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A Theoretical and Experimental Investigation of Efficiency, Equity, and Uncertainty in Tournaments

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A Theoretical and Experimental Investigation of Efficiency, Equity, and Uncertainty in Tournaments

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Abstract

This dissertation consists of three essays centered around labor incentives that arise in relative compensation contracts. Chapter 1 poses the question: if devotion to a core competence were truly optimal, why would firms do otherwise? We argue that the behavior of drifting from the core may be motivated by the competitive incentives faced by managers who seek to rise within a firm. We find competition creates an incentive for a manager to look for less correlated opportunities that pull the firm in a new direction. In a symmetric equilibrium all managers behave this way, leading to lower expected output for the firm. A "stick" that punishes the lowest performing manager in conjunction with a prize to the top performer can deter such behavior. Chapter 2 investigates how common shocks affect the behavior of heterogeneous agents in contracts designed to achieve both equity and efficiency. We show that when a procedurally fair and efficient tournament is desirable and compensates for heterogeneity, common shocks can bias the probability of winning of the agents. Also, the principal is found to have a commitment problem, favoring low risk agents to win, when fair and efficient tournaments are held between agents with different uncertainty distributions. Chapter 3 uses an experiment to examine issues of fairness and efficiency in rank-order tournaments with agents heterogeneous in abilities and random shock distributions. To study these issues, we observe agent effort, contract choice, and role choice from participating in three contracts that

vary in perceptions of equal opportunity: equal access, equal expected earnings, and equal changes of winning. The primary result is that equal pay in the form of equal expected outcomes promotes efficiency and greater perceptions of fairness than does equal access to common contracts.

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Chapter 1

Competition and Core Competency: Risk Independence as a Strategy

1.1 Introduction

Adherence to a strong *core competency* has been argued to be a common trait of successful corporations. Proponents argue that a firm's sustained success is engendered by the discipline of its managers to adhere to a core strength from which the firm derives competitive advantage. But when firms perform poorly, the decline is often associated with a fragmented market strategy and investments into business areas where they take on new risks, are less productive, and do not compete effectively. This argument begs the question of why executives would engage in such behavior. That is, if devotion to a core competence is truly optimal, why would firms do otherwise?

Of course it is not necessarily the case that adhering to core strengths is optimal for any particular firm. Concluding from the observation that firms in decline often exhibit "drift" from their core competence suffers an obvious problem of selection bias. For example, shifting to new markets may be the least-bad alternative when disruptive innovation or market changes erode a firm's competitive position in its traditional markets. Nevertheless, the observation that drift by a firm from its core is often associated with decline raises a very interesting question: might such behavior be the result of a systematic agency problem that exists between firms and their managers under common incentive structures?

We argue that this behavior may be motivated by the competitive incentives faced by managers who seek to rise within a firm. It is well documented in the literature that competition among managers for promotion, bonuses, or other rewards can be efficiently employed to motivate productive effort. Much recent work on competitive incentives has focused on how competition influences other strategic choices, such as risk-taking and cheating. Distinct in the model we develop, a competing manager has discretion that enables him to choose his effort and how to allocate his effort between projects that are inside versus outside the core activities of the firm.

We assume that a manager's expected output is maximized for any level of effort if he adheres to the core of the firm, but by doing so the randomness affecting his output is most correlated with that of the other managers. Conversely, if he shifts his activities away from the core of the firm into other ventures his expected output for a given level of effort declines and his random shocks become less correlated with his competitors. We find competition creates an incentive for a manager to look for less correlated opportunities that pull the firm in a new direction, away from its core activities. In a symmetric equilibrium, all managers behave this way which leads to lower expected output for the firm.

Prahalad and Hamel (1990) observed fragmented market strategies when they originated the concept of core competency. They describe how NEC Corporation successfully employed a strategic advantage in semiconductors and manufacturing to dominate the computing and communication industry in the 1980's; while their rival GTE Corporation, having a comparable starting point, faltered. A key difference was that divisions within GTE were managed as autonomous business units while NEC was organized to develop and leverage their core competencies across all divisions.[1] Business managers at GTE were given the authority to make independent risk and market decisions while contemporaneously competing for internal resources and compensation, an environment which ultimately led to strategies that included business divestitures and collaboration with outsiders — all in leu of investments in the core competency.

We model how competitive compensation schemes used to motivate performance may also motivate inefficient drift behavior in firms like GTE. A manager's autonomy represents the notion that he has control over a division, has discretion over some activities, or can enter new markets on behalf of the firm. To capture this notion, our premise is that autonomy gives managers discretion to pursue independent non-core related ventures that subsequently reduce the common exposure to shocks they share with the other managers within the organization.

We develop a tournament model to represent the incentives that autonomous managers may face as they compete for rewards within a firm (or possibly even across firms in the broader labor market context). The model is based on a Lazear and

[1]The sustainability of NEC's advantage came from investing in the culture and innovative technologies that fostered the next generation competencies of the whole organization—all virtues of the original concept. The advice from Prahalad and Hamel (1990) to a modern corporation is obvious: "Focusing on core competencies creates unique, integrated systems that reinforce fit among your firm's diverse production and technology skills—a systemic advantage your competitors can't copy."

Rosen (1981) winner-take-all rank-order tournament which in this context motivates managers by ranking their observable output and rewarding only the top manager. As is standard in tournament models, we assume each manager's output is determined by a combination of his unverifiable effort and random shocks. A random shock can be common (experienced by all agents within the firm) or idiosyncratic (independently impacting each agent).

Our innovation is to assume that each manager has some discretion regarding how exposed he is to the two types of shocks while simultaneously choosing effort. A key result from this model is that when three or more managers compete, at any symmetric point a single manager can increase his probability of winning by increasing his independence. He does this by increasing his exposure to an idiosyncratic random shock rather than a common shock.

To capture the assumption that firms do best when managers stay close to the firm's core competency, we assume that as a manager becomes more independent he becomes less productive; that is, he increasingly forfeits the benefits that come from the firm's core competency. Hence, as a manager increases his independence, he also increases the cost of effort required to achieve any given level of output. This describes a manager who runs an independent venture and takes on greater burdens as a result of being independent such as reduced production efficiencies and economies of scale or increased costs of information and administration. We also assume that the loss in productivity from core drift is relatively more costly as the core strength of the firm increases.

Within this context, the intuition for our key finding is straightforward. When multiple managers compete in a winner-take-all contest, competition creates an incentive for independence-seeking behavior despite the fact that it reduces managers' productivity. By diverging from the firm's core activities a manager makes his output less correlated with others by separating his source of stochastic risk from the pool of competitors. Holding the behavior of others constant, up to some point this increases his chance of ranking first. By increasing his non-core activities, he makes it less likely he will win if the common shock is positive, but this is more than offset by his increased probability of winning if the common shock is negative.

This leads to a symmetric equilibrium where all managers choose some positive level of independence. We show that all managers wastefully engage in independence-seeking behavior in a symmetric equilibrium despite the fact that this reduces equilibrium expected output. The firm may be hurt by decisions of competing managers who pull the firm in conflicting directions—at odds with its core activity—even though every manager is aware that this drift is costly.

Core drift behavior is also wasteful in the sense that in equilibrium, no agent is better off than before (i.e. his chance of winning is still the same). Core drift can be reduced or eliminated by offering a penalty as a third payoff level. This is akin to the management style of promoting the top managers and firing the bottom managers, a technique made popular by Jack Welch during his tenure as CEO of General Electric between 1981 and 2001. By balancing the first-place prize with an equivalent last-place penalty, the incentive to be independent is eliminated.

A somewhat related problem that has been studied is one of managers choosing between risky assets in financial settings. Kempf and Ruenzi (2008) look at the intra-firm competition for promotions of fund managers and find empirical evidence that "common shock matching" is used as a strategic behavior.[2] A manager who wants

[2]Although not directly modeled in the article, the Kempf and Ruenzi identification presumes a manager who finds himself behind at midyear (a losing manager) in a small competitive firm will increase his risk to improve his chances of winning. Then, as a counter strategy, a winning manager

to preserve his lead may select assets correlated with the pool of other managers. Alternatively, a manager who wants to increase his chance of winning may choose assets that are uncorrelated with the pool. Viewing our model in this context, fund managers can choose the "house" assets that are popular in the firm and share common risk or choose unique assets where risk is less correlated.

Kempf and Ruenzi (2008) also find that managers in large firms who have higher returns by midyear will reduce portfolio risk compared to managers with low returns, as is predicted by Bronars (1987); yet, the opposite behavior occurs in small firms. Kempf et al. (2009) look at how employment risk influences the choice of risk taking for mutual fund managers and find that when employment risk is high, managers reduce the portfolio risk. This supports the conclusions of Gilpatric (2009) who shows that a stick (e.g. being fired) can reduce risk-seeking behavior in agents and motivates the inclusion of penalties for the last place agent in this model.

Lazear and Rosen (1981) introduced the theory of rank-order competition based on the ability of organizers to only observe the ordinal-rank of agents as a function of hidden effort and uncertainty. Since then, a body of literature on risk taking in tournaments has focused on what happens when homogeneous agents can choose their level of idiosyncratic variance in different settings (Bronars (1987); Gaba et al. (2004); Tsetlin et al. (2004); Nieken and Sliwka (2010)). The general finding is that risk taking increases an agent's chances of winning in a one-shot contest among multiple players. However, there are also strategic reasons to act risk-averse to preserve a lead. Bronars (1987) showed how followers in a sequential tournament are risk seeking and

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will mimic the choices of the losing manager (i.e. increase risk) to synchronize his common shock and lock in his win. Then, as a counter action, the losing manager will react to lower his risk to differentiate himself even further. These dynamic reactions raises the concern that this is not an equilibrium since the winning managers could react yet again to lower their risk, and so on..

leaders are risk-averse. This strategic behavior was verified empirically by Knoeber and Thurman (1994) in broiler tournaments.

Our analysis is related to Gaba et al. (2004) who model winner-take-all contests where agents can modify their own distributions and correlations with other contestants. They find that reducing correlation with competitors increases the probability of winning for a single agent. However, their focus is very different insofar as they do not identify an equilibrium of a game in which there is a cost of decreasing correlation (they instead argue contestants' dominant strategy is to choose the least correlated distribution available). Additionally, we add the simultaneous choice of effort and correlation. In an innovation type tournament, Levitt (1995) finds that an organizer can benefit from reducing the correlation between agents in a contest of two agents when he is interested in only the top performers' output. In his model, agents are unaware of the source of their shocks and pay a fixed cost of bearing risk.

The study of risk taking in tournaments has also branched out to model the two-variable choice of effort and the variance of idiosyncratic risk (Hvide (2002); Gilpatric (2009)). The general findings are that when risk taking is costless, agents will wastefully take larger risks and produce less effort. Nieken et al. (2010) conducts a two stage experiment based on the model originally developed by Hvide (2002) where a subject first chooses between two distributions of risk, then chooses effort in a two-player contest. She finds that contrary to two-stage theory, subjects are not strategically risk-seeking to reduce effort in the second stage. She does, however, confirm that when the high risk option is selected, less effort is observed in the second stage. Also contrary to theory, she finds that subjects only take personal risk into account when making strategic decisions, ignoring the strategy the other player may have over risk.

The simultaneous setting proposed by Gilpatric (2009) is close to the model considered here. This model assumes that an agent can simultaneously choose his level of effort and own level of risk at a cost in a winner-take-all tournament. In comparison, we assume instead that an agent can choose his level of effort and his independence from the common shock of the core competency of the firm at a cost. Gilpatric showed that an agent has an incentive to increase his idiosyncratic variation in output as a strategy to increase his chances of winning for 3 or more players, but that he has no incentive to do so for a two player tournament. Gilpatric shows further how a penalty that is symmetric but opposite the prize to the lowest ranked agent eliminates the incentive for agents to pursue risky and more costly strategies when the penalty is of identical magnitude to the winner payout. Although the underlying mechanism is different, we show that using a stick can eliminate incentives to be independent in this model.

The rest of this paper is organized as follows. Section 1.2 1.2describes the model and develops the results to the principal-agent problem. Section 1.3 provides concluding remarks followed by the References. All proofs are in the Appendix.

1.2 Model

Consider a contest among n homogeneous risk-neutral agents who work within a firm where $\lambda \in [0, \infty)$ is an increasing measure of the strength of the core competency of the organization. Unverifiable to a third party, agent i chooses his level of independence $\alpha_i \in [0, 1]$ simultaneously while choosing effort $\mu_i \geq 0$. He can align fully with the core competency by choosing $\alpha_i = 0$ or work completely independently from the core and all other agents by choosing $\alpha_i = 1$, or select partial independence by choosing

α_i between 0 and 1.

By selecting the focus of his work, an agent has two sources of uncertainty that affect output. Let $\varepsilon_i \sim i.i.d.\,(0, \sigma_\varepsilon^2)$ be agent i's draw of an idiosyncratic shock realized after choices are made where f is the probability density function and F is the cumulative distribution function of ε. Let $x \sim (0, \sigma_x^2)$ be a common shock experienced to some extent by all agents who work at least partially within the firm's core competence where g is the probability density function and G is the cumulative distribution function of x. The draws from ε and x are uncorrelated and both f and g are unimodal and symmetric. An agent can choose his level of exposure to x and ε_i by shifting α_i between 0 and 1. Increasing α_i will decrease the effect of x on output while simultaneously increasing the effect of ε_i on output. Therefore, agent i produces output

$$q_i = \mu_i + (1 + \alpha_i)\,\varepsilon_i + (1 - \alpha_i)\,x. \tag{1.2.1}$$

Total output uncertainty for an agent, then, is a function of independence where the total variance is $(1 + \alpha_i)^2\,\sigma_\varepsilon^2 + (1 - \alpha_i)^2\,\sigma_x^2$. An increase in independence shifts the source of uncertainty away from the common shock and increases the influence of the idiosyncratic shock. Only an agent who works within the core in some way (i.e. $\alpha_i < 1$) will experience the core shock x as a factor in his output along with all the other agents who also chose $\alpha_j < 1$ where $j \neq i$ for all i, j. Some level of idiosyncratic risk is present for an agent regardless of α_i which captures his innate level of uncertainty in output; but, the exposure to ε_i increases as α_i increases to represent the greater uncertainty associated with increasingly independent ventures relative to the core.

In the agent's output equation (1.2.1), μ_i increases output linearly and is not directly influenced by α_i; instead, we model the burden of operating independently as impacting effort costs. Agent i has increasing costs from effort described by the function $c(\mu_i)$ where $c^{'}, c^{''} > 0$ and $c(0), c^{'}(0) = 0$. The total costs from effort and independence are captured by the interactive expression$(1 + \lambda z(\alpha_i)) c(\mu_i)$ where $z(\alpha_i)$ increases with independence and $z^{'}, z^{''} > 0$ and $z(0), z^{'}(0) = 0$. The cross derivative of the total cost function is positive for positive values of effort and independence, $\lambda z^{'}(\alpha_i) c^{'}(\mu_i) > 0$. The total cost term represents the notion that achieving a given expected output (i.e. μ_i) entails greater effort as i works more independently or as the strength of the core competency of the organization increases. The core competency parameter λ increases the marginal cost of effort as a result of increasing independence. When there is no core competency in the firm, $\lambda = 0$, there is no distinction made between how costly different levels of independent effort are over the range $\alpha_i \in [0, 1]$ and therefore total costs reduce to $c(\mu_i)$.

The winner-take-all contest of n agents has two prize levels, payoff W_1 to the agent ranked highest and W_2 to all other agents where $W_1 > W_2$. Let $S = W_1 - W_2$ be the spread between the two prizes. Using the agent output equation (1.2.1), agent i is ranked higher than all other agents when $q_i > q_j$ for all agents $i \neq j$. Let P_i be the probability agent i wins the larger prize W_1. Agent i maximizes his expected earnings E_i by choosing independence and effort simultaneously according to

$$\max_{\mu_i, \alpha_i} E_i = P_i S + W_2 - (1 + \lambda z(\alpha_i)) c(\mu_i). \tag{1.2.2}$$

The social optimum is found where $\alpha_i = 0$ by construction of the model; an agent can produce any level of output at the least cost when he leverages the full core competency of the principal.[3]

[3]When $\alpha_i = 0$ for all i, the model collapses to the standard first-best model by Lazear and Rosen

From equation (1.2.1), the probability that i ranks ahead of j given the choices of effort and independence for both agents is

$$prob\left(\left(1+\alpha_j\right)\varepsilon_j < \mu_i - \mu_j + \left(1+\alpha_i\right)\varepsilon_i + \left(1-\alpha_i\right)x - \left(1-\alpha_j\right)x\right)$$

for all agents $j \neq i$. This probability can be expressed in terms of F, the distribution of ε_j.[4] Suppose all agents $j \neq i$ choose a common μ_j and α_j then integrating over the densities of ε_i and x, agent $i's$ probability of winning the tournament by being ranked higher than all other agents is

$$P_i = \iint g\left(x\right) f\left(\varepsilon_i\right) F\left(\frac{\mu_i-\mu_j}{1+\alpha_j} + \frac{1+\alpha_i}{1+\alpha_j}\varepsilon_i + \frac{\alpha_j-\alpha_i}{1+\alpha_j}x\right)^{n-1} \; d\varepsilon_i \, dx. \qquad (1.2.3)$$

We now look at the marginal effects of effort and independence on P_i. Let $\kappa = \left(\frac{\mu_i-\mu_j}{1+\alpha_j} + \frac{1+\alpha_i}{1+\alpha_j}\varepsilon_i + \frac{\alpha_j-\alpha_i}{1+\alpha_j}x\right)$ represent the expression in $F\left(\kappa\right)$ in equation (1.2.3). Agent $i's$ marginal effect of own effort on the probability of winning then becomes

$$\frac{\partial P_i}{\partial \mu_i} = \frac{n-1}{1+\alpha_j} \iint f\left(\varepsilon_i\right) f\left(\kappa\right) F\left(\kappa\right)^{n-2} \; d\varepsilon_i \, g\left(x\right) dx \qquad > 0, \qquad (1.2.4)$$

where the integral portion is everywhere positive; hence, the probability of winning is strictly increasing in effort. We also want to know how α_i will influence the probability of winning. Hence, agent $i's$ marginal effect of independence on the probability of winning is

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{n-1}{1+\alpha_j} \iint \left(\varepsilon_i - x\right) f\left(\varepsilon_i\right) f\left(\kappa\right) F\left(\kappa\right)^{n-2} \; d\varepsilon_i \, g\left(x\right) dx. \qquad (1.2.5)$$

(1981) where $q_i = \mu_i + \varepsilon_i + x$ and the costs are $c\left(\mu_i\right)$, regardless of the value of λ.

[4] Agent $j's$ uncertainty ε_j can be isolated on the left hand side of the inequality by dividing both sides by $\left(1+\alpha_i\right)$.

In this model, we will identify the symmetric Nash Equilibrium of this game. Let $\mu^s = \mu_i$ be a point of symmetric effort and let $\alpha^s = \alpha_i$ be a point of symmetric independence for all i. When behavior is symmetric (μ^s, α^s), the probability of winning in equation (1.2.3) is of course the same for all agents, $P_i = \frac{1}{n}$. Agent $i's$ marginal effect of independence on the probability of winning at any point of symmetry reduces to the integral

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{(n-1)}{1+\alpha^s} \int \varepsilon_i f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} d\varepsilon_i. \tag{1.2.6}$$

This leads to the following result from choosing independence as a strategy.[5] (All proofs are given in the Appendix)

Proposition 1.1. *When agents make symmetric choices (μ^s, α^s), the following results hold:*

(i) For $n = 2$, each agent's probability of winning is unaffected by increasing his independence, $\frac{\partial P_i}{\partial \alpha_i} = 0$;

(ii) For $n > 2$, each agent's probability of winning is increasing in his independence, $\frac{\partial P_i}{\partial \alpha_i} > 0$.

When it is in the firm's best interest to have everyone working in the core activities, the existence of intra-firm competition for $n > 2$ agents may be detrimental because it inherently drives agents toward non-core activities. If all agents are fully engaged in the core activities of the principal where $\alpha^s = 0$, an agent can increase his likelihood of winning the contest by choosing a positive level of independence $\alpha_i > 0$. Notice the incentive to be independent exists regardless of the value of λ in equation

[5] The derivation of equation (1.2.6) is part of the proof of Proposition 1.1.

(1.2.6); however, increasing λ will make his chosen level of independence more costly in equation (1.2.2). The result that the probability of winning does not change when an agent increases his independence when $n = 2$ is a consequence of the symmetry of the distribution of ε.

An agent's incentive to increase α_i from any point of symmetry does not originate from reducing his exposure to the common shock at the margin; rather, it comes from the advantage he gets from increasing his idiosyncratic uncertainty. This is easily verified since equation (1.2.6) is strictly a function of ε_i, but is not a function of x.

Although the common shock has a symmetric effect at the point of symmetry, it does not, however, rule out any advantage that might be gained when an agent increases his exposure to the common shock at a discrete level of independence away from a point of symmetry. At a point *out of symmetry,* exposure to the common shock is no longer experienced equally by all agents and does not cancel out in F and therefore it does influence the marginal effect of independence on the probability of winning.

Even so, an agent will only move off-symmetry if his expected earnings increases from doing so. To insure the symmetric pure strategy Nash equilibrium holds, we specify a global constraint in the next subsection that, if met, is sufficient to rule out such discrete advantages. The next two subsections address the principal-agent problem and identify the strategic behavior of the agents in a Nash equilibrium.

1.2.1 The Agent's Problem

Solving the agent's optimization problem, the first order conditions for agent i from equation (1.2.2) are

$$\frac{\partial E_i}{\partial \mu_i} = \frac{\partial P_i}{\partial \mu_i} S - (1 + \lambda z(\alpha_i)) c'(\mu_i) = 0 \tag{1.2.7}$$

$$\frac{\partial E_i}{\partial \alpha_i} = \frac{\partial P_i}{\partial \alpha_i} S - \lambda z'(\alpha_i) c(\mu_i) = 0 \tag{1.2.8}$$

We impose symmetry (μ^s, α^s) to find the solutions to the response functions of effort and independence. Substitute equations (1.2.4) and (1.2.5) into equations (1.2.7) and (1.2.8) and note that in symmetry the integral terms in equations (1.2.4) and (1.2.5) reduce to the single integral constants $h = (n-1) \int f(\varepsilon)^2 F(\varepsilon)^{n-2} d\varepsilon$ and $k = (n-1) \int \varepsilon_i f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} d\varepsilon_i$ respectively. Both h and k are constants and greater than zero at any point of symmetry and do not depend on μ^s and α^s or x. The best response functions that implicitly describe the Nash equilibrium are

$$(1 + \lambda z(\alpha_i^*)) c'(\mu_i^*) = \frac{Sh}{1 + \alpha_j^*} \tag{1.2.9}$$

$$\lambda z'(\alpha_i^*) c(\mu_i^*) = \frac{Sk}{1 + \alpha_j^*}. \tag{1.2.10}$$

We assume the objective function is concave. To satisfy the second order conditions, the determinant of the hessian matrix is negative semidefinite and the diagonal terms are negative $\frac{\partial^2 P_i}{\partial \mu_i^2} S - (1 + \lambda z(\alpha_i)) c''(\mu_i) \leq 0$ and $\frac{\partial^2 P_i}{\partial \alpha_i^2} S - \lambda z''(\alpha_i) c(\mu_i) \leq 0$ to guarantee a maximum.

Tournaments solve the moral hazard problem when the principal is not able to monitor agent actions directly. However, to avoid shirking (no or little effort) and climbing (too much effort), it must also be the case that a discrete change in choice of μ_i and α_i cannot increase utility when all other agents are choosing μ_j^* and α_j^* for all j—that is, the inequality $E_i\left(\mu_i, \alpha_i, \mu_{-i}^*, \alpha_{-i}^*\right) < E_i\left(\mu_i^*, \alpha_i^*\right)$ is satisfied over the full support of α_i. From equation (1.2.2), the global incentive condition is

$$P_i\left(\mu_i, \alpha_i, \mu_{-i}^*, \alpha_{-i}^*\right) S - \left(1 + \lambda z\left(\alpha_i\right)\right) c\left(\mu_i\right) < \frac{1}{n} S - \left(1 + \lambda z\left(\alpha_i^*\right)\right) c\left(\mu_i^*\right). \qquad (1.2.11)$$

This condition will not hold if the variance of the common shock is sufficiently large compared to variance of the idiosyncratic shock.[6] A large σ_x^2 relative to σ_ε^2 can induce a larger expected earnings for an agent who leverages the full core competency of the firm compared to the expected earnings at the candidate Nash—violating the global constraint in (1.2.11). To prevent this, σ_x^2 must be small enough to insure an agent's probability of winning drops off fast enough to offset the gains in efficiency he incurs from working in the core of the firm. Therefore, for a Nash equilibrium to exist when an agent can choose his relative exposure to the common and idiosyncratic shocks, not only must the variance of the idiosyncratic shock be sufficiently large (as is standard in tournaments), but also the variance of the common shock must be sufficiently small relative to the variance of the idiosyncratic shock.

This is an important finding. Standard tournament models show common shocks are rendered irrelevant, and it has been argued that tournament mechanisms may be desirable for insulating players from these shocks (Lazear and Rosen, 1981; Green and Stokey, 1983). But we find that when common shocks are large, the existence of a pure strategy Nash equilibrium breaks down if players can control their exposure to these shocks. We now look at the agent's strategy at the symmetric Nash.

[6] A discussion of this *Relative Variance Condition* can be found in the Appendix.

Proposition 1.2. *For $n > 2$ and some level of core competency $\lambda > 0$ in a pure strategy Nash equilibrium, agents increase their exposure to the idiosyncratic shock $\alpha^* > 0$ with $\mu^* > 0$ despite the increased effort costs associated with drifting from the organization's core competence.*

To see why this holds, note that the right hand side of equations (1.2.9) and (1.2.10) are both greater than zero; hence the left hand sides are greater than zero too. As a consequence, the agents do not fully leverage the firm's core advantage and first-best effort will not be achieved. Independence-seeking behavior is therefore entirely wasteful to a risk-neutral organizer because it increases the cost of achieving any level of expected output. The principal must anticipate this effect in determining a second-best level of effort from the agents.

We now turn to the principal's problem and identify the conditions for an optimal contract when independence-seeking behavior is present. Henceforth, we assume the global constraint in equation (1.2.11) is satisfied for a Nash Equilibrium to exist. We then look at how adding another prize can control independence-seeking behavior in Subsection 1.2.3.

1.2.2 The Principal's Problem

The principal's problem is to maximize profits resulting from the agents' behavior. Let V be the marginal product of output. The principal has a profit function of $\Pi = nV\mu^* - S - nW$ and can choose the spread and losing payoff as a contract to the agents. Assume the principal holds the agents to their participation constraint $\frac{1}{n}S + W_2 - (1 + \lambda z(\alpha_i^*))\, c(\mu_i^*) = \underline{u}$ where $\underline{u}$ is opportunity cost common to all agents. This expression can be rearranged to describe a binding constraint on the losing payoff.

From equations (1.2.9) and (1.2.10), effort and independence are implicit functions of the spread. Therefore, the principal chooses S to maximize

$$\max_S \Pi = nV\mu_i^*(S) - n\left(\underline{u} + \left(1 + \lambda z\left(\alpha_i^*(S)\right)\right) c\left(\mu_i^*(S)\right)\right) \tag{1.2.12}$$

The first order condition is found to be

$$\frac{\partial \Pi}{\partial S} = V\frac{\partial \mu^*}{\partial S} - \lambda z'\left(\alpha_i^*\right) c\left(\mu_i^*\right)\frac{\partial \alpha^*}{\partial S} - \left(1 + \lambda z\left(\alpha_i^*\right)\right) c'\left(\mu_i^*\right)\frac{\partial \mu^*}{\partial S} = 0. \tag{1.2.13}$$

The above expression from equation (1.2.13) can be analyzed in the following form

$$V - c'\left(\mu_i^*\right) = \lambda\left(z'\left(\alpha_i^*\right) c\left(\mu_i^*\right)\frac{\frac{\partial \alpha^*}{\partial S}}{\frac{\partial \mu^*}{\partial S}} + z\left(\alpha_i^*\right) c'\left(\mu_i^*\right)\right). \tag{1.2.14}$$

When the principal seeks to maximize profits, the first-best outcome is achieved when $\alpha = 0$ and effort is defined by $V = c'(\mu)$. However, when $\lambda > 0$, equation (1.2.14) implies that $V \neq c'(\mu^*)$ at the second best solution which is found where

$$S^* = \frac{V}{h}\left(\frac{1 + \alpha^*}{1 + \frac{k}{h}\frac{\frac{\partial \alpha^*}{\partial S}}{\frac{\partial \mu^*}{\partial S}}}\right).$$

As a result of Proposition 1.2, the principal experiences wasteful costs of $n\lambda z\left(\alpha^*\right) c\left(\mu_i^*\right)$ because he cannot eliminate a positive α^* by motivating agents with simply S^*. This can be resolved by adding another degree of freedom to the principal's optimization problem.

1.2.3 Carrots and Sticks

When independence seeking behavior is undesirable, as is the case when the principal values expected aggregate output $n\mu^*$, such behavior can be controlled by offering more than two prize levels. Including a penalty for finishing last has been shown to be sufficient to reduce or eliminate risk-seeking behavior separately from effort in contests when costs of effort and increased output variance are linearly separable (Gilpatric, 2009). The cost functions in this model are not linearly separable, however an analog finding still obtains.

Consider a contest with three or more agents and three payoffs: $W_1 > W_2 > W_3$. Let $S_1 = W_1 - W_2$ represent the winning spread and $S_2 = W_2 - W_3$ represent the penalty spread. Let P_i represent the probability of ranking first and receiving prize W_1 and let R_i represent the probability of ranking last and receiving the penalty W_3. The expected earnings of the agent is now

$$E_i = P_i S_1 - R_i S_2 + W_2 - (1 + \lambda z(\alpha_i))\, c(\mu_i) \tag{1.2.15}$$

Agents again choose μ_i and α_i to maximize their expected earnings, but this time with a separate probability to be either first or last. The probability of winning W_1 is as before $P_i = \iint f(\varepsilon_i)\, F(\kappa)^{n-1}\ d\varepsilon_i\, g(x)\, dx$, and the probability of being last is $R_i = \iint f(\varepsilon_i)\, [1 - F(\kappa)]^{n-1}\ d\varepsilon_i\, g(x)\, dx$. The probability of finishing last can be shown to be decreasing in effort $\frac{\partial R_i}{\partial \mu_i} < 0$. Increasing the mean of effort increases the probability of being first while simultaneously reduces the chance of being last. In a manner similar to the proof of Proposition 1.1, it can also be shown that the probability of finishing last is increasing in independence from any point of symmetry $\frac{\partial R_i}{\partial \alpha_i} > 0$. Increasing α_i behaves differently than increasing μ_i. Increasing independence increases the probability of being both first and last.

At a Nash equilibrium, symmetry leads to the following properties $P_i = R_i$, $\frac{\partial R_i}{\partial \mu_i} = -\frac{\partial P_i}{\partial \mu_i}$ and $\frac{\partial R_i}{\partial \alpha_i} = \frac{\partial P_i}{\partial \alpha_i}$.[7] The first order conditions are now a function of the two spreads. The equations that implicitly describe the Nash Equilibrium are

$$\left(1 + \lambda z\left(\alpha^*\right)\right) c^{'}\left(\mu^*\right) = \frac{\left(S_1 + S_2\right) h}{1 + \alpha^*} \tag{1.2.16}$$

$$\lambda z^{'}\left(\alpha^*\right) c\left(\mu_i^*\right) = \frac{\left(S_1 - S_2\right) k}{1 + \alpha^*}. \tag{1.2.17}$$

Define the parameter $\hat{\mu} > 0$ to be the desired level of effort and $\hat{\alpha} \in [0, 1]$ to be the desired level of independence in the firm sought by the principal at the Nash equilibrium. Holding agents indifferent to outside opportunities $\underline{u}$, let $m = \underline{u} + \left(1 + \lambda z\left(\hat{\alpha}\right)\right) c\left(\mu^*\right)$ be the expected payoff. The payoffs that lead to the desired level of effort and independence are

$$W_1\left(\hat{\mu}, \hat{\alpha}\right) = m + \frac{\left(n-2\right)\lambda}{2nk}\left(1+\hat{\alpha}\right) z^{'}\left(\hat{\alpha}\right) c\left(\hat{\mu}\right) + \frac{c^{'}\left(\hat{\mu}\right)}{2h}\left(1+\hat{\alpha}\right)\left(1 + \lambda z\left(\hat{\alpha}\right)\right) \tag{1.2.18}$$

$$W_2\left(\hat{\mu}, \hat{\alpha}\right) = m - \frac{\lambda}{nk}\left(1+\hat{\alpha}\right) z^{'}\left(\hat{\alpha}\right) c\left(\hat{\mu}\right)$$

$$W_3\left(\hat{\mu}, \hat{\alpha}\right) = m + \frac{\left(n-2\right)\lambda}{2nk}\left(1+\hat{\alpha}\right) z^{'}\left(\hat{\alpha}\right) c\left(\hat{\mu}\right) - \frac{c^{'}\left(\hat{\mu}\right)}{2h}\left(1+\hat{\alpha}\right)\left(1 + \lambda z\left(\hat{\alpha}\right)\right).$$

[7] The proofs of these properties can be easily developed from the equations of P_i and R_i.

The highest ranked agent wins W_1, the lowest ranked agent receives W_3, and all other $n-2$ agents get the middle payoff W_2. The first-best optimum can be realized with a three-payoff mechanism. When strict loyalty to the core is desired $\hat{\alpha}=0$, all agents except those with the highest and lowest outputs are paid their expected payoff $W_2=m$. However, when a positive level of independence is desired of all the agents $\hat{\alpha}>0$, the middle prize drops $W_2<m$. The difference $-\frac{\lambda}{nk}\left(1+\hat{\alpha}\right)z'\left(\hat{\alpha}\right)c\left(\hat{\mu}\right)$ serves to discourage shirking and make mediocre performance more costly.

The least costly way for the principal to achieve any given level of aggregate output $n\mu^*$is to have $\hat{\alpha}=0$. By adding a penalty, the principal is now able to adjust two separate levels of prizes and control the incentive to be independent and the incentive to be loyal to the firm. When an agent increases σ_{ε_i} by increasing his independence from a point of symmetry, more density shift to the tails of his output distribution relative to competitors which increases his probability of being both first and last. The probability of being first and winning a prize creates the incentive to be independent; and the probability of being last and suffering a loss creates the incentive to be loyal. When these incentives are balanced in the contract, the benefit to be independent cancels out the loss to be independent.

Proposition 1.3. *At a Nash equilibrium, the principal can induce any desired level of effort and any desired level of independence with payoffs* $W_1\left(\hat{\mu},\hat{\alpha}\right)$*,* $W_2\left(\hat{\mu},\hat{\alpha}\right)$*, and* $W_3\left(\hat{\mu},\hat{\alpha}\right)$*; where first-best efficiency is achieved when* $\hat{\alpha}=0$*.*

To reach first-best efficiency and eliminate wasteful independence-seeking, the principal can set $\hat{\mu}=\mu^*$ and $\hat{\alpha}=0$ in the payoff functions and offer the contract $W_1=\underline{u}+c\left(\mu^*\right)+\frac{V}{2h}$, $W_2=\underline{u}+c\left(\mu^*\right)$, and $W_3=\underline{u}+c\left(\mu^*\right)-\frac{V}{2h}$. The difference of the spreads is $S_1-S_2=0$ which eliminates the marginal incentive to increase independence in equation (1.2.17). Agents no longer gain any advantage in winning the contest by being independent from the firm or from each other. Since independence-seeking is costly to effort, the best action for all agents to take is to stay loyal to the core competency of the firm and compete solely on effort. With $\alpha^* = 0$, equation (1.2.16) and the sum of the spreads $S_1 + S_2 = \frac{V}{h}$ delivers the first-best solution $V = c^{'}(\mu^*)$.

Firms may indeed find market conditions drive the need for managers to pursue some level of R&D. If desired, the principal can choose any level of independence and still reach the efficient level of effort μ^* but it will be a second-best solution since the cost to reach this level is higher when $\hat{\alpha} > 0$. Because the cost functions of effort and independence are multiplicative in this model, an upper bound on $\hat{\alpha}$ for any given level of effort is not inherently limited by equations (1.2.16) and (1.2.17). The firm must select the pair $(\hat{\mu}, \hat{\alpha})$, and then set prizes according to the payoffs in (1.2.18).

1.3 Conclusion

While competition for advancement within a firm clearly is a central tool for motivating effort, it is important to consider how competitive mechanisms impact managerial decision-making in other dimensions. We've shown that in a corporate setting where working within the core competency of the firm is most productive, competition and autonomy create the incentive for managers to drift from core activities.

Competition will motivate divergence from the firm's core activities, and this may explain some apparent market behavior of firms. In many organizations independent decisions-makers may be uncertain about what tasks will yield the greatest return to invested effort—it may not be obvious to managers where the organization's greatest strengths lie. But our model suggests that, even when it is clear what tasks will yield the greatest return to effort for the organization, competitive mechanisms may not align manager's incentives to common goals.

The model can also apply to relative consumption theory where individuals seek to differentiate themselves by consuming goods that have uncertainty about how popular the good will become. Early adopters of technology may seek to adopt products less mainstream in the hope the product becomes the new trend and by extension, makes the person more popular. In a different setting, a salesman might deviate from the company's predetermined sales pitch toward a new unproven script to increase the variation in responses he receives to increase his chances of winning a sales bonus.

Chapter 2

Fair Tournaments in Common Environments

2.1 Introduction

In what ways can tournaments be equitable and efficient? Can both be achieved when contestants are heterogeneous in both ability and idiosyncratic shock distributions? Offering all contestants the same payoff contract may seem equitable, but when contestants are heterogeneous, these contracts fail to achieve efficiency. A more tractable path is procedural fairness, which in this context refers to the equity of the process by which labor contracts are selected and verifiably administered. Organizing a fair process is important because tournaments, once played, create both winners and losers and so by their very nature cannot be considered equitable in the ex post distribution of payoffs. Psychology theories suggest that even if the requirements of distributive fairness in outcomes are not met ex post, equitable outcomes can be achieved with a fair procedure ex ante (Tyler, 1989). If we subscribe to this notion, then we have a means to realize both equity and efficiency simultaneously in competitive contracts. However, it is important to understand the characteristics of the contestants and the environment they compete in so that policy is verifiably fair. The focus of this paper is to explore the implications of policies that provide contestants of different backgrounds procedural fairness in tournaments while maintaining optimal productivity, and to consider how fairness is impacted when contestants are subjected to common environmental shocks.

As motivation, consider the broiler tournament. Relative compensation has evolved in the broiler industry in the U.S. to protect poultry integrators from moral hazard but also in part as a means to protect growers' earnings against large common production shocks like swings in temperature, disease, rapid technological progress, and large industry price fluctuations (Knoeber, 1989; Knoeber and Thurman, 1995). Yet, despite these benefits, growers have consistently complained that the tournament scheme is unfair and that new regulations are needed. Growers are typically individual owners that raise broilers (chickens) from chicks and are paid by the pound of final broiler weight. Growers compete each flock for better pay rates against other growers that vary in production capacity from 10,000 chicks to over 100,000 with production risk largely proportional to flock size. To account for the heterogeneity, growers are ranked using a handicap mechanism that converts output into a measure of efficiency called "settlement costs", a calculated value of the ratio of input costs to output broiler weight. Further, practices include weening undesirable birds from the flock prior to weighing, another form of handicap. With such ranking processes in place, it may be that one source of grower discontent comes from the lack of procedural fairness. More specifically, that the complex ranking process designed to make a contest between heterogeneous growers fair, when combined with large environmental shocks, may produce unintended consequences and subsequent discontent in growers.

Using the theory of Lazear and Rosen (1981), we describe a heterogeneous tournament model incorporating a handicapped rank rule to compensate for background differences in participants. The model provides all participants a strategically symmetric opportunity to win and that the process by which a winner is chosen is verifiable. The agents modeled here are risk-neutral and heterogeneous in both production uncertainty and ability and are subject to two forms of common environmental shocks; an additive output common shock that shift agents' production and a multiplicative output common shock that scales agents' production. Second, we explore how common shocks alter the outcomes asymmetrically for specific types of handicap rank rule structures. Lastly, we consider how using fair tournament structures influence the incentives of the principle. A multiplicative shock characterizes common events that effect every unit of production, like extremely cold weather that impacts the growth of all chickens. An additive shock models a common fixed-size event, like how an integrator might take the same fixed amount of birds for testing from all growers.

To facilitate fairness, a linear rank rule is defined that renders the contest strategically symmetric in equilibrium. To satisfy the verifiability requirement, we define fair criteria at the efficient equilibrium that each contract must satisfy to insure symmetry in the contest. Tournament results are typically unaffected by common shocks; however, when the idiosyncratic uncertainty is not identically distributed, this may no longer hold even when the tournament is designed to be procedurally fair. Therefore, we also show how incomplete information about the environment can lead to undesirable biases in the probability of winning which fail to meet the criteria of background fairness. Finally, procedural fairness also demands neutrality of the principal or at least a fair process that is enforceable and verifiable to a third party;

hence, we check for neutral incentives for the principal and do find conditions for a commitment problem.

In addition to broiler tournaments, the linear model can be applied to many existing handicap systems where the purpose is to make the contests more fair. Lavy (2002, 2009) study teachers who compete over student performance in a bonus tournament that is adjusted for school and classroom effects. Also, linear handicaps are prevalent in sports such as in golf where strokes are added to the score of less able golfers, in yacht racing where finish times are multiplied by a handicap that is proportional to the performance of the boats, and in horse races where weights are added to each horse based on past performance to even out the expected finish times.

The notion of background fairness is a complimentary refinement to procedural fairness that seeks adjustment for the different background conditions in ability, environment, or technology between contestants. As Barry (1990) describes, "Procedural fairness rules out one boxer having a piece of lead inside his gloves, but background fairness would also rule out any undue disparity in the weight of the boxers; similarly background fairness would rule out sailing boats or cars of different sizes being raced against one another unless suitably handicapped."[1] Rawls (1971) formalizes procedural justice with the concept that a fair outcome is the result so long as the process that produced that outcome was fair. Fishkin (1983) looks at equal opportunity and states that procedural and background fairness as essential and distinct notions for justice. He defines procedural fairness as the principle of merit and background fairness as the equality of life chances. Konow (2003) highlights that theories of desert (that is, compensation based on what is deserved) point toward effort as the dimension

[1] Holding an unfair contest may rule out participation by some part of the population. Barry captures this sentiment when he writes "One agent demonstrating superior skill is grounds for complaints based on background fairness."

that should determine rewards and that other differences like luck, birth, and other characteristics of individuals should not affect rewards.

The work by Bolton et al. (2005) is a direct application of the notion of procedural fairness to an economic context. They compare how often a fair allocation option is rejected vs. how often a fair procedure option (with unfair allocation) is rejected in ultimatum games and find that a fair procedure may substitute for a fair outcome. Their results support the evidence in the psychology literature that these two forms of fairness are linked, but distinct.

Procedural fairness and background fairness have not been formally modeled in the tournament literature. However, several papers have considered how tournament mechanisms can accommodate heterogeneity to make a contest more fair in some sense. One of the key assumptions for these models is that contestants have identical distributions of idiosyncratic shocks. This originates with Lazear and Rosen (1981) who use an additive handicap to even the differences in the levels of median expected output between agents. O'Keeffe et al. (1984) define "fair contests to be those that are symmetric with respect to permutations of the contestants" and describe how handicaps in the form of translations of the probabilities can preserve player incentives by shifting outputs or prizes as needed. However, they do not model the effects of common shocks. Bhattacharya, Sudipto and Guasch (1988) study promotions awarded over relative effort in hierarchies. Recent work by Gürtler and Kräkel (2010) show that individual prizes dominate single prize contracts and shifting prizes down for the high ability workers will lead to a more even match without the use of handicaps. Wu and Roe (2006) find experimental evidence that inequality in tournaments lowers player's willingness to pay to enter the tournament. This is an important finding because it implies a more stringent participation constraint may be needed. We relax the identical distribution constraint and look at the type of handicaps that arise in rank-order tournaments when contestants have non-identical distributions of idiosyncratic shocks. Our concern for the relationship between fairness and risk is supported by Cappelen et al. (2013) who find experimentally that most people would consider it unfair when agents must be compensated differently based on different exposures to risk even when they contribute identical amounts of efficient effort.

The results of the paper show that with perfect information, a general rank rule that meets the fair criteria when agents are heterogeneous achieves efficiency. When the perfect information assumption regarding the environment is relaxed, common shocks are shown to no longer affect agents symmetrically. The handicaps in the rank rule designed to achieve symmetry now bias the common shocks and can alter the probability of winning when what is observed by the principal is different from the actual shock. Under certain conditions, the marginal probability of winning is increasing in multiplicative common shocks when agents have a higher than average ability-variance ratio and decreasing when it is below average. Also, the marginal probability of winning is increasing in additive common shocks when agents have a variance lower than the harmonic mean and decreasing when it is above the harmonic mean.

When the principal can control the type of agents that enter the contest, the information requirements can be relaxed and the basic assumptions made about the agents dictate the type of rank rule used and biases that might be present. When we investigate how making tournaments fair affects the incentives for the principal, we find that fair tournaments that compensate for non-identically distributed shocks produce a commitment problem for the principal. The principal has an incentive to bias the rank-order in favor of the agents with the lowest output variance. Procedural fairness may be compromised unless the principal can be monitored or the rank evaluation process can be verified.

Procedural fairness is often intertwined with distributive fairness, the concerned with other regarding preferences and fairness in the distribution of contract outcomes. Two popular models, Fehr and Schmidt (1999) model fairness as self centered inequity aversion and Bolton and Ockenfels (2000) model equity in an incomplete information setting. For a discussion on the implications of distributive fairness in the form of inequity aversion in tournaments, see Grund and Sliwka (2005). Bolton et al. (2005) use lottery games to show that procedural fairness is conceptually distinct, but linked to allocation fairness. For information about the link between procedural and distributive justice, see Roch et al. (2007); Krawczyk (2009); Rousseau et al. (2009) and Balafoutas et al. (2012). More recently, Saito (2013) builds on the work of Fehr and Schmidt (1999) to describe a model that bridges the gap between equality of opportunity and equality of outcome. None of these papers look at the refinement of background fairness as a contributing factor of inequity.

The rest of this paper is organized as follows. Section 2.2 develops the tournament model. Section 2.3 defines the key terms and fair criteria that establish background fairness. Section 2.4 presents the major results for policy and for the agent. Section 2.5 discusses the principal's incentives. Section 2.6 concludes. Appendix B contains the proofs.

2.2 Model

Consider a rank-order tournament held between many risk-neutral agents who have different abilities and different variance of uncertainty and who are exposed to common environmental shocks. Agents compete and are then rank-ordered by a linearly handicapped output. The top rank wins. The purpose of handicaps are to impose a symmetric game between agents at the Nash equilibrium. Therefore in our model, the mechanism for fairness takes on the form of a policy handicap rank rule.

Let i, j, k be in the set of n risk-neutral agents. Agents are heterogeneous in ability level $a_i > 0$. More broadly, ability can describe differences in talent, technology, or job characteristics. Agent ability is observable to all. Agents have an individual variance factor $\alpha_i > 0$ which describes differences in risk or productive uncertainty. The idiosyncratic output shock is $\varepsilon_i = \alpha_i \kappa_i$ where $\kappa_i \sim (0, \sigma_\kappa^2)$ is an *i.i.d.* draw from a uni-modal and symmetric distribution that is uncorrelated with effort. Individual variance is $\sigma_i^2 = \alpha_i^2 \sigma_\kappa^2$ for all i where σ^2 is a constant variance level associated with the competitive environment common to all agents.

Agent i produces output $q_i = \theta\mu_i + \varepsilon_i + s$ as a function of his own effort μ_i, own shock, an additive output common shock $s \,\epsilon\, (-\infty, \infty)$ and a multiplicative output common shock $\theta \,\epsilon\, (0, \infty)$. We study the consequences that common shocks have on fairness when they are both perfectly and imperfectly observable by the principal. An agent's ability influences the cost of effort described by the function $a_i C\left(\frac{\mu_i}{a_i}\right)$ where the marginal cost to effort is $C'\left(\frac{\mu_i}{a_i}\right)$, with $C'(\mu_i) > 0$, $C''(\mu_i) > 0$ and further $C(0) = C'(0) = 0$. The assumed cost function insures that ability has a marginal effect on effort in the marginal cost function with the same exponent-order as effort and that this holds for many different cost specifications.[2] Costs functions are known to all, are invertible at all levels, and fixed costs are zero.

Agent $i's$ rank in the tournament is a linear function of his own output according

[2]Examples are k^{th} order functions $a_i\left(\frac{\mu_i}{a_i}\right)^k$ and exponential functions $a_i e^{\frac{\mu_i}{a_i}}$. The respective marginal cost functions are $k\left(\frac{\mu_i}{a_i}\right)^{k-1}$ and $e^{\frac{\mu_i}{a_i}}$ which meet the requirements.

to the rank rule

$$r_i = \phi_i q_i + h_i \tag{2.2.1}$$

where ϕ_i is a ratio handicap and h_i is an additive handicap for player i. Verifiable to a third party, the rank function adjusts both the offset and slope of the agent's productive output depending on the requirements of the evaluation policy. Expanding q_i in equation (2.2.1) yields the agent's competitive rank rule $r_i = \phi_i\theta\mu_i + \phi_i\varepsilon_i + \phi_i s + h_i$. Note that since the ratio handicap is multiplied directly by output, it modifies not only the effort but also the uncertainty term and the common shocks.

Payoff contracts (W_i, L_i) are offered to agent i where W_i is the winning payoff and L_i is the losing payoff. Agents are ranked based on the rank rule and awarded payoffs based on the criteria

$$\text{Player } i's \text{ payoff} = \begin{cases} W_i & if \quad r_i > \ r_j \ for\ all\ j \neq i \\ L_i & if \quad r_i < \ r_j \ for\ at\ least\ one\ j \neq i. \end{cases}$$

The tournament produces n payoff combinations with only one winner and $n-1$ losers.

The probability agent i has a higher rank in equation (2.2.1) than agent j for a given draw of ε_i, θ, and s is

$$prob\left(\phi_j\varepsilon_j < \phi_i\varepsilon_i + \phi_i\theta\mu_i - \phi_j\theta\mu_j + (\phi_i - \phi_j)\, s + h_i - h_j\right)$$

for all agents $j \neq i$. Let f be the probability density function and F be the cumulative distribution function of agent $i's$ handicapped uncertainty $\phi_i\varepsilon_i$. Similarly, let z_j be the probability density function and Z_j be the cumulative distribution function of agent $j's$ handicapped uncertainty $\phi_j \varepsilon_j$ for all $j \neq i$. Agent $i's$ probability of winning the tournament by being ranked higher than all other agents is

$$P_i = \int f(\phi_i \varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\phi_i \varepsilon_i + \phi_i \theta \mu_i - \phi_j \theta \mu_j + (\phi_i - \phi_j) s + h_i - h_j\right) \, d\varepsilon_i. \qquad (2.2.2)$$

The marginal effect of effort on the probability of winning is

$$\frac{\partial P_i}{\partial \mu_i} = \phi_i \theta g_i =$$

$$\phi_i \theta (n-1) \int f(\phi_i \varepsilon_i) \sum_{j \neq i}^{n-1} \Bigg\{ z_j \left(\phi_i \varepsilon_i + \phi_i \theta \mu_i - \phi_j \theta \mu_j + (\phi_i - \phi_j) s + h_i - h_j\right)$$

$$\times \prod_{k \neq j \neq i}^{n-2} Z_k \left(\phi_i \varepsilon_i + \phi_i \theta \mu_i - \phi_j \theta \mu_j + (\phi_i - \phi_j) s + h_i - h_j\right) \Bigg\} \, d\varepsilon_i, \qquad (2.2.3)$$

where g_i is the integral portion and is everywhere positive; hence, the probability of winning is strictly increasing in effort $\frac{\partial P_i}{\partial \mu_i} > 0$.

We are interested in the equilibrium of the tournament that motivates efficient effort from all agents. Adopting the Nash-Cournot assumptions, we find the best-response function for a given agent i by maximizing utility taking the actions of the all other agents as given. With probability of winning from equation (2.2.2), he produces optimal effort according to

$$\mu_i^* = \underset{\mu_i}{argmax} \quad EU_i = P_i (W_i - L_i) + L_i - a_i C\left(\frac{\mu_i}{a_i}\right) \qquad (2.2.4)$$

The first order conditions are

$$\frac{\partial EU_i}{\partial \mu_i} = \phi_i \theta g_i (W_i - L_i) - C'\left(\frac{\mu_i^*}{a_i}\right) \equiv 0. \qquad (2.2.5)$$

The second order conditions to guarantee a maximum are

$$\frac{\partial^2 EU_i}{\partial \mu_i^2} = (\phi_i \theta)^2 g_i^{'} (W_i - L_i) - \frac{1}{a_i} C^{''} \left(\frac{\mu_i}{a_i} \right) < 0.$$

The efficient outcome is reached when the marginal value of product V is equal to the marginal cost

$$V = C^{'} \left(\frac{\mu_i^o}{a_i} \right) \tag{2.2.6}$$

where μ_i^o is defined as the agent's efficient level of effort that satisfies equation (2.2.6).[3] We obtain the efficient Nash equilibrium of the game by evaluating equation (2.2.5) where all agents meet the condition in equation (2.2.6) such that

$$\mu_i = \mu_i^o = a_i \mu^o \qquad\qquad \forall\ i \tag{2.2.7}$$

where μ^o is a normalized efficient level of effort when ability is equal to one. Define the individual payoff spread that satisfies equations (2.2.5), (2.2.6), and (2.2.7) as

$$y_i^o = W_i - L_i|_{\mu_i^* = \mu_i^o} = \frac{V}{\phi_i \theta g_i^o \left(\mu_i^o, \mu_{-i}^o \right)}. \tag{2.2.8}$$

The payoffs are now described that yield efficiency and hold agents indifferent to outside opportunities. Agent $i's$ participation inequality is $EU_i\left(\mu_i^o\right) \geq \underline{u}_i$ for any level of effort where his expected utility is from equation (2.2.4) and $\underline{u}_i$ is his opportunity cost. To insure he is held indifferent to outside opportunities, this inequality binds at

[3]The superscript ^{o} is used to denote variables related to the efficient outcome.

equilibrium and his participation constraint is

$$P_i (W_i - L_i) + L_i - a_i C \left(\frac{\mu_i^o}{a_i} \right) = \underline{u}_i. \tag{2.2.9}$$

Combining equation (2.2.9) with the spread in (2.2.8) obtains individual payoff contracts

$$\begin{aligned} W_i &= \underline{u}_i + a_i C (\mu^o) + (1 - P_i) \tfrac{V}{\phi_i \theta g_i^o} \\ L_i &= \underline{u}_i + a_i C (\mu^o) - P_i \tfrac{V}{\phi_i \theta g_i^o} \end{aligned} \tag{2.2.10}$$

Individual contracts provide a flexible framework with multiple degrees of freedom to construct mechanisms that meet both efficiency and fairness requirements. When the contest is also symmetric, $P_i = \frac{1}{n}$ and g_i^o are the same for everyone, the levels are scaled by $\underline{u}_i$ and a_i, and the spread is scaled solely by $\frac{1}{\phi_i}$ and θ. The firm can compensate for expected multiplicative shocks by adjusting the spread. If $0 < E[\theta] < 1$, a negative environmental shock is expected and the spread can be increased to compensate agents in preparation for adverse conditions. Likewise, reducing the spread in anticipation of a positive shock $1 < E[\theta]$ reduces the incentive for agents to overwork during good times.

We shall explore in Section 2.4 a mechanism to even the variance and ability between contestants thereby inducing a symmetric tournament. This mechanism will be shown to set the multiplicative handicap inversely related to agents' variance factor $\phi_i = \frac{1}{\alpha_i}$. In this case, the spread in a symmetric tournament is increasing in individual variance factor; agents with relatively larger uncertainty will have larger payoff spreads.

In addition to the condition for a local maximum that defines the Nash equilibrium, the condition $EU_i(\mu_i) < EU_i(\mu_i^o)$ ensures that the local maximum is a global maximum to an agent's utility maximization problem. This condition protects against the possibility that agents may have other incentives between some intermediate μ_i between zero and μ_i^o or even above efficiency. An agent still has some probability of winning at zero effort $P_i(0) \geq 0$ based on the idiosyncratic draws from all agents.[4] From equation (2.2.4) and the payoffs from equations (2.2.10), the global incentive inequality is

$$a_i C\left(\frac{\mu_i^o}{a_i}\right) - a_i C\left(\frac{\mu_i}{a_i}\right) < \left(P_i(\mu_i^o) - P_i(\mu_i)\right) y_i^o \qquad \forall\, i, \mu_i \neq \mu_i^o.$$

The discrete change in costs must be less than the discrete change in the expected payoff spread between effort levels.

As an illustration, consider a tournament between $n = 3$ agents who exhibit efficient equilibrium behavior. Agents are of types $i = \{l, m, h\} = \{low,\ meduim,\ high\}$ and have cost functions $\{c_l, c_m, c_h\}$ and expected revenue functions $\{R_{l.}, R_m, R_h\}$ respectively. The cost functions $c_i = \underline{u} + a_i C\left(\frac{\mu_i}{a_i}\right)$ describe where agents are indifferent between the contest and outside opportunities $\underline{u}$. The expected revenue functions $R_i = P_i(\mu_i) y_i^o + L_i$ describe the revenue for agent i holding all other agent effort at the Nash equilibrium. Figure 2.1 describes the case where no policy rules are being imposed $(\phi_i = 1, h_i = 0)$, hence $r_i = q_i$ for all four plates.[5] The horizontal axis is effort, the vertical axis is money. The efficient Nash equilibrium, if it exists, is found

[4] The global participation constraint inO'Keeffe et al. (1984) assumes that zero effort yields a zero probability of winning. In this context with rank rules, we tighten the constraint so that no other level of effort produces higher expected utility than at efficiency.

[5] All figures are modeled in Matlab. The cost function is of the form $1.5 + a_i\left(\frac{\mu_i}{a_i}\right)^{4.3}$ where $\underline{u} = 1.5$. Uncertainty is logistic with $\sigma = 0.147$. Agents of type $\{l, m, h\}$ have variance factors $\{\alpha_l, \alpha_m, \alpha_h\} = \{1, 2, 5\}$. Nash equilibrium, if it were to exist, is $\{(\mu_l^o, c_l(\mu_l^o)), (\mu_m^o, c_m(\mu_m^o)), (\mu_h^o, c_h(\mu_h^o))\} = \{(0.90, 2.13), (1.79, 2.75), (4.48, 4.63)\}$. Common parameters are: $V = 3$, $\theta = 1$, $s = 0$.

where $EU_i = c_i - R_i = 0$ and is depicted by the points $(\mu_i^o, c_i(\mu_i^o))$. In plate 2.1(a), agents are homogeneous of ability type a_m and uncertainty is *i.i.d.* with variance factor α_m. In plate 2.1(b), all agents have ability a_m and different variance factors $\alpha_l < \alpha_m < \alpha_h$. In plate 2.1(c), agents are heterogeneous in ability $a_l < a_m < a_h$ and uncertainty that is *i.i.d.* with variance factor α_m. In plate 2.1(d), agents vary in ability $a_l < a_m < a_h$ and variance factors where $\alpha_i \propto a_i$. Only plate 2.1(a) is symmetric.

The necessary conditions for a pure strategy Nash equilibrium to exist in a tournament are observable in the figures. The first order condition in equation (2.2.5) requires the slope of the expected revenue function to equal the slope of the cost function at μ_i^o. The participation equation (2.2.9) requires the expected revenue function to equal the cost function at μ_i^o; hence the two functions are tangent at the Nash equilibrium. The second order condition insures that the slope of the cost function is increasing at a faster rate than the slope of the expected revenue function at μ_i^o. The global incentives insure that the cost function lies above the expected revenue function at all levels of effort except at μ_i^o where they are equal.

In this example, only Plates 2.1(a) and 2.1(b) meet all the necessary constraints for a pure strategy Nash equilibrium to exist. Because agents are homogeneous in Plate 2.1(a), the optimal payoff contracts and spreads are symmetric. Expected revenue is bounded by the payoffs $R_m \epsilon [L_m, W_m]$, and is everywhere below c_m except at μ_m^o where it is equal and so meets the global constraint. The slope of c_m is increasing faster than the slope of R_m at equilibrium so the second order conditions are satisfied. The equilibrium is located where the probability of winning is symmetric at $\frac{1}{3}$ of the way up R_m between L_m and W_m at point $(\mu_m^o, c_m(\mu_m^o))$. Plate 2.1(b) plots the revenue functions when agents have homogeneous ability a_m, but heterogeneous variance $\alpha_l <$

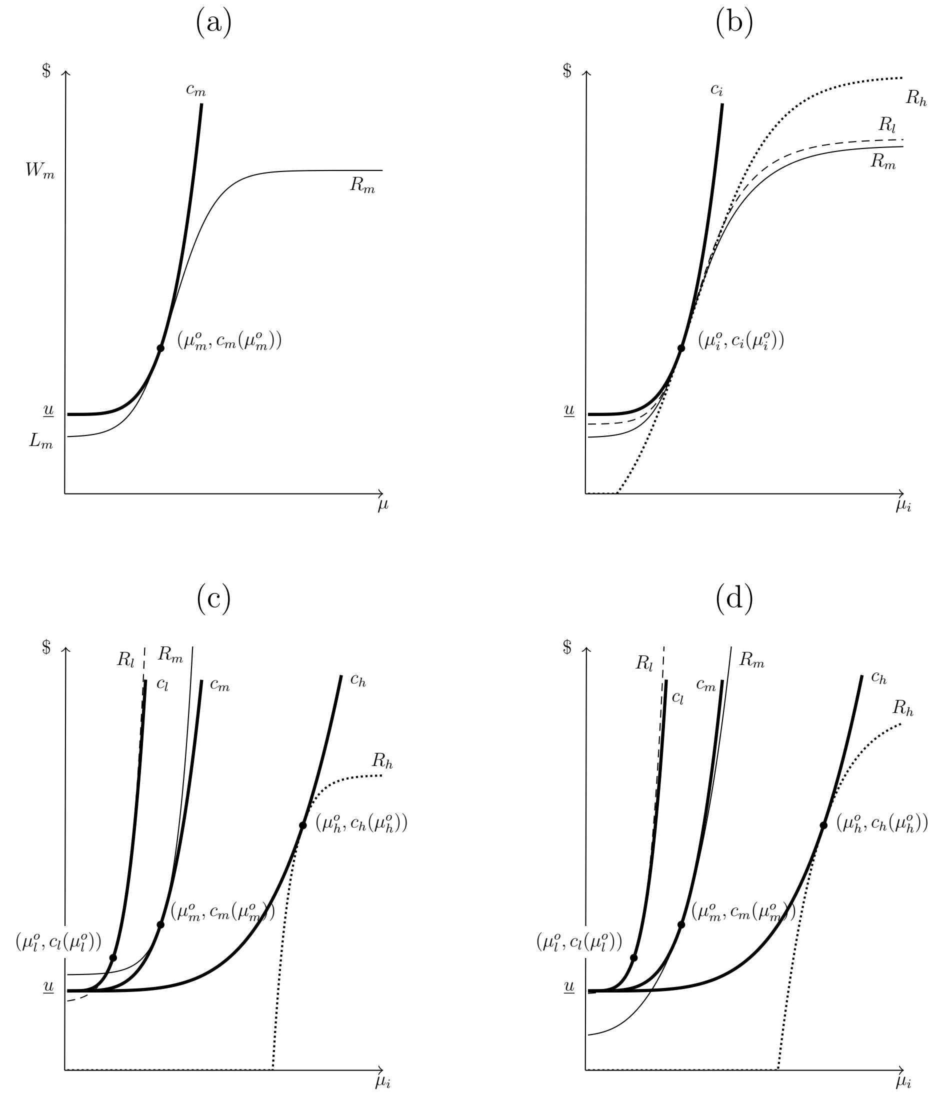

Figure 2.1: **No rank rules.** Cost and revenue vs. effort for an efficient tournament.

$\alpha_m < \alpha_h$. The contest in Plate 2.1(b) meets the global incentive requirements and second order conditions, however the probability of winning for each agent is not symmetric at equilibrium $P_l < P_m < P_h$. The plotted R functions reflect different spreads. When agents are heterogeneous in variance, the payoff contracts have the same expected payoff, but the spreads are increasing in α_i since g_i is decreasing in α_i.

An equilibrium does not exist in Plates 2.1(c) and 2.1(d) because the global participation constraint and second order condition are not met for agent l for the given variance levels α_i. This is visible in plate 2.1(c) where $R_l > c_l$ and also for agent m, $R_m > c_m$. In Plate 2.1(c), agents are heterogeneous in ability $a_l < a_m < a_h$, but are characterized by individual uncertainty that is *i.i.d.* with $\alpha_i = \alpha_m$. Agent h has a very high probability of winning and hence is offered a very small W_h and a highly negative L_h as an incentive to reach efficiency. Both l and m have such low probabilities of winning, their winning payoff must be set extremely high to induce effort. However, neither l nor m meet the global incentive constraints nor the second order conditions. The second order condition is violated because R_l departs the equilibrium point with a slope that increases faster than c_l so the least capable agent has the incentive to overwork, likewise for m. Agent m has the additional incentive to shirk and collect L_m which is greater than $\underline{u}$. The final Plate 2.1(d) depicts the heterogeneous ability and variance case where $\alpha_i \propto a_i$. Agent h is now taking on more risk, hence his spread and W_h are increased to compensate. Agent l is taking on less risk which compresses his spread and worsens his incentives compared to Plate 2.1(c).

We now turn the discussion toward the criteria necessary to achieve fairness. The section after next identifies a rank rule for fair and efficient tournaments, identifies sources of bias, and looks at some special cases of handicaps often found in the literature.

2.3 Fair Criteria

The model in the previous section describes a simple tournament based on the framework of Lazear and Rosen (1981) with the addition of a rank rule. In this section, We explore the question: what criteria characterize an efficient tournament as fair? Prior to a contest, the rank rule can compensate for known differences in capabilities, variance factors, and expected shocks. However, to insure procedural fairness, the final ranking must also account for any asymmetries that might arise from the introduction of the policy instrument. In other words, in order for the policy to be procedurally fair, the implementation must be impartial, neutral, and induce symmetry without creating unintentional or perverse incentives. Therefore we seek a notion of equality that induces strategically symmetric behavior between agents as a policy ideal at equilibrium.

We constrain the discussion to solutions at the efficient equilibrium. In the context of this model, the first concern for fairness is that there should be a means to balance the effect of ability between agents. Second, there should be a means to equalize the distributions of idiosyncratic shocks. The final concern is to insure that common environmental conditions do not effect the agents asymmetrically. The definitions and criteria that follow describe how unfair and fair tournaments are identified and evaluated at the efficient equilibrium

Definition 2.1. *Background differences* are disparities of agent ability a_i, uncertainty variance factor σ_i^2 or handicapped common shocks $\phi_i\theta$ and $\phi_i s$ that influence the probability of winning in a non-symmetric way such that some agents gain an advantage toward winning over others.

Differences in ability lead to non-symmetric levels of effort in the probability equation (2.2.2) at the efficient equilibrium. Differences in variance leads to non-symmetric draws which too can affect the probability of winning Gilpatric (2009). When the rank rule is used to neutralize the non-symmetric effects of ability and uncertainty differences, common shocks may become biased. This leads to the undesirable result that environmental conditions can have non-symmetric influences. Barry (1990) describes the notion that a background fair contest exists when all background differences are made symmetric. We confine our interpretation of Barry to mean that a background fair contest exists when all of the effects from the defined background differences are compensated in the rank rule. We formalize this concept now in the framework of our model.

Definition 2.2. *Background fairness* is the condition that occurs when the influences of the background differences are strategically symmetric at equilibrium.

This does not mean the influences are non-zero, but it does require all of the effects of the differences comparable between agents be equivalent at equilibrium. In the rest of this paper, the terms *fair, fairness,* and *background fair* are synonymous with Definition 2.2.

We now develop a set of requirements that establish the conditions for background fairness and address these concerns. The following *fair criteria* describe the conditions when background differences no longer influence the probability of winning in a non-symmetric way at efficient equilibrium and hence meet the requirements of a background fair tournament. Specifically, in order to be considered efficient and fair, a tournament that uses a rank rule must have equal probabilities, identical uncertainty distributions, and no common shock biases in equilibrium as defined as follows.

Criterion 1 *Equal probabilities.* The probability of winning for any one agent is equal to the probability of winning for all other agents. $P_i = \frac{1}{n}$ for all i in equilibrium.

This only occurs where everyone has the same probability of winning in equilibrium. Hence the background differences in ability that lead to differences in efficient effort must be made symmetric in effort, using the rank rule, without creating any other biases. This criterion addresses the concern that Barry describes in his 1990 document "One agent demonstrating superior skill is grounds for complaints based on background fairness." The next two criteria reenforce this requirement. Criterion 2.3 keeps the distribution of lucky draws symmetric.

Criterion 2 *Equal variance.* The variance of every agent's handicapped uncertainty term is identically distributed. $E\left[(\phi_i\varepsilon_i)^2\right] = \sigma_\kappa^2$ for all i.

This insures no single agent has an intrinsic advantage in luck over any other. We prove in the appendix that an increase in variance increases the probability of winning at the efficient equilibrium for $n > 2$ handicapped agents. This proof is essentially the same used by Gilpatric (2009) who shows that when an agent chooses his own variance, he gains an advantage in the probability of winning.[6] To insure the contest is fair in luck, all agents must have an independent draw from the same effective distribution after the rank rule has compensated for all background differences. Common shocks must also be symmetric to satisfy Criterion 1, which leads to a more formal requirement for common effects.

[6]Gilpatric (2009) also shows that for the case when $n = 2$, the effect of an agent strategically changing his own variance does not effect the probability of winning in equilibrium. This is also the case here for two agents endowed with different variances and the proof is omitted.

Criterion 3 *No common shock bias.* The marginal effect of common shocks on the probability of winning for every agent is zero. $\frac{\partial P_i}{\partial \theta} = 0$ and $\frac{\partial P_i}{\partial s} = 0$ for all i.

This insures that the application of the rank rule does not adversely affect the relative strength of the common shocks between agents. Using ratio handicaps in the rank rule to compensate for differences in ability and variance multiplies the effect of the shocks differently for each agent and makes common shocks uncommon. Resolving this may ultimately require an investment in information for the principal by observation of the common shocks over the course of the tournament. Taken together, the fair criteria establish the symmetry requirements for background fairness.

2.4 Fair Tournaments

The previous section established the criteria for fair and efficient tournaments. This section identifies a general rank rule and the organizer requirements that can produce a fair and efficient outcome with perfect information. Additionally, we relax the *information* assumption that commons shocks are observed perfectly and characterize the bias effects on symmetry. We then move from the general rank rule to more specific combinations of background differences, those common to the literature and new, and the rank rules that both meet and fail to meet the fair criteria for these reduced forms.

2.4.1 Fair Tournaments with Perfect Information

Fair procedural requirements add information demands on the tournament organizer to collect and observe agent and environmental properties. A practical limitation of using a handicap rule like equation (2.2.1) in actual tournaments is the need to observe and compensate for background differences accurately. We first establish fairness when the organizer has perfect knowledge of agent characteristics μ_i^o, a_i, and α_i. Further, we assume the environmental shocks, θ, and s can be measured accurately.

For the case where all background differences are perfectly observable, a general fair and efficient rank rule of the form in equation (2.2.1) can be identified. To be useful, the rank rule must satisfy the fair criteria for each agent even for the case where agents are heterogeneous in ability, have unique uncertainty distributions and experience both multiplicative and additive common shocks. A linear rank rule limits the effective mechanism to a combination of ϕ_i and h_i that can overcome all four differences, so care must be taken when crafting a legitimate rank rule. One solution is to first adjust for the background differences of variance, then ability, then any common shock bias that may be present.

To meet the equal variance criteria, agent $i's$ rank rule in equation (2.2.1) must offset his unique variation term α_i in $\sigma_i^2 = \alpha_i^2\sigma_\kappa^2$ to make his exposure to uncertainty the same as all other agents. However, the idiosyncratic uncertainty is inseparable from observable output hence any attempt to compensate for α_i will also distort the effect on rank from effort and the common shocks. Nevertheless, equating variance can be accomplished with the multiplicative handicap in equation (2.2.1) set to the inverse of the unique variation term $\phi_i = \frac{1}{\alpha_i}$ such that $\frac{1}{\alpha_i}q_i = \frac{\theta\mu_i^o}{\alpha_i} + \frac{s}{\alpha_i} + \frac{\varepsilon_i}{\alpha_i}$. This does, however, create undesirable effects: a distorted effort term $\frac{\theta\mu_i^o}{\alpha_i}$ and a distorted additive common shock term $\frac{s}{\alpha_i}$.

The multiplicative handicap can be used as a mechanism to adjust the uncertainty variation so long as the distortions to effort and shocks are managed in h_i. Setting $h_i = -\frac{\theta\mu_i^o}{\alpha_i} - \frac{s}{\alpha_i}$ accomplishes this task. Individual effort is also inseparable from observable output; however in equilibrium, we can use the property in equation (2.2.7) to describe the handicap as a function of agent $i's$ ability and the level of effort synonymous with having ability equal to unity, namely $\mu_i^o = a_i \mu^o$. The conditions of differences and the rank rule that achieve a fair and efficient outcome are formalized in Proposition 2.1. (all proofs are found in Appendix A).

Proposition 2.1. *Efficient tournaments with agents of heterogeneous ability and unique idiosyncratic uncertainty distributions, who experience common shocks s and θ, are background fair when evaluated using the rank rule*
$r_i = \frac{1}{\alpha_i}(q_i - s - \theta a_i \mu^o)$.

Proposition 2.1 describes the general case where all differences are known and can be used to reach a fair and efficient tournament outcome. In comparison, O'Keeffe et al. (1984) describe how tournaments are made fair between agents heterogeneous in ability where idiosyncratic shocks are drawn from an identical distribution by shifting the probabilities to be equal at the point of efficiency. The rank rule in Proposition 2.1 acts to shift the levels as they described, but unlike O'Keeffe et al., also adjusts the slopes of the probabilities to compensate for the differences in variance.

Recall the example from Figure 2.1 where no rank rule was used. This can be compared to Figure 2.2 where rank rules are used. The plates in Figure 2.1 correspond to the same agent backgrounds as the plates in Figure 2.2.[7] In all cases, $P_i = 1/3$ where the efficient Nash equilibrium should be. However, for this case only plates (a) and (d) meet all the necessary constraints for a Nash equilibrium to exist. First, note the

[7] Just as in Figure 2.1, the backgrounds in Figure 2.2 are: plate (a) describe curves for agents homogeneous of ability type a_m and uncertainty that is *i.i.d.* with variance factor α_m; in plate (b), agents are described by ability a_m and different variance factors $\alpha_l < \alpha_m < \alpha_h$; in plate (c), agents are described by ability $a_l < a_m < a_h$ and uncertainty that is *i.i.d.* with variance factor α_m; in plate (d), agents have ability $a_l < a_m < a_h$ and variance factors $\alpha_i \propto a_i$. All plates are symmetric.

homogeneous case in plate 2.2(a) has the identical outcome to plate 2.1(a); all agents are offered the same payoff contract and the tournament is fair with no handicaps $r_i = q_i$. However in plate 2(b) where agents are homogeneous in ability with different variances, the payoff levels at $(\mu_i^o, c_i(\mu_i^o))$ remain the same for all agents but the spread in equation (2.2.8) is increasing in α_i because $\phi_i = \frac{1}{\alpha_i}$. All strategies satisfy the second order conditions, but one consequence for this example is that the fair contract for agent l does not meet the global incentives requirement. Shirking is the optimal strategy for l because the fair contract would set $L_l > \underline{u}$. In this case, there is insufficient variance in the performance of l to warrant including him in the tournament.

A similar shirking incentive is seen in plate 2(c). However, it is now agent h with incentives to deviate because $L_h > \underline{u}$. The spread is identical for all three agents because the shocks are *i.i.d.* This effect will worsen if ability and variance are inversely proportional, reducing the spread on the high types even more. However, when ability and variance are directly proportional as is shown in plate 2(d), the fair payoff spread is increasing with ability and the result is fair, efficient, and incentive compatible.

To reach this level of fairness, however, information demands on the tournament organizer may be large. While sufficient statistics for α_i, a_i, and μ^o are determined ex ante, the common shocks θ and s must also be observed ex post. This requires a means of disentangling the multiplicative and additive shocks from observable output and inserting them into the rank rules as correcting factors prior to final ranking. The information costs may be impractical or undesirable in many competitive settings. A potential solution is presented by Holmstrom (1982) who suggests the weighted average of productive output can capture the relevant information about the common

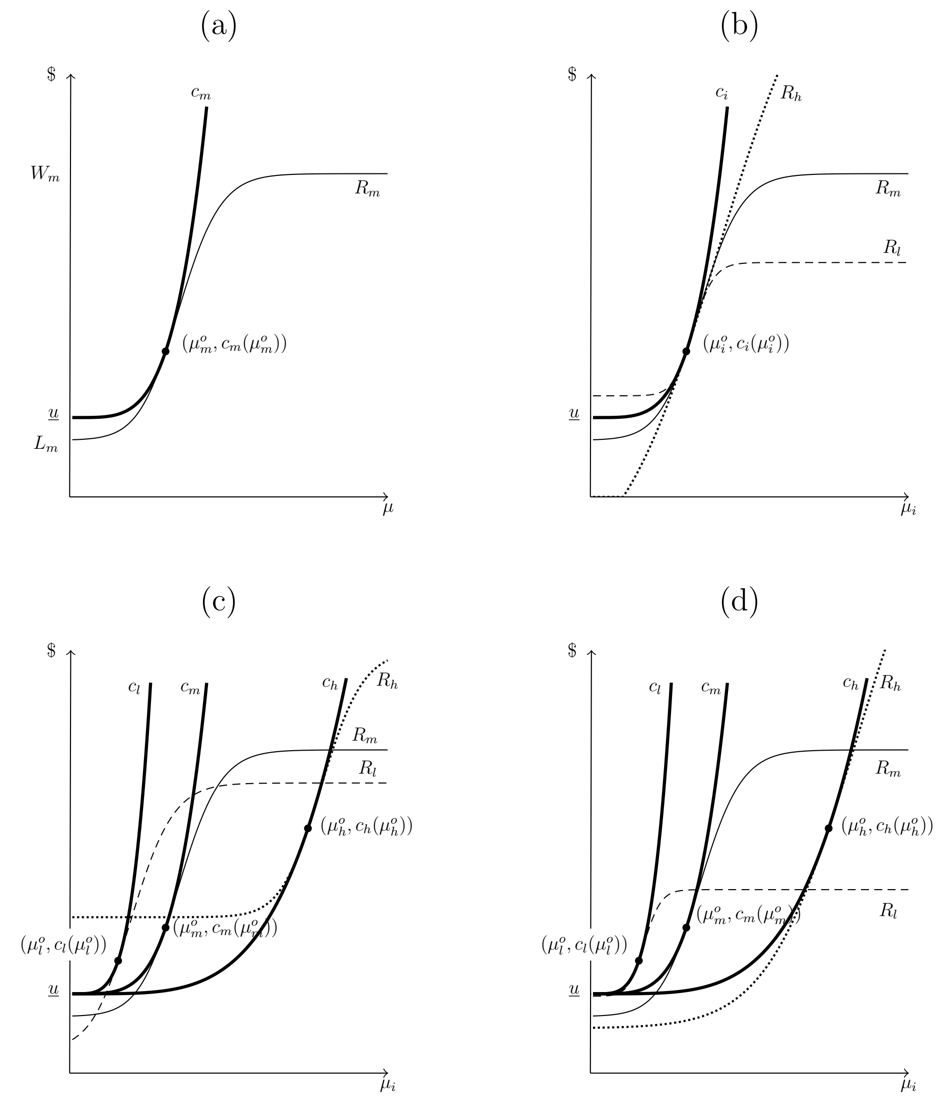

Figure 2.2: **Using a rank rule.** Cost and revenue vs. effort for an efficient tournament that has been made fair.

shocks for very similar specifications.

The need to observe common shocks reveals a conundrum: under certain conditions and rank rules, efficient tournaments can either be unfair and filter out unobserved common shocks or be fair but require observation of the common shocks. Which problem is better to have may depend on the particular circumstances. The next subsection details how the effect of imperfectly observable common shocks can bias the results of an otherwise fair and efficient tournament.

2.4.2 Common shock bias

One of the most desirable characteristics of relative performance compensation highlighted in the literature is the ability to filter out common shocks. We now look at the case where the organizer's expectation of the environmental shocks are normalized to $\theta = 1$, and $s = 0$.

The rank rule in Proposition 2.1 then simplifies to

$$r_i = \frac{1}{\alpha_i}\left(q_i - a_i \mu^o\right). \tag{2.4.1}$$

The effective rule in this reduced form is only a function of individual differences a_i, α_i, and μ^o. At the efficient equilibrium, if the realized common shocks are $s = 0$ and $\theta = 1$, agent $i's$ rank depends solely on a common level of uncertainty $r_i = \kappa_i$ which is the desired result. When $s \neq 0$ or $\theta \neq 1$, agent i will experience a change in r_i ex post.

The bias in the rule due to the additive shock difference is $\frac{\partial r_i}{\partial s} = \frac{1}{\alpha_i}$. If α_i is the same for everyone, the typical result in the literature is achieved when error terms are *i.i.d.*: the bias will shift the rank for everyone by the same amount and not effect the rank order outcome. However, if α_i is unique to individuals, this result no longer holds and the effects of s are no longer balanced across agents.

The bias in the rule due to the multiplicative shock difference $\frac{\partial r_i}{\partial \theta} = \frac{a_i \mu^o}{\alpha_i}$ is made unique to individuals by the ability-variance ratio $\frac{a_j}{\alpha_j}$. If we take high ability or low variance as a signal of quality, then $\frac{a_i}{\alpha_i}$ can be considered the agents' intrinsic signal to noise ratio. Having higher quality can disproportionately increase rank as θ increases. However, when θ is decreasing, rank will decline instead.

When agents are inherently different and rank rules are designed to be fair, common shocks no longer cancel out of the contest. Both types of common shock will alter the probability of winning asymmetrically for each agent in the contest with changes in ability and variance . The result in Proposition 2.2 captures the effect of the multiplicative bias.

Proposition 2.2. *At the efficient equilibrium using the rank rule in Proposition 2.1;*

(i) for the case $\theta = 1$ or $\frac{a_i}{\alpha_i} = \frac{a_j}{\alpha_j}$ *for all* i, j*, the tournament is background fair, no multiplicative common shock exists, and* $\frac{\partial P_i}{\partial \theta} = 0$*;*

(ii) for the case $\theta \neq 1$ and $\frac{a_i}{\alpha_i} \neq \frac{a_j}{\alpha_j}$ *for any one* i *or* j*, the tournament is not background fair and a positive multiplicative shock improves the relative probability of winning for agents with greater than average ability-variance ratio and worsens the probability of winning for agents with a lower than average ability-variance ratio, and vice-verse when a negative multiplicative shock occurs according to*

$$\frac{\partial P_i}{\partial \theta} \gtreqless 0 \quad if \quad \frac{a_i}{\alpha_i} \gtreqless \frac{1}{n}\sum_j^n \frac{a_j}{\alpha_j}.$$

Proposition 2.2 describes the conditions when a multiplicative common shock bias occurs in efficient tournaments that are made fair. Case 2.2.*(i)* states the conditions necessary for background fairness such that no bias is created from the realized multiplicative common shock in the probability of winning for any agent. When organizers accurately observe the shock $\theta = 1$ the probability of winning will be unaffected. Additionally, when variance changes proportional to ability for all agents such that $\frac{a_i}{\alpha_i}$ is the same for all i, the multiplicative common shock is filtered out of the tournament.[8] However, if even one agent does not have the same ability-variance ratio as the rest of the contestants, the probability for everyone will be effected.

Case 2.2.*(ii)* describes what happens when ability-variance ratios are different for one or more agents. If agent $i's$ ability-variance ratio is above the average ratio of all agents, he experiences an increase in his probability of winning with a positive shock $\theta > 1$. The advantage goes to the more precise and able. However, a negative shock difference $\theta < 0$ has the opposite effect on the highly productive and precise. Instead, the agent with an ability-variance ratio below the average will gain an advantage in probability of winning. The advantage goes to the less precise and less able.[9]

Without loss of generality, the results of Proposition 2.2 can be written in terms of a weighted harmonic mean of the variance where the weighted values are either ability or efficient effort. If we compare the level of effort agents put forth at efficiency, the effort-variance ratio $\frac{\mu_i^o}{\alpha_i}$ can be substituted for the ability variance ratio in 2.2.*(ii)*.[10]

[8] A similar result is found in Nalebuff and Stiglitz (1983).

[9] If the settling cost rule in broiler tournaments is susceptible to this effect, growers may have incentives to install new, large capacity houses with the latest technologies to increase production and reduce the effect of ambient temperature variations to gain an advantage over the previous generation of technologies if the integrator systematically under-reports, $\theta > 1$. However, this incentive may reverse, inducing sloppy management practices in low capacity growers if the shock difference is systematically negative $\theta < 1$.

[10] An alternate specification of equation (2.4.1) is $r_i = \frac{1}{\alpha_i}\left(q_i - \mu_i^o\right)$ which leads to a different interpretation of Proposition 2.2, specifically $\frac{\partial P_i}{\partial \theta} \gtreqless 0 \quad if \quad \frac{\mu_i^o}{\alpha_i} \gtreqless \frac{1}{n}\sum_j^n \frac{\mu_j^o}{\alpha_j}$; or in terms of the weighted

Agents that exhibit greater than average efficient effort gain an advantage when $\theta > 1$ and agents that have lower than average efficient effort gain an advantage when $\theta < 1$ holding variance the same.

We now turn our attention to the additive shock term $\frac{s}{\alpha_i}$ in equation (2.4.1). Let $\breve{\alpha} = \left(\frac{1}{n}\sum_j^n \frac{1}{\alpha_j}\right)^{-1}$ be the harmonic mean of variance for all j. When the additive shock is non negative, this term alters the probability of winning asymmetrically for agent i as a function of his variance. The specific formulation of Proposition 2.3 captures the effect of the additive bias when s is non-zero.

Proposition 2.3. *At the efficient equilibrium using the rank rule in Proposition 2.1;*

(i) for the case $s = 0$ or $\alpha_i = \alpha_j$ *for all* i, j*, the tournament is background fair, no additive common shock bias exists, and* $\frac{\partial P_i}{\partial s} = 0$*;*

(ii) for the case $s \neq 0$ and $\alpha_i \neq \alpha_j$ *for any one* i, j *pair, the tournament is not background fair and a positive additive shock improves the relative probability of winning for agents with a variance less than the harmonic mean of variance and worsens the probability of winning for agents with a variance greater than the harmonic mean of variance, and vice-verse when a negative additive shock occurs according to*

$$\frac{\partial P_i}{\partial s} \gtreqqless 0 \quad if \quad \alpha_i \lesseqqgtr \breve{\alpha}.$$

Proposition 2.3 describes the conditions when an additive common shock bias occurs in efficient tournaments made fair. Case 2.3.*(i)* states the conditions necessary for background fairness such that no bias is created from the realized additive common

harmonic mean of variance $\frac{\partial P_i}{\partial \theta} \gtreqqless 0 \quad if \quad \frac{\alpha_i}{\mu_i^o} \lesseqqgtr \left(\frac{1}{n}\sum_j^n \frac{\mu_i^o}{\alpha_j}\right)^{-1}$.

shock in the probability of winning for any agent. When organizers accurately observe the additive shock, then $s = 0$ and the probability of winning will be unaffected by the additive shock. Additionally, when the variance for all agents is identical such that $\alpha = \alpha_j$ and therefore $\sigma_j^2 = \alpha^2\sigma_\kappa^2$ for all j, the additive common shock is filtered out of the tournament. Hence, for a fair and efficient tournament, both homogeneous and heterogeneous agents with identically distributed uncertainty are unaffected by additive common shocks which supports the findings in Lazear and Rosen (1981) and Green and Stokey (1983).

Case 2.3.*(ii)* describes what happens when at least one agent has a unique level of variance from all others in the group. The rank rule can compensate for the asymmetry between agents in both ability and variance, but an additive shock bias will result from an imperfect observation of s. If agent $i's$ variance is below the harmonic mean of all agents' variance, he experiences an increase in his probability of winning when a positive additive shock occurs. The harmonic mean is smaller than the arithmetic mean and will be sensitive to abnormally small values of variance.

Agents who have comparatively high precision in output gain a competitive advantage when faced with positive additive shocks $s > 0$. This incentive counter-acts the gains from increasing own variance as a strategy. In contrast, if $s < 0$, the agent with variance above the harmonic mean will experience an increase in probability of winning. Negative additive shocks help those who have more risk, but hurt those who have it safe.

In general, If tournament organizers are unaware of the common shock biases, then they may create contests that can be gamed.

2.4.3 Special cases

In some settings, entry fees and certifications can act to sort agents into more homogeneous groups. Additionally, the uncertainty of outcomes to agents can be managed with increased monitoring. In such situations, a simplification of the rank rule in Proposition 2.1 can reduce the burden of sufficient statistics for the principal.

We focus on three prevalent combinations of background differences that lead to unique rank rules to highlight their affect in matters of fairness. The first two cases consider environments where agents have identical uncertainty distributions but may or may not be the same in ability. The last case considers an environment where the uncertainty distributions of heterogeneous agents are proportional to ability $\alpha_i \propto a_i$. Uncertainty increasing with ability satisfies the stylized facts of hierarchy as described in Malcomson (1984) and is used later in the discussion of groups in Section **??**. [11] Following are four Corollaries to Proposition 2.1 with a brief discussion for each.

Corollary 2.1. *Efficient tournaments with agents of identical ability and identical idiosyncratic uncertainty distributions, and who experience common shocks s and θ, are background fair when evaluated using the outcome rule $r_i = q_i$.*

Proposition 2.1 describes that the basic homogeneous agent tournament as originally described by Lazear and Rosen (1981), meets the fair criteria. Identical uncertainty distributions give all agents a fair draw. The fair criteria are satisfied as a direct result of the inherent symmetry of the background differences. The rank function naturally meets the equal variance criterion condition $E[(\phi_i \varepsilon_i)^2] = \sigma_\kappa^2$ when

[11] Malcomson (1984) highlights that the variance of earnings increases with experience as a stylized fact of internal hierarchical labor markets.

$\phi_i = 1$.[12] The efficiency requirement in equation (2.2.7) reduces to $\mu_i^o = \mu_j^o$ which also supports setting $\phi_i = 1$. Common shocks cancel when the multiplicative handicaps are identical which meet the no common shock bias criterion and leads to $h_i = 0$. With the background differences accounted for, equation (2.2.2) will satisfy the equal probabilities criterion.

For the second case, the simplified outcome rule in Corollary 2.1 is no longer fair when agents have heterogeneous ability and therefore a different rule is needed. This is because agents of different ability put forth different amounts of efficient effort so the simple comparison of output directly is no longer symmetric and fails to satisfy the equal probabilities criterion.

One possible solution is to use the simplified ratio handicapped rule $r_i = \frac{1}{a_i}(q_i - s)$ that satisfies the efficiency criteria in equation (2.2.7) such that the effort terms are symmetric. However, using any ratio handicap when distributions are identical imposes an asymmetry across agents; hence, $\phi_i = \frac{1}{a_i}$ modifies the uncertainty distributions and the equal variance criterion is violated. Just as is the case in Corollary 2.1, the equal variance criterion is satisfied only when $\phi_i = 1$ for an identical uncertainty distribution.

The solution is to only use an additive handicap to compensate for the differences in effort and therefore satisfies the equal probabilities criterion without adding any other biases. Any additive handicap that satisfies $h_i = r - \theta\mu_i$ where r is common to all agents will provide an adequate solution. For convenience, we set $r = 0$ and describe the conditions that satisfy an efficient and fair solution in Corollary 2.2 and also prove the other rules are not fair.

[12] It would be equally fine to set ϕ_i to any constant to satisfy the equal variance criterion, however any value other than 1 simply scales the values for all agents.

Corollary 2.2. *Efficient tournaments with agents of different ability and identical idiosyncratic uncertainty distributions who experience common shocks s and θ,*

(i) are background fair using $r_i = q_i - \theta\mu_i^o$;

(ii) are not background fair using $r_i = q_i$;

(iii) are not background fair using $r_i = \frac{1}{a_i}(q_i - s)$.

When agents are heterogeneous in ability only, multiplicative common shocks must be observable, but additive common shocks can be ignored as is found in Proposition 2.3.*(i)*. Corollary 2.2.*(i)* describes that the basic handicapped heterogeneous agent tournament as originally described by Lazear and Rosen (1981) meets the fair criteria, but only when the additive handicap fully compensates for the level of efficient effort $h_i = -\theta\mu_i^o$. The notable difference is, however, that θ must be observed by the principal and incorporated into the rank rule to be fair.[13] Rule $r_i = q_i$ fails the equal probabilities criterion because it does not adequately compensate for differences in ability. Rule $r_i = \frac{1}{a_i}(q_i - s)$ fails the equal variance criteria because the ratio handicap term $\frac{1}{a_i}$ distorts the identical distributions and the equal variance criterion is violated.

For the third case, heterogeneous agents with variance proportional to ability are analyzed. Recall that the variance of agent $i's$ uncertainty term is $\sigma_i^2 = a_i^2\sigma_\kappa^2$. The ratio handicap set inversely proportional to ability $\phi_i = \frac{1}{a_i}$ is appropriate to use to satisfy the equal variance criterion in this case. The variance of handicapped uncertainty becomes $E[(\phi_i\varepsilon_i)^2] = \sigma_\kappa^2$ for all agents which meets the equal variance criteria. The rank rule must also satisfy the symmetric effort requirements in the equal probabilities criterion which also occurs when $\phi_i = \frac{1}{a_i}$.

[13]A direct comparison with Lazear and Rosen (1981) requires $\theta = 1$.

Corollary 2.3. *Efficient tournaments with agents of different ability and idiosyncratic uncertainty distributions proportional to ability who experience common shocks* s *and* θ*,*

(i) are background fair using $r_i = \frac{1}{a_i}(q_i - s)$*;*

(ii) are not background fair using $r_i = q_i$*;*

(iii) are not background fair using $r_i = q_i - \theta\mu_i^o$*;*

When agents are heterogeneous in ability and $\alpha_i \propto a_i$, additive common shocks must be observable, but multiplicative common shocks can be ignored. The burden on the principal to observe and report shocks is similar to Corollary 2.2, but opposite in the type of shock and is a direct result of Proposition 2.2.*(i)*.

Corollary 2.3.*(i)* states that the rule $r_i = \frac{1}{a_i}(q_i - s)$ fully compensates for all background differences under these circumstances and is fair. However, as is the case in Corollary 2.2, rule $r_i = q_i$ fails the equal probabilities criterion because it does not adequately compensate for differences in ability. It also fails the equal variance criterion because it does not adjust for heteroskedasticity. Rule $r_i = q_i - \theta\mu_i^o$ solves the symmetric effort problem and therefore adequately compensates for differences in ability, but it fails the equal variance criterion. An additive handicap alone is not sufficient to compensate for differences in variance. To meet the no common shock bias criterion, the additive handicap must take on the form $h_i = \frac{s}{a_i}$.

Section 2.4.1 provided a general solution for fairness with perfect information when agents are highly heterogeneous. Section 2.4.2 demonstrated the bias effects that imperfect information about common shocks can have on the probability of winning and subsequent rank order and how this violates procedural fairness. When a procedurally fair and efficient tournament is required ex ante, imperfectly observed common shocks can bias the rank order of the agents, ex post. Finally, Section 2.4.3

outlined how the rank rule in Proposition 2.1 can be relaxed when the principal can control the type of agents that enter the contest or the exposure to risk. The basic assumptions made about the agents dictate the use of a different rank rule, but still requires diligent awareness of environmental conditions. Hence, there is no policy panacea for fairness, only a recipe for fair practices.

2.5 The Principal

Standard symmetric tournament models indicate that the principal need only commit to awarding the specified prizes, because he has no incentive to prefer awarding the winning prize to one contestant over another. This has been argued to be an important advantage of tournament mechanisms because it may be much easier to make such a commitment then commit to a more complex contract that requires a third party to observe output. However, this result may not obtain in the more general tournament context that we have modeled here in which the organizer employs handicaps and differential prize spreads to achieve efficiency and fairness. We show that the organizer may have an incentive ex post to award the winning prize to some contestants rather than others.

What, if any, incentives exist for the principal when he has already committed to using fair tournaments? Consider a single efficient tournament where the organizer has perfect information and Proposition 2.1 is satisfied. The organizer has expected profits of

$$E\left[\Pi_1\right] = \sum_{i=1}^{n} \left\{ V\mu_i^o - \frac{1}{n}\left(W_i + (n-1)\,L_i\right) \right\}.$$

When the tournament is fair and efficient, the integral portion of the marginal probability of winning in equation (2.2.3) is symmetric for all agents $g^o = g_i^o = (n-1)\int z(\kappa_i)^2 Z(\kappa_i)^{xH-2}\, d\kappa_i$. Let $\bar{\alpha} = \frac{1}{n}\sum_{j=1}^{n}\alpha_j$ be the average variance factor. Using payoffs in equation (2.2.10), actual profits when agent i wins are

$$\Pi_1|_{i\,wins} = \sum_{j=1}^{n}\left\{V\mu_j^o - \underline{u}_j - a_j C(\mu^o)\right\} + \frac{V}{\theta g^o}(\bar{\alpha} - \alpha_i).$$

The summation is equivalent to expected profit. The term $\frac{V}{\theta g^o}(\bar{\alpha} - \alpha_i)$ is a result of organizing fair tournaments between agents with different shock distributions.

Proposition 2.4. *The organizer has an incentive to choose an agent with a below average variance factor according to* $\frac{\partial \Pi_i}{\partial \alpha_i} = -\frac{V}{\theta g^o}$*, and actual profits differ from expectations according to* $\Pi|_{i\,wins} = E[\Pi] + \frac{V}{\theta g^o}(\bar{\alpha} - \alpha_i)$*.*

The organizer is no longer indifferent to who wins. If the organizer has any control over who the winner might be in the contest, he has incentives to choose an agent with a relatively low variance based on the size of the spread according to the comparative static $\frac{\partial \Pi_i}{\partial \alpha_i} < 0$. For the proportional case $\alpha_i \propto a_i$, the firm has incentives to bias the tournament and promote from the low ranks of managers to avoid paying the larger wages associated with senior managers. The magnitude of this effect is increasing in the value of the marginal product, decreasing in the expected value of the multiplicative environmental condition, and increasing in the overall uncertainty level of the organization.

The organizer can create a hidden benefit when environmental conditions are favorable if the agents are naive or cannot negotiate. Propositions 2.2 and 2.3 provide the conditions for when common shocks can favor agents with low variance. When the expectation of additive shocks are positive $E[s] > 0$, agents with small relative variance gain an advantage in both probability of winning and overall ranking. The principal can increase profits by letting nature bias the outcome. Identifying the exact conditions when the principal can benefit from multiplicative environmental shocks is more complicated because the benefit of the bias effect depends on variance weighted by ability. Consider again the case when heterogeneity is proportional $\alpha_i \propto a_i$. The organizer has a higher probability of losing profits by overpaying those with the highest winning payoff when $E[\theta] > 1$ because the most productive and precise agents gain the advantage. In contrast, the principal gains an advantage in profits when $E[\theta] < 1$ because the relatively less precise, less capable, and least productive have an increased probability of winning.

2.6 Conclusion

A first best solution exists for background fairness and efficiency in heterogeneous tournaments under perfect information. When information costs are high for the principal, the second best solution depends greatly on the characteristics of the agents, the environmental conditions, and the verifiability of the principal's ranking process.

To be procedurally fair, a tournament must be designed to be fair ex ante and also insure the rank-order outcomes are not biased by environmental conditions ex post. We define background fairness and the fair criteria necessary to qualify a symmetric contest as fair, then use a linear rank rule to satisfy the conditions of symmetry between heterogeneous agents.

We identify a general rank rule in Proposition 2.1, that meets the fair criteria when agents are heterogeneous in ability and idiosyncratic uncertainty. The necessary assumption behind this result is perfect information about the agents as well as the two environmental shocks. When the perfect information assumption about the environment is relaxed, common shocks no longer affect agents symmetrically. The handicaps in the rank rule designed to achieve symmetry now bias the common shocks and can alter the rank order ex post when what is observed by the principal is different from the actual shock.

Under certain conditions, Proposition 2.2 states the marginal probability of winning is increasing in multiplicative common shocks when agents have a higher than average ability-variance ratio and decreasing when it is below average. We find that the advantage goes to the more capable and precise agents when the principal under reports positive environmental conditions. Yet, when he overestimates the positive shock, the advantage is reversed and now favors the less capable and imprecise agents.

A similar result is found in Proposition 2.3; the marginal probability of winning is increasing in additive common shocks when agents have a variance lower than the harmonic mean and decreasing when it is above the harmonic mean. Capability is not a factor. Agents who face lower overall variation in uncertainty will gain an advantage when the principal under reports positive environmental conditions and conversely, agents who face higher overall uncertainty will gain an advantage when negative environmental conditions are under reported.

When the principal can control the type of agents that enter the contest, the basic assumptions made about the agents dictate the use of a different rank rule. Not surprisingly, the basic homogeneous agent tournament as originally described by Lazear and Rosen (1981) meets the fair criteria and is captured in Corollary 2.1 to Proposition 2.1. When agents are heterogeneous in ability with uncertainty that is *i.i.d*, only an additive handicap is needed to make tournaments fair. However, as shown in Corollary 2.2, the multiplicative common shock can still bias the results if not perfectly monitored. When variance is proportional to ability, only the additive common shock need be monitored as captured in Corollary 2.3.

A final concern captured in Proposition 2.4 is that imperfectly monitored environmental conditions can create a hidden benefit for the principal when common shock bias increases the probability that someone from a low variance group wins. Industries where risk increases with output and that regularly experience negative environmental shocks that scale the effect of effort are particularly susceptible. Fair tournaments used in other industries that are subject to positive additive productive shocks on average also bias in favor of lower risk groups. It is the combination of a contest between groups of different levels of risk, imperfectly monitored environmental shocks, and the policy of a calculated fair rank rule that produce these effects.

Chapter 3

Fairness and workplace incentives: Evidence from a tournament experiment with heterogeneous agents

3.1 Introduction

Workers are inherently different. The challenge for the firm is how to motivate workers who are different using incentive contracts when the workplace is competitive. Employees, for instance, may desire that contracts provide equal opportunity regardless of worker differences. Managers may care because if they treat everyone who is different the same, workers may not produce efficient effort. If a less able worker is competing for a bonus with his colleagues and they are better than him in some way, he may not even try for the bonus; nor might he think it fair. Likewise, the worker who is endowed with some advantage over his colleagues, might perceive that attaining the bonus is a foregone conclusion and not work hard.

Managers are left to choose between overpaying less able workers or run the risk of losing top talent by underpaying them to maintain equality among employees. Managers who care about efficiency, morale, and retention may therefore seek a relative compensation scheme that generates efficient effort from all types of workers and provides equal opportunity at the same time. Using a tournament experiment, this chapter investigates contract options that vary in perceptions of equal opportunity by observing behavior when workers who are heterogeneous in both ability and the distribution of idiosyncratic shock uncertainty compete for high earnings.

An experimental setting provides the necessary control over the heterogeneous characteristics of the workers and specific contract treatments to study worker behavior that would not be practical in a real world setting. The results in this article are applicable to actual economic settings where the characteristics of the workers align with those in the study as well as in a general sense.

Worker heterogeneity is often lumped into a single characteristic—ability. But doing so ignores how differences in precision of execution, individual preparation, or differences in monitoring can asymmetrically effect the distributions of worker uncertainty. Similar to differences in ability, if managers treat workers with different random shock distributions the same, they will not produce efficient effort. The advantage goes to the worker who in comparison to his colleagues is not monitored as closely. The increased spread in his shock uncertainty makes it less likely he will win if his shock is negative, but this is more than offset by his increased probability of winning if his shock is positive. Similarly, a worker endowed with a more precise production process than his equally able peers stands less of a chance of winning the bonus and may become discouraged, all the while a colleague with low precision can slack off expecting to catch a big break. Work groups comprised of complex combinations in ability and uncertainty of random shocks confounds the efficiency and equity tradeoff even more.

To capture these differences in the experiment, we characterize five work groups based on their differences. Unique to this study, two groups include members heterogeneous in both ability and uncertainty. The first of these captures the natural correlation that can occur when ability and uncertainty scale proportionally. Broiler tournaments where uncertainty scales with capacity are a widely known example. The second one captures the correlation that can occur when ability and precision scale proportionally, such as the composition of workers found in service industries or sports competitions like golf where top performers are also characterized with relatively high precision. The other three groups include a homogeneous benchmark, a group heterogeneous in ability only, and a group heterogeneous in uncertainty only. The last group describes a bonus contest between equally able fund managers who manage portfolios with different risk profiles.

When a firm seeks an incentive mechanism that optimizes effort from all workers, what form of equality is most relevant? Equal opportunity in one sense means *equal access*. Programs of affirmative action provide minority groups access to the competitive workplace and the opportunities for the same bonuses and advancement that reside within. However, equal opportunity can also refer to *equal pay* - the same pay for the same work, regardless of worker differences. Even so, in a relative compensation setting characterized by uncertainty in outcomes where many compete but not every worker earns a bonus or promotion, equal pay in final earnings is by its very nature antithetical. In what way, then, can relative compensation contests be considered fair? Workers may consider the differences they receive in distributions to be fair so long as the contest itself, the process by which colleagues are judged, is considered fair. Within the context of procedural fairness, equal pay may take on the form of incentive contracts that equalize expected earnings, regardless of employee differences. In this way, workers enter the contest with equal expectations of outcomes and are more willing to accept differences in actual earnings as a result of competition. Contracts in an equal pay setting are therefore unique to the individual to compensate for heterogeneity and deliver equal expected earnings.

A third perception of equal opportunity relevant in contest settings is an *equal chance* to win. Holding all else constant, the worker with the largest variance in random shocks will have the highest probability of winning. Recall that one argument for equal access is that workers from minority groups have just as much to offer as those in majority groups, but only need access to the same arena. But innate differences between groups in terms of access to education, life long experiences, or preparation may provide an advantage to one worker over another, giving them a higher probability of winning. Likewise, a worker that has been in the workforce longer will have gained valuable experience, have stronger networks and methods that increase his/her probability of winning the promotion or bonus in spite of an incentive contract that offers equal pay. If a fair chance is also important to workers, should the firm seek a contract that can deliver efficiency and all three forms of equality?

To investigate how important equal access, equal pay, and equal chances are to competitive workers, we define three contract types. The first is a homogenous contract that does not vary across agents and as such is not theoretically optimal for the principal. The second and third contracts are made to be optimal and use handicaps to establish procedural fairness (i.e. equate expected earnings across agents). The third contract treatment is strategically symmetric and is distinguished by the fact that it equalizes the probability of winning as well as equating expected earnings.

To explore the importance of equal access, equal pay, and equal chance, we develop a novel linear rank mechanism that allows the firm to compensate for heterogeneous ability and uncertainty. The mechanism allows the firm to choose between different notions of equality by varying a handicap and payoff structure. A firm that can employ individual offset handicaps and has the means to adjust the prize spread and prize levels individually can induce efficiency in a tournament heterogeneous in ability and uncertainty and achieve equal expected payoffs. However, there are insufficient degrees of freedom to also equalize the probability of winning across agents. When equal chance is also desired, the multiplicative handicap provides an additional degree of freedom to accommodate differences in uncertainty.

The main hypotheses of the experiment are that equal pay increases the perception of procedural fairness over the distributional fairness concept of equal access, and that equal chance increases the perception of procedural fairness even further. Additionally, that this result holds for many different group compositions so long as the contest is made strategically symmetric across agents.

The identification method is to observe and compare effort choices, solicit role preferences, contract preferences, and assess fairness. The experimental design is to randomly assign subjects into roles (types) that are endowed with different levels of ability and shock uncertainty, then assign them into groups composed of 2 other agents of different types. In a single session, each group plays two of the four equal opportunity tournament contracts specified in the experiment. After a subject gains experience in a contract from 20 round of play, he is asked to state his preferences for roles. After both contracts are played, a subject is asked to vote for which contract he would prefer to play again. In a group, the contract choice one of the three players is randomly selected and the group plays 5 more rounds of the chosen contract.

Results of the experiment in both effort and contract choice observed at the firm (pooled) level and the individual group level appear neutral even though individual effort contributions and contract choices are not. The general result is that contracts that promote equal pay in the form of equal expected outcomes promote efficiency as well as greater perceptions of fairness than do contracts that promote equal access.

Additionally, the effort performance and contract choices made by the group of agents heterogeneous in ability conform to theory. As expected, both the low ability and medium ability types prefer equal pay over equal access and the high ability type prefers equal access over equal pay. Moreover, we show that fairness preferences are transitive for this group: a homogeneous contract is considered more fair than an equal pay contract which is considered more fair than an equal access contract at the group level. However, introducing complex combinations of ability and uncertainty spoils these results.

The choices made by the individual worker types, for the most part, reflect a preference for having a strategic advantage. As expected, except in one case of indifference, all of the types that had a strategic advantaged in the groups selected equal access over either equal pay or equal chance nearly unanimously. At the same time, those types that had a strategic disadvantage chose exactly the opposite nearly unanimously with only three cases of indifference. Contradictions to our predictions were found in the group heterogeneous in uncertainty. Those with a strategic disadvantage, low uncertainty and medium uncertainty types, chose equal access over equal pay while the advantaged high types were indifferent between all three contracts, suggesting that the advantages that come from differences in uncertainty are less salient than the advantages that come from differences in ability.

Also striking are the results from the fairness solicitation. All of the advantaged types across all the groups considered the equal access contract to be just as fair as the equal pay and equal chance contract. However, most disadvantaged types felt that either equal pay or equal chance were more fair than equal access, supporting the premise that fairness is a matter of perspective.

3.1.1 Literature Review

One of the contributions to the experimental literature is the introduction of a linear rank rule that combines the offset handicap and multiplicative handicap into the same model. Handicaps are a common mechanism to adjust for different ability in tournaments. Lazear and Rosen (1981) originated the idea by introducing the offset handicap and O'Keeffe et al. (1984) improved on the theory and show how offsets can be used to achieve efficiency among heterogeneous agents and how equal expected payoffs can be achieve by adjusting the prize levels individually for each agent. Bhattacharya, Sudipto and Guasch (1988) develop a form of multiplicative handicap used to compare workers across hierarchies of different ability. They also point to a need for more sophisticated models to account for differences in shocks correlated across ability levels. Gürtler and Kräkel (2010) show how individual prize levels can be used in leu of handicaps to make the contest less uneven by shifting the winning prize for the more able worker below that of the less able worker.

Less common in the literature is theory that considers handicaps in tournaments between agents heterogeneous in ability and non-identically distributed shocks. O'Keeffe et al. (1984) and Bhattacharya, Sudipto and Guasch (1988) broach the topic, but do not describe a tournament model with many agents. Chapter 2 in this dissertation describes a tournament model that accounts for heterogeneity in both ability and shock uncertainty and a linear rank rule to compensate for agent differences. The model of Chapter 2 is adopted here and developed for the purpose of the experiment.

Several experiments including Bull et al. (1987),Schotter and Weigelt (1992), Dijk et al. (2001), and Harbring and Lunser (2008) test tournament model predictions with agents heterogeneous in ability. Results of these studies generally support the theory with the exception that low ability agents are found to systematically work more than predicted. The overworking low ability agent is also found in our study. The use of handicaps is less frequent. Experiments that evaluate inequalities using some form of offset handicap include Schotter and Weigelt (1992) and Orrison et al. (2004) who use an offset to describe discrimination and Gürtler and Harbring (2010) who investigate head-starts in contests. Related to the concern for equal pay is Gneezy et al. (2003) who compare the competitive performance of men and women in a gender study.

The experiment most closely related to ours is Wu et al. (2006) who elicit WTP between playing two agent tournaments versus piece rate contracts by comparing bids in a sealed-bid auction after an initial training period. Using the distributional fairness inequity model of Fehr and Schmidt (1999), they show that when subjects are disadvantaged by tournaments, they are willing to pay more to avoid them. Our study differs from Wu et al. by eliciting contract choice in an ex-ante procedural framework (prior to a third stage) and asking a subject to rate the fairness of tournament mechanisms and preferences for different roles.

Hammond and Zheng (2013) model both additive and multiplicative production effects in both ordinal and cardinal tournament models in a real effort experiment. Harbring and Irlenbusch (2005) and more recently Dechenaux et al. (2012) provide a comprehensive summary of tournaments in the experimental literature. The relevant literature Dechenaux et al. review covers handicaps and affirmative action in tournaments and finds that the disadvantaged benefit from affirmative action. Our investigation differs from these experiments on several levels. First, these studies are building a consensus that current models of fairness, risk, and competition in experiments are incomplete and fall short of a comprehensive explanation of ex ante and ex post concerns. This article is the first experimental investigation of behavior when agents are heterogeneous in shock uncertainty, including settings where the shock uncertainty is both positively and negatively correlated with ability differences. Also, this study is the first to investigate the effectiveness of multiplicative handicaps in a tournament experiment.

Most advances in behavioral models of fairness have come in the form of distributive fairness such as Fehr and Schmidt (1999) and Bolton and Ockenfels (2000). Grund and Sliwka (2005) develops a tournament model around Fehr and Schmidt (1999) to show that inequity averse agents produce less effort than self-interested ones. Alternatively, Krawczyk (2009) proposes a model of the interaction of procedural and distributional fairness. He points out that the distributional preferences models of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) only focus on the other regarding preferences of distributional outcomes and disregard the way in which those outcomes came about. He expands the Bolton and Ockenfels model to include differences in expected payoff and average expected payoff as a proxy for perceived procedural fairness. We are testing a different channel of procedural fairness than Krawczyk. His model uses differences in the expected payoffs as a proxy for procedural fairness and does not consider uneven background differences as a source of unrest.

More recent, Saito (2013) develops the expected inequality-averse model that builds on the guilt and envy model of Fehr and Schmidt with preferences for equality of opportunity. The Saito model captures the preferences between expected outcome and distributional outcomes but requires that the agent be inequality averse. For the experiment in this article, efficiency and expected payoff are held constant for two of the contracts. The variation in contracts in our study are derived from compensating for background differences between agents; hence the perceived fairness between contracts comes from comparing probabilities of winning, prize levels, prizes spreads, and handicaps across agent types. Therefore, the Krawczyk and Saito models may not be well suited to capture the nature of procedural fairness we seek to uncover in these contracts.

The next section introduces the theoretical model. Section 3.3 describes the experimental design. Section 3.4 contains the results and Section 3.5 provides concluding remarks. Appendix C contains documents used in the experiment including screen shots.

3.2 Theoretical Model

A risk neutral, profit-maximizing firm employs n heterogeneous agents in production. An agent's output $q_i = \mu_i + \varepsilon_i$ is a linear combination of his effort μ_i and an idiosyncratic shock $\varepsilon_i \sim i.n.i.d\,(0, \sigma_i^2)$. The distribution of ε_i is independent but not identically distributed with other agents; is unimodal and symmetric about zero. Because of the uncertainty introduced by the shock, only q_i is verifiable at the end of the work period. The variation in distributional spread between agents represents the innate differences in uncertainty that come from individual preparation, work environment, or the measurement error in the firm's assessment of performance.

In addition to differences in uncertainty, an agent is endowed with idiosyncratic ability a_i that affects his cost of performance. Effort is costly for an agent and is denoted by $c(\mu_i, a_i)$, where $c_\mu, c_{\mu\mu} > 0$ and $c(0), c_\mu(0) = 0$. As ability increases, the marginal cost of effort is decreasing such that for any given level of effort, agents with higher ability have a lower cost and lower marginal cost to effort. Cost functions are known such that the ability of each agent can be inferred.

The firm earns total revenue from the output of all agents $\sum_{i=1}^{n} = Vq_i$ where V is the marginal product of output. The firm would like to compensate agents based on actual performance, but output does not accurately represent agent effort due to uncertainty. Unverifiable effort creates a moral hazard for the firm that is solved by the rank-order winner-take-all tournament model of Lazear and Rosen (1981).

The firm offers a winning payoff W_i and losing payoff L_i to each agent. When agents are homogeneous, Lazear and Rosen show how a common payoff $W_i = W_H$ and $L_i = L_H$ for all i are sufficient to reach efficiency, but that offering this same contract to agents heterogeneous in ability leads to inefficient effort.

When agents are risk-neutral, the inefficiency can be overcome by adjusting the spread of the individual payoffs, and the levels can be adjusted to equalize expected earnings, but more flexibility is needed in the contract to equalize the chances of winning. This is important when crafting incentives in practical settings. When only payoff adjustments are used, the low ability agent may have a probability of winning so close to zero that he will have to be offered an exorbitant winning payoff just to participate. Conversely, the high ability agent endowed with a probability of winning very close to one may need to be threatened with bankruptcy if he loses. This type of contract may be impractical and constrained in limited liability settings where firing the top agent is the worse punishment that can be delivered. The model we develop introduces two additional degrees of freedom to overcome these problems.

To facilitate equal opportunity and equal probability, the firm ranks agents by a linear rule $r_i(\mu_i, \phi_i, h_i) = \phi_i q_i(\mu_i) + h_i$ where ϕ_i is a multiplicative handicap and h_i is an offset handicap, both used to adjust for differences in ability and uncertainty distributions. Agent rank is the only determinant of earnings and is decoupled from actual effort. Individual payoffs are distributed to agents according to ordinal rank

$$Payoff_i = \begin{cases} W_i & if \qquad r_i > r_j \qquad for\ all\ j \neq i \\ L_i & if \qquad r_i < r_j \qquad for\ at\ least\ one\ j \neq i \end{cases}.$$

Let f_j be the probability density function and let F_j be the cumulative distribution function of ε_j for all j. As a function of effort, ability and standard deviation, the probability agent i ranks higher than j is $F_j = prob\left(\varepsilon_j(\sigma_j) < \frac{1}{\phi_j}\left(-\phi_j \mu_j(a_j) - h_j + \phi_i \mu_i(a_i) + \phi_i \varepsilon_i(\sigma_i) + h_i\right)\right)$. Let μ, ϕ, and h be the set of effort, multiplicative handicaps, and offset handicaps for all agents respectively. In the contest, agent $i's$ probability of being ranked higher than all other agents $j \neq i$ can be written

$$P_i(\mu, \phi, h) = \int f_i(\varepsilon_i) \prod_{j \neq i}^{n-1} F_j\left(\varepsilon_j - \frac{1}{\phi_j}\left(r_j(\mu_j, \phi_j, h_j) - r_i(\mu_i, \phi_i, h_i)\right)\right) d\varepsilon_i. \quad (3.2.1)$$

The marginal probability of effort for i is

$$g_i(\mu, \phi, h) = \frac{\partial P_i(\mu, \phi, h)}{\partial \mu_i}$$

$$= \int f_i(\varepsilon_i) \sum_{j \neq i}^{n-1} \left\{ \frac{\phi_i}{\phi_j} f_j \left(\varepsilon_j - \frac{1}{\phi_j} \left(r_j(\mu_j, \phi_j, h_j) - r_i(\mu_i, \phi_i, h_i) \right) \right) \right.$$

$$\left. \times \prod_{k \neq j \neq i}^{n-2} F_k \left(\varepsilon_k - \frac{1}{\phi_k} \left(r_k(\mu_k, \phi_k, h_k) - r_i(\mu_i, \phi_i, h_i) \right) \right) \right\} d\varepsilon_i$$

and $g_i > 0$ such that the probability of winning is increasing in effort.

The firm will pay one agent his winning payoff W_i with a probability P_i, otherwise he will be paid L_i. Summing up all of the revenue and expected payoffs for the agents, expected profit for the firm is described by

$$\pi = \sum_{i=1}^{n} \left\{ V q_i(\mu_i) - P_i(\mu, \phi, h) W_i - (1 - P_i(\mu, \phi, h) L_i) \right\}.$$

The firm solves the principal-agent problem by backward induction, evaluating the incentives of the agent to choose a contract $(W_i, L_i, r_i(\phi_i, h_i))$ that will deliver optimal effort μ_i^* while holding all agents indifferent to outside opportunities X.

Agent i maximizes his expected utility choosing effort according to

$$\underset{\mu_i}{max} \quad u_i = P_i(\mu, \phi, h) W_i + (1 - P_i(\mu, \phi, h)) L_i - c(\mu_i, a_i). \qquad (3.2.2)$$

Using Nash-Cournot assumptions, the best response function for i that implicitly describes the Nash equilibrium given that all other $n-1$ agents j are playing optimal strategies is

$$S_i = W_i - L_i = \frac{c^{'}(\mu_i^*)}{g_i(\mu^*, \phi, h)}, \tag{3.2.3}$$

where S_i is the payoff spread and the non-subscripted μ^* is the set of optimal effort levels of all agents. The outcome of the contest is efficient when $V = c^{'}(\mu_i^*, a_i)$ in equation (3.2.3) for all i.

The firm can choose *equal access* by offering all agents the same contract $(W, L, r_i(1,0))$ by setting $\phi_i = 1$ and $h_i = 0$, and $S = S_i = W - L$ for all i. However, the firm cannot achieve efficiency with this contract because at efficient effort, $g_i(\mu_i^*, 1, 0) \neq \frac{S}{V}$ in equation (3.2.3) for all heterogeneous agents i.[1]

The contract preference of *equal expected earnings* is achieved by holding agent i indifferent to outside opportunities X_i in equation (3.2.2) at the Nash equilibrium, and satisfying the optimum strategies in equation (3.2.3), the individual payoff functions based on optimal effort μ_i^* and the handicaps are

$$\begin{array}{c} W_i(\mu^*, \phi, h) = X_i + c(\mu_i^*, a_i) + (1 - P_i(\mu^*, \phi, h)) S_i(\mu^*, \phi, h) \\ L_i(\mu^*, \phi, h) = X_i + c(\mu_i^*, a_i) - P_i(\mu^*, \phi, h) S_i(\mu^*, \phi, h) \end{array}. \tag{3.2.4}$$

Individual contracts $\left(W_i(\mu^*, \phi, h), L_i(\mu^*, \phi, h), r_i(\phi_i, h_i)\right)|_{V=c^{'}_{\mu}}$ will deliver the efficient outcome when $V = c^{'}_{\mu}$.[2]

The contract preference of *equal chance* is achieved by selecting the set of ϕ and

[1] For equal access, $g_i(\mu_i^*, 1, 0) = \int f_i(\varepsilon_i) \sum_{j\neq i}^{n-1} f_j\left(\mu_i^* - \mu_j^* + \varepsilon_i\right) \prod_{k\neq j\neq i}^{n-2} F_k\left(\mu_i^* - \mu_k^* + \varepsilon_i\right) d\varepsilon_i$ and is unique for different abilities and standard deviations. This is the same for equal expected earnings with no handicaps $\phi_i = 1$, and $h_i = 0$ for all i.

[2] For equal expected earnings using h only, $g_i(\mu_i^*, 1, h) = \int f_i(\varepsilon_i) \sum_{j\neq i}^{n-1} f_j(\varepsilon_i) \prod_{k\neq j\neq i}^{n-2} F_k(\varepsilon_i)\, d\varepsilon_i$ and is the same for *i.i.d.* distributions but is unique when distributions are *i.n.i.d.* at the efficient equilibrium and $\phi_i = 1$ for all i.

h in the rank rule so that $P_i = \frac{1}{n}$ for all i. One solution to equalize probabilities and simultaneously yield efficient effort, handicapped or otherwise, is to set the expectation of agent rank the same for all agents at the Nash equilibrium. Let r be a target adjusted output for all agents in equilibrium. Equating the target adjusted output to the expectation of the rank rule $E[r_i] = r$, the offset handicap for any agent can be calculated as $h_i = r - \phi_i(\sigma_i)\mu_i^*(a_i)$. When distributions are *i.i.d.*, setting $\phi_i = 1$ and $r = 0$ so that $h_i = -\mu_i^*$ is sufficient to equalize effort across agents and achieve $P_i = \frac{1}{n}$. However, when the distributions of the idiosyncratic shocks are heterogeneous, agents endowed with greater variance gain an advantage in equation (3.2.1) for contests where $n > 2$ (see Equal Variance proof in Chapter 2). To overcome this, the multiplicative handicap can be used to equalize the variance of the distributions. The distributions are normalized by setting $\phi_i = \frac{1}{\sigma_i}$ so that $Dist(\phi_j\varepsilon_j(\sigma_j)) = Dist(\phi_i\varepsilon_i(\sigma_i))$ for all $j \neq i$. With both handicaps and μ_i^*, $F_j = prob\left(\varepsilon_j(\sigma_j) < \frac{\sigma_j}{\sigma_i}\varepsilon_i(\sigma_i)\right) = \frac{1}{2}$ for any two agents.[3] The ratio $\frac{\phi_i}{\phi_j} = \frac{\sigma_j}{\sigma_i}$ is equal to the square root of the variance ratio between i and j.

3.3 Experimental Design

The main objectives of the equal opportunity experiment are to test if the competitive contract of equal pay (equal expected earnings) increases the perception of fairness over the contract of equal access (identical payoffs) and if the contract of equal chance (equal expected earnings and equal probability of winning) increases the perception of

[3] For equal probability, $g_i\left(\mu_i^*, \frac{1}{\sigma_i}, h_i\right) = \int f_i(\varepsilon_i)\sum_{j\neq i}^{n-1}\frac{\sigma_j}{\sigma_i}f_j\left(\frac{\sigma_j}{\sigma_i}\varepsilon_i\right)\prod_{k\neq j\neq i}^{n-2}F_k\left(\frac{\sigma_k}{\sigma_i}\varepsilon_i\right)d\varepsilon_i$ and is unique for different abilities and standard deviations at the efficient equilibrium where $h_i = r - \frac{\mu_i^*}{\sigma_i}$. The incentive for i derived from the marginal probability g_i is identical to what he would experience if he were to play a homogeneous tournament against members of his peer group at equilibrium.

fairness over the contract of equal pay for many different group compositions. We do this by testing the effort decisions made by agents and the choices they make between playing different tournament contracts.

3.3.1 Player Types and Functional Relationships

The experiment focuses on tournaments with groups of $n = 3$ agents $\{i, j, k\}$. Three ability levels are defined: low a_L, medium a_M, and high a_H. Ability levels vary within a group to better understand the effects that may arise from having the perception of being very far ahead or very far behind or in-between. Three standard deviations are defined: low σ_L, medium σ_M, and high σ_H. Uncertainty levels vary within a group to address the advantages and disadvantages that more uncertainty brings to some agents, but not others.

Individual contracts $(W_i, L_i, r_i^* (\phi_i, h_i))$ are calculated by solving equation (3.2.3) for optimum effort μ_i^* based on the group composition and contract objective where $r_i^* (\phi_i, h_i) = \phi_i \mu_i^* + \phi_i \varepsilon_i + h_i$.

Idiosyncratic uncertainty is modeled as a uniform distribution. The pdf of the distribution in terms of rank rule and standard deviation for the opponents of i at the Nash equilibrium is

$$f_j(\varepsilon_j) = \begin{cases} \frac{1}{2\sqrt{3}\sigma_j} & for \quad -\sqrt{3}\sigma_j \leq \varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) \leq \sqrt{3}\sigma_j \\ 0 & otherwise \end{cases} \qquad \forall\, j \neq i.$$

Likewise, the cdf at the Nash equilibrium for all $j \neq i$ is

$$F_j(\varepsilon_j) = \begin{cases} 0 & for \quad \varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) \leq -\sqrt{3}\sigma_j \\ \frac{1}{2\sqrt{3}\sigma_j}\left(\varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) + \sqrt{3}\sigma_j\right) & for \quad -\sqrt{3}\sigma_j \leq \varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) \leq \sqrt{3}\sigma_j \\ 1 & for \quad \sqrt{3}\sigma_j \leq \varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) \end{cases}$$

The mean of the distribution for j relative to the mean of i is determined by $\frac{\phi_i}{\phi_j}\mu_i^* - \mu_j^* + \frac{\phi_i}{\phi_j}\varepsilon_i - \frac{1}{\phi_j}(h_i - h_j)$. Let I be an activation term for $f_i(\varepsilon_i)$ and $F_i(\varepsilon_i)$, let J be an activation term for $f_j(\varepsilon_j)$ and $F_j(\varepsilon_j)$, and let K be an activation term for $f_k(\varepsilon_k)$ and $F_k(\varepsilon_k)$ that correspond to the intervals described by the uniform distribution.

The probability agent i wins at the Nash equilibrium is

$$P_i = \int \left(\frac{1}{2\sqrt{3}\sigma_i}\right)^I \left(\frac{1}{2\sqrt{3}\sigma_j}\left(\varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) + \sqrt{3}\sigma_j\right)\right)^J$$

$$\times \left(\frac{1}{2\sqrt{3}\sigma_k}\left(\varepsilon_k - \frac{1}{\phi_k}\left(r_k^* - r_i^*\right) + \sqrt{3}\sigma_k\right)\right)^K d\varepsilon_i.$$

The marginal probability of effort at the Nash equilibrium is

$$g_i = \int \left(\frac{1}{2\sqrt{3}\sigma_i}\right)^I \left\{\left(\frac{\phi_i}{\phi_j 2\sqrt{3}\sigma_j}\right)^J \left(\frac{1}{2\sqrt{3}\sigma_k}\left(\varepsilon_k - \frac{1}{\phi_k}\left(r_k^* - r_i^*\right) + \sqrt{3}\sigma_k\right)\right)^K\right.$$

$$\left. \times \left(\frac{\phi_i}{\phi_k 2\sqrt{3}\sigma_k}\right)^K \left(\frac{1}{2\sqrt{3}\sigma_j}\left(\varepsilon_j - \frac{1}{\phi_j}\left(r_j^* - r_i^*\right) + \sqrt{3}\sigma_j\right)\right)^J\right\} d\varepsilon_i.$$

Although the distributions are uniform and finite, the probability functions are in-

Table 3.1: Fundamental Functional Parameters

Description	**Parameter**	**Parameter Values**
Cost exponent	k	4
Marginal product	V	32
Low ability	a_L	10
Medium ability	a_M	15
High ability	a_H	30
Low standard deviation	σ_L	13.53175
Medium standard deviation	σ_M	20.29763
High standard deviation	σ_H	40.59525
Opportunity Cost	X	2000 lab dollars

definite integrals. With heterogeneous ability and uncertainty, the widths of the distributions vary as well as the means at the Nash equilibrium causing discontinuities in the probability functions; hence, the integrals are calculated by intervals and programmed using visual basic to automate contract parameter selection.

We define a convex exponential cost function $c\left(\mu_i, a_i\right) = a_i \left(\frac{\mu_i}{a_i}\right)^k$ and $c^{'}\left(\mu_i, a_i\right) = k\left(\frac{\mu_i}{a_i}\right)^{k-1}$ where the parameter k is common to all agents. Using the relationship $V = c^{'}\left(\mu_i^*\right)$, efficient effort is calculated $\mu_i^* = a_i\left(\frac{V}{k}\right)^{\frac{1}{k-1}}$.

A common fixed payment X was included in each payoff function. Agent High Payment and Low Payments are calculated according to

$$\begin{array}{c} W_i = X + a_i\left(\frac{\mu_i^*}{a_i}\right)^k + \left(1 - P_i\right)\frac{V}{g_i} \\ L_i = X + a_i\left(\frac{\mu_i^*}{a_i}\right)^k - P_i\frac{V}{g_i} \end{array}.$$

The fundamental parameters used in the experiment are shown in Table 3.1.

3.3.2 Group Compositions and Contracts

We define five competitive groups $\{g_0, g_1, g_2, g_3, g_4\}$, composed of 3 agents each. The composition of all five groups is shown in Table 3.2. As a benchmark, the homogeneous group g_0 is formed where agents are identical in ability a_M and uncertainty σ_M described by $i_0(a_M, \sigma_M)$, $j_0(a_M, \sigma_M)$, and $k_0(a_M, \sigma_M)$. Group g_1 comprises the most common form of heterogeneity modeled in the literature where agents vary in ability only $i_1(a_L, \sigma_M)$, $j_1(a_M, \sigma_M)$, and $k_1(a_H, \sigma_M)$. Less common, agents within group g_2 vary only by uncertainty $i_2(a_M, \sigma_L)$, $j_2(a_M, \sigma_M)$, and $k_2(a_M, \sigma_H)$.

Table 3.2: Group compositions of heterogeneous backgrounds

Group	**Composition**	**Agent i^***	**Agent j^***	**Agent k^***
0	Homogeneous	$i_0\,(a_M, \sigma_M)$	$j_0\,(a_M, \sigma_M)$	$k_o\,(a_M, \sigma_M)$
1	Heterogeneous Ability	$i_1\,(a_L, \sigma_M)$	$j_1\,(a_M, \sigma_M)$	$k_1\,(a_H, \sigma_M)$
2	Heterogeneous Uncertainty	$i_2\,(a_M, \sigma_L)$	$j_2\,(a_M, \sigma_M)$	$k_2\,(a_M, \sigma_H)$
3	Ability $\propto$ Uncertainty	$i_3\,(a_L, \sigma_L)$	$j_3\,(a_M, \sigma_M)$	$k_3\,(a_H, \sigma_H)$
4	Ability $\propto$ Precision	$i_4\,(a_L, \sigma_H)$	$j_4\,(a_M, \sigma_M)$	$k_4\,(a_H, \sigma_L)$

* *Ability:* $\{a_L, a_M, a_H\} = \{10, 15, 30\}$, *Std. Dev.:* $\{\sigma_L, \sigma_M, \sigma_H\} = \{13.5, 20.3, 40.6\}$

Groups g_3 and g_4 represent previously unexplored cases in the experimental literature in that agents vary in both dimensions of ability and variance. Group g_3 models the composition of agents typically found in the production of goods where the ability to produce a given level of output is positively correlated with the variance of idiosyncratic uncertainty by $i_3(a_L, \sigma_L)$, $j_3(a_M, \sigma_M)$, and $k_3(a_H, \sigma_H)$. Broiler tournaments are a widely known example. Group g_4 depicts the composition of agents found in service industries or sports competitions such as golf where top performers are also characterized with relatively high precision $i_4(a_L, \sigma_H)$, $j_4(a_M, \sigma_M)$, and $k_4(a_H, \sigma_L)$.

When agents are heterogeneous, relative compensation contracts must account for these differences to achieve an efficient outcome. To better understand why this is

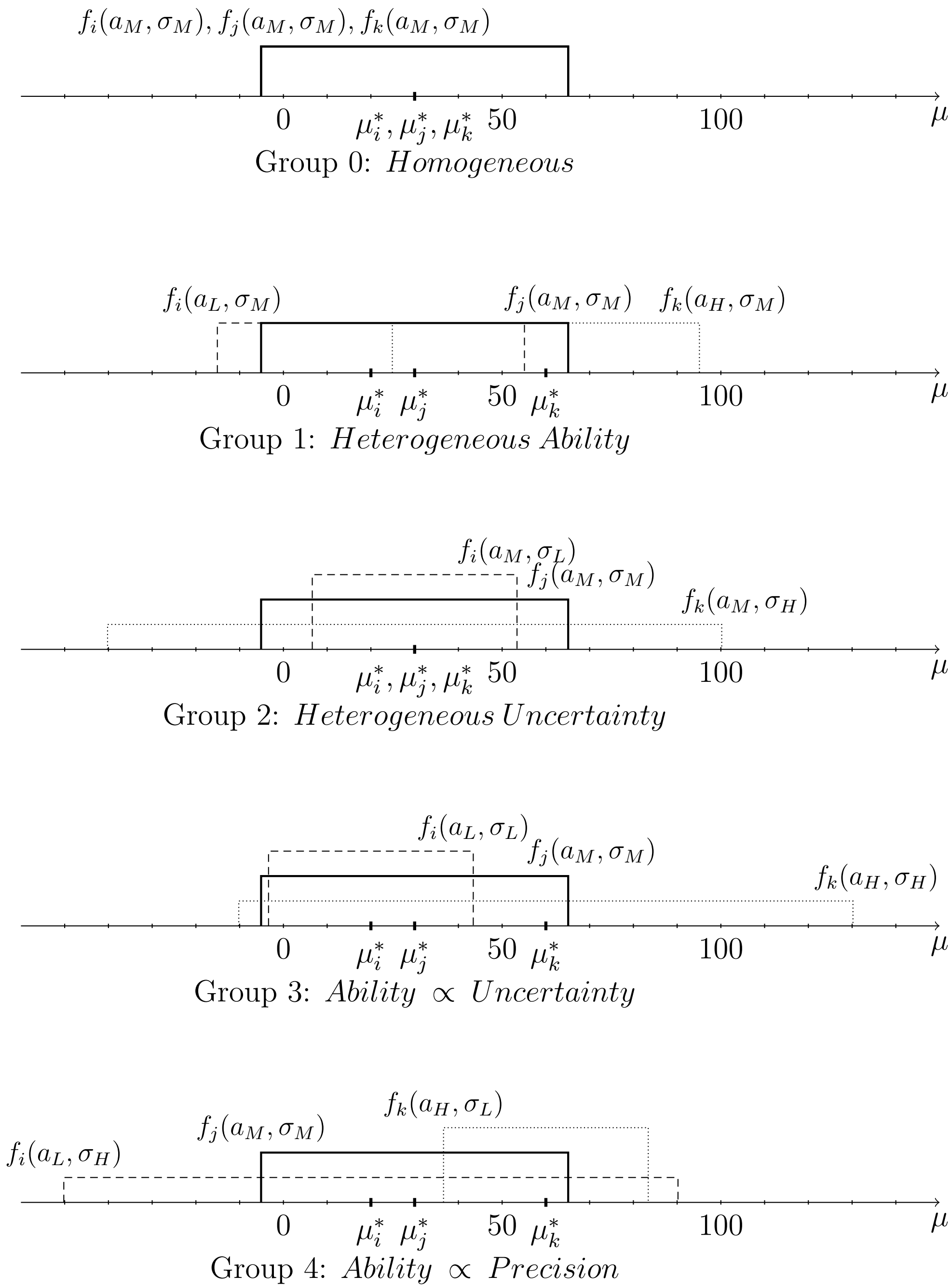

Figure 3.1: Group density graphs for socially optimal strategies

so, the group compositions described in Table 3.2 are graphed in Figure 3.1 at the efficient Nash equilibrium. The uniform density functions f overlap in equilibrium for each group setting. The mean of each density is centered at the efficient Nash equilibrium effort level chosen by the agent and the density limits are $\pm\sqrt{3}\sigma_v$ where $v \in \{L, M, H\}$.

The parameter values are selected to satisfy several design constraints. As shown in Table 3.1, ability levels in the experiment have the following relationship $3a_L = 2a_M = a_H$. Likewise, standard deviations in the experiment have the following relationship $3\sigma_L = 2\sigma_M = \sigma_H$. The ratios are kept the same so that ability and uncertainty scale proportionally in g_3 and inversely proportional in g_4 in order to examine how well multiplicative handicaps perform independent of offset handicaps. Abilities are chosen so that strategies in equilibrium that yield efficient effort were sufficiently separated in the decision interval $[0, 100]$. As depicted in Figure 3.1, the ability levels $\{a_L, a_M, a_H\} = \{10, 15, 30\}$ generate effort levels $\left\{\mu_i^*, \mu_j^*, \mu_k^*\right\} = \{20, 30, 60\}$ when agents work efficiently. Second, the standard deviations were made just large enough to satisfy the second order conditions and the global participation constraint so that all tournaments have a theoretical Nash equilibrium $\{\sigma_L, \sigma_M, \sigma_H\} = \{13.5, 20.3, 40.6\}$.

Table 3.3: Tournament contracts

Contract	**Groups**	**Description**	**Efficient**	ϕ	h	**Payoffs**
H	Homogeneous, g_0	Homogeneous	Yes	No	No	Identical*
A	g_1, g_2, g_3, g_4	Equal access to opportunity	No	No	No	Identical*
B	g_1, g_2, g_3, g_4	Equal expected earnings, $E[u]$	Yes	No	Yes	Individual
C	g_2, g_3, g_4	Equal $E[u]$ and chance to win	Yes	Yes	Yes	Individual

* *The payoffs are the same for all types and for all groups, including the homogeneous group*

To study concepts of fairness and efficiency, we evaluate the performance of the groups and agents in three contracts and one benchmark. The tournament contracts are listed in Table 3.3. The first and second contracts do not use handicaps. The third contract incorporates offset handicaps $h = \{h_i, h_j, h_k\}$ to induce efficient effort $\mu^* = \{\mu_i^*, \mu_j^*, \mu_k^*\}$. The fourth contract employs both offset and multiplicative handicaps $\phi = \{\phi_i, \phi_j, \phi_k\}$.

Contract H is a benchmark case and exhibits all the properties of fairness: equal access to opportunity, equal expected earnings, and equal chance to win. H is designed as an incentive for agents in the homogeneous group g_0 to work efficiently. Since g_0 is homogeneous, handicaps are unnecessary, the probability of winning for each agent is $P = \frac{1}{3}$, and expected earnings are equal to the fixed payment X.

In Contract A, the firm offers agents in the heterogeneous groups equality in pay structure with no handicaps $\phi = \{1, 1, 1\}$ and $h = \{0, 0, 0\}$. The contract is identical to the payoffs offered in Contract H (i.e. all agents in groups g_1, g_2, g_3, g_4 are offered the same winning and losing payoffs). A contest offering equal payoffs is common practice in labor settings; however, because agents are heterogeneous, the best response functions lead to non-uniform strategies, unequal probabilities of winning, and lower than efficient effort. Therefore, expected earnings are unique and no longer equal to X. The concept of fairness in Contract A is equal access of opportunity; even though agents are different, they all have access to the same contest. Differences in ability and uncertainty are ignored in policy.

In Contract B, the firm incorporates only offset handicaps to provide the incentive for all agents to perform efficiently. The Multiplicative handicaps are not used, $\phi = \{1, 1, 1\}$. All agents in Contract B have fairness in expected earnings equal to X. The offset handicap provides the firm with an extra degree of freedom to create the proper incentives for agents to generate efficient effort. Offset handicaps are set $h = \{40, 20, 0\}$ to compensate for differences in expected efficient effort when

$\mu_i^* \neq \mu_j^* \neq \mu_k^*$ in groups g_1, g_3, and g_4. Since ability is the same for agents in group g_2, handicaps are unnecessary $h = \{0, 0, 0\}$ to adjust for differences in effort. To equate expected earnings to X, individual payoff contracts are offered to each agent by adjusting winning and losing payoffs based on individual attributes of ability and uncertainty. The firm does not have enough degrees of freedom in Contract B to equate the probability of winning between agents in g_2, g_3, and g_4 because the offset handicap is not sufficient to compensate for the differences in uncertainty. However, since uncertainty in g_1 is *i.i.d.*, agents have the same probability of winning for this one case in Contract B.

In the final Contract C, the firm induces efficiency, equalizes expected outcomes, and equalizes the probability of winning. This mechanism is unique in the literature and uses a combination of multiplier and offset handicaps as needed to achieve the firm's design goals for each group. Differences in risk contribute to inequalities in expected earnings and the probability of winning in Contract A. To eliminate this effect, the multiplicative handicap is used in Contract C to equate the distributions of uncertainty between agents. The agents endowed with the standard deviations $\{\sigma_L, \sigma_M, \sigma_H\}$ correspond to the set of handicaps $\phi = \{3, 2, 1\}$. The agent with the smallest variance in distribution receives the largest multiple. Since $r_i^* = \phi_i \mu_i^* + \phi_i \varepsilon_i + h_i$, the handicap ϕ has the effect of scaling both the error term and the effort for the agent.

This is a desirable property in g_3 because ability is proportional to uncertainty. The specific cost function in this experiment induces the same proportion in equilibrium effort when agents are working efficiently; therefore in g_3, the multiplicative handicap is sufficient to equate the distributions and effort simultaneously such that the offset handicap is not needed $h = \{0, 0, 0\}$. In the other heterogeneous groups

g_2, and g_4, h is used to compensate for the scaling effect of ϕ. Evaluating the expectation of the rank rule $E[r_i] = r$, the offset handicap for all agents were calculated $h_i = r - \phi_i \mu_i^*$. To avoid inducing loss aversion, the offset handicaps were normalized so that $h_i \geq 0$ by choosing $r = max\left[\phi_i \mu_i^*, \phi_j \mu_j^*, \phi_j \mu_j^*\right]$ and the agent of the group with the smallest offset handicap received $h_i = 0$. In g_1, the handicap $\phi_i = 1$ since the variance is identical across agents. In fact, the optimal contract for group g_1 is to use only h as in Contract B; hence g_1 is omitted from Contract C.

3.3.3 Participant Pool and Procedures

The experiment consists of 216 subjects broken up into 10 sessions of 24, 21 or 18 subjects each depending on how many subjects showed up to the session. Subjects were accepted in groups of three. Any subject who did not match into a group of 3 were paid a \$5 show-up fee and asked sign up again for a later session. A typical 24-subject session has 8 total groups, 2 of each type of group. Undergraduate students from the University of Tennessee economics experimental subject pool volunteer to participate using the ORSEE online registration process (Greiner, 2004). z-Tree was used to conduct the experiment (Fischbacher, 2007). Average earnings per subject is \$34.7. The experiment lasted for 2 hours and 15 minutes.

A session, depicted in Figure 3.2, proceeds as follows. Subjects arrive and are given a randomly assigned seat at one of the computer terminals in the lab. Once all of the subjects have arrived, instructions are read aloud by a moderator and subjects are given a chance to ask questions. Instructions are shown in subsection C.3 of the Appendix. Subjects then complete a training session, play three training rounds, and are again given the opportunity to ask questions. Based upon the random order of arrival, and a pseudo-random group assignment chart that is unique to each session,

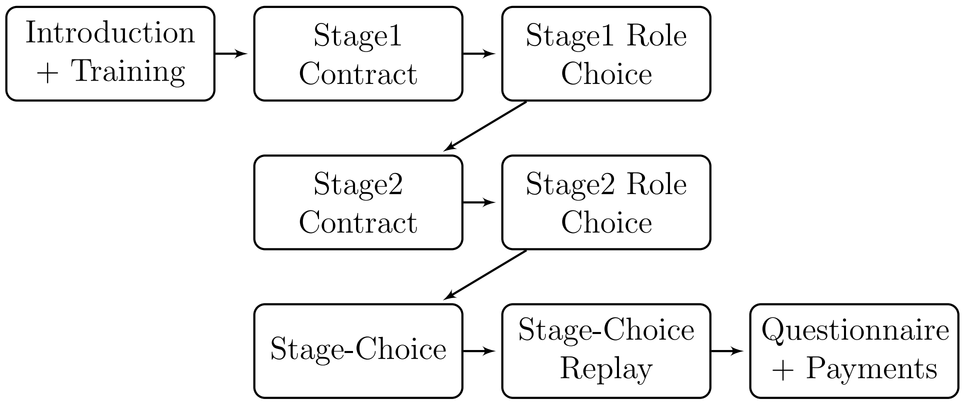

Figure 3.2: Typical session block diagram

subjects are assigned into one of the 5 groups and into an agent type within the group. 180 of the subjects remain in the same group and same type for the entire session. 36 subjects change between groups g_0 and g_1 between stages. The session was split into two Sets to control for order effects that balance the group exposure to contracts across Stage 1 and Stage 2. The only difference between Set 1 and Set 2 is which two of the four contracts will be given and in which order.

At the beginning of Stage 1, subjects play two training rounds in the contract assigned to the stage. Agent High Payment and Low Payments are calculated and displayed on the decision screen for each contract. A typical decision screen is shown in the Appendix in Figure C.3. Subjects then make decisions in 20 paid rounds. During a paid decision round, the cost of effort was shown in a table on the decision screen and was also calculated interactively on the screen as subjects considered different decision numbers. As subjects made decisions, High Earnings and Low Earnings based on actual decision numbers were interactively displayed on the screen.

After each decision round, a unique random number is added to the decision number, then the rank rule is applied to create a total number. The total numbers of each group member are ranked. The rank and total number of each group member are shown on a result screen. A typical result screen is shown in the Appendix in Figure C.3. The random number drawn for the subject is also shown on the result screen as a graphical bar and value.

In the final 21^{st} round of the stage, a questionnaire is distributed to solicit role preferences, then the stage ends. Stage 2 is conducted in exactly the same manner as Stage 1 except the contract changes between stages. To solicit which contract is preferred, subjects are asked which stage they prefer to play again in the Stage Choice block immediately following Stage 2. One of the choices is randomly selected and each group plays 5 more rounds of the chosen stage. The final period solicits fairness preferences and demographic information at the end of the session for all 24 subjects. Results of the risk elicitation and earnings are then shown. Subjects are paid in private using envelopes as they leave the room.

Role preferences are solicited in the 21^{st} paid round for each contract for all sessions except the pilot run. The role preference sheet is shown in subsection C.4 of the Appendix. The question asks subjects to circle the role (type) they would prefer to play again if they were given the opportunity. In the pilot run, a question of role preference was in the questionnaire at the end of the session that asked if subjects would prefer to play the same role (type) or switch to one of the other roles.

3.3.4 Hypotheses

In the equal access contract A, each agent has the same contract with the firm. However, agents k_1, k_2, $k_3\, k_4$, and j_3 have an advantage in either ability, a larger variance in uncertainty, or a combination of both that enhances their probability of winning at the Nash equilibrium and ultimately leads to the highest expected earnings in their group. We expect this to be an overwhelming reason for these types to choose an equal access contract over either equal pay or equal chance. For the same reason, we expect the disadvantaged agents i_1, i_2, i_3, i_4 and j_1, j_2, j_4 to prefer to not play equal access because they have lower probability of winning at the Nash equilibrium that ultimately leads to lower expected earnings in their group as compared to contracts equal pay or equal chance. Agent j_2 is only slightly worse off in expected earnings in contract A and j_3 is slightly better off in expected earnings in contract A than in either B or C, hence these two may be indifferent as these difference may not be perceptible to the agents.

Hypothesis 1. Advantaged types will choose contract *equal access* A over both *equal pay* B and *equal chance* C.

Hypothesis 2. Disadvantaged types will choose contract *equal pay* B or *equal chance* C over *equal access* A.

In the equal pay contract B, agents have individual payoffs and offset handicaps such that everyone has the same expected earnings. However, the offset handicap is only able to make group g_1 strategically symmetric. In the other groups, Agents k_2, k_3, and i_4 have an advantage from a larger variance in uncertainty that enhances their probability of winning to 0.43 at the Nash equilibrium. If equal expected earnings is all that is considered important to an agent ex ante, then having a higher probability of winning should not make a difference and therefore there should be no perceptible preferences for any agent between contract B or C. On the other hand, if strategic symmetry is considered more fair than equal expected earnings are alone ex-ante, then we can expect a higher probability of winning to be a reason for the advantaged types to choose an equal pay contract over equal chance. Likewise, we expect the agents

i_2, i_3 and k_4 with low variance in uncertainty and a relatively lower probability of winning of 0.27 in B to prefer C to B. Agents j_2, j_3, and j_4 all have a slightly lower probability of winning of 0.30 in contract B compared to $P = 1/3$ in contract C and hence these agents may be slightly in favor of contract C.

Hypothesis 3. Types with high probability of winning in B will choose contract *equal pay* B over *equal chance* C.

Hypothesis 4. Types with low probability of winning in B will choose contract *equal chance* C over *equal pay* B.

Role preferences are also tested. The advantages that lead to Hypotheses 1 and 2 also lead to another prediction about which role is most preferred in contract A. Any type should prefer to play the role in his group with the greatest expected earnings.

Hypothesis 5. Types in contract *equal access* A will choose to play the role with the highest expected payoff.

Also, if differences in probability of winning are considered important, then any type should prefer to play the role in his group with the greatest probability of winning in contract B. Finally, if we are to assume that a symmetrically strategic contest is considered most fair in this experiment, then all types should be indifferent between playing any role in a strategically symmetric game. The strategically symmetric games include the homogeneous group $g_0$in contract H, group $g_1$in contract B, and groups g_2, g_3, and g_4 in contract C.

Hypothesis 6. Types in groups g_2, g_3, and g_4 in contract *equal pay* B will choose to play the role with the highest probability of winning.

Hypothesis 7. Types in strategically symmetric contracts H, C and g_1 in B will be indifferent between playing any role.

3.4 Experimental Results

We now look at the results of the experiment for the three categories of contract choice and fairness, role preferences, and effort. Firms that employ equal opportunity contracts may have inadequate resources to monitor the results of such programs in great detail. In such cases, it is likely average results are used to quantify program effectiveness in terms of efficiency and equity. Therefore, we use a top-down approach to view the data at different levels of aggregation at the firm level, group level, and individual agent type level conditional on group composition.

3.4.1 Contract Choice and Fairness

After gaining experience playing contracts in stage 1 and 2, subjects are asked to choose which of the two contracts they prefer to play again. After all group members make a decision, the choice of one group member is selected at random and the group plays 5 more rounds of the chosen stage. The contract choice options are as follows:

○ *I SELECT TO PLAY ACCORDING TO STAGE 1 CONDITIONS*
○ *I SELECT TO PLAY ACCORDING TO STAGE 2 CONDITIONS*

The 216 observations are coded into a single variable $y_i \in [0, 1]$ where 1 indicates contract x is preferred to contract z and 0 indicates contract z is preferred to contract x. In this way, a coefficient will measure what percentage of subjects prefer contract x to contract z. The dependent variable is coded as contract choice pairs $\{x \text{ } or \text{ } z\} = \{A \text{ } or \text{ } H;\ B \text{ } or \text{ } H;\ A \text{ } or \text{ } B;\ A \text{ } or \text{ } C;\ B \text{ } or \text{ } C\}$.

Subjects are also asked on the same screen to indicate the strength of their preferences with the following question.

> *Please indicate how strongly you prefer your chosen Stage over the alternative one (This will not be used to determine the outcome, it is for research purposes only):*
> 1. *I strongly prefer the stage I selected.*
> 2. *I moderately prefer the stage I selected.*
> 3. *I weakly prefer the stage I selected.*
> 4. *I have no preference of one stage over the other.*

Since subjects are forced to choose between the two stages in the contract choice question, selecting "4" in the preference strength question is interpreted as indifference. The preference strength, combined with the binary contract, is used to construct a preference *Decision (gradient)* variable. Specifically, a choice between Contract x and z is coded $\{z1, z2, z3, z4 = x4, x3, x2, x1\} \rightarrow \{0, 0.17, 0.33, 0.50, 0.67, 0.83, 1\}$. In this case, a score of 0.5 represents indifference between the two contracts, a score of 1 indicates strong preference for Contract x and a score of 0.17 represents a moderate preference for Contract z.

A *Fairness* dependent variable in created as the difference between two identical questions about fairness that subjects answer during the questionnaire about either Stage 1 or Stage 2, shown here:

> *In* [STAGE 1 | STAGE 2] *of the experiment, did you feel that the experiment was "fair" in the sense that all players had the same potential to earn money?*
> 1. *Not fair at all*
> 2. *Somewhat fair*
> 3. *Moderately fair*
> 4. *Mostly fair*
> 5. *Perfectly fair*

The *Fairness* variable is constructed as the difference between the two questions and then scaled to the interval $[0, 1]$. A value of 0.5 represents that contract x and z are considered to be of equal fairness by the same subject (however, since *Fairness* represents the difference between Likert scales, it does not indicate at what level of fairness that would be). A value of 1 represents that contract x was considered perfectly fair while contract z was considered not fair at all. A value of 0 represents that contract z was considered perfectly fair while contract x was considered not fair at all.

An *Earnings difference* dependent variable is constructed as the difference in total earnings received in 20 paid rounds from contract x minus total earnings received in 20 paid rounds from contract z for each subject in US dollars. We also count the number of times a subject won in a contract out of 20 rounds of play and calculate the percentage. The *Percent Won difference* dependent variable represent the average difference between winning in contract x minus winning in contract z.

Tables at the pooled level and individual type level are shown in this section. Tables at the Group level are shown in Appendix C.1. The first column of the tables, *Decision*, shows the percentage of the group that prefers contract x to z. The second column shows the same *Decision (gradient)* augmented by the preference strength question. The *Fairness* variable is in the third column, the *Earnings difference* is in column 4, and the *Percent Won difference* in column 5. Significant values on variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50. A significant value on *Earnings difference* indicates an average earnings difference exists for the group in one contract over another where a negative value indicates more was earned in Contract z than x. By design, the earnings difference between contracts B and C is expected to be zero since they both have equal expected earnings. A significant value on *Percent Won difference* indicates subjects won more often in one contract over another, a negative sign indicates subjects won more often in contract z.

Table 3.4: Pooled Contract Choice

Variable	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Win (difference) b/se
Choice was Contract A or B	0.486	0.500	0.481	0.619	-0.004
	(0.059)	(0.051)	(0.022)	(0.936)	(0.036)
Choice was Contract A or C	0.537	0.512	0.505	2.472	-0.002
	(0.068)	(0.058)	(0.030)	(1.597)	(0.035)
Choice was Contract B or C	0.556	0.519	0.525	0.331	0.002
	(0.068)	(0.060)	(0.020)	(0.700)	(0.027)
R-sqr	0.524	0.583	0.879	0.027	0.000
dfres	179	179	179	179	179
BIC	275.8	223.3	-79.4	1299.2	41.2
N	180.0	180.0	180.0	180.0	180.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on Choice variables indicate statistical difference from the midrange value, 0.50.

Of the 216 subjects, Table 3.4 shows the results from the pooled regressions for the 180 who make a choice between contracts without switching roles or group composition. At the aggregate level, there is no statistical preference for one contract type over the other. This result is not entirely unexpected for choices made between Contracts B and C since both contracts are handicapped and have the same expected payoff. However, the result that subjects are indifferent from a handicapped or homogeneous contract and Contract A with no handicaps suggests the votes made by subjects in advantaged roles may be canceling the votes made by those in disadvantaged roles. Overall, only 3.14% of subjects were indifferent over contracts, but were still required to make a choice.[4]

[4]The histogram of contract choice with a preference strength gradient is bimodal (1=x strongly preferred to z is 31.02%, 0.83 is 16.67%, 0.67 is 1.9%, 0.5=no preference is 3.24%, 0.33 is 3.70%, 0.167 is 14.81%, and 0=z strongly preferred to x is 28.70%.

Table 3.5: Tabulation of Group by Contract Choice

Group	Contract A	Choice Contract B	Contract C	Contract H	Total
homogeneous or ability A	9			9	18 subjects
	50.00			50.00	100 %
homogeneous or ability B		9		9	18 subjects
		50.00		50.00	100 %
diff ability	7	11			18 subjects
	38.89	61.11			100 %
diff uncertainty	21	17	16		54 subjects
	38.89	31.48	29.63		100 %
ability $\propto$ uncertainty	15	22	17		54 subjects
	27.78	40.74	31.48		100 %
ability $\propto$ precision	21	17	16		54 subjects
	38.89	31.48	29.63		100 %
Total	73	76	49	18	216 subjects
	33.80	35.19	22.68	8.33	100 %

First number is subject count, second is percent of row.

The group composition was altered on the other 36 subjects to investigate what happens when subjects are exposed to a homogeneous group g_0 and to one with different abilities g_1. The pooled level results of these 36 subjects and a discussion are in Appendix C.1.

3.4.1.1 Aggregated Group Results

This subsection discusses the results of Group level contract choice decisions. Each regression includes twelve dummy variables that indicate contract choices made by each group. Errors are grouped at the subject level. The regression Tables that support this discussion are in Appendix C.1.

Table 3.5 provides the tabulation summary of how often a contract was chosen in the experiment. None of the observations are statistically different from equal expectations in each contract choice. This is easily seen for those who switched between homogeneous and ability groups in the first two rows where subjects preferred being in the homogeneous contract 50% of the time. For groups in rows 4, 5, and 6, none of the percentages are statistically different from $1/3$. Tabulating the binary contract choice, we also find no significant difference in contract preference in the pooled data. This result suggests that a collection of heterogeneous groups in mixed contracts may on average vote to use any of the three contracts with equal probability.

Overall, contract preferences, fairness, and percent won aggregated at the group level are shown to be neutral. There is also no significant difference in average earnings between any contract even though the expected earnings are different for different types in contract A. When averaged at the group level, the *Percent Won* should be $1/3$ for all contracts.

One exception was in Group 0 and Group 1 where contracts were found to display transitive properties in fairness. Table C.2 shows the results of the five regressions for agents in homogeneous Group 0 and Group 1, heterogeneous in ability only. Column 3 shows that the homogeneous contract H is considered more fair than either contract A or contract B and that contract B is considered more fair than contract A. Therefore, for groups of agents with differences in ability, fairness preferences are transitive $A \prec B \prec H$ at the group level. The only other exception to neutral results is found in Group 2 with heterogeneous uncertainty. Contract A is considered slightly more fair than contract B to the 10% level.[5]

3.4.1.2 Individual Type Results

The variation in the contract choice data that is hidden at the firm and group levels becomes more evident at the type level, conditional on group composition. The ex-

[5]Table C.3 shows the results of the five regressions for agents in Group 2.

Table 3.6: Contract Preferences, Type Decisions in Group 0

Group 0 Variables (Ability/Homogeneous)	Type	Choice	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
T14: ability/homo	i : Low	A or H	0.000***	0.167***	0.146***	-6.483	-0.175***
			(.)	(0.09)	(0.08)	(5.08)	(0.03)
T14: ability/homo	i : Low	B or H	0.667	0.639	0.333	1.948	0.142**
			(0.21)	(0.20)	(0.14)	(1.22)	(0.07)
T15: ability/homo	j : Med	A or H	0.500	0.528	0.313	-1.606	-0.158*
			(0.22)	(0.19)	(0.12)	(1.57)	(0.08)
T15: ability/homo	j : Med	B or H	0.500	0.556	0.375	0.020	-0.008
			(0.22)	(0.16)	(0.12)	(0.84)	(0.04)
T16: ability/homo	k : High	A or H	1.000***	1.000***	0.417	8.312***	0.342***
			(0.00)	(0.00)	(0.14)	(1.85)	(0.08)
T16: ability/homo	k : High	B or H	0.333	0.361	0.229***	-0.190	-0.142***
			(0.21)	(0.18)	(0.09)	(1.13)	(0.05)
R-sqr			0.685	0.734	0.872	0.322	0.535
dfres			215	215	215	215	215
BIC			372.4	337.1	77.4	1642.9	51.9
N			216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

periment captures the unique perspectives of subjects endowed with different abilities and shock uncertainty as they face contracts of equal access, equal pay, and equal chance. The same five regressions run for the group level are rerun at the type level and split by group into the next 5 tables. With 216 subjects and 36 contract choices, this provides 6 fairness observations per type per contract choice.

Table 3.6 shows the partial results of the five type-level regressions for agents in Group 0 that switch roles and choose between heterogeneous ability contracts and a homogenous contract. Consistent with Hypothesis 1, a high ability type ($T16 - G0, k$) unanimously prefers equal access over a homogeneous contract in column 1. A reason for this can be seen in column 4 where the high type earned \$8.31 more in contract A than in H and won more often in A as seen in column 5. Consistent with Hypothesis 2, the low ability type ($T14 - G0, i$) unanimously prefers a homogeneous contract to equal access. Although not significant, the low types suffered a loss differential

Table 3.7: Contract Preferences, Type Decisions in Group 1

Group 1 Variables (Different Ability)	Type	Choice	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
T1: ability	i : Low	A or B	0.000***	0.028***	0.271***	-8.634***	-0.208***
			(.)	(0.03)	(0.08)	(3.28)	(0.07)
T2: ability	j : Med	A or B	0.167**	0.139***	0.396	-4.112***	-0.250***
			(0.17)	(0.11)	(0.08)	(1.28)	(0.07)
T3: ability	k : High	A or B	1.000***	1.000***	0.542	7.935***	0.433***
			(0.00)	(0.00)	(0.07)	(0.84)	(0.13)
R-sqr			0.685	0.734	0.872	0.322	0.535
dfres			215	215	215	215	215
BIC			372.4	337.1	77.4	1642.9	51.9
N			216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

of -\$6.48 (5.08). The low type considered the homogeneous contract more fair than equal access, while the high type was indifferent in column 3. The medium type $(T15 - G0, j)$ is neutral in fairness between all three A, B, or H contracts.

Table 3.6 shows the partial results of the five type-level regressions for agents in Group 1 that are endowed with heterogeneous ability and choose between equal access A and equal pay/equal chance contract B. Preferences are strong in this group and consistent with the literature on contests between agents heterogeneous in ability in column 1. Consistent with Hypothesis 2, both the low ability type and medium ability type prefer equal pay over equal access. Both low and medium ability win more often in contract B and lose more money playing A than in B by a differential of -\$8.63 and -\$4.11 respectively and not surprisingly, they both prefer the equal pay contract B. Consistent with Hypothesis 1, the high ability type prefers equal access over equal pay. The low ability type also considers B to be more fair. The high type is able to collect \$7.94 more on average in contract A and votes unanimously for A over B, but does not consider A to be more fair than B.

The type-level decisions from Group 2 that come from agents heterogeneous in

Table 3.8: Contract Preferences, Type Decisions in Group 2

Group 2 Variables (Different Uncertainty)	Type	Choice	Decision [0,1] b/se	Decision (weighted) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
T4: uncertainty	i : Low	A or B	0.833**	0.694	0.542	3.078	-0.008
			(0.17)	(0.12)	(0.07)	(3.53)	(0.06)
T4: uncertainty	i : Low	A or C	0.500	0.444	0.500	-1.504	-0.067
			(0.22)	(0.19)	(0.08)	(0.99)	(0.07)
T4: uncertainty	i : Low	B or C	0.500	0.500	0.500	1.436	0.050
			(0.22)	(0.18)	(0.03)	(1.59)	(0.07)
T5: uncertainty	j : Med	A or B	0.500	0.556	0.667***	-0.076	0.033
			(0.22)	(0.16)	(0.06)	(2.01)	(0.10)
T5: uncertainty	j : Med	A or C	0.500	0.500	0.625*	1.422	0.058
			(0.22)	(0.20)	(0.06)	(1.30)	(0.05)
T5: uncertainty	j : Med	B or C	0.500	0.444	0.458	-0.278	-0.075
			(0.22)	(0.18)	(0.08)	(1.02)	(0.07)
T6: uncertainty	k : High	A or B	0.500	0.556	0.521	3.057	-0.042
			(0.22)	(0.18)	(0.07)	(2.08)	(0.08)
T6: uncertainty	k : High	A or C	0.667	0.639	0.563	9.942	0.025
			(0.21)	(0.18)	(0.13)	(9.12)	(0.06)
T6: uncertainty	k : High	B or C	0.667	0.556	0.563	-0.137	0.025
			(0.21)	(0.18)	(0.05)	(3.66)	(0.11)
R-sqr			0.685	0.734	0.872	0.322	0.535
dfres			215	215	215	215	215
BIC			372.4	337.1	77.4	1642.9	51.9
N			216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

Table 3.9: Contract Preferences, Type Decisions in Group 3

Group 3 Variable (ability $\propto$ uncertainty)	Type	Choice	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
T7: ability $\propto$ uncertainty	i : Low	A or B	0.000***	0.083***	0.396**	-9.920***	-0.350***
			(.)	(0.06)	(0.05)	(2.92)	(0.04)
T7: ability $\propto$ uncertainty	i : Low	A or C	0.000***	0.028***	0.292***	-2.865	-0.317***
			(.)	(0.03)	(0.05)	(3.66)	(0.06)
T7: ability $\propto$ uncertainty	i : Low	B or C	0.500	0.72	0.500	2.151	-0.100
			(0.22)	(0.21)	(0.03)	(3.99)	(0.08)
T8: ability $\propto$ uncertainty	j : Med	A or B	0.167**	0.278	0.521	1.042	0.050
			(0.17)	(0.14)	(0.08)	(1.51)	(0.05)
T8: ability $\propto$ uncertainty	j : Med	A or C	0.500	0.444	0.458	0.466	-0.075
			(0.22)	(0.16)	(0.04)	(2.17)	(0.11)
T8: ability $\propto$ uncertainty	j : Med	B or C	0.500	0.583	0.625*	-0.118	-0.075
			(0.22)	(0.19)	(0.06)	(1.27)	(0.06)
T9: ability $\propto$ uncertainty	k : High	A or B	1.000***	0.972***	0.479	9.543***	0.300***
			(0.00)	(0.03)	(0.07)	(2.39)	(0.06)
T9: ability $\propto$ uncertainty	k : High	A or C	0.833**	0.778**	0.604	6.562**	0.375***
			(0.17)	(0.13)	(0.10)	(3.05)	(0.11)
T9: ability $\propto$ uncertainty	k : High	B or C	0.833**	0.806*	0.500	-1.491	0.167
			(0.17)	(0.16)	(0.10)	(1.60)	(0.12)
R-sqr			0.685	0.734	0.872	0.322	0.535
dfres			215	215	215	215	215
BIC			372.4	337.1	77.4	1642.9	51.9
N			216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

shock uncertainty are reported in Table 3.8. These agents experience all three contract types of equal access, equal pay, and equal chance. In all but one case, all types are indifferent between which contract they play, suggesting differences in uncertainty are not salient enough to matter. The one exception is for the low type where, contrary to predictions, they prefer contract A to B. This result goes away in column 2 when preference strength is included. The medium type considered A more fair than B or C.

Table 3.9 shows the partial results of the five type-level regressions for agents in Group 3 that are endowed with heterogeneous ability proportional to uncertainty and choose between equal access, equal pay, and equal chance contracts. Contract choice by each type is largely as predicted. Consistent with Hypothesis 2, the disadvantaged

Table 3.10: Contract Preferences, Type Decisions in Group 4

Group 4 Variables (ability $\propto$ precision)	Type	Choice	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
T10: ability $\propto$ precision	i : Low	A or B	0.500	0.528	0.438	2.712	-0.208***
			(0.22)	(0.19)	(0.05)	(3.51)	(0.07)
T10: ability $\propto$ precision	i : Low	A or C	0.667	0.611	0.542	9.797	-0.017
			(0.21)	(0.17)	(0.12)	(8.97)	(0.09)
T10: ability $\propto$ precision	i : Low	B or C	0.333	0.361	0.583***	-0.817	0.058
			(0.21)	(0.20)	(0.03)	(1.67)	(0.05)
T11: ability $\propto$ precision	j : Med	A or B	0.167**	0.222*	0.479	-2.860	-0.158*
			(0.17)	(0.16)	(0.09)	(1.83)	(0.08)
T11: ability $\propto$ precision	j : Med	A or C	0.333	0.389	0.438	-4.408*	-0.108
			(0.21)	(0.18)	(0.12)	(2.25)	(0.08)
T11: ability $\propto$ precision	j : Med	B or C	0.833**	0.639	0.500	1.992**	0.033
			(0.17)	(0.16)	(0.09)	(0.88)	(0.06)
T12: ability $\propto$ precision	k : High	A or B	1.000***	0.944***	0.521	5.657***	0.358***
			(0.00)	(0.04)	(0.06)	(1.16)	(0.09)
T12: ability $\propto$ precision	k : High	A or C	0.833**	0.778*	0.521	2.840***	0.108
			(0.17)	(0.16)	(0.05)	(0.59)	(0.09)
T12: ability $\propto$ precision	k : High	B or C	0.333	0.306	0.500	0.237	-0.067
			(0.21)	(0.19)	(0.05)	(1.91)	(0.08)
R-sqr			0.685	0.734	0.872	0.322	0.535
dfres			215	215	215	215	215
BIC			372.4	337.1	77.4	1642.9	51.9
N			216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

type $(T7, i)$ endowed with low ability and low uncertainty, won less often and earned less in A, and unanimously vote for a handicapped contract and consider them to both be more fair than A. However, contrary to Hypothesis 4 they are indifferent between which handicap contract they play. On the other spectrum, the advantaged type $(T9, k)$ endowed with high ability and high uncertainty, won more often and earned more in A, and voted for an equal access contract A over either the equal pay B or equal pay/chance C as predicted in Hypothesis 1. The type $(T8, j)$ endowed with medium ability and medium uncertainty is indifferent between all contracts except does prefer B over A to the 5% level in column 1. Consistent with Hypothesis 3, $(T9, k)$ prefers B over C.

The type-level decisions of Group 4 that come from agents with ability propor-

Table 3.11: Hypotheses Results Summary

Hypothesis	**Preference**	i_1	i_2	i_3	i_4	j_1	j_2	j_3	j_4	k_1	k_2	k_3	k_4
1	A over B	-	-	-	-	-	-	Y	-	Y	I	Y	Y
1	A over C	-	-	-	-	-	-	I	-	-	I	Y	Y
2	B over A	Y	N	Y	I	Y	I	-	Y	-	-	-	-
2	C over A	-	I	Y	I	Y	I	-	I	-	-	-	-
3	B over C	-	-	-	I	-	-	-	-	-	I	Y	-
4	C over B	-	I	I	-	-	-	-	-	-	-	-	I
5	high $E[u]$ role in A	Y	Y	Y	N	N	Y	Y	N	Y	Y	N	Y
6	high P role in B	N	N	N	N	N	N	N	N	Y	Y	N	N
7	role indifference in C	-	Y	Y	N	-	Y	N	N	-	N	Y	N

Y, the hypothesis is statistically satisfied; N, it is not; I, the type was indifferent

tional to precision are reported in Table 3.10. Type $(T12, k)$ endowed with high ability and low uncertainty has the advantage in A and unanimously prefers A over B or C as predicted. This is coincident with earnings premiums in A compared to B or C. Both type $(T10, i)$ and $(T11, j)$ prefer equal pay/chance over equal pay, suggesting that having an equal probability of winning is desirable; although, $(T10, i)$ does find B to be more fair than C, suggesting otherwise. The medium type earned \$1.99 more in B than C and prefers B over C.

The results of all hypotheses are summarized in Table 3.11 including the role preference results from the next subsection. Additionally, all of the advantaged types report neutral fairness between contract A or B and A or C. This result suggests that those endowed with an advantage consider an even playing field just as fair as the one stacked in his favor.

3.4.2 Role Preferences

Role preferences were solicited in the 21^{st}round of each stage with the following question.

If you had the ability to choose, which role would you play? (circle one)

my current role *role of* 1^{st}*group member* *role of* 1^{st}*group member*

Overall, 50.23% reported they would play the same role again. This proportion may be high because subjects may have preferences for the status quo. A subject who switches away from his current role must have preferences strong enough to overcome his preferences for the familiarity of his current role. Similarly, indifference between contracts may appear as a choice to play the same role more frequently.

The next three tables show the tabulation of role preferences chosen by the actual role of the subject type.[6] Role preference variables were coded as Prefer Role i, Prefer Role j, and Prefer Role k and regressed on dummy variables for each type for each contract. The hypothesis results are summarized in Table 3.11.

Table 3.12 shows the role preference tabulation for Type-level decisions in equal access Contract A. As expected, the benchmark case with homogeneous agents does not show a statistical difference in preference, supporting hypothesis 7, since all of the other roles are the same. For the case of equal access, agents with greater ability have a higher likelihood of winning which leads to a higher expected payoff. Consistent with Hypothesis 5, the low ability types in $G1$ prefer to be high ability as do the medium and high ability types. Without handicaps, higher uncertainty increases the chances

[6]The initial session did not include a role preference solicitation. As such, Contract C has 10 respondents per type. Contract B has 11, and Contract A has 11. Since contract H is composed of homogeneous agents, it has 33.

Table 3.12: Equal Access Role Preferences, Type Decisions in Contract A

Variable (Contract A)	Type	Prefer Role i b/se	Prefer Role j b/se	Prefer Role k b/se
G0-Homogeneous		0.333	0.242	0.424
		(0.09)	(0.08)	(0.09)
G1: ability	i : Low	0.167	0.083***	0.750***
		(0.11)	(0.08)	(0.13)
G1: ability	j : Med	0.083***	0.167	0.750***
		(0.08)	(0.11)	(0.13)
G1: ability	k : High	0.000***	0.167	0.833***
		(.)	(0.11)	(0.11)
G2: uncertainty	i : Low	0.455	0.273	0.273
		(0.16)	(0.14)	(0.14)
G2: uncertainty	j : Med	0.182	0.455	0.364
		(0.12)	(0.16)	(0.15)
G2: uncertainty	k : High	0.364	0.000***	0.636**
		(0.15)	(.)	(0.15)
G3: ability $\propto$ uncertainty	i : Low	0.364	0.000***	0.636**
		(0.15)	(.)	(0.15)
G3: ability $\propto$ uncertainty	j : Med	0.182	0.455	0.364
		(0.12)	(0.16)	(0.15)
G3: ability $\propto$ uncertainty	k : High	0.182	0.091***	0.727***
		(0.12)	(0.09)	(0.14)
G4: ability $\propto$ precision	i : Low	0.091***	0.273	0.636**
		(0.09)	(0.14)	(0.15)
G4: ability $\propto$ precision	j : Med	0.364	0.182	0.455
		(0.15)	(0.12)	(0.16)
G4: ability $\propto$ precision	k : High	0.000***	0.091***	0.909***
		(.)	(0.09)	(0.09)
R-sqr		0.419	0.362	0.555
dfres		194	194	194
BIC		641.9	551.4	674.3
N		390.0	390.0	390.0
n		168.0	168.0	168.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on role variables in columns 1, 2, and 3 indicate statistical difference from the even preference value of 0.333. Statistic ”n” represents the sub-sample size of this table, ”N” is the regression size.

of winning holding all else constant; hence, we should expect a similar response that all subjects prefer to by the high uncertainty type. However in equal access contracts, only the high type appears to recognize the benefit of his own endowment such that 63.6% choose to remain in the same role. In the ability $\propto$ uncertainty group, 63.6% of low type respondents recognize the advantages and choose to be high types and 72.7% of high types also choose to remain high types. This pattern is repeated in the ability $\propto$ precision group 4. Overall, the medium type, who is always endowed with the same medium ability and medium uncertainty in all groups, does not show a significant preference for any role except in Group 1.

The role preferences for the equal pay contract B is shown in Table 3.13. Inconsistent with hypothesis 6, 54.5% of low ability types in Group 1 now choose to stay in their own role compared to only 16.7% under equal access. This pattern is repeated for all low types in every group suggesting status quo preferences may be present. Medium types only slightly increase their desire to be in the same role again. High types mostly reduce their preference to stay in the same role again, suggesting the advantage they experienced in A is reduced in B. Taken together, these results suggest equal pay increases the perception of fairness in all group compositions over equal access.

The role preferences for the equal chance (with equal pay) contract is shown in Table 3.14. Much like equal pay, the prediction is that equal expected payoffs with equal probabilities will induce procedural fairness such that all types will be indifferent between playing any role. Interestingly, preferences for the status quo does appear to effect contract C. On average, subjects choose their same role more frequently than the other two options. Although this does not suggest indifference between roles exists in contract C, it does confirm that the incentives that make subjects in disadvantaged

Table 3.13: Equal Pay Role Preferences, Type Decisions in Contract B

Variable (Contract B)	Type	Prefer Role i b/se	Prefer Role j b/se	Prefer Role k b/se
G1: ability	i : Low	0.545	0.091***	0.364
		(0.16)	(0.09)	(0.15)
G1: ability	j : Med	0.455	0.182	0.364
		(0.16)	(0.12)	(0.15)
G1: ability	k : High	0.364	0.000***	0.636**
		(0.15)	(.)	(0.15)
G2: uncertainty	i : Low	0.818***	0.091***	0.091***
		(0.12)	(0.09)	(0.09)
G2: uncertainty	j : Med	0.364	0.455	0.182
		(0.15)	(0.16)	(0.12)
G2: uncertainty	k : High	0.273	0.000***	0.727***
		(0.14)	(.)	(0.14)
G3: ability $\propto$ uncertainty	i : Low	0.455	0.273	0.273
		(0.16)	(0.14)	(0.14)
G3: ability $\propto$ uncertainty	j : Med	0.364	0.545	0.091***
		(0.15)	(0.16)	(0.09)
G3: ability $\propto$ uncertainty	k : High	0.364	0.182	0.455
		(0.15)	(0.12)	(0.16)
G4: ability $\propto$ precision	i : Low	0.545	0.182	0.273
		(0.16)	(0.12)	(0.14)
G4: ability $\propto$ precision	j : Med	0.182	0.636**	0.182
		(0.12)	(0.15)	(0.12)
G4: ability $\propto$ precision	k : High	0.182	0.364	0.455
		(0.12)	(0.15)	(0.16)
R-sqr		0.419	0.362	0.555
dfres		194	194	194
BIC		641.9	551.4	674.3
N		390.0	390.0	390.0
n		132.0	132.0	132.0

* $p<0.10$, ** $p<0.05$, *** $p<0.01$. Significant values on role variables in columns 1, 2, and 3 indicate statistical difference from the even preference value of 0.333. Statistic "n" represents the sub-sample size of this table, "N" is the regression size.

Table 3.14: Equal Chance Role Preferences, Type Decisions in Contract C

Variable (Contract C)	Type	Prefer Role i b/se	Prefer Role j b/se	Prefer Role k b/se
G2: uncertainty	i : Low	0.400	0.400	0.200
		(0.16)	(0.16)	(0.13)
G2: uncertainty	j : Med	0.500	0.300	0.200
		(0.17)	(0.15)	(0.13)
G2: uncertainty	k : High	0.200	0.100**	0.700**
		(0.13)	(0.10)	(0.15)
G3: ability $\propto$ uncertainty	i : Low	0.600	0.200	0.200
		(0.16)	(0.13)	(0.13)
G3: ability $\propto$ uncertainty	j : Med	0.400	0.500	0.100**
		(0.16)	(0.17)	(0.10)
G3: ability $\propto$ uncertainty	k : High	0.600	0.100**	0.300
		(0.16)	(0.10)	(0.15)
G4: ability $\propto$ precision	i : Low	0.300	0.100**	0.600
		(0.15)	(0.10)	(0.16)
G4: ability $\propto$ precision	j : Med	0.200	0.500	0.300
		(0.13)	(0.17)	(0.15)
G4: ability $\propto$ precision	k : High	0.100**	0.500	0.400
		(0.10)	(0.17)	(0.16)
R-sqr		0.419	0.362	0.555
dfres		194	194	194
BIC		641.9	551.4	674.3
N		390.0	390.0	390.0
n		90.0	90.0	90.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on role variables in columns 1, 2, and 3 indicate statistical difference from the even preference value of 0.333. Statistic ”n” represents the sub-sample size of this table, ”N” is the regression size.

roles want to switch into advantages roles are no longer present. Given this evidence, equal chance also improves perceptions of equality over equal access.

3.4.3 Effort

Pooling results for all 216 subjects and 8640 observations, average effort is 33.13 which is not statistically different from the expected average of 32.66 if everyone chooses the Nash. By design, the equal access contract A was expected to deliver lower than efficient effort. The pooled average effort reflects this result and is statistically different from the expected average of 34.44 if everyone were to choose the efficient level of effort given their endowed ability level. For reference, Appendix C.2 contains graphs that show the average effort for each period for each group in each contract.

To evaluate effort, a dependent variable is created as the difference between actual effort and predicted effort for all paid rounds. Four contract dummy variables are created to indicate what contract each subject was playing. Five group dummy variables are created to indicate subject membership in a particular group. Thirteen dummy variables are also created to indicate subject membership in a particular type of role that is specific to a group.[7] Both group and type dummy variables are interacted with contract dummy variables for part of the analysis. For regression tables in this section, the first column *Effort* shows the actual effort from the subjects, the second column shows the *Predicted* level of effort, and the third column shows the *Difference* between actual and predicted. Errors are clustered at the subject level.

Table 3.15 shows the mean effort by contract. Agents overworked when subjected to contract A with no handicaps, significant to the 1% level.[8] In contrast, the handi-

[7] The homogeneous contract counts as a single type since $i_0 = j_o = k_0$,

[8] Values reported in the text are significant to the 1% unless otherwise indicated. Significance levels in the tables are reported as * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.15: Effort by Contract

Contract	Effort b/se	Predicted	Difference b/se
Homogeneous	30.828	30.000	0.828
	(1.21)		(1.21)
Contract A	32.490	29.638	2.852***
	(1.32)		(0.90)
Contract B	33.852	35.000	-1.148
	(1.38)		(1.02)
Contract C	33.801	34.444	-0.643
	(1.49)		(1.04)
R-sqr	0.752		0.013
dfres	215		215
BIC	75465.1		72115.7
N	8640.0		8640.0

* p<0.10, ** p<0.05, *** p<0.01

capped contracts B and C yield both optimal and efficient effort as predicted, a result also found with the homogeneous control group.

Table 3.16 shows the mean effort by group. In Table 3.16 and subsequent tables, a handicap of *none* indicates equal access or Contract A, *offset* indicates equal pay or Contract B, and *full* indicates equal chance to win or Contract C. The homogeneous case serves as a benchmark of the experiment and is shown to be the same as predicted. Generally, we find there is no significant difference in effort from the predicted level. The group characterized by different ability only is observed as inefficient. When subjects are in heterogeneous groups, the benefits of employing fairness rules that equalize the expected output or make the probability of winning the same between agents may go unobserved. Then again, these results may exist simply because data is pooled by groups, so we now look at how each group responded to the treatments.

Table 3.17 shows the mean level of effort by group and contract treatment. Results are organized by group, then by treatment. Using a Wald test to compare the coef-

Table 3.16: Effort by Group

Group	Effort b/se	Predicted	Difference b/se
G0: homogeneous	30.828	30.000	0.828
	(1.21)		(1.21)
G1: diff ability	32.411	33.980	-1.569
	(2.26)		(1.63)
G2: diff uncertainty	28.726	28.774	-0.048
	(1.13)		(1.13)
G3: ability $\propto$ uncertainty	35.282	34.236	1.047
	(1.91)		(1.14)
G4: ability $\propto$ precision	36.642	34.966	1.677
	(2.48)		(1.32)
R-sqr	0.758		0.006
dfres	215		215
BIC	75262.9		72190.1
N	8640.0		8640.0

* p<0.10, ** p<0.05, *** p<0.01 indicate significance from zero of effort minus predicted in the Difference column.

ficients for a particular group, we observe no significant difference in effort between treatments at the group level in the first column. This result suggests that Firms may not observe the impact that handicaps have on effort at the group level when comparing contract performance. A neutral result from a policy to increase contract fairness in an attempt to improve group performance may leave managers unable to observe a difference in output.

Also, all groups produced the predicted level of effort except in three cases. Common to findings in the literature and predictions, the different ability group falls short of the efficient level of effort when no handicaps are applied. However, contrary to predictions, the addition of offset handicaps did not improve the efficiency of the group; the ability group still underperforms by -4.204 at the 10% level.

For the group endowed with ability proportional to uncertainty, having no hand-

Table 3.17: Effort by Group and Contract

Variable contract-group	Handicap	Effort b/se	Predicted b/se	Difference b/se
G0: Homogeneous		30.828	30.000	0.828
		(1.21)	(0.00)	(1.21)
CA-G1: ability	none	32.360	31.293	1.066
		(2.93)	(2.86)	(1.92)
CB-G1: ability	offset	32.463	36.667	-4.204*
		(2.80)	(2.84)	(2.47)
CA-G2: uncertainty	none	28.364	26.323	2.041
		(1.27)	(0.30)	(1.28)
CB-G2: uncertainty	offset	29.664	30.000	-0.336
		(1.84)	(0.00)	(1.84)
CB-G2: uncertainty	full	28.151	30.000	-1.849
		(1.73)	(0.00)	(1.73)
CA-G3: ability $\propto$ uncertainty	none	35.326	29.373	5.953***
		(2.77)	(2.26)	(1.61)
CB-G3: ability $\propto$ uncertainty	offset	36.611	36.667	-0.056
		(2.97)	(2.84)	(1.75)
CB-G3: ability $\propto$ uncertainty	full	33.910	36.667	-2.757
		(2.08)	(2.84)	(2.12)
CA-G4: ability $\propto$ precision	none	33.911	31.563	2.348
		(3.08)	(2.96)	(2.17)
CB-G4: ability $\propto$ precision	offset	36.672	36.667	0.006
		(3.04)	(2.84)	(1.92)
CB-G4: ability $\propto$ precision	full	39.343	36.667	2.676*
		(3.33)	(2.84)	(1.36)
R-sqr		0.759	0.853	0.028
dfres		215	215	215
BIC		75286.0	69771.4	72060.8
N		8640.0	8640.0	8640.0

* p<0.10, ** p<0.05, *** p<0.01

icap induced a significantly larger level of effort than predicted of 5.953. So much so that effort is statistically equal to the efficient level of effort for that group. The reason for this is unclear when viewed at this level of aggregation. Lastly, when ability and precision are proportional in the group, the full handicap case yields significantly higher effort than predicted by 2.676 at the 10% level.

Results from Table 3.17 suggest for any given group type, the firm might not observe a perceptible difference in output when implementing rules that equalize the expected output or make the probability of winning the same between agents. The increased costs of administering a handicapped contest might be too high to implement, especially with no concrete evidence of improved performance. Yet, agent performance is predicted to improve with the use of handicaps, so why don't they? The difference in performance becomes more apparent when we look closer at how subject types perform in each group across treatments.

Some of the aggregated results can be understood by examining the groups from within to see how each type behaved. In all, there are 12 unique types across the 4 different groups in addition to the homogeneous type. Table 3.18 shows the effort delivered by each type, which group they were in, and by contract treatment. Effort is predicted to be below efficient levels for all types in the untreated Contract A (CA), however 6 of the 12 types significantly overworked as is indicated in the Difference column for the "none" treatment cases in Table 3.18.

We can now see that the different ability group in Contract A ($CA - G1$, $none$) behaved as predicted except for the Low ability type who overworked by 6.422 units of effort on average at the 1% level. The High ability type's underperformance for Contract A was not significant enough to be non-optimal at the individual level, but as we saw in Table 3.17, it was enough to mask the climbing behavior of the Low

Table 3.18: Effort by Group 0, 1, 2, Contract, Type Treatment

Variable contract-group	Type	Handicap	Effort b/se	Predicted b/se	Difference b/se
Group 0 (Homogeneous):					
Contract H			30.828	30.000	0.828
			(1.22)	(0.00)	(1.22)
Group 1 (Heterogeneous Ability):					
Contract A	i : Low	none	20.642	14.220	6.422***
			(1.89)	(0.00)	(1.89)
Contract A	j : Med	none	24.717	24.980	-0.263
			(3.32)	(0.00)	(3.32)
Contract A	k : High	none	51.721	54.680	-2.959
			(3.85)	(0.00)	(3.85)
Contract B	i : Low	offset	20.738	20.000	0.738
			(1.27)	(0.00)	(1.27)
Contract B	j : Med	offset	30.546	30.000	0.546
			(1.63)	(0.00)	(1.63)
Contract B	k : High	offset	46.104	60.000	-13.896**
			(6.26)	(0.00)	(6.26)
Group 2 (Heterogeneous Uncertainty):					
Contract A	i : Low	none	26.854	27.580	-0.726
			(2.03)	(0.00)	(2.03)
Contract A	j : Med	none	30.942	27.580	3.362
			(2.31)	(0.00)	(2.31)
Contract A	k : High	none	27.296	23.810	3.486*
			(2.07)	(0.00)	(2.07)
Contract B	i : Low	offset	29.704	30.000	-0.296
			(3.88)	(0.00)	(3.88)
Contract B	j : Med	offset	29.538	30.000	-0.463
			(2.26)	(0.00)	(2.26)
Contract B	k : High	offset	29.750	30.000	-0.250
			(3.23)	(0.00)	(3.23)
Contract C	i : Low	full	25.038	30.000	-4.963
			(3.32)	(0.00)	(3.32)
Contract C	j : Med	full	29.529	30.000	-0.471
			(2.42)	(0.00)	(2.42)
Contract C	k : High	full	29.887	30.000	-0.113
			(2.97)	(0.00)	(2.97)
R-sqr			0.845	1.000	0.091
dfres			215	215	215
BIC			71681.8	.	71681.8
N			8640.0	8640.0	8640.0

* p<0.10, ** p<0.05, *** p<0.01

Table 3.19: Effort by Group 4 & 5, Contract, and Type Treatment

Variable contract-group	Type	Handicap	Effort b/se	Predicted b/se	Difference b/se
Group 3 (Ability ∝ Uncertainty):					
Contract A	i : Low	none	22.346	15.330	7.016**
			(3.11)	(0.00)	(3.11)
Contract A	j : Med	none	29.675	25.170	4.505
			(2.90)	(0.00)	(2.90)
Contract A	k : High	none	53.958	47.620	6.338***
			(2.28)	(0.00)	(2.28)
Contract B	i : Low	offset	22.104	20.000	2.104
			(2.15)	(0.00)	(2.15)
Contract B	j : Med	offset	31.192	30.000	1.192
			(1.43)	(0.00)	(1.43)
Contract B	k : High	offset	56.538	60.000	-3.462
			(4.40)	(0.00)	(4.40)
Contract C	i : Low	full	24.321	20.000	4.321*
			(2.20)	(0.00)	(2.20)
Contract C	j : Med	full	32.604	30.000	2.604
			(1.64)	(0.00)	(1.64)
Contract C	k : High	full	44.804	60.000	-15.196***
			(3.69)	(0.00)	(3.69)
Group 4 (Ability ∝ Precision):					
Contract A	i : Low	none	19.046	15.870	3.176*
			(1.69)	(0.00)	(1.69)
Contract A	j : Med	none	30.238	22.480	7.757***
			(2.34)	(0.00)	(2.34)
Contract A	k : High	none	52.450	56.340	-3.890
			(5.33)	(0.00)	(5.33)
Contract B	i : Low	offset	21.975	20.000	1.975
			(2.76)	(0.00)	(2.76)
Contract B	j : Med	offset	31.863	30.000	1.863
			(1.94)	(0.00)	(1.94)
Contract B	k : High	offset	56.179	60.000	-3.821
			(4.48)	(0.00)	(4.48)
Contract C	i : Low	full	20.888	20.000	0.888
			(1.86)	(0.00)	(1.86)
Contract C	j : Med	full	33.038	30.000	3.038**
			(1.27)	(0.00)	(1.27)
Contract C	k : High	full	64.104	60.000	4.104
			(3.35)	(0.00)	(3.35)
R-sqr			0.845	1.000	0.091
dfres			215	215	215
BIC			71681.8	.	71681.8
N			8640.0	8640.0	8640.0

* p<0.10, ** p<0.05, *** p<0.01

ability type to deliver predicted group level performance. We can also see in Table 3.18 why the addition of offset handicaps did not increase effort to the efficient level of the group in Table 3.17. The Low ability and Med ability types in Contract B ($CB - G1$, *offset*) do perform as predicted, however the High ability type shirked by a large amount -13.896, or 23.2% lower than predicted at the 1% level.

From the model, increasing idiosyncratic uncertainty for an agent increases his probability of winning holding all else constant. Therefore expected output is higher for any given level of effort. Without handicaps, agents with high uncertainty have incentives to work below efficient levels. The types endowed with the most uncertainty in their respective group include the High type in ($CA - G2$), the High type in ($CA - G3$), and the Low type in ($CA - G4$). In all three cases, subjects climbed beyond predicted at varying levels of significance for the *none* handicap case, suggesting having relatively low precision creates an incentive to overwork. If this is true, then we might see the opposite behavior (or at least the absence of climbing) from types endowed with high precision. The types endowed with the least uncertainty in the group include the Low type in ($CA - G2$), the Low type in ($CA - G3$) and the High type in ($CA - G4$). Both the Low type in ($CA - G2$) and the High type in ($CA - G4$) perform as predicted for the *none* case which supports the conjecture. However, we have already noted the curious climbing behavior of the Low type in ($CA - G3$) where low ability may be the dominating trait that drives the incentive in the opposite direction.

We now look at the handicapped contracts (CB/*offset* and CC/*full*) in Table 3.18. Both handicap contracts are designed to equate expected output as well as deliver the efficient level of effort for all types. Contract C also sets the probability of winning at $P = \frac{1}{3}$ for all agents in a group when they play the Nash equilibrium.

Of the 21 handicapped cases, only 4 types performed non-optimally as shown in the *Difference* column: the High type in $(CB-G1)$ under performed, while the Low type in $(CC-G3)$, the High type in $(CC-G3)$, and the Med type in $(CC-G4)$ all overworked at varying levels of significance.

We looked at how effort changed across treatments for each type. Using a Wald test, we find that effort is statistically the same across treatments for all 12 types except for the following two. First, the medium type in the ability group increased effort from 24.7 to 30.5 between Contract A $(CA-G1)$ and B $(CB-G1)$ at the 10% level, this is as predicted. Second, we observe non-predicted behavior from the high type in the ability $\propto$ uncertainty group. Subjects in this type significantly overworked from the predicted Nash by 6.3 in the untreated contract A $(CA-G3)$, yet in the full handicap treatment $(CC-G3)$, underworked by 15.196. This type is endowed with the highest ability and highest uncertainty in the group which is a distinct advantage over the other two group members in the untreated contract; with his chance of winning predicted to be 61.8%, this type was expected to shirk by 20.6%, but instead only shirked by 10%. When this type was treated with full handicaps, shirking increased to 25.3% when it was predicted to be zero. However, this type did behave as predicted during the offset handicap contract.

3.4.4 Design Effects

When contract order is controlled for, the significance of the pooled effort results only hold for the second contract stages. The significance of the pooled effort in the first stage is reversed; effort is statistically higher than the expected Nash and not statistically different from the efficient level of effort. We believe the higher effort present in the first contract stage is due to learning by doing. Splitting the 40 rounds of contract play into sets of 10 rounds each, the average effort of the first quartile is 35.41 and is statistically larger than the subsequent quartiles which are all statistically the same as the expected Nash. Replicating these tests on the third contract stage, average effort is recorded to be 32.66 and is statistically identical to the expected Nash, adding support to this conclusion. The third stage result holds even when controlling for which contract stage is repeated.

The concern preferences for the status quo may be present in the contract choice mechanism was tested. A dummy to indicated the last contract played was created to test if subjects preferred to re-play the most recently played contract more often. This dummy was added to the contract choice, gradient choice, and the fairness difference regressions in section 3.4.1 with insignificant coefficient results of 0.056 (0.06), 0.048 (0.05), and 0.017 (0.03) respectively.

3.5 Conclusion

This study uses experiments to examine issues of fairness and efficiency in rank-order tournaments with heterogeneous agents. We consider settings where agents are different both in their abilities and in their random shocks and evaluate agent performance under three contracts that vary in perceptions of equal opportunity: equal access, equal expected earnings, and equal changes of winning. After gaining experience, agents choose which role they would prefer to play if they had a chance to play again. They also vote on which contract to re-play in a third round while remaining in their current role to elicit preferences for fairness. Results in both effort and contract choice observed at the firm level and the group level appear neutral even though individual contributions and choices are not, suggesting that firms with limited ability to monitor results may have a difficult time evaluating both efficiency and equity in the workforce.

The primary result is that having equal pay in the form of expected outcomes promotes efficiency and well as greater perceptions of fairness than does having equal access. The results for groups composed of homogeneous agents and groups composed of agents with heterogeneous ability support previous findings in the literature. Without handicaps, in groups with agents that differ only in ability, agents perform as predicted and offset handicaps improve performance. Supported by results observed in contract choice, fairness elicitations, and role preferences, both the low ability and medium ability types prefer equal pay over equal access and the high ability type prefers equal access over equal pay. Introducing complex combinations of ability and uncertainty do not lead to the same results and opens the door for further analysis to isolate the competing incentives in Groups 3 and 4.

Differences in shock uncertainty did not have the same level of effect on participants as differences in ability. One reason for this might be that not all agent types perceive the advantages gained with an increase in uncertainty. This reaction may also be due to risk aversion. It may also be the case that when ability is homogeneous, precision is viewed as an advantage in an equal access setting or that the size of the loss that comes from greater uncertainty is undesirable in a loss averse context, even though losing by a little is the same as losing by a lot. However, one exception to this observation are the agents endowed with high uncertainty. High uncertainty agents in Group 2 prefer their role over any other in equal access settings, while the low and medium types do not prefer to be high types.

In general, role preference results suggest equal pay increases the perception of procedural fairness in all group compositions over equal access, and that equal chance also improves perceptions of equality over equal access, but may not have significant benefits in terms of perceived fairness over equal pay.

The disadvantaged types may find it necessary to pay the advantaged types rents to enter into fair contracts. This is because in every group setting, the advantaged types consider the equal access contract to be just as fair as the equal pay or the equal chance contract and in most cases, having an advantage in an equal access contract resulted in higher earnings. In contrast, the disadvantaged types considered equal access to be the least fair contract of all and that having a disadvantage in an equal access contract resulted in lower earnings compared to either the equal pay or equal chance contracts. This contrast supports the premise that fairness is a matter of perspective.

Bibliography

Balafoutas, L., Kerschbamer, R., and Sutter, M. (2012). Distributional preferences and competitive behavior. *Journal of Economic Behavior & Organization*, 83(1):125–135.

Barry, B. M. (1990). *Political argument: a reissue with a new introduction*. University of California Press, Berkeley and Los Angeles, CA.

Bhattacharya, Sudipto and Guasch, J. L. (1988). Heterogeneity, Tournaments, and Hierarchies. *Journal of Political Economy*, 96(4):867 – 881.

Bolton, G. E., Brandts, J., and Ockenfels, A. (2005). Fair Procedures: Evidence from Games Involving Lotteries. *The Economic Journal*, 115(506):1054–1076.

Bolton, G. E. and Ockenfels, A. (2000). ERC: A Theory of Equity. Reciprocity. and Competition. *The American Economic Review*, 90(1):166–193.

Bronars, S. (1987). Risk-Taking in Tournaments. *Working Paper, Austin TX: University of Texas*.

Bull, C., Schotter, A., and Weigelt, K. (1987). Tournaments and Piece Rates : An Experimental Study. *Journal of Political Economy*, 95(1):1–33.

Cappelen, A. W., Konow, J., Sø rensen, E. O., and Tungodden, B. (2013). Just Luck : An Experimental Study of Risk Taking and Fairness. *American Economic Review*, 103(4):1398–1413.

Dechenaux, E., Kovenock, D., and Sheremeta, R. M. (2012). A Survey of Experimental Research on Contests , All-Pay Auctions and Tournaments. *Social Sciences Research Network*.

Dijk, F. V., Sonnemans, J., and Winden, F. V. (2001). Incentive systems in a real effort experiment. *European Economic Review*, 45:187–214.

Fehr, E. and Schmidt, K. M. (1999). A Theory of Fairness, Competition, and Cooperation. *The Quarterly Journal of Economics*, 114(3):817–868.

Fischbacher, U. (2007). Z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, 10:171–178.

Fishkin, J. S. (1983). *Justice, Equal Opportunity, and the Family.* Yale University Press.

Gaba, A., Tsetlin, I., and Winkler, R. L. (2004). Modifying Variability and Correlations in Winner-Take-All Contests. *Operations Research*, 52(3):384–395.

Gilpatric, S. M. (2009). Risk Taking in Contests and the Role of Carrots and Sticks. *Economic Inquiry*, 47(2):266–277.

Gneezy, U., Niederle, M., and Rustichini, A. (2003). Performance in Competitive Environments: Gender Differences. *Quarterly Journal of Economics*, 118(3):1049–1074.

Green, J. R. and Stokey, N. L. (1983). A Comparison of Tournaments and Contracts. 91(3):349–364.

Greiner, B. (2004). An Online Recruitment Sytem for Economic Experiments.

Grund, C. and Sliwka, D. (2005). Envy and Compassion in Tournaments. *Journal of Economics & Management Strategy*, 14(1):187–207.

Gürtler, O. and Harbring, C. (2010). Feedback in tournaments under commitment problems: Experimental evidence. *Journal of Economics and Management Strategy*, 19(3):771–810.

Gürtler, O. and Kräkel, M. (2010). Optimal tournament contracts for heterogeneous workers. *Journal of Economic Behavior & Organization*, 75(2):180–191.

Hammond, R. G. and Zheng, X. (2013). Heterogeneity in tournaments with incomplete information: An experimental analysis. *International Journal of Industrial Organization*, 31(3):248–260.

Harbring, C. and Irlenbusch, B. (2005). Incentives in Tournaments with Endogenous Prize Selection. *Journal of Institutional and Theoretical Economics*, 161(4):636–663.

Harbring, C. and Lunser, G. K. (2008). On the Competition of Asymmetric Agents. *German Economic Review*, 9(3):373–395.

Holmstrom, B. (1982). Moral hazard in teams. 13(2):324–340.

Hvide, H. K. (2002). Tournament Rewards and Risk Taking. *Journal of Labor Economics*, 20(4):877–898.

Kempf, a. and Ruenzi, S. (2008). Tournaments in Mutual-Fund Families. *The Review of Financial Studies*, 21(2):1013–1036.

Kempf, A., Ruenzi, S., and Thiele, T. (2009). Employment risk, compensation incentives, and managerial risk taking: Evidence from the mutual fund industry. *Journal of Financial Economics*, 92(1):92–108.

Knoeber, C. R. (1989). A Real Game of Chicken : Contracts , Tournaments , the Production of Broilers. *Journal of Law, Economics, & Organization*, 5(2):271–292.

Knoeber, C. R. and Thurman, W. N. (1994). Testing the Theory of Tournaments : An Empirical Analysis of Broiler Production. *Journal of Labor Economics*, 12(2):155–179.

Knoeber, C. R. and Thurman, W. N. (1995). "Don't Count Your Chickens...": Risk and Risk Shifting in the Broiler Industry. *American Journal of Agricultural Economics*, 77(3):486–496.

Konow, J. (2003). Which Is the Fairest One of All? A Positive Analysis of Justice Theories. *Journal of Economic Literature*, XLI(December):1188–1239.

Krawczyk, M. W. (2009). A model of procedural and distributive fairness. *Theory and Decision*, 70(1):111–128.

Lavy, V. (2002). Evaluating the Effect of Teachers' Group Performance Incentives on Pupil Achievement. *Journal of Political Economy*, 110(6):1286–1317.

Lavy, V. (2009). Performance Pay and Teachers' Effort, Productivity, and Grading Ethics. *American Economic Review*, 5(99):1979–2011.

Lazear, E. P. and Rosen, S. (1981). Rank-Order Tournaments as Optimum Labor Contracts. *The Journal of Political Economy*, 89(5):841–864.

Levitt, S. D. (1995). Optimal incentive schemes when only the agents' "best" output matters to the principal. *RAND Journal of Economics*, 26(4):744–760.

Malcomson, J. M. (1984). Work Incentives , Hierarchy , and Internal Labor Markets. *Journal of Political Economy*, 92(3):486–507.

Nalebuff, B. J. and Stiglitz, J. E. (1983). Prizes and Incentives: Towards a General Theory of Compensation and Competition. *The Bell Journal of Economics*, 14(1):21–43.

Nieken, P., Irlenbusch, B., and Sliwka, D. (2010). On the Choice of Risk and Effort in Tournaments - Experimental Evidence. *Journal of Economics & Management Strategy*, 19(3):811–840.

Nieken, P. and Sliwka, D. (2010). Risk-taking tournaments - Theory and experimental evidence. *Journal of Economic Psychology*, 31(3):254–268.

O'Keeffe, M., Viscusi, W. K., and Zeckhauser, R. J. (1984). Economic Contests : Comparative Reward Schemes. *Journal of Labor Economics*, 2(1):27–56.

Orrison, A., Schotter, A., and Weigelt, K. (2004). Multiperson Tournaments: An Experimental Examination. *Management Science*, 50(2):268–279.

Prahalad, C. K. and Hamel, G. (1990). The Core Competence of the Corporation. *Harvard Business Review*, May:1–15.

Rawls, J. (1971). *A Theory of Justice: Revised Edition*. Harvard University Press.

Roch, S. G., Sternburgh, A. M., and Caputo, P. M. (2007). Absolute vs Relative Performance Rating Formats: Implications for fairness and organizational justice. *International Journal of Selection and Assessment*, 15(3):302–316.

Rousseau, V., Salek, S., Aubé, C., and Morin, E. M. (2009). Distributive justice, procedural justice, and psychological distress: the moderating effect of coworker support and work autonomy. *Journal of occupational health psychology*, 14(3):305–17.

Saito, K. (2013). Social Preferences under Risk: Equality of Opportunity versus Equality of Outcome. *American Economic Review*, 103(7):3084–3101.

Schotter, A. and Weigelt, K. (1992). Asymmetric tournaments, equal opportunity laws, and affirmative action: some experimental results. *The Quarterly Journal of Economics*, (May).

Tsetlin, I., Gaba, A., and Winkler, R. L. (2004). Strategic Choice of Variability in Multiround Contests and Contests with Handicaps. *The Journal of Risk and Uncertainty*, 2(29):143–158.

Tyler, T. R. (1989). The psychology of procedural justice: A test of the group-value model. *Journal of Personality and Social Psychology*, 57(5):830–838.

Wu, S. and Roe, B. (2006). Tournaments , Fairness , and Risk. *American Journal of Agricultural Economics*, 88(3):561–573.

Wu, S., Roe, B., and Sporleder, T. (2006). Mixed Tournaments, Common Shocks, and Disincentives: An Experimental Study.

Appendices

Appendix A

Competition and Core Competency: Risk Independence as a Strategy

A.1 Proof of Proposition 1.1

For any point of symmetry (μ^s, α^s), the marginal effect of α_i on P_i in equation (1.2.5) can be simplified. The expression in F reduces to the idiosyncratic shock $\kappa = \varepsilon_i$ and

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{n-1}{1+\alpha_j} \iint (\varepsilon_i - x) f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \, d\varepsilon_i \, g(x) \, dx$$

can be simplified by first evaluating the integral of x. The integral is a linear combination and can be separated into two components.

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{n-1}{1+\alpha_j} \iint \varepsilon_i f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \, d\varepsilon_i \, g(x) \, dx$$

$$-\frac{n-1}{1+\alpha_j} \iint f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \; d\varepsilon_i \, xg(x) \, dx.$$

Since ε_i and x are *i.i.d.*, the double integrals can be rewritten as products of single integrals

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{n-1}{1+\alpha_j} \int \varepsilon_i f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \; d\varepsilon_i \int g(x) \, dx$$

$$-\frac{n-1}{1+\alpha_j} \int f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \; d\varepsilon_i \int xg(x) \, dx.$$

The integrals $\int g(x) \, dx = G(x)|_{-\infty}^{\infty} = 1$ and $\int xg(x) \, dx = E[x] = 0$; hence, the marginal effect reduces to the single integral

$$\frac{\partial P_i}{\partial \alpha_i} = \frac{n-1}{1+\alpha} \int \varepsilon_i f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} \; d\varepsilon_i.$$

For case (i), when $n = 2$, the term $F(\varepsilon_i)^{n-2} = 1$ and the expression of the inner integral reduces to the expectation of the idiosyncratic shock weighted by its own symmetric distribution $E[\varepsilon_i f(\varepsilon_i)] = \int \{\varepsilon_i f(\varepsilon_i)\} f(\varepsilon_i) \; d\varepsilon_i = 0$; hence $\frac{\partial P_i}{\partial \alpha_i} = 0$.

For case (ii), when $n > 2$, an inequality across the support of ε_i results from the fact that $F(\varepsilon_i)^{n-2} \in (0,1)$ and is increasing in ε_i. Hence, every negative value of ε_i is outweighed by a corresponding and equally likely positive value of ε_i such that $F(-\varepsilon_i)^{n-2} < F(\varepsilon_i)^{n-2}$ for all i; therefore, the integral is positive. The terms $\frac{n-1}{1+\alpha}$ in front of the integral are always positive; hence $\frac{\partial P_i}{\partial \alpha_i} > 0$.

A.2 The Relative Variance Condition

Assume agent i only changes his level of independence α_i but continues to hold his level of effort constant at $\mu_i = \mu^*$. Recall that the probability of winning at the symmetric Nash is $\frac{1}{n}$. In comparison, the probability of winning for agent i from equation (1.2.3) becomes

$$P_i = \iint g(x)\, f(\varepsilon_i)\, F\left(\frac{1+\alpha_i}{1+\alpha^*}\varepsilon_i + \frac{\alpha^* - \alpha_i}{1+\alpha^*}x\right)^{n-1} d\varepsilon_i\, dx.$$

Because F is raised to $n-1$, increasing the variance in the arguments in F can be shown to increase the probability of winning for $n > 2$ by moving more density to the right tail of the distribution. Note that for an arbitrary function $R = \int s(y)\, S(\theta y)^{n-1}\, dy$ where θ is a scaling parameter and where $y \sim i.i.d.\left(0, \sigma_y^2\right)$ is symmetric and unimodal with pdf of s and cdf of S, then $\frac{\partial R}{\partial \theta} = (n-1) \int ys(y)\, s(\theta y)\, S(\theta y)^{n-2}\, dy$. Increasing θ increases the variance of the argument of S, $var = \theta^2 \sigma_y^2$. For $n = 2$, $\frac{\partial R}{\partial \theta} = 0$. For $n > 2$, R is increasing in θ, $\frac{\partial R}{\partial \theta} > 0$; hence, the function R is increasing with variance of the argument. We use this result to simplify the problem. We analyze the variance of the argument in F in what follows to show when $P_i \leq \frac{1}{n}$.

Since ε_i and x are $i.i.d.$, the variance of the argument in F is $\left(\frac{1+\alpha_i}{1+\alpha^*}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\alpha^*-\alpha_i}{1+\alpha^*}\right)^2 \sigma_x^2$. Noting that the variance of the argument at the Nash is σ_ε^2, we can solve for a ratio of $\sigma_x^2 : \sigma_\varepsilon^2$ that describes when $P_i \leq \frac{1}{n}$.

$$\frac{\sigma_x^2}{\sigma_\varepsilon^2} \leq \frac{2 + \alpha^* + \alpha_i}{\alpha^* - \alpha_i}. \tag{A.2.1}$$

The upper bound of this requirement is infinite as $\alpha_i \to \alpha^*$, meaning σ_x^2 can be very large compared to σ_ε^2 for a local region around α^* and the Nash will still hold. However, the lower bound requirement will depend upon μ_i and the explicit cost functions in (1.2.11), but it will be found where ${}^{\sigma_x^2}/_{\sigma_\varepsilon^2} \ll 3$ for the extreme case when the Nash is at $\alpha^* = 1$ and an agent chooses $\alpha_i = 0$. Equation (A.2.1), therefore, is a necessary but not sufficient condition for a Nash Equilibrium.

To see why this is not a sufficient condition, consider the case of equality when $\sigma_x^2 = 3\sigma_\varepsilon^2$ at the lower bound, agent i can maintain his probability of winning $P_i = \frac{1}{n}$ while holding his level of effort constant at $\mu_i = \mu^*$ by adhering to the firm's core competency even when all other agents are fully independent. Since his costs of effort are less at $\alpha_i = 0$, his expected utility is greater than at the Nash which violates equilibrium. If he can also re-optimize $\mu_i > \mu_i^*$, he will increase $P_i > \frac{1}{n}$ and further increase his expected utility by working harder than everyone else because he is also now more efficient, which imposes an even stronger restriction on ${}^{\sigma_x^2}/_{\sigma_\varepsilon^2}$.

Consequently, the cost functions in the global constraint in equation (1.2.11) require the ratio of σ_x^2 and σ_ε^2 to be even smaller than shown in equation (A.2.1) to insure P_i is small enough at $\alpha_i = 0$ to offset any gains from increased effort $\mu_i > \mu_i^*$.

A.3 Proof of Proposition 1.2

From the model assumptions, at a point $\mu_i = 0$ and $\alpha_i = 0$, costs $c(0) = c'(0) = 0$ and $z(0) = z'(0) = 0$; i.e. there is no marginal cost to increase effort or independence from zero. The incentive to increase effort from zero comes from equation (1.2.4), namely $\frac{\partial P_i}{\partial \mu_i} > 0$ and the incentive to increase independence from zero is shown in Proposition 1.1, $\frac{\partial P_i}{\partial \alpha_i} > 0$. At a Nash equilibrium with the constraint $S > 0$, the right hand side of both equations (1.2.9) and (1.2.10) are positive. It then follows that

$\mu^* > 0$ and $\alpha^* > 0$.

A.4 Proof of Proposition 1.3

From equations (1.2.18), calculate the first spread $S_1 = W_1 - W_2$ to be

$$S_1\left(\hat{\mu},\hat{\alpha}\right) = m + \frac{\lambda\left(n-2\right)}{2nk}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right) + \frac{V}{2h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)$$

$$-m + \frac{\lambda}{nk}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right)$$

$$S_1\left(\hat{\mu},\hat{\alpha}\right) = \frac{\lambda}{2k}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right) + \frac{V}{2h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)$$

From equations (1.2.18), calculate the second spread $S_2 = W_2 - W_3$ to be

$$S_2\left(\hat{\mu},\hat{\alpha}\right) = m - \frac{\lambda}{nk}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right)$$

$$-m - \frac{\lambda\left(n-2\right)}{2nk}\left(1+\alpha^*\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right) + \frac{V}{2h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)$$

$$S_2\left(\hat{\mu},\hat{\alpha}\right) = -\frac{\lambda}{2k}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right) + \frac{V}{2h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)$$

The difference of the spreads is $S_1 - S_2 = \frac{\lambda}{k}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right)$. Then the first order condition in equation (1.2.17) is

$$\lambda z^{'}\left(\alpha^{*}\right)c\left(\mu_{i}^{*}\right)=\frac{\left(\frac{\lambda}{k}\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right)\right)k}{1+\alpha^{*}},$$

which reduces to the first symmetric equation

$$\left(1+\alpha^{*}\right)z^{'}\left(\alpha^{*}\right)c\left(\mu^{*}\right)=\left(1+\hat{\alpha}\right)z^{'}\left(\hat{\alpha}\right)c\left(\hat{\mu}\right).$$

The sum of the spreads is then $S_1+S_2=\frac{c^{'}(\hat{\mu})}{h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right).$ Then the first order condition in equation (1.2.16) is

$$\left(1+\lambda z\left(\alpha^{*}\right)\right)c^{'}\left(\mu^{*}\right)=\frac{\left(\frac{c^{'}(\hat{\mu})}{h}\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)\right)h}{1+\alpha^{*}},$$

which reduces to the second symmetric equation

$$\left(1+\alpha^{*}\right)\left(1+\lambda z\left(\alpha^{*}\right)\right)c^{'}\left(\mu^{*}\right)=\left(1+\hat{\alpha}\right)\left(1+\lambda z\left(\hat{\alpha}\right)\right)c^{'}\left(\hat{\mu}\right).$$

Given the two symmetric equations and that z is a monotonically increasing function of α, and that c is a monotonically increasing function of μ, it follows that $\alpha^{*}=\hat{\alpha}$ and $\mu^{*}=\hat{\mu}$ simultaneously.

Appendix B

Fair Tournaments in Common Environments

B.1 Equal Variance

The fair criterion 2. For the case $n > 2$, the outcome that unequal variance leads to a non-symmetric probability of winning is shown for the case of no common shocks and when uncertainty is distributed proportional to ability $a_i = \alpha_i$ and hence $\sigma_i^2 = a_i^2 \sigma^2$. We show that from a symmetric equilibrium, a change in one agent's variance changes the probability of winning.

Proof. Let all agents have ratio handicaps equal to their inverted ability $\phi_i = \frac{1}{a_i}$. At efficient equilibrium, $\frac{\mu_i^o}{a_i} = \frac{\mu_j^o}{a_j}$ from equation (2.2.7), effort terms cancel in the probabilities of equation (2.2.2). The variance of the handicapped distribution is $E\left[(\phi_i \varepsilon_i)^2\right] = E\left[\left(\frac{\varepsilon_i}{a_i}\right)^2\right] = \sigma_\kappa^2$ and idiosyncratic uncertainty becomes $\frac{\varepsilon_i}{a_i} = \kappa_i$ for all i; hence $f = z_j$ and $F = Z_j$ for all j. The probability of winning for agent i is then $P_i = \int f(\kappa_i) F(\kappa_i)^{n-1} d\kappa_i$,

$$P_i = \frac{1}{n} F(\kappa_i)^n \bigg|_{-\infty}^{\infty} = \frac{1}{n}[1-0]^n = \frac{1}{n}.$$

To find out what happens to the probability of winning when the variance increases, take the comparative static of probability with respect to the standard deviation $\frac{\partial P_i}{\partial \sigma_i} = \int \frac{\partial f(\kappa_i)}{\partial \sigma_i} F(\kappa_i)^{n-1} d\kappa_i$. Since f is symmetric about zero, $\frac{\partial f(\kappa_i)}{\partial \sigma_i} = \frac{\partial f(-\kappa_i)}{\partial \sigma_i}$ and $F(\kappa_i) = 1 - F(-\kappa_i)$. Then the comparative static can be rewritten as

$$\frac{\partial P_i}{\partial \sigma_i} = \int_0^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} \left[F(\kappa_i)^{n-1} + \left(1 - F(\kappa_i)^{n-1}\right)\right] d\kappa_i.$$

Define $b(\kappa_i) = \left[F(\kappa_i)^{n-1} + \left(1 - F(\kappa_i)^{n-1}\right)\right]$, then

$$\frac{\partial b(\kappa_i)}{\partial \kappa_i} = \frac{(n-1)}{a_i} f(\kappa_i) \left[F(\kappa_i)^{n-2} + \left(1 - F(\kappa_i)^{n-2}\right)\right]$$

For $\kappa_i \epsilon [0, \infty)$, $F(\kappa_i) \geq (1 - F(-\kappa_i))$, therefore $\frac{\partial b(\kappa_i)}{\partial \kappa_i} > 0$. The comparative static can then be written $\frac{\partial P_i}{\partial \sigma_i} = \int_0^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} b(\kappa_i) d\kappa_i$. The term $\frac{\partial f(\kappa_i)}{\partial \sigma_i}$ is decreasing for κ_i close to zero and increasing for values further from zero at both ends of the distribution. Therefore, define k such that $\frac{\partial f(\kappa_i)}{\partial \sigma_i} < 0$ for $-k < \kappa_i < k$ and $\frac{\partial f(\kappa_i)}{\partial \sigma_i} > 0$ otherwise.

Since $\int_0^{\infty} f(\kappa_i) d\kappa_i = \frac{1}{2}$ for any σ_i^2, then $\int_0^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} d\kappa_i = 0$ and it must be the case that $\int_0^{k} \frac{\partial f(\kappa_i)}{\partial \sigma_i} d\kappa_i = \int_k^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} d\kappa_i$. The comparative static can be rewritten again as

$$\frac{\partial P_i}{\partial \sigma_i} = \int_0^{k} \frac{\partial f(\kappa_i)}{\partial \sigma_i} b(\kappa_i) d\kappa_i + \int_k^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} b(\kappa_i) d\kappa_i.$$

The first term is negative and the second term is positive. Since $b(\kappa_i)$ is strictly increasing on $\kappa_i \epsilon [0, \infty)$, then $\int_0^{k} \frac{\partial f(\kappa_i)}{\partial \sigma_i} b(\kappa_i) d\kappa_i < \int_k^{\infty} \frac{\partial f(\kappa_i)}{\partial \sigma_i} b(\kappa_i) d\kappa_i$ and $\frac{\partial P_i}{\partial \sigma_i} > 0$. $\square$

An increase in variance increases the probability of winning at the efficient equilibrium for $n > 2$ handicapped agents.

B.2 Proof of Proposition 2.1

The proof that efficient tournaments in equilibrium with agents of heterogeneous ability and unique idiosyncratic uncertainty distributions who experience common shocks s and θ are background fair when evaluated using the outcome rule $r_i = \frac{1}{\alpha_i}\left(q_i - s - \theta a_i \mu^o\right)$ is as follows.

Proof. From the hypothesis, a fair and efficient tournament using the evaluation rule $r_i = \frac{1}{\alpha_i}\left(q_i - s - \theta a_i \mu^o\right)$ must satisfy the fair criteria of equal probabilities, equal variance, and no common shock bias. Given $\phi_i = \frac{1}{\alpha_i}$, $h_i = -\frac{1}{\alpha_i}\left(s + \theta a_i \mu^o\right)$, $\sigma_i^2 = \alpha_i^2 \sigma_\kappa^2$, and expanding $r_i = \frac{1}{\alpha_i}\left(\theta \mu_i^o + s + \varepsilon_i - s - \theta a_i \mu^o\right)$, then

Criterion 2 - Equal variance: $E\left[\left(\phi_i \varepsilon_i\right)^2\right] = E\left[\left(\frac{\varepsilon_i}{\alpha_i}\right)^2\right] = \frac{\alpha_i^2 \sigma^2}{\alpha_i^2} = \sigma_\kappa^2$ is constant for all i.

Criterion 1 - Equal probabilities: from equation (2.2.2),

$$P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) \prod_{j \neq i}^{n-1} Z_j \left(\frac{\varepsilon_i}{\alpha_i} + \frac{\theta \mu_i^o}{\alpha_i} - \frac{\theta \mu_j^o}{\alpha_j} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j}\right)(s - s) - \theta \mu^o \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j}\right)\right) \, d\varepsilon_i.$$

Satisfying $\mu_i^o = a_i \mu^o$ from equation (2.2.7), the terms $\frac{\theta \mu_i^o}{\alpha_i} = \frac{\theta a_i \mu^o}{\alpha_i}$ cancel for all i. The additive common shock is perfectly observed and symmetric and cancels out. Since the equal variance criterion is satisfied above, the terms $\frac{\varepsilon_i}{\alpha_i}$ and $\frac{\varepsilon_j}{\alpha_j}$ are *i.i.d.*, then $f = z_j$ and $F = Z_j$ for all j which leads to $P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) F\left(\frac{\varepsilon_i}{\alpha_i}\right)^{n-1} d\varepsilon_i$,

$$P_i = \frac{1}{n} F\left(\frac{\varepsilon_i}{\alpha_i}\right)^n \Bigg|_{-\infty}^{\infty} = \frac{1}{n}\left[1 - 0\right]^n = \frac{1}{n}.$$

Criterion 3 - No common shock bias: Using the probability from the proof of Criterion 1 above, $\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s} \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) F\left(\frac{\varepsilon_i}{\alpha_i}\right)^{n-1} d\varepsilon_i = 0$ and
$\frac{\partial P_i}{\partial \theta} = \frac{\partial}{\partial \theta} \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) F\left(\frac{\varepsilon_i}{\alpha_i}\right)^{n-1} d\varepsilon_i = 0$ □

B.3 Proof of Proposition 2.2

Proof. From equation (2.2.2) and the rank rule $r_i = \frac{1}{\alpha_i}\left(q_i - a_i \mu^o\right)$, the probability of i winning with a single draw from the distribution of the common shock at the efficient equilibrium is

$$P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) \prod_{j \neq i}^{n-1} Z_j \left(\left(\frac{\varepsilon_i}{\alpha_i} + \frac{1}{\alpha_i}\theta\mu_i^o + \frac{1}{\alpha_i}s - \frac{a_i}{\alpha_i}\mu^o\right) - \left(\frac{1}{\alpha_j}\theta\mu_j^o + \frac{1}{\alpha_j}s - \frac{a_j}{\alpha_j}\mu^o\right)\right) d\varepsilon_i$$

recall $\mu_i^o = a_i \mu^o$in equation (2.2.7),

$$P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) \prod_{j \neq i}^{n-1} Z_j \left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j}\right) s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j}\right)(\theta - 1)\mu^o\right) d\varepsilon_i$$

where the term $\frac{a_i}{\alpha_i}$ is the ability to variance ratio for agent i. Taking the comparative static of the probability with respect to θ yields

$$\frac{\partial P_i}{\partial \theta} = \sum_{j \neq i}^{n-1} \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j} \right) \mu^o$$

$$\times \int f\left(\frac{\varepsilon_i}{\alpha_i}\right) \sum_{j \neq i}^{n-1} \left\{ z_j \left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j} \right) s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j} \right) (\theta - 1) \mu^o \right) \right.$$

$$\left. \times \prod_{k \neq j \neq i}^{n-2} Z_k \left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j} \right) s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j} \right) (\theta - 1) \mu^o \right) \right\} d\varepsilon_i.$$

The integral is everywhere positive and the summation of the differences in the ability to variance ratios in the first term will determine the sign of $\frac{\partial P_i}{\partial \theta}$. The coefficient term can be rearranged to $(n-1)\mu^o \frac{a_i}{\alpha_i} - \mu^o \sum_{j \neq i}^{n-1} \frac{a_j}{\alpha_j}$. Add and subtract the term $\mu^o \frac{a_i}{\alpha_i}$ yields $n\mu^o \left(\frac{a_i}{\alpha_i} - \frac{1}{n} \sum_j^n \frac{a_j}{\alpha_j} \right)$. For case (i), when $a_j = \alpha_j$ for all j, $n\mu^o \left(1 - \frac{n}{n}\right) = 0$ and $\frac{\partial P_i}{\partial \theta} = 0$. For case (ii), when $a_j \neq \alpha_j$ for at least one j, $\frac{\partial P_i}{\partial \theta} > 0$ if $\frac{a_i}{\alpha_i} > \frac{1}{n} \sum_j^n \frac{a_j}{\alpha_j}$. Generalizing, the probability for player i changes with a change in multiplicative common shock difference according to

$$\frac{\partial P_i}{\partial \theta} \begin{cases} > 0 & if \quad \frac{a_i}{\alpha_i} > \frac{1}{n} \sum_j^n \frac{a_j}{\alpha_j} \\ = 0 & if \quad \frac{a_i}{\alpha_i} = \frac{1}{n} \sum_j^n \frac{a_j}{\alpha_j} \\ < 0 & if \quad \frac{a_i}{\alpha_i} < \frac{1}{n} \sum_j^n \frac{a_j}{\alpha_j} \end{cases}.$$

$\square$

B.4 Proof of Proposition 2.3

The proof of proposition 2.3 is as follows.

Proof. From equation (2.2.2) and the rank rule$r_i = \frac{1}{\alpha_i}\left(q_i - a_i\mu^o\right)$, the probability of i winning with a single draw from the distribution of the common shock at the efficient equilibrium is

$$P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right)\prod_{j\neq i}^{n-1} Z_j\left(\left(\frac{\varepsilon_i}{\alpha_i} + \frac{1}{\alpha_i}\theta\mu_i^o + \frac{1}{\alpha_i}s - \frac{a_i}{\alpha_i}\mu^o\right) - \left(\frac{1}{\alpha_j}\theta\mu_j^o + \frac{1}{\alpha_j}s - \frac{a_j}{\alpha_j}\mu^o\right)\right) d\varepsilon_i$$

recall $\mu_i^o = a_i\mu^o$in equation (2.2.7),

$$P_i = \int f\left(\frac{\varepsilon_i}{\alpha_i}\right)\prod_{j\neq i}^{n-1} Z_j\left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j}\right)s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j}\right)(\theta - 1)\,\mu^o\right) d\varepsilon_i$$

where the term $\frac{a_i}{\alpha_i}$ is the ability to variance ratio for agent i. Taking the comparative static of the probability with respect to s yields

$$\frac{\partial P_i}{\partial s} = \sum_{j\neq i}^{n-1}\left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j}\right) \qquad \times$$

$$\int f\left(\frac{\varepsilon_i}{\alpha_i}\right)\sum_{j\neq i}^{n-1}\left\{z_j\left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j}\right)s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j}\right)(\theta - 1)\,\mu^o\right) \qquad \times\right.$$

$$\prod_{k\neq j\neq i}^{n-2} Z_k \left(\frac{\varepsilon_i}{\alpha_i} + \left(\frac{1}{\alpha_i} - \frac{1}{\alpha_j} \right) s + \left(\frac{a_i}{\alpha_i} - \frac{a_j}{\alpha_j} \right) (\theta - 1)\, \mu^o \right) \Bigg\} d\varepsilon_i.$$

The integral is everywhere positive, and the summation of the differences in handicaps will determine the sign of $\frac{\partial P_i}{\partial s}$. The first term can be rearranged to $(n-1)\frac{1}{\alpha_i} - \sum_{j\neq i}^{n-1} \frac{1}{\alpha_j}$. Add and subtract the term $\frac{1}{\alpha_i}$ yields $n\left(\frac{1}{\alpha_i} - \frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}\right)$. For the identically distributed case, let $\alpha = \alpha_j$ for all j, then $n\left(\frac{1}{\alpha} - \frac{1}{\alpha}\right) = 0$ and $\frac{\partial P_i}{\partial s} = 0$. When $\alpha \neq \alpha_j$ for at least one j, $\frac{\partial P_i}{\partial s} > 0$ if $\frac{1}{\alpha_i} > \frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}$. For convenience, the inequality can be inverted such that $\alpha_i < \left(\frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}\right)^{-1}$. Generalizing, the probability for player i changes with a change in additive common shock difference according to

$$\frac{\partial P_i}{\partial s} \begin{cases} > 0 & if \quad \alpha_i < \left(\frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}\right)^{-1} \\ = 0 & if \quad \alpha_i = \left(\frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}\right)^{-1} \\ < 0 & if \quad \alpha_i > \left(\frac{1}{n}\sum_{j}^{n}\frac{1}{\alpha_j}\right)^{-1} \end{cases}.$$

□

B.5 Proof of Corollary 2.1

The proof that efficient tournaments with agents of identical ability and identical idiosyncratic uncertainty distributions who experience common shocks s and θ are background fair when evaluated using the outcome rule $r_i = q_i$ is as follows.

Proof. Given $\phi_i = 1$ and $h_i = 0$, *2.1.1. Criterion 1:* From equation (2.2.2),

$$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \left(\mu_i^o - \mu_j^o\right)\theta + s - s\right) d\varepsilon_i.$$

From equation (2.2.7), $\mu_i^o = \mu_j^o$ and the effort and common shock terms are symmetric. Since ε_i and ε_j are *i.i.d.*, then $f = z_j$ and $F = Z_j$ which reduces the probability to $P_i = \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i$,

$$P_i = \frac{1}{n} F(\varepsilon_i)^n \Big|_{-\infty}^{\infty} = \frac{1}{n}[1-0]^n = \frac{1}{n}.$$

2.1.2. Criterion 2: Using the probability from the proof of *2.1.1* above,
$\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s} \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i = 0$ and
$\frac{\partial P_i}{\partial \theta} = \frac{\partial}{\partial \theta} \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i = 0.$.

2.1.3. Criterion 3: is met by hypothesis. □

B.6 Proof of Corollary 2.2

The proof that efficient tournaments with agents of different ability and identical idiosyncratic uncertainty distributions who experience common shocks s and θ, (i) are background fair using $r_i = q_i - \theta\mu_i^o$; (ii) are not background fair using $r_i = q_i$; (iii) are not background fair using $r_i = \frac{1}{a_i}(q_i - s)$ is as follows.

Proof. 2.2.(i) Using the evaluation rule $r_i = q_i - \theta\mu_i^o$ produces a fair outcome: Given $\phi_i = 1$ and $h_i = -\theta\mu_i^o$, *2.2.(i).1. Criterion 1:* From equation (2.2.2),

$$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \left(\mu_i^o - \mu_j^o\right)\theta + s - s - \theta\mu_i^o + \theta\mu_j^o\right) d\varepsilon_i$$

From equation (2.2.7), the effort and common shock terms are symmetric. Since ε_i and ε_j are *i.i.d.*, then $f = z_j$ and $F = Z_j$ which reduces the probability to $P_i = \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i$,

$$P_i = \frac{1}{n} F(\varepsilon_i)^n \Big|_{-\infty}^{\infty} = \frac{1}{n}[1-0]^n = \frac{1}{n}.$$

2.2.(i).2. Criterion 2: Using the probability from the proof of *2.2.(i).1* above,
$\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s} \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i = 0$ and
$\frac{\partial P_i}{\partial \theta} = \frac{\partial}{\partial \theta} \int f(\varepsilon_i) F(\varepsilon_i)^{n-1} d\varepsilon_i = 0.$

2.2.(i).3. Criterion 3: is met by hypothesis.

2.2.(ii) Using the evaluation rule $r_i = q_i$ does not produces a fair outcome: Given $\phi_i = 1$ and $h_i = 0$,

2.2.(ii).1. Criterion 1: From equation (2.2.2),

$$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \left(\mu_i^o - \mu_j^o\right)\theta + s - s\right) d\varepsilon_i.$$

All of the common shock terms are symmetric. Since ε_i and ε_j are *i.i.d.*, then $f = z_j$ and $F = Z_j$ for all j. Also, given $a_i \neq a_j$, rearrange equation (2.2.7) to $\mu_j^o = \frac{a_j}{a_i}\mu_i^o$, then substitute into the probability

$$P_i = \int f(\varepsilon_i) F\left(\varepsilon_i + \mu_i^o\left(1 - \frac{a_j}{a_i}\right)\theta\right)^{n-1} d\varepsilon_i \neq \frac{1}{n}.$$

Term $\mu_i^o\left(1-\frac{a_j}{a_i}\right)\theta$ represents a unique non-zero bias in equilibrium for each instance of F for every i.

2.2.(ii).2. Criterion 2: Using the probability from the proof of *2.2.(ii).1* above, $\frac{\partial P_i}{\partial s}=\frac{\partial}{\partial s}\int f(\varepsilon_i)\,F\left(\varepsilon_i+\mu_i^o\left(1-\frac{a_j}{a_i}\right)\theta\right)^{n-1}d\varepsilon_i=0$. However,
$\frac{\partial P_i}{\partial \theta}=\mu_i^o\left(1-\frac{a_j}{a_i}\right)(n-1)$
$\times\int f(\varepsilon_i)\,f\left(\varepsilon_i+\mu_i^o\left(1-\frac{a_j}{a_i}\right)\theta\right)F\left(\varepsilon_i+\mu_i^o\left(1-\frac{a_j}{a_i}\right)\theta\right)^{n-2}d\varepsilon_i$ and $\frac{\partial P_i}{\partial \theta}\neq 0$.

2.2.(ii).3. Criterion 3: is met by hypothesis.

2.2.(iii) Using the evaluation rule $r_i=\frac{1}{a_i}(q_i-s)$ does not produces a fair outcome: Given $\phi_i=\frac{1}{a_i}$ and $h_i=-\frac{s}{a_i}$,

2.2.(iii).1. Criterion 1: (is not met) From equation (2.2.2),

$$P_i=\int f\left(\frac{\varepsilon_i}{a_i}\right)\prod_{j\neq i}^{n-1}Z_j\left(\frac{\varepsilon_i}{a_i}+\left(\frac{\mu_i^o}{a_i}-\frac{\mu_j^o}{a_j}\right)\theta+\left(\frac{1}{a_i}-\frac{1}{a_j}\right)(s-s)\right)\,d\varepsilon_i.$$

Satisfying equation (2.2.7), all of the terms $\frac{\mu_i^o}{a_i}-\frac{\mu_j^o}{a_j}$ cancel. Common shock terms are symmetric. Since ε_i and ε_j are *i.i.d.* and given $a_i\neq a_j$, then $\frac{\varepsilon_i}{a_i}\neq\frac{\varepsilon_j}{a_j}$ for all $i\neq j$ and $Z_j\left(\frac{\varepsilon_i}{a_i}\right)\neq Z_k\left(\frac{\varepsilon_i}{a_i}\right)$ for all $i\neq j\neq k$, hence $P_i=\int f\left(\frac{\varepsilon_i}{a_i}\right)\prod_{j\neq i}^{n-1}Z_j\left(\frac{\varepsilon_i}{a_i}\right)\,d\varepsilon_i\neq\frac{1}{n}$.

2.2.(iii).2. Criterion 2: Using the probability in *2.2.(iii).1,*

$\frac{\partial P_i}{\partial s}=\frac{\partial}{\partial s}\int f\left(\frac{\varepsilon_i}{a_i}\right)\prod_{j\neq i}^{n-1}Z_j\left(\frac{\varepsilon_i}{a_i}\right)\,d\varepsilon_i=0$ and

$\frac{\partial P_i}{\partial \theta}=\frac{\partial}{\partial \theta}\int f\left(\frac{\varepsilon_i}{a_i}\right)\prod_{j\neq i}^{n-1}Z_j\left(\frac{\varepsilon_i}{a_i}\right)\,d\varepsilon_i=0.$

2.2.(iii).3. Criterion 3: The variance $E\left[\left(\frac{\varepsilon_i}{a_i}\right)^2\right]=\frac{\sigma^2}{a_i^2}$ is not identical for all i. See the equal variance proof of asymmetry. □

B.7 Proof of Corollary 2.3

The proof using Efficient tournaments with agents of different ability and idiosyncratic uncertainty distributions proportional to ability who experience common shocks s and θ, (i) are background fair using $r_i = \frac{1}{a_i}(q_i - s)$; (ii) are not background fair using $r_i = q_i$; (iii) are not background fair using $r_i = q_i - \theta\mu_i^o$ is as follows.

Proof. 2.3.(i) Using the evaluation rule $r_i = \frac{1}{a_i}(q_i - s)$ produces a fair outcome: Given $\phi_i = \frac{1}{a_i}$ and $h_i = -\frac{s}{a_i}$,

2.3.(i).3. Criterion 3: $E\left[\left(\frac{\varepsilon_i}{a_i}\right)^2\right] = \frac{a_i^2\sigma^2}{a_i^2} = \sigma_\kappa^2$ is identical for all i.

2.3.(i).1. Criterion 1: From equation (2.2.2),

$$P_i = \int f\left(\frac{\varepsilon_i}{a_i}\right)\prod_{j\neq i}^{n-1} Z_j\left(\frac{\varepsilon_i}{a_i} + \left(\frac{\mu_i^o}{a_i} - \frac{\mu_j^o}{a_j}\right)\theta + \left(\frac{1}{a_i} - \frac{1}{a_j}\right)(s - s)\right)\, d\varepsilon_i.$$

Satisfying equation (2.2.7), the terms $\frac{\mu_i^o}{a_i} = \frac{\mu_j^o}{a_j}$ cancel. Common shock terms are symmetric. From the result in *2.3.(i).3*, the terms $\frac{\varepsilon_i}{a_i}$ and $\frac{\varepsilon_j}{a_j}$ are *i.i.d.*, then $f = z_j$ and $F = Z_j$ for all j and $P_i = \int f\left(\frac{\varepsilon_i}{a_i}\right) F\left(\frac{\varepsilon_i}{a_i}\right)^{n-1} d\varepsilon_i$,

$$P_i = \frac{1}{n} F\left(\frac{\varepsilon_i}{a_i}\right)^n\Bigg|_{-\infty}^{\infty} = \frac{1}{n}[1-0]^n = \frac{1}{n}.$$

2.3.(i).2. Criterion 2: Using the probability from the proof of Criterion 1 above,
$\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s}\int f\left(\frac{\varepsilon_i}{a_i}\right) F\left(\frac{\varepsilon_i}{a_i}\right)^{n-1} d\varepsilon_i = 0$ and
$\frac{\partial P_i}{\partial \theta} = \frac{\partial}{\partial \theta}\int f\left(\frac{\varepsilon_i}{a_i}\right) F\left(\frac{\varepsilon_i}{a_i}\right)^{n-1} d\varepsilon_i = 0.$

2.3.(ii) Using the evaluation rule $r_i = q_i$ does not produces a fair outcome: Given $\phi_i = 1$ and $h_i = 0$,

2.3.(ii).3. Criterion 3: $E\left[(\varepsilon_i)^2\right] = a_i^2\sigma_\kappa^2$ is not identical for all i.

2.3.(ii).1. Criterion 1: From equation (2.2.2),

$$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \theta \mu_i^o - \theta \mu_j^o + s - s \right) d\varepsilon_i.$$

The common shocks cancel. Rearranging equation (2.2.7) to $\mu_j^o = \frac{a_j}{a_i} \mu_i^o$ and inserted into P_i, the term $\mu_i^o \left(1 - \frac{a_j}{a_i}\right) \theta$ emerges and is a unique non-zero bias in each instance of Z_j. Additionally, ε_i and ε_j are different distributions and $Z_j(\varepsilon_i) \neq Z_k(\varepsilon_i)$ for all $i \neq j \neq k$, hence $P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \mu_i^o \left(1 - \frac{a_j}{a_i}\right) \theta \right) d\varepsilon_i \neq \frac{1}{n}$.

2.3.(ii).2. Criterion 2: Using the probability from the proof of *2.3.(ii).1* above, $\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s} \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \mu_i^o \left(1 - \frac{a_j}{a_i}\right) \theta \right) d\varepsilon_i = 0$. However,

$\frac{\partial P_i}{\partial \theta} = \mu_i^o \left(1 - \frac{a_j}{a_i}\right)(n-1)$

$\times \int f(\varepsilon_i) f\left(\varepsilon_i + \mu_i^o \left(1 - \frac{a_j}{a_i}\right) \theta \right) F\left(\varepsilon_i + \mu_i^o \left(1 - \frac{a_j}{a_i}\right) \theta \right)^{n-2} d\varepsilon_i$ and $\frac{\partial P_i}{\partial \theta} \neq 0$.

2.3.(iii) Using the evaluation rule $r_i = q_i - \theta \mu_i^o$ does not produces a fair outcome: Given $\phi_i = 1$ and $h_i = -\theta \mu_i^o$,

2.3.(iii).3. Criterion 3: $E\left[(\varepsilon_i)^2\right] = a_i^2 \sigma_\kappa^2$ is not constant for all i.

2.3.(iii).1. Criterion 1: From equation (2.2.2),

$$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j \left(\varepsilon_i + \theta \mu_i^o - \theta \mu_j^o + s - s - \theta \mu_i^o + \theta \mu_j^o \right) \, d\varepsilon_i.$$

From equation (2.2.7), the effort terms cancel. The error terms ε_i and ε_j are different distributions and $Z_j(\varepsilon_i) \neq Z_k(\varepsilon_i)$ for all $j \neq k$ and *2.3.(iii).3* above, hence

$P_i = \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j(\varepsilon_i) \, d\varepsilon_i \neq \frac{1}{n}$.

2.3.(iii).2. Criterion 2: The marginal probability due to the shock

$\frac{\partial P_i}{\partial s} = \frac{\partial}{\partial s} \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j(\varepsilon_i) \, d\varepsilon_i = 0$ and

$\frac{\partial P_i}{\partial \theta} = \frac{\partial}{\partial \theta} \int f(\varepsilon_i) \prod_{j \neq i}^{n-1} Z_j(\varepsilon_i) \, d\varepsilon_i = 0$. $\square$

B.8 Proof of Proposition 2.4

Proof. Individual payoffs from equation (2.2.10) for a fair and efficient tournament for agent i are given by

$$W_i = \underline{u}_i + a_i C\left(\mu^o\right) + \left(1 - P_i\right) \frac{V}{\phi_i \theta g^o}$$

$$L_i = \underline{u}_i + a_i C\left(\mu^o\right) - P_i \frac{V}{\phi_i \theta g^o}$$

Expected profits are

$$E\left[\Pi\right] = \sum_{i=1}^{n} \left\{ V \mu_i^o - \frac{1}{n}\left(W_i + (n-1) L_i\right) \right\}$$

$$= \sum_{i=1}^{n} \left\{ V \mu_i^o - \frac{1}{n}\left(\underline{u}_i + a_i C\left(\mu^o\right) + \left(\frac{n-1}{n}\right) \frac{V}{\phi_i \theta g^o} \right) - \left(\frac{n-1}{n}\right) \left(\underline{u}_i + a_i C\left(\mu^o\right) - \frac{1}{n} \frac{V}{\phi_i \theta g^o} \right) \right\}$$

$$E\left[\Pi\right] = \sum_{i=1}^{n} \left\{ V \mu_i^o - \frac{1}{n}\left(\frac{V}{\phi_i \theta g^o} \right) - \left(\underline{u}_i + a_i C\left(\mu^o\right) - \frac{1}{n} \frac{V}{\phi_i \theta g^o} \right) \right\}$$

$$E\left[\Pi\right] = \sum_{i=1}^{n} \left\{ V \mu_i^o - \underline{u}_i - a_i C\left(\mu^o\right) \right\}$$

Actual profits when agent i wins,

$$\Pi|_{i\,wins} = V \mu_i^o - \left(\underline{u}_i + a_i C\left(\mu^o\right) + \left(\frac{n-1}{n}\right) \frac{V}{\phi_i \theta g^o} \right) + \sum_{j=1 \neq i}^{n-1} \left\{ V \mu_j^o - \left(\underline{u}_j + a_j C\left(\mu^o\right) - \frac{1}{n} \frac{V}{\phi_j \theta g^o} \right) \right\}$$

$$\Pi|_{i\,wins} = \sum_{j=1}^{n} \left\{ V \mu_j^o - \underline{u}_j - a_j C\left(\mu^o\right) \right\} - \left(\frac{n-1}{n}\right) \frac{V}{\phi_i \theta g^o} + \sum_{j=1}^{n-1} \left\{ \frac{1}{n} \frac{V}{\phi_j \theta g^o} \right\}$$

$$\Pi|_{i\,wins} = E\left[\Pi\right] - \frac{V}{\phi_i \theta g^o} + \frac{1}{n} \frac{V}{\phi_i \theta g^o} + \sum_{j=1}^{n-1} \left\{ \frac{1}{n} \frac{V}{\phi_j \theta g^o} \right\}$$

$$\Pi|_{i\,wins} = E\left[\Pi\right] - \frac{V}{\phi_i \theta g^o} + \sum_{j=1}^{n} \left\{ \frac{1}{n} \frac{V}{\phi_j \theta g^o} \right\}$$

$$\Pi|_{i\,wins} = E\left[\Pi\right] + \frac{V}{\theta g^o} \left(\frac{1}{n} \sum_{j=1}^{n} \frac{1}{\phi_j} - \frac{1}{\phi_i} \right)$$

Given that $\frac{1}{\phi_j} = \alpha_j$ and $\bar{\alpha} = \frac{1}{n} \sum_{j=1}^{n} \alpha_j$, $\Pi|_{i\,wins} = E\left[\Pi\right] + \frac{V}{\theta g^o}\left(\bar{\alpha} - \alpha_i\right)$ results.

The comparative static of profit with respect to α_i is $\frac{\partial \Pi_i}{\partial \alpha_i} = -\frac{V}{\theta g^o}$. □

Appendix C

Fairness and workplace incentives: Evidence from a tournament experiment with heterogeneous agents

C.1 Contract Choice Tables

This subsection contains the results of the contract choice regressions at the pooled level and the group level. Averaging at the pooled level and the group level, the *Percent Won* should be $1/3$ for all contracts in the following tables.

Table C.1: Pooled Contract Choice for the Benchmark

Variable	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Win (difference) b/se
Choice was Contract A or Homogeneous	0.500	0.565	0.292***	0.074	0.003
	(0.121)	(0.105)	(0.069)	(2.310)	(0.069)
Choice was Contract B or Homogeneous	0.500	0.519	0.313***	0.593	-0.003
	(0.121)	(0.103)	(0.065)	(0.628)	(0.041)
R-sqr	0.500	0.615	0.547	0.004	0.000
dfres	35	35	35	35	35
BIC	59.4	48.3	16.4	249.2	4.9
N	36.0	36.0	36.0	36.0	36.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on Choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

The pooled level results of the 36 subjects that switched groups between stages played the homogeneous contract H in Group 0 as well as a different ability contract in Group 1 are shown in Table C.1. Of the 36 subjects, 12 switched roles between $i_0\,(a_M, \sigma_M)$ and $i_1\,(a_L, \sigma_M)$, and 12 switched roles between $k_0\,(a_M, \sigma_M)$ and $k_1\,(a_H, \sigma_M)$. The other 12 retained the same ability and uncertainty parameters, but switched between the two groups $j_{0,1}\,(a_M, \sigma_M)$. Half of these subjects had to choose between Group 1 with no handicap in Contract A or Group 0 playing Contract H. The other half had a choice to make between Group 1 with offset handicaps in Contract B or Group 0 playing Contract H. In both cases, contract H was considered more fair than either B or C.

The results that support the group level contract choice decisions in subsection 3.4.1.1 are shown in the next four tables beginning with TableC.2. Each table shows the results from 5 regressions where the results of each regression continue in the same column of each table. Each regression includes twelve dummy variables that indicate contract choices made by each group. Errors are grouped at the subject level. At the group level, there is no significant difference in average earnings between any contract even though the expected earnings are different for different types in contract A. Averaging at the group level, the *Percent Won* should be 1/3 for all contracts in the following tables.

The following four tables support the discussion on group level results

Table C.2: Contract Preferences, Group 0 and Group 1 Decisions

Variable	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Win (difference) b/se
Group 0 (Hetero Ability or Homogeneous):					
Choice of Contract A or Homogeneous	0.500	0.565	0.292***	0.074	0.003
	(0.12)	(0.11)	(0.07)	(2.31)	(0.07)
Choice of Contract B or Homogeneous	0.500	0.519	0.313***	0.593	-0.003
	(0.12)	(0.10)	(0.06)	(0.63)	(0.04)
Group 1 (Heterogeneous Ability):					
Choice of Contract A or B	0.389	0.389	0.403**	-1.604	-0.008
	(0.12)	(0.11)	(0.05)	(2.04)	(0.09)
R-sqr	0.530	0.600	0.852	0.037	0.000
dfres	215	215	215	215	215
BIC	372.6	307.1	-20.2	1589.7	88.4
N	216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

Table C.3: Contract Preferences, Group 2 Decisions

Variable Group 2 (Heterogeneous Uncertainty)	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
Choice of Contract A or B	0.611	0.602	0.576*	2.020	-0.006
	(0.12)	(0.09)	(0.04)	(1.47)	(0.05)
Choice of Contract A or C	0.556	0.528	0.563	3.287	0.006
	(0.12)	(0.11)	(0.05)	(3.13)	(0.03)
Choice of Contract B or C	0.556	0.500	0.507	0.340	0.000
	(0.12)	(0.10)	(0.03)	(1.30)	(0.05)
R-sqr	0.530	0.600	0.852	0.037	0.000
dfres	215	215	215	215	215
BIC	372.6	307.1	-20.2	1589.7	88.4
N	216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

Table C.4: Contract Preferences, Group 3 Decisions

Variable Group 3 (Ability ∝ Uncertainty)	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
Choice of Contract A or B	0.389	0.444	0.465	0.222	0.000
	(0.12)	(0.10)	(0.04)	(2.31)	(0.07)
Choice of Contract A or C	0.444	0.417	0.451	1.387	-0.006
	(0.12)	(0.10)	(0.05)	(1.89)	(0.09)
Choice of Contract B or C	0.611	0.620	0.542	0.181	-0.003
	(0.12)	(0.11)	(0.04)	(1.45)	(0.06)
R-sqr	0.530	0.600	0.852	0.037	0.000
dfres	215	215	215	215	215
BIC	372.6	307.1	-20.2	1589.7	88.4
N	216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

Table C.5: Contract Preferences, Group 4 Decisions

Variable Group 4 (Ability ∝ Precision)	Decision [0,1] b/se	Decision (gradient) b/se	Fairness [0,1] b/se	Earnings (difference) b/se	Percent Won (difference) b/se
Choice of Contract A or B	0.556	0.565	0.479	1.837	-0.003
	(0.12)	(0.11)	(0.04)	(1.55)	(0.08)
Choice of Contract A or C	0.611	0.593	0.500	2.743	-0.006
	(0.12)	(0.10)	(0.06)	(3.22)	(0.05
Choice of Contract B or C	0.500	0.435	0.528	0.471	0.008
	(0.12)	(0.11)	(0.03)	(0.89)	(0.04)
R-sqr	0.530	0.600	0.852	0.037	0.000
dfres	215	215	215	215	215
BIC	372.6	307.1	-20.2	1589.7	88.4
N	216.0	216.0	216.0	216.0	216.0

* p<0.10, ** p<0.05, *** p<0.01. Significant values on choice variables in columns 1, 2, and 3 indicate statistical difference from the midrange value, 0.50.

C.2 Effort by Round

The following figures show the average effort for each period for each group in each contract. Effort is averaged by round. The 20 paid rounds are shown.

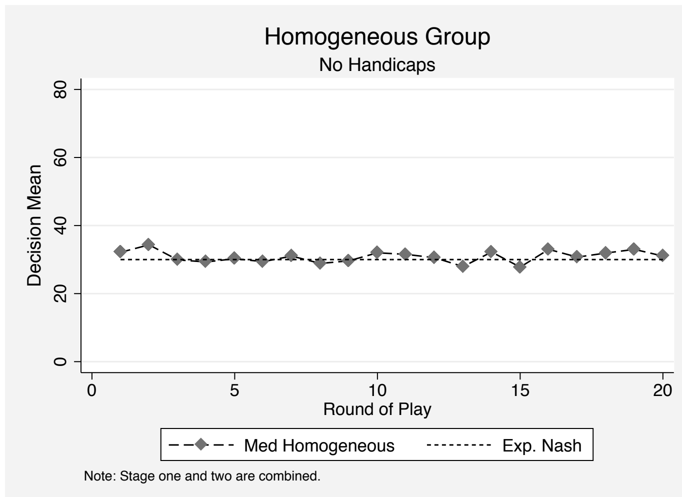

Figure C.1: Homogeneous group, average effort for each period.

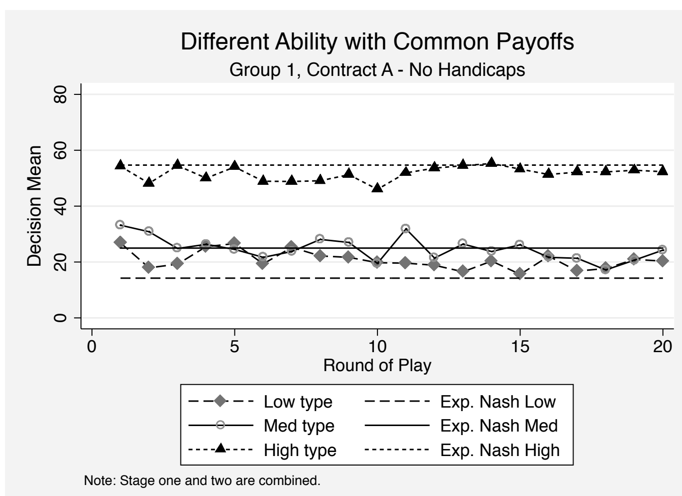

Figure C.2: Group 1- heterogeneous ability, Contract A, average effort for each period.

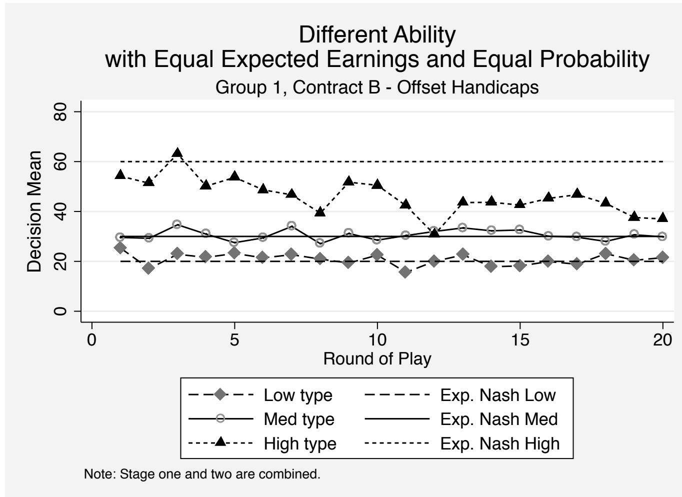

Figure C.3: Group 1- heterogeneous ability, Contract B , average effort for each period.

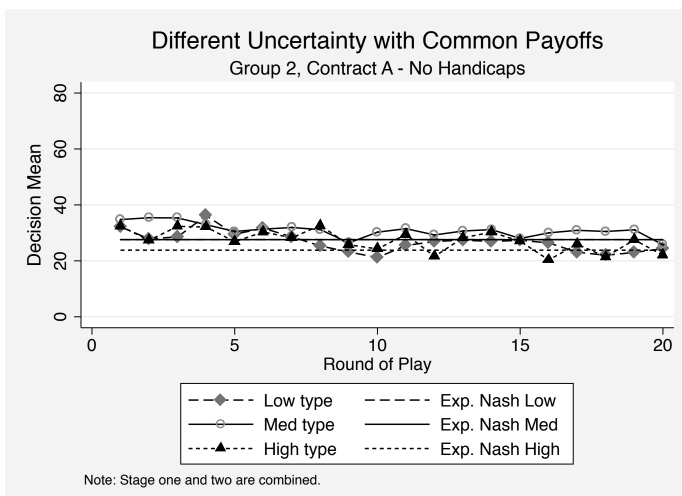

Figure C.4: Group 2- heterogeneous uncertainty, Contract A, average effort for each period.

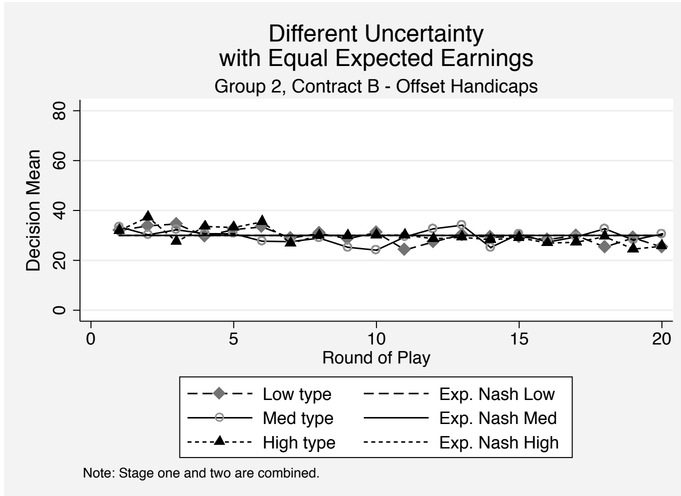

Figure C.5: Group 2- heterogeneous uncertainty, Contract B, average effort for each period.

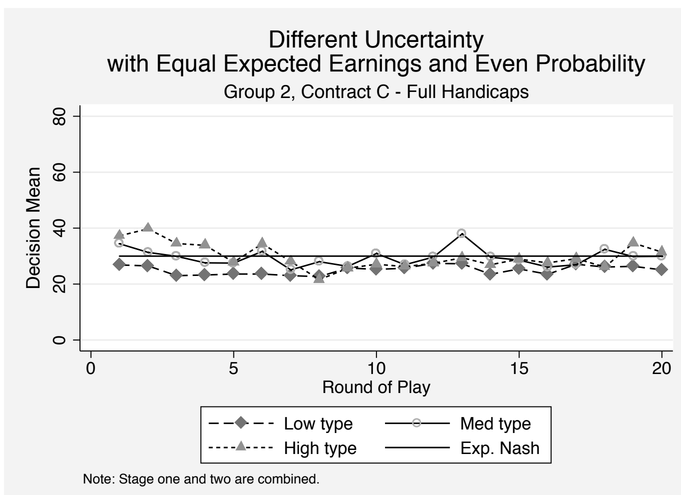

Figure C.6: Group 2- heterogeneous uncertainty, Contract C, average effort for each period.

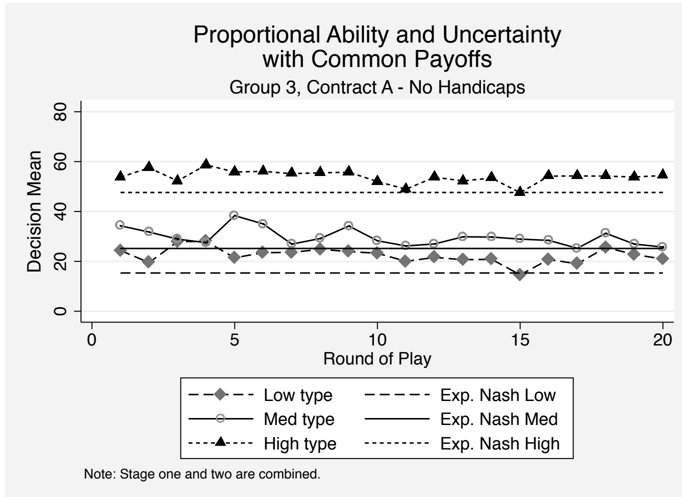

Figure C.7: Group 3- ability $\propto$ uncertainty, Contract A, average effort for each period.

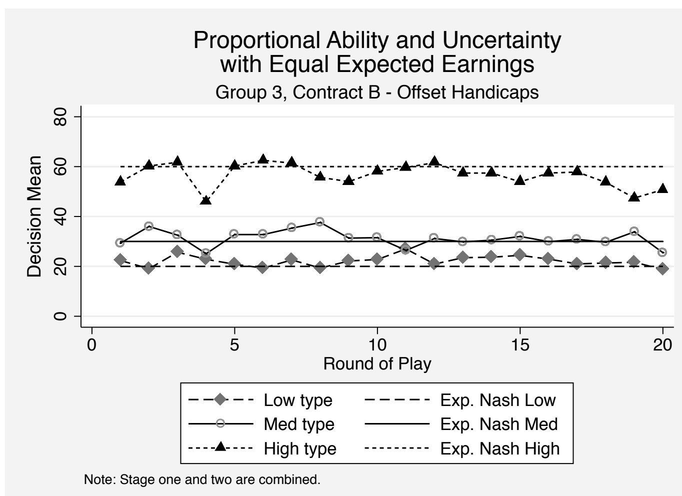

Figure C.8: Group 3- ability $\propto$ uncertainty, Contract B, average effort for each period.

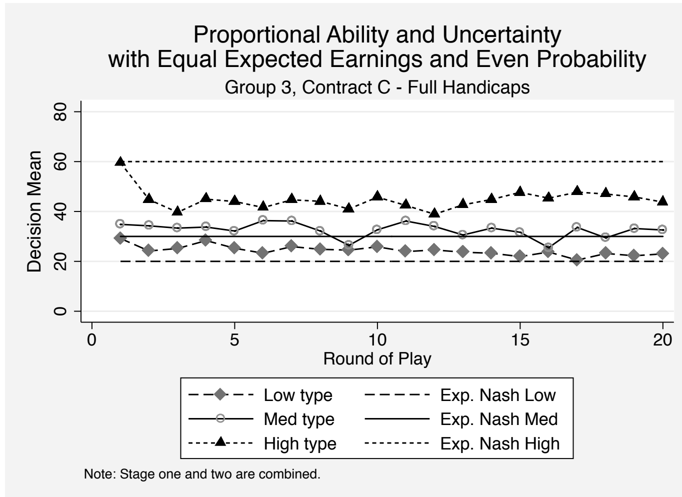

Figure C.9: Group 3- ability $\propto$ uncertainty, Contract C, average effort for each period.

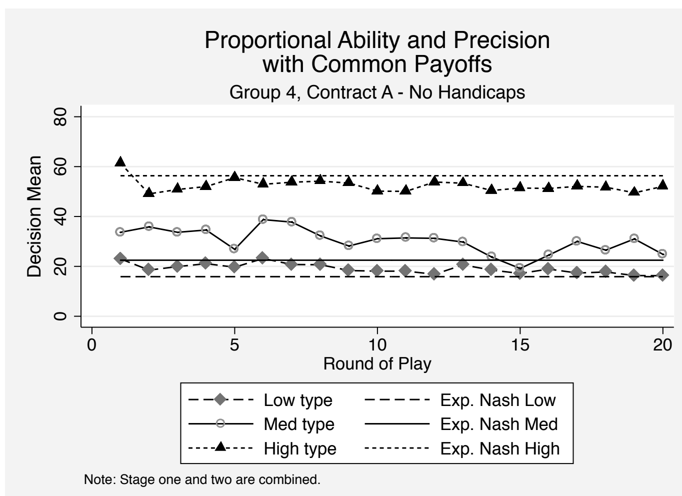

Figure C.10: Group 4 - ability $\propto$ precision, Contract A, average effort for each period.

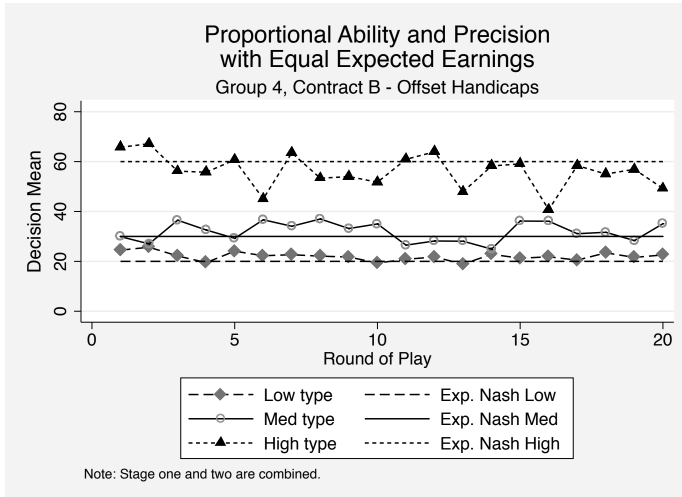

Figure C.11: Group 4 - ability $\propto$ precision, Contract B, average effort for each period.

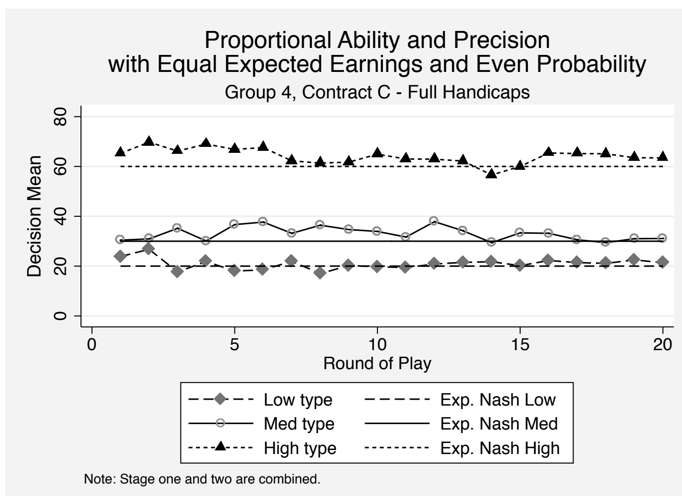

Figure C.12: Group 4 - ability $\propto$ precision, Contract C, average effort for each period.

C.3 Experiment Instructions

The instructions are found on the next page. The instructions were placed at each terminal prior to the subjects entering the room. After all subjects were in the room and sitting at their terminal, subjects were informed the instructions were to be read out loud. The instructions were read while all subjects followed along. Subjects were given several opportunities to ask questions while the instructions were read and during the training segment of the experiment.

Experiment Instructions

Introduction

This is an experiment in economic decision-making. Please follow the instructions carefully. If you make good decisions you can earn a considerable amount of money. At the end of the session, you will be paid your earnings in private and in cash. Please do not communicate with other participants during the experiment unless instructed. Importantly, please refrain from verbally reacting to events that occur.

You will be randomly matched into a group with two other players that are sitting in the room. The identity of your group members will not be revealed to you. You will remain in the same group for the entire experiment.

The experiment will last for many decision "rounds". In each decision round, you and your group members will independently choose a Decision Number. These choices will be converted to Total Numbers. The person in your group with the highest Total Number will receive a high fixed payment; the other two will receive a low fixed payment. Your earnings for a round are equal to your fixed payment minus the cost associated with your Decision Number.

All earnings are denominated in lab dollars. At the end of the experiment your earnings in lab dollars will be converted to U.S. dollars at an exchange rate of 2500 to 1.

The Decision Setting

In this experiment you will perform a simple task. The computer will ask you to choose a Decision Number, which will be your only choice in a round. This Decision Number is a choice between 0 and 100. Tied to each Decision Number is a Decision Cost: the higher the Decision Number chosen, the higher the cost. On your computer you will see a table that provides information on this relationship. You will select a Decision Number using a slider bar on your computer screen. When you are ready to make your decision, please make sure to position the slider on the chosen Decision Number, and then click the "SUBMIT DECISION" button.

After you have submitted your decision, the computer will draw a Random Number for you. Your Random Number can be a positive or negative value and will be added to your Decision Number. The range of possible values is centered on the number zero (0). You will know the range of possible random numbers that will be drawn for you. Know that each number in your random number range has an equal chance of being selected.

Calculation of "Total Numbers"

Your Decision Number will be converted into a Total Number. This calculation involves two adjustments using a multiplier and an offset. Specifically, the following calculation is used:

Total Number = multiplier × (Decision Number + Random Number) + offset

If you choose a higher Decision Number, you will have a higher Total Number, on average. Since the Random Number is a draw from a range of possible numbers, for any Decision Number you choose there will likewise be a range of possible Total Numbers associated with it.

Earnings

After the Total Numbers for you and your group members have been calculated, the computer will compare them. If your Total Number is the highest in the group, you will receive a high fixed payment; otherwise you will receive a low fixed payment. If there happens to be a tie for the highest Total Number, the computer will randomly break the tie.

Your earnings for the decision round will be calculated as the difference between the fixed payment you receive and your Decision Cost:

Round Earnings = (High or Low) Fixed Payment − Decision Cost

It is important to know that, although the Random Number can play a role in determining whether you have the highest Total Number, it has no impact on your Decision Cost.

Example

At this time, please click to "continue" past the current computer screen. Once you see the example decision screen, do NOT click "continue" until instructed to do so. Now, please look closely at the decision screen. According to the Total Number calculation near the top of your screen, notice that the multiplier is 2 and the offset is 30. What does this mean?

- Since the multiplier is **2**, this means that both the Decision Number and Random Number are multiplied by **2** when calculating the Total Number.
- The offset is **30**, such that **30** will be added-in automatically to determine your Total Number. The offset does not depend on your Decision Number or the Random Number.

For illustration, move your slider bar at the bottom of your screen to indicate a Decision Number of ***55***. Continuing with the example, notice that the Random Number range, found near the bottom of your screen, is from **-35** to **35**. How does this relate to your Total Number?

- On average, if you enter a Decision Number of 55, your Total Number equals $2 \times (55 + \mathbf{0}) + 30 = 140$. This is because the Random Number equals **0** on average.
- With a Decision Number of 55, the highest possible Total Number is $2 \times (55 + \mathbf{35}) + 30 = 210$. This is because the highest possible Random Number in this example is **35**.
- With a Decision Number of 55, the lowest possible Total Number is $2 \times (55 - \mathbf{35}) + 30 = 70$. This is because the lowest possible Random Number in this example is $-\mathbf{35}$.
- As illustrated near the top of your screen, if you submitted a Decision Number of 55, the range of possible Total Numbers would be 70 to 210, and is centered on 140.

Next, please click the "SUBMIT DECISION" decision. This will draw a Random Number, and to the right of your screen you will see an example Total Number calculation. You can click the button multiple times, and in each case you will notice that the Total Number does in fact lie in the Total Number range illustrated near the top of your screen. Please know that in actual decision rounds, if you click the "SUBMIT DECISION" button this will submit your decision. You will not know the Random Number draw until after your decision is submitted.

You will notice that the computer provides information on Decision Cost, and displays what your earnings for the round will be for any selected Decision Number. In this example, you will see that the high fixed payment is 3740 and the low fixed payment is 1490. With a Decision Number of 55, the Decision Cost is 2711. Thus, if you submitted this decision and had the highest Total Number in your group, you would earn 3740 – 2711 = 1029. Otherwise you would earn 1490 – 2711 = –1221. Notice that in this example, if you received the low fixed payment this would result in negative earnings. This does mean that you would in fact lose money. Additionally, if both the high fixed payment and low fixed payment amounts are negative, then you are guaranteed to lose money if you submit the indicated Decision Number.

Please move your slider to a different Decision Number. If you increase the Decision Number, you will notice that this increases the Total Number on average, and also increases your Decision Cost. The opposite is true as you decrease the Decision Number. The Random Number range will not change, nor will the high or low fixed payment as these amounts do not depend on your Decision Number.

Other Group Members

The other members of your group may face different conditions than you. In particular, one or more of the following may be different: (1) the Decision Cost associated with each Decision Number; (2) the random number range; (3) the multiplier; (4) the offset; (5) the high and low fixed payment amounts.

Given the possible differences between the conditions faced by you and your group members, and how Total Numbers are calculated, there are a few important things to know:

- Random Numbers will be determined separately for each group member. It is very unlikely that you will have the same Random Number as any other group member in a round.
- If your Decision Cost is higher than another group member's Decision Cost for any given Decision Number, then it will cost you more to achieve the same Total Number, on average (assuming the multiplier and offset are the same).
- If you have a higher multiplier than another, you will have a higher Total Number if you both chose the same Decision Number, on average (assuming the same offset)
- If you have a higher offset than another, you will have a higher Total Number if you both chose the same Decision Number, on average (assuming the multiplier is the same).

As you proceed to the decision rounds, on your decision screen you will be provided complete information about the conditions faced by your other two group members. This will allow you to compare your relative situation. In addition, you have the opportunity to speculate by hypothetically choosing Decision Numbers for the other two members, and seeing the corresponding range of possible Total Numbers that would result along with earnings outcomes. If you make hypothetical choices for the other group members, this will have absolutely no effect on your earnings or the choices made by the other group members! This is for your use only.

Results

After all participants submit their Decision Numbers for a round, you will see a results screen. The results screen will show your Random Number, Total Number and earnings for the round. You will also see Total Numbers of your other group members, and who received the high fixed payment (who ranked 1^{st}) and who received the low fixed payment (who ranked 2^{nd} and 3^{rd}). You will not know the Decision Numbers or the Random Numbers for the other group members – only the Total Numbers.

Summary

- You will be randomly matched with two other participants in this room for the entire experiment. You will not know the identity of the other two participants.
- In each decision round you choose a Decision Number. The higher the Decision Number you choose, the higher is your Decision Cost.
- Your Decision Number will be converted into a Total Number based on Random Number, multiplier and an offset. The member of your group with the highest Total Number receives the high fixed payment. Other members receive the low fixed payment.
- Your earnings in a round are equal to the difference between the fixed payment you receive and your decision cost.

Experiment Organization

The decision rounds are arranged into multiple stages. You will not know the number of stages until the experiment ends. Within a stage, you will have the same decision cost schedule, the same multiplier and offset, and the same random number range from one round to the next. You will not know the number of rounds in a stage until the stage is finished.

Your computer will alert you before a new stage begins. At the beginning of a new stage, please look carefully at the information on your decision screen, as this is likely to have changed.

The experiment begins with 2 training rounds. You will not have the opportunity to earn money in these rounds. Aside from training rounds, your decision in each round will determine your earnings: your earnings from the experiment will be the total of your earnings from each paid round. After the second training round, you will have a final opportunity to ask questions.

Timers

Please know that there is a timer on the decision screen. If the timer goes to zero before you click the "SUBMIT DECISION" button, the computer will record whatever Decision Number you have indicated when the timer hits zero. The amount of time given will decrease as the experiment progresses. This is just to help make sure the experiment finishes as scheduled.

Please click "continue" to move past the example decision screen. Then, read the instructions for the training rounds and then click "continue" to begin the first training round.

C.4 In Game Role Surveys

The in game surveys are shown on the next page. The surveys were used in the 21^{st}round of each stage to solicit which role(s) a subject would prefer to play if they had the ability to choose. Additional data was collected as to what factors were most important in determining the choice they made.

Participant ID __________

You will know your ID at the end of the session. Please fill in the ID number at that time.

Stage 1 Questions

Please look carefully at your decision screen, at the conditions you face as well as the conditions faced by your "1st group member" and "2nd group member".

1. If you had the ability to choose, which role would you play? (circle one)

   my current role   role of 1st group member   role of 2nd group member

2. How important were the following factors when determining your choice above? (circle a response to each)

	Not at all important		Moderately Important		Very Important
a. Decision cost	1	2	3	4	5
b. Random number range	1	2	3	4	5
c. Multiplier	1	2	3	4	5
d. Offset	1	2	3	4	5
e. High payment	1	2	3	4	5
f. Low payment	1	2	3	4	5

3. What would be your second preferred role? (circle one)

   my current role   role of 1st group member   role of 2nd group member

After you are finished, please submit your choice for this decision round on your computer.

Participant ID __________

You will know your ID at the end of the session. Please fill in the ID number at that time.

Stage 2 Questions

Please look carefully at your decision screen, at the conditions you face as well as the conditions faced by your "1^{st} group member" and "2^{nd} group member".

1. If you had the ability to choose, which role would you play? (circle one)

   my current role   role of 1^{st} group member   role of 2^{nd} group member

2. How important were the following factors when determining your choice above? (circle a response to each)

	Not at all important		Moderately Important		Very Important
a. Decision cost	1	2	3	4	5
b. Random number range	1	2	3	4	5
c. Multiplier	1	2	3	4	5
d. Offset	1	2	3	4	5
e. High payment	1	2	3	4	5
f. Low payment	1	2	3	4	5

3. What would be your second preferred role? (circle one)

   my current role   role of 1^{st} group member   role of 2^{nd} group member

After you are finished, please submit your choice for this decision round on your computer.

C.5 Performance Notice

The performance notice is shown on the next page. The nature of the game did allow subjects to severely hurt themselves by choosing very large decision numbers that ultimately generated very large costs for them. This document was presented to subjects that incurred large costs to make them aware their decisions are very costly. A notice was given for any subject who incurred $10 or more in overall costs.

Your Attention Please

We would like to bring to your attention that, with the exchange rate of 2500 lab dollars to 1 U.S. dollar, you have lost over _______ U.S. dollars so far in the experiment.

Please notice that when you submit a Decision Number you will receive either the earnings indicated under the "High Fixed Payment" or the "Low Fixed Payment" scenarios presented on your screen. When these indicate negative numbers this does mean this amount will be subtracted from your earnings. If, based on your chosen Decision Number, the amounts under both the "High Fixed Payment" and "Low Fixed Payment" scenarios are negative, then you are guaranteed to lose money if you submit this Decision Number. Choosing a lower Decision Number will decrease the possibility of negative earnings.

The experiment will proceed for many additional rounds, and we want you to have the opportunity to earn money (and not owe us money!). Therefore, we will add _______ U.S. dollars to your overall earnings to help offset your current losses.

This is a one-time action. Please continue with the experiment and good luck.

C.6 Decision and Results Screen Snapshots

The snapshots of the decision screen and results screen are shown on the following pages. Static games of complete information assume players have knowledge of the strategic reaction functions of their opponents and are further able to deduce their strategic solutions. Given the complex nature of the games in this experiment, a novel decision screen was developed to provide subjects with tools to speculate on the strategic behavior of their opponents. The decision screen incorporated novel techniques to provide subjects with as much information as possible about their opponents. First, subjects used slider-bars to choose their decision number between 0 and 100 (as opposed to typing a decision number into a text box field). This novel approach has several advantages: It provided visual feedback cues as to where a subject's decision number lies along the number line between 0 and 100. In Z-Tree, each time a slider-bar is moved, the input is made available to the programing environment. This gave us the opportunity to calculate expected total numbers, costs, and high and low earnings and provide instant feedback interactively on the screen.

Also, two extra slider-bars were provided to allow subjects the opportunity to "try-out" different decision numbers from the perspective of the other group members, to "see" what their opponents see and speculate as to what strategy their opponents might choose. A subject could use the speculative slider bars to speculate how an opponent's decision number would translate into the expected total numbers, costs, and high and low earnings for the opponent.

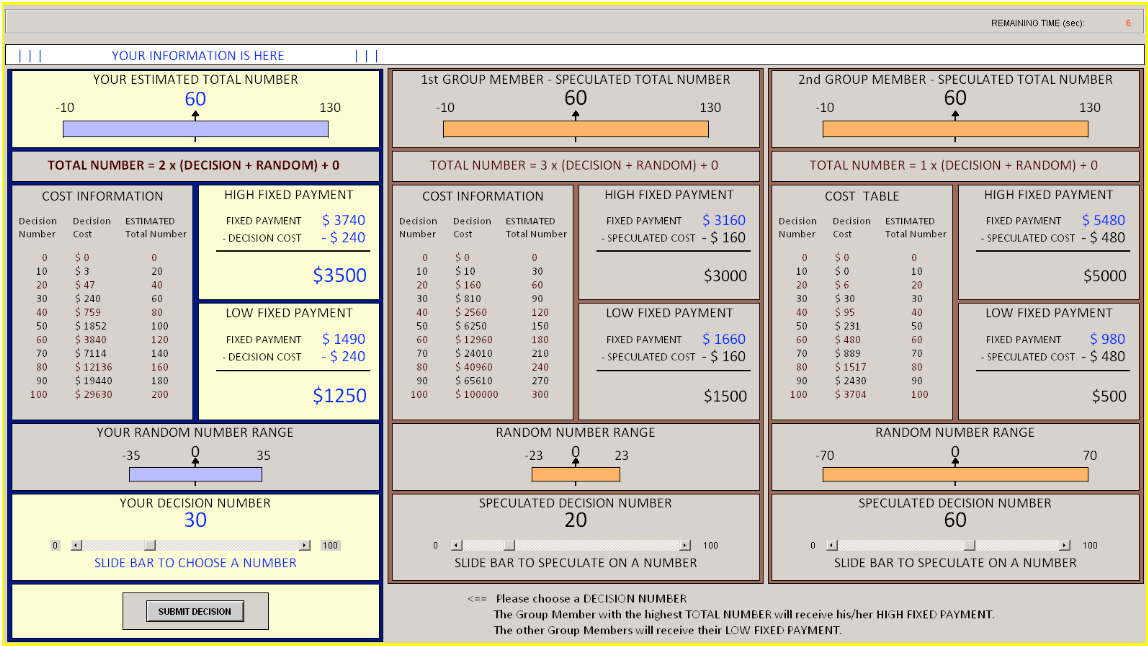

Figure C.13: Decision Screen Example

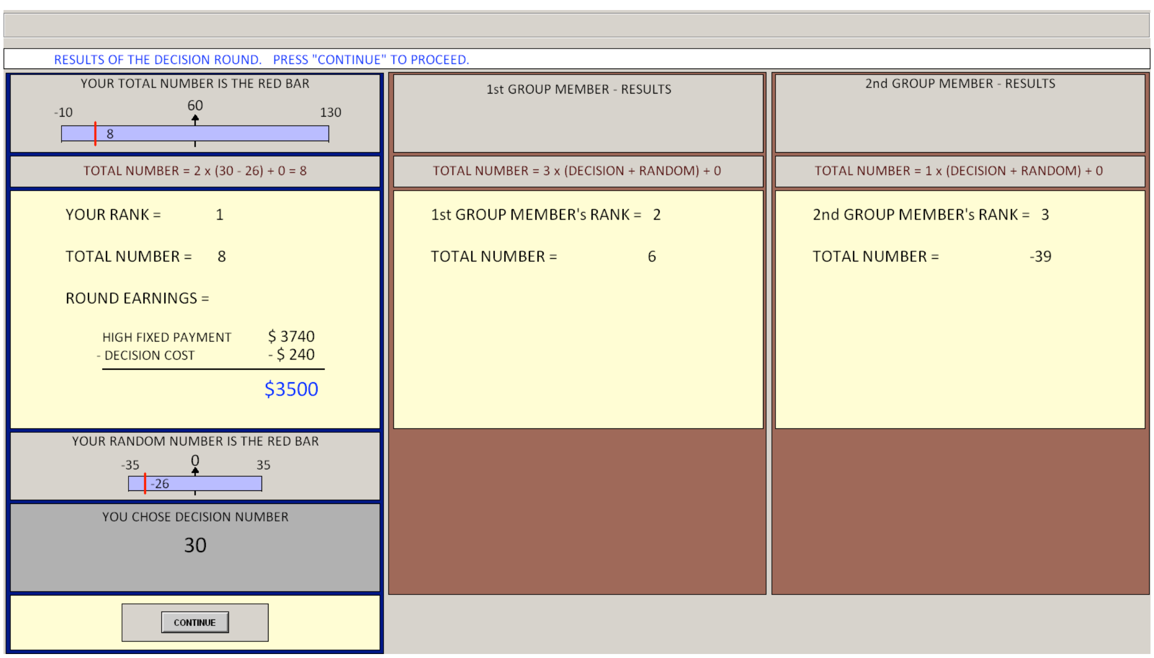

Figure C.14: Results Screen Example

Vita

Nicholas Busko received a liberal arts education from Maryville College and served as captain three of four years on the MC soccer team. He earned his BS in Electrical Engineering and Computer Engineering from the University of Tennessee, Knoxville in 1990 with honors and went on to work at several startup companies as a systems engineer where he designed robotic control systems, vision hardware systems, and application specific integrated circuits for many diverse products. During this time he contributed to 8 U.S. Patents in the application fields of image processing and battery energy management. As a Senior Director at Flextronics International, Nicholas led several international teams in IC product development. Nicholas is also a competitive soccer coach, holds a commercial pilot license, and is a former flight instructor. Nicholas earned his MA in Economics from the University of Tennessee in 2010 and his PhD in Economics from the University of Tennessee with the acceptance of this dissertation in 2015. He studies in the fields of Industrial Organization and Environmental Economics with research interests in applied game theory, global resource management, and experimental economics. Nicholas is currently faculty at Salisbury University in Maryland.