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The Generalized DEA Model of Fundamental Analysis of Public Firms, with Application to Portfolio Selection

Xin Zhang
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Xin Zhang entitled "The Generalized DEA Model of Fundamental Analysis of Public Firms, with Application to Portfolio Selection." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Management Science.

Chanaka Edirisinghe, Major Professor

We have read this dissertation and recommend its acceptance:

Charles E. Noon, Russell Zaretzki, Bruce Behn

Accepted for the Council:

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Vice Provost and Dean of the Graduate School

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A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Xin Zhang
December 2007

Dedication

This dissertation is dedicated to my husband, Yingfeng Guan, my parents, Baojun Zhang and Peimei Sun, great role models and friends, and the rest of my family.

Acknowledgments

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I would like to thank Dr. Bruce Behn, Dr. Charles Noon, and Dr. Russell Zaretzki for serving in my dissertation committee. Their advices and suggestions led me to consider various aspects that made this research more comprehensive. I would also like to thank Jane Moser for her kindness and the encouraging words throughout my stay at the Management Science program.

Last but not least, my family has been a constant source of strength and wisdom during my Ph.D. study. I want to thank them for their care, encouragement, and patience at every opportunity they had.

Abstract

Fundamental analysis is an approach for evaluating a public firm for its investment-worthiness by looking at its business at the basic or fundamental financial level. The focus of this thesis is on utilizing financial statement data and a new generalization of the Data Envelopment Analysis, termed the GDEA model, to determine a relative financial strength (RFS) indicator that represents the underlying business strength of a firm. This approach is based on maximizing a correlation metric between GDEA-based score of financial strength and stock price performance. The correlation maximization problem is a difficult binary nonlinear optimization that requires iterative re-configuration of parameters of financial statements as inputs and outputs. A two-step heuristic algorithm that combines random sampling and local search optimization is developed. Theoretical optimality conditions are also derived for checking solutions of the GDEA model. Statistical tests are developed for validating the utility of the RFS indicator for portfolio selection, and the approach is computationally tested and compared with competing approaches.

The GDEA model is also further extended by incorporating Expert Information on input/output selection. In addition to deriving theoretical properties of the model,

a new methodology is developed for testing if such exogenous expert knowledge can be significant in obtaining stronger RFS indicators. Finally, the RFS approach under expert information is applied in a Case Study, involving more than 800 firms covering all sectors of the U.S. stock market, to determine optimized RFS indicators for stock selection. Those selected stocks are then used within portfolio optimization models to demonstrate the superiority of the techniques developed in this thesis.

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Chapter 1

Introduction

This thesis is concerned with the development of a new methodology termed Generalized Data Envelopment Analysis (GDEA) that enables ranking of a firm's fundamental business strength, relative to other firms, and using such ranking in the design of financial portfolios. Portfolio design in an uncertain environment is of paramount importance in the management of mutual funds, retirement and pension funds, bank and insurance portfolio management, for instance. Such problems involve, first, choosing individual firms, industries, or industry groups that are expected to display strong performance in a competitive market, thus, leading to successful investments in the future; second, it also requires a decision analysis of how best to periodically rebalance such funds to account for evolving general and firm-specific economic conditions. It is the success of both these functions that allows a portfolio manager to maintain the risk-level of the fund within acceptable limits, as specified by regulatory and other policy and risk considerations. This thesis aims to provide a significant contribution to the former function. The work in this thesis also complements the latter function by the development and computational testing of a stochastic programming investment optimization model that determines optimal portfolio dynamic allocations satisfying risk and policy specifications. These models are empirically tested using real-world data from the U.S. stock market.

1.1 Conceptual Framework

In determining the financial strength of a given firm/industry/sector, this research focuses on the financial data that are made public through balance sheet, income statement, etc, on a quarterly basis. The central premise of this research is that market (stock) prices have factored in publicly-available information about the firm, but the future expectations of price performance are determined by the perceived business strength of the firm. This notion is consistent with the "efficient market hypothesis"(EMH) [41], where the price of a stock is assumed to reflect the knowledge and expectations of all investors since everyone has the same information about the stock. It must be noted that EMH does not imply that investors have perfect (or identical) powers of prediction; all it means is that the current stock price is an unbiased estimate of the firm's true economic value based on the information revealed.

A firm's business (or economic) strength can be evaluated by factors that are internal as well as external to the firm. From the perspective of internal factors, a publicly traded firm is in the business of producing marketable outputs, which are products and services, using an input supply of raw materials, labor, and other resources. Such a business is typified, in microeconomics, by a production process that transforms or converts inputs into outputs, and a productivity or efficiency metric can be associated with such a transformation process. A firm's internal business strength is directly related to its productivity or efficiency in the conversion of inputs to outputs. For example, if a firm increases its productivity, it is likely that this firm can produce products with lower production cost, thus, resulting in higher profits. Then, such productivity gains will be reflected in the financial statement data revealed by the firm.

On the other hand, from the perspective of external factors, a firm's business success often depends on whether the firm produces to growth or matured markets and also on market factors such as product competition, substitution effects, and market supply/demand imbalance, for instance. Therefore, it is the relative business strength of a firm, relative to competition with other firms in a similar business segment, that influences the firm's overall financial success. Consequently, in this thesis, the basic underlying concept is that the stock price performance of a firm is dictated by both the internal productivity/efficiency considerations as well as external relative valuations in the presence of other firms. In the absence of strong competition, the lack of internal productivity may not significantly affect the financial well-being of the firm, and thus, the stock returns. However, in the presence of strong market participants, productivity losses can lead to severe internal financial ill-health, and thus, diminished stock market performance for the firm. This triad notion of

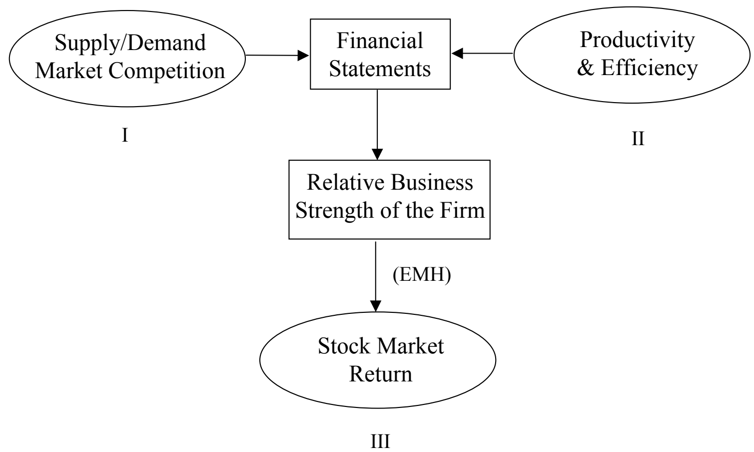

Figure 1.1: Schematic of the framework

productivity, competition, and market returns is conceptualized in the schematic in Figure 1.1. This conceptual framework is the basic building block of this thesis.

In general, financial statements provide the basic data that reflects both internal and external influences on a firm's financial performance. Consequently, analysis of financial statements is a common approach for gauging a firm's business strength. For instance, many accounting models, such as the free cash flow model [58] and the residual income valuation model [29], have been developed to determine a firm's intrinsic value, based on information (or forecasts thereof) obtained from the financial statements of the firm. Such an intrinsic value can then be compared with the firm's current market value with the hope of finding investments where the intrinsic value exceeds the market value. The notable drawback of this approach is that such an intrinsic value has only implicitly accounted for business strength of other competing firms. This lack of explicit relative valuation makes it difficult for fund managers to allocate portfolio dollars to firms with similar value/price ratios. In contrast, our approach strives to determine a strength metric for the firm that provides an explicit reference to the operation of other firms in the same business segment. Then, the resulting financial strength metric for the firm is computed relative to other firms, instead of computing a firm-specific absolute intrinsic value.

In this thesis, a firm's financial strength is, thus, measured by taking both productivity and strengths of other firms into consideration. The aim is to provide a measurable (objective) metric that is highly correlated with stock price performance under the assumption of "Efficient Market Hypothesis". This metric represents the performance of firms on a quarterly basis, and thus, it pertains to a relatively short-term of financial strength analysis. Such a metric can then be used as a proxy for gauging a firm's expected financial performance, and hence the firm's future (quarterly) stock price performance. The basic modeling tool employed for this purpose is the so-called Data Envelopment Analysis (DEA) methodology.

DEA is a ranking technique, which estimates a firm's efficiency, by comparing the firm to many other firms operating under a similar environment. A detailed description of DEA methodology will be provided in Chapter 2. To the best of our knowledge, this thesis is the first instance where DEA-based methodology has been incorporated for fundamental analysis towards portfolio selection.

1.2 Fundamental Analysis

Fundamental analysis and technical analysis are two basic methodologies that are typically used to make stock selection decisions. Fundamental analysis is the study of economic, industry specific, and firm specific conditions in order to determine the underlying value of a (public) company's stock. At the economic level, fundamental analysis studies if overall macroeconomic conditions, as measured by interest rate, inflation rate, unemployment, etc., are favorable for the stock market. At the industry level, fundamental analysis examines the underlying factors of supply and demand for the products offered in a given industry and determines how strong that industry is for investment. At the company level, it evaluates a public firm for its investment-worthiness by looking at its business at the basic or fundamental financial level, see [62]. It involves examining a firm's financials and operations, especially, sales, earnings, growth potential, assets, debt, management, products, and competition. The end goal of performing fundamental analysis is to understand the business strength of a firm, identify the intrinsic or fundamental value of its stock shares, and hence, determine an investment position to take in the security market, see [23] [48], for instance. Thus, fundamental analysis takes on a longer-term perspective in determining which firms are most likely to perform well in the future, based on their fundamental business strength.

On the other hand, technical analysis focuses on analyzing actual market price behavior of a security, rather than directly evaluating fundamental business strength of the firm. Strategies based on technical analysis generally utilize a series of calculations designed to detect when a price change is likely to occur. Then, an investor can use such detections to manage market positions in the short-term, such as the case in highly leveraged derivative markets. One plausible argument for technical analysis is that historical (price and volume) charts represent the past behavior of the pool of investors. Since that pool does not change rapidly, one might expect to see similar chart patterns in the future. A second argument in favor of technical analysis is that the chart patterns display the action inherent in an auction market. Since not everyone reacts to the information instantly, technical chart analysis can provide some predictive value in the short-term.

This thesis focuses on the longer term perspective as contemplated by the fundamental analysis. In particular, the focus in fundamental analysis in this thesis is limited to the two dimensions provided by the industry- and firm-specific conditions, thus leaving out macroeconomic conditions. Consequently, our use of the term fundamental analysis refers to financial statement analysis, which involves the use of various financial statements of firms in a given market segment.

1.2.1 Financial statements

Financial statement analysis is a standard practice in understanding the underlying internal business strength of a firm. Financial statements typically include balance sheet and income statement that are released to the public at regular time intervals. A balance sheet summarizes the book value of all assets, liabilities, and shareholder's equity of a business at a specific point in time, usually the end of a year or a quarter. The purpose of the balance sheet is to examine what a company owns and owes at that point in time. The balance sheet must follow the following basic expression:

$$\textit{Assets - Liabilities} = \textit{Stockholder's Equity}$$

Each of three components in the above formula has many subcomponents, referred to as accounts, and each account corresponds to a value at a specific point in time. For instance, accounts such as cash, accounts receivable, inventory and property are on the asset side of the balance sheet, while on the liability side there are accounts such as accounts payable and long-term debt. Under stockholder's equity, accounts such as common stock and retained earnings are present. However, not all accounts are present in every balance sheet, and they may differ by company and by industry.

The income statement is another important financial information issued by the company. It summarizes revenues and expenses in a particular period of time, usually the end of a year or a quarter. The purpose of an income statement is to show investors how much revenue and profit a company has generated over the given period. The items in the income statement include revenue, operating and non-operating activities, interest expenses, and net income before and after tax, etc.

Using the above two types of statements, balance sheet and income statement, certain derivative statements can be produced, such as statement of retained earnings and cash flow statement [34]. Retained earnings are earnings not paid out as dividends, but retained by the company to be reinvested in itself or to pay debt. Statement of retained earnings explains the changes in a company's retained earnings over the reporting period. The cash flow statement reports on a company's cash flow activities, particularly its operating, investing and financing activities [30]. These statements, including the balance sheet, the income statement, the cash flow statement, and statement of retained earnings, provide an overview of a firm' s profitability and financial condition, in a quarterly, semiannual, or annual basis.

1.3 Some Existing Methodologies for Firm Valuation

As stated earlier, in financial statement analysis, one attempts to understand the firm's business via its industry position, sales, costs, earnings, etc., see [58]. It is the combination of many data elements in financial statements that point to a metric of the value of the firm. In the literature, many methods have been developed to assess a firm's value, based upon cash flows, growth, and risk. For instance, the dividend discount model, the free cash flow model, and the residual income valuation model are standard methods for firm valuation.

1.3.1 Dividend Discount Model (DDM)

The DDM is the simplest and the oldest present value approach to valuing an equity. The value of a stock is the present value of the future expected dividends produced by the firm. When an investor buys a share of the stock, he/she generally expects to get two types of cash flow – expected dividends during the period of holding the stock and an expected price at the end of the holding period. If the holding period is finite, for an n-period model, the value of the stock is the present value of the expected dividends for the n-periods plus the present value of the expected price at the end of period n, see, for instance, [58]. The formula for the finite-period case is then given by

$$V = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}, \tag{1.1}$$

where V is the present value of a share of stock, D_t is the expected dividends per share in period t, P_n is the expected price per share in period n, and r is the cost of equity. Cost of equity (COE) is the rate of return required by shareholders for investing in a stock. It generally reflects the dividends paid on the shares and the appreciation in the market value of the stock. If an investor invests in a more risky stock, then COE will be higher because the investor would expect a higher return to compensate for the increased risk.

The underlying concept of the equation in (1.1) is that cash flows in different time periods cannot be directly compared since investors prefer to receive a payment of a fixed amount of money today rather than an equal amount in the future, all else being equal. This is because the cash today could be deposited in an interest-bearing bank account.

If the holding period of a stock is extended indefinitely, the stock's value becomes

$$V = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}, \tag{1.2}$$

which is the present value of all expected future dividends. Such an infinite sequence of cash flow is the basis of the two valuation models presented next.

1.3.2 Free cash flow to equity (FCFE) model

FCFE is the cash flow available to a firm's shareholders after all operating expenses, interest, and principal payments have been paid and necessary investment in working capital (i.e., inventory) and fixed capital (i.e., equipment) have been made [58].

FCFE can be computed as the cash flow from operations minus capital expenditures minus payments to debt-holders, as given by the expression

$$FCFE = NI + NCC - FCInv - WCInv + NB, \tag{1.3}$$

where NI is net income available to common shareholders, NCC is net non-cash charges, such as depreciation or depletion, FCInv is investment in fixed capital, WCInv is investment in working capital, and NB is net borrowing, which is net debt issued less debt repayments over the period of calculating the free cash flow.

The value of equity (V) is then computed by discounting FCFE at the cost of equity (r) as given by

$$V = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}. \tag{1.4}$$

FCFE is the cash flow remaining for shareholders after all claims are satisfied, thus, discounting FCFE by r gives the present value of the firm's equity. Dividing the total value of equity by the number of outstanding shares gives the value per share, see [58]. Consider the special case where FCFE grows at a constant rate g for every future period. Then,

$$FCFE_t = FCFE_{t-1} \times (1+g). \tag{1.5}$$

Therefore, the current value of equity is given by

$$V = \frac{FCFE_1}{1+r} + \frac{FCFE_1(1+g)}{(1+r)^2} + \ldots + \frac{FCFE_1(1+g)^{n-1}}{(1+r)^n} + \ldots, \tag{1.6}$$

which is an infinite sequence. Each term in the above expression is equal to the previous term times the constant $\bar{r} = \frac{1+g}{1+r} < 1$,

$$V = \frac{FCFE_1}{(1+r)(1-\bar{r})} = \frac{FCFE_1}{r-g} = \frac{FCFE_0(1+g)}{r-g}. \tag{1.7}$$

1.3.3 Residual Income Valuation Model (RIV)

The RIV is based on the valuation models that are deducted from the theory of capital value [58]. The RIV analyzes the intrinsic value of a firm in two components: the current book value of a firm, and the present value of expected future residual income. The residual income is defined as the difference between reported income and the cost of equity capital multiplied by reported book value at the beginning of a period [2]. It represents the net dividends being paid to a firm's shareholders for a given period of time. In RIV, given the expected total dividend paid to the shareholders (D) and the cost of equity capital (r), a stock's fundamental value at the present time t is equal to the present value of its future (expected) dividend payments, i.e.,

$$V_t = \sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+r)^{\tau}}. \tag{1.8}$$

Furthermore, RIV assumes that the clean surplus relation of accounting holds in each period $(t+\tau)$, that is,

$$D_{t+\tau} = B_{t+\tau-1} + I_{t+\tau} - B_{t+\tau} \tag{1.9}$$

must hold, where $I_{t+\tau}$ is the accrued net income and $B_{t+\tau}$ is the book value of owner's equity. This assumption allows future dividends to be expressed in terms of future earnings and book value. It implies that the net dividend being paid at the end of a period equals to the difference between the net income and the change in the book value of shareholder's equity during that period. Clean surplus relation of accounting requires all gains and losses that affect book value to be included in the earnings. Combining the clean surplus relation in (1.9) with (1.8) yields

$$V_t = B_t + \sum_{\tau=1}^{\infty} \frac{B_{t+\tau-1} + I_{t+\tau} - B_{t+\tau}}{(1+r)^{\tau}}. \tag{1.10}$$

By simple algebraic transformation, (1.10) can be rewritten as:

$$V_t = B_t + \sum_{\tau=1}^{\infty} \frac{I_{t+\tau} - r \cdot B_{t+\tau-1}}{(1+r)^{\tau}}. \tag{1.11}$$

Equation (1.11) is also equivalent to

$$V_t = B_t + \sum_{\tau=1}^{\infty} \frac{v_{t+\tau} - r}{(1+r)^{\tau}} \cdot B_{t+\tau-1}, \tag{1.12}$$

where $v_{t+\tau}$ is the (book) return on owner's equity given by $v_{t+\tau} = I_{t+\tau}/B_{t+\tau-1}$. According to the above expression, the fundamental value of a stock equals its book value per share plus the present value of expected future per-share residual income. Note that when the present value of future per-share residual income is positive, the value of the stock is always greater than the book value per share [58]. The present capital V_t is then determined by first expanding (1.8) to T terms (for a chosen time horizon T), and then taking the remaining terms in the expansion as a perpetuity, under a long-term constant growth rate g of earnings. The RIV with $T = 3$ future periods is then given by, see [2],

$$V_t = B_t + \frac{(v_t - r)}{(1+r)} B_t + \frac{(v_{t+1} - r)}{(1+r)^2} B_{t+1} + \frac{(v_{t+2} - r)}{(1+r)^2 r} B_{t+2}. \tag{1.13}$$

To apply (1.13), book value of owner's equity (B) and earning (I) must be estimated, and then, return on equity is forecasted as suggested in [29]:

$$v_t = \frac{I_t}{(B_{t-1} + B_{t-2})/2}, \tag{1.14}$$

$$v_{t+1} = \frac{I_{t+1}}{(B_t + B_{t-1})/2}, \tag{1.15}$$

and

$$v_{t+2} = \frac{I_{t+1}(1+g)}{(B_{t+1} + B_t)/2}, \tag{1.16}$$

assuming the future earnings are expected to grow at a constant rate g.

1.3.4 Comparison of valuation models and drawbacks

Valuation models based on discount dividends, free cash flow, and residual income determine a firm's intrinsic value, using different approaches. For example, both DDM and FCFE models require forecasts of future expected cash flows generated by the firm and find the value of the corresponding stock by discounting the cash flows back to the present using the required return on equity. In the DDM model, dividends are used as a measure of the cash flows returned to the shareholder. In contrast, RIV starts with a value based on the financial statements, book value of common equity, and adjusts this value by adding the present values of expected future residual income [58]. In practice, a firm with positive and predictable cash flow that pays dividends to shareholders are suited for DDM and FCFE models. However, if a firm has near-term negative free cash flow or the future cash flow is uncertain, a RIV model may be more appropriate [58].

The purpose for using the above valuation models is to estimate the intrinsic value of securities so that investments in the firm's stock returns a true value that exceeds its current market value. When forecasts of cash flow are made, as required by the valuation models, there is no formal mechanism for objectively considering the relative strength of the firm in the presence of other competing firms. Thus, absolute intrinsic value so-computed for a firm is likely to be a weak metric due to a lack of influence from other firms. This renders the valuation models to be highly sensitive to forecasts that the analyst may use. For instance, if the future net income or long-term growth rate of a firm is inappropriately estimated, the intrinsic value of the firm, determined by RIV model, may be overweighted. This will give investor a sign to buy the firm's stock, which may result in financial loss in the stock market. In contrast, the methodology in this thesis advocates a relative ranking system, which is based upon measuring relative productivity/efficiency, and it is likely to be more stable under such scenarios.

1.4 Productivity and Technical Efficiency

A firm's business strength is related to its productivity and efficiency to a great extent. For instance, if a firm achieves economies of scale, this firm may lower the average cost per product through increased production. As a result, the firm's revenue, net income, and cash balance, etc., which are the accounts listed in balance sheet and income statements, may be greatly improved. Thus, the firm might be considered with better financial circumstance by investors and hence its stock may be considered as a favorable candidate for investment.

Consider a firm as a production system that transforms inputs to marketable outputs (i.e., goods and services). This transformation process may involve manufacturing, storing, shipping, and packaging, etc. A production plan with a given input-output combination is called feasible if the specified amount of output can be produced by the specified input amount. Usually a firm's resource utilization or performance can be characterized by two concepts: productivity and efficiency. They are often treated the same in the sense that if firm A is more productive than firm B, then firm A is also believed to be more efficient. However, these two concepts are fundamentally different, although they are closely related. Productivity and efficiency are different measures of performance of a firm. Productivity is the amount of output created (in terms of goods produced or services rendered) per unit input used. Efficiency is used to determine whether a production process is optimal. A production process is said to be efficient if its productivity index is optimal, that is, a given quantity of inputs produces the maximum amount of outputs. Similarly, a production process is called inefficient when there exists another feasible production plan that produces higher outputs using the same quantity of inputs. The difference between productivity and efficiency can be easily understood using an example of two firms from a single-input, single-output industry, as discussed next.

1.4.1 The Single-Input and Single-Output case

Suppose firms A and B are in a similar type of production, using the same input to produce the same output. Firm A uses an input amount of x_A to produce an output amount of y_A and firm B uses x_B units of input to produce y_B units of output. The average productivity for firm A is $P_A = \frac{y_A}{x_A}$ and that for firm B is $P_B = \frac{y_B}{x_B}$. If $P_A > P_B$, we say firm A is more productive than firm B. The productivity index of firm A *relative* to firm B can be measured by $\frac{P_A}{P_B} = \frac{y_A}{x_A} / \frac{y_B}{x_B} = m$. If m is greater than 1, then we can say firm A is more productive than firm B, or more specifically, firm A is m times as productive as firm B.

Let the production function $y = f(x)$ represent the input-output conversion process for firms A and B, where x is the input units, y is the output units, and f represents the technology or the transformation process. Let $y_A^* = f(x_A)$, i.e., the maximum level of output that can be produced by using x_A. Similarly, let $y_B^* = f(x_B)$ be the maximum level of output that can be produced by using x_B. The efficiencies for firm A can be measured by comparing its actual output with the maximum producible output when the input is fixed, i.e., efficiency $E_A = \frac{y_A}{y_A^*}$ and the efficiency of firm B is $E_B = \frac{y_B}{y_B^*}$. Then $P_A^* = \frac{y_{A^*}}{x_A}$ is the average productivity when the maximum output is produced, and that for firm B is $P_B^* = \frac{y_B^*}{x_B}$. The efficiencies for firms A and

B can also be represented by their average productivities. That is,

$$E_A = \frac{y_A}{y_A^*} = \frac{y_A/x_A}{y_A^*/x_A} = \frac{P_A}{P_A^*},$$
$$E_B = \frac{y_B}{y_B^*} = \frac{y_B/x_B}{y_B^*/x_B} = \frac{P_B}{P_B^*}.$$

Therefore, the efficiency of a firm is its productivity index (under input x) relative to the productivity of a virtual firm that produces the efficient production plan $(x, f(x))$, see, for example, [51].

1.4.2 The Multiple-Input and Multiple-Output case

In a world of multiple inputs and multiple outputs, the average productivity concept introduced in the preceding section is no longer valid to compare productivities of firms. Let us consider the case when two inputs (indexed by 1 and 2) are used to produce a single output. Suppose firm A uses input levels x_{1A} and x_{2A} to produce an output level y_A and firm B uses inputs x_{1B} and x_{2B} to produce output y_B. Then, two sets of average productivities can be computed for firm A: $P_{1A} = \frac{y_A}{x_{1A}}$ and $P_{2A} = \frac{y_A}{x_{2A}}$. Similarly, those for firm B are $P_{1B} = \frac{y_B}{x_{1B}}$ and $P_{2B} = \frac{y_B}{x_{2B}}$. In this case, we cannot say firm A is more productive than firm B if $P_{1A} > P_{1B}$ because it is possible that $P_{2A} < P_{2B}$.

The average productivity of a firm relative to one input depends on the quantity of other inputs. Therefore, it is inappropriate to measure a firm's productivity relying on only one input but disregarding all the others. Thus, in the above single-output, two-inputs case, we need to find a representative input. We may consider

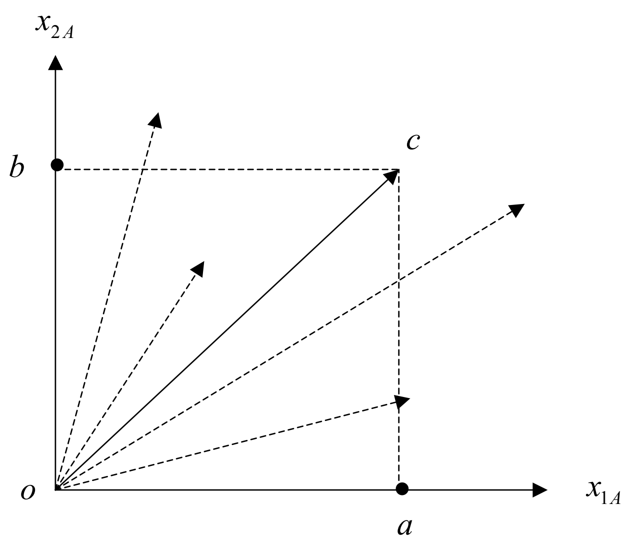

Figure 1.2: Aggregation of two inputs

an aggregate of the two input values into a composite input [51]. Let v_{1A} and v_{2A} denote the aggregation multipliers for inputs x_{1A} and x_{2A}, respectively. Thus, the aggregated input for firm A is presented as follows: $X_A = v_{1A}x_{1A} + v_{2A}x_{2A}$. Similarly, for firm B, it is $X_B = v_{1B}x_{1B} + v_{2B}x_{2B}$.

The aggregation of two inputs can be illustrated in Figure 1.2. Suppose a is the input quantity for x_{1A} and b is the quantity for x_{2A}. The aggregation of inputs x_{1A} and x_{2A} without the inclusion of aggregation multipliers is represented by the vector oc. With the inclusion of multipliers (v_{1A}, v_{2A}), the aggregated input can be represented by, for example, the vectors with broken lines. It must be noted that the aggregation multipliers must be nonnegative, and thus, the aggregation of two inputs will be in the domain of positive quadrant.

Then, the average productivity for firms A and B can be measured by the ratio of output quantity to the quantity of the aggregated input. That is,

$$P_A = \frac{y_A}{v_{1A}x_{1A} + v_{2A}x_{2A}},$$
$$P_B = \frac{y_B}{v_{1B}x_{1B} + v_{2B}x_{2B}}.$$

Now let $y_A^* = f(x_{1A}, x_{2A})$ be the maximum level of output that can be produced by using inputs x_{1A} and x_{2A}. Similarly, let $y_B^* = f(x_{1B}, x_{2B})$ be the maximum level of output that can be produced by using x_{1B} and x_{2B}. Following the argument of the single-input and single-output case, the efficiency for firm A is $E_A = \frac{y_A}{y_A^*} = \frac{y_A/X_A}{y_A^*/X_A} = \frac{P_A}{P_A^*}$ and the efficiency for firm B is $E_B = \frac{y_B}{y_B^*} = \frac{y_B/X_B}{y_B^*/X_B} = \frac{P_B}{P_B^*}$.

This two-inputs case can also be generalized to multiple-inputs case. If there are more than one output for a firm, we can aggregate all the output values in a similar manner. Such a view of aggregated inputs and outputs form the basis of Data Envelopment Analysis of efficiency evaluation.

1.5 Data Envelopment Analysis (DEA)

It is evident from the preceding discussion that a firm's efficiency measures whether the firm's average productivity is optimal, that is, whether a firm can produce the maximum level of (aggregated) output by using a certain level of (aggregated) input. This is the basic idea behind DEA methodology. However, in DEA, the notion of efficiency of a firm is measured relative to a reference set of other firms that also use the same input/output set. If the evaluated firm can produce the maximum amount of output using a certain level of input, compared to the firms in the reference set, we say the firm achieves its 100%-efficiency relative to the others. In this sense, it is a relative efficiency score, where every firm must use the same aggregation multipliers. If one or more firms in the reference set produce a larger amount of output than that of the evaluated firm, using the same level of input, we conclude that the firm being evaluated is not 100%-efficient, or inefficient. The DEA model that maximizes outputs for given inputs is termed an output-oriented DEA model.

In DEA methodology, an alternative view of measuring efficiency of a firm is to check if the firm can produce a certain level of output using the minimum possible level of input, hence, termed input-oriented DEA. If the firm can use the minimum amount of input to produce the same amount of output, compared to the firms in the reference set, the firm is 100%-efficient relative to the other firms. If one or more firms in the reference set can produce the same amount of output using less input, we conclude that the evaluated firm is inefficient. Therefore, DEA provides a performance measure for each unit, relative to other units under consideration. The key feature that makes the units comparable is that all firms use the same set of inputs and outputs and typically all these firms operate under a similar business environment. For example, when comparing performance of banks, one approach is to view banks as institutions that use capital and labor to produce loans and deposit account services. In this case, labor, capital, and operating costs can be considered as inputs and the number of accounts and transactions can be treated as outputs [61].

The DEA model was first introduced by Charnes, Cooper, and Rhodes in 1978 [16]. It has been extensively used in performance appraisal in a wide range of applications including financial performance as well as non-financial performance measurement. In the non-financial area, DEA has been applied in industry performance ranking [3], hospital performance comparison [6], university selection [14], and in electricity distribution districts [44], for instance. In the financial applications of DEA methodology, DEA has been applied to evaluate performance of banks [64], CRAF participants [11], defense business segments [12], and credit unions [47], etc. A detailed discussion on applications of DEA will be provided in Chapter 2.

The DEA model is a nonlinear optimization problem of maximizing the output to input ratio. The basic preliminaries of mathematical optimization are summarized in the next section for convenience of the reader.

1.6 Mathematical Optimization: Review

A mathematical programming model is an optimization problem of the form

$$\begin{aligned} &\textit{minimize} && f(x) \\ &\textit{subject to} && g_i(x) \leq 0,\ i = 1, \ldots, m \\ &&& x \in X, \end{aligned} \tag{1.17}$$

where X is a nonempty subset of R^n and is in the domain of the real-valued functions, $f : R^n \to R^1$, and $g : R^n \to R^1$ for $i = 1, \ldots, m$. The relation, $g_i(x) \leq 0$ is called a constraint, and $f(x)$ is called the objective function, see [9] for details.

A given $x \in R^n$ is feasible if it is in the domain of X and satisfies the constraints $g_i(x) \leq 0, \forall i$. A point x^* is said to be a global optimum if it is feasible and if the value of the objective function is not more than that of any other feasible solution: $f(x^*) \leq f(x)$ for all feasible x. A point $\hat{x}$ is said to be a local optimum if there exists an ϵ-neighborhood $N_\epsilon(\hat{x})$, i.e., a ball of radius ϵ with center at $\hat{x}$, such that $f(\hat{x}) \leq f(x)$ for each $x \in X \bigcap N_\epsilon(\hat{x})$.

A set S is said to be convex if the line segment connecting any two points in the set belongs to the set. That is, if x_1 and x_2 are any two points in the set S, then a linear combination of these two points, denoted by $\lambda x_1 + (1-\lambda)x_2$, is in S for any $\lambda \in [0,1]$. A function f is said to be convex on S if $f\left[\lambda x_1 + (1-\lambda)x_2\right] \leq \lambda f(x_1) + (1-\lambda)f(x_2)$, for each x_1 and x_2 in S and for each $\lambda \in [0,1]$. The function f is said to be quasiconvex if for each $x_1, x_2 \in S$, the following inequality holds: $f\left[\lambda x_1 + (1-\lambda)x_2\right] \leq maximum \left\{f(x_1), f(x_2)\right\}$. Let E be a nonempty set and f is differentiable on E. The function f is said to be pseudoconvex if for each $x_1, x_2 \in S$ with $\nabla f(x_1)^{'}(x_2 - x_1) \geq 0$ we have $f(x_2) \geq f(x_1)$. A convex function is also pseudoconvex, but not vice versa; a pseudoconvex function is also quasiconvex; but not vice versa. The negative of a convex function is a concave.

The Karush-Kuhn-Tucker (KKT) conditions, see [9], are usually used to verify if a solution x^* is (global/local) optimal to an optimization problem. The KKT conditions for the problem in (1.17) are given as follows:

$$\left.\begin{array}{l}\nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla_x g_i(x) = 0 \\ \lambda_i g_i(x) = 0 \; i = 1, \ldots, m \\ g_i(x) \leq 0, \, i = 1, \ldots, m \\ \lambda_i \geq 0, \; i = 1, \ldots, m,\end{array}\right\} \tag{1.18}$$

where $\nabla_x f$ represents the gradient vector with respect to x. If x^* is a local minimum for the problem (1.17) and Constraint Qualification holds at x^*, then x^* satisfies the KKT conditions in (1.18). In addition, if $f(x)$ and $g_i(x)$ are differentiable, $f(x)$ is pseudoconvex and $g_i(x), i = 1, \ldots m$ is quasiconvex, x^* is a global minimum solution to the problem (1.17) if x^* satisfies the KKT conditions in (1.18).

The Generalized DEA (GDEA) model developed in this research has a form similar to problem (1.17). However, the GDEA model is a difficult problem because the objective function is neither convex, pseudoconvex, nor quasiconvex, and also, it is non-differentiable. This thesis develops a direct search method to solve this problem and use the conditions that are similar to (1.18) to test the optimality of a solution.

1.7 The Scope of Research

The scope of this research is limited to determining a relative financial strength metric for publicly-traded firms using financial statement data and to demonstrate the use of that metric in portfolio selection. In the sequel, this thesis develops both the detailed theoretical framework and the methodological solution approach to accomplish the above tasks. In particular, the standard DEA modeling paradigm is generalized in which the input/output selection process is defined under an optimization criterion. Such an approach has not been taken in the existing literature of DEA theory and applications. Moreover, statistical tests are developed for validation of the approach.

Through an application involving more than 800 firms from the U.S. stock markets, the developed procedures are empirically tested, and portfolios are analyzed and compared with standard stock selection methodologies. Furthermore, this thesis also introduces the notion of expert information into DEA modeling. However, the expert information considered in this thesis is limited to those that are concerned with input/output selection for DEA models.

It must be stated that no attempt is made in this thesis to develop methodologies to determine if a given stock is over- or under-valued. Furthermore, the view of fundamental analysis as treated in this thesis is limited to that provided by the collective set of financial statements of firms in an industry or sector of the market. No attempt has been made to factor macroeconomic information into the analysis in this thesis.

1.8 Outline of the Thesis

The thesis is organized in 8 chapters. The topics covered in each chapter of this thesis are outlined below.

- Chapter 2 provides the mathematical formulation of a standard DEA model, along with some pertinent properties.

- Chapter 3 develops a new DEA-based financial strength measure based on financial statements of public firms. An application to the U.S. Technology sector is provided for illustration.

- Chapter 4 develops a new generalization of the DEA approach, where inputs/outputs are specified endogenously. An optimization model is developed to maximize the correlation between the DEA-based financial strength and the firms (stock) market performance and a two-stage (heuristic) solution scheme is developed for its solution. The method is applied to the Technology sector and compared to the standard DEA.

- Chapter 5 identifies several drawbacks of the GDEA-based strength measure. The model is enhanced by developing a Corrected GDEA model.

- Chapter 6 discusses the question of input/output selection under Expert Information (EI). The utility of such EI is tested via the developed Value of Expert Information measure. Pertinent theoretical properties of the model, as well as its optimality conditions, are derived.

- Chapter 7 provides a detailed Case Study of the GDEA-based strength measure (under expert information) to portfolio optimization using about 800 firms in 9 sectors that collectively span the major stocks in the U.S. stock markets, including S&P 500 firms.

- Chapter 8 concludes this dissertation and presents directions for future research.

Chapter 2

DEA Methodology

DEA is a nonparametric method for measuring the relative efficiencies of a set of similar decision making units (DMUs) by relating their outputs to their inputs and categorizing the DMUs into managerially efficient and managerially inefficient. Nonparametric models differ from parametric models in that the model structure is not specified a priori but is instead determined from data. Therefore, nonparametric methods require very few assumptions about the form of the population distribution from which the data are sampled, see [59].

The idea of DEA was germinated by Farrell [28] who was motivated by the need for developing better methods and models for evaluating productivity. In 1978, Charnes, Cooper, and Rhodes introduced the first DEA model [16] by combining Farrell's idea and multidimensional engineering efficiency. This model is called the CCR ratio model, which is based on a nonlinear programming formulation that seeks to optimize the ratio of a linear combination of outputs to a linear combination of inputs and subject to production constraints to determine the (managerial) DEA-efficiency of a given DMU relative to other DMUs. Thus, the application of a DEA model presupposes that a certain set of inputs and outputs are clearly identified for the case in hand. Therefore, depending on the application, these input/output sets of performance measurement can be very different. Various applications from both financial and nonfinancial areas will be considered next to highlight such choices in inputs and outputs.

2.1 The applications of the DEA model

DEA models have been widely applied in both financial and non-financial areas. In the non-financial DEA applications, Ali and Nakosteen [3] rank the economic performance of different industries. Barua and Brockett et al. [8] compare the performance between internet dot com companies that produce only physical products and those that produce only digital products. Banker [6] examines the performance of 117 North Carolina hospitals. Moreover, Carrico and Hogan et al. [14] develop a decision-making process for university selection, and Miliotis [44] measures the efficiency for 45 electricity distribution districts in Greece.

In the financial applications of DEA methodology, one particularly appealing idea is to measure managerial efficiency of a company by using its financial statements. In such cases, data from financial reports of a given firm can be used as inputs and outputs for a DEA model. For example, using certain financial ratios as inputs and outputs, Yeh [64] uses the DEA model to evaluate performance of banks. Bowlin [11] compares the importance of United States Department of Defense's Civil Reserve Air Fleet (CRAF) participants. Bowlin [12] also examines the financial performance of defense-oriented business segments, compared to non-defense business segments. Pille [47] investigates the weakness of credit unions in Ontario. Ozcan [45] derives an aggregate metric, termed "financial performance index (FPI)", using DEA and compares it with various financial ratios to indicate performance levels. Seiford [55] develops a two-stage DEA model to examine the profitability and marketability of large banks. DEA has also been investigated as a viable modeling tool in comparison to alternative methods. For instance, Cielen [17] compares DEA and decision tree models in the classification of performance of bankruptcy predictions. Gregoriou [31] compares DEA-based efficiencies and Sharpe ratios [56] to evaluate performance of different hedge funds. Alam and Robin [1] compute relative technical efficiencies for firms in the airline industry and analyze their association with corresponding stock price returns. However, their work is based upon input and output variables that are generally non-financial in nature and they are typically not found in the publicly available financial statements. The corresponding input and output selections for the above DEA models are summarized in Table 2.1.

2.2 Graphical example of DEA

The basic idea of DEA can be demonstrated using a simple two-dimensional example. Assume there is a group of workers (i.e., group of DMUs), denoted by A, B, C, D, and E, each using the same amount of a single input (denoted by x) and producing

Table 2.1: Inputs/outputs selection of the DEA model

Application Area	*Inputs*	*Outputs*
Industry Ranking	cost of materials, production hour, and capital expenditures	value added and value of shipments
Dot Com Companies	IT capital, NIT capital, labor, and number of years in business	sales and gross margins
Rate Department	total costs of rates collection	non-council hereditaments, rate rebates granted, summonses issued and distress warrants obtained, and net present value of non-council rates collected
University Selection	entry points to university	student/staff ratio, library spending, accommodation, teaching assessment, and proportion of first class degrees awarded, and research
Electricity Distribution Districts	network, capacity of installed transformation points, general expenses, administrative labor, and technical labor	number of customers and energy supplied
Hospital	nursing service hours, general service hours, ancillary service hours, and number of beds	patient days for patients less than 14 years old, between 14 and 65 years in age, and more than 65 years old
Bank	number of employees, assets, and equity	revenue and profit
CRAF Participants	operating cash flow, free cash flow, operating income, net income, sales, market value, and market returns	total assets, operating expenses, number of shares of common stock outstanding, number of employees, and property, plant, and equipment
Defense Business Segment	operating expenses and identifiable assets	operating profit, operating cash flow, and sales
Ontario Credit Unions	non-interest expense, and deposits	loans, cash, and investment, equity, and net interest income and other incomes
Mutual Fund	transaction costs, operational expenses, management fees, and market and administrative expenses	mean portfolio return of mutual fund
Hedge Fund	lower mean monthly semi-skewness, lower mean monthly semi-variance, and mean monthly lower return	upper mean monthly semi-skewness, upper mean monthly semi-variance, and mean monthly upper return

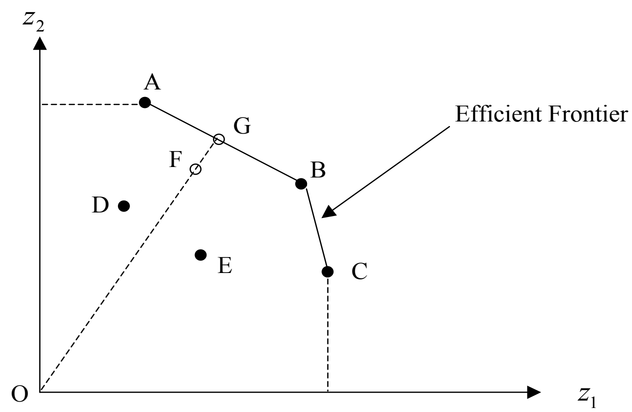

Figure 2.1: Graphical example of DEA with one input and two outputs

different amounts of two outputs (denoted by y_1 and y_2) at a production facility. The ratio of output to input is computed for a given worker and it is compared with that of every other worker. Figure 2.1 shows a plot of the output/input ratios, $z_1 = y_1/x$ and $z_2 = y_2/x$, for each of the workers.

A given point (DMU) is a nondominated DMU if and only if there does not exist any other DMU having a higher value in each coordinate $z_k, \forall k$. Thus, DMU$_i$ dominates DMU$_j$, denoted by $DMU_i \succ DMU_j$, if the following is satisfied:

Definition 2.2.1 *$DMU_i \succ DMU_j \Leftrightarrow z_{ik} \geq z_{jk}, \forall k,$ and for at least one k, $z_{ik} > z_{jk}$ holds, where k is the index number of dimensions K, $k = 1, \ldots, K$.*

From Figure 2.1, it is clear that workers D and E are dominated by worker B because B is higher in both coordinates than those of either D or E. Thus, D and E are in the dominated set. In contrast, A, B, and C are not dominated by any other worker, hence, A, B, and C are in the non-dominated set. Connecting points in the non-dominated set forms the efficient frontier, which is a piecewise linear curve, stretches around the periphery of all workers, forming an envelope around the set of all points. Any points on the line segments that connect two non-dominated points are on the efficient frontier. Workers on the efficient frontier are considered to be efficient, i.e., A, B, and C. Workers that are not on the efficient frontier, such as D and E, are termed inefficient in the presence of A, B, and C.

Suppose we add another worker F in the worker group. It should be noted that worker F is not dominated by workers A, B, and C. However, if we extend the line OF to the efficient frontier and let the point G be the intersection, it is clear that G dominates F. Thus, F is inefficient compared to the virtual worker G and the efficiency of F is determined by the ratio OF/OG. The efficiencies of D and E can be computed in a similar manner. In the case when there are two inputs and only one output, a similar analysis to the above can be performed.

When considering the case of multiple inputs $(x_1, \ldots, x_n)$ and only two outputs (y_1 and y_2), a representative input is needed and it can be obtained by aggregating the input values into a composite input, i.e., $x(u) = \sum_{i=1}^{n} u_i x_i$, where $u_i, i = 1, \ldots, n$ are input multipliers, also see Section 1.4.2. For a given multiplier vector $u \in R^n$, define the output to input ratios $z_1(u) = y_1/x(u)$ and $z_2(u) = y_2/x(u)$. These ratios for all workers are plotted in Figure 2.2 for two given vectors $\bar{u} \in R^n$ and $\hat{u} \in R^n$, where Figure 2.2 (a) uses $u = \bar{u}$ and Figure 2.2 (b) uses $u = \hat{u}$.

When $u = \bar{u}$, notice that workers B, E, and F are not on the efficient frontier, thus, B, E and F are inefficient, while workers A, C, and D are efficient. However, as u is changed from $\bar{u}$ to $\hat{u}$, see Figure 2.2 (b), the position of each worker changes

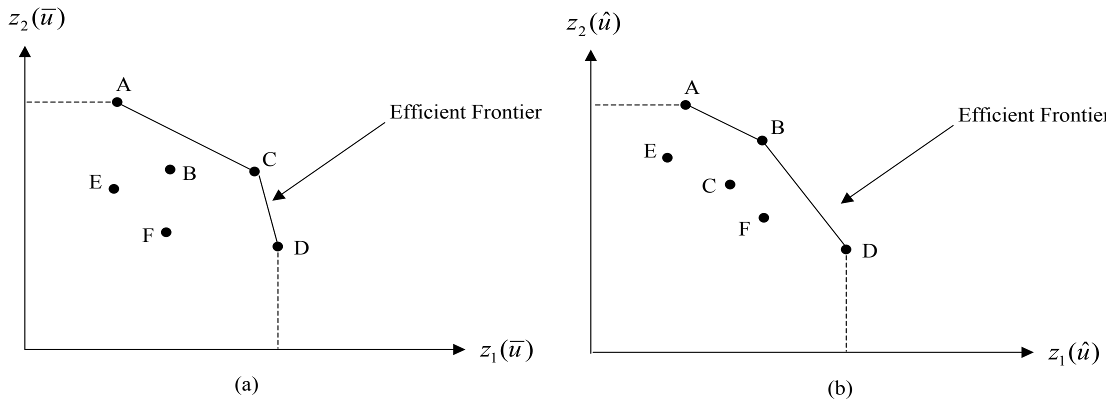

Figure 2.2: Graphical example of DEA with two outputs and multiple inputs

as shown. For example, the worker B is now brought on to the efficient frontier, and thus, B now becomes efficient.

This idea that by changing the aggregating multipliers, a given DMU, (such as B,) can attempt to be as efficient as possible, in the presence of other DMUs, is the basis of the DEA methodology. That is, a DMU under evaluation is given the freedom to choose its own multipliers so as to bring the DMU to the efficient frontier as close as possible. If the DMU can be brought to the efficient frontier by a certain choice of multipliers, then it is termed DEA-efficient; however, if such a set of multipliers does not exist, the DMU is said to be inefficient.

The above basic concept can be extended to multiple inputs and multiple outputs, where one identifies a vector of input multipliers (u) as well as a vector of output multipliers (v). However, the identification of DMUs on the efficient frontier becomes significantly more complicated as the dimension of inputs and outputs increases. Therefore, the graphical analysis above must be formalized for this purpose. The mathematical programming model provided by DEA is a step in this direction. Such a model allows one to compute the efficient frontier for a large number of DMUs with multiple inputs and outputs with relative ease.

2.3 CCR model

To explain the basic premise of a DEA model, let there be J independent DMUs (firms) whose performance (or efficiency) must be evaluated relative to each other. One begins with a given set of inputs parameters (say, M) and a given set of output parameters (say, N) which are measured for all J firms. For a given firm j, $j = 1, \ldots, J$, let ξ^i_{mj} be the (measured) level of input parameter m, $m = 1, \ldots, M$, while that of the output parameter n is denoted by ξ^o_{nj} for $n = 1, \ldots, N$. The input and output nonnegative multipliers for firm j are denoted by the variables u_m and v_n, $m = 1, \ldots, M$ and $n = 1, \ldots, N$, respectively. Then, by taking the nonnegative linear combination of the M inputs, the inputs are aggregated as $\sum_{m=1}^{M} \xi^i_{mj} u_m$, and the outputs can be aggregated as $\sum_{n=1}^{N} \xi^o_{nj} u_n$ using a similar manner, see Section 1.4.2. The productivity (for firm j) that measures how well the firm j (in the group of J firms) converts its M inputs to the N outputs, can be computed as the ratio of the aggregated output measure to the aggregated input measure, i.e., $\sum_{n=1}^{N} \xi^o_{nj} u_n / \sum_{m=1}^{M} \xi^i_{mj} u_m$. By choosing its own input and output multipliers, the productivity of firm $j(= 1, \ldots, J)$ is to be brought on to the efficient frontier as much as possible.

Following this idea, the relative efficiency f_k of a given firm k (from the set of J firms), is then defined as its maximized productivity, determined over all possible aggregating multipliers such that no firm in the group will attain a relative performance measure greater than unity. This results in the so-called CCR model [16], as follows:

$$\begin{aligned} f_k := \ \max_{u,v} \quad & \frac{\sum_{n=1}^{N} \xi_{nk}^{o} v_n}{\sum_{m=1}^{M} \xi_{mk}^{i} u_m} \\ \text{s.t.} \quad & \frac{\sum_{n=1}^{N} \xi_{nj}^{o} v_n}{\sum_{m=1}^{M} \xi_{mj}^{i} u_m} \leq 1, \quad j = 1, \ldots, J \\ & u_m, v_n \geq 0, \ m = 1, \ldots, M, \ n = 1, \ldots, N. \end{aligned} \tag{2.1}$$

The model in (2.1) yields the maximum achievable efficiency for firm k, denoted by f_k, provided every other firm is also applying the same aggregating nonnegative multipliers in computing their input to output conversion ratios, and it is termed the DEA efficiency score of firm k. An efficiency score of less than one is indicative of that it may be possible to decrease the level of input for the same level of output, while a score of 1 indicates the firm is DEA-efficient. By applying (2.1) to each firm independently, the respective (maximum) relative efficiency score for each firm is computed.

In order to simplify the CCR ratio model in (2.1), Charnes and Cooper [15] employed a linear transformation method, which results in a linear programming (LP) formulation. The equivalent linear programming formulation of model (2.1) is, see [16]:

$$\begin{aligned} \hat{f}_k := \max_{u,v} \quad & \sum_{n=1}^{N} \xi^o_{nk} v_n \\ \text{s.t.} \quad & \sum_{m=1}^{M} \xi^i_{mk} u_m = 1 \\ & -\sum_{m=1}^{M} \xi^i_{mj} u_m + \sum_{n=1}^{N} \xi^o_{nj} v_n \leq 0, \ j = 1, \ldots, J \\ & u_m, v_n \geq 0, \ m = 1, \ldots, M, \ n = 1, \ldots, N. \end{aligned} \tag{2.2}$$

It is straightforward to show that $\hat{f}_k = f_k$ holds under the nonnegativity of the observed data. More precisely, if $\xi^i_{mk} > 0$ for some $m = 1, \ldots, M$, then, $\hat{f}_k = f_k$ holds.

One property of the CCR model is that as the number of inputs and outputs increased, the efficiency score calculated by (2.2) will have a better chance to reach the efficient frontier. This is caused by model saturation, a phenomenon attributed to over-specifying parameters in a model. A similar situation also occurs in statistical regression modeling, where the coefficient of determination R^2 can be arbitrarily increased towards 1 by adding (or saturating) more independent variables. In this case, R^2 fails to capture the predictive power of the model. Likewise, the choice of parameters in a DEA model is of paramount interest.

To show the explicit dependence of $\hat{f}_k$ in (2.2) on the number of inputs/outputs, let $\hat{f}_k(\Pi_I, \Pi_O)$ be the efficiency score when using the input set Π_I and the output set Π_O. The following properties holds for the CCR model.

Proposition 2.3.1 *1.* $\hat{f}_k(\Pi^1_I, \Pi_O) \leq \hat{f}_k(\Pi^2_I, \Pi_O)$ *if* $\Pi^1_I \subseteq \Pi^2_I$*;*

2. $\hat{f}_k(\Pi_I, \Pi_O^1) \leq \hat{f}_k(\Pi_I, \Pi_O^2)$ *if* $\Pi_O^1 \subseteq \Pi_O^2$*;*

3. $\hat{f}_k(\Pi_I^1, \Pi_O^1) \leq \hat{f}_k(\Pi_I^2, \Pi_O^2)$ *if* $\Pi_I^1 \subseteq \Pi_I^2$ *and* $\Pi_O^1 \subseteq \Pi_O^2$*.*

Proof.

1. $\Pi_I^1 \subseteq \Pi_I^2$ implies that model (2.2) with input set Π_I^2 is a relaxed problem compared to model (2.2) having input set Π_I^1. Thus, the maximized objective value is greater under the input set Π_I^2.

2. Similarly, $\Pi_O^1 \subseteq \Pi_O^2$ implies that model (2.2) with output set Π_O^2 is a relaxed problem compared to model (2.2) with output set Π_O^1. Given the maximization objective of the model, the result follows.

3. Combining the results of 1 and 2, the result follows. ■

Proposition 2.3.2 $\hat{f}_k$ *in (2.2) is positively homogeneous of degree zero in* ξ_{mk}^i *and* ξ_{nk}^o *jointly and separately, for* $m = 1, \ldots, M$*,* $n = 1, \ldots, N$*.*

Proof. Write the optimal value of (2.2) as a function of an input/output pair (m, n) of firm k as $\hat{f}_k(\xi_{mk}^i \xi_{nk}^o)$. For the joint homogeneity (of degree zero), we must show that $\hat{f}_k(\lambda\xi_{mk}^i, \lambda\xi_{nk}^o) = \hat{f}_k(\xi_{mk}^i, \xi_{nk}^o)$ for all scalars $\lambda > 0$. Let an optimal solution of model (2.2) be given by (u^*, v^*), i.e., $\hat{f}_k(\xi_{mk}^i, \xi_{nk}^o) = \sum_{n=1}^N \xi_{nk}^o v_n^*$. When the input and output data is increased by λ, construct the solution $(\hat{u}, \hat{v})$ such that $\hat{u}_m = \lambda u_m^*$ and $\hat{v}_n = \lambda v_m^*$. Then, this "hat" solution can be verified to be feasible in (2.2) for $(\lambda\xi_{mk}^i, \lambda\xi_{mk}^o)$ with the associate objective value $\hat{f}_k(\xi_{mk}^i, \xi_{nk}^o)$. Thus, $\hat{f}_k(\lambda\xi_{mk}^i, \lambda\xi_{nk}^o) \geq \hat{f}_k(\xi_{mk}^i, \xi_{nk}^o)$. Similarly, considering an optimal solution u^{**}, v^{**} of the model (2.2) with $(\lambda\xi_{mk}^i, \lambda\xi_{nk}^o)$, one can claim analogously $\hat{f}_k(\xi_{mk}^i, \xi_{nk}^o) \geq \hat{f}_k(\lambda\xi_{mk}^i, \lambda\xi_{nk}^o)$. Thus, $\hat{f}_k(\lambda\xi_{mk}^i, \lambda\xi_{nk}^o) = \hat{f}_k(\xi_{mk}^i, \xi_{nk}^o)$ follows.

Table 2.2: Financial characteristics for three firms in the example

	Parameter	Company A	Company B	Company C
Output	Net profit margin	0.05	0.3	0.8
	Earnings per share	12	18	20
Input	Current ratio	4.7	2.2	3.6
	Inventory Turnover	6.8	5.6	12.1
	Leverage Ratio	1.3	1.8	6.5

The positive homogeneity (of degree zero) in (ξ^i_{mk}, ξ^o_{nk}) separately can also be shown in an analogous manner, i.e., $\hat{f}_k(\lambda\xi^i_{mk}, \xi^o_{nk}) = \hat{f}_k(\xi^i_{mk}, \xi^o_{nk})$ and $\hat{f}_k(\xi^i_{mk}, \lambda\xi^o_{nk}) = \hat{f}_k(\xi^i_{mk}, \xi^o_{nk})$ for $\lambda > 0$. ■

The above homogeneity property will be utilized in the generalization of the DEA model, termed the GDEA model, see Section 4.1.1. In particular, this property enables an efficient description of the feasible domain for the GDEA model.

2.4 A numerical example of CCR model

Consider three companies A, B, and C, each with the financial characteristics given in Table 2.2.

Suppose v_i, $i = 1, 2$ represents the multipliers for outputs and u_j, $j = 1, 2, 3$ represents the multipliers for inputs. The DEA models for evaluating each company are listed below.

$$
\begin{aligned}
\textit{Company A: maximize}\quad & 0.05v_1 + 12v_2 \\
\text{s.t}\quad & 4.7u_1 + 6.8u_2 + 1.3u_3 \leq 1 \\
& -4.7u_1 - 6.8u_2 - 1.3u_3 + 0.05v_1 + 12v_2 \leq 0 \\
& -2.2u_1 - 5.6u_2 - 1.8u_3 + 0.3v_1 + 18v_2 \leq 0 \\
& -3.6u_1 - 12.1u_2 - 6.5u_3 + 0.8v_1 + 20v_2 \leq 0 \\
& u_1, u_2, u_3, v_1, v_2 \geq 0.
\end{aligned}
$$

$$
\begin{aligned}
\textit{Company B: maximize}\quad & 0.3v_1 + 18v_2 \\
\text{s.t}\quad & 2.2u_1 + 5.6u_2 + 1.8u_3 \leq 1 \\
& -4.7u_1 - 6.8u_2 - 1.3u_3 + 0.05v_1 + 12v_2 \leq 0 \\
& -2.2u_1 - 5.6u_2 - 1.8u_3 + 0.3v_1 + 18v_2 \leq 0 \\
& -3.6u_1 - 12.1u_2 - 6.5u_3 + 0.8v_1 + 20v_2 \leq 0 \\
& u_1, u_2, u_3, v_1, v_2 \geq 0.
\end{aligned}
$$

$$
\begin{aligned}
\textit{Company C: maximize}\quad & 0.8v_1 + 20v_2 \\
\text{s.t}\quad & 3.6u_1 + 12.1u_2 + 6.5u_3 \leq 1 \\
& -4.7u_1 - 6.8u_2 - 1.3u_3 + 0.05v_1 + 12v_2 \leq 0 \\
& -2.2u_1 - 5.6u_2 - 1.8u_3 + 0.3v_1 + 18v_2 \leq 0 \\
& -3.6u_1 - 12.1u_2 - 6.5u_3 + 0.8v_1 + 20v_2 \leq 0 \\
& u_1, u_2, u_3, v_1, v_2 \geq 0.
\end{aligned}
$$

Table 2.3: Summary of the optimal results of the DEA models in the example

Firm	Objective Value	Variable				
	($\hat{f}$)	u_1	u_2	u_3	v_1	v_2
A	0.923	0	0	0.769	0	0.077
B	1	0	0	0.056	0	0.056
C	1	0.278	0	0	0.688	0.022

The optimal solutions from the above LPs are summarized in Table 2.3.

The objective value column in Table 2.3 represents the relative efficiency of each firm. It is evident that firm B and C have efficiency of 1, implying these two firms are DEA-efficient. The efficiency of firm A is 0.923, indicating DEA-inefficient relative to the two efficient firms B and C.

The variables u and v in Table 2.3 represent the multipliers of inputs and outputs in the model. A nonzero multiplier indicates an input or output that contributes to increasing the efficiency of the evaluated firm. If a multiplier is zero, the corresponding input or output is non-contributive. For example, for firm C, the multipliers for current ratio, net profit margin, and earnings per share are nonzero. This indicates that the most preferred way for firm C to measure its productivity, in comparison to firm A and B, is to measure only two outputs, net profit margin and earnings per share, with respect to a single input, current ratio. In this sense, firm C emerges DEA-efficient when the same input and output multipliers are applied to the A and B as well. In contrast, firm A which chooses the sole input, leverage ratio, and the sole output, earnings per share, finds it impossible to be DEA-efficient and its best efficiency is still only 92.3%.

2.5 Returns to scale and DEA Modeling

2.5.1 Basic concepts on returns to scale

Returns to scale (RTS) is a key concept in economics. It indicates how production changes if we increase all inputs by a constant multiplicative amount λ, where $\lambda > 1$. If a given proportionate increase in all inputs leads to the same proportionate increase in production, the production system is said to display Constant Returns to Scale (CRS). If a given proportionate increase in all inputs leads to even a greater proportionate increase in output, we have Increasing Returns to Scale (IRS). If the proportionate increase in output is less than a given proportionate increase in all inputs, the system has Decreasing Returns to Scale (DRS). The latter two cases are commonly referred to as Variable Returns to Scale (VRS) in production.

Consider a production process in which a firm uses M input amounts $(x_1, \ldots, x_M)$ to produce a single output amount (y). Let $x = \sum_{m=1}^{M} x_m u_m$ be the aggregated input, where $u_m, m = 1, \ldots, M$ are input multipliers. The production function, denoted by $f(x)$, is defined as the maximum quantity of output (y) that can be produced by the aggregated input (x), i.e., $y = f(x)$. The production function has the following properties, see [18].

- Nonnegativity: the production function is defined only for nonnegative values of the input and output levels;

- Weak Essentiality: a positive minimum level of input is required to produce positive output;

- Nondecreasing in x: the production function is normally assumed to be non-decreasing;

- Concave in x: any linear combination of input vectors x^1 and x^2 will produce an output that is no less than the same linear combination of $f(x^1)$ and $f(x^2)$, see Section 1.17 for the definition of concavity.

As discussed in Section 1.4.1, the average productivity is computed by $P(x) = \frac{f(x)}{x}$, at some given point x on the production function. By the definitions of different types of RTS introduced above, it is clear that the production is IRS, DRS, or CRS depending on whether $P(x)$ increases, decreases, or remains the same, respectively, as x increases. It should be noted that the concavity property of $f(x)$, plus x being linear (and $x > 0$ for positive output), implies that the average productivity $P(x)$ is a pseudoconcave function, see [42] for details. Therefore, when the first derivative $P^{'}(x) = 0$, the average productivity $P(x)$ reaches a maximum at a finite level of x, see [9]. Furthermore,

$$P^{'}(x) = \frac{xf'(x) - f(x)}{x^2} = \frac{f(x)}{x^2}\left[\frac{xf'(x)}{f(x)} - 1\right]. \tag{2.3}$$

Both $f(x)$ and x are positive for a production system that produces positive outputs, thus, when $P^{'}(x) = 0$, we have

$$\frac{xf'(x)}{f(x)} = 1 \Rightarrow f'(x) = \frac{f(x)}{x} = P(x). \tag{2.4}$$

Let the input x be increased by a multiplicative amount λ to λx, where $\lambda > 1$, and $\hat{P}(\lambda x) = \frac{f(\lambda x)}{\lambda x}$ is the corresponding productivity. By applying the definition of the derivative of the function f at x, at maximum productivity,

$$P(x) = f^{'}(x) = lim_{\lambda \to 1} \frac{f(\lambda x) - f(x)}{\lambda x - x}. \tag{2.5}$$

By contradiction, suppose $f(\lambda x) > \lambda f(x)$. Then, $lim_{\lambda \to 1} \frac{f(\lambda x) - f(x)}{\lambda x - x} > lim_{\lambda \to 1} \frac{\lambda f(x) - f(x)}{\lambda x - x}$ holds, which implies $f^{'}(x) > P(x)$. On the other hand, if $f(\lambda x) < \lambda f(x)$, then, $lim_{\lambda \to 1} \frac{f(\lambda x) - f(x)}{\lambda x - x} < lim_{\lambda \to 1} \frac{\lambda f(x) - f(x)}{\lambda x - x}$ holds, and thus, $f^{'}(x) < P(x)$. Therefore, it is clear that $f^{'}(x) = P(x)$ necessarily implies that $f(\lambda x) = \lambda f(x)$. That is, at the maximum productivity, the production level displays CRS.

It should be noted that when $f(\lambda x) > \lambda f(x)$, we have $\frac{f(\lambda x)}{\lambda x} > \frac{\lambda f(x)}{\lambda x}$, thus, $P > \hat{P}$, which indicates the production system exhibits IRS. Similarly, we can show that the system displays DRS when $f(\lambda x) < \lambda f(x)$. Therefore, we have the following properties: when $\lambda > 1$,

$$\begin{aligned} \text{if } f(\lambda x) &> \lambda f(x), \text{ then } f^{'}(x) > P(x) \qquad (IRS); \\ \text{if } f(\lambda x) &< \lambda f(x), \text{ then } f^{'}(x) < P(x) \qquad (DRS); \\ \text{if } f(\lambda x) &= \lambda f(x), \text{ then } f^{'}(x) = P(x) \qquad (CRS). \end{aligned} \tag{2.6}$$

Note that if $f(\lambda x) = \lambda f(x)$,

$$P(\lambda x) = \frac{f(\lambda x)}{\lambda x} = \frac{\lambda f(x)}{\lambda x} = P(x), \tag{2.7}$$

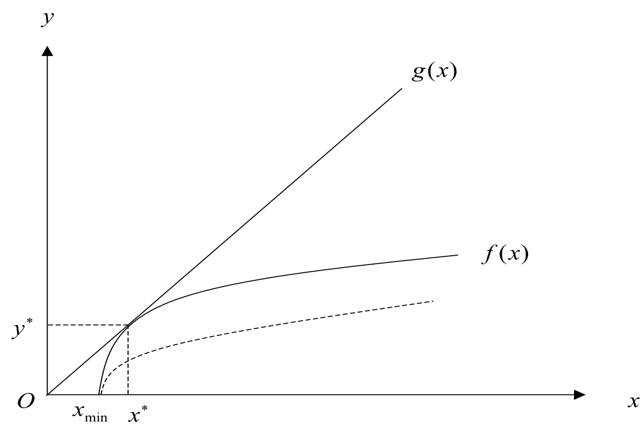

Figure 2.3: Production function under returns to scale

which implies that the productivity is positively homogeneous of degree zero. The CCR model introduced in Section 2.3 has the same property, thus, the CCR model assumes CRS.

A production process may allow multiple returns to scale, i.e., IRS, CRS, and DRS. Figure 2.3 shows various returns to scale in a production function ($f(x)$) with multiple inputs and a single output, where x_{min} is the minimum input level below which production cannot occur. It is observed that from x_{min} to x^*, $f'(x) > P(x)$, indicating an IRS. Beyond input level x^*, $f'(x) < P(x)$, exhibiting DRS in this region. At the point x^*, $f'(x) = P(x)$, hence CRS holds.

Suppose the single output (in the above case) is extended to N outputs, i.e., $(y_1, \ldots, y_N)$, where $N > 1$. The multi-output production function is defined as $F = [f_1, \ldots, f_N]'$, where $y_1 = f_1(x), \ldots,$ and $y_N = f_N(x)$, where $x = \sum_{m=1}^{M} x_m u_m$. The same aggregation technique discussed in Section 1.4.2 can be applied to find an composite production function, i.e., $\hat{f}(x) = \sum_{i=1}^{N} f_i(x) v_i$. The new productivity can then be computed by $\frac{\hat{f}(x)}{x}$. The RTS cases discussed above can also be shown in a similar manner.

2.5.2 Returns to scale approach with DEA model

If a firm's production curve overlaps with the production function $f(x)$, as shown in Figure 2.3, it indicates that the firm produces the maximum level of output using a certain level of input. If a firm's operating curve is below the production function $f(x)$, as presented by the broken curve, it shows that this firm does not manage its resources as efficiently as a firm that produces the maximum output. This type of efficiency is termed "technical efficiency", which is determined by a firm's managerial and operational capability.

On the other hand, a firm's production system may exhibit IRS, DRS, or CRS. If the system exhibits IRS, it is more efficient to produce with a larger plant. The firm may increase production scale to provide cheaper goods. However, if the system displays DRS, a smaller plant may be more preferable in order to increase productivity. If the production system presents CRS, it indicates that the firm is operating at the optimal scale size. Such a measurement that is based on a firm's actual scale conditions under which the firm is operating is termed "scale efficiency". For example, in Figure 2.3, $g(x)$, a ray from origin, is a tangent line to the production function $f(x)$. $g(x)$ is called CRS frontier and the scale efficiency is then determined by $\frac{f(x)}{g(x)}$. At the operating point (x^*, y^*), the firm's scale efficiency is 1, thus, the average productivity of the firm reaches its maximum.

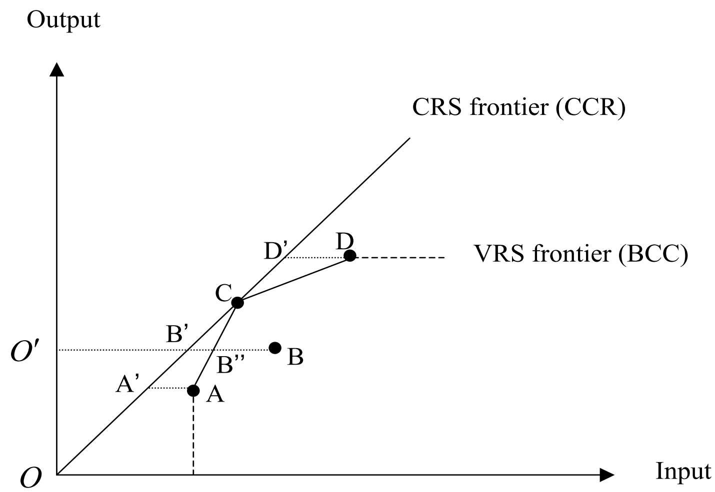

Figure 2.4: CRS vs VRS frontier

Both technical efficiency and scale efficiency are measured in the DEA models. As discussed in the previous section, the CCR model assumes a CRS relationship between inputs and outputs, see [20]. On the other hand, DEA models that are based on VRS assumption have been proposed, see the BCC model [7], for instance. Figure 2.4 shows the comparison of efficient frontier for both CCR model and BCC model in a single input single output case.

Suppose we have four firms, A, B, C, and D as shown in Figure 2.4. Ray OC is the CRS frontier (or CCR frontier). AC and CD constitute the VRS frontier (or BCC frontier). In order to be efficient under CRS assumption, a firm has to be operating at its optimal scale, that is, the scale efficiency must be 1. In addition, the firm must be technically efficient as well. It can be observed in Figure 2.4 that only firm C can be described as CRS-efficient because it is both technically efficient and scale efficient. However, when imposing VRS, a firm maybe technically efficient but not operating at its optimal scale, i.e., firm A and D are on the VRS frontier but not on CRS frontier. Furthermore, on the line segment AC, IRS prevails to the left of C, and on the line segment CD, DRS prevails to the right of C. At the point C, CRS is exhibited. Let the efficiencies that are evaluated under both CCR and BCC models be termed CCR-efficiency and BCC-efficiency, respectively. The difference between these two efficiencies can be illustrated by the following example. In Figure 2.4, firm B is neither CCR-efficient nor BCC-efficient. The technical efficiency under CRS assumption of firm B is TE-CRS=$\frac{O'B'}{O'B}$. The technical efficiency under VRS assumption is TE-VRS=$\frac{O'B''}{O'B}$ and the scale efficiency is SE=$\frac{O'B'}{O'B''}$. The relationship between CCR-efficiency and BCC-efficiency can be summarized as follows:

$$\text{CCR-efficiency} = \text{TE-CRS} = \text{TE-VRS} \times \text{SE} = \text{BCC-efficiency} \times \text{SE}$$

Same can be applied to firms A and D. For both of these two firms, BCC-efficiency=1; however, for firm A, CCR-efficiency is determined by $\frac{O'A'}{O'A}$, and that for firm D is $\frac{O'D'}{O'D}$. It is clear that BCC-efficiency will always be greater than equal to CCR-efficiency because the scale efficiency is always bound by 1. Also note that firm C is both CCR-efficient and BCC-efficient.

Therefore, if the BCC model is used to evaluate a firm's performance, only productivity inefficiency that is caused by managerial and operational drawbacks in the firm's activities is captured, while the production scale condition of the firm is not taken into consideration. Hence, an inefficient firm is only compared to efficient ones of similar scale, thus the efficiency score only represents the pure technical efficiency. However, if the CCR model is used, both technical efficiency and scale efficiency are under consideration, thus, the efficiency score captures not only the productivity inefficiency of a firm at its actual scale size, but also any inefficiency due to its actual scale size being different from the optimal scale size [6].

In the financial application of DEA in this thesis, the objective is to screen companies within a given market segment based on their business performance attributes, although these firms may be of different scale sizes. In this sense, the CCR model is seen to provide a viable approach (over BCC) for measuring the underlying fundamental business strength in order to make successful investment decisions.

2.6 DEA model and financial statements

In order to apply the CCR model, the input parameters and output parameters are required to be explicitly identified *a priori*. While this may be possible in evaluating the efficiency of production processes where input (such as raw materials and labor) to output (such as product and services) conversion mechanism are well-understood, our case is different. Instead of focusing on examining the internal managerial level of a firm using inputs and outputs collected from the production process, we use publicly available financial statement information as a proxy so that financial parameters derived from financial statements can be used as inputs and outputs of the DEA model. Thus, the underlying strength of a business can be measured, relative to other businesses in the same market segment. By doing so, a relative ranking system can be established to determine how strong/weak a firm is, in the presence of other competing firms, both in terms of if the firm is operating with optimal productivity and optimal scale size. Therefore, it provides a form of relative ranking that cannot be directly obtained from other valuation models. The way of identifying inputs and outputs from financial statements and how they can be incorporated with DEA model will be discussed in the next chapter.

Chapter 3

DEA-based financial strength measure

This chapter focuses on using financial information obtained from publicly available (quarterly) financial statements as inputs and outputs in the DEA model to compute relative efficiency scores for firms in a given market segment, such as an industry group. In DEA, all required input and output parameters must be identified *a priori*, and in particular, when these parameters are obtained from financial statements, the resulting DEA scores may be interpreted to provide a relative measure of the concerned firm's financial strength. The aim here is to verify if such a measure can be strongly (positively) correlated with the stock market returns, and to develop statistical tests that are necessary to establish the significance of that correlation. Indeed, the latter correlation depends on the chosen inputs and outputs. The basic premise is that a financial strength indicator computed using an appropriate choice of inputs/outputs would be expected to have high correlation with the stock market returns under the Efficient Market Hypothesis (EMH). Can such a financial strength indicator have predictive power of the direction of (quarterly) stock prices? If so, well-informed assessments can be made for stock selection in investment portfolios.

3.1 Financial statement parameters

The input and output selection in the DEA model is of paramount importance for assessing the financial strength of a firm. In the context of comparing the financial health of commercial air carriers using DEA, Bowlin [11] stated that the specified inputs and outputs should correspond to financial measures used in analyzing the actual performance of a firm. Thus, the main concern is to choose metrics that are generally used by the accounting community to establish financial performance of firms and use those metrics as inputs/outputs of the DEA model.

A number of different approaches might be used in assessing the financial performance a firm. For many investors, financial statements are the only sources of financial information they can obtain for the firm. Thus, information from that firm's financial statements needs to be utilized in order to assess the financial condition and performance of the firm. In a *direct* approach of choosing inputs and outputs for the DEA model, raw parameters obtained from financial statements may be directly used, for instance, accounts receivables (AR), inventory (IN), total assets (TA), total liabilities (TL), long-term debt (LD), revenue (RV), net income (NI), and shareholder's equity (SE). Inputs and outputs for the DEA model may then be selected from these raw data.

On the other hand, ratio analysis is another approach to investigate the financial performance of a firm. However, no one ratio in itself is sufficient for realistic assessment of financial performance of a firm. With a group of ratios, however, reasonable judgments can be made, see [34]. Thereby, various financial ratios can be computed from financial statements in order to help an individual understand a firm's strength or weakness relative to those of competitors.

3.1.1 Ratio Definition

In order to assess a firm's business strength, we consider 18 financial ratios (or parameters) and the descriptions of these parameters are given below. These are standard descriptions, and they can be found in [21] and [52].

1. *Return on Equity* (P1) $= \frac{\textit{Net Income}}{\textit{Shareholders' Equity}}$

   Return on Equity measures how much profit a firm earned in comparison to shareholders' book-value investment. It shows how well a firm uses investment dollars to generate earnings growth.

2. *Return on Assets* (P2) $= \frac{\textit{Net Income}}{\textit{Total Assets}}$

   Return on Assets tells how profitable a firm is relative to its available assets.

It gives an idea of the effectiveness of the firm's management in using its assets to generate earnings.

3. *Net Profit Margin* (P3) $= \frac{\textit{Net Income}}{\textit{Sales}}$

   Net Profit Margin measures profitability with respect to sales generated. This number is an indication of how effective a firm is at cost control.

4. *Earnings per Share* (P4) $= \frac{\textit{Net Income - Dividends on Preferred Stock}}{\textit{Outstanding Shares}}$

   Earnings per Share serves as an indicator of a firm's profitability. It represents how much of earnings each share is entitled to.

5. *Receivables Turnover* (P5) $= \frac{\textit{Sales}}{\textit{Receivables}}$

   Receivables Turnover measures how many times the receivables have been turned over (into cash) within a given financial reporting period. It provides insight into quality of the receivables.

6. *Inventory Turnover* (P6) $= \frac{\textit{Cost of Goods Sold}}{\textit{Inventory}}$

   Inventory Turnover measures the number of times that the inventory has been turned over (sold) within a given financial reporting period. It is a good indicator of inventory quality and inventory management.

7. *Asset Turnover* (P7) $= \frac{\textit{Sales}}{\textit{Total Assets}}$

Asset Turnover measures a firm's efficiency of using its total assets to generate sales. It is an indicator on performance of the assets, whether they under performing or over performing.

8. *Current Ratio* (P8) $= \frac{\textit{Current Assets}}{\textit{Current Liabilities}}$

   Current Ratio determines if a firm is able to pay its short-term obligations with current assets. It tells us the current financial strength of the firm, primarily in terms of the cash and credit standing of the firm.

9. *Quick Ratio* (P9) $= \frac{\textit{Current Assets - Inventories}}{\textit{Current Liabilities}}$

   Quick Ratio is also called Acid Test Ratio, which is a more conservative measure of liquidity of a firm. It investigates the ability of a firm to meet short-term obligations with most liquid current assets, i.e, cash and cash equivalents, accounts receivables, etc.

10. *Debt to Equity Ratio* (P10) $= \frac{\textit{Long-term Debt}}{\textit{Shareholders' Equity}}$

    A measure of a firm's financial leverage calculated by dividing long-term debt by stockholder equity. This measure tells us the relative importance of long-term debt to the financial structure of a firm.

11. *Leverage Ratio* (P11) $= \frac{\textit{Total Assets}}{\textit{Shareholders' Equity}}$

    This ratio shows the percentage of assets centered in fixed assets compared to total equity. A higher percentage indicates that capital is frozen in the form of machinery and the margin for operating funds becomes too narrow for day-to-day operations.

12. *Solvency Ratio - I* (P12) $= \frac{\textit{Total Liability}}{\textit{Total Assets}}$

    This debt ratio highlights the relative importance of debt financing to the firm by showing the percentage of the firm's assets that is supported by debt financing.

13. *Solvency Ratio - II* (P13) $= \frac{\textit{Total Liability}}{\textit{Shareholders' Equity}}$

    This ratio serves a similar purpose to Solvency Ratio - I. It a measure of the extent to which a firm's debt financing is used relative to equity financing.

14. *Price to Earnings Ratio* (P14) $= \frac{\textit{Market Value per Share}}{\textit{Earnings per Share}}$

    Price to Earnings (P/E) Ratio examines the relationship between the stock price and the firm's earnings. It is sometimes referred to as the "multiple" because it shows how much investors are willing to pay per dollar of earnings.

15. *Price to Book Ratio* (P15) $= \frac{\textit{Market Capitalization}}{\textit{Book Value}}$

    Price to Book Ratio compares the market's valuation of a firm to the value of that firm as indicated on its financial statements. If it is below 1.0, then it means that the firm is selling below book value and theoretically below its liquidation value.

16. *Revenue Growth Rate* (P16) $= \dfrac{\textit{Current Quarter's Revenue}}{\textit{Previous Quarter's Revenue}} - 1$

    This ratio measures the growth potential of a firm's revenue.

17. *Net Income Growth Rate* (P17) $= \dfrac{\textit{Current Quarter's Net Income}}{\textit{Previous Quarter's Net Income}} - 1$

    This ratio measures the growth potential of a firm's net income.

18. *Earnings per Share Growth Rate* (P18) $= \dfrac{\textit{Current Quarter's EPS}}{\textit{Previous Quarter's EPS}} - 1$

    This ratio measures the growth potential of a firm's earnings per share.

These financial parameters represent a firm's underlying performance from different perspectives, such as profitability, asset utilization, liquidity, leverage, valuation, and growth perspectives. By applying these parameters (as inputs and outputs) to the DEA model, a measure of the firm's business strength, relative to its competitors, can be determined.

3.2 Possible input/output selection of DEA model

There are multiple ways of categorizing the available financial parameters as inputs and outputs for the DEA model. On one hand, the raw data that are directly obtained from the financial statements can be used as inputs and outputs of the model. On the other hand, the computed financial ratios based on the raw data can also be considered as input/output candidates.

3.2.1 Direct selection of financial data

As discussed in the preceding section, inputs and outputs for the DEA model can be formed by directly using raw parameters obtained from financial statements, for example, accounts receivables (AR), inventory (IN), total assets (TA), total liabilities (TL), long-term debt (LD), revenue (RV), net income (NI), and shareholder's equity (SE). Thus, inputs and outputs for the DEA model must then be selected from these raw data. NI and RV are certainly measures of output, while TA, TL, LD, IN, and AR are typically concerned with input control. Since SE=TA-TL, shareholder's equity cannot be treated as an independent input or output in the presence of TA and TL. Accordingly, we define the *Direct* Selection (DS) of inputs/outputs as the following categorizations of the raw data, denoted by input set Π_I^d and output set Π_O^d.

$$\Pi_I^d = \{AR, IN, TA, TL, LD\} \tag{3.1}$$

$$\Pi_O^d = \{RV, NI\}\,. \tag{3.2}$$

Therefore, the DS-based DEA model has 6 inputs and 2 outputs.

3.2.2 Ratio-based inputs and outputs

In this section, a firm's relative business strength is measured using a DEA model specified with the 18 financial parameters presented in Section 3.1.1. These ratios are first categorized into various perspectives of functionality based on generally-accepted knowledge. That is, the inputs/outputs selection process is driven by the knowledge and understanding of how these parameters are related to the general operation of a firm. Such “a priori” knowledge is hereby termed **expert information** on parameter selection, which is a subject of extended discussion in Chapter 6.

Return-on-Equity (P1), Return-on-Assets (P2), Net Profit Margin (P3), and Earnings per Share (P4) represent a firm's profitability perspective. Receivables Turnover (P5), Inventory Turnover (P6), and Asset Turnover (P7) measure the degree of asset utilization of a firm. Current Ratio (P8) and Quick Ratio (P9) examine liquidity level of a firm. Debt to Equity Ratio (P10), Leverage Ratio (P11), Solvency Ratios - I and II (P12, P13) are estimators of a firm's leverage condition. Price to Earnings Ratio (P14) and Price to Book Ratio (P15) are indicators of a firm's valuation perspective. Revenue Growth Rate (P16), Net Income Growth Rate (P17), and Earnings per Share Growth Rate (P18) measure the growth potential.

It is commonly known that profitability and growth perspectives are measures of outputs because revenue or income generation is a major objective criterion for a firm. On the other hand, asset utilization, liquidity, and leverage perspectives are considered as inputs because they are concerned with the planning and operational strategies of a firm. In contrast, valuation perspective is concerned with how well the equity markets perceive ”success” of a firm, and thus, it is not concerned with a firm's input strategy; however, its inclusion in the output set must depend on the degree of predictive power the Valuation perspective offers for stock returns.

Basic and Augmented selection

First, by dropping the valuation perspective (due to its ambiguity as an output), we define the following input/output categorization of the financial parameters, which is referred to as *Basic* Selection (BS), where Π_I^b and Π_O^b denote the input and output sets, respectively,

$$\Pi_I^b = \{P5, P6, P7, P8, P9, P10, P11, P12, P13\} \tag{3.3}$$

$$\Pi_O^b = \{P1, P2, P3, P4, P16, P17, P18\} . \tag{3.4}$$

The *Basic* selection results in 9 inputs and 7 output parameters. Next, by incorporating the valuation perspective to the output set, an *Augmented* Selection (RS) is defined by the input/output pair (Π_I^b, Π_O^a), where

$$\Pi_O^a = \{P1, P2, P3, P4, P14, P15, P16, P17, P18\} . \tag{3.5}$$

Let the efficiency score calculated by model (2.2) for firm k using *Basic* selection be denoted by $\hat{f}_k(\Pi_I^b, \Pi_O^b)$, and that using *Augmented* selection is $\hat{f}_k(\Pi_I^b, \Pi_O^a)$. Due to Proposition 2.3.1, we have $\hat{f}_k(\Pi_I^b, \Pi_O^b) \leq \hat{f}_k(\Pi_I^b, \Pi_O^a)$ because $\Pi_O^b \subset \Pi_O^a$. However, it is neither implied nor asserted that the *Augmented* selection-based DEA score leads to an increased predictive power for stock price returns over the *Basic* selection-based model. Indeed, the choice between *Augmented* selection and *Basic* selection has to be made in the context of the predictive power they yield for stock price returns across the market, as we shall consider computationally.

3.3 Negative data in DEA model

As stated in Section 2.5.2, in order to measure the strength due to both technical efficiency and scale efficiency, the CCR-based DEA model in (2.2) is used. The CCR model in (2.2) is based on the assumption that all input and output parameters are positive [16]. However, in our case, it is possible that all of the input parameters for a given firm have non-positive observations (financial data), depending on how the input parameters are chosen from financial statements. For instance, if "return on assets" and "return on equity" are chosen as the only input parameters, and if these two parameters are negative for a given firm, then the linear model in (2.2) is infeasible, i.e., constraint $\sum_{m=1}^{M} \xi_{mk}^{i} u_m = 1$ in (2.2) cannot be satisfied. While it makes no sense to assign an efficiency score in such a case, for computing an underlying strength index for the firm, a value must be assigned - the least possible efficiency of zero. Consequently, the model below is presented, which is a slight generalization from (2.2), and it is hereby referred to as the DEA model of financial strength and its value is simply called the Relative Performance Score (RPS).

$$\begin{aligned} f_k^* := \max_{u,v} \quad & \sum_{n=1}^{N} \xi_{nk}^{o} v_n \\ \text{s.t.} \quad & \sum_{m=1}^{M} \xi_{mk}^{i} u_m \leq 1 \\ & -\sum_{m=1}^{M} \xi_{mj}^{i} u_m + \sum_{n=1}^{N} \xi_{nj}^{o} v_n \leq 0, \ j = 1, \ldots, J \\ & u_m, v_n \geq 0, \ m = 1, \ldots, M, \ n = 1, \ldots, N. \end{aligned} \tag{3.6}$$

The only difference in (3.6) from (2.2) is that the equality in the first constraint is now replaced with an inequality. The two models are equivalent if at least one of the input metrics is strictly positive for the firm , as claimed in Proposition (3.3.1) below. The fact that the resulting efficiency is zero for a firm whose all of the input metrics are non-positive is shown in Proposition (3.3.2).

Proposition 3.3.1 *Suppose $\xi^i_{mk} > 0$, for some m, where $m = 1, 2, \ldots M$. Then $f^*_k = \hat{f}_k$ holds.*

Proof. Since the constraints in (3.6) provide a relaxation to those in (2.2) - due to the inequality replacing the strict equality - it follows that $f^*_k \geq \hat{f}_k$. Conversely, suppose an optimal solution of (3.6) is given by $(\hat{u}_m, \hat{v}_n)$. If $\sum_{m=1}^{M} \xi^i_{mk}\hat{u}_m = 1$ for this optimal solution, the result $f^*_k = \hat{f}_k$ follows trivially. Otherwise, if $\sum_{m=1}^{M} \xi^i_{mk}\hat{u}_m < 1$, then for an index p such that $\xi^i_{pk} > 0$, define

$$\bar{u}_p = \hat{u}_p + \frac{\left(1 - \sum_{m=1}^{M} \xi^i_{mk}\bar{u}_m\right)}{\xi^i_{pk}},$$

and thus, $\bar{u}_p > \hat{u}_p$, and set $\bar{u}_m = \hat{u}_m$ for $m \neq p$. Therefore, $\sum_{m=1}^{M} \xi^i_{mk}\bar{u}_m = 1$ holds. Moreover, since $\bar{u}_p > \hat{u}_p$ holds, it follows that

$$\sum_{n=1}^{N} \xi^o_{nj}\hat{v}_n \leq \sum_{m=1}^{M} \xi^i_{mj}\hat{u}_m < \sum_{m=1}^{M} \xi^i_{mj}\bar{u}_m,$$

and thus, $(\bar{u}, \hat{v})$ is feasible in problem (2.2). Therefore, $\hat{f}_k \geq f^*_k$ holds. Combining with $f^*_k \geq \hat{f}_k$, the proof is completed. ■

Proposition 3.3.2 *If $\xi^i_{mk} \leq 0$, for all $m = 1, 2, \ldots M$, then $f^*_k = 0$.*

Proof. $\xi^i_{mk} \leq 0$ implies $\sum_{m=1}^{M} \xi^i_{mk} u_m \leq 0 < 1$ because $u_m \geq 0\ \forall m$. Therefore, the second constraint for $j = k$ implies $\sum_{n=1}^{N} \xi^o_{nj} v_n \leq \sum_{m=1}^{M} \xi^i_{mj} u_m$, and thus, $f^*_k \leq 0$ must hold. On the other hand, the solution $u_m = 0$ for all $m = 1, 2, \ldots, M$, along with $v_n = 0$, $n = 1, 2, \ldots, N$, is feasible in (3.6), which leads to the trivial lower bound $f^*_k \geq 0$. Therefore, $f^*_k = 0$ follows. ■

For a detailed discussion on DEA models that involve negative inputs/outputs, see [38] and [49], for instance. Proposition 3.3.1 and 3.3.2 treated negative data in the inputs. On the other hand, if all outputs are negative for a firm, it follows that the computed RPS in (3.6) is zero. In addition, the following properties can also be shown in a straightforward manner.

Proposition 3.3.3 *Given a firm k under evaluation in (3.6), if there exists an output parameter that is positive for k and there exists an input parameter that is positive for all firms, then the DEA efficiency computed in (3.6) is strictly positive;*

Proof. Suppose the positive output for firm k is ξ^o_{qk} and the positive input for all firms is ξ^i_{pj}, where $j = 1, \ldots, J$. Construct the solution $(\hat{u}, \hat{v})$ such that $\hat{u}_p > 0$ and $\hat{u}_m = 0, \forall\, m \neq p$, and $\hat{v}_q > 0$ and $\hat{v}_n = 0, \forall\, n \neq q$. Then, $(\hat{u}, \hat{v})$ can be verified feasible in (3.6). In particular, $\hat{u}_p = 1/\xi^i_{pk} > 0$ and $0 \leq \hat{v}_q \leq \xi^i_{pj} \hat{u}_p / \xi^o_{qj},\ \forall j$. Furthermore, the objective value is $f^*_k = \xi^o_{qk} \hat{v}_q$. Given the maximization objective of the model, $f^*_k > 0$. ■

Proposition 3.3.4 *Given a firm k under evaluation in (3.6), if there exists an output parameter q such that $\xi^o_{qk} > 0$, but $\xi^o_{qj} \leq 0$, $j \neq k$, and there is an input* *parameter p that is positive for all firms, then the DEA efficiency computed in (3.6) is one, i.e., firm k is DEA-efficient;*

Proof. Given $\xi^o_{qk} > 0$, but $\xi^o_{qj} \leq 0$, $j \neq k$, and since $\xi^i_{pj} > 0$, where $j = 1, \ldots, J$, we construct the solution $(\hat{u}, \hat{v})$ such that $\hat{u}_p = \frac{1}{\xi^i_{pk}} > 0$ and $\hat{u}_m = 0, \forall\ m \neq p$, and $\hat{v}_q = \frac{1}{\xi^o_{qk}} > 0$ and $\hat{v}_n = 0, \forall\ n \neq q$. Then, it is straightforward to verify that the pair $(\hat{u}, \hat{v})$ is feasible in (3.6). This implies $f^*_k \geq \sum_{n=1}^{N} \xi^o_{nk}\hat{v}_n = \xi^o_{qk}\hat{v}_q$=1. Since $f^*_k \leq 1$ must also hold, we have $f^*_k = 1$. ■

Proposition 3.3.5 *Given a firm k under evaluation in (3.6), if there exists an output parameter n such that $\xi^o_{nk} < 0$, and $\xi^o_{nj} \geq 0$, $j \neq k$, then in the optimal solution of (3.6), the multiplier for this output parameter is zero, i.e., $v_n = 0$.*

Proof. Let $(\hat{u}, \hat{v})$ be one of the solutions to model (3.6). The second constraint in (3.6) can be written as $\sum_{n=1}^{N} \xi^o_{nk}\hat{v}_n \leq \sum_{m=1}^{M} \xi^i_{mk}\hat{u}_m,\ j = k$, which is denoted by $l1$ and $\sum_{n=1}^{N} \xi^o_{nj}\hat{v}_n \leq \sum_{m=1}^{M} \xi^i_{mj}\hat{u}_m,\ j \neq k$, denoted by $l2$. When $\hat{v}_n > 0$, $l1$ is easier to be satisfied than $l2$ due to the given conditions. When $\hat{v}_n = 0$, the problem is equivalent to that output parameter n is removed from the model, which is also feasible to model (3.6). The objective value is $f^*_k = \sum_{n=1}^{N} \xi^o_{nk}\hat{v}_n$. Since $\xi^o_{nk} < 0$ and given the maximization objective of the model, $\hat{v}_n = 0$. ■

3.4 Correlation between RPS and Stock Return

By computing an RPS using various input/output selections, as discussed in Section 3.2, we pursue the question as to which selection is the most representative of the underlying business strength of a firm, as supported by stock price action. For this purpose, correlation between the RPS and the stock price returns is examined under various input/output selections by considering all firms within a given market segment, such as an industry. Finding such high correlations allows RPS to have high predictive ability of stock price returns, which is valuable when designing equity portfolios.

For a given market segment, i.e., industry, consider the RPS model in (3.6) in which inputs and outputs are specified either with the 18 financial ratio parameters discussed in Section 3.1.1, or with the *Direct* selection in Section 3.2.1. The rate of return (RoR) for each stock for different time periods can also be computed from market data. RoR is defined as the percentage of gain (positive) or loss (negative) generated from a $1 investment over the specified period. We denote this RoR variable by r_{jt} for firm j and period t. Here, a period refers to a quarter of financial information. The RPS in (3.6) computed under the input/output categories given by the *Direct* model (Π_I^d, Π_O^d) is denoted by $\eta_{jt}^d := f_j^*(\Pi_I^d, \Pi_O^d)$. Similarly, $\eta_{jt}^b := f_j^*(\Pi_I^b, \Pi_O^b)$ refers to RPS under *Basic* selection and $\eta_{jt}^a := f_j^*(\Pi_I^b, \Pi_O^a)$ refers to RPS under *Augmented* selection.

Let F_h denote the set of firms in industry h and $J_h := |F_h|$ is the number of firms. To compute the required correlations, collect market price RoR values r_{jt} for each period $t = 1, ..., T$ for firm j, in industry h, and form the RoR vector (or sequence) for firm j, denoted by $R_j(h)$. This process is repeated for all $j \in F_h$ and for all $h = 1, ..., H$. Similarly, collecting η_{jt}^b (for the *Basic* selection) for each period $t = 1, ..., T$, the RPS vector (or sequence) $E_j^b(h)$ is formed. This process is repeated for all $j \in F_h$ and for all industries $h = 1, ..., H$. The correlation coefficient between the two vectors $E_j^b(h)$ and $R_j(h)$ is $\rho_S^b(E_j^b(h), R_j(h))$, or simply referred to as $\rho_S^b(h, j)$. The subscript S indicates it is a *synchronous* correlation for firm j in industry h and it is called synchronous because firm j's efficiency scores are correlated with stock returns of similar (or contemporaneous) time periods. The term synchronous is used to indicate that under EMH, financial strength of a firm in a certain period is correlated with the stock return of the firm in the same period. The significance of this synchronous correlation will be investigated using statistical tests in the next section.

3.5 Statistical tests of correlations

3.5.1 Correlation test

For a given firm j in industry h, a computed synchronous correlation $\rho_S^b(h, j)$ - for *Basic* selection, for instance - is checked for its significance using the following hypothesis test:

$$\left.\begin{aligned} H_0 &: \ \tilde{\rho}_S^b(h, j) = 0 \\ H_1 &: \ \tilde{\rho}_S^b(h, j) \neq 0, \end{aligned}\right\} \tag{3.7}$$

where $\tilde{\rho}_S^b(h, j)$ is the (true) population correlation coefficient. The null hypothesis H_0 indicates that there is no correlation between DEA-based strength and RoR of the same quarter. Under H_0, it can be shown that the test statistic,

$$\zeta_S^b(h,j) = \frac{\rho_S^b(h,j)\sqrt{T-2}}{\sqrt{1-\left[\rho_S^b(h,j)\right]^2}} \tag{3.8}$$

is *student-t* distributed, see [59], with $T-2$ degrees of freedom (*d.f.*). Given a significant level α, for the two-sided test, if the critical *student-t* value $\zeta_{cr}(T-2,\alpha/2)$ is smaller than the computed sample statistic $\zeta_S^b(h,j)$, then the null hypothesis is rejected in favor of accepting that there is predictive power in firm j's efficiency score on its stock price RoR.

It should be noted that when *student-t* test is used to examine the correlation between two random variables, these two random variables must come from a bivariate normal distribution, see [59]. The stock returns are often assumed to be normally distributed, see for instance [4]. However, the distribution of RPS scores remains largely unknown. In the event, RPS values are non-normally distributed, to the best of the author's knowledge, there does not exist a computable test statistic for the correlation hypothesis test. In Chapter 5, transformations to obtain near-normality are considered. In the ensuing discussion, we proceed with the test assuming normality of RPS.

Using the RPS model in (3.6), the above synchronous-test statistics are computed for the *Direct*, *Basic*, and *Augmented* selections, denoted by $\zeta_S^d(h,j)$, $\zeta_S^b(h,j)$, and $\zeta_S^a(h,j)$, respectively, for each firm $j \in F_h$ and for all industries $h = 1,...,H$. If any of these test statistics are consistently large across the firms in an industry, then the corresponding RPS measure can be interpreted to have a strong predictive power in the industry. Furthermore, if such strength is also strong across all industry groups, the corresponding DEA input/output selection may be deemed to provide a significant measure of underlying relative financial strength of firms. First, in order to establish a given input/output selection leads to consistently-significant correlations across firms in an industry, the statistical test in the following section is applied.

3.5.2 Identifying industries lacking RPS-based predictability

Let us consider the *Basic* RPS model with synchronous correlations for industry h. The same procedure is applied to the *direct* and *augmented* versions of the RPS model. Consider the null hypothesis H_0 in (3.7) for all firms $j = 1, \ldots, J_h$, i.e.,

$$\left.\begin{array}{l} H_0 :\ \tilde{\rho}^b_S(h,j) = 0,\ j = 1, \ldots, J_h \\ \\ H_1 :\ \tilde{\rho}^b_S(h,j) \neq 0,\ \text{for some } j. \end{array}\right\} \tag{3.9}$$

If H_0 in (3.9) holds, then each test statistic $\zeta^b_S(h,j)$ in (3.8), for $j = 1, \ldots, J_h$, has a *student-t* distribution with $T - 2$ *d.f.* Thus, it follows that the collection $Q^b_S(h) := \left\{\zeta^b_S(h,j)\right\}_{j=1}^{J_h}$ of the test statistics is a sample drawn from a *student-t* distribution with $(T-2)$ *d.f.* under H_0 in (3.9).

In order to test if the sample $Q^b_S(h)$ comes from a *student-t* distribution with $(T-2)$ *d.f.*, we employ a Chi-Square (χ^2) goodness-of-fit test. Using a grid of l intervals, the resulting goodness-of-fit test statistic, denoted by $\omega^b_S(h)$, is χ^2 -*distributed* with $(l-1)$ *d.f.*, see [59]. For the one-sided test and a significant level of α, if the critical

χ^2 value

$$\vartheta_{cr}(l-1,\alpha) \geq \omega_S^b(h), \tag{3.10}$$

we fail to reject the null hypothesis H_0 in (3.9) and conclude that the sample $Q_S^b(h)$ must have come from a *student-t* distribution with $(T-2)$ *d.f.* That is, this industry h does not support using the *basic* selection-based DEA model as a proxy for stock price returns (of the same quarter). Indeed, if

$$\vartheta_{cr}(l-1,\alpha) < \omega_S^b(h), \tag{3.11}$$

then rejecting H_0 in (3.9) does not necessarily imply that RPS is a strong predictor for every firm.

The basic methodology described so far is illustrated with an application to the technology sector of the U.S. stock market, in the next section.

3.6 Application of RPS to the Technology Sector

The validity of the DEA-based RPS as a predictor for stock returns (RoR) is demonstrated using publicly traded U.S. firms. Only the technology sector is used for the experimentation, of which six broad industry groups are formed: Computer Software ($h = 1$), Communication Equipment ($h = 2$), Computer Hardware ($h = 3$), Electronics ($h = 4$), Semiconductors ($h = 5$), and Computer Services ($h = 6$) industries. Firms belonging to each of these 6 industry groups were identified using public information available from the website: http://biz.yahoo.com/p/8conameu.html. In some cases, several individual industries of similar type are aggregated to form one industry group, as shown in Table 3.1.

Quarterly financial statements for the period 1996 to 2002 for all firms in our six industry groups are electronically obtained from the WRDS (Wharton Research Data Services) database. The financial statement data, as well as quarterly stock prices, are checked for completeness and only those firms with complete data are chosen in each industry group. Thus, the usable number of firms (i.e., sample size) in each industry group is limited. For each group h, the usable set of firms, denoted, is unique, and the number of firms, is shown in Table 3.1. Thus, there is a total of 313 firms, referred to as the *universe* of firms.

First, using the stock prices from 96Q1 to 02Q4, quarterly RoR for each firm in each industry group is calculated for all 27 quarters, i.e., 96Q2 to 02Q4. Second, RPS values using model (3.6) are calculated for each firm in each industry group, for every quarter from 96Q2 to 02Q4. The first quarter (i.e., 96Q1) is set aside for the computations of RoR and the 18 financial parameters presented in Section 3.1.1. The quarterly RPS values so-computed are plotted in Figure 3.1, where η_{jt}^b are averaged over all J_h firms for industry h and over all 27 quarters, and denoted by $\bar{\eta}_h^b = \frac{1}{27J_h} \sum_{t=1}^{27} \sum_{j=1}^{J_h} \eta_{jt}^b$, where $t = 1$ refers to the 96Q2 quarter. In the same graph, $\bar{\eta}_h^d$ and $\bar{\eta}_h^a$, for *Direct* and *Augmented* selections, are also plotted for each industry h. It is evident that the *Augmented* selection produces larger average efficiency scores than the *Basic* selection (as supported by Proposition 2.3.1). Also, it appears that the *Direct* RPS model that uses raw financial parameters yield larger average RPS scores compared to the *Basic* RPS model. Is this increased RPS value of a firm

Table 3.1: Industries and number of firms

Industry Group	Industries Included	# firms
Software ($h = 1$)	Application software, Multimedia & graphics software, Technical & systems software	42
Communications ($h = 2$)	Communication equipment, Processing systems & products	49
Hardware ($h = 3$)	Computer peripherals, Data storage devices, Networking & communication devices	43
Electronics ($h = 4$)	Diversified electronics, Printed circuit boards, Scientific & technical instruments	74
Semiconductors ($h = 5$)	Semiconductor equipment & materials, Semiconductor-Broad line, Integrated circuits, Specialized, Memory chips	69
Services ($h = 6$)	Information technology services, Internet software & services, Business software & services, Telecom services	36

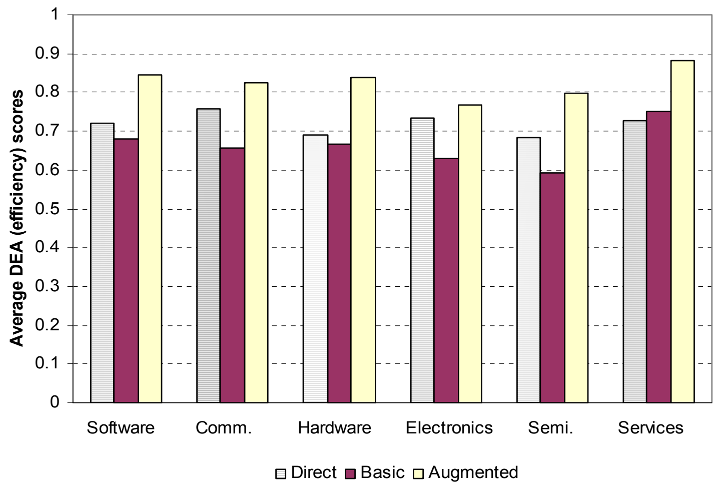

Figure 3.1: Average Efficiency Scores for each industry

representative of higher relative financial strength for the firm? The question will be answered by analyzing the correlation between firms' RPS and their stock returns.

3.6.1 Synchronous versus lagged correlations

A high synchronous correlation would indicate that stock returns are influenced by firms' underlying financial strength in a contemporaneous manner, thus representing the case for market informational efficiency (or, EMH). However, the implementation of the RPS as a proxy of stock returns is complicated due to two reasons. First, a quarter's financial information is not available prior to the beginning of a quarter, and thus, any implementation requires forecasting RPS to the immediate quarter that follows. Second, quarterly financial information typically is not released to the public immediately after a quarter ends, but could take as much as an additional month or more. Therefore, any useful application of the RPS concept needs to examine if the financial strength in a given quarter (as determined by RPS) is significantly correlated with stock returns that occur a month or many months beyond the quarter. For this purpose, we define "Lagged Correlations" associated with RPS.

"Lagged Correlations" measure the influence of business strength observed at the end of quarter t on the RoR in a 3-month period starting τ months from the beginning of quarter t. We set τ to be 1, 2, or 3 months. A pictorial representation of the synchronous and lagged concept is Figure 3.2. Suppose RPS is calculated for a firm for quarter 1, i.e., January to March, which is denoted by RPSQ1. Then the "synchronous" correlation measures the correlation between the RPSQ1 with stock returns that is also from January to March. However, "one-month Lagged" (Lag1) correlation ($\tau = 1$) examines the correlation between the RPSQ1 and the stock returns from February to April. "Two-month Lagged" (Lag2) correlation ($\tau = 2$) examines the correlation between the RPSQ1 and the stock returns from March to May. "Three-month Lagged" (Lag3) correlation ($\tau = 3$) examines the correlation between the RPSQ1 and the stock returns from April to June. Thus, the case of $\tau = 3$ measures the *lagged* correlation between financial strength in quarter t with RoR of quarter $t + 1$.

These three forms of lagged correlations are denoted by $\rho^b_{L,\tau}(h, j)$ for $\tau = 1, 2, 3$ (for the *Basic* selection), and thus, $\rho^b_{L,3}(h, j)$ is the one-quarter lagged correlation for firm j. This procedure is repeated for the *Direct* and *Augmented* selections to compute the corresponding *synchronous* and the three versions of the *lagged* correlation.

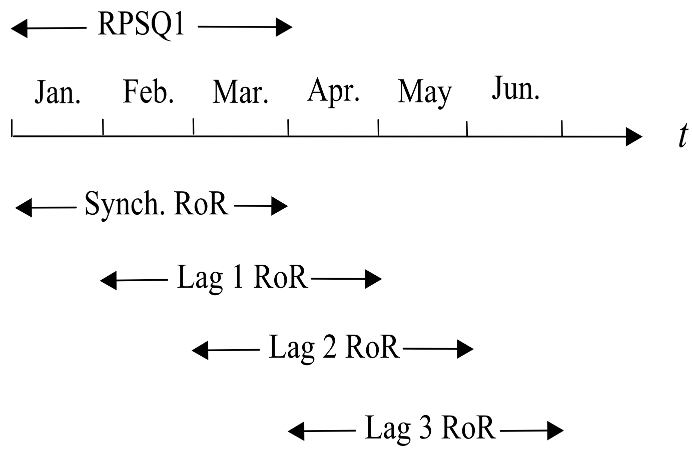

Figure 3.2: Synchronous versus lagged concept of correlations

In order to find RPS values that are highly correlated with stock market returns, statistical tests are provided for all versions of these correlations, as discussed next.

3.6.2 Comparison of RPS

The proportions of significant correlations (over all firms) in each industry for all three input/output selections are presented in Figure 3.3, where each “vertical bar” represents the cumulative proportion over all industries. Note that, in this figure, all four cases of synchronous, one-month lagged, two-month lagged, and three-month lagged correlations are presented. In particular, the *Basic* RPS is the strongest in the synchronous and lagged correlations, while *Direct* RPS is the weakest. Are these strong correlations largely positive or negative? In Figure 3.4, we plot average t-test statistics for each industry for the *Basic* RPS, where synchronous test statistics

$\rho_S^b(h,j)$, as well as the lagged test statistics $\rho_{L\tau}^b(h,j)$, for $\tau = 1,2,3$ months, are averaged over all firms in an industry. If the average test statistic is positive and large, it is indicative of significant (positive) correlation. Generally, with increased time lag between RPS measurement and stock returns, correlations seem to lose their strength, although 1-month lagged case appears quite significant.

For determining the consistency of high correlations across all firms, the goodness-of-fit tests (see Section 3.5.2) are performed for each industry group for synchronous and lagged cases, using the all three versions of input/output selection. These χ^2-values are reported in Table 3.2. A few observations come to light from the results in Tables 3.2. First, the *Direct* Selection of inputs/outputs from raw financial data does not yield significant correlations in at least 3 of the 6 industry groups, for either the synchronous or the lagged cases. Consequently, it is concluded that the *Direct* RPS is not a significant proxy for financial strength of a firm.

In contrast, the *Basic* and *Augmented* selections for the RPS model in (3.6) are seen to provide significant correlations across many industries. In particular, the *Basic* RPS is significant across 5 out of the 6 industries under either the synchronous or lagged modes, with the exception of the 3-month lagged case. Recall that financial information almost surely is not available in a contemporaneous manner; instead, quarterly financial information for a given quarter is generally made public only with a certain time delay, e.g., 30 days or more. In lagged correlations, referring to Table 3.2, it is evident that the *Basic* RPS is the most significant for both 1- and 2-month lags. The *Augmented* RPS is significant only for 1-month lag (in 3 of the 6 industry

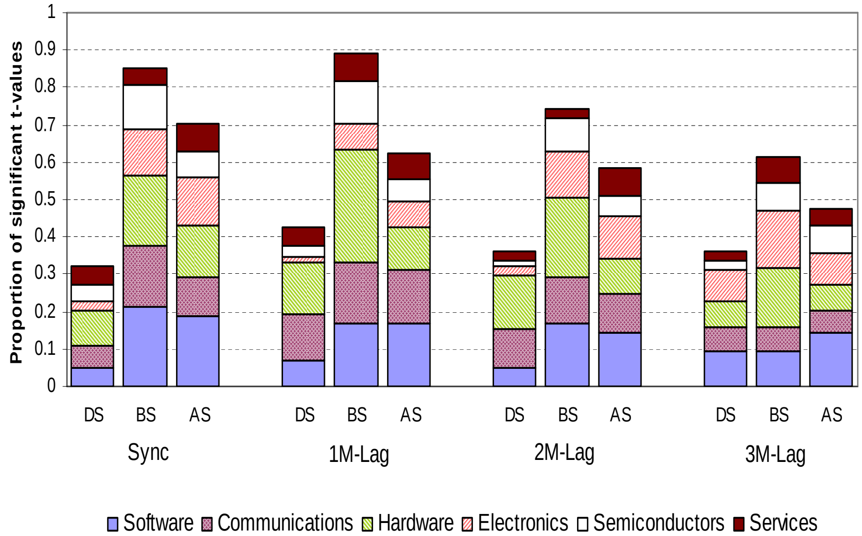

Figure 3.3: Proportion of significant correlations in *Direct* (DS), *Basic* (BS), and *Augmented* (AS) models

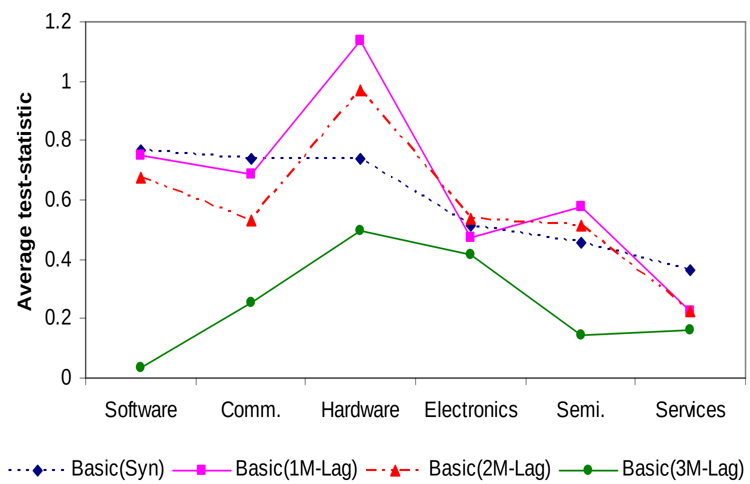

Figure 3.4: Average t-test statistics (of correlation) for the *Basic* model

Table 3.2: Goodness-of-fit χ^2-values for each industry using RPS model in (3.6)

Model Combination		Software	Comm.	Hardware	Elec.	Semi.	Services
Synchronous	Direct	12.286	29.163	29.326	1.889	5.928	6.222
	Basic	39.429	27.939	34.442	29.111	18.681	13.444
	Augm.	37.524	27.122	22.349	28.278	23.029	20.667
Lagged (1 month)	Direct	18.476	24.0674	47.930	6.611	14.044	10.111
	Basic	28.952	41.000	59.093	21.333	24.768	11.222
	Augm.	29.905	17.327	20.023	9.389	15.493	12.333
Lagged (2 month)	Direct	18.000	17.417	24.191	26.889	9.406	6.778
	Basic	20.381	22.000	43.714	21.056	28.826	9.000
	Augm.	8.000	8.250	14.191	9.667	10.565	11.222
Lagged (3 month)	Direct	24.667	5.898	14.442	14.111	33.174	9.556
	Basic	7.048	3.857	28.861	18.000	9.696	6.778
	Augm.	10.857	8.347	7.000	5.222	7.087	7.333

groups), although even in this case, the *Basic* RPS generally outperforms the *Augmented* RPS with stronger statistics. For a 3-month delay in quarterly information, none of these two input/output selections provides a measure of underlying financial strength to predict stock returns 3 months later. Thus, it is concluded that RPS computed by the model in (3.6) using the *Basic* Selection of inputs/outputs in (3.3)-(3.4) qualify as a proxy for relative financial strength of a firm when applied with one or two-month lag for stock selection. The efficiency score of the *Basic* selection-based RPS model in (3.6) is hereby termed the Basic-RPS, or BRPS, indicator.

We test the BRPS indicator, under 1-, 2-, and 3-month lagged forms, using the (in-sample) historical quarterly data from 96Q1-02Q4, for selection of firms with a favorable potential for investments. These firms are then used in a portfolio weighting model for portfolio optimization. To benchmark the BRPS indicator, the RIV model presented in Section 1.3.3 is also used for stock selection and compared with portfolios determined under BRPS.

3.6.3 BRPS- and RIV-based stock selection

The set of 313 firms in the 6 industry groups, presented in Table 3.1, forms the universe of stocks for the experiments in this section. However, under the BRPS indicator, only 5 of the 6 industry groups display significant predictive power of stock returns (when quarterly information is made public with a time delay), and thus, Services group is eliminated from the BRPS-based stock selection. For each of the 277 firms in the remaining 5 industry groups, the BRPS indicator is forecasted for the (future) quarter 03Q1, based on the already computed historical BRPS series that use the financial reports up to 02Q4. A simple forecast given by the two-quarter moving average, i.e., the arithmetic average of BRPS for 02Q3 and 02Q4. Then, we apply:

BRPS-based Stock selection rule: a firm is considered *investment-worthy* only if its predicted $BRPS \geq 0.90$.

The above rule results in a total of $J_{BRPS} = 78$ firms for possible inclusion in the portfolio. The distribution of these 78 firms chosen by the BRPS selection across the 5 industry groups is also indicated in Table 3.3.

The Residual Income Valuation (RIV) model, as introduced in Section 1.3.3, is used to compare with the BRPS-based portfolio selections. RIV model is chosen, instead of DDM and FCFE models, because the latter two models are usually used

Table 3.3: Selection of firms using DEA and RIV models

Indicator	Industry Group					
	Software	Comm.	Hardware	Elec.	Semi.	Services
BRPS	17	14	19	15	13	-
RIV	15	15	16	20	8	9
Common	5	6	11	6	2	0

to compute a firm's intrinsic value when the firm has positive and predictable cash flow that pays dividends to stockholders. However, not all of the firms that are used in our application are guaranteed to have positive and predictable cash flow. The RIV model, on the other hand, is more suited for firms with negative or uncertain cash flow.

As for applying the RIV model, book value of shareholder's equity (B_t) and earnings (I_t) must be forecasted for 03Q1, 03Q2, and 03Q3. We use a simple estimation procedure using past quarters up to 02Q4; the details are omitted for brevity. Long term (5-year) growth rates for each firm were taken from the publicly available data and geometric-adjusted for quarterly periods to obtain the required growth rate g. The cost of capital r values are not readily available for each firm, and their computations require much more information than discussed in this thesis so far. Consequently, a representative r value was obtained for each industry from published sources, and within a given industry, this value was set fixed for each firm. It is noted that for a more rigorous calculation of RIV, additional information such as volatility measurement, beta, and coupon rate of issued bond will be necessary for each firm in each industry. Accordingly, the present capital value V_{0j} is calculated using (1.13) for each firm j in the universe of 313 firms, and compared to the stock price as of the end of 2002Q4, denoted by P_j. The following **RIV-based Stock selection rule** is used:

RIV-based Stock selection rule: a firm is considered *investment-worthy* only if its *value-to-price ratio* $V_{0j}/P_j \geq 1.05$.

This procedure resulted in a total of $J_{RIV} = 83$ firms for possible inclusion in the portfolio. The distribution of these 83 firms across the 6 industry groups is indicated in Table 3.3. Observe that the number of firms that are common between the BRPS and RIV selections is 30.

The BRPS- and RIV-based stock selections are compared with the case when portfolio optimization uses the entire universe of 313 stocks, referred to as the ALL case. No quarterly financial performance information is utilized in the ALL case, and thus, any one of the $J_{ALL} = 313$ firms is a potential candidate for investment within the portfolio selection model.

3.6.4 Application of BRPS in a Portfolio Selection Model

As stated earlier, quarterly financial information is made public with a certain time delay, typically a month or two after the quarter ends. Furthermore, the BRPS indicator was shown to be statistically significant for 1 or 2-month lagged cases while a 3-month lagged case is not significant. To ascertain the value of this in portfolio optimization, the BRPS (and RIV) stock selections are applied over an investment horizon of 3 months under three cases: from Feb 2003 to Apr 2003, from Mar 2003 to May 2003, and from Apr 2003 to June 2003, representing 1-month lagged, 2-month lagged, and 3-month lagged investments, respectively - these 3 cases are simply referred to as Lag-1, Lag-2, and Lag-3 investment horizons. A monthly-rebalancing strategy is applied in each case where portfolio allocations are optimally adjusted at the beginning of each of the 3 months in a given investment horizon. Portfolio allocations are determined using a static mean-variance framework, see [43], where portfolio expected return is traded off with portfolio variance, as follows.

$$\begin{aligned} \textit{Maximize} \quad & \sum_{i=1}^{J} \mu_i x_i - \sum_{i=1}^{J} L(y_i) - \lambda \sum_{i=1}^{J} \sum_{j=1}^{J} \sigma_{ij} x_i x_j \\ \textit{Subject to} \quad & \sum_{i=1}^{J} x_i \leq C^0 \\ & y_i = \left| x_i - x_i^0 \right|, \ i = 1, \ldots, J \\ & x_i \geq 0, \ i = 1, \ldots, J. \end{aligned} \tag{3.12}$$

Optimal portfolio allocations x_i (\$ investment), for firm $i = 1, \ldots, J$, are determined by solving the quadratic portfolio optimization model in (3.12) for the three cases: $J = J_{BRPS}$, $J = J_{RIV}$, and $J = J_{ALL}$, with an initial budget of $\$C^0$. Observe that a slippage loss function $L(.)$ is incorporated in the objective of (3.12) to account for possibly investing in stocks with relatively light trading volume, see [27]. The loss function applied here has the general form

$$L(y_i) = a y_i + b(y_i)^2 / vol_i, \tag{3.13}$$

where vol_i is the (estimated) market total daily trading dollar volume, y_i is the dollar volume of shares purchased/sold in stock i, and a, b are constants. Two levels of values for (a, b) are applied depending on whether the stock has very low trading volume or not. For very low volume (under 50K shares a day), $(a, b) = (0.10, 5.0)$ is set while for others, $(a, b) = (0.05, 1.0)$. For details, see [25]. The initial budget is set at $C^0 = 100,000$ and the initial stock positions $x_i^0 = 0$ in all stocks for the first month of investments. For the remaining two rebalancing periods, C^0 is automatically adjusted to the cash position carried forward in the portfolio and x_i^0 is set to the beginning stock positions at the rebalancing time. Expected (monthly) rate of return is μ_i, covariance of RoR between stocks i and j is σ_{ij}, and $\lambda > 0$ is a risk tolerance parameter where larger λ implies an increased *risk-aversion*.

The required statistical parameters are estimated using historical stock price data of the year 2002, using the same estimation techniques for all three cases: BRPS, RIV, and ALL. Under the monthly rebalancing strategy, such estimations are needed at the beginning of each month in the horizon, conditional upon the data available prior to that point in time. This approach results in dynamically evolving monthly portfolios, and these portfolios are (out-of-sample) simulated using the (actual) realized price series during the horizon. The portfolio model executions and out-of-sample simulations are all performed using ©MiSOFT software, see Edirisinghe [26].

Standard & Poor 500 index-tracking stock ticker SPY is used as the market barometer to track the (overall) market performance. The market volatility during the investment horizon is given by the annualized standard deviation of SPY, which is about 21.6%, 21.2%, and 18.0%, respectively, for Lag-1, Lag-2, and Lag-3 investment horizons. Thus, for the purposes of relative portfolio performance comparisons, when the portfolio model in (3.12) is executed for stock selections in BRPS, RIV, and ALL

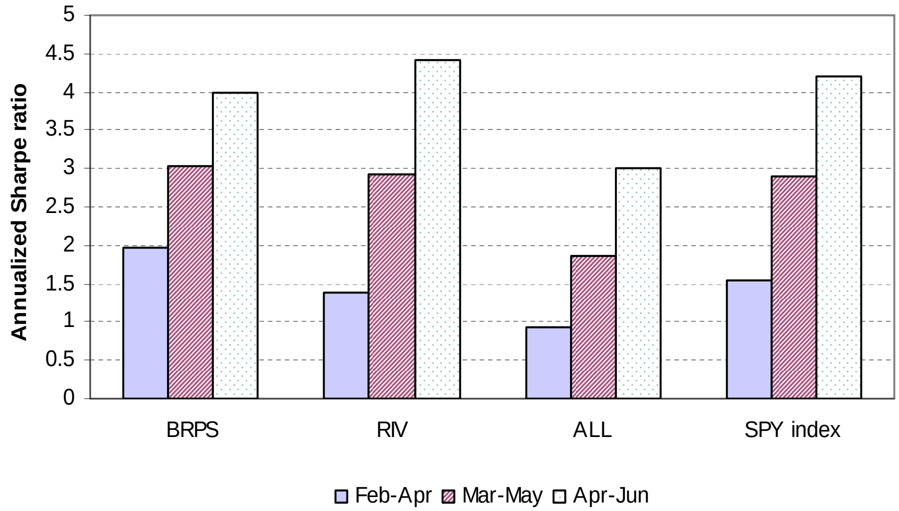

Figure 3.5: Portfolio performances under 1-, 2-, and 3-month lagged investments

versions, risk tolerance level λ is adjusted such that the resulting portfolio annualized standard deviation is roughly coincide with that of SPY in each investment horizon. Hence, the market and the portfolios obtained by solving (3.12) have approximately the same volatility. Portfolio performance is then measured by the (annualized) Sharpe Ratio, see [56], which is the annualized RoR (less the risk-free rate, which is zero in our case) divided by the annualized volatility. These Sharpe ratios are in Figure 3.5 for each investment horizon using the three model versions of (3.12), along with the market performance.

Observe that the BRPS indicator has the strongest performance under Lag-1 investment. For Lag-2 investment, BRPS is still the best stock selection criterion, while RIV is a close second. However, Lag-3 investment is relatively a weak proposition for the BRPS selection, as it was also evident in the statistical significance tests reported in Table 3.2. In Lag-1 and Lag-2 cases, BRPS selection outperforms both the RIV selection and the ALL option. The market has the strongest performance in the Lag-3 investment horizon, and only the RIV selection is able to significantly outperform the market. The relative weak performance of the ALL case may be partially explained by the fact that BRPS or RIV has smaller dimensions of uncertainty (78 or 83) compared to the ALL case with 313 stocks. The smaller dimension may possibly avoid unnecessary estimation biases, which help in increasing the accuracy of the diversification afforded by the portfolio optimization model.

3.7 Noteworthy Issues

Although the DEA-based BRPS indicator provides a new stock selection approach for portfolio investment and the results show that BRPS indicator outperforms the RIV-based stock selection, there are still a few issues that need to be highlighted.

First, Proposition 2.3.1 implies that the model saturation problem will occur in the DEA model if too many inputs and outputs are chosen for the model. In this case, the DEA model will lose its ability to discriminate the underlying firms. The generally-accepted "Rule of Thumb" is that the sample size should be at least twice the product of the number of inputs and number of outputs, see [24]. In the foregoing application, this rule of thumb is violated by the *Basic* and the *Augmented* selections. For the *Basic* RPS, at least 126 firms are required. The number of firms required for the *Augmented* selection is even larger. However, the sample sizes for the industry groups that are used in the previous application are fixed and none of them exceeds the required number. Thus, an approach that can reduce the number of inputs and outputs in DEA-based strength analysis is highly desirable.

Second, the performance scores computed by the RPS model (3.6) are in the range of 0 to 1, which implies that firms with score of 1 cannot differentiate themselves, and the firms with score 0 cannot be differentiated either. Thus, the computed correlation may be affected by this "truncation phenomenon". For illustration, the RPS values computed using the *Basic* model for each industry group are plotted in Figure 3.6 - Figure 3.11. The frequency of RPS being 1 is quite evident from these figures. Most notably, Services industry shows a high incidence of such a case. These frequencies under the *Basic* model (considering all 27 quarters in each industry) is plotted in Figure 3.12. Given the apparent high proportion of RPS=1, a further ranking of the DEA-efficient firms may be desirable towards a modified RPS value.

Third, when the *student-t* test is used to examine the correlation between firms' efficiency scores (that are computed using model (3.6)) and stock RoRs, the normality assumption was made for both the efficiency scores and RoR. The histogram of RPS values, computed using the *Basic* model, for a given firm across all quarters in Electronics industry is plotted in Figure 3.13. It is observed that the normality assumption does not hold for the RPS values. Hence, a transformation of the DEA-based scores is needed to ensure a closer satisfaction of the required normality.

The foregoing issues will be addressed in the subsequent chapters. More specifically, the first issue is addressed in Chapter 4, while the second and third issues are further discussed and developed in Chapter 5.

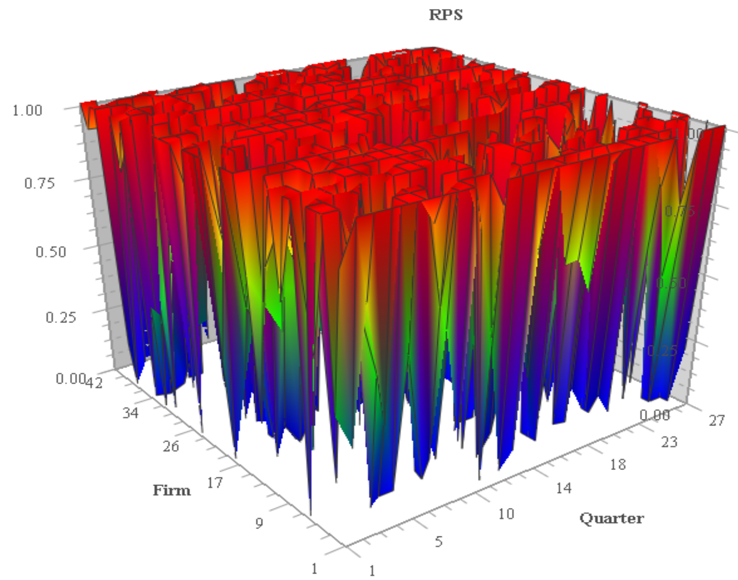

Figure 3.6: RPS values using Basic model for Software industry

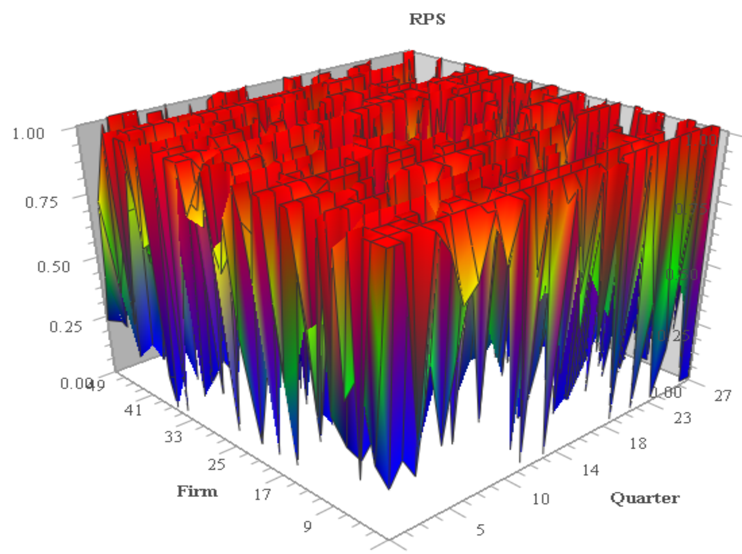

Figure 3.7: RPS values using Basic model for Communication industry

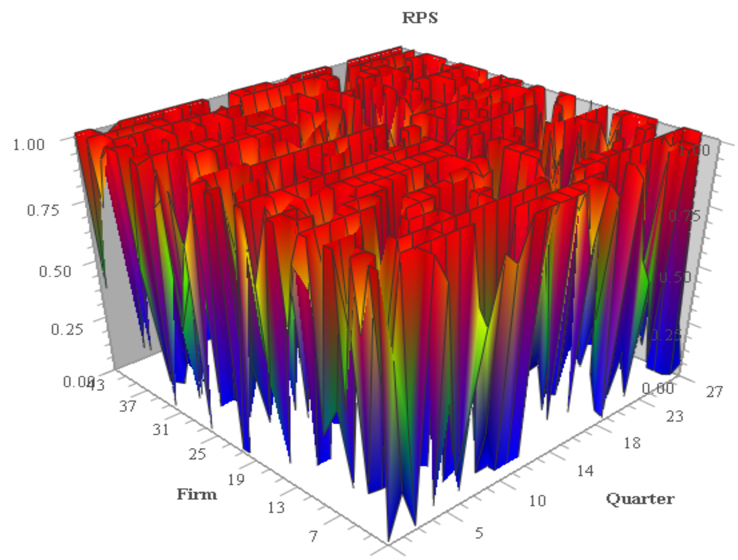

Figure 3.8: RPS values using Basic model for Hardware industry

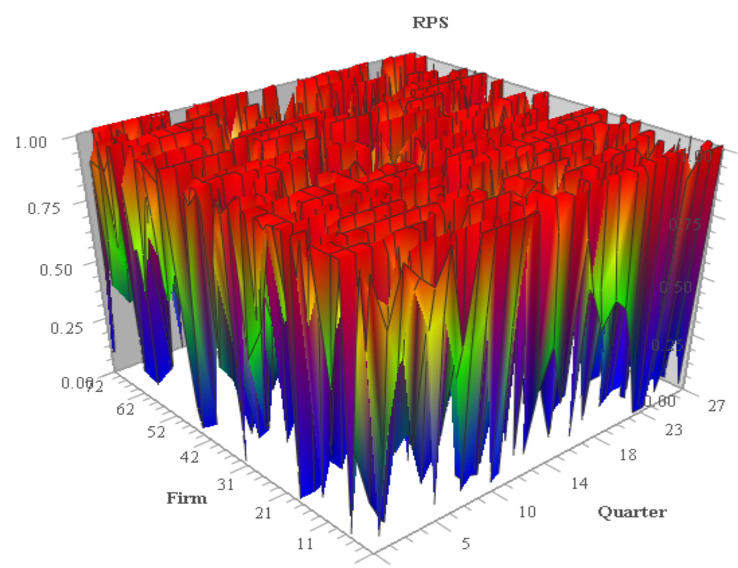

Figure 3.9: RPS values using Basic model for Electronics industry

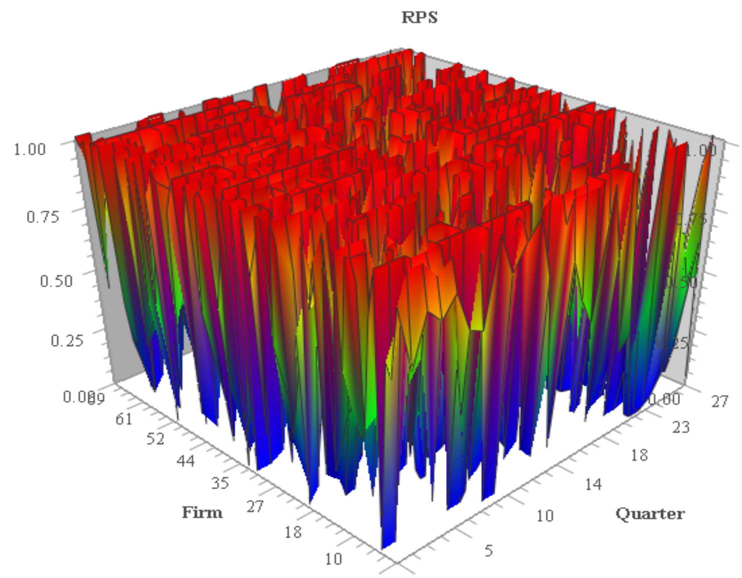

Figure 3.10: RPS values using Basic model for Semiconductors industry

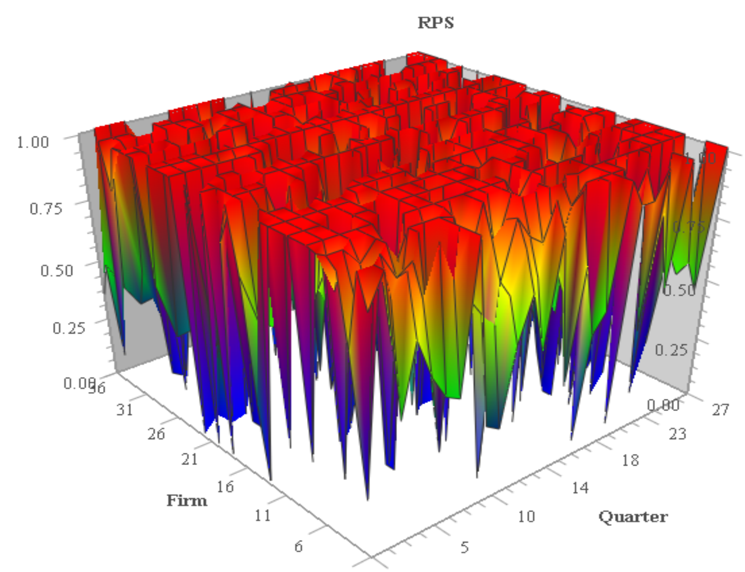

Figure 3.11: RPS values using Basic model for Services industry

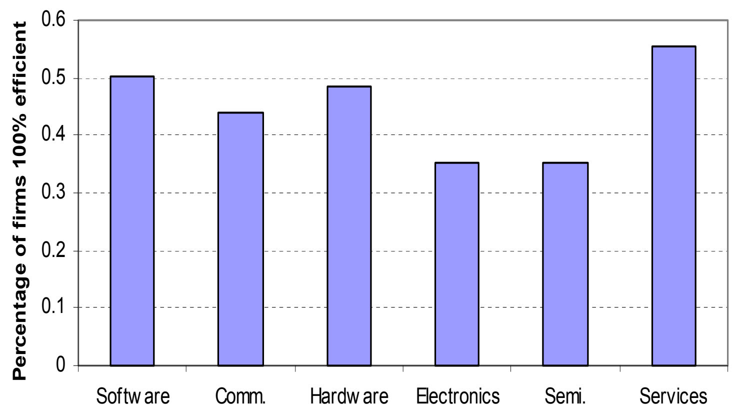

Figure 3.12: Percentage of 100%-efficient firms in each industry using *Basic* selection

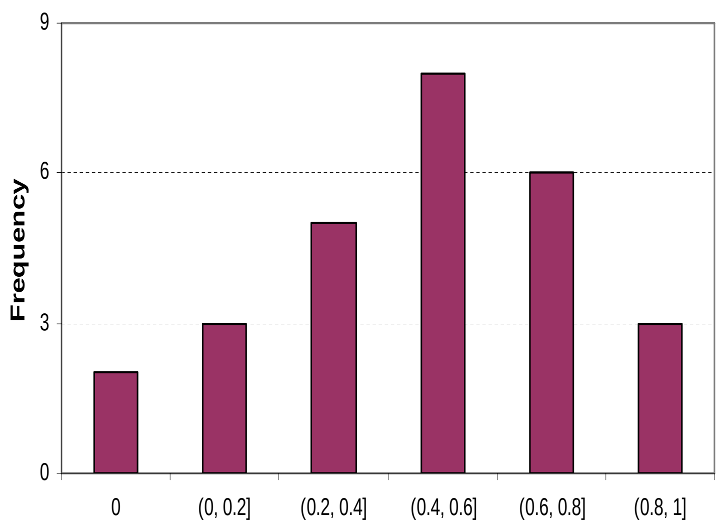

Figure 3.13: Histogram of the efficiency scores for firm SPEC in Electronics industry

3.8 Comments on using linear regression

When considering the influence of RPS on stock returns, one may be tempted to apply a regression model. That is, stock returns of all firms over all quarters are regressed on RPS scores of all firms and quarters. Consider stock RoR r_{jt} and RPS η_{jt}, for $j = 1, \ldots, J$ and $t = 1, \ldots, T$. Let $p = J \times T$ and construct the two vectors $R \in \Re^p$ and $E \in \Re^p$, where

$$R = \{r_{11}, \ldots, r_{J1},\ \ r_{12}, \ldots, r_{J2}, \ldots, r_{1T}, \ldots, r_{JT}\}' \quad \text{and}$$

$$E = \{\eta_{11}, \ldots, \eta_{J1},\ \ \eta_{12}, \ldots, \eta_{J2}, \ldots, \eta_{1T}, \ldots, \eta_{JT}\}'.$$

Consider the simple linear regression model

$$R = \beta_0 + \beta_1 E, \tag{3.14}$$

where β_0 and β_1 are scalars. The overall relationship between stock returns, R, and financial strength, E, over all firms and quarters is examined by model (3.14). However, such an overall treatment fails to capture the possibility of RPS being not predictive on RoR for certain firms, since (3.14) is only concerned with the existence of a linear trend across all firms in an industry. However, under EMH, the focus should be to verify that RPS would be a strong predictor across all (or most) firms in the industry. This was the underlying premise of the correlation analysis in Section 3.5. To be consistent, under regression analysis, a set of separate J simple linear regressions must be performed and those separate "slope" coefficients must

Table 3.4: F ratios for Basic model using simple linear regression in (3.14)

		Software	Comm.	Hardware	Elec.	Semi.	Services
Sync.	F Ratio	16.444	2.637	18.004	21.773	21.993	5.544
	P-value	<.0001	0.105	<.0001	<.0001	<.0001	0.019
Lagged	F Ratio	18.416	16.350	47.664	23.336	21.581	1.627
(1 mon.)	P-value	<.0001	<.0001	<.0001	<.0001	<.0001	0.203
Lagged	F Ratio	11.624	1.100	32.787	14.054	10.692	0.080
(2 mon.)	P-value.	0.0007	0.295	<.0001	0.0002	0.0011	0.777
Lagged	F Ratio	1.923	0.405	13.089	13.187	5.205	0.003
(3 mon.)	P-value	0.166	0.525	0.0003	0.0003	0.023	0.957

be tested for significance. This would then be essentially equivalent to what was performed in the statistical test in Section 3.5.2.

The results of the regression model in (3.14) are reported in Table 3.4. In this case, the results indicate an overall agreememnt with those in Table 3.2, except for Communications and Services industries. However, this similarity should be considered a manifestation of the data used in this case, rather than an agreement of the conceptual basis of the two approaches.

Chapter 4

The Generalization of DEA Model

The purpose of selecting inputs and outputs in the DEA model is to develop a relative financial strength (of a firm) that will be highly correlated with stock return. Given the 18 financial parameters (in Section 3.1.1), it still remains unknown how these parameters should be allocated into input/output sets so that the resulting correlation between a firm's relative performance score (RPS) and its stock market return will be maximized. If the inputs and outputs are set "a priori", it is unlikely that these selected inputs and outputs will produce the highest correlation. Furthermore, if the number of selected inputs and outputs exceeds a certain threshold, where the sample size is less than twice the product of the number of inputs and number of outputs, see [24], model saturation problem may occur. In this case, the DEA model will lose its ability to discriminate the relative performance of firms, thus, a firm with weak performance may be falsely treated as a strong firm. Subsequently, the computed RPS values will have weak correlation with stock market returns.

The focus is to find a set of inputs/outputs that provides a relative strength measure that has the highest correlation with stock market returns. A generalized DEA model is developed to determine such a configuration of input/output that maximizes the correlation between the DEA-based RPS scores and the stock market performance. This maximization involves a difficult binary nonlinear program that requires iterative re-configuration of parameters of financial statements as inputs and outputs. A two-step heuristic algorithm that combines random sampling and local search optimization is utilized for this purpose. A statistical test is developed and it is used to validate the maximized correlation. A predictor termed the "Relative Financial Strength Indicator (RFSI)" is developed, which is representative of the stock market returns. The methodology is tested in the U.S. Technology sector to determine RFSI indicators for stock selection. Then, those selected stocks are used within portfolio optimization models to demonstrate the usefulness of the scheme for portfolio risk management. It should be noted that the "model saturation" issue, raised in Section 3.7, will be corrected under the GDEA approach.

4.1 The Generalized DEA Approach

When the DEA-based RPS model in (3.6) is used, the M input parameters and N output parameters are required to be explicitly identified *a priori*, i.e., an exogenous specification. While this may be possible in certain applications (such as production) where input to output conversion mechanisms are well-understood, our case is different. We must select a set of input and output parameters from the universe of 18 financial parameters describing a firm's financial health (see Section 3.1.1). The objective of such a selection is that the resulting RPS score of a firm can be interpreted as providing a measure of its underlying financial strength. Such financial strength measures are required to be strongly correlated with the market price process, under the efficient market hypothesis. If the inputs and outputs for the RPS model are inappropriately chosen, the resulting RPS values for firms may not be representative of the fundamental financial strengths that are rewarded by the financial markets. The generalized DEA approach (GDEA) developed in this chapter leaves the selection of inputs and outputs as flexible as possible in the sense that a proper selection of the latter is sought iteratively to maximize the correlation of the DEA-based strength evaluation and the stock market performance. This process is best-explained in Figure 4.1, and thus, the GDEA model is based on an endogenous input/output specification.

The GDEA process of input/output selection in Figure 4.1 can be described as follows. Suppose a group of firms in a specific industry is under consideration. For a given fixed input/output categorization, solve the DEA model to obtain the RPS for each firm, then, the correlation between the RPS and the stock market returns will be measured and checked if it has the maximum value. If not, reconfigure inputs and outputs according to some criterion and re-solve the DEA model. This iterative process that reconfigures the input/output sets, leading to the highest correlation, is termed the Generalized DEA approach.

More specifically, let us consider the universe of I (=18) parameters that are potential inputs and outputs. Suppose a given parameter i may be used as an

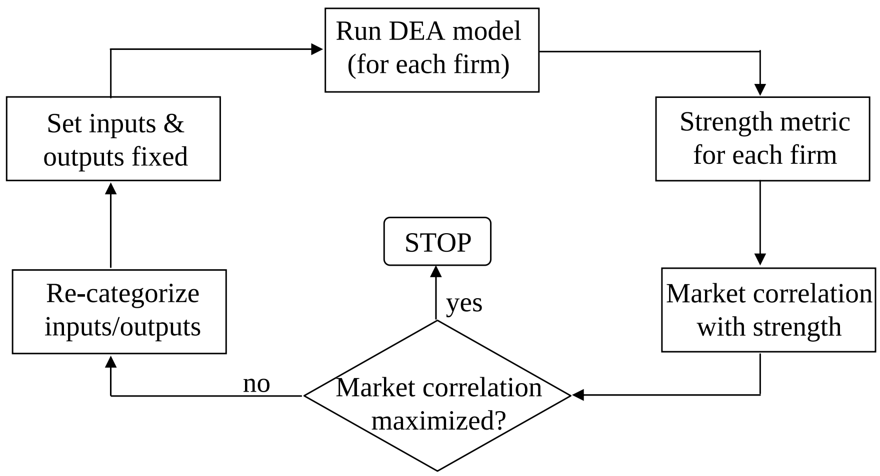

Figure 4.1: Schematic of the Generalized DEA approach

input and/or output, or not used at all. Furthermore, suppose the level at which a parameter must be specified in the DEA model in (3.6) is treated as unknown. Consequently, for a parameter i with an observed (data) value ξ_{ij} for firm j, the level at which it enters the model as an *input* is denoted by $y_i \xi_{ij}$, where the input scaling variable $y_i \geq 0$. Similarly, the level at which the parameter i enters as an *output* for firm j is $z_i \xi_{ij}$ and the output scaling variable $z_i \geq 0$. Collecting the y_i and z_i components for all parameters, we define an input scaling parameter vector by $y \in \Re^I$ and an output scaling vector by $z \in \Re^I$. An appropriate selection of values for the pair $(y, z) \in \Re^{2I}$ is not a firm-specific issue. Rather, it must be chosen as a property of the industry, so that RPS of firms can be compared to each other within the same industry. More importantly, such a performance score must represent the fundamental financial strength of a firm that is predictive of (or highly correlated with) the stock price action. Therefore, the vector (y, z) is to be held fixed when computing performance scores of all J firms in the group. Under the scaling vector parameterization (y, z), the resulting DEA model is

$$\begin{aligned}
\eta_k(y,z) := \max_{u,v} \quad & \frac{\sum_{i=1}^{I}(z_i\xi_{ik})v_i}{\sum_{i=1}^{I}(y_i\xi_{ik})u_i} \\
\text{s.t.} \quad & \frac{\sum_{i=1}^{I}(z_i\xi_{ij})v_i}{\sum_{i=1}^{I}(y_i\xi_{ij})u_i} \leq 1, \quad j = 1, \ldots, J \\
& u_i, v_i \geq 0, \ i = 1, \ldots, I,
\end{aligned} \tag{4.1}$$

where y is chosen such that $\sum_{i=1}^{I} y_i > 0$. $\eta_k(y, z)$ is the relative performance score (RPS) of firm k corresponding to the input/output scaling vector pair (y, z). The following equivalent linear programming model can be used to compute $\eta_k(y, z)$.

$$\begin{aligned}
\eta_k(y,z) := \max_{u,v} \quad & \sum_{i=1}^{I}(z_i\xi_{ik})v_i \\
\text{s.t.} \quad & \sum_{i=1}^{I}(y_i\xi_{ik})u_i \leq 1 \\
& -\sum_{i=1}^{I}(y_i\xi_{ij})u_i + \sum_{i=1}^{I}(z_i\xi_{ij})v_i \leq 0, \quad j = 1, \ldots, J \\
& u_i, v_i \geq 0, \ i = 1, \ldots, I.
\end{aligned} \tag{4.2}$$

In DEA, the issue of setting a given parameter in both the input and output sets simultaneously has been addressed in, for instance, see [10] and [19]. In the case of a CCR model, when a parameter is used both in inputs and outputs, the resulting DEA score is 1 for each firm. That is,

Proposition 4.1.1 *For some parameter $i \in \{1, \ldots, I\}$, let $y_i > 0$ and $z_i > 0$. For a firm k being evaluated, suppose the measured value of parameter i satisfies $x_{ik} > 0$. Then, $\eta_k(y, z) = 1$ holds.*

Proof. Considering the model (4.2), construct the solution $(\hat{u}, \hat{v})$: $\hat{u}_r = \frac{1}{y_r \xi_{rk}} > 0$, $\hat{u}_i = 0$ if $i \neq r$ and $\hat{v}_r = \frac{1}{z_r \xi_{rk}} > 0$, $\hat{v}_i = 0$ if $i \neq r$. The solution $(\hat{u}, \hat{v})$ satisfies the constraints of (4.2) since $(y_r \xi_{rk})\hat{u}_r = 1$ and

$$-(y_r \xi_{rj})\hat{u}_r + (z_r \xi_{rj})\hat{v}_r = 0, \quad j = 1, \ldots, J.$$

Thus, $\eta_k(y, z) \geq (z_r \xi_{rk})\hat{v}_r = 1$. Since $\eta_k(y, z) \leq 1$ also must hold, we conclude that $\eta_k(y, z) = 1$. ■

For example, when the parameter i is the "Current Ratio", the data is always positive for all firms. If "Current Ratio" is chosen as both input and output in the DEA model, the performance scores for all the firms under evaluation will be 1. Thus, the model fails to uncover a firm's business strength. In such a case, correlation between the computed RPS and the stock market performance is zero, and thus, such a choice on (y, z) will not maximize the desired strength-market correlation, see Figure 1. Consequently, to reduce the search space for (y, z) in the correlation maximization, for every component pair (y_i, z_i), we must specify $y_i z_i = 0$ for all $i = 1, \ldots, I$. This prohibits a given financial parameter i from being in the inputs and outputs simultaneously.

Definition 4.1.2 *A given vector-pair (y, z) is said to satisfy the* **complementarity** *condition if and only if $y_i z_i = 0$ for all $i = 1, \ldots, I$. In this case, such a pair is simply referred to as a complementary pair (y, z).*

Therefore, a complementary pair (y, z) allows the categorization of the universe of I parameters as distinct inputs and outputs. In contrast, Cook *et al.* [19] introduced the notion of flexible measures whereby a new parameter can be considered in the presence of existing input/output sets. Their model then determines if this new parameter should be an input or an output in order to improve the (maximized) DEA efficiency. In our case, the objective is to have the highest correlation between DEA-based RPS and the stock market returns. Therefore, we take a different approach that allows the complementary vector pairs (y, z) to play the role of flexible measures in a more generalized setting. For this purpose, the domain of (y, z) must be appropriately chosen to force which parameters should never (or must) be in inputs/outputs.

Recall that in Chapter 3, the term "expert information on parameter selection" was used to indicate the existence of "expert knowledge" in specifying inputs and outputs in the DEA model. For example, the parameters of asset utilization, liquidity, and leverage perspectives can generally be interpreted as inputs because activities that are measured by these parameters depend on the planning and operational strategies of a firm. On the other hand, the parameters of profitability and growth perspectives are generally considered as outputs because revenue/income generation is a major objective criterion for a firm. The valuation parameters measure how well the equity markets perceive "success" of a firm, and they are generally not concerned with a firm's input strategy. By considering the above "a priori" knowledge, the *Augmented* selection of inputs and outputs (see (3.3) and (3.5)) is developed by including all 18 parameters in the model. By incorporating the concept of complementary pair (y, z), the *Augmented* selection can be expressed using the following complementary pair,

$$\left.\begin{aligned} y^a &:= \{0,0,0,0,1,1,1,1,1,1,1,1,1,0,0,0,0,0\} \\ z^a &:= \{1,1,1,1,0,0,0,0,0,0,0,0,0,1,1,1,1,1\}\,. \end{aligned}\right\} \tag{4.3}$$

However, the purpose of this chapter is to find a DEA-based relative performance strength for a firm such that the correlation between the firm's RPS and stock market returns is maximized. Although the *Augmented* selection uses some expert knowledge to partition inputs and outputs, is it really necessary to include all these parameters in the model? Will the correlation increase by reducing the number of inputs and the number of outputs? What input/output category will produce the highest correlation? To answer these questions, an input/output search needs to be conducted in the inputs and outputs feasible domain. In this domain, input parameters are only chosen from the perspectives of asset utilization, liquidity, and leverage, while the output parameters are chosen only from the profitability, growth, and valuation perspectives.

4.1.1 Feasible Domain of Scaling Vectors

Appealing to Proposition 2.3.2, it follows that the DEA model in (4.2) is positively homogeneous of degree 0 in (y, z) jointly and separately. The main implication of this result is that it restricts the domain of feasible *complementary* vector pairs (y, z)

to a binary space. Along with the complementarity condition in Definition 4.1.2, thus, the feasible domain of the scaling vectors (y, z) in (4.2) must satisfy,

$$y_i z_i = 0, \;\; y_i, z_i \in \{0, 1\}, \;\; i = 1, \ldots, I. \tag{4.4}$$

An equivalent linear transformation of (4.4), along with the condition that $\sum_i y_i > 0$ and expert information, is considered. This yields the following Restricted Binary Complementary Domain (RBCD), denoted by Ω^*, for the feasible choices for (y, z).

$$\textbf{RBCD :} \quad \Omega^* := \left\{ (y, z) \;\middle|\; \begin{array}{l} \sum_{i=1}^{I} y_i \geq 1, \; y_i + z_i \leq 1, \; \sum_{i=1}^{4} y_i + \sum_{i=14}^{18} y_i = 0, \\ \sum_{i=5}^{13} z_i = 0, \; y_i, z_i \in \{0, 1\}, \; i = 1, \ldots, I \end{array} \right\}. \tag{4.5}$$

The reason the above domain is called *Restricted* binary complementary domain is that it is restricted by some expert information. Accordingly, for every firm k in the industry, the corresponding relative performance score $\eta_k(y, z)$ is determined by the model in (4.2) for a specified binary complementary vector pair $(y, z) \in \Omega^*$. The goal is to search for $(y, z) \in \Omega^*$ such that the relative performance score so-computed would be a suitable metric of the underlying financial strength of a given firm, relative to all firms in the group.

When the model in (4.2) is specified using parameters under the RBCD condition in (4.5) that requires choosing $(y, z) \in \Omega^*$, it is herein referred to as GDEA under *Restricted* BCD, or simply, *Restricted* GDEA version.

On the other hand, a relaxed version of the *Restricted* binary complementary domain is also considered when the prior information is not applied in parameter categorization. This leads to the following *Unrestricted* Binary Complementary Domain (or UBCD).

$$\textbf{UBCD :} \quad \Omega := \left\{ (y,z) \; : \; \sum_{i=1}^{I} y_i \geq 1, \; y_i + z_i \leq 1, \; y_i, z_i \in \{0,1\}, \; i = 1, \ldots, I \right\}. \tag{4.6}$$

If the model in (4.2) uses parameters under the UBCD condition in (4.6) that requires choosing $(y,z) \in \Omega$, it is herein referred to as GDEA under *Unrestricted* BCD, or simply, *Unrestricted* GDEA version. Performance of the *Restricted* and *Unrestricted* GDEA versions will be compared within portfolio optimization using the application reported in Section 4.5. In the sequel, the results using GDEA approach will also be compared with the *Basic* input/output selection in (3.3) and (3.4).

4.2 Relative Financial Strength Indicator (RFSI)

The process of determining an RFSI requires, first, determining a correlation metric for the DEA-based RPS scores and the stock price returns, for the industry as a whole, for a given vector pair (y,z), and second, designing a suitable iterative procedure to choose $(y,z) \in \Omega$ (or Ω^*) in an attempt to maximize the latter correlation metric (see Figure 4.1).

In Chapter 3, the basic idea behind determining correlation between a DEA-based financial strength and its stock price returns for each firm was presented. In the development here, this idea will be more formalized. The discussion here pertains to the unrestricted Ω; for the restricted version of RFSI, Ω is simply replaced with Ω^*.

Let the DEA-based performance score for a firm k in a given industry be determined according to the model in (4.2) as $\eta_k(y, z)$, for a specified categorization $(y, z) \in \Omega$. Solving (4.2) requires the realized values ξ_{ij} of all financial parameters for all firms. The future value of a parameter i for firm j is a random variable, denoted by Ξ_{ij}. The collection of random variables Ξ_{ij} for $i = 1, \ldots, I = 18$ and $j = 1, \ldots, J$ is $\boldsymbol{\Xi}$. Realizations of Ξ_{ij} are observed as ξ_{ij} in (published) financial statements of a given period (i.e., quarter). For a future period t of uncertain financial performance, the collection of random variables is the vector $\boldsymbol{\Xi}^t := \{\Xi^t_{ij} \ : \ \forall i, \ \forall j\}$. Let the DEA-based relative performance score (RPS) for the industry is represented by the collection of random variables $\boldsymbol{\eta}^t(y, z) := \{\eta_j(y, z; \boldsymbol{\Xi}^t) \ : \ j = 1, \ldots, J\}$. Once the period t financial statements are observed, with $\boldsymbol{\Xi}^t$ realized as $\boldsymbol{\xi}^t$, the random vector $\boldsymbol{\eta}^t(y, z)$ is realized as the vector of values $\left\{\eta_j(y, z; \boldsymbol{\xi}^t) \ : \ j = 1, \ldots, J\right\}$. In addition, let R^t_j denote the stock price rate of return (RoR) random variable (for future period t) of firm j, and those for all firms are represented by the random J-vector $\boldsymbol{R}^t := \{R^t_j \ : \ j = 1, \ldots, J\}$. Observed realizations of period t RoR is the vector $\boldsymbol{r}^t := \{r^t_j \ : \ j = 1, \ldots, J\}$. Consider the pairwise correlations between the two random vectors $\boldsymbol{\eta}^t(y, z)$ and $\boldsymbol{R}^t$, denoted by the correlation vector $\boldsymbol{\Gamma}^t(y, z) \in \Re^J$. Its

j^{th} component, for firm j, is given by

$$\Gamma_j^t(y,z) := Corr\left\{\eta_j(y,z;\boldsymbol{\Xi}^t)\,,\; R_j^t\right\}, \tag{4.7}$$

where $j = 1,\ldots,J$. The correlation vector $\mathbf{\Gamma}^t(y,z)$ is, therefore, a measure of the predictive power of the DEA-based RPS value on stock price returns for the chosen industry. Indeed, a positive and significant correlation vector $\mathbf{\Gamma}^t(y,z)$ implies that the DEA-based performance score is a valuable proxy of the stock market performance of the industry. Observe that $\mathbf{\Gamma}^t(y,z)$ for period t depends on the chosen binary complementary vector $(y,z) \in \Omega$. The best industry correlation is thus obtained when one searches for $(y,z) \in \Omega$ such that an appropriate metric of the vector $\mathbf{\Gamma}^t(y,z)$ is maximized. Vector norms cannot be used as appropriate metrics here because the goal is to seek positive (and large) correlations across all firms in the industry. While more complicated formulae are possible, we use the simple average metric,

$$\bar{\Gamma}^t(y,z) := \frac{1}{J}\sum_{j=1}^{J}\Gamma_j^t(y,z), \tag{4.8}$$

herein termed the *industry correlation metric*, to search for the highest positive correlations industry-wide. Note that the correlation vector $\mathbf{\Gamma}^t(y,z)$ is unknown for the future period t, and thus, it must be forecasted. To estimate $\mathbf{\Gamma}^t(y,z)$, we use the historical (observed) sample $\boldsymbol{\xi}^\ell$, $\ell = 1,\ldots,t-1$. Using a history length of t_0 periods,

$\Gamma_j^t(y,z)$ is estimated by the sample correlation coefficient, given by

$$\gamma_j^t(y,z) := \text{Correlation coefficient between } \left\{\eta_j(y,z;\boldsymbol{\xi}^\ell)\right\}_{\ell=t-t_0}^{t-1} \text{ and } \left\{r_j^\ell\right\}_{\ell=t-t_0}^{t-1}. \tag{4.9}$$

Then, the industry correlation metric $\bar{\Gamma}^t(y,z)$ in (4.8) is estimated as

$$\bar{\gamma}^t(y,z) := \frac{1}{J}\sum_{j=1}^{J}\gamma_j^t(y,z). \tag{4.10}$$

Observe that the statistic $\bar{\gamma}^t(y,z)$ for period t depends on the chosen binary complementary vector $(y,z) \in \Omega$. The best industry-correlation metric is thus obtained when one searches for $(y,z) \in \Omega$ such that $\bar{\gamma}^t(y,z)$ is maximized, i.e., solve the industry-correlation maximization model

$$\begin{aligned} \texttt{(CORMAX)} \ : \ \bar{\gamma}^0 := \quad & \max_{y,z} \quad \bar{\gamma}^t(y,z) \\ & \text{s.t.} \quad (y,z) \in \Omega. \end{aligned} \tag{4.11}$$

Let the objective value for model (4.11) when Ω is replaced with Ω^* be denoted by $\bar{\gamma}^*$. Then, we have the following result:

Proposition 4.2.1 $\bar{\gamma}^0 \ \geq \ \bar{\gamma}^* := \max_{y,z}\left\{\bar{\gamma}^t(y,z) \ : \ (y,z) \in \Omega^*\right\}$.

Proof. The model in (4.11) with domain Ω is relaxed problem of model in (4.11) with domain Ω^*. Given the maximization objective, the result holds. ■

Let an optimal binary complementary pair solving the above maximization (in (4.11)) be denoted by (y^*, z^*), and that for using domain Ω^* is $(\hat{y}^*, \hat{z}^*)$. Note that dependence of this pair on the period index t is suppressed. Although the in-sample correlation that is obtained from GDEA under *Unrestricted* BCD is higher than that from GDEA under *Restricted* BCD, will it imply that the out-of-sample portfolio test using stocks selected based on $(\hat{y}^*, \hat{z}^*)$ will outperform that based on (y^*, z^*)? This will be pursued in Section 4.6. Although the maximized industry correlation metric $\bar{\Gamma}^t(y^*, z^*)$ in (4.8) is estimated by solving the model in (4.11), it is required to be statistically significant, for if not, the use of the DEA-based financial strength indicator for the given industry cannot be validated for investment decision making. Statistical tests for this purpose are discussed in Section 4.3. When this industry correlation metric is verified to be statistically significant, the Relative Financial Strength Indicator (RFSI) for a given firm in the industry is defined as follows.

Definition 4.2.2 *Suppose $\bar{\Gamma}^t(y^*, z^*)$ is statistically significant for a given industry, where (y^*, z^*) is an optimal solution of (4.11). Then, the Relative Financial Strength Indicator (RFSI) of firm j for (a future) period t, given the observed financial statement data $\boldsymbol{\xi}^\ell$ for $t - t_0 \leq \ell \leq t - 1$ for the industry, is defined by*

$$RFSI(t, j) := E\left[\eta_j(y^*, z^*; \boldsymbol{\Xi}^t) \mid \eta_j(y^*, z^*; \boldsymbol{\xi}^{t-t_0}), \ldots, \eta_j(y^*, z^*; \boldsymbol{\xi}^{t-1})\right], \qquad (4.12)$$

where $\eta_j(y^*, z^*; \boldsymbol{\xi}^\ell)$ is computed according to the DEA model in (4.2) for the input/output categorization (y^*, z^*), and $E[.]$ denotes the conditional expectation given the RPS scores of the historical t_0 periods.

To simplify the computation of RFSI, the expectation in (4.12) is estimated by the simple moving average forecast (of $\hat{t}$ periods, $\hat{t} \leq t_0$), given as

$$RFSI(t, j; \hat{t}) = \frac{1}{\hat{t}} \sum_{\ell=t-\hat{t}}^{t-1} \eta_j(y^*, z^*; \boldsymbol{\xi}^\ell). \tag{4.13}$$

$RFSI(t, j; \hat{t})$ is bounded within 0 and 1, where a value of unity indicates the highest possible relative financial strength indicator for firm j, relative to the industry concerned. Also note that a single input/output categorization (y^*, z^*) of the 18 financial parameters in Section 3.1.1 is used in computing the RFSI for all firms in the industry, for the future period t. For future periods beyond t, it may be necessary to adapt RFSI to new financial statement observations, by resolving (4.11) for a revised optimal input/output categorization.

4.2.1 Solution method

The `CORMAX` model in (4.11) is a difficult optimization problem because evaluation of the objective function (statistic) $\bar{\gamma}^t(y, z)$ in (4.10) requires the solution of a sequence of linear optimization models (4.2) so that each of the sample correlation coefficients $\gamma_j^\ell(y, z)$ in (4.9) can be computed. Therefore, the objective function in (4.11) cannot be explicitly written in closed-form nor can it be verified to be concave (or pseudo-concave) in the $2I$-dimensional decision variable-vector (y, z). Nonconvex optimization is known to be computationally tedious, see for instance, [35]. Moreover, Ω is a binary solution space, i.e., (4.11) is a binary nonconvex optimization model. Global optimality conditions for discrete nonconvex optimization have been studied, e.g. see [36]. However, efficient methods are available only for specially structured problems and/or without integer restrictions, e.g. see [60] and [65]. Alternatively, we employ the following heuristic solution scheme: Random Sampling with Local Optimization (RSLO).

Random Sampling

The method is a two-step procedure, which is based on, first, sampling a set of initial (y, z) points from the feasible domain Ω, and then, performing a local-search optimization in Ω for each of those initial sample points. Consider a random sample of (vector) points $\omega^s := (y^s, z^s) \in \Omega \subset \Re^{2I}$, for $s \in \mathcal{S}$, where $\mathcal{S}$ denotes the index set of the sample points. For each sample point, the objective criterion is calculated and the sample of industry correlation metric values

$$\left\{\bar{\gamma}^t(y^s, z^s) \ : \ s \in \mathcal{S}\right\} \tag{4.14}$$

is collected. Then, each sample value $\bar{\gamma}^t(y^s, z^s)$ is improved locally by employing a non-gradient based local search procedure, starting from the point $\omega^s \in \Omega$. The corresponding locally improved solution is denoted by $\tilde{\omega}^s := (\tilde{y}^s, \tilde{z}^s)$, which is termed a *pseudo-optimal* solution. (Local optimality of pseudo-optimal solutions is pursued in Chapter 6). Then, an approximation for the *best* input/output categorization for the industry is determined by

$$(y^*, z^*) = \arg\max_{s \in S} \left\{ \ \bar{\gamma}^t(\tilde{y}^s, \tilde{z}^s) \ \right\}. \tag{4.15}$$

The following procedure is applied to generate a (random) sample point $\omega^s \in \Omega$: randomly draw a set of $2I$ values from a continuous uniform distribution in $[-1, 1]$. The first I values are collected to form the I-vector α^s. The last I values are collected to form the I-vector β^s. Then, the sample point $\omega^s = (y^s, z^s)$ is defined by the solution of the binary linear program

$$(y^s, z^s) := \arg\max_{(y,z)} \left\{ (\alpha^s)' y + (\beta^s)' z \ : \ (y, z) \in \Omega \right\}, \tag{4.16}$$

where a prime denotes the transposition of a vector. This process is then repeated for each $s \in \mathcal{S}$.

Local search

The non-gradient-based local search procedure applied here is a modification from the Hooke and Jeeves (HJ) method, see [9]. Given a current solution ω^p, at some iteration p of the local search procedure, the original HJ method performs an *exploratory* search along the coordinate directions. Coordinate directions that improve the objective function are used to define a new iterate. The direction to the new iterate from the starting solution ω^p is used to perform a *pattern* search. The basic idea about local search is plotted in Figure 4.2, where $(y^0,\ z^0)$ is the starting location and $(\hat{y},\ \hat{z})$ is the local optimal. This modified HJ method is adapted to the binary model in (4.11).

For some candidate $\omega^p \in \Omega$, the i^{th} coordinate ω_i^p is either 0 or 1. If $\omega_i^p = 0$, then an exploratory move is allowed only in the positive (ω_i) coordinate direction. If $\omega_i^p = 1$, then an exploratory move is allowed only in the negative (ω_i) coordinate

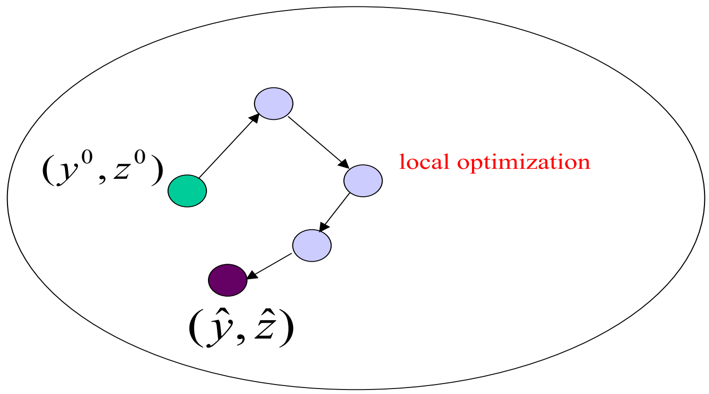

Figure 4.2: Local optimization steps

direction. Once, a new iterate $\hat{\omega}^p$ is so-determined, a pattern (line) search is not necessary in our case since the search point $\omega^p + \lambda(\hat{\omega}^p - \omega^p) \notin \Omega$ for $\lambda \notin \{0, 1\}$. The resulting algorithmic steps are as follows:

<u>Algorithm-LS</u>

`Initialization:` Given $\omega^s = (y^s, z^s) \in \Omega$, see (4.16), determine $\bar{\gamma}^t(\omega^s)$.

Set $p = 1$, $f(p) = \bar{\gamma}^t(\omega^s)$, and $\omega(p) = \omega^s$.

`Step 1:` For $i = 1, \ldots, 2I$ and denoting the i^{th} *elementary* coordinate direction by e_i, let

$$
\begin{array}{lll}
 & \omega^i := \omega(p) + e_i & \text{if } (\omega(p) + e_i) \in \Omega \text{ and } f(p) < \bar{\gamma}^t(\omega(p) + e_i) \\
\text{else,} & \omega^i := \omega(p) - e_i & \text{if } (\omega(p) - e_i) \in \Omega \text{ and } f(p) < \bar{\gamma}^t(\omega(p) - e_i) \\
\text{else,} & \omega^i := \omega(p). &
\end{array}
$$

Compute ω^{2I+1} by the XOR (`"exclusive or"` or `"not equal to"`) operation:

$$\omega^{2I+1} := \omega^1 \textbf{ xor } \omega^2 \textbf{ xor } \cdots \textbf{ xor } \omega^{2I}.$$

If $\omega^{2I+1} \notin \Omega$, set $\omega^{2I+1} = \omega(p)$. If $\omega^{2I+1} \in \Omega$, compute $\bar{\gamma}^t(\omega^{2I+1})$.

`Step 2:` Let $\omega(p+1) := \arg\max\{\bar{\gamma}^t(\omega^i) \ : \ i = 1\ldots, 2I+1\}$.

If $\bar{\gamma}^t(\omega(p+1)) = \bar{\gamma}^t(\omega(p))$: `Terminate the local search` and set $\bar{\omega}^s = \omega(p)$.

Else, if $\bar{\gamma}^t(\omega(p+1)) > \bar{\gamma}^t(\omega(p))$, let $f(p+1) = \bar{\gamma}^t(\omega(p+1))$

set $p \leftarrow p+1$ and go to Step 1.

4.3 Statistical tests of Correlations

Suppose given H industries, we are concerned with identifying industries that do not provide statistical evidence for RFSI-based predictability of stock returns, i.e., the industry correlation metric $\bar{\Gamma}^t_h(y^{h*}, z^{h*})$ is not a significant positive value. Consider the following hypothesis test for a minimum positive correlation (ρ_0) for a given industry h,

$$\left.\begin{aligned} H_0 &: \ \bar{\Gamma}^t_h(y^{h*}, z^{h*}) \leq \rho_0 \\ H_1 &: \ \bar{\Gamma}^t_h(y^{h*}, z^{h*}) > \rho_0. \end{aligned}\right\} \tag{4.17}$$

The above *null* hypothesis H_0 indicates that DEA-based relative financial strength is not consistent with the efficiency market hypothesis (EMH) for industry h. Note that $\bar{\Gamma}^t_h(y^{h*}, z^{h*}) := \frac{1}{J_h}\sum_{j=1}^{J}\Gamma^t_{j,h}(y^{h*}, z^{h*})$, see (4.8), and firm-correlations $\Gamma^t_{j,h}(y^{h*}, z^{h*})$ are estimated by $\gamma^t_{j,h}(y^{h*}, z^{h*})$ in (4.9). Consider the following arctan hyperbolic transformation of $\gamma^t_{j,h}(y^{h*}, z^{h*})$:

$$\psi_{j,h} := \tanh^{-1} \gamma^t_{j,h}(y^{h*}, z^{h*}) = \frac{1}{2} \log_e \left[\frac{1 + \gamma^t_{j,h}(y^{h*}, z^{h*})}{1 - \gamma^t_{j,h}(y^{h*}, z^{h*})} \right]. \tag{4.18}$$

Using the results that $E[\gamma^t_{j,h}] \approx \Gamma^t_{j,h}$ and $Var(\gamma^t_{j,h}) \approx \left[1 - (\Gamma^t_{j,h})^2\right]^2$, R.A. Fisher (1890-1962) showed that $\psi_{j.h}$ is approximately normally distributed,

$$\psi_{j,h} \approx Normal\left(\frac{1}{2} \log_e \left[\frac{1 + \Gamma^t_{j,h}(y^{h*}, z^{h*})}{1 - \Gamma^t_{j,h}(y^{h*}, z^{h*})} \right] , \frac{1}{t_0 - 3} \right), \tag{4.19}$$

see [59], where t_0 is the number of periods used in the estimation in (4.9). Next, define the industry-average statistic by

$$\bar{\psi}_h := \frac{1}{J_h} \sum_{j=1}^{J_h} \psi_{j,h} , \tag{4.20}$$

which is the sum of J_h normal random variables, and thus, $\bar{\psi}_h$ is normally distributed:

$$\bar{\psi}_h \approx Normal\left(\frac{1}{2J_h} \sum_{j=1}^{J_h} \log_e \left[\frac{1 + \Gamma^t_{j,h}(y^{h*}, z^{h*})}{1 - \Gamma^t_{j,h}(y^{h*}, z^{h*})} \right] , \hat{\sigma}^2 \right), \tag{4.21}$$

where the variance of $\bar{\psi}_h$ is given by

$$\hat{\sigma}^2 := \frac{1}{J_h(t_0 - 3)} + \frac{2}{(J_h)^2} \sum_{j<k} Cov\left(\psi_{j,h}, \psi_{k,h}\right). \tag{4.22}$$

$Cov\left(\psi_{j,h}, \psi_{k,h}\right)$ in (4.22) depends on the covariance between $\gamma^t_{j,h}(y^{h*}, z^{h*})$ and $\gamma^t_{k,h}(y^{h*}, z^{h*})$. The latter two correlations are the point estimates of the firm-correlations $\Gamma^t_{j,h}(y^{h*}, z^{h*})$

and $\Gamma^t_{k,h}(y^{h*}, z^{h*})$, for firms j and k. Firm-correlation measures the degree of association between a firm's financial strength and its stock price return, a process that may be expected to be fairly consistent across all firms in the industry. Therefore, when firms j and k operate independent of each other, the point estimates $\gamma^t_{j,h}(y^{h*}, z^{h*})$ and $\gamma^t_{k,h}(y^{h*}, z^{h*})$ of the firm-correlations $\Gamma^t_{j,h}(y^{h*}, z^{h*})$ and $\Gamma^t_{k,h}(y^{h*}, z^{h*})$, respectively, can be expected to be independent of each other as well. This independence assumption results in $Cov\,(\psi_{j,h}, \psi_{k,h}) = 0$, and thus, the parameters of distribution of $\bar{\psi}_h$ are (approximately) known once the value of $\Gamma^t_{j,h}(y^{h*}, z^{h*})$ is known.

Observe that under the equality sign in the null hypothesis in (4.17), one has reference only to the industry correlation metric, i.e., $\bar{\Gamma}^t_h(y^{h*}, z^{h*}) = \rho_0$; however, we need knowledge of the individual firm-correlations $\Gamma^t_{j,h}(y^{h*}, z^{h*})$. Let the latter correlations be given by

$$\Gamma^t_{j,h}(y^{h*}, z^{h*}) = \rho_0 \theta_{j,h}\ , \quad \text{for } j = 1, \ldots, J_h, \tag{4.23}$$

where $\theta_{j,h}$ must satisfy the requirements:

$$\sum_{j=1}^{J_h} \theta_{j,h} = J_h \quad \text{and} \quad -\frac{1}{\rho_0} \leq \theta_{j,h} \leq \frac{1}{\rho_0}\,, \forall j = 1, \ldots, J_h. \tag{4.24}$$

The constraints in (4.24) follow from the fact that the industry-correlation metric is ρ_0 (specified as a positive value) and that firm-correlations are bounded within -1

and +1. Then, denoting

$$\psi_0(\theta) := \frac{1}{2J_h}\sum_{j=1}^{J_h}\log_e\left[\frac{1+\rho_0\theta_{j,h}}{1-\rho_0\theta_{j,h}}\right] \quad \text{and} \quad \sigma^2 := \frac{1}{J_h(t_0-3)}, \tag{4.25}$$

it follows that $\bar{\psi}_h \approx Normal\,(\psi_0(\theta),\sigma^2)$. For finiteness of the mean, $\psi_0(\theta)$, the inequalities in (4.24) must be satisfied as strict inequalities, i.e., $-\frac{1}{\rho_0} < \theta_{j,h} < \frac{1}{\rho_0}$, $j = 1,\ldots,J_h$. Then, for α-significance level and for the one-sided test, H_0 is accepted if

$$\sqrt{J_h(t_0-3)}\left(\bar{\psi}_h - \psi_0(\theta)\right) \leq \mathcal{Z}^{-1}(1-\alpha), \tag{4.26}$$

or,

$$\bar{\psi}_h \leq \psi_0(\theta) + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{J_h(t_0-3)}}, \tag{4.27}$$

where $\mathcal{Z}^{-1}(.)$ is the inverse *c.d.f.* of a standard normal random variable. To test H_0 to conclude that the DEA-based relative strength does not provide sufficient explanatory power for stock price returns in industry h, therefore, specific θ values are required. Such information is not available, nor can it be estimated. However, if H_0 is accepted for the smallest (threshold) value of the right hand side in (4.27) over all possible θ, then, indeed H_0 is accepted for the industry h. Define,

$$\psi_0^{\min} := \inf_{\theta}\left\{\psi_0(\theta)\ :\ \sum_{j=1}^{J_h}\theta_{j,h} = J_h\,,\ -\frac{1}{\rho_0} < \theta_{j,h} < \frac{1}{\rho_0}\,, j = 1,\ldots,J_h\right\}. \tag{4.28}$$

Hence, if $\bar{\psi}_h \leq \psi_0^{\min} + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{J_h(t_0-3)}}$, then H_0 is accepted for the industry h.

Proposition 4.3.1 *For* $0 < \rho_0 < 1$,

$$\psi_0^{\min} = \frac{1}{2} \log_e \left(\frac{1+\rho_0}{1-\rho_0} \right). \tag{4.29}$$

Proof. Note that $\psi_0(\theta)$ is nonconvex - it is convex in the positive orthant and concave in the negative orthant. Consider the (relaxed) minimization problem:

$$Z^* := \min_{\theta} \left\{ \psi_0(\theta) \ : \ \sum_{j=1}^{J_h} \theta_{j,h} = J_h \right\}, \tag{4.30}$$

and thus, $Z^* \leq \psi_0^{\min}$. Since the constraints of (4.30) are linear, "Constraint Qualification" (CQ) is satisfied at all feasible solutions, thus implying that every optimal solution of (4.30) must be a Karush-Kuhn-Tucker (KKT) point, see [9]. Denoting the Lagrange multiplier associated with the equality constraint by λ, the KKT conditions yield,

$$\frac{2\rho_0}{1-(\rho_0 \theta_{j,h})^2} + \lambda = 0\,, \quad \forall j = 1, \ldots, J_h. \tag{4.31}$$

Therefore, $\theta_{j,h} = \nu_j a$ must hold for all $j = 1, \ldots, J_h$, where ν_j is $+1$ or -1 and a is a positive constant. The equality constraint thus implies that $\sum_{j=1}^{J_h} \nu_j = J_h/a > 0$, which is the net count of positive values in $\theta_{j,h}$ for $j = 1, \ldots, J_h$, and thus, $\sum_{j=1}^{J_h} \nu_j = 1, 2, \ldots, J_h$. That is, a can take on the set of possible discrete values $\{1, \frac{J_h}{J_h-1}, \frac{J_h}{J_h-2} \ldots, \frac{J_h}{2}, J_h\}$. Each of these values for a defines a distinct KKT point provided the objective function is well-defined at those points, which is given by

$$\begin{aligned}
&\frac{1}{2J_h}\left[\sum_{j:\nu_j=+1}\log_e\left(\frac{1+a\rho_0}{1-a\rho_0}\right)+\sum_{j:\nu_j=-1}\log_e\left(\frac{1-a\rho_0}{1+a\rho_0}\right)\right] \\
&= \frac{1}{2J_h}\left(\sum_{j=1}^{J_h}\nu_j\right)\log_e\left(\frac{1+a\rho_0}{1-a\rho_0}\right)
\end{aligned}$$

since the following identity holds:

$$\log_e\left[\frac{1+\rho_0(a)}{1-\rho_0(a)}\right]+\log_e\left[\frac{1+\rho_0(-a)}{1-\rho_0(-a)}\right]=0.$$

Therefore, the objective value associated with each KKT-point is given by $\frac{1}{2J_h}\left[\frac{J_h}{a}\log_e\frac{1+a\rho_0}{1-a\rho_0}\right]$ provided $a \leq \frac{1}{\rho_0}$. Then, the minimum in (4.30) is obtained by

$$Z^* = \min\left\{\frac{1}{2a}\log_e\left(\frac{1+a\rho_0}{1-a\rho_0}\right)\ :\ a=1,\frac{J_h}{J_h-1},\frac{J_h}{J_h-2}\ldots,\min\{J_h,1/\rho_0\}\right\}. \tag{4.32}$$

Next, noting that the function $f(x)=\frac{1}{x}\log_e\left(\frac{1+x\rho_0}{1-x\rho_0}\right)$ is monotonically nondecreasing in x for $x\in[1,1/\rho_0]$, it follows that $a=1$ is indeed the optimal solution in (4.32), which thus implies that each $\nu_j=+1$ at the optimum. That is, $\theta_{j,h}=1$, $\forall j$, solves the relaxed problem in (4.30), which yields $Z^*=\frac{1}{2}\log_e\left(\frac{1+\rho_0}{1-\rho_0}\right)$. But, for $0<\rho_0<1$, we have $-\frac{1}{\rho_0}<\theta_{j,h}=1<\frac{1}{\rho_0}$ for all j. This leads to the feasible point upper bound on the infimum in (4.28) as $\psi_0^{\min}\leq Z^*$. Combining with $Z^*\leq\psi_0^{\min}$, the proof is completed. ■

4.4 Selection criteria for portfolio optimization

The foregoing statistical analysis can be used to determine an industry partition for investment. Given a set of industries $h = 1, \ldots, H$ for consideration in period t, an investment-worthy (screened) set $\mathcal{H}$ is determined under the *Industry Selection Criterion* given by

$$\textsf{(ISC)}: \quad \mathcal{H} := \left\{ h \ : \ \bar{\psi}_h > \frac{\kappa}{2} \log_e \left(\frac{1+\rho_0}{1-\rho_0} \right) + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{J_h(t_0-3)}}, \ h = 1, \ldots, H \right\}, \tag{4.33}$$

where $\kappa \geq 1$ is a user-specified (safety) factor. For each industry $h \in \mathcal{H}$, individual stocks j are chosen from the given firms $j = 1, \ldots, J_h$ by using RFSI as a selection discriminator. Under Definition 4.2.2 and referring to the computation of RFSI in (4.13), for a given industry $h \in \mathcal{H}$, evaluate the moving average forecast of $\hat{t}$ periods,

$$RFSI(t, j; \hat{t}) = \frac{1}{\hat{t}} \sum_{\ell = t - \hat{t}}^{t-1} \eta_j(y^{h*}, z^{h*}; \boldsymbol{\xi}^{\ell}). \tag{4.34}$$

The subset (of firms) $\mathcal{J}_h$ from industry $h \in \mathcal{H}$ is chosen for portfolio analysis under the *Stock Selection Criterion* given by

$$\textsf{(SSC)}: \quad \mathcal{J}_h := \left\{ j \ : \ RFSI(t, j; \hat{t}) \geq R^*, \ j = 1, \ldots, J_h \right\}, \tag{4.35}$$

where R^* is a prespecified threshold, where $0 < R^* \leq 1$. The stocks in the subset $\mathcal{J}_h$, for $h \in \mathcal{H}$, are then expected to perform well in the stock market with high confidence. The universe of securities for portfolio analysis is thus given by

$$\mathcal{N} := \bigcup_{h \in \mathcal{H}} \mathcal{J}_h, \tag{4.36}$$

which is a subset of the original universe of stocks, i.e., $|\mathcal{N}| \leq J_0 := \sum_{h=1}^{H} J_h$. Investment weight to be attached to each stock $j \in \mathcal{N}$ is then determined by a portfolio optimization model. There are several models in the literature for this purpose, and the choice of a model is primarily guided by risk/return considerations. Risk specifications are multi-pronged and portfolio optimization models are multi-faceted. For instance, when there are transactions and slippage costs of trading, portfolio drawdown characteristics are a major concern of risk. Also, when market evolutionary dynamics are nonstationary, multiperiod sequential stochastic decision optimization is shown to yield superior performance compared to static one period models, see [25] for details.

The focus here is to demonstrate the usefulness of the preceding selection criteria, (ISC) and (SSC), over the *unscreened* set of J_0 stocks. We will use the portfolio optimization model in (3.12) to construct two portfolios: one using all J_0 stocks, and the other using $\mathcal{N}$ stocks that is selected by using ISC and SSC. The performance of these two portfolios are compared in the next section.

4.5 Application of RFSI in the Technology Sector

The preceding relative financial strength indicator, RFSI, is applied in portfolio optimization using the data set in Section 3.6, where several (publicly-traded) U.S.

companies in various industries are considered. The objective is to validate the use of RFSI-based stock selection as a means of improving risk/return performance of optimized portfolios. Quarterly financial statements of firms during the period 1996 to 2002 are used. Reported results pertain to a time window of $t_0 = 27$ quarters for industry-correlation maximization in (4.11). The data set involves only the technology sector, as identified by the industry groups listed in Table 3.1: Computer Software ($h = 1$), Communication Equipment ($h = 2$), Computer Hardware ($h = 3$), Electronics ($h = 4$), Semiconductors ($h = 5$), and Computer Services ($h = 6$). Thus, the total number of firms is 313.

Consider the concept of synchronous and lagged correlations introduced in Section 3.6.1. Also, recall that lagged correlations are important from the standpoint of implementations because quarterly financial information is made public with a certain time delay, typically a month after the quarter ends. Thus, the maximum synchronous correlation, along with one-month lagged maximum correlation, are computed through the GDEA process for each industry group in the *Technology* sector.

In order to determine the optimal input/output categorization of the 18 financial parameters in Section 3.1.1, the two-step heuristic solution method in Section 4.2.1 is applied, where an initial sample (y^s, z^s), $s \in \mathcal{S}$, is determined using a sample of size $|\mathcal{S}| = 20$. The corresponding sample of objective correlations in (4.14) is then computed. For each sample point, the objective industry-correlation value $\bar{\gamma}^t_{hS}(y, z)$ is improved via the local search procedure in Algorithm-LS, see 4.2.1. Note that the subscript S indicates it is a synchronous correlation. The optimal vector pairs

(y_S^{h*}, z_S^{h*}) corresponding to the largest correlation are then obtained, see Table 4.1. This process is repeated to obtain one-month lagged optimal correlation, which is denoted by $\bar{\gamma}_{hL}^t(y_L^{h*}, z_L^{h*})$, where (y_L^{h*}, z_L^{h*}) is the corresponding optimal input/output pair, see Table 4.2. The notation 'in', 'out', or '-' represent a given financial statement parameter i is an *input*, *output*, or it is *not considered*, respectively, in an industry h. These results pertain to the *Unrestricted* version that uses Ω in (4.6). Those for the *Restricted* version Ω^* in (4.5) are in parentheses in Table 4.1 and 4.2.

The local search trajectory corresponding to the sample point that leads to the reported optimal input/output categorization is plotted, for the *Unrestricted* domain Ω and the *Restricted* domain Ω^* for both synchronous and one-month lagged cases, in Figures 4.3, 4.4, 4.5, and 4.6, respectively, for each industry.

For industry-optimal (y_S^{h*}, z_S^{h*}) categorization, for both cases of *Unrestricted* and *Restricted* domains, industry-correlation metric $\bar{\gamma}_{hS}^t(y_S^{h*}, z_S^{h*})$ is estimated according to (4.9), and each industry is tested for statistical significance using the hypothesis test in (4.17). The test statistic $\bar{\psi}_{hS}$ in (4.20) is computed and reported in Table 4.1. The same process is repeated for one-month lagged case and the corresponding test statistic $\bar{\psi}_{hL}$ is reported in Table 4.2. The minimum positive correlation is set to $\rho_0 = +0.10$, which yields $\psi_0^{\min} = 0.1003$, see (4.29). Setting the level of significance $\alpha = 5\%$ and the safety factor $\kappa = 1.2$, the resulting critical values for the ISC criterion in (4.33) are also reported in Table 4.1 and 4.2. Observe that optimized industry-correlation metric under *Unrestricted* parameter domain Ω and the *Restricted* domain Ω^*, for the synchronous case, all six industries are chosen by the ISC criterion. However, for the one-month lagged case, under both *Unrestricted*

Table 4.1: Optimal input/output categorization (y_S^{h*}, z_S^{h*}) for (synchronous) RFSI in each industry

Financial parameter (i)	Industry (h) Software	Communic.	Hardware	Electro.	Semicond.	Services
1	in (out)	out (-)	- (-)	out (out)	- (-)	out (out)
2	out (-)	- (-)	- (-)	- (-)	- (-)	- (-)
3	in (out)	in (out)	- (-)	in (-)	- (-)	- (-)
4	- (-)	out (in)	- (-)	- (in)	out (-)	in (in)
5	- (in)	in (in)	- (-)	- (-)	- (-)	out (-)
6	in (in)	- (-)	- (-)	- (-)	out (-)	- (-)
7	- (in)	- (in)	in (in)	in (in)	out (-)	in (in)
8	out (-)	- (-)	- (-)	- (-)	in (in)	- (-)
9	in (in)	- (-)	- (-)	- (-)	- (-)	- (in)
10	- (in)	- (in)	in (in)	- (-)	out (-)	in (-)
11	- (-)	- (in)	in (in)	- (-)	in (in)	- (-)
12	in (-)	- (-)	- (-)	- (-)	in (-)	in (-)
13	- (-)	- (-)	- (-)	- (-)	in (-)	out (out)
14	out (out)	out (out)	out (out)	out (out)	- (out)	out (out)
15	in (out)	- (out)	out (out)	- (-)	- (-)	out (out)
16	out (-)	- (-)	- (-)	- (-)	out (out)	out (out)
17	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
18	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
Max.Corr. $\bar{\gamma}_{hS}^t(y_S^{h*}, z_S^{h*})$:						
Unrestricted domain	0.247	0.330	0.234	0.283	0.233	0.200
Restricted domain	(0.245)	(0.249)	(0.234)	(0.278)	(0.225)	(0.198)
Test statistic $\bar{\psi}_h^S$:						
Unrestricted domain	0.261	0.365	0.250	0.308	0.244	0.212
Restricted domain	(0.258)	(0.263)	(0.250)	(0.301)	(0.242)	(0.209)
Basic categorization :						
Corr. $\bar{\gamma}_{hS}^t(y_S^0, z_S^0)$	0.142	0.137	0.139	0.098	0.085	0.058
Test statistic	0.149	0.144	0.144	0.101	0.089	0.059
ISC Critical Value	0.172	0.169	0.172	0.160	0.161	0.175

Table 4.2: Optimal input/output categorization $(y_{L_1}^{h*}, z_{L_1}^{h*})$ for (one-month lagged) RFSI in each industry

Financial parameter (i)	Industry (h) Software	Communic.	Hardware	Electro.	Semicond.	Services
1	in (-)	out (-)	out (out)	out (out)	out (-)	- (-)
2	out (out)	in (-)	- (-)	- (-)	out (-)	- (-)
3	out (out)	- (-)	out (out)	- (-)	in (-)	- (-)
4	- (-)	out (in)	in (-)	- (-)	- (-)	- (in)
5	out (in)	in (in)	out (-)	in (in)	out (-)	- (in)
6	in (-)	- (-)	- (-)	- (-)	- (-)	- (-)
7	- (in)	out (-)	in (in)	in (in)	- (-)	in (in)
8	- (-)	- (-)	- (-)	- (-)	in (in)	in (in)
9	- (-)	- (-)	- (-)	- (-)	- (-)	- (in)
10	- (in)	- (-)	- (-)	- (-)	- (-)	in (-)
11	- (-)	- (-)	in (in)	- (-)	in (in)	out (-)
12	in (in)	- (-)	- (-)	- (-)	- (in)	out (-)
13	- (-)	- (-)	- (-)	- (-)	- (-)	out (out)
14	out (out)	out (out)	- (out)	out (out)	- (out)	out (out)
15	- (out)	- (out)	out (out)	- (-)	- (-)	out (out)
16	- (out)	- (out)	out (out)	- (-)	out (out)	- (out)
17	out (-)	- (out)	- (-)	- (-)	- (-)	- (-)
18	in (out)	- (-)	- (-)	- (-)	- (-)	- (-)
Max.Corr. $\bar{\gamma}_{hL}^{t}(y_L^{h*}, z_L^{h*})$:						
Unrestricted domain	0.208	0.278	0.264	0.220	0.212	0.156
Restricted domain	(0.185)	(0.194)	(0.252)	(0.220)	(0.201)	(0.125)
Test statistic ψ_h^L:						
Unrestricted domain	0.220	0.303	0.283	0.239	0.224	0.161
Restricted domain	(0.194)	(0.206)	(0.272)	(0.239)	(0.214)	(0.130)
Basic categorization :						
Corr. $\bar{\gamma}_{hL}^{t}(y_L^0, z_L^0)$	0.141	0.130	0.207	0.090	0.108	0.034
Test statistic	0.147	0.134	0.220	0.093	0.113	0.034
ISC Critical Value	0.172	0.168	0.172	0.160	0.161	0.175

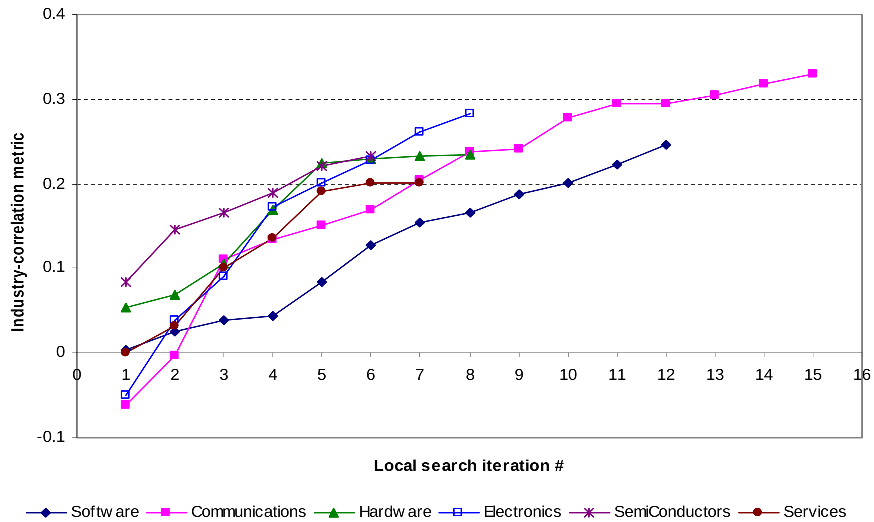

Figure 4.3: Trajectories of maximizing synchronous industry-correlation - over Unrestricted BCD

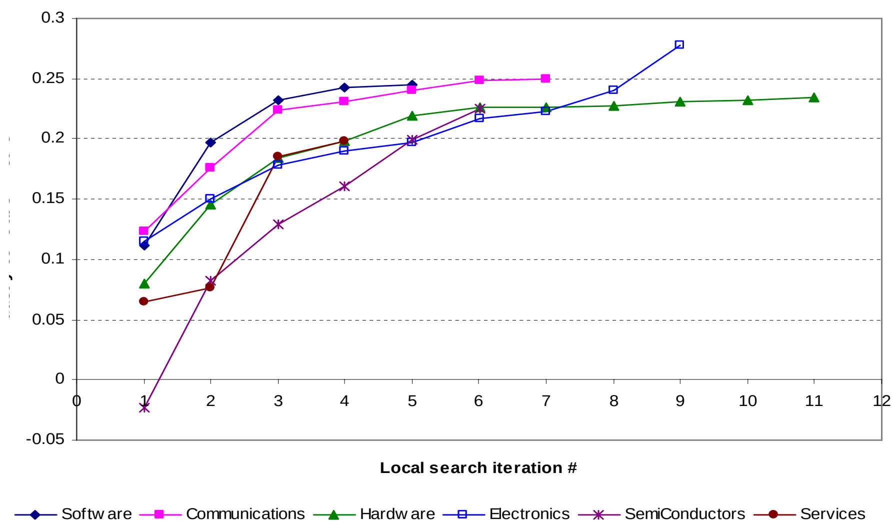

Figure 4.4: Trajectories of maximizing synchronous industry-correlation - over Restricted BCD

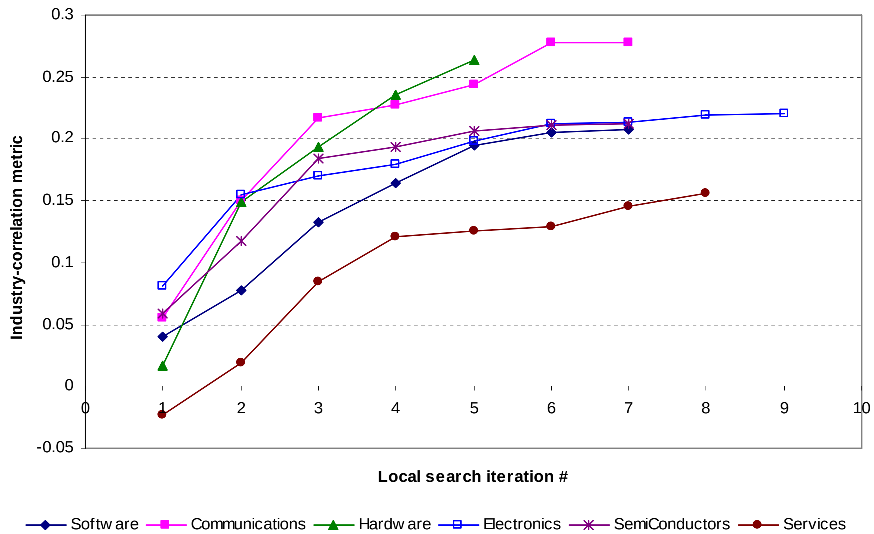

Figure 4.5: Trajectories of maximizing one-month lagged industry-correlation - over Unrestricted BCD

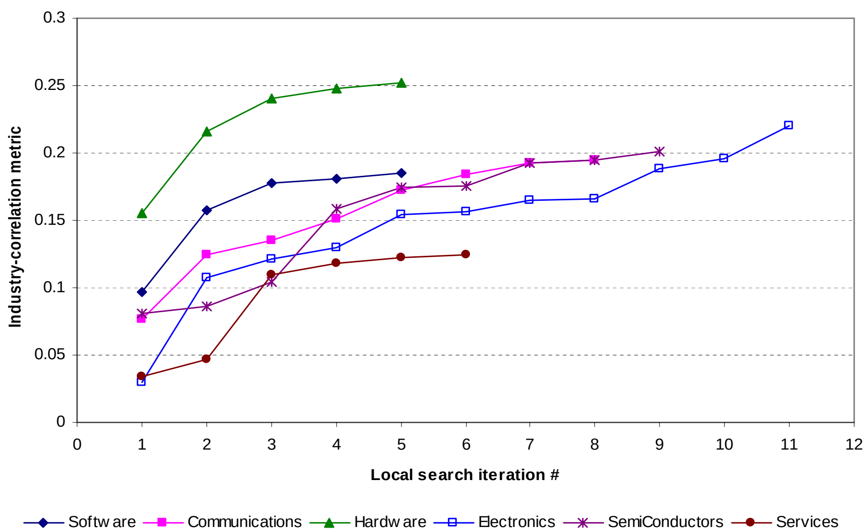

Figure 4.6: Trajectories of maximizing one-month lagged industry-correlation - over Restricted BCD

and *Restricted* domains leads to rejecting one industry, Services ($h = 6$). As stated earlier, synchronous correlation is not appropriate to be used for industry/stock selection due to the time delay of quarterly financial statements going public, thus, only the one-month lagged correlation is used as a metric for industry/stock selection.

Correlations corresponding to the exogenous *Basic* selection of input/output in (3.3) and (3.4) are also reported in Table 4.1 and 4.2, respectively, for synchronous and one-month lagged cases. Note that the maximized correlations are strictly better than those resulting from the *Basic* selection of the 18 financial parameters. Also note that the *Basic* selection fails to pick a single industry for investment for synchronous case, while for the one-month lagged case, only Hardware industry ($h = 3$) is selected, based on DEA-based predictability.

For the industries chosen as above using one-month lagged correlation, RFSI indicator is computed according to (4.34), with $\hat{t} = 2$. That is, the most recent two quarter moving average is computed for predicting RFSI for quarter 1 of 2003. These RFSI predictions are plotted in Figures 4.7 and 4.8, respectively, for *Unrestricted* and *Restricted* cases. Specifying the threshold $R^* = 0.60$ for the stock selection criterion in (4.35), stocks are chosen for portfolio optimization. As evident from Figures 4.7 and 4.8, only a small fraction of the universe of 313 securities are chosen by the SSC; for the *Unrestricted* case, 51 securities are selected ($|\mathcal{N}| = 51$) and for the *Restricted* case, 60 securities are selected ($|\mathcal{N}| = 60$).

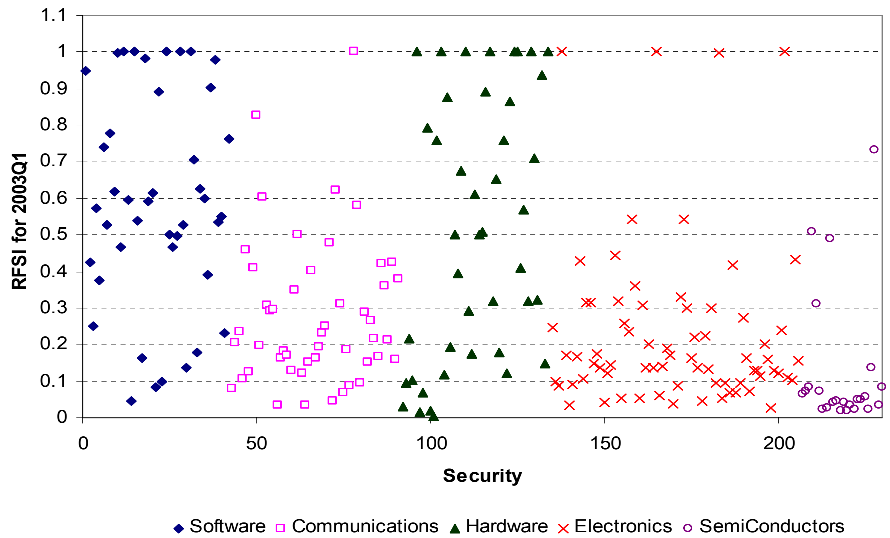

Figure 4.7: RFSI predictions for chosen industries - *Unrestricted* case

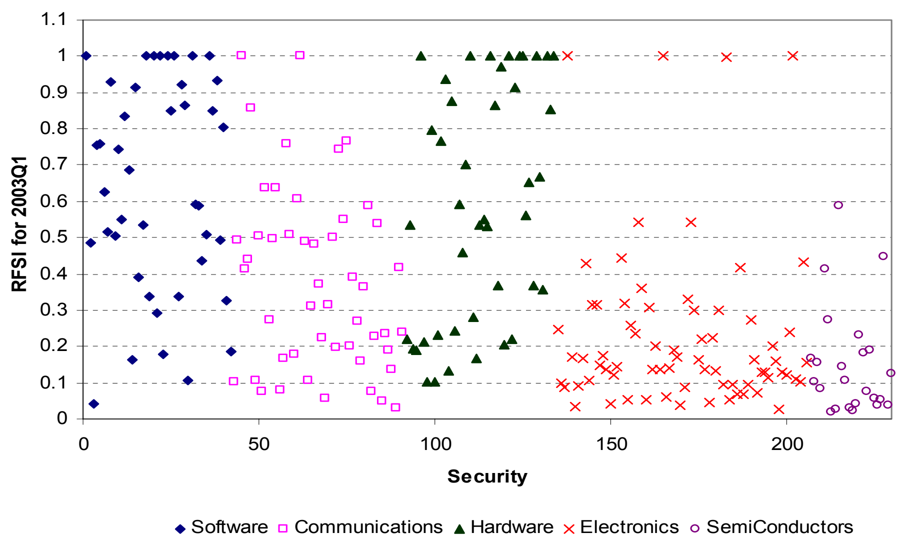

Figure 4.8: RFSI predictions for chosen industries - *Restricted* case

4.6 Portfolio optimization

The portfolio optimization model in (3.12) is executed for several risk tolerance levels using the screened subsets of stocks, under the stock selection criterion. Portfolio allocations (i.e., weights) are determined by solving the appropriate quadratic programming models, specified with a model-time period of 3-months from Feb 01-Apr 30, 2003, herein referred to as the investment horizon. A *monthly-rebalancing* strategy is applied where portfolio allocations are optimally adjusted at the beginning of each of the 3 months in the investment horizon. Consequently, the resulting portfolios are evaluated (i.e., out-of-sample simulated) on a daily basis by using the actual price realizations from the investment horizon. That is, portfolio allocations determined at the end of 2002 by the model are simulated using prices from Feb 01-Apr 31, 2003 to determine the *actual* portfolio performance characteristics. All portfolio computations are carried out within ©MiSOFT software, see [26].

Performance characteristics are compared for the following four cases: stock selection over *Unrestricted* domain, stock selection over *Restricted* domain, stock selection using Basic model that is introduced in Chapter 3, and stock selection using the RIV model in Section 1.3.3. By varying the value of the risk tolerance parameter λ in each case, efficient frontiers are traced and plotted in Figure 4.9. It is evident that the RFSI-based stock selection outperforms the the case that uses the *Basic* selection and RIV selection, with the *Unrestricted* RFSI version showing better portfolio gains than the *Restricted* version.

During the same investment horizon, the market barometer index, S&P-500 index, displays an annualized standard deviation of 22.4%. The *Unrestricted* RFSI, *Basic*, and RIV versions of portfolio investment are set such that each version will provide a portfolio with an annualized standard deviation of (approximately) 22.4%. Portfolio evolutions corresponding to this case are depicted in Figure 4.10, where the performance of S&P-500 index is also indicated. It is observed that the cumulative return reaches the highest level at the end of the investment horizon in the case of the RFSI-based stock selection using the *Unrestricted* selection of input/output parameters.

4.6.1 Concluding remarks

This chapter developed a new quantitative metric, termed the Relative Financial Strength Indicator (RFSI), which is designed to have high correlation with stock price returns. The underlying methodology is based on using a generalized version of data envelopment analysis, coupled with selecting inputs and outputs from financial statements via a well-defined optimization process. From Table 4.1 and 4.2, it is observed that the number of inputs and outputs selected by this optimization process is much less than that chosen by *Basic* selection, which includes 9 inputs and 7 outputs. For example, for the one-month lagged case, in Communications industry, 2 inputs and 4 outputs are chosen using both *Unrestricted* BCD and *Restricted* BCD. In Semiconductor industry, 3 inputs and 4 outputs are chosen using *Unrestricted* domain and 3 inputs and 2 outputs are chosen using *Restricted* domains. Thus, the model saturation problem is fixed to a great extent, while increasing the required correlations significantly.

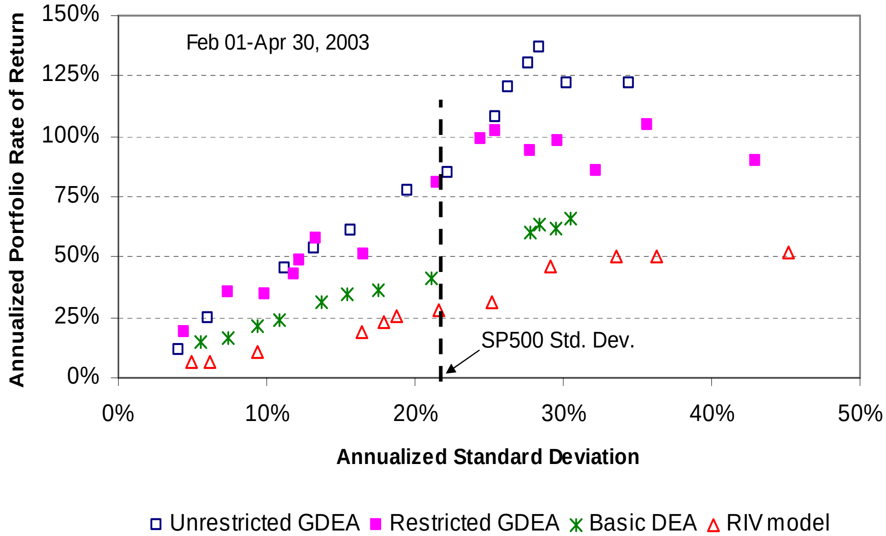

Figure 4.9: Portfolio (actual) efficient frontiers under RFSI-based stock selection and universe

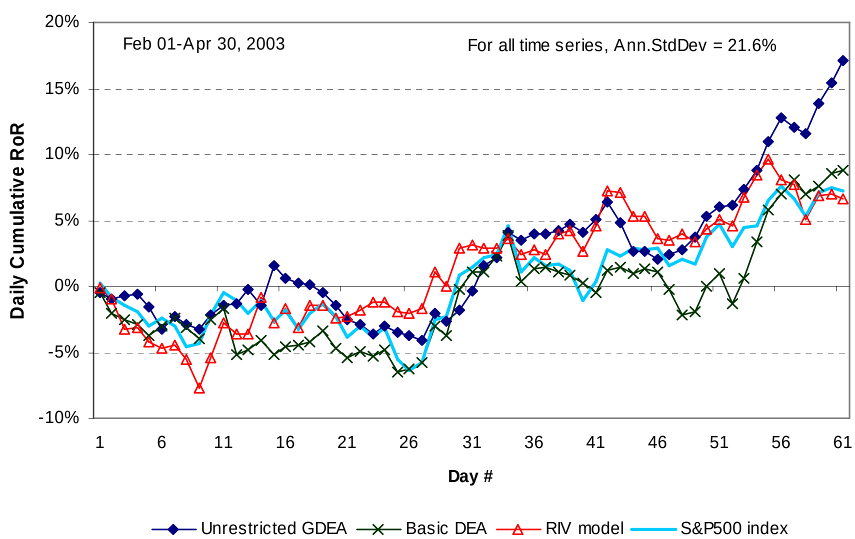

Figure 4.10: Portfolio RoR under RFSI-based stock selection and universe

Chapter 5

Improvements on the GDEA Approach

In Section 3.7, three specific noteworthy issues were discussed to improve the DEA-based strength evaluation. The issue of model saturation effect was rectified via the GDEA approach given in Chapter 4. That is, by optimally choosing those subsets of inputs and outputs that lead to maximized correlation, GDEA model is not subject to parameter saturation. However, the issue of Relative Performance Score (RPS) being restricted between 0 and 1 remains a major concern. That is, firms with RPS of 1 do not differentiate themselves, and also since firms with RPS of 0 are not further discriminated either. Hence, the computed correlations may be adversely affected by this truncation phenomenon. Furthermore, in the third issue raised, the assumption of normality for the RPS values may very well be violated as demonstrated earlier. This assumption is necessary when statistical significance tests are performed, and therefore, the raw RPS values may need to be transformed. Such a transformation is investigated so that a near-normal strength metric can be developed, which has high correlation with market returns. These are the topics of discussion in this chapter.

5.1 Discrimination of efficient firms

The problem of non-discrimination of firms with RPS of 1 is exemplified using the *Basic* selection of inputs and outputs in Section 3.7. Figure 3.12 shows that there is a high percentage of firms labeled RPS=1 across almost all industry groups. This artificial (upper) truncation of RPS by 1 may result in a lack of discrimination of firms that are labeled strong, hence, its correlation with market return may be adversely affected. Below, a modified RPS score, denoted by $\eta_k^c(y, z)$, is developed for firm k that mitigates this truncation error.

The method developed here is essentially an extension of the idea proposed by Anderson and Peterson [5], where they provided a fix for the truncation error of VRS-based DEA models, such as the BCC model. However, in our case, the DEA model is CRS, and the following correction is made, where C is a constant greater than unity.

$$\begin{aligned}
\eta_k^c(y,z) := \max_{u,v} \quad & \sum_{i=1}^{I} (z_i \xi_{ik}) v_i \\
\text{s.t.} \quad & \sum_{i=1}^{I} (y_i \xi_{ik}) u_i = 1 \\
& -\sum_{i=1}^{I} (y_i \xi_{ij}) u_i + \sum_{i=1}^{I} (z_i \xi_{ij}) v_i \leq 0, \ \ \forall j, \ j \neq k \\
& -C \sum_{i=1}^{I} (y_i \xi_{ik}) u_i + \sum_{i=1}^{I} (z_i \xi_{ik}) v_i \leq 0, \\
& u_i, v_i \geq 0, \ i = 1, \ldots, I.
\end{aligned} \tag{5.1}$$

The basic idea of the corrected GDEA model in (5.1) is to check how much further an efficient firm can increase its inputs proportionately without sacrificing the firm's efficiency. Note that the proportionate constant C ($C > 1$) is introduced in the model only for firm k that is being evaluated. The model in (5.1) has the following properties:

Properties:

1. $0 \ \leq \ \eta_k^c(y,z) \ \leq \ C$

2. $\eta_k^c(y,z) \ \geq \ \eta_k(y,z), \quad \forall k$

3. $\eta_k^c(y,z) = \eta_k(y,z)$ if $\eta_k(y,z) < 1$

4. $\eta_k^c(y,z) \ > \ 1$ only if $\eta_k(y,z) = 1$

Whether the corrected DEA model leads to improving the differentiation among firms that are under evaluation will be addressed in the computational results in Section 5.3 of this chapter.

5.2 Transformation of RPS Values

There does not exist, to the best of the author's knowledge, computable statistical analysis of correlation when the underlying random variables are non-normally distributed. RPS (or the corrected RPS (CRPS)) scores are generally non-normally distributed, see Figure 3.13. In this case, a transformation is needed to obtain approximately normally distributed CRPS scores. Generally, stock returns are assumed to be normally distributed, see [4]. Thus, this section attempts to transform CRPS to assure near-normality. In the sequel, such a transformed CRPS score is referred to as the relative financial strength of a firm.

The transformation function B should have the following two properties: first, B must be nondecreasing, which indicates that a firm's high CRPS will correspond to a high measure of relative financial strength; second, the range of B must be $(-\infty, \infty)$, representing the range of a normal distribution. The Box-Cox transformation [13] is one of the most commonly used nonlinear transformations that satisfy these two properties, see [39].

The Box-Cox transformed CRPS score η, denoted by $B_\alpha(\eta)$, is

$$B_\alpha(\eta) = \begin{cases} \frac{\eta^\alpha - 1}{\alpha} & \text{when } \alpha \neq 0 \\ log(\eta) & \text{when } \alpha = 0, \end{cases} \tag{5.2}$$

where α is a scalar parameter, see [63] for details. Since (5.2) is not well defined for $\eta = 0$, we set a lower bound to CRPS as $\eta \geq \epsilon$ for some small $\epsilon > 0$. The transformed value $B_\alpha(\eta)$ has the following properties:

<u>Properties:</u>

1. $B_{\bar{\alpha}}(\eta) \geq B_{\hat{\alpha}}(\eta)$ if $\bar{\alpha} \geq \hat{\alpha}$

2. if $\alpha > 1,\ B_\alpha(\eta)$ is convex

3. if $\alpha < 1,\ B_\alpha(\eta)$ is concave

4. if $\alpha = 1,\ B_\alpha(\eta)$ is linear

5. if $\alpha > 0,\ B_\alpha(\eta) \leq log(\eta)$

6. if $\alpha < 0,\ B_\alpha(\eta) \geq log(\eta)$

The Box-Cox method suggests that for some α values, $B_\alpha(\eta)$ is normally distributed, see [50].

The Box-Cox normality transformation can be applied to the performance scores, $\eta_k^c(y,z)$, that are computed by the modified GDEA model in (5.1). In this case, the range of $\eta_k^c(y,z)$ is between ϵ and C, i.e., $0 < \epsilon \leq \eta_k^c \leq C$. It is necessary to choose an α value such that the transformed scores $B_\alpha(\eta_k^c(y,z))$ become near-normally distributed. This will be addressed in the following section using firms from the *Technology* sector in the U.S. market.

5.3 Computational Results

The same industry groups in *Technology* sector that are used for experimentation in Chapters 3 and 4, see Table 3.1, are employed in this chapter to validate the methods proposed in Sections 5.1 and 5.2. By applying the corrected DEA model in (5.1), the CRPS values are obtained for the *Basic* model using $C = 100$. The histogram of CRPS values that are greater or equal to 1 are plotted in Figure 5.1-5.6, for each industry group. It is evident that firms with RPS value of 1 are now further differentiated and their corrected values, in most cases, remain between 1 and 3. With this additional layer of differentiation of firms with RPS value of 1, the resulting CRPS must be transformed under the Box-Cox method to test if more *normally* distributed relative strength scores can result under a suitably-chosen α value. To test this computationally, CRPS values are computed using the optimal inputs and outputs obtained using the GDEA approach in Chapter 4.

Using the optimal input/output vector (y_S^{h*}, z_S^{h*}) obtained using the GDEA model in Chapter 4 for industry h under both *Unrestricted* and *Restricted* domains, see Table 4.1, the CRPS values, denoted by $\eta_{jt}^c(y_S^{h*}, z_S^{h*})$, are computed for $j = 1, \ldots, J_h$ and $t = 1, \ldots, t_0$. Note that the subscript S indicates that (y_S^{h*}, z_S^{h*}) corresponds to the maximized synchronous correlation.

For a given firm j, consider the sample of t_0 values of the CRPS $\eta_{jt}^c \equiv \eta_{jt}^c(y_S^{h*}, z_S^{h*})$ for $t = 1, \ldots, t_0$. The focus here is on whether the transformed sample, $B_\alpha\left(\eta_{jt}^c\right)$, $t = 1, \ldots, t_0$, can be near-normally distributed for a certain α specified in (5.2). To verify this *normality*, the Chi-square (χ^2) goodness-of-fit test is employed. Using a grid of 10 intervals, the resulting goodness-of-fit test statistic, denoted by $\Psi_j^c(\alpha)$, is

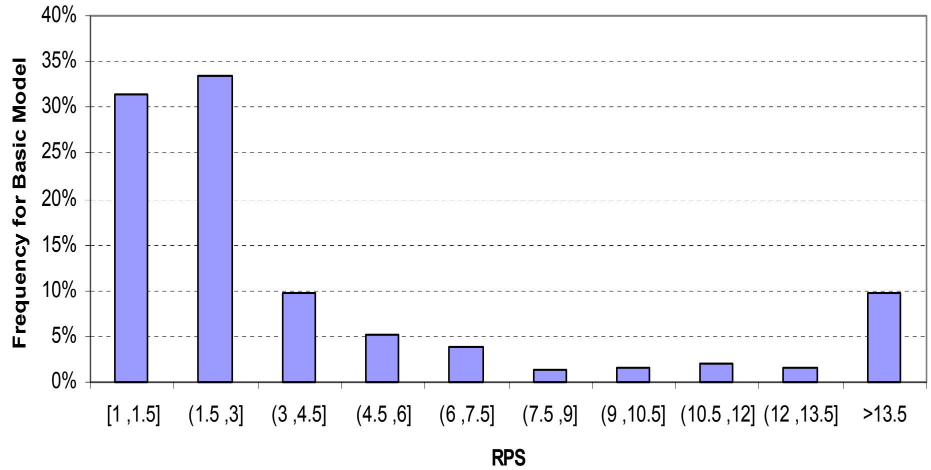

Figure 5.1: Histogram of corrected RPS values for 100%-efficient firms in Software industry–*Basic* model

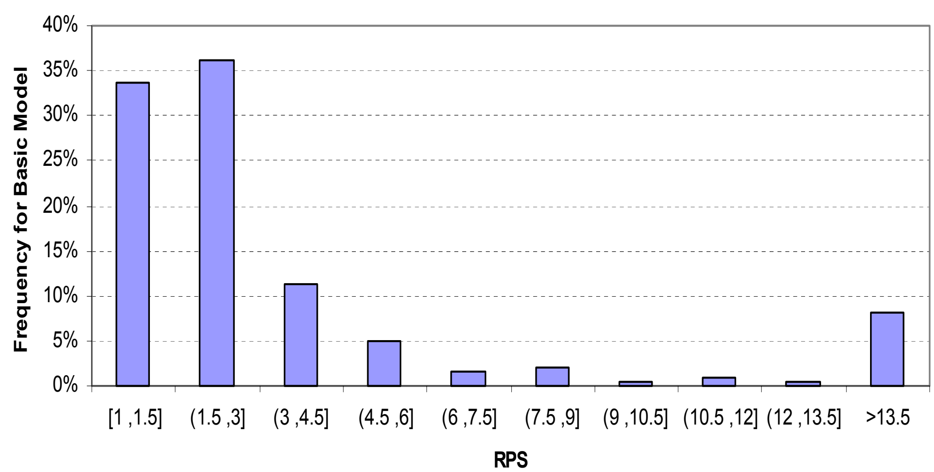

Figure 5.2: Histogram of corrected RPS values for 100%-efficient firms in Communication industry–*Basic* model

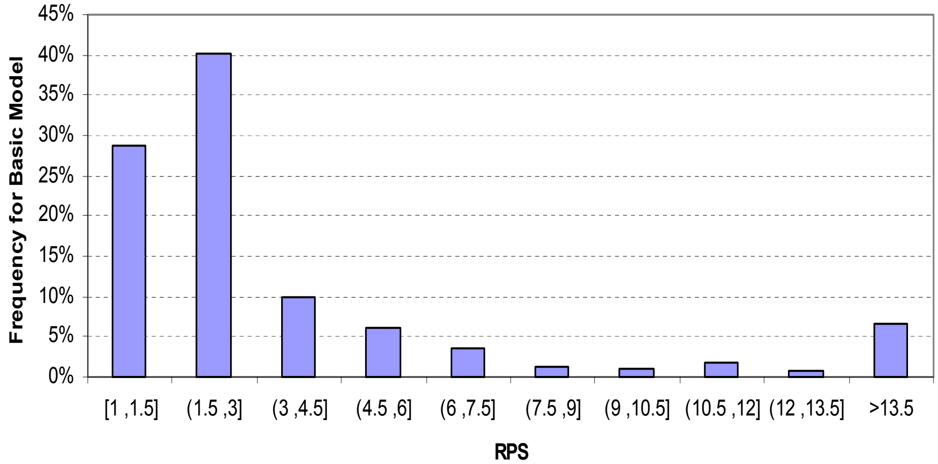

Figure 5.3: Histogram of corrected RPS values for 100%-efficient firms in Hardware industry–*Basic* model

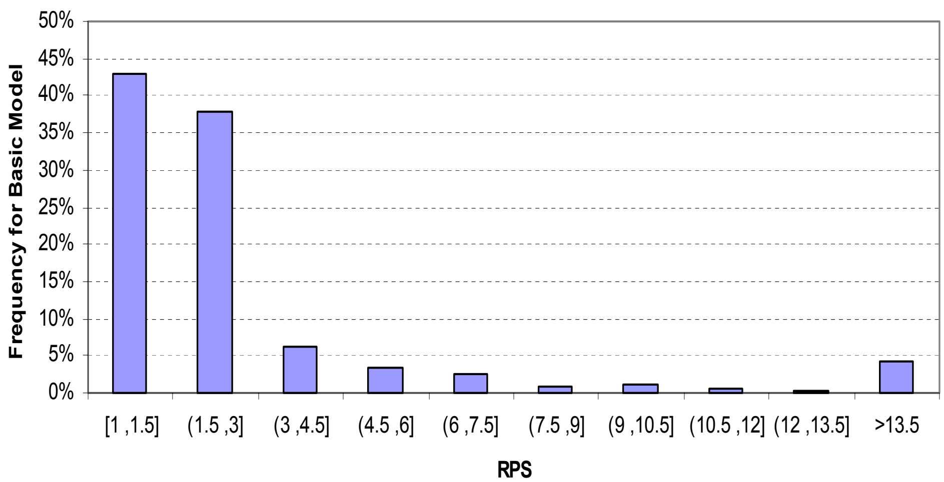

Figure 5.4: Histogram of corrected RPS values for 100%-efficient firms in Electronics industry–*Basic* model

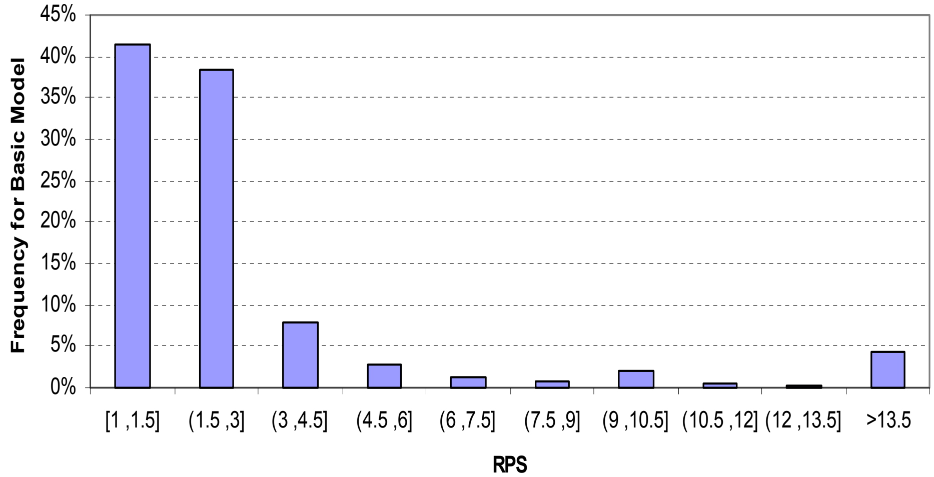

Figure 5.5: Histogram of corrected RPS values for 100%-efficient firms in Semiconductors industry–*Basic* model

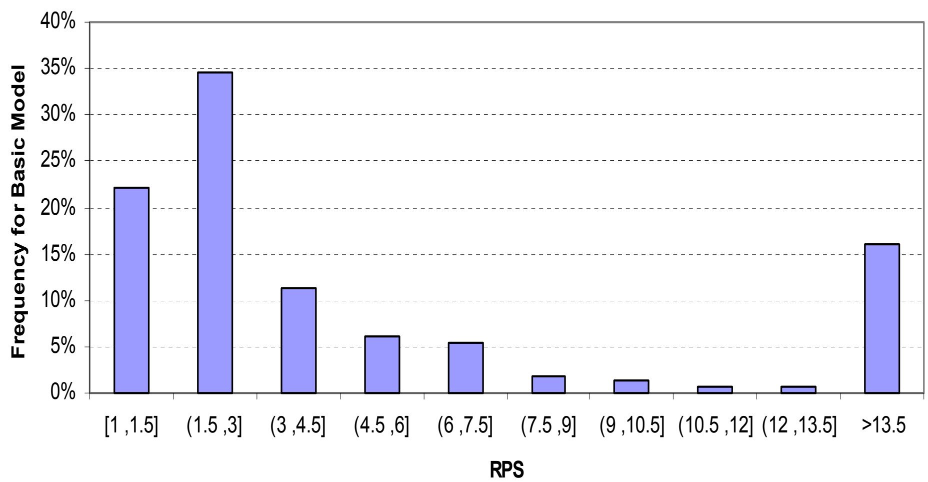

Figure 5.6: Histogram of corrected RPS values for 100%-efficient firms in Services industry–*Basic* model

χ^2-distributed with 9 *d.f.* For the one-sided test and a significance level of 5%, the critical value is $\chi^2_{9,0.05} = 16.919$. Thus if $\Psi^c_j(\alpha) \leq 16.919$, it is concluded that the sample $B_\alpha\left(\eta^c_{jt}\right)$, $t = 1, \ldots, t_0$, comes from a *normal* distribution. Otherwise, there is no statistical evidence for the required normality. This process is repeated for each firm $j = 1, \ldots, J_h$, in the chosen industry $h = 1, \ldots, H$. Accordingly, the measure of *Normality Satifaction Degree* is defined for each industry as follows.

Definition 5.3.1 *(***Normality Satisfaction Degree: NSD***)*
The Normality Satisfaction Degree, for given α, is defined for a chosen industry h by $p^h(\alpha) = \sum_{j=1}^{J_h} w^h_j(\alpha)/J_h$, where

$$w^h_j(\alpha) := \begin{cases} 1 & \textit{if } \Psi^c_j(\alpha) \leq 16.919 \\ 0 & \textit{otherwise.} \end{cases}$$

With C value in (5.1) set to 100, the degree of normality measure $p^h(\alpha)$ is computed for $\alpha = -0.2, -0.1, 0, 0.1, 0.2, 1$, and they are given in Table 5.1 and 5.2, respectively, for *Unrestricted* and *Restricted* cases for each industry. Note that $\alpha = 1$ indicates that no Box-Cox transformation is applied to the efficiency scores. In the same tables, the last column represent the NSD measure for the case of $C = 1$ and $\alpha = 1$, that is, no further differentiation of efficient firms nor any nonlinear transformation of the efficiency scores are applied.

It must be observed that NSD values are improved in most cases under the Box-Cox transformation on $\eta^c_{jt}(y^{h*}_S, z^{h*}_S)$, and the improvement varies with the chosen α value. An appropriate choice of α is made under the following two rules of thumb:

Industry Group	$C = 100$						$C = 1$
	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 1$	$\alpha = 1$
Software	42.86%	35.71%	33.33%	16.67%	14.29%	11.90%	28.57%
Communication	16.33%	38.78%	63.27%	57.14%	65.31%	46.94%	57.14%
Hardware	88.37%	86.05%	86.05%	86.05%	86.05%	58.14%	41.86%
Electronics	88.89%	88.89%	88.89%	83.33%	80.56%	41.67%	41.67%
Semiconductors	59.42%	57.97%	50.72%	46.38%	49.28%	44.93%	28.12%
Services	84.21%	86.84%	84.21%	71.05%	73.68%	33.33%	53.53%
Average NSD	63.35%	65.71%	67.75%	60.10%	61.53%	39.49%	41.82%

Table 5.1: *Normality Satisfaction Degree* for different α values for each industry under *Unrestricted* domain

Industry Group	$C = 100$						$C = 1$
	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 1$	$\alpha = 1$
Software	80.95%	85.71%	85.71%	76.19%	76.19%	47.62%	42.86%
Communication	73.47%	79.59%	83.67%	79.59%	79.59%	55.10%	40.82%
Hardware	88.37%	86.05%	86.05%	86.05%	86.05%	58.14%	41.86%
Electronics	86.11%	88.89%	90.28%	88.89%	88.89%	56.94%	58.33%
Semiconductors	86.96%	82.61%	81.16%	81.16%	75.36%	26.87%	33.33%
Services	89.47%	89.47%	89.47%	76.32%	73.68%	38.89%	26.32%
Average NSD	84.22%	85.39%	86.06%	81.37%	79.96%	47.26%	40.59%

Table 5.2: *Normality Satisfaction Degree* for different α values for each industry under *Restricted* domain

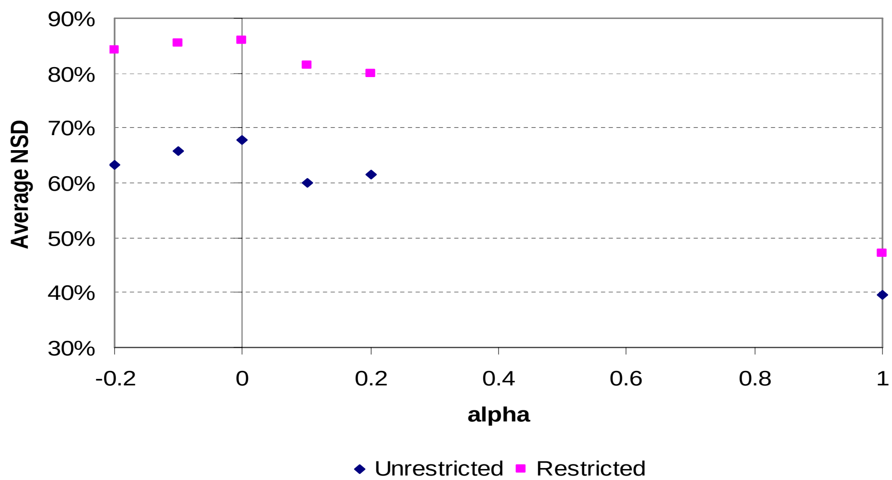

Figure 5.7: Average NSD for various α values for $C = 100$ under *Unrestricted* and *Restricted* domains

(i) NSD value under the chosen α must yield a significant increase over that under $\alpha = 1$ for every industry.

(ii) NSD value under the chosen α must have the highest average over all industries.

By the above rules of thumb, it is concluded that $\alpha = 0$ is an appropriate choice for the transformation of CRPS scores to obtain near-normal distributions, see Figure 5.7.

Chapter 6

Value of Expert Information under GDEA

The concept of "Expert Information (EI)" was introduced in Chapter 4 where an outside expert or specialist (of an industry/market) may provide additional (exogenous) information on input/output selection for the GDEA model. Such information may be based on his/her knowledge or experience with respect to the ability of the chosen universe of financial parameters to represent the underling operational successes of the firms. For example, the *Restricted* BCD version, see (4.5), of the GDEA model employs such an approach where the 18 financial parameters are categorized into broader input and output sets. Then, the GDEA model determines an optimal set of inputs and outputs, within those categories of expert judgment, to maximize the correlation metric between the relative strength so-determined and the market returns. Literature on incorporating expert judgment with input/output selection in DEA or analyzing the value of such information is non-existent to the best of our knowledge. This chapter provides a framework for objectively evaluating the value of such expert information. By comparing the correlation metric with that due to not utilizing the expert knowledge at all, i.e., the *Unrestricted* version of GDEA in (4.6), or not fully utilizing the expert information, the value of EI can be gauged. However, direct comparisons of the correlation metrics are data-specific, and such point-wise comparisons cannot be used to reach a definitive conclusion regarding the Value of EI, or VEI. That is, VEI requires statistical justification with respect to the (population-based) correlation metrics. In the first part of this chapter, this is the main focus. The second half of the chapter is concerned with developing theoretical optimality conditions for the GDEA optimization model under expert information.

6.1 Model under Expert Information

Recall that the GDEA approach considers an optimal partitioning of the parameters from the *Unrestricted* binary complementary domain Ω, mathematically expressed as

$$\Omega = \left\{ x \in \{0,1\}^{2I} \ : \ \sum_{i=1}^{I} x_i \geq 1, \ x_i + x_{I+i} \leq 1, \ i = 1, \dots, I \right\}, \tag{6.1}$$

where the binary complementary vector pair (y, z), see (4.6), is represented by the $2I$-dimensional binary vector x with $x_i = y_i$ and $x_{I+i} = z_i$ for $i = 1, \dots, I$. Then, the GDEA model in (5.1) can be rewritten as:

$$
\begin{aligned}
\eta^c_{k\tau}(x) := \max_{u,v} \quad & \sum_{i=1}^{I}(x_{I+i}\xi_{ik\tau})v_{i\tau} \\
\text{s.t.} \quad & \sum_{i=1}^{I}(x_i\xi_{ik\tau})u_{i\tau} \leq 1 \\
& -\sum_{i=1}^{I}(x_i\xi_{ij\tau})u_{i\tau} + \sum_{i=1}^{I}(x_{I+i}\xi_{ij\tau})v_{i\tau} \leq 0, \;\; \forall j, \;\; j \neq k \\
& -C\sum_{i=1}^{I}(x_i\xi_{ik\tau})u_{i\tau} + \sum_{i=1}^{I}(x_{I+i}\xi_{ik\tau})v_{i\tau} \leq 0, \\
& u_{i\tau}, v_{i\tau} \geq 0, \; i = 1, \ldots, I,
\end{aligned}
\tag{6.2}
$$

where $\eta^c_{k\tau}(x)$ is the corrected RPS (CRPS) value for firm k and period τ, where in period τ, financial data $\xi_{ij\tau}$ is realized for each firm j, $j = 1, \ldots, J$.

In Section 5.2, the Box-Cox transformation is applied to the computed CRPS values in order to make the distribution of CRPS values more *normally* dispersed. While it was concluded in Section 5.3 that $\alpha = 0$ is the appropriate choice to ensure near normality, the development in this chapter is for a general α. Therefore, the CRPS value after the Box-Cox transformation is defined by $\hat{\eta}_{j\tau}(x) := B_\alpha(\eta^c_{j\tau}(x))$ if $\eta^c_{j\tau}(x) > 0$. If $\eta^c_{j\tau}(x) = 0$, $\hat{\eta}_{j\tau}(x) := B_\alpha(1/C)$, and thus, $B_\alpha(1/C) \leq \hat{\eta}_{j\tau}(x) \leq B_\alpha(C)$. Note that these transformed CRPS values are also referred to by the term "RPS".

As in Section 4.2, define the correlation

$$\gamma_j(x) := \mathit{Correlation} \; \{(\hat{\eta}_{j\tau}(x), r_{j\tau}) \mid \tau = 1, \ldots, t_0\},$$

which is the computed correlation between the RPS scores and stock returns over the considered t_0 time periods of historical data for firm j. This is a sample-computed value of the population correlation parameter, denoted by $\Gamma_j(x)$. Then, the vector of correlations $\Gamma(x) := [\Gamma_1(x), \ldots, \Gamma_J(x)]$ is a measure of how good the GDEA-based relative financial strength measure will be for predicting the stock returns, and then, the Sector Correlation Metric is defined as follows.

Definition 6.1.1 (**Sector Correlation Metric: SCM**) *For the sector identified by the firms $j = 1, \ldots, J$, SCM is defined by $\bar{\Gamma}(x) := \frac{1}{J}\sum_{j=1}^{J} \Gamma_j(x)$, for a given $2I$-dimensional binary vector x.*

Thus, SCM is the average of components of the vector $\bar{\Gamma}(x)$, and it is sample-estimated by $\bar{\gamma}(x) = \frac{1}{J}\sum_{j=1}^{J} \gamma_j(x)$. For more details, see Section 4.2. In the GDEA approach, the maximized SCM is estimated by searching over an appropriate set $\bar{\Omega}$ of *binary complementary* solutions such that for some $\bar{x} \in \bar{\Omega}$, $\bar{\gamma}(x)$ is maximized - see (4.11) -, and its statistical significance is verified using the statistical test in Section 4.3. According to the feasible domain $\bar{\Omega}$, a relative financial strength indicator, herein referred to as RFS, for a given firm in a sector, is defined as follows.

Definition 6.1.2 (**Relative Financial Strength: RFS**) *Suppose $\bar{\Gamma}^t(\bar{x})$ is statistically significant for a given sector, where $\bar{x}$ is an optimal solution of the sector correlation metric (SCM) maximization problem. Then, the Relative Financial Strength (RFS) of firm j for (a future) period t is defined by*

$$RFS(t,j) := E\left[\hat{\eta}_{jt}(\bar{x}) \mid \hat{\eta}_{j,t-t_0}(\bar{x}), \ldots, \hat{\eta}_{j,t-1}(\bar{x})\right], \tag{6.3}$$

where $\hat{\eta}_{j\tau}(\bar{x}))$ for $t - t_0 \leq \tau \leq t-1$ are computed according to the modified GDEA model in (6.2) for the input/output categorization $\bar{x}$, and $E[.]$ denotes the conditional expectation given the RPS scores of the historical t_0 periods.

In particular, under the *Unrestricted* binary complementary domain Ω, a sample statistic of the correlation metric $\bar{\Gamma}^0$ is determined by solving the nonlinear binary optimization model, see Section 4.1 and 4.2:

$$\begin{aligned} \bar{\gamma}^0 := \quad & \max_x \quad \bar{\gamma}(x) \\ & \text{s.t.} \quad x \in \Omega. \end{aligned} \tag{6.4}$$

Note that an optimal input/output solution vector x^0 of the above correlation maximization model in (6.4) is determined in an "unrestrictive" manner in that no prior knowledge of expert information on x has been utilized. That is, the feasible vectors x are only required to satisfy the required binary complementarity. However, in most practical situations, an expert or specialist may provide additional (exogenous) information regarding which parameters are appropriate or inappropriate as inputs and outputs, based on his/her knowledge or experience. Availability of such expert information (EI) would essentially restrict the binary feasibility domain Ω of the maximization in (6.4). This section considers how the existence of exogenous information would modify the feasible set Ω, and thus the so-computed RFS, and also how one would model violation of such exogenous information.

Expert knowledge may be presented in a very general format with regard to parameters, as indicated below, in K distinct information categories.

Description of EI set:

$\mathcal{I}_1$: Given a set G of parameters ($G \subset \{1, \ldots, I\}$), each $i \in G$ is an input.

$\mathcal{I}_2$: Given a set G of parameters ($G \subset \{1, \ldots, I\}$), each $i \in G$ is an output.

$\mathcal{I}_3$: Given a set G of parameters ($G \subset \{1, \ldots, I\}$), not all of $i \in G$ are inputs.

$\mathcal{I}_4$: Given a set G of parameters ($G \subset \{1, \ldots, I\}$), not all of $i \in G$ are outputs.

$\mathcal{I}_5$: Given two sets G_1, G_2 of parameters ($G_1, G_2 \subset \{1, \ldots, I\}$), if $i \in G_1$ are inputs, then $i \in G_2$ cannot be outputs.

$\vdots$

$\mathcal{I}_K$: etc.

Information categories $\mathcal{I}_k$, $k = 1, \ldots, K$ collectively represent the expert's knowledge. Clearly, each $\mathcal{I}_k$ can be associated with a set of complementary binary vectors $x \in \Omega$ satisfying the conditions in $\mathcal{I}_k$. That is, there exists a mapping $\mathcal{M}_k : \mathcal{I}_k \to \Omega_k$ where $\Omega_k \subset \{0, 1\}^{2I}$. Then, all information corresponding to the expert knowledge is represented by the set

$$\Omega^{\mathcal{I}} := \bigcap_{k=1}^{K} \Omega_k,$$

and thus, maximizing the correlation metric subject to expert information requires solving the problem:

$$\begin{aligned} \bar{\gamma}^* := \ \ \max_x \quad & \bar{\gamma}(x) \\ \text{s.t.} \quad & x \in \Omega^* := \Omega \bigcap \Omega^{\mathcal{I}}, \end{aligned} \tag{6.5}$$

where $\bar{\gamma}^*$ is the sample estimate of $\bar{\Gamma}^*$. Ω^* is the feasible domain of all binary complementary vectors satisfying the specified expert information. Hence, the above problem represents fully utilizing EI. Let the optimal solution of model (6.5) be denoted by x^*, i.e., $\bar{\gamma}^* = \bar{\gamma}(x^*) = \frac{1}{J}\sum_{j=1}^{J}\gamma_j(x^*)$. While $\bar{\gamma}^0 \geq \bar{\gamma}^*$ holds, the focus here is on the value of (or lack thereof) EI. Can the expert information be violated to some degree without losing the predictive ability of the RPS so-computed? The notion of Value of Expert Information (VEI) is thus introduced, under possible violations of the EI when RPS is determined. Toward this, we impose penalty for violating expert's knowledge. Violation of expert information (of the category $\mathcal{I}_k$) by some $\bar{x} \in \{0,1\}^{2I}$ is measured by the *distance* from the set Ω_k, defined hereby as

$$\mathcal{D}(\bar{x}, \Omega_k) := \min_{x \in \Omega_k} ||x - \bar{x}||_1,$$

where *norm-1* distance metric $||x||_1 = \sum_i |x_i|$. See the illustration in Figure 6.1. Such violation in the information category $\mathcal{I}_k$ is penalized by a (cost) function $\Pi_k\left(\mathcal{D}(\bar{x}, \Omega_k)\right)$ where $\Pi_k : \Re \rightarrow \Re$ is a convex, increasing function such that $\Pi_k(0) = 0$, for all $k = 1, \ldots, K$. Then, the overall penalty on violating the expert's information is given by

$$\Pi(x) := \sum_{k=1}^{K} \Pi_k\left(\mathcal{D}(x, \Omega_k)\right).$$

The EI-penalty function $\Pi : \{0,1\}^{2I} \rightarrow \Re$ is defined to be convex, increasing, and it satisfies $\Pi(x) = 0$ for $x \in \Omega^{\mathcal{I}}$ and $\Pi(x) > 0$ for $x \notin \Omega^{\mathcal{I}}$.

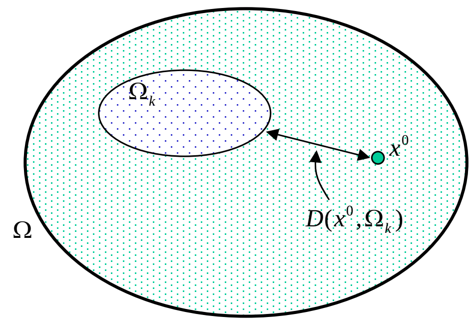

Figure 6.1: Violation of Expert Information

6.2 Hypothesis of VEI

How much importance should be given to exogenous information that represents an expert's knowledge? Indeed, a complete disregard of the EI results in the *unrestricted* version of the GDEA model, see (6.4), which yields the sample estimate $\bar{\gamma}^0$ for the population correlation metric $\bar{\Gamma}^0$. In contrast, under a certain "degree of violation" of EI, as measured by $\Pi(x) = \pi$ (for $\pi > 0$), let the resulting maximized SCM be denoted by $\bar{\Gamma}^\pi$, which is estimated by solving the model

$$\begin{aligned} \bar{\gamma}^\pi := \ \max_x \ & \bar{\gamma}(x) \\ \text{s.t.} \ & \Pi(x) = \pi \\ & x \in \Omega, \end{aligned} \tag{6.6}$$

whose solution is denoted by x^π, and thus, $\bar{\gamma}^\pi = \bar{\gamma}(x^\pi)$. In particular, when $\pi \to 0$, one has strict satisfaction of EI, and thus, $\bar{\Gamma}^\pi \to \bar{\Gamma}^*$, see (6.5), which is the correlation metric under full EI.

Definition 6.2.1 *For a given degree of EI-violation, denoted by* π*, the Value of Expert Information, VEI, is defined by* $\mathcal{V}(\pi) := \bar{\Gamma}^{\pi} - \bar{\Gamma}^{*}$.

Thus, $\mathcal{V}(\pi)$ represents the relative benefit due to the specified degree of violation π of EI (relative to its strict satisfaction). For some finite π, suppose $\mathcal{V}(\pi)$ is significantly positive. Then, one may conclude that the EI is not valuable relative to a violation of size π, i.e., VEI is not significant at violation level π. Note that this is a *local* property for VEI at $\Pi(x) = \pi$. On the other hand, if $\mathcal{V}(\pi)$ is significantly positive for all $\pi \geq 0$, then, VEI is declared unimportant *globally.* To test the local property of EI, consider the following hypothesis test, for a given threshold $\nu_0(> 0)$, at a fixed violation level π:

$$\left.\begin{aligned} H_0 &: \ \mathcal{V}(\pi) \leq \nu_0 \\ H_1 &: \ \mathcal{V}(\pi) > \nu_0. \end{aligned}\right\} \tag{6.7}$$

Definition 6.2.2 *If* H_0 *is not rejected for some* $\pi > 0$*, then we say, VEI holds locally at* π*. If* H_0 *is not rejected for all* $\pi > 0$*, then we say, VEI holds globally.*

The statistical procedures for the preceding hypothesis test will be provided in Section 6.3. Next, we provide the specific set of expert information that will be analyzed in the Case Study of Chapter 7, where statistical estimates of the VEI are computed for various market sectors.

6.2.1 Specific case of EI

For the purposes of illustration in this section, the following general view of the 18 (financial) parameters given in Section 3.1.1 is considered as the expert information:

"*Profitability and growth perspectives are typically considered as outputs because revenue or income generation is a major objective criterion for a firm. On the other hand, asset utilization, liquidity, and leverage perspectives are considered as inputs because they are concerned with the planning and operational strategies of a firm. In contrast, valuation perspective is concerned with how well the equity markets perceive success of a firm, and thus, it is not concerned with a firm's input strategy.*"
Accordingly, the EI can be presented as follows.

$\mathcal{I}_1$: Each Profitability parameter is an output

$\mathcal{I}_2$: Each Valuation parameter is an output

$\mathcal{I}_3$: Each Growth parameter is an output

$\mathcal{I}_4$: Each Asset Utilization parameter is an input

$\mathcal{I}_5$: Each Liquidity parameter is an input

$\mathcal{I}_6$: Each Leverage parameter is an input.

Then, the corresponding Ω_k sets are given by,

$$\left.\begin{aligned}
\Omega_1 &= \{x \in \{0,1\}^{36} \mid x_1 = x_2 = x_3 = x_4 = 0\} \\
\Omega_2 &= \{x \in \{0,1\}^{36} \mid x_{14} = x_{15} = 0\} \\
\Omega_3 &= \{x \in \{0,1\}^{36} \mid x_{16} = x_{17} = x_{18} = 0\} \\
\Omega_4 &= \{x \in \{0,1\}^{36} \mid x_{23} = x_{24} = x_{25} = 0\} \\
\Omega_5 &= \{x \in \{0,1\}^{36} \mid x_{26} = x_{27} = x_{28} = 0\} \\
\Omega_6 &= \{x \in \{0,1\}^{36} \mid x_{29} = x_{30} = x_{31} = 0\}\,.
\end{aligned}\right\} \qquad (6.8)$$

The corresponding distance functions are then given by

$$\left.\begin{array}{ll} \mathcal{D}(x,\Omega_1)=x_1+x_2+x_3+x_4, & \mathcal{D}(x,\Omega_2)=x_{14}+x_{15}, \\ \mathcal{D}(x,\Omega_3)=x_{16}+x_{17}+x_{18}, & \mathcal{D}(x,\Omega_4)=x_{23}+x_{24}+x_{25}, \\ \mathcal{D}(x,\Omega_5)=x_{26}+x_{27}+x_{28}, & \mathcal{D}(x,\Omega_6)=x_{29}+x_{30}+x_{31}. \end{array}\right\} \tag{6.9}$$

Defining a quadratic penalty function on violation in each information category, we have $\Pi_k(.) = (.)^2$, $\forall k$, and thus, it follows that

$$\begin{aligned} \Pi(x) = {} & (x_1+x_2+x_3+x_4)^2 + (x_{14}+x_{15})^2 + (x_{16}+x_{17}+x_{18})^2 \\ & + (x_{23}+x_{24}+x_{25})^2 + (x_{26}+x_{27}+x_{28})^2 + (x_{29}+x_{30}+x_{31})^2 . \end{aligned} \tag{6.10}$$

We will employ the above EI penalty function in the Case Study reported in Chapter 7.

6.3 Statistical Tests for Value of EI

For the correlation metric maximized under expert information in model (6.5), where $\bar{\gamma}^* = \frac{1}{J}\sum_{j=1}^{J}\gamma_j(x^*)$, consider the following arctan hyperbolic transformation of the firm-correlations $\gamma_j(x^*)$:

$$\psi_j^* := \tanh^{-1}\gamma_j(x^*) = \frac{1}{2}\,\log_e\left[\frac{1+\gamma_j(x^*)}{1-\gamma_j(x^*)}\right]. \tag{6.11}$$

Then,

$$\psi_j^* \approx Normal\left(\frac{1}{2}\,\log_e\left[\frac{1+\Gamma_j(x^*)}{1-\Gamma_j(x^*)}\right],\ \frac{1}{t_0-3}\right), \tag{6.12}$$

see Section 4.3. Defining the average-statistic by $\bar{\psi}^* := \frac{1}{J}\sum_{j=1}^{J}\psi_j^*$, it can be shown that $\bar{\psi}^*$ is approximated normally distributed:

$$\bar{\psi}^* \approx Normal\left(\frac{1}{2J}\sum_{j=1}^{J}\log_e\left[\frac{1+\Gamma_j^*}{1-\Gamma_j^*}\right], \frac{1}{J(t_0-3)}\right), \tag{6.13}$$

see Section 4.3 for details. Similarly, for the correlation maximizing model in (6.6) under violation of EI by a given level $\pi > 0$, $\bar{\gamma}^\pi = \frac{1}{J}\sum_{j=1}^{J}\gamma_j(x^\pi)$, and defining

$$\psi_j^\pi := \tanh^{-1}\gamma_j(x^\pi) = \frac{1}{2}\log_e\left[\frac{1+\gamma_j(x^\pi)}{1-\gamma_j(x^\pi)}\right]. \tag{6.14}$$

Then, by defining $\bar{\psi}^\pi := \frac{1}{J}\sum_{j=1}^{J}\psi_j^\pi$, it follows that

$$\bar{\psi}^\pi \approx Normal\left(\frac{1}{2J}\sum_{j=1}^{J}\log_e\left[\frac{1+\Gamma_j^\pi}{1-\Gamma_j^\pi}\right], \frac{1}{J(t_0-3)}\right). \tag{6.15}$$

Since $\bar{\psi}^*$ is calculated under the full use of EI and $\bar{\psi}^\pi$ is calculated under a certain violation level π of EI, the sample statistics $\bar{\psi}^*$ and $\bar{\psi}^\pi$ are independent. Then, by defining the difference

$$\bar{\psi}(\pi) := \bar{\psi}^\pi - \bar{\psi}^*, \tag{6.16}$$

$$\bar{\psi}(\pi) \approx Normal\left(\frac{1}{2J}\left\{\sum_{j=1}^{J}\log_e\left[\frac{1+\Gamma_j^\pi}{1-\Gamma_j^\pi}\right] - \sum_{j=1}^{J}\log_e\left[\frac{1+\Gamma_j^*}{1-\Gamma_j^*}\right]\right\}, \frac{2}{J(t_0-3)}\right). \tag{6.17}$$

Also, see Section 4.3 for details. Consequently, $\bar{\psi}(\pi)$ will be used as a test statistic for the hypothesis test on $\mathcal{V}(\pi) = \bar{\Gamma}^\pi - \bar{\Gamma}^*$ given in (6.7).

Under the equality sign in the null hypothesis in (6.7), one has reference only to the overall difference $\bar{\Gamma}^{\pi} - \bar{\Gamma}^{*} = \nu_0$; however, we need knowledge of the individual firm-correlations under both the full use of EI (i.e., Γ_j^*) and under EI violation level π (i.e., Γ_j^{π}). Let the firm-correlations under the full use of EI be given, for unknown coefficients θ_j, by

$$\Gamma_j^* = \nu_0 \theta_j \ , \quad \text{for } j = 1, \ldots, J, \tag{6.18}$$

and those under EI violation level π be given, for unknown coefficients β_j, by

$$\Gamma_j^{\pi} = \nu_0 \beta_j \ , \quad \text{for } j = 1, \ldots, J, \tag{6.19}$$

where θ_j and β_j must satisfy the restrictions:

$$\sum_{j=1}^{J} (\theta_j - \beta_j) = J \quad \text{and} \quad -\frac{1}{\nu_0} \leq \theta_j, \beta_j \leq \frac{1}{\nu_0}, \forall j = 1, \ldots, J. \tag{6.20}$$

Then, $\bar{\psi}(\pi) \approx Normal\,(\psi_0(\theta, \beta), \sigma^2)$, where

$$\psi_0(\theta, \beta) := \frac{1}{2J} \sum_{j=1}^{J} \left[\log_e \left(\frac{1 + \nu_0 \theta_j}{1 - \nu_0 \theta_j} \right) - \log_e \left(\frac{1 + \nu_0 \beta_j}{1 - \nu_0 \beta_j} \right) \right] \quad \text{and} \quad \sigma^2 := \frac{2}{J(t_0 - 3)}. \tag{6.21}$$

For finiteness of the mean, $\psi_0(\theta, \beta)$, the inequalities in (6.20) must be satisfied as strict inequalities, i.e., $-\frac{1}{\nu_0} < \theta_j, \beta_j < \frac{1}{\nu_0}$, $j = 1, \ldots, J$. Then, for α-significance level and for the one-sided test, H_0 in (6.7) is accepted if

$$\sqrt{\frac{J}{2}(t_0 - 3)} \left[\bar{\psi}(\pi) - \psi_0(\theta, \beta) \right] \leq \mathcal{Z}^{-1}(1 - \alpha), \tag{6.22}$$

or, the computed sample value $\bar{\psi}(\pi)$, for a given EI violation level $\pi > 0$, must satisfy

$$\bar{\psi}(\pi) \leq \psi_0(\theta, \beta) + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{\frac{J}{2}(t_0 - 3)}}, \tag{6.23}$$

where $\mathcal{Z}^{-1}(.)$ is the inverse *c.d.f.* of the standard normal random variable. To test H_0 to conclude that the value of expert information (VEI) is significant at a given violation level π, therefore, specific values for θ and β vectors are required. Such information is not available, nor can it be estimated. However, if H_0 is accepted for the smallest (threshold) value of the right hand side in (6.23), over all possible θ and β, indeed H_0 is accepted. To this end, define:

$$\psi_0^{\min} := \inf_{\theta, \beta} \left\{ \psi_0(\theta, \beta) \; : \; \sum_{j=1}^{J} (\theta_j - \beta_j) = J \, , \; -\frac{1}{\nu_0} < \theta_j, \beta_j < \frac{1}{\nu_0} \, , j = 1, \ldots, J \right\}. \tag{6.24}$$

Hence, if $\bar{\psi}(\pi) \leq \psi_0^{\min} + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{\frac{J}{2}(t_0-3)}}$, then H_0 is accepted at the specified violation level π.

Proposition 6.3.1 *For* $0 < \nu_0 < 2$,

$$\psi_0^{\min} = \log_e \left(\frac{2 + \nu_0}{2 - \nu_0} \right). \tag{6.25}$$

Proof. Note that $\psi_0(\theta, \beta)$ is a nonconvex function. Consider the (relaxed) minimization problem:

$$Z^* := \min_{\theta, \beta} \left\{ \psi_0(\theta, \beta) \; : \; \sum_{j=1}^{J} (\theta_j - \beta_j) = J \right\}, \tag{6.26}$$

and thus, $Z^* \leq \psi_0^{\min}$. Since the constraints of (6.26) are linear, "Constraint Qualification" (CQ) is satisfied at all feasible solutions, which implies that every optimal solution of (6.26) must be a Karush-Kuhn-Tucker (KKT) point, see Bazaraa *et al.* [9]. Denoting the Lagrange multiplier associated with the equality constraint by λ, the KKT conditions yield,

$$\frac{2\nu_0}{2J\left[1-(\nu_0\theta_j)^2\right]} + \lambda = 0\,, \quad \forall j = 1, \ldots, J, \tag{6.27}$$

and

$$\frac{2\nu_0}{2J\left[1-(\nu_0\beta_j)^2\right]} + \lambda = 0\,, \quad \forall j = 1, \ldots, J, \tag{6.28}$$

which imply that $|\theta_j| = |\beta_j|, \forall j$. Therefore, $\theta_j = p_j a$ and $\beta_j = q_j a$ must hold for all $j = 1, \ldots, J$, where p_j and q_j are each $+1$ or -1 and a is a positive constant. The equality constraint then implies that $\sum_{j=1}^{J} r_j = \frac{J}{2a}$, where $r_j = \frac{1}{2}(\theta_j - \beta_j)$ can take on values -1, 0, or 1. Therefore, $a = \frac{J}{2b}$ where $b = \sum_{j=1}^{J} r_j$ can take on values $-J, -J+1, ..., 0, 1, 2, ..., J$. Since $a > 0$, b must be positive and thus, $a \in \{\frac{1}{2}, \frac{J}{2(J-1)}, \frac{J}{2(J-2)}, \ldots, \frac{J}{2}\}$. Each of these values for a defines a distinct KKT point, provided the objective function is well-defined at those points, which is given by

$$
\begin{aligned}
&\frac{1}{2J}\left[\sum_{j:p_j=+1}\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)+\sum_{j:p_j=-1}\log_e\left(\frac{1-\nu_0 a}{1+\nu_0 a}\right)\right.\\
&\qquad\left.-\sum_{j:q_j=+1}\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)-\sum_{j:q_j=-1}\log_e\left(\frac{1-\nu_0 a}{1+\nu_0 a}\right)\right]\\
&= \frac{1}{2J}\left[\sum_{j=1}^{J}(p_j-q_j)\right]\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)\\
&= \frac{b}{J}\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)\\
&= \frac{1}{2a}\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)
\end{aligned}
$$

Then, the minimum in (6.26) is obtained by

$$
Z^* = \min\left\{\frac{1}{2a}\log_e\left(\frac{1+\nu_0 a}{1-\nu_0 a}\right)\ :\ a=\frac{1}{2},\frac{J}{2(J-1)},\frac{J}{2(J-2)},\ldots,\frac{J}{2}\right\}. \tag{6.29}
$$

Next, noting that the function $f(x)=\frac{1}{x}\log_e\left(\frac{1+x\nu_0}{1-x\nu_0}\right)$ is monotonically nondecreasing in x for $x\in\left[\frac{1}{2},\frac{J}{2}\right]$, it follows that $a=\frac{1}{2}$ is indeed the optimal solution in (6.29), which thus implies that each $p_j=+1$ and $q_j=-1$ at the optimum. That is, $\theta_j=\frac{1}{2}$ and $\beta_j=-\frac{1}{2}$, $\forall j$, solve the relaxed problem in (6.26), which yields $Z^*=\log_e\left(\frac{2+\nu_0}{2-\nu_0}\right)$. But, for $0<\nu_0<2$, we have $-\frac{1}{\nu_0}<\theta_j-\beta_j=1<\frac{1}{\nu_0}$ for all j. This leads to the feasible point upper bound on the infimum in (6.24) as $\psi_0^{\min}\le Z^*$. Combining with $Z^*\le\psi_0^{\min}$, the proof is completed. ■

The most conservative form of the test in (6.23) is then given by

$$\bar{\psi}(\pi) \leq \log_e \left(\frac{2+\nu_0}{2-\nu_0} \right) + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{\frac{J}{2}(t_0-3)}}. \tag{6.30}$$

For a given $\pi > 0$, if (6.30) is satisfied, then VEI is significant at the EI-violation level π. However, if (6.30) is violated at a given π, it does not necessarily imply that H_0 is rejected. For this purpose, we introduce a rejection tolerance $\kappa \geq 1$, where if

$$\bar{\psi}(\pi) > \kappa \log_e \left(\frac{2+\nu_0}{2-\nu_0} \right) + \frac{\mathcal{Z}^{-1}(1-\alpha)}{\sqrt{\frac{J}{2}(t_0-3)}} \tag{6.31}$$

holds, then it is concluded that VEI is not significant at the violation level π.

It is of interest to determine how the significance of VEI changes as π changes. As the violation level π is decreased towards zero, $\bar{\Gamma}^{\pi} - \bar{\Gamma}^{*} \to 0$ holds, and thus H_0 will not be rejected. Alternatively, it is of interest to determine whether it is more likely to reject H_0 as π is increased. These concerns will be pursued empirically using the Case Study in Chapter 7.

6.4 Model for Computing VEI

The model in (6.6) is difficult to compute because the maximization involves choosing a binary vector (in $2I = 36$ dimensions) with a complicated nonlinear objective function and linear and nonlinear constraints. An alternative, but equivalent, formulation is utilized below, where any EI-violation is traded-off with the correlation metric using the tolerance parameter $w(\geq 0)$:

$$f(w) := \max_{x \in \Omega} \left\{ \bar{\gamma}(x) - w\Pi(x) \right\}. \tag{6.32}$$

When $w = 0$, note that (6.32) is the (unrestricted) maximum correlation under no expert information $\bar{\gamma}^0$ in (6.4), i.e., $f(0) = \bar{\gamma}^0$. The model in (6.32) can be solved using the direct search technique under sampling from the feasible domain, as described in Section 4.2.1, for a specified value of w. However, since (6.32) is a nonlinear binary mathematical program, there is no guarantee that the solutions so-obtained would be globally optimal. To this end, we derive certain theoretical properties that are useful in checking whether additional sampling from the feasible domain would be needed to obtain an optimal solution.

6.4.1 Properties of $f(w)$

We will discuss several pertinent properties of the optimal value function $f(w)$. First,

Theorem 6.4.1 *For $\bar{\gamma}^0$ in (6.4) and for an optimal solution $x(w)$ of (6.32) for some $w > 0$, it holds that $\bar{\gamma}^0 \geq \bar{\gamma}(x(w))$, i.e., $\bar{\gamma}^0$ is the highest-achievable correlation.*

Proof. Noting that an optimal solution of (6.32) is feasible in (6.4), the proof follows in a straightforward manner. ∎

Theorem 6.4.2 *For all $w \in \Re$, $f(w) \geq \bar{\gamma}^*$ holds. Moreover, $\lim_{w \to \infty} f(w) = \bar{\gamma}^*$ holds.*

Proof. Let an optimal solution of the correlation maximization problem in (6.5) be denoted by x^*. Then, $\bar{\gamma}^* = \bar{\gamma}(x^*)$ and $x^* \in \Omega \bigcap \Omega^{\mathcal{I}}$, and thus, $\Pi(x^*) = 0$ holds. Since $x^* \in \Omega$, x^* is feasible in (6.32), and thus, $f(w) \geq \bar{\gamma}(x^*) - w\Pi(x^*) = \bar{\gamma}(x^*)$ for any w, which proves the first assertion. Moreover, taking limits on both sides, $\lim_{w\to\infty} f(w) \geq \bar{\gamma}^*$, and thus, $\lim_{w\to\infty} f(w) \geq -1$.

Given an optimal solution $x(w)$ of (6.32), since $f(w) = \bar{\gamma}(x(w)) - w\Pi(x(w)) \leq 1 - w\Pi(x(w))$,

$$-1 \leq \lim_{w\to\infty} f(w) \leq 1 - \lim_{w\to\infty} \left[w\Pi(x(w))\right],$$

implying that $M := \lim_{w\to\infty} [w\Pi(x(w))]$ is finite (and $M \geq 0$ since $w, \Pi \geq 0$). Define $\pi^\infty := \lim_{w\to\infty} \Pi(x(w))$, and thus, $\pi^\infty \geq 0$. Suppose (by contradiction) $\pi^\infty > 0$. Then, for small $\varepsilon > 0$ (such that $\varepsilon < \pi^\infty$) it follows that there exists $w(\varepsilon)$ such that for all $w > w(\varepsilon)$, $|\Pi(x(w)) - \pi^\infty| < \varepsilon$. Thus,

$$w(\pi^\infty - \varepsilon) < w\Pi(x(w)) < w(\pi^\infty + \varepsilon)\ , \qquad \text{for } w > w(\varepsilon),$$

and thus, by taking limits, $\lim_{w\to\infty} [w\Pi(x(w))] = M \to +\infty$, which violates the previous conclusion that M is finite. Therefore, $\pi^\infty = 0$ must hold.

Denoting $x^\infty := \lim_{w\to\infty} x(w)$, since, Π is a continuous function, $\Pi(x^\infty) = 0$ follows. Moreover, $x^\infty \in \Omega^*$ since Ω^* is closed. Thus,

$$\begin{aligned} \lim_{w\to\infty} f(w) &= \lim_{w\to\infty} \bar{\gamma}(x(w)) - \lim_{w\to\infty} [w\Pi(x(w))] \\ &= \bar{\gamma}(x^\infty) - M \\ &\leq \bar{\gamma}(x^\infty). \end{aligned}$$

Since $x^\infty \in \Omega$, we have $f(w) \geq \bar{\gamma}(x^\infty) - w\Pi(x^\infty) = \bar{\gamma}(x^\infty)$, and also, $\bar{\gamma}^* \geq \bar{\gamma}(x^\infty)$. Combining with the previous inequality, we get

$$\bar{\gamma}(x^\infty) \geq \lim_{w\to\infty} f(w) \geq \bar{\gamma}^* \geq \bar{\gamma}(x^\infty),$$

and thus, $\lim_{w\to\infty} f(w) = \bar{\gamma}^* = \bar{\gamma}(x^\infty)$. This completes the proof. $\blacksquare$

In particular for $w = 0$, we have the *Relaxed* maximum of correlation under no expert information $\bar{\gamma}^0$, i.e., $f(0) = \bar{\gamma}^0$, and due to Theorem 6.4.2, $\bar{\gamma}^0 \geq \bar{\gamma}^*$.

Theorem 6.4.3 *The function $f(w)$ is monotonically nonincreasing in $w \in \Re$.*

Proof. For $w_1 \in \Re$ and $w_2 \in \Re$ such that $w_1 < w_2$, let the associated optimal solutions (input/output partitions) of the mathematical program in (6.32) be denoted by x^1 and x^2, respectively. Then,

$$\begin{aligned} f(w_2) &= \bar{\gamma}(x^2) - w_2\Pi(x^2) \\ &\leq \bar{\gamma}(x^2) - w_1\Pi(x^2) \qquad \text{(since } w_1 < w_2 \text{ and } \Pi \geq 0\text{)} \\ &\leq \bar{\gamma}(x^1) - w_1\Pi(x^1) \qquad \text{(since } x^2 \text{ is feasible in (6.32) for } w = w_1\text{)} \\ &= f(w_1). \qquad \blacksquare \end{aligned}$$

Furthermore,

Theorem 6.4.4 *The function $f(w)$ is convex in $w \in \Re$.*

Proof. For $w_1, w_2 \in \Re$, and $\alpha \in [0,1]$, let $w_\alpha := \alpha w_1 + (1-\alpha)w_2$. Then,

$$\begin{aligned} f(w_\alpha) &= \max_{x\in\Omega} \{\bar{\gamma}(x) - [\alpha w_1 + (1-\alpha)w_2]\Pi(x)\} \\ &= \max_{x\in\Omega} \{\alpha[\bar{\gamma}(x) - w_1\Pi(x)] + (1-\alpha)[\bar{\gamma}(x) - w_2\Pi(x)]\} \\ &\leq \max_{x\in\Omega} \{\alpha[\bar{\gamma}(x) - w_1\Pi(x)]\} + \max_{x\in\Omega} \{(1-\alpha)[\bar{\gamma}(x) - w_2\Pi(x)]\} \\ &= \alpha f(w_1) + (1-\alpha)f(w_2), \end{aligned}$$

and thus, f is convex. ■

The above properties confirm that $f(w)$ is convex, nonincreasing and its maximum is at $w = 0$ with the function value $\bar{\gamma}^0$, and f reaches $\bar{\gamma}^*$ for sufficiently large w (certainly as $w \to \infty$), see the illustration in Figure 6.2. The convexity of f may be used to check if f values obtained for a given set of w values satisfy (global) optimality. This idea will be utilized when the two-stage heuristic, see Section 4.2.1, is applied for solving the problem in (6.32) in the Case Study in Chapter 7.

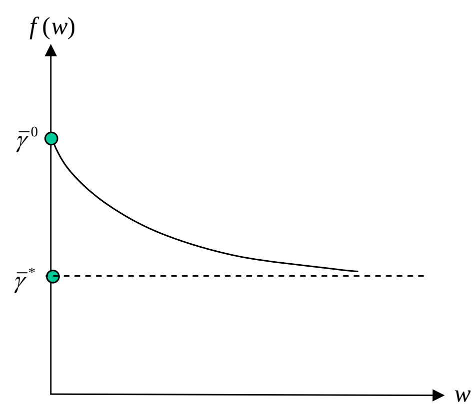

Figure 6.2: Optimal value function under Expert Information

6.4.2 Marginal Value of Expert Information (MVEI)

Considering VEI in Definition 6.2.1, the focus here is on the incremental gain in VEI per unit violation of expert information, termed the Marginal VEI (MVEI). That is, MVEI is given by $\frac{d\mathcal{V}(\pi)}{d\pi}$, and its maximum value is associated with the EI violation level $\pi^{**} = \arg\max_{\pi \geq 0} \dfrac{d\mathcal{V}(\pi)}{d\pi}$. Determining this population-parameter is difficult, if not impossible. Instead, the maximum MVEI is estimated by solving the following fractional mathematical program:

$$w^* := \sup_x \left\{ \frac{\bar{\gamma}(x) - \bar{\gamma}^*}{\Pi(x)} \;:\; \Pi(x) > 0,\, x \in \Omega \right\}. \tag{6.33}$$

Then, w^* is termed the (estimated) maximum marginal value of expert information per unit violation of the EI.

Theorem 6.4.5 *$f(w) = \bar{\gamma}^*$ for $w \geq w^*$.*

Proof. Since

$$w^* \geq \frac{\bar{\gamma}(x) - \bar{\gamma}^*}{\Pi(x)}\ , \quad \forall x \in \Omega \text{ and } \Pi(x) > 0,$$

it follows that

$$\begin{aligned}
\bar{\gamma}^* &\geq \bar{\gamma}(x) - w^*\Pi(x)\ , \quad \forall x \in \Omega \text{ and } \Pi(x) > 0 \\
&\geq \bar{\gamma}(x) - w\Pi(x)\ , \quad \forall x \in \Omega,\, \Pi(x) > 0 \text{ and } w \geq w^* \\
&\geq \sup_{x \in \Omega, \Pi(x) > 0} \{\bar{\gamma}(x) - w\Pi(x)\}\ , \quad \forall w \geq w^* \\
&= f(w)\ , \quad \forall w \geq w^*.
\end{aligned}$$

Moreover, in the proof of Theorem 6.4.2, it was shown that $f(w) \geq \bar{\gamma}(x^*)$ for any $w \geq 0$. This along with $f(w) \leq \bar{\gamma}(x^*)$ for any $w \geq w^*$ proves the assertion. $\blacksquare$

Theorem 6.4.6 *$f(w) > \bar{\gamma}^*$ for $w < w^*$.*

Proof. By contradiction, suppose for some $\hat{w}$ such that $\hat{w} < w^*$, we have $f(\hat{w}) = \bar{\gamma}^*$. Then,

$$\begin{aligned} \bar{\gamma}^* &= \max_{x \in \Omega} \{\bar{\gamma}(x) - \hat{w}\Pi(x)\} \\ &\geq \bar{\gamma}(x) - \hat{w}\Pi(x) \quad \forall x \in \Omega \text{ s.t. } \Pi(x) > 0 \\ &> \bar{\gamma}(x) - w^*\Pi(x) \quad \forall x \in \Omega \text{ s.t. } \Pi(x) > 0, \end{aligned}$$

and thus,

$$w^* > \frac{\bar{\gamma}(x) - \bar{\gamma}^*}{\Pi(x)} \quad \forall x \in \Omega \text{ s.t. } \Pi(x) > 0.$$

This violates the optimality of the objective value w^* in (6.33). $\blacksquare$

Theorem 6.4.7 *For all $w \geq w^*$, $\Pi(x(w)) = 0$ must hold, where $x(w)$ is an optimal solution of (6.32), i.e., $f(w) = \bar{\gamma}(x(w)) - w\Pi(x(w))$.*

Proof. By contradiction, suppose for some $\hat{w} > w^*$, we have $\Pi(x(\hat{w})) > 0$. Then, by Theorem 6.4.5, $f(\hat{w}) = \bar{\gamma}^*$ and

$$\begin{aligned} \bar{\gamma}^* = f(\hat{w}) &= \bar{\gamma}(x(\hat{w})) - \hat{w}\Pi(x(\hat{w})) \\ &< \bar{\gamma}(x(\hat{w})) - w^*\Pi(x(\hat{w})) \quad \text{(since } \hat{w} > w^* \text{ and } \Pi(x(\hat{w})) > 0), \end{aligned}$$

and thus,

$$w^* < \frac{\bar{\gamma}(x(\hat{w})) - \bar{\gamma}^*}{\Pi(x(\hat{w}))}$$

holds for $x(\hat{w}) \in \Omega$, which violates the optimality of the objective value w^* in (6.33). This completes the proof. ■

The maximum MVEI, w^*, can be determined by the following computational procedure for solving fractional mathematical programs (see, Lasdon [37]):

Procedure-MVEI

Step-0: (Initialization) Solve the model in (6.4) to determine an optimal solution x^0, and define, $w_0 = \frac{\bar{\gamma}(x^0) - \bar{\gamma}^*}{\Pi(x^0)}$.

Set iteration count $k = 0$.

Step-1: Solve the model in (6.32) for $w = w_k$ and obtain its solution x^{k+1}.

Step-2: Define, $w_{k+1} = \frac{\bar{\gamma}(x^{k+1}) - \bar{\gamma}^*}{\Pi(x^{k+1})}$.

If $w_{k+1} \leq w_k$, Stop (and set $w^* := w_k$).

Otherwise, set $k \leftarrow k + 1$, and go to Step-1.

It is neither asserted nor proven that w^* is finite. In order to obtain the maximized MVEI value, the model in (6.32) needs to be solved iteratively. This is a nonlinear binary problem and the objective function is neither concave nor pseudo-concave, and it is also non-differentiable. Thus, it is necessary to verify whether the chosen (heuristic) solution procedure yield input/output categorizations that satisfy optimality conditions. These optimality conditions are developed in the next section.

6.5 First-order conditions of optimality

The feasible set (Binary Complementary Domain - BCD) Ω in (4.5) has the following continuous representation:

$$\Omega_c := \left\{ x \in \Re^{2I} \mid \sum_{i=1}^{I} x_i \geq 1,\ x_i x_{I+i} \leq 0,\ x_i, x_{I+i} \geq 0, \forall i = 1, \ldots, I \right\}, \tag{6.34}$$

which implies that Ω_c is a convex set in $\Re^{2I}$. Therefore, the problem in (6.32) has the following equivalent representation:

$$f(w) := \max_{x \in \Omega_c} \{\bar{\gamma}(x) - w\Pi(x)\}. \tag{6.35}$$

Although Ω_c is convex, its interior is not non-empty, and thus, (Slater's) regularity condition does not hold. It can be shown that not all feasible solutions in Ω_c are *regular*, i.e., the Constraint Qualification (CQ) may not hold everywhere on Ω_c. Consequently, the first order conditions for (6.35) may not even be necessary for optimality. To see this, denoting the constraints by $g(x) = \sum_{i=1}^{I} x_i - 1 \geq 0$, $h_i^1(x) = x_i x_{I+i} \leq 0$ and $h_i^2(x) = x_i \geq 0$, for an arbitrary feasible $x \in \Omega_c$, define the set of directions $d \in \Re^{2I}$, let

$$\begin{aligned} \bar{D}(x) &:= \left\{ d \ : \ \nabla g(x)' d \geq 0,\ \nabla h_i^1(x)' d \leq 0,\ \nabla h_i^2(x)' d \geq 0 \right\} \\ &= \left\{ d \ : \ \sum_{i=1}^{I} d_i \geq 0,\ x_{I+i} d_i + x_i d_{I+i} \leq 0,\ d_i \geq 0,\ \forall i \right\}. \end{aligned}$$

Now, consider the set of *feasible directions* in Ω_c, denoted by $\hat{D}(x)$ for some given $x \in \Omega_c$, as given by

$$\begin{aligned}\hat{D}(x) &:= \left\{d \, : \, \exists \mu > 0 \text{ s.t. } g(x+\mu d) \geq 0, \; h_i^1(x+\mu d) \leq 0, \; h_i^2(x+\mu d) \geq 0\right\} \\ &= \left\{d \, : \, \sum_{i=1}^{I} d_i \geq 0, \; x_{I+i}d_i + x_i d_{I+i} + \mu d_i d_{I+i} \leq 0, \; d_i \geq 0, \; \forall i\right\}.\end{aligned}$$

For the CQ to hold, we must have $\bar{D}(x) \subseteq \hat{D}(x)$ for all $x \in \Omega_c$. But, when a parameter k is not chosen in the DEA model, we have $x_k = x_{I+k} = 0$, and thus, choose $\hat{d} \in \bar{D}(x)$ such that $\hat{d}_k > 0$ and $\hat{d}_{I+k} > 0$. Hence, $\hat{d}_k \hat{d}_{I+k} > 0$, which violates a condition in the definition of the set $\hat{D}(x)$, i.e., $\hat{d} \notin \hat{D}(x)$.

If each and every parameter is included in the DEA model, and thus, $x_i + x_{I+i} > 0$, $\forall i$, it can be shown that CQ holds. However, since some parameters may have to be dropped when maximizing the correlation metric, one has to proceed differently in order to formulate first order conditions of optimality.

6.5.1 Continuous model and discontinuities

The complementary conditions $x_i x_{I+i} = 0$ are merely incorporated to ensure that a given parameter is not chosen both as input and output. By doing so, the binary search space has been significantly reduced. However, in the event $x_i x_{I+i} > 0$, the resulting DEA-based RPS score is not representative of the firm's fundamental performance, as RPS may turn out to be identically C across all (or many) firms, as claimed next.

Theorem 6.5.1 *For some parameter $i \in \{1, \dots, I\}$, let $x_i > 0$ and $x_{I+i} > 0$. For some company j being evaluated (in time period t), suppose the measured value of parameter i satisfies $\xi_{ijt} > 0$. Then, for the optimal value in (6.2), $\eta^c_{jt}(x) = C$ holds.*

Proof. Set $\hat{u}_{kjt} = 0$ and $\hat{v}_{kjt} = 0$ for all $k \neq i$. Given $x_i > 0$ and $x_{I+i} > 0$, let $\hat{u}_{ijt} = 1/(x_i \xi_{ijt})$ and $\hat{v}_{ijt} = C/(x_{I+i}\xi_{ijt})$. This $(\hat{u}, \hat{v})$ solution is feasible in (6.2) with the objective value C, and thus, $\eta^c_{jt}(x) \geq C$. Since $\eta^c_{jt}(x) \leq C$ must also hold, the result follows. ■

Therefore, under the conditions in Theorem 6.5.1, many firms can end up having constant scores of C, which would then result in very low correlation, i.e., $\bar{\gamma}(x) \approx 0$. Therefore, choices of x such that $x_i x_{I+i} > 0$ would not emerge as optimal solutions of the correlation maximization problem. Consequently, without loss of generality, the correlation maximization model under DEI can be restated as:

$$f(w) = \max_{x \geq 0} \{\bar{\gamma}(x) - w\Pi(x)\}, \tag{6.36}$$

with the revised objective function definition:

$$\bar{\gamma}(x) := \begin{cases} \bar{\gamma}(x) & \text{if } \sum_{i=1}^{I} x_i > 0 \\ -\infty & \text{if } \sum_{i=1}^{I} x_i = 0. \end{cases} \tag{6.37}$$

While the constraint set of the alternative formulation in (6.36) is simply the nonnegativity restrictions (and thus, CQ holds everywhere!), its objective function is still discontinuous. To see this, note that each function $\gamma_{jt}(x)$ is the correlation between the (transformed) DEA-based score $\eta^c_{jt}(x)$ and the stock return random variable r_{jt}. The function $\eta^c_{jt}(x)$ is discontinuous at boundaries where $x_i = 0$. For example, given a point $\bar{x}$ such that $\bar{x}_i = 0$ for some $i \leq I$, consider $\eta^c_{jt}(\bar{x}+\mu e_i) = a(\mu)$, where e_i is the i^{th} elementary vector, and μ is a scalar. It is possible that $a(1) > a(0)$ if including the parameter i as an input can strictly improve the DEA score. Thus, $a(\mu) > a(0)$ for $\mu > 0$, and there exists a discontinuity at $\mu = 0$, noting that $a(\mu)$ is a constant for all $\mu > 0$.

In the preceding example, the discontinuity can be removed by modifying the function description of η^c_{jt} in the interval of $\mu \in [0, 1]$ such that it is replaced by a polyhedral (in the case of the above example, by a piecewise linear) function $q_{jt}(.)$ satisfying:

$$q_{jt}(\bar{x}) = \eta^c_{jt}(\bar{x})$$
$$q_{jt}(\bar{x} + \mu e_i) = \eta^c_{jt}(\bar{x} + \mu e_i), \quad \mu \geq 1$$
$$q_{jt}(\bar{x} + \mu e_i) = \eta^c_{jt}(\bar{x}) + \mu \left[\eta^c_{jt}(\bar{x} + e_i) - \eta^c_{jt}(\bar{x})\right], \quad \mu \in (0, 1).$$

See the illustration in Figure 6.3 where the concerned polyhedral function is shown in 2-dimensions. For our purposes here, it is neither necessary to understand a polyhedral construction nor is it relevant how this polyhedral function behaves in areas where x_i is between 0 and 1. The main reason for this is, as it will become clear later, that the optimality conditions are eventually stated in terms of only binary x solutions. However, the polyhedral nature of $q(.)$ implies that it is nondifferentiable

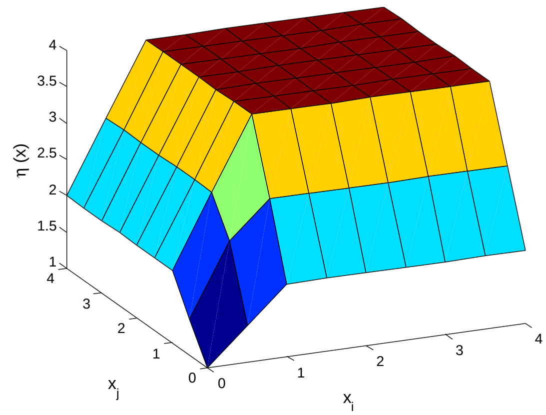

Figure 6.3: Continuous approximation for discontinuous function

precisely at those binary x points, and thus, we have to derive optimality conditions for the nondifferentiable optimization problem:

$$g(w) := \max_{x \geq 0} \{\bar{\gamma}_q(x) - w\Pi(x)\} \tag{6.38}$$

where $\bar{\gamma}_q$ function represents the correlation metric computed under the $q(.)$ function, instead of the $\eta^c(.)$ function. Clearly, $f(w) \leq g(w)$ holds since an optimal solution defining f is a binary solution. The approach is to verify the optimality of a binary solution to (6.38) so that $g(w) \leq f(w)$ holds as well, and thus, $f(w) = g(w)$ holds.

6.5.2 Nondifferentiable Optimization

Although $q_{jt}(x)$ is not differentiable at locations where $x_i = 1$, it is subdifferentiable. As such, we have to resort to optimality conditions for subdifferentiable functions, see [53].

Definition 6.5.2 *A vector $a \in \Re^n$ is called a subgradient of $f : \Re^n \to \Re$ at $\bar{x} \in dom(f)$ if for all $x \in dom(f)$, $f(x) \geq f(\bar{x}) + a'(x - \bar{x})$. If f is convex and differentiable, then its gradient at $\bar{x}$ is a subgradient. But a subgradient can exist even when f is not differentiable at $\bar{x}$. A function f is called subdifferentiable at $\bar{x}$ if there exists at least one subgradient at $\bar{x}$. The set of subgradients of f at the point $\bar{x}$ is called the subdifferential of f at $\bar{x}$, and is denoted $\partial f(\bar{x})$. If f is convex and differentiable, then $\partial f(\bar{x}) = \{\nabla f(\bar{x})\}$.*

Theorem 6.5.3 *At some $x \geq 0$, let the i^{th} component of a subgradient $\partial_i q_{jt}(x) \in S_i$. If $x_i = 1$, then $S_i = [a_{ijt}(x)\ ,\ 0]$, where*

$$a_{ijt}(x) = \eta^c_{jt}(x - e_i) - \eta^c_{jt}(x)\ \leq 0. \tag{6.39}$$

If $x_i = 0$, then $S_i = \{b_{ijt}(x)\}$, where

$$b_{ijt}(x) := \eta^c_{jt}(x + e_i) - \eta^c_{jt}(x)\ \geq 0. \tag{6.40}$$

Moreover, the subdifferential at x is given by $S := \times_{i=1}^{2I} S_i$.

Proof. Straightforward by appealing to the positive homogeneity result in Proposition 2.3.2. ■

The generalization of the Karush-Kuhn-Tucker (KKT) conditions for the (generic) subdifferentiable optimization problem in

$$\begin{array}{ll} \min_x & f_0(x) \\ \text{s.t.} & f_i(x) \leq 0,\ i = 1, \ldots, m, \end{array}$$

under Constraint Qualification (CQ) are given by, see [53],

$$\begin{aligned} &f_i(x) \leq 0, \ \lambda_i \geq 0 \\ &0 \in \partial f_0(x) + \textstyle\sum_{i=1}^m \lambda_i \partial f_i(x) \\ &\lambda_i f_i(x) = 0, \ i = 1, \ldots, m. \end{aligned} \tag{6.41}$$

Specialization of (6.41) to the problem in (6.38) yields the following first order conditions (since CQ is satisfied for the nonnegativity constraints) where $\lambda \in \Re^{2I}$ is a vector of dual multipliers:

$$\left. \begin{aligned} 0 \in \partial \bar{\gamma}_q(x) - w \nabla \Pi(x) - \lambda \\ \lambda_i x_i = 0, \quad \lambda_i, \, x_i \geq 0, \quad \forall i = 1, \ldots, 2I. \end{aligned} \right\} \tag{6.42}$$

Every local optimum of (6.38) must satisfy the above conditions. The subgradients of $\bar{\gamma}_q$ are determined by noting $\bar{\gamma}_q(x) = \frac{1}{N} \sum_{j=1}^N \gamma_{jq}(x)$, and the firm-correlations (computed over T time periods) between $\hat{q}_{jt}(x) := \frac{[q_{jt}(x)]^\alpha - 1}{\alpha}$ and r_{jt} are given by

$$\gamma_{jq}(x) = \frac{1}{AB} \left[\sum_{t=1}^{t_0} \hat{q}_{jt}(x) r_{jt} - \frac{1}{t_0} \left(\sum_{\tau=1}^{t_0} \hat{q}_{j\tau}(x) \right) \left(\sum_{\tau=1}^{t_0} r_{j\tau} \right) \right], \tag{6.43}$$

where

$$A = \left[\sum_{t=1}^{t_0} \left(\hat{q}_{jt}(x) \right)^2 - \frac{1}{t_0} \left(\sum_{t=1}^{t_0} \hat{q}_{jt}(x) \right)^2 \right]^{\frac{1}{2}} \tag{6.44}$$

and

$$B = \left[\sum_{t=1}^{t_0} \left(r_{jt} \right)^2 - \frac{1}{t_0} \left(\sum_{t=1}^{t_0} r_{jt} \right)^2 \right]^{\frac{1}{2}}. \tag{6.45}$$

Then, partial derivatives $\frac{\partial \gamma_{jq}}{\partial \hat{q}_{jt}}$ can be obtained from (6.43) as

$$\frac{\partial \gamma_{jq}}{\partial \hat{q}_{jt}}(x) = \frac{1}{AB}\left(r_{jt} - r_j^{av}\right) - \frac{\gamma_{jq}(x)}{A^2}\left[\hat{q}_{jt}(x) - \hat{q}_j^{av}(x)\right] \tag{6.46}$$

and r_j^{av} is the average of returns over t_0 periods and $\hat{q}_j^{av}$ is the average of (modified) scores $\hat{q}_{jt}(x)$ over t_0 periods for firm j. Therefore,

$$\frac{\partial \bar{\gamma}_q(x)}{\partial x_i} = \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{t_0}\left(\frac{\partial q_{jt}}{\partial x_i}\right)\left(q_{jt}^{\alpha-1}\frac{\partial \gamma_{jq}}{\partial \hat{q}_{jt}}\right) \tag{6.47}$$

Observing that $q_{jt}(x)$ is not differentiable w.r.t. x_i everywhere, its subgradient are given in the Theorem 6.5.3 for binary x values.

6.5.3 Optimality conditions under violation of EI

Consider $x \in \Omega$, and thus, x is a binary vector. For such x, define

$$\alpha_j(x) = \left[\sum_{t=1}^{t_0}\left(\frac{\left(\eta_{jt}^c(x)\right)^\alpha - 1}{\alpha}\right)^2 - t_0\left(\bar{\eta}_j(x)\right)^2\right]^{\frac{1}{2}} \text{ and } \beta_j = \left[\sum_{t=1}^{t_0}\left(r_{jt}\right)^2 - t_0\left(\bar{r}_j\right)^2\right]^{\frac{1}{2}} \tag{6.48}$$

where

$$\bar{\eta}_j(x) := \frac{1}{t_0}\left(\sum_{t=1}^{t_0}\frac{\left(\eta_{jt}^c(x)\right)^\alpha - 1}{\alpha}\right) \text{ and } \bar{r}_j := \frac{1}{t_0}\left(\sum_{t=1}^{t_0} r_{jt}\right). \tag{6.49}$$

Define the partial derivative of γ_j, for $j = 1, \ldots, N$, by

$$\vartheta_j(x) := \frac{1}{\alpha_j(x)\beta_j}\left(r_{jt} - \bar{r}_j\right) - \frac{\gamma_j(x)}{\left(\alpha_j(x)\right)^2}\left(\frac{\left(\eta_{jt}^c(x)\right)^\alpha - 1}{\alpha} - \bar{\eta}_j(x)\right). \tag{6.50}$$

For $x \in \Omega$, if $x_i = 1$, define the (univariate interval) set $S_i(x)$, $i = 1, \ldots, 2I$, by

$$S_i(x) := \left\{ \frac{1}{J} \sum_{j=1}^{J} \sum_{t=1}^{t_0} \vartheta_j(x) \theta_{ijt} \left(\eta_{jt}^c(x) \right)^{\alpha-1} \quad : \quad \theta_{ijt} \in [a_{ijt}(x) \ , \ 0] \right\}, \tag{6.51}$$

where $a_{ijt}(x)$ is defined in (6.39), and if $x_i = 0$, define the scalar $\delta_i(x)$, $i = 1, \ldots, 2I$, by

$$\delta_i(x) = \frac{1}{J} \sum_{j=1}^{J} \sum_{t=1}^{t_0} \vartheta_j(x) b_{ijt}(x) \left(\eta_{jt}^c(x) \right)^{\alpha-1}, \tag{6.52}$$

where $b_{ijt}(x)$ is defined in (6.40). Under the notation $\vartheta_j^-(x) = \min\{\vartheta_j(x), 0\}$ and $\vartheta_j^+(x) = \max\{\vartheta_j(x), 0\}$, notice that (6.51) can be simplified as

$$\left.\begin{aligned} S_i(x) &= \left[s_i^l(x) \ , \ s_i^u(x) \right], \quad \text{where} \\ s_i^l(x) &:= \frac{1}{J} \sum_{j=1}^{J} \sum_{t=1}^{t_0} a_{ijt} \vartheta_j^+(x) \left(\eta_{jt}^c(x) \right)^{\alpha-1} \\ s_i^u(x) &:= \frac{1}{J} \sum_{j=1}^{J} \sum_{t=1}^{t_0} a_{ijt} \vartheta_j^-(x) \left(\eta_{jt}^c(x) \right)^{\alpha-1}. \end{aligned}\right\} \tag{6.53}$$

Theorem 6.5.4 *Suppose an optimal solution x^* of (6.38) satisfies $x^* \in \Omega$, i.e., x^* is a (binary) input/output choice-vector for a given $w \geq 0$. Then, x^* is also an optimal solution of the model in (6.32) and the following condition must hold at $x^* \in \Omega$:*

$$w \, \nabla_x \Pi(x^*) \leq \omega(x^*), \tag{6.54}$$

where $\omega_i(x^*)$ *is defined, for every* $i = 1, \ldots, 2I$, *by*

$$\omega_i(x^*) := \begin{cases} s_i^u(x^*) & \text{if } x_i^* = 1 \\ \delta_i(x^*) & \text{if } x_i^* = 0. \end{cases} \tag{6.55}$$

Proof. When $x_i^* = 1$, referring to the conditions in (6.42), $\lambda_i = 0$ holds, and thus, $w\partial\Pi(x)/\partial x_i \in S_i(x^*)$, and thus,

$$s_i^l(x^*) \leq w\frac{\partial\Pi}{\partial x_i}|_{x^*} \leq s_i^u(x^*).$$

Since $a_{ijt} \leq 0$, $s_i^l(x^*) \leq 0$ holds. Moreover, $\Pi(.)$ is a nondecreasing convex function implying that $\partial\Pi(x)/\partial x_i \geq 0$. Thus, $w \geq 0$ yields $w\partial\Pi(x)/\partial x_i \leq \omega_i(x^*)$ for $x_i^* = 1$. When $x_i^* = 0$, we have $0 \leq \lambda_i = \partial_i\bar{\gamma}_q(x) - w\nabla_i\Pi(x)$, which yields

$$w\frac{\partial\Pi}{\partial x_i}|_{x^*} \leq \delta_i(x^*).$$

Noting the definition of $\omega(x^*)$, the proof is completed. ∎

Corollary 6.5.5 *For a given* $w = \hat{w} \geq 0$, *suppose* $x^* \in \Omega$ *is a KKT point of (6.38). Then,* x^* *is also a KKT point of (6.38) for any* $w \in [0, \hat{w}]$.

6.5.4 Conditions for the specific case of EI

For the specific case of Expert Information considered in Section 6.2.1, recall the penalty function $\Pi(x)$ in (6.10). The following simplified results hold:

1. For some $w \in \Re$, if an optimal solution $x(w) \in \Omega$ satisfies $\Pi(x(w)) = 0$, then the condition in (6.5.4) reduces to

$$\delta_i(x(w)) \geq 0 \quad \text{if } x_i(w) = 0.$$

2. If $w = 0$, then the condition in (6.5.4) reduces to

$$\delta_i(x(0)) \geq 0 \quad \text{if } x_i(0) = 0.$$

3. If $w > 0$ and x_i is one of the variables included in the definition of the function $\Pi(x)$ in (6.10), then the condition in (6.5.4) reduces to

$$\begin{cases} w \leq 0.5 s_i^u(x(w)) & \text{if } x_i(w) = 1 \\ \delta_i(x(w)) \geq 0 & \text{if } x_i(w) = 0. \end{cases}$$

Chapter 7

Case Study of the U.S. Market Sectors

The Expert Information (EI)-based GDEA approach is applied to the U.S. stock markets involving more than 800 publicly-traded firms. These firms span 9 major market sectors and they include all stocks of the Standard & Poors 500 index. These 9 sectors are Technology ($h = 1$), Health Care ($h = 2$), Financial ($h = 3$), Energy ($h = 4$), Utilities ($h = 5$), Consumer Discretionary ($h = 6$), Consumer Staples ($h = 7$), Basic Materials ($h = 8$), and Industrial Goods ($h = 9$). The industries that are included in each sector are obtained from http://biz.yahoo.com/p/ and they are listed in Table A.1 - Table A.9, see Appendix. The objectives of this case study include, first, determining which market sectors support the expert knowledge presented in Section 6.2.1 for input/output selection; second, determining the value of that EI for determining optimal Relative Financial Strength (RFS) indicators for each sector; and finally, to use those RFS indicators in stock screening for portfolio optimization and comparing performances of such portfolios with the market (S&P 500 index) itself. In the sequel, the input/output solutions of the GDEA-based optimization model obtained via the heuristic methodology are checked for local optimality using the first-order conditions developed in Section 6.5.3.

Quarterly financial statements of firms during the period 1997 to 2004 are used in the case study. Of the 32 consecutive quarters, the first quarter is set aside for the initial calculations of RoR, growth rates etc. Quarterly data for all firms are electronically obtained from the WRDS (Wharton Research Data Services) database. The financial statement data, as well as quarterly stock price information, are checked for completeness and only those firms with complete data are chosen within each sector. Thus, the usable number of firms (J_h) in each sector are $J_1 = 159$, $J_2 = 107$, $J_3 = 86$, $J_4 = 49$, $J_5 = 128$, $J_6 = 110$, $J_7 = 68$, $J_8 = 58$, and $J_9 = 62$. This leads to a unique set of $\sum_{h=1}^{9} J_h = 827$ firms.

In the case study, both synchronous and lagged predictions of stock returns are examined. In the lagged case, GDEA-based RFS value of a firm in a given quarter is expected to influence the return on its stock price within a one month offset after the beginning of the quarter. Such a case is of paramount interest due to a possible delay of one month in releasing quarterly financial results to the public. Portfolio selection using a "lagged" measure of RFS score can lead to quite different investments, relative to that using a "contemporaneous" or synchronous measure of the RFS score. These cases are compared and discussed towards the end of this chapter.

7.1 VEI under Synchronous Case

The case of synchronous RFS measures is addressed, and the subscript S is used to identify the correlations computed in this case. First, the unrestricted model in (6.4) was solved to determine an optimal input/output categorization of the 18 financial parameters in Section 3.1.1 using the two-step heuristic solution method in Section 4.2.1, using a sample size of 20 runs. Referring to Section 6.4.1, see Theorem 6.4.1, if the computed $\bar{\gamma}_S^0$ is less than that computed under some $w > 0$, i.e., $\bar{\gamma}_S^0 < \bar{\gamma}_S(x(w))$, then additional solution sample runs are conducted to better-estimate $\bar{\gamma}_S^0$. Maximized sector correlation metrics (SCM) $\bar{\gamma}_S^0$, along with their associated EI-penalty $\pi_S^0 = \Pi(x_S^0)$, are reported in Table 7.1. Next, the model in (6.5) under the full use of EI is executed to determine SCM values $\bar{\gamma}_S^*$, which thus have $\pi_S^* = \Pi(x_S^*) = 0$, and reported in the same table. Then, running the GDEA model under EI violations, under various choices of the tolerance parameter w, see (6.32), the maximized SCM values $\bar{\gamma}_S^\pi$ for EI-penalty levels $\pi = 1, 2, 3$ are also given in Table 7.1.

The distances between the input/output parameters with $\pi = 0$ (under fully complying with EI) and those with $\pi = 1, 2,$ and 3 are also listed in Table 7.1. The distance is evaluated by norm-1 distance, that is,

$$d_S^\pi = \|x_S^\pi - x_S^*\|_1 = \sum_{i=1}^{36} |(x_S^\pi)_i - (x_S^*)_i|. \tag{7.1}$$

It can be observed, generally, that d_S^π is small when π is small, i.e., for small violations of EI, and the optimal input/output categorization has many common parameters with that resulting under the full use of EI. As the violations become larger (e.g.

Table 7.1: Maximum synchronous correlations for each sector under different levels of EI violation

Sector name	Unrestricted $\bar{\gamma}_S^0$, π_S^0	$(\pi = 0)$ $\bar{\gamma}_S^*$	$\pi = 1$ $\bar{\gamma}_S^\pi$	d_S^π	$\pi = 2$ $\bar{\gamma}_S^\pi$	d_S^π	$\pi = 3$ $\bar{\gamma}_S^\pi$	d_S^π	MVEI w_S^*
Technology	0.310, 1	0.299	0.310	3	0.196	10	0.171	12	0.011
Health Care	0.200, 1	0.192	0.200	1	0.166	6	0.131	10	0.008
Financial	0.252, 2	0.121	0.247	9	0.252	8	0.247	12	0.126
Energy	0.180, 1	0.153	0.180	2	0.152	9	0.127	10	0.027
Utilities	0.206, 3	0.184	0.205	5	0.136	6	0.120	5	0.016
Cons. Discr.	0.236, 1	0.208	0.236	5	0.235	5	0.231	9	0.028
Cons. Stap.	0.268, 0	0.268	0.219	6	0.220	5	0.198	10	-
Basic Mat.	0.186, 3	0.169	0.181	4	0.180	10	0.186	10	0.006
Ind. Goods	0.246, 3	0.210	0.211	8	0.216	7	0.246	9	0.012

$\pi = 3$), the optimal input/output sets tend to become more different from that under fully complying with the EI.

The optimal correlations $\bar{\gamma}_S^\pi$ over a wide range of π is plotted in Figure 7.1 for all sectors. Note that the maximum achievable correlation metric is computed by model (6.4), herein denoted by $\bar{\gamma}_S^{0,h}$ for industry h, and the corresponding EI-violation and the input/output vector are denoted by $\pi_S^{0,h}$ and $x_S^{0,h}$, respectively. It is evident from Figure 7.1 that in most sectors, such as Technology, Health care, Basic materials, Energy, Industrial goods, Consumer discretionary, and Utilities, the correlation metric changes only marginally when the expert information is violated slightly. For example, in Technology sector, correlation is improved from 0.299 for $\pi = 0$ to 0.310 for $\pi = 1$, indicating that EI violation does not lead to any significant gains on the predictive value of RFS. Note that Technology sector has the highest correlation under all levels of EI violation. However, in Financial sector, significant gains in correlation

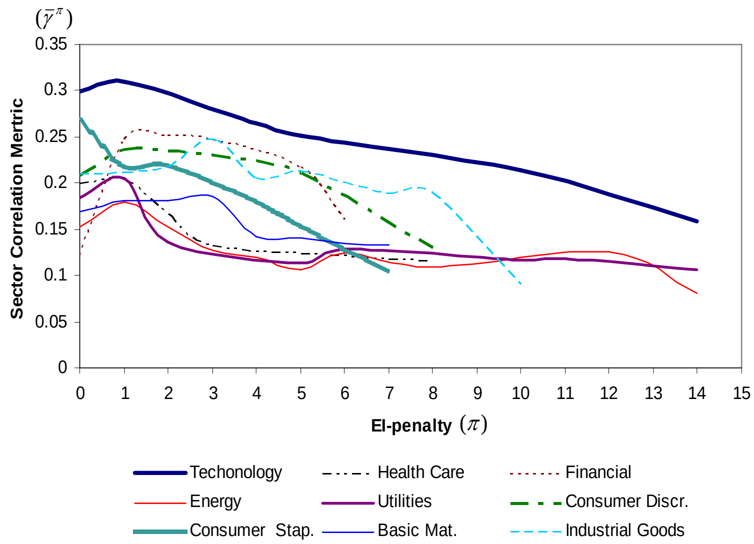

Figure 7.1: Maximum sector-correlations vs. EI-penalty (synchronous)

can be achieved under moderate violation of EI. On the other hand, for instance in Consumer Staples sector, violation of EI leads to significant decay in predictive value of the Relative Financial Strength (RFS). Nevertheless, excessive violation of EI tend to weaken the predictive value of RFS across all sectors. This supports the view that the input/output perspectives of financial ratios from accounting statements, as described under EI, do carry value in representing the underlying performance of a firm.

The maximum marginal value of expert information (MVEI), see w_S^* in (6.33), is also computed for each sector h using the Procedure-MVEI, see Section 6.4.2. These values are in the last column in Table 7.1. w_S^* measures the relative importance of EI. For example, in Financial sector, correlation can be improved by approximately 13% by not complying with the expert in at least one input/output parameter. MVEI for

Consumer Staples sector is not defined since the trade-off curve of $\bar{\gamma}_S^\pi$, as a function of π, is negatively sloped at $\pi = 0$. In the remaining sectors, MVEI values are relatively insignificant.

The value of EI is verified using the statistical test developed in Section 6.3. The hypothesis test given in (6.7) can be used to test the significance of correlation improvement as EI is violated in each sector. Of the 9 sectors, Consumer staples is excluded from the test because, as Figure 7.1 indicates, correlation strictly decreases as π increases from 0. In the remaining 8 sectors, only correlations (associated with $\pi = 1, 2, \text{and } 3$) that are larger than $\bar{\gamma}_S^*$ are tested for VEI using (6.31). The threshold of VEI is determined for each sector by requiring a minimum of 10% relative improvement over the correlation $\bar{\gamma}_S^{*,h}$ for sector h, i.e., $(\bar{\gamma}_S^{\pi,h} - \bar{\gamma}_S^{*,h})/\bar{\gamma}_S^{*,h} \geq 10\%$. Thus, threshold in (6.7) for industry h is $\nu_{0,S}^h = 0.10\bar{\gamma}_S^{*,h}$. The test results of (6.7) are shown in Table 7.2.

If VEI is statistically significant with a certain π value, we can conclude that the expert information can be violated by the corresponding amount in order to improve the correlation between RFS and stock returns. On the other hand, if the improvement of the correlation is not statistically significant, it indicates that expert information should not be violated when inputs and outputs are selected to compute the GDEA-based RFS. Results in Table 7.2 show that with the exception of the Financial sector, none of the sectors provide statistical evidence for violating the expert information in input/output selection. Indeed, Consumer staples sector provides the strongest evidence for supporting expert information in input/output selection for computing the RFS in that sector. Thus, we can conclude that as

Table 7.2: VEI significance results (synchronous)

Sector name	$\pi = 1$		$\pi = 2$		$\pi = 3$		Critical value	$\nu_{0,S}$
	Statistic	Signi.	Statistic	Signi.	Statistic	Signi.		
Technology	0.015	No	-	-	-	-	0.070	0.030
Health Care	0.008	No	-	-	-	-	0.065	0.019
Financial	0.132	Yes	0.137	Yes	0.10	Yes	0.072	0.012
Energy	0.028	No	-	-	-	-	0.081	0.015
Utilities	0.022	No	-	-	0.023	No	0.077	0.018
Cons. Discr.	0.029	No	-	-	-	-	0.067	0.021
Cons. Stap.	-	-	-	-	-	-	-	-
Basic Mat.	0.015	No	-	-	0.019	No	0.068	0.017
Ind. Goods	0.003	No	0.010	No	0.044	No	0.064	0.021

the degree of EI violation is increased, the relative gain in predictive value of RFS is statistically insignificant in all sectors except for Financial sector. Accordingly, optimal input/output vectors are computed using no violation of EI for all sectors, except for Financial sector where it is optimal to set $\pi = 1$ (since $\pi = 2$ does not yield significantly better correlation). The corresponding optimal input/output vectors, denoted by x_S^h, $h = 1, \ldots, 9$, are reported in Table 7.3. Note that in Table 7.3, "EI" indicates the inputs/outputs under full EI condition, while "VEI" indicates the inputs/outputs that utilize the value of expert information.

7.2 VEI under Lagged Case

The lagged correlation measures the influence of financial strength observed at the end of quarter t on the stock return in a 3-month period starting one month from the beginning of the quarter t, see Section 3.6.1 for details. Lagged correlation is computed for the investment purpose since the quarterly financial statements are

Table 7.3: Optimal input/output vector x_S^h in each industry (synchronous)

Financial	Tech.		Health Care		Financials		Energy		Utilities		Cons. Disc.		Cons. Stap.		Basic Mat.		Indus. Goods	
parameter	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI
1	-	-	out	out	-	in	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	out	out	-	-	-	-	-	-	-	-	out	out	-	-
3	-	-	-	-	-	out	-	-	out	out	-	-	-	-	-	-	-	-
4	-	-	-	-	-	in	-	-	in	in	-	-	-	-	-	-	-	-
5	-	-	-	-	in	in	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	in	-	in	in	-	-	-	-	in	in	-	-	in	in
7	-	-	in	in	in	-	in	in	-	-	in	in	-	-	in	in	-	-
8	in	in	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	in	in	in	-	-	-	-	-	-	-	in	in	-	-	-	-
10	in	in	-	-	in	-	-	-	-	-	-	-	-	-	in	in	-	-
11	-	-	-	-	-	-	-	-	-	-	in	in	-	-	in	in	in	in
12	in	in	-	-	-	-	-	-	in	in	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	out	out	-	-	-	-	-	-
14	out	out	out	out	-	out	out	out	out	out	out	out	out	out	out	out	out	out
15	out	out	-	-	-	-	out	out	-	-	-	-	-	-	-	-	-	-
16	-	-	-	-	out	-	-	-	-	-	-	-	out	out	-	-	out	out
17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 7.4: Maximum one-month lagged correlations for each sector under different levels of EI violation

Sector name	Unrestricted $\bar{\gamma}_L^0$, π_L^0	($\pi = 0$) $\bar{\gamma}_L^*$	$\pi = 1$ $\bar{\gamma}_L^\pi$	d_L^π	$\pi = 2$ $\bar{\gamma}_L^\pi$	d_L^π	$\pi = 3$ $\bar{\gamma}_L^\pi$	d_L^π	MVEI w_L^*
Technology	0.224, 1	0.209	0.224	2	0.199	7	0.195	8	0.001
Health Care	0.152, 4	0.141	0.148	9	0.143	8	0.143	10	0.003
Financial	0.214, 4	0.125	0.194	5	0.162	6	0.180	9	0.069
Energy	0.238, 1	0.163	0.238	8	0.192	7	0.157	5	0.026
Utilities	0.183, 3	0.149	0.170	7	0.182	2	0.183	3	0.011
Cons. Disc.	0.178, 5	0.161	0.176	6	0.162	8	0.145	10	0.004
Cons. Stap.	0.192, 2	0.130	0.192	3	0.193	6	0.072	9	0.061
Basic Mat.	0.216, 2	0.143	0.190	6	0.216	6	0.110	5	0.047
Ind. Goods	0.189, 3	0.172	0.147	5	0.186	5	0.189	6	0.004

often released to the public one month after the quarter ends. Therefore, one-month lagged correlation is utilized in this section to examine the value of EI.

The testing in Section 7.1 is repeated for the one-month lagged case. The maximized SCM $\bar{\gamma}_L^0$ computed using model in (6.4), $\bar{\gamma}_L^*$ computed using model in (6.5), and $\bar{\gamma}_L^\pi$ obtained using model in (6.32) with $\pi = 1, 2, 3$ for all sectors are reported in Table 7.4. The subscript L corresponds to lagged correlation.

It is clear that the lagged correlations are lower than the synchronous correlations, in general. For example, only Energy and Basic Materials have higher unrestricted correlations in the lagged case. In addition, it is observed that lagged case allows more violation on EI in order to obtain the maximum SCM. For instance, penalties π^0 corresponding to the unrestricted SCM $\bar{\gamma}_L^0$ increase for all sectors except for Basic Materials. In Health Care, Financial, Consumer Discretionary, and Consumer Staples, the penalties increase by at least 2. Especially in Consumer Staples sector,

$\pi^0 = 0$ corresponds to the highest SCM in the synchronous case, however, $\pi^0 = 2$ relates to the highest SCM in the lagged case. As far as the amount of improvement on SCM is concerned when violation on EI is allowed, it is clear that in sectors such as Technology, Health Care, Consumer Discretionary and Industrial Goods, the correlation metric increases only slightly when the EI is violated. On the other hand, the improvement in correlation is relatively large with EI violation in Financial, Energy, Utilities, Consumer Staples, and Basic Materials. However, whether these improvements are significant have to be examined using the statistical test developed in Section 6.3.

The distances between the input/output parameters with $\pi = 0$ and those with $\pi = 1, 2, 3$ are computed (using the norm-1 distance) and reported in Table 7.4, as well as the maximum marginal value of expert information (MVEI) w_L^*. Unlike the synchronous case, where the distance d_S^π increases when π increases from 1 to 3, there is no such clear trend on distance d_L^π in the lagged case. For example, only Technology, Financial, Consumer Discretionary, and Consumer Staples sectors have that tendency as in the synchronous case. In other sectors, the distances either remain in a close range or decrease when π increases. The value of MVEI, w_L^*, is listed in the last column of Table 7.4 for each sector. It is evident that for those sectors that show slight improvement on correlation metric when EI is violated, w_L^* is also small. This happens in Technology, Health Care, Consumer Discretionary, and Industrial Goods. For example, the correlation is increased by 0.015 in Technology sector when EI is violated, and its corresponding w_L^* is 0.005. On the other hand, in the sectors with relatively large gains on correlation when EI is violated, w_L^* is also relatively

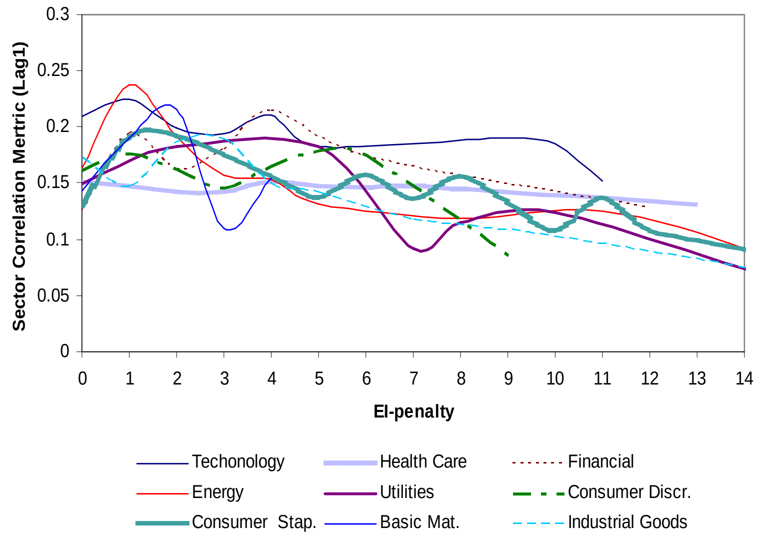

Figure 7.2: Maximum sector-correlations vs. EI-penalty (one-month lagged)

large, see Financial, Energy, Utilities, Consumer Staples, and Basic Materials, for instance. It should be noted that Financial sector has the highest w_L^* among all U.S. sectors, which also happens in the synchronous case although $w_L^* < w_S^*$ in Financial sector. The optimal correlations $\bar{\gamma}_L^{\pi}$ over a wide range of π is plotted in Figure 7.2. It is evident from Figure 7.2 that similar to the synchronous case, under sufficiently large violation of the expert information, correlations diminish in value across all sectors.

The significance of correlation improvement as EI is violated in each sector is tested using the statistical test in Section 6.3. Only correlations (associated with $\pi = 1, 2, 3$) that are larger than $\bar{\gamma}_L^*$ are tested for VEI. The threshold of VEI is computed in the same way as it is determined in the synchronous case. That is, a minimum of 10% relative improvement over the correlation $\bar{\gamma}_L^*$ is required for each

Table 7.5: VEI significance results (one-month lagged)

Sector name	$\pi = 1$		$\pi = 2$		$\pi = 3$		Critical value	$\nu_{0,L}$
	Statistic	Signif.	Statistic	Signif.	Statistic	Signif.		
Technology	0.017	No	-	-	-	-	0.060	0.021
Health Care	0.007	No	0.002	No	0.001	No	0.059	0.014
Financial	0.073	Yes	0.037	No	0.057	No	0.073	0.012
Energy	0.080	No	-	-	-	-	0.082	0.016
Utilities	0.025	No	0.040	No	0.040	No	0.074	0.015
Cons. Disc.	0.014	No	< 0.001	No	-	-	0.061	0.016
Cons. Stap.	0.063	No	0.063	No	-	-	0.069	0.013
Basic Mat.	0.048	No	0.077	Yes	-	-	0.065	0.014
Ind. Goods	-	-	0.018	No	0.021	No	0.060	0.017

sector, i.e., $(\bar{\gamma}_L^{\pi,h} - \bar{\gamma}_L^{*,h})/\bar{\gamma}_L^{*,h} \geq 10\%$. Thus, threshold in (6.7) for industry h is $\nu_{0,L}^h = 0.10\bar{\gamma}_L^{*,h}$. The test results of hypothesis test given in (6.7) are shown in Table 7.5.

Table 7.5 shows that significant improvements occur in Financial and Basic materials sectors when EI is violated, i.e., π=1 in Financial sector and $\pi = 2$ in Basic Materials sector, which indicates that EI-violation should preferably be allowed in input/output selection. In the remaining sectors, none of them provide statistical evidence for violating the expert information in input/output selection. The corresponding input/output vector, denoted by x_L^h, $h = 1, \ldots, 9$, are reported in Table 7.6. Note that "EI" indicates the optimal input/output categorization that does not violate the expert information, while "VEI" indicates the optimal input/output categorization that is determined by the value of expert information.

Table 7.6: Optimal input/output vector x_L^h in each industry (lagged)

Financial	Tech.		Health Care		Financials		Energy		Utilities		Cons. Disc.		Cons. Stap.		Basic Mat.		Indus. Goods	
parameter	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI	EI	VEI
1	out	out	out	out	-	in	out	out	-	-	-	-	out	out	out	out	-	-
2	out	out	-	-	out	-	-	-	-	-	-	-	out	out	out	out	-	-
3	-	-	-	-	out	out	-	-	out	out	-	-	-	-	-	-	-	-
4	in	in	in	in	in	in	-	-	in	in	-	-	-	-	in	-	-	-
5	-	-	in	in	-	in	in	in	-	-	in	in	-	-	-	-	-	-
6	-	-	-	-	in	-	in	in	-	-	-	-	-	-	in	in	in	in
7	-	-	-	-	-	-	-	-	-	-	in	in	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	in	in	-	-	-	-	-	-	in	in	in	in	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	in	in	in	in	in	in	-	out	in	in
12	in	in	in	in	in	-	-	-	in	in	-	-	in	in	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	out	-	out	out
14	out	out	out	out	out	out	out	out	out	out	out	out	out	out	out	out	out	out
15	out	out	-	-	-	-	-	-	-	-	-	-	-	-	out	out	-	-
16	-	-	out	out	-	-	-	-	-	-	out	out	out	out	out	out	out	out
17	-	-	-	-	-	-	out	out	-	-	-	-	-	-	-	out	-	-
18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	in	-	-

Overall, in the lagged case, the maximum correlation allows more violation on the expert information, in comparison to the synchronous case. Hence, for contemporaneous prediction of stock returns, EI appears more important. Does this lead to making more successful investments in portfolio selection using expert information? To answer this question, performance of the synchronous and lagged cases will be compared within portfolio optimization using the 9 sectors in the U.S. stock market.

7.3 Stock Selection Criterion

The purpose of applying EI-based input/output selection is to determine whether the resulting RFS measure enables stock selection that would improve risk/return performance of optimized portfolios. This process involves two steps: first, determining if RFS has predictive power in a given market sector; second, selecting individual stocks based on predicted high RFS values to be included in stock portfolios from the chosen sectors.

Sector selection is based on the significance of the industry correlation metric (ICM) as discussed in Section 4.3. Sectors that are chosen for RFS-based investment are indicated in Table 7.7 and 7.8, respectively, for synchronous and lagged cases.

For the synchronous case, only the Energy sector escapes any investment under the RFS measure. However, for the lagged case, Health Care, Utilities, and Consumer Staples are not selected for portfolio investment due to low correlations. For the chosen sectors in both cases, individual stock selection is performed as given below.

Table 7.7: Sector selection using correlation metric (synchronous)

Sector	Correlation	Test Statistic	Critical value	Selection
Technology	0.299	0.322	0.145	Yes
Health Care	0.192	0.202	0.150	Yes
Financial	0.247	0.260	0.161	Yes
Energy	0.153	0.159	0.165	No
Utilities	0.184	0.191	0.160	Yes
Consumer Discre.	0.208	0.216	0.150	Yes
Consumer Staple	0.268	0.286	0.158	Yes
Basic Mat.	0.169	0.175	0.154	Yes
Industrial Goods	0.210	0.216	0.148	Yes

Table 7.8: Sector selection using correlation metric (one-month lagged)

Sector	Correlation	Test Statistic	Critical value	Selection
Technology	0.209	0.219	0.145	Yes
Health Care	0.141	0.147	0.150	No
Financial	0.194	0.204	0.161	Yes
Energy	0.163	0.171	0.165	yes
Utilities	0.149	0.156	0.160	No
Consumer Discre.	0.161	0.168	0.150	Yes
Consumer Staple	0.130	0.137	0.158	No
Basic Mat.	0.216	0.227	0.154	Yes
Industrial Goods	0.172	0.179	0.148	Yes

For each chosen sector h, using the optimal input/output vector x^h, the RPS value $\hat{\eta}_{jt}(x^h_S)$ (for synchronous case) is computed, see Section 6.1, for each stock $j = 1, \ldots, J_h$ and for the last τ $(< t_0)$ quarters, $t = t_0 - \tau + 1, \ldots, t_0$. τ represents the historical period of RPS values to predict an RFS for stock j for the future quarter $t_0 + 1$. This forecast is denoted by $RFS_j(\tau)$. A stock is chosen for investment in quarter $t_0 + 1$ only if this predicted RFS value is no less than prespecified threshold R^*. That is, the set of stocks $\mathcal{J}_h$ selected for possible portfolio analysis from sector h is given by the *Stock Selection Criterion* (SSC):

$$\textsf{(SSC)}: \quad \mathcal{J}_h := \{j \ : \ RFS_j(\tau) \geq R^*, \ j = 1, \ldots, J_h\}, \tag{7.2}$$

for all sectors h, except Energy sector $(h = 4)$. We set $\tau = 4$ (i.e., from 04Q1 to 04Q4) and use a moving average forecasting method for computing $RFS_j(\tau)$. The predicted RFS values are plotted in Figure 7.3 for all stocks in the chosen 8 sectors. Setting $R^* = -0.45$ in (7.2), stocks are chosen to be included in the portfolio, which results in 13 stocks from Technology, 13 stocks from Health Care, 15 stocks from Financial, 13 stocks from Utilities, 5 stocks from Consumer Discretionary, 4 stocks from Consumer Staples, 18 stocks from Basic Materials, and 8 stocks from Industrial Goods sectors. Therefore, a total of 89 stocks are selected under the synchronous case. Such a universe of stocks under RFS is denoted by the generic set $\mathcal{N}$. This stock selection involves using the inputs/outputs that incorporate the value of expert information, which allows a certain degree of violation, see Table 7.3, hence, it is referred to as the case *under violated EI condition.*

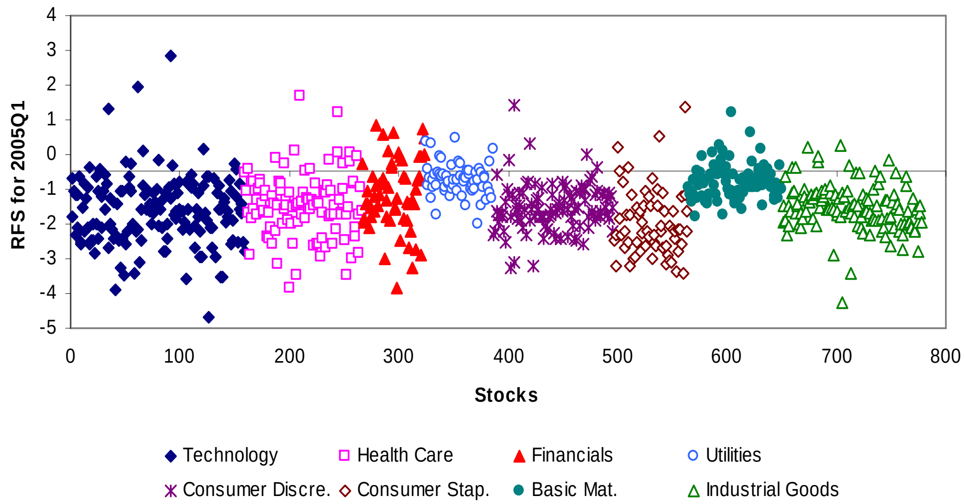

Figure 7.3: Predicted 2005Q1 RFS-values for stocks (under violated EI, synchronous)

In order to examine whether the value of expert information is useful in input/output selection, stock selection using inputs/outputs that fully comply with the expert information is also performed for the above 8 chosen sectors. The corresponding RFS values are plotted in Figure 7.4. By applying the same stock selection rule in (7.2), a total of 88 stocks are selected for investment. This stock selection is thus referred to as the case *under full EI condition*. Note that only the Financial sector has a different selection of stocks for investment, relative to the case under violated EI.

The same stock selection criterion is applied to the chosen sectors for the lagged case. First, RFS are computed under violated EI condition for the 6 chosen sectors. Note that Health Care ($h = 2$), Utilities ($h = 5$), and Consumer Staples ($h = 7$) are excluded from any investment. Using stock selection rule in (7.2), a total of

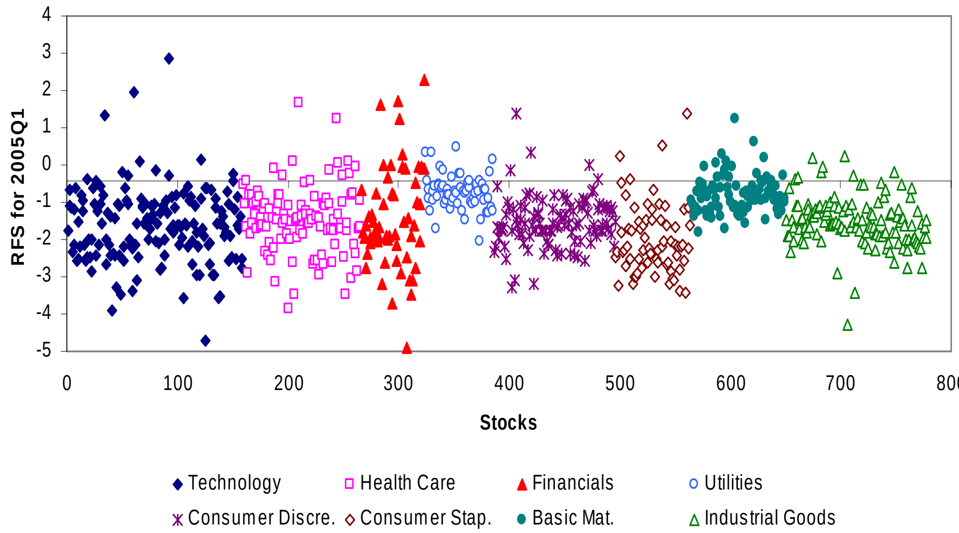

Figure 7.4: Predicted 2005Q1 RFS-values for stocks (under full EI, synchronous)

90 stocks are selected with 4 stocks from Technology, 11 stocks from Financial, 14 stocks from Energy, 22 stocks from Consumer Discretionary, 25 stocks from Basic Materials, and 14 from Industrial Goods. The RFS values from these 6 chosen sectors are plotted in Figure 7.5. Next, RFS scores are computed under the full EI condition for the 6 chosen sectors and they are plotted in Figure 7.6. This results in a total of 88 stocks selected for investment, with 4 stocks from Technology, 17 stocks from Financial, 14 stocks from Energy, 22 stocks from Consumer Discretionary, 17 stocks from Basic Materials, and 14 from Industrial Goods. Note that only Financial and Basic Materials sectors have different stocks for investment.

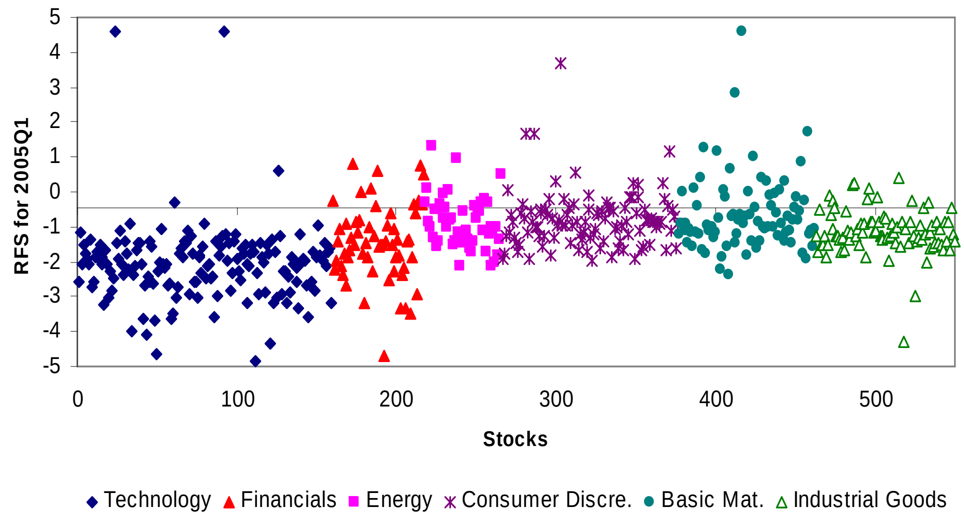

Figure 7.5: Predicted 2005Q1 RFS-values for stocks (under violated EI, lagged)

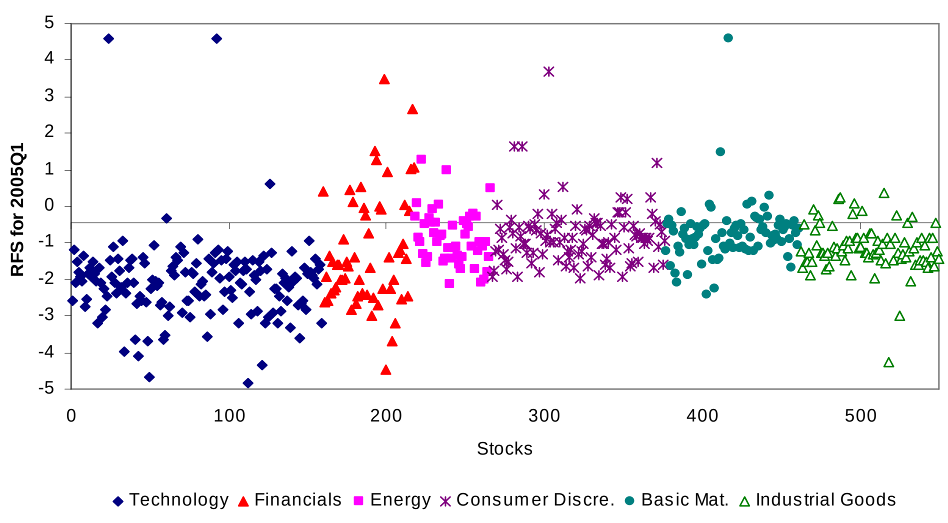

Figure 7.6: Predicted 2005Q1 RFS-values for stocks (under EI, lagged)

7.4 Comments on Theoretical Properties

The model in (6.32) can be used to obtain the value of expert information for the following cases: (i) expert information is completely disregarded; (ii) expert information is fully complied; and (iii) expert information is partially used (or partially violated). When $w = 0$, the model in (6.32) is equivalent to the model in (6.4), which pays no attention to the expert information. When $w \geq w^*$, where w^* is the maximum marginal value of EI, the model in (6.32) is equivalent to the model in (6.5), in which expert information must be fully satisfied. If $w < w^*$, expert information is allowed to be violated with a certain degree. The w^* for each sector are reported in Table 7.1 and 7.4, respectively, for the synchronous and lagged cases. However, obtaining globally optimal solutions to (6.32) is computationally tedious, and certainly, the heuristic employed for solution is not guaranteed to provide such a solution. Several properties are developed for $f(w)$ in Section 6.4.1, where $f(w)$ is shown to be convex and monotonically nonincreasing in w, and $f(w) \geq \bar{\gamma}^*$, for all $w \in \Re$. These properties can guide us to determine if the sample size need to be increased when the two-step heuristic solution method is applied to solve the model. Using various w values, the trade-off between $f(w)$ and w for each sector for synchronous case is plotted in Figure 7.7, and those for lagged case are plotted in 7.8 and 7.9. These figures show that the objective values obtained from model (6.32) follow the afore-mentioned theoretical properties of $f(w)$.

However, it must be noted that the heuristic solution scheme in 4.2.1 is at best a direct search technique without any guarantee of satisfying the first order conditions of optimality, let alone global optimal solutions. It is neither asserted nor proven that

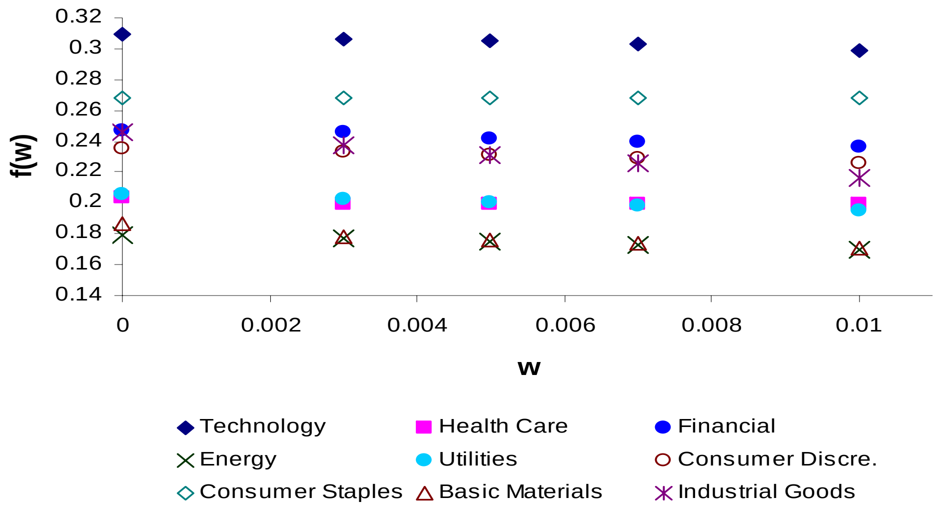

Figure 7.7: trade-off curve under expert information (synchronous)

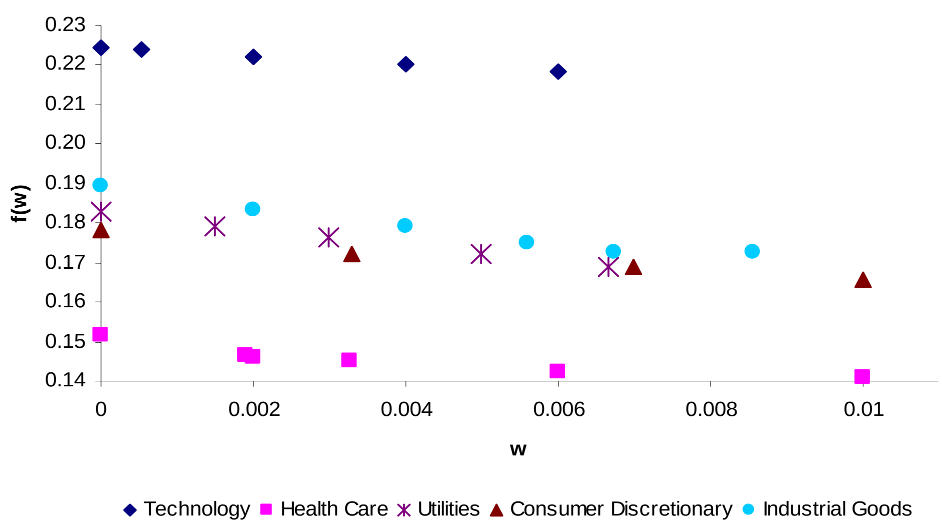

Figure 7.8: trade-off curve under expert information (lagged) (a)

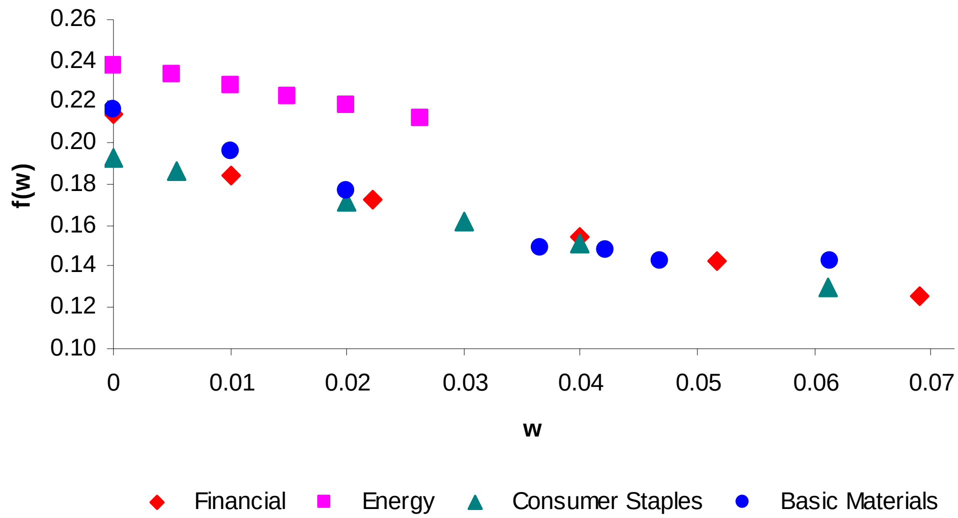

Figure 7.9: trade-off curve under expert information (lagged) (b)

this solution method provides locally maximum correlations. Towards verifying the local optimality, the first order conditions in Section 6.5.3 are numerically checked. For this, we define the term *optimality satisfaction degree* as follows.

Definition 7.4.1 *(***Optimality Satisfaction Degree: OSD***) Given a binary input/output vector $x \in \Re^{36}$, let $\sigma_i = 1$ if (6.54) is satisfied for coordinate i, $i = 1, \ldots, 36$; $\sigma_i = 0$ otherwise. Then, $\phi = \sum_{i=1}^{36} \sigma_i/36$ is defined as the measure of the optimality satisfaction degree of solution x.*

The OSD for the optimal inputs/outputs obtained under violated EI condition and under full EI condition for both synchronous and lagged cases are reported in Table 7.9. It is evident that none of the inputs/outputs obtained using the heuristic solution method in Section 4.2.1 completely satisfy the first order optimality conditions. That

Table 7.9: Optimality Satisfaction Degree for each sector

Sector	Synchronous		Lagged	
name	Under violated EI	Under EI	Under violated EI	Under EI
Technology	38.89%	38.89%	58.33%	58.33%
Health Care	30.56%	30.56%	77.78%	77.78%
Financial	72.22%	52.78%	66.67%	50.00%
Energy	50.00%	50.00%	72.22%	72.22%
Utilities	50.00%	50.00%	47.22%	47.22%
Consu. Discre.	88.89%	88.89%	47.22%	47.22%
Consu. Stap.	75.00%	75.00%	47.22%	47.22%
Basic Mat.	63.89%	63.89%	41.67%	83.33%
Indus. Goods	58.33%	58.33%	38.89%	38.89%

is, the solutions obtained are not Karush-Kuhn-Tucker (KKT) points, hence, they are not locally optimal.

7.5 Portfolio Optimization

The foregoing RFS-based stock selections are applied over an investment horizon of 3 months in 2005. As mentioned earlier, synchronous case is often not practically implementable in investment because quarterly financial information usually will not be released to the public until one month after the quarter ends. Therefore, the lagged case is the practically viable means of implementing RFS for investment purposes. Consider the following three portfolio strategies:

Strategy 1: Use the optimal inputs/outputs that correspond to the synchronous correlation to compute RFS and rank firms by the end of December 2004.

Then, form portfolios at the beginning of January 2005. Thus, the investment horizon is from January 2005 to March 2005.

Strategy 2: Use the optimal inputs/outputs that correspond to the synchronous correlation to compute RFS and rank firms by the end of December 2004. Then, form portfolios at the beginning of February 2005. Thus, the investment horizon is from February 2005 to April 2005.

Strategy 3: Use the optimal inputs/outputs that corresponds to the one-month lagged correlation to compute RFS and rank firms by the end of January 2005. Then, form portfolios at the beginning of February 2005. Thus, the investment horizon is from February 2005 to April 2005.

Observe that Strategies 2 and 3 represent the fact that public knowledge of quarterly financial information is delayed by one month. Also, note that Strategies 1 and 2 assume that a firm's underlying financial strength influences its stock return in the same quarter in an efficient market; however, Strategy 2 implements with a month's delay. In contrast, Strategy 3 assumes that a firm's underlying financial strength influences its stock return with a one-month time delay, being efficient up to the availability of financial information. The above strategies are applied under both violated EI condition and under full EI condition, and performances are compared.

A *monthly-rebalancing* strategy is applied where portfolio allocations are optimally adjusted at the beginning of each of the 3 months in the investment horizon. There are several models in the literature for determining portfolio allocations based upon various risk expressions, see Edirisinghe [25] for instance. For the illustration here, the *tracking risk control* model developed in the latter reference, see [25, Section 4.1.1], is employed to determine the portfolio allocations. This form of risk control has been demonstrated to track a market index in "good" times, and stay "neutral" during other times, and it is shown to outperform the pure mean-variance model of Markowitz [43].

To present the tracking risk expression, let $y \in \Re^{|\mathcal{N}|}$ be a vector of portfolio weights for each stock, and R be the random variable representing the market rate of return. Then, denote the market expected return by $m = E[R]$, its variance by $M = Var(R)$, and for each stock j, define its "beta" by

$$\beta_j := \frac{Cov(r_j, R)}{M}.$$

Note that β_j is a measure of volatility of stock return r_j in comparison to the market return (say, using a market index). Moreover, $\beta_j = 1$ indicates that stock j moves in-sync with the market. $\beta_j < 1$ means that the stock j is less volatile than the market. $\beta_j > 1$ indicates that stock return is more volatile than the market return. Then, the tracking risk control is expressed by $y'Qy$, where the matrix

$$Q = V + (\mu - m\mathbf{1})(\mu - m\mathbf{1})' + M(\beta - \mathbf{1})(\beta - \mathbf{1})', \tag{7.3}$$

and $\mu \in \Re^{|\mathcal{N}|}$ is the vector of expected rate of return for each stock, $\beta \in \Re^{|\mathcal{N}|}$ is the stock beta-vector, and $\mathbf{1} \in \Re^{|\mathcal{N}|}$ is a vector of 1's. The first term, V, of (7.3) is the variance-covariance matrix that accounts for risks due to inherent stock correlations, the second term accounts for risk due to not tracking benchmark mean return, and the third term is for risk when portfolio *beta* is not aligned with the market. In a static one period setting, the portfolio investment model under tracking risk control is as follows:

$$\begin{aligned} \max_{y} \quad & \mu^{'} y - \lambda y^{'} Q y \\ \text{s.t.} \quad & y^{'} \mathbf{1} \leq C^0 \\ & |\beta^{'} y| \leq b \\ & y \geq 0, \end{aligned} \tag{7.4}$$

where the initial budget is set at $C^0 = \$1m$ and the initial stock positions are zero in all stocks for the first month of investments. For the remaining two rebalancing periods, C^0 is automatically adjusted to the cash position carried forward in the portfolio. λ is a user-specified risk tolerance parameter where larger λ implies an increased risk-aversion. Also, b is a user-specified control on portfolio beta, $\beta' y$, to keep it within prespecified bounds.

All required statistical parameters are estimated using historical stock price data of the years 2003 and 2004. Under the monthly rebalancing strategy, such estimations are needed at the beginning of each month in the horizon, conditional upon the data available prior to that point in time. This approach results in a dynamically-evolving portfolio, and the resulting portfolios are (out-of-sample) simulated using the (actual) realized price series during the horizon. In the portfolio selection model in (7.4), risk is controlled by both the portfolio variance trade-off parameter λ and the portfolio beta parameter b. Larger the value of λ is, more risk-averse the portfolio allocations would be, while higher the value of b is, more volatile the portfolio return would be

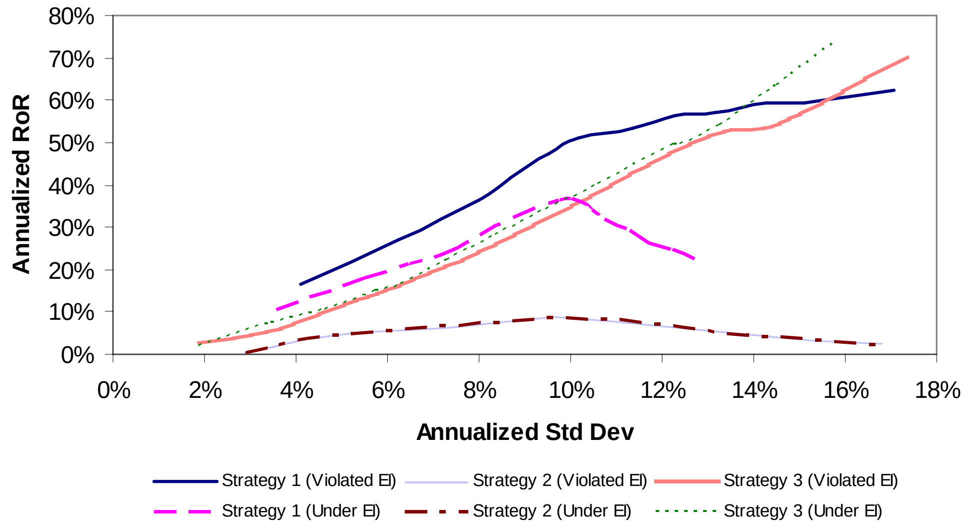

Figure 7.10: RFS-based portfolio efficient frontier

relative to the market. All estimations, portfolio optimizations, and simulations are performed using ©MiSOFT software, see [26].

Solving the model (7.4) by changing the pair of values (λ, b), one determines the efficient frontier of portfolio investments. There are 6 possible efficient frontiers under consideration - Strategy k under EI, or under violated EI, for $k = 1, 2, 3$. These frontiers are depicted in Figure 7.10. Evidently, Strategy 1, under violated EI, has the highest performance, notably when the annualized standard deviation is below 14%. However, for increased portfolio volatility, Strategy 3, under EI, performs better. Strategy 2 remains a fairly-weak proposition for portfolio investment.

Standard & Poor 500 index-tracking stock ticker SPY is used as the market barometer to track the (overall) market performance. The market volatility during the investment horizon is given by the annualized standard deviation of SPY, which is about 10.2% from January, 2005 to March, 2005, and 12.2% from February, 2005 to April, 2005. Thus, for the purposes of relative portfolio performance comparisons, efficient portfolios having standard deviations as same as the market were chosen using the efficient frontiers in Figure 7.10. That is, for the Strategy 1, standard deviation is set at 10.2%, while that for the Strategies 2 and 3 are set at 12.2%. The daily performances of these portfolios are compared for the cases of violating EI and fully satisfying EI in Figures 7.11, 7.12, and 7.13, respectively, for each of the strategies 1, 2, and 3. As can be seen from these figures, the market index is well outperformed by Strategy 1 and 3, while Startegy 2's performance is only modest relative to the market. The Sharpe ratios for the 3 strategies under both violated EI and full EI conditions are shown in Figure 7.14. Observe that strategy 1 under violated EI condition has the strongest performance while its performance under full EI condition cannot perform as strong as strategy 3 under both violated EI and full EI conditions. As is expected from the daily performance curves, Strategy 2 has the weakest Sharpe ratio, compared to the other two strategies.

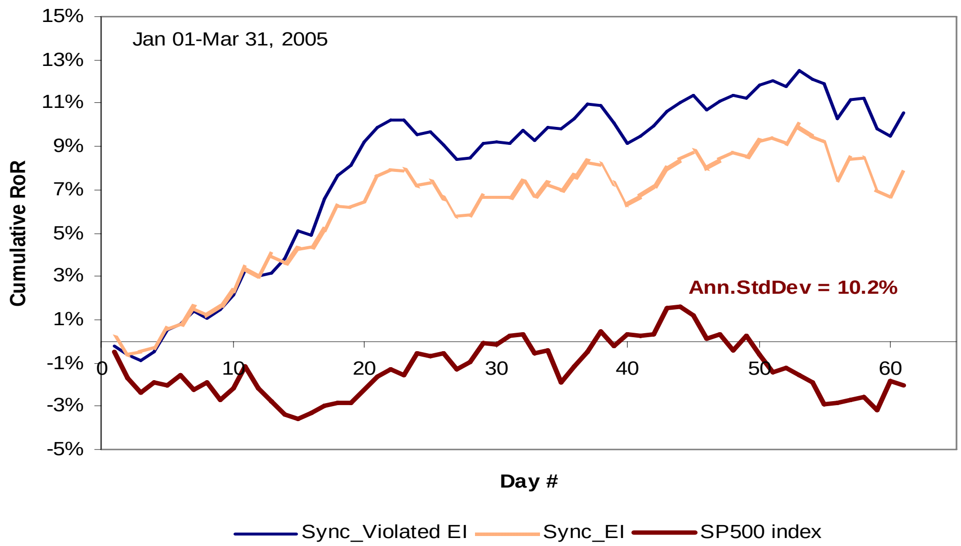

Figure 7.11: RFS-based stock portfolio and S&P 500 index performances (Strategy 1)

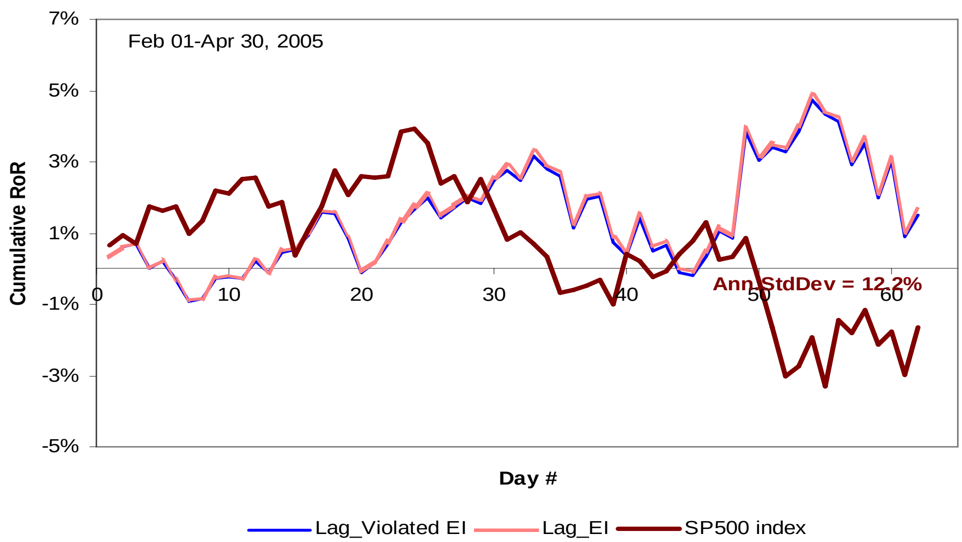

Figure 7.12: RFS-based stock portfolio and S&P 500 index performances (Strategy 2)

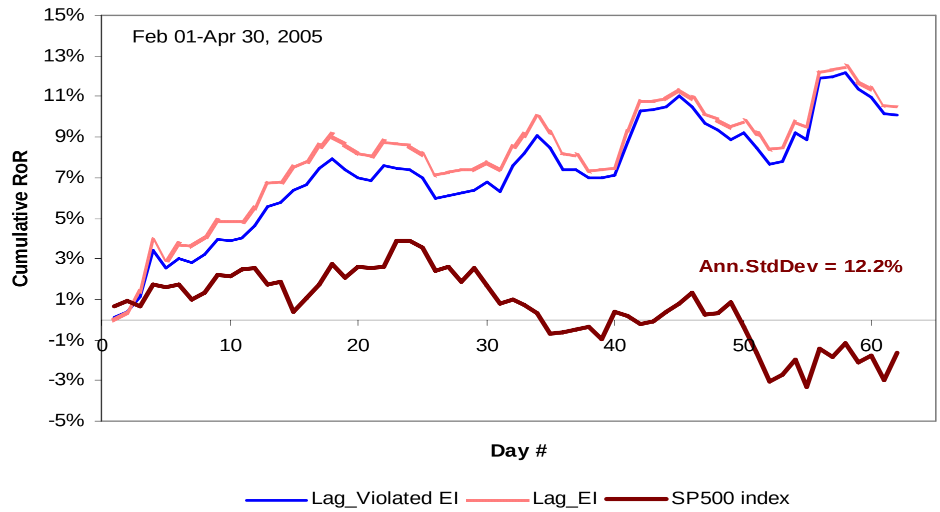

Figure 7.13: RFS-based stock portfolio and S&P 500 index performances (Strategy 3)

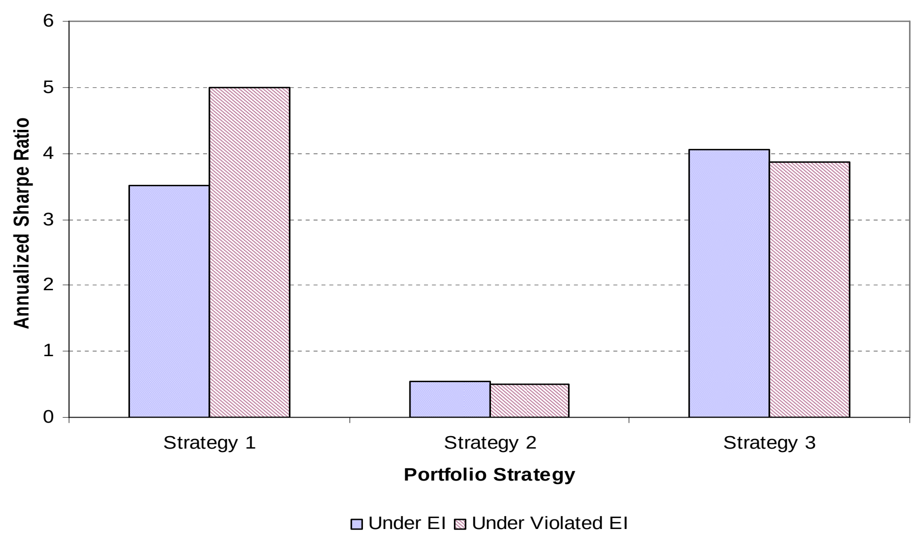

Figure 7.14: Portfolio performance under three strategies

Chapter 8

Concluding Remarks

This thesis makes several contributions, both in the development of DEA-based methodology, as well as in the application in financial investments. Most notably, the thesis provides a generalization framework for Data Envelopment Analysis. In the standard DEA, a set of input and output parameters of performance measurement must be identified to rank overall performance of one firm relative to other firms. The Generalized DEA (GDEA) method developed in the thesis allows for an iterative process for selecting inputs/outputs such that the resulting strength metric of firms will have high correlation with an obervable reward metric, in this case, stock returns. In this sense, the GDEA model may be used in areas outside financial applications. With the developed statistical testing methodology, such correlations can be verified to be sufficiently significant as the particular application may require. Moreover, theoretical optimality conditions are also developed for the GDEA model solutions.

The application of the GDEA for fundamental analysis provides a novel use of mathematical optimization techniques in stock selection for investments, as opposed to using the standard accounting models in the literature. In particular, this thesis develops a computable objective metric, termed Relative Financial Strength (RFS), that uses only the financial information that is publicly available. By applying the RFS indicator for stock selection in a lagged-format, it is successfully demonstrated that portfolio optimization can benefit immensely with such techniques. The novelty of the RFS approach is that it is a measure of relative strength of a firm, rather than an absolute intrinsic measurement (or forecast) of a firm's share value.

The method of RFS is further complemented by considering expert information on input/output selection. This idea is particularly useful in cases where an outside expert's knowledge is being sought to improve the analysis. This is the first instance of using expert information in the context of DEA analysis. The required theoretical insight as well as statistical tests are developed for objectively verifying the value of such expert information. As the case study involving over 800 firms indicated, such expert knowledge can sometimes be accurate (as in certain market sectors in this case study), and at other times, it is best violated to improve the predictive power of the indicator. The degree of violation required to obtain the maximum predictive power was determined objectively.

While there have been numerous research in applying optimization methodology for risk-return trade off for portfolio rebalancing, selecting individual assets for this purpose has not been addressed sufficiently in the literature using mathematical programming methods. In that regard, the optimization-based framework for stock selection developed in this thesis can become a strong complement to other fundamental analysis tools used by fund managers.

8.1 Directions for Future Research

The GDEA optimization model developed in this thesis is a difficult mathematical problem because the objective function is neither concave nor pseudoconcave. Furthermore, there is no closed form expression for the objective function, i.e., evaluation of the objective function requires the solution of a sequence of linear programming models. The two-step heuristic algorithm proposed for solving this model does not guarantee a local optimum solution, let alone a global optima. While the algorithm yielded substantial correlations that are significant, a better solution methodology could improve upon these correlations. A first step in this direction is utilizing the first-order optimality conditions to guide a solution approach. More specifically, when the heuristic fails to satisfy the optimality conditions, a new search direction may be formed to improve the current iterate. This would certainly be a valuable avenue to follow in future research.

Finally, while the GDEA approach for fundamental analysis focused solely on inputs/outputs from public financial statements, there is no reason why the model cannot consider factors that are not directly financial in nature as input/output parameters. For instance, computable metrics representing a firm's CEO-strength/expertise, or the quality of the workforce of the firm (as measured by, for instance, educational/training level, prior experience). The GDEA model has the ability to objectively verify if such parameters add value for an indicator of a firm's relative strength in predicting its stock returns.

Bibliography

Bibliography

[1] I. M. S. Alam, and C. S. Robin. The relationship between stock market returns and technical efficiency innovations: evidence from the US airline industry. *Journal of Productivity Analysis*, 9: 35-51, 1998.

[2] A. Ali, L. Hwang, and M. A. Trombley. Residual income based valuation predicts future stock returns: evidence on mispricing vs. risk explanations. *The Accounting Review*, 78: 377-396, 2003.

[3] A. I. Ali, and R. Nakosteen. Ranking industry performance in the US. *Socio-Economic Planning Science*, 39: 11-24, 2005.

[4] T. G. Andersen, T. Bollerslev, F. Diebold and H. Ebens. The distribution of realized stock return volatility. *The Journal of Financial Economics*, 61: 42-55, 2001.

[5] P. Andersen, and N. C. Petersen. A procedure for ranking efficient units in Data Envelopment Analysis. *Management Science*, 39: 1261-1264, 1993.

[6] R. D. Banker. Estimating most productive scale size using data envelopment analysis. *European Journal of Operational Research*, 17: 35-44, 1984.

[7] R. D. Banker, A. Charnes, and W. W. Cooper. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, 30: 1078-1092, 1984.

[8] A. Barua, P. L. Brockett, W. W. Cooper, H. Deng, B. R. Parker, T. W. Ruefli, and A. Whinston. DEA evaluations of long- and short-run efficiencies of digital vs. physical product "dot com" companies. *Socio-Economic Planning Sciences*, 38: 233-253, 2004.

[9] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty. Nonlinear programming. John Wiley, 1993.

[10] J. Beasley. Determining teaching and research efficiencies. *Journal of the Operational Research Society*, 46: 441-452, 1995.

[11] W. F. Bowlin. Financial analysis of civil reserve air fleet participants using data envelopment analysis. *European Journal of Operational Research*, 154: 691-709, 2004.

[12] W. F. Bowlin. An analysis of the financial performance of defense business segments using data envelopment analysis. *Journal of Accounting and Public Policy*, 18: 287-310, 1999.

[13] G. E. P. Box, D. R. Cox. An analysis of transformations. *Journal of Royal Statistical Society*, Series B, 26: 211-246, 1964.

[14] C. S. Carrico, S. M. Hogan, R. G. Dyson, and A. D. Athanassopoulos. Data envelopment analysis and university selection. *Journal of Operational Research Society*, 48: 1163-1177, 1997.

[15] A. Charnes, W. W. Cooper. Programming with linear fractional functionals. *Naval Research Logistics Quarterly*, 9: 181-185, 1962.

[16] A. Charnes, W. W. Cooper, and E. Rhodes. Measuring the efficiency of decision making units. *European Journal of the Operational Research*, 2: 429-444, 1978.

[17] A. Cielen, L. Peeters, and K. Banhoof. Bankruptcy prediction using a data envelopment analysis. *European Journal of Operational Research*, 154: 526-532, 2004.

[18] T. J. Coelli, D. S. P. Rao, C. J. O'Donnell, and G. E. Battese. An introduction to efficiency and productivity analysis. Springer Science and Business Media, US, 2005.

[19] W. D. Cook, R. H. Green, and J. Zhu. Dual role factors in data envelopment analysis. *IIE Transactions*, 38: 105-115, 2006.

[20] W. W. Cooper, L. M. Seiford, and J. Zhu. Handbook on data envelopment analysis. Kluwer Academic Publishers, Massachusettes, 2004.

[21] P. R. Daves, M. C. Ehrhardt, and R. E. Shrieves. Corporate valuation: a guide for managers and investors. South-Western, Thomson, 2004.

[22] G. J. Deboeck. Using GAs to optimize a trading system. Trading On the Edge. New York, Wiley, 174-188, 1994.

[23] J. T. Doyle, R. J. Lundholm, and M. T. Soliman. The predictive value of expenses excluded from 'pro forma' earnings. *Review of Accounting Studies*, 8: 145-174, 2003.

[24] R. G. Dyson, R. Allen, A. S. Camanho, V. V. Podinovski, C. S. Sarrico, and E. A. Shale. Pitfalls and protocols in DEA. *European Journal of Operational Research*, 132: 245-259, 2001.

[25] N. C. P. Edirisinghe. Integrated risk control using stochastic programming ALM models for money management. In: Zenios, S.A., Ziemba, W.T.(Eds.), In: Handbook of Asset and Liability Management, vol. 2. Elsevier Science BV, 2007.

[26] N. C. P. Edirisinghe. MiSOFT User's Guide: Multistage interactive Stochastic Optimizer for Financial Trading, Version 3.6, 2006.

[27] N. C. P. Edirisinghe, and E. I. Patterson. Multi-period stochastic portfolio optimization: block separable decomposition. *Annals of Operations Research*, 152: 367-394, 2007.

[28] M J. Farrell. The measurement of productive efficiency. *Journal of the Royal Statistical Society*, 120: 253-279, 1957.

[29] R. Frankel, and C. M. C. Lee. Accounting valuation, market expectation, and cross-sectional stock returns. *Journal of Accounting and Economics*, 25: 283-319, 1998.

[30] M. S. Fridson. Financial statement analysis. Wiley, New York. Prentice Hall, NJ, 2002.

[31] G. N. Gregoriou, K. Sedzro, and J. Zhu. Hedge fund performance appraisal using data envelopment analysis. *Journal of the Operational Research Society*, 164: 555-571, 2005.

[32] J. M. Henderson, R. E. Quandt. Microeconomic theory: a mathematical approach. McGraw-Hill Book Company, 1980.

[33] J. H. Holland. Adaptation in natural and artificial systems. University of Michigan Press, Ann Arbor, 1975.

[34] J. C. V. Horne and J. M. Wachowicz. Fundamentals of financial management. Prentice Hall, NJ, 2001.

[35] R. Horst, R. M. Pardalos, and N. V. Thoai. Introduction to global optimization. Series in: Nonconvex Optimization and its Applications. Kluwer Academic Publishers, 1995.

[36] T. Larsson, M. Patrikssony. Global optimality conditions for discrete and nonconvex optimization - with applications to Lagrangian heuristic and column generation. Working Paper, Linkoping University, SE-581 83 Linkoping, Sweden, 2005.

[37] L.S. Lasdon. Optimization Theory for Large Systems. MacMillan-London, 1970.

[38] C. A. K. Lovell, J. T. Pastor. Units invariant and translation invariant DEA models. *Operational Research Letters*, 18: 147–151, 1995.

[39] D. MacKinnon. Estimation and inference in econometrics. Oxford University Press, New York, 1993.

[40] S. Mahfoud, G. Mani. Genetic algorithms for predicting individual stock performance. *Proceedings of the Third International Conference on Artificial Intelligence Applications on Wall Street*, 174-181, 1995.

[41] B. G. Malkiel. Efficient market hypothesis. *The new palgrave: a dictionary of economics*, 2: 120-123, 1987.

[42] O. L. Mangasarian. Convexity, pseudo-convexity and quasi-convexity of composite functions. Mathematics of optimization: smooth and nonsmooth case. Elsevier, 2004.

[43] H. M. Markowitz. Portfolio Selection. *Journal of Finance* 7: 77–91, 1952.

[44] P. A. Miliotis. Data envelopment analysis applied to electricity distribution districts. *Journal of the Operational Research Society*, 43: 549-555, 1992.

[45] Y. A. Ozcan, and M. J. McCue. Development of a financial performance index for hospitals: DEA approach. *Journal of the Operational Research Society*, 47: 18-26, 1996.

[46] C. Phillips, C. L. Karr, G. Walker. Helicopter flight control with fuzzy logic and genetic algorithm. *Engineering Applications of Artificial Intelligence*, 9: 175-184, 1996.

[47] P. Pille, and J. C. Paradi. Financial performance analysis of Ontario (Canada) Credit Unions: an application of DEA in the regulatory environment. *Journal of the Operational Research Society*, 139: 339-350, 2002.

[48] J. D. Piotroski. Value investing: the use of historical financial statement information to separate winners from losers. *Journal of Accounting Research*, 38: 1-41, 2000.

[49] M. C. A. S. Portela, E. Thanassoulis, and G. Simpson. Negative data in DEA: a directional distance approach applied to bank. *Journal of the Operational Research Society*, 55: 1111-1121, 2004.

[50] J. O. Rawlings. Applied regression analysis: a research tool. Wadsworth Inc., CA, 1988.

[51] S. C. Ray. Data Envelopment Analysis-theory and techniques for economics and operations research. Cambridge University Press, Cambridge, 2004.

[52] F. K. Reilly. Investments. the Dryden Press, New York, 1982.

[53] A. Ruszczynski. Nonlinear Optimization. Princeton University Press, 2006.

[54] A. Santos, A. Dourado. Global optimization of energy and production in process industries: a genetic algorithm application. *Control Engineering Practice*, 7: 549-554, 1999.

[55] L. M. Seiford, and J. Zhu. Profitability and marketability of the top 55 U.S. commercial banks. *Management Science*, 45: 1270-1288, 1999.

[56] W. F. Sharpe. A simplified model for portfolio analysis. *Management Science*, 9: 277-293, 1963.

[57] K. S. Shin, Y. J. Lee. A genetic algorithm application in bankruptcy prediction modeling. *Expert Systems with Applications*, 23: 321-328, 2002.

[58] J. D. Stowe, T. R. Robinson, J. E. Pinto, and D. W. McLeavey. Analysis of equity investments: valuation. United Book Press, Baltimore, 2002.

[59] A. C. Tamhane, and D. D. Dunlop. Statistics and data analysis from elementary to intermediate. Prentice-Hall, New Jersey, 2000.

[60] M. Tawarmalani, N. V. Sahinidis. Global optimization of mixed-integer nonlinear programs: A theoretical and computational study. *Mathematical Programming*, 99: 563-591, 2004.

[61] E. Thanassoulis. Introduction to the theory and application of data envelopment analysis-a foundation text with integrated software. Kluwer Academic Oublishers, Massachusetts, 2001.

[62] M. C. Thomsett. Mastering Fundamental Analysis. Dearborn, Chicago, 1998.

[63] G. B. Wetherill. Regression analysis with applications. Chapman and Hall, New York, 1986.

[64] Q. J. Yeh. The application of data envelopment analysis in conjunction with financial ratios for bank performance evaluation. *Journal of the Operational Research Society*, 47: 980-988, 1996.

[65] L. S. Zhang, F. Gao, W. X. Zhu. Nonlinear integer programming and global optimization. *Journal of Computational Mathematics*, 17: 179-190, 1990.

Appendix A

Sector	Industries Included
Technology Number of Firms = 159	Application Software Business Software & Services Communication Equipment Computer Based Systems Computer Peripherals Data Storage Devices Diversified Communication Services Diversified Computer Systems Diversified Electronics Healthcare Information Services Information & Delivery Services Information Technology Services Internet Information Providers Internet Service Providers Internet Software & Services Long Distance Carriers Multimedia & Graphics Software Networking & Communication Devices Personal Computers Printed Circuit Boards Processing Systems & Products Scientific & Technical Instruments Security Software & Services Semiconductor-Broad Line Semiconductor-Integrated Circuits Semiconductor-Specialized Semiconductor Equipment & Materials Semiconductor-Memory Chips Technical & System Software Telecom Services-Domestic Telecom Services-Foreign Wireless Communications

Table A.1: Industries included in Technology sector

Sector	Industries Included
Health Care Number of Firms = 107	Biotechnology
	Diagnostic Substances
	Drug Delivery
	Drug Manufacturers-Major
	Drug Manufacturers-Other
	Drug Related Products
	Drugs-Generic
	Health Care Plans
	Home Health Care
	Hospitals
	Long-Term Care Facilities
	Medical Appliances & Equipment
	Medical Instruments & Supplies
	Medical Laboratories & Research
	Medical Practitioners
	Specialized Health Services

Table A.2: Industries included in Health Care sector

Sector	Industries Included
Basic Materials Number of Firms = 86	Agricultural Chemicals
	Aluminum
	Chemicals-Major Diversified
	Copper
	Gold
	Industrial Metals & Minerals
	Nonmetallic Mineral Mining
	Silver
	Specialty Chemicals
	Steel & Iron
	Synthetics

Table A.3: Industries included in Basic Materials sector

Sector	Industries Included
Energy Number of Firms = 49	Independent Oil & Gas Major Integrated Oil & Gas Oil & Gas Drilling & Exploration Oil & Gas Equipment & Services Oil & Gas Pipelines Oil & Gas Refining & Marketing

Table A.4: Industries included in Energy sector

Sector	Industries Included
Industrial Goods Number of Firms = 128	Aerospace/Defense - Major Diversified Aerospace/Defense Products & Services Cement Diversified Machinery Farm & Construction Machinery General Building Materials General Contractors Heavy Construction Industrial Electrical Equipment Industrial Equipment & Components Lumber Wood Production Machine Tools & Accessories Manufactured Housing Metal Fabrication Pollution & Treatment Controls Residential Construction Small Tools & Accessories Textile Industrial Waste Management

Table A.5: Industries included in Industrial Goods sector

Sector	Industries Included
Consumer Discretionary Number of Firms = 110	Appliances Auto Manufacturers-Major Auto Parts Business Equipment Electronic Equipment Home Furnishings & Fixtures Housewares & Accessories Office Supplies Packaging & Containers Photographic Equipment & Supplies Recreational Goods - Other Recreational Vehicles Rubber & Plastics Sporting Goods Textile-Apparel Clothing Textile-Apparel Footwear & Accessories Toys & Games Trucks & Other Vehicles

Table A.6: Industries included in Consumer Discretionary sector

Sector	Industries Included
Consumer Staples Number of Firms = 68	Beverages-Brewers Beverages-Soft Drinks Beverages-Wineries & Distillers Cigarettes Cleaning Products Confectioners Dairy Products Farm Products Food-Major Diversified Meat Products Paper & Paper Products Personal Products Processed & Packaged Goods Tobacco Products-Other

Table A.7: Industries included in Consumer Staples sector

Sector	Industries Included
Financial Number of Firms = 58	Accident & Health Insurance Asset Management Closed-End Fund-Debt Closed-End Fund-Equity Closed-End Fund-Foreign Credit Services Diversified Investments Foreign Money Center Banks Foreign Regional Banks Insurance Brokers Investment Brokerage - National Investment Brokerage - Regional Life Insurance Money Center Banks Mortgage Investment Property & Casualty Insurance Property Management Real Estate Development Regional - Mid-Atlantic Banks Regional - Midwest Banks Regional - Northeast Banks Regional - Pacific Banks Regional - Southeast Banks Regional - Southwest Banks REIT - Diversified REIT - Healthcare Facilities REIT - Hotel/Motel REIT - Industrial REIT - Office REIT - Residential REIT - Retail Savings & Loans

Table A.8: Industries included in Financial sector

Sector	Industries Included
Utilities Number of Firms = 62	Diversified Utilities Electric Utilities Foreign Utilities Gas Utilities Water Utilities

Table A.9: Industries included in Utilities sector

Vita

Xin Zhang was born in Changsha, China on October 22, 1976. She graduated from the High School attached to Central South University in 1994 and entered the College of Business at Central South University (CSU), China, in the same year. In 1998, she completed her undergraduate study and received her Bachelor of Science degree in Finance. In 2000, she completed her Masters of Science degree at the same school.

In 2002, Xin joined the master program in Management Science Department at the University of Tennessee, Knoxville. Following completion her Masters of Science degree in 2004, she pursued a Ph.D. degree in Management Science at the University of Tennessee. She received her doctoral degree in December 2007.