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Eliminating Harmonics in a Cascaded H-Bridges Multilevel Inverter Using Resultant Theory, Symmetric Polynomials, and Power Sums

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To the Graduate Council:

I am submitting herewith a thesis written by Keith Jeremy McKenzie entitled "Eliminating Harmonics in a Cascaded H-Bridges Multilevel Inverter Using Resultant Theory, Symmetric Polynomials, and Power Sums." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Leon M. Tolbert, Major Professor

We have read this thesis and recommend its acceptance:

Chiasson, Lawler

Accepted for the Council:
Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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Eliminating Harmonics in a Cascaded H-Bridges Multilevel Inverter Using Resultant Theory, Symmetric Polynomials, and Power Sums

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Keith Jeremy McKenzie

May 2004

Dedication

I wish to dedicate this thesis to my parents, Dr. Don McKenzie and Sue McKenzie. Their unending love, support, and guidance made the completion of this thesis possible.

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I wish to thank all those who helped me complete my M.S. in Electrical Engineering at the University of Tennessee. I would like to thank Dr. Tolbert for providing advice, serving as my major professor, and funding my research. I would like to thank Dr. Chiasson for providing advice, serving on my committee, and funding my research. I would like to thank Dr. Lawler for providing advice and serving on my committee. Finally, I would like to thank Zhong Du for helping me collect experimental data.

Abstract

This thesis studies a multilevel converter with assumed equal dc sources. The multilevel fundamental switching scheme is used to control the needed power electronics switches. Also, a method is presented where switching angles are computed such that a desired fundamental sinusoidal voltage is produced while at the same time certain higher order harmonics are eliminated.

Using Fourier Series theory, the transcendental equations eliminating certain higher order harmonics were derived in terms of the switching angles. Furthermore, these transcendental equations were transformed into polynomial equations by making some simple changes of variables. Resultant theory was used to solve the polynomial equations. Furthermore, using the ideas of Symmetric Polynomials and Power Sums, these polynomials were reduced further to form smaller degree polynomials, which are much easier to solve. This approach will find all solutions. Numerical techniques, such as Newton-Raphson, will find only one solution.

The computer algebra software package Mathematica was used to symbolically solve the above polynomials. When five dc sources were used, it was found that quite often the switching angles could be selected such that the output voltage Total Harmonic Distortion (THD) was less than 7%. When six dc sources were used, quite often the switching angles could be selected such that the output voltage THD was less than 6%.

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1 Introduction

1.1 Chapter Overview

The following chapter serves several purposes. Section 1.2 provides a brief discussion of the research presented in this thesis. In Section 1.3, the general idea of the multilevel converter is presented. Also contained is an introduction to the "multilevel fundamental switching scheme." Section 1.4 discusses some of the applications being considered for the multilevel converter.

Section 1.5 compares on a general level the aforementioned multilevel fundamental switching scheme to more traditional Pulse-Width Modulation (PWM) methods. Section 1.6 discusses some benefits of harmonic elimination, which is the main idea behind the multilevel fundamental switching scheme presented in this thesis. In Section 1.7, some additional advantages and disadvantages of multilevel converters are presented.

1.2 Thesis Research

In this thesis, a multilevel converter with assumed equal dc sources is studied. The multilevel fundamental switching scheme is used to control the needed power electronics switches. Also, a method is presented where switching angles are computed such that a desired fundamental sinusoidal voltage is produced while at the same time certain higher order harmonics are eliminated.

Using the idea of the Fourier Series (which will be discussed in Chapter 3), the equations eliminating certain harmonics were derived in terms of the switching angles. In fact, one will see these equations are transcendental equations. By making some simple substitutions, these transcendental equations were transformed into polynomial equations.

After forming these polynomial equations, Resultant theory was used to solve the polynomial equations. Furthermore, using the ideas of Symmetric Polynomials and Power Sums, these polynomials were reduced further to form smaller degree polynomials, which are much easier to solve. What makes this approach appealing is that all solutions were found. Numerical techniques, such as Newton-Raphson, will find only one solution.

Using the computer algebra software package Mathematica, the aforementioned equations were derived and solved for four different cases. In each case, a different number of dc sources were used with the multilevel converter. The cases consisted of using three, four, five, and six dc sources.

1.3 Multilevel Converters

There are several types of multilevel converters. The three main types of multilevel converters are: diode-clamped multilevel converters, flying-capacitor (also referred to as capacitor-clamped) multilevel converters, and cascaded H-bridges multilevel converters [1].

At this point, it seems appropriate to discuss the difference between the terms "multilevel converter" and "multilevel inverter." The term "multilevel converter" refers to the converter itself. Furthermore, the connotation of the term is that power can flow in one of two directions. Power can flow from the ac side to the dc side of the multilevel converter. This method of operation is called the rectification mode of operation. Power can also flow from the dc side to the ac side of the multilevel converter. This method of operation is called the inverting mode of operation. The term "multilevel inverter" refers to using a multilevel converter in the inverting mode of operation. Chapter 2 will discuss cascaded H-bridges multilevel inverters in more detail.

The main function of a multilevel inverter is to produce a desired ac voltage waveform from several levels of dc voltages. These dc voltages may or may not be equal to one another. The ac voltage produced from these dc voltages approaches a sinusoid [1].

As an example of a multilevel inverter, consider the staircase waveform in Figure 1.1. In this figure, five 12 V dc sources produce a staircase waveform with a peak-to-

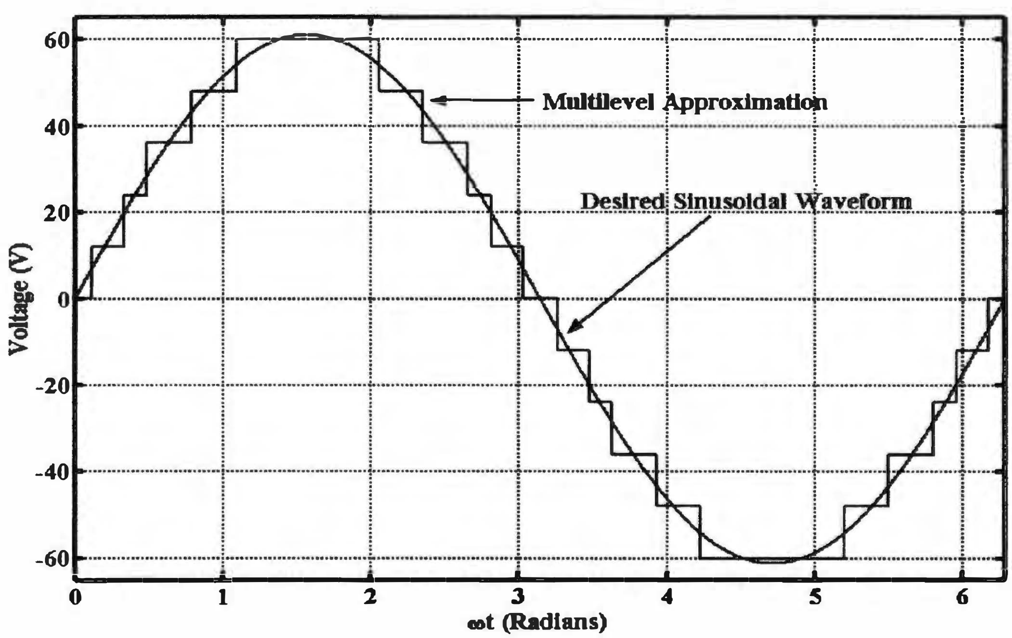

Figure 1.1: Multilevel inverter using five equal dc sources.

peak voltage of 120 V. In this case, the multilevel inverter produces a fair approximation to a sinusoidal waveform. As one increases the number of dc sources, this approximation will get better and better. Ideally, as the number of dc sources approaches infinity, the staircase waveform will approach the desired sinusoid.

Figure 1.1 also illustrates the "multilevel fundamental switching scheme." This scheme simply refers to determining the switching angles of the multilevel inverter such that a staircase waveform can be produced that approximates a sinusoid. Furthermore, the fundamental frequency of the produced staircase waveform and the frequency of the desired sinusoid are the same.

There are other switching schemes that can be implemented on a multilevel inverter but do not produce a staircase waveform. Some examples include Bipolar Programmed PWM, Unipolar Programmed PWM, and Virtual Stage PWM. These switching schemes, along with some other switching schemes, are discussed further in Chapter 2.

One pitfall of using multilevel inverters to approximate sinusoidal waveforms concerns harmonics. As one can see in Figure 1.1, the staircase waveform produced by the multilevel inverter contains sharp transitions. From Fourier Series theory, this phenomenon results in harmonics, in addition to the fundamental frequency of the sinusoidal waveform.

However, by altering the times at which these sharp transitions occur, one can reduce and/or eliminate some of the unwanted harmonics. Furthermore, by increasing the number of dc sources, more harmonic content can be eliminated.

1.4 Applications of Multilevel Converters

As mentioned earlier, multilevel inverters utilize several dc voltages to synthesize a desired ac voltage. For this reason, multilevel inverters can be implemented using distributed energy resources such as photovoltaics and fuel cells. Energy storage devices like ultracapacitors and batteries can also be used with multilevel inverters [2].

Many people feel that distributed energy resources will become increasingly prevalent in the future. As a result, one notable application of multilevel inverters being considered is connecting the aforementioned energy resources with an ac power grid [2].

If a multilevel converter is made to either draw or supply purely reactive power, then the multilevel converter can be used as a reactive power compensator. For example, a multilevel converter being used as a reactive power compensator might be placed in parallel with a load connected to an ac system. Using a multilevel converter as a reactive power compensator can help to improve the power factor of a load [3].

It was mentioned earlier that it is possible to determine the switching angles of the multilevel converter such that certain higher order harmonics are either minimized or eliminated altogether. The switching angles can also be varied in order to inject certain harmonics into an ac system. For example, consider once again a multilevel converter placed in parallel with a load connected to an ac system. If the load draws a current containing a high amount of harmonic distortion, the multilevel converter can be used to provide some of these harmonics. As a result, the ac system can provide a more sinusoidal current [4].

If the dc sources of the multilevel converter are banks of batteries or capacitors, the multilevel converter can also be used to provide ride-through capability under emergency conditions. This application is extremely useful when voltage sags or load swings are experienced at the utility connection [5].

Multilevel converters can also be used to construct a high voltage dc back-to-back intertie. For example, two diode-clamped multilevel converters can be used to produce such a system. One multilevel converter acts as a rectifier for the utility interface. The other multilevel converter acts as an inverter to supply the desired ac load. One idea behind using a back-to-back intertie is to connect two asynchronous systems. The intertie can be used as a frequency changer, a phase shifter, or a power flow controller [3].

Since the back-to-back intertie system can be used as a frequency changer, it would seem reasonable that a multilevel converter can also be used as an Adjustable Speed Drive (ASD). The input from the ac source can be a constant, defined frequency. The output of the ASD can be connected to an ac load whose frequency can vary [3].

Another possible application of multilevel converters is their use in Electric Vehicles (EVs) and Hybrid Electric Vehicles (HEVs). One reason is that multilevel converters, EVs, and HEVs are all ideally suited for utilization of a large number of relatively small-sized energy sources, such as batteries and fuel cells. Also, multilevel converters generally allow for smaller components, thus reducing weight.

1.5 Multilevel Fundamental Switching vs. Traditional PWM

When considering the application of the multilevel fundamental switching scheme, one might ask the question: Why use the multilevel fundamental switching scheme when traditional PWM schemes can be used? One answer to this question refers to the switching frequencies employed by these schemes. Traditional PWM methods employ switching frequencies on the order of several kHz. The multilevel fundamental switching scheme employs switching frequencies on the order of 60 Hz.

One benefit of traditional PWM methods employing much higher switching frequencies concerns harmonics. Undesirable harmonics occur at much higher frequencies. Thus, filtering is much easier and less expensive. Also, the generated harmonics might be above the bandwidth of some actual systems, which means there is no power dissipation due to these harmonics [6].

Given a specified switch duty ratio, switch conduction losses are approximately independent of switching frequency [7]. Therefore, the multilevel fundamental switching scheme will lead to switch conduction losses comparable to typical PWM schemes. However, switching losses increase as the switching frequency increases [7]. As a result, it is desirable to make the switching frequency as low as possible. In this case, switching at the desired fundamental frequency would seem to make the most sense. In other words, the multilevel fundamental switching scheme will lead to lower switching losses. Therefore, using the multilevel fundamental switching scheme will result in increased efficiency.

One disadvantage of using the multilevel fundamental switching scheme is that the created harmonics occur at much lower levels. However, determining the appropriate switching angles can result in eliminating some of these harmonics. The other harmonics can then be filtered.

Traditional PWM schemes also have the inherent problems of producing Electromagnetic Interference (EMI). Rapid changes in voltages (dv/dt) are a source of EMI [7]. The presence of a high dv/dt can cause damage to electrical motors. For example, a high dv/dt produces common-mode voltages across the motor windings. Furthermore, high switching frequencies can make this problem worse due to the increased number of times these common-mode voltages are applied to the motor during each fundamental cycle. Problems such as motor bearing failure and motor winding insulation breakdown can result due to circulating currents and voltage surges [8]. Also, long current-carrying conductors connecting equipment can result in a considerable amount of EMI.

Multilevel converters inherently tend to have a smaller dv/dt due to the fact that switching involves several smaller voltages [8]. Furthermore, switching at the fundamental frequency will also result in decreasing the number of times these voltage changes occur per fundamental cycle.

Another consequence of having a high dv/dt concerns device ratings. Using typical PWM switching schemes will result in an increased "oversizing" of devices in order to prevent voltage and current surges from destroying components. Furthermore, using PWM switching schemes might require the use of more snubber circuits and EMI filters.

1.6 Harmonic Elimination

The multilevel fundamental switching scheme inherently provides the opportunity to eliminate certain higher order harmonics by varying the times at which certain switches are turned "on" and turned "off" (i.e. varying the switching angles). Before doing so, one might ask: Why perform harmonic elimination?

One reason concerns EMI. Quite simply, harmonics are a source of EMI. As mentioned earlier, EMI can create voltage and current surges. Without harmonic elimination, designed circuits would need more protection in the form of snubbers and EMI filters. As a result, designed circuits would cost more.

EMI can also interfere with other "message" signals, such as the control signals used to control power electronics devices. Radio signals are another form of "message" signal that might be affected by the unwanted EMI.

Harmonics can also create losses in power equipment. For example, harmonic currents in an electrical induction motor will dissipate power in the motor stator and rotor windings. There will also be additional core losses due to harmonic frequency eddy currents [7].

Harmonics can also lower the power factor of a load. The power factor of a load is proportional to the ratio of the magnitude of the fundamental of the load current to the magnitude of the load current [7]. Increased harmonic content may decrease the magnitude of the fundamental relative to the magnitude of the entire current. As a result, the power factor would decrease.

It was mentioned earlier that an increase in the number of dc sources in a multilevel inverter results in a better approximation to a sinusoidal waveform. Furthermore, the increased number of dc sources provides the opportunity to eliminate more harmonic content. Eliminating harmonic content will make it easier to filter the remaining harmonic content. As a result, filters will be easier to design and build. Also, filters will be smaller and cheaper.

1.7 Further Advantages/Disadvantages of Multilevel Converters

One additional advantage of multilevel converters concerns switch ratings. Since multilevel converters usually utilize a large number of dc sources, switches are required to block smaller voltages. Since switch stresses are reduced, required switch ratings are lowered. As a result, cost is reduced.

The ability of multilevel converters to utilize a large number of dc sources provides another advantage. Utilization of a large number of dc sources allows for multilevel converters to produce high voltages and thus high power ratings. One distinct benefit is the idea that no transformers are needed to produce these high voltages, whereas traditional 12, 24, and 48-pulse inverters require transformers. Transformers are bulky and expensive. Furthermore, complicated connections of these transformers are sometimes required.

Another advantage of multilevel converters concerns the idea of reliability. If a component fails on a multilevel converter, most of the time the converter will still be usable, albeit at a reduced power level. Furthermore, multilevel converters tend to have switching redundancies. In other words, there might be more than one way to produce the desired voltage.

Another advantage of multilevel converters concerns application practicality. As an example, consider designing an inverter for a large HEV. Such an application would require excessively large components to deal with the relatively large working voltages and currents. These large components are expensive, bulky, and generally not reliable [1]. However, multilevel converters allow for the utilization of smaller, more reliable components.

One disadvantage of multilevel converters is that they require more devices than many traditional converters. The system cost will increase (although this increased cost might be offset by the fact switches with lower ratings are being used). The idea of using more devices also means the probability of a device failure will increase.

Another disadvantage of multilevel converters concerns the idea of controlling the switches. The increased number of switches will result in more complicated control.

1.8 Chapter Summary and Thesis Outline

In this chapter, several topics were discussed. A brief summary of the research to be presented in this thesis was first provided. Also discussed was some introductory material on multilevel converters. A general definition of the multilevel converter was given along with some advantages and disadvantages. Also, some applications of the multilevel converter were given. The multilevel fundamental switching scheme was introduced and compared to typical PWM schemes. The benefits of harmonic elimination were also given.

In Chapter 2, the cascaded H-bridges multilevel inverter will be discussed in more detail. Also, in addition to the multilevel fundamental switching scheme, some other switching schemes being applied to multilevel inverters will be discussed.

Chapter 3 will discuss some of the theory behind the research presented in this thesis. A brief summary of the Fourier Series will be presented. The idea of the Fourier Series will then be used to derive the harmonic equations corresponding to the multilevel fundamental switching scheme. Resultant theory will also be presented. It will then be shown how Resultant theory can be used to find the solutions to the aforementioned harmonic equations.

Chapter 4 will discuss more theory behind the research presented in this thesis. The ideas of Symmetric Polynomials and Power Sums will be presented. Using these ideas, it will be shown how the derived harmonic equations can be simplified.

The main purpose of Chapter 5 will be to discuss theoretical and experimentation results. The experimentation setup will first be discussed. Then, theoretical calculations and experimental results will be presented.

In Chapter 6, a brief summary of the thesis will be given. From this summary, some conclusions regarding the research will be made. Finally, some suggestions on possible future research will be given.

2 Background

2.1 Chapter Overview

In the previous chapter, some introductory material on multilevel converters/inverters was presented. The purpose of this chapter is to provide some background material on other research pertaining to the multilevel inverter. In Section 2.2, the cascaded H-bridges multilevel inverter will be discussed in more detail. Section 2.3 will present some other switching schemes involving harmonic elimination besides multilevel fundamental switching.

In Section 2.4, the idea of using unequal dc sources with multilevel inverters will be discussed. Section 2.5 will discuss the idea of "duty cycle swapping" as it applies to multilevel inverters. Section 2.6 will briefly discuss the use of numerical iterative techniques in solving nonlinear equations.

2.2 Cascaded H-Bridges Multilevel Inverter

The cascaded H-bridges multilevel inverter is a relatively new inverter structure [3]. A cascaded H-bridges multilevel inverter is simply a series connection of multiple H-bridge inverters. Each H-bridge inverter has the same configuration as a typical single-phase full-bridge inverter [1].

The cascaded H-bridges multilevel inverter introduces the idea of using separate dc sources to produce an ac voltage waveform. Each H-bridge inverter is connected to its own dc source V_{dc}. By cascading the ac outputs of each H-bridge inverter, an ac voltage waveform is produced. Figure 2.1 provides an illustration of a single-phase cascaded H-bridges multilevel inverter using five dc sources.

By closing the appropriate switches, each H-bridge inverter can produce three different voltages: $+V_{dc}$, 0, and $-V_{dc}$. When switches S_1 and S_4 of one particular H-bridge inverter in Figure 2.1 are closed, the output voltage is $+V_{dc}$. When switches S_2 and S_3 are closed, the output voltage is $-V_{dc}$. When either the switches S_1 and S_2 or the switches S_3 and S_4 are closed, the output voltage is 0.

As mentioned earlier, each H-bridge inverter produces an ac voltage v_i, where the i stands for one particular H-bridge inverter. Figure 2.1 contains five such H-bridges, one for each dc source. Therefore, to obtain the total ac voltage produced by the multilevel inverter, these five distinct ac voltages are added together. Figure 2.2 provides an illustration of these ideas. In the figure, the multilevel fundamental switching scheme is used.

Figure 2.2 also illustrates the idea of "levels" in a cascaded H-bridges multilevel inverter. In the figure, one notices that five distinct dc sources can produce a maximum of 11 distinct levels in the output phase voltage of the multilevel inverter. More generally, a cascaded H-bridges multilevel inverter using s separate dc sources can produce a maximum of $2s + 1$ distinct levels in the output phase voltage.

Some of the advantages and disadvantages of cascaded H-bridges multilevel inverters are the following:

Advantages:

1. To achieve the same number of voltage levels, this type of inverter requires the least number of components [3].

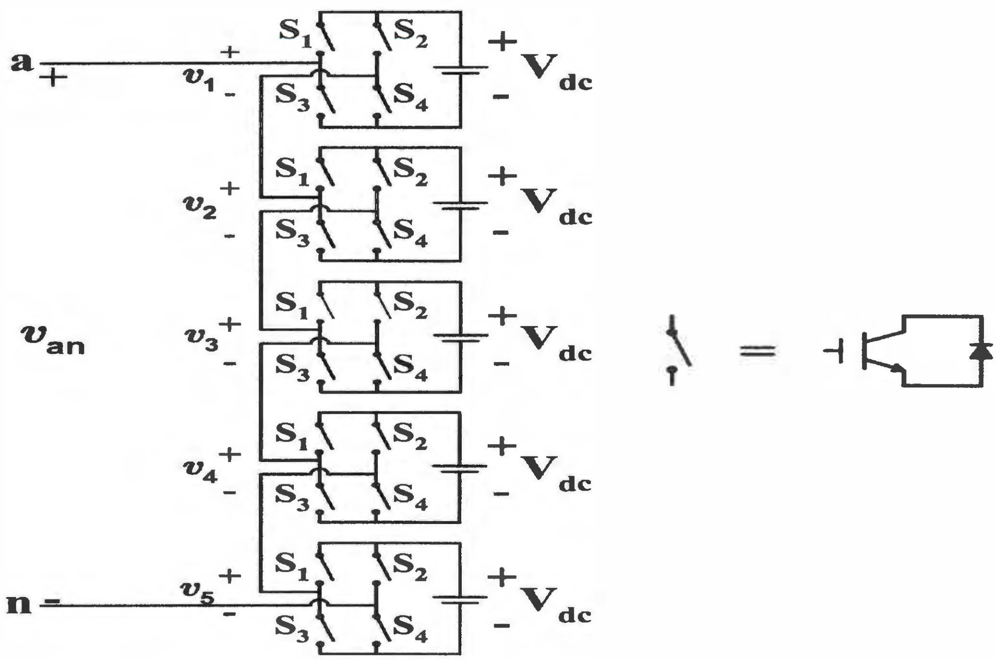

Figure 2.1: Cascaded H-bridges multilevel inverter using five dc sources.

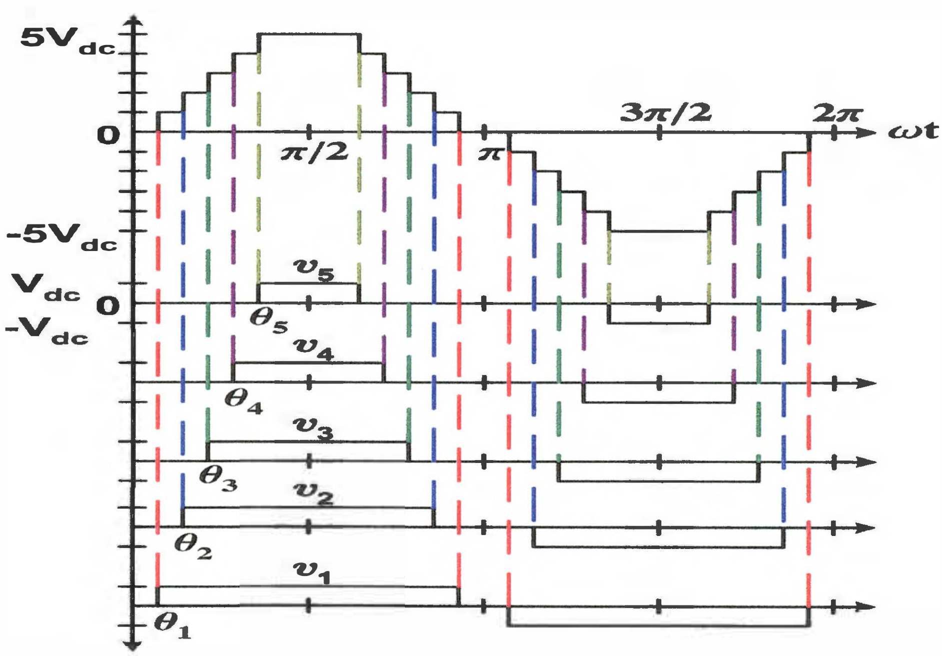

Figure 2.2: Voltage output of cascaded H-bridges multilevel inverter.

2. Unlike diode-clamped and flying-capacitor multilevel inverters, no extra clamping diodes or voltage balancing capacitors are needed [3].
3. Since each H-bridge has the same structure, modularized circuit layout and packaging are possible [3].
4. Since the total output phase voltage is a summation of voltages produced by each H-bridge inverter, switching redundancies exist [1].
5. Smaller dc sources are usually involved, resulting in fewer safety issues.

Disadvantages:

1. Separate dc sources are required, resulting in limited applicability [3].
2. For a three-phase system, this type of inverter will require more switches than a more traditional inverter.

It is important to mention that the research conducted for this thesis was done using a cascaded H-bridges multilevel inverter. Therefore, for consistency and simplicity, all future discussions of multilevel inverters will refer to this type of multilevel inverter unless stated otherwise.

2.3 Harmonic Elimination Switching Schemes

This section will present some other switching schemes involving harmonic elimination besides multilevel fundamental switching. More specifically, Bipolar Programmed PWM, Unipolar Programmed PWM, and Virtual Stage PWM will be discussed. The Unified Approach switching scheme will be discussed as well.

2.3.1 Bipolar Programmed PWM

One switching scheme involving harmonic elimination that has been around for many years is Bipolar Programmed PWM. In Bipolar Programmed PWM, the output voltage is either $+V_{dc}$ or $-V_{dc}$. Figure 2.3 illustrates the Bipolar Programmed PWM switching scheme using three switching angles and a V_{dc} equal to 12 V.

As one can see from the figure, Bipolar Programmed PWM uses predetermined switching angles to cut notches into an otherwise square-wave output. These notches take the voltage either from $+V_{dc}$ to $-V_{dc}$ or from $-V_{dc}$ to $+V_{dc}$. The number of notches cut per fundamental cycle is equal to twice the number of switching angles used [7].

By using Fourier Series theory, these switching angles can be used to eliminate certain harmonics. For example, three switching angles can be used to eliminate the fifth and seventh order harmonics while at the same time controlling the value of the

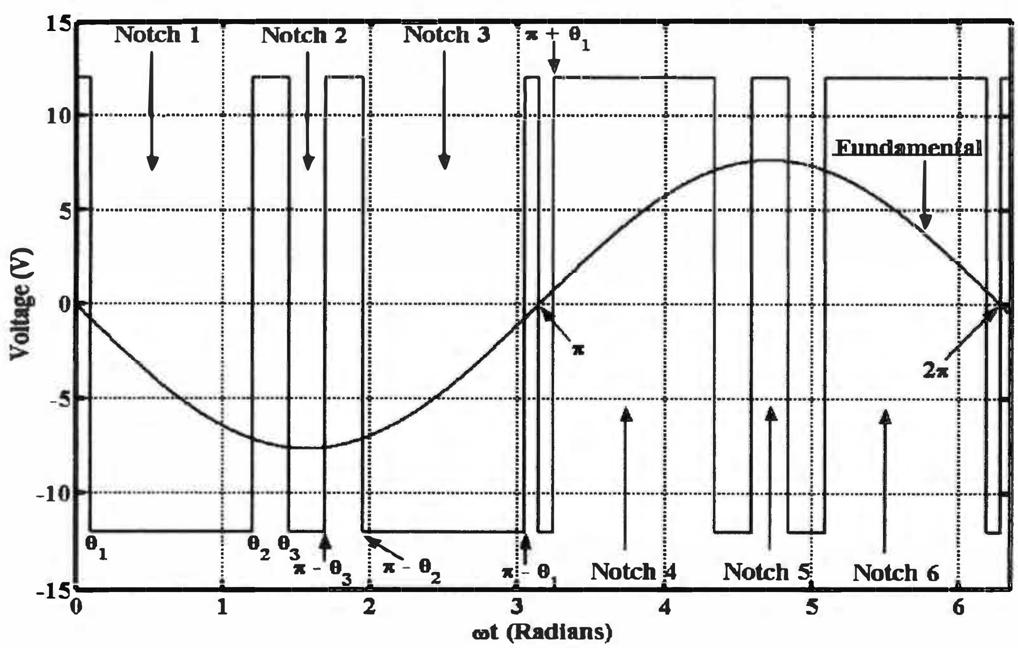

Figure 2.3: Bipolar Programmed PWM using three switching angles.

fundamental.

One of the main advantages of using Bipolar Programmed PWM concerns its applicability when low modulation indices are used. When low modulation indices are used, Chapter 5 will show that one may not be able to use the fundamental multilevel switching scheme to perform the desired harmonic elimination process. For example, considering again the three switching angles case above, one might not be able to use the fundamental multilevel switching scheme to eliminate both the fifth and seventh order harmonics and still control the value of the fundamental. However, it can be shown that Bipolar Programmed PWM can still be used with low modulation indices [6].

When a multilevel inverter utilizes Bipolar Programmed PWM for a low modulation index, typically one H-bridge is used. Therefore, another advantage is redundancy. If one H-bridge fails, another H-bridge can be used to provide the necessary voltage. Also, the desired voltage can be achieved by rotating the use of each H-bridge inverter for short periods of time.

Bipolar Programmed PWM can be used for higher modulation indices in addition to low modulation indices. For example, if a multilevel inverter needs to use two or more H-bridges in order to produce a desired voltage, one can choose a lower modulation index and use Bipolar Programmed PWM on multiple H-bridges.

Another advantage of Bipolar Programmed PWM is that control is not as complicated as some other switching schemes. For example, consider one of the H-bridges in Figure 2.1. Neglecting blanking time, switches S_1 and S_4 are switched "on" and "off" together. Similarly, switches S_2 and S_3 are switched "on" and "off" together.

Bipolar Programmed PWM also has some disadvantages. One disadvantage concerns EMI. As mentioned earlier, Bipolar Programmed PWM produces voltage changes equal to $2V_{dc}$. Therefore, a large V_{dc} can produce a considerable amount of EMI. Furthermore, Bipolar Programmed PWM inherently increases the effective switching frequency. For example, the multilevel fundamental switching scheme can result in each switch being turned "on" and "off" once per cycle. However, if Bipolar Programmed PWM is implemented using three switching angles, each switch is turned "on" and "off" seven times. Therefore, the effective switching frequency of each switch is increased by a factor of seven.

Another disadvantage of Bipolar Programmed PWM concerns harmonic distortion. For low modulation indices, using Bipolar Programmed PWM may still lead to a high amount of harmonic content in the output. In fact, the Total Harmonic Distortion (THD) may be over 100% for certain modulation indices.

2.3.2 Unipolar Programmed PWM

Unipolar Programmed PWM is another switching scheme involving harmonic elimination that has been around for many years. In Unipolar Programmed PWM, the output voltage is $+V_{dc}$, $-V_{dc}$, or 0. Furthermore, a voltage change is from $\pm V_{dc}$ to 0 and vice versa. Figure 2.4 illustrates the Unipolar Programmed PWM switching scheme using three switching angles and a V_{dc} equal to 12 V.

As one can see from the figure, Unipolar Programmed PWM uses predetermined switching angles to produce an output consisting of multiple pulses of varying widths. For the positive half of the fundamental cycle, these pulses have a voltage equal to $+V_{dc}$.

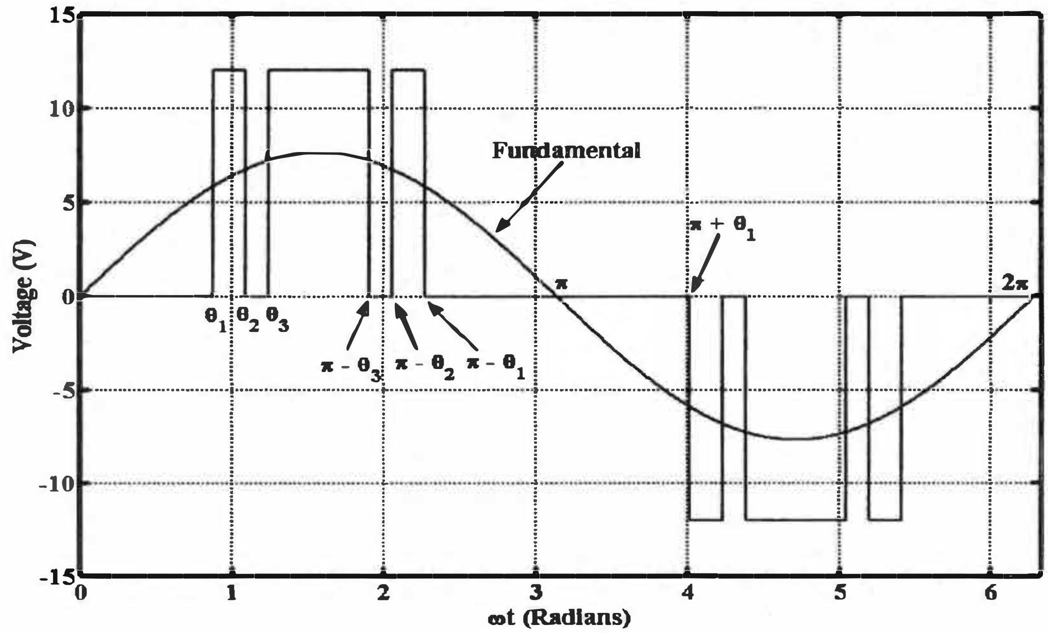

Figure 2.4: Unipolar Programmed PWM using three switching angles.

For the negative half of the fundamental cycle, these pulses have a voltage equal to $-V_{dc}$. The number of pulses per fundamental cycle is equal to twice the number of switching angles used.

Similar to Bipolar Programmed PWM, Fourier Series theory can be used to determine the switching angles such that certain harmonics are eliminated. In fact, these two switching schemes produce almost identical equations to solve. The only differences between the two sets of equations are that the Bipolar Programmed PWM equations contain a few extra numerical constants.

Unipolar Programmed PWM shares many of the advantages of Bipolar Programmed PWM. For example, it was mentioned earlier that Bipolar Programmed PWM can still be used with low modulation indices, even when one may not be able to use the multilevel fundamental switching scheme. This statement holds true for Unipolar

Programmed PWM as well.

Unipolar Programmed PWM also shares the advantage of redundancy when low modulation indices are used. As with Bipolar Programmed PWM, if one H-bridge fails, another H-bridge can be used to provide the necessary voltage. Furthermore, Unipolar Programmed PWM will still allow for the desired voltage to be achieved by rotating the use of each H-bridge inverter for short periods of time. Unipolar Programmed PWM can also be used for higher modulation indices in a way similar to the process described above with Bipolar Programmed PWM.

Like Bipolar Programmed PWM, one disadvantage of Unipolar Programmed PWM concerns harmonic distortion. For low modulation indices, using Unipolar Programmed PWM may still lead to a high output THD. However, Unipolar Programmed PWM tends to produce a lower THD than Bipolar Programmed PWM. One possible explanation can be given by referring to Figure 2.3 and Figure 2.4. From these two figures, one can see that Unipolar Programmed PWM seems to provide a more natural approximation to a sinusoidal waveform.

Unipolar Programmed PWM will also tend to produce less EMI than Bipolar Programmed PWM. Bipolar Programmed PWM produces voltage changes equal to $2V_{dc}$. However, Unipolar Programmed PWM produces voltage changes equal to only V_{dc}. Furthermore, Unipolar Programmed PWM increases the effective switching frequency by a smaller factor. For example, if Unipolar Programmed PWM is implemented using three switching angles, each switch can be made to turn "on" and "off" three times per cycle. Therefore, the effective switching frequency of each switch is increased by a factor of three, instead of the factor of seven increase caused by using Bipolar

Programmed PWM.

However, Unipolar Programmed PWM does require more complicated control compared to Bipolar Programmed PWM. For example, consider once again one of the H-bridges in Figure 2.1. Switches S_1 and S_4 are no longer switched “on” and “off” together. They must now be controlled independently. The same scenario occurs with switches S_2 and S_3.

2.3.3 Virtual Stage PWM

Virtual Stage PWM is another switching scheme involving harmonic elimination. This switching scheme appears to be relatively new. Shyu and Lai are two of the most recent people to have done research using this scheme on multilevel inverters [9].

Virtual Stage PWM is a mix between Unipolar Programmed PWM and the multilevel fundamental switching scheme. When Unipolar Programmed PWM is employed on a multilevel inverter, typically one dc source is involved, where the switches connected to the dc source are switched “on” and “off” several times per fundamental cycle. Furthermore, Unipolar Programmed PWM refers to exactly one switching pattern. In other words, given the switching angles, one should know exactly what the output voltage waveform looks like (See Figure 2.4).

When the multilevel fundamental switching scheme is used, all of the dc sources are typically involved, where all of the switches are turned “on” and “off” only once per fundamental cycle. The multilevel fundamental switching scheme also refers to exactly one switching pattern (See Figure 2.2).

However, when Virtual Stage PWM is used, the number of dc sources used varies. In other words, this particular switching scheme does not refer to any one specific switching pattern. Figure 2.5 provides an example of one particular switching pattern. In this figure, only two dc sources are used, whereas there are four switching angles. The third switching angle forces the second H-bridge to produce a zero output voltage. Effectively, the switching pattern in Figure 2.5 is comparable to using the multilevel fundamental switching scheme on one H-bridge with θ_1 as the switching angle and Unipolar Programmed PWM on a second H-bridge with θ_2, θ_3, and θ_4 as the switching angles.

Figure 2.6 provides an illustration of another Virtual Stage PWM switching pattern. In this figure, three dc sources are used, whereas once again there are four switching angles. The fourth switching angle forces the third H-bridge to produce a zero output voltage. Effectively, the switching pattern in Figure 2.6 is comparable to using the multilevel fundamental switching scheme on two H-bridges with θ_1 and θ_2 as the switching angles and Unipolar Programmed PWM on a third H-bridge with θ_3 and θ_4 as the switching angles.

One of the main reasons for considering Virtual Stage PWM concerns THD. For some modulation indices, using Virtual Stage PWM on a multilevel inverter will result in a lower THD than if the multilevel fundamental switching scheme were used.

One explanation for this phenomenon can be given by looking at the harmonic content that is eliminated by each switching scheme. For example, consider a multilevel inverter using three dc sources. If the multilevel fundamental switching scheme is used, typically the fifth and seventh order harmonics are eliminated. However, the Virtual Stage PWM switching scheme described above utilizes four switching angles. In this

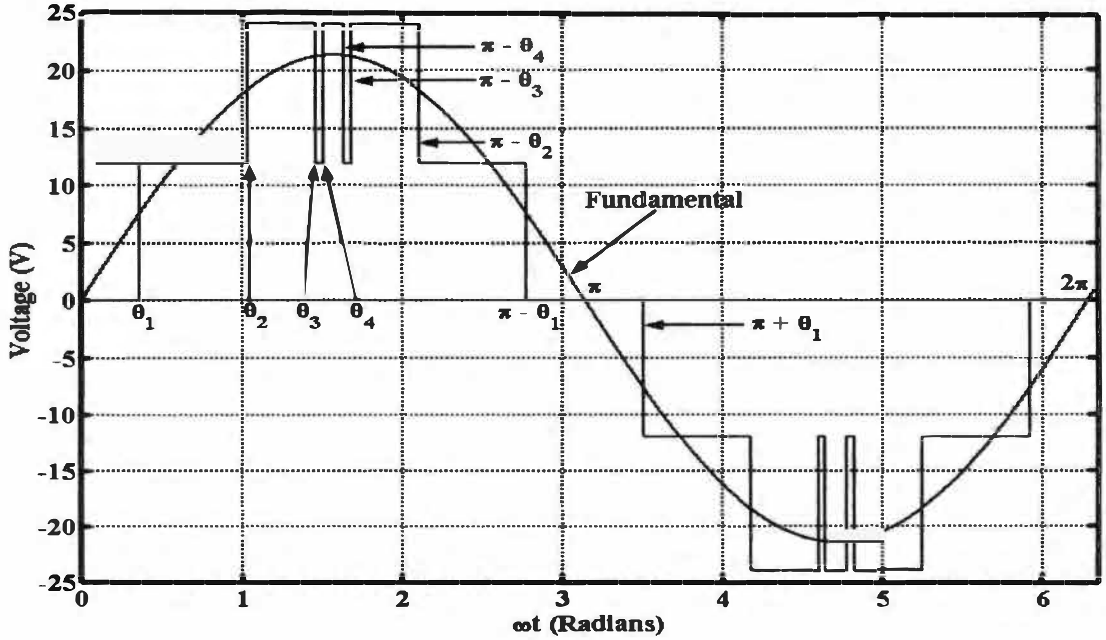

Figure 2.5: Virtual Stage PWM using two dc sources.

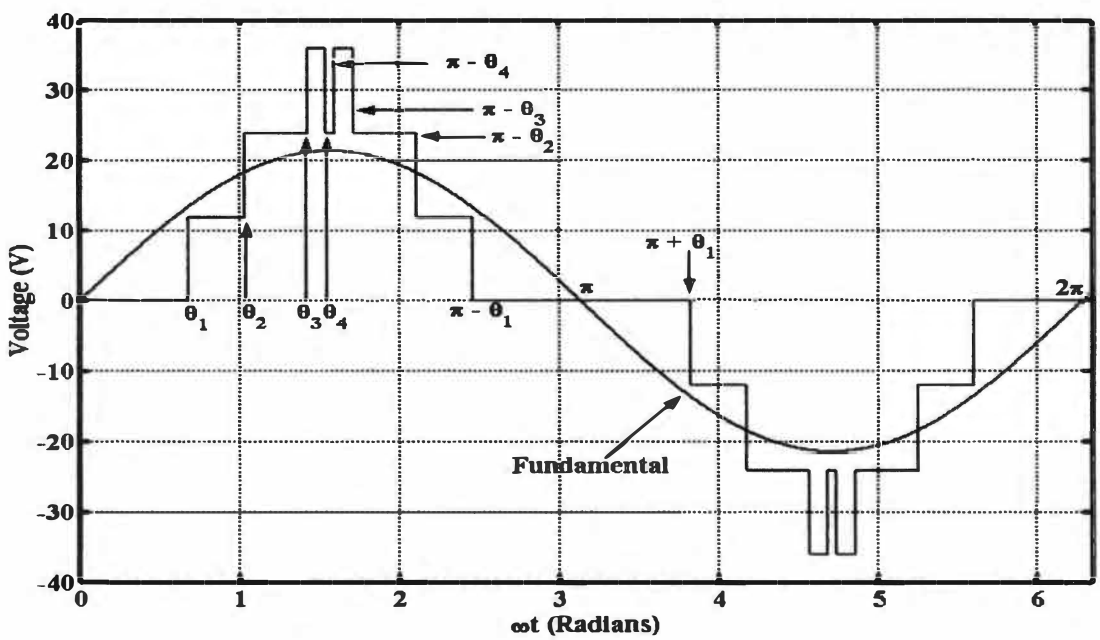

Figure 2.6: Virtual Stage PWM using three dc sources.

case, the eleventh order harmonic can also be eliminated in addition to the fifth and seventh order harmonics. As a result, it would seem reasonable that for certain modulation indices the Virtual Stage PWM switching scheme will produce an output waveform with a lower THD.

It was mentioned earlier that one of the advantages of Bipolar Programmed PWM and Unipolar Programmed PWM was that they could be used for modulation indices too low for the applicability of the multilevel fundamental switching scheme. Virtual Stage PWM can also be used for many of these low modulation indices. Furthermore, Virtual Stage PWM will most of the time produce output waveforms with a lower THD. As a result, Virtual Stage PWM provides another alternative to Bipolar Programmed PWM and Unipolar Programmed PWM.

2.3.4 Unified Approach

Thus far, four different switching schemes involving harmonic elimination have been discussed. After comparing these four different schemes, one might ask the question: Why not find a way to use all four schemes? In other words, why not pick the scheme that produces the best results at the given time?

The Unified Approach switching scheme tries to answer this question [10]. The Unified Approach switching scheme applied to a multilevel inverter makes use of Unipolar Programmed PWM, Virtual Stage PWM, and multilevel fundamental switching. Given a particular modulation index for a multilevel inverter using s dc sources, the Unified Approach switching scheme considers: Unipolar Programmed PWM with $s + 1$ switching angles, Virtual Stage PWM with $s + 1$ switching angles, and multilevel fundamental switching with s switching angles. After finding all possible solutions, the Unified Approach will pick the scheme that produces the lowest THD in the output waveform.

One disadvantage associated with the Unified Approach switching scheme concerns control of the switches. Since the Unified Approach switching scheme is a combination of several switching schemes, controlling the switches in the multilevel inverter will be more difficult. The reason is due to the fact each switching scheme contains its own particular switching pattern.

2.4 Multilevel Inverters with Varying DC Sources

The previously discussed switching schemes assumed that the dc sources used by the multilevel inverter were all equal to one another. However, quite often these dc sources are not equal to one another. Even if one tries to keep the various dc sources equal to one another, it is quite difficult to accomplish. As a result, research has been conducted where multilevel inverters are used with unequal dc sources. For example, Cunnyngham has worked on finding appropriate switching angles for multilevel inverters with unequal dc sources [1]. Also, the author has worked with Chiasson and Tolbert on the idea of applying Resultant theory to this same problem [11, 20].

One reason why one will have the problem of unequal dc sources deals with the idea that each dc source will charge and discharge differently from another dc source. For example, a multilevel inverter may use batteries as its dc sources. However, one battery will have a different internal resistance than a different battery. This factor alone will contribute to different charging/discharging rates.

Another reason why one might have the problem of unequal dc sources can be seen from observing Figure 2.2. In this figure, one notices that the H-bridge on the bottom is producing an output voltage for a longer period of time than the rest of the H-bridges. If real power flow is required, this particular H-bridge will transfer more power than the other H-bridges. As a result, the energy contained within that particular dc source will decrease more rapidly.

Figure 2.7 gives an example of implementing the multilevel fundamental switching scheme on a multilevel inverter using three unequal dc sources. Even when the dc sources are not equal, the switching angles can still be determined such that the fifth and seventh order harmonics are eliminated while at the same time controlling the value of the fundamental. In the figure, the nominal dc voltage of each dc source is 12 V.

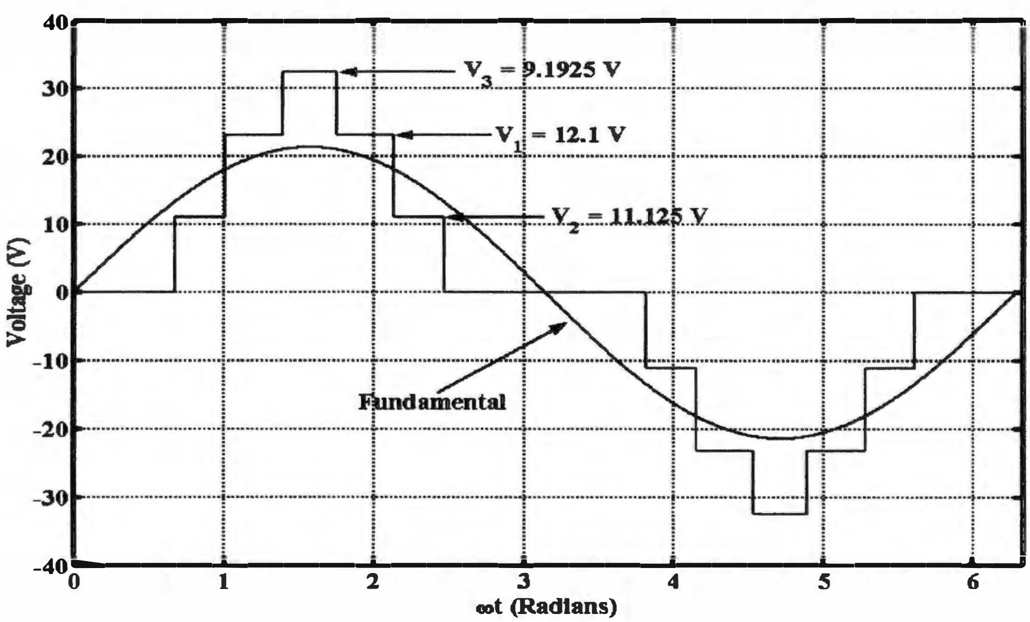

Figure 2.7: Multilevel inverter using three unequal dc sources.

Also, the voltages of the three dc sources are 12.1 V, 11.125 V, and 9.1925 V. Therefore, the three dc sources are operating at 100.8%, 92.7%, and 76.6% of their nominal values.

2.5 Duty Cycle Swapping

When a multilevel inverter is used for applications requiring real power flow, it can be undesirable to have a particular H-bridge produce a particular output voltage for an extended period. As explained above, the dc sources could become unequal. One way to combat this problem is to perform what is called "duty cycle swapping."

Figure 2.8 gives an example of the utilization of duty cycle swapping on a multilevel inverter using five dc sources. When duty cycle swapping is used, after each half cycle the switching angle for a particular H-bridge is effectively rotated to a different H-bridge. The result is that every half cycle a single H-bridge produces a pulse of

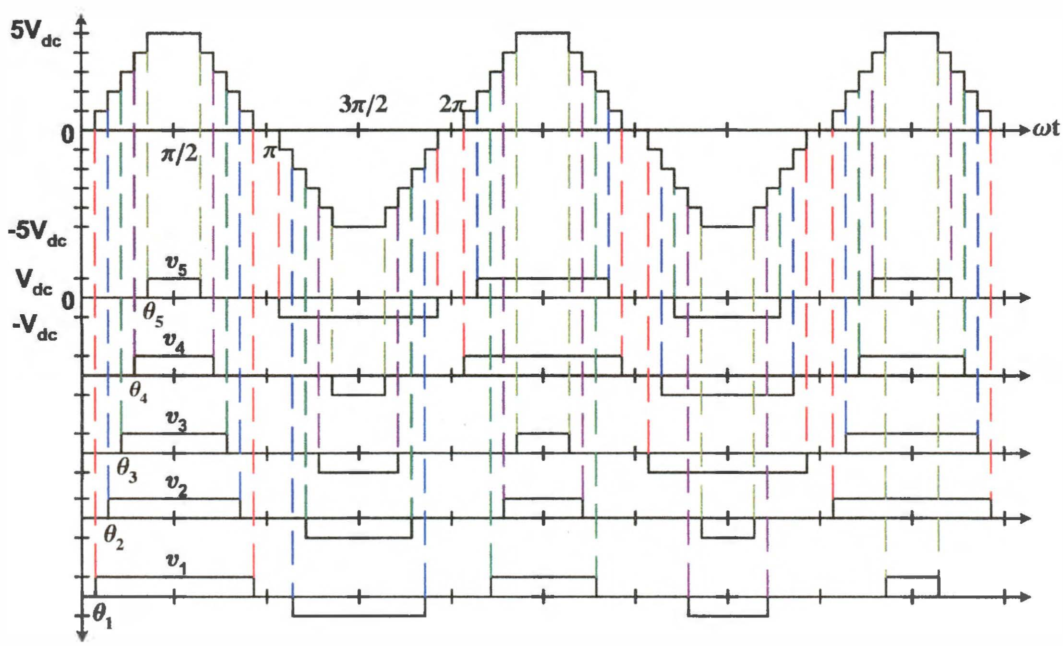

Figure 2.8: Duty cycle swapping using five dc sources.

different time duration than the previous half cycle. By performing duty cycle swapping, each dc source will be utilized equally [12].

2.6 Numerical Methods for Solving Nonlinear Equations

It has been discussed that when switching schemes involving harmonic elimination are used, the derived equations are nonlinear equations. As a result, many people have utilized numerical iterative techniques in order to solve these equations. For example, Cunnyngham used the Newton-Raphson numerical technique in his research [1]. Another numerical technique one might use is Gauss-Seidel, although this particular numerical technique is not as robust as Newton-Raphson.

Unfortunately, numerical iterative techniques have their drawbacks. One drawback is that these techniques require an initial guess in order to work. However, if the initial guess is not good enough, a solution will not be found. Another drawback is that numerical iterative techniques will only find one solution, if one exists. The obvious drawback here is that more than one solution might exist to the problem at hand.

Until recently, numerical iterative techniques seemed to be the only viable method to solve the aforementioned nonlinear harmonic equations. However, Chapter 3 will introduce Resultant theory. Using Resultant theory, all solutions to these nonlinear equations can be found without the need for an initial guess.

2.7 Chapter Summary

In this chapter, several topics were discussed. The cascaded H-bridges multilevel inverter was first discussed in more detail. Following the discussion on cascaded H-bridges multilevel inverters, some other switching schemes involving harmonic elimination besides multilevel fundamental switching were discussed. More specifically, the Bipolar Programmed PWM, Unipolar Programmed PWM, Virtual Stage PWM, and Unified Approach switching schemes were presented. The idea of using unequal dc sources with multilevel inverters was then discussed, followed by the concept of "duty cycle swapping." Finally, the use of numerical iterative techniques in solving nonlinear equations was briefly discussed.

The purpose of the previous two chapters was to provide both an introduction to multilevel inverters as well as some background information regarding other research concerning the multilevel inverter. However, the next chapter will provide some of the theory behind the research conducted for this thesis. A brief summary of the Fourier Series will be presented. The idea of the Fourier Series will then be used to derive the harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations will be written in terms of the switching angles of the multilevel inverter. After these equations are derived, Resultant theory will be presented. It will then be shown how Resultant theory can be used to find the solutions (if they exist) to the aforementioned harmonic equations.

3 Fourier Series and Resultant Theory

3.1 Chapter Overview

In the previous chapter, some background material on multilevel inverters was presented. The purpose of this chapter is to discuss some of the theory behind the research presented in this thesis. In Section 3.2, the idea of the Fourier Series will be discussed briefly. Section 3.3 will then use Fourier Series theory to derive the transcendental harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations will be written in terms of the switching angles of the multilevel inverter.

In Section 3.4, Resultant theory will be introduced. Section 3.5 will show how to transform the aforementioned transcendental harmonic equations into polynomial equations. It will then be shown how Resultant theory can be used to solve these polynomial equations. In particular, an example application of Resultant theory will be given by considering a cascaded H-bridges multilevel inverter using three equal dc sources. In this example, the value of the output voltage fundamental will be controlled while the fifth and seventh order harmonics are eliminated.

3.2 Fourier Series

The Fourier Series is named after Jean Baptiste Joseph Fourier (1768 – 1830). Fourier discovered that it is possible to represent periodic functions by an infinite sum of sine and/or cosine functions that are harmonically related. In other words, each trigonometric term in this infinite series has a frequency equal to an integer multiple of the fundamental frequency of the original periodic function. To express these ideas in mathematical form, Fourier showed that a periodic function $f(t)$ can be expressed as

$$f(t) = a_v + \sum_{n=1}^{\infty} a_n \cos\left(2\pi n f_o t\right) + b_n \sin\left(2\pi n f_o t\right), \tag{3.1}$$

where n is the set of natural numbers $1,2,3,\ldots,\infty$ [13].

In (3.1), a_v, a_n, and b_n are called the Fourier coefficients. These terms are determined from $f(t)$. The term f_o is the fundamental frequency of the periodic function $f(t)$. The integer multiples of f_o, such as $2f_o$ and $3f_o$, are known as the harmonic frequencies of $f(t)$. Therefore, the term nf_o is the nth harmonic of $f(t)$ [13].

3.2.1 Fourier Coefficients

Once $f(t)$ is known over one fundamental period, the terms a_v, a_n, and b_n can be determined from the following relationships:

$$a_v = \frac{1}{T} \int_{t_o}^{t_o+T} f(t)\,dt\ , \tag{3.2}$$

$$a_n = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \cos\left(2\pi n f_o t\right) dt\ , \tag{3.3}$$

and

$$b_n = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \sin\left(2\pi n f_o t\right) dt\ . \tag{3.4}$$

In these equations, t_o is any arbitrary time reference. T is the fundamental period of $f(t)$, determined by taking the reciprocal of f_o [13].

From observing (3.2), it is obvious that a_v is simply the average value of $f(t)$. This result can be obtained by integrating both sides of (3.1) over one fundamental period:

$$\int_{t_o}^{t_o+T} f(t)dt = \int_{t_o}^{t_o+T} \left(a_v + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_o t) + b_n \sin(2\pi n f_o t) \right) dt \tag{3.5}$$

$$= \int_{t_o}^{t_o+T} a_v dt + \sum_{n=1}^{\infty} \int_{t_o}^{t_o+T} (a_n \cos(2\pi n f_o t) + b_n \sin(2\pi n f_o t)) dt \tag{3.6}$$

$$= a_v T + 0 . \tag{3.7}$$

Equation (3.2) follows directly from (3.7) [13].

One can obtain the value for the kth value of a_n by multiplying (3.1) by $\cos(2\pi k f_o t)$ and then integrating both sides over one fundamental period:

$$\int_{t_o}^{t_o+T} f(t)\cos(2\pi k f_o t)dt = \int_{t_o}^{t_o+T} (a_v \cos(2\pi k f_o t))dt$$

$$+ \sum_{n=1}^{\infty} \int_{t_o}^{t_o+T} (a_n \cos(2\pi n f_o t)\cos(2\pi k f_o t))dt$$

$$+ \sum_{n=1}^{\infty} \int_{t_o}^{t_o+T} (b_n \sin(2\pi n f_o t)\cos(2\pi k f_o t))dt \tag{3.8}$$

$$= 0 + a_k(\frac{T}{2}) + 0 . \tag{3.9}$$

Equation (3.3) follows directly from (3.9). Similarly, one can obtain the value for the kth value of b_n by multiplying (3.1) by $\sin(2\pi k f_0 t)$ and then integrating both sides over one fundamental period [13].

3.2.2 Odd Symmetry

If a periodic function contains certain symmetries, the calculation of the Fourier coefficients can be greatly simplified. For example, a periodic function might possess odd symmetry. An odd function exhibits the property

$$f(t) = -f(-t) . \tag{3.10}$$

Functions satisfying (3.10) are called odd because polynomials containing only terms with odd exponents have this particular symmetry. For periodic functions with odd symmetry, the equations determining the Fourier coefficients can be simplified to the following expressions [13]:

$$a_v = 0 , \tag{3.11}$$

$$a_n = 0 \qquad \text{for all } n, \tag{3.12}$$

and

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin\left(2\pi n f_0 t\right) dt . \tag{3.13}$$

When a periodic function is odd, one should notice three facts. The first fact is that the average value is zero. The second fact is that the periodic function can be written in terms of an infinite series of only sine functions. Furthermore, in order to determine the amplitude of these sine functions, one only needs to integrate over half of a fundamental period.

3.2.3 Half-Wave Symmetry

Another symmetry a periodic function might possess is half-wave symmetry. A periodic function that possesses half-wave symmetry exhibits the property

$$f(t) = -f(t - T/2)\,. \tag{3.14}$$

For periodic functions with half-wave symmetry, it can be shown that the equations determining the Fourier coefficients simplify to the following expressions [13]:

$$a_v = 0\,, \tag{3.15}$$

$$a_n = 0 \qquad \text{for } n \text{ even,} \tag{3.16}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos\left(2\pi n f_o t\right)dt \qquad \text{for } n \text{ odd,} \tag{3.17}$$

$$b_n = 0 \qquad \text{for } n \text{ even,} \tag{3.18}$$

and

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin\left(2\pi n f_o t\right)dt \qquad \text{for } n \text{ odd.} \tag{3.19}$$

When a periodic function possesses half-wave symmetry, one should notice three facts. The first fact is that the average value is zero. The second fact is that the even harmonics are zero. Also, in order to determine the amplitudes of the odd harmonics, one only needs to integrate over half of a fundamental period.

3.2.4 Odd Quarter-Wave Symmetry

Another symmetry a periodic function might possess is odd quarter-wave symmetry. Odd quarter-wave symmetry simply means the periodic function possesses both odd and half-wave symmetries [7]. For periodic functions with odd quarter-wave symmetry, it can be shown that the equations determining the Fourier coefficients simplify to the following expressions [13]:

$$a_v = 0\,, \tag{3.20}$$

$$a_n = 0 \qquad \text{for all } n, \tag{3.21}$$

$$b_n = 0 \qquad \text{for } n \text{ even}, \tag{3.22}$$

and

$$b_n = \frac{8}{T}\int_0^{T/4} f(t)\sin\left(2\pi n f_o t\right)dt \qquad \text{for } n \text{ odd.} \tag{3.23}$$

When a periodic function possesses odd quarter-wave symmetry, the average value is zero. The reason is due to the fact the function is odd. Also, odd symmetry results in all of the cosine harmonics being zero. The half-wave symmetry of the periodic function forces the even sine harmonics to be zero. Furthermore, in order to determine the amplitude of the odd sine harmonics, one only needs to integrate over one-fourth of a fundamental period.

3.3 Application of Fourier Series to Multilevel Inverter

The purpose of this section is to utilize Fourier Series theory to derive the transcendental harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations will be written in terms of the switching angles of the multilevel inverter.

Figure 2.2 provides an illustration of the multilevel fundamental switching scheme being implemented on a multilevel inverter using five equal dc sources. Before calculating the Fourier coefficients corresponding to this particular periodic function, one should first see if the function possesses any symmetry. As one can see from the figure, it is evident that the periodic function possesses odd quarter-wave symmetry.

Since the output of the multilevel inverter is odd quarter-wave, (3.20) thru (3.23) can be used in calculating the Fourier coefficients. As one can see from these equations, only the odd sine harmonics can be nonzero. Making the change of variable $\omega t = 2\pi f_o t$, (3.23) becomes

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} f\left(\frac{\omega t}{2\pi f_o}\right)\sin(n\omega t)d(\omega t). \tag{3.24}$$

Using the output of the multilevel inverter given in Figure 2.2, (3.24) then becomes

$$b_n = \int_{\theta_1}^{\theta_2}\frac{4}{\pi}(V_{dc})\sin(n\omega t)d(\omega t) + \int_{\theta_2}^{\theta_3}\frac{4}{\pi}(2V_{dc})\sin(n\omega t)d(\omega t)$$

$$+ \int_{\theta_3}^{\theta_4}\frac{4}{\pi}(3V_{dc})\sin(n\omega t)d(\omega t) + \int_{\theta_4}^{\theta_5}\frac{4}{\pi}(4V_{dc})\sin(n\omega t)d(\omega t)$$

$$+ \int_{\theta_5}^{\pi/2}\frac{4}{\pi}(5V_{dc})\sin(n\omega t)d(\omega t), \tag{3.25}$$

where $\theta_1, \theta_2, \theta_3, \theta_4,$ and θ_5 are the switching angles and V_{dc} is the voltage of each dc source. Performing the required integrations, (3.25) then becomes

$$b_n = -\frac{4}{\pi n}V_{dc}\left[\cos(n\omega t)\right]_{\theta_1}^{\theta_2} - \frac{4}{\pi n}(2V_{dc})\left[\cos(n\omega t)\right]_{\theta_2}^{\theta_3}$$

$$-\frac{4}{\pi n}(3V_{dc})\left[\cos(n\omega t)\right]_{\theta_3}^{\theta_4} - \frac{4}{\pi n}(4V_{dc})\left[\cos(n\omega t)\right]_{\theta_4}^{\theta_5}$$

$$-\frac{4}{\pi n}(5V_{dc})[\cos(n\omega t)]_{\theta_5}^{\pi/2} \tag{3.26}$$

$$= \frac{4}{\pi n}V_{dc}[\cos(n\theta_1)-\cos(n\theta_2)]+\frac{4}{\pi n}(2V_{dc})[\cos(n\theta_2)-\cos(n\theta_3)]$$

$$+\frac{4}{\pi n}(3V_{dc})[\cos(n\theta_3)-\cos(n\theta_4)]+\frac{4}{\pi n}(4V_{dc})[\cos(n\theta_4)-\cos(n\theta_5)]$$

$$+\frac{4}{\pi n}(5V_{dc})\left[\cos(n\theta_5)-\cos\left(n\frac{\pi}{2}\right)\right]. \tag{3.27}$$

Finally, using the fact that $\cos\left(n\frac{\pi}{2}\right) = 0$ when n is an odd integer and simplifying, (3.27) becomes

$$b_n = \frac{4}{\pi n}V_{dc}[\cos(n\theta_1)+\cos(n\theta_2)+\cos(n\theta_3)+\cos(n\theta_4)+\cos(n\theta_5)]. \tag{3.28}$$

To briefly summarize the results in this section, the multilevel fundamental switching scheme produces an output voltage that possesses odd quarter-wave symmetry. As a result, this particular waveform only possesses odd sine harmonics. Equation (3.28) expresses the peak values of these odd harmonics in terms of the switching angles $\theta_1, \theta_2, \theta_3, \theta_4$, and θ_5. Furthermore, the harmonic equations produced from (3.28) are transcendental equations.

3.4 Resultant Theory

The purpose of this section is to discuss Resultant theory. When the multilevel fundamental switching scheme is implemented using s switching angles, (3.28) can be used to derive s different harmonic equations. In other words, s switching angles will be used to control the values of s different harmonics.

Unfortunately, these harmonic equations are transcendental equations, making them difficult to solve without making use of some sort of numerical iterative technique, such as Newton-Raphson. However, by making some simple changes of variables and simplifying, these transcendental equations can be transformed into a set of polynomial equations. Then, Resultant theory can be utilized to find all solutions to the harmonic equations.

3.4.1 Coprime Fractions

Consider a proper rational function (i.e. a fraction where the degree of the numerator is less than or equal to the degree of the denominator)

$$\hat{g}(s) = \frac{N(s)}{D(s)}, \tag{3.29}$$

where $N(s)$ and $D(s)$ are polynomial functions in the variable s. If $\hat{g}(s)$ is a coprime fraction, then $N(s)$ and $D(s)$ do not have any common factors of degree one or higher. In other words, the greatest common divisor of $N(s)$ and $D(s)$ is a nonzero constant α [14].

As an example of a proper rational function where the numerator and denominator are coprime, consider the function

$$\frac{N(s)}{D(s)} = \frac{(s+4)(s-2)}{(s+1)(s+3)}. \tag{3.30}$$

In (3.30), it is evident that $N(s)$ and $D(s)$ do not have any common factors of degree one or higher. Therefore, $N(s)$ and $D(s)$ are coprime.

However, as an example of a proper rational function where the numerator and denominator are not coprime, consider the function

$$\frac{N(s)}{D(s)} = \frac{(s+5)(s-6)}{(s+5)(s+13)}. \tag{3.31}$$

In this example, $N(s)$ and $D(s)$ share the common factor $(s+5)$. Therefore, $N(s)$ and $D(s)$ are not coprime. Instead, the fraction in (3.31) can be reduced to the fraction

$$\frac{N(s)}{D(s)} = \frac{\overline{N}(s)}{\overline{D}(s)} = \frac{(s-6)}{(s+13)}, \tag{3.32}$$

where $\overline{N}(s)$ and $\overline{D}(s)$ are coprime.

3.4.2 Sylvester Resultant Matrix

In this section, the Sylvester Resultant Matrix will be discussed. Suppose one has the following two polynomials in x_1 and x_2:

$$a(x_1, x_2) = a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1) \tag{3.33}$$

and

$$b(x_1, x_2) = b_3(x_1)x_2^3 + b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1), \tag{3.34}$$

where $a_3(x_1)$, $a_2(x_1)$, $a_1(x_1)$, $a_0(x_1)$, $b_3(x_1)$, $b_2(x_1)$, $b_1(x_1)$, and $b_0(x_1)$ are polynomials in x_1. Also, define the following two polynomials in x_2:

$$\alpha(x_2) = \alpha_2 x_2^2 + \alpha_1 x_2 + \alpha_0 \tag{3.35}$$

and

$$\beta(x_2) = \beta_2 x_2^2 + \beta_1 x_2 + \beta_0 . \tag{3.36}$$

When a specific numerical value is assigned to x_1, $a(x_1,x_2)$ and $b(x_1,x_2)$ become polynomials in only one variable, x_2. Using the above definition of coprimeness, it is clear these new polynomials are not coprime if and only if there exist polynomials $\alpha(x_2)$ and $\beta(x_2)$ such that

$$\frac{a(x_1,x_2)}{b(x_1,x_2)}=\frac{\alpha(x_2)}{\beta(x_2)}, \tag{3.37}$$

or, equivalently [14],

$$b(x_1,x_2)(-\alpha(x_2))+a(x_1,x_2)\beta(x_2)=0. \tag{3.38}$$

Using the definitions of $a(x_1,x_2)$, $b(x_1,x_2)$, $\alpha(x_2)$, and $\beta(x_2)$ given above, (3.38) can be rewritten in the matrix form

$$S(x_1)V=\bar{0}, \tag{3.39}$$

where the coefficients of x_2^k, $k=0,1,\ldots,5$, are equated to zero. The resulting matrix equation is [14]:

$$\begin{bmatrix} b_0(x_1) & a_0(x_1) & 0 & 0 & 0 & 0 \\ b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) & 0 & 0 \\ b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) \\ b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) \\ 0 & 0 & b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) \\ 0 & 0 & 0 & 0 & b_3(x_1) & a_3(x_1) \end{bmatrix} \begin{bmatrix} -\alpha_0 \\ \beta_0 \\ -\alpha_1 \\ \beta_1 \\ -\alpha_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3.40}$$

The columns of $S(x_1)$ are formed from the coefficients of $a(x_1,x_2)$ and $b(x_1,x_2)$ arranged in ascending powers of x_2. The $6\ X\ 6$ square matrix $S(x_1)$ in (3.40) is called the Sylvester Resultant Matrix. In general, the Sylvester Resultant Matrix will be $(d_1+d_2)\ X\ (d_1+d_2)$, where d_1 and d_2 are the degrees of the two polynomials

$a(x_1, x_2)$ and $b(x_1, x_2)$ in x_2 [14].

Equation (3.40) is a homogeneous linear algebraic equation. If a particular numerical value for x_1 results in a nonsingular $S(x_1)$, only the trivial solution $V = \bar{0}$ solves (3.40). In other words, the polynomials $a(x_1, x_2)$ and $b(x_1, x_2)$ (now only functions of the variable x_2) are coprime. However, if a particular numerical value for x_1 results in a singular $S(x_1)$, then there are nonzero solutions to (3.40). In other words, $a(x_1, x_2)$ and $b(x_1, x_2)$ are not coprime. Furthermore, the determinant of $S(x_1)$ is zero [14]:

$$R(x_1) = \det\left(\begin{bmatrix} b_0(x_1) & a_0(x_1) & 0 & 0 & 0 & 0 \\ b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) & 0 & 0 \\ b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) \\ b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) \\ 0 & 0 & b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) \\ 0 & 0 & 0 & 0 & b_3(x_1) & a_3(x_1) \end{bmatrix}\right) = 0. \tag{3.41}$$

In (3.41), $R(x_1)$ is called the Resultant Polynomial. One should notice that $R(x_1)$ only depends on x_1.

In conclusion, the procedure to see if two polynomials $a(x_1, x_2)$ and $b(x_1, x_2)$ have any common roots consists of four steps [15]:

1. Compute the roots x_{1k}, $k = 0,1,\ldots, n_R$, of $R(x_1) = 0$, where n_R is the degree of $R(x_1)$.
2. Substitute these roots into $a(x_1, x_2)$.
3. For $k = 0,1,\ldots, n_R$, solve the equation $a(x_{1k}, x_2) = 0$ to get the roots x_{2kl},

$l = 0,1,\ldots, n_a$, where n_a is the degree of $a(x_1, x_2)$ in x_2.

4. The common zeros of $a(x_1, x_2)$ and $b(x_1, x_2)$ are then the values (x_{1k}, x_{2kl}) that satisfy the equation $b(x_{1k}, x_{2kl}) = 0$.

3.5 Application of Resultant Theory to Multilevel Inverter

In this section, an example application of Resultant theory will be given by considering a cascaded H-bridges multilevel inverter using three equal dc sources. In this example, the value of the output voltage fundamental will be controlled while the fifth and seventh order harmonics are eliminated.

3.5.1 Transcendental Harmonic Equations

Consider (3.28). This equation gives the values of the odd sine harmonics corresponding to the multilevel fundamental switching scheme using five switching angles. If three switching angles are used instead, it can be shown that the corresponding equation is

$$b_n = \frac{4}{\pi n} V_{dc} \left[\cos(n\theta_1) + \cos(n\theta_2) + \cos(n\theta_3)\right]. \tag{3.42}$$

If one wants to control the peak value of the output voltage to be V_1 and eliminate the fifth and seventh order harmonics, the resulting harmonic equations are:

$$\frac{4}{\pi} V_{dc} \left[\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3)\right] = V_1, \tag{3.43}$$

$$\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) = 0, \tag{3.44}$$

and

$$\cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) = 0 . \tag{3.45}$$

One can also write (3.43) as

$$\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) = m , \tag{3.46}$$

where

$$m = \frac{V_1}{\left(\frac{4}{\pi} V_{dc}\right)} . \tag{3.47}$$

With this definition of the parameter *m*, the modulation index m_a is given by

$$m_a = \frac{m}{s} = \frac{V_1}{\left(\frac{4}{\pi} s V_{dc}\right)} , \tag{3.48}$$

where *s* is the number of separate dc sources. It should be pointed out that a square wave of amplitude sV_{dc} results in the maximum peak value of the fundamental [16]:

$$V_{1_{\max}} = \frac{4}{\pi} s V_{dc} . \tag{3.49}$$

When V_1 is equal to the value in (3.49), the resulting modulation index m_a is

$$m_a = \frac{V_{1_{\max}}}{\left(\frac{4}{\pi} s V_{dc}\right)} = \frac{\frac{4}{\pi} s V_{dc}}{\left(\frac{4}{\pi} s V_{dc}\right)} = 1 . \tag{3.50}$$

In other words, the square wave mode operation corresponds to a modulation index equal to one.

At this point, one might ask the question: Why not eliminate the third order harmonic? The reason is that the third order harmonic is a triplen harmonic. A harmonic is a triplen harmonic if its frequency is an integer multiple of three times the fundamental frequency. For balanced three-phase systems, each phase voltage will contain triplen harmonics equal in both magnitude and phase to the triplen harmonics of the other two phases. Therefore, all triplen harmonics will cancel in the line-to-line voltages. For this reason, when multilevel fundamental switching is employed, the selected harmonics to be eliminated are usually not triplen harmonics.

3.5.2 Polynomial Harmonic Equations

For the transcendental harmonic equations given in (3.44) thru (3.46), consider the following changes of variables:

$$x_1 = \cos(\theta_1), \tag{3.51}$$

$$x_2 = \cos(\theta_2), \tag{3.52}$$

and

$$x_3 = \cos(\theta_3). \tag{3.53}$$

Also, consider the following trigonometric identities:

$$\cos(5\theta) = 5\cos(\theta) - 20\cos^3(\theta) + 16\cos^5(\theta), \tag{3.54}$$

and

$$\cos(7\theta) = -7\cos(\theta) + 56\cos^3(\theta) - 112\cos^5(\theta) + 64\cos^7(\theta). \tag{3.55}$$

Applying the results given in (3.51) thru (3.55) to the transcendental harmonic equations above, one obtains

$$p_1(x_1, x_2, x_3) = 0 = x_1 + x_2 + x_3 - m, \tag{3.56}$$

$$p_5(x_1, x_2, x_3) = 0 = \sum_{n=1}^{3}\left(5x_n - 20x_n^3 + 16x_n^5\right), \tag{3.57}$$

and

$$p_7(x_1, x_2, x_3) = 0 = \sum_{n=1}^{3}\left(-7x_n + 56x_n^3 - 112x_n^5 + 64x_n^7\right). \tag{3.58}$$

It should be noted that multilevel fundamental switching requires

$$0 \le \theta_1 \le \theta_2 \le \theta_3 \le \frac{\pi}{2}, \tag{3.59}$$

where the units of the switching angles are radians. Therefore, the new variables x_1, x_2, and x_3 must satisfy

$$0 \le x_3 \le x_2 \le x_1 \le 1. \tag{3.60}$$

Equations (3.56) thru (3.58) are polynomial equations in the variables x_1, x_2, and x_3. Resultant theory can now be used to solve polynomials p_1, p_5, and p_7 for the common roots of these three equations.

3.5.3 Solutions to Polynomials Using Resultant Theory

The polynomials p_1, p_5, and p_7 are functions of the variables x_1, x_2, and x_3. Using p_1 to solve for x_1 in terms of the other two variables, one gets

$$x_1 = m - x_2 - x_3. \tag{3.61}$$

Substituting this result into the other two polynomials, one gets

$$\begin{aligned} p_5(x_2, x_3) = &\left(5(m - x_2 - x_3) - 20(m - x_2 - x_3)^3 + 16(m - x_2 - x_3)^5\right) \\ &+ \left(5x_2 - 20x_2^3 + 16x_2^5\right) + \left(5x_3 - 20x_3^3 + 16x_3^5\right) \end{aligned} \tag{3.62}$$

and

$$p_7(x_2, x_3) = \left(-7(m - x_2 - x_3) + 56(m - x_2 - x_3)^3 - 112(m - x_2 - x_3)^5 + 64(m - x_2 - x_3)^7\right) + \left(-7x_2 + 56x_2^3 - 112x_2^5 + 64x_2^7\right) + \left(-7x_3 + 56x_3^3 - 112x_3^5 + 64x_3^7\right). \tag{3.63}$$

After x_1 has been trivially eliminated, one can now apply Resultant theory to eliminate x_2. For the research presented in this thesis, all Resultant calculations were found by using the *Resultant* command in the software package Mathematica. After factoring and then eliminating redundant factors and unnecessary numerical constants, the Resultant of the two polynomials in (3.62) and (3.63) was found to be

$$\begin{aligned} res(x_3) = {} & \left(6125\, m^2 - 49000\, m^4 + 137200\, m^6 - 179200\, m^8 + 116480\, m^{10} \right. \\ & \left. - 35840\, m^{12} + 4096\, m^{14}\right) + \left(-18375\, m + 269500\, m^3 - 1019200\, m^5 \right. \\ & \left. + 1691200\, m^7 - 1361920\, m^9 + 501760\, m^{11} - 65536\, m^{13}\right) x_3 \\ & + \left(12250 - 588000\, m^2 + 3234000\, m^4 - 7156800\, m^6 + 7293440\, m^8 \right. \\ & \left. - 3261440\, m^{10} + 491520\, m^{12}\right) x_3^2 + \left(637000\, m - 5782000\, m^3 \right. \\ & + 17875200\, m^5 - 23385600\, m^7 + 12902400\, m^9 \\ & \left. - 2293760\, m^{11}\right) x_3^3 + \left(-269500 + 6370000\, m^2 - 28694400\, m^4 \right. \\ & \left. + 49324800\, m^6 - 34298880\, m^8 + 7454720\, m^{10}\right) x_3^4 \\ & + \left(-4410000\, m + 30184000\, m^3 - 71500800\, m^5 + 63974400\, m^7 \right. \\ & \left. - 17776640\, m^9\right) x_3^5 + \left(1470000 - 20776000\, m^2 + 72441600\, m^4 \right. \\ & \left. - 84940800\, m^6 + 31539200\, m^8\right) x_3^6 + \left(9800000\, m - 50176000\, m^3 \right. \end{aligned}$$

$$+80281600\ m^5 - 40857600\ m^7\Big)x_3^7 + \Big(-2744000\ + 21952000\ m^2$$

$$-53939200\ m^4 + 36556800\ m^6\Big)x_3^8 + \Big(-6272000\ m$$

$$+25088000\ m^3 - 20070400\ m^5\Big)x_3^9 + \Big(1568000\ - 6272000\ m^2$$

$$+5017600\ m^4\Big)x_3^{10}\ . \tag{3.64}$$

Since the polynomial *res* is only a function of one variable, one can begin the process of finding the appropriate switching angles. This process consists of nine steps [16]:

1. Given the value for the parameter *m*, solve for the roots of $res(x_3) = 0$.
2. Keep the roots for which $0 \le \mathrm{Re}(x_3) \le 1$, where Re refers to the real part of a possibly complex root. Denote these roots as $\{x_{3k}\}$.
3. For each member of the set $\{x_{3k}\}$, substitute it into $p_5(x_2, x_3)$ and solve for the roots of $p_5(x_2, x_{3k}) = 0$.
4. Keep the roots for which $0 \le \mathrm{Re}(x_{3k}) \le \mathrm{Re}(x_2) \le 1$. Denote the set of remaining roots as $\{(x_{2l}, x_{3l})\}$.
5. For each member of the set $\{(x_{2l}, x_{3l})\}$, compute $m - x_{2l} - x_{3l}$ to find the values for x_1.
6. Keep the roots for which $0 \le \mathrm{Re}(x_{3l}) \le \mathrm{Re}(x_{2l}) \le \mathrm{Re}(x_1) \le 1$. Denote the set of remaining roots as $\{(x_{1n}, x_{2n}, x_{3n})\}$.
7. For each member of the set $\{(x_{1n}, x_{2n}, x_{3n})\}$, keep just the real parts of x_{1n}, x_{2n} and x_{3n}. Denote these triples as $\{(\hat{x}_{1n}, \hat{x}_{2n}, \hat{x}_{3n})\}$.

8. Using (3.57) and (3.58), compute

$$\sqrt{\left(\left(\frac{p_5(\hat{x}_{1n},\hat{x}_{2n},\hat{x}_{3n})}{5}\right)^2+\left(\frac{p_7(\hat{x}_{1n},\hat{x}_{2n},\hat{x}_{3n})}{7}\right)^2\right)}. \tag{3.65}$$

9. If the result is less than some arbitrarily small tolerance level ε, the switching angles are given by

$$\{(\theta_{1n},\theta_{2n},\theta_{3n})\}=\{\cos^{-1}(\hat{x}_{1n}),\cos^{-1}(\hat{x}_{2n}),\cos^{-1}(\hat{x}_{3n})\}. \tag{3.66}$$

One should notice above that complex roots to the polynomial equations are being considered as candidates for switching angles. The reason is due to the fact that the imaginary part of the root may be small enough such that the real part of the root may still lead to a viable switching angle.

Equation (3.65) gives an indication of the harmonic distortion due to the fifth and seventh order harmonics. Theoretically, (3.65) should always be zero since one is supposed to be eliminating the fifth and seventh order harmonics. However, it was just mentioned that complex roots might be considered where the imaginary part is infinitesimally small. Nevertheless, these complex roots will lead to a small but nonzero harmonic distortion. Also, numerical round off in the computation of the roots will lead to a small harmonic distortion.

The values given by (3.65) can be controlled such that they are always below some arbitrarily small number ε. For the research presented in this thesis, this tolerance level was set at 0.001 times the current value of m.

Two more important points should be made. In general, the algorithm for finding the desired switching angles can theoretically be applied to the more general case of s

switching angles. It should also be pointed out that the above algorithm can be applied to the Unified Approach switching scheme. However, the process is more complicated. For a multilevel inverter using s dc sources, recall that the Unified Approach switching scheme considers: Unipolar Programmed PWM with $s + 1$ switching angles, Virtual Stage PWM with $s + 1$ switching angles, and multilevel fundamental switching with s switching angles. Therefore, two algorithms need to be implemented.

The first algorithm finds the solutions corresponding to multilevel fundamental switching with s switching angles. The second algorithm finds the solutions corresponding to Unipolar Programmed PWM and Virtual Stage PWM with $s + 1$ switching angles. Using the algorithm above, one must replace all occurrences of the term $\mathrm{Re}(x)$ with the term $|\mathrm{Re}(x)|$, where x refers to the variables $x_1, x_2, \ldots, x_{s+1}$. In other words, the real parts of the roots can be negative.

3.6 Chapter Summary

In this chapter, several topics were discussed. The idea of the Fourier Series was first discussed. It was then illustrated how Fourier Series theory could be used to derive the transcendental harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations were written in terms of the switching angles of the multilevel inverter.

Resultant theory was then introduced. After the aforementioned transcendental harmonic equations were transformed into polynomial equations, it was shown how Resultant theory could be used to solve these polynomial equations. Finally, an example application of Resultant theory was given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered. In this example, the value of the output voltage fundamental was controlled while the fifth and seventh order harmonics were eliminated.

The purpose of this chapter was to introduce some of the theory behind the research presented in this thesis. The next chapter will discuss some additional ideas that were needed for the research. The idea of Symmetric Polynomials will be discussed first. For the special case of when the multilevel inverter is using equal dc sources, it will be shown how the idea of Symmetric Polynomials can be utilized to transform the set of polynomial equations above into a new set of polynomials of lower degree. As a result, it will be easier to solve these equations. The idea of Power Sums will also be discussed. Power Sums theory provides another way of transforming the set of polynomial equations above into a new set of polynomials of lower degree.

4 Symmetric Polynomials and Power Sums

4.1 Chapter Overview

The previous chapter introduced some of the theory behind the research presented in this thesis. In this chapter, some additional ideas that were needed for the research will be discussed. In Section 4.2, the idea of Symmetric Polynomials will be presented. Section 4.3 will discuss an example application of Symmetric Polynomials. In this example, a cascaded H-bridges multilevel inverter using three equal dc sources will once again be considered. For the special case of when a multilevel inverter is utilizing equal dc sources, it will be shown how the idea of Symmetric Polynomials can be utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree. As a result, it will be easier to solve these equations.

Section 4.4 will discuss the idea of Power Sums. Power Sums theory provides another way of transforming a set of symmetric polynomials into a new set of polynomials of lower degree. In Section 4.5, Power Sums theory will be applied to the aforementioned multilevel inverter using three equal dc sources.

4.2 Symmetric Polynomials

In Chapter 3, it was mentioned that Resultant theory alone could, in principle, be utilized to find the desired switching angles for the general case of a multilevel inverter utilizing s switching angles. However, increasing the number of switching angles causes the degrees of the corresponding harmonic polynomial equations to increase as well.

Thus, the symbolic Resultant calculations become more time consuming and complex. In fact, for a multilevel inverter using five equal dc sources, the original set of five equations in five unknowns could not be solved.

In this section, the idea of Symmetric Polynomials will be introduced. For the special case of when a multilevel inverter is utilizing equal dc sources, the idea of Symmetric Polynomials can be utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree. The definition of a symmetric polynomial will first be given. A subset of the symmetric polynomials, the elementary symmetric polynomials will then be defined. After the elementary symmetric polynomials are defined, The Fundamental Theorem of Symmetric Polynomials will be presented.

4.2.1 Symmetric Polynomial Definition

The idea of Symmetric Polynomials arises when one studies the roots of a polynomial. For example, consider the polynomial

$$f(x) = (x - r_1)(x - r_2)(x - r_3), \tag{4.1}$$

where r_1, r_2, and r_3 are the roots of the polynomial. Expansion of the right-hand side of (4.1) yields [17]

$$f(x) = x^3 + bx^2 + cx + d, \tag{4.2}$$

where

$$b = -(r_1 + r_2 + r_3), \tag{4.3}$$

$$c = (r_1 r_2 + r_1 r_3 + r_2 r_3), \tag{4.4}$$

and

$$d = -(r_1 r_2 r_3). \tag{4.5}$$

Equations (4.3) thru (4.5) show that the coefficients of $f(x)$ are polynomials in its roots. Furthermore, since reordering the roots does not change $f(x)$, the coefficients $b, c,$ and d will be unchanged if the roots $r_1, r_2,$ and r_3 are reordered. Therefore, the polynomials given in (4.3) thru (4.5) are said to be *symmetric* [17].

In general, a polynomial $f \in k[x_1,..., x_n]$ is said to be *symmetric* if

$$f(x_{i_1},..., x_{i_n}) = f(x_1,..., x_n) \tag{4.6}$$

for every possible permutation $x_{i_1},...x_{i_n}$ of the variables $x_1,..., x_n$. The term $k[x_1,..., x_n]$ refers to the set of all polynomials in $x_1,..., x_n$ with coefficients in the field *k*. A field is a set where addition, subtraction, multiplication, and division can be defined with the usual properties. Two common examples of fields are the real numbers and the complex numbers [17].

4.2.2 Elementary Symmetric Polynomials

Given the variables $x_1,..., x_n$, define $s_1,..., s_n \in k[x_1,..., x_n]$ by the following expressions:

$$s_1 = x_1 + ... + x_n, \tag{4.7}$$

.

.

.

$$s_r = \sum_{i_1 < i_2 < \ldots < i_r} \left(x_{i_1} x_{i_2} \ldots x_{i_r} \right), \tag{4.8}$$

$$\vdots$$

$$s_n = x_1 x_2 \ldots x_n . \tag{4.9}$$

In (4.7) thru (4.9), each polynomial s_r, $1 \le r \le n$, is the sum of all monomials that are products of *r* distinct variables. In other words, each term in s_r has total degree *r* [17].

To see if these polynomials are indeed symmetric, consider the polynomial

$$f(x) = (x - x_1)(x - x_2)\ldots(x - x_n), \tag{4.10}$$

where $x_1, \ldots, x_n$ are the roots of the polynomial. Expansion of the right-hand side of (4.10) yields

$$f(x) = x^n - s_1 x^{n-1} + s_2 x^{n-2} + \ldots + (-1)^{n-1} s_{n-1} x + (-1)^n s_n . \tag{4.11}$$

Suppose the roots $x_1, \ldots, x_n$ are now rearranged. The order of the factors on the right-hand side of (4.10) is different. However, $f(x)$ remains unchanged, implying the coefficients $(-1)^r s_r$ of $f(x)$ are symmetric polynomials. Therefore, each polynomial s_r is symmetric. The symmetric polynomials $s_1, \ldots, s_n$ are called the *elementary symmetric polynomials* [17].

4.2.3 The Fundamental Theorem of Symmetric Polynomials

The elementary symmetric polynomials are perhaps the most important symmetric polynomials. The following theorem, called The Fundamental Theorem of

Symmetric Polynomials, explains why:

⇒ Every symmetric polynomial in $k[x_1,..., x_n]$ can be written uniquely as a polynomial in the elementary symmetric polynomials $s_1,..., s_n$.

For a proof of this theorem, see [17].

For a simple example of the above theorem, consider the symmetric polynomial

$$f(x, y, z) = x^2y^2 + x^2yz + x^2z^2 + xy^2z + xyz^2 + y^2z^2, \tag{4.12}$$

where $x, y,$ and z are the independent variables. It can be shown that the above polynomial can be rewritten as

$$f(x, y, z) = \left(x^2y^2 + y^2z^2 + x^2z^2\right) + 2\left(x^2yz + xy^2z + xyz^2\right)$$

$$- \left(x^2yz + xy^2z + xyz^2\right) \tag{4.13}$$

$$= (xy + xz + yz)(xy + xz + yz) - (x + y + z)(xyz) \tag{4.14}$$

$$= s_2^2 - s_1s_3, \tag{4.15}$$

where $s_1, s_2,$ and s_3 are the elementary symmetric polynomials corresponding to the variables $x, y,$ and z [17].

4.3 Application of Symmetric Polynomials to Multilevel Inverter

In this section, an example application of Symmetric Polynomials will be presented. In this example, a cascaded H-bridges multilevel inverter using three equal dc sources will once again be considered.

4.3.1 Symmetric Polynomial Reduction

Equations (3.56) thru (3.58) provide the polynomial harmonic equations for the case when one wants to control the peak value of the output voltage to be V_1 while eliminating the fifth and seventh order harmonics. These equations are repeated below:

$$p_1(x_1,x_2,x_3)=0=x_1+x_2+x_3-m\,, \tag{4.16}$$

$$p_5(x_1,x_2,x_3)=0=\sum_{n=1}^{3}\left(5x_n-20x_n^3+16x_n^5\right), \tag{4.17}$$

and

$$p_7(x_1,x_2,x_3)=0=\sum_{n=1}^{3}\left(-7x_n+56x_n^3-112x_n^5+64x_n^7\right). \tag{4.18}$$

In (4.16), m is given by

$$m=\frac{V_1}{\left(\frac{4}{\pi}V_{dc}\right)}, \tag{4.19}$$

where V_{dc} represents the voltage of each dc source. For the variables $x_1, x_2,$ and x_3, the corresponding elementary symmetric polynomials are

$$s_1=x_1+x_2+x_3\,, \tag{4.20}$$

$$s_2=x_1x_2+x_1x_3+x_2x_3\,, \tag{4.21}$$

and

$$s_3=x_1x_2x_3\,. \tag{4.22}$$

Recall that multilevel fundamental switching requires that

$$0\le x_3\le x_2\le x_1\le 1\,. \tag{4.23}$$

As a result, the new variables $s_1, s_2,$ and s_3 must satisfy

$$0 \le s_1 \le 3, \tag{4.24}$$

$$0 \le s_2 \le 3, \tag{4.25}$$

and

$$0 \le s_3 \le 1. \tag{4.26}$$

One should notice that $p_1, p_5,$ and p_7 are symmetric polynomials. Therefore, The Fundamental Theorem of Symmetric Polynomials states that one can express these polynomials in terms of $s_1, s_2,$ and s_3. The resulting symmetric polynomials are:

$$p_1(s_1) = 0 = s_1 - m, \tag{4.27}$$

$$p_5(s_1, s_2, s_3) = 0 = 5s_1 - 20s_1^3 + 16s_1^5 + 60s_1s_2 - 80s_1^3s_2 + 80s_1s_2^2$$

$$- 60s_3 + 80s_1^2s_3 - 80s_2s_3, \tag{4.28}$$

and

$$p_7(s_1, s_2, s_3) = 0 = -7s_1 + 56s_1^3 - 112s_1^5 + 64s_1^7 - 168s_1s_2 + 560s_1^3s_2$$

$$- 448s_1^5s_2 - 560s_1s_2^2 + 896s_1^3s_2^2 - 448s_1s_2^3 + 168s_3$$

$$- 560s_1^2s_3 + 448s_1^4s_3 + 560s_2s_3 - 1344s_1^2s_2s_3$$

$$+ 448s_2^2s_3 + 448s_1s_3^2. \tag{4.29}$$

It should be noted that these equations were created using the *SymmetricReduction* command in Mathematica. Since (4.27) says that $s_1 = m$, p_5 and p_7 can be rewritten as

$$p_5(s_2, s_3) = 5m - 20m^3 + 16m^5 + 60ms_2 - 80m^3s_2 + 80ms_2^2 - 60s_3$$

$$+80m^2s_3-80s_2s_3 \tag{4.30}$$

and

$$p_7(s_2,s_3)=-7m+56m^3-112m^5+64m^7-168ms_2+560m^3s_2$$

$$-448m^5s_2-560ms_2^2+896m^3s_2^2-448ms_2^3+168s_3$$

$$-560m^2s_3+448m^4s_3+560s_2s_3-1344m^2s_2s_3$$

$$+448s_2^2s_3+448ms_3^2. \tag{4.31}$$

For (3.62) and (3.63) in Chapter 3, it turns out that the degrees of $p_5(x_2,x_3)$ and $p_7(x_2,x_3)$ in the variable x_2 are four and six, respectively. As a result, the Sylvester Resultant Matrix corresponding to these two polynomials is a 10 X 10 matrix. On the other hand, the degrees of $p_5(s_2,s_3)$ and $p_7(s_2,s_3)$ in the variable s_3 are one and two, respectively. As a result, the Sylvester Resultant Matrix corresponding to these two polynomials is a 3 X 3 matrix. In other words, the idea of Symmetric Polynomials has been effectively utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree, thus making them much easier to solve.

4.3.2 Solutions to Symmetric Polynomials

After s_1 has been eliminated, the next step is to eliminate the variable s_3 by calculating the Resultant of $p_5(s_2,s_3)$ and $p_7(s_2,s_3)$. After factoring and then eliminating unnecessary numerical constants, the Resultant of the two polynomials in (4.30) and (4.31) was found to be

$$res(s_2) = \left(-1575 + 9800\,m^2 - 24080\,m^4 + 28160\,m^6 - 15360\,m^8 + 3072\,m^{10}\right) + \left(-10500 + 56000\,m^2 - 103040\,m^4 + 78080\,m^6 - 20480\,m^8\right)s_2 + \left(-19600 + 89600\,m^2 - 116480\,m^4 + 46080\,m^6\right)s_2^2 + \left(-11200 + 44800\,m^2 - 35840\,m^4\right)s_2^3 . \quad (4.32)$$

Since the polynomial *res* is only a function of one variable, one can now solve for the variables s_1, s_2, and s_3.

Once values for s_1, s_2, and s_3 are found, it is necessary to use these values to find the corresponding values of x_1, x_2, and x_3 in order to obtain the switching angles. The following three equations need to be solved in order to obtain the values of x_1, x_2, and x_3 :

$$f_1(x_1, x_2, x_3) = 0 = s_1 - (x_1 + x_2 + x_3), \quad (4.33)$$

$$f_2(x_1, x_2, x_3) = 0 = s_2 - (x_1x_2 + x_1x_3 + x_2x_3), \quad (4.34)$$

and

$$f_3(x_1, x_2, x_3) = 0 = s_3 - (x_1x_2x_3). \quad (4.35)$$

One can now eliminate x_1 by calculating the Resultant of f_1 and f_2 as well as the Resultant of f_1 and f_3. The resulting equations are

$$g_1(x_2, x_3) = -s_2 + s_1x_2 - x_2^2 + s_1x_3 - x_2x_3 - x_3^2 \quad (4.36)$$

and

$$g_2(x_2, x_3) = -s_3 + s_1x_2x_3 - x_2^2x_3 - x_2x_3^2 . \quad (4.37)$$

One can now eliminate x_2 by calculating the Resultant of g_1 and g_2. After factoring and then eliminating redundant factors, the resulting equation is

$$h(x_3) = s_3 - s_2 x_3 + s_1 x_3^2 - x_3^3 . \tag{4.38}$$

Since the polynomial h is only a function of one variable, one can now solve for the variables $x_1, x_2,$ and x_3.

It is now possible to begin the process of finding the appropriate switching angles. This process consists of 14 steps:

1. Set s_1 to be equal to the parameter m.
2. Solve for the roots of $res(s_2) = 0$.
3. Keep the roots for which $0 \le \mathrm{Re}(s_2) \le 3$, where Re refers to the real part of a possibly complex root. Denote these roots as $\{s_{2k}\}$.
4. For each member of the set $\{s_{2k}\}$, substitute it into $p_5(s_2, s_3)$ and solve for the roots of $p_5(s_{2k}, s_3) = 0$.
5. Keep the roots for which $0 \le \mathrm{Re}(s_3) \le 1$. Denote the set of remaining roots as $\{(s_{2l}, s_{3l})\}$.
6. For each member of the set $\{(s_{2l}, s_{3l})\}$, substitute it along with the value for s_1 into $h(x_3)$ and solve for the roots of $h(x_3) = 0$.
7. Keep the roots for which $0 \le \mathrm{Re}(x_3) \le 1$. Denote the set of remaining roots as $\{(s_{2n}, s_{3n}, x_{3n})\}$.
8. For each member of the set $\{(s_{2n}, s_{3n}, x_{3n})\}$, substitute it along with the value for s_1 into $g_1(x_2, x_3)$ and solve for the roots of $g_1(x_2, x_{3n}) = 0$.

9. Keep the roots for which $0 \le \mathrm{Re}(x_{3n}) \le \mathrm{Re}(x_2) \le 1$. Denote the set of remaining roots as $\{(s_{2r}, s_{3r}, x_{2r}, x_{3r})\}$.

10. For each member of the set $\{(s_{2r}, s_{3r}, x_{2r}, x_{3r})\}$, compute $m - x_{2r} - x_{3r}$ to find the value for x_1.

11. Keep the roots for which $0 \le \mathrm{Re}(x_{3r}) \le \mathrm{Re}(x_{2r}) \le \mathrm{Re}(x_1) \le 1$. Denote the set of remaining roots as $\{(s_{2s}, s_{3s}, x_{1s}, x_{2s}, x_{3s})\}$.

12. For each member of the set $\{(s_{2s}, s_{3s}, x_{1s}, x_{2s}, x_{3s})\}$, keep just the real parts of x_{1s}, x_{2s}, and x_{3s}. Denote these triples as $\{(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})\}$.

13. Using (4.17) and (4.18), compute

$$\sqrt{\left(\left(\frac{p_5(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})}{5}\right)^2 + \left(\frac{p_7(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})}{7}\right)^2\right)}. \tag{4.39}$$

14. If the result is less than some arbitrarily small tolerance level ε, the switching angles are given by

$$\{(\theta_{1s}, \theta_{2s}, \theta_{3s})\} = \{\cos^{-1}(\hat{x}_{1s}), \cos^{-1}(\hat{x}_{2s}), \cos^{-1}(\hat{x}_{3s})\}. \tag{4.40}$$

Two important points should be made. In general, the algorithm for finding the desired switching angles can theoretically be applied to the more general case of s switching angles. It should also be pointed out that the above algorithm can be applied to the Unified Approach switching scheme. However, two algorithms need to be implemented.

For a multilevel inverter using s dc sources, the first algorithm finds the solutions corresponding to multilevel fundamental switching with s switching angles. The second algorithm finds the solutions corresponding to Unipolar Programmed PWM and Virtual Stage PWM with $s + 1$ switching angles. Using the algorithm above, one must replace all occurrences of the term $\mathrm{Re}(x)$ with the term $|\mathrm{Re}(x)|$.

4.4 Power Sums

In this section, the idea of Power Sums will be introduced. For the special case of a multilevel inverter using equal dc sources, it was shown in Section 4.3 how the idea of Symmetric Polynomials could be utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree. Power Sums simply provides another way of transforming a set of symmetric polynomials into a new set of polynomials of lower degree.

In general, given the variables $x_1,..., x_n$, the *power sums* polynomials are defined as

$$t_k = x_1^k + x_2^k + ... + x_n^k, \tag{4.41}$$

where k is a natural number. One should notice that t_k is a symmetric polynomial. Therefore, The Fundamental Theorem of Symmetric Polynomials says that t_k can be written in terms of the elementary symmetric polynomials $s_1,..., s_n$ [17].

More importantly, however, the following theorem says that any arbitrary symmetric polynomial can be written in terms of $t_1,..., t_n$:

$\Rightarrow$ If k is a field containing the set of rational numbers, then every symmetric polynomial in $k[x_1,..., x_n]$ can be written as a polynomial in the power sums

$t_1,\ldots, t_n$.

Since any symmetric polynomial can be expressed in terms of the elementary symmetric polynomials $s_1,\ldots, s_n$, a sufficient proof of this theorem consists of showing that $s_1,\ldots, s_n$ can be written in terms of $t_1,\ldots, t_n$. In order to prove this statement, the equations

$$t_r - s_1 t_{r-1} + \ldots + (-1)^{r-1} s_{r-1} t_1 + (-1)^r r s_r = 0, \quad \text{for } 1 \le r \le n, \tag{4.42}$$

and

$$t_r - s_1 t_{r-1} + \ldots + (-1)^{n-1} s_{n-1} t_{r-n+1} + (-1)^n s_n t_{r-n} = 0, \quad \text{for } r > n, \tag{4.43}$$

are needed. These equations are referred to as the *Newton Identities* [17].

It will now be proved by induction on r that s_r can be written in terms of $t_1,\ldots, t_n$. For the case of when $r = 1$, it is obvious that $s_1 = t_1$. Assume now that the above claim is true for $1,2,\ldots, i-1$, where $2 \le i \le n$. For the case of $r = i$, the Newton Identities state that

$$s_i = (-1)^{i-1} \frac{1}{i}\left(t_i - s_1 t_{i-1} + \ldots + (-1)^{i-1} s_{i-1} t_1\right). \tag{4.44}$$

Since it was assumed earlier that the rational numbers were contained within the coefficient field k, it is acceptable to divide by i in (4.44). The expression in (4.44) shows that the polynomial s_i can be written in terms of the power sums $t_1,\ldots, t_n$. Therefore, the proof is now complete [17].

4.5 Application of Power Sums to Multilevel Inverter

In this section, an example application of Power Sums will be presented. In this example, a cascaded H-bridges multilevel inverter using three equal dc sources will once again be considered.

4.5.1 Power Sums Reduction

For the variables x_1, x_2, and x_3, define the power sums polynomials $t_1, t_2, \ldots, t_7$ as follows:

$$t_1 = x_1 + x_2 + x_3, \tag{4.45}$$

$$t_2 = x_1^2 + x_2^2 + x_3^2, \tag{4.46}$$

$$t_3 = x_1^3 + x_2^3 + x_3^3, \tag{4.47}$$

$$\vdots$$

$$t_7 = x_1^7 + x_2^7 + x_3^7. \tag{4.48}$$

Using the expressions for $t_1, t_2, \ldots, t_7$, the polynomials p_1, p_5, and p_7 given in (4.16) thru (4.18) can be rewritten as

$$p_1(t_1) = 0 = t_1 - m, \tag{4.49}$$

$$p_5(t_1, t_3, t_5) = 0 = 5t_1 - 20t_3 + 16t_5, \tag{4.50}$$

and

$$p_7(t_1, t_3, t_5, t_7) = 0 = -7t_1 + 56t_3 - 112t_5 + 64t_7. \tag{4.51}$$

From the Newton Identities given in (4.42) and (4.43), t_5 and t_7 can be rewritten in terms of t_1, t_2, and t_3. The resulting expressions are

$$t_5 = \frac{1}{6}\left(t_1^5 - 5t_1^3 t_2 + 5t_1^2 t_3 + 5t_2 t_3\right) \tag{4.52}$$

and

$$t_7 = \frac{1}{36}\left(t_1^7 - 21t_1^3 t_2^2 + 7t_1^4 t_3 + 21t_2^2 t_3 + 28t_1 t_3^2\right). \tag{4.53}$$

Using these new expressions, p_5 and p_7 can now be rewritten as

$$p_5(t_1, t_2, t_3) = \frac{1}{3}\left(15t_1 + 8t_1^5 - 40t_1^3 t_2 - 60t_3 + 40t_1^2 t_3 + 40t_2 t_3\right) \tag{4.54}$$

and

$$p_7(t_1, t_2, t_3) = \frac{1}{9}\Big(-63t_1 - 168t_1^5 + 16t_1^7 + 840t_1^3 t_2 - 336t_1^3 t_2^2 + 504t_3$$

$$- 840t_1^2 t_3 + 112t_1^4 t_3 - 840t_2 t_3 + 336t_2^2 t_3 + 448t_1 t_3^2\Big). \tag{4.55}$$

Since (4.49) says that $t_1 = m$, p_5 and p_7 can now be rewritten in terms of only t_2 and t_3. The resulting equations are

$$p_5(t_2, t_3) = \frac{1}{3}\left(15m + 8m^5 - 40m^3 t_2 - 60t_3 + 40m^2 t_3 + 40t_2 t_3\right) \tag{4.56}$$

and

$$p_7(t_2, t_3) = \frac{1}{9}\Big(-63m - 168m^5 + 16m^7 + 840m^3 t_2 - 336m^3 t_2^2 + 504t_3$$

$$- 840m^2 t_3 + 112m^4 t_3 - 840t_2 t_3 + 336t_2^2 t_3 + 448mt_3^2\Big). \tag{4.57}$$

Notice that the degrees of $p_5(t_2, t_3)$ and $p_7(t_2, t_3)$ in the variable t_2 are one and two, respectively. As a result, the Sylvester Resultant Matrix corresponding to these two polynomials is a 3 X 3 matrix. As with Symmetric Polynomials, the idea of Power

Sums has been effectively utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree, thus making them much easier to solve.

4.5.2 Solutions to Power Sums

After t_1 has been eliminated, the next step is to eliminate the variable t_2 by calculating the Resultant of $p_5(t_2,t_3)$ and $p_7(t_2,t_3)$. After factoring and then eliminating unnecessary numerical constants, the Resultant of the two polynomials in (4.56) and (4.57) was found to be

$$\begin{aligned} res(t_3) = & \left(-4725\,m^4 + 25200\,m^6 - 5040\,m^8 + 256\,m^{12}\right) + \left(4725\,m\right. \\ & \left. - 12600\,m^3 - 95760\,m^5 + 20160\,m^7 - 4096\,m^9\right)t_3 + \left(-12600\right. \\ & \left. + 100800\,m^2 + 80640\,m^4 + 3840\,m^6\right)t_3^2 + \left(-100800\,m\right. \\ & \left. - 44800\,m^3\right)t_3^2 + \left(44800\right)t_3^4 \,. \end{aligned} \tag{4.58}$$

Since the polynomial *res* is only a function of one variable, one can now solve for the variables t_1, t_2, and t_3.

Once values for t_1, t_2, and t_3 are found, it is necessary to use these values to find the corresponding values of x_1, x_2, and x_3 in order to obtain the switching angles. The following three equations need to be solved in order to obtain the values of x_1, x_2, and x_3 [19]:

$$f_1(x_1,x_2,x_3) = 0 = t_1 - (x_1 + x_2 + x_3), \tag{4.59}$$

$$f_2(x_1, x_2, x_3) = 0 = t_2 - \left(x_1^2 + x_2^2 + x_3^2\right), \tag{4.60}$$

and

$$f_3(x_1, x_2, x_3) = 0 = t_3 - \left(x_1^3 + x_2^3 + x_3^3\right). \tag{4.61}$$

One can now eliminate x_1 by calculating the Resultant of f_1 and f_2 as well as the Resultant of f_1 and f_3. The resulting equations are

$$g_1(x_2, x_3) = -t_1^2 + t_2 + 2t_1x_2 - 2x_2^2 + 2t_1x_3 - 2x_2x_3 - 2x_3^2 \tag{4.62}$$

and

$$g_2(x_2, x_3) = t_1^3 - t_3 - 3t_1^2x_2 + 3t_1x_2^2 - 3t_1^2x_3 + 6t_1x_2x_3 - 3x_2^2x_3 + 3t_1x_3^2 - 3x_2x_3^2. \tag{4.63}$$

The variable x_2 can now be eliminated by calculating the Resultant of g_1 and g_2. After factoring and then eliminating redundant factors, the resulting equation is

$$h(x_3) = t_1^3 - 3t_1t_2 + 2t_3 - 3t_1^2x_3 + 3t_2x_3 + 6t_1x_3^2 - 6x_3^3. \tag{4.64}$$

Since the polynomial h is only a function of one variable, one can now solve for the variables $x_1, x_2,$ and x_3.

It is now possible to begin the process of finding the appropriate switching angles. This process consists of 12 steps:

1. Set t_1 to be equal to the parameter m.
2. Solve for the roots of $res(t_3) = 0$. Denote these roots as $\{t_{3k}\}$.
3. For each member of the set $\{t_{3k}\}$, substitute it into $p_5(t_2, t_3)$ and solve for the roots of $p_5(t_2, t_{3k}) = 0$. Denote the set of remaining roots as $\{(t_{2l}, t_{3l})\}$.

4. For each member of the set $\{(t_{2l}, t_{3l})\}$, substitute it along with the value for t_1 into $h(x_3)$ and solve for the roots of $h(x_3) = 0$.

5. Keep the roots for which $0 \le \operatorname{Re}(x_3) \le 1$. Denote the set of remaining roots as $\{(t_{2n}, t_{3n}, x_{3n})\}$.

6. For each member of the set $\{(t_{2n}, t_{3n}, x_{3n})\}$, substitute it along with the value for t_1 into $g_1(x_2, x_3)$ and solve for the roots of $g_1(x_2, x_{3n}) = 0$.

7. Keep the roots for which $0 \le \operatorname{Re}(x_{3n}) \le \operatorname{Re}(x_2) \le 1$. Denote the set of remaining roots as $\{(t_{2r}, t_{3r}, x_{2r}, x_{3r})\}$.

8. For each member of the set $\{(t_{2r}, t_{3r}, x_{2r}, x_{3r})\}$, compute $m - x_{2r} - x_{3r}$ to find the value for x_1.

9. Keep the roots for which $0 \le \operatorname{Re}(x_{3r}) \le \operatorname{Re}(x_{2r}) \le \operatorname{Re}(x_1) \le 1$. Denote the set of remaining roots as $\{(t_{2s}, t_{3s}, x_{1s}, x_{2s}, x_{3s})\}$.

10. For each member of the set $\{(t_{2s}, t_{3s}, x_{1s}, x_{2s}, x_{3s})\}$, keep just the real parts of x_{1s}, x_{2s}, and x_{3s}. Denote these triples as $\{(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})\}$.

11. Using (4.17) and (4.18), compute

$$\sqrt{\left(\left(\frac{p_5(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})}{5}\right)^2 + \left(\frac{p_7(\hat{x}_{1s}, \hat{x}_{2s}, \hat{x}_{3s})}{7}\right)^2\right)}. \tag{4.65}$$

12. If the result is less than some arbitrarily small tolerance level ε, the switching angles are given by

$$\{(\theta_{1s}, \theta_{2s}, \theta_{3s})\} = \{\cos^{-1}(\hat{x}_{1s}), \cos^{-1}(\hat{x}_{2s}), \cos^{-1}(\hat{x}_{3s})\}. \tag{4.66}$$

Similar to the case of Symmetric Polynomials, the algorithm for finding the desired switching angles can theoretically be applied to the more general case of s switching angles. It should also be pointed out that the above algorithm can be applied to the Unified Approach switching scheme. However, two algorithms need to be implemented.

For a multilevel inverter using s dc sources, the first algorithm finds the solutions corresponding to multilevel fundamental switching with s switching angles. The second algorithm finds the solutions corresponding to Unipolar Programmed PWM and Virtual Stage PWM with $s + 1$ switching angles. Using the algorithm above, one must replace all occurrences of the term $\mathrm{Re}(x)$ with the term $|\mathrm{Re}(x)|$.

4.6 Chapter Summary

In this chapter, several topics were discussed. The idea of Symmetric Polynomials was first presented. For the special case of when a multilevel inverter is utilizing equal dc sources, it was shown how the idea of Symmetric Polynomials could be utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree. As a result, these equations were easier to solve. An example application of Symmetric Polynomials theory was also given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered.

The idea of Power Sums was then introduced. It was shown that Power Sums theory provides another way of transforming a set of symmetric polynomials into a new set of polynomials of lower degree. As with Symmetric Polynomials, an example application of Power Sums theory was given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered.

The purpose of this chapter was to introduce more of the theory behind the research presented in this thesis. The next chapter will present some of the theoretical and experimental results. A general discussion of each step in the research process will be provided first. Some of the theoretical and experimental results obtained for a cascaded H-bridges multilevel inverter using a different number of dc sources will then be provided. Two cases will be considered in detail. These cases consist of using five and six dc sources. For the five dc sources case, the switching angles of the inverter were determined such that the 5^{th}, 7^{th}, 11^{th}, and 13^{th} order harmonics were eliminated while at the same time controlling the value of the fundamental. For the six dc sources case, the additional switching angle was used to eliminate the 17^{th} order harmonic.

5 Theoretical and Experimental Results

5.1 Chapter Overview

The previous two chapters introduced the theory behind the research presented in this thesis. In this chapter, some of the theoretical and experimental results will be presented. In Section 5.2, a general discussion of each step in the research process will be provided. Section 5.3 will discuss the theoretical results obtained for a cascaded H-bridges multilevel inverter using five and six equal dc sources.

Section 5.4 will discuss some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing five equal dc sources. The switching angles of the inverter were determined such that the 5^{th}, 7^{th}, 11^{th}, and 13^{th} order harmonics were eliminated while at the same time controlling the value of the fundamental. Section 5.5 looks at the case of using six equal dc sources. The additional switching angle was used to eliminate the 17^{th} order harmonic.

5.2 Steps Performed in Research

In this section, a description of the research process will be given. The research process consisted of three main steps: theoretical calculations, simulations, and experimentation.

5.2.1 Theoretical Calculations

The first part of the theoretical calculations dealt with finding the appropriate switching angles. Using the ideas presented in Chapter 3, the appropriate polynomial harmonic equations were first derived. For the five dc sources and six dc sources cases, the idea of Symmetric Polynomials was then used in conjunction with Resultant theory to solve these polynomials for all possible switching angles. The computer software package Mathematica was used to perform all of the above calculations. Mathematica was used since many of the above calculations were symbolic, not numerical. As one will see in Section 5.3, the algorithm used to find the switching angles resulted in some modulation indices having more than one set of corresponding switching angles.

The second part of the theoretical calculations involved organizing and analyzing all of the collected switching angles. For this purpose, the software package MATLAB was utilized. Using MATLAB, the collected switching angles were organized into look-up tables to be used later in simulations and experiments. Also, MATLAB was used to generate plots of the switching angles and THD versus the modulation index.

5.2.2 Simulations of Multilevel Inverter

After all of the theoretical calculations were completed, simulations of the multilevel inverter were then conducted. These simulations were necessary in order to make sure the computed switching angles were actually eliminating the desired harmonics while at the same time controlling the value of the fundamental. Simulink, an extension to MATLAB, was used to perform the simulations of the multilevel inverter.

5.2.3 Experiments on Multilevel Inverter

A three-phase, wye-connected, 11-level (five dc sources) cascaded H-bridges multilevel inverter was used for all of the conducted experiments. The power electronics switches used for this particular multilevel inverter were MOSFETs with voltage ratings of 100 V and current ratings of 70 A. Rechargeable batteries were used for the separate dc sources.

The RT-LAB software package from Opal-RT-Technologies was used to interface the computer generating the control signals for each switch to the gate driver board on the multilevel inverter. Use of the RT-LAB software allowed for the development of the switching algorithm in Simulink. The switching model was then converted into C code using Real-Time Workshop (RTW). The RT-LAB software provided icons to interface the Simulink model with a digital I/O board. Also, the C code was converted into executable code [18].

Two types of experiments were performed. The first experiments consisted of measuring the open-circuit voltages of the multilevel inverter. However, for brevity, these results will not be presented in this thesis.

After measurements were made under open-circuit conditions, a load was then connected to the output of the multilevel inverter. For the five dc sources case, the multilevel inverter was connected to a three-phase induction motor. The induction motor had a rated horsepower of 1/3 hp, a rated speed of 1725 rpm, a rated current of 1.5 A, and a rated voltage of 208 V (RMS line-to-line voltage at a frequency of 60 Hz) [19]. No additional physical load was placed on the shaft of the motor. Measurements were taken of the output phase voltages and line currents. For the six dc sources case, six of the H-bridge inverters on the multilevel inverter were cascaded to form a single-phase system. With a RL (resistor-inductor) load connected to the output, measurements were taken of the output voltage and current. Figure 5.1 provides a picture of the experimental setup.

5.3 Theoretical Results

5.3.1 Five DC Sources

For the case of a multilevel inverter using five equal dc sources, the five switching angles were determined such that the 5th, 7th, 11th, and 13th order harmonics were eliminated while at the same time controlling the value of the fundamental. The idea of Symmetric Polynomials was used in conjunction with Resultant theory to solve the corresponding set of polynomial harmonic equations for all possible switching angles.

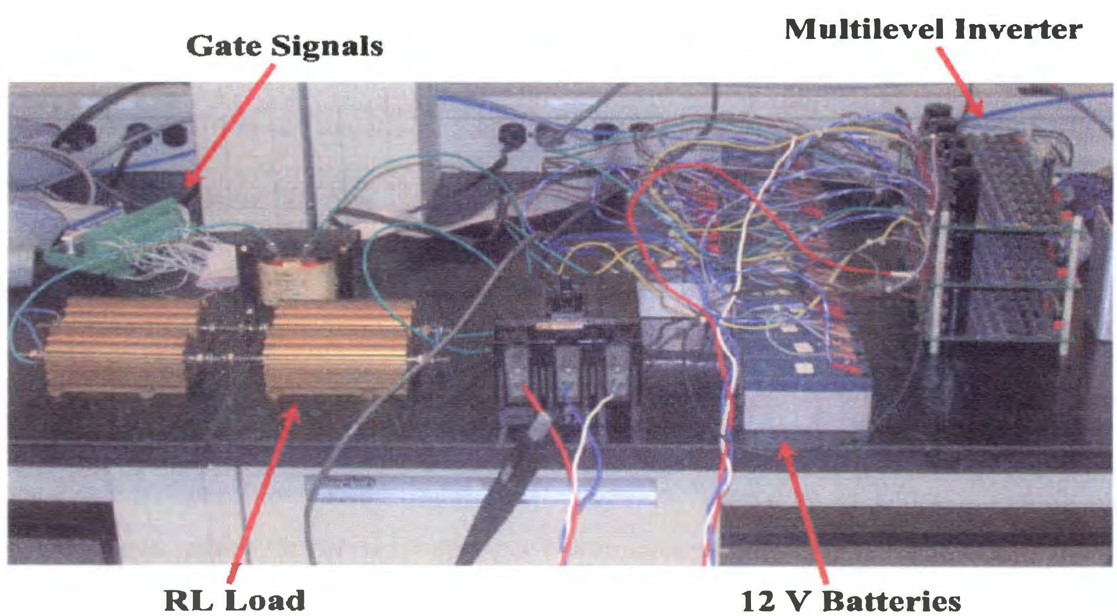

Figure 5.1: Experimental setup.

The solutions to the polynomial equations are plotted in Figure 5.2 versus the modulation index m_a. For purposes of simplicity, only modulation indices in increments of 0.01/5 were considered.

Neglecting the isolated points in Figure 5.2, at least one set of solutions exists when m_a is in the intervals $[0.442, 0.728]$ and $[0.748, 0.846]$. Furthermore, multiple solutions exist when m_a is in the subintervals $[0.506, 0.580]$ and $[0.610, 0.700]$. For modulation indices that do not have any solutions, a different switching scheme must be used. When $m_a \leq 0.732$, it can be shown that the Unified Approach switching scheme will have at least one set of solutions to the aforementioned polynomial harmonic equations.

For each set of switching angles, Figure 5.3 provides a plot of the output phase voltage THD. The voltage THD was calculated thru the 49th harmonic using the equation

$$THD_v(\%) = \sqrt{\frac{V_{17}^2 + V_{19}^2 + V_{23}^2 + V_{25}^2 + \ldots + V_{49}^2}{V_1^2}} * 100, \tag{5.1}$$

where V_1 is the peak value of the fundamental and $V_{17}, V_{19}, \ldots, V_{49}$ are the peak values of the harmonics [19]. Notice in (5.1) that the 5th, 7th, 11th, and 13th order harmonics were omitted. The reason is the switching angles were determined such that these harmonics were eliminated. Also notice that only odd, nontriplen harmonics were considered. As discussed in Chapter 3, the even harmonics are zero due to the symmetry of the multilevel fundamental switching scheme. The triplen harmonics cancel in the line-to-line voltages. In Figure 5.3, the black points correspond to the switching angles that give the lowest THD. For modulation indices having multiple solutions, the blue and red

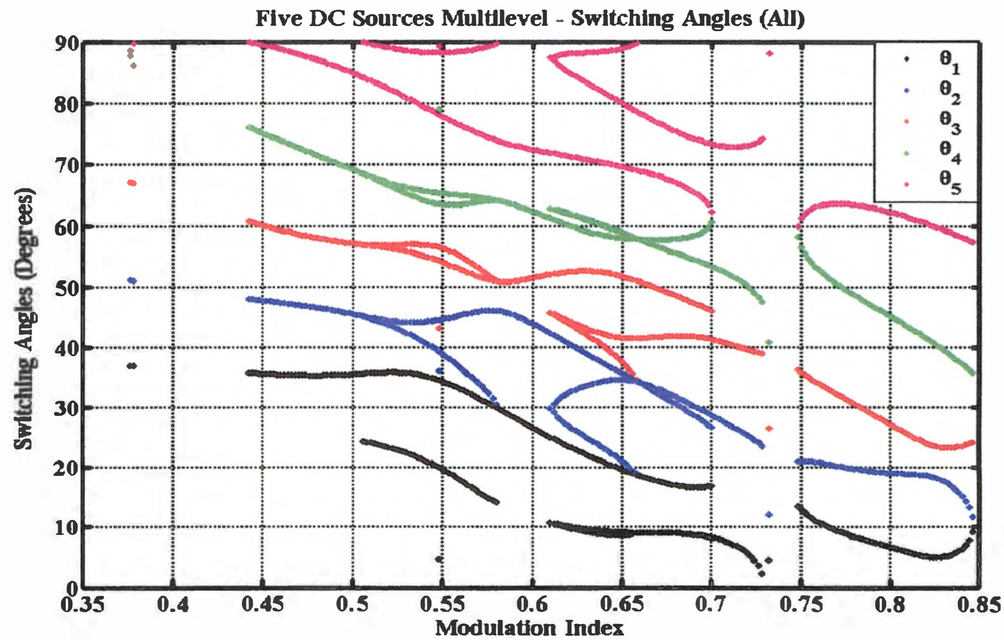

Figure 5.2: Switching angles for an 11-level multilevel inverter.

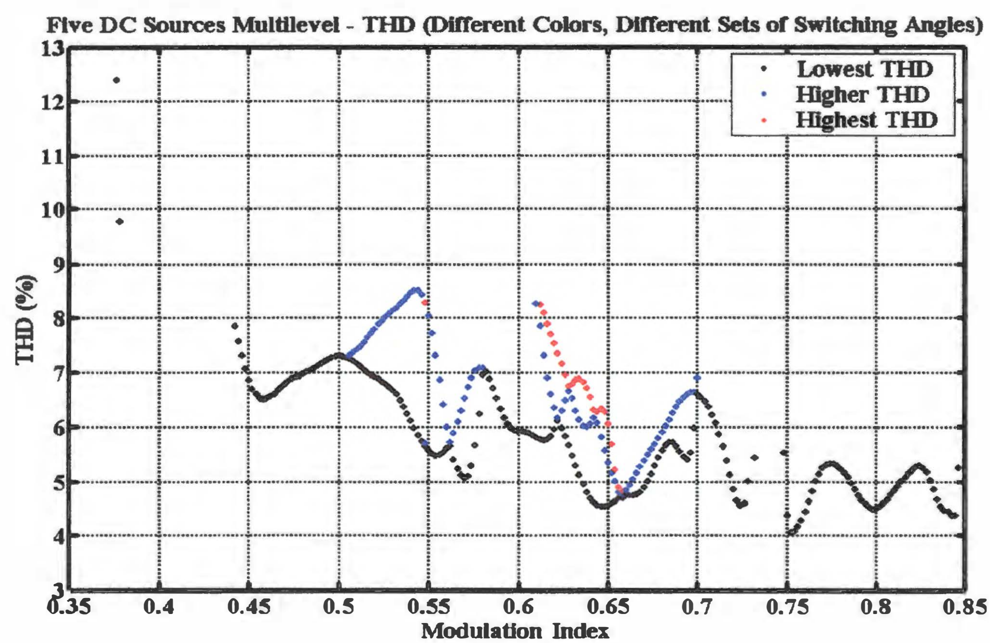

Figure 5.3: Voltage THD from an 11-level multilevel inverter.

points correspond to switching angles that give a higher THD.

When multiple solutions exist, one possibility would be to choose the switching angles that give the lowest THD. Figure 5.4 is a plot of these switching angles versus m_a. Figure 5.5 provides a plot of the corresponding voltage THD. For the modulation indices having at least one set of solutions, notice from Figure 5.5 that most of the time the switching angles can be picked such that the THD is less than 7%. For some of the modulation indices having multiple solutions, it is also obvious from Figure 5.3 that one set of switching angles may give a much higher THD than another set. For example, consider the case $m_a = 0.548$. This particular m_a has three sets of solutions. One set gives a THD of 8.29%. However, a second set gives a THD of 5.64%, a difference of 2.65%.

Notice from Figure 5.2 that the switching angles can be picked such that they form a relatively continuous function when plotted versus m_a. These switching angles are the solutions that a numerical iterative algorithm would most likely produce. However, as Figure 5.4 illustrates, it should be pointed out that these same switching angles might not give the lowest THD. Hence, unlike numerical iterative procedures, the method of determining switching angles presented in this thesis allows for one to always obtain switching angles giving the lowest THD. Tables 1 thru 3 in the Appendix provide a list of selected modulation indices along with their corresponding switching angles and THD for a multilevel inverter using three, four, and five equal dc sources, respectively.

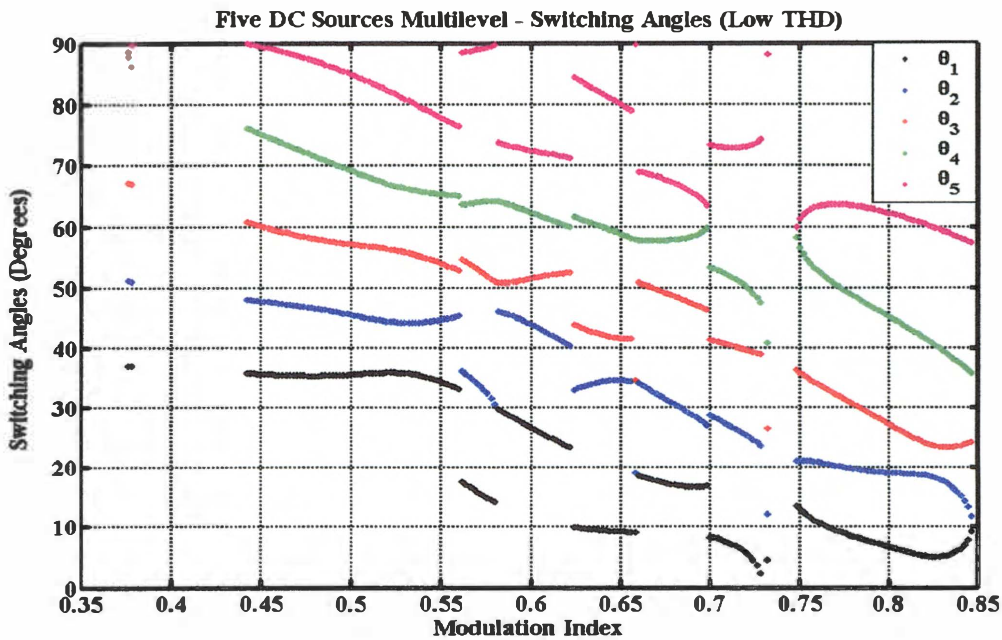

Figure 5.4: 11-level multilevel inverter switching angles giving the lowest THD.

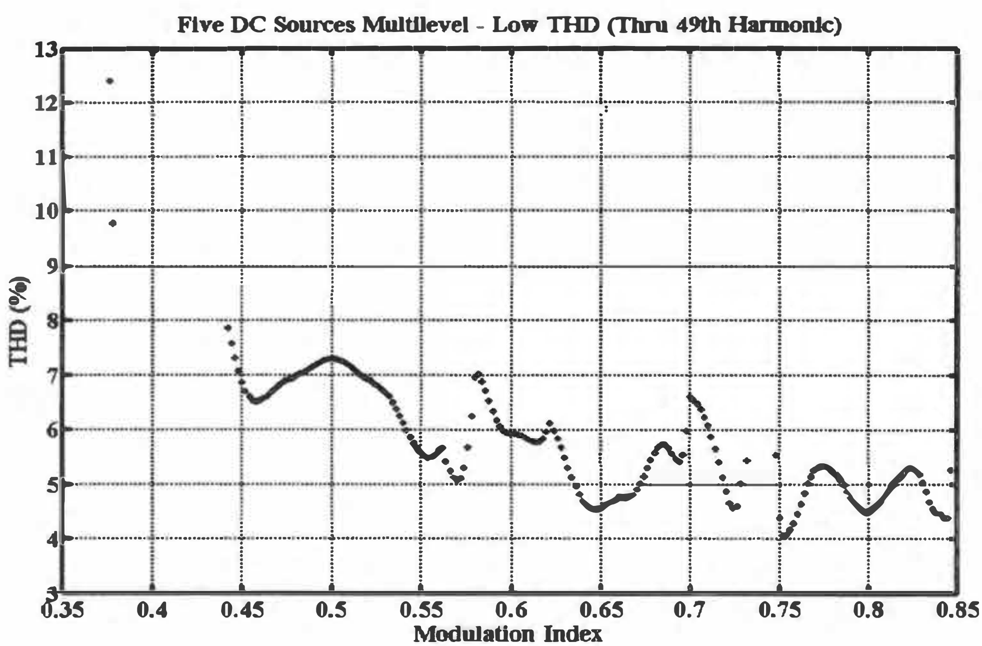

Figure 5.5: Lowest voltage THD from an 11-level multilevel inverter.

5.3.2 Six DC Sources

For the case of a multilevel inverter using six equal dc sources, the six switching angles were determined such that the 5^{th}, 7^{th}, 11^{th}, 13^{th}, and 17^{th} order harmonics were eliminated while at the same time controlling the value of the fundamental. The idea of Symmetric Polynomials was used in conjunction with Resultant theory to solve the corresponding set of polynomial harmonic equations for all possible switching angles. The solutions to the polynomial equations are plotted in Figure 5.6 versus the modulation index m_a. Similar to the five dc sources case, only modulation indices in increments of 0.01/6 were considered.

Neglecting the isolated points in Figure 5.6, at least one set of solutions exists when m_a is in the intervals $[0.455, 0.503]$, $[0.530, 0.647]$, $[0.650, 0.752]$, and $[0.778, 0.828]$. Furthermore, multiple solutions exist when m_a is in the subintervals $[0.530, 0.572]$, $[0.608, 0.633]$, and $[0.667, 0.722]$. For modulation indices that do not have any solutions, a different scheme must be used. For $m_a \leq 0.760$ (with the exception $m_a = 0.330$), it can be shown that the Unified Approach switching scheme will have at least one set of solutions to the aforementioned polynomial harmonic equations.

For each set of switching angles, Figure 5.7 provides a plot of the voltage THD. Similar to the five dc sources case, the THD was calculated using the equation

$$THD_v(\%) = \sqrt{\frac{V_{19}^2 + V_{23}^2 + V_{25}^2 + \ldots + V_{49}^2}{V_1^2}} * 100 . \quad (5.2)$$

Notice in (5.2) that the 17^{th} order harmonic was ignored in the calculation. In Figure 5.7,

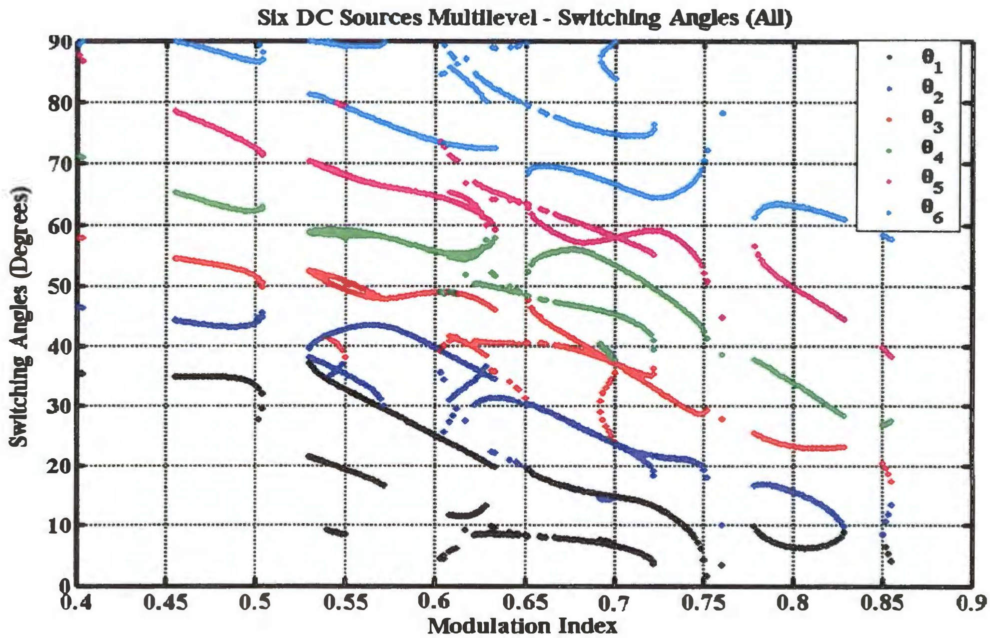

Figure 5.6: Switching angles for a 13-level multilevel inverter.

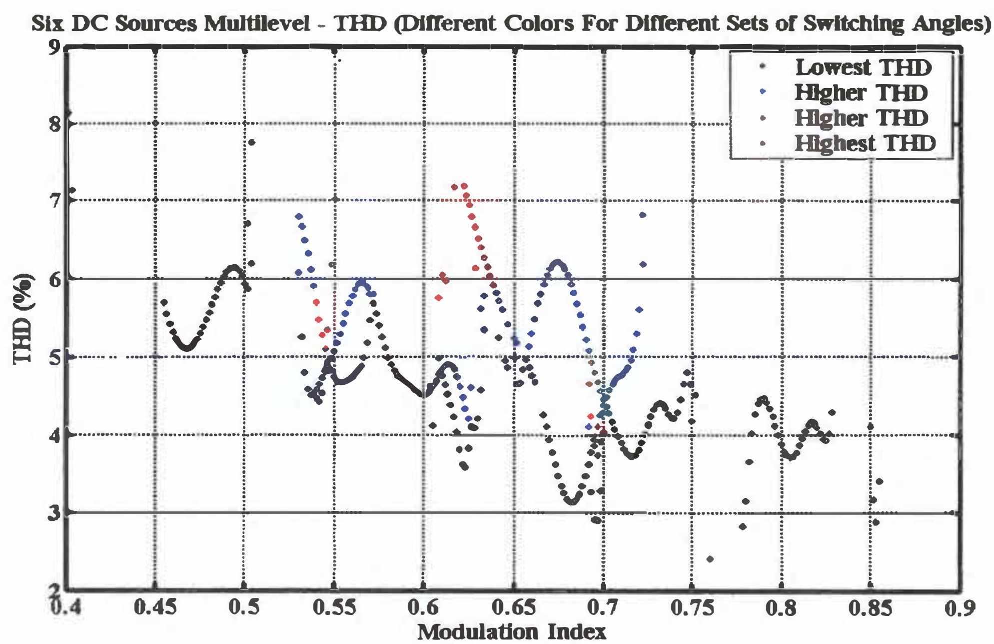

Figure 5.7: Voltage THD from a 13-level multilevel inverter.

the black points correspond to the switching angles that give the lowest THD. For modulation indices having multiple solutions, the blue, red, and green points correspond to switching angles that give a higher THD.

Figure 5.8 provides a plot of the switching angles giving the lowest THD versus m_a. Figure 5.9 is a plot of the corresponding THD. For the modulation indices having at least one set of solutions, notice from Figure 5.9 that most of the time the switching angles can be picked such that the THD is less than 6%. As with the five dc sources case, for modulation indices having multiple solutions, one set of switching angles may give a much higher THD than another set. For example, consider the case $m_a = 0.622$. This particular modulation index has three sets of solutions. One set gives a THD of 7.19%. However, a second set gives a THD of 3.62%, a difference of 3.57%.

Notice from Figure 5.6 that the switching angles can be picked such that they form a relatively continuous function when plotted versus m_a. These switching angles are the solutions that a numerical iterative algorithm would most likely produce. However, similar to the five dc sources case, these same switching angles might not give the lowest THD. Table 4 in the Appendix provides a list of selected modulation indices along with their corresponding switching angles and THD for a multilevel inverter using six dc sources.

5.4 Experimental Results: Five DC Sources

This section will discuss some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing five equal dc sources. A three-phase,

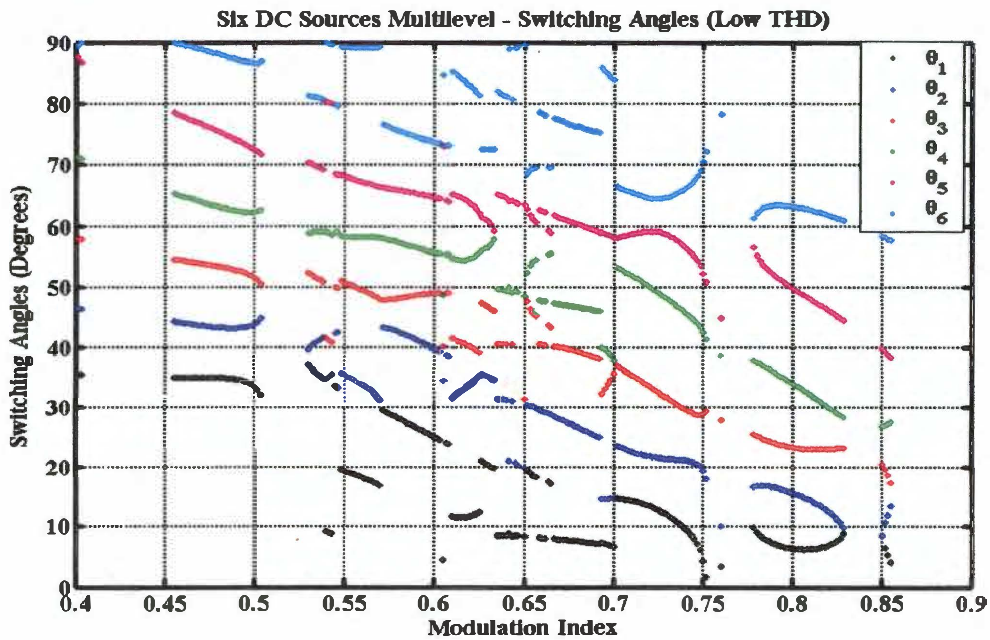

Figure 5.8: 13-level multilevel inverter switching angles giving the lowest THD.

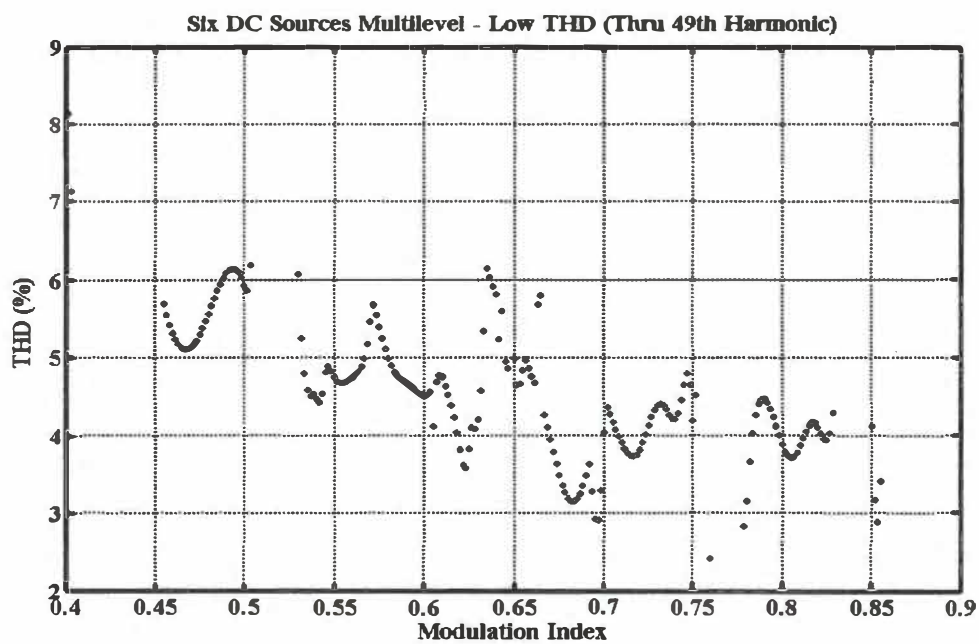

Figure 5.9: Lowest voltage THD from a 13-level multilevel inverter.

wye-connected, 11-level cascaded H-bridges multilevel inverter was connected to a three-phase induction motor with no additional physical load attached to the shaft of the motor. Measurements were then taken of the output phase voltages and line currents. Within the RT-LAB software, a step size of 32 microseconds was used for the real time implementation.

Each dc source was comprised of three 12 V (nominal) batteries connected in series. For phase A of the multilevel inverter, the voltages of the dc sources were measured to be 38.20 V, 38.24 V, 38.14 V, 38.22 V, and 38.24 V. The voltages of the dc sources for phase B were measured to be 37.79 V, 37.82 V, 38.01 V, 37.82 V, and 38.27 V. For phase C, the voltages of the dc sources were measured to be 37.85 V, 37.64 V, 37.73 V, 37.84 V, and 38.18 V. Although the dc sources above are not equal to one another, they were considered equal in the experimentation.

5.4.1 $m_a = 0.640$: Lowest THD

For the case $m_a = 0.640$, one can see from Figure 5.2 that there are three different solution sets. In the experimentation, the switching angles giving the lowest voltage THD were used. Furthermore, the frequency of the fundamental voltage was set at 60 Hz.

Figure 5.10 illustrates the phase voltages measured at the output of the multilevel inverter. Figure 5.11 provides the corresponding normalized FFT of the phase A output voltage. The term "normalized" simply refers to the idea that the magnitudes of all harmonics were found relative to the magnitude of the fundamental.

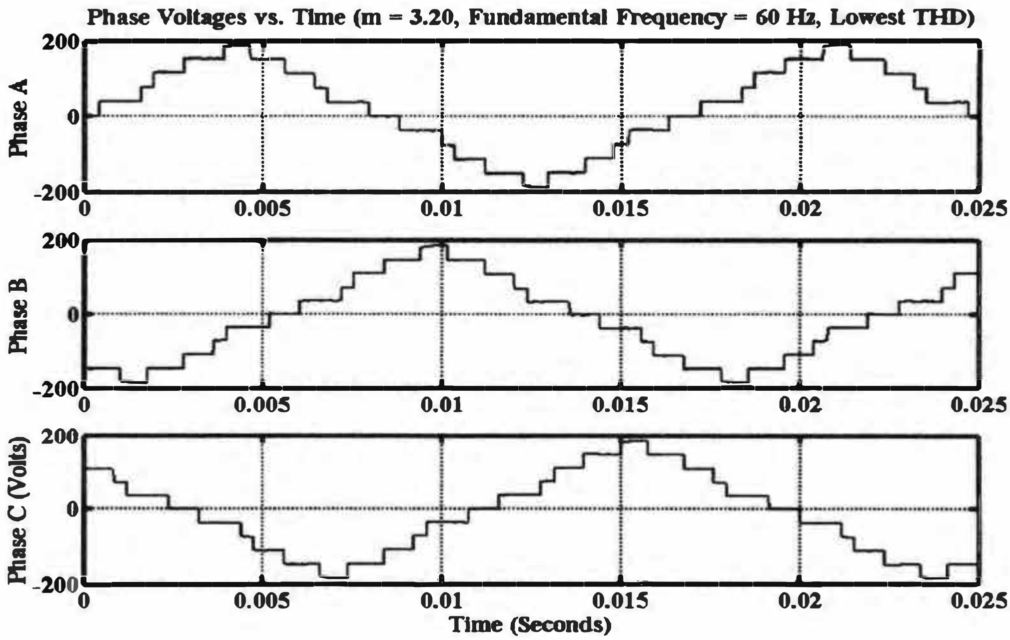

Figure 5.10: Phase voltage waveforms for $m_a = 0.640$ (lowest THD).

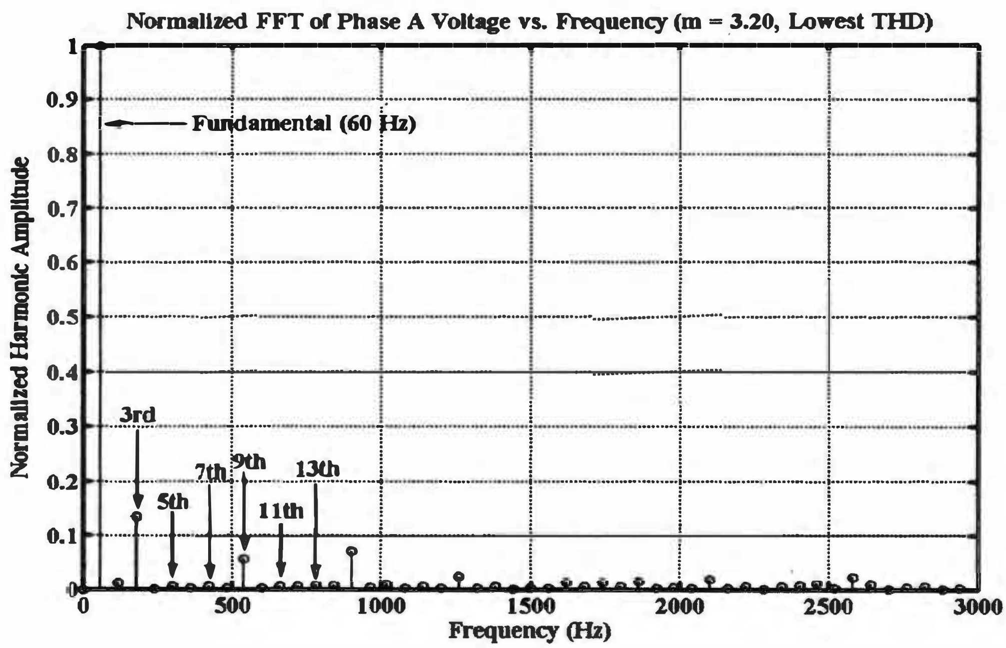

Figure 5.11: Normalized FFT of phase A voltage for $m_a = 0.640$ (lowest THD).

Notice from Figure 5.11 that the magnitudes of the 5th, 7th, 11th, and 13th harmonics are quite small. One reason they are not precisely zero is the battery voltages are not exactly equal to one another. Also, the quantization error of the oscilloscope will lead to some error. Using (5.1), the voltage THD was determined from Figure 5.11 to be 4.26%. When this value is compared to the theoretical value of 4.69%, the resulting percent difference is

$$\%\ Difference = \frac{|4.69 - 4.26|}{4.69} * 100 = 9.15\%. \tag{5.3}$$

Figure 5.12 illustrates the measured phase A output current. Figure 5.13 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.13 that the magnitudes of the 5th, 7th, 11th, and 13th harmonics are quite small. Furthermore, notice that the odd, triplen harmonics are practically zero. Using the equation

$$THD_i(\%) = \sqrt{\frac{I_{17}^2 + I_{19}^2 + I_{23}^2 + I_{25}^2 + \ldots + I_{49}^2}{I_1^2}} * 100, \tag{5.4}$$

the current THD was determined from Figure 5.13 to be 1.36%, which is less than the measured voltage THD. The inductance of the three-phase induction motor acts as a lowpass current filter.

5.4.2 $m_a = 0.548$: Lowest THD

For the case $m_a = 0.548$, one can see from Figure 5.2 that there are three different solution sets. In one experiment, the switching angles giving the lowest voltage THD were used. Furthermore, the frequency of the fundamental voltage was set at 51

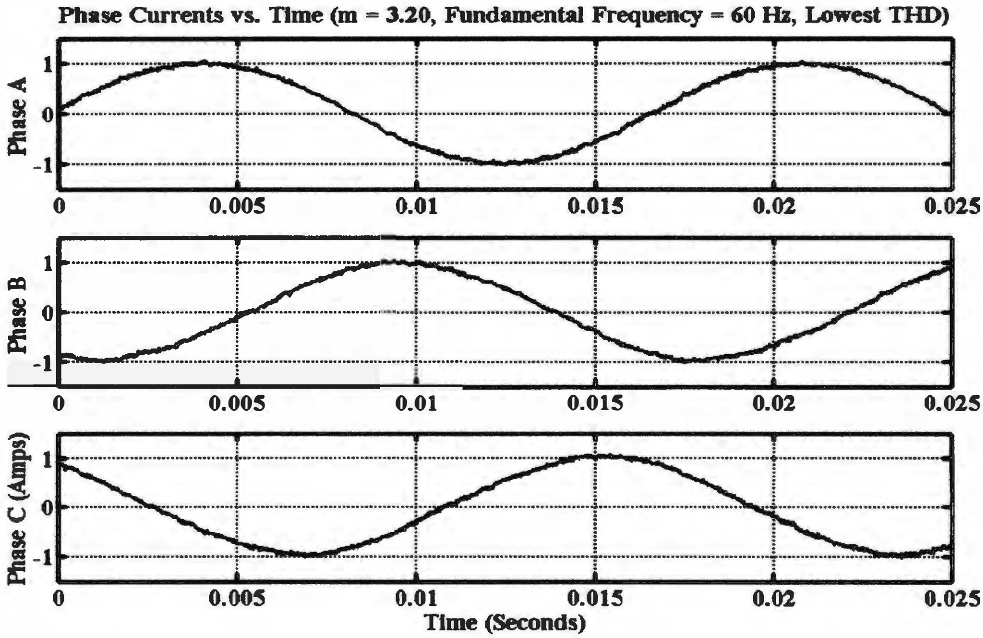

Figure 5.12: Phase current waveforms for m_a = 0.640 (lowest THD).

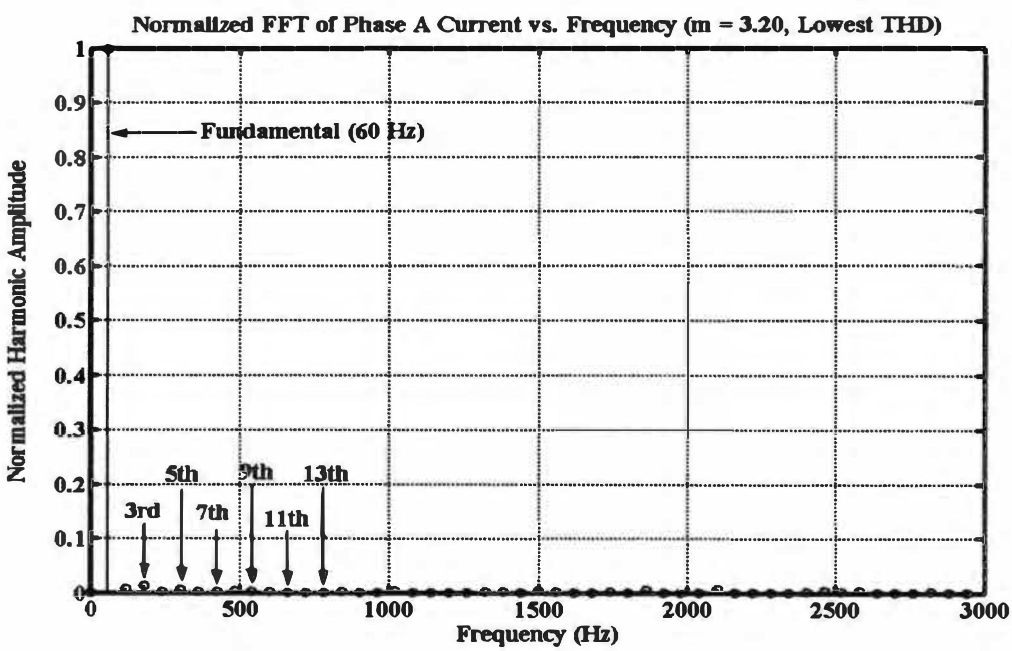

Figure 5.13: Normalized FFT of phase A current for m_a = 0.640 (lowest THD).

Hz. The fundamental frequency was lowered in order to maintain a constant V/Hz ratio.

Figure 5.14 illustrates the phase voltages measured at the output of the multilevel inverter. Figure 5.15 provides the corresponding normalized FFT of the phase A output voltage. Notice from Figure 5.15 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, and 13^{th} harmonics are quite small. Using (5.1), the voltage THD was determined from Figure 5.15 to be 5.09%. Comparing this value to the theoretical value of 5.64%, the resulting percent difference is

$$\%\ Difference \ = \frac{|5.64 - 5.09|}{5.64} * 100 = 9.77\%. \tag{5.5}$$

Figure 5.16 illustrates the measured phase A output current. Figure 5.17 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.17 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, and 13^{th} harmonics are quite small. Furthermore, notice that the odd, triplen harmonics are small. Using (5.4), the current THD was determined from Figure 5.17 to be 1.59%, which is less than the measured voltage THD.

5.4.3 m_a = 0.548: Highest THD

For the case $m_a = 0.548$, another experiment was conducted where the switching angles giving the highest voltage THD were used. As with the experiment discussed in Section 5.4.2 above, the frequency of the fundamental voltage was set at 51 Hz.

Figure 5.18 illustrates the phase voltages measured at the output of the multilevel inverter. Figure 5.19 provides the corresponding normalized FFT of the phase A output voltage. Notice from Figure 5.19 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, and 13^{th}

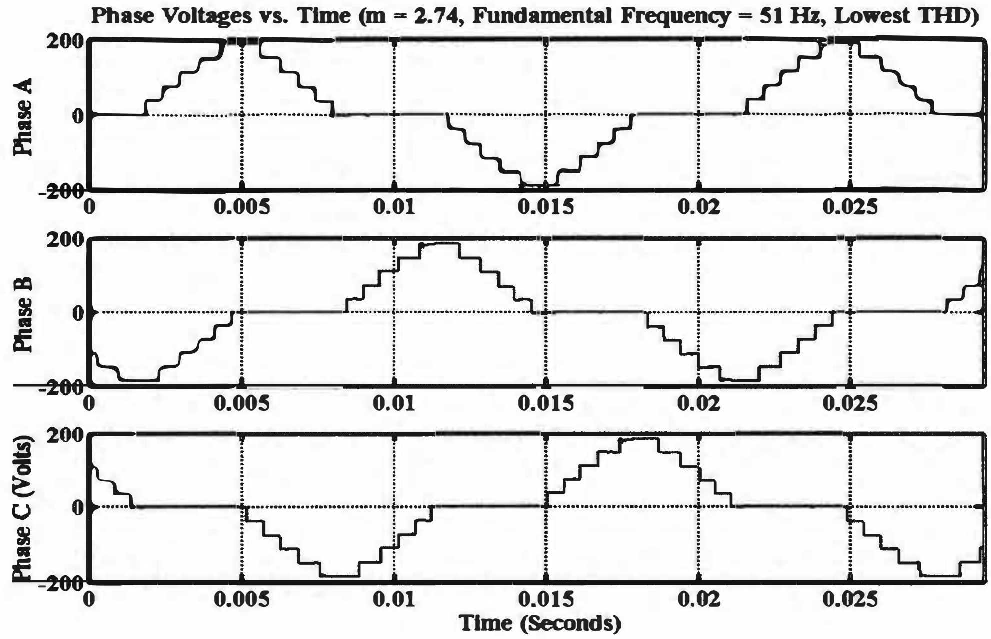

Figure 5.14: Phase voltage waveforms for $m_a = 0.548$ (lowest THD).

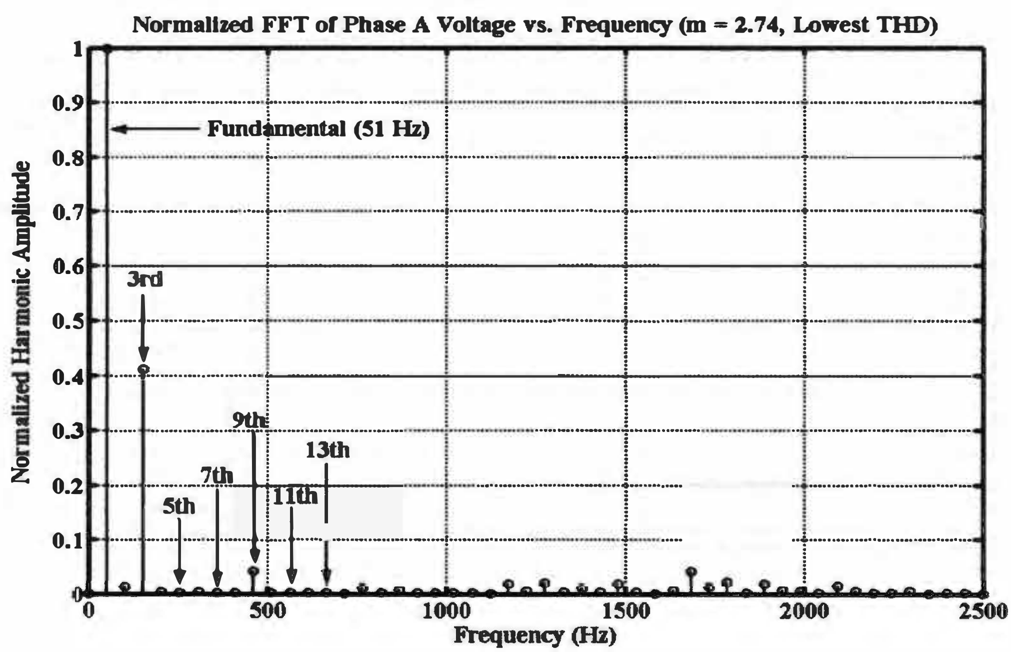

Figure 5.15: Normalized FFT of phase A voltage for $m_a = 0.548$ (lowest THD).

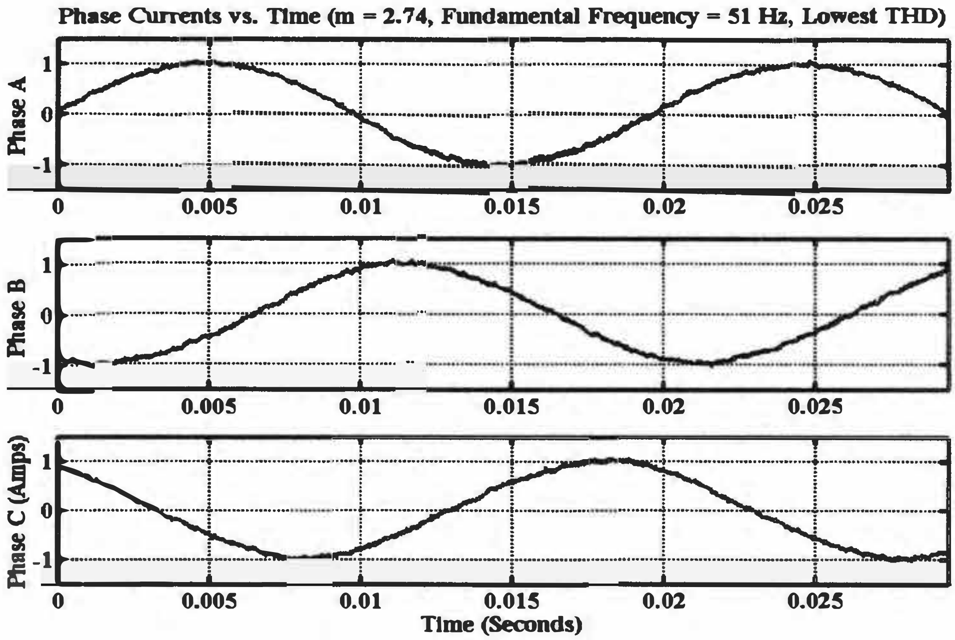

Figure 5.16: Phase current waveforms for $m_a = 0.548$ (lowest THD).

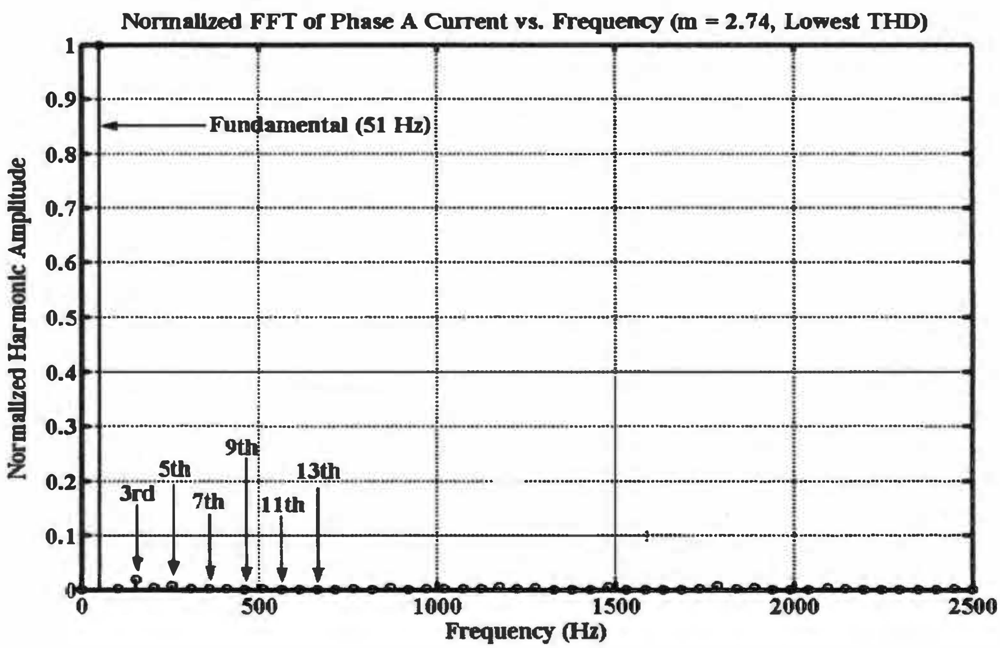

Figure 5.17: Normalized FFT of phase A current for $m_a = 0.548$ (lowest THD).

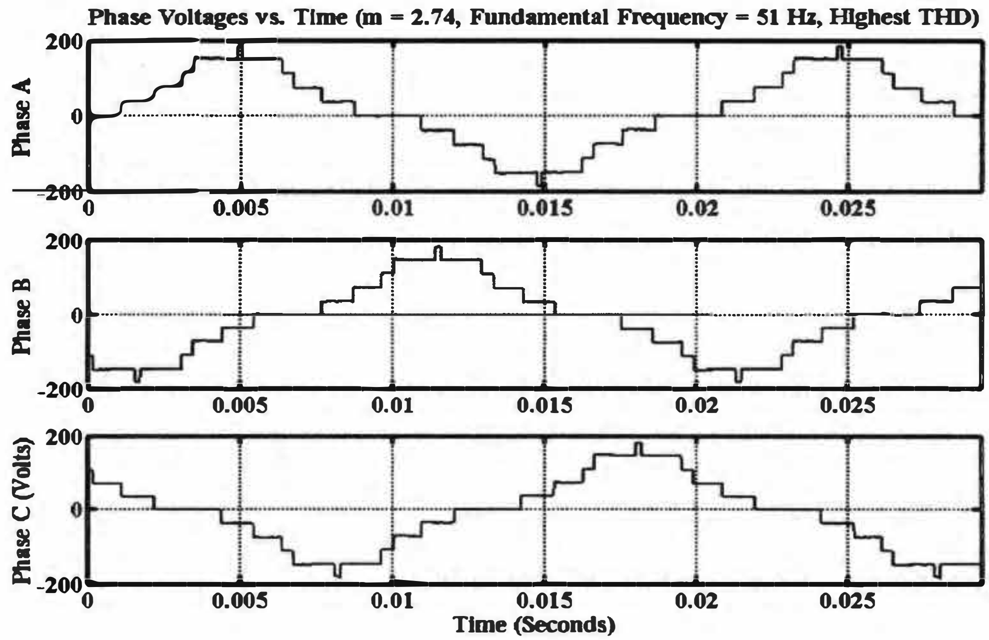

Figure 5.18: Phase voltage waveforms for $m_a = 0.548$ (highest THD).

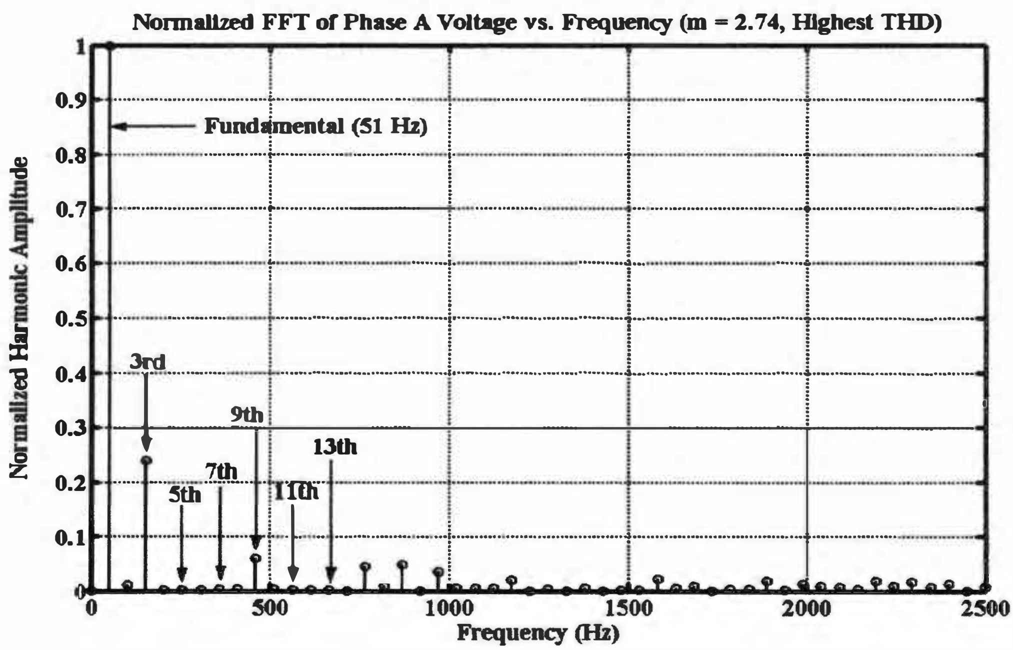

Figure 5.19: Normalized FFT of phase A voltage for $m_a = 0.548$ (highest THD).

harmonics are quite small. Also notice that the magnitudes of the 17th and 19th order harmonics in Figure 5.19 are larger than the corresponding harmonics in Figure 5.15. Using (5.1), the voltage THD was determined from Figure 5.19 to be 7.68%. Comparing this value to the theoretical value of 8.29%, the resulting percent difference is

$$\%\ Difference = \frac{|8.29 - 7.68|}{8.29} * 100 = 7.30\%. \tag{5.6}$$

Figure 5.20 illustrates the measured phase A output current. Figure 5.21 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.21 that the magnitudes of the 5th, 7th, 11th, and 13th harmonics are quite small. Furthermore, the odd, triplen harmonics are small. One should also notice that the magnitudes of the 17th and 19th order harmonics in Figure 5.21 are larger than the corresponding harmonics in Figure 5.17. Using (5.4), the current THD was determined from Figure 5.21 to be 3.18%, which is less than the measured voltage THD.

5.5 Experimental Results: Six DC Sources

This section will discuss some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing six equal dc sources. It was mentioned earlier that experiments were conducted on a single-phase system comprised of six cascaded H-bridge inverters. With an inductive load connected to the output of this single-phase system, measurements were taken of the output voltage and current. The inductive load consisted of a 10 mH inductor connected in series with a measured equivalent resistance of 61.1 Ω. Within the RT-LAB software, a step size of 32 microseconds was used for the real time implementation. Furthermore, the fundamental

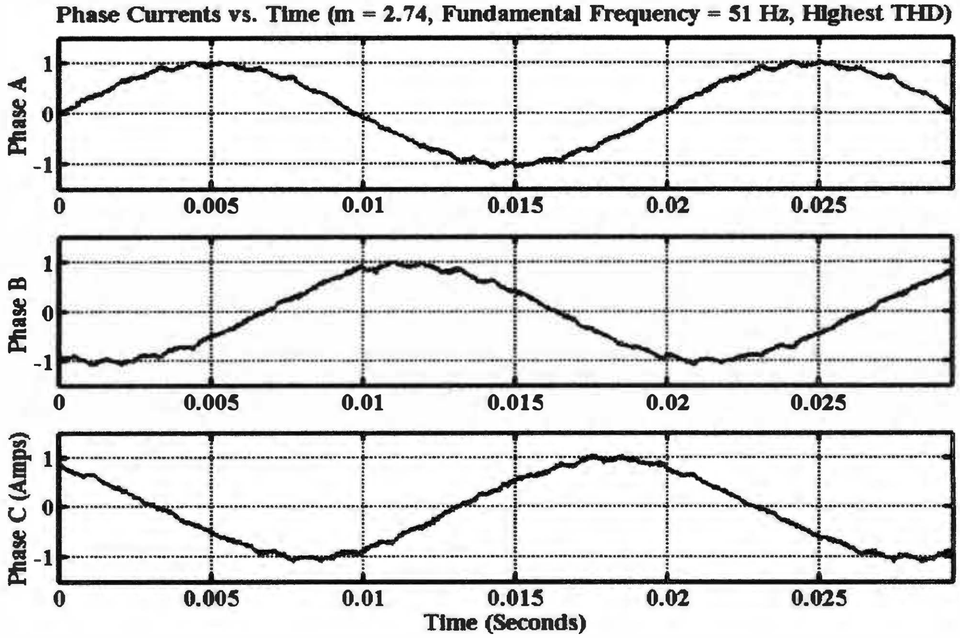

Figure 5.20: Phase current waveforms for $m_a = 0.548$ (highest THD).

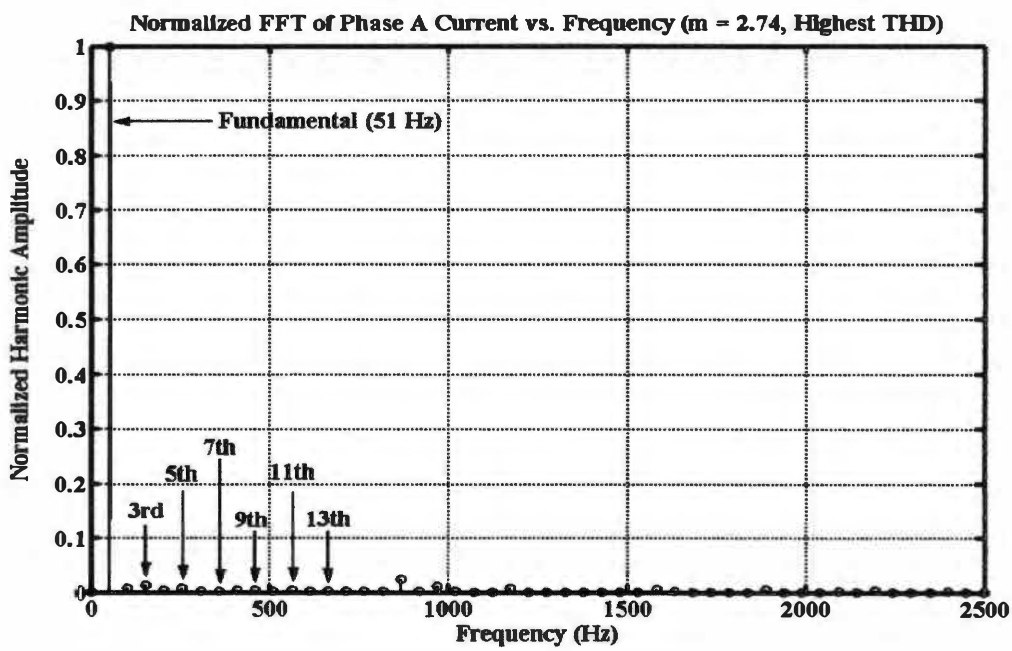

Figure 5.21: Normalized FFT of phase A current for $m_a = 0.548$ (highest THD).

frequency was set at 60 Hz for all experiments.

Similar to the five dc sources case, each dc source was comprised of three 12 V (nominal) batteries connected in series. The voltages of the dc sources were measured to be 38.65 V, 38.65 V, 38.55 V, 38.63 V, 38.48 V, and 38.65 V. Although the dc sources above are not equal to one another, they were considered equal in the experimentation.

5.5.1 $m_a = 0.760$

For the case $m_a = 0.760$, one can see from Figure 5.6 that there exists only one solution. However, out of all modulation indices that have solutions, it is evident from Figure 5.9 that this particular modulation index will give the lowest output voltage THD.

Figure 5.22 illustrates the voltage measured at the output of the multilevel inverter. Figure 5.23 provides the corresponding normalized FFT of the output voltage. Notice from Figure 5.23 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, 13^{th}, and 17^{th} harmonics are quite small. Using (5.2), the voltage THD was determined from Figure 5.23 to be 2.49%. Comparing this value to the theoretical value of 2.41%, the resulting percent difference is

$$\%\ \textit{Difference} = \frac{|2.41 - 2.49|}{2.41} * 100 = 2.95\%. \tag{5.7}$$

It should be noted that the peak-to-peak value of the output voltage in Figure 5.22 is about 40 V less than the corresponding value under open-circuit conditions. One explanation is the internal resistance of each battery leads to a voltage drop proportional to the current. Another explanation is the on-state voltage drop of each MOSFET is proportional to the current.

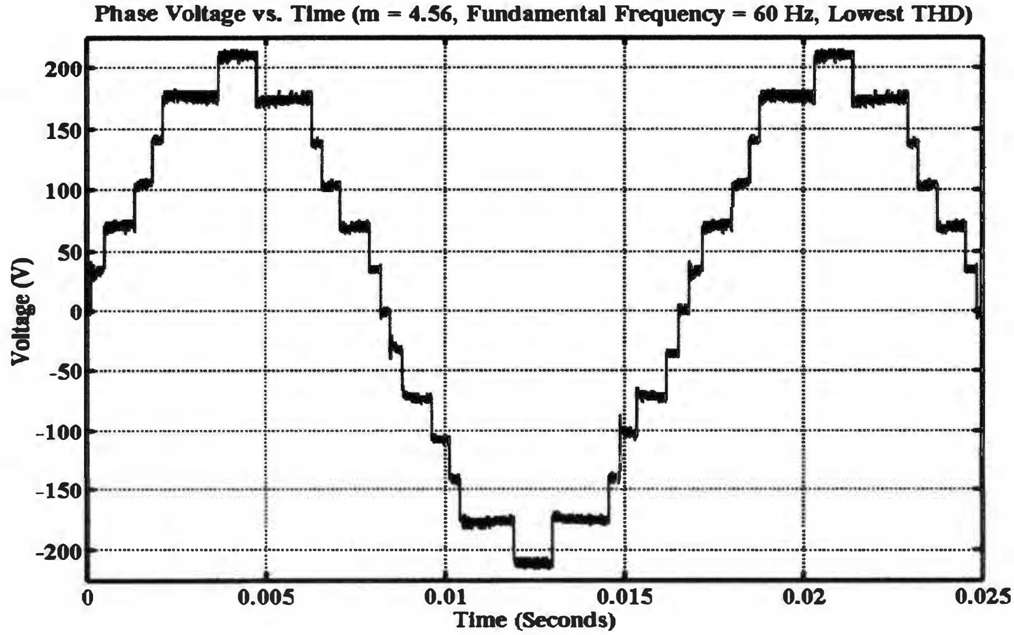

Figure 5.22: Voltage waveform for $m_a = 0.760$.

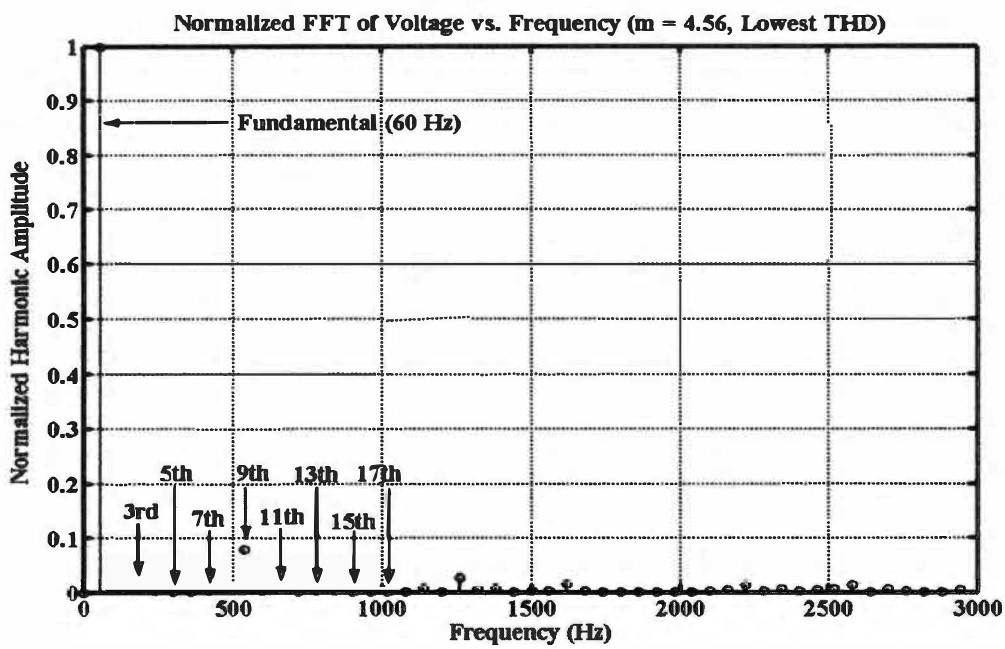

Figure 5.23: Normalized FFT of voltage for $m_a = 0.760$.

Figure 5.24 illustrates the measured output current. Figure 5.25 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.25 that the magnitudes of the 5th, 7th, 11th, 13th, and 17th harmonics are small. However, notice that the odd, triplen harmonics are not small. The reason is due to the fact that a single-phase system was considered instead of a balanced three-phase system. A balanced three-phase system does not have any odd, triplen harmonics. Therefore, the current THD was determined using the equation

$$THD_i(\%) = \sqrt{\frac{I_{19}^2 + I_{23}^2 + I_{25}^2 + \ldots + I_{49}^2}{I_1^2}} * 100. \tag{5.8}$$

Notice in (5.8) that the odd, triple harmonics were neglected. Using (5.8), the current THD was determined from Figure 5.25 to be 0.99%, which is less than the measured voltage THD. The 10mH inductor acts as a lowpass current filter. To the nearest 0.01%, the measured current THD is equal to the theoretical value of the current THD.

5.5.2 $m_a = 0.622$: Lowest THD

For the case $m_a = 0.622$, one can see from Figure 5.6 that there are three different solution sets. In one experiment, the switching angles giving the lowest voltage THD were used.

Figure 5.26 illustrates the voltage measured at the output of the multilevel inverter. Figure 5.27 provides the corresponding normalized FFT of the output voltage. Notice from Figure 5.27 that the magnitudes of the 5th, 7th, 11th, 13th, and 17th harmonics are quite small. Using (5.2), the voltage THD was determined from Figure 5.27 to be

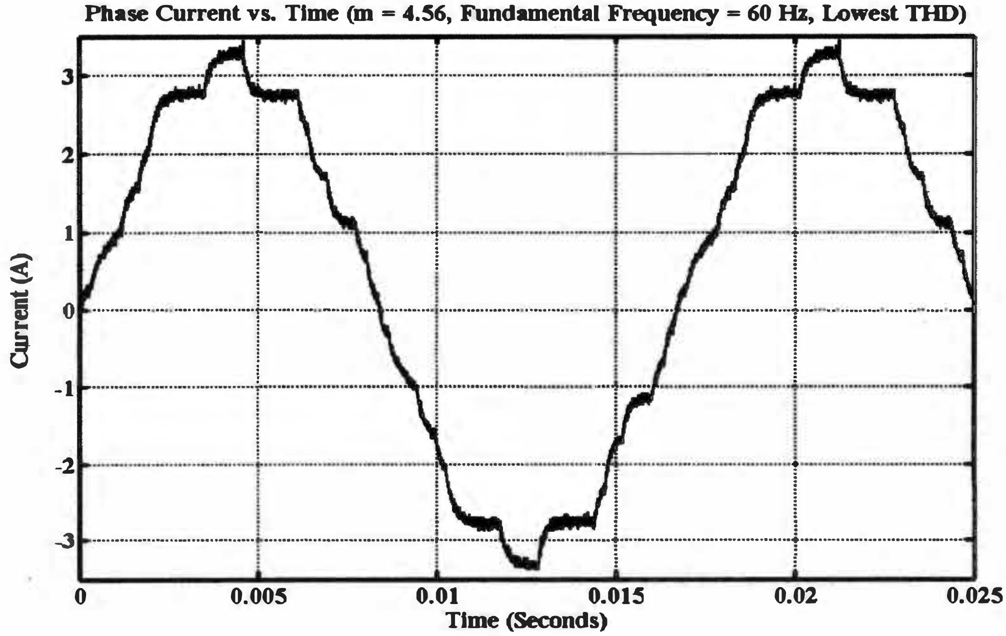

Figure 5.24: Current waveform for $m_a = 0.760$.

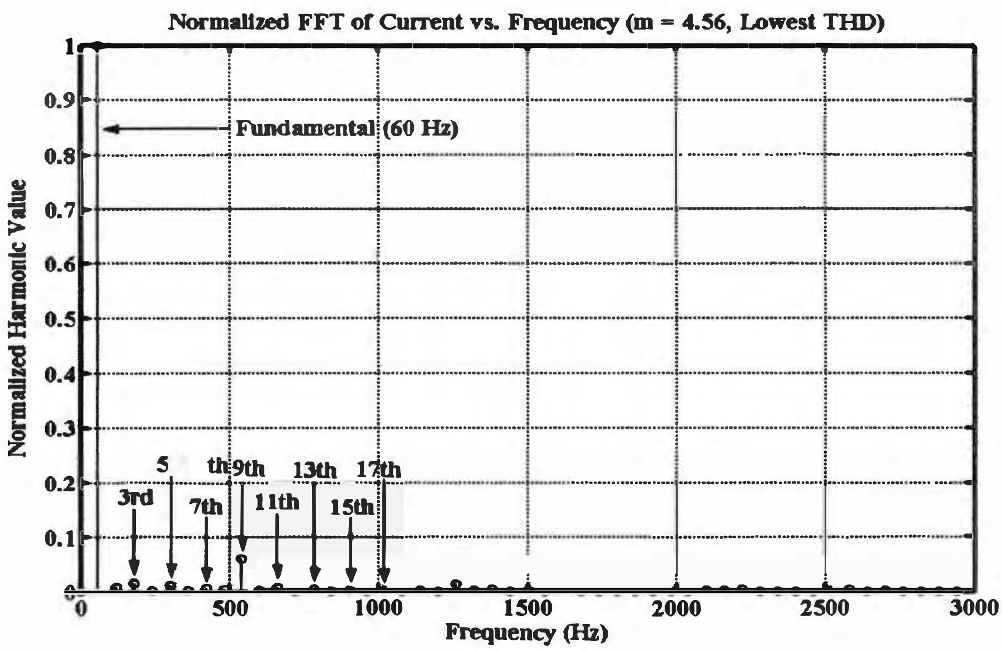

Figure 5.25: Normalized FFT of current for $m_a = 0.760$.

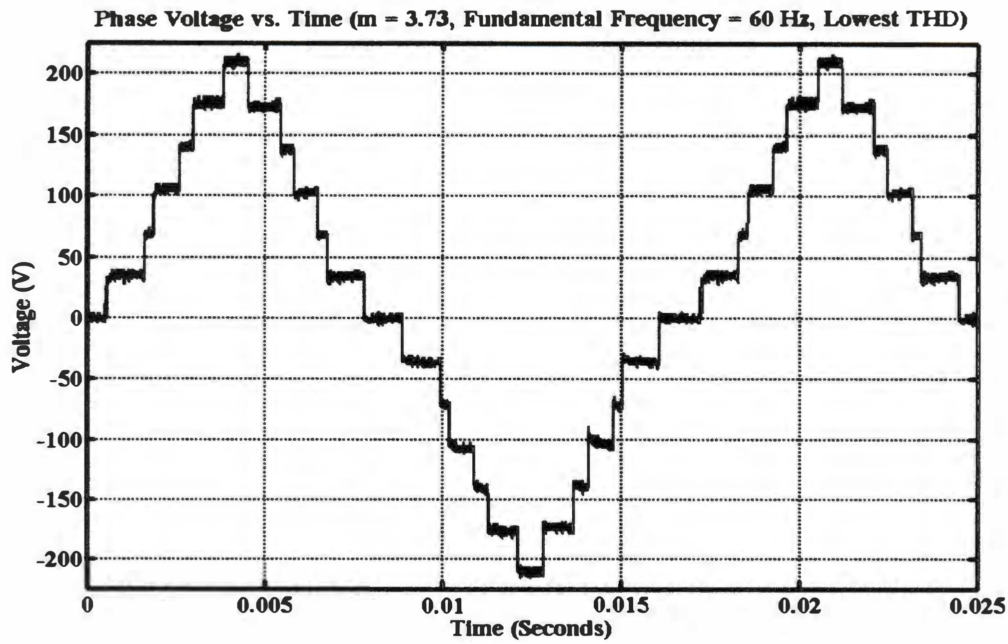

Figure 5.26: Voltage waveform for $m_a = 0.622$ (lowest THD).

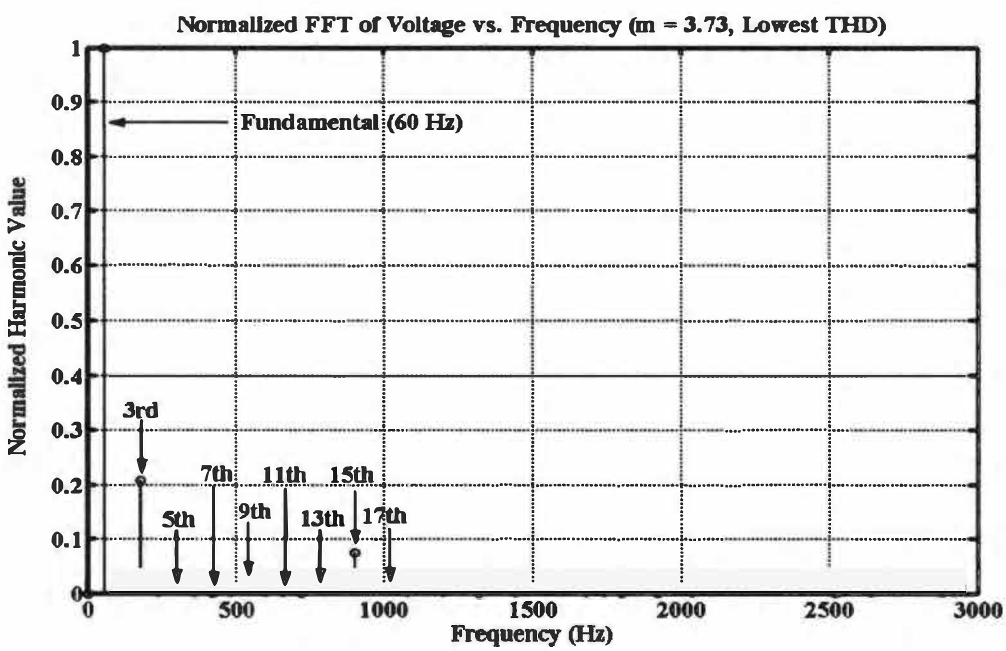

Figure 5.27: Normalized FFT of voltage for $m_a = 0.622$ (lowest THD).

3.19%. Comparing this value to the theoretical value of 3.62%, the resulting percent difference is

$$\%\ Difference = \frac{|3.62 - 3.19|}{3.62} * 100 = 11.85\%. \tag{5.9}$$

As with the experiment discussed in Section 5.5.1, the peak-to-peak value of the output voltage in Figure 5.26 is about 40 V less than the corresponding value under open-circuit conditions.

Figure 5.28 illustrates the measured output current. Figure 5.29 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.29 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, 13^{th}, and 17^{th} harmonics are quite small. However, notice that the odd, triplen harmonics are not small. As discussed earlier, the reason is due to the fact that a single-phase system was considered instead of a balanced three-phase system. A balanced three-phase system does not have any odd, triplen harmonics. Therefore, neglecting the odd, triplen harmonics and using (5.8), the current THD was determined from Figure 5.29 to be 1.24%, which is less than the measured voltage THD. Comparing this value to the theoretical value of 1.45%, the resulting percent difference is

$$\%\ Difference = \frac{|1.45 - 1.24|}{1.45} * 100 = 14.33\%. \tag{5.10}$$

5.5.3 $m_a = 0.622$: Highest THD

For the case $m_a = 0.622$, another experiment was conducted where the switching angles giving the highest voltage THD were used. Figure 5.30 illustrates the voltage

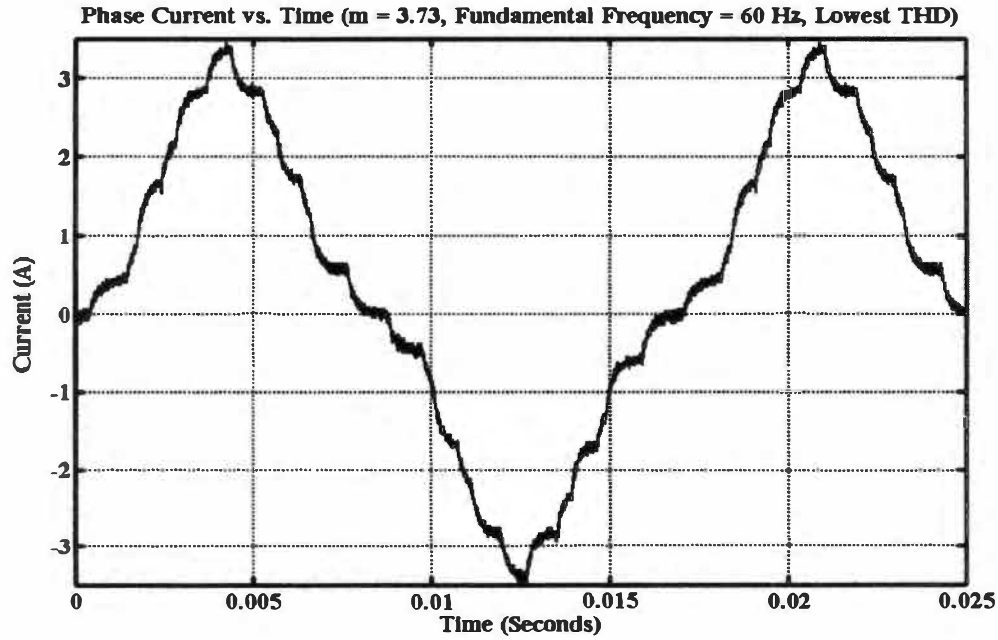

Figure 5.28: Current waveform for $m_a = 0.622$ (lowest THD).

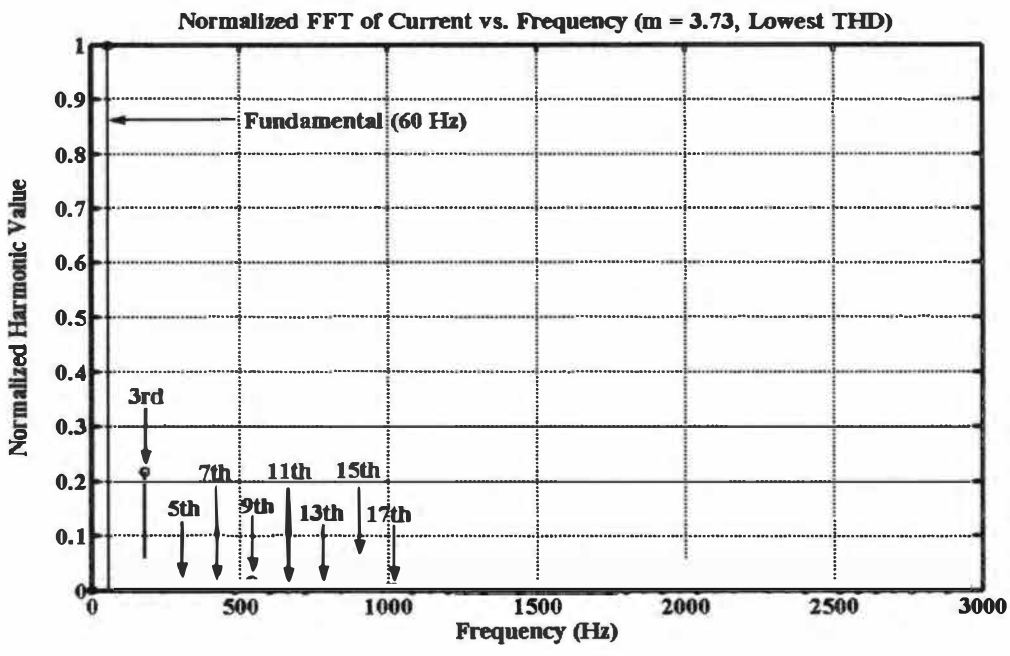

Figure 5.29: Normalized FFT of current for $m_a = 0.622$ (lowest THD).

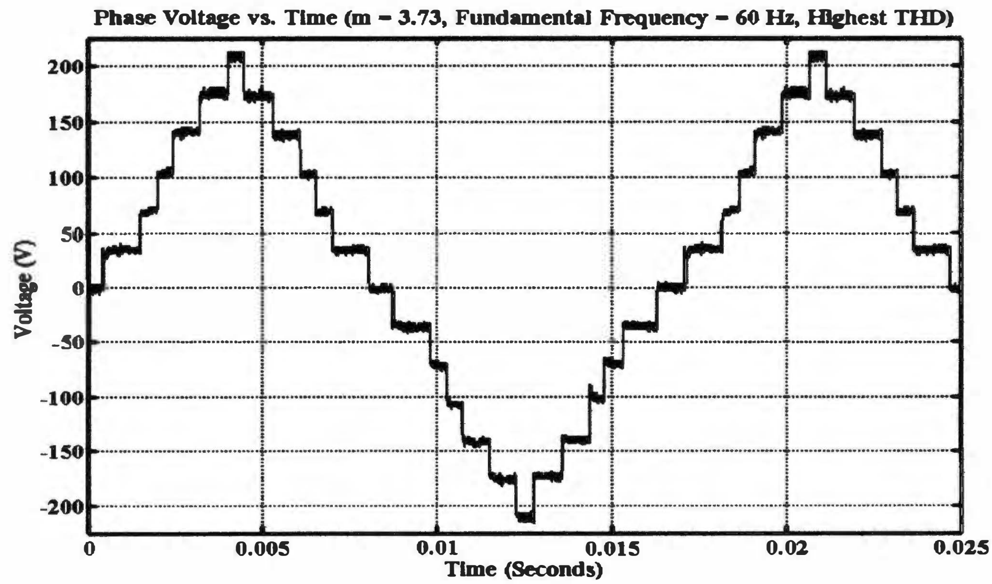

Figure 5.30: Voltage waveform for $m_a = 0.622$ (highest THD).

measured at the output of the multilevel inverter. Figure 5.31 provides the corresponding normalized FFT of the output voltage. Notice from Figure 5.31 that the magnitudes of the 5th, 7th, 11th, 13th, and 17th harmonics are small. Also notice that the magnitudes of the 19th and 23rd order harmonics in Figure 5.31 are larger than the corresponding harmonics in Figure 5.27. Using (5.2), the voltage THD was determined from Figure 5.31 to be 6.77%. Comparing this value to the theoretical value of 7.19%, the resulting percent difference is

$$\%\ Difference = \frac{|7.19 - 6.77|}{7.19} * 100 = 5.93\%. \tag{5.11}$$

As with the experiments discussed in Sections 5.5.1 and 5.5.2, the peak-to-peak value of the output voltage in Figure 5.30 is about 40 V less than the corresponding value under open-circuit conditions.

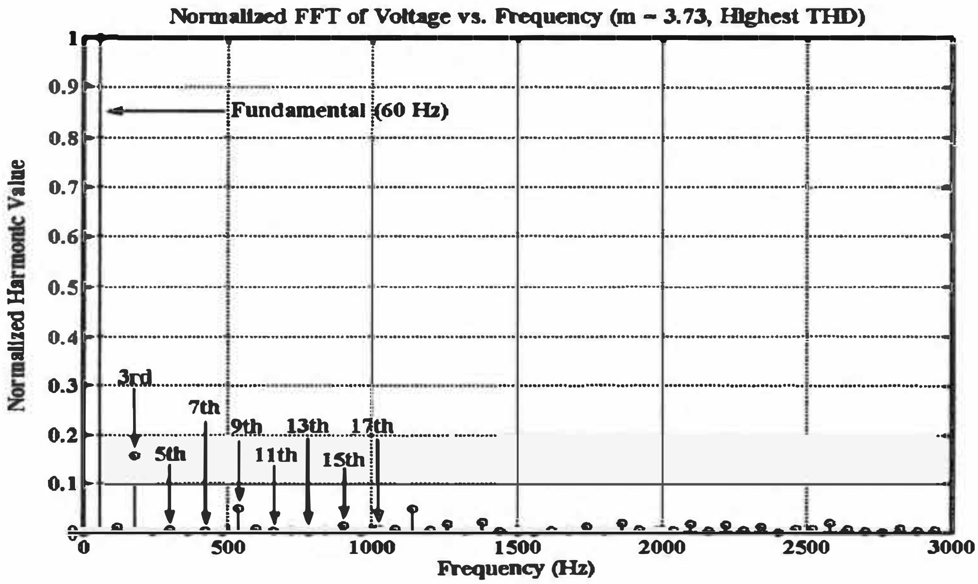

Figure 5.31: Normalized FFT of voltage for $m_a = 0.622$ (highest THD).

Figure 5.32 illustrates the measured output current. Figure 5.33 provides a plot of the corresponding normalized FFT. As with the output voltage, notice from Figure 5.33 that the magnitudes of the 5^{th}, 7^{th}, 11^{th}, 13^{th}, and 17^{th} harmonics are small. One should also notice that the magnitudes of the 19^{th} and 23^{rd} order harmonics in Figure 5.33 are larger than the corresponding harmonics in Figure 5.29. However, notice that the odd, triplen harmonics are not small. As discussed earlier, a single-phase system was considered instead of a balanced three-phase system. A balanced three-phase system does not have any odd, triplen harmonics. Therefore, neglecting the odd, triplen harmonics and using (5.8), the current THD was determined from Figure 5.33 to be 3.37%, which is less than the measured voltage THD. Comparing this value to the theoretical value of 4.04%, the resulting percent difference is

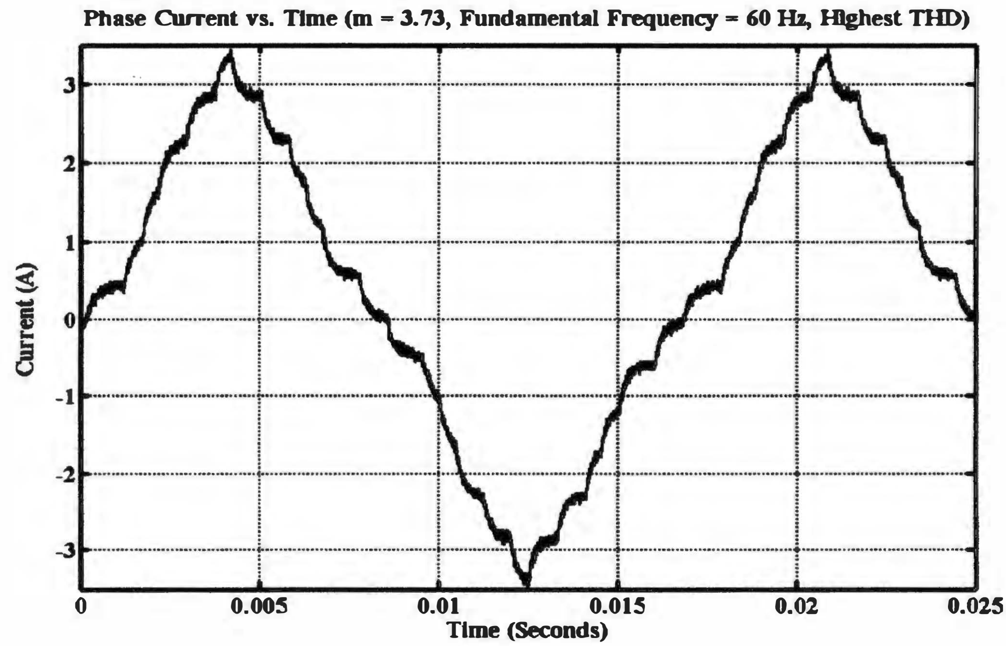

Figure 5.32: Current waveform for $m_a = 0.622$ (highest THD).

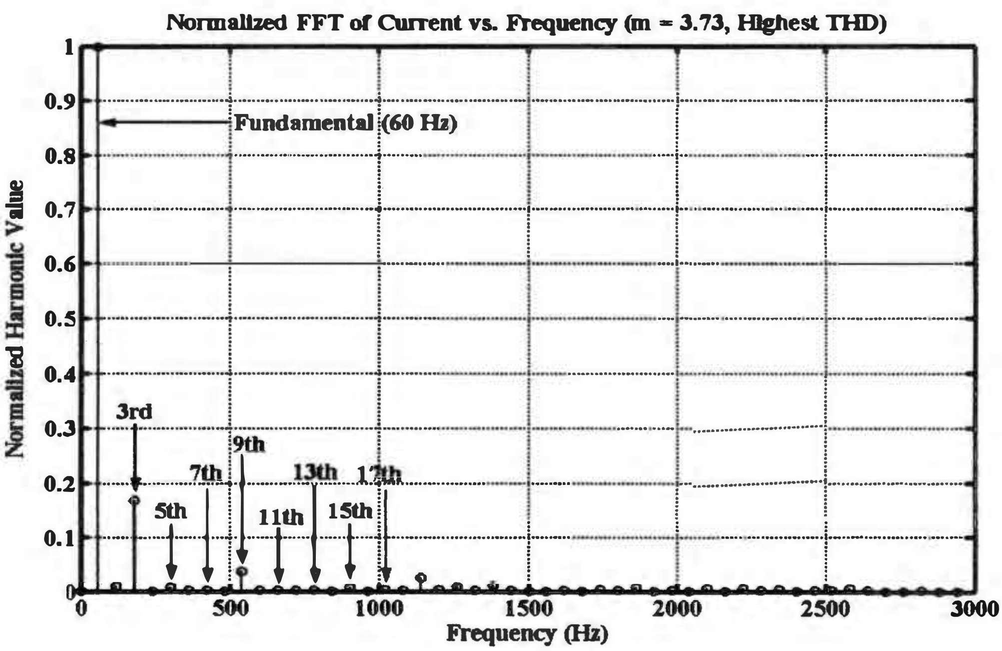

Figure 5.33: Normalized FFT of current for $m_a = 0.622$ (highest THD).

$$\% \; Difference = \frac{|4.04 - 3.37|}{4.04} * 100 = 16.59\%. \tag{5.12}$$

5.6 Chapter Summary

In this chapter, several topics were discussed. A general discussion of each step in the research process was presented first. The research process consisted of three main steps: theoretical calculations, simulations, and experimentation. Theoretical results obtained for a cascaded H-bridges multilevel inverter using five and six equal dc sources were then presented.

Some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing five equal dc sources were then presented. In the experimentation, the switching angles of the inverter were determined such that the 5^{th}, 7^{th}, 11^{th}, and 13^{th} order harmonics were eliminated while at the same time controlling the value of the fundamental. Some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing six equal dc sources were also discussed. In these experiments, the additional switching angle was used to eliminate the 17^{th} order harmonic.

In the next chapter, a brief summary of the thesis will be given. From this summary, some conclusions regarding the research will be made. Finally, some suggestions on possible future research will be given.

6 Conclusions

6.1 Chapter Overview

The purpose of this chapter is to provide some concluding remarks. In Section 6.2, a brief summary of the thesis will be given. From this summary, some conclusions regarding the research will be made in Section 6.3. Section 6.4 will provide suggestions for possible future research in the area of multilevel inverters.

6.2 Thesis Summary

Chapters 1 and 2 served to provide both an introduction to multilevel converters/inverters as well as some background information regarding other research concerning the multilevel inverter. In Chapter 1, a brief summary of the research to be presented in the thesis was first provided. A general definition of the multilevel converter was then given along with some advantages and disadvantages. Also, some applications of the multilevel converter were given. The multilevel fundamental switching scheme was introduced and compared to typical PWM schemes. The benefits of harmonic elimination were also given.

In Chapter 2, the cascaded H-bridges multilevel inverter was first discussed in more detail. Following the discussion on cascaded H-bridges multilevel inverters, some other switching schemes involving harmonic elimination besides multilevel fundamental switching were discussed. More specifically, the Bipolar Programmed PWM, Unipolar Programmed PWM, Virtual Stage PWM, and Unified Approach switching schemes were presented. The idea of using unequal dc sources with multilevel inverters was then discussed, followed by the concept of "duty cycle swapping." Finally, the use of numerical iterative techniques in solving nonlinear equations was briefly discussed.

Chapters 3 and 4 discussed some of the theory behind the research presented in the thesis. In Chapter 3, the idea of the Fourier Series was discussed first. It was then illustrated how Fourier Series theory could be used to derive the transcendental harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations were written in terms of the switching angles of the multilevel inverter.

Resultant theory was then introduced. After the aforementioned transcendental harmonic equations were transformed into polynomial equations, it was shown how Resultant theory could be used to solve these polynomial equations. Finally, an example application of Resultant theory was given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered. In this example, the value of the output voltage fundamental was controlled while the fifth and seventh order harmonics were eliminated.

In Chapter 4, the idea of Symmetric Polynomials was presented. For the special case of when a multilevel inverter is utilizing equal dc sources, it was shown how the idea of Symmetric Polynomials could be utilized to transform the set of polynomial equations derived in Chapter 3 into a new set of polynomials of lower degree. As a result, these equations were easier to solve. An example application of Symmetric Polynomials theory was also given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered.

The idea of Power Sums was then introduced. It was shown that Power Sums theory provides another way of transforming a set of symmetric polynomials into a new set of polynomials of lower degree. As with Symmetric Polynomials, an example application of Power Sums theory was given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered.

The purpose of Chapter 5 was to present some of the collected theoretical and experimental results. A general discussion of each step in the research process was presented first. The research process consisted of three main steps: theoretical calculations, simulations, and experimentation. Theoretical results obtained for a cascaded H-bridges multilevel inverter using five and six equal dc sources were then presented.

Some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing five equal dc sources were then presented. In the experimentation, the switching angles of the inverter were determined such that the 5^{th}, 7^{th}, 11^{th}, and 13^{th} order harmonics were eliminated while at the same time controlling the value of the fundamental. Some of the experimental results obtained from a cascaded H-bridges multilevel inverter utilizing six equal dc sources were also discussed. In these experiments, the additional switching angle was used to eliminate the 17^{th} order harmonic.

6.3 Conclusions From Research

Compared to typical PWM switching schemes, multilevel fundamental switching will lead to lower switching losses. As a result, using the multilevel fundamental switching scheme will lead to increased efficiency. One drawback of using the multilevel fundamental switching scheme is that the created harmonics occur at frequencies around the fundamental. However, appropriate switching angles can be determined such that some of these harmonics are eliminated. As a result, smaller filters can be used to eliminate the remaining harmonics.

This thesis presented a procedure to selectively eliminate certain harmonics in a multilevel inverter utilizing the multilevel fundamental switching scheme. For a given modulation index, this procedure will produce all solutions (if a solution exists) to the corresponding harmonic equations. In comparison, numerical techniques, such as Newton-Raphson, will produce only one solution. Furthermore, unlike numerical techniques, the procedure presented in this thesis does not require an initial guess in order to find a solution.

One drawback of the above procedure should be noted. Increasing the number of switching angles will lead to polynomial harmonic equations of higher degree. As a result, these equations could become too complex to solve using conventional computer algebra software tools.

For some modulation indices, it was shown that multiple solutions exist. As a result, one can utilize the set of switching angles giving the smallest voltage THD. For a cascaded H-bridges multilevel inverter utilizing five equal dc sources, most of the time the switching angles can be selected such that the output voltage THD is less than 7%. For a cascaded H-bridges multilevel inverter utilizing six equal dc sources, most of the time the switching angles can be selected such that the output voltage THD is less than 6%.

However, a large percentage of the modulation indices do not have any solutions. For a cascaded H-bridges multilevel inverter utilizing five equal dc sources, about 61% of the modulation indices do not have any solutions. If the Unified Approach switching scheme is used, only about 17% of the modulation indices do not have any solutions. Therefore, to cover a wider range of modulation indices, the Unified Approach switching scheme is preferred.

6.4 Future Research

One suggestion for future research would be to extend the multilevel fundamental switching scheme to include more than six equal dc sources. For example, perhaps the ideas presented in this thesis could be used to determine the switching angles for a cascaded H-bridges multilevel inverter utilizing seven equal dc sources. In this case, the seventh switching angle would be used to eliminate the 19^{th} order harmonic. However, as mentioned above, increasing the number of switching angles will lead to polynomial equations of higher degree. Even if the ideas of Symmetric Polynomials and Power Sums are used to simplify these equations, they could become too complex to solve.

A second suggestion for future research concerns the Unified Approach switching scheme. The Unified Approach switching scheme makes use of Unipolar Programmed PWM, Virtual Stage PWM, and multilevel fundamental switching. As mentioned above, one advantage of using the Unified Approach switching scheme is that more modulation indices will have solutions to the corresponding harmonic equations. It was also mentioned in Chapters 3 and 4 that Resultant theory, Symmetric Polynomials theory, and Power Sums theory could all be applied to the Unified Approach switching scheme.

Therefore, it would seem natural that the next step would be to try to implement the Unified Approach switching scheme. This switching scheme has already been implemented on a cascaded H-bridges multilevel inverter utilizing three equal dc sources [10]. As with multilevel fundamental switching, it should be noted that increasing the number of switching angles could eventually lead to polynomial equations that are too difficult to solve.

All of the research presented in this thesis assumed that the dc sources used by a multilevel inverter were all equal to one another. A third suggestion for future research concerns the idea of utilizing dc sources that are not equal to one another. Resultant theory has already been used to implement the multilevel fundamental switching scheme on a cascaded H-bridges multilevel inverter using three unequal dc sources [11, 20]. However, using the ideas in this thesis, the polynomial equations corresponding to four unequal dc sources cannot be solved using conventional computer algebra software tools. Furthermore, neither Symmetric Polynomials theory nor Power Sums theory can be used since the derived harmonic polynomial equations are no longer symmetric. In future research, perhaps one can find a way to simplify these polynomial equations such that they can be solved using Resultant theory.

In Chapter 5, when experiments were conducted on a single-phase system, the odd, triplen harmonics were not small. Therefore, for single-phase applications, a fourth suggestion for future research is to use the ideas presented in this thesis to eliminate all odd harmonics (triplen or otherwise). However, one drawback is that some of the previously eliminated harmonics will not be eliminated. As an example of the above ideas, consider a multilevel inverter using three equal dc sources. Instead of eliminating the fifth and seventh order harmonics, one might instead eliminate the third and fifth order harmonics.

6.5 Chapter Summary

The purpose of this chapter was to provide some concluding remarks. A brief summary of the thesis was first provided. Following this summary, some conclusions regarding the research were then made. Some topics concerning future research in the area of multilevel inverters were also discussed.

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List of References

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Appendix

Table 1: Switching angles and THD for a 7-level multilevel inverter.

m_a	Switching Angles (Degrees) and THD (%)							
	Set 1				Set 2			
	θ_1	θ_2	θ_3	THD	θ_1	θ_2	θ_3	THD
0.410	40.26	64.18	88.21	16.95	No Solution			
0.420	40.01	63.24	87.49	16.64	No Solution			
0.430	39.81	62.31	86.72	15.95	No Solution			
0.440	39.65	61.39	85.92	14.83	No Solution			
0.450	39.54	60.47	85.07	13.62	No Solution			
0.460	39.47	59.57	84.17	12.78	No Solution			
0.470	39.43	58.69	83.22	12.36	No Solution			
0.480	39.42	57.84	82.23	12.08	No Solution			
0.490	39.43	57.02	81.19	11.80	No Solution			
0.500	39.43	56.25	80.10	11.66	20.45	56.12	89.68	12.01
0.510	39.39	55.55	78.96	11.89	20.12	55.06	88.95	13.10
0.520	39.30	54.95	77.77	12.43	19.70	53.97	88.27	14.29
0.530	39.11	54.47	76.53	12.72	19.19	52.83	87.62	15.23
0.540	38.79	54.12	75.26	12.52	18.60	51.65	87.03	15.82
0.550	38.33	53.93	73.94	12.23	17.90	50.40	86.50	16.11
0.560	37.70	53.89	72.58	11.97	17.09	49.06	86.05	16.28
0.570	36.89	53.99	71.20	11.67	16.14	47.61	85.69	16.67
0.580	35.91	54.21	69.80	11.17	15.01	45.98	85.46	16.96
0.590	34.78	54.49	68.42	10.54	13.62	44.08	85.43	15.47
0.600	33.50	54.76	67.10	10.28	11.83	41.71	85.72	12.67
0.610	9.22	38.30	86.67	9.66	32.09	54.91	65.92	10.49
0.620	30.57	54.81	64.99	10.95	No Solution			
0.630	28.96	54.33	64.41	10.55	No Solution			
0.640	27.30	53.40	64.20	9.49	No Solution			
0.650	25.62	52.12	64.26	8.92	No Solution			
0.660	23.97	50.61	64.43	8.73	No Solution			
0.670	22.39	48.98	64.59	9.10	No Solution			
0.680	20.92	47.33	64.65	9.65	No Solution			
0.690	19.55	45.70	64.58	10.30	No Solution			
0.700	18.30	44.12	64.36	11.37	No Solution			
0.710	17.16	42.57	64.02	11.76	No Solution			
0.720	16.12	41.06	63.56	10.85	No Solution			
0.730	15.17	39.57	63.00	9.65	No Solution			
0.740	14.30	38.09	62.35	8.78	No Solution			
0.750	13.53	36.62	61.63	7.78	No Solution			
0.760	12.85	35.12	60.85	6.85	No Solution			
0.770	12.28	33.61	60.00	6.56	No Solution			
0.780	11.85	32.04	59.09	7.06	No Solution			
0.790	11.58	30.42	58.13	7.98	No Solution			
0.800	11.50	28.72	57.11	8.01	No Solution			
0.810	11.68	26.89	56.03	7.63	No Solution			
0.820	12.18	24.86	54.89	7.60	No Solution			

Table 2: Switching angles and THD for a 9-level multilevel inverter.

m_a	Switching Angles (Degrees) and THD (%)														
	Set 1					Set 2					Set 3				
	θ_1	θ_2	θ_3	θ_4	THD	θ_1	θ_2	θ_3	θ_4	THD	θ_1	θ_2	θ_3	θ_4	THD
0.420	37.98	53.64	72.84	89.78	11.87	No Solution					No Solution				
0.430	37.88	52.97	71.75	89.12	11.31	No Solution					No Solution				
0.440	37.76	52.36	70.66	88.42	10.64	No Solution					No Solution				
0.450	37.61	51.82	69.56	87.68	9.87	No Solution					No Solution				
0.460	37.39	51.38	68.45	86.90	9.25	No Solution					No Solution				
0.470	37.08	51.05	67.30	86.11	8.91	No Solution					No Solution				
0.480	36.63	50.87	66.12	85.32	8.69	No Solution					No Solution				
0.490	35.97	50.89	64.85	84.56	8.61	No Solution					No Solution				
0.500	34.91	51.21	63.40	83.93	9.33	26.00	51.91	62.75	88.48	11.69	No Solution				
0.510	31.45	52.87	60.84	84.49	12.46	No Solution					No Solution				
0.550	36.06	47.92	61.03	76.29	8.08	15.38	39.81	62.60	89.58	9.45	No Solution				
0.560	14.78	38.74	61.54	89.05	8.34	34.05	48.98	59.49	75.67	8.66	No Solution				
0.570	14.12	37.56	60.48	88.59	7.49	32.57	49.41	58.40	74.77	9.53	No Solution				
0.580	13.39	36.22	59.38	88.21	6.91	31.21	49.48	57.62	73.77	9.81	No Solution				
0.590	12.58	34.62	58.26	88.00	6.74	29.89	49.21	57.12	72.73	9.31	No Solution				
0.600	11.67	32.24	57.08	88.20	7.49	28.56	48.60	56.91	71.67	8.09	No Solution				
0.610	27.22	47.70	56.90	70.63	7.25	No Solution					No Solution				
0.620	25.86	46.58	56.98	69.64	7.23	No Solution					No Solution				
0.630	24.46	45.31	57.05	68.75	7.27	No Solution					No Solution				
0.640	23.05	43.92	56.99	68.00	7.38	No Solution					No Solution				
0.650	21.62	42.44	56.72	67.44	7.84	No Solution					No Solution				
0.660	20.19	40.91	56.18	67.10	8.38	No Solution					No Solution				
0.670	18.76	39.34	55.36	66.96	7.74	No Solution					No Solution				
0.675	18.04	38.56	54.86	66.95	6.99	3.33	30.70	45.17	82.14	8.50	2.33	28.93	44.86	83.30	9.25
0.680	17.34	37.77	54.29	66.97	6.32	4.86	33.75	44.74	79.52	6.37	No Solution				
0.685	5.90	35.24	44.28	77.72	5.45	16.63	37.00	53.66	67.01	5.95	1.60	21.48	40.36	87.26	9.40
0.688	6.44	35.76	44.15	76.86	5.47	16.27	36.61	53.32	67.05	5.90	3.73	19.42	38.78	88.31	7.87
0.690	6.51	16.48	36.60	89.73	5.60	7.01	36.14	44.13	75.99	5.83	15.91	36.23	52.96	67.09	5.90
0.700	14.31	34.82	51.16	67.48	6.36	9.79	35.90	45.79	72.11	8.32	No Solution				
0.730	13.09	29.50	49.58	64.71	7.85	No Solution					No Solution				
0.740	12.02	28.22	47.71	64.67	8.03	No Solution					No Solution				
0.750	11.29	26.87	46.13	64.26	7.29	No Solution					No Solution				
0.760	10.75	25.51	44.62	63.69	6.56	No Solution					No Solution				
0.770	10.37	24.16	43.11	63.01	6.05	No Solution					No Solution				
0.780	10.11	22.84	41.59	62.23	6.02	No Solution					No Solution				
0.790	9.95	21.56	40.03	61.36	6.33	No Solution					No Solution				
0.800	9.84	20.38	38.41	60.42	5.92	No Solution					No Solution				
0.810	9.67	19.38	36.70	59.40	4.93	No Solution					No Solution				
0.820	9.29	18.69	34.88	58.30	4.72	No Solution					No Solution				
0.830	8.47	18.52	32.88	57.12	4.97	No Solution					No Solution				
0.840	7.01	19.03	30.59	55.86	6.08	No Solution					No Solution				
0.850	4.53	20.56	27.62	54.49	6.97	No Solution					No Solution				

Table 3: Switching angles and THD for an 11-level multilevel inverter.

m_a	Switching Angles (Degrees) and THD (%)											
	Set 1						Set 2					
	θ_1	θ_2	θ_3	θ_4	θ_5	THD	θ_1	θ_2	θ_3	θ_4	θ_5	THD
0.450	35.62	47.75	60.08	75.15	89.43	6.86	No Solution					
0.460	35.46	47.39	59.33	73.96	88.73	6.52	No Solution					
0.470	35.35	47.00	58.66	72.76	87.94	6.78	No Solution					
0.480	35.32	46.56	58.08	71.56	87.05	6.97	No Solution					
0.490	35.38	46.06	57.60	70.37	86.05	7.16	No Solution					
0.500	35.53	45.49	57.21	69.20	84.92	7.30	No Solution					
0.510	35.72	44.91	56.85	68.10	83.68	7.16	24.09	44.74	56.99	67.96	89.61	7.42
0.520	35.84	44.41	56.44	67.12	82.33	6.91	23.38	43.68	56.98	66.63	89.01	7.80
0.530	35.73	44.13	55.90	66.32	80.89	6.67	22.40	42.38	57.03	65.33	88.55	8.13
0.540	35.25	44.20	55.14	65.74	79.40	6.11	21.15	40.80	56.97	64.21	88.27	8.46
0.548	34.56	44.52	54.35	65.43	78.18	5.64	4.70	36.03	43.21	78.91	89.22	5.71
0.550	34.35	44.63	54.12	65.37	77.88	5.56	19.59	38.90	56.44	63.54	88.21	8.05
0.560	33.09	45.29	52.93	65.09	76.42	5.61	17.74	36.63	55.13	63.54	88.41	5.99
0.570	15.79	34.05	53.17	63.95	88.82	5.06	31.60	45.90	51.76	64.76	75.10	6.52
0.580	14.02	30.54	51.06	64.21	89.72	6.96	29.97	46.03	51.01	64.23	74.01	7.08
0.590	28.30	45.29	51.03	63.43	73.16	6.33	No Solution					
0.600	26.64	43.93	51.53	62.40	72.50	5.92	No Solution					
0.610	25.05	42.33	52.12	61.25	71.95	5.81	10.69	29.83	45.71	62.89	87.37	8.26
0.614	24.43	41.67	52.32	60.80	71.74	5.76	10.44	31.19	45.16	62.54	86.28	7.30
0.620	23.53	40.67	52.55	60.14	71.42	5.96	10.10	32.35	44.35	61.99	85.07	6.35
0.630	9.63	33.61	43.10	61.01	83.31	5.48	9.22	25.03	42.09	61.15	88.15	6.51
0.636	9.42	34.13	42.47	60.39	82.30	4.95	8.92	23.91	40.90	60.54	88.27	6.02
0.640	9.31	34.38	42.11	59.96	81.64	4.69	8.76	23.13	40.05	60.11	88.38	6.05
0.650	9.12	34.57	41.54	58.87	80.00	4.57	19.55	35.66	51.78	58.07	69.66	5.35
0.656	9.07	34.36	41.43	58.20	79.03	4.69	8.80	19.46	35.53	58.25	89.49	4.81
0.660	18.45	33.99	50.84	57.87	68.97	4.77	9.04	34.09	41.45	57.76	78.39	4.84
0.670	17.53	32.29	49.74	57.81	68.16	4.89	9.02	33.07	41.66	56.66	76.84	5.38
0.680	16.87	30.54	48.56	57.94	67.10	5.57	8.96	31.72	41.85	55.60	75.42	5.90
0.690	16.58	28.67	47.34	58.49	65.55	5.55	8.75	30.22	41.77	54.55	74.24	6.47
0.700	8.24	28.66	41.31	53.44	73.39	6.60	16.73	26.64	46.00	60.69	62.34	6.90
0.710	7.31	27.07	40.51	52.10	72.96	6.07	No Solution					
0.720	5.64	25.36	39.56	50.20	73.13	4.86	No Solution					
0.750	12.79	21.02	35.82	56.60	61.32	4.36	No Solution					
0.760	10.76	20.73	33.93	52.89	63.31	4.45	No Solution					
0.770	9.40	20.16	32.22	50.56	63.68	5.25	No Solution					
0.780	8.36	19.62	30.56	48.64	63.46	5.24	No Solution					
0.790	7.45	19.20	28.88	46.87	62.95	4.76	No Solution					
0.800	6.57	18.94	27.18	45.14	62.24	4.50	No Solution					
0.810	5.74	18.79	25.51	43.36	61.40	4.84	No Solution					
0.820	5.10	18.51	24.04	41.50	60.43	5.22	No Solution					
0.830	5.06	17.52	23.25	39.49	59.35	5.03	No Solution					
0.840	6.37	15.05	23.54	37.23	58.16	4.44	No Solution					

Table 3. Continued.

m_a	Switching Angles (Degrees) and THD (%)					
	Set 3					
	θ_1	θ_2	θ_3	θ_4	θ_5	THD
0.450	No Solution					
0.460	No Solution					
0.470	No Solution					
0.480	No Solution					
0.490	No Solution					
0.500	No Solution					
0.510	No Solution					
0.520	No Solution					
0.530	No Solution					
0.540	No Solution					
0.548	19.92	39.31	56.61	63.62	88.20	8.29
0.550	No Solution					
0.560	No Solution					
0.570	No Solution					
0.580	No Solution					
0.590	No Solution					
0.600	No Solution					
0.610	No Solution					
0.614	10.34	28.36	44.99	62.59	87.78	8.09
0.620	9.87	26.95	43.93	62.08	87.99	7.53
0.630	22.11	39.00	52.68	59.17	70.87	6.79
0.636	21.30	38.00	52.58	58.72	70.52	6.83
0.640	20.78	37.33	52.43	58.48	70.29	6.54
0.650	8.60	21.00	37.55	58.98	88.88	6.06
0.656	18.87	34.66	51.24	57.93	69.26	4.90
0.660	No Solution					
0.670	No Solution					
0.680	No Solution					
0.690	No Solution					
0.700	No Solution					
0.710	No Solution					
0.720	No Solution					
0.750	No Solution					
0.760	No Solution					
0.770	No Solution					
0.780	No Solution					
0.790	No Solution					
0.800	No Solution					
0.810	No Solution					
0.820	No Solution					
0.830	No Solution					
0.840	No Solution					

Table 4: Switching angles and THD for a 13-level multilevel inverter.

m_a	Switching Angles (Degrees) and THD (%)													
	Set 1							Set 2						
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	THD	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	THD
0.460	34.93	44.07	54.43	64.92	77.95	89.58	5.32	No Solution						
0.470	34.95	43.64	53.98	63.99	76.71	88.84	5.13	No Solution						
0.480	34.90	43.31	53.48	63.17	75.44	88.03	5.57	No Solution						
0.490	34.61	43.21	52.82	62.55	74.11	87.20	6.08	No Solution						
0.500	33.46	43.85	51.60	62.36	72.53	86.60	5.94	No Solution						
0.530	37.11	39.58	52.38	58.90	70.43	81.39	6.08	21.37	38.08	52.55	58.76	70.48	89.95	6.80
0.540	9.11	34.75	41.49	59.10	80.40	89.91	4.47	34.42	41.78	50.70	59.22	69.06	80.40	4.58
0.545	33.56	42.35	50.02	59.21	68.49	79.83	4.80	8.78	35.82	40.38	58.48	79.87	89.40	4.84
0.550	19.32	35.35	50.80	58.37	68.27	89.20	4.73	32.77	42.82	49.40	59.14	67.97	79.23	5.07
0.560	18.14	33.67	49.56	58.33	67.26	89.02	4.75	31.25	43.43	48.38	58.81	67.10	78.01	5.78
0.570	16.82	31.16	48.10	58.18	66.43	89.30	5.46	29.73	43.40	47.92	58.23	66.42	76.80	5.80
0.580	28.19	42.57	48.13	57.44	65.88	75.64	4.99	No Solution						
0.590	26.63	41.26	48.64	56.55	65.35	74.61	4.67	No Solution						
0.600	25.06	39.75	49.03	55.76	64.70	73.76	4.51	No Solution						
0.605	4.54	34.37	40.07	48.78	73.12	84.66	4.11	24.28	38.95	49.07	55.50	64.30	73.41	4.62
0.610	11.63	31.61	41.42	54.68	65.24	85.29	4.76	23.50	38.15	48.95	55.37	63.83	73.12	4.83
0.620	11.70	33.89	40.07	54.68	63.75	82.82	3.81	21.94	36.53	48.21	55.59	62.63	72.69	4.63
0.622	11.87	34.31	39.80	54.92	63.30	82.35	3.62	21.68	36.27	48.02	55.69	62.39	72.64	4.48
0.625	12.37	35.32	39.17	55.74	62.10	81.30	3.83	21.15	35.74	47.58	55.97	61.85	72.56	4.20
0.630	20.32	34.99	46.76	56.67	60.78	72.53	4.20	8.59	31.33	40.51	50.13	65.75	82.82	6.53
0.640	8.46	31.23	40.40	49.36	64.59	81.15	5.81	No Solution						
0.650	8.28	19.74	31.41	48.59	63.49	89.55	4.99	8.25	30.43	40.45	48.48	63.53	79.71	5.24
0.660	8.04	29.31	40.42	47.65	62.49	78.41	4.76	17.73	29.49	44.37	55.08	59.89	69.63	5.29
0.670	7.84	28.02	40.00	47.11	61.38	77.24	3.94	16.64	28.03	42.41	55.86	58.26	69.29	6.13
0.680	7.58	26.64	39.27	46.61	60.30	76.22	3.19	15.85	26.54	40.66	56.01	57.23	68.62	6.04
0.690	7.19	25.20	38.27	46.06	59.22	75.38	3.48	15.26	25.07	38.92	54.98	57.41	67.73	5.21
0.695	7.17	14.56	33.03	39.59	58.82	85.23	2.92	7.43	14.16	26.69	39.20	58.83	88.69	3.87
0.700	6.65	14.73	35.65	37.71	58.15	83.79	4.04	6.71	14.62	24.00	37.33	58.15	89.84	4.28
0.705	14.55	23.07	36.25	52.74	58.38	66.06	4.17	6.24	22.93	36.53	44.78	57.57	74.61	4.65
0.710	14.25	22.52	35.32	51.90	58.74	65.47	3.90	5.79	22.11	35.95	44.15	56.99	74.53	4.77
0.715	13.87	22.06	34.38	51.01	59.05	64.95	3.73	5.22	21.20	35.44	43.33	56.35	74.60	4.95
0.720	13.36	21.71	33.42	50.07	59.20	64.58	3.81	4.39	19.96	35.15	42.00	55.63	75.07	5.61
0.730	11.86	21.31	31.47	48.02	58.58	64.75	4.39	No Solution						
0.740	9.58	21.03	29.64	45.73	56.60	66.24	4.21	No Solution						
0.750	4.30	19.24	28.90	42.49	52.20	70.56	4.17	No Solution						
0.760	3.39	10.06	27.89	38.66	44.85	78.26	2.41	No Solution						
0.780	9.08	16.75	25.17	37.47	55.26	62.21	3.16	No Solution						
0.790	7.14	16.50	23.98	35.75	51.95	63.43	4.46	No Solution						
0.800	6.38	15.61	23.25	33.93	49.89	63.23	3.88	No Solution						
0.810	6.25	14.32	22.92	32.04	48.02	62.65	3.86	No Solution						
0.820	6.87	12.51	22.94	30.06	46.17	61.83	4.10	No Solution						
0.850	8.57	8.57	20.39	26.75	39.71	58.48	4.12	No Solution						

Table 4. Continued.

m_a	Switching Angles (Degrees) and THD (%)													
	Set 3							Set 4						
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	THD	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	THD
0.460	No Solution							No Solution						
0.470	No Solution							No Solution						
0.480	No Solution							No Solution						
0.490	No Solution							No Solution						
0.500	No Solution							No Solution						
0.530	No Solution							No Solution						
0.540	20.39	36.79	51.81	58.48	69.36	89.53	5.69	No Solution						
0.545	19.87	36.09	51.33	58.41	68.81	89.35	5.09	No Solution						
0.550	8.44	38.07	38.07	57.86	79.35	88.88	7.02	No Solution						
0.560	No Solution							No Solution						
0.570	No Solution							No Solution						
0.580	No Solution							No Solution						
0.590	No Solution							No Solution						
0.600	No Solution							No Solution						
0.605	4.96	25.63	39.96	49.10	72.70	89.23	4.62	No Solution						
0.610	5.95	28.35	41.20	48.96	70.95	87.15	6.05	No Solution						
0.620	No Solution							No Solution						
0.622	8.33	30.21	40.84	50.33	67.08	84.71	7.19	No Solution						
0.625	8.50	30.87	40.70	50.32	66.49	83.86	6.94	No Solution						
0.630	No Solution							No Solution						
0.640	No Solution							No Solution						
0.650	No Solution							No Solution						
0.660	No Solution							No Solution						
0.670	No Solution							No Solution						
0.680	No Solution							No Solution						
0.690	No Solution							No Solution						
0.695	6.93	24.46	37.71	45.72	58.68	75.06	3.94	15.02	24.36	38.05	54.28	57.69	67.21	4.79
0.700	6.61	23.71	37.12	45.30	58.14	74.79	4.37	14.79	23.69	37.16	53.53	58.02	66.64	4.46
0.705	No Solution							No Solution						
0.710	No Solution							No Solution						
0.715	No Solution							No Solution						
0.720	No Solution							No Solution						
0.730	No Solution							No Solution						
0.740	No Solution							No Solution						
0.750	No Solution							No Solution						
0.760	No Solution							No Solution						
0.780	No Solution							No Solution						
0.790	No Solution							No Solution						
0.800	No Solution							No Solution						
0.810	No Solution							No Solution						
0.820	No Solution							No Solution						
0.850	No Solution							No Solution						

Vita

Keith Jeremy McKenzie was born in Knoxville, TN in 1977. He graduated with a B.S. in Electrical Engineering from the University of Tennessee in 2001. He graduated with a M.S. in Electrical Engineering from the University of Tennessee in 2004. He is a coauthor of 14 publications. His area of interest is Power Quality. In the future, he plans to pursue a Ph.D. in this area.