma photons to escape. However, the material does not attenuate spontaneous fission neutrons. This is given as a reason to combine neutron and gamma detectors.

The fourth characteristic is shielding. This is identical to the particle size characteristic above respect to high-Z material attenuation of gammas.

The fifth characteristic is background. There is a square root relationship between the background level and detection limit. Depending on elevation and environment, neutron and/or gamma background may be too large or varying to allow accurate measurement. For example, neutron portals at 2100 m elevation have detection limits typically two to three times larger than at sea level for similar operating parameters due to the increased background.

Background radiation at higher elevations is higher due to the decreasing attenuation of the earth's atmosphere and the greater effective source strength of cosmic rays and other high-energy particles. Background radiation varies from environment to environment due to the presence or absence of radioactive materials in the earth's crust at that location.

The sixth characteristic is the detector geometry and setup. These of course affect the detection efficiency and system response.

The final characteristic is the counting time and/or travel speed. The portals are intended for use as a vehicle monitor, meaning a moving vehicle passes through the portals at some non-negligible speed. Canberra's SNM portals use specialized algorithms to reduce the susceptibility to variations in vehicle and/or personnel speed.

SNM Pedestrian and Vehicle Monitors sense a radiation intensity increase by comparing short monitoring measurements with an alarm threshold derived from previous unoccupied background measurements. Once the portal is occupied, the portal controller examines small count intervals. This short-term occupation of the portal creates an effective transient departure from steady-state (which would ideally only measure background).

Each interval is analyzed using the SPRT and compared to a background and background plus transient threshold. This method improves sensitivity levels in high or fluctuating backgrounds with a reduced number of nuisance alarms compared to analysis with fixed ratemeter alarm points. Using the SPRT also allows portals to meet performance requirements if passage times are faster or slower compared to calibration.

The monitors continuously checks variations in the background and the background level. If either metric is too large (or small) to allow accurate measurements, the portal monitor will indicate so. This prevents diversion through intentional background alteration.

In order to meet the above criterion for sensitivity, the system uses a micro-processor based smart controller to allow operation in a pass-through or wait-in mode. To meet the above criterion for decreased nuisance, the system uses a single-channel analyzer (SCA) and a lower-level dial (LLD) so that the user may set lower and upper discrimination to minimize counts from cosmic rays. To meet the above criterion for tamper-resistance, the monitors have battery backups and are housed in a rugged, weatherproof low-Z material. The enclosures have keyed locks and tamper indicating devices (TIDs) and a secured communication port that allows only authorized personnel to alter calculation and threshold parameters stored in non-volatile RAM.

The portal monitors come in two varieties. The first is the gamma-ray portal monitor. This monitor has shielded, large-area plastic scintillators to detect gammas emitted by HEU and/or Pu. These scintillators are lead-shielded on three sides to reduce background and increase sensitivity. Plastic detectors are preferred over NaI(Tl) due to their efficiency at fast neutron and prompt-fission gammas and because they are large enough to intercept more radiation than equal-cost NaI(Tl) detectors.

The second is the neutron portal monitor. These monitors use ^{3}He proportional counters inside a hollow, high-density polyethylene enclosure to detect thermalized neutrons from spontaneous fissions in small quantities of shielded Pu. Neutron portals have two major advantages over gamma portals. First, neutron measurements can be made in background of high or fluctuating gammas, and neutrons emitted by Pu easily penetrate high-Z containers, making Pu difficult to shield.

A third type of monitor exists – a combination portal monitor. These portal monitors provide both gamma and neutron detection capabilities. Although these portals are less efficient at both gamma and neutron, they provide two separate measurements for detection purposes.

The test results reported by Canberra indicate that these portal monitors using the SPRT are capable of detecting low-burnup Pu and HEU material between one kilogram and hundredths of a gram. Depending on whether the monitor is vehicle or pedestrian, the portal monitors are especially effective at detecting low-burnup Pu.

The common factor in the SNM portal monitors and this research is that both use the SPRT to make decisions about whether to accept or reject a null hypothesis and one or more alternative hypotheses. The portal monitors simply seek to determine whether the radiation measured in a specific time interval is more (or less) than expected. This research is more detailed, but the basic problems encountered in the portal monitors are more than likely pertinent to this research. In relation to this project, however, these characteristics apply very weakly. These are examined in detail below.

First, the only radiation characteristic measured in this research is gamma activity within a specific energy window (60 – 90 keV, chapter 3). This nullifies any discussion of the neutron spectrum. Also, the primary gamma energy does not change appreciably within the practical lifetime of the research due to the static nature of the material activity (chapter 3).

Second, all material examined in the research has similar isotopic and chemical composition. All material is high-enriched uranium (HEU) in oxide form. This cancels the need to separate Pu from U in the gamma spectrum or activity resolution. Daughter products will themselves produce a separate gamma spectrum, but this activity is negligible for the practical lifetime of the research, again due to the static nature of the material activity.

Third, the physical form of material is not best described as particulate. There are no large metallic lumps with which to contend. This still allows self-shielding; however, the self-shielding is constant with respect to each sample. This can create the situation of material physically distant from the detector not affecting the total count rate and therefore not accounted for, but other factors minimize the ramifications of that situation.

Specifically, the mass characteristic of each sample is continually measured and logged; therefore if any amount of material not measured radioactively is removed, the mass measurement will indicate its removal.

Fourth, there is minimal shielding between the sample and the detector. The canister containing the sample is made of a slightly attenuating material, but its attenuation remains constant. There is no other material between the canister and the detector.

Fifth, the background should remain essentially constant if not nonexistent. The physical setup of the vault system (canisters surrounded by concrete with a dedicated detector placed in immediate vicinity to the sample) minimizes gamma leakage from one sample to the next. Possible changes in background are addressed in concurrent research performed by Joseph Bowling [Bowling, 2004].

Sixth, the detector is placed directly beneath the sample for measurement and then effectively isolated from any other nearby samples. The detector will not change its position in relation to the sample. There does exist the possibility that any seismic activity (to include any collision involving the storage palette) could shift the position of the detector; however, this seismic activity would ostensibly affect all detectors within a palette. This is a systemic change, and Bowling's research addresses these changes through examination of all data available from all weight and radiation sensors simultaneously. That is, any change in state that would affect all components simultaneously (such as seismic activity) will register with Bowling's expert system.

Finally, the entire characteristic is static. No material is moving and no detectors are moving. The detector time resolution is less than one second, meaning any gross change in sample attributes would be detected in around a second. However, the sample collection time is variable depending on system requirements.

2.1.5 PWR Applications

The SPRT also has other applications within the nuclear industry. The SPRT has become more common in signal validation and process monitoring arenas. Much of this research was conducted at Argonne National Laboratory by Gross, Humenik, and Singer [Gross, 1997; Humenik, 1990; Singer, 1997].

The SPRT, as mentioned earlier, is the optimal testing method for Gaussian distributions. Within industrial settings, including nuclear power plants, many processes do in fact follow Gaussian distributions. This characteristic makes the SPRT particularly useful for signal validation.

The SPRT is useful in signal validation because it is capable of monitoring several statistical characteristics, such as the variance and through indirect methods the skewness and kurtosis. Other methods (such as control charts, below) are incapable of integrated hypothesis testing. This flexibility, when coupled with its sensitivity, makes it very powerful.

For instance, the SPRT is capable of detecting slow, subtle drifts well before other techniques would be able. This rapid detection capability provides operators and maintenance personnel ample time to plan for and implement cost-saving maintenance. The SPRT therefore finds use in flowmeter, flow sensor, and pressure transmitter signal validation within nuclear power plants.

2.1.6 Other Applications

Another application of the SPRT is given in other research by Gross, Bhardwaj, and Bickford [Gross, 2002]. Their research uses the SPRT to perform proactive maintenance for detecting software aging mechanisms in performance-critical computers, including computers responsible for control of military weapons. This application does not detect or fix "bugs"; it instead helps diagnose the possible onset of such problems as memory leaks, unreleased file locks, fragmentation, and data corruption.

The SPRT is useful in this application because of its sensitivity to small anomaly magnitudes. In this application, the SPRT also exhibited zero false alarms. Although this application is different from the research presented in this thesis, it demonstrates the wide range of the SPRT's applicability.

2.2 Control Charts

The previous sections described the SPRT and examples of its use. These sections describe control charts and their uses.

A control chart (also referred to as a run chart or a Shewhart chart) of a process characteristic (such as mean or variation) consists of values plotted sequentially over time, and it includes a centerline as well as a lower control limit (LCL) and an upper control limit (UCL) [Triola, 1998]. The centerline represents a central value of the characteristic measurements, whereas the control limits are boundaries used to separate and identify any points considered to be unusual.

When plotted on a control chart, measured values can graphically reveal significant information about the underlying distribution of the process being measured.

For example, plotted sequential data may show shifts or trends in mean or variance as well as cyclical patterns.

The objective of using control charts is to determine whether a process is statistically stable (or within statistical control). A statistically stable process has only natural variation with no patterns, cycles, or any unusual points. Control charts are important in this research because the statistical parameters of the measured variable (radiation) do not change over time. Therefore a control chart would be useful within this context.

2.2.1 The R Chart

The R (for "range") chart is a tool for measuring the variation in a process. In order to construct a control chart for monitoring variation, the sample ranges are plotted instead of the individual value [Triola, 1998]. The sample range is simply the maximum sample value minus the minimum sample value. The R chart uses the following notation: Given process data consisting of a sequence of samples all the same size with an essentially normal distribution

n is the size of each sample

$\bar{\bar{x}}$ is the mean of the sample means (equivalent to the grand mean of all the data)

$\bar{R}$ is the mean of the sample ranges.

With these values, the plot is generated in this manner:

Plot the sample ranges with centerline $\bar{R}$. The upper control limit (UCL) is $D_4\bar{R}$ and the lower control limit (LCL) is $D_3\bar{R}$, where D_3 and D_4 are taken from control chart tables and are dependent upon the number of observations.

2.2.2 The $\bar{x}$ Chart

In constructing a control chart for monitoring means, this approach applies the central limit theorem by locating the control limits at $\bar{\bar{x}} \pm \frac{3\bar{s}}{\sqrt{n}}$ [Triola, 1998]. The 3 in the numerator is the familiar number of standard deviations above and below the mean in order to provide a 99% confidence interval (since $\bar{s}$ is the sample standard deviation).

The plot is then generated by plotting the sample means with centerline $\bar{\bar{x}}$. The UCL is given by $\bar{\bar{x}} + A_2\bar{R}$ and the LCL is given by $\bar{\bar{x}} - A_2\bar{R}$ where A_2 is given in control chart tables.

2.2.3 The Cumulative Sum (CUSUM) Control Charts

These control charts are effective in detecting special causes [Pouliezos, 1994]. The CUSUM chart is usually maintained by taking samples at fixed time intervals and plotting a cumulative sum of differences between the sample means and the target value ordered in time. The process mean is considered to be on target as long as the CUSUM statistic does not fall into the signal region of the chart. If a value falls into the signal region, it is an indication the process mean has changed and the possible causes should be investigated.

CUSUM charts are often used in place of standard control charts when detection of small changes in a parameter is important. For comparable average run lengths (ARLs), CUSUM charts can be designed to perform better than the standard control charts.

2.2.4 The Exponentially Weighted Moving Average Control Chart

Another control chart is the exponentially weighted moving average (EWMA) control chart [Pouliezos, 1994]. While other control charts assume that the mean of the process is static, the EWMA makes no such assumption. This chart is not as widely used, but it bears mentioning.

The EWMA is a statistic with the characteristic that it gives less and less weight to data as they get older and older. A plotted point on an EWMA chart can be given a long memory and emulate a CUSUM chart or a short memory and emulate a standard control chart. That is, as memory increases, the EWMA becomes a cumulative test and as memory decreases the EWMA becomes an independent sample test.

The EWMA is best plotted one time position ahead of the most recent observation since it may act as a forecast. The EWMA is equal to the present predicted value plus λ times the present observed error of prediction. Thus,

$$EWMA = \hat{y}_{t+1} = \hat{y}_t + \lambda e_t = \hat{y}_t + \lambda(y_t - \hat{y}_t) = \lambda y_t + (1-\lambda)\hat{y}_t \qquad (6)$$

where $\hat{y}_{t+1}$ is the predicted value at time t+1 (the new EWMA), y_t is the observed value at time t, $\hat{y}_t$ is the predicted value at time t (the old EWMA), $e_t = y_t - \hat{y}_t$ is the observed error at time t and λ is a constant ($0<\lambda<1$) that determines the depth of the memory of the EWMA.

The EWMA can be written as

$$y_{t+1} = \sum_{i=0}^{t} w_i y_i \qquad (7)$$

where the w_i are weights defined by

$w_i = \lambda(1-\lambda)^{t-i}$ with sum $\sum_{i=0}^{t} w_i = 1$. The constant λ determines the memory of the EWMA. As $\lambda \rightarrow 1$, $w_1 \rightarrow 1$ and $\hat{y}_{t+1}$ practically equals the most recent observation y_t. As $\lambda \rightarrow 0$, the most recent observation has small weight and previous observations near equal weights. Thus, low λ approximates CUSUM and high λ approximates standard charts.

The choice of λ is left to the judgment of quality control analyst or estimated using an iterative least squares procedure. The analyst would consider the data as new data arriving sequentially and for different values of λ compute the corresponding sequential set of predicted values $\hat{y}$ based on the EWMA; the value of λ corresponding to the smallest error sum of squares is preferred, although this choice is based on limited evidence.

The variance of the EWMA is

$$\sigma^2_{EWMA} = \frac{\lambda\sigma^2}{2-\lambda} \tag{8}$$

An estimate of σ^2 can be obtained from the minimum error sum of squares

$$\hat{\sigma}^2 = \sum_{t=1}^{T} \frac{e_t^2}{T-1} \tag{9}$$

and the equation for the estimated EWMA variance becomes

$$\hat{\sigma}^2_{EWMA} = \frac{\lambda\hat{\sigma}^2}{2-\lambda} \tag{10}$$

and the corresponding 3σ control limits become $\tau \pm 3\hat{\sigma}_{EWMA}$

2.2.5 Decision Making with Control Charts

Now that the data are plotted into the control charts as desired, some rules are applied in order to find departures from the null hypothesis of mean μ_0. These rules are laid out as follows [Triola, 1998].

Reject the null hypothesis if:

1) There is a pattern, trend, or cycle that is obviously not random (such as sinusoidal behavior, step behavior, or other visually identifiable characteristics).
2) There is a point lying beyond the upper or lower control limits.
3) There are eight consecutive points all above or all below the center line. This is also known as the Run of 8 Rule.
4) There are six consecutive points all increasing or all decreasing
5) There are 14 consecutive points all alternating between up and down.
6) Two out of three consecutive points are beyond control limits that are 1 standard deviation away from the center line
7) Four out of five consecutive points are beyond control limits that are 2 standard deviations away from the centerline.

The methods presented above represent the most common control chart feature extraction methods. Most have well-established roles in industrial settings; however, the nature of the research does not lend itself easily to simple control charts.

The main concern is that these control charts use multiple simultaneous (or near simultaneous) sample sets. This research concentrates on each sensor individually. While this would not preclude the use of control charts, the SPRT has already been established as the optimal method for detecting departures from the null hypothesis of normality.

2.3 Selected Feature Extraction Methods

The research does retain some aspects of control charts. The Run of 8 Rule is retained as well as a modified version of "x out of y." The specific features extracted are:

1. point indices (and therefore times) of SPRT alternative hypothesis alarms over the last 100 and 1000 data points; this calculates the running SPRT alternative hypothesis frequency (similar to the "x out of y" in control charts)
2. interval between successive SPRT alternative hypothesis alarms for each hypothesis
3. number of same-sign residuals (similar to the Run of 8 Rule in control charts)
4. variance of the last 5 points

These features are explained in detail in chapter 4.

The next chapter presents background information on radioactive decay and basic statistics necessary to perform the analyses.

3.0 Radioactive Decay

The radiation measured in the Y-12 storage vaults fall in the 60- to 90-keV x-ray range. These x-rays come from the decay of the uranium and its (grand)-daughters. The CAVIS radiation sensor is the RADSiP detector developed by Joe A. Williams at Oak Ridge National Laboratory [y12.doe.gov]. This device monitors gross gamma activity from the material in storage and produces a pulse rate proportional to the gamma and x ray dose. Variations in the pulse rate may be indicative of changing material characteristics and/or natural statistical fluctuations of a stochastic process.

3.1 Statistics

Radioactive decay follows simple and well-known statistical behavior. This section outlines and explains the basics of this statistical behavior as formulated into statistical distributions.

3.1.1 The Binomial

Radioactive decay by nature follows a binomial process – an atom either does or does not decay in a given time interval. The equation describing a binomial process is:

$$p(y) = \frac{n!}{y!(n-y)!}\pi^{y}(1-\pi)^{n-y} \tag{11}$$

where p(y) is the probability of success at y, n is the number of trials, π is the probability of success on a single trial, and y is the number of successes in n trials [Tsoulfanidis, 1995]. For a given binomial distribution, the mean is the product of n and π. A plot of sample binomial distributions with identical means and different parameters appears below in Figure 1.

As seen in Figure 1, the distributions are skewed. As the mean increases, the distribution becomes more symmetric as seen in Figure 2.

As seen in Figure 2, with a mean of sufficient size, the distribution becomes symmetric. This characteristic of distributions also appears in the Poisson distribution described below.

3.1.2 The Poisson

The calculations necessary for a large number of atoms (or trials) present in a sample become unwieldy. For example, 235 g of ^{235}U contains Avogadro's number, or $6.022 * 10^{23}$ atoms (or trials). Also, the probability of decay (a success) π is very small. Therefore radioactive decay is more easily modeled as a Poisson process. In a Poisson distribution, the mean and variance are numerically equivalent and its equation is given by:

$$p(y) = \frac{\mu^y e^{-\mu}}{y!} \qquad (12)$$

where p(y) is the probability of success at y and μ is the mean of the distribution [Tsoulfanidis, 1995]. A set of examples of Poisson distributions with different means appears in Figure 3.

3.1.3 The Gaussian

For low means, the Poisson results in a skewed distribution. As the mean grows sufficiently large (approximately 20 and above) the distribution becomes less skewed and

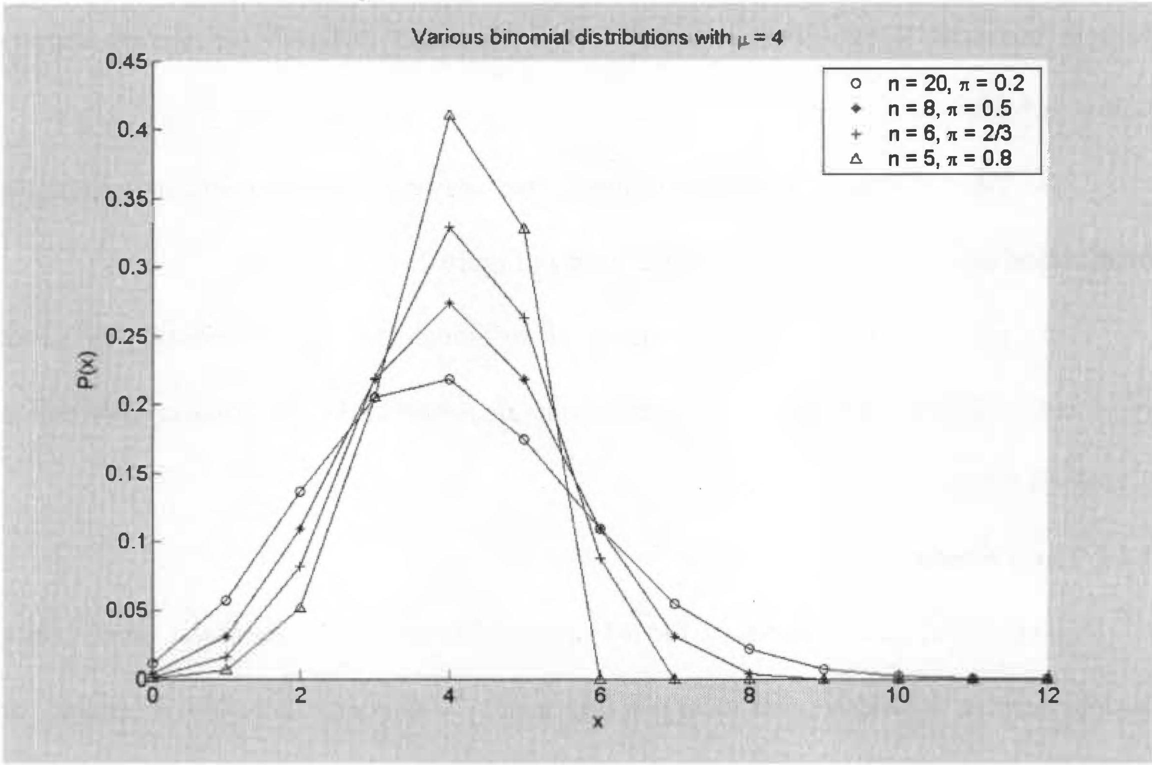

Figure 1 – Binomial Distribution: Small Mean

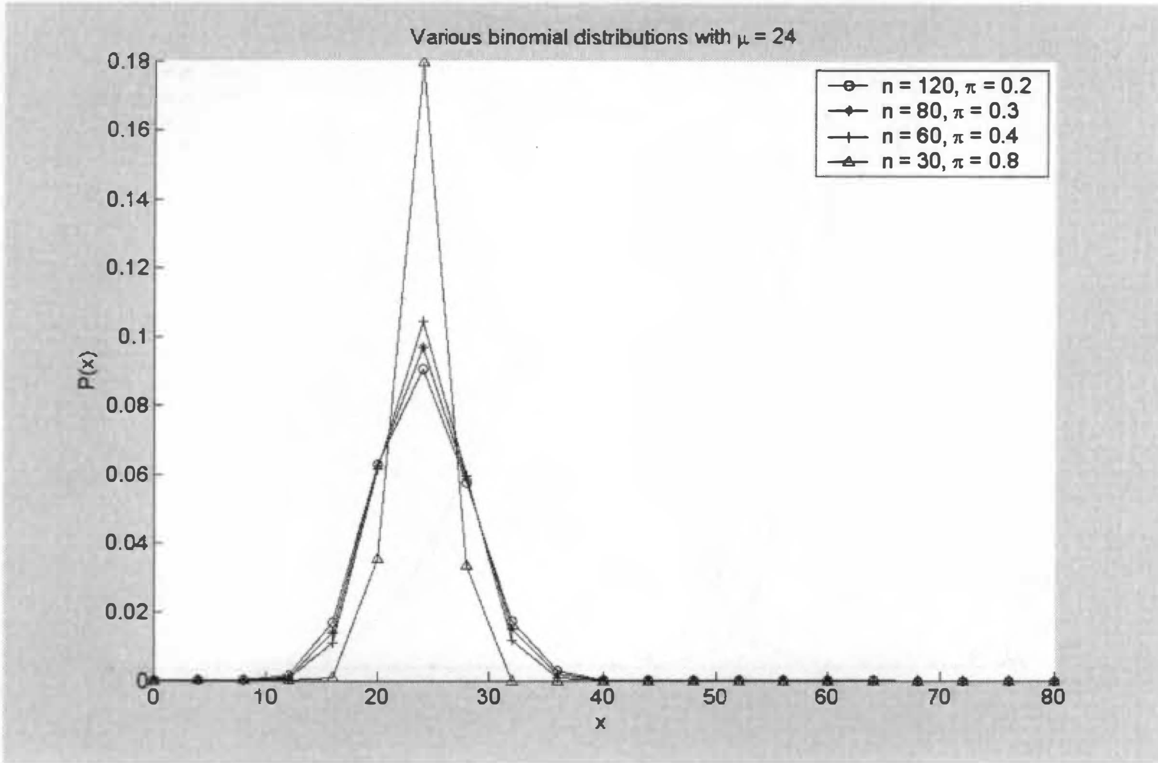

Figure 2 – Binomial Distribution: Larger Mean

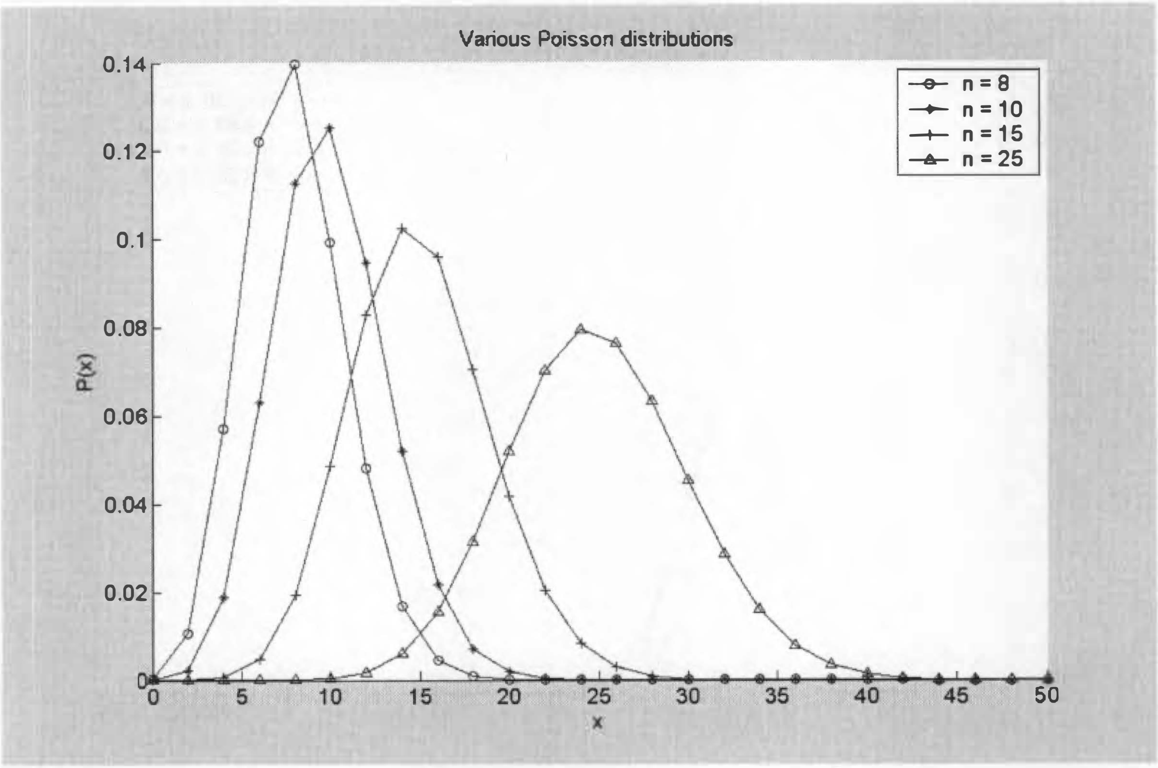

Figure 3 – Poisson Distributions

thus approaches a normal, or Gaussian, distribution [Tsoulfanidis, 1995]. This density function is given by the equation:

$$P(y \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \tag{13}$$

where μ is the mean and σ^2 is the variance. This can also be written as $N(\mu, \sigma^2)$ when μ and σ^2 are known. A comparison of the Poisson and Gaussian distributions appears in Figure 4.

This Gaussian distribution holds for any number of collected counts for a specific time interval. However, radioactive decay decreases exponentially over time and this

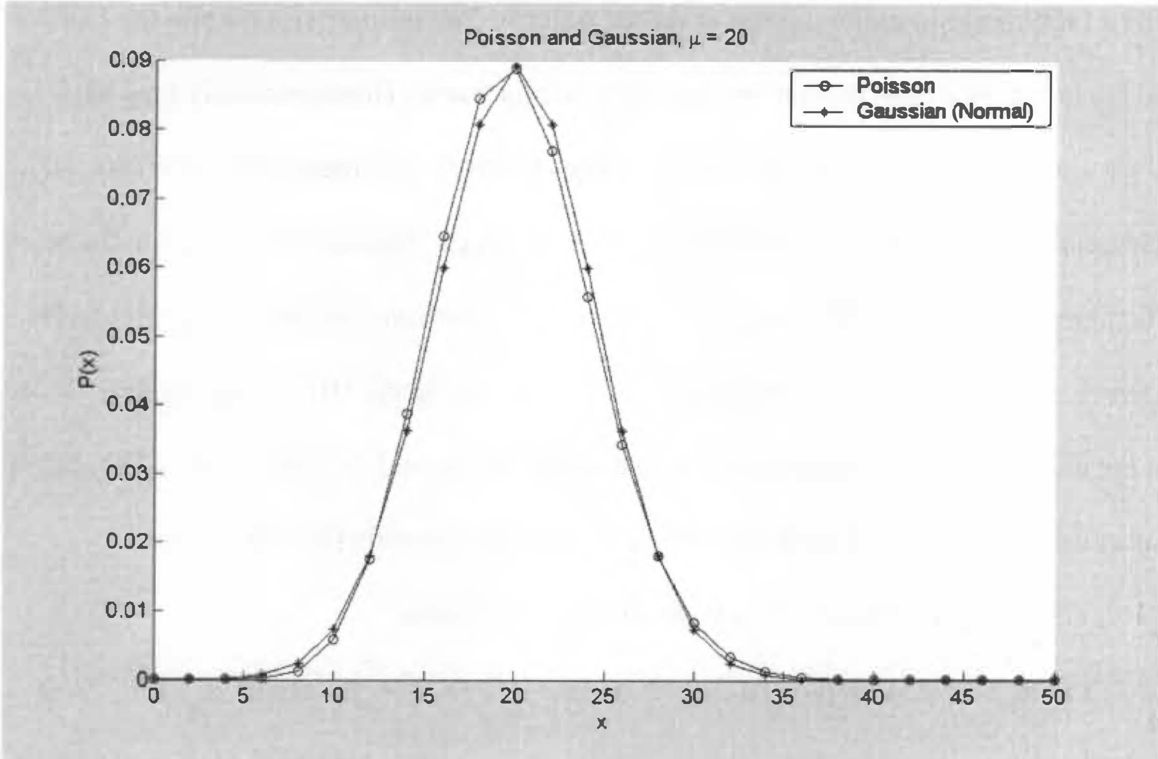

Figure 4 – Gaussian and Poisson Distributions

decrease in activity changes the Gaussian distribution which describes the radioactive decay. This implies that the distribution is in fact a function of time.

3.2 Radioactive Decay

3.2.1 Uranium Basics

In general, HEU is any amount of uranium in which the fissile isotope ^{235}U has been enriched from its natural 0.7% by weight to over 20% by weight. In nuclear weapon terminology, HEU refers to uranium in which ^{235}U has been enriched to over 90%. The remainder of the uranium is generally the non-fissile isotope ^{238}U. The half-life of the fissile uranium isotope is on the order of 700 million years while the half-life of the non-fissile isotope is on the order of 4 billion years. These extremely long half-lives give the material an essentially static activity, and therefore essentially static Gaussian distribution, over the lifetime of its storage. This assumption of a Gaussian distribution with a definite mean and variance forms the basis for the null hypothesis H_0. Before characterizing the radioactive profile of the stored HEU, the nuclear decay schemes of the two primary radioactive materials should be discussed. The below schemes disregard low-probability decays such as spontaneous fissions.

The exact decay of ^{235}U follows this general scheme:

U-235 → α + Th-231 → β- + Pa-231 → α + Ac-227 → β- + Th-227 → α + Ra-223 → α + Rn-219 → α + Po-215 → α + Pb-211 → β- + Bi-211 → α + Tl-207 → β- + Pb-207

This decay scheme (including low-probability decays) appears in Figure 5 [nuclides.net].

The exact decay of ^{238}U (excluding low-probability decays) follows this general scheme:

U-238 → α + Th-234 → β- + Pa-234 → β- + U-234 → α + Th-230 → α + Ra-226 → α + Rn-222 → α + Po-218 → α + Pb-214 → β- + Bi-214 → β- + Po-214 → α + Pb-210 → β- + Bi-210 → β- + Po-210 → α + Pb-206

This decay scheme (including low-probability decays) appears in Figure 6 [nuclides.net].

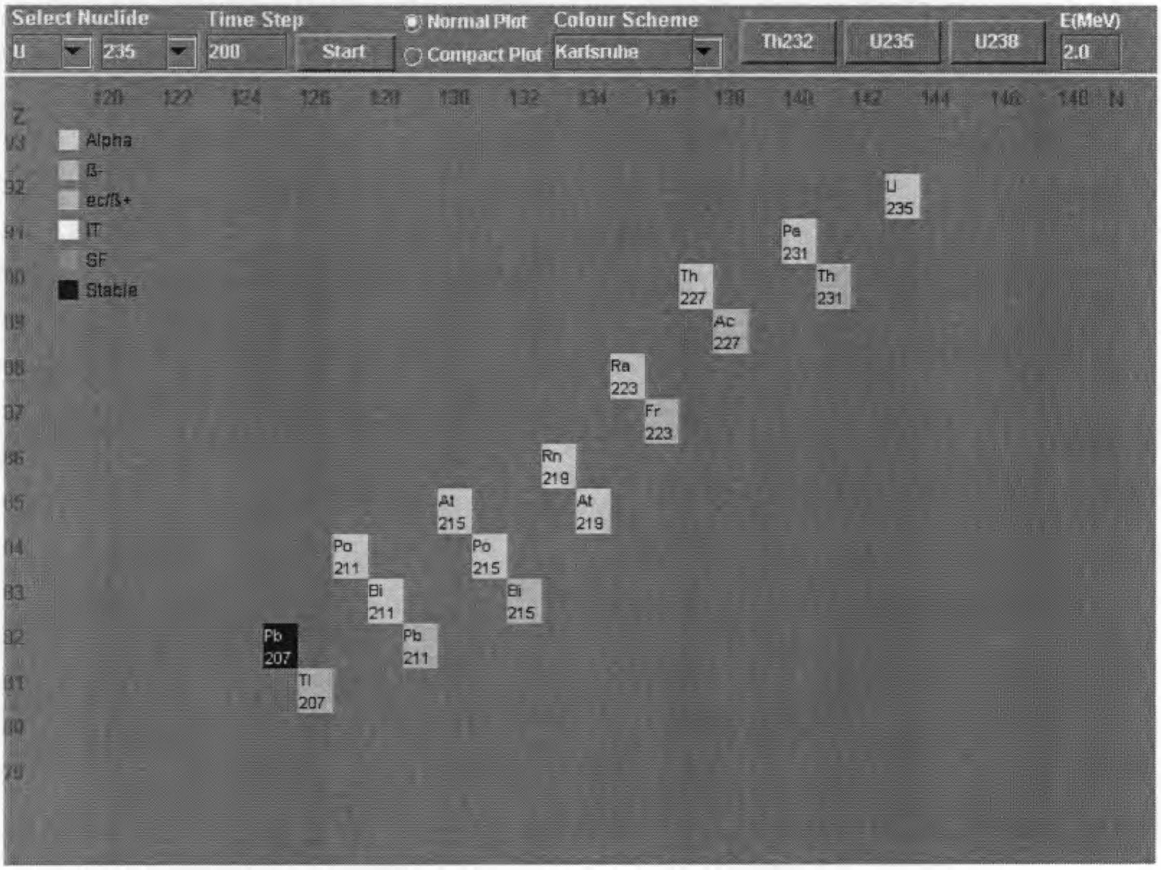

Figure 5 – ^{235}U Decay

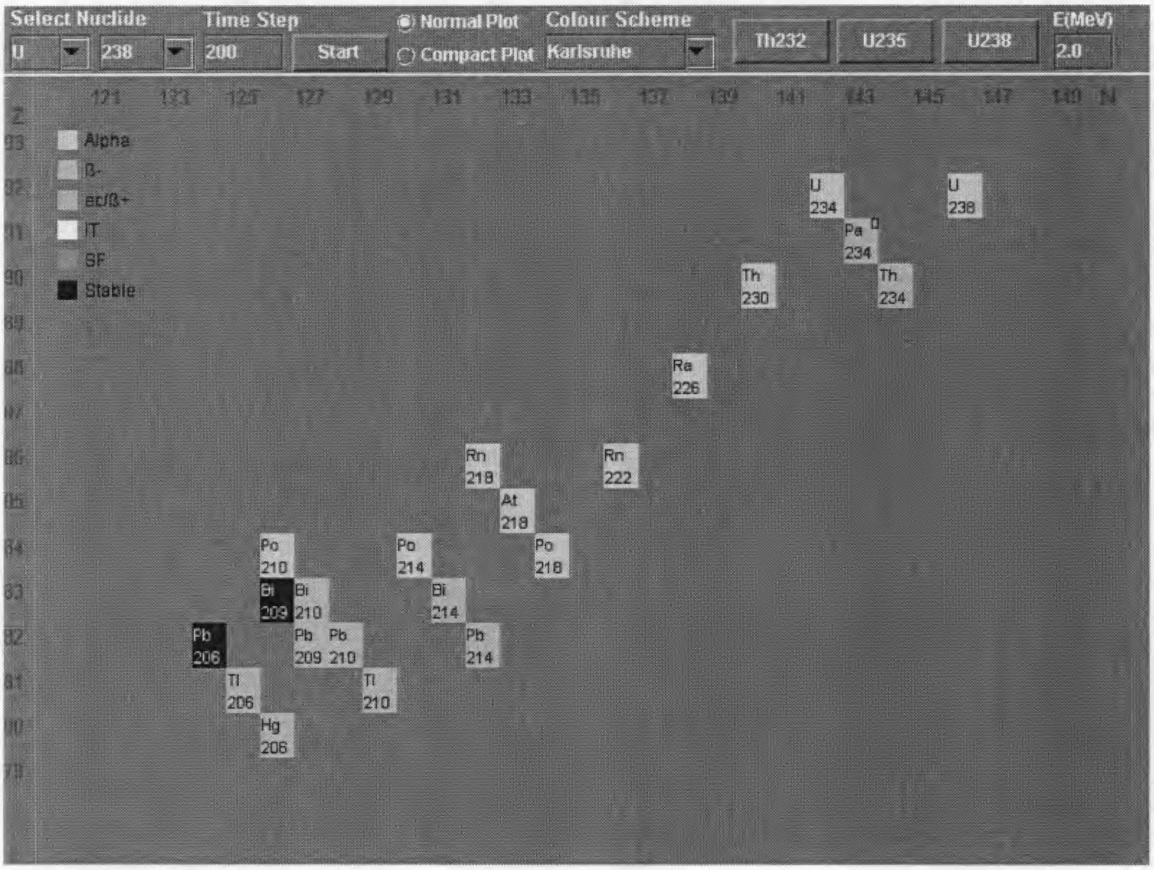

Figure 6 – ^{238}U Decay

3.2.2 Activities

It is reasonable to assume the HEU to have a ^{235}U to ^{238}U ratio of at least 9:1, and since the half-life for ^{235}U is one-sixth the half-life for ^{238}U (meaning that nearly all the ^{235}U will decay before half the ^{238}U decays), it is reasonable to assume the activity from the ^{235}U chain to be greater than the ^{238}U chain and effectively ignore the ^{238}U.

Most of these daughter and granddaughter products have activities of their own, resulting in a buildup of residual activities in the range of interest. However, these daughter and granddaughter products have half-lives far shorter than the parent. This vast difference in half-lives forces the daughters and granddaughters to come into equilibrium with the ^{235}U.

The activity of ^{235}U does not change appreciably due to its large half-life, while ^{231}Th (half-life: 25.52 hours) reaches equilibrium with ^{235}U quickly (roughly 3.5 days). ^{231}Pa (half-life: 32760 years) does not reach equilibrium with ^{235}U within 100 years and will in fact require over 108 000 years to do so. In this time frame, the activity of ^{235}U is at least three orders of magnitude greater than the activity of ^{231}Pa. All other radionuclides in the decay chain must reach equilibrium with ^{231}Pa before reaching equilibrium with ^{235}U; therefore, the emissions from stored ^{235}U and its entire decay chain will not change appreciably in this time frame. Thus the assumption of static Gaussian count distributions is valid.

3.2.3 The RADSiP

The RADSiP is a radiation (RAD) detector using a silicon photodiode (hence SiP) [y12.doe.gov]. The silicon diode is a Hamamatsu S3590-01 Si PIN Photodiode, 1cm square and 200 microns thick. Gamma and X radiation incident on the reversed-biased diode ionizes the silicon wafer, producing approximately 1 electron-hole pair per 3.5 eV deposited energy. This charge is separated within the wafer by an applied electric field (created by connection to the bias power supply), and collected at the anode and cathode of the diode. The charge pulse is converted to a voltage pulse by a shaping amplifier and the voltage pulse is converted to a digital pulse by a threshold circuit. The threshold circuit is adjusted so that diodes 3 mm, 5 mm, and 10 mm in diameter produce approximately 500, 1500, and 5000 pulses per second per R/hr, respectively.

The full specification sheet for the RADSiP appears in the appendix.

3.3 Statistical Analysis of the Count Rate

Since the radiation detector signal has Poisson characteristics that can be approximated by a Gaussian distribution, the SPRT will directly apply. As described and referenced in chapter 2, Abraham Wald developed the SPRT specifically for the Gaussian distribution, and therefore this physical situation (that is, the collection of Gaussian data) proves to be a perfect opportunity to apply it. Modeling the process as a Gaussian distribution instead of a Poisson or binomial distribution thus has the distinct advantage of using the Sequential Probability Ratio Test (SPRT) to test for changes in the underlying probabilistic parameters, specifically its mean μ and variance σ^2.

Also included in this section is an overview of standard statistical analysis methods such as Student's t-test on the mean and the χ^2 test on the variance. These methods individually test the mean and variance of a data set (as their descriptions imply). As will be shown, the SPRT tests both the mean and variance of a data set on its own.

3.3.1 Assumptions

The first assumption is that the radiation measured by the RADSiP has a definite calculable mean μ and variance σ^2. As demonstrated in the preceding section, the radiation statistical parameters will not change appreciably over any reasonable lifetime for storage; therefore the definite calculable mean and variance should not change. These parameters form the null hypothesis (H_0) for statistical tests.

Any departure from the null hypothesis can be characterized by a change in either of the two parameters μ and σ^2. These parameters can either increase or decrease and can occur independently or simultaneously. That is, the distribution mean can decrease, remain static, or increase while the variance can decrease, remain static, or increase. This results in nine possible situations – a continuation of the null hypothesis (mean and variance remain static) and eight alternative hypotheses. However, it is unnecessary to test eight alternate distributions, as shown below.

Another common assumption about the radiation data is that it follows a normal distribution. As will be shown directly, this is a valid assumption when dealing with radiation of a sufficiently high mean.

3.3.2 Standard Statistical Test Methods

Standard methods that test for changes in distributions require off-line batch testing. Standard hypothesis tests for the mean require the calculation of the sample mean and the computation of a t-statistic. This t-statistic is then compared to the level of significance of the test. The test is explained below and more detail can be found in Ott and Longnecker.

The t-test (the method of using the t-statistic) has two varieties when dealing with distributions with both a mean and a variance. The first (the standard t-test) assumes that the sample variance σ^2 is known, while the other (Student's t-test) makes no such assumption.

For review, Type I errors are rejection of the null hypothesis H_0 when the null hypothesis is true. Type II errors are the acceptance of the null hypothesis H_0 when the null hypothesis is false. The Type I and Type II error probabilities, or rates, are denoted α and β, respectively.

3.3.2.1 The Standard t-test for the Mean, Known Variance

Given a sample of n data points from a Gaussian distribution with unknown mean μ and known variance σ^2, there are three possible cases [Ott, 2001]. The null hypothesis H_0 is explained to the right of each case.

Case 1: H_0: $\mu \le \mu_0$ vs H_1: $\mu > \mu_0$ (mean $\mu \le$ hypothesized mean μ_0)

Case 2: H_0: $\mu \ge \mu_0$ vs H_1: $\mu < \mu_0$ (mean $\mu \ge$ hypothesized mean μ_0)

Case 3: H_0: $\mu = \mu_0$ vs H_1: $\mu \neq \mu_0$ (mean $\mu =$ hypothesized mean μ_0)

The test statistic (T.S.) is given by:

$$\text{T.S.:} \quad z = \frac{\bar{y} - \mu_0}{\sigma / \sqrt{n}} \qquad (14)$$

where σ is the population standard deviation, n is the sample size, μ_0 is the hypothesized mean, and $\bar{y}$ is the sample mean.

For a given probability α of a Type I error, each case is rejected below as described.

Case 1: Reject H_0 if $z \geq z_\alpha$

Case 2: Reject H_0 if $z \leq -z_\alpha$

Case 3: Reject H_0 if $|z| \geq z_{\alpha/2}$

where z_α is defined as the z at which, given a standard normal distribution N(0, 1), 100(1-α)% of all observations will fall below z.

As seen above, the standard t-test requires knowledge of the population variance σ^2. In order to relax that assumption, the hypothesis test becomes Student's t-test.

3.3.2.2 Student's t-test for the Mean, Unknown Variance

This test is very similar to the above test except that the population standard deviation σ is replaced by the sample standard deviation s and the t-statistic z is replaced by the t-statistic t. This t comes from Student's t-distribution. By removing the assumption of a known variance, the test becomes more robust when applied to real data sets where the number of data points may be limited. The t-distribution has the following properties [Ott, 2001]:

1. There are many different t distributions. Particular distributions are specified by the parameter called degrees of freedom (df). For a sample of size n, there are (n-1) degrees of freedom.
2. The t distribution is symmetrical about 0 (mean of 0).
3. The t distribution has variance df/(df-2). This variance is greater than the standard t-test variance of 1.
4. As df increases, the t distribution approaches the z distribution (used in the standard t-test). That is, as n $\rightarrow \infty$, the variance $\rightarrow$ 1.

A visual comparison of the t-distribution and the standard normal distribution appears in Figure 7.

As seen in Figure 7, the t-distribution has a shorter maximum and "fatter" tails, meaning that more of the t-distribution's values fall farther away from the mean than in the normal distribution. As the degrees of freedom increases, the distribution will tend more and more towards the standard normal.

Written explicitly, the t-distribution density function is given by:

$$P(x \mid \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{x^2}{2}\right)^{\frac{\nu+1}{2}}} \tag{15}$$

where ν is the degrees of freedom and Γ is the gamma function. For reference, the gamma function is given by:

$$\Gamma(a) = \int_0^\infty e^{-t} t^{a-1}\, dt \tag{16}$$

The Student's t-test is therefore

Case 1: H_0: $\mu \leq \mu_0$ vs H_1: $\mu > \mu_0$ (mean $\mu \leq$ hypothesized mean μ_0)

Case 2: H_0: $\mu \geq \mu_0$ vs H_1: $\mu < \mu_0$ (mean $\mu \geq$ hypothesized mean μ_0)

Case 3: H_0: $\mu = \mu_0$ vs H_1: $\mu \neq \mu_0$ (mean $\mu =$ hypothesized mean μ_0)

The test statistic (T.S.) is given by:

T.S.: $$t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} \tag{17}$$

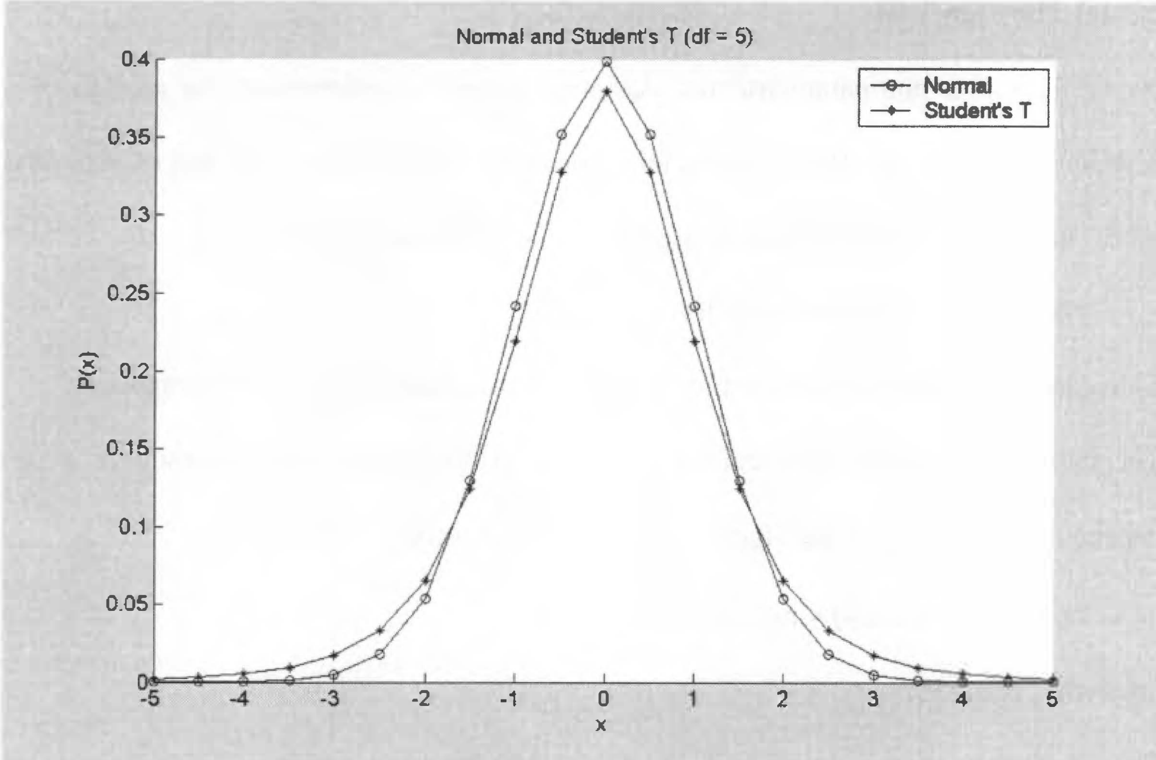

Figure 7 – Student's t- and Normal Distributions

where s is the sample standard deviation, n is the sample size, μ_0 is the hypothesized mean, and $\bar{y}$ is the sample mean.

For a given probability α of a Type I error and df = n-1, each case is rejected below as described.

Case 1: Reject H_0 if $t \geq t_\alpha$

Case 2: Reject H_0 if $t \leq -t_\alpha$

Case 3: Reject H_0 if $|t| \geq t_{\alpha/2}$

3.3.2.3 The Sign Test

This is a non-parametric test designed to test hypotheses on the median of a distribution [Pouliezos, 1994]. Since for a normal distribution the mean and median (and mode) are identical, this test can be used for mean hypotheses as well.

The test is performed as follows:

Calculate the residuals of the sample. That is, subtract the mean from all samples.

The number of positive residuals is counted and compared to two thresholds which depend on sample size n and significance level α. Thus, if

$n_1 <$ (# positive residuals) $< n_2$, accept H_0: $\mu = 0$

otherwise, reject H_0

The values for n_1 and n_2 come from provided tables based on a binomial distribution and the sample size.

The above tests are designed for testing the mean μ. They give no information about the variance σ^2. However, this research also requires information on the variance. Therefore, the standard test for variance is examined.

3.3.2.4 χ^2 Test for the Variance

This is a statistical test on the variance of a data set. It can be used to determine whether a data set's variance is increasing, decreasing, or remaining constant to within a certain confidence.

The sample variance s^2 can be used for inferences concerning a population variance σ^2 and for a sample of n measurements drawn from a population with mean μ

and variance σ^2, s^2 is an unbiased estimator of σ^2 [Ott, 2001]. If the population distribution is normal, then the sampling distribution of s^2 can be specified as follows.

From repeated samples of size n from a normal population whose variance is σ^2, calculate the statistic $\frac{(n-1)s^2}{\sigma^2}$ and plot the histogram for these values. Sample histograms appear in Figure 8.

As seen above, as the degrees of freedom increases, the χ^2 distribution approaches the normal distribution. Explicitly, the χ^2 distribution density function is given by:

$$P(x \mid \nu) = \frac{x^{(\nu-2)/2} e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)} \tag{18}$$

where ν is the degrees of freedom and Γ is the gamma function (defined earlier).

The χ^2 distribution as the following properties [Ott, 2001]:

1. The χ^2 distribution is positively skewed with values between 0 and ∞ (as seen above).
2. There are many different χ^2 distributions. A particular χ^2 distribution is labeled by its degrees of freedom.
3. The mean and variance of a χ^2 distribution are given by μ = df and σ^2 = 2df. For example, if df = 30, μ = 30 and σ^2 = 60.

Similar to the tests for the mean as described above, the test for the variance has three cases outlined below.

Case 1: H_0: $\sigma^2 \le \sigma_0^2$ vs H_1: $\sigma^2 > \sigma_0^2$ (variance $\sigma^2 \le$ hypothesis variance σ_0^2)

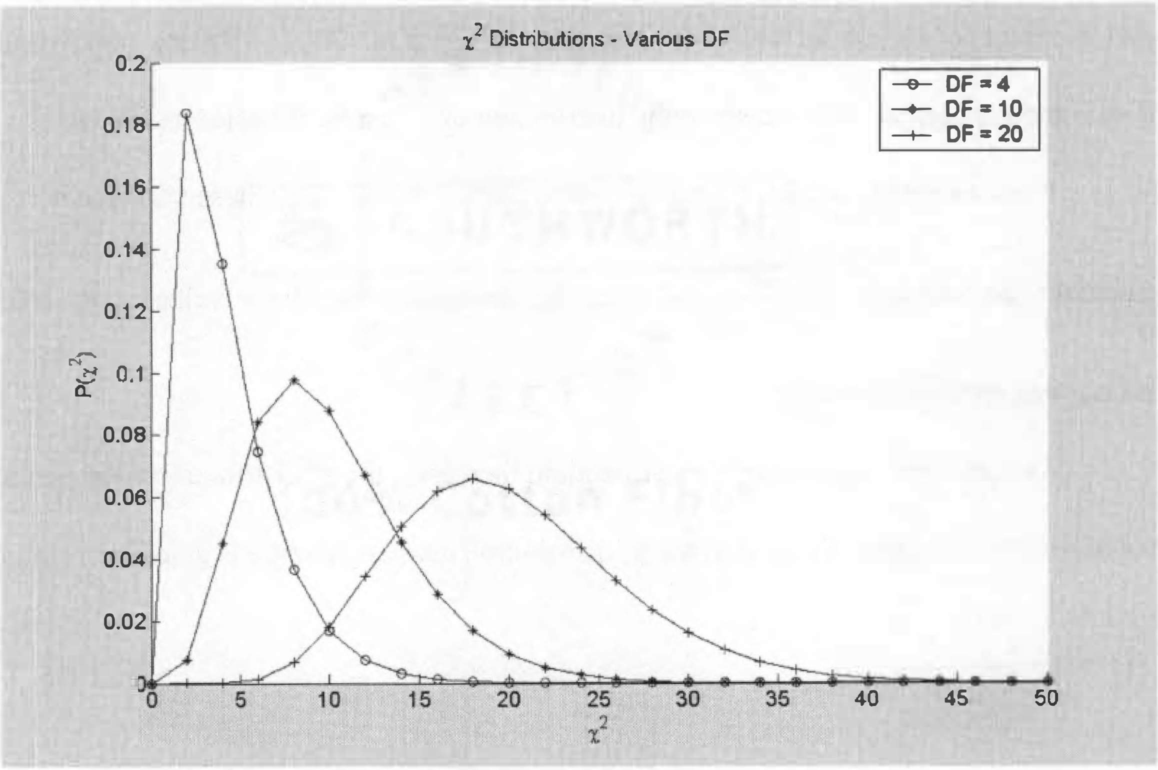

Figure 8 – χ^2 Distributions

Case 2: H$_0$: $\sigma^2 \geq \sigma_0^2$ vs H$_1$: $\sigma^2 < \sigma_0^2$ (variance $\sigma^2 \geq$ hypothesis variance σ_0^2)

Case 3: H$_0$: $\sigma^2 = \sigma_0^2$ vs H$_1$: $\sigma^2 \neq \sigma_0^2$ (variance $\sigma^2 =$ hypothesis variance σ_0^2)

The test-statistic (T.S.) is given by:

T.S.: $$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \tag{19}$$

For a given α

Case 1: Reject H$_0$ if χ^2 is greater than χ_u^2, the upper-tail value for a = α, df = n-1

Case 2: Reject H$_0$ if χ^2 is less than χ_l^2, the lower-tail value for a = 1-α, df = n-1

Case 3: Reject H_0 if χ^2 is greater than χ_u^2 based on a = $\alpha/2$, df = n-1, or less than χ_l^2 based on a = 1-$\alpha/2$, df = n-1

where a is the area under the χ^2 distribution.

Most of the above tests give three cases; however, this research deals with only one of the three cases, Case 3. Case 1 tests whether the variance is greater than an expected value and Case 2 tests whether the variance is less than an expected value. While it is useful to know whether the value is greater or less than the expected value, this research examines departures – both above and below – the expected values of the mean and variance for the radiation detector signal.

3.3.3 The SPRT

Since it is assumed the count rate distributions are normal due to the central limit theorem and the Poisson approximation, the SPRT, a test designed specifically for the Gaussian distribution, is the test of choice. This test has the distinct advantage of being an online test; that is, instead of examining batch data, the SPRT processes sequential data (hence the name sequential probability ratio test).

Also as shown earlier, the SPRT requires on average a smaller sample size n, or fewer collected data points, to determine a departure from the null hypothesis. When combined with the online nature of the SPRT, the ability to perform real-time analysis with less data becomes the greatest strength of the SPRT.

3.3.3.1 Hypotheses

The SPRT examines the null hypothesis H_0 and four other hypotheses: H_1, H_2, H_3, and H_4. The first two deal with shifts in the mean (positive for H_1, negative for H_2) and the second two deal with shifts in the variance (increase for H_3 and decrease for H_4).

All possible situations fall into these categories. For example if the radiation distribution changes to a distribution with a higher mean and lower variance, such as the radiation detector fouling and reporting identical high scores repeatedly, then the SPRT will alarm for hypotheses 1 and 4. All hypotheses appear in Table 1.

The variables M, V^+, and V^- are defined as:

- M is the amount by which the mean shifts either up or down;
- V^+ is the factor by which the variance increases; and
- V^- is the factor by which the variance decreases.

These different hypotheses yield different distribution shapes. Examples appear in Figure 9.

The following derivation of the SPRT is included for completeness. All equations are taken (and modified slightly when necessary) from Herzog [Herzog] and Gross [Gross, 2002].

Table 1 – SPRT Hypotheses

Hypothesis	Meaning	Variables	Example
0	Null Distribution	μ, σ^2	Normal
1	Mean Increase	$\mu_1=(\mu + M)$, σ^2	Presence of external source; Environmental effects
2	Mean Decrease	$\mu_2=(\mu - M)$, σ^2	Shift in material proximity to detector; Material removal
3	Variance Increase	μ, $\sigma_3^2=(V^+*\sigma^2)$	Loose wiring; Power surge
4	Variance Decrease	μ, $\sigma_4^2=(V^-*\sigma^2)$	"Stuck" detector

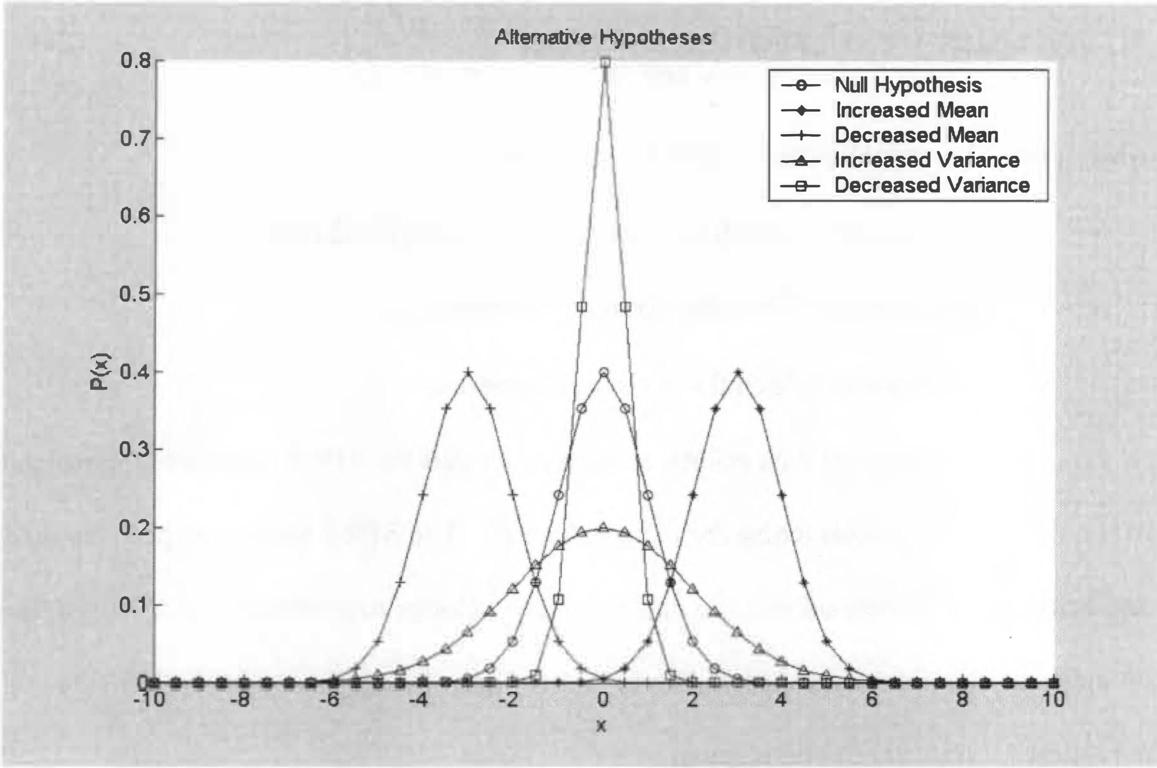

Figure 9 – Hypothesis Examples

The five hypotheses (null hypothesis and four alternative hypotheses) are defined as:

$$H_0: \quad \mu = \mu, \sigma^2 = \sigma^2, P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \tag{20}$$

$$H_1: \quad \mu = \mu + M, \sigma^2 = \sigma^2, P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-(\mu+M))^2}{2\sigma^2}\right] \tag{21}$$

$$H_2: \quad \mu = \mu - M, \sigma^2 = \sigma^2, P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-(\mu-M))^2}{2\sigma^2}\right] \tag{22}$$

$$H_3: \quad \mu = \mu, \sigma^2 = V^+\sigma^2, P(x) = \frac{1}{\sqrt{2\pi V^+\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2V^+\sigma^2}\right] \tag{23}$$

$$H_4: \quad \mu = \mu, \sigma^2 = V^{-}\sigma^2, P(x) = \frac{1}{\sqrt{2\pi V^{-}\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2V^{-}\sigma^2}\right] \tag{24}$$

where again the variables M, V^+, and V^- are defined as:

- M is the amount by which the mean shifts either up or down;
- V^+ is the factor by which the variance increases; and
- V^- is the factor by which the variance decreases.

For a sequence of data points, at each data point the SPRT calculates the residual of the data point by subtracting the expected mean. The SPRT then uses these residuals and calculates a likelihood that the data sequence belongs to a different distribution (one or more of the four hypotheses above) rather than the original distribution (the null hypothesis). That likelihood is given by:

$$L_n = \frac{p(Y_n \mid H_j)}{p(Y_n \mid H_0)} \tag{25}$$

where $p(Y_n|H_x)$ is the probability of an observed sequence of residuals Y_n given that H_x is true. This is a linear transformation of variables ($y = x-\mu$) and changes the hypotheses to

$$H_0: \quad \mu = \mu, \sigma^2 = \sigma^2, P(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y)^2}{2\sigma^2}\right] \tag{26}$$

$$H_1: \quad \mu = \mu + M, \sigma^2 = \sigma^2, P(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y-M)^2}{2\sigma^2}\right] \tag{27}$$

$$H_2: \quad \mu = \mu - M, \sigma^2 = \sigma^2, P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y+M)^2}{2\sigma^2}\right] \tag{28}$$

$$H_3: \quad \mu = \mu, \sigma^2 = V^{+}\sigma^2, P(x) = \frac{1}{\sqrt{2\pi V^{+}\sigma^2}} \exp\left[\frac{-(y)^2}{2V^{+}\sigma^2}\right] \tag{29}$$

$$H_4: \quad \mu = \mu, \sigma^2 = V^- \sigma^2, P(x) = \frac{1}{\sqrt{2\pi V^- \sigma^2}} \exp\left[\frac{-(y)^2}{2V^- \sigma^2}\right] \tag{30}$$

where all (x-μ)'s have been replaced by y.

The probability for a sequence of data points for any given distribution is simply the product if individual probabilities, that is,

$$p(y_1, y_2, \ldots, y_n \mid H_i) = \prod_{k=1}^{n} P(y_k \mid H_i) \tag{31}$$

The probabilities for the null hypothesis and the alternate hypotheses are (after algebra) therefore given by:

$$p(y_1, y_2, \ldots, y_n \mid H_0) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left[-\frac{1}{2\sigma^2} \sum_{k=1}^{n} y_k^2\right] \tag{32}$$

$$p(y_1, y_2, \ldots, y_n \mid H_1) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left[-\frac{1}{2\sigma^2} \left(\sum_{k=1}^{n} y_k^2 - 2\sum_{k=1}^{n} y_k M + \sum_{k=1}^{n} M^2\right)\right] \tag{33}$$

$$p(y_1, y_2, \ldots, y_n \mid H_2) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left[-\frac{1}{2\sigma^2} \left(\sum_{k=1}^{n} y_k^2 + 2\sum_{k=1}^{n} y_k M + \sum_{k=1}^{n} M^2\right)\right] \tag{34}$$

$$p(y_1, y_2, \ldots, y_n \mid H_3) = \frac{1}{(2\pi V^+ \sigma^2)^{n/2}} \exp\left[-\frac{1}{2V^+ \sigma^2} \sum_{k=1}^{n} y_k^2\right] \tag{35}$$

$$p(y_1, y_2, \ldots, y_n \mid H_4) = \frac{1}{(2\pi V^- \sigma^2)^{n/2}} \exp\left[-\frac{1}{2V^- \sigma^2} \sum_{k=1}^{n} y_k^2\right] \tag{36}$$

By substituting these equations into the L_n equation, the likelihood ratio for the alternate hypotheses appear as:

$$L_1 = \exp\left[\frac{-1}{2\sigma^2} \sum_{k=1}^{n} M(M - 2y_k)\right] \tag{37}$$

$$L_2 = \exp\left[\frac{-1}{2\sigma^2}\sum_{k=1}^{n} M(M + 2y_k)\right] \quad (38)$$

$$L_3 = (V^+)^{-n/2} \exp\left[\frac{-1}{2\sigma^2}\left(\frac{1-V^+}{V^+}\right)\sum_{k=1}^{n} y_k^2\right] \quad (39)$$

$$L_4 = (V^-)^{-n/2} \exp\left[\frac{-1}{2\sigma^2}\left(\frac{1-V^-}{V^-}\right)\sum_{k=1}^{n} y_k^2\right] \quad (40)$$

The SPRT then takes the natural logarithm of this likelihood ratio to get:

$$LR_1 = \left[\frac{-1}{2\sigma^2}\sum_{k=1}^{n} M(M - 2y_k)\right] \quad (41)$$

$$LR_2 = \left[\frac{-1}{2\sigma^2}\sum_{k=1}^{n} M(M + 2y_k)\right] \quad (42)$$

$$LR_3 = -\frac{n}{2}\ln(V^+) - \frac{1}{2\sigma^2}\left(\frac{1-V^+}{V^+}\right)\sum_{k=1}^{n} y_k^2 \quad (43)$$

$$LR_4 = -\frac{n}{2}\ln(V^-) - \frac{1}{2\sigma^2}\left(\frac{1-V^-}{V^-}\right)\sum_{k=1}^{n} y_k^2 \quad (44)$$

Finally, the SPRT compares the log-likelihood ratios for each hypothesis to both an upper limit and a lower limit. The upper limit is based on a value B and the lower limit is based on a value A. These are defined as:

$A = \beta/(1-\alpha)$ (45)

$B = (1-\beta)/\alpha$ (46)

where α and β are the Type I and Type II error rates, respectively, as defined earlier. These rates are usually set to 0.001 and 0.1, respectively.

The actual upper and lower limits are upper limit lnB (the natural log of B) and lower limit lnA (the natural log of A). The limits are thus given by:

$$\text{LL} = \ln A = \ln\left[\frac{\beta}{1-\alpha}\right] \tag{47}$$

$$\text{UL} = \ln B = \ln\left[\frac{1-\beta}{\alpha}\right] \tag{48}$$

If a sequence has an LR greater than lnB, the sequence has a greater probability of belonging to the respective hypothesis. If a sequence has an LR less than lnA, the sequence has a greater probability of belonging to the null hypothesis. If a sequence has an LR between lnA and lnB, no decision about its parent distribution can be made and more data must be collected. Once an LR has gone outside the limits, the sequence is cleared and the LR is reset to 0.

Each hypothesis operates independently of the other hypotheses, that is, each hypothesis uses the same data point as collected but may be operating on completely different sequences depending on the last data point which cleared the sequence.

For review, the variables M, V^+, and V^- are defined as:

- M is the amount by which the mean shifts either up or down;
- V^+ is the factor by which the variance increases; and
- V^- is the factor by which the variance decreases.

For this research, the M, V^+ and V^- are set as follows:

$$M = 3\sigma \tag{49}$$

$$V^+ = \frac{\sigma_3^2}{\sigma^2} = \frac{\mu_1}{\mu} = \frac{\mu + M}{\mu} = 1 + \frac{3}{\sigma} \tag{50}$$

$$V^{-} = \frac{\sigma_4^2}{\sigma^2} = \frac{\mu_2}{\mu} = \frac{\mu - M}{\mu} = 1 - \frac{3}{\sigma} \quad (51)$$

For a Gaussian distribution, 99% of all data falls within 3 standard deviations above and below the mean. The setting for M means that a fault hypothesis mean is outside this 99% interval, and the settings for V^+ and V^- then correspond to these new means. In other words, these settings allow for the faulted hypotheses H_1 and H_2 to have means outside a window +/- 3σ from the null hypothesis mean, and for hypotheses H_3 and H_4 to have variances based on H_1 and H_2 and their Poisson natures.

The next chapter gives a description of the physical setup, which corresponds to the analyzed system. It than delves into the methodology of the statistical analysis of the detector data and demonstrates the use of the SPRT as described above. It also presents parametric studies which form the basis for the expert system alarm thresholds.

3.3.4 Chien's One-Sided Test

Another SPRT is Chien's one-sided test. As the name implies, this SPRT uses only one boundary limit to make its decision. The mathematics of this SPRT are described below.

The decision boundary B_1 satisfies the following relation:

$$\exp(B_1) - B_1 - 1 = -\left[B + A\frac{\exp(B) - 1}{1 - \exp(A)}\right] \quad (52)$$

where A and B in brackets are identical to the A and B in the earlier SPRT derivation.

For $\exp(B_1) >> B_1 - 1$, the above equation simplifies to:

$$\exp(B_1) \approx -\left[B + A\frac{\exp(B) - 1}{1 - \exp(A)}\right] \quad (53)$$

and is solved as:

$$B_1 \approx \ln\left(-\left[B + A\frac{\exp(B)-1}{1-\exp(A)}\right]\right) \tag{54}$$

When evaluated, the current value of the SPRT (the LR's from above) are then compared to the B_1. Chien's method therefore seems like a simpler method.

However, Chien's test has a major drawback. While Wald's two-sided SPRT tests for either mean or variance shifts, Chien's one-sided SPRT can only test for mean shifts. This arises from the lack of appropriate expression for the missed alarm criterion for the case of variance change [Harrison, 2002].

3.3.5 Relaxed Gaussian Behavior

A major assumption of measuring a normally distributed random variable characteristic of a process at steady state is that each data point measures the process in question and includes a noise component that is distributed identically to and independently from the noise components of every other data point. When violating the assumption of independence, such as through serially correlated changes in noise or process values, the measured signal then departs from its assumed Gaussian behavior. That is, the signal loses an important aspect of its randomness; while a histogram of the signal would show a Gaussian distribution, its sequential behavior would have definite and easily identifiable trends. Since the SPRT was developed for Gaussian behavior, this departure increases the false alarm rate.

Research performed by Kenny Gross and Kristin Hoyer [Gross, 1992] has demonstrated the ability to relax the necessity of strict Gaussian behavior when dealing with serially correlated noise contaminations. This research used data from EBR-II which displayed definite serially correlated noise in its collected data. By performing a Fourier series expansion and calculating the power spectrum density (PSD) for each constituent frequency, they found the modes that contributed the most to the serial behavior. These modes were retained and reincorporated into the expected value.

That is, instead of assuming the expected value of the measured signal remained exactly at the mean, the expected value shifts according to the periodic components. The residual (actual minus expected) values then approach a more Gaussian behavior.

This method allows the SPRT to operate as designed on the residuals, now in a Gaussian distribution and acting with Gaussian sequential characteristics. However the main drawback to this method is its batch calculation characteristic. The PSD must be calculated from all the collected data instead of online without *a priori* knowledge of future frequency components. The application of this method to SPRT hypotheses other than mean shifts is also unclear. Therefore this method is useful for post-operation analysis, but has limited use for real-time monitoring.

3.3.6 Relaxed Gaussian Assumption

Instead of relaxing only the assumption of non-Gaussian noise, another approach is the relaxing of the assumption of Gaussianity altogether. Research performed by Chenggang Yu and Bingjing Su [Yu, 2001] has led to the development of a non-parametric SPRT (or NSPRT). Their research requires only a continuous symmetric distribution of any type (such as certain binomials, certain Poissons, Student's T, etc.)

This method calculates its likelihood ratios (Eqn's 37 – 40) without assuming a Gaussian distribution. In fact the test assumes no particular distribution. Therefore they use the probability provided by the Wilcoxon signed rank test, a standard non-parametric statistics test. The likelihood then depends on the rank statistic from the Wilcoxon test.

Their research shows that for actual Gaussian distributions, the SPRT remains the optimal test, as mentioned in chapter 2. However, for other distributions the NSPRT outperforms the SPRT.

This method would therefore be of great use in situations where the exact distribution of collected data was unknown but assumed to be symmetric and continuous. This assumption is a weak assumption and is most likely easily met in most situations.

The drawback to this method is that it also concentrates on mean shifts. The variance issue is not addressed, and without a definite distribution with which to calculate variance, it may be difficult to resolve.

4.0 Methodology

A system featuring SPRT-based feature extraction and an expert system was developed and optimized to monitor the CAVIS system in order to eliminate costly alarm responses and unnecessary inventory checks. The system is named ESKIMO, an acronym for "*expert system keeping important material on-shelf*". As previously mentioned, the SPRT is a statistical method used to detect changes in the statistical characteristics of a data stream. The SPRT is used to extract features, which are used by an expert system. An expert system is a rule-based system designed to perform functions similar to those of an expert.

4.1 The Physical Setup

Eskimo operates by extracting features from the radiation signal for each sensor and then the expert system analyzes the extracted features to detect root causes of alarms in the CAVIS system. The CAVIS setup is shown in Figure 10 and the Feature Extraction/Expert System setup is shown in Figure 11.

In Figure 10, an arbitrary number of RADSiP detectors feed into one sensor concentrator (Concntrtr) box. Multiple sensor concentrators feed into one Power Communication/Distribution Unit (PCDU), and multiple PCDUs feed into the controlling computer.

In future applications, this setup may be altered as necessary. However, this research used this particular system architecture. The final setup will affect the expert system rules base, but will not affect the feature extraction system.

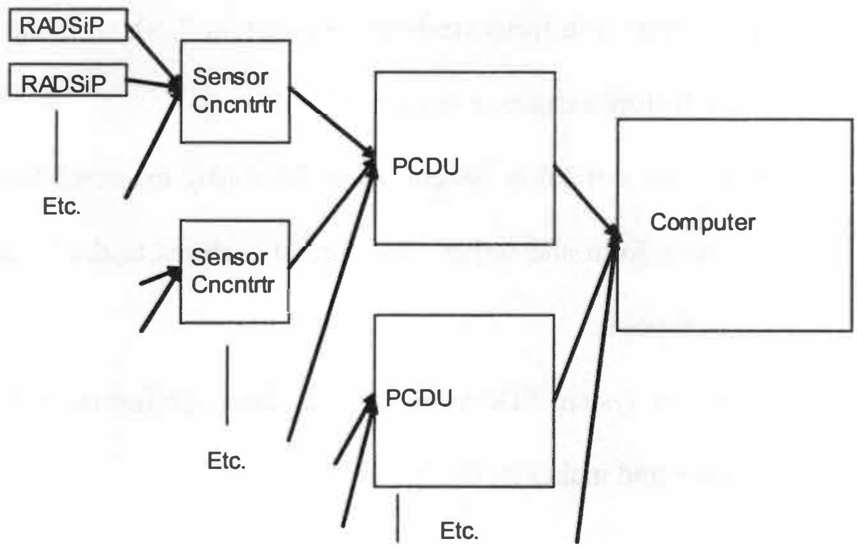

Figure 10 – The CAVIS System

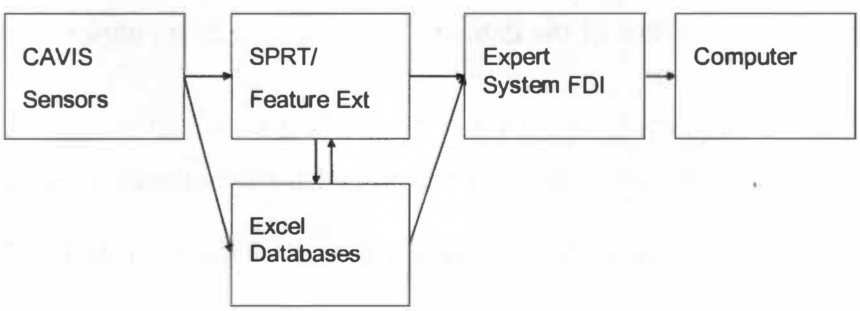

Figure 11 – Feature Extraction/Expert System

The basic scheme of the system in Figure 11 is as follows:

1) The CAVIS detectors send their count rates to the CAVIS system. The CAVIS system in turns sends those counts to both an Excel database system and the feature extraction system.

2) The feature extraction system reads previously extracted features and current conditions from and writes new current features to the Excel databases with each new point.

3) The expert system FDI receives input from the feature extraction and Excel databases and makes its decisions.

4.2 Application of the SPRT to Count Rate Data

An example of the SPRT appears in Figure 12. The figure was generated in MATLAB using its normally distributed random number generator and modifying it to have the desired μ and σ. It was then analyzed using seqprob2.m as included in the Appendix. This example uses fabricated data, mean and variance of 50, without faults. The top subplot is a plot of the data as fabricated; the four subplots correspond to each fault hypothesis.

Note that the SPRT does not alarm, and for hypotheses 1 and 2 in fact tends to reach the lower limit regularly, indicating a strong adherence to the null hypothesis mean. The numbers along the horizontal axis are, from left to right, the total alarm rate, hypothesis 1 alarm rate, hypothesis 2 alarm rate, hypothesis 3 alarm rate, and hypothesis 4 alarm rate. Since there were no alarms, all alarm rates are 0.

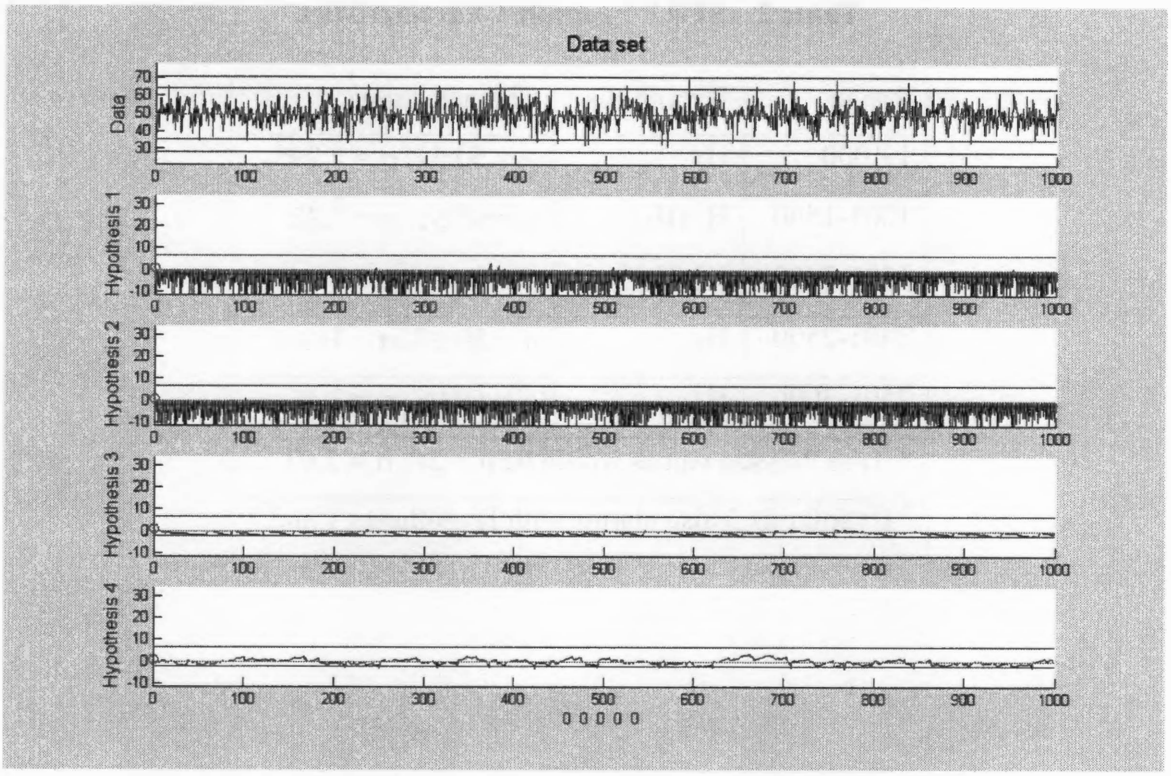

Figure 12 – First SPRT Example

Next, a faulted data set in constructed. In this case, four different faults, each corresponding to a particular hypothesis, are induced in fabricated data. The data characteristics appear in Table 2.

The plot appears in Figure 13. Note the high number of alarms in each region as fabricated. Note also that hypothesis 3 alarms with hypotheses 1 and 2, and conversely hypotheses 1 and 2 alarm to some extent with hypothesis 3.

In this SPRT, the total alarm rate is 0.116, or 11.6%. Most of the alarms are spread evenly among hypotheses 1 – 3 with hypothesis 4 contributing only a minor share.

Table 2 – SPRT Example Characteristics

Range	Hypothesis	Characteristics
1-1000	H_0	$\mu = 50.32$, $\sigma = 7.29$*
1001-1500	H_1 $(H_3)^{\#}$	$\mu = 65.51$, $\sigma = 7.32$
1501-2000	H_2 $(H_3)^{\#}$	$\mu = 34.74$, $\sigma = 7.16$
2001-2500	H_3	$\mu = 50.37$, $\sigma = 10.64$
2501-3000	H_4	$\mu = 50.06$, $\sigma = 3.46$
* True Poisson values would be $\mu = 50$, $\sigma = 7.07$		
# Hypothesis 3 also alarms with Hypotheses 1 and 2		

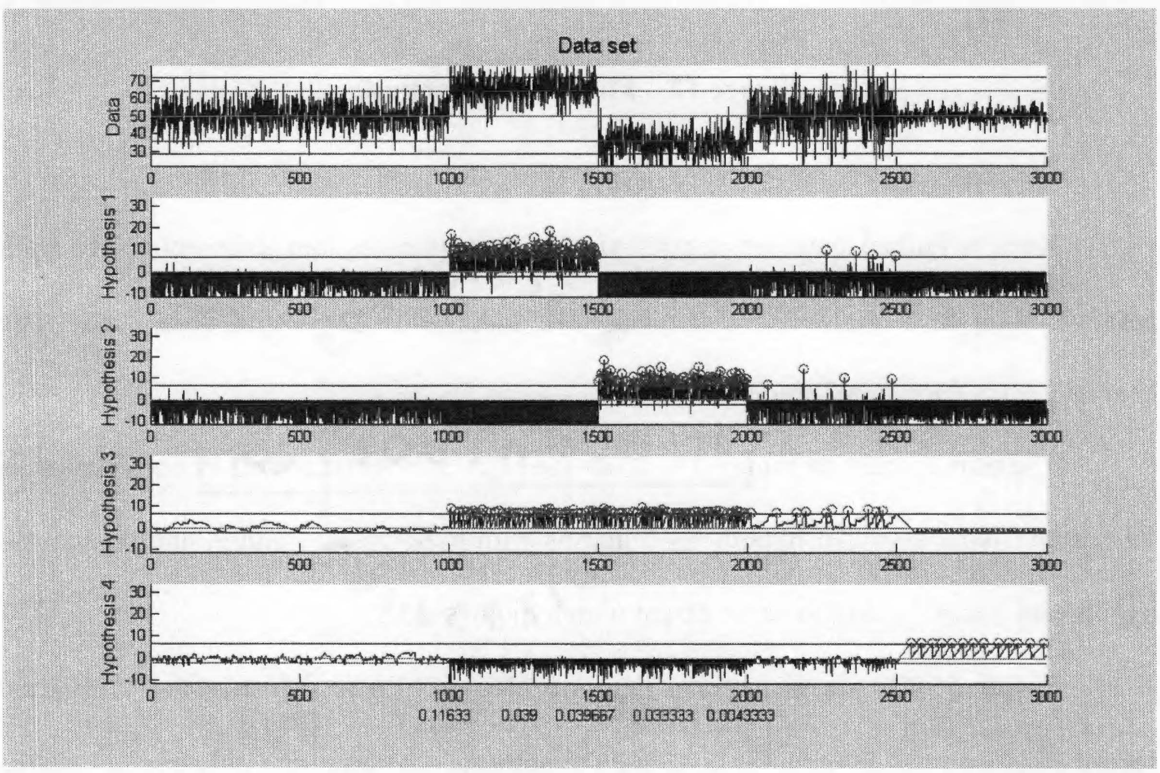

Figure 13 – Second SPRT Example

4.3 Feature Extraction System

The feature extraction system (FES) acts as a pre-processor for the collected data. The FES acts as custodian and interpreter for the count rate database and extracts information useful to the expert system. The FES then stores this extracted information in another database for expert system use.

The FES uses the SPRT and runs tests to calculate, track, and communicate trends within the collected count rates for use in the expert system. The expert system uses these extracted features to isolate and diagnose system faults.

The FES:

1. tracks the indices of all alternative hypothesis alarms over the last 1000 and 100 data points and uses this index list to calculate a running alarm frequency for each hypothesis. The system then reports the running frequencies to the expert system database for use in its inference engines.
2. tracks the interval since the last alarm for each hypothesis. The system reports these intervals directly to the expert system database for its use.
3. maintains the SPRT values for each hypothesis. These values determine whether a hypothesis alarms or not and get updated with each data point. The expert system never sees this data.
4. tracks the number of same-sign residuals. This number goes directly to the expert system database for a "runs test".

5. retains an explicit record of the last 50 data points. It calculates a "most recent" variance of the last 5 data points and sends this variance to the expert system database.

The result is that the FES continually reads from and overwrites its own database and overwrites the expert system database.

The reasoning behind each feature extraction task is explained as follows. By counting faults over a fixed number of data points, this effectively measures the alarm rate. Experimentally, an unfaulted distribution will produce a total number of faults as tabulated in a parametric study as reported later. The expected total alarm rate for means ranging from 25 to 145 is around $2 * 10^{-4}$, while the alarm rates for hypotheses 1 and 2 are around $0.75 * 10^{-4}$ and hypotheses 3 and 4 are around $0.25 * 10^{-4}$. The maximum number of SPRT alarms in the last 100 or 1000 data points is set through both experimentally and practicality.

However, if the fault tolerances were set at levels that low, one random data point outside the 3σ bands sufficient to cause any SPRT alarm will cause continuing alarms until the total number of data points has increased enough to cause the rate to drop to expected levels. Therefore in the actual fault detection/isolation algorithm, different values are used. These values and the results of their implementation may be found in Joseph Bowling's thesis [Bowling, 2004].

Since the FES post-processes the SPRT to prevent continuous alarm frequency alarms, other metrics are necessary. The FES examines the alarm interval as one of those other metrics. This therefore serves as a surrogate alarm threshold for finding drifting or wildly varying systems.

The consecutive residuals feature (the runs test) is a standard statistical controls test as outlined earlier. The probability of any residual being above or below the mean is 0.5. Therefore 9 consecutive same-sign residuals occur every 2^9, or every 512, sets of 9 consecutive data points.

For example, a set of 12 data points has 4 sets of 9 consecutive data points (1 through 9, 2 through 10, 3 through 11, and 4 through 12). Simple algebra yields the relation of when given n points, there are (n-8) sets of 9 consecutive data points. Thus, there should be one run of 9 consecutive same-sign residuals in 520 points.

Finally the current variance over the last 5 data points acts as another surrogate measure in lieu of true expected alarm rate. The variance of the last 5 quickly diagnoses "stuck" detectors that may be stuck around the mean. This "sticking" phenomenon occurs frequently in data sets provided by Y-12 and is most likely indicative of a communications problem from concentrator to computer.

Two other features used by the expert system include the current count rate and a built-in system status signal from the CAVIS hardware. The FES supplies the current count rate directly to the expert system database without processing, but the FES does not extract the system status signal.

4.4 Parametric Studies

This research required several parametric studies in MATLAB to ensure the robustness and applicability of the SPRT for this system. Those studies are summarized below. For each study the SPRT is used with the above-defined M, V^+, and V^- and the assumption that the mean and variance are identical. The calculations were performed in MATLAB with the function seqprob2a.m as included. The specific coding for each study is also included.

4.4.1 The SPRT with Varying α, β, and μ (paramplot.m)

This study verifies that the SPRT does not vary too wildly over the expected counting range, approximately 20 – 150. This is an important property to verify so that the same SPRT and feature extraction model may be used throughout the range of the expected counting range. Here, α, β, and μ refer to false alarm rate, missed alarm rate, and mean, respectively.

The α's took the values 0.0005, 0.001, 0.005, 0.01, and 0.05 while the β's took the values 0.05, 0.1, 0.15, and 0.2. Clearly, these two sets include the research-used values of 0.001 and 0.1 for α and β, respectively. The α's and β's were kept low due to the practical purpose of keeping these values low. As these values increase the effectiveness of the SPRT (as any other statistical test) is diminished. The mean μ was then varied from 20 to 80 in steps of 10. As will be seen below, varying the mean above 80 is unnecessary. This was performed 50 times with data sets generated with a normally distributed random number generator with set length 500, and the results from each run were averaged to produce a more stable prediction for the alarm rate.

The result of this parametric study is that the SPRT behaves similarly if not identically for different means. That is, with a given α and β, the same behavior can be expected for any mean. Therefore the SPRT is not restricted to a certain range of means. In fact, as the mean increases, the total alarm rate slightly decreases. As mentioned above, varying the mean above 80 is unnecessary due to this tendency for alarm rate decrease. This is examined in another parametric study.

Example plots from the parametric study appear in Figure 14. For this, the mean μ is 20 and α and β vary as described above.

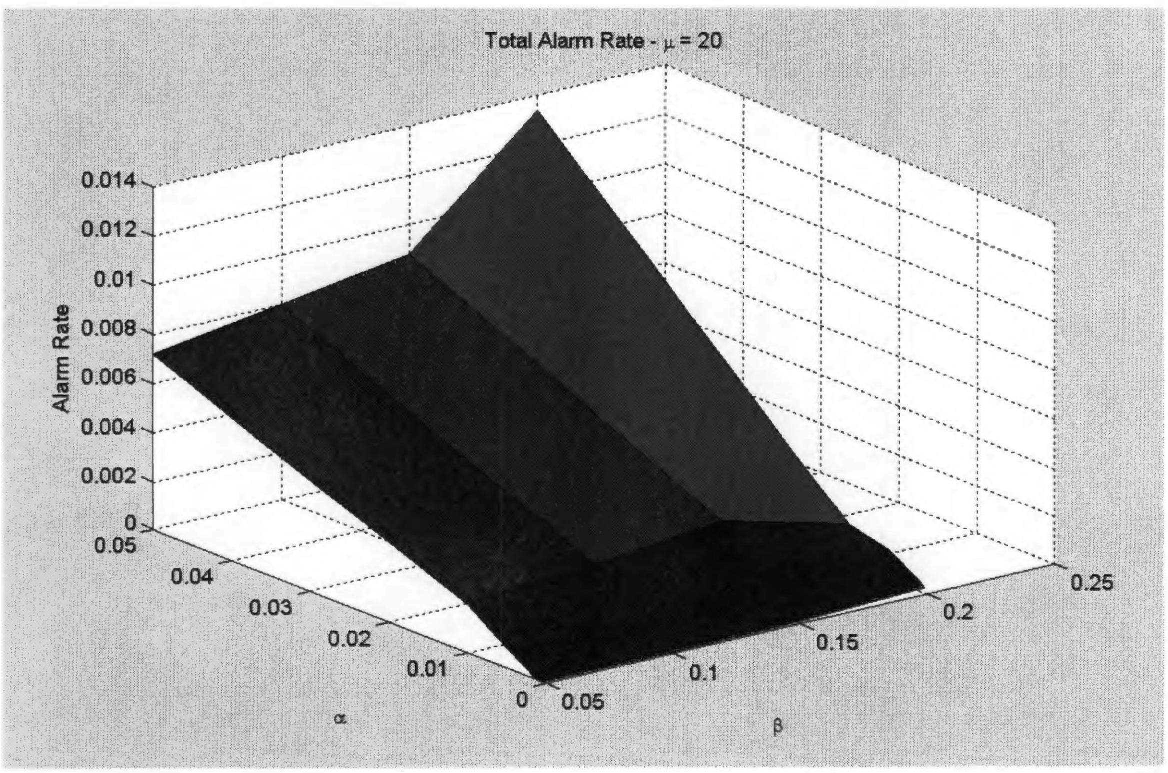

Figure 14 – Total Alarm Rate, μ = 20, Varying α and β

All plots from all the means could be examined; however, a better way of viewing this trend is the following plot. The x-axis represents data set number. For the study, 140 data sets (representing 7 μ's, 5 α's, and 4 β's) were used. Each different combination can be clearly seen in Figure 15, but for completeness are given:

$\alpha = 0.0005, 0.001, 0.005, 0.01, 0.05$

$\beta = 0.05, 0.1, 0.15, 0.2$

$\mu = 20, 30, 40, 50, 60, 70, 80$

The tallest curve represents the total alarm rate, and there are obviously 7 "spikes", each spike representing the end of one set of μ variation. That is, the first spike

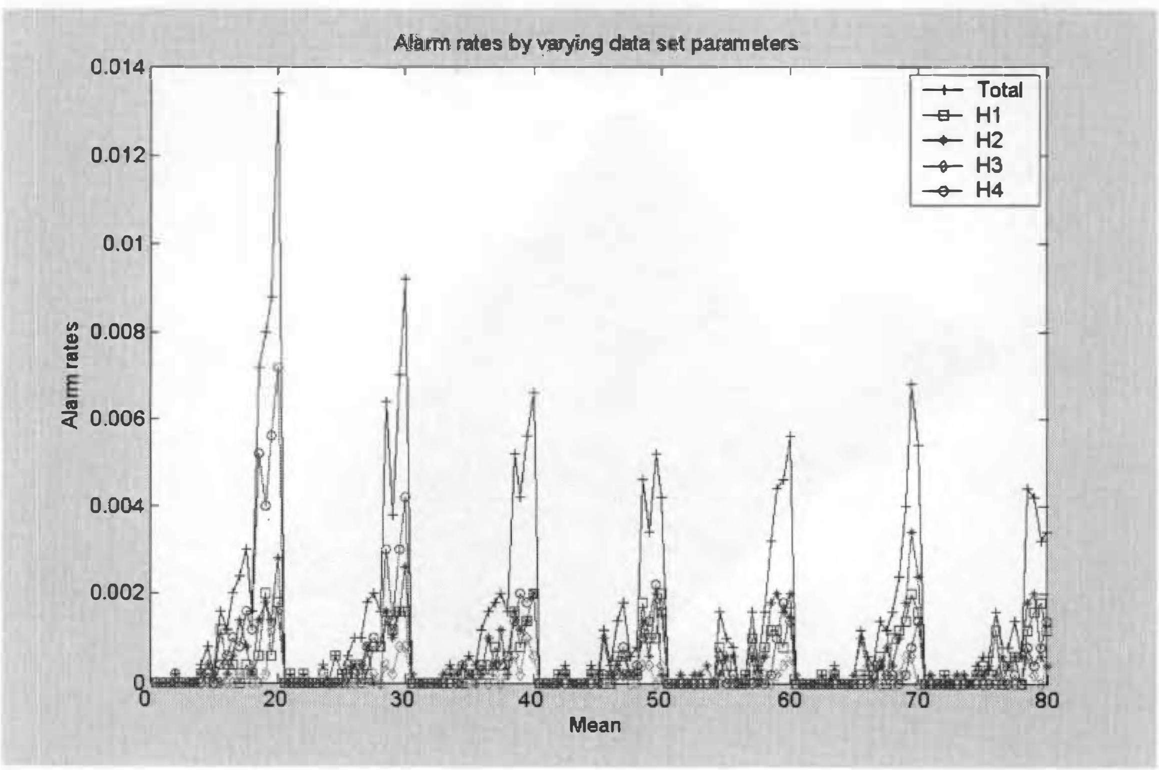

Figure 15 – Alarm Rates with All Varied Parameters

represents the variation of α and β with fixed $\mu = 20$. The second spike represents variation with $\mu = 30$, and so on, as can be correlated with the horizontal axis.

The second fixed parameter is α. Thus, as each smaller section continues to the right, this represents an increase in α with varying β. Figure 16 demonstrates.

In this plot, α increases to the right with the data sets. This plot shows that the most sensitive parameter for a given m is α. This result is expected since α directly affects the number of alarms by causing false alarms whereas β allows true alarms to slip through undetected.

The net result of this parametric study is that varying α and β has similar effects for any given μ within the research's expected regime. This result means that using the SPRT across a fairly wide range of μ's will yield similar and predictable results with different α's and β's, allowing future statements about the SPRT to be made with respect to the research in general.

4.4.2 Expected Alarm Rates with Fixed α's and β's (samplesa.m and sampletest.m)

After the previous parametric study demonstrated the effects of varying α and β across the μ spectrum, the next step is to fix $\alpha = 0.001$ and $\beta = 0.1$ and find the general trend of alarms across the same spectrum. To do this, the SPRT is applied to data sets which only vary by μ. Each data set is generated by a normally distributed random number generator with set length 10000. The test for each μ is performed 50 times and averaged to decrease the variance of the expected value. The μ's vary from 25 to 145 by steps of 3 for a total of 41 μ's. The resulting data is plotted in Figure 17 below.

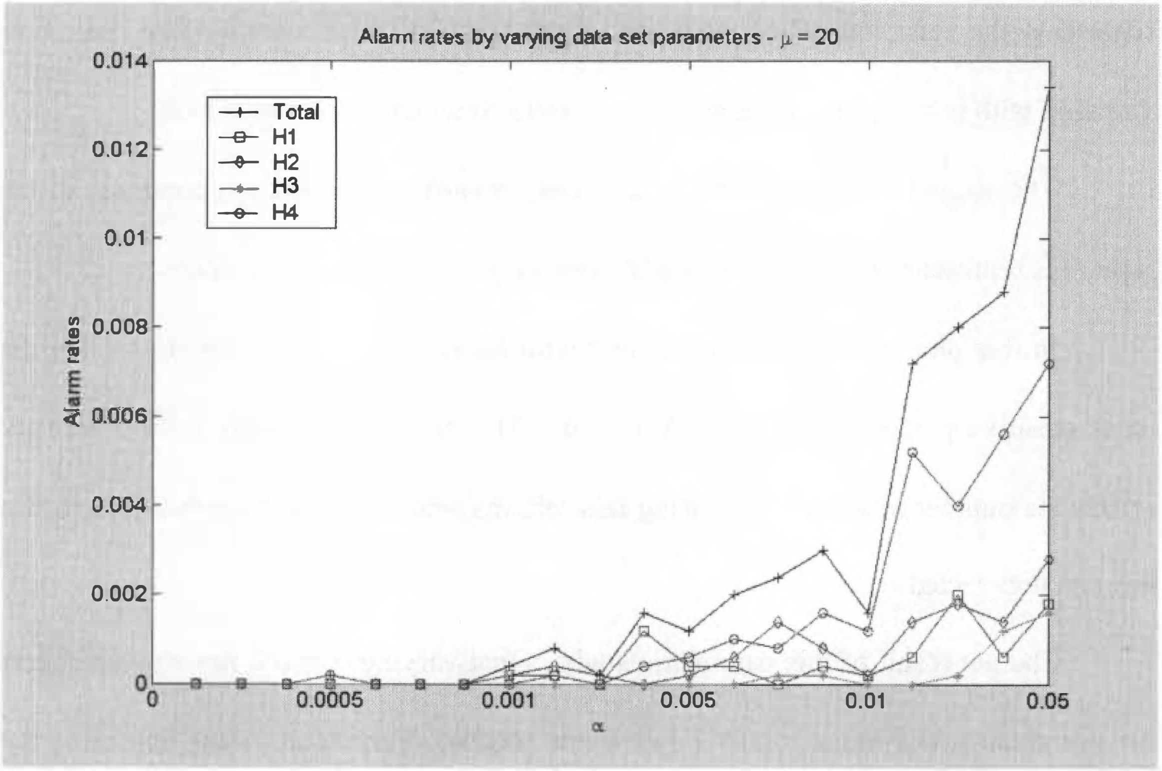

Figure 16 – Alarm Rates, $\mu = 20$, Varying α and β

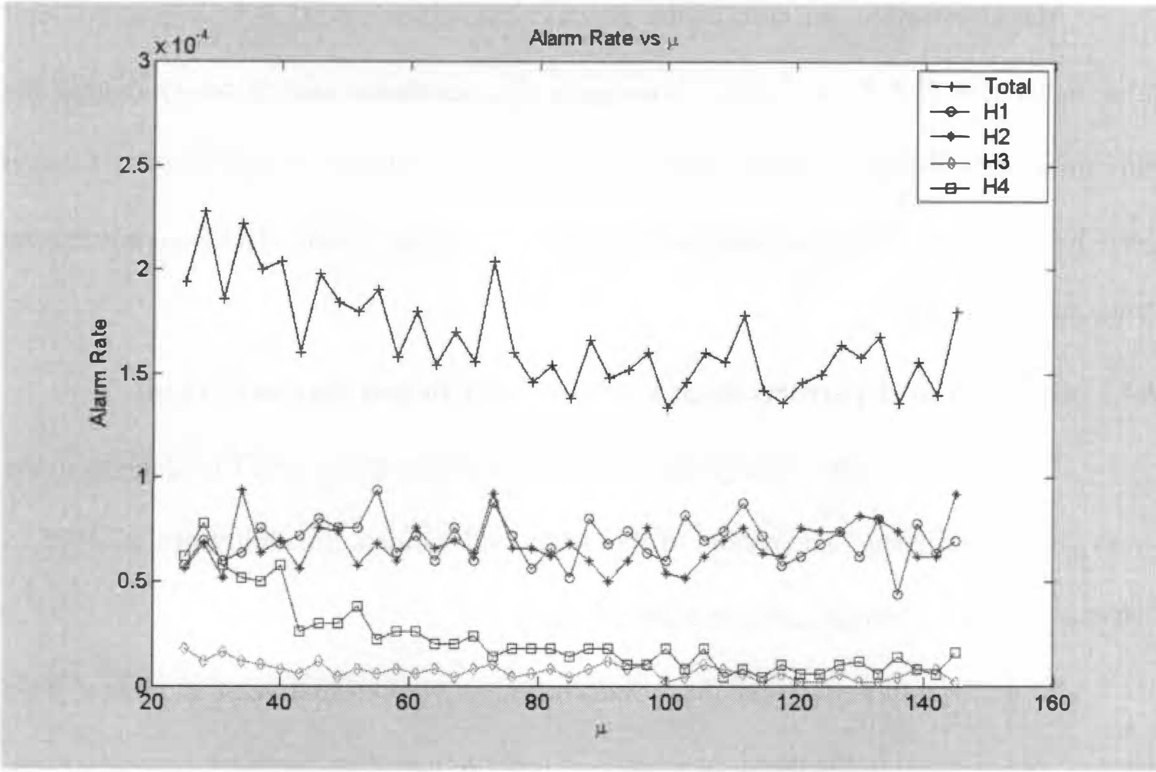

Figure 17 – Alarm Rate vs μ

This parametric study shows that the alarm rates are rather steady across the range of means. The total alarm rate is generally between 1.5 and 2 * 10^{-4}. Hypotheses 1 and 2 remain around 0.75 * 10^{-4} and hypotheses 3 and 4 are generally less than 0.5 * 10^{-4}. As in the previous parametric study, the total alarm rates decrease as the mean increases. This is therefore consistent with earlier results.

The importance of this study is to demonstrate the validity of expecting the same behavior of the SPRT at all expected μ's. Since none of the alarm rates vary drastically from mean to mean, any assumptions made at one μ is essentially valid at any other μ.

Mathematically, the ratio of the greatest total alarm rate (2.3 * 10^{-4}) to the least total alarm rate (1.3 * 10^{-4}) is 1.7, meaning the maximum rate is nearly double the minimum rate. However, due to the 10^{-4} factor, this amounts to a difference of 1 alarm per 10000 points. Thus, the assertion that there is no appreciable difference among the different μ's is valid.

4.4.3 Sensitivity to Departures from Expected Distribution (paramplot2.m)

This final parametric study examines the sensitivity of the SPRT to changes in the mean and the variance. The results of this study will validate the ability of the SPRT to detect quickly the changes in those characteristics.

To perform this study one mean was chosen as a representative of all means. This decision is validated in the previous study. In order to maintain complete representation, the mean is chosen to be near the center of the expected means. In this case, this value is 95. Thus, the sample distribution has mean and variance of 95. This study used the normally distributed random number generator with data set length 500. It was performed 50 times and averaged to provide a more robust estimate.

The sample distribution was altered in three cases in four ways each. First, the mean was increased, second the mean was decreased, third the variance was increased, and fourth the variance was decreased. The difference among the three cases is the amount by which the values were altered. In the first case, the amount of altering was 1%, in the second, 10%, and in the third, 50%.

For the mean increase or decrease, the mean was shifted by adding or subtracting a value arrived at multiplying the mean by the parameter. For example, with parameter = 1%, the mean increased or decreased by 1%. For the variance increase or decrease, the standard deviation (not the variance) was multiplied or divided by 1 plus the parameter. For example, with parameter = 1%, the standard deviation multiplier (or divisor) was 1.1. In effect, that increased or decreased the variance by the square of that amount.

The results of this study are shown in Figure 18. The alarm rate increases with the parameter. This result is expected – as the parameter increases the departure from the expected distribution increases. As seen in the figure above, the mean shift sensitivity is the greatest. In the top subplot, there appear to be only three lines; the fourth line (H2) and fifth line (H4) remain 0. In the second subplot, Hl and H4 remain 0. In the third, H4 again remains 0. In the bottom subplot, the total follows the H4 line and the H1, H2, and H3 lines remain at 0.

The parametric study shows that hypothesis 3 – increase in σ^2 – is effective in every situation of departure from the expected distribution. It also shows that even with a drastic decrease in variance, hypothesis 4 – decrease in σ^2 – is rather ineffective. Finally, the study shows that the total alarm rate increases much quicker with shifts in the mean, not the variance.

This final result is important since the mean is the most important measurement. The mean is important because it is a direct measure of the amount of material being measured – if the amount of material varies, the mean will vary with it while signal variance may be affected by other sources of noise.

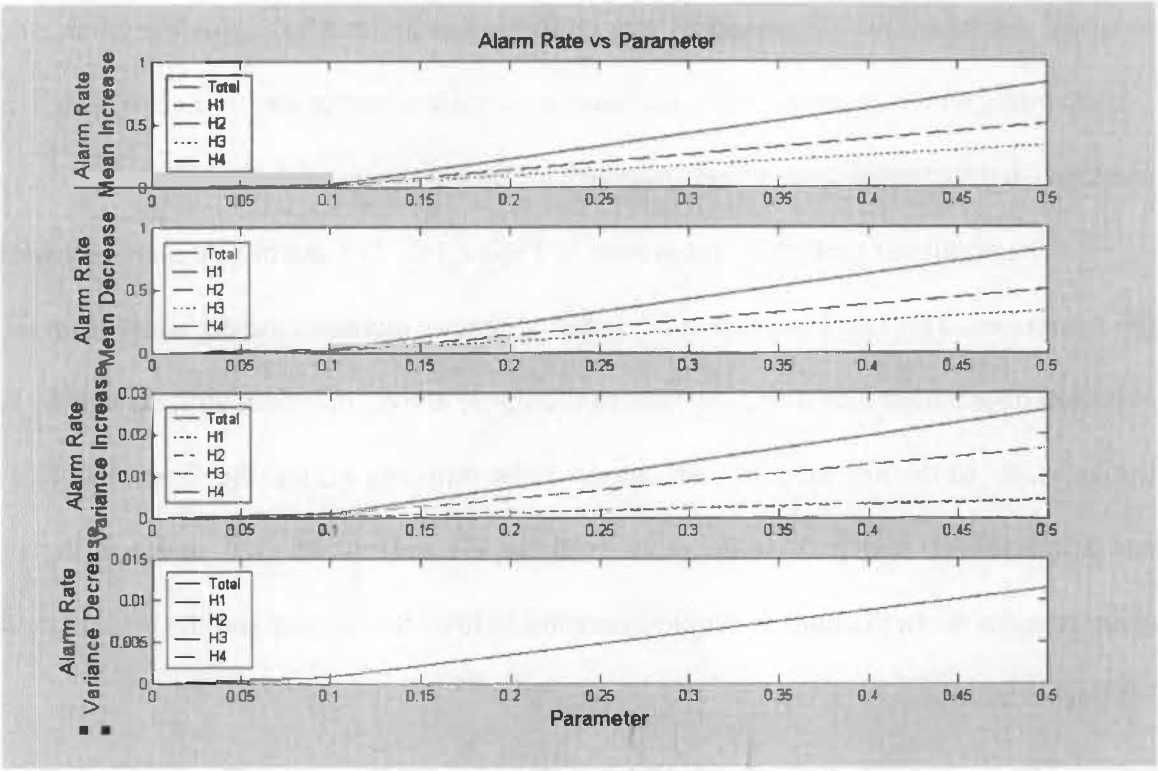

Figure 18 – Variation of Alteration Parameter

4.5 Application to the Y-12 Data Sets

Y-12 has provided data sets for analysis, and these have a substantial amount of data contained within them. The data sets are arranged in groups of 20. Each group of 20 is broken down into four subgroups denoted by the letters A – D, and each of these subgroups has five detectors contained within it. There are 21 of these groups taking the letters A – Z excluding D, O, U, W, and Y. Every file starts with the letter C. Therefore, an example data file would be CFB5, meaning the fifth detector in subgroup B from group F. The physical setup would put five detectors going into one concentrator, five concentrators going into each PCDU, and 21 PCDUs going into the collection computer.

Each detector has two data sets associated (except for K and Z which only have one). The first data set is "slow" data with counts/min recorded at intervals of one hour while the second set is "fast" data recorded at intervals of one minute. The two data sets are remarkably dissimilar; the "slow" data has systematic errors throughout while the "fast" data is relatively well-behaved. Chronologically speaking, the slow data predate the fast data.

Analysis of each data set is unnecessary since all data sets are similar to each other. The slow sets can be regarded as simply a collection of bona fide data points while the fast sets have utility as being available for SPRT application to real-life data.

While the opportunity to check the SPRT with real-life data is vital, the data sets do have some dissimilarity to the expected distribution. Due to the collection algorithm, the variance of the data set is not equal to the mean.

The collection algorithm is as follows:

1. the counter logs the count rate over 200 ms
2. 5 200 ms counts are logged and summed to generate a 1 s count
3. 40 of these 1 s counts are summed and divided by 4 to generate a 10 s count

This algorithm generates data with a normal distribution with mean of 10λ and variance 2.5λ, where λ is the true count rate [Harrison, 2002]. The difference in mean and variance come from the fact that the collection performs a linear transformation on the data in order to collect it but does not restore it to its original state through a second linear transformation. That is, the division by 4 at the end stage does not negate any other transformation.

However, a simple linear transformation can remedy that. This linear transformation is achieved in this manner:

1. Calculate the data mean and variance.
2. Subtract the mean from the data (also known as mean-centering) to find the residuals.
3. Divide the residuals by the calculated standard deviation (also known as standardizing).
4. Multiply the standardized residuals by the new standard deviation (the square root of the mean in this case).
5. Add the mean.

The new data set has variance equal to the mean while keeping the information from the old distribution.

Now that the feature extraction system has been developed, it will be applied to the Y-12 data and data from the experimental setup. The results are presented in the following section.

5.0 Results

The results of this research are somewhat abstract unless viewed in context and in conjunction with simultaneous research reported in Bowling [Bowling, 2004]. The SPRT and feature extraction system as described contain no real decision-making abilities and only provide information to be used in a decision-making process in a final step of processing. The effectiveness of this research is addressed in Bowling. With that framework, the results of this research are as follows.

5.1 Physical Reality

This research has demonstrated that the physical situation, that is, the storage of HEU, has a theoretical basis for application of the SPRT. The radiation profile as collected and analyzed should follow a normal or Gaussian distribution with a calculable definite mean.

Y-12 has provided a multitude of data sets for analysis. Upon examination of these data sets, many contain radical departures from the expected distribution. The collection algorithm for these data sets included a filtering process that changed the variance statistic. However, this filtering process has been eliminated from the collection algorithm and now provides a distribution that falls very closely to the expected distribution.

The most significant departures from the expected distribution arose from system faults. As discussed earlier, different faults manifest themselves in different ways, such as increased variance induced by loose wiring or increased mean induced by an external source. This research does not diagnose the faults; it merely provides tools for diagnosis.

5.2 Statistical Reality

When performing the SPRT on the Y-12 data sets, the departure from the expected variance is a problem. However, this problem is easily fixed by rescoring the data as described in the methodology. The results of applying the SPRT to a few of the data sets can be seen below in the following figures.

Figure 19 is detector CNA2. Of the slow data, this one is relatively well-behaved. This data set has a few obvious flaws (examined later with the SPRT). Figure 20 is a histogram of the data, and figure 21 is a normal probability plot of the data. These show that the data does in fact follow a normal distribution. Nearly all data sets provided by Y-12 are normal. As mentioned, many sets have data problems of some kind that tend to cause a departure from normality. However, all data sets selected for these examples have normal distributions, as will future data sets.

Figure 22 shows the SPRT application to the data. At points 4215, 8270, 9987, and 10374, (marked by arrows) the detector reads 0. Since its mean is almost exactly 120, statistically speaking these points should not exist. The SPRT alarms in both hypotheses 2 and 3 on all four of these occasions.

However, the detector "sticks" around point 8700 and remains "stuck" until nearly point 9200. Since the stick is at 108 and the standard deviation is sqrt(120) or nearly 11, this is at almost exactly one standard deviation from the mean. The test is defined to look for a 3σ change in the mean; sticks that close to 1σ with a mean that high

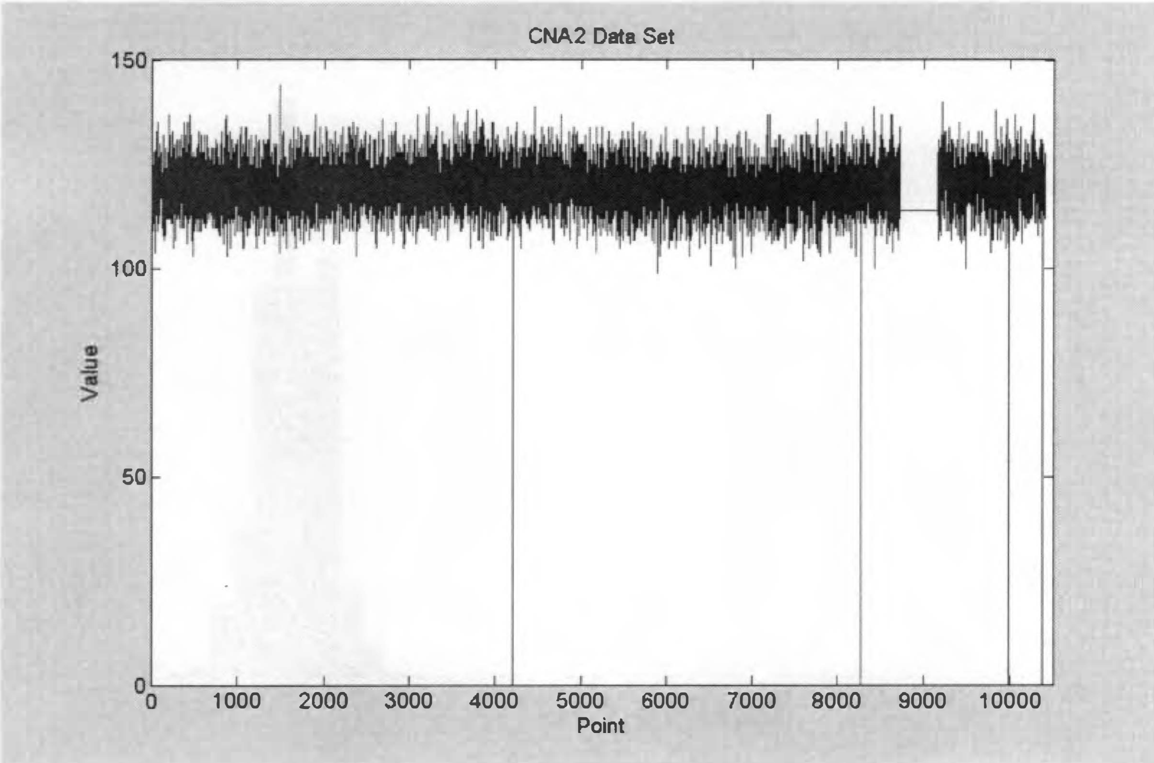

Figure 19 – CNA2 “Slow” Data Set

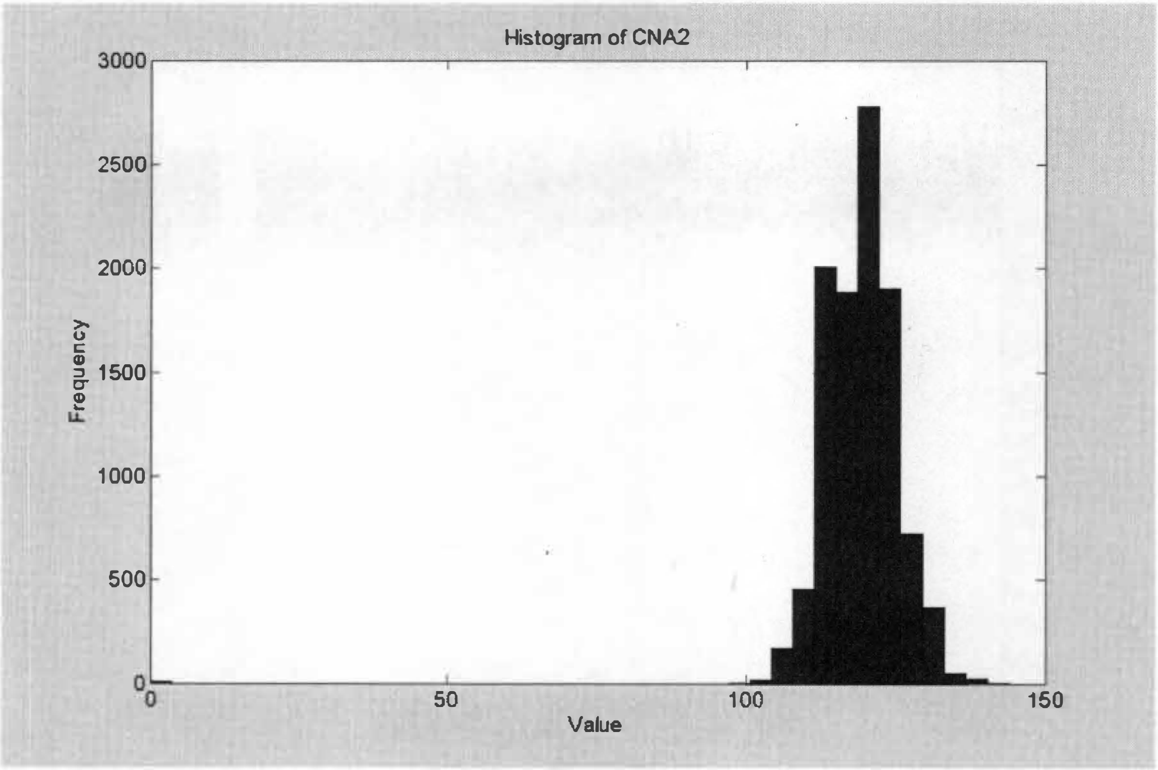

Figure 20 – Histogram of CNA2

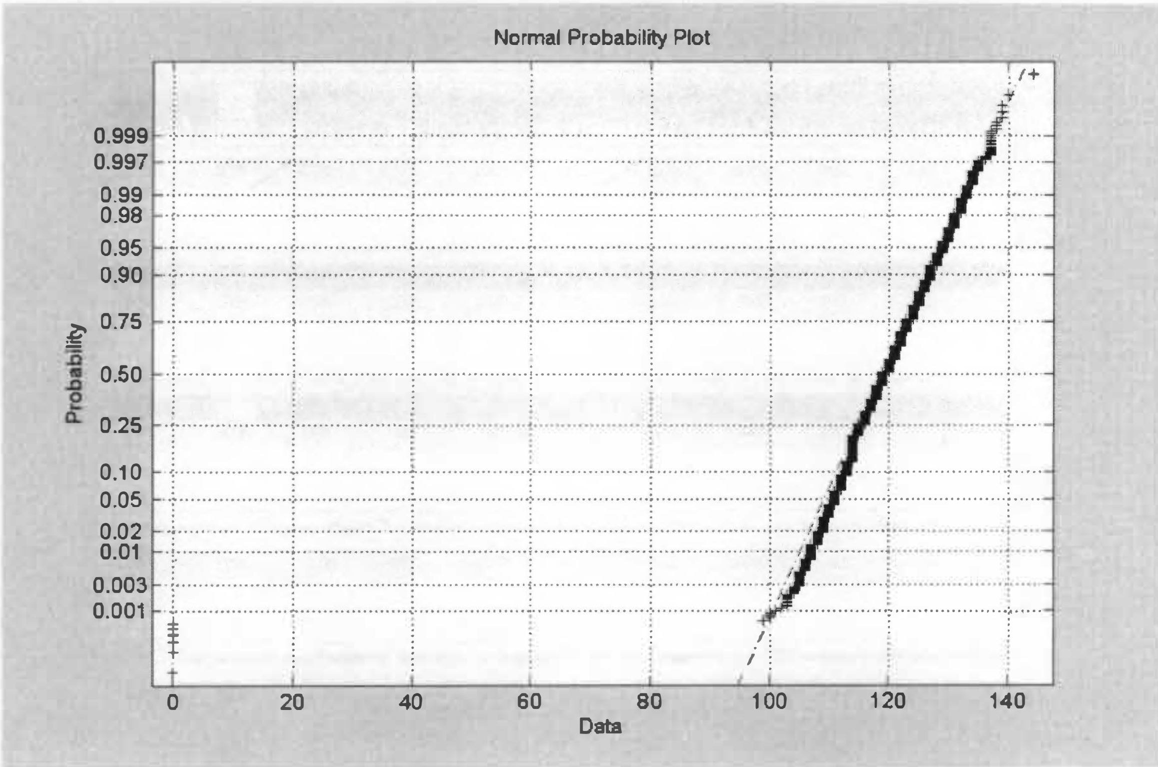

Figure 21 – Normal Probability Plot of CNA2 Data

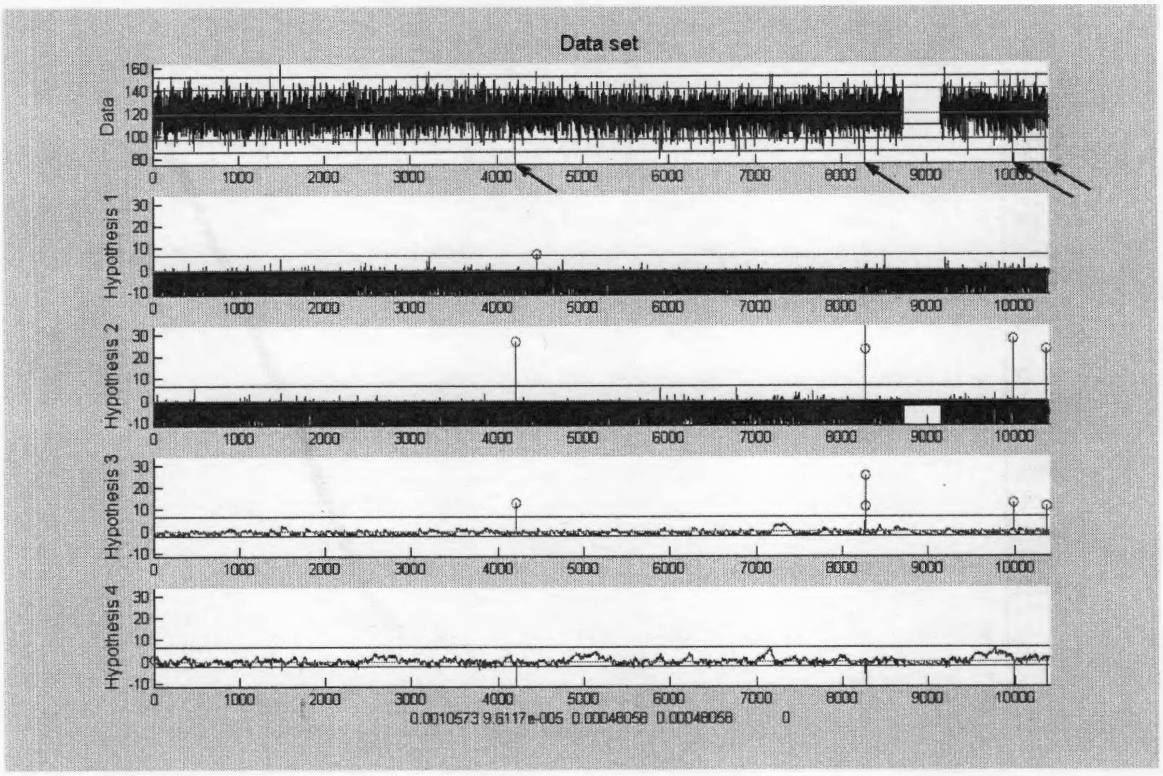

Figure 22 – Data Set CNA2 – "Slow"

are difficult to differentiate using only the SPRT. And as demonstrated earlier, the SPRT variance tests – especially the decrease in variance – take longer to alarm. The SPRT does not catch it. However, the other feature extraction methods catch it after 5 points (the last-5 variance test) or 9 points (the runs test).

The runs test shows that there are 35 runs of 9 or more points above the mean and 439 runs of 9 or more points below the mean. This high number of runs test alarms is due to the "stick" less than the mean. Data points can be counted twice in these figures, that is, if 12 consecutive points are above the mean, this counts as four runs of 9 (terminating in 9, 10, 11, and 12).

As mentioned earlier, there should be approximately one run of 9 per 500 data points. In a data set of 10000, there should be around 20. The expert system must not generate an alarm based on these normal statistical events, and so the expert system knowledge base must be robust to these happenings.

Figure 23 is the same detector but with fast data. The figure shows absolutely no alarms at all. This is consistent with a visual examination of the data. The runs test shows one run of 9 below the mean with an expected number of runs around 2 or 3.

This may indicate a process less random than expected, but having only one instead of 2 or 3 is not very conclusive (as opposed to one runs test instead of an expected 15, for example). For all intents and purposes, this data set is perfect.

The next slow data set in Figure 24 has a couple of slow, subtle drifts at the beginning and a stick towards the end, similar to the first slow data set. (In fact, all the slow data sets have a stick in them).

The slow subtle drift is picked up as an increase in the mean by hypothesis 1 alarms as well as by the variance increase alarm in hypothesis 3. The most interesting feature is that the stick that eluded the SPRT in the first example is caught in this one. The mean for this data set is a little more than 56, and the sensor sticks at a little under 54. The standard deviation is sqrt(56) or around 7.5, so this is far less than one standard deviation. The SPRT picks it up almost instantaneously. The runs test for consecutive same-sign residuals shows 56 runs of 9 or more points above the mean and 540 runs of 9 or more points below it.

Figure 25 is the same detector but from the fast data collection.

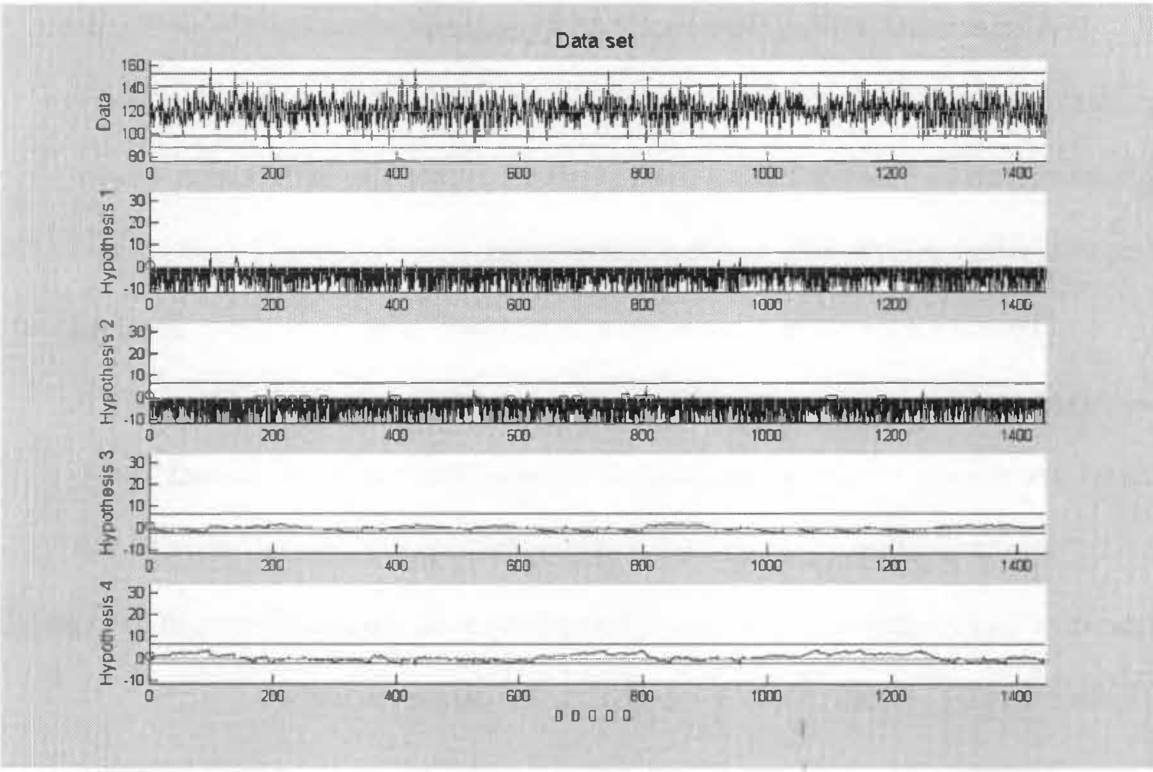

Figure 23 – CNA2 Data – "Fast"

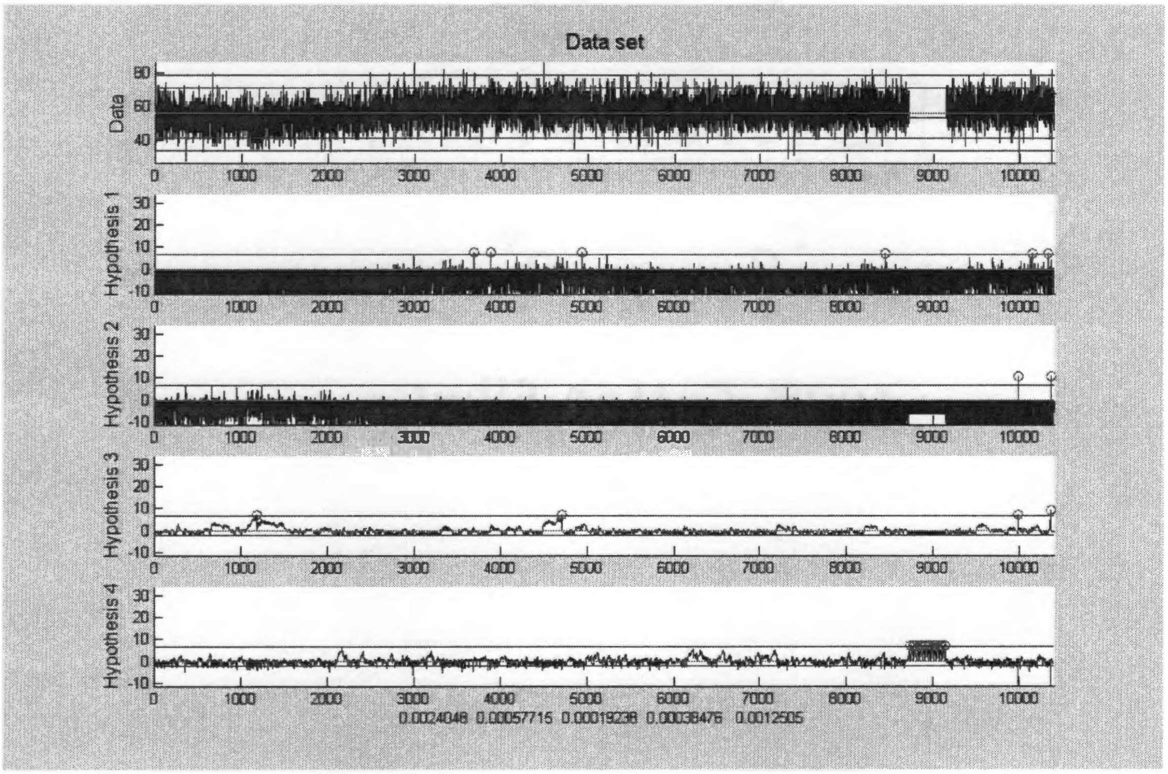

Figure 24 – CAD4 Data – "Slow"

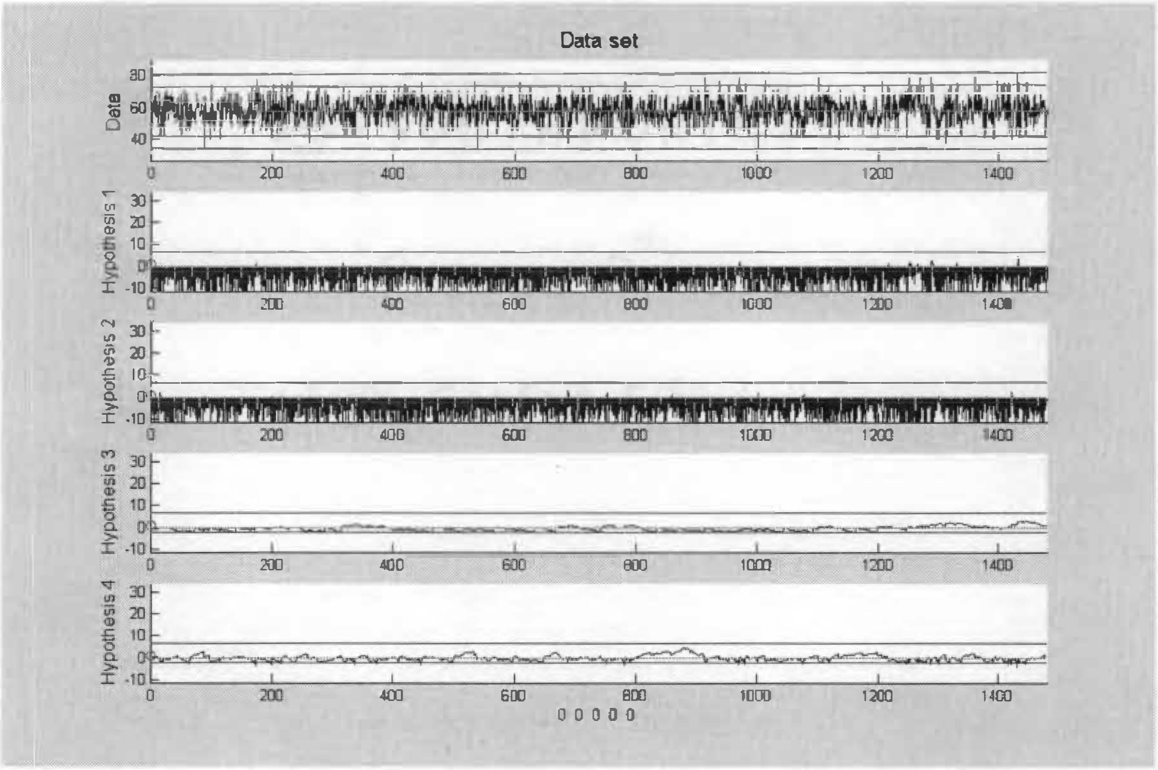

Figure 25 – CAD4 Data – "Fast"

Again, the fast version of the data set has no obvious faults and the SPRT confirms that hypothesis. However, the runs test for consecutive same signs yields 1 positive run and 12 negative runs. This total of 14 runs is far higher than the expected 2 or 3 and indicates some unknown characteristic. The distribution has a skewness of -0.0227 and bias-adjusted kurtosis of -0.3135 (perfectly normal distributions have 0 for both). The mean is 58.0108 and the variance is 57.8755, indicating that not only is this distribution very normal but also very Poisson. The underlying cause of the runs tests alarms remains unknown.

The examples above demonstrate the SPRT, the variance test (5-point variance) and the runs test. The other feature extraction tasks are more easily viewed dynamically since the tests (running frequencies and intervals) are themselves dynamic.

Table 3 presents the summary statistics for hypothesis testing on each card from the fast data sets. Due to the poor quality of the slow data sets, a statistical analysis of those cards would yield marginally useful information. Each card represents 20 radiation detectors as mentioned earlier in this chapter.

As seen in the table, the total alarm rate for all the cards was around $4 * 10^{-4}$. The parametric studies showed that the expected total alarm rate for "perfect" data is around $2 * 10^{-4}$. As demonstrated earlier, the expected values are around $0.6 * 10^{-4}$ for hypotheses 1 through 3 and around $0.2 * 10^{-4}$ for hypothesis 4. Several cards approach the expected total alarm rate, but none approach each individual alarm rate.

Now the thesis will present conclusions and touch on possible future work. The conclusions and future work represent the most feasible and applicable routes to which this research may flow.

Table 3 – Card Analysis

Card	Total Points	H1	Rate	H2	Rate	H3	Rate	H4	Rate	Total	Rate
FCA	1479	4	1.35E-04	3	1.01E-04	0	0	0	0	7	2.37E-04
FCB	1372	8	2.92E-04	0	0	1	3.64E-05	0	0	9	3.28E-04
FCC	1493	9	3.01E-04	2	6.70E-05	0	0	1	3.35E-05	12	4.02E-04
FCE	1503	8	2.66E-04	12	3.99E-04	2	6.65E-05	1	3.33E-05	23	7.65E-04
FCF	1508	10	3.32E-04	6	1.99E-04	0	0	0	0	16	5.31E-04
FCG	1458	9	3.09E-04	3	1.03E-04	0	0	3	1.03E-04	15	5.14E-04
FCH	1471	8	2.72E-04	2	6.80E-05	4	1.36E-04	13	4.42E-04	27	9.18E-04
FCI	1459	6	2.06E-04	3	1.03E-04	0	0	0	0	9	3.08E-04
FCJ	1434	6	2.09E-04	1	3.49E-05	2	6.97E-05	5	1.74E-04	14	4.88E-04
FCL	1452	4	1.38E-04	0	0	0	0	0	0	4	1.38E-04
FCM	1253	8	3.19E-04	0	0	2	7.98E-05	0	0	10	3.99E-04
FCN	1444	4	1.39E-04	3	1.04E-04	0	0	0	0	7	2.42E-04
FCP	1475	1	3.39E-05	1	3.39E-05	0	0	1	3.39E-05	3	1.02E-04
FCQ	1449	3	1.04E-04	3	1.04E-04	1	3.45E-05	0	0	7	2.42E-04
FCR	1507	9	2.99E-04	0	0	2	6.64E-05	0	0	11	3.65E-04
FCS	1456	7	2.40E-04	9	3.09E-04	2	6.87E-05	1	3.43E-05	19	6.52E-04
FCT	1513	18	5.95E-04	2	6.61E-05	4	1.32E-04	2	6.61E-05	26	8.59E-04
FCV	1475	10	3.39E-04	1	3.39E-05	0	0	0	0	11	3.73E-04
FCX	1472	7	2.38E-04	7	2.38E-04	0	0	0	0	14	4.76E-04
	27673	**139**	**2.51E-04**	**58**	**1.05E-04**	**20**	**3.61E-05**	**27**	**4.88E-05**	**244**	**4.41E-04**

6.0 Conclusions and Future Work

6.1 Conclusions

Since the radiation characteristics of stored HEU have an essentially static Gaussian distribution the SPRT is the optimal choice for hypothesis testing to determine departures from the expected measurement distribution. The SPRT on average will detect any changes in μ and σ^2 quicker than any other hypothesis testing procedure. The SPRT also examines both the mean μ and variance σ^2 of the distribution so that only one test is necessary.

Although the SPRT has these optimal properties it is incapable of finding all departures from expectation. Therefore other information must be gleaned, and thus a second level of analysis – the feature extraction system (FES) – is necessary. The FES also presents the important information (the alarm, the type of alarm, and at what data point the alarm occurred) as provided by the SPRT while allowing the user to extract more information from the data than with the SPRT alone.

The FES also allows the testing procedure to be artificially desensitized and resensitized during post-processing to prevent nuisance alarms and to accentuate or attenuate other aspects of the data as needed.

When applied to the Y-12 data sets, the SPRT features show that none of the radiation detectors exhibit completely "normal" behavior. In light of the highly variable nature of some of the data sets, this is not an unexpected finding.

The information stored in the feature extraction system also cannot act independently. Thus, a final processing step is required, and this step will make the decisions based on the characteristics of the data set. This final step is a knowledge-based expert system as featured in Joseph Bowling's thesis.

6.2 Future Work

The recommended future work of the project is to assimilate mass sensor data into the detector and feature extraction system. The characteristics of mass sensor data may not have a Gaussian distribution and thus would require a different version of the SPRT and FES.

Other future possibilities could include the application of the SPRT and FES to other measurement problems. In any situation with normally distributed values, the SPRT and the FES would perform well.

Finally future work can include tweaking and expansion of extracted features or modification of the SPRT alarm rates as necessary. The values presented in this thesis are experimentally and empirically derived and set; their optimal values may be different depending on use and circumstances.

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Appendix

paramplot.m

This m-file performs the first parametric study.

```
clear all;
close all;
option1 = 2;
index = 0;
alphaA = [0.0005 0.001 0.005 0.01 0.05];
betaB = [0.05 0.1 0.15 0.2];
for mu = 20:10:80
   for billy = 1:5
      alpha = alphaA(billy);
      for sammy = 1:4
         beta = betaB(sammy);
         index = index + 1
         bravo = zeros(1,5);
         for terry = 1:50
            x = sqrt(mu)*randn(1,100) + mu;
            [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x,option1,mu,alpha,beta);
            bravo = bravo + z;
         end
         bravo = bravo/50;
         charlie(index,1:5) = bravo;
         charlie(index,6) = beta;
         charlie(index,7) = alpha;
         charlie(index,8) = mu;
      end
   end
end
jimi = 0;
for iota = 1:7
   jimi = jimi + 1;
   index = 20*(iota-1);
   for x = 1:5
      for y = 1:4
         index = index + 1;
         art(x,y) = charlie(index,1);
         ar1(x,y) = charlie(index,2);
         ar2(x,y) = charlie(index,3);
         ar3(x,y) = charlie(index,4);
         ar4(x,y) = charlie(index,5);
      end
   end
   figstart = 5*(jimi - 1);
   figure(figstart+1);
```

```
    surf(betaB,alphaA,art);
    xlabel('\beta');
    ylabel('\alpha');
    zlabel('Alarm Rate');
    mum = 20 + (jimi-1)*10;
    TTL = ['Total Alarm Rate - \mu = ' num2str(mum)];
    title(TTL);
    figure(figstart+2);
    surf(betaB,alphaA,ar1);
    xlabel('\beta');
    ylabel('\alpha');
    zlabel('Alarm Rate');
    TTL = ['H1 Alarm Rate - \mu = ' num2str(mum)];
    title(TTL);
    figure(figstart+3);
    surf(betaB,alphaA,ar2);
    xlabel('\beta');
    ylabel('\alpha');
    zlabel('Alarm Rate');
    TTL = ['H2 Alarm Rate - \mu = ' num2str(mum)];
    title(TTL);
    figure(figstart+4);
    surf(betaB,alphaA,ar3);
    xlabel('\beta');
    ylabel('\alpha');
    zlabel('Alarm Rate');
    TTL = ['H3 Alarm Rate - \mu = ' num2str(mum)];
    title(TTL);
    figure(figstart+5);
    surf(betaB,alphaA,ar4);
    xlabel('\beta');
    ylabel('\alpha');
    zlabel('Alarm Rate');
    TTL = ['H4 Alarm Rate - \mu = ' num2str(mum)];
    title(TTL);
end
figure(36);
plot(charlie(:,1:5));
xlabel('Data set');
ylabel('Alarm rates');
title('Alarm rates by varying data set parameters');
legend('Total','H1','H2','H3','H4');
figure(37);
plot(charlie(1:20,1:5));
xlabel('Data set');
```

```
ylabel('Alarm rates');
title('Alarm rates by varying data set parameters - \mu = 20');
legend('Total','H1','H2','H3','H4');
```

samplesa.m

This m-file is used to create the data sets for the second parametric study.

```
clear all;
for n = 1:50
index = 0;
for mu = 25:3:145
   index = index + 1;
   x = randn(1,10000)*sqrt(mu) + mu;
   [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x,2,mu,0.001,0.1);
   thing(index,:) = z;
   qs(index) = q;
   al1{index} = alarm1;
   al2{index} = alarm2;
   al3{index} = alarm3;
   al4{index} = alarm4;
end
TTL = ['sample' num2str(n)];
save(TTL);
end
```

sampletest.m

This m-file performs the second parametric study.

```
clear all;
close all;
avg = zeros(41,5);
for n = 1:50
   TTL = ['sample' num2str(n)];
   load(TTL);
   avg = avg + thing;
end
mu = 25:3:145;
avg = avg/50;
plot(mu',avg);
xlabel('\mu');
ylabel('Alarm Rate');
title('Alarm Rate vs \mu');
legend('Total','H1','H2','H3','H4');
```

paramplot2.m

This m-file performs the third parametric study.

```
clear all;
close all;
option1 = 2;
alpha = 0.001;
beta = 0.1;
parama = [0.01 0.1 0.5];
indexa = 0;
for mu = 25:7:165
   indexa = indexa + 1;
   indexb = 0;
   for kappa = 1:3;
      param = parama(kappa);
      z = zeros(4,5);
      indexb = indexb + 1;
      for p = 1:50
         x1 = sqrt(mu)*randn(1,500) + mu + param*mu;
         x2 = sqrt(mu)*randn(1,500) + mu - param*mu;
         x3 = (1+param)*sqrt(mu)*randn(1,500) + mu;
         x4 = 1/(1+param)*sqrt(mu)*randn(1,500) + mu;
         [z1,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x1,option1,mu,alpha,beta);
         [z2,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x2,option1,mu,alpha,beta);
         [z3,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x3,option1,mu,alpha,beta);
         [z4,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(x4,option1,mu,alpha,beta);
         z(1,1:5) = z(1,1:5) + z1;
         z(2,1:5) = z(2,1:5) + z2;
         z(3,1:5) = z(3,1:5) + z3;
         z(4,1:5) = z(4,1:5) + z4;
      end
      z = z/50;
      z(:,6) = param;
      z(:,7) = mu;
      archie{indexa,indexb} = z;
      indexa
      indexb
   end
end
pra1 = archie{11,1};
pra2 = archie{11,2};
pra3 = archie{11,3};
p1(1,1:5) = pra1(1,1:5);
p1(2,1:5) = pra2(1,1:5);
p1(3,1:5) = pra3(1,1:5);
p2(1,1:5) = pra1(2,1:5);
p2(2,1:5) = pra2(2,1:5);
p2(3,1:5) = pra3(2,1:5);
```

```
p3(1,1:5) = pra1(3,1:5);
p3(2,1:5) = pra2(3,1:5);
p3(3,1:5) = pra3(3,1:5);
p4(1,1:5) = pra1(4,1:5);
p4(2,1:5) = pra2(4,1:5);
p4(3,1:5) = pra3(4,1:5);
subplot(4,1,1);
plot(parama,p1);
legend('Total','H1','H2','H3','H4');
title('Alarm Rate vs Parameter');
ylabel('Alarm Rate');
subplot(4,1,2);
plot(parama,p2);
ylabel('Alarm Rate');
subplot(4,1,3);
plot(parama,p3);
ylabel('Alarm Rate');
subplot(4,1,4);
plot(parama,p4);
xlabel('Parameter');
ylabel('Alarm Rate');
```

seqprob2a.m

This m-file performs the SPRT.

```
function [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(data,option1,mu,alpha,beta)
%seqprob2a.m
%  This is a Sequential Probability Ratio Test function.
%
%  The function is of the form
%  [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2a(data,option1,mu,alpha,beta)
%
%  The inputs are
%  1)  a data vector
%  2)  option1 (whether the data vector is filtered or unfiltered)
%      1 = filtered (for use with a specific set of data vectors)
%      2 = unfiltered
%  3)  the mean of the data vector
%      if not supplied in the function call, the function will prompt for the value
%  4)  the false alarm rate
%  5)  the missed alarm rate
%
%  The six outputs are
%  1)  a 5x1 vector of alarm rates
%      total alarm rate
%      mean increase alarm rate
```

```
%      mean decrease alarm rate
%      variance increase alarm rate
%      variance decrease alarm rate
%  2)  the size of the input vector
%  3) - 6) vectors of the individual alarm indices
x = data;
q = length(x);
if nargin < 3
   mu = input('Enter data mean:  ');
else
end
if nargin > 1
   if option1 == 1
      sig = sqrt(mu)/2;
      Vplus = 1 + 0.75/sig;
      Vminus = 1 - 0.75/sig;
   elseif option1 == 2
      sig = sqrt(mu);
      Vplus = 1 + 3/sig;
      Vminus = 1 - 3/sig;
   end
end
M = 3*sig;
apos = 1;
aneg = 1;
anom = 1;
ainv = 1;
%alpha = 0.001;
%beta = 0.1;
A = beta/(1-alpha);
B = (1-beta)/alpha;
logA = log(A);
lowlim = 5*logA;
logB = log(B);
uplim = 5*logB;
SPRTpos = zeros(1,q);
SPRTneg = zeros(1,q);
SPRTnom = zeros(1,q);
SPRTinv = zeros(1,q);
apos = 1;
aneg = 1;
anom = 1;
ainv = 1;
k1 = 1;
k2 = 1;
```

```
k3 = 1;
k4 = 1;
com1 = 1;
com2 = 1;
com3 = 1;
com4 = 1;
alarm1 = 0;
alarm2 = 0;
alarm3 = 0;
alarm4 = 0;
numalarm1 = 1;
numalarm2 = 1;
numalarm3 = 1;
numalarm4 = 1;
for index = 2:q
   ctpos = x(apos:index);
   ctcent = ctpos - mu;
   ctdif1 = ctcent - M/2;
   SPRTpos(index) = (M/sig^2)*sum(ctdif1);
   if (SPRTpos(index - 1) <= logA)
      SPRTpos(index) = 0;
      apos = index;
   elseif (SPRTpos(index - 1) >= logB)
      SPRTpos(index) = 0;
      apos = index;
      alarm1(k1) = index - 1;
      numalarm1(k1) = SPRTpos(index - 1);
      k1 = k1 + 1;
   end
   ctneg = x(aneg:index);
   ctcent = ctneg - mu;
   ctdif2 = - ctcent - M/2;
   SPRTneg(index) = (M/sig^2)*sum(ctdif2);
   if (SPRTneg(index - 1) <= logA)
      SPRTneg(index) = 0;
      aneg = index;
   elseif (SPRTneg(index - 1) >= logB)
      SPRTneg(index) = 0;
      aneg = index;
      alarm2(k2) = index - 1;
      numalarm2(k2) = SPRTneg(index - 1);
      k2 = k2 + 1;
   end
   ctnom = x(anom:index);
   ctcent = ctnom - mu;
```

```
    ctcent2 = ctcent.^2;
    SPRTnom(index) = (1/(2*sig^2))*((Vplus - 1)/Vplus)*sum(ctcent2) -
(log(Vplus))*length(ctcent2)/2;
    if (SPRTnom(index - 1) <= logA)
       SPRTnom(index) = 0;
       anom = index;
    elseif (SPRTnom(index - 1) >= logB)
       SPRTnom(index) = 0;
       anom = index;
       alarm3(k3) = index - 1;
       numalarm3(k3) = SPRTnom(index - 1);
       k3 = k3 + 1;
    end
    ctinv = x(ainv:index);
    ctcent = ctinv - mu;
    ctcent2 = ctcent.^2;
    SPRTinv(index) = (1/(2*sig^2))*(1 - 1/Vminus)*sum(ctcent2) +
(log(1/Vminus))*length(ctcent2)/2;
    if (SPRTinv(index - 1) <= logA)
       SPRTinv(index) = 0;
       ainv = index;
    elseif (SPRTinv(index - 1) >= logB)
       SPRTinv(index) = 0;
       ainv = index;
       alarm4(k4) = index - 1;
       numalarm4(k4) = SPRTinv(index - 1);
       k4 = k4 + 1;
    end
end
hyp1 = (k1 - 1)/q;
hyp2 = (k2 - 1)/q;
hyp3 = (k3 - 1)/q;
hyp4 = (k4 - 1)/q;
hyp = hyp1 + hyp2 + hyp3 + hyp4;
z = [hyp hyp1 hyp2 hyp3 hyp4];
alarm = [alarm1 alarm2 alarm3 alarm4];
```

seqprob2.m

This is a function similar to seqprob2a.m. This function does not give the option of changing α or β, but does plot the SPRT.

```
function [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2(data,option1,mu)
%seqprob2.m
%  This is a Sequential Probability Ratio Test function.
%
%  The function is of the form
```

```
% [z,q,alarm1,alarm2,alarm3,alarm4] = seqprob2(data,option1,mu)
%
% The inputs are
% 1) a data vector
% 2) option1 (whether the data vector is filtered or unfiltered)
%    1 = filtered (for use with a specific set of data vectors)
%    2 = unfiltered
% 3) the mean of the data vector
%    if not supplied in the function call, the function will prompt for the value
%
% The six outputs are
% 1) a 5x1 vector of alarm rates
%    total alarm rate
%    mean increase alarm rate
%    mean decrease alarm rate
%    variance increase alarm rate
%    variance decrease alarm rate
% 2) the size of the input vector
% 3) - 6) vectors of the individual alarm indices
%
% The function also plots the data set in a subplot.
% 1) The data set
% 2) Hypothesis 1
% 3) Hypothesis 2
% 4) Hypothesis 3
% 5) Hypothesis 4
%
% The xlabel of the subplot is the same as the 5x1 vector alarm rate output

x = data;
q = length(x);
if nargin < 3
   mu = input('Enter data mean:  ');
else
end
if nargin > 1
   if option1 == 1
      sig = sqrt(mu)/2;
      Vplus = 1 + 0.75/sig;
      Vminus = 1 - 0.75/sig;
   elseif option1 == 2
      sig = sqrt(mu);
      Vplus = 1 + 3/sig;
      Vminus = 1 - 3/sig;
   end
```

```
end
M = 3*sig;
apos = 1;
aneg = 1;
anom = 1;
ainv = 1;
alpha = 0.001;
beta = 0.1;
A = beta/(1-alpha);
B = (1-beta)/alpha;
logA = log(A);
lowlim = 5*logA;
logB = log(B);
uplim = 5*logB;
SPRTpos = zeros(1,q);
SPRTneg = zeros(1,q);
SPRTnom = zeros(1,q);
SPRTinv = zeros(1,q);
apos = 1;
aneg = 1;
anom = 1;
ainv = 1;
k1 = 1;
k2 = 1;
k3 = 1;
k4 = 1;
com1 = 1;
com2 = 1;
com3 = 1;
com4 = 1;
alarm1 = 0;
alarm2 = 0;
alarm3 = 0;
alarm4 = 0;
numalarm1 = 1;
numalarm2 = 1;
numalarm3 = 1;
numalarm4 = 1;
for index = 2:q
   ctpos = x(apos:index);
   ctcent = ctpos - mu;
   ctdif1 = ctcent - M/2;
   SPRTpos(index) = (M/sig^2)*sum(ctdif1);
   if (SPRTpos(index - 1) <= logA)
      SPRTpos(index) = 0;
```

```
      apos = index;
   elseif (SPRTpos(index - 1) >= logB)
      SPRTpos(index) = 0;
      apos = index;
      alarm1(k1) = index - 1;
      numalarm1(k1) = SPRTpos(index - 1);
      k1 = k1 + 1;
   end
   ctneg = x(aneg:index);
   ctcent = ctneg - mu;
   ctdif2 = - ctcent - M/2;
   SPRTneg(index) = (M/sig^2)*sum(ctdif2);
   if (SPRTneg(index - 1) <= logA)
      SPRTneg(index) = 0;
      aneg = index;
   elseif (SPRTneg(index - 1) >= logB)
      SPRTneg(index) = 0;
      aneg = index;
      alarm2(k2) = index - 1;
      numalarm2(k2) = SPRTneg(index - 1);
      k2 = k2 + 1;
   end
   ctnom = x(anom:index);
   ctcent = ctnom - mu;
   ctcent2 = ctcent.^2;
   SPRTnom(index) = (1/(2*sig^2))*((Vplus - 1)/Vplus)*sum(ctcent2) -
(log(Vplus))*length(ctcent2)/2;
   if (SPRTnom(index - 1) <= logA)
      SPRTnom(index) = 0;
      anom = index;
   elseif (SPRTnom(index - 1) >= logB)
      SPRTnom(index) = 0;
      anom = index;
      alarm3(k3) = index - 1;
      numalarm3(k3) = SPRTnom(index - 1);
      k3 = k3 + 1;
   end
   ctinv = x(ainv:index);
   ctcent = ctinv - mu;
   ctcent2 = ctcent.^2;
   SPRTinv(index) = (1/(2*sig^2))*(1 - 1/Vminus)*sum(ctcent2) +
(log(1/Vminus))*length(ctcent2)/2;
   if (SPRTinv(index - 1) <= logA)
      SPRTinv(index) = 0;
      ainv = index;
```

```
    elseif (SPRTinv(index - 1) >= logB)
        SPRTinv(index) = 0;
        ainv = index;
        alarm4(k4) = index - 1;
        numalarm4(k4) = SPRTinv(index - 1);
        k4 = k4 + 1;
    end
end
hyp1 = (k1 - 1)/q;
hyp2 = (k2 - 1)/q;
hyp3 = (k3 - 1)/q;
hyp4 = (k4 - 1)/q;
hyp = hyp1 + hyp2 + hyp3 + hyp4;
z = [hyp hyp1 hyp2 hyp3 hyp4];
alarm = [alarm1 alarm2 alarm3 alarm4];
Up = logB*ones(1,q);
Down = logA*ones(1,q);
Zer = zeros(1,q);
figure(1);
subplot(5,1,1);
title('Data set');
hold on;
plot(x);
mm = mu*ones(q,1);
plot(mm,'g-');
mp2sig = (mu + 2*sig)*ones(q,1);
mm2sig = (mu - 2*sig)*ones(q,1);
plot(mp2sig,'m-');
plot(mm2sig,'m-');
mp3sig = (mu + 3*sig)*ones(q,1);
mm3sig = (mu - 3*sig)*ones(q,1);
plot(mm3sig,'r-');
plot(mp3sig,'r-');
wert1 = mu + 4*sig;
wert2 = mu - 4*sig;
axis([0 q wert2 wert1]);
ylabel('Data');
subplot(5,1,2);
hold on;
ylabel('Hypothesis 1');
axis([0 q lowlim uplim]);
plot(SPRTpos);
plot(Up,'r');
plot(Down,'r');
plot(Zer,'g');
```

```
plot(alarm1,numalarm1,'mo');
subplot(5,1,3);
hold on;
ylabel('Hypothesis 2');
axis([0 q lowlim uplim]);
plot(SPRTneg);
plot(Up,'r');
plot(Down,'r');
plot(Zer,'g');
plot(alarm2,numalarm2,'mo');
subplot(5,1,4);
hold on;
ylabel('Hypothesis 3');
axis([0 q lowlim uplim]);
plot(SPRTnom);
plot(Up,'r');
plot(Down,'r');
plot(Zer,'g');
plot(alarm3,numalarm3,'mo');
subplot(5,1,5);
hold on;
ylabel('Hypothesis 4');
axis([0 q lowlim uplim]);
plot(SPRTinv);
plot(Up,'r');
plot(Down,'r');
plot(Zer,'g');
plot(alarm4,numalarm4,'mo');
xlabel(num2str(z));
```

samesign.m

This m-file extracts the number of 9-consecutive-same-sign runs a data string has.

```
function [G,posu,posd] = samesign(data,mean);
%samesign.m
%  [G,posu,posd] = samesign(data,mean)
%  This is a runs test. It counts the number of
%  consecutive same-sign residuals.
%  The input is a data vector and its mean
%  The output is the number of same-sign runs
%  of nine or more and the starting position of
%  each run for up and down.
G = 0;
p = length(data);
dcent = data - mean;
for i = 1:p
```

```
   if dcent(i) > 0
      sgnp(i) = 1;
   elseif dcent(i) < 0
      sgnp(i) = -1;
   end
end
iu = 1;
id = 1;
for i = 1:(p-8)
   sammy = sgnp(i:(i+8));
   S = sum(sammy);
   if S == 9
      G = G + 1;
      posu(iu) = i;
      iu = iu + 1;
   elseif S == -9
      G = G + 1;
      posd(id) = i;
      id = id + 1;
   end
end
```

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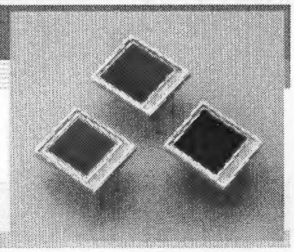
PHOTODIODE

Si PIN photodiode

S3590 series

Large area sensors for scintillation detection



Features

- Higher sensitivity and low dark current than conventional type
- Sensitivity matching with BGO and CsI (Tl) scintillators
- High quantum efficiency: QE=85 % (λ=540 nm)
- Low capacitance
- High-speed response
- High stability
- Good energy resolution

Applications

- Scintillation detectors
- Calorimeters
- Hodoscopes
- TOF counters
- Air shower counters
- Particle detectors, etc.

■ General ratings / Absolute maximum ratings

Type No.	Window material	Active area (mm)	Absolute maximum ratings: Reverse voltage VR Max.	Power dissipation P (mW)	Operating temperature Topr (°C)	Storage temperature Tstg (°C)
S3590-01	Epoxy resin	10 × 10	50	100	-20 to +60	-20 to +80
S3590-02	Window-less					
S3590-05	Epoxy resin	9 × 9	150			
S3590-06	Window-less					
S3590-08	Epoxy resin	10 × 10	100			
S3590-09	Window-less					

■ Electrical and optical characteristics (Typ. Ta=25 °C, unless otherwise noted)

Type No.	Spectral response range λ (nm)	Peak sensitivity wavelength λp (nm)	Photo sensitivity S: λ=λp (A/W)	LSO 420 nm (A/W)	BGO 480 nm (A/W)	CsI(Tl) 540 nm (A/W)	Short circuit current Isc 100 lx (μA)	Dark current ID Typ. (A/W)	Dark current ID Min. (A/W)	Temp. coefficient of ID TCID (times/°C)	Cut-off Frequency fc (MHz)	Terminal capacitance Ct f= 1MHz (pF)	NEP VR=70V (W/Hz^1/2)
S3590-01	320 to 1060	920	0.58	0.19	0.26	0.31	80	1.5 *1	5 *1	1.12	35 *1	75 *1	3.9×10^{-14}
S3590-02			0.62	0.23	0.32	0.39							
S3590-05	320 to 1120	980	0.62	0.19	0.25	0.30	77	8 *2	30 *2		20 *2	25 *2	8.4×10^{-14}
S3590-06			0.64	0.23	0.32	0.39							
S3590-08	320 to 1100	960	0.66	0.20	0.30	0.36	100	2 *3	6 *3		40 *3	40 *3	3.8×10^{-14}
S3590-09			0.66	0.22	0.33	0.41							

*1: VR=30 V
*2: VR=100 V
*3: VR=70 V

SOLID STATE DIVISION

HAMAMATSU

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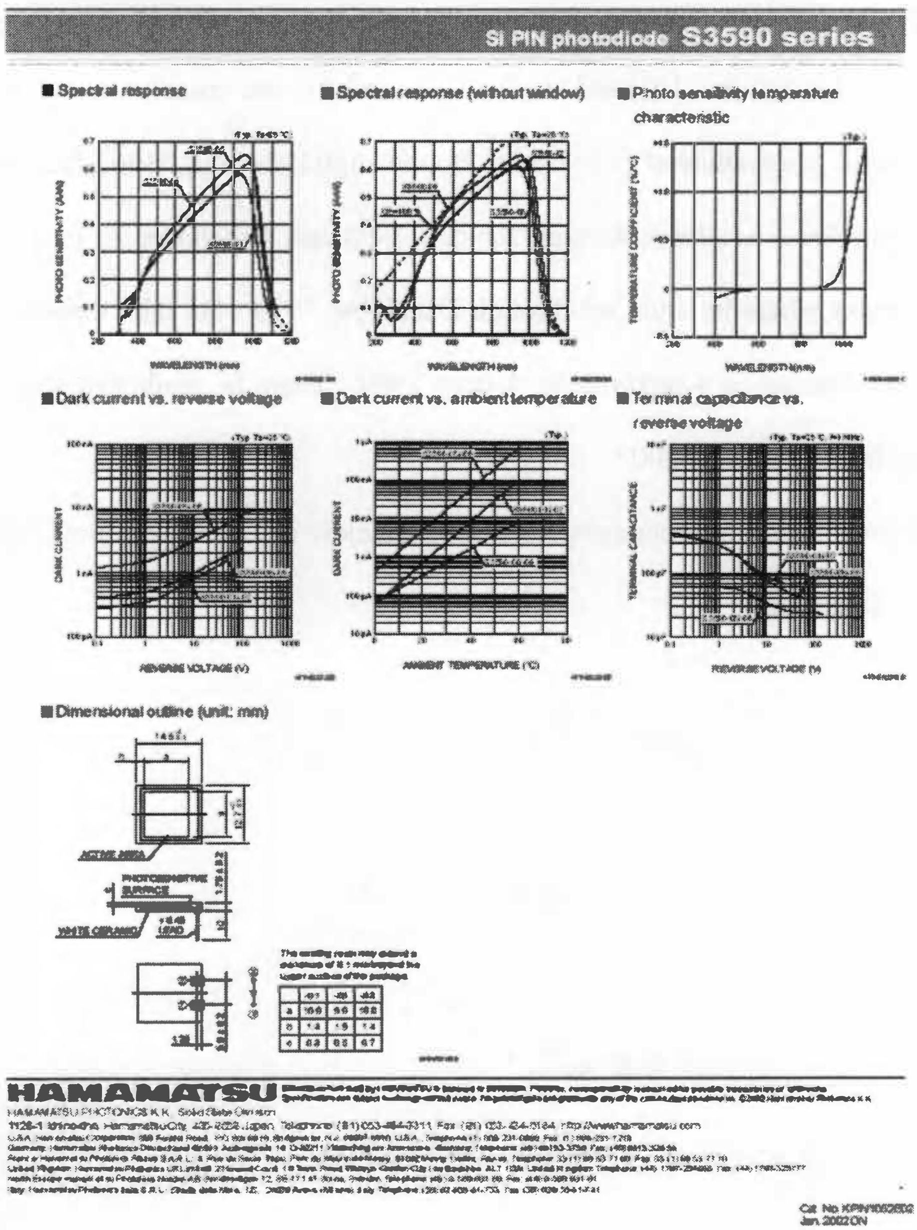

Vita

Thomas Jay Harrison was born in Johnson City, TN, on October 16, 1978. He was raised in Limestone, TN, and went to elementary and middle school at Sulphur Springs School. He graduated from Daniel Boone High School in 1996. He attended the United States Naval Academy through January 1998 and transferred to East Tennessee State University where he attended through December 1998. He then transferred to the University of Tennessee, Knoxville, in August 1999. There he graduated with a B.S. in nuclear engineering in May 2002.

Thomas is currently pursuing his M.S. in nuclear engineering at the University of Tennessee, Knoxville.