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Demand Estimation at Manufacturer-Retailer Duo: A Macro-Micro Approach

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Recommended Citation

To the Graduate Council:

I am submitting herewith a dissertation written by Sirisha Saripalli entitled "Demand Estimation at Manufacturer-Retailer Duo: A Macro-Micro Approach." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Industrial Engineering.

Rapinder Sawhney, Major Professor

We have read this dissertation and recommend its acceptance:

Xueping Li, Frank M. Guess, Ramon V. Leon

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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Demand Estimation at Manufacturer-Retailer Duo: A Macro-Micro Approach

A Dissertation

Presented for the

Doctor of Philosophy Degree

The University of Tennessee, Knoxville

Sirisha Saripalli

May 2008

DEDICATION

This dissertation is dedicated:

To

My grandfather *Late Mr. S.V.Ramana*

My grandmother Mrs. S.Suryakantham.

and

To

My *Husband*, Dr. *Phani K Nukala*,

My Mother, Mrs. S.Aruna, and

My Father, Mr. S.Y.Narayana Rao

My brother, Mr. S.V. R. Karthik

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I would like to thank God for being so kind to me all throughout my life. If not for his constant blessings and grace I wouldn't have been able to accomplish this task.

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ABSTRACT

This dissertation is divided into two phases. The main objective of this phase is to use Bayesian MCMC technique, to attain (1) estimates, (2) predictions and (3) posterior probability of sales greater than certain amount for sampled regions and any random region selected from the population or sample. These regions are served by a single product manufacturer who is considered to be similar to newsvendor. The optimal estimates, predictions and posterior probabilities are obtained in presence of advertising expenditure set by the manufacturer, past historical sales data that contains both censored and exact observations and finally stochastic regional effects that cannot be quantified but are believed to strongly influence future demand. Knowledge of these optimal values is useful in eliminating stock-out and excess inventory holding situations while increasing the profitability across the entire supply chain.

Subsequently, the second phase, examines the impact of Cournot and Stackelberg games in a supply-chain on shelf space allocation and pricing decisions. In particular, we consider two scenarios: (1) two manufacturers competing for shelf space allocation at a single retailer, and (2) two manufacturers competing for shelf space allocation at two competing retailers, whose pricing decisions influence their demand which in turn influences their shelf-space allocation. We obtain the optimal pricing and shelf-space allocation in these two scenarios by optimizing the profit functions for each of the players in the game. Our numerical results indicate that (1) Cournot games to be the most profitable along the whole supply chain whereas Stackelberg games and mixed games turn out to be least profitable, and (2) higher the shelf space elasticity, lower the wholesale price of the product; conversely, lower the retail price of the product, greater the shelf space allocated for that product.

PREFACE

Accurate prediction of demand is crucial to an integrated supply chain. Knowing how much to supply in order to reduce stock-outs and surplus inventory is critical for achieving optimum profits. However, accurate prediction/estimation of optimal demand of a product is complicated due to the presence of censored demand data, which depends crucially on various factors such as the advertising budget, price of the product and its substitute, product's shelf space allocation, and competition among the product manufacturers and the retailers. In addition, optimal demand estimation is complicated by the stochastic nature of random store/regional effects, which include such factors as geographical location, population density, and weather that have significant impact on sales but are difficult to quantify for modeling purposes. Despite an extensive research on accurate modeling of supply chains over the last two decades with special emphasis on demand and inventory estimation models, there still remain many practical/implementation issues related to these models that have not received due attention.

This dissertation presents a macro-micro estimation of demand faced by a manufacturer-retailer duo. The macroscopic phase is defined as demand estimation at regional level for a single product manufacturer and the microscopic phase is defined as demand estimation at store level for a retailer selling competing substitutable products. For a given product, the manufacturer is interested in a macroscopic estimation of regional demand in the presence of stochasticity introduced by random regional effects such as geographical location and population density and the advertising budget, which is considered to have the most dominant effect on demand for a product. To address this macroscopic phase of regional demand estimation, in this dissertation, a Bayesian Markov Chain Monte Carlo (MCMC) technique is used to predict and estimate optimal

regional demand for a product in the presence of censored and exact demand data subjected to certain random regional effects and a fixed advertising budget.

From a microscopic demand estimation point of view, the stochasticity in macroscopic demand arises due to random amounts of demand at the store/retailer level. These random demand levels at the store/retailer are influenced by competition among local retailers, competition among substitutable products, shelf space allocated for a particular product and the pricing strategies adopted by the retailers. Consequently, at the microscopic (store/retailer) level of demand estimation, this study considers the following two scenarios: (1) a single retailer, two manufacturer scenario to understand the influence of competing substitutable products on local demand estimation; and (2) two competing retailers and two competing manufacturers scenario to understand the influence of competition among retailers through their pricing and shelf space allocation strategies. Cournot and Stackelberg game theory models are used to study competition among retailers and manufacturers to obtain optimal pricing and shelf space allocation strategies at Nash equilibrium. The optimal (local) demand at each of the retailers is derived based on these optimal pricing and shelf space allocation strategies. Finally, these optimal local demands at each of the retailers are then summed up to obtain the stochastic regional demand at the macroscopic level. Repeating this procedure with a different value of microscopic stochastic parameter each time, a macroscopic demand distribution for the stochastic effects on regional demand is obtained.

This dissertation presents a unique hierarchical demand estimation methodology that takes into account censored/missing historical data, competition and cooperation among the supply chain members, pricing and shelf space allocation strategies, and stochastic (random) regional effects. Demand estimation models that account for censored data, competition among supply chain

partners, and random regional effects are especially important for perishable goods and those with short life spans in order to eliminate variability or *"bullwhip effect"*, reduce stock-outs and carry forward inventories.

TABLE OF CONTENTS

Chapter III

Pricing decisions at competing retailers for competing substitutable products-

Microscopic Level

Chapter IV

Discussion of Findings

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

CHAPTER II

D = Exact and censored demand faced by a manufacturer serving various regions

y= inventory on hand

β=shape parameter

η=scale parameter

$D \sim Weibull(\beta,\eta)$

F(d) = cumulative distribution of demand

f(d)=probability density of demand

γ_1=advertising expenditure elasticity of demand

B=advertising expenditure

γ_0 = constant

$\psi \sim N(0, \tau)$ = random regional effects

τ= precision$\rightarrow$ inverse of variance $\rightarrow$ $\tau = \dfrac{1}{\sigma^2}$

Var(D)=variance of demand

λ = demand rate

c_f = Fixed Cost

c_v = Variable Cost

c_s = Shortage Cost

H = Holding Cost

x=Initial inventory

q=quantity produced

E(q-D,0)=Expected excess stock at the manufacturer

E(D-q,0)=Expected shortage at the manufacturer

$q_{optimal}$ =Optimal demand percentile

O_t =(O_{t1} , O_{t2}) = demand observation during time period t

O_{t1} = sales quantity which is known exactly and cannot be negative

O_{t2} = observed status that can be exact or censored

$\pi_t(\theta)$= prior distribution

π_{t+1}=posterior distribution

$l(O_t | \theta)$ =likelihood function

CHAPTER III

Scenario 1

$d_1^{tot} = d_1^{(1)} + d_1^{(2)}$ *represents the total demand @ product*1 *for both the retailers*;

$d_1^{(1)} = demand \ for \ product$1 $@ R$1 ; $d_1^{(2)} = demand \ for \ product$1 $@ R2$

$d_2^{tot} = d_2^{(1)} + d_2^{(2)}$ *represents the total demand for product*2 *@ both the retailers*;

$d_2^{(1)} = demand \ for \ product$2 $@ R$1 ; $d_2^{(2)} = demand \ for \ product$2 $@ R2$

$Q_1 = Supply \ of \ product \ 1 \ from \ manufacturer$1 $to \ retailer \ (only \ 1 \ retailer) in \ case$2

$Q2 = Supply \ of \ product \ 2 \ from \ manufacturer$2 $to \ retailer \ (only \ 1 \ retailer) in \ case$2

$\alpha = cons \tan t;$

$\mu_i = direct \ price \ elasticity$

$\varepsilon_i = cross \ price \ elasticity$

$\gamma_1 = direct \ space \ elasticity$

$\gamma_2 = cross \ space \ elasticity$

$N = number \ of \ products \ in \ that \ category$

$S = total \ available \ shelf \ space \ within \ the \ product \ category \ (inches)$

$S_i = total \ available \ shelf \ space \ for \ product \ i$

$P_i = selling \ price \ of \ product \ i \ (\$)$

$W_i = wholesale \ price \ of \ product \ i(\$)$

$\pi_R = \Pr ofit \ of \ the \ retailer$

$\pi_M = \Pr ofit \ of \ the \ retailer$

Scenario 2

$P_{11} = price\ of\ product1@retailer1$

$P_{12} = price\ of\ product1@retailer2$

$P_{21} = price\ of\ product2@retailer1$

$P_{22} = price\ of\ product2@retailer2$

$\mu_1 = Cross\ elasticity$

$\mu_2 = Cross\ elasticity$

$\varepsilon_{11} = Direct\ price\ elasticity$

$\varepsilon_{12} = Cross\ elasticity$

$\varepsilon_{21} = Cross\ elasticity$

$\varepsilon_{22} = Direct\ price\ elasticity$

$\gamma_1 = Shelf\ space\ elasticity@retailer\ 1$

$\gamma_2 = Shelf\ space\ elasticity@retailer3$

$W_1 = Wholesale\ price\ product\ 1@Manufacturer1$

$W_2 = Wholesale\ price\ product\ 2@Manufacturer2$

$C_1 = Pr\ oduction\ \cos t\ for\ product1@Manufacturer1$

$C_2 = Pr\ oduction\ \cos t\ for\ product2@Manufacturer2$

$S_1 = shelf\ space\ allocated\ to\ product1@retailer1$

$1-S_1 = shelf\ space\ allocated\ to\ product2@retailer1$

$S_2 = shelf\ space\ allocated\ to\ product1@retailer2$

$1-S_2 = shelf\ space\ allocated\ to\ product2@retailer2$

Chapter I

Demand Estimation in an Integrated Supply Chain - Generalized View

1.1 Introduction

A supply chain is, broadly speaking, "the network of retailers, distributors, transporters, storage facilities and suppliers that participate in the sale, delivery and production of a particular product" [1]. From the definition above, it is clear that supply-chain management is a cross-functional approach to management of the movement of raw materials into an organization, certain aspects of the internal processing of materials into finished goods, and the subsequent movement of finished goods out of the organization towards the end consumer. The related term "supply-chain execution" refers to the process of managing and coordinating the movement of materials, information, and funds across the supply chain. Common problems encountered in executing a typical supply chain often occur in the following areas:

➢ *Demand estimation and/or inventory management.* The main issue here is the accurate estimation of demand quantity and the location of inventory (including raw materials, work in process and finished goods) in order to avoid either excess inventory holding or stock-out situations.

➢ *Information sharing and cash flow*: The integration of systems and processes throughout the supply chain is critical to enable the sharing of valuable information, including demand signals, forecasts, inventory and transportation etc. Arranging for payment terms and methods for exchanging funds across the entities within the supply chain is an important issue.

➢ *Distribution network configuration and strategies.* The effective design of the distribution network including the number and location of suppliers, production facilities, distribution centers, warehouses, and customers, is critical to smooth production along with perfect strategies. Significant decisions that must be made include whether to adopt a centralized or decentralized models, as well as whether to use direct shipment, cross docking, push/pull strategies. 3PL also needs to be addressed.

The key issues at all levels of activity—strategic, tactical and operation—are the accurate estimation and forecasting of demand. Accurate prediction of the demand for a product plays a crucial role in governing strategies for amplified cash flow, superior distribution strategies and network configuration, improved information sharing and enhanced profits for all the supply chain partners. Strategic questions about demand continually perplex both retailers and manufacturers: Do current conditions show signs of too much market supply? Too little demand? Vice versa? How much supply or demand is "too much"? When is the right balance achieved? All these questions are dependent on demand estimation in one way or another.

In general, there are two ways to view market share: by supply or by demand (that is, from either the manufacturer's or the retailer's perspective). Analysis of consumer spending reveals market demand, or retail potential. Examining business revenues, advertising budgets, sales (censored) or demand data (exact), and competition reveals market supply. However, the accurate estimation of demand and its forecasting in an integrated supply chain is a complicated problem for various reasons:

➢ Presence of too many influential factors that directly or indirectly sway the demand curve[2];

➢ Methods used to analyze the data are not appropriate for the purpose, resulting in incorrect results and interpretations [**3**] ;

➢ Missing data for a certain time period or periods;

➢ Competition among supply-chain partners and different supply chains.

➢ Availability of multiple substitutable items in the same product category.

Various researchers have attacked the problem of accurate forecasting. Meredith (2006) [**2**] has presented a list of variables that make demand estimation complicated at various levels of the supply chain and cause the most serious estimation errors. Robert (1985) [**3**] in his article reviewed 29 different research methodologies used for measuring the demand for a telecommunication product. There were several mistakes in how the studies had been designed and implemented, and he suggested methods for improving the accuracy of the forecasts. Radchenko and Tsurumi (2006) [**4**] estimate parameters and a supply demand equations of the US gasoline market using Bayesian MCMC techniques because of the limited nature of the available information. Harish et al. (2008) [**5**] present a replenishment model for retail stores incorporating direct and cross space elasticities while considering demand rate to be linearly dependent on the shelf space allocated to a product by a multiple product retailer, using mathematical optimization techniques. Herr' an et al. (2006)[**6**] consider two competing manufacturers fighting for shelf space at a single retailer and use game theory to resolve this dilemma. All the above mentioned models consider some variables and try to estimate demand using different methods for different sections of the supply chain partners. But none of them consider integration or looking at the problem of demand estimation from the retailers' as well as the manufacturers' perspective.

1.2 Motivation

Over the last three decades, numerous demand estimation models have been proposed in order to gain a better understanding of inventory management and demand estimation in a supply chain. Until recently, however, many of these studies have concentrated on estimating demand at an isolate level(whether retailer or manufacturer) of a supply chain. That is these studies consider demand estimation at individual levels of a supply chain and did not consider interactions among retailers and manufacturers through their pricing and strategic decisions. In a pioneering work, Edward Hawkins (1957) [7] presented on of the earliest demand estimation models to identify demand at an isolated level of a supply chain. In this article Hawkins pointed out that statistical analysis of the demand data yielded reliable results in estimating the price elasticity of demand. Even today, most retailers, even large ones such as Wal-Mart, use simple forecasting techniques based on regression analysis and exponential smoothing methods to analyze, interpret and forecast demand. However, these simple regression and exponential smoothing methods suffer from shortcomings that arise from the presence of censored and missing data in the historical data sets on which demand forecasting depends [8]

Choi et al. (2003) [9] investigated an optimal two-stage ordering policy for seasonal products. Market information is collected at the first stage and is used to update the demand forecast at the second stage by using a Bayesian approach. A two-stage dynamic optimization problem is formulated, and an optimal policy is derived using dynamic programming. The service level and profit uncertainty level under the optimal policy are discussed. Extensive numerical analyses are carried out to study the performance of the optimal policy. Yang et al. (2003) [10] developed a Bayesian method to address the computational challenge of estimating simultaneous demand and

supply models that can be applied to both the analysis of household panel data and aggregated demand data. These models use Bayesian methods to evaluate demand from a supplier's perspective and do not investigate the hidden potential of these Bayesian methods.

Urban (1998) [11]suggested a heuristic that allocates shelf space by removing at each iteration the item in the assortment with the lowest contribution to profits. The procedure stops when profits start to decrease. Building on Corstjens and Doyle (1981)[12], Borin et al. (1994) [13] used an elaborated version of the same demand function to allow for simultaneous decisions about assortment selection and shelf space allocation. Yang and Chen (1999) [14] used a simplified version of the same demand formulation by not including the cross-elasticities and assumed that a product's profit is linear within a small number of facings which are constructed by the products lower and upper bound of the number of facings. The term facing is defined as "the number of identical products (or same SKUs) on a shelf turned out toward the customer"[15]. If used as verb it can also be defined as : "the act of pulling each product to the front edge of a shelf with the label turned forward. This gives the store an appearance of being full with merchandise"[15]. This is commonly used terminology while implementing certain shelf space allocation strategies using common software's while cleaning up the store and making it user friendly for the customers.

On the other hand, Chintagunta and Jain (1992) [16] adopted a differential game strategy to examine the effect of channel dynamics on the difference in profits resulting from following coordinated rather than uncoordinated strategies; they also identify situations in which this profit differential provides an incentive for channel members to coordinate their marketing efforts. Subsequently, Jorgensen and Zaccour (1999) [17] proposed a differential game model for analyzing the relationship between two firms under conditions of conflict and of coordination;

6

they also include pricing and advertising strategies for both the firms under consideration. The models described in this paragraph all try to evaluate demand from retailers' perspectives while introducing competition. But none of these models actually account for manufacturers' decisions on shelf-space allocation, which affects the demand experienced by the retailer.

Despite an extensive amount of research on demand estimation models, there still remain practical and implementation issues that have not been addressed. Some of those issues are outlined below:

- ➢ how to deal with the presence of censored demand (historical) data;
- ➢ the role of marketing/advertising expenditures;
- ➢ competition among the supply chain partners in terms of pricing, substitutable products, and shelf-space allocation strategies;
- ➢ stochastic/random regional effect (geographical location, population density, weather, household income, etc.).

1.3 Problem Statement

The main aim of this research is to develop a hierarchical demand estimation methodology that takes into account censored and exact historical data, the influence of marketing/advertising expenditure on demand, competition and cooperation among the supply chain members, the pricing decisions of the partners, shelf-space allocation strategies, and stochastic (random) regional effects.

The definition of "supply chain" in Section 1.1 clearly states that all supply chains consist of different tiers, each tier representing a supply-chain partner or member. As a result, the factors that influence the demand at each tier are also different. Therefore, demand models at different tiers of a supply chain are different because of disparate influencing factors. Consequently, this dissertation investigates a macro-micro level [2] estimation of the demand faced by manufacturers and retailers of a supply chain in order to obtain optimal demand, pricing, and shelf-space allocation values at different tiers of the supply chain.

The macro-level model (Phase I) of this dissertation deals with demand estimation at the manufacturing level of a supply chain. This model estimates regional demand for a single product at a manufacturer in the presence of censored and exact sales data, while taking into consideration the influence of factors such as advertising expenditure and stochastic regional effects on the demand for that product. On the other hand, the micro-level model deals with demand estimation at a retailer level of a supply chain. This model estimates demand at a retailer level considering the effect of shelf-space allocation, pricing decisions, and the competition from substitutable products.

The micro-level model (Phase II) of this dissertation is further divided into two scenarios that are defined below:

➢ *Scenario 1:* This scenario is an extension of an already built demand estimation model by Herr'an et al. (2006) [6], consisting of two competing manufacturers of homogenous substitutable products battling for shelf space at a single retailer by means of their pricing decisions. The manufacturers' competition for shelf space influences the demand for each of the products.

> *Scenario 2:* This scenario extends the demand model formulation of Scenario 1 to consider two competing retailers as well as the two competing manufacturers. As in Scenario 1, the manufacturers compete with one another through their substitutable products, thereby influencing the shelf-space allocation at each of the retailers. The shelf-space allocation in turn influences the demand for each of the products. In addition, the retailers compete with one another through their pricing strategies for each of the substitutable products.

Demand estimation at the manufacturer level (macro-level model) is achieved using the Bayesian Markov Chain Monte Carlo (MCMC) method. The Bayesian MCMC method is suitable for estimating demand in the presence of censored demand data. In addition, game theory-based models are used to incorporate the factor of competition among supply-chain partners (manufacturers and retailers) for estimating demand at the retailer level (micro-level model).

1.4 Organization

Chapter 2 presents the macroscopic approach to demand estimation at the manufacturer level in an integrated supply chain. In this chapter we discuss in detail the Bayesian MCMC methodology used to solve the demand estimation problem for a single product manufacturer supplying the product to several regions. In particular, the Bayesian MCMC method accounts for :

> historical sales data (which may contain both censored and exact observations),
> the influence of advertising expenditure on demand, and

> ➤ stochastic regional effects that mimic such features as the effect of geographical location, population density, and weather on regional demand for a product.

The specific aims of this macroscopic level regional demand estimation model are the following:

> ➤ To estimate the posterior percentile of 90th percentile of demand for the sampled regions and a random region selected from the population or sample.
>
> ➤ To predict the posterior percentile of 90th percentile of demand for the sampled regions and a random region selected from the population or sample.
>
> ➤ To compute posterior probability of demand greater than a certain amount given random advertising budgets for the sample regions and for a random region selected from the population or sample.

These 90th posterior percentiles are termed "optimal point predictions" and "estimates of demand." for the future demand observations and future parameter estimates for unknown parameters respectively. The novelty of this macro level model is that it provides demand forecasts in the presence of censored demand, as well as using Bayesian MCMC techniques.

Chapter 3 presents the microscopic approach to demand estimation at the store/retailer level in an integrated supply chain. In particular, this phase focuses on demand estimation at the retailer/store level in the presence of competition and cooperation among the supply-chain partners (manufacturer-retailers) through optimal competitive pricing and shelf-space allocation strategies attained at Nash equilibrium using game theory methodology.

Specifically, this microscopic level is further split into two manageable scenarios for store-level demand estimation. The first scenario considers two manufacturers competing for shelf space at a single retailer, and the second scenario consists of two competing manufacturers battling for shelf space and pricing decisions at two competing retailers. In each of these scenarios, all the members (retailers as well as the manufacturers) of the supply chain use different game strategies in order to obtain optimal pricing and shelf space allocation strategies. The specific goals of this microscopic level local demand estimation model are as follows:

> To formulate demand models for both scenarios and to optimize the profits for all the supply-chain members in both scenarios in order to derive optimal pricing and shelf-space allocation strategies.

> To introduce competition among the retailers in Scenario 2.This is the first time that the competitors' pricing for the same product is considered in the context of shelf-space allocation.

> To demonstrate the real potential of this methodology and investigate the profits across the supply chain for both static and sequential games.

This microscopic-level approach utilizes the tools of game theory models and optimization techniques in order to attain these goals. The novelty of this phase lies in the second scenario, where competition among the retailers is accounted for by inserting their competitor's price for the same product into their respective demand models.

Chapter 4 discusses the findings, provides general conclusions, and reflects on the limitations of this research and future directions for research beyond this dissertation. Pictorial representation of the organization is presented in Figure 1 for better understanding.

11
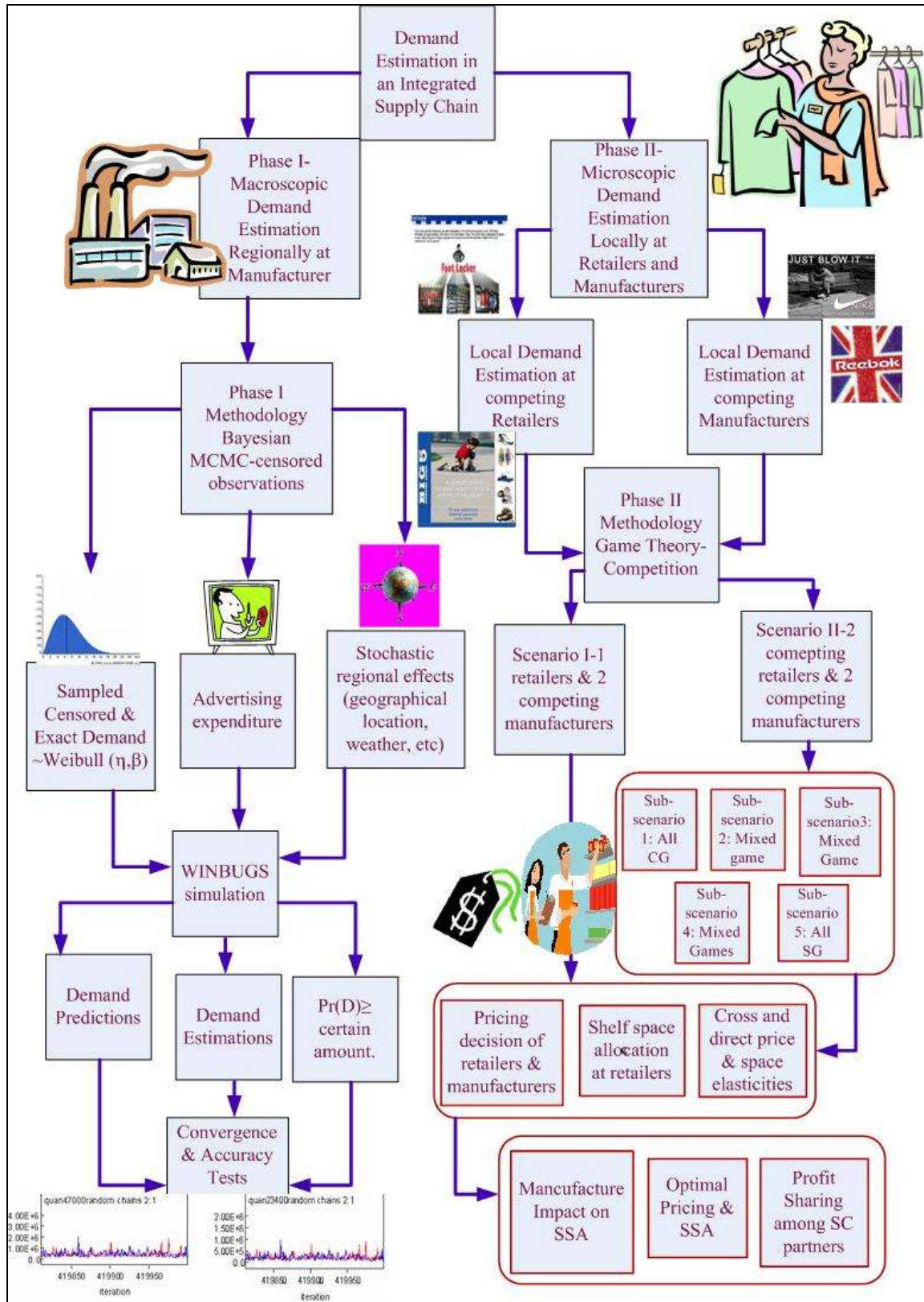
Figure 1: Dissertation Roadmap

Chapter II

Phase I: Demand Estimation at Manufacturer – Macroscopic Level

Abstract

The main objective of this study is to use the Bayesian MCMC technique to obtain several important pieces of information: (1) estimates; (2) predictions; and (3) posterior probabilities of sales for optimal percentiles of demand for a certain product supplied to different regions across North America. The manufacturer serving these regions is similar to an individual newsvendor. The optimal estimates, predictions and probabilities are obtained in the presence of an advertising budget set by the manufacturer for that product, from historical censored and exact sales data, and from random region effects that cannot be quantified but that significantly impact the future demand for the product. Knowing these optimal estimates and predictions helps the manufacturer reduce stock-outs and the retention of excessive stock; this translates to increased profitability across the entire supply chain.
.

Keywords: supply chain, demand, censored demand, inventory, prediction intervals, credibility intervals, Bayesian modeling, advertising and Weibull.

2.1 Introduction

Executing a supply-chain complicated because of the uncertainties associated with each cross functional component involved. The measurement of uncertainty is a complicated task due to the presence of many influential factors,[2] such as censored and exact demand data, advertising, pricing decisions, shelf space allocation, competition among supply chain partners and any random effects (weather, geographical location, etc) that are believed to significantly impact future demand but cannot be quantified. A plethora of research has been conducted in this area of demand estimation using various statistical, mathematical and analytical models. These models provide interesting insights and valuable tools for obtaining future demand by estimating inventory or predicting demand. In general, there are two ways to measure market share: through (1) inventory models and (2) demand distribution models.

Researchers such as Karlin (1960)[18], Scarf (1960)[19] and Iglehart (1964)[20] have studied updating schemes for dynamic inventory policies with exponential family of demand density distributions having unknown parameters. These studies considered order up to policy as the optimal policy in which critical values depend on path of historical data (distributions followed) through sufficient statistic. Azoury (1985)[21] extended the work of Karlin [18] and Iglehart [20] to more general demand distributions and provided conditions under which finding the optimal policy using Bayesian updating reduces to solving a stochastic dynamic programming problem in one dimension.

Whereas Conrad (1976)[22] was the first to explicitly distinguish between sales and demand in demand censored inventory systems and used a Poisson demand distribution to investigate the

effect of censored demand and proposed a Maximum Likelihood Estimate (MLE) of the Poisson

parameter. Subsequently, Nahmias (1994)[23] considered a censored demand model with

normal demand distribution and proposed a procedure for sequentially updating estimates of

normal parameters. This study also addressed the difficulties associated with classical statistical

approaches in parameter estimation. Agarwal and Smith (1996)[24] showed that a negative

binomial distribution provides a better fit for discrete data than a Poisson or normal distribution

and proposed a parameter estimation method. But in the presence of unknown demand

distributions and historical data, a Bayesian updating approach provides better and more reliable

demand estimates.

Bayesian statistics is a significant tool that has been used previously to estimate demand. Despite

their computational complexity, Bayesian methods have been in common use for more than five

decades. Scarf (1959) [25] pioneered the empirical Bayesian approach to the problem, in which

the retailer optimally manages his or her inventory levels by observing demand distribution over

time. Assuming a normal distribution for both stated variables and unknown parameters,

Florentine (1962) [26] observed that optimal control determined by Bayesian updating methods is

better than that determined by non-Bayesian methods. Following a conjugate family concept,

Branden and Friemer (1991)[27] developed the so-called "newsboy distributions" (Weibull) that

allow for a parsimonious updating process of the prior distribution for the case of unobserved lost

sales with identically independently distributed (iid) demand realizations with a known shape and

unknown scale parameters. Using Branden and Friemer's framework [27], Lariviere and Porteus

(1999) [28] examined an empirical Bayesian inventory problem in which unmet demand is lost

and unobserved. Using base stock inventory systems with truncated sales, Ding (2001) [29]

developed Bayesian and maximum likelihood estimates for a Poisson and zero-inflated Poisson

16

demand distributions for a newsvendor and (s, S) inventory system with censored and exact demand data.

This chapter presents the Bayesian MCMC technique as a tool to estimate and predict demand for the optimal demand percentile given censored and exact information, advertising expenditure set by the manufacturer of the product as well as random regional effects for all the sampled regions and any random region picked from the population. The manufacturer sells his product to different regions all across North America. He is considered to be an individual newsvendor.

The hypothetical scenario is as follows: A manufacturer that sells a certain product to different regions across North America and initiates to conduct an experiment in order to estimate and predict demand at some sampled regions in the presence of certain advertising expenditure set by the manufacturer and a stochastic regional -effects component (weather, geographical location, population density, income level of the families, etc., in that region) along with historical censored and exact demand data for that product. A Bayesian MCMC technique is utilized to meet this goal of estimating and predicting future demand for that product. This experiment is conducted to evaluate the efficacy of Bayesian MCMC techniques as well as to set the optimal advertising budget per region so as to optimize his cost structure, while trying to eliminate a bull-whip effect across the supply chain partners. The manufacturer advertises for this product twice at the sample regions, and he maintains a centralized database that contains the historical sales data for that product. All the partners in the supply chain have to make decisions on the amount of inventory to carry in a given time period t. Limited inventory causes lost sales that are neither observed nor accounted for [**30, 28**]. Therefore, the purpose of this study is to estimate and predict demand in the presence of a limited inventory for a given advertising expenditure and stochastic regional effects.

17

Models for allocating advertising expenditure to market territories under sales uncertainty were developed using an exponential distribution where parameter values needed for the implementation of the model can be derived from the seemingly unrelated regression model [**31**]. Advertising expenditure is considered a key variable since empirical studies in the past have shown that advertising significantly influences demand for a product[**32**]. Sampled regions are selected randomly from a large population of regions across North America to which the manufacturer sells his product. Each sampled region is allocated an advertising expenditure per period for 't' periods. The time periods 't" for allocating the expenditure are chosen such that promotional periods or holiday periods are not included . Inclusion of promotional variables further complicates the analysis of the current problem, as we then need to include bumpy demand data along with censored and exact data. The sales data are represented as censored demand when sales are equal to inventory as well as exact demand when the sales are less than on-hand inventory.

Though, previously cited studies involving censored data are based on either normally distributed demands (a common assumption in inventory modeling) or negative binomial demands, which has been observed in many real world environments, but for the censored data in this chapter we consider Weibull distribution. The Weibull Distribution belongs to the family of "newsboy distributions" characterized by Braden and Freimer (1991)[**27**]. It has a fixed dimensional sufficient statistic under exact and right-censored observations. In this chapter, the manufacturer is considered to be a "newsvendor" or "newsboy." Using the framework of Braden and Freimer (1991)[**27**], the historical demand dataset obtained at these manufacturers is considered to contain both exact and right-censored observations that follow a Weibull distribution with two unknown parameters . It is a significantly more difficult case than Braden and Freimer (1991)[**27**], because

it requires a two-dimensional joint prior distribution of the Weibull parameters known as shape and scale parameters. In this chapter, a diffused prior belief exists on the shape parameter β of the Weibull demand distribution, and the scale parameter follows a log linear relationship with advertising expenditure and stochastic regional effects. Furthermore, these two parameters are not constrained by the conjugate family concept. Using this demand distribution and Bayesian MCMC strategy, this chapter (1) estimates the 90th percentile of demand; (2) predicts the 90th percentile of demand; (3) computes the posterior and predictive probability distribution of sales given the advertising expenditure, demand data, and stochastic regional effects for any sampled region or randomly picked region from the population.

This chapter is organized as follows: Section 2.2 reviews the literature on various categories of demand estimation models. This section provides us with the deficiencies of these models presented in Section 2.3. In an attempt to overcome most of the deficiencies, Section 2.4 presents the problem statement and objectives. To solve the current problem of demand estimation at the manufacturer, Section 2.5 presents the methodology adopted to achieve the objectives presented in Section 2.4. Sections 2.6 and 2.7 explain in detail the MCMC and Bayesian techniques that are cited in the methodology section. Section 2.8 contains the model formulation for this solution of the problem, followed by a case study and numerical example to reveal the real potential of the Bayesian MCMC techniques. Section 2.10 presents the results and discussions followed by Section 2. 11, containing conclusions.

2.2 Literature Review

To the best of my knowledge, classical forecasting models are used to predict future demand and maximum likelihood estimation techniques (statistical models) are used for estimating demand parameters. As explained in introduction, demand can be measured through classical inventory management models or demand distribution (statistical) models. But the tools used to estimate and predict demand through are different which makes these studies and results interesting. Therefore the entire literature review section is categorized into four different models as follows:

2.2.1. Mathematical models

These are often categorized as operations research or optimization models. Gurnani and Tang (1999) [33] determined the profit maximizing ordering strategies for a retailer who has two instances of ordering optimal quantities of seasonal products from a manufacturer. While the demand is uncertain, he can definitely improve his forecasts from the first instance to the second by observing the market signals. Here they present a nested newsboy problem for determining optimal order quantity at each instance. Fisher and Rajaram (2000) [34] use a K-median model to cluster the regions of the chain based on a region-similarity measure defined by sales history and then choose test regions from each cluster for the purpose of this study. Subsequently, a linear programming model is used to fit a formula that is then used to predict sales from the test sales. These methods have been used on real-time apparel retailers and shoe retailers; the methods showed a significant increase in profits at the apparel retailer. Lau et al. (2002) [35] do not

explicitly estimate or predict demand at the manufacturer but instead study the effect of reducing demand uncertainty in a manufacturer-retailer channel.

Most of the models expressed in this section are complicated mathematical models and seldom find applicability to real-world problems. Again, not many researchers have focused their attention on manufacturers' estimating demand at the regional level. Due to the lower practical significance, most of the retailing industry uses typical forecasting methods described in the Section 2.2.2 below for estimating demand

2.2.2. Forecasting/Statistical Models:

Lordahl and Bookbinder (1994) [36] proposed a method in which they used an ordered? static approach and made no assumptions on the underlying lead time demand distribution or its parameters; they also used a only small amount of lead-time demand data to estimate the reorder point for an (s,s) inventory system. In a recent paper, Lee, So, and Tang (2000) [37] showed that in a two-level supply chain with non-stationary AR(1) end demand, the manufacturer benefits significantly when the retailer shares point-of-sale (POS) demand data. Raghunathan (2001) [38] shows, both analytically and through simulation, that the manufacturer's benefit is insignificant when the parameters of the AR(1) process are known to both parties, as in Lee, So, and Tang (LST) (2000) [37]. The key reason for the difference between these results and those of LST is that LST assumes that the manufacturer also uses an AR(1) process to forecast the retailer order quantity. However, the manufacturer can reduce the variance of its forecast further by using the entire order history to which it has access. Thus, where there is sufficiently intelligent use of already available internal information (order history), there is no need to invest in inter-

organizational systems for information sharing. Chen et al. (2000) [**39**] consider a two-stage supply chain with a single retailer and single manufacturer and demonstrate that an exponential smoothing forecast of a retailer can cause a bull-whip effect; they contrast these results with the increase in variability due to the use of a moving average forecast. Kim and Ryan (2003) [**40**] present an extension of the basic newsvendor model that allows us to quantify the value of the observed demand data and the impact of suboptimal forecasting on the expected costs at the retailer. This method is demonstrated using an exponential smoothing technique. The model is also used to quantify the value of information and information sharing for a decoupled supply chain in which both the retailer and the manufacturer must forecast demand.

All these models focus their attention on retail or distribution centers while predicting optimal inventory levels. None of the above models use censored demand information for their predictions.

2.2.3. Programming Models:

Due to the advancement of technology, there have been many software packages available on the market to estimate demand. These packages are designed according to certain heuristics and aim to provide good demand predictions for a future period given certain values. Following is the list of demand estimation software vendors:[**8, 41**]

> ➢ Analytica is available through Lumina Decision Systems, Inc., and is a manually driven software package for forecasting demand.

- AUTOBOX can be used as automatic, semi-automatic or manual forecasting software. It works on DOS, Linux, UNIX, and Windows operating systems.

- Crystal Ball is an automatic forecasting software. It works only on Windows.

- Demand Works DP and Demand Works Smoothie are two software packages supplied by Demand Works and work on Windows only. The DP version can be used for automatic, semiautomatic, or manual forecasting, whereas the Smoothie version is automatic only.

- Forecast Pro Unlimited and Forecast Pro XE are two versions available through Business Forecast Systems, Inc. These also work on Windows only and are usable for automatic, semi-automatic, and manual forecasting.

- ForecastX Wizard is available through John Galt Solutions and can be used only on the Windows platform. Again, this can be used for generating forecasts automatically, semi-automatically and manually.

- GAUSS is available through Aptech Systems, Inc., and works on Windows, Linux, and Mac OSX and Sun Sparc platforms. The drawback is that it is available only in the manual-forecasting version.

- PEER planner and PEER Forecaster are available through Dalphus, Inc, and work only on the Windows platform. This can be used for automatic, semi-automatic, and manual forecasting.

- Minitab, NCSS, SPSS,JMP and SAS These all are statistical software packages and have the ability to predict demand when trend, seasonality, cyclical and irregularity data are available. These packages have built-in functions that make them automatic; some parameters and some methods can be tweaked to make them semi-automatic; the user can also create his or her own values for deseasonalising the data and detrending the data and finally get the predictions, which is essentially manual forecasting.

> ➢ Time Trends Forecast Warehouse Demand Forecasting System is available through ALT-C Systems, Inc., and works only for Windows. It has automatic, semi-automatic, and manual forecasting capabilities.

Most of the software packages mentioned above are built on a statistical understanding of forecasting models, such as simple linear and non-linear regression models and exponential smoothing techniques for trends and seasonality. Decomposition models are used to tackle difficult data sets that don't show obvious trends and seasonality and have cyclical and irregular components in them. ARIMA or Box-Jenkins methods are used for dealing with datasets that will not respond to any of the above simpler methods. The drawback of all these models is that they cannot include censored information. With censored demand information, it is not easy to tackle the data set using traditional forecasting techniques. It is possible to use Cox-regression models to implement censored information, but normally these are not as highly recommended in demand estimation as they are in lifetime failure data analysis.

2.2.4. Bayesian Models

Most of the literature in supply-chain demand estimation focuses on retailers. Bayesian models have gained importance and seen increased practical application recently, even though they have been researched since the 1950s. Most of the literature on Bayesian models is focused on developing optimal inventory models. Therefore, in this section we will highlight all the studies that lead to our ultimate goal.

Choi et al. (2003) [9] investigate an optimal two-stage ordering policy for seasonal products. Before the selling season, a retailer can place orders for a seasonal product from his or her supplier at two distinct stages in order to satisfy the lead-time requirement. Market information is collected at the first stage and is used to update the demand forecast at the second stage by using a Bayesian approach. The ordering cost at the first stage is known, but the ordering cost at the second stage is uncertain. A two-stage dynamic optimization problem is formulated and an optimal policy is derived using dynamic programming. The optimal ordering policy exhibits attractive structural properties and can easily be implemented by a computer program. The detailed implementation scheme is proposed. The service level and profit uncertainty level under the optimal policy are discussed. Extensive numerical analyses are carried out to study the performance of the optimal policy.

Yang et al. (2003) [10] develop a Bayesian method to address the computational challenge of estimating simultaneous demand and supply models that can be applied to both the analysis of household panel data and to aggregated demand data. The method is developed within the context of a heterogeneous discrete choice model coupled with pricing equations derived from either specific competitive structures or linear equations of the kind used in instrumental variable estimation; the method is then applied to a scanner panel dataset of light beer purchases. But again, this study is not solely aimed at the manufacturer, nor does it consider random effects and advertising budgets that can act as catalysts in demand estimation. Wu (2005) [42] considers a decentralized supply chain consisting of two independent players, a manufacturer and a retailer. Under a quantity flexibility contract, the retailer first proposes an initial forecast as the production reference to the manufacturer. Then he or she uses the Bayesian procedure to update demand information and makes the ultimate purchase commitment, which is constrained by the negotiated

flexibility and the manufacturer's production. The incentives of both parties are modeled, and the effects of flexibility, transfer cost, and the number of Bayesian updates on the performance of the two parties is investigated. Results show that more flexibility always benefits the retailer, while the manufacturer can only benefit from a very small range of flexibility. In addition, this contract allows them to share the benefits from information updating.

2.3 Limitations of Literature

In the light of the literature review presented above, this section discusses the limitations of previous models as follows:

- ➢ Classical operation research and forecasting techniques cannot predict demand using censored historical data.
- ➢ Commercial software gives an output for what is fed into the computer and does not account for random regional effects such as weather, geographical location, population density, etc.
- ➢ Frequentist approaches cannot estimate demand with two unknown parameters and known demand distribution.
- ➢ None of the models referenced in the literature, whether mathematical or statistical, consider random effects that can influence future demand.

2.4 Problem Statement and Objectives

In this chapter, the problem of demand estimation at the manufacturer, also termed "macroscopic demand estimation," is addressed. The macro model estimates regional demand for a single product at a manufacturer in the presence of censored and exact sales data while taking into consideration the influence of factors such as advertising expenditure and stochastic regional effects on the demand for that product. Random regional effects such as regional weather, geographical location, population density, family income, etc., are factors that are considered to significantly impact the future demand for that product.

In this chapter, the manufacturers demand estimation problem is addressed. A demand estimation model is formulated in section 2.5 followed by the explanation of the methodology adopted to provide accurate estimates and predictions of demand for a single product manufacturer in section 2.6 , 2.7 and 2.8. In order to evaluate the real potential of this Bayesian MCMC technique, a case study and numerical analysis is presented in sections 2.9. Brief discussion of the numerical results of the case study are presented in section 2.10 followed by conclusions in section 2.11.

Figure 2 below is a pictorial representation of this macroscopic level manufacturers' demand estimation model.
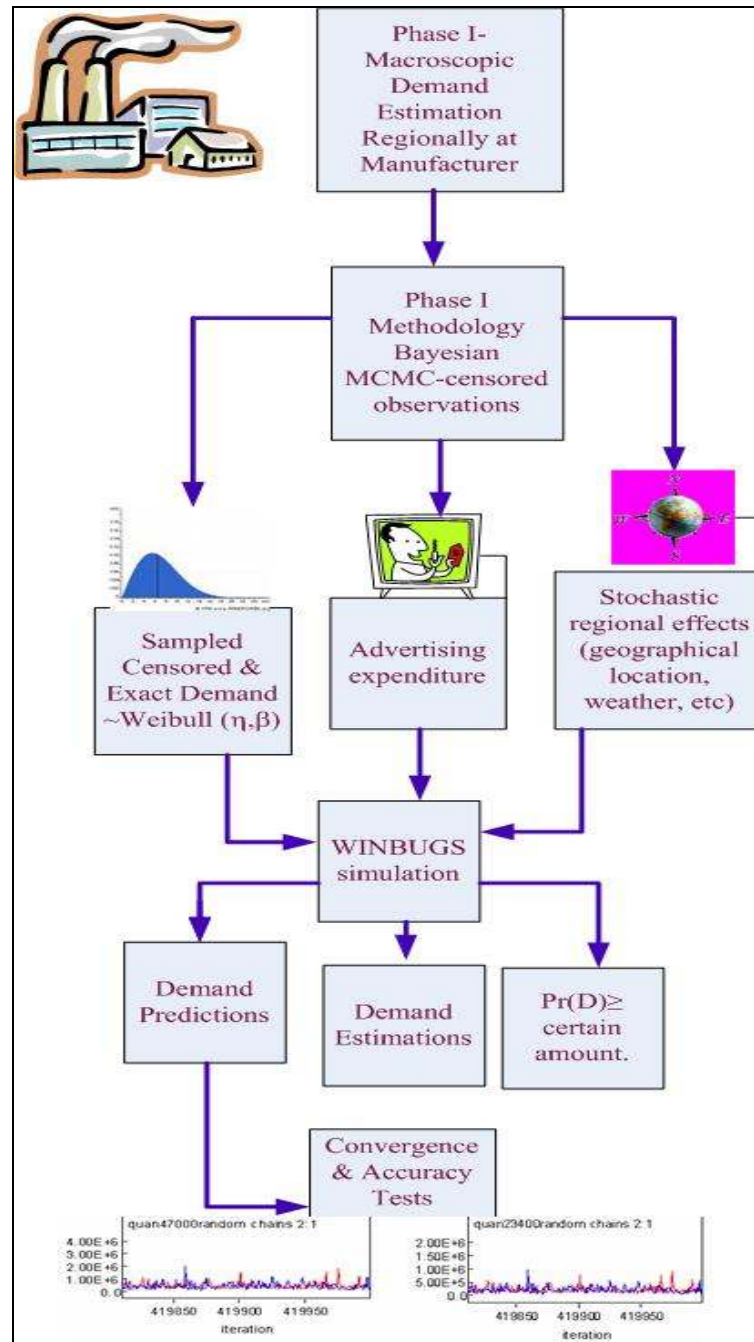
Figure 2: Describes the situation of a single product manufacturer who serves several regions in order to increase .

28

2.5 Model Formulation

The model considered in this study assumes the following: (1) demand data follows a Weibull distribution with two unknown (shape and scale) parameters; (2) the shape parameter of the demand distribution is not fixed or known, but we have a diffused prior belief about that parameter. This makes our model a more generalized version of the problem considered in [**27**], thereby enabling us to avoid the conjugate family concept.

At this point, some common notation that will be used throughout the chapter should be introduced. Let 'y' represent the inventory and 'D' represent the exact and censored demand at particular sampled regions. If the sales are less than the inventory on hand, then we have exact demand information; alternatively, the demand is at least 'y' and is censored. The demand is described by a Weibull distribution function with unknown shape and scale parameters β and η, respectively [**27, 28**]. The scale parameter reflects the size of the market and shape parameter reflects both market size and the precision with which the underlying distribution is known. The cumulative distribution of demand is given by

$$F(d) = P(D \leq d) = 1 - exp\left[-\left(\frac{d}{\eta}\right)^{\beta}\right]; d \geq 0; \beta > 0; \eta > 0 \quad . \qquad (2.6)$$

2.5.1 Scale Parameter

Let 'B' be the allocated advertising expenditure at a particular region. Empirical studies in the past have shown that advertising significantly influences demand for a product [**32**]. Using the

concept of advertising expenditure elasticity of demand similar to price elasticity of demand, we introduce a log-linear regression-like relation between the advertising expenditure and the demand. This log-linear relation is well supported by the real market data of demand dependency on the advertising expenditure. The advertising expenditure elasticity (γ_1) of demand is defined as [**43**]

$$\gamma_1 = \frac{\% \ change \ in \ market \ size}{\% \ change \ in \ adverti\sin g \ budget} = \frac{\partial \eta / \eta}{\partial B / B}$$

Integrating on both sides yields:

$$log(\eta) = \gamma_0 + \gamma_1 \, log(B) \tag{2.7}$$

where, γ_0 is a constant and γ_1 is the advertising expenditure elasticity variable.

While advertising elasticities are less commonly estimated, they can be used to determine whether advertising is predatory (rearranging market shares) or cooperative (shifting demand out). In addition to this relation, the demand at a region is further influenced by the random regions effects. The random regions effect may be associated with the dependence of market size on factors such as pricing, availability of substitutable products, geographical location of the manufacturer and retailers who sell this product, and many other factors that we don't consider specifically but believe would have a significant impact on the sales of the items from that region. Often, these are also called marketing mix variables. The impact of these random regional effects on market size is taken into account by adding a random variable ψ to the above log-linear relation. That is,

$$log(\eta) = \gamma_0 + \gamma_1 log(B) + \psi \tag{2.8}$$

where $\psi \sim N(0, \tau)$ follows a normal distribution with a mean of zero and precision τ which is the

inverse of variance; i.e., $\tau = \dfrac{1}{\sigma^2}$. This is a common notation used in Bayesian literature.

Figure 3 below represents the log linear equation (3) for scale parameter.

2.5.2. Shape Parameter

The variance of demand D is related to the shape parameter as [**44**]:

$$Var(D) = \eta^2 \left[\Gamma \left(1 + \frac{2}{\beta} \right) - \Gamma^2 \left(1 + \frac{1}{\beta} \right) \right] \tag{2.9}$$
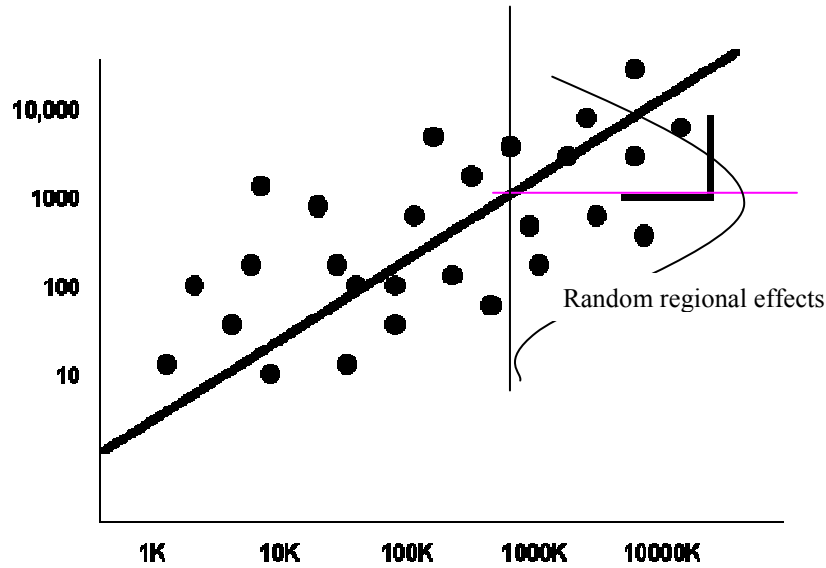
Figure 3: Represents log linear relationship of market size(η) with advertising expenditure(B) and random regional effects(ψ).

31

In particular, the variance of demand decreases with increasing β values. Moreover, the shape parameter β is intimately related to "*demand rate, (λ)*, defined as the probability that a demand is seen in a specified interval Δd given that there is no demand less than d. Mathematically, the demand rate λ is defined as:

$$\lambda = \frac{\Pr(D < d + \Delta d \mid D > d)}{\Delta d}$$

(2.10)

Expanding the above conditional probability, we have:

$$\Pr(D < d + \Delta d \mid D > d) = \frac{\Pr((D < d + \Delta d) \wedge (D > d))}{\Pr(D > d)} = \frac{f(d)\,\Delta d}{1 - F(d)}$$

where F(d) is the cumulative demand distribution and f(d) is the demand density. Hence, the demand rate λ is expressed as:

$$\lambda = \frac{f(d)}{1 - F(d)}\,.$$

(2.11)

The demand rate λ decreases with time when β < 1; alternatively, λ increases with time when β > 1. For β = 1, the demand rate remains constant and the demand distribution becomes exponential. Physically, β < 1 represents an *instant success demand* scenario (fashion goods, video games etc.), β > 1 represents a *late success demand* scenario, and β = 1 represents *random demand* for a product. [**45**],[**44**]

32

2.5.3 Cost Model for Optimal Percentiles

Assuming that the '"newsvendor" is in fact a small company or manufacturer, as in this case, who wants to produce goods for an uncertain market, I use the familiar extended newsvendor cost model with holding costs for selecting optimal demand percentile that should be produced in order to minimize cost for the manufacturer. This is also termed, "cost-based optimization of inventory levels." A generalized version of the cost function for the "newsvendor" (manufacturer) is given below[46]:

$$M(q) = c_f + c_v(q-x) + c_s * E[max(D-q,0)] + H * E[max(q-D,0)] \text{ where}$$

$c_f = Fixed\ Cost$

$c_v = Variable\ Cost$

$c_s = Shortage\ Cost$

$H = Holding\ Cost$

$D = Demand\ seen / predicted\ from\ region$

$x = Initial\ Inventory$

$q = Quantity\ produced$

$E(q-D,0) = Expected\ excess\ stock\ at\ the\ retailer$

$E(D-q,0) = Expected\ shortage\ at\ the\ retailer$

On the basis of the cost function, the determination of the optimal inventory level is a minimization problem. So in the long run, the amount of the optimal demand percentile to produce can be calculated using the following equation:

$$q_{optimal} = F^{-1}\left(\frac{c_s - c_v}{c_s + H}\right) \qquad\qquad (2.12)$$

For the purpose of concreteness, we assume that the costs associated with the above equation are such that we obtain 90th percentile of demand as the optimal percentile that the manufacturer should consider to minimize costs. This would also be close to reality, as this means that we are trying to meet the 90th percent of demand and losing only 10% of sales, assuming that the penalty for shortages is less compared to the holding costs.

2.6 Methodology

Accurate demand estimation is very important in an integrated supply chain, mostly because it helps avoid bull-whip effects and reduces excess holding and stock-outs. There has been ample research conducted in this area of demand estimation and inventory models, as seen in Section 2.2. All of this research provides valuable tools and interesting demand models. In the present study, utilizing the available tools and demand models from literature, a refined demand model is formulated and evaluated. The current micro level demand model attempts to overcome the limitations of traditional literature that were presented in Section 2.3. The novelty of this research lies in utilizing the well known Bayesian MCMC techniques to solve this demand estimation problem in the presence of a combination of factors (advertising expenditure per sampled region, random regional effects and a historical data set that contains both exact and censored demand data) that has not been considered to date by any of the above referenced researchers.

In order to address the issue of censored or missing data, there have been enormous advances in the use of the Bayesian methodology for the analysis of inventory management (see section 2.1 and section 2.2.4) and identifying the optimal policies with known demand distributions and parameters. There exist many practical advantages to the Bayesian approach over the classical forecasting techniques and optimization models discussed in the section 2.2.4. A great advantage of Bayesian models is that they can accommodate unobserved variables, in this case censored demand data. The use of prior probability distribution represents a powerful mechanism for incorporating information from previous studies and for controlling for confounding data. Posterior probabilities can be used as easily interpretable alternatives to p values. Recent developments in Markov Chain Monte Carlo methodology facilitate the implementation of Bayesian analysis of complex data sets that contain censored and exact values. The goal of this chapter is to highlight some advantages and distinct features of the Bayesian analysis of censored demand data in the presence of advertising budget and random regional effects in order to take advantage of this powerful approach.

2.7 Markov Chain Monte Carlo Technique

Markov Chain Monte Carlo (MCMC) methods, sometimes called random walk Monte Carlo methods, are a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its stationary distribution [47]. The state of the chain after a large number of steps is then used as a sample from the desired distribution. The quality of the sample improves as a function of the number of steps. Usually it is not hard to construct a Markov chain (Ross (2003)[48]) with the desired properties. The more difficult problem is to determine how many steps are needed to converge to the stationary

distribution within an acceptable error. A good chain will have rapid mixing---the stationary distribution is reached quickly starting from an arbitrary position. Tools for proving rapid mixing include arguments based on conductance and the coupling method. Typical use of MCMC sampling can only approximate the target distribution, as there is always some residual effect of the starting position. More sophisticated MCMC-based algorithms such as coupling from the past can produce exact samples, at the cost of additional computation and an unbounded (though finite on average) running time. The most common application of these algorithms is the numerical calculation of multi-dimensional integrals. In these methods, an ensemble of "walkers" moves around randomly. At each point where the walker steps, the integrand value at that point is counted towards the integral. The walker then may make a number of tentative steps around the area, looking for a place with a reasonably high contribution to the integral to move into the next. Random walk methods are a kind of random simulation or Monte Carlo method. However, whereas the random samples of the integrand used in a conventional Monte Carlo integration are statistically independent, those used in MCMC are *correlated*. A Markov chain is constructed in such a way as to have the integrand as its equilibrium distribution. Surprisingly, this is often easy to do.

2.7.1 Markov Monte Carlo Techniques [49]

- ➤ Metropolis-Hastings algorithm: Generates a random walk using a proposal density and a method for rejecting proposed moves[50].
- ➤ Gibbs sampling: Requires that all the conditional distributions of the target distribution can be sampled exactly. Gibbs sampling has the advantage that it does not display random walk behavior. However, it can run into problems when variables are strongly

correlated. When this happens, a technique called simultaneous over-relaxation can be used[**50**],[**51**].

➢ Hybrid Markov Chain Monte Carlo: Tries to avoid random walk behavior by introducing an auxiliary momentum vector and implementing Hamiltonian dynamics where the potential function is the target density. The momentum samples are discarded after sampling. The end result of Hybrid MCMC is that proposals move across the sample space in larger steps and are therefore less correlated and converge to the target distribution more rapidly.

➢ Slice sampling: Depends on the principle that one can sample from a distribution by sampling uniformly from the region under the plot of its density function. This method alternates uniform sampling in the vertical direction with uniform sampling from the horizontal "slice" defined by the current vertical position[**52**].

➢ Reversible Jump.

2.7.2 Weaknesses of MCMC techniques

Maximum-likelihood algorithms may not always be able to find the true global maximum of the likelihood function. Similarly, MCMC techniques can fail to converge to the stationary distribution of the posterior probabilities. Failure to visit all highly probable regions of the parameter space because of local maxima in the likelihood curve can be a possible reason for this. However poor proposal mechanisms and/or failure to run the chain long enough are usually the main cause of sample defect.

- ➢ MCMC techniques coupled with Bayesian Estimation are used to solve many inventory problems in supply-chain management by estimating demand. We update the demand information to the central system and then use MCMC techniques to iterate the demand values a large number of times, finally obtaining a demand value that is very close to the actual demand.

- ➢ Many times there are unobserved lost sales (censored demand) in all the supply chains; these MCMC techniques and Bayesian inferences can significantly aid in estimating the cost as well as the inventory levels in any kind of inventory system, including the dynamic newsvendor model, (S, s) and (Q, r).

- ➢ MCMC techniques can also be used in application areas such as like fashion goods, which involve perishable items

2.8 Bayesian Statistics

In this section, we briefly introduce Bayesian analysis concepts and develop a procedure for calculating the posterior distributions of the unknown parameters and point estimates of the demand . Posterior distributions are critical for all the analysis in Bayesian statistics since these distributions contain all the relevant information regarding the unknown parameters for a given set of observations on demand. The Bayesian inferences and point estimates are derived from these posterior distributions[**53, 54, 55, 56**]. The main focus of this chapter is to determine the demand point estimates along with the lower and upper credibility and prediction intervals [**55**].

2.8.1 Components of Bayesian Inference

➢ *Prior information:* the belief or informal estimate we have about the unknown parameters. It can be either non-informative or informative.

➢ *Unknown parameters*: the posterior percentiles, shape parameters, scale parameters, etc.

➢ *Missing or censored* observations.

➢ *Posterior* is critical as it contains all the relevant information regarding unknown parameters.

2.8.2 Prior Information

In many real-world scenarios, there is minimal prior information on demand, on inventory, and on their respective distributions. This uncertainty in prior information is handled in Bayesian statistical analysis through the use of non-informative priors. For example, a non-informative prior on the variance of a normal random variable is given by N(0, t), where t is a small number, usually taken as 0.001, to represent the uncertainty of the prior information on the precision which is the inverse of the variance represented as $\tau = 1/s^2$ (or large variance) [**53, 54**] of the random variable. We do not consider N (0, 0) for non-informative priors since this represents a flat response over the entire number line; consequently, it is "improper" in the sense that there is infinite area under the curve. Use of non-informative priors also reduces the task of sensitivity analysis as we are not using informative priors and hence are sure that our prior beliefs are not

unduly impacting our analysis and results. In this context, we introduce some prior selection methods [**53, 54**]:

- ➢ The classical Bayesian approach assumes that the prior is a necessary evil and the prior chosen should interject the least information possible.

- ➢ A modern Parametric Bayesian approach acknowledges prior information as a useful convenience and chooses prior distributions with desirable properties (e.g. conjugacy). Given a distributional choice, prior parameters are chosen to interject the least information possible.

- ➢ Subjective Bayesian approach considers the prior as a summary of old beliefs and chooses prior distributions based on previous knowledge—either the results of earlier studies or non-scientific opinion.

The model considered in this chapter for the macroscopic problem of estimating demand at the manufacturer has four unknown parameters; namely, γ_0, γ_1, ψ, and β, of which γ_0, γ_1 and ψ are unknown parameters with non-informative priors and β is an unknown with some amount of prior information. We represent the minimal prior information on γ, and γ_1 using N (0.0001,0.001).

The random regional variable ψ is modeled as $\psi \sim N(0, \tau)$, where τ is precision. In order to represent a lack of sufficient prior information on τ, a gamma distribution is used for τ. The gamma distribution applies to unknown quantities that take values between 0 and ∞ (for example, the unknown precision τ of an unknown quantity). Complete ignorance about a positive-valued

unknown quantity is generally represented as a Gamma (0, 0) distribution. Since this distribution is improper, Gamma (ε, ε) is used in practice, with ε a small number such as 0.001. The inverse gamma is a conjugate prior distribution for the normal variance, which is defined as the inverse of the precision parameter that follows a non-informative proper prior gamma in (0.001,0.001):

$$\tau = (1 / \sigma^2) \sim Gamma(0.001, 0.001).$$

As mentioned in Section 2, a Weibull distribution is used for modeling demand $O_t \sim Weibull(\eta, \beta)$ [Azoury (1985) [21] and Lariviere (2002)[28]]. To represent the prior known information on the parameter β, we use $\beta \sim Gamma(1, 0.2)$.

2.8.3 Bayesian Updating

Let $O_t = (O_{t1}, O_{t2})$ denote the demand observation during time period t, where O_{t1} is the sales quantity, which is known exactly and cannot be negative, while O_{t2} is the observed status that can be exact or censored. Let us assume that the inventory be y_t and demand be D_t during time period t. If $y_t > D_t$ then sales is D_t and the observation is exact. Otherwise, the sales quantity is y_t and observation is censored. Thus O_t is determined by y_t and D_t, which we represent mathematically as follows:

$$O_t = y_t \otimes D_t \cong \begin{cases} (D_t, e) & if \quad y_t \geq D_t \\ (y_t, c) & if \quad y_t \leq D_t \end{cases} \tag{2.1}$$

where e and c represent exact and censored information, respectively.

41

Let $\pi_t(\theta)$ denote the prior distribution of unknown demand parameters $\theta = (\beta, \eta)$. The corresponding posterior distribution [55]π_{t+1} is given by

$$\pi_{t+1}(\theta \mid O_t) = \frac{l(O_t \mid \theta) * \pi_t(\theta)}{\int\limits_{\theta \in \Theta} l(O_t \mid \theta') * \pi_t(\theta')d\theta'} \tag{2.2}$$

where the likelihood function $l(O_t \mid \theta)$ of the demand is given by

$$l(O_t \mid \theta) = \begin{cases} f(O_{t1} \mid \theta) & \text{if} \quad O_{t2} = e \\ 1 - F(O_{t1} \mid \theta) & \text{if} \quad O_{t2} = c \end{cases}. \tag{2.3}$$

In the above expression, $f(O_t \mid \theta)$ and $F(O_t \mid \theta)$ denote the probability density and cumulative distributions of demand with two unknown parameters $\theta = (\beta, \eta)$. Using the posterior distributions of the unknown parameters, we can estimate the posterior predictive distribution of the demand as [57]:

$$\pi_{t+1}(D_{new} \mid O_t) = \int \pi_{t+1}(D_{new} \mid \theta, O_t) * \pi_{t+1}(\theta \mid O_t)d\theta . \tag{2.4}$$

Using posterior distributions we can calculate the point estimates. For example if we are interested in calculating the point estimate of the unknown parameters[57]:

$$\tilde{\theta} = \int f(\theta) * \pi_{t+1}(\theta \mid O_t)d\theta \tag{2.5}$$

where $f(\theta)=\theta$ would represent the mean point estimate; similarly, if $f(\theta)=\theta^2$ it would form the second moment point estimate.

In many cases, evaluation of these integrals analytically is not possible. Therefore, Markov Chain Monte Carlo (MCMC) techniques [47];[49];[51];[58] are in common use to evaluate these integrals numerically. The basic principle underlying MCMC technique is that a Markov chain can be constructed with a stationary distribution that is the joint posterior probability distribution of the parameters of the model. These Markov chains are defined using standard algorithms such as Gibbs sampling or Metropolis Hasting algorithms[57]. Using these algorithms, it is possible to implement posterior simulations that form the foundations for all the Bayesian statistical inferences.

2.9 Why use Bayesian MCMC ?

Since there exists a historical demand data set which is considered as present data, and the goal is to estimate/predict the future demand, this follows the Markovian property that states, "the future is independent of the past given the present"; therefore, this forms a Markov chain.

The historical data set evaluated for future demand contains censored observations. As a result, the only statistical tool that can forecast demand is Bayesian statistics.

The posterior integrals are complicated and pose difficultly in solving manually; consequently, I adopt the well known Monte Carlo simulation techniques.

For all the reasons enumerated above, the Bayesian MCMC technique is considered the most apposite tool to solve this problem of demand estimation for the manufacturer.

2.10 Case Study/Numerical Example

To evaluate the model indicated in this chapter we generate synthetic data and perform Bayesian MCMC simulations on the data using the prior information and historical demand data.

2.10.1 Synthetic Data Generation

For the purpose of this research, we considered 30 regions, five advertising expenditure levels, and ten weeks of time period, as this would form a large enough data set of 300 demand points. We used a percentage sales method [59] to obtain the advertising expenditure and chose 90th percentile of demand to represent the censoring point, which is commonly the amount of inventory carried during off-peak seasons. We chose the log-linear regression model presented in Section 2 for the dependence of mean demand on the advertising expenditure. The coefficients of the regression model were selected such that they were best representative of the real-world scenarios. The random regional effects of demand on local advertising expenditure were considered to be normally distributed with zero mean and a standard deviation of 0.5 indicative of the region-to-region variation of 0.25. Since we are using synthetic data, we used non-informative priors on all the unknown parameters in order to minimize their impact on further analysis.

2.10.2 Prior Information

In many real-world scenarios, there exists minimal prior information on demand/inventory, and their respective distributions. This uncertainty in prior information is handled in Bayesian statistical analysis through the use of non-informative priors. For example, a non-informative prior on the variance of a normal random variable is given by $N(0, \tau)$, where τ is a small number, usually taken as 0.001, to represent the uncertainty of the prior information on the precision which is inverse of the variance represented as $\tau = 1/\sigma^2$ (or large variance) of the random variable. We do not consider $N(0, 0)$ for non-informative priors since this represents a flat response over the entire number line; consequently, it is "improper" in the sense that there is infinite area under the curve (since the area under the curve for a pdf should integrate to 1).

The model considered in Section 2 has four unknown parameters, namely, γ_0, γ_1, ψ, and β, of which γ_0, γ_1, and ψ are unknown parameters with non-informative priors, and β is an unknown with some amount of prior information. We represent the minimal prior information on γ_0, and γ_1 using a normal distribution as shown below:

$$\gamma_0 \sim N(0, 0.001) \qquad ; \qquad \gamma_1 \sim N(0, 0.001)$$

The random regional variable ψ is modeled as $\psi \sim N(0, \tau)$, [60] where τ is precision. In order to represent the lack of sufficient prior information on τ, a gamma distribution is used for τ. The gamma distribution applies to unknown quantities that take values between 0 and ∞ (for example, the unknown precision τ of an unknown quantity). Complete ignorance about a positive-valued unknown quantity is generally represented as a Gamma (0, 0) distribution. Since this

distribution is improper, Gamma(ε, ε) is used in practice, with ε a small number such as 0.001. The inverse gamma is a conjugate prior distribution for the normal variance, which is defined as the inverse of a precision parameter that follows a non-informative proper prior gamma in (0.001,0.001)[**50**].

$$\tau = (\frac{1}{\sigma^2}) \sim Gamma(0.001,0.001)$$

As mentioned in Section 2, a Weibull distribution is used for modeling demand $O_t \sim Weibull(\eta, \beta)$ [**21**] and [**30**]. To represent the prior known information on the parameter β, we use $\beta \sim Gamma(1,0.2)$.

2.10.3 Simulation

As explained in Section 3, it is very difficult and sometimes impossible to evaluate the integrals analytically using common mathematical techniques; therefore, we resort to using Monte Carlo simulations. Since our output is dependent on the joint probability distributions generated one stage ahead, it forms a Markov chain since the principle underlying Markov chains states that the future is independent of the past given the present. Hence, these simulations are called Markov Chain Monte Carlo simulations. Since we use a Bayesian paradigm to utilize the present information and predict/estimate the future, these simulations are termed Bayesian MCMC .

Using the synthetic data, the priors described above, and the demand model described in Section 2 we ran the MCMC simulation that used Gibbs algorithm and produced simulated values that

were, after a suitable burn-in period, approximately distributed from the posterior distribution of the parameter of interest or the predictive distribution of the new observation. To provide more precise results, we ran our program five times for one million iterations with a burn-in period of 10,000 iterations [**50**]. This number of iterations was justified as sufficient to obtain samples from the posterior and predictive distributions of interest by using trace plots that showed excellent convergence patterns for the parameters of interest. We also ran the programs for two million iterations and found no appreciable difference in the estimates obtained.

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2.11 Results and Discussions

Using the code in WINBUGS (http://www.mrc-bsu.cam.ac.uk/bugs/) and the simulated data, posterior and predictive distributions were obtained. These posterior distributions help us in attaining the point estimates, credibility intervals and prediction intervals at optimal demand percentiles for the next time period and also for any random regions and random advertising expenditure from either the sampled regions or from the population of regions. We use 90th demand percentiles (section 2.3) so as to avoid lost sales or excess inventory carryovers. Since we do not consider any promotional effects, the model predictions would reflect the time periods where there isn't any promotional activity. Promotional effects can be incorporated in the demand equation as a function of sales data but this would create more complexity to the already complex censored demand problem. Since we use a Bayesian approach, interpretation of the bounds will be slightly different compared to the frequentist approach.

The tables below indicate that we are 95% sure that the 90th percentile of posterior demand lies between 2.5% and 97.5%. Since the posterior densities are right skewed heavy tailed distributions

we consider that the optimal 90th posterior percentile is the median indicated in the tables for simplicity. For all practical purposes using some other cost model or some other criteria, if we expect our 90th posterior percentile to be anything else above or below the median , this could be easily computed from the simulation with the built in capability of the WINBUGS software.

The credibility intervals for the results shown in following tables, between 2.5% and 97.5% imply we have 95% credibility that can be interpreted as : we would give odds of 19 to 1 that the true value of the parameter (percentile) is contained within these particular limits.

If the table is reporting 90th percentile point estimates then the interval is called credibility interval. On the other if the table is reporting 90th percentile predictions for future demand, then the interval is termed prediction interval. Either ways, the interpretation is very similar for both the intervals as described above.

Table 1 presents 90th percentile point estimates for two assigned advertising expenditure. $23400 advertising budget was chosen since it was less than the lowest advertising expenditure value shown in historical data. Similarly $47000 advertising expenditure was chosen because it is the closest larger number than the maximum advertising money spent as seen in the historical data.

Similarly table 2 presents 90th percentile posterior predictions at $23400 and $47000 advertising expenditure for all the sampled regions.

Table 3 and 4 report probability that predicted demand would be less than $100,000 and $170,000 respectively. This kind of information is important while making inventory management decisions by manufacturers in anticipation of future demand.

48

Table 1:Point estimates for 90th percentile demand at (a) 23400 random advertising expenditure & (b) 47000 random advertising expenditure.

Region	(a) Demand Median	Lower CL^2 (2.5%)	Upper CL (97.5%)	(b) Demand Median	Lower CL (2.5%)	Upper CL (97.5%)
1	140200	1.22E+05	1.67E+05	334900	290200	401200
2	152900	132900	1.82E+05	365400	315500	438200
3	216800	1.89E+05	256600	517900	449300	616900
4	85750	74140	102800	204800	175800	247700
5	176600	153900	209500	421900	366200	503300
6	200300	173500	239800	478500	413100	575900
7	140200	121800	167100	334800	290100	401100
8	156300	135900	1.86E+05	373400	322500	447600
9	4.20E+05	355900	519600	1003000	843700	1251000
10	86750	75010	1.04E+05	207200	178100	250300
11	199600	173800	236700	476900	412300	570100
12	126200	109600	150500	301500	260500	361900
13	269600	233400	321900	644100	552700	776100
14	147600	128400	175400	352600	305200	421900
15	150700	1.31E+05	179500	360100	311200	431700
16	151100	131500	179600	361000	313000	431400
17	217100	188100	259600	518400	448200	622800
18	498300	423100	614400	1191000	1003000	1479000
19	313500	2.70E+05	378200	749000	640000	911300
20	268300	231100	324200	640900	548900	779700
21	206400	1.80E+05	244500	493100	428700	587200
22	359300	306900	438300	858400	726700	1056000
23	229300	198700	274300	547900	471800	659900
24	140100	121600	166600	334600	288900	401000
25	460300	3.81E+05	593900	1100000	904300	1428000
26	344700	297100	415400	823400	703900	1001000
27	306700	2.62E+05	374700	732500	621900	901100
28	135100	117500	160900	322800	279400	386600
29	208300	181400	246900	497700	429800	595200
30	150800	131100	179100	360100	312900	429200

Table 2:Bayesian fit for a random effects model with Gamma (1, 0.2) prior on the shape parameter of the Weibull distribution for Demand. Prediction Intervals for the demand generated from each of the 30 sampled regions and a random region selected from the population

Region	(a) Demand Median	Lower PL 2.5%	Upper PL 97.5%	(b) Demand Median	Lower PL 2.5%	Upper PL 97.5%
1	104800	46210	163700	211100	92880	330600
2	114100	50430	178500	230300	101500	360700
3	161800	71480	252500	326700	1.44E+05	509700
4	64020	28290	100400	129100	56870	202900
5	131900	58080	205700	2.66E+05	117100	415400
6	149700	66040	234400	3.02E+05	133300	473200
7	104700	46150	163500	2.11E+05	93150	330200
8	116600	51510	182400	235400	103700	368500
9	313500	138200	4.98E+05	632800	277600	1.01E+06
10	64740	28590	101500	130600	57690	205200
11	149100	65860	232700	300600	132500	469800
12	94290	41720	147400	190100	83800	297500
13	201300	88640	314900	405800	178500	636900
14	110300	48710	172200	222300	98130	347900
15	112500	49670	1.76E+05	2.27E+05	99790	354900
16	112900	49780	176300	227600	100400	356100
17	162200	71470	253700	3.27E+05	143900	5.12E+05
18	371800	163500	590700	750700	3.30E+05	1.19E+06
19	2.34E+05	103200	367500	4.72E+05	207700	7.44E+05
20	200400	88280	3.15E+05	404200	178300	636400
21	154200	67970	240200	3.11E+05	137300	485500
22	268200	118300	423400	540800	238400	856500
23	171200	75550	268200	345200	152200	541600
24	104600	46180	163300	2.11E+05	93080	3.30E+05
25	343600	150700	557700	6.93E+05	304100	1.13E+06
26	257400	113400	404300	519100	228700	818200
27	2.29E+05	101100	361600	461800	203500	731300
28	100900	44520	157600	203700	89890	318100
29	155500	68740	242700	313600	138300	490600
30	112600	49700	175900	2.27E+05	99980	354500

Table 3: Probability of Demand at a given time period will be greater than $100000 for each of the thirty

regions at $23400 advertising budget standard deviation for stochastic regional effects.

Store	Lower CL 2.5%	Median probability of Demand	Upper CL 97.5%
1	0.2445	0.4365	0.6417
2	0.1788	0.3308	0.514
3	0.04489	0.09115	0.1634
4	0.8724	0.9869	0.9997
5	0.1028	0.199	0.3321
6	0.06017	0.1238	0.22
7	0.2455	0.4372	0.6424
8	0.1656	0.3074	0.4822
9	0.002298	0.006265	0.01473
10	0.8598	0.9839	0.9995
11	0.06281	0.1256	0.2198
12	0.3518	0.5868	0.7945
13	0.01732	0.03824	0.07511
14	0.2055	0.3715	0.5637
15	0.1894	0.3471	0.5342
16	0.1864	0.3439	0.5321
17	0.04315	0.09065	0.1657
18	0.001114	0.003109	0.007584
19	0.008821	0.02072	0.04287
20	0.01734	0.03896	0.07625
21	0.05504	0.1103	0.1945
22	0.004754	0.01188	0.02605
23	0.03454	0.073	0.1349
24	0.2477	0.4382	0.6447
25	0.001381	0.004303	0.01087
26	0.006001	0.01409	0.02941
27	0.009367	0.02269	0.04756
28	0.2807	0.4876	0.6961
29	0.0529	0.1065	0.1888
30	0.1883	0.3464	0.5365

Table 4:Probability of Demand at a given time period will be greater than $170000 for each of the thirty

regions at $47000 advertising budget for stochastic regional effects.

Store	Lower CL 2.5%	Median probability of Demand	Upper CL 97.5%
1	0.1271	0.2468	0.4063
2	0.0905	0.18	0.3093
3	0.02168	0.04614	0.08805
4	0.6345	0.8827	0.9817
5	0.05081	0.1039	0.1866
6	0.02952	0.06326	0.1191
7	0.1285	0.2473	0.4043
8	0.08363	0.1659	0.2862
9	0.00108	0.003101	0.007768
10	0.6184	0.8703	0.9776
11	0.03038	0.06418	0.1208
12	0.1896	0.354	0.5512
13	0.008178	0.01908	0.04018
14	0.1053	0.2051	0.3437
15	0.09665	0.1901	0.3221
16	0.09481	0.188	0.3211
17	0.02101	0.04589	0.08838
18	5.22E-04	0.001537	0.003998
19	0.004156	0.0103	0.02268
20	0.008313	0.01945	0.04009
21	0.02683	0.05611	0.105
22	0.002231	0.00589	0.01377
23	0.01664	0.03677	0.07217
24	0.1286	0.2478	0.4091
25	6.57E-04	0.002128	0.005663
26	0.002827	0.006989	0.01551
27	0.004465	0.01128	0.02499
28	0.1484	0.2814	0.4513
29	0.0254	0.05409	0.1033
30	0.09677	0.1897	0.3209

2.11.1 Posterior Density

In this section we graphically express smoothed kernel posterior estimates of the parameters of interest for random regions only. (Graphical posterior densities for all the sampled regions and other tables will be provided on request). These posterior densities play a major role in determining the optimal values for the 90th percentile of demand. Since these densities are right skewed heavy tailed distributions we presume that the optimal value for the 90th percentile would be median. Figures 4 and 5 are indicative of the posterior right skewed densities of the 90th percentiles.
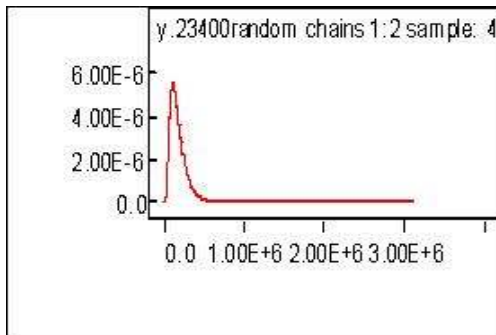
Figure 4: Heavily right skewed posterior density for a random region selected from the population of regions at $23400 advertising	**Figure 5:** Heavy tailed right skewed posterior density distribution for parameter estimates at 90th percentile demand for $23.4K
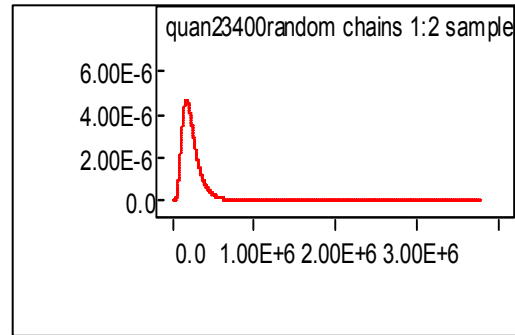
2.11.2 Sensitivity Analysis

A sensitivity analysis is performed using different region-to-region variation. To demonstrate the effect of increasing region-to-region variation table 5, table 6, table 7 and table 8present the 90th percentile credibility and prediction intervals for a randomly selected region from the population of regions to which the manufacturer sells his product. These tables compare the intervals for two different stochastic regional effects of 0.5 and 1.5 standard deviation.

In all the tables above "Random region" corresponds to a region selected randomly from the population of regions nationwide from this retail chain with a region to region standard deviation of 0.5 .

Median demand corresponds to the 90th percentile posterior estimate that should be produced in order to avoid excess inventory holding costs or stock-out situations at the manufacturers end.

The CL stands for credibility limits interpreted according to the Bayesian paradigm, similar to confidence limits in frequentist literature. These Lower and Upper CL correspond respectively to the 2.5th and 97.5th percentiles of the posterior distribution. The credibility intervals in table 5, table 7 and table 8 are extremely wide for larger variation. This is a clear indication of direct relation of wider intervals to increasing region-to-region variation finally leading to less reliable predictions.

PL stands for prediction limits. Lower PL is the 2.5th percentile and Upper PL is the 97.5th percentile of posterior prediction distribution. The region between this range can be interpreted as: 95% probability of seeing future demand to fluctuate between the lower and upper prediction

limits for the given advertising budgets. Once again, the prediction intervals indicated in table 6 are large for larger variation indicating no sufficient evidence in favor of the predictions.

Table 7 and Table 8 report 90th percentile posterior point estimate for the probability of demand greater than $100000 when the advertising expenditure is lower end of the horizon $23400 and $170000 while the advertising expenditure is upper end of the horizon $47000 respectively.

Both tables 7 and 8 indicate that the probabilities attained when the standard deviation associated with stochastic regional effects is higher the probabilities attained are flaky prediction.

All the tables below indicate that there exists significant region-to region variation that translates to unsatisfactory predictions, probabilities and estimates.

Table 5:Point estimates for random region at 90th percentile demand at (a) $23.4K ; and (b) $47K advertising budgets for region to region variation of 0.5 and 1.5 respectively.

Region	(a) Demand Median	Lower CL 2.5%	Upper CL 97.5%	(b) Demand Median	Lower CL 2.5%	Upper CL 97.5%
Random region($\sigma = 0.5$)	202100	77410	528400	482900	184800	1.27E+06
Random region ^{3} ($\sigma = 1.5$)	184700	5569	6.2E+06	450400	13590	1.52E+07

Table 6:Bayesian fit demand model with Gamma (1, 0.2) prior on the shape parameter of the Weibull . Prediction Intervals for random regions with region to region variation of 0.5 and 1.5 for (a) $23400 (b) $47000 advertising budgets

Region	(a) Demand Median	Lower PL 2.5%	Upper PL 97.5%	(b) Demand Median	Lower PL 2.5%	Upper PL 97.5%
Random region($\sigma = 0.5$)	145600	43800	429300	293700	88170	866800
Random region^3 ($\sigma = 1.5$)	127200	3575	4.5E+6	258700	7218	9.21E+6

Table 7: Probability of Demand .for a random region with region to region variation of 0.5 and 1.5 in a given time period will be greater than $100000 at $23400 advertising budget

Region	Lower CL 2.5%	Median	Upper CL 97.5%
Random region($\sigma = 0.5$)	0.002409	0.1205	0.9986
Random region^3 ($\sigma = 1.5$)	3.78E-07	0.2041	1

Table 8:Probability of Demand .for a random region with region to region variation of 0.5 and 1.5 in a given time period will be greater than $170000 at $47000 advertising budget

Region	Lower CL 2.5%	Median	Upper CL 97.5%
Random region($\sigma = 0.5$)	0.001176	0.06162	0.9617
Random region ($\sigma = 1.5$)	1.93E-07	0.111	1

2.11.3 Convergence of the Simulation

Since MCMC simulation is applied to synthetic data, a check for convergence is performed that indicates the reliability of this approach. In order to check for convergence, trace plots of the posterior observations are graphed. Check for convergence is important in case of simulations especially using synthetic data as it shows us that we have reached the optimal solution, using the simulation techniques and data. Below are the some trace plots which show zero convergence for random regions and also some of the sampled regions. In this chapter two chain are run simultaneously[**50**]. The trace plots show each chain in a different color. Figure 5 and 6 indicate convergence due to overlapping trace plots for both the chains.

Looking at figures 6 and 7, I am reasonably confident that convergence has been achieved since both the chains appear to be overlapping one another[**50**].
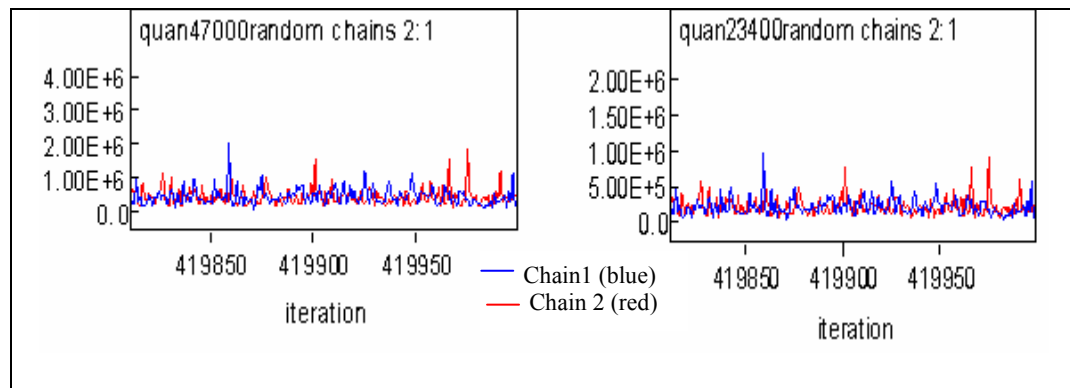
Figure 6 : Trace plot for checking convergence for 90th percentile point estimates of demand for a random region selected from the population of regions at $23.4K & $47K advertising expenditure.
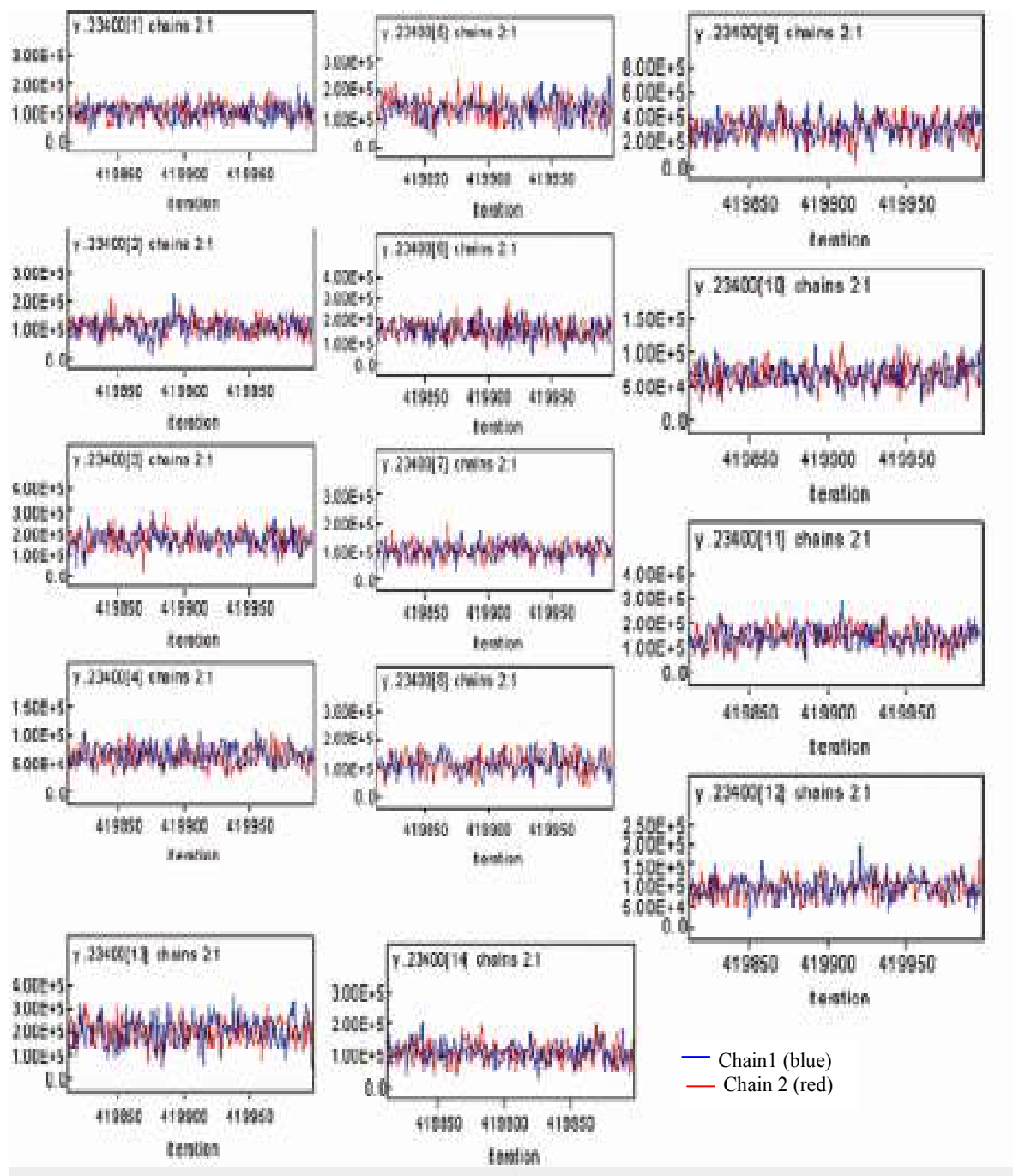
Figure 7: Predictions trace plots for checking convergence for 15 of the 30 sampled regions at $47000 advertising expenditure

2.11.4 Accuracy Evaluation

Once it is ensured that convergence has been achieved, we need to run the simulation for a further number of iterations to obtain samples that can be used for posterior inference. The more samples we save, the more accurate we will be about our posterior estimates. Hence we run the simulation for one million iterations five times and two million iterations twice and did not find appreciable difference.

One way to assess the accuracy of the posterior estimates is by calculating the Monte Carlo error(MC error) for each parameter [**50**]. This is an estimate of the difference between the mean of the sampled values (which we are using as our estimate of the posterior mean for each parameter) and the true posterior mean. As a rule of thumb, the simulation should be run until the MC error for each parameter of interest is less than about 5% of the sample standard deviation [**50**]. Tabulated below are some of the results indicating MC error is less than 5% of the sample standard deviation for estimations, and predictions at \$23,400 advertising budget for random regions.

Table 9 clearly indicates MC error is less than 5% of the sample standard deviation (sd) at one millionth iteration for the 90th percentile estimate of demand. MC error from table 10 is also below 5% of the sample standard deviation which means that the accuracy of this simulation is very good hence one can be certain about the predictions even though we have wider prediction intervals.

Table 9 : Estimation accuracy for 90th percentile demand verified for a random region selected from the population of regions at $23,400 advertising expenditure .

Region	sd	MC error	2.50%	median	97.50%
Random region	120200	491.5	77410	202100	528400

Table 10: Prediction accuracy for 90th percentile demand verified for all the thirty sampled regions at $47000 advertising expenditure.

Region	Sd	MC error	Demand Median	Lower PL 2.5%	Upper PL 97.5%
1	60780	145.4	211100	92880	330600
2	66430	186.4	230300	101500	360700
3	93820	244.6	326700	1.44E+05	509700
4	37370	114.8	129100	56870	202900
5	76450	179.9	2.66E+05	117100	415400
6	87190	192.2	3.02E+05	133300	473200
7	60810	130.7	2.11E+05	93150	330200
8	67720	189.5	235400	103700	368500
9	1.8E+05	685.4	632800	277600	1.01E+06
10	37820	108.6	130600	57690	205200
11	86530	257.2	300600	132500	469800
12	54830	139	190100	83800	297500
13	117700	420.5	405800	178500	636900
14	64050	150.9	222300	98130	347900
15	65320	164.5	2.27E+05	99790	354900
16	65450	155.4	227600	100400	356100
17	94210	199.2	3.27E+05	143900	5.12E+05
18	221400	813.3	750700	3.30E+05	1.19E+06
19	137400	463.7	4.72E+05	207700	7.44E+05
20	117600	317.7	404200	178300	636400
21	89260	190.9	3.11E+05	137300	485500
22	158900	583	540800	238400	856500
23	99980	275.6	345200	152200	541600
24	60790	157.1	2.11E+05	93080	3.30E+05
25	210500	718.1	6.93E+05	304100	1.13E+06
26	150800	506.3	519100	228700	818200
27	135200	383.3	461800	203500	731300
28	58500	131.7	203700	89890	318100
29	90350	290.2	313600	138300	490600
30	65330	130	2.27E+05	99980	354500

2.12 Conclusions

When analyzing data, parameters of statistical models are typically not the quantities of interest. Indeed, we are interested in functions of parameters, such as predicted values [**61, 62**] Using Bayesian methods makes it easy to compute any type of quantities of interest, and propagate the uncertainty about parameters into uncertainty about these quantities. Current study focuses on how to make credible demand estimations and reliable predictions from censored and uncensored data which is collected from different regions that are different from each other which is translated to random regional effects or regions to regions variation presented in section 2 of the demand model.

In this application the credibility intervals obtained are fairly narrow compared to the prediction intervals which is always true since we are trying to predict optimal demand in a follow up period for an exact regions which is more uncertain compared to predicting average demand for all the regions combined. Therefore we have wider prediction intervals compared to credibility intervals.

Random regional effects (ψ−presented in section 2.2) also plays a significant role in estimation and prediction of demand values at 90th percentile. In order to see the effect of regions to regions variation we performed a sensitivity analysis by increasing the standard deviation associated with regions to regions variation to 1.5 which produced very large credibility and prediction intervals. This calls into question our ability to make statements about the reliability of the predictions and estimations of demand in the future period given this advertising budget and demand for the present for large regional variations. However, the results discussed in this paper give us an idea of how to apply tools like Bayesian MCMC techniques to real world scenarios which is very

important. This kind of analysis might be of interest to manufacturers of single products, who would like to predict their demand or to retail chains for different parameters of interest in order to increase their fill rates and customer service levels.

Chapter III

Pricing decisions at competing retailers for competing substitutable

products-Microscopic Level

Abstract

Second phase of this dissertation examines the influence of competition among the supply chain partners on demand for a product. Competition among the manufacturers and retailers is introduced through the product pricing decisions and the shelf space allocated for each of the products and their substitutes, which have significant impact on product's demand. In this chapter, game theory based methodologies are employed since competition within and among supply chain partners is considered critical. In particular, we consider Cournot and Stackelberg games to account for competing pricing decisions and shelf space allocation strategies. The impact of shelf space allocated to substitutable products on demand is studied by considering a game between two manufacturers who are competing for shelf space allocation at a single retailer. Subsequently, the combined effect of pricing competition among retailers and the manufacturers through their substitutable products is modeled by considering a game that includes two manufacturers competing for shelf space allocation at two competing retailers. In each of the games, Nash equilibrium is achieved by optimizing the profit functions, which results in optimal pricing and shelf space allocations. Numerical results presented in this chapter indicate that (1) Cournot games are most profitable along the whole supply chain than Stackelberg games, and (2) Lower the retail price translates to lower wholesale price that implies greater the shelf space allocated for that product (3) Manufacturers can use their prices as valuable tools to intervene into the process of decision making at the retailers.

Keywords: Nash equilibrium, Cournot game, Stackelberg game, shelf space allocation, pricing decisions, cooperative, non-cooperative, direct and cross elasticities, retailers and manufacturers.

3.1 Introduction

In Chapter 2, a macro-level demand estimation model at the manufacturer level in a supply chain is presented. This model is capable of estimating regional demand for a single product at a manufacturer in the presence of censored and exact sales data, while taking into consideration the influence of factors such as advertising/marketing expenditure and stochastic regional effects. This chapter presents a micro-level demand estimation model that calculates optimal demand values at the retailer level of a supply chain.

Demand estimation for a product at the retail level of a supply chain is crucial for a variety of management decisions, such as the ideal product assortment to carry, how much of each item should be stocked, and how often the stock should be replenished. Studies such as those done by Gruen et al. (2002) [**63**], who examined consumer response to stock-outs across eight categories at retailers worldwide, point to the importance of demand estimation for retailers. This study reported that during a stock-out, 45% of customers will substitute, i.e., buy one of the available items from that category; 15% will delay purchase; 31% will switch to another store; and 9% will not buy any item at all. That is, this study suggests that when faced with stock-outs, consumers will often either buy substitute items at the same retailer or switch to another store. These numbers also highlight the importance of competition among substitutable products and competition among retailers in terms of their pricing decisions on demand estimation.

Typically, competition among equally viable (from a product pricing and substitutability points of view) products is achieved at a retailer level by the amount of shelf space allocated for each of the products. Since the amount of shelf space allocated for a product significantly impacts the sales

volume of that product, retailers regularly make decisions about the types of products to display (product assortment) and the amount of shelf space to allocate for each of the products thereby influencing customers' purchasing decisions [64].

Another critical aspect of a product's demand curve is the price elasticity of the product, i.e., how much of that product is demanded when the price changes. The price elasticity of demand underlines the importance of price in estimating demand for a product [65]. Typically, price elasticity of demand is negative implying that consumers might buy more of a product at lower prices and less at higher prices, all other things being equal. Warehouse/retail outlets such as Costco and Sam's Club are examples of this. On the other hand, price elasticity of demand may occasionally become positive implying that higher the price higher the demand. Such an anomaly can be traced back to the perceived notion that the product with a higher price either offers higher quality or more operational features in which case substitutability of the competing products is not well understood. In such cases, personal preferences and brand name management come to forefront. The demand estimation for these types of products is not considered in this chapter

The purpose of this chapter/study is to introduce a microscopic local demand estimation model at the retailer level. This micro model estimates the optimal demand faced by the retailer when attempting to make optimal pricing and shelf-space allocation decisions. In particular, demand in the micro phase is evaluated in the presence of competition among retailers and manufacturers through their competing substitutable products, shelf-space allocation for their respective products, and pricing decisions.

3.2 Literature Review

Most retailers face challenges such as how to respond to consumer's ever-changing demands and how to adapt themselves to keen competition in dynamic market. Therefore retailers need to frequently make decisions about which products to display (assortment) and how much shelf space to allocate these products. Shelf space is an important resource for retail stores since great quantities of products compete for the limited shelf space at display. Shelf space along with pricing decisions play a vital role in increasing or decreasing sales volume or future demand for a product as well as its substitute. Plethora of research has been conducted in the areas of demand estimation using various tools and techniques. Therefore the literature review section has been categorized on basis of the tools and techniques used to estimate demand mostly at retailers. Below are the summaries of each categories.

3.2.1. Statistical Models

The business of a retailer is not only to provide services to his customers but also space to his suppliers/manufacturers while making appropriate pricing decisions in order to withstand competition from other retailers selling same products. Cairns (1962)[66] in his study indicates that models of market behavior assume that the customer gets his goods directly from the manufacturer and stresses that there has been little attention given to the relation between manufactures, retailers, customers and sales of a product.. Cairns (1963)[67] proposed a graphical solution for the problem of allocating shelf space to two products in order to maximize total gross profit, taking space elasticity into account. His method may be extended to the case of many products. Some basic rules for the allocation of shelf space are used in the industry. In the best

known of these rule the proportion of the available shelf space allocated to a product equals its contribution to some measure of profit. But this was unable to provide us with the responses of sales to changes in shelf space. Subsequently, Cox (1964) [68] tested whether food sales are responsive to shelf space or not. In particular sales of impulses are effected more than staples. Using a 6x6 Latin square design for a large supermarket chain Cox (1964) [68] rejected the original hypothesis that sales of impulses is more effected than staples. This study also indicated how distortion of variation between regions and time periods can be statistically eliminated. But this study couldn't give us interaction effects and competition for shelf space between manufacturers. Similarly, Banks (1965) [69] uses a 4x4 Latin square design as, it was felt that a mechanical dispenser was necessary for sales to reach the highest volume possible when this product was offered in drug regions or other sources of fountain sales.

Cox (1964) [68]and Banks (1965)[69] used single and double Latin square design for conducting experiments. They use single square design because it avoided repeated measures through time. The double Latin square design involves one replication of the single square. It was used to increase the power of the design and to establish the reliability of the results. But these couldn't not be used for different retail chain like a drug regions as they don't have enough degrees of freedom. Hence Kotzan and Evanson (1969) [70]considered a replicated balanced Latin square design and conducted a study to examine the sales effect of altering product shelf facings of four selected products in a chain drug store environment. The specific objective of this study was to isolate statistically the sales to shelf facing relationship and ascertain whether this relationship was significant. But Kotzan et al (1969) did not attempt to determine why a significant relationship existed. Subsequently, Frank and Massy (1970)[71] report that a cross-sectional analysis of the effects of shelf position and space on sales for a frequently purchased branded grocery product. This study uses the ideas of design of experiments and is concerned with

answering questions like effect of varying the number of facings of a brands shelf display. In this study it was found that regular or best selling size items were affected by space changes more than were off size (largest or smallest size) items and only a modest effect to varying the shelf level at which a product was normally sold and they noted no significant interaction between the effects of shelf space on sales. Curhan (1972)[72] hypothesized that space elasticity was a function of products physical properties, merchandising characteristics and use characteristics. However, eleven independent variables explained but 1% of the variance in space elasticity for 493 experimental changes. Still elasticity was higher for private brands that for store brands and for impulse as opposed to staple items. Space allocation in UK grocery retailing is the topic of interest to many researchers, therefore they used other statistical tool available like PCA (principal component analysis). Davies & Ward (2000) [73] in their study indicated the of principal component analysis techniques over six variables that were identified on basis of questionnaires returned to the authors. The final model developed using PCA and structured equation modeling approaches had two clusters of externally related variables and four clusters of internally related variables and this model was able to capture 70% of the total richness in the full 48 variable set of responses. The expert retail opinion given on this final model by four willing to participate retailers was there is no evidence of the use of a systematic algorithmic support for the interrelationships uncovered in this study. Thus by having systematically examined management insight the generated model should be uniquely well placed to represent the interaction of variables at fixture category and store level. However the present form of the model does not allow the development of decision support.

The limitations encountered in frequentist statistical approaches such as expensive models due to replication, complication in setting up experiments, difficulties associated with data collection and the lack of generalized conclusions for a group or class of products led to the next category of

research models in shelf space allocation literature known as Operations research and/or optimization models also known as mathematical models.

3.2. 2. Optimization and Operation Research

Urban (1969) [74] in his paper develops an interesting mathematical model of the interaction among products for normative strategy recommendations. The model includes aggregate product group marketing mix, product interdependency, and competitive brand effects. Although reasonable descriptive adequacy was found it would be useful to have an information system that builds a data band on the products performance to obtain more accurate input estimates and to test more complex response forms. This kind of models lead to the invention of models that could also optimize brand selection along with optimal shelf space allocation. Anderson and Amato (1974)[75] realizing the importance came up with a mathematical model for simultaneously determining optimal brand selection and shelf space allocation. They addressed a short term resource allocation problem faced by a retail distribution and breaks up the total market demand according to various levels of brand preferences that could conceivably exist in final markets and then employ an algorithm similar to the one used to solve fixed charge problem to find optimal brand mix and display area allocation, but this methodology did not address assortment issues or elasticities involved. Hansen and Heinsbroek (1979) [76]were the first few to develop an optimization model and an algorithm for the simultaneous optimal selection among a given set of products of the assortment of products to be sold in a supermarket and the allocation of shelf space to these products. They used a nonlinear demand function which incorporates individual space-elasticities but disregards cross-elasticities from similar products. Binary variables for handling assortment decisions are included but again there is no cross-elasticity. Subsequently,

Crostjens and Doyle (1981,1983)[**12, 77**] criticized the models developed by Hansen and Heinsbroek (1979)[**76**] and Anderson and Amato (1974) [**75**]they ignore the interactions that can exist between different products in the store, that is, substitution or complementary effects. These authors suggested a model that takes into account main and cross-space elasticities, different product profit margins, and inventory-management costs. Their results indicate that retailers are better off when they allow more space to the products with higher margins and higher space elasticity when the products are substitutes. A drawback in Corstjens and Doyle (1981, 1983)[**12, 77**] is that their models do not provide integer solutions as requested in such problems. Zufreyden (1986) [**78**] suggested an integer-programming model with only main elasticity effects of shelfspace and nonspace variables. The author did not indicate any rule suggesting an optimal allocation of shelf space. However, the reported simulation results show that the higher the space elasticity, the higher the allocated shelf space. Bultez and Naert (1988) [**79**] in their *S.H.A.R.P's* model formulate the shelf-space-allocation problem as a standard mathematical programming one and provide an allocation rule that gives priority to items whose displays are the most profitable. Dreze et. al (1994) [**80**] performed a series of field studies to measure the effectiveness of two shelf management techniques "space-to-movement" and "product reorganization" as they realized that shelf space allocation followed rules of thumb to guide them in practice. Using the results of their field study they concluded that location of the item had larger impact on sales of the product relative to changes in the number of facings allocated to a brand as long as minimum threshold (to avoid stock-outs) was maintained in a retail store. However, Urban (1998) [**11**]suggested a heuristic that allocates the shelf space by removing at each iteration the item in the assortment with the lowest contribution to profits. The procedure stops when profits start to decrease. Building on Corstjens and Doyle (1981)[**12**], Borin et al (1994) [**13**] used an elaborated version of the same demand function to allow for simultaneous decisions about assortment selection and shelf space allocations. Even Yang and Chen (1999) [**14**]used the simplified version of the same

demand formulation by not including the cross-elasticities and assumed that a products profit is linear within a small number of facings which are constructed by the products lower and upper bound of the number of facings. Both the models developed in[**13, 14**] have the following disadvantages : they concentrate on the revenue aspect and ignore the cost of the operations explicitly. For example smaller the shelf space allocated to a product greater the frequency of restocking that item, which thereby increases the restocking cost. Bookbinder and Zaccour (2001) [**81**] proposed an optimization model that provides the percentage of space allocated to each item based on direct product profitability. Yang (2001) [**82**] built an algorithm similar to the one used for solving the knapsack problem. Shelf space is allocated according to brand weight, measured by the ratio of sales profits per display area. The allocation is done after the satisfaction of the space availability constraint.

Irion et. al. (2004) [**83**] also developed a demand model similar to Crostjens and Doyle (1981,1983) [**12, 77**] by introducing certain marketing variables in the demand function through general production terms. They use piecewise linearization techniques to solve for the optimal shelf space allocation through retailers profit function. Most of the studies addressed in this section do not account for the pricing competition as well as substitution issues. Moreover most of the models described here are rarely implementable in real world. Therefore some big corporations came forward to develop some commercial software that could solve the problem of shelf space allocation and assortment strategies based on heuristics.

3. 2. 3. Commercial Software

Since computers gained importance there have been many computer codes written so as to ease this process of shelf space allocation. But not all were really successful. Zurfryden (1986) ([**84**] developed a program that applies modeling principles and have gained many customers within the retailing industry due to their general simplicity and easily implementable decisions. The use of planogram software however, enables a user to do much more advanced and detailed analysis. Most planogram programs even automatically add product images to products, in addition to providing dynamic shading and labeling to better show opportunities in the set. One step further is to automate the production of planograms where a retailer or merchandiser requires many planograms to be produced at once, based on store specific data.Consequently most retailers use these tools to planogram accounting which would help reduce manual labor hours spent manipulating the shelves Today there are several PC-based software programs that can provide the retailers with realistic view of the shelves and are capable of allocating shelf space according to simple heuristics such as turnover, gross profit or margin, using handling and inventory costs as constraints. There are a number of companies offering planogram creation software

➢ *QUANT:* This is a new generation of complex space management solutions. It is built on innovative planogram creation software *Quant Studio* (available for Windows, Macintosh and Linux) allows automating the creation of many planograms respecting real measures of shelves at once. Internet portal *iQuant* and connected technologies allow dynamic distribution of planograms to stores, fast feedback from stores to merchandiser and presentation of analyzed data.

➢ *JDA INTACTIX* : This software offers a variety of space management solutions including

Space Planning, formerly known as Pro Space.

➢ *PLANO GRAPHICS*: This planogram software is one of the least expensive, most innovative planogramming solutions available. Plano Graphics is widely used in the US, Europe and Asia and is available for both Windows and Macintosh.

➢ *DESIGNER WORKSTATION : I*s automated planogram production software developed by IRI and distributed in the UK by Retail Smart Ltd. Retail smart offer a range space planning solutions where range management decisions are made according to essentially derived data - based on the performance of 'space to sales'.

➢ *SPACEMAN* developed by ACNielsen offers a wide range of space management solutions.

➢ *SHELF LOGIC* sells three versions of its low-cost, dedicated planogramming software and has a user base spanning the full range of businesses from the smallest to the largest and includes manufacturers, retailers, distributors, sales forces, importers and more.

➢ *MARKETMAX:* Provider of merchandise analysis, planning, and optimization solutions. Their Planogram Manager is a PC-based application for developing planogram.

➢ *SPACEMATE:* Ingen Spacemate is software for creating, viewing and editing planogram. Various rules can be applied to the products in order to produce an optimal planogram

➢ *METERMAN:* This is an example of a trend, where the manufacturer, in this case Sandberg, offers a planogram service to their own dealers. Meter Man is unique due to the fact that it is free of charge, it designs the planogram automatically from given parameters, it's switchable between boxes and photographic product facings, and the same software combines automated reordering plus an advances but easy-to-use statistics module. Meter Man does also take care of price tag printing, design of reordering lists etc. But at present it does only work with Sandberg's own products.

➢ *COSMOS:* This is based on a rule developed by Buzzell, Salmon and Vancil (1965) [**85**]

that removes the least profitable item and allocated the space to most profitable ones.

➢ *APPOLLO:* The Apollo Space Management System is powerful software to help retailers and manufacturers increase profits by managing shelf space wisely. Using it, retailers can directly assess the financial performance of a recommended shelf set. Combined with InfoScan data, Apollo is a powerful tool to manage distribution and shelf space. TotalStore™ applies Apollo algorithms to the entire store, enabling retailers to improve space productivity for stores, departments, aisles, gondolas, sections, shelves or items.

The drawback of such computer programs or systems arises from their incompetence to capture the demand effects. Since these softwares suffered some drawbacks, corporate America funded researchers to conduct research in the area of shelf space allocation, which lead to the famous Beer and diapers study. Researchers called this area artificial intelligence or data mining. From that time on the field of data mining fast spread its wings into retail world.

3. 2. 4.Artificial Intelligence/data Mining Models

During the past decade, there have been a variety of significant developments in data mining techniques. Some of these developments are implemented in customized service to develop customer relationship. Customized service is actually crucial in retail markets. Marketing managers can develop long-term and pleasant relationships with customers if they can detect and predict changes in customer behavior. In the dynamic retail market, understanding changes in customer behavior can help managers to establish effective promotion campaigns. Not until the famous example of beers and diapers hit the retailing ground was data mining popular. This example that is cited everywhere created a revolution in shelf space allocation literature and

exposed certain consumer buying behavior patterns that lead to further analysis in this area using data mining techniques. The most frequented approaches are market basket analysis, association rules, support vector machines a few to start with. There have been occasions where researchers used wavelet theories and other complex tools of data mining to deal with shelf space allocation problems. Russell and Pertersen (2000) [86] present a new approach to market basket analysis by assuming that choice in one category impacts choice in other categories. This actually implies that the researcher can specify the probability that the consumer will actually pick another item given he has picked a certain item and finally show that by using these condition choice models, it is possible to infer the market basket distribution that explains purchasing in all categories. This paper adds competition to its dimension which helps us understand the placement of items on shelf for betterment of retailers profits. Chen, Chiu and Chang (2005) [87] in their article integrate customer behavioral variables, demographic variables, and transaction database to establish a method of mining changes in customer behavior. For mining change patterns, two extended measures of similarity and unexpectedness are designed to analyze the degree of resemblance between patterns at different time periods. The proposed approach for mining changes in customer behavior can assist managers in developing better marketing strategies. These would help in identifying the placement of products on shelves.

The more sophisticated mining techniques, like the effect of spatial relationship such as shelf space adjacencies of distinct items, are superior to traditional approaches in retail knowledge discovery, such as the market basket analysis or frequent-buyer program [88]. In some cases, the fact that items sell well together is obvious, such as laundry detergent and fabric softener, greeting cards and seasonal candy, or coffee and coffee makers. Occasionally, however, the fact that certain items would sell well together is far from obvious, such as in the case of diapers and beer [89]or bottled juice and cold remedies . The true reason behind such purchase patterns

remain unclear; it may be due to their close proximity in shelf location or other consumer behavior we have yet to discover. In this regard, the market basket analysis or frequent-buyer program is unable to provide satisfactory results. The proposed scheme attempts to dig for obscure clues by introducing the spatial relationship and transaction time information into the mining techniques. The visual effect of adjacency can stimulate impulse purchases that account for 70% of buying decisions in a supermarket [**14**]. In light of this potential, Chen, Chen and Tung (2006) [**90**] in their paper attempt to discover the implicit, yet meaningful, relationship between the relative spatial "distance" of displayed products and the items' unit sales in a retail store using data mining techniques. Special focus is placed on building a novel representation scheme for the historical transaction data and on developing an efficient and robust algorithm for knowledge mining. The proposed approaches measure and classify the effects of spatial adjacency of distinct items on increased sales. Recently, Chen and Lin (2007) [**91**] utilize a popular data mining approach, association rule mining, instead of space elasticity to resolve the product assortment and allocation problems in retailing. In this paper, the multi-level association rule mining is applied to explore the relationships between products as well as between product categories. Because association rules are obtained by directly analyzing the transaction database, they can generate more reliable information to shelf space management.

Even though data mining is fairly new field, it still suffers drawbacks. One obvious limitation is none of these models address the issue of competition among supply chain partners. They either look at the retailers or manufacturers profit functions but not both for making decisions.

3. 2. 5. Game Theory models

Marketing science literature has often adopted Game theory as a possible technique to deal with possible issues of cooperation, non-cooperation and competition among the members of a marketing channel. Most these papers cited in this section address the issues such as pricing, marketing expenditures, and profit sharing, that could induce cooperation into the marketing channel. A stream of literature has adopted a differential game formalism to analyze conflicts and coordination in marketing channels. Chintagunta and Jain (1992) [16]adopted differential game strategy to examine the effect of channel dynamics on the difference in profits resulting from following coordinated as opposed to uncoordinated strategies and identify situations in which this profit differential provides an incentive for channel members to coordinate their marketing efforts. Subsequently, Jorgensen and Zaccour (1999) [17] proposed a differential game model for analyzing the relationship between the two firms under conflict and coordination and also conclude on pricing and advertising strategies for both the firms under consideration. Moscarini and Ottaviani (2001) [92] in their paper investigate price competition in presence of private information on demand side. In this article two retailers are selling different brand of the same product to a customer endowed with a private binary signal on their relative quality. This model provides a platform to differentiation in Hotelling's price competition game. The results of this study (1) competition is fierce when the prior strongly favors one seller and private signals are relatively uninformative. (2) Sellers equilibrium profits may fall with the revelation of public information and are non-monotonic in the prior belief. Although there exists abundant literature applying game theory to demand, advertising, pricing and coordination, it is only recently that a few studies have concentrated on the issue of shelf-space allocation using game theory concepts.

Wang and Gerchak (2001) [**64**], the shelf-space-allocation issue is examined as a coordinating tool in the marketing channel. Two kinds of channel structures (a channel composed of a single manufacturer selling its product through an exclusive dealer, a bilateral monopoly, and a channel with a monopolist manufacturer and two competing retailers) are studied in order to determine if a holding-cost subsidy, designed by the manufacturer to push retailers into allocating more shelf space to their products can maximize total channel profits. Despite the interesting insights given by this study, the fact that each retailer handles the product of a single manufacturer makes the issue of shelf-space allocation less interesting than in channel structures in which competing manufacturers battle to acquire the highest share of the space. Furthermore, in Wang and Gerchak (2001)[**64**], retail and wholesale prices are exogenous. proposed a differential game formalism for determining the shelf-space allocation by a retailer for her two competing brands. To be able to characterize feedback Stackelberg equilibrium, they assumed that the two brands' manufacturers behave myopically, i.e. each player observes only the evolution of her brand goodwill stock.

Jorgensen, Taboubi and Zaccour (2003)[**93**] consider a single manufacturer and a single retailer, where the manufacturer advertises on national media and retailer promotes the brand locally. They develop a differential game model with an infinite time horizon to see what would happen if the manufacturer pays a part of the price of advertising for the retailer. Final results of this study show that cooperative program is implementable if the level of initial brand image is small or if the level of the initial brand image is intermediate and promotion is not too damaging to the brand image in certain cases. This paper doesn't stress on the shelf space aspect of our interest but focuses mostly on promotional effects using game theory which adds dimension to our array of knowledge. Chen et al (2004)[**39**] refer to a horizontal market with multiple newsvendors who face stochastic price-dependent demand. They make their pricing decisions and use this value to compete for the demand in that category. Here they use a linear and logit demand models with

substitutions among firms demand to prove the existence of NE and uniqueness of the solution by making he game super modular and showing that the best response function is a contraction respectively. Herr´an and Taboubi (2005) [**94**]and Herr´an et al. (2005) [**95**] examined the issue of shelf-space allocation in a dynamic game setting for a channel composed of two manufacturers and an exclusive retailer. In Herr´an and Taboubi (2005) [**94**] , the information structure is Markovian and the manufacturers are assumed to act myopically, while in Herr´an et al. (2005) [**95**], shelf-space and advertising decisions are time-dependent and the hypothesis of myopia is removed. In Herr´an and Taboubi (2005)[**94**] and Herr´an et al. (2005) [**95**]the authors assume that the retail and wholesale prices are constant. Borger and Dender (2006) [**96**] in their recent article study a duopolistic interaction between congestible facilities that supply perfect substitutes and make sequential decision on capacities and prices and compare the results to monopoly and first best out comes. The results of this study were there is more congestion at NE in duopoly compared to social optimum which is similar to monopoly situation. The higher congestion levels in duopoly are dies to strategic pricing responses to capacity investments. Also higher marginal costs increase profits in duopoly and lastly when capacity is cheap or demand inelastic stable asymmetric NE may result where the high capacity facility offers low time costs at high price and the smaller facility offers lower service levels at lower price.

Most of the papers in marketing science literature look at the shelf space allocation problem as a potential source of conflict between the retailers and manufacturers. Subsequently, Herr'an et al. (2006) [**6**] in their article examine shelf space allocation and pricing decisions in a market channel as a result of static game played as a Stackelberg between two manufacturers of competing brands and a retailer. The main results of this study are: (1) Lower the unit cost or greater the price elasticity the greater the shelf space allocated to that brand. (2) The higher the shelf space elasticity the lower are the wholesale prices and profits of all the members in the

supply chain. This paper forms the basis to our research and serves as a benchmark to the empirical results and analytical results obtained further in the dissertation.

3.3 Limitations of Previous Literature

Although, there have been many interesting studies in the area of demand estimation, there still exist some drawbacks with each of the categories examined in the literature review section3.2. Some of drawbacks are listed below:

- ➢ DOE: expensive models due to replication, complication in setting up experiments, difficulties associated with data collection and the lack of generalized conclusions for a group or class

- ➢ Mathematical Models: Often complicated to interpret and implement. Most of them used profits maximization at a retailer only for estimating optimal demand. Cannot examine competition.

- ➢ Software programs: The drawback of such computer programs or systems arises from their incompetence to capture the demand effects. Demand is a RV and highly stochastic in nature. Most of the commercial software discussed above are programmed based on heuristics and hence cannot accommodate the rapid change in demand that occurs due to stochasticity and competition.

- ➢ Artificial Intelligence: Drawback with such kind of models is that none of them really address the issue from the marketing channel point of view. They either look at the retailers or manufacturers profit functions but not both for making decisions. These are

81

mostly classical statistical methods as well. The models developed till date cannot address competition.

➢ General Limitation: Most of the models discussed in section 3.2 fail to include explicitly other subjective variables such as retailer to retailer competition and impact of manufacturers response that greatly influence future demand predictions.

3.4 Problem statement and Objectives

In this chapter the problem of demand estimation at the retailers also termed as microscopic demand estimation is addressed. The micro demand model estimates demand at a retailer level considering the effect of shelf-space allocated for each of the substitutable products, the price competition among these substitutable products at the retailer, and the pricing competition for these products at competing retailer locations. Therefore the main objective of this study is to evaluate demand in presence of competition within retailers, manufacturers and between retailers and manufacturers.

3.5 Methodology

Accurate demand estimation is an important aspect of integrated supply chain that is used to avoid bull whip effect, reduce excess holding of inventory and stock outs. There has been ample research conducted in this area of demand estimation and inventory models as seen in section 3.2. All this research provides us with valuable tools and interesting demand models. In this chapter, a

refined demand model that alleviates most of the limitations addressed in Section 3.3 of the existing demand models is presented. The current micro level demand model accounts for

➢ Competition among Manufacturers: A manufacturer influences a product's demand through his/her wholesale price that determines the product margins. In many of the shelf-space-optimization models, (as well as the user-friendly commercial tools) the amount of shelf-space allocated for a product depends on the product margins, which significantly affect the relative profitability achievable for a set of competing products. Since the amount of shelf-space allocated has a significant effect on the product's demand, a manufacturer can influence a product's demand through his/her wholesale price. Consequently, competition for different manufacturer's products is achieved at the retailer level by the amount of shelf-spce allocated at the retailer level that is in turn dictated by the manufacturer's wholesale prices [6].

➢ Competition among retailers: As stated by Gruen (2002) [63] 31% of the customers go to another store to get certain product when they don't find it in the current store. This statement explicitly notifies us of the inherent competition among retailers. Besides unavailability of the product at a certain store, factors like price difference can also be considered possible reasons for competition among retailers. Empirical studies suggest that different retailers can charge different prices for the same product and the substitutes[97]. This difference in prices is termed as price discrimination. There is a vast literature in economics on the theory of price discrimination (see, for example, Varian 1989 [98]for a review of this literature). In addition, a number of important papers in marketing (e.g., Moorthy 1984[99], Narasimhan 1984 [100]) have discussed different forms of price discrimination and how they might be implemented in practice. Due to

83

price discrimination at retailers as well as manufacturers, there exists competition among different retailers selling same products (e.g. Nike at Foot Locker vs. Nike at Big 5) and substitutes at different prices (e.g. Nike and Reebok running shoes).

To better understand the competition among multiple retailers/manufacturers, it is necessary to understand economics concerning markets and pricing. When retailers/ manufacturers sell a product they face a demand/supply curve under which customers are willing to purchase a certain quantity that is dependent upon the price. Depending on the model of the market, the supply chain members either produce a specific quantity or set a specific price. The supply chain members make these choices dependent on possible outcomes of what they expect their competitors to do [101]. In strategic decision making situations, a supply chain member chooses his strategies such that he will maximize his returns, given the strategic choices of other supply chain members. The idea of the micro model discussed earlier is to incorporate these options encountered and interactions among the supply chain members into a formal demand estimation model. This is done in order to obtain optimal demand for retailers when costs and benefits of each option depend on the choices of competing retailers and manufacturers. The best tool to use such strategic decision making situations is Game theory.

Game theory studies decisions made in environments in which supply chain members interact. The situations in which game theory has been applied in reality reveal its selective usefulness for the problems and solutions of distinctive and competitive nature. The two significant areas of application of this viable tool have been economics and war. The current micro model can be categorized as economic model and the members of this supply chain can be reckoned as real oligopolies. The term oligopoly denotes a situation where there are few sellers for a product or

service. The members of an oligopoly change the nature of a free market. While they can't dictate price and availability like a monopoly can, they often turn into friendly competitors, since it is in all the members' interest to maintain a stable market and profitable prices [**102**].

In order to develop an understanding of the workings of real oligopolies a simple demand model that accounts for competition among manufacturers is considered first. Later, this model is augmented to account for competition among retailers. These two scenarios are described below:

- ➢ *Scenario 1:* This scenario is an extension of an already built demand estimation model from Herr' an et al (2006). It consists of two competing manufacturers of homogenous substitutable products battling for shelf space at a single retailer based on their pricing decisions. This scenario models competition among the manufacturers through their substitutable products.

- ➢ *Scenario 2:* This scenario extends the demand model formulation of Scenario 1 to incorporate competition among the retailers while retaining the competition among the manufacturers. This scenario considers two competing retailer in addition to two existing competing manufacturers. As in Scenario 1, the manufacturers compete with one another through their substitutable products while the retailers compete through their pricing strategies.

Figure 8 is a pictorial representation of this microscopic demand estimation model clearly explaining the two scenarios, sub-scenarios, methodology adopted and finally the variables that impact the whole process. In the next section, a brief description of game theoretic concepts is presented.
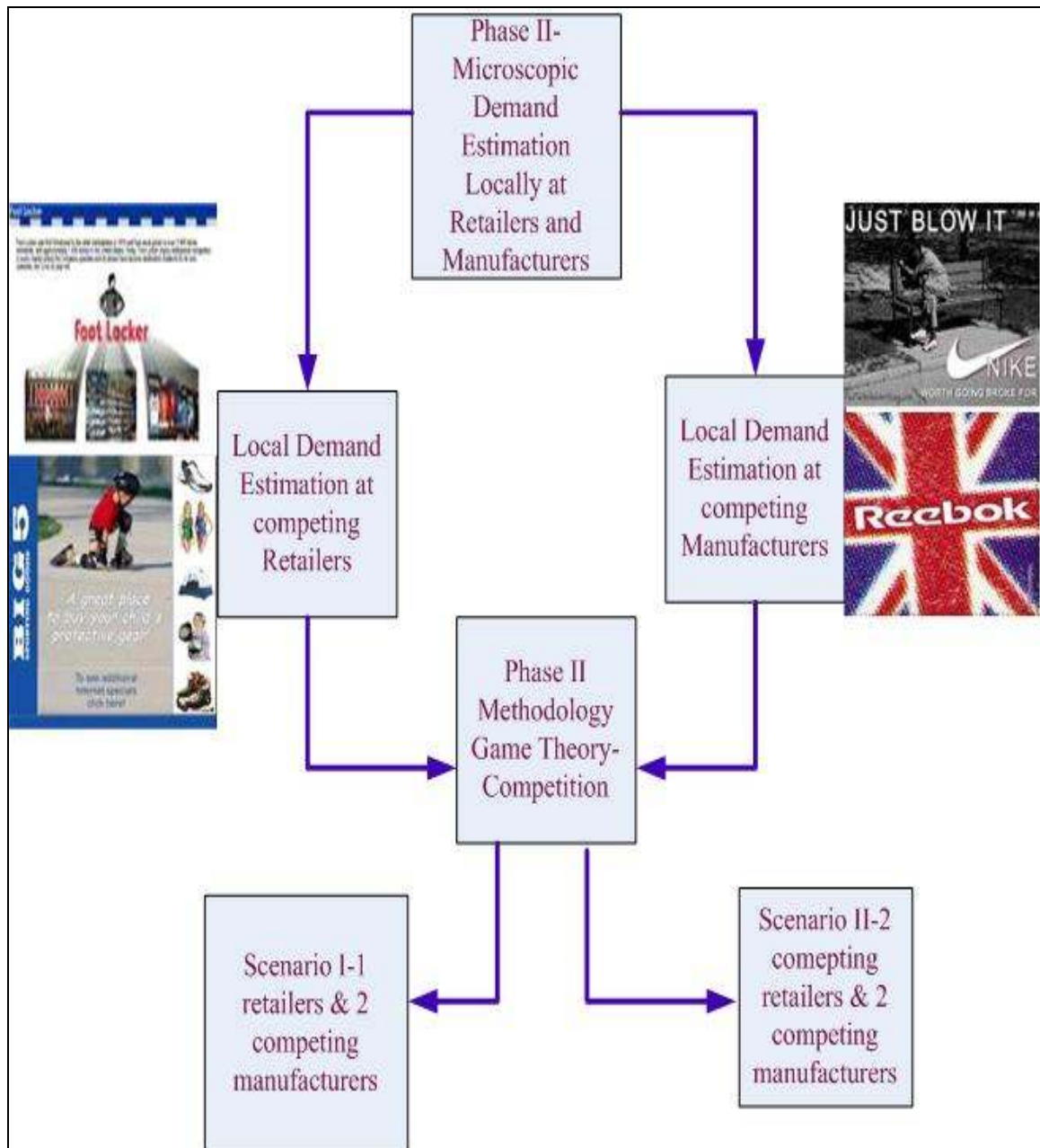
Figure 8: Pictorial representation of micro-level demand estimation model.

3.6 Important definitions

A two firm oligopoly is examined below in order to demonstrate game theoretic concepts such as Cournot, Stackelberg games, Nash equilibriums and sub-game perfect equilibra [101]

Definition 1: A Nash equilibrium (for two players) is a pair of strategies (S_1*, S_2*) such that

$$\pi_1\left(S_1^*, S_2^*\right) \geq \pi_1\left(S_1, S_2^*\right) \quad \forall S_1 \in \Sigma_1$$
and
$$\pi_2\left(S_1^*, S_2^*\right) \geq \pi_2\left(S_1^*, S_2\right) \quad \forall S_2 \in \Sigma_2$$

where Σ_1 and Σ_2 define the strategy spaces, π_1 and π_2 define the payoffs of the two players as a function of chosen strategies. In other words, given the strategy adopted by the other player neither player could do strictly better (i.e. increase their payoff) by adopting another strategy. It is clear from definition 1 that a Nash equilibrium never includes strictly dominated strategies, but it may include weakly dominated strategies.

When a player tries to choose the "best" strategy among a multitude of options, that player may compare two strategies A and B to see which one is better. The result of the comparison is one of:

➢ B dominates A: choosing B always gives at least as good an outcome as choosing A.
  There are 2 possibilities:
  o B strictly dominates A: choosing B always gives a better outcome than choosing A, no matter what the other player(s) do.

- o B weakly dominates A: There is at least one set of opponents' action for which B is superior, and all other sets of opponents' actions give B at least the same payoff as A.
- ➢ B and A are intransitive: B neither dominates, nor is dominated by, A. Choosing A is better in some cases, while choosing B is better in other cases, depending on exactly how the opponent chooses to play. For example, B is "throw rock" while A is "throw scissors" in Rock, Paper, Scissors.
- ➢ B is dominated by A: choosing B never gives a better outcome than choosing A, no matter what the other player(s) do. There are 2 possibilities:
  - o B is weakly dominated by A: There is at least one set of opponents' actions for which B gives a worse outcome than A, while all other sets of opponents' actions give A at least the same payoff as B. (Strategy A weakly dominates B).
  - o B is strictly dominated by A: choosing B always gives a worse outcome than choosing A, no matter what the other player(s) do. (Strategy A strictly dominates B).

This notion can be generalized beyond the comparison of two strategies.

- ➢ Strategy B is strictly dominant if strategy B *strictly dominates* every other possible strategy.
- ➢ Strategy B is weakly dominant if strategy B *dominates* all other strategies, but some are only weakly dominated.
- ➢ Strategy B is strictly dominated if some other strategy exists that strictly dominates B.
- ➢ Strategy B is weakly dominated if some other strategy exists that weakly dominates B.

Mathematical definition weakly and strictly dominated strategies is as follows:

In mathematical terms, for any player 'i', a strategy $s^* \in S_i$ weakly dominates another strategy

$s' \in S_i$ if: $\forall s_{-i} \in S_{-i}[u_i(s^*, s_{-i}) \geq u_i(s', s_{-i})]$ (With at least one strict inequality)

Remember that S_{-i} represents the product of all strategy sets other than i's

On the other hand s* strictly dominates s' if: $\forall s_{-i} \in S_{-i}[u_i(s^*, s_{-i}) > u_i(s', s_{-i})]$

Definition 2: A strategy for player 1, $\hat{S}_1$ is a best response to some (fixed) strategy for player 2, S_2 if:

$$\hat{S}_1 \in \arg\max_{S_1 \in \Sigma_1} \pi_1(S_1, S_2)$$

Similarly, $\hat{S}_2$ is a best response to some S_1 if:

$$\hat{S}_2 \in \arg\max_{S_2 \in \Sigma_2} \pi_2(S_1, S_2)$$

Therefore, an equivalent form of the definition of a Nash equilibrium which focuses on the strategies rather than the payoffs, is that S_1^* is a best response to S_2^* and vice versa. [**103**]

Definition 3: A pair of strategies (S_1^*, S_2^*) is a Nash equilibrium if:

$$S_1^* \in \arg\max_{S_1 \in \Sigma_1} \pi_1(S_1, S_2^*)$$

and

$$S_2^* \in \arg\max_{S_2 \in \Sigma_2} \pi_2(S_1^*, S_2)$$

It is clear that strictly dominated strategy is never a best response to any strategy, whereas weakly dominated strategy may be a best response to some strategy. This is why weakly dominated strategies may appear in Nash equilibra but strictly dominated strategies do not. To use this

definition to find Nash equilibra we find for each player, the set of best responses to every possible strategy of the other player. We then look for pairs of strategies that are best responses to each other.

The extension of the theory of games with more than two players is straight forward. Let us label the players by $i \in \{1,2,....,n\}$. Each player has a set of pure strategies S_i and a corresponding set of mixed strategies $\sum_i$. The payoff to player 'i' depends on a list of strategies $S_1, S_2,........,S_n$ -one for each layer. For the definition of a Nash equilibrium we will need to separate out the strategy for each of the players, so we denote by S_{-i} the list of strategies used by all the players except the ith player.

Definition 4: A Nash equilibrium in a n-player game is a list of mixed strategies $S_1^*, S_2^*, S_3^*, ..., S_n^*$ such that $S_i^* \in \underset{S_i \in \Sigma_i}{arg\,max}\, \pi_i \left(S_i, S_{-i}^* \right) \quad \forall i \in \{1,2,....,n\}$ [103]

Definition 5: In game theory, a sub-game perfect equilibrium is a refinement of a Nash equilibrium used in dynamic games. A strategy profile is a sub-game perfect equilibrium if it represents equilibrium of every sub-game of the original game. More informally this means that if (1) the players played any smaller game that consisted of only one part of the larger game and (2) their behavior represents a Nash equilibrium of that smaller game, then their behavior is a sub-game perfect equilibrium of the larger game. A common method for determining sub-game perfect equilibrium is backward induction.

Definition 6:: A static game is one in which a single decision is made by each player and each player has no knowledge of the decisions made by the other players before making their own

decisions. Sometimes such games are referred to as simultaneous decision games because any actual order in which the decisions are made is irrelevant. [**103**]

Definition 7: Cournot competition is an economic model used to describe industry structure. It has the following features:

- ➤ There is more than one firm and all firms produce homogenous products
- ➤ Firms do not cooperate
- ➤ Firms have the market power
- ➤ The number of firms is fixed
- ➤ Firms compete in quantities and choose quantities simultaneously
- ➤ There is strategic behavior by the firms

An essential assumption of this model is that each firm aims to maximize profits based on the expectation that its own output decision will not have an effect on the decisions of its rivals. Price is commonly a known decreasing function of the total output. Each firm has a cost function $C_i(Q_i)$. Normally the cost functions are treated as common knowledge. The cost functions may be the same or different among firms. The market price is set at a level such that demand equals the total quantity produced by both firms. Each firm takes the quantity set by its competitors as given, evaluates its residual demand and then behaves as a monopoly. [**104**]

Let the demand faced by firms be characterized by demand-price curve $Q_D(p)$, in which Q_D is the quantity demanded of the firms at a set price p. For simplicity's sake, let the demand is a linear function of the price:

$$Q_D(p) = a - bp$$

Firms face costs of production of two forms, fixed and variable. For this example let the fixed cost be zero and the marginal cost be a constant value c. This assumption means that costs are linear in q_s that quantity supplied. So, the cost function is of the form:

$$C(q) = cq_s$$

As per this example, there exist two firms competing and their strategy is based on quantity rather than price. The two firms set their production quantities q_1 *and* q_2 . Because these are two suppliers in the market , the market price is a function of the sum of the two firms production , $(q_1 + q_2)$, which is the total production in the market. As with the previous case, a linear demand curve and a constant marginal cost of production is employed. The outcome from this strategy decision is the firms profit, which is a function of the market price, the quantity sold and the marginal cost:

$$\pi(q_1) = q_1 * P(q_1 + q_2) - q_1 * c$$

The above equation simply states that a firms profit is always equal to the quantity sold times the price at which it is sold minus, the costs associated with production. By substituting the inverse demand function, $P(q_1 + q_2)$ which is the linear demand curve solved for price, the new profit function is:

$$\pi(q_1) = q_1 * \left(\frac{a}{b} - \frac{q_1 + q_2}{b} \right) - q_1 * c$$

Any firm chooses its quantity of production so as to maximize its own profits indicating that firms profit is dependent on its own production quantity and competitors production quantity as well. The profit maximizing quantity is found by setting the derivative of the profit function for firm1 equal to zero.

$$\frac{\partial \pi(q_1)}{\partial q_1} = \frac{a}{b} - \frac{2q_1 + q_2}{b} - c = 0$$

$$\Rightarrow q_1 = \frac{a - cb - q_2}{2}$$

The same process is repeated for firm 2 resulting in a equation for q_2

$$q_2 = \frac{a - cb - q_1}{2}$$

Above two equations for the quantities are known as reaction curves or best response functions. They predict what quantity each firm would produce, given the quantity that the other firm is producing. Solving these two equations presents the optimal values of each quantity to be:

$$q_1 = q_2 = \frac{a - cb}{3}$$

The quantities found in the above equation are substituted into the original profit function, to calculate the profits of each firm as shown:

$$\pi_1(q_1) = \frac{1}{9}(a - cb)^2$$

Since the firms are assumed to be identical, their strategies are also identical. Because there is perfect information, both firms know the other firms strategy. By solving the two reaction curves simultaneously, the equilibrium levels of production known as the Cournot equilibrium are presented. It is interesting to note that the Cournot equilibrium is in between the monopoly levels of production and the competitive levels. In addition, the profits earned by the two firms are less than half the monopoly profits.[101].

Implications of Cournot Games[104]

➢ Output is greater with Cournot duopoly than monopoly, but lower than perfect competition.

➢ Price is lower with Cournot games than monopoly but not as low as with perfect competition.

Definition 8: Stackelberg leadership model is a strategic game in which the leader firm moves first and then the follower firms move sequentially. Firms can engage in Stackelberg competition if one has some sort of advantage enabling it to move first. Moving observably first is the most obvious means of commitment: once the leader has made its move, he cannot undo the action. Moving first may be possible if the leader is an incumbent monopoly of the industry and the follower is a new entrant. Holding excess capacity is another means of commitment. [**105**]

Example for Dynamic Multi-firm Competition

The model is essentially identical to Cournot model except that it is a dynamic game. This means that there is a sequence to the course of events in the decision making process. Rather than both firms setting quantities at the same time, one firm is able to set a quantity first. For both firms the profit function looks identical to the one in Cournot example. However rather than maximizing profit with firm 2's quantity as an unknown, firm 1 uses its knowledge of Firm 2's reaction curve to gain further profits. Firm 1's function now is solely a function of q_1:

$$\pi(q_1) = q_1 * \left(\frac{a}{b} - \frac{q_1 + q_2(q_1)}{b} \right) - q_1 * c$$

Substituting the value of q2 (q1) from the Cournot reaction curve, we have:

$$\pi(q_1) = q_1 * \left(\frac{a}{b} - \frac{q_1 + \dfrac{(a - cb - q_1)}{2}}{b} \right) - q_1 * c$$

Further simplification of the above profit equation would yield:

$$\pi(q_1) = q_1 * \left(\frac{a - q_1}{2b} - \frac{c}{2} \right)$$

Again, maximizing profits over quantity. Firm 1 produces a Stackelberg equilibrium quantity and Firm 2 produces a quantity that maximizes its profits given firm 1's quantity as defined by its reaction curves. To find Firm 1's production, the derivative of its profit function with respect to its quantity is calculated and set it equal to zero as shown below:

$$\frac{\partial \pi(q_1)}{\partial q_1} = \frac{a - 2q_1}{2b} - \frac{c}{2} = 0$$

$$\Rightarrow q_1 = \frac{a - cb}{2}$$

Firm 2 has no option but to react to Firm 1's production level in a way that maximizes its profits. From the original Cournot model, it is common knowledge that this strategy is determined by Firm 2's reaction curve. Subtracting the value for q_1 given by the above equation , value for q_2 is calculated as:

$$q_2 = \frac{a - cb}{4}$$

Thus, by having the first mover or the leader advantage, Firm 1 captures more of the market share. In addition increasing its own profits as well. [101]

➢ Cournot and Stackelberg games compete on quantity and hence have similar models. T

➢ Stackelberg leader has a crucial advantages as he moves first.

➢ Perfect information sharing is an important assumption in Stackelberg game. This means that the follower must observe the quantity chosen by the leader else the game reduces to Cournot.

➢ Stackelberg can also be a classical example of too much information hurting a player (follower) most of the times since these are sequential games

➢ Simultaneous Cournot games prove to advantageous to all the players.

3.7 Scenario 1

This scenario defines competition among manufacturers through availability of homogeneous substitutable products at a retailer. The demand model for this scenario depicts the competition between manufacturers by using cross space and price elasticities.

Figure (9) below is a pictorial representation of this scenario 1 describing the two competing manufacturers scenario along with the factors the factors that influence demand and the results of this scenario.
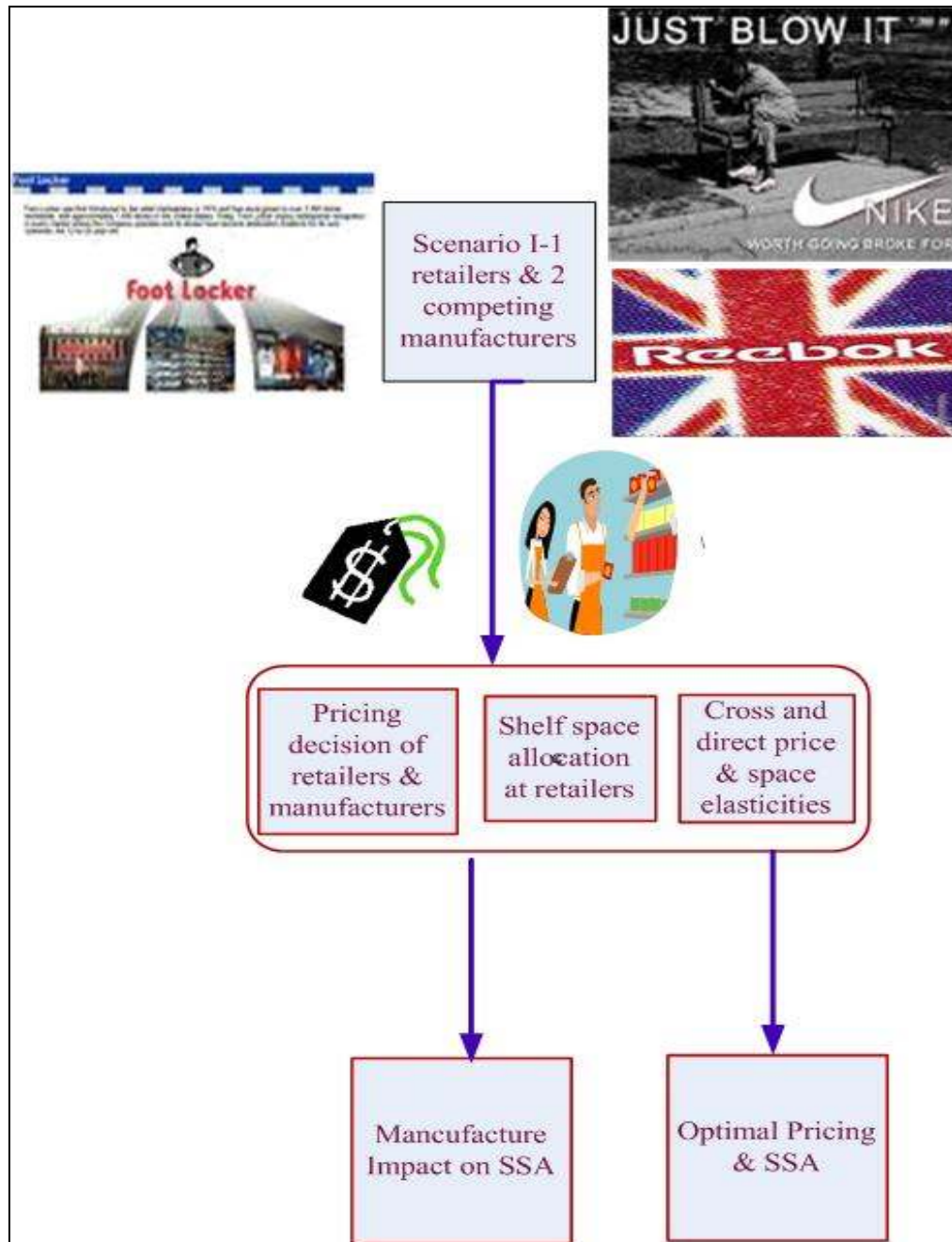
Figure 9: Pictorial representation of Scenario1 for this micro level model where competition among manufacturers is considered.

3.8 Scenario 1 Model Formulation

It is assumed in here that the prices of substitutable products are different and the shelf space is normalized to 1. Let S_1 denote the shelf space allocated to manufacturer 1 and S_2 the shelf space allocated to manufacturer 2 such that

$$\sum_{i=1}^{K} S_i = 1$$

It is also assumed that there are no trade promotions, which means that the cost for shelf space is constant. This is clearly a simplifying assumption for product categories and brands where the manufacturers have to pay retailers to gain access to their shelf space.

At the retailer level, the demand Q_k for a product k is influenced by: 1) the elasticity (γ_k) of shelf-space (S_k) allocated for that product, 2) the cross-elasticity (γ_l) of shelf-space (S_l) allocated for substitutional products, 3) the price elasticity (μ_k) of the product with price (P_k), and 4) the cross price elasticity (ε_k) of the substitutable (competing) product with price (P_l) [**12, 77, 106**].

In economics and business studies, the price elasticity of demand (PED) is an "elasticity that measures the nature and degree of the relationship between changes in quantity demanded of a good to changes in its price". By definition, price elasticity (μ_k) of demand is given by :

$$\mu_k = -\frac{\%\,change\,in\,quantity\,demanded}{\%\,change\,in\,price} = -\frac{\Delta Q_k / Q_k}{\Delta P_k / P_k}$$

Using differential calculus:

$$\mu_k = -\frac{dQ}{dP} * \frac{P}{Q}$$

By integrating this equation, you will get the power law form given by:

$$Q \propto P^{-\mu_k} \tag{3a}$$

The cross price elasticity is defined as "the measure of responsiveness of the quantity demand of a good to a change in the price of a competing substitutable product from another manufacturer". By definition, cross elasticity (ε_k) of demand is given by:

$$\varepsilon_k = \frac{\% \, change\,in\,the\,quantity\,demanded\,of\,product\,k}{\% \, change\,in\,price\,of\,product\,l} = \frac{\Delta Q_k / Q_k}{\Delta P_l / P_l}$$

Using differential calculus:

$$\varepsilon_k = \frac{dQ_K}{dP_l} * \frac{P_l}{Q_k}$$

By integrating this equation, you will get the power law form given by:

$$Q_k \propto P_l^{\varepsilon_k} \tag{3b}$$

Similarly, the elasticity (γ_k) of shelf-space (S_k) is defined as: "elasticity that measures the nature and degree of the relationship between changes in quantity demanded of a good to changes in the amount of shelf space allocated to that product". By definition, direct shelf space elasticity (γ_k) of demand is given by :

$$\gamma_k = \frac{\% \, change \, in \, quantity \, demanded}{\% \, change \, in \, shelf \, space \, allocated \, to \, that \, product} = -\frac{\Delta Q_k / Q_k}{\Delta S_k / S_k}$$

Using differential calculus:

$$\gamma_k = \frac{dQ_k}{dS_k} * \frac{S_k}{Q_k}$$

By integrating this equation, you will get the power law form given by:

$$. \; Q_k \propto S_k^{\gamma_k} \qquad\qquad\qquad (3c)$$

The cross shelf space elasticity is defined as "the measure of responsiveness of the quantity demand of a good to a change in the shelf space allocated to a competing substitutable product from another manufacturer". By definition, cross elasticity (γ_l) of demand is given by:

$$\gamma_l = \frac{\% \, change \, in \, the \, quantity \, demanded \, of \, product \, k}{\% \, change \, in \, shelf \, space \, allocated \, to \, product \, l} = \frac{\Delta Q_k / Q_k}{\Delta S_l / S_l}$$

Using differential calculus:

$$\gamma_l = \frac{dQ_K}{dS_l} * \frac{S_l}{Q_k}$$

By integrating this equation, you will get the power law form given by:

$$Q_k \propto S_l^{\gamma_l} \qquad\qquad (3d)$$

Combining the above four equations, the demand Q_k for a product k is given by

$$Q_k = \alpha * S_k^{\gamma_k} * S_l^{\gamma_l} * P_k^{-\mu_K} * P_l^{\varepsilon_K} \qquad \text{for k, l} = 1,2 \qquad\qquad (3.1)$$

where $\alpha > 0$, $\mu_k \geq 0$, $\varepsilon_k \geq 0, 0 < \gamma < 1$.

The demand function in (3.1) also known as Cobb-Douglas equation in economics literature. The power law formulation of the demand model accounts for the interaction between the variables and also has the property that elasticities are constant. Unlike the models proposed by Zufreyden (1986)[78]and Herr'an et al (2006)[6], our demand/supply/sales function specifically includes the cross-shelf-space elasticities between products within the same product category.

It should be noted that:

> $Q_k \geq 0$: The proof of this is simple and intuitive. There cannot be negative amounts of any product supplied or demanded. Therefore the quantity of sales >0 which is strictly positive.

➤ $\dfrac{\partial Q_k}{\partial S_k} \geq 0$

Proof 1: Using equation (3.1) the first order derivative with respect to shelf space allocated to manufacturer k is calculated as:

$$Q_k = \alpha S_k^{\gamma_k} S_l^{\gamma_l} P_k^{-\mu_k} P_l^{\varepsilon_k}$$

$$\dfrac{\partial Q_k}{\partial S_k} = \alpha \gamma_k S_k^{\gamma_k-1} S_l^{\gamma_l} P_k^{-\mu_k} P_l^{\varepsilon_k} \geq 0$$

The above equation is not negative because, all the prices involved are positive and also shelf space allocation cannot be negative, the shelf space direct elasticity value is also observed to be between 0 and 1 which restricts its negativity and the constant alpha is strictly positive. Similarly,

➤ $\dfrac{\partial Q_k}{\partial P_l} \geq 0$

Proof 2: A first order derivative of quantity is taken with same restrictions on shelf space and prices as proof 1. The resulting derivative is a demand function multiplied by cross space elasticity. All the terms in the resulting derivative are positive, therefore the derivative is also positive.

$$\dfrac{\partial Q_k}{\partial P_l} = \alpha \varepsilon_k S_k^{\gamma_k} S_l^{\gamma_l} P_k^{-\mu_k} P_l^{\varepsilon_k-1} \geq 0$$

Finally,

➤ $\dfrac{\partial Q_k}{\partial P_k} \leq 0$

Proof 3: Again a first order partial derivative on quantity demanded with similar restrictions as proof1 on shelf space and price is applied. The resulting derivative is multiplied by direct shelf space elasticity that carries a negative sign. Therefore the whole derivative is negative and less than zero.

$$\frac{\partial Q_k}{\partial P_k} = \alpha(-\mu_k)S_k^{\gamma_k} S_l^{\gamma_l} P_k^{-\mu_k-1} P_l^{\varepsilon_k} \leq 0$$

The above proofs indicate that sales of brand k are nonnegative, increase with the shelf space allocated to that brand and with the competing brand's retail price, and decrease with its own retail price and competing brand's shelf space. The inequality $\mu_k \geq \varepsilon_k$ reflects the accepted assumption in economics, stating that own-price elasticity is higher in absolute value than cross-price elasticity. This assumption is made in oligopoly models [**6**] and it has also been empirically validated in the context of price promotions models [**107**].

REMARK 1: Although the direct and cross shelf-space elasticities are defined earlier as two independent quantities, the normalization constraint on the shelf-space imposes a relation between these two elasticities. Consider the demand of two products at a retailer as

$$Q_1 = \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_1^{-\mu_1} * P_2^{\varepsilon_1}$$
$$Q_2 = \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2}$$

(3.2)

Since $S_1 + S_2 = 1$,

$$\frac{\partial S_2}{\partial S_1} = -1 \;; \tag{3.3}$$

$$\frac{\partial S_1}{\partial S_2} = -1; \tag{3.4}$$

$$\frac{\partial Q_1}{\partial S_1} = Q_1 * \left[\frac{\gamma_1}{S_1} - \frac{\gamma_2}{S_2} \right]$$

$$\frac{\partial Q_1}{\partial S_1} = \frac{Q_1}{S_1} * \left[\gamma_1 - \gamma_2 * \frac{S_1}{S_2} \right]$$

$$\frac{\partial Q_1 / Q_1}{\partial S_1 / S_1} = \left[\gamma_1 - \gamma_2 * \frac{S_1}{S_2} \right] \rightarrow Direct \quad Shelf \quad space \quad elasticity \tag{3.5}$$

Similarly solving for cross shelf space elasticity

$$\frac{\partial Q_1}{\partial S_2} = Q_1 * \left[\frac{\gamma_2}{S_2} - \frac{\gamma_1}{S_1} \right]$$

$$\frac{\partial Q_1}{\partial S_2} = \frac{Q_1}{S_2} * \left[\gamma_2 - \gamma_1 * \frac{S_2}{S_1} \right]$$

$$\frac{\partial Q_1 / Q_1}{\partial S_2 / S_2} = \left[\gamma_2 - \gamma_1 * \frac{S_2}{S_1} \right] \rightarrow Cross \quad Shelf \quad space \quad elasticity \tag{3.6}$$

Simple algebraic manipulations applied to (3.6) would yield the result indicated below:

$$\frac{\partial Q_1 / Q_1}{\partial S_2 / S_2} = \left[\gamma_2 - \gamma_1 * \frac{S_2}{S_1} \right]$$

The above equation could be rewritten as :

$$\frac{\partial Q_1 / Q_1}{\partial S_2 / S_2} = -\frac{S_2}{S_1} * \underbrace{\left[\gamma_1 - \gamma_2 * \frac{S_1}{S_2} \right]}_{Direct\ shelf\ space\ elasticity} \tag{3.6}$$

$$\therefore Cross\ shelf\ space\ elasticity = -\frac{S_2}{S_1} * Direct\ shelf\ space\ elasticity \tag{3.7}$$

Thus, cross-elasticity is proportional to shelf-space ratio, with the proportionality factor being minus the direct elasticity.

3.9 Scenario 1: Stackelberg Game

In the two manufacturer competition scenario, the optimal demand/supply values are obtained by maximizing each player's profit functions (Cournot game). Additionally, in a Stackelberg game scenario, manufacturers become leaders and the retailer is the follower. The optimal demand values in a Stackelberg equilibrium are obtained by following the two-step procedure outlined below:

> ➢ First, optimize the follower's (retailer) profit equation to get her reaction functions (i.e., shelf space allocation and sale price) for a given leaders' strategies (wholesale prices). The leaders strategies are given by their responses to a Cournot game scenario.

> Next, these reaction functions are inserted into the manufacturers' objectives and Nash equilibrium is calculated.

This is best illustrated in the two manufacturers scenario in which manufacturers compete for shelf-space allocation at a given retailer. The retailer's optimization problem is given by:

$$\text{Max: } \Pi_R = (P_1 - W_1)Q_1 + (P_2 - W_2)Q_2 \tag{3.8}$$

$$\text{Subject to: } \sum_{i=1}^{2} S_i = 1 \tag{3.9}$$

where

$$Q_1 = \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_1^{-\mu_1} * P_2^{\varepsilon_1}$$
$$Q_2 = \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2} \tag{3.2}$$

Substituting these (3.2) in the profit function described in (3.9) would yield:

$$\Pi_R = (P_1 - W_1) * \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_1^{-\mu_1} * P_2^{\varepsilon_1} +$$
$$(P_2 - W_2) * \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2}$$
$$= P_1^{1-\mu_1} * \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_2^{\varepsilon_1} - W_1 * P_2^{\varepsilon_1} * \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_1^{-\mu_1} \tag{3.10}$$
$$+ P_2^{1-\mu_2} * \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_1^{\varepsilon_2} - W_2 * \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2}$$

Similarly, the manufacturer M_k optimization problem is given by:

$$\underset{W_K}{\text{Max }} \Pi_{M_K} = (W_K - C_K)Q_K$$
$$\text{where } Q_K = \alpha * S_K^{\gamma_K} * S_1^{\gamma_1} * P_K^{-\mu_k} * P_1^{\varepsilon_K}$$

where c_k is the unit-constant-production cost. Using equations (3.2) the objective function becomes:

$$Max_{W_K} \Pi_{M_K} = (W_K - C_K) * \alpha * S_K^{\gamma_K} * S_l^{\gamma_l} * P_K^{-\mu_k} * P_l^{\varepsilon_K} \tag{3.11}$$

3.9.1 Retailers Response Functions

The following proposition characterizes the retailers reaction functions for shelf space allocation and sales prices at the equilibrium which are obtained by solving the first order optimality conditions for the retailers profit maximization.

Let us assume $A = \left[\dfrac{(P_2 - W_2) * P_2^{-\mu_2} * P_1^{\varepsilon_2}}{(P_1 - W_1) * P_1^{-\mu_1} * P_2^{\varepsilon_1}} \right]$

Proposition 1

(a) Assuming there is an interior solution, the retailer's reaction function for the shelf space allocated to the two products is given as follows:

$$S_1^* = \left[\frac{-\gamma_2}{\gamma_1} * A * S_2^{\gamma_1 - \gamma_2} \right]^{\frac{1}{\gamma_1 - \gamma_2}} \tag{3.12}$$

108

$$S_2^* = \left[\frac{-\gamma_1}{\gamma_2} * A * S_1^{\gamma_2-\gamma_1} \right]^{\frac{1}{\gamma_2-\gamma_1}}$$

(3.13)

(b) The retailers sales prices for both the products at the equilibrium are give as follows:

$$P_1^{1-\mu_1-\varepsilon_2}[(1-\mu_1)+W_1 P_1^{-1}] = -\varepsilon_2 (P_2-W_2)P_2^{-\mu_2-\varepsilon_1} S_1^{\gamma_2-\gamma_1} S_2^{\gamma_1-\gamma_2}$$

(3.14)

$$P_2^{1-\mu_2-\varepsilon_1}[(1-\mu_2)+W_2 P_2^{-1}] = -\varepsilon_1 (P_1-W_1)P_1^{-\mu_1-\varepsilon_2} S_1^{\gamma_1-\gamma_2} S_2^{\gamma_2-\gamma_1}$$

(3.15)

On substituting the optimal values of shelf space from equations (3.12) and (3.13) in equations (3.14) and (3.15) a more complex equation for the sales prices is obtained. Therefore numerical analysis is opted for in this situation.

Proof 4: Assuming interior solution, first order optimality condition for retailer's maximization problem with respect to shelf space allocation as well as price is given by:

$$\frac{\partial \Pi_R}{\partial S_1} = (P_1-W_1)*\gamma_1 *\alpha * S_1^{\gamma_1-1} * S_2^{\gamma_2} * P_1^{-\mu_1} * P_2^{\varepsilon_1} +$$
$$(P_2-W_2)*\gamma_2 *\alpha * S_2^{\gamma_1} * S_1^{\gamma_2-1} * P_2^{-\mu_2} * P_1^{\varepsilon_2} = 0$$

(3.16)

$$\frac{\partial \Pi_R}{\partial S_2} = (P_1-W_1)*\gamma_2 *\alpha * S_1^{\gamma_1} * S_2^{\gamma_2-1} * P_1^{-\mu_1} * P_2^{\varepsilon_1} +$$
$$(P_2-W_2)*\gamma_1 *\alpha * S_2^{\gamma_1-1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2} = 0$$

$$\frac{\partial \Pi_R}{\partial P_2} = (1 - \mu_2)P_2^{1-\mu_2}(\alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_1^{\varepsilon_2} + \mu_2 * W_2 * \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_1^{\varepsilon_2})$$
$$+ (P_1 - W_1) * \varepsilon_1 * \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_1^{-\mu_2} * P_2^{\varepsilon_2 - 1} = 0$$

$$\frac{\partial \Pi_R}{\partial P_1} = (1 - \mu_1)P_1^{-\mu_1}(\alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_2^{\varepsilon_1} + \mu_1 * W_1 * \alpha * S_1^{\gamma_1} * S_2^{\gamma_2} * P_2^{\varepsilon_1})$$
$$+ (P_2 - W_2) * \varepsilon_2 * \alpha * S_2^{\gamma_1} * S_1^{\gamma_2} * P_2^{-\mu_2} * P_1^{\varepsilon_2 - 1} = 0$$

Solving (3.16 and 3.17) subject to the shelf-space normalization constraint results in optimal shelf-space allocation expressions presented in (3.12 and 3.13). However, given the functional forms of the retail prices in equations (3.14 and 3.15), an analytical solution for the optimal price values at Stackelberg equilibrium is cumbersome. Therefore, Newton Raphson's method is employed to numerically evaluate the optimal values at equilibrium.

Item (a) in Proposition 1 indicates that the shelf space allocated to brand 1, and similarly to brand 2, is a function of all the model's parameters (direct- and cross price elasticities and shelf-space elasticity). It also depends on wholesale prices set by the manufacturers as well as the retail prices for both the products. In retailer shelf-space-optimization models, the wholesale prices are assumed to be exogenous. But at Nash equilibrium the optimal wholesale prices are obtained. Here, the wholesale price can be interpreted as a mechanism used by a manufacturer to obtain the desirable share of the shelf.

Notice also that these reaction functions satisfy $0 < S_1^*, S_2^* < 1$, for all parameters and wholesale price values. Hence the solution is indeed interior. The shelf space allocated to each brand is decreasing in its wholesale price and increasing in the competitive brand's wholesale price. Indeed, the partial derivatives of S_1^*, S_2^* are given by:

$$\frac{\partial S_1^*}{\partial W_1} = \left[\frac{-\gamma_2 * P_2^{-(\mu_2+\varepsilon_1)} * P_1^{(\mu_1+\varepsilon_2)} * (P_2 - W_2)}{\gamma_1 * (P_1 - W_1)} \right]^{\frac{1}{\gamma_1-\gamma_2}}$$
$$* \left(\frac{(P_1 - W_1)}{\gamma_1 - \gamma_2} \right) * S_2 \tag{3.20}$$

$$\frac{\partial S_2^*}{\partial W_1} = \left[\frac{-\gamma_1 * P_2^{-(\mu_2+\varepsilon_1)} * P_1^{(\mu_1+\varepsilon_2)} * (P_1 - W_1)}{\gamma_2 * (P_2 - W_2)} \right]^{\frac{1}{\gamma_1-\gamma_2}}$$
$$* \left(\frac{(P_2 - W_2)}{\gamma_2 - \gamma_1} \right) * S_1 \tag{3.21}$$

Similarly, the retail prices are also increasing in the competitive brands wholesale prices and decreasing in their own prices. The partial derivatives of the optimal retail prices with respect to their wholesale prices are given below:

$$\frac{\partial P_1^*}{\partial W_1} = \frac{-\mu_1 * P_1^{-(1+\mu_1)} * P_2^{\varepsilon_1} * S_1^{\gamma_1} * S_2^{\gamma_2}}{\left(\begin{array}{l} \mu_1 * P_1^{-(\mu_1+1)} * P_2^{\varepsilon_1} * S_1^{\gamma_1} * S_2^{\gamma_2} (-1 + \mu_1 - (1+\mu_1) * P_1^{-1} * W_1 - \\ \varepsilon_2(\varepsilon_2 - 1) * P_2^{-\mu_2} * P_1^{\varepsilon_1} * S_1^{\gamma_2} * S_2^{\gamma_1} (P_2 - W_2) \end{array} \right)} \tag{3.22}$$

111

$$\frac{\partial P_2^*}{\partial W_1} = \frac{-\varepsilon_1 * P_1^{-\mu_1} * P_2^{\varepsilon_1-1} * S_1^{\gamma_1} * S_2^{\gamma_2}}{\begin{pmatrix} \mu_2 * P_2^{-(\mu_2+1)} * P_1^{\varepsilon_2} * S_1^{\gamma_2} * S_2^{\gamma_1} (-1+\mu_2-(1+\mu_2) * P_2^{-1} * W_2) \\ -\varepsilon_1(\varepsilon_1-1) * P_1^{-\mu_1} * P_2^{\varepsilon_2} * S_1^{\gamma_1} * S_2^{\gamma_2} (P_1-W_1) \end{pmatrix}}$$

(3.23)

Item (b) in Proposition 1 indicates that the sale price set by the retailer is non-linear and depends on the direct- and cross-price elasticities of sales and shelf space allocated along with the sale price of the substitutable product and its own wholesale price. The sale price of a product increases when the price elasticity for the brand decreases and when its cross-price elasticity increases. This result is intuitive because it confirms that the retailer will increase their sale price or retail margin for those brands whose sales are less sensitive to a variation in their own price, but also when the brand's sales are very sensitive to the price variations of the competing brand. Although the final equilibrium expression of the shelf space allocated by the retailer to each brand will be obtained after computing the retail and whole sale prices, we can nevertheless characterize the ratio of shelf space.

Let us assume $x = \frac{1}{(\gamma_1-\gamma_2)}$

Proposition 2

The equilibrium ratio of brands respective shelf space is given as the ratio of revenues generated with respect to substitutable product:

$$\frac{S_1^*}{S_2^*} = \left(\frac{Q_1 * (P_2-W_2)}{Q_2 * (P_1-W_1)}\right)$$

(3.24)

$$\frac{S_1^*}{S_2^*} = \left(\frac{P_2 - W_2}{P_1 - W_1}\right)^x * P_2^{-(\mu_2 + \varepsilon_1)x} * P_1^{(\mu_1 + \varepsilon_2)x} \tag{3.25}$$

Ratio of brand sales is given by:

$$\frac{Q_1}{Q_2} = \left(\frac{S_1^*}{S_2^*}\right)^{\gamma_1 - \gamma_2} * P_1^{-(\mu_1 + \varepsilon_2)} * P_2^{(\mu_2 + \varepsilon_1)} \tag{3.26}$$

Therefore, proposition 2 provides us with an optimal allocation rule that articulates, the brands' relative shares of the shelf space must be equal to their substitutable profits.

3.9.2 Manufacturers Response Functions

The manufacturers take into account the retailer's reaction functions and play a Nash (Stackelberg) game. Therefore, both manufacturers' wholesale prices at equilibrium are obtained by replacing the retailer's reaction functions into their profit functions and maximizing them. That is, manufacturer 'k' fixes his wholesale price by maximizing his objective function or profit function with respect to the whole sale price subject to retailer's reaction functions of shelf space and price given in section 3.9.1. The profit function of a manufacturer M_k is given by

$$\underset{W_K}{Max}\ \Pi_{M_K} = (W_K - C_K) * \alpha * S_K^{\gamma_K} * S_l^{\gamma_l} * P_K^{-\mu_k} * P_l^{\varepsilon_K}$$

Since a closed form structure is not available for the retail prices, these values are not substituted

in the manufacturer's profit function. The derivatives of the profit function are solved numerically using Newton Raphson's method to obtain optimal demand/supply values. Proposition 3 given below characterizes the manufacturers equilibrium wholesale prices that act as tools through which manufacturer can intervene in the decision making processes.

Proposition 3:

Assuming there exists an interior solution, the manufacturer's equilibrium wholesale prices are given as:

$$(W_K - C_K) \begin{bmatrix} \gamma_K P_K^{-\mu_K} P_1^{\varepsilon_1} S_K^{\gamma_K - 1} S_1^{\gamma_1} \dfrac{\partial S_K^*}{\partial W_K} + \gamma_1 P_K^{-\mu_K} P_1^{\varepsilon_L} S_K^{\gamma_K} S_1^{\gamma_{1-1}} \dfrac{\partial S_1^*}{\partial W_K} + \\ \varepsilon_k P_k^{-\mu_K} P_1^{\varepsilon_1 - 1} S_k^{\gamma_K} S_1^{\gamma_1} \dfrac{\partial P_1^*}{\partial W_K} - \mu_k P_k^{-\mu_K - 1} P_1^{\varepsilon_1} S_k^{\gamma_K} S_1^{\gamma_1} \dfrac{\partial P_K^*}{\partial W_K} \end{bmatrix} + P_K^{-\mu_K} P_1^{\varepsilon_K} S_K^{\gamma_K} S_1^{\gamma_1} = 0$$

(3.27)

$$0 < \gamma_1 < 1; \ \ 0 < \gamma_2 < 1; \ K, l = 1,2 \ \ and \ K \neq l$$

Proof 5: Assuming interior solution, the first order optimality condition for each of the manufacturer's profit maximization problem is given as follows:

$$\frac{\partial \Pi_{M_1}}{\partial W_1} = 0$$ (3.28)

$$(W_1 - C_1) \left[\begin{array}{l} \gamma_1 P_1^{-\mu_1} P_2^{\varepsilon_1} S_1^{\gamma_1-1} S_2^{\gamma_2} \dfrac{\partial S_1^*}{\partial W_1} + \gamma_2 P_1^{-\mu_1} P_2^{\varepsilon_1} S_1^{\gamma_1} S_2^{\gamma_2-1} \dfrac{\partial S_2^*}{\partial W_1} + \\[2mm] \varepsilon_1 P_1^{-\mu_1} P_2^{\varepsilon_1-1} S_1^{\gamma_1} S_2^{\gamma_2} \dfrac{\partial P_2^*}{\partial W_1} - \mu_1 P_1^{-\mu_1-1} P_2^{\varepsilon_1} S_1^{\gamma_1} S_2^{\gamma_2} \dfrac{\partial P_1^*}{\partial W_1} \end{array} \right]$$
$$+ P_1^{-\mu_1} P_2^{\varepsilon_1} S_1^{\gamma_1} S_2^{\gamma_2} = 0 \tag{3.29}$$

$$W_1^* = C_1 +$$
$$\cfrac{P_1^{-\mu_1*} P_2^{\varepsilon_1*} S_1^{\gamma_1*} S_2^{\gamma_2*}}{\left(\begin{array}{l} \gamma_1 P_1^{-\mu_1*} P_2^{\varepsilon_1*} S_1^{\gamma_1-1*} S_2^{\gamma_2*} \dfrac{\partial S_1^*}{\partial W_1} + \gamma_2 P_1^{-\mu_1*} P_2^{\varepsilon_1*} S_1^{\gamma_1*} S_2^{\gamma_2-1} \dfrac{\partial S_2^*}{\partial W_1} + \\[3mm] \varepsilon_1 P_1^{-\mu_1*} P_2^{\varepsilon_1-1*} S_1^{\gamma_1*} S_2^{\gamma_2*} \dfrac{\partial P_2^*}{\partial W_1} - \mu_1 P_1^{-\mu_1-1*} P_2^{\varepsilon_1*} S_1^{\gamma_1*} S_2^{\gamma_2*} \dfrac{\partial P_1^*}{\partial W_1} \end{array} \right)} \tag{3.30}$$

Similarly, for Manufacturer 2, we have

$$\frac{\partial \Pi_{M_2}}{\partial W_2} = 0 \tag{3.31}$$

$$(W_2 - C_2) \left[\begin{array}{l} \gamma_2 P_2^{-\mu_2} P_1^{\varepsilon_2} S_1^{\gamma_2-1} S_1^{\gamma_1} \dfrac{\partial S_1^*}{\partial W_2} + \gamma_1 P_2^{-\mu_2} P_1^{\varepsilon_2} S_1^{\gamma_2} S_1^{\gamma_1-1} \dfrac{\partial S_2^*}{\partial W_2} \\[2mm] + \varepsilon_2 P_2^{-\mu_2} P_1^{\varepsilon_2-1} S_1^{\gamma_2} S_1^{\gamma_1} \dfrac{\partial P_1^*}{\partial W_2} - \mu_2 P_2^{-\mu_2-1} P_1^{\varepsilon_2} S_1^{\gamma_2} S_1^{\gamma_1} \dfrac{\partial P_2^*}{\partial W_2} \end{array} \right]$$
$$+ P_2^{-\mu_2} P_1^{\varepsilon_2} S_1^{\gamma_2} S_1^{\gamma_1} = 0 \tag{3.32}$$

$$W_2^* = C_2 + \cfrac{P_2^{-\mu_2*} P_1^{\varepsilon_2*} S_1^{\gamma_2*} S_1^{\gamma_1*}}{\left(\begin{array}{l} \gamma_2 P_2^{-\mu_2*} P_1^{\varepsilon_2*} S_1^{\gamma_2-1*} S_1^{\gamma_1*} \dfrac{\partial S_1^*}{\partial W_2} + \gamma_1 P_2^{-\mu_2*} P_1^{\varepsilon_2*} S_1^{\gamma_2*} S_1^{\gamma_1-1*} \dfrac{\partial S_2^*}{\partial W_2} + \\[3mm] \varepsilon_2 P_2^{-\mu_2*} P_1^{\varepsilon_2-1*} S_1^{\gamma_2*} S_1^{\gamma_1*} \dfrac{\partial P_1^*}{\partial W_2} - \mu_2 P_2^{-\mu_2-1*} P_1^{\varepsilon_2*} S_1^{\gamma_2*} S_1^{\gamma_1*} \dfrac{\partial P_2^*}{\partial W_2} \end{array} \right)} \tag{3.33}$$

Unfortunately the system of equations identified for the retailer in Section 3.9.1 do not have a closed form solution. Consequently, closed-form analytical solutions for optimal values of wholesale prices are unattainable. Therefore, Newton Raphson's method is employed to evaluate optimal values.

3.9.3 Importance of propositions

> ➤ Proposition 1: answers the question regarding the impact of manufacturer's decision in pricing and shelf-space allocation.
> ➤ Proposition 2: provides allocation ratio based on revenues made.
> ➤ Proposition 3: summarizes results for the case where both manufacturers are active players. Provides the manufacturers with tools through which they can intervene in the decision making process.

3.10 Case Study/Numerical Analysis

A generalized analytical equilibrium solution for the optimal demand could not be obtained due to the open loop structure of the retail prices. Therefore numerical evaluation techniques such as Newton Raphson's method are employed to gain insights into the optimal behavior of retailers and manufacturers under different strategies. Initial starting values and fixed values for elasticities in order to start the simulation for obtaining optimal shelf space and prices are taken from Herr' an et al (2006) [6] and are tabulated in table 11. The simulation was run in R-software.

The maximum number of iterations to reach convergence for this method was set to 10,000. The optimal values are reported in the results section 3.11.

3.11 Scenario 1 : Results & Discussion

This section presents the results when the supply chain members interact with each other through the combination of options they choose. The results are believed to be best for the equilibrium strategies and deviation from these strategies indicates deviation from equilibrium which would prove to be non-beneficial to any of the supply chain members. Tabulated below are the results for this scenario.

Table 11 depicts the initial values used for starting the Newton Raphson's method in R-software. The values as indicated are adopted from Herr' an et al (2006) [6]

Table 12 shows us that lower the retail price more is the shelf space allocated, this result is similar to Herr' an et al (2006)[6] for scenario 1.

Table 13 provides optimal wholesale prices through which the manufacturers can intervene into the decision making process. The manufacturer with lower wholesale prices gets more shelf space since that is translated into lower retail price. Hence this is a clear indication that demand is more or less price driven.

Results shown in table 14 indicate prices and profits for leaders are more than those for followers. Lower the prices, higher the shelf space allocated to the products. Therefore the manufacturers can actually compete with each other in order to provide the retailers with lowest price for

substitutable products thereby attracting more shelf space for their product. This way, the manufacturers can influence the shelf space allocation strategies at the retailers.

Table 15 numerically validates the proofs 1 to 5. This table clearly proves that demand increases with the price of the substitutable items whereas demand decreases with its own price. Similarly, shelf-space increases with decreasing prices and vice versa.

Table 11: Initial values taken for obtaining optimal values of retailer

Elasticities	Values
Direct price elasticity	$\mu_1=4.5$; $\mu_2=5$
Cross price elasticity	$\varepsilon_1=1$; $\varepsilon_2=1.5$
Shelf space elasticity	$\gamma_1=0.5$; $\gamma_2=0.25$
Manufacturer 1	$W_1=1$
Manufacturer 2	$W_2=1.1$

Table 12: Optimal Values for the retailer playing a Cournot game.

Retailers Optimal	Values
Sales price of product1	$P_1*=1.85$
Sales price of product2	$P_2*=2.13$
Shelf space to manufacturer1	$S_1*=0.71$
Shelf space to manufacturer 2	$S_2*=0.29$

Table 13: Optimal value for manufacturers at Cournot Equilibrium.

Manufacturer Optimal	Values
Wholesale price of manufacturer 1	$W_1* = 1.143848$
Wholesale price of manufacturer 2	$W_2* = 1.330885$

Table 14: Optimal values for all the marketing channel members at Stackelberg Equilibrium.

Optimal	Values
Manufacturer 1	$W_1* = 1.05$
Manufacturer 2	$W_2* = 1.330885$
Retail price of product 1	$P_1* = 1.9$
Retail price of product2	$P_2* = 2.05$
Manufacturer 1 shelf space	$S_1* = 0.715$
Manufacturer 2 shelf space	$S_2* = 0.285$

Table 15: Numerical Analysis results of the proofs

Proofs	Values	Proofs	Values
$\partial Q_1 / \partial P_1 \leq 0$	-0.2010834	$\partial Q_2 / \partial P_1 \geq 0$	0.02300344
$\partial Q_1 / \partial P_2 \geq 0$	0.03881110	$\partial Q_2 / \partial P_2 \leq 0$	-0.06659836
$\partial Q_1 / \partial S_1 \geq 0$	0.05821664	$\partial Q_2 / \partial S_1 \geq 0$	0.009989755
$\partial Q_1 / \partial S_2 \geq 0$	0.0712652	$\partial Q_2 / \partial S_2 \geq 0$	0.04891535

3.12 Scenario 2

This scenario extends the demand model formulation of scenario 1 to incorporate competition among the retailers while retaining the manufacturers competition. This scenario considers two competing retailer as well as two competing manufacturers. As in Scenario 1, the manufacturers compete with one another through their substitutable products, which influences the shelf-space allocation at each of the retailers. The shelf-space allocation in turn influences the demand for each of the products. In addition, the retailers compete with one another through their pricing strategies for each of the substitutable products.

Figure (10) is a diagrammatic representation of two competing manufacturers and two competing retailers scenario. In particular, the figure shows:

➢ Examples of two competing manufacturers such as Nike and Reebok shoe companies

➢ Examples of two competing retailers selling the goods from competing manufacturers such as Big 5 and Foot Locker.

➢ Breakdown structure of this scenario into five sub-scenarios because of various gaming strategies followed by the supply chain partners

➢ Factors effecting this scenario such as pricing, shelf space and competition among various levels of supply chain.

➢ Finally it also reflects on the results that are achieved using different gaming strategies.
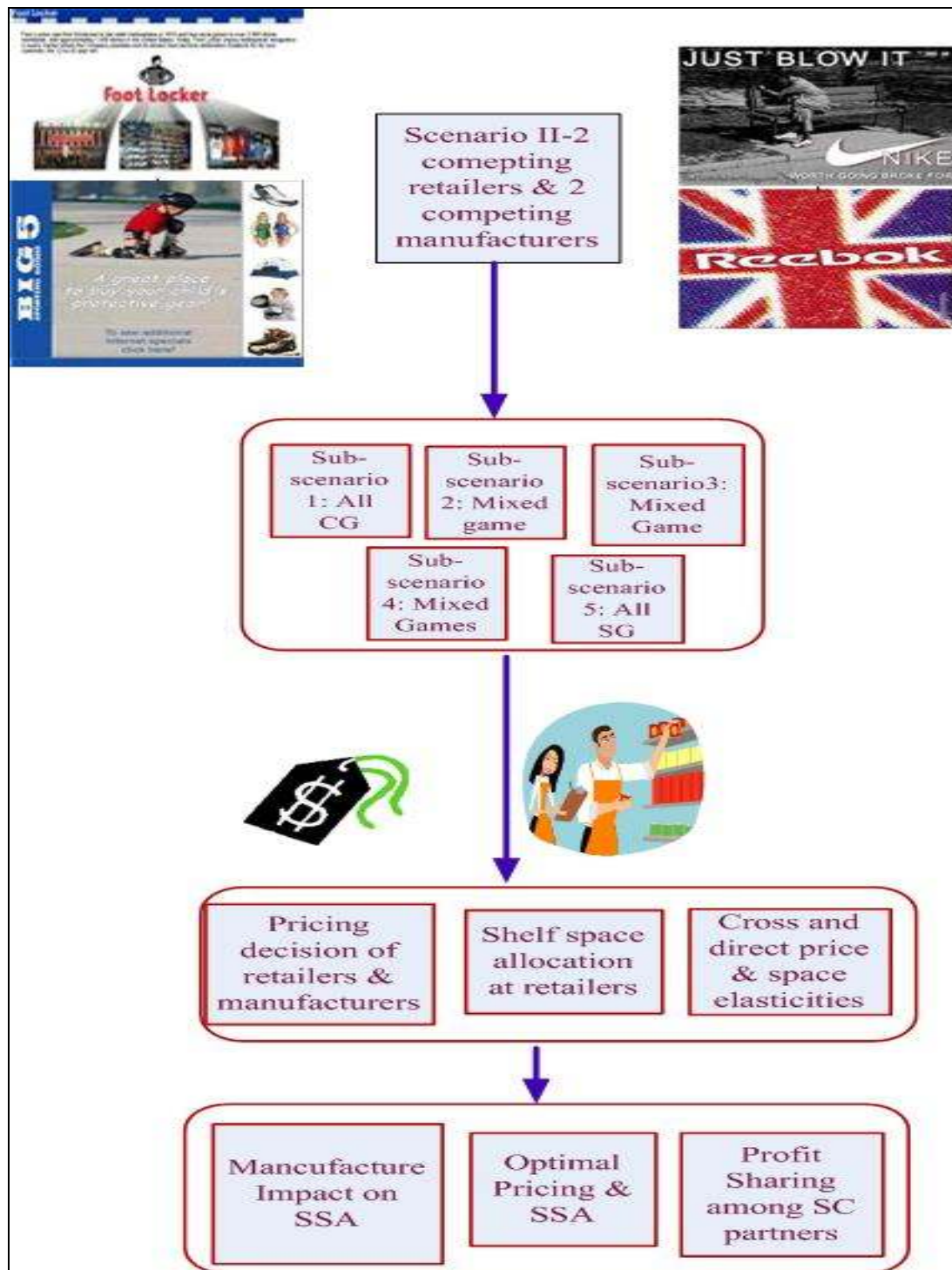
Figure 10: Represents Scenario II where competition among retailers is considered along with competing manufacturers.

3.13 Scenario 2 Model Formulation

In this section a retail shelf space allocation model for a competitive distribution channel is proposed. This model is examined to see if competitive pricing decisions are profitable for such channels. While previous studies indicated that coop programs increase the total channel profits there has been no evidence incase of bilateral monopolies. In this section the demand model presented consists of two competing retailers and two competing manufacturers. In particular, this model acounts for brand (competing manufacturers) and store (competing retailers) substitution effects generated by the shelf space and pricing strategies while playing Cournot, Stackelberg and Mixed games.

Each player maximizes her own profit. Since there are two retailer's, their objective function is given by:

$$\text{Max: } \Pi_i = (P_k - W_k)Q_k + (P_l - W_l)Q_l \tag{3.34}$$

$$Q_k = \alpha S_k^{\gamma_1} (1 - S_k^{\gamma_1}) P_k^{\varepsilon_{ki}} P_l^{\varepsilon_{li}} P_{ki}^{\mu_i} \tag{3.35}$$

$$Q_l = \alpha S_l^{\gamma_j} (1 - S_l^{\gamma_j}) P_l^{\varepsilon_{jl}} P_k^{\varepsilon_{jk}} P_{li}^{\mu_j} \tag{3.36}$$

Similarly, the manufacturer *Mk* optimization problem is given by :

$$\underset{W_K}{Max}\ \Pi_{M_K} = (W_{Ki} - C_K)Q_{Ki} + (W_{kj} - C_K)Q_l$$

where Q_K and Q_l are as seen in equation (3.35) and (3.36)

where *ck* is the unit-constant-production cost. Using equations (3.35,3.36) the objective function becomes:

$$\underset{W_K}{Max}\ \Pi_{M_K} = (W_{Ki} - C_K) * \alpha S_k^{\gamma_1}(1 - S_k^{\gamma_1})P_k^{\varepsilon_{ki}}P_l^{\varepsilon_{li}}P_{ki}^{\mu_i} + (W_{kj} - C_K) * \alpha S_l^{\gamma_j}(1 - S_l^{\gamma_j})P_l^{\varepsilon_{jl}}P_k^{\varepsilon_{jk}}P_{li}^{\mu_j}$$

(3.37)

The full demand model described in this scenario accounts for substitution effects and is of interest to mass distribution industries where substitutable products/brands are sold in competing stores. However, this is not the most general model in the sense that a symmetry assumption has been made regarding substitution effects. Indeed its readily seen that :

$Q_k \geq 0$: The proof of this is very simple and intuitive. There cannot be negative amounts of any product supplied or demanded. Therefore the quantity of sales >0 which is strictly positive.

$$\frac{\partial Q_k}{\partial S_k} \geq 0 ;\quad \frac{\partial Q_k}{\partial P_l} \geq 0 ;\quad \frac{\partial Q_k}{\partial P_k} \leq 0 ;\quad \frac{\partial Q_k}{\partial P_{kj}} \geq 0$$

The above proofs indicate that sales of brand *k* are nonnegative, increasing in the shelf space allocated to that brand and in the competing brand's retail price, and decreasing in its own retail price and competing brands shelf space. This is applicable to both the retailers and manufacturer goods. It is also believed that these assumptions are acceptable in the context of consumer products belonging to the same category and where the stores carrying them are of the same type.

Bergen and John (1997) [**108**] and Trivedi (1998) [**109**] deal with symmetric substitutability effect in competitive marketing channels. But one should note that the above assumptions do not imply symmetric elasticities.

The game of pricing and shelf-space allocation for attaining optimal demand is played as a Cournot ,Stackelberg and Mixed in five stages. Scenario 2 is more complex compared to scenario 1, because there exists competition within and between retailers and manufacturers. In order to obtain optimal demand values, several types of games have to be played between members and among different levels of the supply chain. This leads us to the classification of scenario 2 into five sub-scenarios as follows:

- ➢ <u>Sub-scenario1:</u> Cournot game played at both retailers and manufacturers level considering elasticities fixed values

- ➢ <u>Sub-scenario2:</u> Retailers play a Stackelberg game considering the wholesale prices are known.

- ➢ <u>Sub-scenario 3:</u> Manufacturer1 plays a SG individually with both the competing retailers as followers. The optimal retail prices are obtained from sub-scenario 2. Cournot game is played within the manufacturers group.

- ➢ <u>Sub-scenario 4:</u> Manufacturer2 plays SG individually with both the competing retailers as followers. The optimal retail prices are obtained from sub-scenario 2. Cournot game is played within the manufacturers group.

> ➤ Sub-scenario 5: Stackelberg played at both retailers and manufacturers to find optimal wholesale prices, the retail prices are obtained from sub-scenario 2.

3.14 Scenario 2: Stackelberg Game

In the two manufacturer two retailers competition scenario, the optimal demand/supply values are obtained by maximizing each player's profit functions (Cournot game). Additionally, in a Stackelberg game scenario, manufacturers become leaders and the retailers are the followers. The optimal demand values in a Stackelberg equilibrium are obtained by following the two-step procedure outlined below:

> ➤ First, optimize the follower's (retailer) profit equation to get her reaction functions (i.e., shelf space allocation and sale price) for a given leaders' strategies (wholesale prices). The leaders strategies are given by their responses to a Cournot game scenario.

> ➤ Next, these reaction functions are inserted into the manufacturers' objectives and Nash equilibrium is calculated.

This is best illustrated in the two manufacturers two retailers scenario in which manufacturers compete for shelf-space allocation at a given retailer while the retailers compete through their retail pricing strategies.

3.15 Retailers Response Functions

The following proposition characterizes the competing retailers reaction functions for shelf space allocation and retail prices at Nash equilibrium. The optimal prices and allocations are obtained by solving the first order optimality conditions for the retailers profit maximization.

Proposition 1:

(a) Assuming there is an interior solution, both the retailers reaction function for the shelf space allocated to the two products is given as follows:

Retailer 1:

$$S_1^* = \frac{\left[(P_{21} - W_{21}) * P_{21}^{\varepsilon_{22}} * P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2}\right]^{\frac{1}{\gamma_1 - 1}}}{\left[(P_{11} - W_{11}) * P_{11}^{\varepsilon_{11}} * P_{21}^{\varepsilon_{12}} * P_{12}^{\mu_1}\right]^{\frac{1}{\gamma_1 - 1}} + \left[(P_{21} - W_{21}) * P_{21}^{\varepsilon_{22}} * P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2}\right]^{\frac{1}{\gamma_1 - 1}}} \tag{3.38}$$

Retailer 2:

$$S_2^* = \frac{\left[(P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2}\right]^{\frac{1}{\gamma_2 - 1}}}{\left[(P_{12} - W_{12}) * P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1}\right]^{\frac{1}{\gamma_2 - 1}} + \left[(P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2}\right]^{\frac{1}{\gamma_2 - 1}}} \tag{3.39}$$

(b) The retailers sales prices for both the products at the equilibrium are give as follows:

Retailer 1: Sales price of product 1 at retailer 1 is P_{11}:

$$P_{11}^{\varepsilon_{21}-\varepsilon_{11}} = \frac{P_{21}^{\varepsilon_{11}} * P_{12}^{\mu_1} * S_1^{\gamma_1}}{\varepsilon_{21} * (P_{21} - W_{21})(1 - S_1)^{\gamma_1} * P_{21}^{\varepsilon_{22}} * P_{22}^{\mu_2}} * \left(W_{11} * \varepsilon_{11} - (\varepsilon_{11} + 1) * P_{11}\right) \qquad (3.40)$$

Sales price of product 2 at retailer 1 is given as P_{21}:

$$P_{21}^{\varepsilon_{12}-\varepsilon_{22}} = \frac{P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2} * (1 - S_1)^{\gamma_1}}{(P_{11} - W_{11}) * S_1^{\gamma_1} * P_{11}^{\varepsilon_{11}} * P_{12}^{\mu_1} * \varepsilon_{12}} * \left(\varepsilon_{22} * W_{21} - (\varepsilon_{22} + 1) * P_{21}\right) \qquad (3.41)$$

Retailer 2: Sales price of product 1 at retailer 2 is given as P_{12}

$$P_{12}^{\varepsilon_{21}-\varepsilon_{11}} = \frac{P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1} * S_2^{\gamma_2}}{(P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_1} * (1 - S_2)^{\gamma_2} * \varepsilon_{21}} * \left(\varepsilon_{11} * W_{12} - (\varepsilon_{11} + 1) * P_{12}\right) \quad (3.42)$$

Sales price of product 2 at retailer 2 is given as P_{22}

$$P_{22}^{\varepsilon_{12}-\varepsilon_{22}} = \frac{P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2} * (1 - S_2)^{\gamma_2}}{(P_{12} - W_{12}) * P_{12}^{\varepsilon_{11}} * P_{11}^{\mu_1} * S_2^{\gamma_2} * \varepsilon_{22}} * \left(\varepsilon_{22} * W_{22} - (\varepsilon_{22} + 1) * P_{22}\right) \qquad (3.43)$$

Substituting the optimal values of shelf space from equations (3.38) and (3.39) into optimal retail prices would complicate the already complex equations for the four retail prices. Therefore a closed form analytical solution is not available.

Proof 6 _:_ Assuming interior solution first order optimality condition for retailers maximization problem with respect to shelf space allocation as well as price is as follows:

Retailer 1:

$$\frac{\partial \Pi_1}{\partial S_1} = (P_{11} - W_{11}) * P_{11}^{\varepsilon_{11}} * P_{21}^{\varepsilon_{12}} * P_{12}^{\mu_1} \gamma_1 S_1^{\gamma_1 - 1} - (P_{21} - W_{21}) * P_{21}^{\varepsilon_{22}} * P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2} * \gamma_1 * (1 - S_1)^{\gamma_1 - 1} = 0$$

(3.44)

$$\frac{\partial \Pi_1}{\partial P_{11}} = P_{11}^{\varepsilon_{21} - \varepsilon_{11}} + \frac{P_{21}^{\varepsilon_{11}} * P_{12}^{\mu_1} * S_1^{\gamma_1}}{\varepsilon_{21} * (P_{21} - W_{21})(1 - S_1)^{\gamma_1} * P_{21}^{\varepsilon_{22}} * P_{22}^{\mu_2}} * \left((\varepsilon_{11} + 1) * P_{11} - W_{11} * \varepsilon_{11}\right) = 0 \quad (3.45)$$

$$\frac{\partial \Pi_1}{\partial P_{21}} = P_{21}^{\varepsilon_{12} - \varepsilon_{22}} + \frac{P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2} * (1 - S_1)^{\gamma_1}}{(P_{11} - W_{11}) * S_1^{\gamma_1} * P_{11}^{\varepsilon_{11}} * P_{12}^{\mu_1} * \varepsilon_{12}} * \left((\varepsilon_{22} + 1) * P_{21} - \varepsilon_{22} * W_{21}\right) = 0 \quad (3.46)$$

Retailer 2

$$\frac{\partial \Pi_2}{\partial S_2} = (P_{12} - W_{12}) * P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1} \gamma_2 S_2^{\gamma_2 - 1} - (P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2} * \gamma_2 * (1 - S_2)^{\gamma_2 - 1} = 0$$

(3.47)

$$\frac{\partial \Pi_2}{\partial P_{12}} = P_{12}^{\varepsilon_{21} - \varepsilon_{11}} + \frac{P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1} * S_2^{\gamma_2}}{(P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_1} * (1 - S_2)^{\gamma_2} * \varepsilon_{21}} * \left((\varepsilon_{11} + 1) * P_{12} - \varepsilon_{11} * W_{12}\right) \quad (3.48)$$

$$\frac{\partial \Pi_2}{\partial P_{22}} = P_{22}^{\varepsilon_{12}-\varepsilon_{22}} + \frac{P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2} * (1-S_2)^{\gamma_2}}{(P_{12}-W_{12})*P_{12}^{\varepsilon_{11}}*P_{11}^{\mu_1}*S_2^{\gamma_2}*\varepsilon_{22}} * \left((\varepsilon_{22}+1)*P_{22}-\varepsilon_{22}*W_{22}\right) \qquad (3.49)$$

The analytical solutions presented in this section for retail prices are not closed form solutions. Hence a numerical optimization technique called Newton Raphon's method is used to arrive at the optimal retail prices while trying to maximize retailers profit functions.

Most of the literature on retailer demand optimization models assumes wholesale prices to be exogenous. But this chapter considers wholesale prices as a tools used by a manufacturer to obtain the desirable share of the shelf. Notice also that the retailers reaction functions satisfy $0 < S_1^*, S_2^* < 1$, for all parameters and wholesale price values. Hence the solution is indeed interior. The shelf space allocated to each brand is decreasing in its wholesale price and increasing in the competitive brand's wholesale price. Below are the partial derivatives of the optimal prices and shelf spaces with respect to wholesale prices offered to retailer for products 1 and 2.

Retailer 1:

$$\frac{\partial S_1^*}{\partial W_{11}} = \frac{(P_{21}-W_{21})^c (P_{11}-W_{11})^{c-1} * P_{11}^{\varepsilon_{11}+\varepsilon_{21}} * P_{21}^{\varepsilon_{12}+\varepsilon_{22}} * P_{12}^{\mu_1} * P_{22}^{\mu_2}}{[(P_{21}-W_{21})^c * P_{21}^{\varepsilon_{22}} * P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2} + (P_{11}-W_{11})*P_{11}^{\varepsilon_{11}}*P_{21}^{\varepsilon_{12}}*P_{12}^{\mu_1}]^2} \qquad (3.50)$$

$$\frac{\partial S_1^*}{\partial W_{21}} = \left(\frac{c*(P_{21}-W_{21})^{2c-1}(P_{11}^{\varepsilon_{21}}*P_{21}^{\varepsilon_{22}}*P_{22}^{\mu_2})^2}{[(P_{11}-W_{11})*P_{11}^{\varepsilon_{11}}*P_{21}^{\varepsilon_{12}}*P_{12}^{\mu_1}+(P_{21}-W_{21})^c*P_{21}^{\varepsilon_{22}}*P_{11}^{\varepsilon_{21}}*P_{22}^{\mu_2}]^2} - c*(P_{21}-W_{21})^{c-1} \right) *$$
$$\frac{1}{[(P_{11}-W_{11})*P_{11}^{\varepsilon_{11}}*P_{21}^{\varepsilon_{12}}*P_{12}^{\mu_1}+(P_{21}-W_{21})^c*P_{21}^{\varepsilon_{22}}*P_{11}^{\varepsilon_{21}}*P_{22}^{\mu_2}]} \qquad (3.51)$$

$$\frac{\partial P_{11}^*}{\partial W_{11}} = \left(\frac{b_1 * \varepsilon_{11}}{a_1 * \varepsilon_{12}}\right) \div \left[(\varepsilon_{21} - \varepsilon_{11}) * P_{11}^{\varepsilon_{21}-\varepsilon_{11}-1} + \frac{(1+\varepsilon_{11})*b_1}{\varepsilon_{21}*a_1}\right] \qquad (3.52)$$

$$\frac{\partial P_{21}^*}{\partial W_{11}} = \left[\frac{((\varepsilon_{22}*W_{21}) - (\varepsilon_{22}+1)*P_{21})*b_2}{\varepsilon_{12}*a_2*(P_{11}-W_{11})}\right] \div \left[(\varepsilon_{12} - \varepsilon_{22}) * P_{11}^{\varepsilon_{12}-\varepsilon_{22}-1} + \frac{(\varepsilon_{22}+1)*b_2}{\varepsilon_{12}*a_2}\right] \quad (3.53)$$

$$\frac{\partial P_{11}^*}{\partial W_{21}} = \left(\frac{y_2 - x_2 * P_{11}}{(P_{21}-W_{21})^2}\right) \div \left[(\varepsilon_{21} - \varepsilon_{11}) * P_{11}^{\varepsilon_{21}-\varepsilon_{11}-1} + \frac{(\varepsilon_{22}+1)*b_2}{(P_{21}-W_{21})}\right] \qquad (3.54)$$

$$\frac{\partial P_{21}^*}{\partial W_{21}} = \left(\frac{b_2 * \varepsilon_{22}}{a_2 * \varepsilon_{12}}\right) \div \left[(\varepsilon_{12} - \varepsilon_{11}) * P_{21}^{\varepsilon_{12}-\varepsilon_{11}-1} + \frac{(\varepsilon_{22}+1)*b_2}{\varepsilon_{12}*a_2}\right] \qquad (3.55)$$

Where :

$a_1 = (P_{21} - W_{21}) * (1 - S_1)^{\gamma_1} * P_{21}^{\varepsilon_{22}} * P_{22}^{\mu_2}$

$b_1 = P_{21}^{\varepsilon_{111}} * P_{12}^{\mu_1} * S_1^{\gamma_1}$

$a_2 = (P_{11} - W_{11}) * S_1^{\gamma_1} * P_{11}^{\varepsilon_{11}} * P_{12}^{\mu_1}$

$b_2 = P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_2} * (1 - S_1)^{\gamma_1}$

$x_2 = (\varepsilon_{11}+1) * b_1 * (1-S_1)^{-\gamma_1} * P_{21}^{-\varepsilon_{22}} * P_{22}^{-\mu_2} \Big/ \varepsilon_{21}$

$y_2 = b_1 * W_{11} * \varepsilon_{11}(1-S_1)^{\gamma_1} * P_{21}^{\varepsilon_{22}} * P_{22}^{\mu_2} \Big/ \varepsilon_{22}$

$c = 1 \Big/ \gamma_1 - 1$

Retailer 2:

$$\frac{\partial S_2^*}{\partial W_{12}} = \frac{d * (P_{22} - W_{22})^d * (P_{12} - W_{12})^{d-1} * (P_{12}^{\varepsilon_{21}+\varepsilon_{11}} * P_{22}^{\varepsilon_{22}+\varepsilon_{12}} * P_{11}^{\mu_1} * P_{21}^{\mu_2})^d}{\left(\left[(P_{22} - W_{22})^d P_{12}^{\varepsilon_{21}} * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_2}\right]^d + \left[(P_{12} - W_{12})^d P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1}\right]^d\right)^2}$$ (3.56)

$$\frac{\partial S_2^*}{\partial W_{22}} = \frac{- d * (P_{22} - W_{22})^{d-1} * P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2}}{(P_{22} - W_{22})^d (P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2})^d + (P_{12} - W_{12})^d (P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1})^d} +$$

$$\left(\frac{d * (P_{22} - W_{22})^{2d-1}\left(P_{12}^{\varepsilon_{21}} * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_2}\right)^{2d}}{\left((P_{22} - W_{22})^d (P_{12}^{\varepsilon_{21}} * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_2})^d + (P_{12} - W_{12})^d (P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1})^d\right)^2}\right)$$ (3.57)

$$\frac{\partial P_{22}^*}{\partial W_{12}} = \left(\frac{(\varepsilon_{22} + 1) * P_{22} - a_5 * W_{22} * \varepsilon_{22}}{\varepsilon_{12} * a_5 * (P_{12} - W_{12})}\right) \div \left[(\varepsilon_{12} - \varepsilon_{22}) * P_{22}^{\varepsilon_{12}-\varepsilon_{22}-1} - \frac{(\varepsilon_{22} + 1)}{\varepsilon_{12} * b_5}\right]$$ (3.58)

$$\frac{\partial P_{22}^*}{\partial W_{22}} = \left(\frac{\varepsilon_{22} * b_5}{\varepsilon_{12} * a_5}\right) \div \left[(\varepsilon_{12} - \varepsilon_{22}) P_{22}^{\varepsilon_{12}-\varepsilon_{22}-1} + \frac{(\varepsilon_{22} + 1) * b_5}{\varepsilon_{12} * a_5}\right]$$ (3.59)

$$\frac{\partial P_{12}^*}{\partial W_{12}} = \left(\frac{\varepsilon_{11} * a_4}{\varepsilon_{21} * b_4}\right) \div \left[(\varepsilon_{21} - \varepsilon_{11}) * P_{12}^{\varepsilon_{21}-\varepsilon_{11}-1} + \frac{(\varepsilon_{11} + 1) * a_4}{\varepsilon_{21} * b_4}\right]$$ (3.60)

$$\frac{\partial P_{12}^*}{\partial W_{22}} = \left(\frac{y_3 - x_3 * P_{12}}{(P_{22} - W_{22})^2}\right) \div \left[(\varepsilon_{21} - \varepsilon_{11}) * P_{12}^{\varepsilon_{21}-\varepsilon_{11}-1} + \frac{x_3}{(P_{22} - W_{22})}\right]$$ (3.61)

Where :

$$a_5 = P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2} * (1 - S_2)^{\gamma_2}$$

$$b_5 = (P_{12} - W_{12}) * P_{12}^{\varepsilon_{11}} * P_{11}^{\mu_1} * S_2^{\gamma_2}$$

$$a_4 = P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_1} * S_2^{\gamma_2}$$

$$b_4 = (P_{22} - W_{22}) * P_{22}^{\varepsilon_{22}} * P_{21}^{\mu_1} * (1 - S_2)^{\gamma_2}$$

$$x_3 = \frac{(\varepsilon_{11} + 1) * a_4}{\varepsilon_{21}} * P_{22}^{-\varepsilon_{22}} * P_{21}^{-\mu_2} * (1 - S_2)^{-\gamma_2}$$

$$y_3 = \frac{\varepsilon_{11} * a_4 * W_{12} * P_{22}^{-\varepsilon_{22}} * P_{21}^{-\mu_2} * (1 - S_2)^{-\gamma_2}}{\varepsilon_{21}}$$

$$d = \frac{1}{\gamma_2} - 1$$

Item (ii) in Proposition 1 indicates that the sale price set by the retailer depend on the direct- and cross-price elasticities of sales and shelf space allocated along with the sale price of the substitutable product and its whole sale price. The sale price of a product increases when the price elasticity for the brand decreases and when its cross-price elasticity increases. This result is intuitive because it confirms that the retailer will increase their sale price or retail margin for those brands whose sales are less sensitive to a variation in their own price, but also when the brand's sales are very sensitive to the price variations of the competing brand. Although the final equilibrium expression of the shelf space allocated by the retailer to each brand will be obtained after computing the retail and whole sale prices , we can nevertheless characterize the ratio of shelf space.

Let us assume $c = \frac{1}{\gamma_1} - 1$

Proposition 2:

The equilibrium ratio of brands respective shelf space is given as the ratio of revenues generated with respect to substitutable product:

$$\frac{S_1^*}{1 - S_1^*} = \left(\frac{Q_{11}^c * (P_{11} - W_{11})^c}{Q_{21}^c * (P_{21} - W_{21})^c} \right) \tag{3.62}$$

$$\frac{S_1^*}{1 - S_1^*} = \left(\frac{(P_{11} - W_{11})^c}{(P_{21} - W_{21})^c} \right) * P_{11}^{\varepsilon_{11} - \varepsilon_{21}} * P_{21}^{\varepsilon_{12} - \varepsilon_{22}} * P_{12}^{\mu_1} * P_{22}^{-\mu_2} \tag{3.63}$$

Ratio of brand sales is given by:

$$\frac{Q_1}{Q_2} = \left(\frac{S_1^*}{1 - S_2^*} \right)^{\gamma_1} * P_{11}^{\varepsilon_{21} - \varepsilon_{11}} * P_{21}^{\varepsilon_{22} - \varepsilon_{12}} * P_{12}^{-\mu_1} * P_{22}^{\mu_2} \tag{3.64}$$

Therefore, the allocation rule in equation (3.62) tells us that the brands' relative shares of the shelf space must be equal to their own profits. This proposition provides us an answer for the equilibrium allocation rule. Proposition 1 as a whole answers our questions regarding the influence of manufacturers prices in the decision making processes. But the actual influence is observed only when a Stackelberg game played by manufacturers with retailers.

3.16 Manufacturers Response Functions

The manufacturers take into account both the retailer's reaction functions and play a Nash game. Therefore, both manufacturers' wholesale prices at equilibrium are obtained by replacing the retailer's reaction functions into their profit functions and maximizing them. The main assumption in this scenario is that each manufacturer offers a different price to each retailer. This means that the whole sale price for product 1 or 2 are not the same for both the retailers. There could be several reasons behind this price discrimination. Some of them could be the quantity ordered, relations maintained (personal communication with Wal-Mart) and visibility offered for certain products over others.

Manufacturer 'k' fixes his wholesale price by maximizing his objective function or profit function with respect to the whole sale prices subject to retailers optimal of shelf space allocations and prices given by equations in section 3.16. The profit function of the 'k^{th}' manufacturer is :

$$\underset{W_K}{Max}\ \Pi_{M_K} = (W_K - C_K) * \alpha * P_{Ki}^{\varepsilon_K}\ P_{li}^{\varepsilon_l}\ S_K^{\gamma_K - 1} P_{lj}^{\mu_l}$$

The retail prices are not substituted in the profit function of the manufacturer above since there exists no closed form solutions for the retail prices. Next step is to evaluate the profit function of manufacturers at Cournot and Stackelberg equilibriums in order to obtain the optimal wholesale prices of their products. The next proposition summarizes results for the case where both manufacturers are active players and offer cooperation to the retailers as leaders. In such a scenario, the game is a two-stage sequential one. Nash equilibrium is determined recursively by

first obtaining both the retailers reaction functions and then determining the manufacturers optimal participation prices.

Proposition 3:

Assuming there exists an interior solutions, the manufacturers equilibrium wholesale prices are given as:

$$
(W_{Ki} - C_{Ki}) \begin{bmatrix} \gamma_{Ki} P_{Ki}^{\varepsilon_K} P_{li}^{\varepsilon_l} S_K^{\gamma_K - 1} P_{lj}^{\mu_l} \dfrac{\partial S_{Ki}^*}{\partial W_{Ki}} + \mu_j P_{Ki}^{\varepsilon_K} P_{li}^{\varepsilon_L} S_K^{\gamma_K} P_{jl}^{\mu_j} \dfrac{\partial P_{lj}^*}{\partial W_{Ki}} \\ + \\ \varepsilon_{kj} P_{lj}^{\mu_K} P_{li}^{\varepsilon_l - 1} S_k^{\gamma_K} P_{ki}^{\varepsilon_l} \dfrac{\partial P_{li}^*}{\partial W_{Ki}} + \mu_j P_{lj}^{\mu_j - 1} P_{li}^{\varepsilon_l} S_k^{\gamma_K} P_{ki}^{\varepsilon_l} \dfrac{\partial P_{Ki}^*}{\partial W_{Ki}} \end{bmatrix}
$$
$$
+ P_{Ki}^{\varepsilon_K} P_{li}^{\varepsilon_l} S_K^{\gamma_K} P_{lj}^{\mu_l} = 0
$$

(3.65)

$0 < \gamma_1 < 1; \ 0 < \gamma_2 < 1; \ K, l = 1,2 \ \text{and} \ K \neq l, i, j = 1,2 \ \text{and} \ i \neq j$

<u>Proof 7</u>: Assuming interior solution the first order optimality condition for each of the manufacturers profit maximization problem is given as follows:

(i) Partial derivative of manufacturer 1 profit with respect to whole sale price offered to first retailer for product1

$$
\frac{\partial \Pi_{M_1}}{\partial W_{11}} = 0
$$

(3.66)

$$P_{11}^{\varepsilon_{11}} * P_{21}^{\varepsilon_{12}} * P_{12}^{\mu_1} * S_1^{\gamma_1} * [1 + (W_{11} - C_{11}) *$$

$$\left(\frac{\varepsilon_{11}}{P_{11}} * \frac{\partial P_{11}}{\partial W_{11}} + \frac{\varepsilon_{12}}{P_{21}} * \frac{\partial P_{21}}{\partial W_{11}} + \frac{\gamma_1}{S_1} * \frac{\partial S_1}{\partial W_{11}} \right) = 0 \qquad (3.67)$$

$$W_{11}^* = C_{11} + \frac{1}{\left(\dfrac{\varepsilon_{11}}{P_{11}} * \dfrac{\partial P_{11}}{\partial W_{11}} + \dfrac{\varepsilon_{12}}{P_{21}} * \dfrac{\partial P_{21}}{\partial W_{11}} + \dfrac{\gamma_1}{S_1} * \dfrac{\partial S_1}{\partial W_{11}} \right)} \qquad (3.68)$$

(ii) Partial derivative of profit with respect to whole sale price offered to second retailer for product1

$$\frac{\partial \Pi_{M_1}}{\partial W_{12}} = 0 \qquad (3.69)$$

$$P_{12}^{\varepsilon_{11}} * P_{22}^{\varepsilon_{12}} * P_{11}^{\mu_2} * S_2^{\gamma_2} * [1 + (W_{12} - C_{11}) *$$

$$\left(\frac{\varepsilon_{11}}{P_{12}} * \frac{\partial P_{12}}{\partial W_{12}} + \frac{\varepsilon_{12}}{P_{22}} * \frac{\partial P_{22}}{\partial W_{12}} + \frac{\gamma_2}{S_2} * \frac{\partial S_2}{\partial W_{12}} \right) = 0 \qquad (3.70)$$

$$W_{12}^* = C_{11} + \frac{1}{\left(\dfrac{\varepsilon_{11}}{P_{12}} * \dfrac{\partial P_{12}}{\partial W_{12}} + \dfrac{\varepsilon_{12}}{P_{22}} * \dfrac{\partial P_{22}}{\partial W_{12}} + \dfrac{\gamma_2}{S_2} * \dfrac{\partial S_2}{\partial W_{12}} \right)} \qquad (3.71)$$

136

Similarly for Manufacturer 2

(i) Partial derivative of manufacturers profit with respect to whole sale price offered to first retailer for product2

$$\frac{\partial \Pi_{M_2}}{\partial W_{21}} = 0 \tag{3.72}$$

$$P_{21}^{\varepsilon_{22}} * P_{11}^{\varepsilon_{21}} * P_{22}^{\mu_1} * (1 - S_1)^{\gamma_1} * [1 + (W_{21} - C_{21}) *$$
$$\left(\frac{\varepsilon_{22}}{P_{21}} * \frac{\partial P_{21}}{\partial W_{21}} + \frac{\varepsilon_{21}}{P_{11}} * \frac{\partial P_{11}}{\partial W_{21}} - \frac{\gamma_1}{1 - S_1} * \frac{\partial S_2}{\partial W_{21}} \right) = 0 \tag{3.73}$$

$$W_{21}^* = C_{21} + \frac{1}{\left(\frac{\varepsilon_{22}}{P_{21}} * \frac{\partial P_{21}}{\partial W_{21}} + \frac{\varepsilon_{21}}{P_{11}} * \frac{\partial P_{11}}{\partial W_{21}} - \frac{\gamma_1}{1 - S_1} * \frac{\partial S_2}{\partial W_{21}} \right)} \tag{3.74}$$

(ii) Partial derivative of manufacturers profit with respect to whole sale price offered to second retailer for product2

$$\frac{\partial \Pi_{M_2}}{\partial W_{22}} = 0 \tag{3.75}$$

137

$$P_{22}^{\varepsilon_{22}} * P_{12}^{\varepsilon_{21}} * P_{21}^{\mu_2} * (1 - S_2)^{\gamma_2} * [1 + (W_{22} - C_{21}) *$$

$$\left(\frac{\varepsilon_{22}}{P_{22}} * \frac{\partial P_{22}}{\partial W_{22}} + \frac{\varepsilon_{21}}{P_{12}} * \frac{\partial P_{12}}{\partial W_{22}} - \frac{\gamma_2}{1 - S_2} * \frac{\partial S_2}{\partial W_{22}} \right) = 0 \qquad (3.76)$$

$$W_{22}^* = C_{21} + \cfrac{1}{\left(\cfrac{\varepsilon_{22}}{P_{22}} * \cfrac{\partial P_{22}}{\partial W_{22}} + \cfrac{\varepsilon_{21}}{P_{12}} * \cfrac{\partial P_{12}}{\partial W_{22}} - \cfrac{\gamma_2}{1 - S_2} * \cfrac{\partial S_2}{\partial W_{22}} \right)} \qquad (3.77)$$

Due to the open loop structure of the retail prices and partial derivatives, numerical analysis is performed. The complexity of retail price substitution and simplification reduces our capability to derive the upper and lower bounds on the wholesale price of the manufacturer like [6]. Hence we are unable to comment on the concavity of the whole sale prices and sensitivity analysis of the lower and upper bounds.

3.17 Case Study/Numerical Analysis

Once the strategies to be played are laid out carefully for scenario 2 using all the sub-scenarios, generalized analytical equilibrium solution for the optimal demand could not be obtained due to the open loop structure of retail prices. Therefore numerical evaluation techniques such as Newton Raphson's method are employed to gain insights into the optimal behavior of retailers and manufacturers under different strategies. Initial starting values and fixed values for elasticities in order to start the simulation for obtaining optimal shelf space and prices are taken from Herr' an et al (2006) [6] and are tabulated in table 16. The simulation was run in R-software.

The maximum number of iterations to reach convergence for this method was set to 10,000. The optimal values are reported in the results section 3.18.

3.18 Scenario 2: Results and Discussions

This section presents the results when the supply chain members interact with each other through the combination of options they choose. The results are believed to be best for the equilibrium strategies and deviation from these strategies indicates deviation from equilibrium which would prove to be non-beneficial to most of the supply chain members. Tabulated below are the results for this scenario.

Table 16 values are used to as fixed values for elasticities and cost prices. The wholesale prices are considered for evaluating retail prices in Cournot games.

Table 17 depicts results of a pure Cournot game played among the retailers considering the wholesale prices as exogenous and given. The shelf space allocated reveal the similarity to scenario 1. Lower the price higher the shelf space allocated. These can be categorized as sub-scenario 1 results or Cournot equilibrium results.

Table 18 considers the values of retailers as exogenous (taken from table 17) and then calculates the values of the manufacturers while playing a pure Cournot game at both levels.

Since there exist two competing retailers, one is considered the leader like a veteran in that area and other is a follower who is a new entrant. The leader fixes the prices of his product first and

the follower observes and makes his moves. The results in table 19 are indicative of the leader-follower relationship among retailers while considering the leaders prices from table 17 and wholesale prices from table 18. The follower values are then computed using the same routine of Newton Raphson's method.

Results of table 20 are obtained by playing different games at different levels of the supply chain. This strategy is called Mixed games. The retailers for this strategy have already play Stackelberg and their values are obtained from table 17. These retail prices are considered to be exogenous variables while optimizing the manufacturers profit function.

Result shown in table 21 are indicative of Stackelberg game between all parties of the supply chain. The retail prices and manufacturer's 1 prices are considered exogenous for obtaining optimal values for manufacturer 2. These exogenous values for retailers are taken from Table 19 and for the Manufacturer 1 are chosen from Table 20.

The profits/losses shown in table 22 indicate coop programs as non-beneficial. Though this result contradicts the literature, the rationale for such behavior is the uniqueness of the demand function followed in scenario 2, that considers competition among retailers. The demand model considers the retail price of one product from the competing retailer to influence the demand. On the other hand, the profits and prices for the Cournot are greater compared to Stackelberg and Mixed games. These results are consistent with the classical economic literature. Table 22 also presents us opportunities for future research as it indicates that there are some other factors that strongly influence the profits which are not considered.

Table 16: Fixed parameters for calculating the two retailers optimal values.

Parameter	Values	Parameter	Values	Parameters	Values	Parameters	Values
ε_{12}	1.5	μ_1	4.5	ε_{11}	1.1	γ_1	0.75
ε_{22}	1.7	μ_2	5	ε_{21}	1.3	γ_2	0.75
W_{11}	\$1/unit	W_{21}	\$1.1/unit	W_{12}	\$1.2/unit	W_{22}	\$1.3/unit
C_{11}	\$0.79	C_{12}	\$0.89	C_{21}	\$0.79	C_{22}	\$0.89

Table 17: Optimal values for both the retailer in Cournot game

Sales prices at retailers	Values ($/unit)	Shelf space	Values (%)
Price at retailer1 for product 1	$P_{11}^*=1.24$	Space allocated to product 1 at retailer 1	$S_1^*=72$
Price at retailer1 for product 2	$P_{21}*=1.723$	Space allocated to product 2 at retailer 1	$(1-S_1^*)=28$
Price at retailer2 for product 1	$P_{12}*=1.832$	Space allocated to product 1 at retailer 2	$S_2^*=69$
Price at retailer2 for product 2	$P_{22}*=2.336$	Space allocated to product 2 at retailer 2	$(1-S_2^*)=31$

Table 18: Optimal Values for Manufacturers playing Cournot with retailers

Wholesale prices	Values	Whole Sale prices	Values
Price of product 1 given to retailer 2	$W_{12}*=3.3$	Price of product 1 given to retailer 1	$W_{11}^*=0.9$
Price of product 2 given to retailer 2	$W_{22}*=1.8458$	Price of product 2 given to retailer 1	$W_{21}*=1.93$

Table 19: Optimal values for both retailers in a Stackelberg game with Retailer 1 as leader.

Sales prices at retailers	Values ($/unit)	Shelf space	Values (%)
Price at retailer1 for product 1	$P_{11}^* = 1.24$	Space allocated to product 1 at retailer 1	$S_1^* = 72$
Price at retailer1 for product 2	$P_{21}^* = 1.723$	Space allocated to product 2 at retailer 1	$(1-S_1^*) = 28$
Price at retailer2 for product 1	$P_{12}^* = 0.7$	Space allocated to product 1 at retailer 2	$S_2^* = 76.82$
Price at retailer2 for product 2	$P_{22}^* = 1.1423$	Space allocated to product 2 at retailer 2	$(1-S_2^*) = 23.18$

Table 20: Optimal Values for Manufacturers playing Cournot & Stackelberg retailers

Wholesale prices	Values	Whole Sale prices	Values
Price of product 1 given to retailer 2	$W_{12}^* = 1.17$	Price of product 1 given to retailer 1	$W_{11}^* = 1.51$
Price of product 2 given to retailer 2	$W_{22}^* = 0.5$	Price of product 2 given to retailer 1	$W_{21}^* = 1.5$

Table 21: Optimal values of M1, M2 in a Stackelberg game at 2 levels with M1 as the leader

Wholesale prices	Values	Whole Sale prices	Values
Price of product 1 given to retailer 2	$W_{12}^* = 1.17$	Price of product 1 given to retailer 1	$W_{11}^* = 1.51$
Price of product 2 given to retailer 2	$W_{22}^* = 0.6356$	Price of product 2 given to retailer 1	$W_{21}^* = 1.08$

Table 22: Optimal values of M1, M2 in a Stackelberg game at 2 levels with M1 as the leader

Players	Cournot Game Profits	Mixed Game Profits	Stackelberg Game Profits	Leader/Follower for Stackelberg game
Retailer 1	-$17,928.90	$143.30	$641.99	Follower
Retailer 2	$9,168.93	$1,724.92	$1,183.86	Leader
Manufacturer 1	$39,826.85	$1,001.43	$1,001.43	Leader
Manufacturer 2	$228,119.37	-$831.87	-$789.50	Follower
Overall Supply Chain	$259,186.25	$2,037.79	$2,037.79	
Overall profits and individual profits	Cournot > Mixed games and Stackelberg			

3.19 Conclusions

This chapter examined the competition among supply-chain members and its influence on demand estimation models at a retailer using a game theoretic approach. A systematic analysis of Cournot and Stackelberg oligopolies with increasingly complex demand functions is conducted in order to obtain optimal demand values. These optimal demand values are translated into optimal shelf-space allocation rules and pricing decisions strategies for the various members of the supply chain. Interesting insights based on factors and strategies affecting allocation rules and profits were also revealed in this process.

This game-theory approach to optimal demand estimation in the presence of competition allows us to make the following inferences:

➢ When a set of homogeneous substitutable products exist, manufacturers can influence a product's demand through their control on wholesale prices, which effects the amount of shelf-space allocated at a retailer in turn influencing the product's demand. Lower wholesale prices translate to lower retail prices, which in turn result in more shelf space allocated for that product and more demand.

➢ The Stackelberg game played among the supply-chain members is very close to a real-world scenario, in which the leader represents a tycoon in that area and the follower represents a new entrant into the business.

♦ The prices and profits of the leaders are in general more than those of the follower;

♦ Similarly, the shelf-space allocated for the follower's product is typically less than that allocated for the leader's product.

➢ Prices and profits are less for sequential simultaneous Stackelberg games compared with those present in Cournot games for all the members of the supply chain. This is consistent with the established results in classical economic literature.

➢ Coordination in terms of knowing the price of the leader might not be beneficial for improving profits. However, it is crucial for reducing the bull-whip effect along the entire supply chain.

➢ Overall, the analysis of various strategies applied across the two scenarios suggests the necessity to confirm the level and intensity of information-sharing among the members of an integrated supply chain.

Chapter IV

Discussion of Findings

4.1 Contributions of this Research

Demand planning is the coordination of the flow of dependent demand through companies in the supply chain. This integrated process is similar to traditional sales and operations planning processes that occur within firms that plan for the internal flow of products within a firm. However, demand planning integrates the processes across firms in the supply chain. In recent decades, increasing globalization, outsourcing, and developments in information technology have enabled many organizations, such as Dell and Hewlett Packard, to successfully operate solid collaborative supply networks in which each specialized business partner focused on only a few key strategic activities (Scott, 1993)[110]. However, it is not clear what kind of performance impacts these different supply network structures may have on businesses, and little is known about the coordination conditions and trade-offs that may exist among the players. From a system point of view, a complex network structure can be decomposed into individual component firms (Zhang and Dilts, 2004)[111]. Using Zhang and Dilts' (2004) [111] idea of decomposition, this project presents a hierarchical demand estimation model that is capable of predicting and estimating demand in the presence of 1) censored and exact demand data, 2) advertising expenditure data, 3) stochastic regional effects, 4) competition among the retailers and manufacturers, 5) shelf-space allocation to certain products and their substitutes, and 6) the pricing decisions of supply chain partners. Since the manufacturers and the retailers are at opposite ends of a supply chain, demand models developed for these two entities are different. Therefore, the entire dissertation is divided into macro-micro demand estimation strategies that estimate demand at both the manufacturer and retailer levels, respectively.

In general, my research has focused on how demand estimation models at different tiers of the supply chain influenced by disparate factors can predict future demand for a product. Most of the analysis conducted in this macro-micro estimation models focuses on the unique set of factors associated with each tier and the novelty of the methodologies used to predict demand. Thus, this investigation consolidates and adds new perspectives to describe the theory of demand estimation at various levels of a supply chain. To the best of my knowledge, the association of these unique sets of factors with these respective methodologies has not been exploited. As such, this dissertation contributes to the understanding of demand estimation theory within the supply chain literature. My work explores multidisciplinary research involving analytical demand estimation models utilizing statistical and optimization techniques to demonstrate the potential of methodologies. These models can help managers to make critical decisions about inventory management, resource allocation, and other strategic activities that drive the profits of an organization. As a result my work is significant in its potential impact on both industry and academia, where demand estimation has been a very important issue for decades.

Concepts and techniques dealing with demand estimation at different tiers of a supply chain have been researched extensively for the past two decades. However, none of these pieces of literature brought the perspective together into one study. Finally, this investigation brings together the manufacturers' perspective (the macro demand estimation model) and the retailers' perspective (micro demand estimation model) into one study. The macroscopic demand estimation model predicts future regional demand for a manufacturer in the presence of censored demand data, advertising expenditure data, and regional stochastic effects. Additionally, this model provides a theoretical understanding of the complexities of demand estimation in the presence of censored data using Bayesian MCMC techniques. The microscopic model, on the other hand, estimates demand in the presence of competition among retailers and manufacturers. This micro-model

147

utilizes game theory and optimization techniques in order to obtain information about optimal demand experienced by retailers.

Summarized below are the specific contributions of each of these macro-micro models.

- *Macroscopic regional demand estimation model contributions:*

In this phase, a demand estimation model for estimating and predicting demand at the manufacturer is formulated. The contributions of this phase are as follows:

➢ It proposes Bayesian MCMC methods for solving demand estimation problem at a single product manufacturer in the presence of past censored and exact demand data, demand distribution with unknown parameters, regional advertising expenditure, and regional stochastic effects.

➢ It calculates the posterior demand percentile for using the familiar "Newsvendor Optimal Cost" model.

Specifically, the contributions of this macro demand estimation model are that Bayesian MCMC method can be used to:

➢ estimate and predict the nth percentile of demand for all the sampled regions and a random region selected from population.

➢ compute the probability that the posterior demand at the nth percentile would be less than a certain amount when a given amount of money is spent on advertising.

> ➢ provide trace plots and an MC errors table that evaluate the convergence and accuracy of this example in order to demonstrate the potential of the proposed method.

- *Microscopic local demand estimation model contributions:*

This micro-model estimates the optimal demand faced by the retailer in making optimal pricing and shelf-space allocation decisions. In particular, demand in the micro phase is evaluated in the presence of competition among retailers and manufacturers through their competing substitutable products, shelf-space allocation for respective products, and pricing decisions. The main contributions of this micro model are that it:

> ➢ proposes game theory and optimization techniques to solve two demand estimation models in the presence of pricing decisions, shelf-space allocation- and competition.
> ➢ utilizes Static Cournot, mixed, and sequential Stackelberg games to obtain optimal pricing and shelf-space allocation strategies.

Specifically, the contributions of this micro demand estimation model are that game theory and optimization techniques can be used to:

> ➢ Introduce competition among manufacturers serving a single retailer in Scenario 1.
> ➢ Extend Scenario 1 to incorporate competition among the retailers while retaining the competition between the manufacturers; this is called Scenario 2.

- ➢ Obtain analytically optimal demand values at Nash equilibrium for both scenarios by playing static Cournot, mixed, and sequential Stackelberg games.

- ➢ Employ Newton Raphson's method to obtain numerically optimal pricing and shelf-space allocation values due to the open-form structure of the analytical solutions and also to evaluate the applicability of these models to industry.

- ➢ Calculate profits across the supply chain for Cournot, Stackelberg and mixed game strategies for Scenario 2.

Both these demand estimation models contribute significantly to the existing supply chain literature by providing a 360° view of looking at demand estimation, both from a retailer's and a manufacturer's point of view. This dissertation formulated and evaluated demand models at two tiers of supply chains using two unique solution strategies for different sets of influential factors. Numerical examples evaluated at each level successfully demonstrate the potential of the two methodologies discussed in Chapters 2 and 3. This perspective not only holds promise for opening up new avenues of scholarly inquiry into demand estimation and supply-chain management, but also offers guidance to managers who are searching for tools fitted for the task of increasing the profits of the entire supply chain and maintaining appropriate inventory levels in presence of various factors that cannot be handled by frequently used techniques.

4.2 Conclusions

This research provides significant insights into several aspects of inter-firm integration and their impact on the entire supply chain's profits and performances. Relationships between demand

estimation and sets of influential factors at various levels of supply chain were investigated in order to improve the supply chain's ability to match demand with supply so as to reduce the "bull-whip" effect, holding costs, and stock-out situations.

Explicitly, this macro demand estimation phase provides a Bayesian MCMC methodology for obtaining optimal regional demand predictions and estimates. This is especially beneficial for manufacturers, as it enables them to:

➢ Choose a minimum advertising budget such that a further increase in advertising budget would not necessarily increase the demand for that product.

➢ Choose a minimum stock level in order to satisfy the end customers.

Similarly, the micro demand estimation model is also a customer-focused model that attempts to balance the tradeoffs of competition and profits through optimal pricing and shelf-space allocation strategies. In particular, the micro demand estimation model at the retailer level reveals that

➢ optimal pricing decisions and shelf-space allocation strategies offer greater opportunity to increase profits at the retail level.

➢ of all the various strategies explored, static Cournot competition among the supply chain partners proved to be the most profitable scenario compared with the mixed or sequential Stackelberg games among the retailers and manufacturers.

➢ Cournot competition allows for higher retail and wholesale prices compared with mixed or sequential Stackelberg games-based competition.

4.3 Limitations

Limiting the first phase of this dissertation to a single product and a single manufacturer served the research purpose by minimizing extraneous variation, thereby limiting the complexity of the numerical model. A more generic real-world scenario with many heterogeneous and homogenous groups of goods and multiple manufacturers would have been a better representation of factors and competition affecting demand. The mathematical and simulated models depended greatly on academic research and synthetic data. Although the numerical analysis was validated using accuracy and convergence models, it would be enlightening and beneficial to use external real-time data in order to evaluate the authenticity and reliability of the proposed models accurately.

4.4 Future directions for research

Future studies that include multiple manufacturers, multiple retailers, and multiple products would provide the opportunity to explore the boundaries within which theoretical relationships hold. As a first step, it will be interesting to revisit the data sections in phases one and two with real-time data and to examine the optimal predictions, estimations, pricing decisions and shelf-space allocations by using training and holdout samples. It will also be interesting to find settings that introduce multiple stochastic regional effects with different prior distributions from the Weibull demand model formulated in phase one. Another possible extension of this study would be to consider three-parameter Weibull, where the location parameter reflects other marketing mix variables such as competition, prices, etc.

The second phase of this study purposely excluded the examination of the effects of trade promotions on the profits of the retailers and manufacturer in order to simplify the existing

supply-demand models. A possible extension of the second phase of this study could be to explore the multi-directional effect of trade promotions, pricing, shelf space, and a random store variable on quantity supplied and demanded. Finally, the justification for this research is the increasing pressure on both retailers and manufacturers to respond to the globalization of supply networks. As supply chains become more global, it is important to extend this research to cross-cultural and international settings in order to examine how differences in culture, currency valuations, open/closed market systems, national trade policies, and political environments influence demand estimation models.

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Appendix

APPENDIX (A) CODES USED FOR THIS STUDY

CODE 1: WINBUGS Code for random store effects of standard deviation 0.5

Weibull Demand with Advertising Budgets and Random Store Effects

```
###########################################################################
###   This program represents the collaborative efforts of Sirisha Nukala and Ramon Leon. # #
##This program contains initial values, advertising budgets, demands and stores which are
randomly distributed#
###########################################################################
                    MODEL    FORMULATION
###########################################################################
# Weibull Demand model has a shape parameter that has a Gamma Prior and Advertising
Budgets that are normal with random store effects which are also normal with a standard
deviation of 0.5.
##############        Prior probability values generated          ##############
{

taub ~ dgamma(0.001,0.001)      # Precision parameter for normal random store effects
sigma <-pow( 1 /  taub,1 / 2)      # Sigma reparameterization
intercept ~ dnorm(0,0.001)        # Intercept component in the regression model
beta.advbud ~ dnorm(0,0.001)    # Fixed effects advertising budgets
r ~ dgamma(1,0.2)                 # Shape parameter for Weibull demand distribution which
# represents the market size. Inverse scale parameter is used by WINBUGS for Gamma
#parameterization#


###########################################################################
##############          RANDOM NORMAL STORE EFFECTS          ##############

for(i in 1:N ) {                                              # N is the number of Stores (30)
  b[i] ~ dnorm(0,taub)                                    # random effect of Stores
  }
##############        End of loop                          ##############
###########################################################################
########## This part reads data and calculates Weibull scale parameter   ##############

for(j in 1:M ) {                  # M represents the total data points (300)

log(eta[j]) <- intercept + beta.advbud * log(advbud[j]) + b[Stores[j]]
# This equation calculates the scale parameter for the Weibull demand distribution in relation to
#log of advertising budgets

lambda[j] <- pow(eta[j],-r)
# WINBUGS program recognizes the reparametrized scale parameter in Weibull distribution.
  }
##############            End of loop                          ##############
###########################################################################
###   This part consists of likelihood of Demand as exact or censored data point  ######
for(j in 1:M ) {
  t[j] ~ dweib(r,lambda[j])I(cen[j],)
# Actual Demand is the exact values when sales is < inventory censored demand when the
```

#sales > the inventory. Both exact &censored values are distributed as Weibull
   }
############### **End of loop**   ###############
###
This part calculates demand quantiles & prediction intervals for all 30 sampled stores

for(i in 1:N) {

eta23400[i] <- exp(intercept + beta.advbud * log(23400) + b[i])
# Scale parameter estimate eta values at $23.4k advertising budget for each sampled store#

quan23400[i] <- eta23400[i] * pow((-log(1 - 0.90)),(1/r))
# 90th percentile at $23400 advertising budget for each sampled store#

lambda23400[i] <- pow(eta23400[i],-r)
# Reparameterization of Weibull scale parameter for WINBUGS to recognize it.

y.23400[i] ~ dweib(r,lambda23400[i])
Predicted sales at $23400 advertising budgets for each of the sampled store

 probability23400[i] <- 1 - exp(-(pow((100000/eta23400[i]),r)))
# Prob of demand greater than $100000 for each of the sampled stores

eta47000[i] <- exp(intercept + beta.advbud * log(47000) + b[i])
# Scale parameter estimate eta values at $47Kadvertising budget for each of the sampled stores

quan47000[i] <- (eta47000[i] * pow((-log(1 - 0.99)),(1/r)))
# 99th percentile at $47K advertising budget for each of the sampled store

lambda47000[i] <- pow(eta47000[i],-r)
#Reparameterization of eta into lambda for WINBUGS to recognize the parameter.

y.47000[i] ~ dweib(r,lambda47000[i])
Predicted sales for $47000 advertising budget for each of the sampled stores

probability47000[i] <- 1 - exp(-(pow((170000/eta47000[i]),r)))
# Prob of demand greater than $170000 for each of the sampled stores
   }
###############   **End of loop**   ###############
###
Estimates & Predictions for a random store from the population of stores nationwide

random ~ dnorm(0,taub)
Chooses one store at random from population of stores nationwide###

eta23400random <- exp(intercept + beta.advbud * log(23400) + random)
 # Weibull scale parameter equation in relation with advertising budget & random store effects #

quan23400random <- eta23400random * pow((-log(1 - 0.9)),(1/r))
# 90th percentile at $23400 advertising budget for a randomly selected store###

lambda23400random <- pow(eta23400random,-r)
# Reparameterization of the weibull scale parameter for WINBUGS to recognize the value##

```
y.23400random ~ dweib(r,lambda23400random)
# Predicted sales at $23.4K advertising budget random store selected from population/sample.

probability23400random <- 1 - exp(-(pow((100000/eta23400random),r)))
# Prob. (demand) > $100K for a random store selected from population of stores nationwide at
$23.4K advertising budget


eta47000random <- exp(intercept + beta.advbud * log(47000) + random)
 # Estimate of Weibull scale parameter at $47K adv. bud for a random store from the population
##of stores##

quan47000random <- (eta47000random * pow((-log(1 - 0.99)),(1/r)))
# Estimating the 99th percentile of demand at  $47K advertising budget for a random store from
#the population of stores nationwide.

lambda47000random <- pow(eta47000random,-r)
#Reparameterization of the weibull scale parameter for WINBUGS to recognize#

y.47000random ~ dweib(r,lambda47000random)
# Predicted sales for a random store at $47K advertising budget##

probability47000random <- 1 - exp(-(pow((170000/eta47000random),r)))
# Prob(demand) >$170K for a random store at $47K advertising budget###


   }
###############          End of program model              ###############
#################################################################################
# DATA GENERATED for random Normal store effects with μ= 0 and σ= 0.5 #
#################################################################################

list(N = 30, M = 300,
     advbud =
c(5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000
,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,5000,
5000,15000,5000,5000,15000,15000,5000,5000,5000,15000,15000,25000,5000,15000,5000,500
0,5000,5000,15000,5000,35000,15000,35000,5000,15000,15000,15000,15000,15000,15000,150
00,5000,5000,5000,5000,15000,5000,15000,25000,5000,15000,35000,25000,15000,25000,5000
,5000,5000,25000,25000,35000,15000,15000,45000,15000,25000,15000,15000,15000,
15000,15000,15000,35000,5000,15000,25000,15000,25000,5000,15000,15000,25000,5000,150
00,25000,25000,45000,15000,15000,45000,15000,5000,35000,25000,15000,15000,45000,2500
0,35000,35000,15000,25000,15000,25000,15000,45000,15000,25000,25000,25000,25000,3500
0,25000,15000,45000,15000,25000,15000,25000,15000,25000,25000,15000,15000,25000,4500
0,35000,35000,15000,25000,35000,35000,15000,25000,25000,45000,35000,25000,35000,1500
0,35000,35000,25000,15000,35000,25000,25000,45000,35000,45000,35000,35000,25000,1500
0,35000,25000,35000,35000,35000,45000,45000,45000,35000,25000,25000,25000,35000,3500
0,25000,25000,25000,25000,45000,15000,35000,35000,15000,25000,35000,25000,15000,3500
0,25000,35000,45000,35000,45000,25000,15000,45000,35000,25000,35000,45000,45000,4500
0,45000,45000,15000,35000,15000,35000,25000,45000,25000,45000,45000,25000,35000,3500
0,45000,45000,25000,45000,25000,35000,45000,35000,45000,45000,35000,25000,45000,3500
0,35000,35000,45000,25000,35000,25000,15000,25000,45000,45000,45000,35000,45000,3500
0,45000,25000,35000,45000,45000,35000,45000,35000,25000,25000,25000,35000,35000,3500
0,35000,35000,35000,35000,35000,45000,45000,45000,45000,45000,45000,45000,45000,4500
0,45000,45000,45000,45000,45000,45000,45000,45000,45000,45000),
```

170

Stores =
c(12,4,10,21,17,24,30,7,10,4,30,7,28,16,14,16,1,28,1,5,24,14,11,2,17,8,6,15,6,12,8,3,15,3,21,10,
2,5,4,4,20,29,23,30,10,10,19,16,23,27,11,29,28,13,14,1,10,20,5,11,1,12,15,7,15,26,9,25,27,14,2
2,28,10,13,12,1,16,30,4,26,25,19,12,28,8,2,6,2,22,7,17,24,21,2,13,16,4,22,7,12,27,24,18,14,8,21
,9,8,30,4,4,23,24,4,5,18,7,2,6,13,10,15,12,4,23,7,29,8,27,1,20,29,6,5,14,30,1,11,24,21,11,3,28,1
9,24,2,29,17,15,10,12,10,19,30,17,1,20,8,3,29,24,14,5,9,16,15,17,3,24,29,27,16,30,5,28,2,1,26,2
8,22,3,7,14,2,14,23,13,6,11,17,5,15,20,16,9,3,28,22,16,9,26,5,29,23,25,8,21,29,8,13,12,20,25,16
,6,19,23,8,21,3,24,11,9,2,18,23,13,11,23,12,15,13,11,11,7,1,26,28,22,21,6,20,7,14,6,19,15,21,3,
19,30,26,17,27,18,18,30,19,13,27,3,26,29,18,22,5,17,26,21,20,9,25,25,18,19,22,25,9,18,25,27,9,
13,18,20,22,25,26,27,6,9,17,18,19,20,22,23,25,26,27),
cen =
c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,394000,394000,394000,394000,394000,394000,394000,394000,39400
0,394000,394000,394000,394000,394000,394000,394000,394000,394000,394000,394000,3940
00,394000,394000,394000,394000,394000,394000,394000,394000,394000),

  t =
c(7616.65182,11463.1929,13513.5374,15793.718,15968.2296,16014.1405,16850.8048,18961.4
672,19156.7892,19289.4012,19365.0875,20534.0901,20592.2633,22073.378,22142.6848,22358
.9199,22397.0702,22608.0307,22701.7079,23775.8793,23934.9098,24552.1288,24843.2581,25
124.4165,25991.6691,26360.2769,26840.882,27304.6421,27360.7689,27392.5839,27748.3041,
27973.8749,28033.5507,28763.0693,30426.1535,30605.2559,30972.8218,31261.2767,31297.86
73,33330.6995,33766.5483,34816.0405,35497.4645,36034.6107,36095.2095,37060.6692,39348
.6072,41421.298,42203.9115,42339.779,44533.6445,45094.9476,46071.6972,47871.7262,4802
4.8457,48778.3477,50024.122,50198.0937,50436.0275,53066.0885,55081.8122,57304.0893,57
829.75,57963.0273,58226.045,58793.9539,59268.3916,60164.0974,60237.4852,62041.1848,63
104.9468,63636.6138,63877.2793,64109.0952,65276.0896,65442.688,65752.6332,66932.3521,
68158.0686,70855.061,71658.9889,73520.0418,73550.6336,73731.1674,74247.8832,75887.996
1,76404.541,76424.7747,76862.9472,77068.4087,77120.8198,78798.4274,79356.5509,79497.3
907,79694.0791,79804.234,80154.4835,80343.4639,81437.3781,81647.0342,81678.3688,82776
.8057,84090.3393,87288.7254,90213.3905,90676.0734,90676.4354,91093.3436,91293.7691,91
638.2653,92365.3072,92602.2719,93072.7133,95133.3723,96372.0945,96636.7563,98337.4454
,99160.7656,100839.86,101488.157,104990.675,105329.144,106161.568,106411.155,112918.9
81,114153.004,114818.498,116126.087,116139.896,118486.138,118753.645,119192.668,11969
1.952,120581.148,121770.407,122349.422,123836.53,124088.277,125807.573,126075.331,127
778.314,127943.366,128019.986,129902.861,131228.133,131545.65,132458.689,132821.549,1
33569,134252.801,135065.966,136557.581,137542.624,138243.052,138453.015,138826.835,13
8833.614,139630.976,139751.724,141670.551,141752.809,146514.727,147529.735,149386.039
,149656.964,150918.491,151278.887,152863.047,154021.201,155471.699,157308.292,162911.
542,163357.973,163635.818,163844.098,168272.826,168300.084,168553.367,168807.048,1710
60.931,171773.475,173030.286,174479.755,174542.926,178747.958,179117.569,179874.797,1
81865.474,181924.962,182089.555,183555.735,186241.128,186299.287,190733.257,192447.86
8,192864.254,193286.891,194384.129,194809.38,195768.806,196374.2,196889.994,204193.12
1,204796.093,205605.756,207340.474,207744.066,208954.889,212256.353,213257.286,213775
.239,214380.548,216505.264,218670.134,219004.092,219358.774,220324.376,222675.481,226
954.816,232902.941,234802.239,234859.444,235734.435,237073.922,240021.47,242101.314,2
42574.625,242722.633,243328.739,245946.826,250861.199,251871.963,252371.111,252486.75
2,255223.321,261140.077,265821.961,269266.735,275357.17,275942.311,277090.67,277656.8,
277880.134,278848.721,279002.333,279175.967,279677.434,285896.444,286532.226,290730.4

73,295394.549,298095.005,310330.057,314588.814,318371.275,318830.849,325162.469,32575
1.033,325936.632,326640.968,330623.355,336413.941,343382.219,355313.376,356456.814,35
8884.704,369293.456,371072.086,373828.17,377175.869,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA))

###############   **End of data**   ##############
###
###############   **INITIAL VALUES**   ##############
###
list(intercept = 2.5, beta.advbud = 1.2, r = 4.2, taub = 4.5)   # initial values based on simulations performed
list(intercept = 1, beta.advbud = 1,r = 1, taub = 1)
###############   **End of initial values**   ##############
###

CODE 2: R-Software Sample Code used for optimization purposes.

This is for scenario 2 retailer 2 while playing a Cournot Game

optim(c(1,0.7541),sc2r2,sc2r2grad,method="BFGS",hessian=T)

*The above line of code represents an optimization for scenario 2 for a Cournot game.

BFGS in the code above refers to Newton Raphsons methods and the initial values

represent the two retail prices for both the products.*

Results:

$par

[1] 0.700195 1.142310

* The results above represent the optimal retail prices at retailer 2 in scenario 2 while

playing a Cournot game*

$value

[1] 0.3722328

$counts

function gradient

    51   5

$convergence

[1] 0

* Convergence is equal to zero implies that our simulation has converged which indicates that the optimal values are perfect*

$hessian

       [,1]   [,2]

[1,] -0.1120951 -0.025325200

[2,] -0.0253252 0.006281819

VITA

Sirisha Saripalli completed her BE from Andhra University, India, majoring in Mechanical Engineering. Upon completion of her Bachelors she came to USA to further her education in Fall 2001. She graduated from Clemson University in 2003 with MS in Industrial Engineering. After her masters with support from her husband and her strong commitment to pursue a PhD to fulfill the dreams of her parents she joined the department of Industrial and Information Engineering in Fall 2003 as a PhD candidate to pursue her interests in Supply Chain management. While conducting research SCM, she encountered several problems that needed vast knowledge of Statistics which encouraged her to pursue her second masters in Statistics. This degree was awarded to her in August 2006.

Despite her hectic schedule as a full time student, teaching / research assistant and a full time house wife, she believes in giving back to the society. She is involved in organizing Relay for Life every year on behalf of ORNL and UT BATTELLE. She is an active volunteer for charity organizations like ASHA & AID who help under privileged children in India. During Katrina, she sheltered several University of New Orleans students at her home. Apart from all these social activities she was a National Roller Skating Champion for three years from 1992-1995 . She has been an exceptional woman of strength and character with an outstanding academic performance all throughout her PhD and Masters. Her GPA of 4.0 is a direct result of her hard work and strong focus on her career and goals. Her journey towards attaining PhD was more or less tensed. Despite all the obstacles, she was focused and had a strong determination to fulfill her parents desire. She graduated with a PhD in I&IE specializing is supply chain modeling and simulation

in December 2007 and currently looking into jobs as a quantitative analyst, consultant or supply chain modeling engineer.