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## Nuclear Structure Studies in $^{90}\text{Y}$

Ronald E. Goans

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Carrol R. Bingham, Major Professor

We have read this thesis and recommend its acceptance:

Robert J. Lovell, Robert W, Lide

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

December 2, 1969

To the Graduate Council:

I am submitting herewith a thesis written by Ronald E. Goans entitled "Nuclear Structure Studies in <sup>90</sup>Y." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

Carrol R. Bingham  
Major Professor

We have read this thesis  
and recommend its acceptance:

Robert J. Lovell  
Robert W. Lide

Accepted for the Council

Arthur A. Smith  
Vice Chancellor for  
Graduate Studies and Research

NUCLEAR STRUCTURE STUDIES IN  $^{90}\text{Y}$

---

A Thesis  
Presented to  
the Graduate Council of  
The University of Tennessee

---

In Partial Fulfillment  
of the Requirements for the Degree  
Master of Science

---

by  
Ronald E. Goans  
December 1969

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## ABSTRACT

Differential cross sections ( $12.5^\circ$  to  $47.5^\circ$ ) were measured for the  $^{89}\text{Y}(d,p)$  reaction with 33.3 Mev deuterons from the Oak Ridge Isochronous Cyclotron. Proton spectra up to 5 Mev excitation in  $^{90}\text{Y}$  were observed in nuclear emulsions exposed in the focal plane of the broad range spectrograph. The resolution was approximately 25 Kev. The excitation energies agreed within experimental error with (d,p) experiments at 12 Mev and 15 Mev. Comparisons were made with DWBA calculations and  $l$ -transfers were assigned to 25 levels. The  $l$ -transfers and spectroscopic factors are in satisfactory agreement with previous papers for most strong transitions. However, the states at 1.962 and 2.245 Mev are found to be populated by  $l=5$  transitions, and if given spins of  $5^+$  and  $6^+$ , respectively, have spectroscopic factors of 0.28 and 0.26. Several other previously unreported states were observed and  $l$ -transfers were assigned to some of them.

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## CHAPTER I

### INTRODUCTION

The single particle shell structure of nuclei is conveniently studied by means of single-nucleon transfer reactions. In this experiment the shell structure of  $^{90}\text{Y}$  is investigated by means of the  $^{89}\text{Y}(d,p)$  reaction with 33.3 Mev deuterons.

The nucleus  $^{89}\text{Y}$  has a closed shell of 50 neutrons and 39 protons coupled to spin  $1/2$ . On the basis of simple shell model structure, one would expect the transferred neutron to be captured into the single particle levels of the next shell, namely, the  $2d_{5/2}$ ,  $2d_{3/2}$ ,  $3s_{1/2}$ ,  $1g_{7/2}$  and  $1h_{11/2}$  orbits. The proton configurations available are  $2p_{1/2}$ ,  $1g_{9/2}$ ,  $1f_{5/2}$ ,  $1p_{3/2}$  and  $1d_{5/2}$ . The ground state of  $^{89}\text{Y}$  is usually considered to be a  $2p_{1/2}$  proton state outside of a closed proton shell. From this consideration, it is expected that each single particle level of the neutron configuration will give rise to a doublet of levels in  $^{90}\text{Y}$ . Each doublet arises from the quantum-mechanical addition of the proton angular momentum  $J_p = 1/2$  to the neutron angular momentum  $J_n$ . The two levels will have spins of  $J_n+1/2$  and  $J_n-1/2$ . Since the stripping

angular distribution is indicative of the orbital angular momentum  $l_n$  of the captured neutron, it is expected that these doublets will be identifiable. This expectation is borne out by experimental results.

The structure of  $^{90}\text{Y}$  has been investigated using the (d,p) reaction by several groups (Hamburger and Hamburger, 1963, 1965; Watson, et al., 1964; Black, et al., 1966; Mishra, et al., 1968), and also by the (n, $\gamma$ ) reaction (Bartholomew, et al., 1959). The results of this experiment agree rather well with the later work of those mentioned above.

None of these (d,p) experiments, all of which were performed at energies below 15 Mev, isolated states populated by  $h_{11/2}$  transfers. Recent (d,p) and ( $\alpha$ ,  $^3\text{He}$ ) experiments (Bingham and Halbert, 1968a, 1968b, 1969) on Zr isotopes populated  $h_{11/2}$  states at low excitation energies. The primary motivation for the present work was to look for  $h_{11/2}$  transitions to  $^{90}\text{Y}$ . Since large  $l$ -transfers are more highly favored relative to small  $l$ -transfers at higher bombarding energies, it was desirable to use a somewhat larger energy than was previously used. The choice of 33.3 Mev was made because of the availability of optical model parameters in this energy range.

The  $Zr(\alpha, {}^3He)$  studies have shown that  $l = 5$  is the most favored transition for reactions induced with 65 Mev alpha particles. As a further experimental test of spins and spectroscopic factors assigned here,  ${}^{89}Y(\alpha, {}^3He)$  spectra were measured at  $15^\circ$  and  $20^\circ$  laboratory. An alpha particle bombarding energy of 65 Mev was used.

## CHAPTER II

### DISTORTED WAVE ANALYSIS

#### Applicability of Distorted Wave Theory to Direct Reactions

The term, direct reaction, has been defined in many ways. It has been said (Austern, 1963) that direct reactions involve only a few internal degrees of freedom of the colliding systems. These internal degrees of freedom can be described by either single particle or collective modes of motion. Another definition (Satchler, 1965) describes a direct process as one having a good overlap between the incident and exit channel wave functions. Elastic scattering is a good example of this since the entrance and exit channels are identical (except for possible spin-flip).

The (d,p) reaction is a one-step process. This was first recognized (Oppenheimer and Phillips, 1935) in analyzing low-energy (d,p) reactions. It was observed that (d,p) reactions were more frequent than (d,n) reactions. This is completely opposite from what would be expected through the formation of a compound nucleus. Due to the absence of the Coulomb barrier, one would have expected many more (d,n) reactions than (d,p) reactions.

Early investigation (Butler, 1950, 1951) on the (d,p) reaction showed that the first peak of the angular distribution

could be fitted by the square of the spherical Bessel function of order  $\ell$ , where  $\ell$  is the angular momentum of the state containing the captured neutron. This discovery allowed many  $\ell$  transfers to be identified in a qualitative fashion. In this theory, the reaction mechanism was treated using a plane wave approximation (Born approximation) which did not accurately describe the process.

Current reaction theories are based upon the distorted-wave Born approximation (DWBA). This type of analysis gives a more accurate description of direct reactions such as stripping, pickup, and inelastic scattering. In the DWBA approximation, the relative motion of the colliding pair before and after the scattering event is described by distorted waves. These distorted waves are calculated by a partial wave solution of the Schrodinger equation using an optical model potential. The optical model potential represents the average effect of all nucleons within the nucleus and adequately describes the elastic scattering process. In the DWBA theory, one assumes a weak coupling representation in which elastic scattering is the most important process that occurs. Inelastic or rearrangement events are treated as perturbations caused by a residual interaction. The transition is then one between elastic scattering states.

### The Born Approximation for Potential Scattering

The problem of a particle scattering from a short-range, local central potential has been treated in many texts (see, for example, Merzbacher, 1961; Eisele, 1969;). The wave function of such a particle satisfies the Schrodinger equation:

$$(\nabla^2 + k^2) \psi(\vec{r}) = \frac{2\mu}{\hbar^2} V(\vec{r}) \psi(\vec{r}) \quad (1)$$

A particular solution with outgoing boundary conditions is

$$\psi^+(\vec{r}) \underset{r \rightarrow \infty}{\sim} e^{i\vec{k} \cdot \vec{r}} + \frac{f(\theta) e^{i\vec{k} \cdot \vec{r}}}{r} \quad (2)$$

where  $k^2 = \frac{2\mu E}{\hbar^2}$ . In the above expression,  $\mu$  is the reduced mass of the system,  $k$  is the wave number, and  $E$  is the total energy in the center of mass system.  $\psi^+(\vec{r})$  refers to the asymptotic form of the wave function which satisfies the usual boundary conditions, namely a sum of the incident plane wave and an outgoing spherical wave. The ordinary Green's function solution of (1) yields the following integral equation for the scattering process:

$$\psi^+(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} - \frac{\mu}{2\pi\hbar^2} \int d^3r' \frac{e^{ik|r-r'|}}{|r-r'|} V(r') \psi^+(r') \quad (3)$$

or

$$\psi^+(\vec{r}) = \phi - G_0^+ V \psi^+ \quad (4)$$

where  $V'(\vec{r}) = \frac{\mu}{2\pi\hbar^2} V(\vec{r})$  for convenience.  $\phi$  represents the plane wave and  $G_0^+$  symbolizes the integral operator in equation (3). Equation (4) is the ordinary scattering equation written in modern symbolic form (McCarthy, 1968). From equation (3), it is easy to show that the scattering amplitude is

$$f(\theta) = \int d^3r' \ e^{-i\vec{k}' \cdot \vec{r}'} V'(r') \psi^+(r') \equiv \langle k' | V' | \psi^+ \rangle \quad (5)$$

The differential cross section is then given by:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad (6)$$

#### The Distorted-Wave Born Approximation

For several years, direct reaction theories were based upon plane wave interactions in the Born approximation. This older method of calculation assumed that elastic scattering was also small and furthermore, one usually excluded the nuclear interior because of strong absorption. These assumptions are basically inconsistent. The DWBA calculation automatically takes care of strong absorptions and also predicts polarization effects which are not allowed in the plane wave theories. The DWBA theory has been presented in several places (Bassel, Drisko, and Satchler, 1962, 1966; Satchler, 1964, 1965, 1966). The following presentation will be a brief review of the theory

applied to the direct reaction  $A(a,b)B$ . Furthermore, the review will be restricted to only those aspects pertinent to single-nucleon transfer reactions utilizing the shell model.

According to equation (5) in the previous section, the stripping amplitude for an outgoing particle with wave vector  $\vec{k}_b$  is:

$$f = \langle \chi_b^{(-)}(\vec{k}_b, \vec{r}_b) \psi_B | V_{bx} + V_b | \psi^{(+)} \rangle \quad (7)$$

where  $\psi^{(+)}$  is the solution with ingoing boundary conditions of the complete many-body problem and  $\chi_b^{(-)}$  is the solution with outgoing boundary conditions of the Schroedinger equation for particle  $b$  according to the asymptotic potential.  $\psi_B$  is the wave function of the final nucleus  $B$ .  $x$  represents the transferred nucleon ( $a=b+x$ ) and can always be identified as a neutron for the purposes of this work.  $V_{bx}$  represents the transferred nucleon-outgoing particle potential and  $V_b$  represents the outgoing particle-residual core potential. In practice, the interaction potential is usually taken to be  $V_{bx}$  for stripping reactions. The use of the appropriate optical model potential for the exit channel effectively compensates for neglect of  $V_b$  in the transition amplitude.



It is convenient to approximate  $\psi^{(+)}$  by

$$\psi^{(+)} \approx \phi_a(\mathbf{r}_{bX}) \chi_a^{(+)}(\vec{k}_a, \vec{r}_a) \psi_A(\xi) \quad (8)$$

where  $\phi_a(\mathbf{r}_{bX})$  is the incident particle internal wave function,  $\chi_a^{(+)}(\vec{k}_a, \vec{r}_a)$  is an elastic distorted wave and  $\psi_A(\xi)$  is the target wave function.  $\xi$  represents the appropriate internal coordinates for the nucleus A. Substitution into equation (7) yields

$$f_{DWBA} = J \int d^3\vec{r}_a d^3\vec{r}_b \chi_b^{(-)*}(\vec{k}_b, \vec{r}_b) \langle B, b | v_{bX} | A, a \rangle \chi_a^+(\vec{k}_a, \vec{r}_a) \quad (9)$$

where  $\vec{r}_a$  is the displacement vector of a from A and  $\vec{r}_b$  the displacement vector of b from B. J is the Jacobian of the transformation to these relative coordinates. The functions  $\chi_b$  and  $\chi_a$  are elastic distorted waves calculated from the Schroedinger equation in an optical model approximation

$$\left\{ \nabla^2 + k^2 - \frac{2\mu}{\hbar^2} U(r) \right\} \chi(\vec{k}, \vec{r}) = 0 \quad (10)$$

where  $\mu$  is the reduced mass of the colliding system and  $U(r)$  is the optical-model potential. This potential also includes the Coulomb potential and a spin-orbit term. From a practical standpoint, the distorted waves are calculated by means of the

usual partial-wave analysis. The quantity  $\langle B,b|V_{bx}|A,a\rangle$  plays the role of an effective interaction which is, in general, nonlocal. The stripping amplitude  $f_{DWBA}$  is then the matrix element of this effective interaction taken between the elastic scattering states  $\chi_a$  and  $\chi_b$ .

The differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{2J_B+1}{(2J_A+1)(2S_a+1)} |f_{DWBA}|^2 \quad (11)$$

where  $k_b$  and  $k_a$  are the wave numbers of particles a and b and  $J_i$  is the total angular momentum of particle i.  $f_{DWBA}$  contains the overlap matrix of the target and residual nuclei. The other quantities entering the calculation are assumed to be known. Hence by comparison of these calculations with experimental data one can deduce the overlap of the target and residual nuclei. Various computer codes have been developed to calculate the differential cross section in the DWBA approximation.

The application of DWBA analysis to stripping reactions assumes that  $V_{bx}$  is central, so that b and x are in an s state of relative motion. One also normally assumes that the wave function for the projectile is factorable. For the (d,p) reaction, this approximation is equivalent to neglecting the small amount of D state strength in the deuteron wave function.

The integral in the expression for  $f_{\text{DWBA}}$  can be simplified by using the zero-range approximation. This approximation is obtained by saying that

$$D(\mathbf{r}_{\text{bx}}) = V_{\text{bx}}(\mathbf{r}_{\text{bx}}) \phi_{\text{a}}(\mathbf{r}_{\text{bx}}) = D_0 \delta(\mathbf{r}_{\text{bx}}) \quad (12)$$

where  $V_{\text{bx}}$  is the interaction potential and  $\phi_{\text{a}}(\mathbf{r}_{\text{bx}})$  is the space part of the incident projectile wave function. A value of  $D_0$  of about  $(1.5)^{1/2} \times 10^2 \text{ Mev } f_{\text{m}}^{1/2}$  is obtained when the Hulthen wave function for the deuteron is used. The zero-range assumption has the physical meaning that particle b is emitted at the same point at which particle a was absorbed. It is also justifiable on the grounds that both  $V_{\text{bx}}$  and the internal wave function have short ranges. In this approximation, it can be shown that

$$\delta(\mathbf{r}_{\text{bx}}) = \frac{1}{J} \delta\left(\mathbf{r}_{\text{b}} - \frac{A}{B} \mathbf{r}_{\text{a}}\right) \quad (13)$$

where  $J$  is the Jacobian of the transformation to relative variables and  $A, B$  are the masses of target and residual nuclei respectively. This equation reduces the six-dimensional integral in equation (9) to a single three-dimensional one which considerably facilitates the computation of the transition amplitude. Further refinements consider the finite range of the interaction and the inclusion of nonlocality effects. These will be considered in the next section.

### Finite Range Corrections

The finite range corrections employed in this work have been calculated in the local energy approximation (LEA) (Buttle and Goldfarb, 1964; Bencze and Zimanyi, 1964; Perey and Saxon, 1964; and Dickens, et al., 1965). This approximation takes into account that the interaction does take place over a finite distance. The zero-range form factor is multiplied by a correction which essentially reduces the contribution from the nuclear interior. For the reaction  $A(a,b)B$ , the correction factor is given by

$$\Lambda(\vec{r}) = 1 - [V_a(\vec{r}) - V_x(\vec{r}) - V_b(\frac{\vec{r}A}{B}) - \epsilon] \frac{mR^2bx}{2a\hbar^2} \quad (14)$$

where  $R$  is the range of the Gaussian interaction,  $V_i$  is the appropriate optical model potential of particle  $i$ ,  $\epsilon$  is the binding energy of  $b$  and  $x$  to form  $a$ , and  $m$  is the atomic mass unit.

### Nonlocality Corrections

The theory of nonlocality effects in the entrance and exit channels was first investigated in the context of neutron scattering (Perey and Buck, 1962; Perey and Saxon, 1964). A nonlocal potential operating on a wave function has

the form

$$V\psi(r) = \int d^3r' V(r,r')\psi(r') \quad (15)$$

The function  $V(r,r')$  is symmetric; this follows from the fact that the potential matrix  $V_{ij}$  is Hermitian. The Schroedinger differential equation becomes an integro-differential equation when nonlocality effects are taken into account.

$$\left(\frac{\hbar^2}{2\mu} \nabla^2 + E\right) \psi(\vec{r}) = \int d^3r' V(r,r')\psi(r') \quad (16)$$

To facilitate computation, the nonlocal potential is chosen to have the form

$$V(r,r') = U\left(\frac{|\mathbf{r} + \mathbf{r}'|}{2}\right) H(|\mathbf{r} - \mathbf{r}'|) \quad (17)$$

where  $U$  is the ordinary optical potential.  $H$  is chosen to have a Gaussian form with nonlocality parameter  $\beta$ .

$$H(|\mathbf{r} - \mathbf{r}'|) = \exp\left[-\frac{|\mathbf{r} - \mathbf{r}'|^2}{\beta^2}\right] / \pi^{3/2} \beta^3 \quad (18)$$

It can be shown (Perey and Saruis, 1965) that the nonlocal wave function is related to the local wave function by

$$\psi_n(\vec{r}) = \frac{\psi_L(\vec{r})}{1 - \frac{\mu\beta^2 V(\vec{r})}{2\hbar^2}} \quad (19)$$

where  $V(\vec{r})$  is the equivalent local potential. A local potential

can always be found which is equivalent to the nonlocal potential in the sense that the scattering cross sections can be reproduced. However, these local potentials are seen to vary with energy. This means that potentials having different forms of nonlocality can be found so that they give the same scattering data; these are equivalent on the energy shell but they are unequivalent off the energy shell. Hence, they have different wave functions.

### Optical Model Analysis

A detailed description of the optical model has been presented in many texts (see, for example, Hodgson, 1963; McCarthy, 1968; Nigam, 1967). The optical model is phenomenological in origin. Basically, it replaces the many-body problem of two interacting nuclei with a simpler two-body problem using a complex potential to describe the interaction. The real part accounts for refractive effects and the imaginary part accounts for absorption of the incident nucleons. The optical potential usually includes a spin-orbit term which describes polarization effects due to the coupling of spin and orbital angular momenta. The form of the potential is

$$U(r) = U_C - \left[ V_0 - \left( \frac{\hbar}{m_\pi c} \right)^2 V_S (\vec{L} \cdot \vec{S}) \frac{1}{r} \frac{d}{dx} \right] (1 + e^x)^{-1} - i \left[ W_0 - 4W_D \frac{d}{dx} \right] (1 + e^x)^{-1} \quad (20)$$

where  $U_c(r)$  is the Coulomb potential due to a uniformly charged sphere of radius  $R_c = r_0 A^{1/3}$

$$U_c = \frac{zz'e^2}{r}, \quad r \geq R_c \quad (21)$$

$$U_c = \frac{zz'e^2}{2R_c} \left( 3 - \frac{r^2}{R_c^2} \right), \quad r \leq R_c \quad (22)$$

The spin-orbit term is of the Thomas type where  $\vec{L}$  and  $\vec{S}$  refer to the orbital and spin angular momentum, respectively, of the incident particle.

The radial function  $(1 + e^x)^{-1}$ , where  $x = (r - r_0 A^{1/3})/a$ , is the Woods-Saxon form factor. This form factor represents a diffuse well of mean radius equal to  $r_0 A^{1/3}$  and diffuseness equal to  $a$ . The form factor is down to one-half of its maximum value at  $r = r_0 A^{1/3}$  and decreases from 0.9 to 0.1 of its maximum value in a radial distance equal to  $4.4a$ . A derivative surface absorption term is also included in the imaginary part of the potential. The form of the optical potential given in equation 20 includes the possibility of using different parameters for the real and imaginary form factors.

One normally obtains optical model parameters by fitting observed elastic scattering data with a computer search routine. Such a code is Hunter written by R. M. Drisko. This

code varies selected parameters in the optical model to obtain a fit to the experimental angular distribution. The criterion used in obtaining the fit is the minimization of the quantity

$$\chi^2 = \frac{1}{N} \sum_i \left[ \frac{\sigma_{th}(i) - \sigma_{exp}(i)}{\Delta\sigma_{exp}(i)} \right]^2 \quad (23)$$

where  $N$  is the number of experimental data points,  $\sigma_{exp}(i)$  and  $\sigma_{th}(i)$  are the experimental and theoretical cross sections at center of mass angle  $\theta_i$ , and  $\Delta\sigma_{exp}$  is the error assigned to the experimental value. Specific values of the optical model parameters for the incident and exit channels will be given in the next section.

#### Entrance and Exit Channel Parameters

In order to perform DWBA calculations for the  $^{89}\text{Y}(d,p)$  reaction, it was necessary to have a knowledge of the distorting potentials in the entrance and exit channels. These potentials were taken as optical model potentials which reproduced the elastic scattering in both channels.

The entrance channel parameters were taken from previous work describing deuteron elastic scattering at 34.4 Mev (Newman, et al., 1967). A suitable set of average parameters was found to be the following:

$$V = 90.2 + 0.89Z/A^{1/3} \quad (\text{Mev}) \quad (24)$$

$$r_0 = 0.968 + 0.029A^{1/3} \quad (\text{fm}) \quad (25)$$



$$a = 0.814 \quad (\text{fm}) \quad (26)$$

$$W_0 = 4.33 + 2.20A^{1/3} \quad (\text{Mev}) \quad (27)$$

$$r_0' = 1.09 + 0.80A^{-1/3} \quad (\text{fm}) \quad (28)$$

$$a' = 0.554 + 0.059A^{1/3} \quad (\text{fm}) \quad (29)$$

$$V_S = 7.0 \quad (\text{Mev}) \quad (30)$$

$$W_0 = 0 \quad (31)$$

These parameters were used to describe the deuteron elastic scattering in the  $^{89}\text{Y}(d,p)$  reaction.

The exit channel parameters were taken from previous work describing the elastic scattering of 30 Mev and 40 Mev protons (Fricke, et al., 1966, 1967). The real potential  $V_0$  can be obtained from the empirical formula

$$V_0 = 49.9 - 0.22E + 0.4Z/A^{1/3} + 26.4(N-Z)/A \quad (32)$$

For  $^{90}\text{Y}$ , this reduces to

$$V_0 = 56.6 - 0.22E \quad (33)$$

The values for the volume and surface absorptions  $W_0$  and  $W_D$ , respectively, were scaled for energy dependence and were based on  $^{90}\text{Zr}$ . These formulae are:

$$W_0 = 10.93 - 0.156E \quad (\text{Mev}) \quad (34)$$

$$W_D = -1.62 + 0.127E \quad (\text{Mev}) \quad (35)$$

where E is the outgoing proton energy. A list of entrance and exit channel parameters is given in Table I.

TABLE I  
ENTRANCE AND EXIT CHANNEL  
SCATTERING PARAMETERS

Entrance Channel $E_d = 33.3$ Mev			Exit Channel $E_p = 37.74$ Mev		
V	(Mev)	98.0			48.3
$W_0$	(Mev)	0			5.05
$W_D$	(Mev)	14.1			3.17
$r_0$	(fm)	1.098			1.16
a	(fm)	0.814			0.75
$V_S$	(Mev)	7.0			6.04
$r'_0$	(fm)	1.269			1.37
$a'_0$	(fm)	0.817			0.63
$r_S$	(fm)	1.098			1.064
$a_S$	(fm)	0.814			0.738
$r_C$	(fm)	1.25			1.25

### Applications to Shell Model Analysis

In the application of DWBA theory to shell model analysis, it is necessary to know the bound state wave function of the captured neutron in order to calculate the transition amplitude  $f_{\text{DWBA}}$ . This wave function is calculated by solving the Schrodinger equation with a Wood-Saxon potential. Spin-orbit effects are included by using a potential of the Thomas type with  $\lambda = 25$ . The form of the potential is given below

$$U(r) = \frac{V_0}{1+e^x} + \frac{\lambda \hbar^2}{4m_0^2 c^2} \frac{V_s}{r} \frac{d(1+e^{x'})^{-1}}{dx'} \quad (36)$$

where  $x = \frac{r-r_0 A^{1/3}}{a}$  and  $m_0$  is the electronic mass. The bound state parameters are  $r_0 = 1.24$  fm,  $a = 0.65$  fm,  $r' = 1.14$  fm and  $a' = 0.65$  fm. The computer code used to calculate the wave function was developed by R. M. Drisko. Two options are allowed in the solution of the wave equation. The first choice allows one to specify  $\lambda$  and the neutron binding energy. The code then chooses a real potential  $V_0$  which reproduces the neutron binding energy. The other choice allows one to specify  $V_0$  specifically in the input.

On the basis of a simple shell model theory, it is expected that the transferred nucleon in a single-nucleon stripping reaction will go into one of the available single

particle levels. However, the single particle strength can often be split up among various excited states in the residual nucleus. The spectroscopic factor  $S_{\ell j}$  gives the content of the single particle strength in the nuclear wave function and for a (d,p) reaction is given by

$$S_{\ell j} = \frac{2J_i+1}{1.5(2J_f+1)} \frac{(d\sigma/d\Omega)_{\text{exp}}}{(d\sigma/d\Omega)_{\text{JULIE}}} \quad (37)$$

In the above equation,  $\ell, j$  denote a specific shell model state,  $(\frac{d\sigma}{d\Omega})_{\text{exp}}$  and  $(\frac{d\sigma}{d\Omega})_{\text{JULIE}}$  represent the experimental and DWBA differential cross sections respectively, and  $J_i$  and  $J_f$  the spins of the target and residual nuclei. For the ground state of  $^{89}\text{Y}$ ,  $J_i^\pi$  is  $1/2^-$ .

## CHAPTER III

### EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental data were taken with a 33.3 Mev deuteron beam at the Oak Ridge Isochronous Cyclotron (ORIC). The ORIC is a fixed frequency cyclotron with an azimuthally varying magnetic field. The cyclotron and the associated optics system have been described elsewhere (Jones, et al., 1962; Oak Ridge National Laboratory, 1963). The arrangement of the experimental equipment is shown in Figure 1. The extracted beam from the cyclotron was focused on the 4-jaw collimator at the entrance to the  $153^\circ$  analyzing magnet by two quadrupole magnets Q-1 and Q-2. The  $153^\circ$  magnet acts as an energy analyzer for the extracted deuteron beam and directs this beam into an experiment room. The  $153^\circ$  analyzing magnet is double focusing ( $n=1/2$ ) and has a 72-inch radius of curvature. The energy resolution of the incident beam is provided by slits placed at the focal points of this magnet. The beam is then brought to a focus at the center of the scattering chamber by two other quadrupole magnets Q-3 and Q-5.

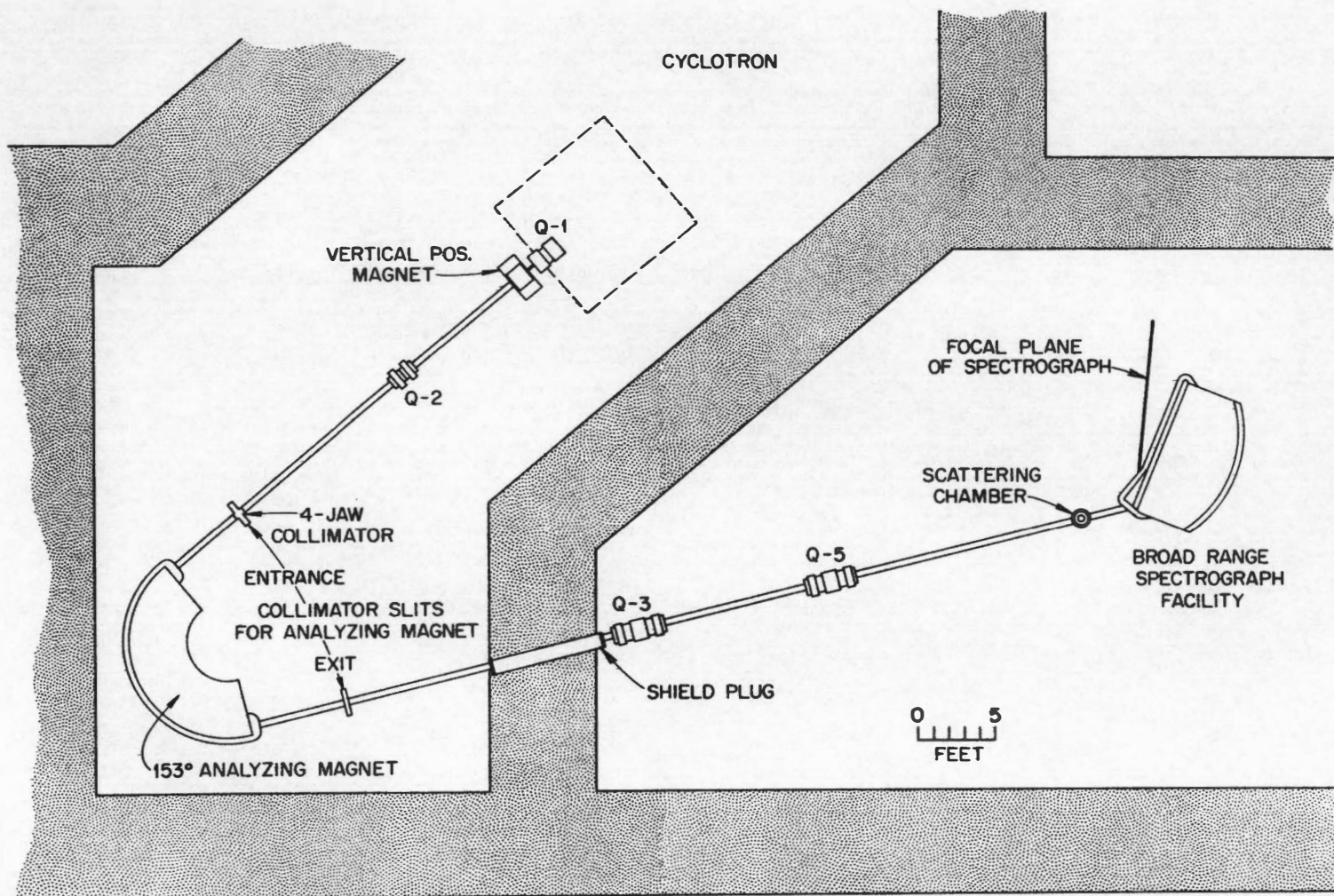


Figure 1. Layout of the ORIC cyclotron and magnetic spectrograph.

The outgoing protons in the  $^{89}\text{Y}(d,p)^{90}\text{Y}$  reaction were detected in nuclear emulsions placed in the focal plane of the broad-range spectrograph. The spectrograph has also been described in previous papers (Borggreen, Elbek, and Nielsen, 1963; Ball, 1966). The spectrograph at ORIC consists of a homogeneous field sector magnet capable of analyzing most reaction products. It was designed to cover a range of radii from 30 to 63 inches and to operate with fields as large as 13.5 kilogauss. The outgoing reaction products enter the spectrograph at an angle of  $35^\circ$  with respect to the normal to the magnetic field and exit at an angle of  $-36.3^\circ$ . The total deflection angle of  $107.4^\circ$  degrees is independent of the radius of curvature of the particle in the magnetic field. The fringing field produces a focusing effect in the focal plane of the spectrograph.

The momentum  $p$  of a particle in a magnetic field is given by

$$p = BqR \quad (38)$$

where  $B$  is the magnetic induction,  $q$  is the charge on the particle, and  $R$  is the radius of curvature. With a given

spectrograph field B, it is possible to calculate the excitation energy of a certain level in  $^{90}\text{Y}$  by observing the position of that peak in the proton spectrum. Each peak in the spectrum corresponds to observation of the proton in the reaction  $^{89}\text{Y}(d,p)^{90}\text{Y}^*$ . By knowing the peak position, one also knows the radius of curvature and by standard kinematic analysis, it is possible to determine the excitation energy.

In order to take full advantage of the resolution capabilities of the spectrograph, it is necessary to use thin targets. The incident projectile and outgoing reaction products have different rates of energy loss in the target and from this consideration, one would expect a small energy spread in the spectrum of the outgoing particles. This spread is due to the fact that the reaction can take place at various depths in the target. In addition, another contribution to the energy spread comes from the statistical process of straggling.

The target was a thin self-supporting  $^{89}\text{Y}$  foil prepared in the Isotopes Division at ORNL. The thickness of the target was measured by observing the energy loss of the



5.470 Mev  $\alpha$  particle from  $^{241}\text{Am}$ . From the energy loss, the thickness was determined by interpolation in the tables of Whaling (1958). The target thickness was found to be  $0.41 \pm 0.04 \text{ mg/cm}^2$ . The uncertainty in target thickness is the main uncertainty in the determination of absolute cross sections since the method is only accurate to within 10 percent. The target was exposed to beam currents ranging from 40 NA to 80 NA. The total integrated charge was measured with a Faraday cup and varied from 147  $\mu$  coul at  $12.5^\circ$  laboratory to 315  $\mu$  coul at  $42.5^\circ$ .

The nuclear emulsions employed in the detection of the outgoing protons were 50 micron thick, 2 inch x 10 inch Kodak type NTB plates. These emulsions could be loaded directly into the spectrograph in subdued light consisting of the emergency light in the experimental room with a red lens and portable safelights with yellow filters. It has been observed by other experimenters that these emulsions can stand a considerable amount of red or yellow light without harmful effects.

After exposure, the emulsions were developed for 30 minutes in a solution consisting of one part D-19 Kodak developer and six parts of tap water. The solution was held

at 68°F throughout development and then the plates were rinsed in running tap water for five to ten minutes. Next, the plates were fixed using Kodak fixer for approximately one hour and following this, they were rinsed in running tap water for about an hour and allowed to dry. The plates could usually be scanned after three to four hours.

The data were obtained by scanning the plates with a standard Bausch and Lomb microscope with a modified mechanical stage. The position of a peak on the plate was found by using a dial indicator on the microscope which reads from 0 to 7 cm in 0.01 mm intervals. Such a small increment facilitates accurate positioning of the plate.

The experimental differential cross section  $\frac{d\sigma}{d\Omega}$  was calculated using the following expression:

$$\frac{d\sigma}{d\Omega} = \frac{n}{NQ\Delta\Omega} \quad (39)$$

where  $n$  is the integrated number of counts under the peak in the proton spectrum corresponding to the desired level in  $^{90}\text{Y}^*$ ,  $N$  is the number of incident deuterons,  $Q$  is the number of target nuclei per  $\text{cm}^2$ , and  $\Delta\Omega$  is the solid angle subtended by the emulsion plate. The kinematic analysis and differential cross section calculation was performed with the aid of the

computer program MAGDAC IV. This program was written by J. B. Ball and R. L. Auble at ORNL to facilitate data reduction from the ORIC broad range spectrograph.

## CHAPTER IV

### EXPERIMENTAL RESULTS AND INTERPRETATION

Proton spectra up to 5 Mev excitation in  $^{90}\text{Y}$  were observed in this experiment and it was possible to obtain an experimental angular distribution for many levels. The angular distributions were compared with DWBA calculations made with the code JULIE in order to identify the  $\ell$ -transfer. This code is able to calculate differential cross sections in various levels of approximation making it possible to include finite range and nonlocality effects in addition to performing the zero-range approximation. Figure 2 shows the various levels of approximation used in calculating  $\frac{d\sigma}{d\Omega}$  JULIE. FRNL represents the calculation with finite range and nonlocality, LZR represents the ordinary local zero-range calculation, and NLZR represents the zero-range case with nonlocality included. The range parameter R was taken to be 1.54 fm. The nonlocality parameter  $\beta$  is 0.85 fm for nucleons and 0.54 fm for deuterons. Nonlocality was not included in the bound state due to lack of theoretical justification. The various levels of approximation effectively change only the magnitude of the spectroscopic factor  $S_{\ell j}$ .

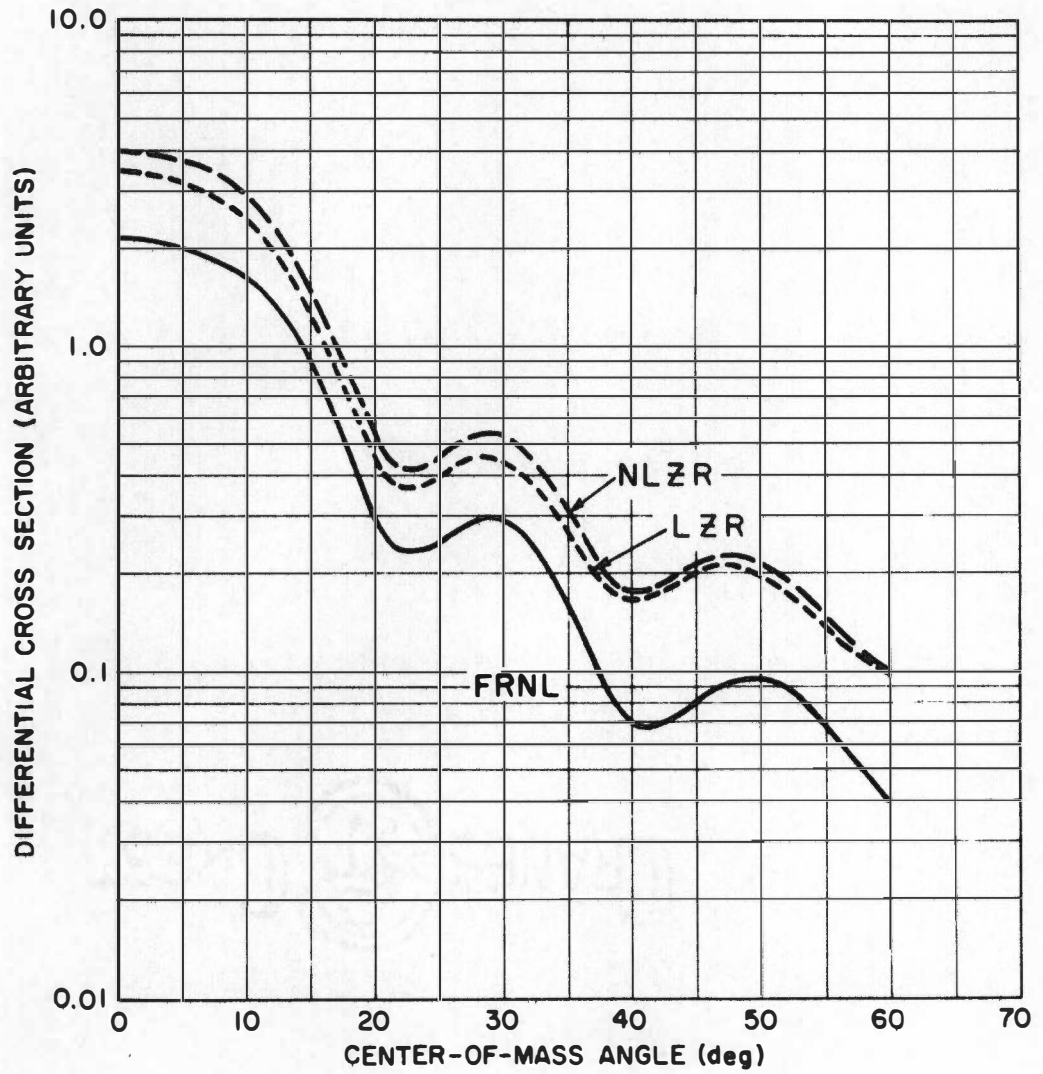


Figure 2. Effect of finite range and nonlocality on the ground state differential cross section.

The proton spectrum at  $32.5^\circ$  laboratory is shown in Figure 3. The ordinate gives the number of proton tracks per quarter millimeter interval on the nuclear emulsion plate while the abscissa gives the position along the plate. The resolution is defined as the width of the peak at half maximum and is approximately 25 Kev.

The angular distributions are given in Figures 4, 5, 6, and 7. The data points represent the experimental differential cross section while the solid line is the DWBA FRNL fit to the data. Only the normalization is adjustable and this gives the spectroscopic factor defined in equation 36. In general, the fits are rather good.

On the basis of a simple stripping mechanism, one would expect the transferred neutron to be captured into the available single particle states, namely the  $2d_{5/2}$ ,  $2d_{3/2}$ ,  $3s_{1/2}$ ,  $1g_{7/2}$  and  $1h_{11/2}$  orbits. The ground state of  $^{89}\text{Y}$  is considered to be a  $2p_{1/2}$  proton outside of a closed proton shell. The proton particle configurations available are  $2p_{1/2}$ ,  $1g_{9/2}$ ,  $1f_{5/2}$ ,  $1p_{3/2}$  and  $1d_{5/2}$  orbitals. If the proton remains in a  $2p_{1/2}$  orbit, then it is expected that each single particle level of the neutron configuration will give rise to a doublet of levels in  $^{90}\text{Y}$ . Each doublet arises from the quantum

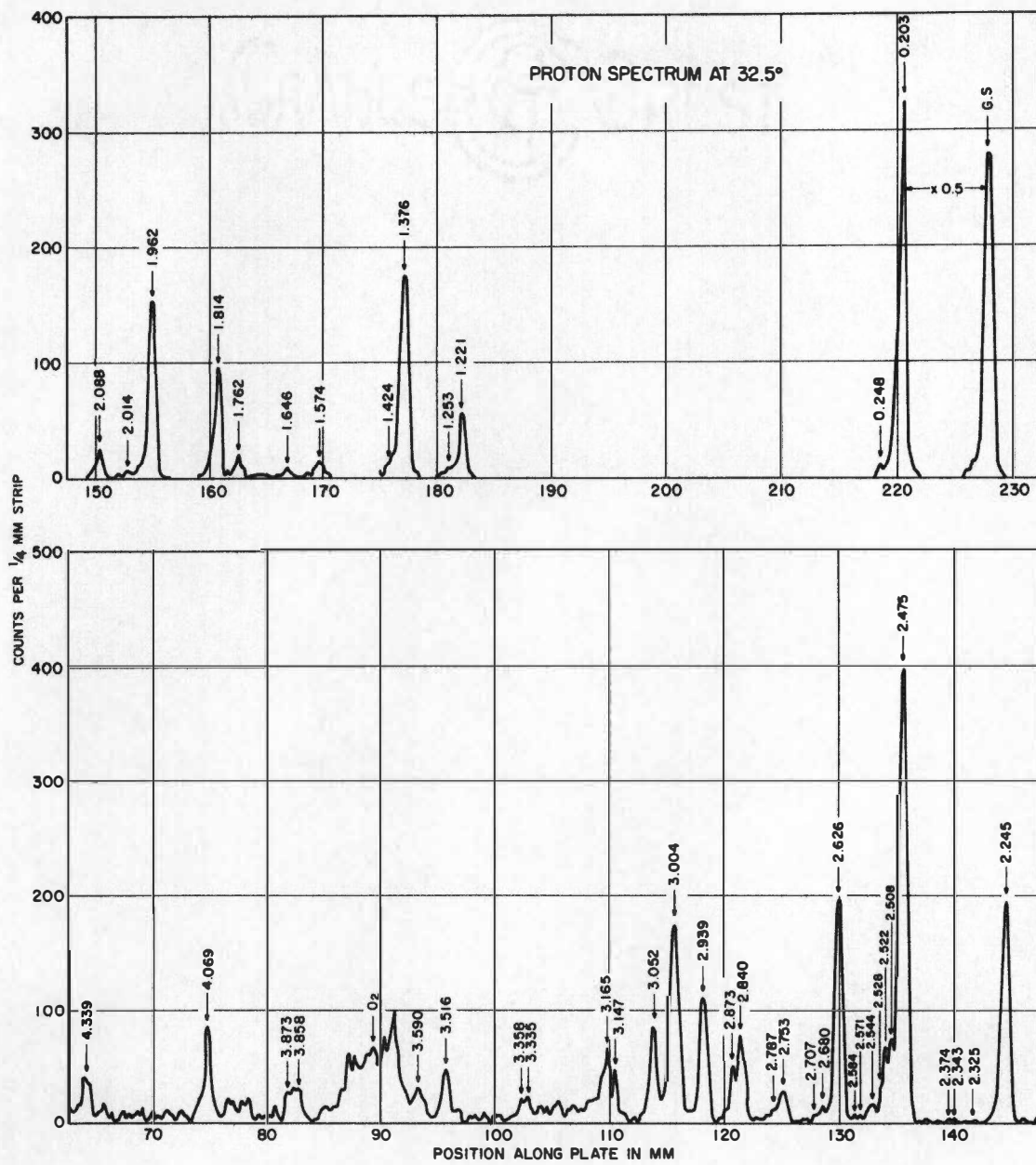


Figure 3. Proton spectrum at 32.5°.

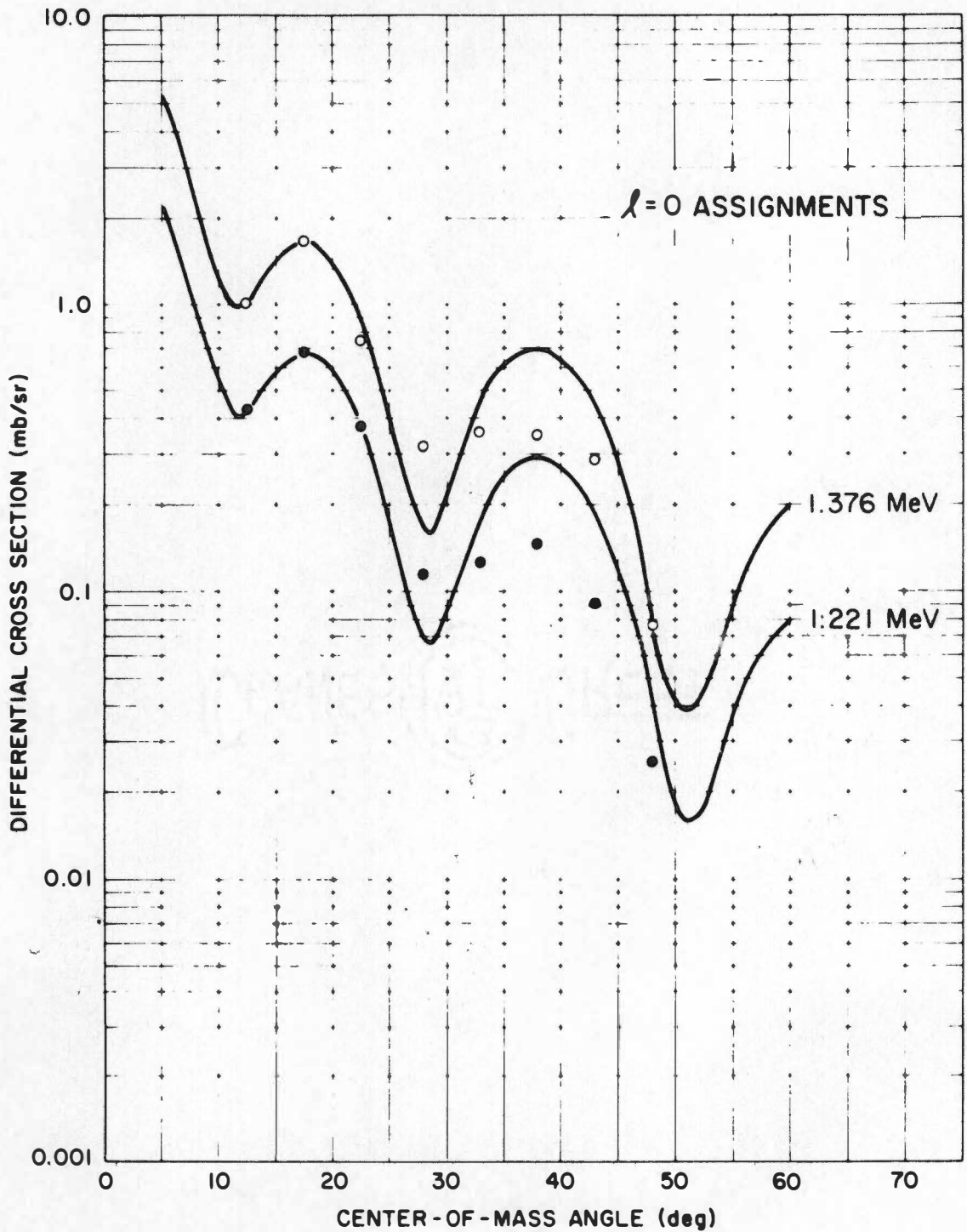


Figure 4. Differential cross sections for  $l=0$  neutron states.



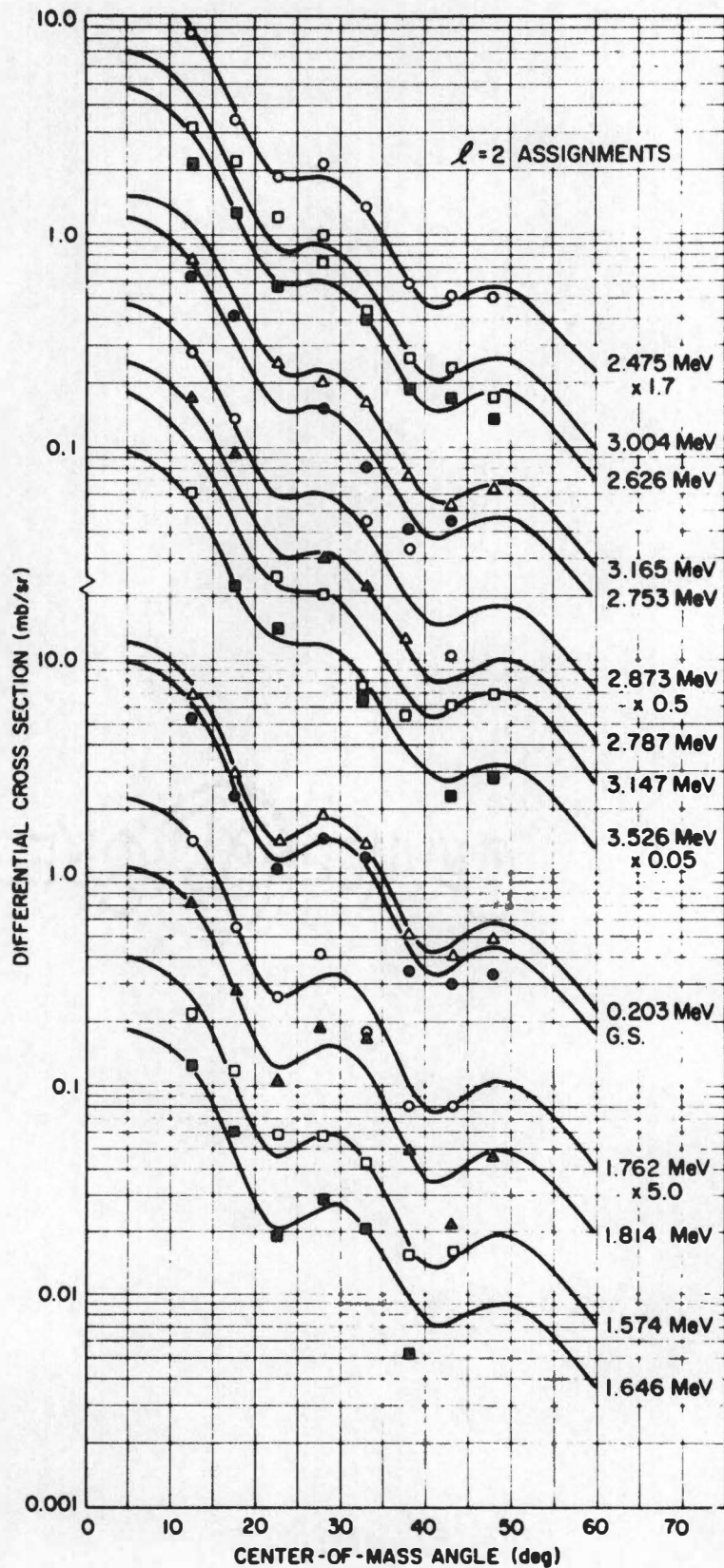


Figure 5. Differential cross sections for  $l=2$  neutron states.

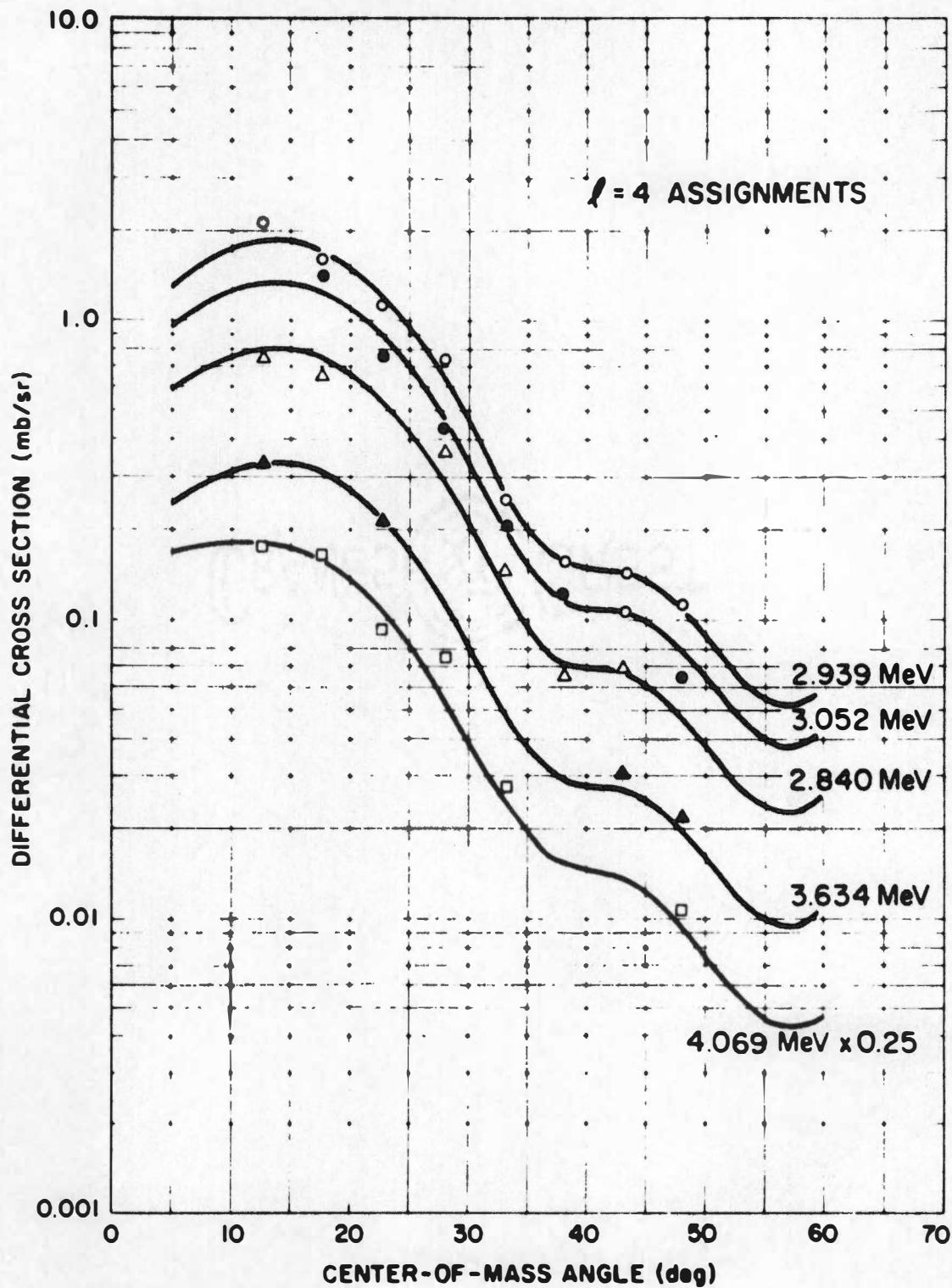


Figure 6. Differential cross sections for  $l=4$  neutron states.

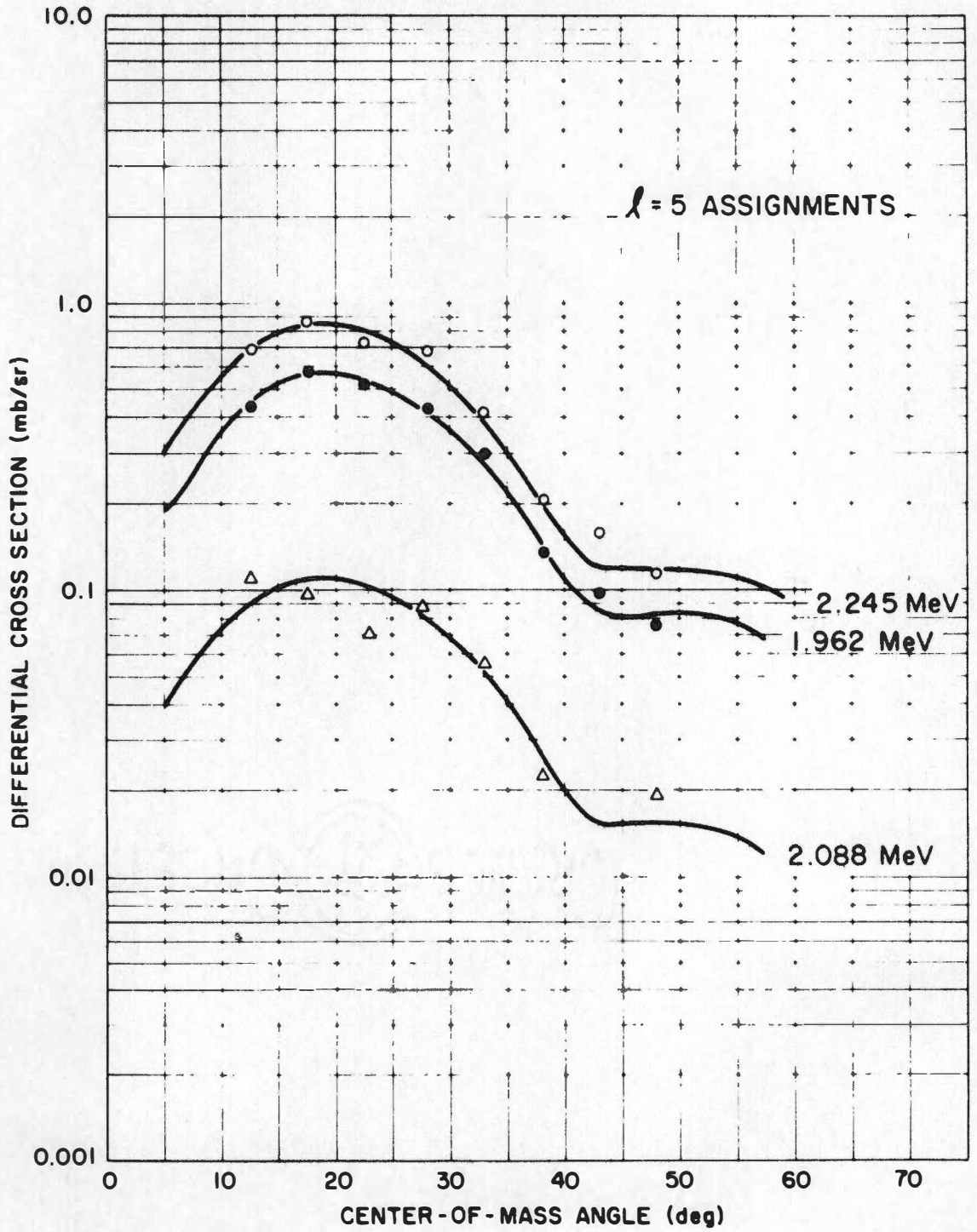


Figure 7. Differential cross sections for  $l=5$  neutron states.

mechanical addition of the proton angular momentum  $J_p = 1/2$  to the neutron angular momentum  $J_n$ . Groups with a different proton configuration could only be populated by second order processes involving proton excitation. A summary of strong transition is given in Table 2. It is interesting also to compare selected levels in  ${}^{91}\text{Zr}$  (Bingham, Unpublished) with those in  ${}^{90}\text{Y}$ . Such a comparison is given in Figure 8. The spectroscopy of  ${}^{91}\text{Zr}$  is expected to be rather simple since, in the ground state, it has 51 neutrons coupled to spin  $5/2^+$  and a closed subshell of 40 protons. Thus, it should be easy to observe the fractionation of neutron single particle levels. If a simple shell model interpretation is correct, each level in  ${}^{91}\text{Zr}$  should correspond to a doublet of levels in  ${}^{90}\text{Y}$ . This splitting comes about from the residual interaction of the  $p_{1/2}$  proton and  $J_n$  neutron in  ${}^{90}\text{Y}$ . If the proton shell is excited, then more levels would be expected from the quantum mechanical addition of  $J_p$  and  $J_n$ .

The expectation of doublet structure in  ${}^{90}\text{Y}$  is borne out by experimental results. It has long been known that the ground state and the 0.203 Mev level correspond to a  $2d_{5/2}$  doublet. Spectroscopic factors of 1.27 and 1.12, respectively, were obtained in this work. This indicates that all of the

TABLE 2  
SUMMARY OF RESULTS FOR SELECTED STATES

$E^*$ (Mev)	$\ell$	$J^\pi$	(2J+1)S	S	Proton-Neutron Configuration
0.000	2	$2^-$	6.35	1.27	$p_{1/2}; d_{5/2}$
<b>0.203</b>	2	$3^-$	<b>5.60</b>	<b>1.12</b>	
1.221	0	$0^-$	1.30	1.30	$p_{1/2}; s_{1/2}$
<b>1.376</b>	0	$1^-$	<b>3.24</b>	<b>1.08</b>	
1.574	2		0.25		
1.646	2		0.12		
1.762	2		0.29		
<b>1.814</b>	2		<b>0.64</b>		
1.962	5	$5^+$	3.08	0.28	$p_{1/2}; h_{11/2}$
<b>2.245</b>	5	$6^+$	<b>3.38</b>	<b>0.26</b>	
<b>2.088</b>	5	$5^+$	<b>0.52</b>	<b>0.047</b>	$p_{1/2}; h_{11/2}$
2.475	2	$2^-$	4.30	0.86	$p_{1/2}; d_{3/2}$
<b>2.626</b>	2	$1^-$	<b>2.31</b>	<b>0.77</b>	
2.753	2		0.57		
2.787	2		0.12		
2.873	2		0.53		
3.004	2		3.43		
3.147	2		0.76		
3.165	2		0.88		

TABLE 2(continued)

$E^*$ (Mev)	$l$	$J^\pi$	$(2J+1)S$	$S$	Proton-Neutron Configuration
2.840	4		2.05		
2.939	4	$4^-$	4.60	0.51	$P_{1/2}; 8_{7/2}$
3.052	4	$3^-$	3.36	0.48	
3.634	4		0.82		
4.069	4		1.80		

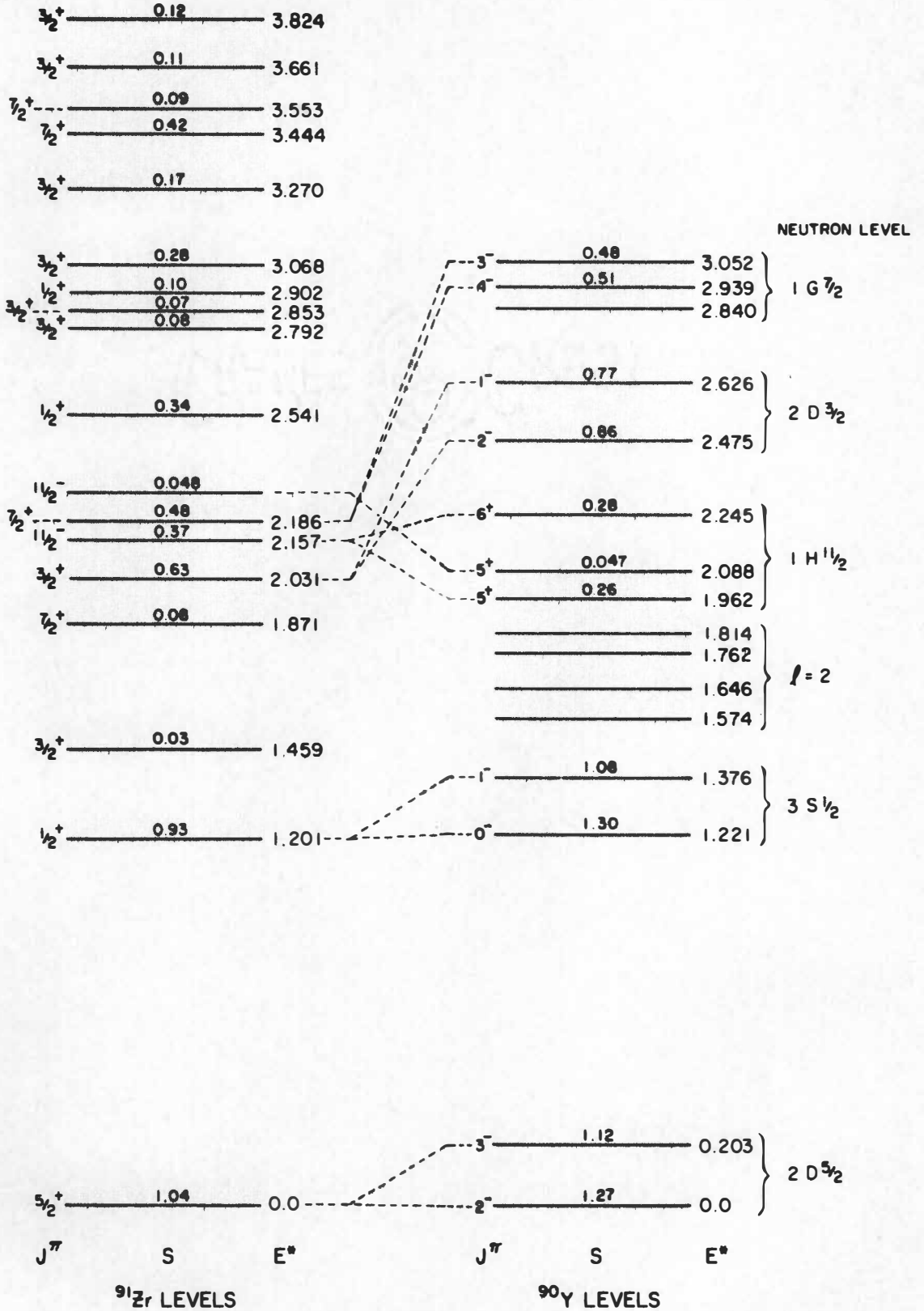


Figure 8. Comparison of selected levels in <sup>91</sup>Zr and in <sup>90</sup>Y.



$d_{5/2}$  strength is concentrated in this doublet. If second order processes are considered, these should be a group of states resulting from the coupling of a  $g_{9/2}$  proton with the  $d_{5/2}$  neutron at about 0.7 Mev. The spins should range from  $2^+$  to  $7^+$ . Very weak states corresponding to some of these energies were observed, but they were not excited to more than one part in 1500 of the intensity observed for the ground state. However, these states have been observed in other processes (Black, et al., 1966; Bartholomew, et al., 1959; Riley, et al., 1964). In particular, the  $7^+$  state is well known since it has an  $M4$  gamma transition.

The  $s_{1/2}$  doublet at 1.221 and 1.376 Mev has been previously identified by other investigators. This doublet could be thought of as corresponding to the splitting of the level in  $^{91}\text{Zr}$  at 1.201 Mev. The spectroscopic factors were found to be 1.30 and 1.08 respectively for these two levels. The  $^{89}\text{Y}(d,p)$  reaction indicates that the  $s_{1/2}$  strength is mostly contained in this doublet. However, other  $1/2^+$  states have been found at higher excitation energies in  $^{91}\text{Zr}$  and this may indicate that the  $s_{1/2}$  strength is more fractionated than previously believed. The spectroscopic factor of 1.30



seems to be too high, but the value of 1.08 is in reasonable agreement with  $^{91}\text{Zr}$  results. The spectroscopic factor of 1.08 compares favorably with the value of 1.11 obtained by Mishra (1968).

Four levels near 1.5 Mev were found to be populated by  $\ell=2$  transitions. This group of states has been observed before, but there seems to be some doubt as to the correct shell model assignments. Recently, Mishra conjectured that these levels represented the coupling of the  $f_{5/2}$  and  $p_{3/2}$  proton states with a  $d_{5/2}$  neutron. If this analysis is correct, then one would expect to see a group of eight states corresponding to the second order process. However, the splitting within the configuration could be small enough for the other four states to be unresolvable. It is also possible that other weak states nearby could arise from this configuration, but these states have thus far not been amenable to analysis. Another interpretation is possible since a weak  $d_{5/2}$  state with  $S = 0.03$  was found at 1.459 Mev in  $^{91}\text{Zr}$ . The levels at 1.574 Mev and 1.646 Mev could represent a  $(p_{1/2}, d_{5/2})$  doublet. If these levels are assigned spins of  $3^-$  and  $2^-$  respectively, then the spectroscopic factors are found to be 0.05 and 0.04. These values are in fair agreement with the spectroscopic factor of 0.03 found for the level in  $^{91}\text{Zr}$ . The other two levels could be the  $2^-$  and  $3^-$

levels of the ( $f_{5/2}$ ,  $d_{5/2}$ ) configuration. These would be populated (d,p) if these states mixed with the ( $p_{1/2}$ ,  $d_{5/2}$ ) configuration which is principally contained in the ground state and the 0.203 Mev levels.

The states at 1.962 and 2.245 Mev were analyzed carefully and were found to be populated by  $\ell = 5$  transitions. If these levels are assigned spins of  $5^+$  and  $6^+$  respectively, the the spectroscopic factors are found to be 0.28 and 0.26. Previous investigators using the (d,p) reaction at 12 Mev and 15 Mev had made either  $\ell = 4$  or  $\ell = 4+5$  assignments to this doublet. However, in this experiment, the higher beam energy of 33.3 Mev greatly enhanced the large spin transfer and the angular distribution were found to fit that for an  $h_{11/2}$  transition. The reaction  $^{89}\text{Y}(\alpha, ^3\text{He})$  at 65 Mev indicated that if these two states were an  $h_{11/2}$  doublet, then they would have spectroscopic factors of 0.25 and 0.27 respectively. These results are in good agreement with the (d,p) reaction. The energy level scheme of  $^{91}\text{Zr}$  indicates the presence of a weak  $h_{11/2}$  state at 2.309 Mev. One member of the expected doublet in  $^{90}\text{Y}$  has been found at 2.088 Mev. If this state is assigned a spin of  $5^+$ , then the spectroscopic factor is found to be 0.047. This value is in excellent agreement with the value of 0.048 found for the state in  $^{91}\text{Zr}$ . The other member of the doublet was not found, but considering the expected

level splitting, it would probably lie around 2.54 Mev. A state was found at 2.544 Mev but it was not amenable to analysis. The small values of the spectroscopic factor for  $\ell = 5$  states in both  $^{90}\text{Y}$  and  $^{91}\text{Zr}$  indicate that the  $h_{11/2}$  strength must be very fractionated.

The next available neutron state is the  $2d_{3/2}$  level. Approximately 80% of the  $d_{3/2}$  strength is contained in the doublet at 2.475 and 2.626 Mev. These levels have spins of  $2^-$  and  $1^-$  respectively, which give them spectroscopic factors of 0.86 and 0.77. These values of  $S$  are in agreement with Mishra ( $S = 0.77, 0.84$ ) and with Hamburger ( $S = 0.81, 0.67$ ). It is expected that this doublet corresponds to the splitting of the  $d_{3/2}$  state at 2.031 Mev in  $^{91}\text{Zr}$ . Six other states near 3.0 Mev were given  $\ell = 2$  assignments but it was not possible to assign spins. Mishra notes that the states at 3.004 and 3.165 Mev can be fitted by an admixture of  $\ell = 2$  and  $\ell = 0$ . His data indicate that  $S = 0.25$  for the  $d_{3/2}$  contribution and  $S = 0.26$  for the  $s_{1/2}$  contribution. These levels were fitted by a pure  $\ell = 2$  distribution in this work, but this might be expected since the  $s_{1/2}$  cross sections are small at 33.3 Mev. Such  $\ell = 0$  transitions are expected since  $s_{1/2}$  levels were found at 2.541 and 2.902

Mev in  $^{91}\text{Zr}$  which have spectroscopic factors of 0.34 and 0.10 respectively. Several  $d_{3/2}$  states above 2.5 Mev were also found in  $^{91}\text{Zr}$ . Their excitation energies are 2.541, 2.792, 2.853 and 3.068 Mev and they have spectroscopic factors of 0.34, 0.08, 0.07 and 0.28 respectively. It was not possible to observe the states in  $^{90}\text{Y}$  corresponding to these levels because of resolution problems.

The last neutron single particle state observed was the  $1g_{7/2}$  state. The levels at 2.939 and 3.052 Mev were found to be  $\ell = 4$  transitions. They were assigned spins of  $4^-$  and  $3^-$  respectively which gives spectroscopic factors of 0.51 and 0.48. From these considerations, approximately one half of the  $g_{7/2}$  strength is contained in this doublet. Other  $\ell = 4$  transitions were found at 2.840, 3.634 and 4.069 Mev. However, it was not possible to assign spins to any of these levels.

Many other levels were seen but, in general, it was not possible to obtain a recognizable angular distribution for them. However, one interesting level was observed at 0.248 Mev. The existence of this state was inferred from the  $(n, \gamma)$  spectroscopy experiment of Bartholomew, but it has not been reported in a  $(d,p)$  experiment before. It was not possible to assign an  $\ell$ -transfer from the  $(d,p)$  data, so the nature of this level is still very much in question.

## CHAPTER V

### SUMMARY AND CONCLUSIONS

Distorted wave analysis of the  $^{89}\text{Y}(d,p)^{90}\text{Y}$  reaction at 33.3 Mev provides information concerning the spins and parities of the neutron states in  $^{90}\text{Y}$ . In particular, it was possible to assign  $\ell$ -transfers by fitting the experimental angular distribution with DWBA calculations with finite range and nonlocality effects included. On the basis of a simple stripping mechanism, it is expected that the transferred neutron will be captured into the available single particle states. If the proton in  $^{89}\text{Y}$  remains in the  $2p_{1/2}$  orbit, then it is expected that each single particle level of the neutron configuration will give rise to a doublet in  $^{90}\text{Y}$ . This expectation is borne out in general.

Uncertainties exist when DWBA analysis is used to obtain the absolute value of the spectroscopic factor for various levels. In general, there is an uncertainty of approximately 15% on all values of  $S$ . Even with such a large uncertainty, valuable information was obtained regarding the spectroscopy of  $^{90}\text{Y}$ . Essentially all of the  $d_{5/2}$  neutron strength is contained in the ground state and the first excited state. Almost all of the  $s_{1/2}$  strength is contained in the doublet at 1.221 and 1.376 Mev, although it is possible that there is some  $s_{1/2}$  strength involved in configuration

mixing near 3.0 Mev. This possibility has been voiced by other investigators and it is consistent with results found in  $^{91}\text{Zr}$ . Approximately 28% of the  $h_{11/2}$  strength has been isolated in the doublet at 1.962 and 2.245 Mev and another 5% has been found in a level at 2.088 Mev. The spectroscopic factors for these levels compare favorably with those obtained for corresponding levels in  $^{91}\text{Zr}$  and with some results obtained with the  $^{89}\text{Y}(\alpha, ^3\text{He})$  reaction at 65 Mev. At higher excitation energies, approximately 80% of the  $d_{3/2}$  strength is contained in the doublet at 2.475 and 2.626 Mev while 50% of the  $g_{7/2}$  strength is in the doublet at 2.939 and 3.052 Mev. Many other levels were identified and  $\ell$ -transfers were assigned to some of them. The state at 0.248 Mev is of particular interest since it has not been previously reported in a (d,p) experiment. The nature of this state is still in doubt since it was not possible to assign an  $\ell$ -transfer.

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## VITA

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