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Strategies and Skills Used by Middle School Students During the Solving of Non-routine Mathematics Problems

Terry D. Rose
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Terry D. Rose entitled "Strategies and Skills Used by Middle School Students During the Solving of Non-routine Mathematics Problems." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Education, with a major in Education.

Jerry J. Bellon, Major Professor

We have read this dissertation and recommend its acceptance:

Phyllis Huff, Kermit Blank, Tom Mathews

Accepted for the Council:
Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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Accepted for the Council:

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Associate Vice Chancellor
and Dean of The Graduate School

STRATEGIES AND SKILLS USED BY MIDDLE SCHOOL STUDENTS DURING THE SOLVING OF NON-ROUTINE MATHEMATICS PROBLEMS

A Dissertation
Presented for the
Doctor of Education
Degree
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Terry D. Rose

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ABSTRACT

This study was conducted to determine the processes and strategies selected middle school students use during the solving of non-routine mathematics problems. Qualitative research methods were used to identify the cognitive and metacognitive skills and processes used in problem solving and to determine the affective influences on the problem solving process.

Six middle grade students were selected to participate in the study. Each student was interviewed four times. The first interview was conducted in order to develop student profiles by obtaining information about each student's family, school, and mathematics background. The second and third interviews consisted of two phases. First, students solved problems for twenty minutes and verbally explained their thoughts and work. Afterwards, a follow-up interview was conducted in order to clarify and enhance information collected during the twenty minute problem solving session. The fourth and final interview was conducted using a grid technique in order to determine student perceptions of the problem solving process.

The interviews were audiotaped, and the problem solving sessions were videotaped. The transcriptions were analyzed using a constant comparative method. Themes emerged from the data analysis, and findings were

identified. The themes and findings led the researcher to the following conclusions.

1. Students are not aware of the various alternatives available to help them understand a non-routine mathematics problem when they first read it.

2. The only skills which students perceive as mathematics skills are the basic computations of addition, subtraction, multiplication, and division.

3. Students are unwilling to take risks when presented with a problem solving situation. They are hesitant to try a strategy unless they have seen a teacher use that particular strategy.

4. Students have been told that various heuristics exist to help them solve problems. Even though they have been instructed to use them, they have not been adequately informed concerning how and when to use the heuristics.

5. Students model the problem solving strategies and behaviors of their teachers.

The study demonstrates that teachers need to concentrate on fostering students' self-esteem and positive attitudes toward problem solving in mathematics. Non-routine problems should become a regular part of the mathematics which students are exposed to in school, and teachers should focus on modeling their successful problem solving behaviors.

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CHAPTER I

INTRODUCTION

In the 1980 publication, An Agenda For Action: Recommendations for School Mathematics of the 1980s, the National Council of Teachers of Mathematics (NCTM) organization's first recommendation was that "problem solving be the focus of school mathematics in the 1980s" (1980, p. 1). The document stressed the importance of organizing the mathematics curriculum around problem solving. Specifically, suggestions made by NCTM for teachers included creating an environment conducive to problem solving and involving students at all grade levels in problem solving. NCTM also recommended that researchers and funding agencies give priority to problem solving studies in the 1980s.

Almost ten years later, in the recently published Curriculum Standards for School Mathematics (1989), NCTM was still calling for an emphasis on problem solving in mathematics education. The existing K-4 mathematics curriculum was criticized in the Standards because it was narrow in scope; it failed to foster mathematical insight, reasoning, and problem solving; and it emphasized rote activities (National Council of Teachers of Mathematics, 1989). Also expressed as a concern in the Standards was that the 5-8 mathematics curriculum had "emphasized computational facility at the expense of a broad, integrated view of mathematics" (National Council of Teachers of Mathematics, 1989, p. 65). In addition, a shift in the role of the teacher at the secondary level was called for by NCTM. This shift would involve the teacher's role changing from that of "dispensing information to facilitating learning, from that of director to that of catalyst and coach" (1989, p. 128).

A comparison of the concerns and recommendations expressed in the 1980 Agenda and the 1989 Standards suggests that not much has changed over the last ten years. Accomplishments in the area of problem solving in mathematics education have been limited in spite of NCTM's continuing efforts. While there is agreement among mathematics educators as to the importance of focusing on problem solving in the classroom, there is little help available for teachers in terms of how to improve students' problem solving abilities. Unfortunately, while much has been written about problem solving in areas other than mathematics over the years, little has been studied or discovered concerning how best to teach problem solving in the mathematics classroom (Romberg & Carpenter, 1986).

The lack of research has been attributed to the nature of problem solving not lending itself well to the quantitative methods typically used in mathematics research (Romberg & Carpenter, 1986; Garofalo & Lester, 1987; Eisenhart, 1988). In order to learn more about problem solving, the more traditionally quantitative methods of mathematics researchers may need to be replaced or enhanced by qualitative methods. Eisenhart (1988) explained that while many mathematics education researchers are asking questions which could be addressed by ethnographic studies, few mathematics education researchers are using ethnographic techniques. Likewise, Eisenhart claimed that ethnographers rarely pay attention to the cognitive factors and developmental theories focused on by mathematics researchers. Eisenhart insisted that a joining of mathematics education researchers and educational ethnographers could produce a new and potentially useful type of study for problem solving research.

Another limitation of the research on problem solving in mathematics is that traditionally mathematics researchers have studied only the cognitive processes involved in problem solving. Most of the reports have focused on sets of steps that are so general or vague that they do not help students become better problem solvers. Other reports have described either algorithms (finite sets of steps for solving a problem) or heuristics (methods of discovery of solutions to a problem). The algorithms and heuristics recommended are usually applicable only in specific problems. Researchers are currently beginning to study metacognitive and affective issues as well as the cognitive aspects of problem solving (Garofalo & Lester, 1985; Romberg & Carpenter, 1986; Borodkin, 1987; McLeod, 1988). However, these studies have had limited impact on the improvement of problem solving due to the lack of a foundation or framework from which to build (Schoenfeld, 1981). Researchers have attempted to study ways of improving the teaching of mathematics problem solving without first identifying and agreeing on which skills and processes are used during problem solving.

Statement of the Problem

With the current emphasis being placed on improving mathematics students' problem solving ability, it has become even more important to find ways to teach problem solving in the classroom. Much of the recent research has focused on whether or not problem solving can be taught or how to improve the teaching of problem solving. However, the research has not thoroughly addressed the initial question of what skills and processes are actually used during the solving of mathematics problems. In order to systematically improve the teaching of problem solving, these skills and processes must be identified. Until recently, most of the research done in mathematics in the area of problem solving has been conducted using quantitative methods. In order to determine exactly what skills and processes students use during problem solving situations, mathematics researchers need to conduct studies designed to discover and describe phenomena as well as those which test or confirm hypotheses. The descriptive methods used in qualitative research could provide a more complete picture of the problem solving process.

Purpose of the Study

The purpose of this study was to determine what processes and/or strategies selected middle school students use during the solving of non-routine mathematics problems. This information could eventually lead to a base of ideas from which teachers will be able to improve students' problem solving abilities. The study was guided by the following questions:

1. What cognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?

2. What metacognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?
3. What affects, beliefs, or attitudes influence middle school students during the solving of non-routine mathematics problems?

Significance of the Study

Problem solving has received a great deal of attention among mathematics educators in the last few years. There is agreement among those in the field of mathematics that problem solving should be stressed in mathematics classes. While there is agreement about the importance of problem solving, there is little help for teachers concerning how to improve students' problem solving abilities. Little research has been conducted in the area of mathematics problem solving, and the research which has been done has lacked a foundation or framework from which to build.

Most of the research that has been conducted in the area of problem solving in mathematics has focused on cognitive aspects and has ignored the metacognitive skills and affective issues involved. Research needs to be done which studies problem solving holistically. Once the skills and processes used during mathematics problem solving have been identified, it can then be determined whether or not problem solving ability can be improved through instruction. If so, then researchers can begin to focus on instructional methods to improve problem solving abilities.

The intent of this study was to identify and describe the skills and processes used by selected middle school students during problem solving situations. The results of this study and similar studies which may follow can be used to help present a holistic view of the problem solving process and eventually lead to the improvement of problem solving instruction.

This study was designed to differ from most of the existing studies of problem solving in four ways. First, most of the studies have focused on one aspect of problem solving such as cognitive processes or metacognitive processes. This study was designed to examine problem solving holistically.

Second, nearly all of the studies on problem solving have used audiotaped interviews of subjects pieced together with their paper and pencil work in order to study the process of problem solving. This study involved using videotapes of the subjects as they worked on an overhead projector in order to simultaneously study their verbal as well as non-verbal and written responses.

Third, most of the studies of problem solving to date have focused entirely on the labels and connections formed by researchers from observing students work problems. In this study, student perceptions of their problem solving strategies and skills were examined along with the observations of the researcher.

Finally, most of the studies of problem solving conducted by mathematics researchers have involved the use of quantitative methods of analysis (Eisenhart, 1988). However, to answer the question of what skills and processes are used in problem solving, the rich descriptions and thematic analyses used in qualitative research are needed. This study involved the use of the ethnographic techniques of observation, description, and thematic analysis in mathematics education research.

Assumptions

Some middle school students exhibit more "seemingly natural" successful problem solving ability than others. It is this researcher's belief, grounded in an interpretist theory (Erickson, 1986), that these students were not born to solve problems, but that they bring with them to problem solving situations certain experiences, values, and beliefs from which they draw to help them successfully solve problems. According to Erickson (1986), interpretive research is designed to identify specific ways in which social and cultural experiences relate to the activities of specific persons in making choices. It is not possible to change the background of a student who has difficulty solving problems. However, this study was based on the assumption that there are certain processes which successful problem solvers learn from experiences which can be studied, isolated, and taught to other students to help improve their own problem solving abilities.

Limitations and Delimitations

Because qualitative research methods were used in this study, a small sample of students was studied. It is difficult, therefore, to generalize the processes and strategies used during problem solving reported by these six students to other populations. The researcher imposed delimitations on the study by requiring that the participating students be middle school students reading on grade level.

Definitions

The following definitions were important in this study:

<u>problem solving</u>: determining a solution to a non-routine problem

<u>routine problem</u>: a problem for which one readily sees a solution or a method of solution

<u>non-routine problem</u>: a problem for which one does not readily see a solution or a method of solution

<u>metacognition</u>: a combination of what one knows about the amount and kind of knowledge one possesses and the regulation or control of that knowledge (Garofalo & Lester, 1985; Brown, 1978)

<u>cognition</u>: formal as well as informal mathematical knowledge (Lester & Garofalo, 1987)

<u>affect</u>: feelings, attitudes, and emotions (Lester & Garofalo, 1987)

<u>skill</u>: "a mental activity that can be applied to specific learning tasks" (Jones, Palincsar, Ogle, & Carr, 1987, p. 14)

strategies: "specific procedures or ways of executing a given skill" (Jones et al., 1987, p. 15)

process: a sequence of skills

algorithm: a set of specified rules for performing a computation or solving a problem

heuristic: a method or methods by which solutions to problems can be discovered

Methods and Procedures

A combination of qualitative and quantitative research methods were used to conduct the study. The study was designed and methods and procedures were pilot tested with middle school students who were not participants in the actual study. Two graduate students were also used during the pilot testing of the grid interview technique (Kelly, 1955). The graduate students were used to help determine the clarity of the directions developed for the repertory grid interviews before pilot testing with the middle school students. The problems used in the study were selected from Problem Solving: A Handbook for Classroom Teachers (Krulik & Rudnick, 1988). The mathematics curriculum of a Tennessee county school system was a factor in problem selection. The problems were pilot tested with two of the middle school students who were not participants in the actual study.

After the initial pilot testing was completed, six subjects were selected, and written consent was obtained from the six subjects and their parents. The subjects selected were middle school students (grades 6,7,8). The ability levels of the students were not a factor. However, it was decided that the students should all be reading on grade level or above.

The students selected for the study participated in four interviews. The first interview was conducted with each student in order to gather data on the student's family, school, and mathematics history. Problem solving sessions were set up and conducted with each of the students. The sessions lasted approximately one hour. During that time, the students were given non-routine problems to solve for twenty minutes. Students worked the problems on the overhead projector and were asked to explain their thoughts on the problem as they worked. The students were videotaped while they worked the problems. The second and third interviews took place after the two problem solving sessions. The students were interviewed concerning their problem solving strategies. The tapes from the first session and interview were analyzed in order to determine the focus of the second session and to begin to develop possible categories for data analysis. The last interview consisted of each student completing a repertory grid (Kelly, 1955) categorizing their problem solving skills and processes as they perceived them. Each child then viewed their videotapes and tallied their skills and processes used according to their repertory grid.

Data Analysis

The data from the initial interviews were organized by analyzing transcriptions of the audiotaped interviews and developing student profiles. The data from the individual problem solving sessions were organized by studying the students' work, their facial expressions, their actions, and their verbal comments as shown by the videotape along with their verbal comments from interviews. Transcriptions from the tapes were used to develop a time chart. The data from the grid interviews were already organized on the form completed by each student and the researcher. After the data had been organized, they were analyzed qualitatively using the ethnographic research techniques for data analysis as described by Spradley (1980): domains (categories) were selected and analyzed, a "taxonomic analysis chart" of the processes and strategies used by middle school students in order to solve problems was formed, and finally a "componential analysis" was made for each set of data. A constant comparative method of qualitative data analysis (Glaser & Strauss, 1967) was then used to complete the qualitative analysis procedure by identifying themes and initial findings.

After the initial qualitative analysis had been completed, categories determined by the analysis were then used to quantitatively analyze the data. Student tallies were used to determine percentages and means for the taxonomic analysis charts prepared by the researcher. The data from the initial interviews, the problem solving sessions, and the repertory grids were then analyzed by comparing the categories and tallies determined by the students to the categories determined by the researcher. Themes and findings generated from the initial qualitative analysis were then compared to the quantified data to complete the development of findings from the study. Conclusions were then drawn from those findings.

Organization of the Study

Chapter I contains an introduction and the statement of the problem, the purpose of the research, the questions to be answered, the significance of the study, the underlying assumptions, limitations and delimitations, definitions of important terms, and the methods and procedures used.

Chapter II is a review of related literature which provides the background information and basis for the study.

Chapter III identifies and explains the methods and procedures used for data collection and analysis.

Chapter IV is a presentation and analysis of the data.

Chapter V contains a summary, major findings of the study, conclusions drawn from the research and findings, and recommendations for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

Problem solving has received a great deal of attention in the last few years, particularly in the area of mathematics education. Several reports reflect a national concern for the need to improve problem solving abilities of students (National Council of Teachers of Mathematics, 1980; National Research Council, 1989; National Council of Teachers of Mathematics, 1989; Willoughby, 1990). The National Council of Teachers of Mathematics (NCTM) organization's recently published Curriculum Standards For School Mathematics (March, 1989) offers a new and quite promising direction for mathematics education. The standards reflect an emphasis on actively involving students in doing mathematics. Problem solving is stressed and the use of manipulatives, cooperative work, discussion, and more justification of student thinking are promoted. NCTM suggested new goals for students which include learning to become mathematical problem solvers and learning to communicate and reason mathematically. These goals de-emphasize rote practice and memorization, computations out of context, drill, and the dispensing of knowledge.

Many teachers share a common concern that the emphasis on basic skills over the last few years has resulted in students' fragmented knowledge of mathematics and students' inability to apply the mathematics they have learned. Reports such as the Third National Assessment of Educational Progress (1983) reflect the teachers' concerns by indicating that the majority of students at all grade levels have difficulty with non-routine problems that require any analysis or thinking (Carpenter, Matthews, Lindquist, & Silver, 1984).

While teachers agree with NCTM's philosophy and would be willing to address the new recommendations in their classrooms, progress toward integrating problem solving into mathematics classes has been slow. Three main reasons are suggested by authors for this slow progress. First, teachers' lessons are often being dictated by an opposing philosophy exhibited by supervisors, curriculum writers, and others. There is a current national push to compete with other countries as well as frequent media reports that we lag behind other countries on standardized tests. These events have led to an emphasis on basic computational skills and accountability resulting in increases in standardized testing. Also, more of the mathematics is being pushed down into earlier grades leaving little time for problem solving (National Council of Teachers of Mathematics, 1989; Willoughby, 1990; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). Finally, Burns and Lash (1988) reported that teachers are reluctant to integrate problem solving into their lessons because of the difference in pedagogical skills required to teach problem solving. The authors described the contrast between basic skills instruction and problem solving instruction. Most basic skills instruction stresses automation of isolated skills through extended drill and practice on daily computation assignments. Problem solving instruction focuses on higher order skills and the development of flexible cognitive ability which means teachers face a different as well as more difficult set of pedagogical concerns when teaching problem solving.

Teachers must find the time to satisfy testing and curriculum requirements yet still help their students become effective problem solvers. Teachers can begin to address this issue by looking at the existing research and literature available on problem solving in order to determine more efficient and effective methods for addressing problem solving in the classroom. Unfortunately, while much has been written about problem solving over the years, little has been written concerning how to best teach problem solving in the mathematics classroom. A study of the existing research and literature on problem solving raises as many questions as it answers.

The following review of the research and literature on problem solving examines problem solving from a historical perspective. This perspective includes the popular thinking about problem solving, in general and specifically in mathematics education, the role of problem solving in mathematics education, and research in the area of mathematics problem solving. Future implications are also examined.

Current Thinking About Problem Solving

While there have been many definitions of problem solving presented in research and literature, most authors agreed that problem solving involves choosing a solution, from at least two options, to a problem in which a solution is not immediately apparent (Polya, 1957; NCTM, 1981; Hayes, 1981; Charles, Lester, & Daffer, 1987; Shuell, 1988). This view of problem solving rejects the idea that solving a routine single step mathematics problem is actually problem solving. Even so, Cawley and Miller (1986) reported that of three types of problem solving activities that prevail in schools, the single step word problems often seen in mathematics texts make up 97% of students' problem solving experiences in schools. The other two types of problems are problems focusing on applications in a certain content area (which require a need for specific knowledge) and activities which stress data collection and analysis (which lead to decision making). The authors explained that these two types of problems are rarely if ever included in students' school experiences. Carpenter and others (Carpenter, Corbitt, Kepner, Lindquist, & Reys; 1980) explained that too much emphasis on the single step problems will teach students only how to routinely solve cue-word type problems, rather than teach them to think about or analyze problems in detail. In order to improve our students' thinking and problem solving experiences in school, much has been studied and written about problem solving, particularly in areas other than mathematics.

Shuell (1988) explained:

> problem solving is a goal directed activity that requires an active search for (and generation of) possible alternative actions and decision making as to which course of action to follow next. As a part of this process, the individual must mentally evaluate the viability of various alternatives and then verify the effectiveness of the one selected by trying it out to see if it

works. Problem solving is clearly an active process! (p. 4).

In the field of psychology, Hayes (1981) defined a sequence of actions characteristic of problem solving:

1. Finding the problem: recognizing that there is a problem to be solved.
2. Representing the problem: understanding the nature of the gap to be crossed.
3. Planning the solution: choosing a method for crossing the gap.
4. Carrying out the plan.
5. Evaluating the solution: asking "How good is the result?" once the plan is carried out.
6. Consolidating gains: learning from the experience of solving (p. 1).

Hayes maintained that successful problem solving depends on the effectiveness of a person's carrying out of each step. He described the second step, representing a problem, as one of the more crucial steps in the process. At this stage, a problem solver imagines objects and relations in their mind which correspond to objects and relations described in the problem (the problem solver's internal representation of the problem). Often, problem solvers will make external representations of the problems by drawing sketches or diagrams or by writing down symbols or equations to represent the problems. Hayes suggested four basic methods for selecting problem solution methods: trial and error, proximity methods (selecting a step at a time), fractionation methods (breaking a problem up into parts), and knowledge-based methods (using methods already stored in memory).

Shuell (1988) explained that problem solving involves more than applying general strategies or following steps. The knowledge that a person brings to the situation when they solve a problem is an important factor in determining the way a person approaches finding the solution to a problem as well as their potential for finding the correct solution to the problem.

There are several different opinions as to the types of prior knowledge needed for problem solving. Riley et. al. (1983) reported three types of knowledge which are used in problem solving: problem schemata (for understanding various semantic relations involved in problems), action schemata (for representing actions involved in problem solving), and strategic knowledge (for planning methods of solutions for problems). Mayer (1983) suggested that there are five types of knowledge needed for problem solving: linguistic knowledge (knowledge of the problem language), semantic knowledge (knowledge about the context of the problem), schema knowledge (knowledge about types of problems), procedural knowledge (knowledge about how to perform operations), and strategic knowledge (knowledge of techniques for solving problems). Regardless of the different classifications of types of knowledge needed for problem solving, Shuell (1988) generalized that just having the knowledge is not at all sufficient. He maintained that students must know how to select from their existing knowledge the knowledge which is relevant to the particular problem being solved.

Shuell also reported that problem solving has been viewed in terms of general strategies for all subjects. It has been thought that if a person was a good problem solver in one content area, they would also be good in other areas. Recently, more emphasis is being placed on the domain-specific nature of problem solving (Shuell, 1988; Lippert, 1988). Shuell (1988) explained that success in problem solving in a particular area is highly dependent on knowledge specific to that content area to the extent that it cannot be expected that any transfer of problem solving ability will exist across content areas. Lippert (1988) explained that "research to date has failed to clearly identify either the cognitive mechanisms or the pedagogical approaches that cultivate problem solving skill within contexts, let alone transfer to other contexts" (p. 1). Most of the past as well as the current thinking concerning mathematics problem solving has been based on the work of George Polya.

Polya wrote several books on problem solving, the most well known of which is How To Solve It (1957). How To Solve It is a book of suggestions and ideas about problem solving for teachers and students. Polya presented four steps to problem solving: "read the problem", "devise a plan for solving the problem", "carry out the plan", and "look back or reflect on the plan" (Polya, 1957, pp. xvi-xvii). Polya's ideas on problem solving focused on practice and repetition to develop problem solving ability. One of Polya's most recurring suggestions for solving a problem is to relate the problem to one that has been done or seen before. He recommended going so far as to use the solution to another problem to help solve the present one. Polya included what he called a dictionary of heuristics in which he pointed out what others have suggested about problem solving heuristics. They included the following:

Analogy is important in problem solving.

Using auxiliary elements is helpful.

Decomposition of the whole into parts can be useful.

Determination and emotions play an important role.

Drawing figures or diagrams is an important heuristic.

Use generalizations where possible.

Is the problem similar to one that has been seen?

Working backwards from a solution is often helpful.

Use indirect proof if helpful.

Choose notation carefully.

Study the unknown carefully.

Use lemmas or auxiliary theorems if possible.

Have "brains" and "good luck" and "wait for a bright idea" (Polya, 1957).

For the most part, Polya's beliefs all involved the idea that practice is all important in developing one's problem solving skills. However, his inclusion of the notion of good luck, brains, and bright ideas suggests the possibility of one or several factors in problem solving that Polya and the earlier investigators of problem solving may not have been able to detect or define.

In The Handbook of Research on Teaching (1986), Romberg and Carpenter pointed out that past thinking and research has focused on heuristics, algorithms, and Polya's steps to problem solving. Willoughby (1990) and Schoenfeld (1987) reported that merely teaching students strategies or steps for problem solving is not an effective way to improve problem solving skill. Current researchers are beginning to look at cognition and metacognition as they pertain to problem solving in mathematics. In "Metacognition, Cognitive Monitoring and Mathematical Performance" (1985), Garofalo and Lester point out that purely cognitive analyses of performances in mathematics are inadequate because they overlook important metacognitive processes.

Metacognition refers to one's knowledge of one's own thinking (Romberg and Carpenter, 1986). Metacognition was described by Garofalo and Lester (1985) as a combination of what one knows about the amount and kind of knowledge one possesses and the regulation or control of that knowledge. Metacognition includes, but is not limited to, skills such as planning, choosing among alternatives, monitoring one's performance, changing one's choice of activities, and checking one's choice of plan or heuristic (Borodkin, 1987; Garofalo & Lester, 1985). For example, students engage in metacognitive skill use during problem solving when they engage in self-talk to check their understanding of the problem, acknowledge and organize existing data concerning the problem, weigh alternative choices of plans or heuristics, change their choice of plan during their working of the problem, and when they check or test their solutions for being reasonable or correct.

Along with the cognitive and metacognitive aspects of problem solving, researchers are beginning to look at the affective aspects of problem solving. Baroody (1987) reported that students' beliefs help explain why some children excel in mathematics while others are so anxious they become defensive and unable to successfully solve problems. McLeod (1988) explained that feelings of frustration, panic, muscle tension, satisfaction, and even joy can all be important factors in problem solving performance. He also states that little is known about affective issues in problem solving. The research in the area has proceeded slowly because the research on affective factors is more complex and difficult to conduct than the research on cognition (McLeod, 1988).

Problem Solving in Mathematics Education

Early ideas on teaching problem solving came from Polya's How To Solve It (1957). Polya maintained that the emphasis in teaching problem solving should be placed on practice and imitation with the teacher providing opportunities for practice, working many problems so that students can see how they should be worked, and asking students leading questions to help them choose the correct heuristic to use in solving the problem. Polya's four steps have often been suggested as the key to teaching problem solving. However, Polya's four steps are not enough to assure successful problem solving attempts. Teachers have been unsure of appropriate methods for teaching problem solving and along with pressure to emphasize basic skills and computation this has led them to often omit or neglect problem solving in the mathematics classroom. Burns and Lash (1988) offered several reasons why problem solving instruction may cause difficulties for teachers. Pedagogical content knowledge issues may be more difficult for problem solving instruction. Teachers tend to agree that drill and practice is acceptable for basic skills instruction, but they do not seem to know the best approach to teaching problem solving. Also, the more difficult nature of problem solving materials tends to affect the methods teachers select to teach problem solving. Teachers tend to teach problem solving using approaches that produce the fewest difficulties for students as well as the fewest management problems for themselves.

Romberg and Carpenter (1986) reported that research on teaching shows that mathematics classroom instruction has not changed much over the last fifty years. The basic pattern has been that of grading homework, teacher presentation, and seatwork with emphasis on computation with little or no time spent on problem solving. In this traditional mathematics instructional approach, the teaching and learning of mathematics is viewed as a passive process. However, with the current emphasis on problem solving by NCTM and others, the focus of mathematics instruction will have to change. Shuell (1988) described problem solving as a highly active process, that is best taught throughout the year in every lesson rather than as an isolated unit. Brandt (1990) cautioned against what he refers to as "the central myth of teaching thinking, which says that to get students to think better, you get them to think more" (p. 51). Teachers often provide students with more opportunities to solve problems, however, the teachers rarely discuss the thinking strategies behind the solutions, therefore, the students do not become aware of them. Brandt maintained that teachers should provide ample time during the lesson to discuss thinking behind both student and teacher solutions to problems.

Several authors have begun to look at ways to incorporate problem solving into mathematics instruction. In the NCTM 1980 Yearbook, Schoenfeld emphasized the importance of teaching students to use heuristics in problem solving. He stressed that training in each of the individual strategies (drawing a diagram, working backwards, etc.) is important but not significant unless teachers give students help in selecting the right strategy for a particular problem. Schoenfeld (1980) stressed that this can be achieved by pointing out cues in the form of problems themselves and by pointing out organization among the heuristics.

In another article in the NCTM Yearbook (1980), Suydam discussed clues from research on problem solving that can be used to teach problem solving. Suydam also wrote about clues that can be untangled from research on problem solving which can lead to improvement in mathematics education. These clues are of three types: "clues about children as problem solvers, clues about the problems themselves, and clues about problem solving strategies" (Suydam, 1980, p. 35). Suydam maintained that all three of these areas must be addressed in order to improve problem solving teaching and learning. In contrast to Polya's suggestions for teaching problem solving, Suydam stressed that practice in problem solving should not consist of repeated experiences in solving the same or similar problems over and over using the same techniques. Instead, she suggested that the way to improve problem solving skills is to practice many different problems using the same techniques and to practice the application of different techniques to the same problems. Two other important points made by Suydam were: the focus should be on the child's understanding of the problem and its solution rather than just on its solution, and children should be encouraged to detect and discuss their errors when they make them.

Butts (1980) reported that one key to successful instruction in problem solving is to pose problems properly. He suggested that teachers learn to pose problems so that the students will be motivated to solve the problems, understand and remember the concepts involved in the solution of the problems, and learn something about the process of problem solving.

Garofalo (1989) stressed the importance of beliefs and attitudes of students toward problem solving. He claimed that students have developed inaccurate beliefs about mathematics (i.e., only the mathematics that is to be tested is important, etc.) and that these beliefs are reinforced by the way mathematics is taught. He points out that mathematics classes should emphasize exploration, discussion, reflection, and interaction with a focus on problem solving and mathematical reasoning.

While many of the authors cited in this chapter have written about ways to improve mathematics instruction in problem solving, none of them have approached the issue of problem solving from a theoretical base. Writers who appear to have done so are Branca (1980) and Gadanidis (1988). Branca (1980) approached the subject of problem solving based on three different interpretations of the term: problem solving as a goal, problem solving as a process, and problem solving as a basic skill. Branca claimed that when problem solving is considered as a goal, it is not thought of in terms of specific problems, procedures, or methods. Learning how to solve problems is seen as the primary reason for studying mathematics and this influences the mathematics curriculum and the teacher's classroom instruction. When problem solving is seen as a process; procedures, methods, and strategies which the student uses becomes the focus. Finally, when problem solving is seen as a basic skill, the focus becomes the problem content and type and the solution method. Branca concludes his article by presenting his view that problem solving should be approached in respect to all three interpretations. Too often instruction is based on one of the interpretations, thus omitting aspects of problem solving.

Gadanidis (1988) presented another integrated approach to mathematics teaching. Gadanidis separated mathematics instruction into three components: understanding, problem solving, and facts and skills. Gadanidis maintained that all three components should be emphasized in order to give students a more holistic view of mathematics. Gadanidis also presented examples of how this integration can take place. While the recent focus on problem solving is important, integrated approaches and viewpoints such as Gadanidis's and Branca's can provide students with a more meaningful presentation of mathematics.

Research on Mathematics Problem Solving

In the past, there has been little research conducted in the area of problem solving (Romberg &

Carpenter, 1986). The lack of research has been attributed to the nature of problem solving not lending itself well to quantitative methods used in mathematics research (Romberg & Carpenter, 1986; Garofalo & Lester, 1987). Garofalo and Lester (1987) explained that, until recently, the nature of mathematics led studies to focus on the purely cognitive aspects of problem solving. The metacognitive and affective aspects of problem solving were viewed as more difficult to study. The most current research in the area of problem solving is beginning to focus on the metacognitive as well as the cognitive aspects of problem solving. Some attention is also being paid to the affective influences on problem solving.

The current research design for problem solving studies seems to follow the same pattern. Subjects are put in problem solving situations and are taped and interviewed concerning their cognitive and metacognitive processes. Subjects either work individually or in pairs. Data are then analyzed according to the researchers' specific interests (Schoenfeld, 1981; Lester & Garofalo, 1987; Clark & Dennis, 1988; McLeod, 1988; Brandau & Dossey, 1979).

Brandau and Dossey (1979) conducted a study of thirty ninth grade students from different high schools. The students were given five open-ended problem solving situations, then a think aloud interview was conducted.

The researchers set up forty categories of problem solving skills of which twenty-five were verbal, ten were non-verbal, and five were transitional. They separated the categories into six classes ranging from analytical and interpretive to operational or procedural and the students received points for statements in each class. Brandau and Dossey were able to determine that certain types of processes were used more than others and that different mathematical situations elicited different types of processes from students. The researchers also noted that students were highly individualistic in their processes. However, the researchers found some types of processes were more commonly preceded or succeeded by certain other types of processes indicating patterns of thinking and behaving in students. Brandau (1979) then studied the five highest scoring students for their creativity in problem solving. Brandau noted some similarities among the five students for creativity even though each student was judged to be highly individualistic.

Schoenfeld (1981) studied "expert" and "novice" problem solvers in order to categorize and describe the impact of the use of metacognitive skills on success or failure in problem solving. He labeled the metacognitive skills as managerial decisions. He found that "expert" problems solvers have vigilant managers to help them strive for efficiency and accuracy, while the novices are not able to make efficient use of their problem solving resources because they do not possess such managers. In addition to this finding, his research produced a framework for assessing cognitive and metacognitive skill use during problem solving. This is important for future research since the framework includes a subjective as well as an objective component. The objective aspect is for recording what happens, and the subjective component includes determining whether and how well decisions are made by the students. Schoenfeld concluded that metacognitive or managerial skills were an extremely important component of problem solving.

Lester and Garofalo (1987) also conducted a study to determine the importance of metacognition during problem solving. They also wanted to determine whether or not these skills could be taught to students who were lacking them. Their subjects were pretested, put into problem solving situations, taped and interviewed, and then given twelve weeks of instruction with an emphasis on the teacher facilitating, monitoring, and modeling the desired skills. The researchers were hesitant about their results because they did not base their results on a framework, however, they were convinced that metacognitive skills are important in problem solving and can be taught to students who have been lacking those skills.

Clark and Dennis (1988) concluded that the evidence in their study supports the idea of being able to train or teach students to monitor themselves during problem solving. They studied sixty fifth and sixth graders using an experimental/control group, pretest/posttest design. The group that received instruction in using metacognitive skills significantly outscored the control group on the problem solving posttest.

Other findings from research on metacognition include that metacognitive processes are quite susceptible to affective influences, such as confidence and anxiety, and that metacognitive processes are affected by students' perceptions of the causes of their successes or failures. While research on metacognition in the area of mathematics is scarce, the findings have shown that future research in this area will be worthwhile and indeed necessary if students are to become successful problem solvers.

Many of the current studies of problem solving make suggestions for future research in the area. In the Handbook of Research on Teaching (1986), Romberg & Carpenter pointed out that more research needs to be done in the areas of cognition, metacognition, and affective influences on problem solving in mathematics. Brandau and Dossey (1979) suggested that more research needs to be done across students of all ages and ability levels. Brandau (1979) added that the area of creativity in mathematics needs more research in relation to problem solving. She also suggested that more research be conducted on problem solving in the classroom setting.

Quite a few of the current researchers pointed to a need for frameworks and methods of measuring problem solving ability (McLeod, 1988; Garofalo & Lester, 1987). Studies described previously, such as McLeod's (1988) study in which he presented a theoretical framework for analyzing affective issues in mathematics, are two of the few studies on problem solving which considered working from frameworks.

Schoenfeld is another researcher who has suggested a framework for research on problem solving. Schoenfeld (1983) suggested three separate categories for analysis of students' problem solving performance: resources, or the knowledge brought to the situation by each individual; control, which is the monitoring, decision-making and metacognitive acts used by an individual; and belief systems, which include both conscious and unconscious beliefs about one's self, the environment, the topic, and mathematics in general.

In yet another attempt to provide a framework for problem solving research, Duffin (1983) described three stages of problem solving that were defined during a mathematics education conference called "Skills and Procedures of Mathematics Problem Solving". Duffin explained that the three stages are: an entry period when a person plays around with a problem and jots down relevant points; an attack period when one begins to employ specific strategies; and a review or extension period when the person formally writes a solution for the problem, tests it, and sometimes generalizes and extends it.

As for measuring problem solving ability, Malone et al. (1980) recommended using the Rasch Approach to measurement. With this approach:

> Problems appropriate to the background of the student population are collected.
>
> The problems are administered to a representative sample of the students and responses for each problem are scored.
>
> A statistical test of the conformity of the responses on each problem to the assumptions of a model is applied.
>
> The item difficulty of each problem is established.
>
> The appropriate problems are selected and administered to the students whose problem solving ability are to be measured.
>
> Responses are marked and scored according to item difficulty (Malone et al., 1980).

Relatively few researchers have developed and used frameworks or measurement scales in their research on problem solving. Perhaps more needs to be known about problem solving processes and behaviors before categories, frameworks, and measurements can be successfully implemented. In the future, in order to learn more about problem solving itself, the more traditionally quantitative methods of mathematics researchers may need to be replaced or enhanced by qualitative methods. Eisenhart (1988) defined ethnographic research as either "the holistic depiction of group interaction over a period of time, accurately representing participant views and meanings" (p. 51) or "the disciplined study of what the world is like for people who have learned to see, hear, speak, think, and act in ways that are different" (p. 51).

Eisenhart (1988) claimed that many mathematics education researchers are asking questions for which ethnographic research would be appropriate, however, relatively few of these researchers are using ethnographic techniques. Likewise, Eisenhart explained that ethnographers rarely pay attention to the cognitive factors and developmental theories focused on by mathematics researchers. Eisenhart suggested that a joining of mathematics education researchers and educational ethnographers could produce a new and potentially useful type of study. This type of study could be important for future problem solving research.

Summary

While much has been written about problem solving over the years, relatively little is known about problem solving processes and teaching methods. Researchers agree that problem solving is an active process which involves evaluation, decision making, verification, and reflection. However, current problem solving instruction is explained in the literature as consisting of repeated practice in solving cue-word type problems which routinizes problem solving rather than teaching students to think or analyze problems. Current instruction is still influenced by past thinking and research which focused on repeated practice of cue-word problems, Polya's four steps to problem solving, and cognitive processes and skills involved in problem solving.

While the quantitative nature of mathematics problem solving research in the past led studies to focus on the purely cognitive aspects of problem solving, current researchers are beginning to study the metacognitive skills and processes as well as the affective influences on problem solving. Research in these areas has proceeded slowly because the research on these factors is more complex and difficult to conduct than the research on cognition.

In the research studies which have been conducted on the metacognitive and affective aspects of problem solving, there have been reports which suggest that the teaching and modeling of metacognitive skills and processes can help improve students problem solving abilities. It was also reported by several researchers that metacognitive processes are quite susceptible to affective influences, such as confidence and anxiety or student perceptions of themselves as problem solvers.

Due to the recent attention that has been given to problem solving in mathematics education, more needs to be learned in order to help teachers meet the new standards for classroom instruction. A review of past and current research in the area of problem solving points to a need for more research in the areas of cognition, metacognition, and affective issues in problem solving as well as the possible integration of classroom mathematics instruction. It has been suggested that a combination of ethnographic research methods and traditional research methods may prove to be helpful in learning more about problem solving.

CHAPTER III

METHODS AND PROCEDURES

Introduction

Most of the past research in mathematics has been conducted using quantitative methods (Romberg & Carpenter, 1986). Romberg and Carpenter also maintained that there is a lack of research in the area of problem solving in mathematics due to the nature of problem solving not lending itself well to the quantitative methods commonly used in mathematics research (Romberg & Carpenter, 1986). Current interest in the cognitive, metacognitive, and affective issues in problem solving is leading to questions which could be answered using qualitative methods or a combination of qualitative and quantitative methods (Eisenhart, 1988). Therefore, the decision was made that a combination of qualitative and quantitative methods would be appropriate for this study.

In order to answer the posed research questions, a case study analysis was chosen for the design. The nature of the research questions required the researcher to closely examine the skills and processes used by middle school students when they solve mathematics problems. According to Goetz and LeCompte (1984), "Case study analysis is appropriate for intensive, in-depth examination of one or a few instances of some phenomena" (p. 47).

The sample population from which the subjects for the study would be selected was identified. Criteria were determined for selecting the subjects for the study and subjects were chosen from the sample population.

Each of the six subjects in this study participated in four interviews. The first interview consisted of questions about the subjects' homes, schools, and mathematics backgrounds. For the second and third interviews, the subjects worked non-routine mathematics word problems for twenty minutes and were instructed to read, work, and think out loud. The problem solving sessions were videotaped and were played back for the subjects during a follow-up audiotaped interview. The final interview consisted of the students and the researcher developing a repertory grid (Kelly, 1955) based on the students' perceptions of their problem solving experiences. Fransella and Bannister (1977) maintain that Kelly devised the repertory grid technique as a method for exploring the categories that a person uses to make sense of their world or their construct system. Munby (1982) modified the grid approach in order to insure that the perspective of the person being interviewed, not the interviewer, is understood. Using Munby's modification of the Kelly grid as a type of unstructured interview in this study would help to explain problem solving as middle school students view the process.

The audiotapes and videotapes were transcribed and analyzed. A subject profile was developed from the initial interviews conducted with the subjects. A time chart was used to organize the data collected during the problem solving sessions and interviews. The repertory grids were developed during the final interviews by each student and the researcher. The grid forms were left in their original form for the data analysis. A content analysis was performed on the existing data. Domains (categories) were selected and analyzed according to Spradley (1980). Continuing the research process described by Spradley (1980), a taxonomic analysis chart of the processes and strategies used by middle school students in order to solve problems was formed. From the chart, a componential analysis was made and themes were identified. After the initial content analysis was completed, the videotapes for each individual student were studied along with each student's tallies. Categories made by the researcher were then compared to the categories and tallies made by the students. Student tallies were used to determine percentages and means for the taxonomic analysis charts prepared by the researcher. The quantified data were then compared to the findings from the initial qualitative analysis in order to complete the identification of findings and themes from which conclusions were drawn.

The Subjects

The Sample Population

The population from which subjects were chosen consisted of middle school students from a Tennessee county school system. Middle school students were chosen as the sample population for several reasons. The research design required that rapport be established early in the study between the students and the researcher. It was necessary for the students to feel comfortable enough to work problems and think out loud in front of the researcher. The researcher had seven years of teaching experience at the middle school level and was confident that rapport could be established easily with students of this age. Also, at the middle school level, students have been exposed to most of the mathematics skills required for many of the non-routine problems presented in the literature. Finally, Willoughby (1990) reported that by the time the students are in middle school most of them have become mature enough to think about their own thinking processes (metacognition).

Since data were collected in the summer, a middle school student was considered to be a student who had just completed the sixth, seventh, or eighth grade. No preference was given to ability level in mathematics, however, the sample population included only students who were reading on at least grade level according to standardized test reports.

Selection of the Sample

The subjects chosen were six middle school students from a county school system in Tennessee. A decision was made by the researcher that six subjects would make an appropriate sample due to the nature of the study. A small sample would allow for more detailed descriptions and data analyses. Subjects were selected from three different schools on a voluntary basis. It was decided that subjects should be selected from more than one school so that they would not have identical scholastic backgrounds. Written permission was obtained from both the students and their parents. Copies of the consent forms may be found in Appendix A.

Composition of the Sample

Two eighth graders, two seventh graders, and two sixth graders participated in the study. There were three male and three female subjects. While no restrictions were designated by the researcher for socioeconomic levels of the participants, the subjects were from primarily middle to upper class families.

The Pilot Test

Problems and Procedures

The problems and procedures were tested prior to the actual study. Forty problems were selected by the researcher from Problem Solving: A Handbook for Classroom Teachers (Krulik & Rudnick, 1988), a book of non-routine mathematics problems for all grade levels. The problems were given individually to two seventh grade students who would not be participating in the actual study. The students worked the problems in three one hour sessions while the researcher recorded the time it took for the students to complete each problem, whether or not the students knew all of the words and terms in the problem, and if they answered the problem correctly. After all forty problems had been worked by the students, the researcher and the students rated the problems 1 through 5 in terms of difficulty with 1 being the easiest and 5 being the most difficult. Based on observations of the students and student ratings, the researcher selected twenty-five problems from the forty tested. Five problems from each difficulty level (1-5) were selected. The twenty-five problems were then arranged in groups of five. Each group of five problems contained one problem from all five of the difficulty levels. The groups were then arranged so that the problems would be encountered in an ascending level of difficulty. The problems were then numbered 1 through 25 and pasted on index cards to be used in the actual study. The twenty-five problems are listed in Appendix B. As the problems were drawn from a stack, students would encounter a problem rated 1, then 2, then 3, etc. up to 5. When the students completed the first set of five problems they were given the opportunity to work the next set. Each set of problems was arranged in the same order.

Initial Interviews and Problem Solving Sessions

The initial interview questions were developed based on the information to be collected. Questions were written to gather information about the subjects' families, schools, and mathematics histories. The initial interview questions were tested with the same two seventh grade students who were used to test the problems. The questions were revised after the pilot test interviews based on the students' understanding of the questions and the usefulness of the information obtained by each question. The copies of all interview questions may be found in Appendix C.

The methods and procedures for the problem solving sessions were also tested with two seventh graders. Two new test subjects were used since the previous two students had seen the problems. The videotaping procedure was tested for lighting, sound, and required space. The follow-up interview questions were written so as to enhance information gained during the videotaping sessions. The questions focused on selected strategies and skills, thoughts or feelings experienced, and self-evaluation procedures used during the problem solving sessions. During the pilot test, the questions were screened for clarity, appropriateness of order, and information collected.

Repertory Grid Construction

The construction of the repertory grid was tested on five different subjects including two adults and three middle school students. The two adults were graduate students in education. The pilot test focused on the clarity and understanding of the directions for completing the grid.

For the pilot test, the five individuals were asked to write on separate index cards everything they could think of that they thought or did during problem solving situations. After they had exhausted all their ideas, they were instructed to take the cards and group them in any way they liked. To design the grid, students were then asked to label each group. Students could name the groups whatever they wanted based on the similarities of the cards in each group (e.g. "things I feel", "things I say to myself", etc.). As the test subjects named the groups, the researcher wrote the name of each category along a horizontal axis. As the cards in each category were read out loud, the researcher wrote the constructs along a vertical axis to complete the grid. Appendix E contains a completed grid form.

Data Collection

Initial Interviews

During the first interview, background information was collected and both the students and their parent(s) were introduced to the nature and procedures of the study. The concept of non-routine problems was introduced and stressed to the students and parents in an effort to relieve any anxiety about performance. They were told that the researcher was more interested in how the students attacked problems in which a solution was not automatically apparent than in how many times they were correctly able to answer problems. Permission slips were secured from parents and students at the initial meeting.

During the first interview, the researcher asked the students questions pertaining to their background. Questions were asked about the students' family histories, general school experiences, and mathematics backgrounds. The interviews were audiotaped, transcribed, and analyzed. The audiotapes were kept in a locked file cabinet drawer and the transcripts were kept in each subject's individual folder in the file cabinet.

The Problem Solving Sessions

A camcorder, an overhead projector, a screen, a television, and a video cassette recorder were set up in an isolated classroom or office. The students were brought in groups of three to be familiarized with the equipment and the process.

The students were then brought in one at a time to conduct the first session. Students were given twenty minutes to work problems on the overhead. They were told that it did not matter how many problems they were able to complete in that length of time. The students were also instructed to do all reading and thinking out loud and to write all of their work on the overhead projector. The problems were put in order numbered 1 through 25, as explained earlier, and stacked upside down. The students would draw problems from the stack to work during the problem solving sessions. Students were to work the problems in order and were told that if they drew a complete blank after reading a problem that the researcher and the student would decide to skip the problem. The students then began to work while the researcher videotaped the session. The videotaping allowed the researcher to record students' written work, facial expressions, actions, and verbal comments simultaneously. Each problem solving session lasted twenty minutes.

After each twenty minute session was over, the researcher conducted the second interview with the students. These interviews were conducted primarily to clarify and supplement information gathered during the videotapings. Students were questioned about selected strategies and skills, thoughts or feelings they may have experienced, and self-evaluation procedures used during the problem solving sessions. While conducting the interviews, the researcher showed the students their videotapes. The students and/or the researcher could stop the tape at any point and add comments or thoughts to what had already been recorded. The second interviews were audiotaped. The tapes were transcribed and a preliminary analysis was performed on all six interviews to help focus or guide the next interview.

After a new focus was determined for the third interview, the next problem solving session was conducted. The procedures were identical to the first session. After all six students' sessions were completed, the audiotapes were transcribed and all tapes and transcriptions were filed with each individual student's folder.

Repertory Grid Interviews

The last interview conducted with the students involved having each individual student complete a repertory grid (Kelly, 1955). Students were first asked to list on individual cards everything they could think of that they thought or did during problem solving situations.

After the students had exhausted all ideas, the researcher then instructed the students to take the cards and group them in any way they liked. Students were allowed to move cards around until they were satisfied with their groups. To develop the grid, subjects were then asked to name each group whatever they felt was appropriate (e.g., "things I wrote," "things I did," "things I thought"). As the students named a group, the researcher wrote down the name of the category along a horizontal axis. As the students read what was in each category, the researcher wrote the constructs along a vertical axis to complete the grid. A copy of a completed grid can be found in Appendix E.

Students were then asked to look at each construct and compare it to each category and rate their relationship as:

1) not related;
2) sometimes related; or
3) definitely related.

Finally, students were shown the videotapes of both their problem solving sessions. They were asked to place a tally mark by each construct when they saw themselves do that particular thing on the tape. The grid construction interviews were audiotaped and tapes, grids, tallies, and cards were filed with each student's individual folder.

Data Analysis

The data from the initial interviews were analyzed by studying the transcripts of the audiotaped interviews. A student profile was developed for each participant. The data from the individual problem solving sessions were analyzed by studying the students' work, their facial expressions, their actions, and their verbal comments as shown by the videotape along with their verbal comments from interviews. A time chart was developed from the transcriptions of the tapes. A copy of a time chart can be found in Appendix D. The grid interviews were analyzed by studying each student's grid along with transcriptions of the audiotape of the interview.

The data from the four interviews were compared and analyzed following a process described by Spradley (1980). Domains, or categories, were selected and analyzed first. According to Spradley (1980), domains consist of three basic elements: the cover term, included terms, and a semantic relationship. The cover term is the name of the category. The included terms are the names of the smaller categories inside each domain. Finally, the semantic relationship is the linking of two or more categories by comparison (Spradley, 1980). Continuing Spradley's research process, a taxonomic analysis chart of the processes and strategies used by middle school students in order to solve problems was formed. A taxonomic analysis chart consists of sets of categories organized on the basis of relationships between them. A componential analysis was then made by defining the attributes of the separate categories of skills and processes used by the students during problem solving situations.

After the initial componential analysis was completed, categories determined by the analysis were used to qualitatively analyze the data using a constant comparative method (Glaser & Strauss, 1967). Specifically, two stages of the procedure described by Glaser and Strauss (1967) were used. First, incidents applicable to each category developed were compared and then, categories and their properties were integrated (Glaser & Strauss, 1967). Themes were identified and initial findings were developed from the constant comparing of categories and their properties.

After the initial findings were developed, categories determined in the original analysis were used to quantitatively analyze the data. The videotapes for each individual student were studied along with the tallies made by the students. Percentages and means were determined for the occurrence of the use of cognitive and metacognitive skills and processes, as well as the occurrence of affective influences as perceived by the students. Data from the initial interviews, the problem solving sessions, and the repertory grids were then analyzed by comparing the categories and tallies determined by the students to the categories determined by the researcher.

Findings generated by the qualitative analysis of the initial interviews, the problem solving interviews, and the grid interviews were then compared with the results of the quantitative analysis to complete the development of findings. These findings, along with current literature and research in the area of problem solving, were the basis for the development of the conclusions presented in the final chapter.

CHAPTER IV

DATA PRESENTATION AND ANALYSIS

Introduction

This study was designed to determine the skills and processes used by middle school students during mathematics problem solving situations. Six students from three different middle schools participated in four interviews. The first interview concerning their family, school, and mathematics history was audiotaped and transcribed. Then, a student profile was developed for each participant.

The second interview consisted of students working problems for twenty minutes using an overhead projector. Students were videotaped while working and then interviewed about their work after the problem solving session was over. The videotapes and audiotapes were transcribed, and time sheets were developed for each student. A preliminary analysis was conducted in order to determine the focus for the next problem solving interview. The third interview was conducted in the same manner as the second interview, and time sheets were developed for each student.

The fourth interview consisted of each student developing a repertory grid (Kelly, 1957) categorizing the skills and processes they use during the solving of non-routine mathematics problems. The students then watched their videotapes and tallied the skills and processes listed in their grids as they saw themselves use them on the tapes.

Presentation of the data and data analysis includes examples of responses to interview questions as well as examples of student responses during the problem solving sessions and grid development. Themes identified through constant comparison of the data are also presented.

The data analysis procedure was guided by the following questions:

1. What cognitive skills or processes do middle school students use during the solving of non-routine mathematics problems?
2. What metacognitive skills or processes do middle school students use during the solving of non-routine mathematics problems?
3. What affects, beliefs, or attitudes influence middle school students as they solve non-routine mathematics problems?

Cognitive Skills And Processes

In order to determine what cognitive skills and processes middle school students use during the solving of non-routine mathematics problems, the data collected during the second and third interviews as well as the grid interviews were used. Triangulation of data was achieved by comparing what students said about their cognitive skills and processes during the actual solving of problems to their responses in a separate, final interview with observations made by the researcher during the problem solving sessions.

Reading

Reading the problem more than once was a cognitive skill used often by students. The amount of time spent reading varied with the student and the problems. Certain themes emerged from the analysis of the problem solving sessions concerning the reading of word problems. Students read the problems more than once for three reasons. They reread the problem when they did not understand it the first time they read it. In nearly all of the cases, when students read a problem for the first time and did not understand it, they just continued to reread it until they either understood the problem or decided to skip it. In very few instances, the students used a chart, diagram, or drawing to help them make sense of the problem, but in most cases they just reread the problem. It was found that students also reread the problem to locate important pieces of information. Students reread the problem many times to pick out the numbers given in the problem so that they could perform a mathematical computation. Sometimes the students would reread for information such as "the Sharks won the game" so that they could label their answers. A final reason for students rereading a problem was to be certain of the question they were asked. Often, the students reported having forgotten what they were trying to find or determine. They reread the problem to be sure they were answering the right question.

Students did not use reading for two purposes identified in the literature on problem solving. They did not use reading to check their work once they had finished a problem. That is, they did not check their answers against criteria given in the problem or to see if their answers were reasonable. None of the students reported looking back at the problem and/or their work once they had arrived at an answer. They also did not use reading to help them discover the meaning of words they did not know such as ratio, sum, addend, units, etc.

For example, Problem No. 1 contained the word ratio.

> The ratio of boys to girls on the camp volleyball team is 3 to 2. There are four more boys than girls on the team. How many girls are on the team (Krulik & Rudnick, 1988, p. 112)?

None of the students remembered what the word ratio meant although they all reported having done ratio problems in school the past year. None of the students reported looking at other words or information in the problem to help them determine what ratio meant. When asked what they were thinking about during the period of time before they skipped this problem, all of the students explained that they were thinking back to school the past year and trying to remember what they had been taught about the meaning of the word ratio.

"Unit's" was another word which students did not try to determine the meaning of from context.

> What's my number?
>
> (a) I am a two-digit number.
> (b) I am a multiple of 6.
> (c) The sum of my digits is 9.
> (d) My ten's digit is one-half of my unit's digit (Krulik & Rudnick 1988, p. 102).

After reading this problem, none of the students knew what the unit's digit meant. One student skipped the problem because she did not know what it meant. The other students chose to ignore (d) and just work the problem based on the other three criteria. When questioned, the students all knew what two digit number and tens digit meant, but none of them were able to determine that if they had a two digit number and knew which one was the ten's digit, then the number in the one's place must be the unit's digit. Two students answered 18 rather than the correct 36 because they chose to skip (d) in the problem.

Mathematics Skills or Knowledge

The four basic mathematics computations (addition, subtraction, multiplication, and division) were used frequently by students in solving the problems. In fact, in several instances, these computations were performed on the numbers in the problem inappropriately because the students could not think of anything else to do. Problem No. 4 involved multiplication and division as well as knowing how many feet are in a mile.

> Two girls wish to find the speed of a moving freight train as it passes by their town. They find that 42 railroad cars pass by the corner in 1 minute. The average length of a railroad car is 60 feet. At what speed is the train moving in miles per hour (Krulik & Rudnick 1988, p. 117)?

One student read the problem over several times and then subtracted 42 from 60 and got 18 miles per hour as her answer. When asked how she decided on that strategy, she replied that she did not know. She explained that she did not understand the problem after reading it several times, and she could not pick out a key word to tell her which operation to use. She could not explain why she chose subtraction.

During the second and third interviews, when students were asked what mathematics skills they used in each problem, the students reported only the four basic computations. However, other mathematics skills were used by the students. Other mathematics skills or knowledge used by students were converting fractions to decimals, sequencing and identifying patterns, and identifying the value of coins in order to determine how many of each type were needed.

In terms of heuristics, students basically used five: checking to see if they had worked or seen a similar problem before, drawing a diagram, making a chart, identifying key words for computations, and trial and error. In almost every case, the students began each problem by determining whether or not they had seen a similar problem or had previously worked a similar problem. The students usually based their thoughts about a problem's difficulty on whether or not they had seen their teacher work a similar problem. Likewise, students were very hesitant to even attempt a problem unless they had seen their teacher work a problem like it. When asked what led one student to skip a certain problem she replied, "our teacher had never shown us how to work one like that before." In one case, a student explained that she always assigned a one to ten rating to problems before she worked them. When asked why she did this, she responded "I don't really know, my teacher just always did that with the problems he worked in class."

Drawing diagrams was used very little by any of the students. A problem involving the removal of toothpicks was the only problem for which they all drew a picture. Then, they just copied a figure which was already drawn for them as part of the problem. Several students drew diagrams on a problem involving perimeter and fence posts and another problem involving a baker dividing dough into pieces. The students drew diagrams only when they already understood the problem. In no case did a student draw a diagram to help them understand the problem.

The students often used forms of charts during the solving of the problems in order to sort and organize the information they were given in the problems. The charts, like the diagrams, were used only after the students understood the problems. Charts were not used to help the students make sense of the problems. Three of the students' charts were more formal than others, but all of the students reported using the charts to organize information or to help them remember important facts or parts of the problem.

During the initial interviews five of the students reported having been instructed to solve word problems by identifying key words in order to perform computations. Even though many of the problems used in this study did not lend themselves to key word/computation solving, the students did use this heuristic when possible. One student missed a problem due to improper use of a key word. The following problem involves placing a fence around some property (perimeter).

> A farmer has a plot of land in the shape of a rectangle that is 32 feet long by 24 feet wide. He wishes to put a fence around the plot of land. If fence posts are to be placed every 8 feet, how many fence posts will he use (Krulik & Rudnick 1988, p. 112)?

The student explained that she multiplied 32 by 24 because "by always means multiply in math." The problem however, required that the students should be concerned with the perimeter of the property rather than the area.

Students who used the trial and error heuristic used it quite often. Three students did not use trial and error at all except for the problem where the removal of toothpicks was involved. In all but two of the instances where trial and error were used, the students' first trials were mere guesses with no basis for their starting guess. But after the first guess was made, all of the students who used trial and error method were able to narrow down the answer by labeling their trials as "too high or too low" or "too much or too little".

Compared to the heuristics described in the literature on problem solving, the students used relatively few of the heuristics that are available to them. Among the heuristics which were reported frequently in problem solving literature but not used by the students in this study were making a simpler problem by temporarily changing the data in the problem, working backwards from information, breaking the problem up into smaller pieces, and adding new elements to the problem temporarily.

The cognitive skills used by students during the two problem solving sessions consisted of mostly reading skills and mathematics computations. Besides mathematics knowledge of words, concepts, and algorithms, the knowledge used by the students was mostly every day knowledge such as what a washer is and how many of each type of coin make up one dollar.

Student Perceptions of Cognitive Skills and Processes

When constructing the repertory grids, students were asked to write on separate index cards everything they could think of that they did, thought, or that influenced them during problem solving situations. In their grids, five of the students included thinking about a possible strategy for solving a problem. All of the students wrote that they always decided whether or not they had seen a similar problem before. When asked about other strategies, the students began to list mathematics skills. Addition, subtraction, multiplication, and division were listed first by all of the students. These were the only mathematics skills mentioned by students as being used during the second and third interviews, even though the students actually used others. However, students listed other mathematics skills when they were completing their grids such as finding a pattern, drawing a picture, using a formula, measurement, graphs and charts, estimation and rounding, changing fractions to decimals, and finding area or perimeter. The students listed these as things they did when they solved problems. They did not label them as mathematics skills. Strategies were often viewed by the students as mere computations to be decided on and performed. If they did not use any of the computations, the students often reported that they did not use a specific strategy to solve a problem.

During the grid constructions only two students reported other possible strategies. One student reported "breaking the problem up into smaller pieces" as a possible strategy, and another student reported "replacing the numbers in the problem with smaller, easier numbers temporarily in order to determine how to solve it." However, neither one of the students used either strategy during the problem solving sessions.

All of the students included reading and rereading the problem in their grids. Several students broke rereading into reading for important information and reading to pull the numbers out. Four of the students reported that they wrote down the important information or numbers to help them remember them while they worked the problem. Five of the students included thinking in their lists. When asked to explain what they meant, the students reported that they would "think about the problem, the numbers, or the question" by merely repeating them over and over in their head.

During the grid construction interviews, students listed a total of eighty-three different ideas concerning things they do or things that affect them when they solve problems. These eighty-three constructs can be found in Appendix E. Forty-three of the constructs were listed by more than one student but only counted once by the researcher. Of the eighty-three constructs, thirty-five were identified by the researcher as cognitive skills or processes. When students were asked to tally their behaviors and thoughts from the two videotaped sessions on their grids, only twenty-three of the thirty-five cognitive constructs received any tallies, and only sixteen of those (or 46% of the total cognitive constructs listed) received four or more tallies. Students are evidently aware of more of the cognitive skills that they have available to them for use than they are actually using or are aware of using.

Of seven hundred seventy-four total tallies (including cognitive, metacognitive, and affective constructs), five hundred eighty-four tallies were made next to cognitive constructs. Therefore, seventy-five percent of the total skills and processes perceived by the students as being used during the problem solving sessions were cognitive skills or processes. The metacognitive constructs received 22% of the total tallies, and the affective constructs received 3% of the tallies.

Of the thirty-five cognitive constructs listed during the grid interviews, seven were listed and received tallies by all of the students. Those seven cognitive constructs with the mean number of tallies they received by the six students are listed in Table I. The seven cognitive constructs listed in Table I were agreed upon by all six of the students as being involved in the solving of problems and in particular, the problems they solved during their two twenty minute sessions.

Table I: Seven cognitive constructs listed by all six students during their repertory grid constructions.

Construct	Mean No. of Tallies
Addition, subtraction, multiplication, or division	19
Read the problem (first time)	12
Reread the problem	10
Draw a picture or diagram	1.5
Write down information or numbers from the problem	5.2
Think about the question, the problem, or the numbers	11.17
Decide if I've seen a similar problem before	7.67

Metacognitive Skills And Processes

The metacognitive skills and processes used by middle school students participating in this study were determined by analyzing the data collected during the second and third interviews as well as the results of the grid interviews. Triangulation of data was achieved by comparing what students reported about their metacognitive skills and processes during the actual solving of problems, the results of the grid interviews, and observations made by the researcher during the problem solving sessions.

Monitoring

Students reported very little monitoring of themselves as they worked. Occasionally, a student would say "Yes, that's right" or "No, that can't be it." Very few times did any of the students check their strategy as they were working to see if they were on the right track. When students selected a strategy they stayed with the selected strategy even when it was leading them to an obviously incorrect solution. For example, one student reported that half of the way through a computation, he realized that the particular strategy he chose would not give him the correct answer to the problem. However, he continued carrying out the same strategy. When asked about changing the strategy, he replied that he rarely ever changed a strategy once he began working a problem.

Several students monitored themselves in terms of their computations. One student slowed down because he "tends to make mistakes when he rushes," and two other students checked certain parts of their work on a problem because it involved a computation that they reported as particularly difficult for them. Several of the students who used trial and error as a strategy monitored their trials as either "too much or too little" and "too high or too low".

During the repertory grid construction, while students were listing things they did when carrying out a strategy, only one student reported correcting himself during problem solving. This same student reported slowing himself down while working a problem. Three of the students included thinking they either had the correct answer or not. Five of the students also reported asking themselves questions such as "is there any missing or extra information," "do I understand the problem," "or does this answer look right?" One student added instinct and common sense to his list.
When asked to explain those he said "stuff just pops into my head and I don't know where it comes from."

Three of the students mentioned getting "stuck" while working problems. When asked what were some of the things they did when they got stuck, students included "looking back in the book"; "asking their parents, the teacher, or another student for help"; "skipping the problem and going back later"; or "skipping the problem all together." They also reported that sometimes they change their strategy or start completely over though none of them did so during the problem solving sessions.

Checking

Students did very little checking of their work while solving problems. Time was not an issue, because they had been told the number of problems they worked was not important. When asked about checking their work, the students explained that they rarely checked their work any time. The only reports of checking from the students were when the problem involved a computation which was particularly difficult for them or when they guessed at their numbers for trial and error.

Once students had arrived at their final answer, they did not check their answer against any criteria or information given in the problem. In a few instances, the students checked to see if their answers were reasonable. Students rarely looked back at the problem after they had answered it. They would make mistakes during the problem solving sessions that they would not notice at the time. The students would often catch their mistakes immediately upon viewing the videotape after the session was over.

There were few instances of checking either work or answers in the problem solving sessions. During their grid constructions, five of the students listed checking their answers as something they do when they engage in problem solving. One said she checks by rereading the problem, two said they check in their mind as they work the problem, and only one mentioned that he checked to see if the answer was reasonable. The remaining student had reported in an earlier interview that he never checked his work.

Guessing

Guessing was used frequently to designate a starting point for trial and error. Four of the students often guessed at their first numbers and then worked from that initial guess to narrow down possibilities until they found their answer. When asked about the nature of their guesses, three of the students explained that there was no basis for their guesses. One student, however, replied that his guesses were educated guesses based on real life knowledge such as how much a baseball and bat cost and how fast a train realistically might travel.

Two of the students used guessing to write a final answer. Even though students were told that how many problems they missed or how many/few problems they worked did not matter, four of the students chose to skip problems as opposed to using guessing as a strategy. During the repertory grid construction, three of the students reported guessing as something they did when they solved problems.

During the grid construction interviews, thirty-seven of the eighty-three different constructs which the students listed for problem solving were identified by the researcher as metacognitive skills or processes. When students were asked to tally their behaviors and thoughts from the two videotaped sessions on their grids, only twenty-six of the thirty-seven metacognitive constructs received tallies and fourteen of those (or 38% of the total number of metacognitive constructs) received four or more tallies. As with the cognitive skills, students appear to be aware of the metacognitive skills available to them, but they did not use or were not able to identify them in their problem solving situations.

Of the seven hundred seventy-four total tallies, including cognitive, metacognitive, and affective constructs, one hundred sixty-five or about 22% of the total skills and processes perceived to be used by the students were metacognitive skills or processes. It could not be determined from the information collected in this study whether the students were not sure how or when to use the metacognitive skills or whether the metacognitive skills are not as important in the overall problem solving process.

Of the thirty-seven metacognitive skills and processes named by the students during the grid interviews, only one was tallied by all six of the students during the viewing of the videotapes. "Deciding to skip a problem" received tallies by all of the students with the mean number of responses by students being 3.3. The other metacognitive constructs listed during the grid interviews received tallies from either one or two of the students while viewing their videotapes.

Affects, Beliefs, and Attitudes

In order to determine what affects, beliefs, and attitudes influence middle school students during the solving of non-routine problems, all four interviews with each student were analyzed. Triangulation of the data was achieved by the constant comparing of student responses during the first interview with their actions and responses during the second and third interviews as well as the grid results.

Beliefs/Attitudes

All six of the students believed they were good students overall, and in particular good mathematics students. When asked what made them good students in math, their replies were very similar to what made them good students in general (i.e., "I do my work," "I behave," "I help the teacher," etc.). However, three of the students explained that they had never really been good at math. This indicates that how well the students solved problems had little to do with their perceptions of themselves as good math students and vice versa.

The reasons for enjoying or liking a subject varied among the students. Three students believed that a teacher was the dominant factor in determining whether they liked a subject or not. Two students reported that interesting material and activities were the most important consideration in determining favorite subjects. The sixth student indicated that the grades he made in each subject determined his favorite classes. Of the six students, two reported that math was one of their two favorite subjects. Three of the students reported that they liked math, although it was not one of their favorite subjects, either because of the teacher or because their grades were not as good in math as other subjects. One student reported that she did not like math.

When asked about word problems, all of the students believed that word problems were harder than the "regular math" and that word problems (especially non-routine word problems) were for extra credit, for those who finished the assignments early, or for special contests.

As mentioned previously, several students believed that only strategies which they had seen their teacher use in math class could be applied to problems. This was indicated by students' analyzing each problem first as to whether or not they had seen one like it before; their unwillingness to risk trying to solve a problem unless they could remember their teacher solving one like it in class before; and by their applying strategies without knowing why, except that they had seen their teacher use that strategy before.

Two of the students strongly believed that if a problem was about a concept they disliked or knew little about (e.g., baseball), they would not be able to work the problem, regardless of whether or not the concept had anything to do with the solution. One student, who had done particularly well on most of the problems she had attempted, claimed that she knew when she read the following problem that she could not work it because it involved baseball, and she knew nothing about baseball.

> What was the final score of the Tigers-Sharks baseball game?
>
> (a) The sum of their scores was 8.
> (b) The product of their scores was 15.
> (c) The Sharks won the game (Krulik & Rudnick, 1988, p. 99).

The same student reported on another problem about the cost of two items that she thought she could not work the problem because the two items were a baseball and a bat. Another student believed a problem was going to be difficult for him because it involved metrics. However, conversion of metrics is not involved in the problem. The metric unit of a gram is used only as a label in the problem.

> A penny weighs approximately 3 grams. A nickel weighs approximately 5 grams. About how much more does $5 in pennies weigh than $5 in nickels (Krulik & Rudnick, 1988, p. 105)?

On the contrary, if these students encountered a problem involving something they were interested in or liked, such as money, they believed that the problem would be easy when they read it, even if the mathematics involved was complicated.

Affects/Feelings

Student feelings were rarely reported during the study. It is not understood at this point whether feelings have such little influence on problem solving or whether students cannot or choose not to describe how they feel.

When asked how they felt when they came to the problem solving sessions, students often replied "I didn't know what to expect" or "I wasn't sure if today would be harder than the first time." Even when questioned specifically about any feelings such as nervousness or anxiety, the students did not indicate that they felt anything. Similarly, when students were asked how they felt after reading or while working a certain problem, the students responded by saying "I thought it would be hard" or "I thought it was easy" as opposed to reporting any type of feelings.

The only reports of feelings were in the form of feelings toward self. When asked how they felt after the first session was over, one student responded "I felt bad because I wasn't able to get any more than I did" or "I felt terrible 'cause I had to skip so many."

Student Perceptions of Affects, Beliefs, And Feelings

As previously mentioned, the students did not report many affective concerns during the problem solving sessions even when asked specifically about their feelings in the follow-up interviews. Likewise, very little was reported during the grid constructions. One student listed feeling relieved when he finished a problem and nervous or tired after he had been working on a problem for a long time. Three of the students reported feelings of frustration during problem solving. One student reported feeling lost or confused while solving some problems. Another student reported feeling good about herself when she got an answer right.

During the grid construction interviews, in the list of eighty-three total constructs named by students, eleven of those constructs were in the affective domain. When students were asked to tally their behaviors and thoughts from the two videotaping sessions on their grids, six of the eleven constructs received tallies from students, and two of those six (or 19% of the eleven total) received four or more tallies. Of seven hundred seventy-four total tallies (cognitive, metacognitive, and affective constructs), twenty-five tallies (3% of the total number of tallies made) were made by the students next to affective constructs. None of the affective constructs received tallies by all six of the students. As with metacognitive skills and processes, it cannot be determined by the information collected in this study whether affective influences are not as important as other skills and processes or if students have trouble identifying their feelings.

Holistic View Of Problem Solving

One objective of this study was to examine problem solving holistically. Therefore, besides analyzing the data in terms of the separate categories of cognitive/ metacognitive skills or processes and affective influences, the data were also analyzed in order to depict the process of problem solving in general. In reviewing the literature on problem solving, two methods of describing the problem solving process in mathematics were evident. First, the various stages or steps for problem solving were described. While there were different terms and different numbers of steps used by authors, most of the stages were related to Polya's (1957) four steps of reading the problem, devising a plan for solving the problem, carrying out the plan, and reflecting about the problem and it's solution.

Another method of describing the process of problem solving was by its components: cognitive skills and processes, metacognitive skills and processes, and affective influences. The researcher categorized the students' lists of constructs both ways: by stages and by components. In both analyses, everything the students listed as skills or influences on problem solving fell under one of the three components and one of the four stages.

When students were asked to categorize the constructs listed on their cards in any way they wanted, the researcher expected some similarities to exist between how the researcher and the authors viewed the entire process of problem solving and the way the students viewed it. It was expected that only the words or terms used by the students would differ from those used by mathematicians. However, the students did not categorize their listings in the same manner in which the researcher and various authors viewed the process of problem solving.

In general, the students had trouble with the exercise and often had one or two groups that contained only one construct. For example, one student had a category labeled "things I feel," but the card which read "feel relieved" was the only card placed in that group. Some of the students named one or two categories with almost identical labels. See Appendix E for a listing of student categories. Students were also asked to compare each construct with each of their categories and rate them as (1) not related, (2) somewhat related, or (3) definitely related. All but one of the students rated 50% or more or their constructs and categories as (1) or not related. The students did not appear to view problem solving as a holistic process, but as bits of isolated, often unrelated skills.

Summary

The constant comparison of categories identified through the data analysis process revealed major themes which were the basis for the findings and conclusions presented in the last chapter. Themes emerged from an analysis of the data collected concerning cognitive and metacognitive skills and processes used by the students as well as affective influences on the students during the solving of non-routine mathematics problems.

Analysis of the data collected revealed the following themes concerning the cognitive and metacognitive skills and processes used by the students:

1. A lack of understanding of words or how to use context to discover word meanings causing students to skip or miss problems.
2. Reporting only addition, subtraction, multiplication, and division as mathematics skills used during problem solving.
3. A limited use of heuristics.
4. The lack of risk taking during problem solving.
5. Little changing of strategies while solving a problem.
6. Little or no monitoring, checking, and guessing while solving problems.

Analysis of the data revealed the following major themes concerning the affective influences on students' solving of non-routine problems:

1. Viewing of word problems as "different from regular math."
2. The subject of a word problem affecting students' attitudes toward and abilities to work a problem.
3. The unableness/unwillingness to report feelings.

Additional themes revealed by the analysis of the data were:

1. The role of the teacher as a model for students in terms their problem solving strategies.
2. Viewing problem solving as bits of isolated or unrelated skills rather than as a holistic process.

CHAPTER V

SUMMARY, FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE RESEARCH

Summary

Problem solving has received increased attention in recent years in mathematics education. Mathematics educators agree that teaching students to become proficient problem solvers should be a top priority goal in mathematics education. However, no one appears to have determined the best method or methods for teaching problem solving. There has not been much research on problem solving in the area of mathematics until recently, and the research that has been done was conducted using mostly quantitative methods. Problem solving is a complex process which lends itself to the rich descriptions found in qualitative research techniques.

The research conducted on problem solving in mathematics has focused on techniques to integrate more problem solving into mathematics lessons with little help for teachers concerning how to teach problem solving. In order to improve the teaching and learning of problem solving, more needs to be learned about the actual skills and strategies involved in the process.

The Problem

With the current emphasis being placed on problem solving, it is important that teachers become informed concerning how best to teach students to become proficient problem solvers. In order to improve the teaching of problem solving, more needs to be learned about the skills and processes involved in problem solving.

This study was designed to investigate the skills and processes involved in, as well as any affective influences on, middle school students' solving of non-routine problems. The research was guided by the following questions:

1. What cognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?
2. What metacognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?
3. What affects, beliefs, or attitudes influence middle school students during the solving of non-routine mathematics problems?

Procedures

The study was conducted using a combination of qualitative and quantitative research methods. Six students from three different middle schools were selected on a voluntary basis to participate in the study. The ability levels of the students were not considered except that all of the students were required to be reading on grade level or above.

The students selected for the study participated in four interviews. The first set of interviews was conducted in order to gather data concerning the students' families, schools, and mathematics histories. The interviews were audiotaped and transcribed, and a student profile was developed for each participant.

The second and third interviews consisted of students solving previously selected and tested problems for twenty minutes. Students worked problems on an overhead projector and were videotaped. Students were instructed to think and work out loud. After the twenty minutes expired, the researcher and the student viewed the videotape while the researcher interviewed the student concerning their work. After the first videotaping sessions were completed, a preliminary analysis was done in order to determine a focus for the third set of interviews and problem solving sessions. The third set of interviews was conducted in the same manner as the second. The videotapes and the audiotaped interviews for each session were transcribed and time sheets were developed for each student.

In the fourth and final set of interviews, each student completed a repertory grid (Kelly, 1957) categorizing their problem solving skills and processes as they perceived them. Each student then viewed both of their videotapes and placed a tally by each construct whenever they observed its occurrence on the videotape.

The data were organized and analyzed qualitatively using the ethnographic technique described by Spradley (1980). Domains, or categories, were selected by the researcher and analyzed, taxonomic analysis charts of the skills and processes used by middle school students were developed, and a componential analysis was made for each set of data. A constant comparative method of qualitative data analysis (Glaser and Strauss, 1967) was then used to complete the procedure by identifying themes across categories.

After the initial qualitative analysis was completed, categories determined by the analysis were then used to quantitatively analyze the data. The tallies made by the students while watching their videotapes were totaled, and percentages were figured for the occurrence of the use of cognitive and metacognitive skills and processes as well as the occurrence of affective influences as perceived by the students. Themes and findings generated from the initial qualitative analysis were then compared to the quantitative analysis to complete the development of findings. Conclusions were then drawn from the findings.

Findings

Research Question 1: What cognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?

Of the total skills and processes perceived by students as being used during the problem solving sessions, 75% of them were cognitive. Most of the cognitive processes used by the students were reading skills and mathematics skills or heuristics.

Reading the problem over several times was a cognitive skill used often by students. It was found that students reread a problem for three main reasons. They reread a problem when they did not understand it the first time they read it. In most instances, the students kept rereading a problem until they understood it or skipped it as opposed to making a chart or diagram to help them make sense of the problem. They also reread problems to help them locate important pieces of information such as numbers or criteria specified in the problem. And finally, they reread a problem to help them remember the question being asked in the problem.

Students did not use reading to help determine the meaning of words they did not know. When students encountered a word they did not know, rather than try to use context to try to figure out the meaning of the word, they either skipped that part of the problem or skipped the problem entirely. When students encountered a word they did not know, it often affected their ability to attempt to solve the problem. Students also did not reread the problem after they had arrived at an answer to determine if their solution was reasonable.

The other cognitive skills or knowledge used by the students during the problem solving sessions were mathematics skills and heuristics. When asked specifically what mathematics skills the students used to solve each problem, they reported the four basic computations (addition, subtraction, multiplication, and division) even though they used others.

When questioned more specifically about strategies, students responded by describing heuristics. The heuristics which students reported using were identifying whether or not they had seen a similar problem before, identifying key words in the problem, drawing diagrams, making charts, and trial and error.

The first strategy used by all the students was to decide if they had seen a similar problem before. In particular, students determined if they had seen their teacher work a problem like it. Students rarely attempted strategies unless they were certain they understood the problem and could apply a strategy they had seen their teacher use.

Compared to the heuristics described in the literature on problem solving the students used relatively few of the heuristics that are available to them. However, when constructing their repertory grids, the students listed more mathematics skills and heuristics as things they do when they solve problems than they actually used in their problem solving sessions. Therefore, the students were aware of more of the cognitive skills which they have available to them for use than they actually use or were aware of using.

The majority of cognitive skills and processes used by the students were reading or mathematics skills. Other cognitive knowledge or skills used by students consisted of everyday knowledge such as what a washer is and how much of each type of coin make up one dollar.

Research Question 2: What metacognitive processes and/or strategies do middle school students use during the solving of non-routine mathematics problems?

Students reported very little monitoring of themselves during their problem solving sessions. Of the total skills and processes perceived by the students as being used during the problem solving sessions, 22% of them were metacognitive skills or processes.

Once students had selected a strategy they seldom reported monitoring themselves to see if the strategy was working. In the few cases where students became aware that their strategy was not going to lead them to the correct solution, they did not change their strategy or try to determine what was wrong with their original strategy.

Students seldom used formal methods of checking their problems. Most of the time they reported that their answer "just looked right." On several occasions, students reported checking their computations as they finished them to identify careless errors. However, when they finished a problem, none of the students reported looking back at the problem or their answer in order to determine if their solution was correct or even reasonable.

As with the cognitive skills, students were evidently aware of more of the metacognitive skills which they have available for use than they actually used or were aware of using during the problem solving sessions. Students reported a very limited amount of metacognitive skills as being used in the actual problem solving sessions. However, they listed other metacognitive skills such as checking their work and breaking the problem up into smaller pieces in their repertory grids as being available for them to use.

Research Question 3: What affects, beliefs, or attitudes influence middle school students during the solving of non-routine mathematics problems?

During the interviews, students described the beliefs they held about themselves, mathematics, and the solving of non-routine problems. Analysis of the interviews revealed beliefs which the six students had in common.

When asked about word problems, all of the students believed that word problems were harder than the "regular math" and that word problems (especially non-routine word problems) were for extra credit, for those who finished the regular assignments early, or for special contests.

How well the students solved problems had very little to do with their perceptions of themselves as good math students and vice versa. When asked, all of the students reported they were good math students, but only two of them regarded themselves as being good at math. When asked what made them good math students, all six of the students responded in the same manner as they did when they were asked what made them good students in general (i.e., "I work hard," "I help the teacher," "I complete all of the assignments").

Several students strongly believed that if a problem involved a concept they disliked or knew little about (e.g., baseball or football), they would not be able to work the problem regardless of whether or not the concept had anything to do with solving the problem. Likewise, if the problem was about something they liked, they believed that the problem would be easy.

Students could not or did not describe their feelings during the solving of non-routine mathematics problems. Some of the students did list some feelings such as nervousness or frustration in their repertory grid, but they did not report any feelings during the problem solving sessions.

Additional Findings

Students did not appear to view problem solving as a holistic process, as reported in the literature, but as bits of isolated, often unrelated skills. This was evident in the students' constructions of their repertory grids. Students were asked to group their constructs in any way they liked and then to name each group whatever they felt was appropriate. The students had difficulty with the task, and their category names did not depict problem solving as a holistic process.

Finally, it was evident from student responses that teacher modeling was an important factor in the way the students selected and carried out strategies. While the students were aware of some heuristics and monitoring skills which were available to them, they only used strategies that they had seen their teacher use in class.

Conclusions

The examination of the data, the themes, and the findings of the study led the researcher to the following conclusions.

<u>Students are not aware of the various alternatives they have available to them to help them understand a mathematics problem when they first read it</u>. For example, when students encountered a problem they did not understand, they reread the problem over and over until they understood it or they skipped it. The students did not use heuristics such as dividing the problem into small parts, putting information into a chart or diagram, determining the meaning of unknown words through context, or altering the numbers or information temporarily in the problem in order to make the problem easier to

understand. Teachers could help improve students' understanding of problems by explaining and modeling the different techniques that exist for making sense of a difficult problem.

The only skills which students perceive as math skills are the basic computations of addition, subtraction, multiplication, and division. While they are aware of other skills (e.g., measuring, converting, trial and error, etc.), they do not classify them as mathematics skills. This narrow classification of mathematics skills could cause students to overlook possible useful skills when solving problems. Again, teacher explanation and modeling of mathematics skill use in non-routine problems could help students make the connection between the skills and problem solving.

Students are unwilling to take risks when presented with a problem solving situation. This is revealed by the fact that the students were often hesitant to try a strategy unless they had seen their teacher use the strategy in class. Also, students were often insistent that they could not try a problem because they had never seen their teacher work one even similar to it before. This lack of risk taking is reinforced by the students' beliefs that non-routine problem solving is for extra credit or for students who finish the regular assignments early or compete in contests. Teachers should provide a problem solving atmosphere in their classrooms which would encourage students to take risks and try problems that are new and different for them. Non-routine problems should become a regular part of the mathematics which students are exposed to in school, and students should be encouraged to consider sharing their methods of solution as important as getting the correct answer. Students will then feel more comfortable and be more willing to risk attempts at difficult problems.

Students have been told that various heuristics exist to help them solve problems and have been instructed to use them, but they have not been adequately informed concerning how and when to use them. This is shown by the students using only a limited number of heuristics during their problem solving sessions, although they had listed other heuristics in their repertory grids as being available for them to use during problem solving. Similarly, the students were aware of more metacognitive skills and processes available to them than they actually used in their problem solving sessions. Students seldom monitored themselves or checked their work while solving problems. When reporting the heuristics and metacognitive processes they knew of but did not use, the students often wrote the skills in teacher terms as though the students were copying a definition they did not really understand. While teachers need to tell students about possible heuristics or skills they could use, teachers also need to model the use of these skills during their lessons so that students can learn how and when to use the skills.

Finally, as shown by the students' responses in all four interviews, teachers are important in students' perceptions and beliefs about themselves, mathematics, and problem solving. Teachers need to be aware of their influence and concentrate on fostering students' self-esteem and positive attitudes toward mathematics, particularly problem solving, and focus on modeling the problem solving behaviors which they themselves use of which students are not often aware.

Implications

Implications for both preservice and inservice teachers are suggested from this study. Preservice teachers should be exposed to more problem solving experiences during their training in order to help their students become more proficient problem solvers.

Preservice teachers should also be encouraged to think out loud when explaining or working a problem and to model the behaviors which lead them to successful problem solving.

Similar experiences would benefit inservice teachers. Besides training in modeling problem solving behaviors, inservice teachers also need suggestions on how to integrate problem solving into an already crowded curriculum.

A final implication of this study concerns evaluation. Currently, standardized tests measure students' ability to perform routine computations and algorithms. While teachers may want to incorporate higher level thinking and problem solving into their lessons, many teachers have chosen to focus on basic skills and increasing standardized test scores to satisfy public demand (Brown, 1990). Methods of evaluation need to be restructured to include the measurement of students' problem solving abilities. Placing more emphasis on the evaluation of problem solving could increase the time spent on problem solving in mathematics classrooms.

Recommendations For Future Research

Examination of the findings and conclusions identified in this study lead to the following recommendations for future research:

1. This study should be replicated with different subjects from the same grade levels as well as with subjects from other grade levels.
2. Further study of problem solving holistically and how students perceive the relationship of the skills and processes involved when they solve problems is suggested.
3. Similar studies need to be conducted with teachers of all grade levels in order to determine their perceptions of problem solving.
4. Mathematics classrooms should be observed in order to study teacher modeling of problem solving, the ways problem solving is taught to students, and how problem solving is integrated into the mathematics curriculum.

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APPENDIXES

APPENDIX A

PARENT CONSENT FORM

I, ________________________, do hereby give permission for my child,____________________, to serve as a subject in a study entitled "Strategies and Skills Used by Middle School Students During the Solving of Non-routine Mathematics Problems." and conducted by Terry D. Rose in order to fulfill the requirements for a doctoral dissertation for the University of Tennessee and to advance the knowledge in the area of problem solving.

I understand that my child will spend one hour each week for five weeks in problem solving sessions with the researcher, and that he/she will be asked to solve problems both verbally and on paper. I understand that the sessions will be videotaped.

I understand that the study will be reported in a dissertation and that anonymity will be maintained in any reporting or publishing of the study.

I understand that Terry Rose will provide transportation to and from the University of Tennessee for my child during the course of the study.

____
(Parent Signature)

____
(Date)

STUDENT CONSENT FORM

I, ____________________________, do hereby agree to serve as a subject in a study entitled "Strategies and Skills Used by Middle School Students During the Solving of Non-routine Mathematics Problems" and conducted by Terry D. Rose in order to fulfill the requirements for a doctoral dissertation for the University of Tennessee and to advance the knowledge in the area of problem solving.

I understand that I will spend one hour each week in problem solving sessions with the researcher, and that I will be asked to solve problems both verbally and on paper. I understand that the sessions will be videotaped.

I understand that the study will be reported in a dissertation and that anonymity will be maintained in any reporting or publishing of the study.

____
(Student Signature)

____
(Date)

APPENDIX B

PROBLEMS

1. The ratio of boys to girls on the camp volleyball team is 3 to 2. There are 4 more boys than girls on the team. How many girls are on the team?

2. Find the next three numbers in the sequence 2,3,5,8,12.

3. Two girls wish to find the speed of a moving freight train as it passes by their town. They find that 42 railroad cars pass by the corner in 1 minute. The average length of a railroad car is 60 feet. At what speed is the train moving in miles per hour?

4. Lonny has 2 bats and 1 ball that cost her $11. Andy has 1 bat and 2 balls that cost him $7. How much does 1 bat and 1 ball cost?

5. Mike has 15 coins that total $1.00. What are the coins and how many of each does he have?

6. What was the final score of the Tigers-Sharks baseball game?

(a) The sum of their scores was 8.
(b) The product of their scores was 15.
(c) The Sharks won the game.

7. A penny weighs approximately 3 grams. A nickel weighs approximately 5 grams. About how much more does $5 in pennies weigh than $5 in nickels?

8. A baker rolls out his dough in the morning and cuts it into 8 equal pieces, which he seasons. He then cuts each of these seasoned pieces into 4 equal parts. He bakes each of these into a loaf of bread that is 3/4 of a foot long. If we were to place all of these loaves end-to-end, how long would the total length be?

9. Mary has scored 98,65,63, and 80 on 4 tests this term. What must she score on the next test, if her average is to be 80 for all 5 tests?

10. During the recent census, a man told the census-taker that he had three children. When asked their ages, he replied, "The product of their ages is 72. The sum of their ages is the same as my house number." The census-taker ran to the door and looked at the house number. "I still can't tell," she complained. The man replied, "Oh, that's right. I forgot to tell you that the oldest one likes chocolate pudding." The census-taker promptly wrote down the ages of the three children. How old are they?

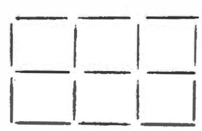
11. Three boys stood on a scale and put a nickel in the slot. The scale showed 390 pounds as their total weight. One boy stepped off the scale. It then showed 255 pounds. The second boy stepped off the scale, and it then showed 145 pounds. Find the weights of all three boys.

12. What's my number?

(a) I am a two-digit number.
(b) I am a multiple of 6.
(c) The sum of my digits is 9.
(d) My ten's digit is one-half of my unit's digit.

13. A pail with 40 washers in it weighs 175 grams. The same pail with 20 washers in it weighs 95 grams. How much does the pail weigh alone? How much does each washer weigh?

14. The figure below is an array of 17 toothpicks forming 6 squares. By removing exactly 6 of the toothpicks, leave exactly 2 squares.



15. A farmer has some pigs and some chickens. He finds that together they have 70 heads and 200 legs. How many pigs and how many chickens does he have?

16. A farmer has a plot of land in the shape of a rectangle that is 32 feet long by 24 feet wide. He wishes to put a fence around the plot of land. If fence posts are to be placed every 8 feet, how many fence posts will he use?

17. A football team won 3 more games than it lost. The team played 11 games. How many games did they lose?

18. A ball drops from a height of 96 feet and rebounds one-half of the total distance it has just fallen each time it bounces. What is the total distance it has traveled when it hits the ground the third time?

19. A can filled with fruit juice weighs 20 ounces. When one-half of the juice is spilled out, the can and the remaining juice weigh 11 and one-half ounces. How much did the can weigh?

20. A grocer has three pails: an empty pail that holds 5 liters, an empty pail that holds 3 liters, and an 8-liter pail that is filled with apple cider. Show how the grocer can measure exactly 4 liters of apple cider with the help of the 5-liter and 3-liter pails.

25. Three missionaries and three cannibals wish to cross a river. There is a boat that can carry up to three people, and either the missionaries or the cannibals can operate the boat. However, it is never permissible for the cannibals to outnumber the missionaries, either in the boat or on either shore. What is the smallest number of trips needed to make the crossing?

Source: Krulik, S., & Rudnick, J. (1987). Problem solving: A handbook for teachers. Boston: Allyn and Bacon.

APPENDIX C

INITIAL INTERVIEW QUESTIONS

1. What is your age? When is your birthday?

2. What grade will you be going into next year?

3. Where do you go to school?

4. Have you always attended your present school? If not where did you go?

5. Did you attend pre-school?

6. In general, how do you feel about school?

7. What is/are your favorite subject(s) in school? Why?

8. What is/are your least favorite subject(s) in school? Why?

9. Would you describe yourself as being a good student? Why or why not?

10. Do you plan to go to college?

11. If so, is there a particular course of study you are in interested in? If not, is there a particular career that you are interested in?

12. How do you feel about homework?

13. On the average, how many hours have you spent doing homework each night since you have been in middle school?

14. Do your parents help you or check your progress on your homework? How?

15. What do each of your parents do?

16. Do you have any brothers or sisters? How many and how old are they?

17. How do you feel about mathematics?

18. Would you describe yourself as being a good mathematics student? Why or why not?

19. What kinds of grades have you made in mathematics in school?

20. How many hours or minutes on the average have you spent doing mathematics homework each night since you have been in middle school?

21. How do you feel when you are given a mathematics word problem to solve when you do not immediately see a solution for it? How do you feel when you are given a mathematics word problem to solve when you do immediately know how to solve it?

22. Of the time spent on math during the school year, approximately how much time would you say you have spent on problem solving (word problems, puzzles, etc.)?

23. Have any of your teachers ever taught you how to solve word problems?

24. Did your teacher that you had last year have a particular pattern to his/her lessons that you could describe? Describe that pattern.

25. Were you "grouped" for math at your school? If so, do you know if you were ever in a certain group such as a compacted class, etc. ?

26. What is your favorite area or topic in mathematics to study?

27. Is there anything you would like to add to our interview about yourself, school, mathematics, or problem solving in general?

FOLLOW-UP INTERVIEW INSTRUMENT FOR SESSION ONE

1. Describe to me any feelings or thoughts you had as you came in for today's session.

2. Describe any feelings or thoughts you had about yourself or this problem after you read it.

3. Did you choose a particular strategy for solving this problem? If so, what did you do? What were you thinking at this particular point in your work?

4. At what point (if any) were you fairly certain that you had the correct solution? How did you know?

5. Describe your feelings when you finished the problem.

6. Describe your feelings when you finished the problem or the allotted time was up.

7. Is there anything you would like to add concerning today's session?

FOLLOW-UP INTERVIEW INSTRUMENT FOR SESSION TWO

1. How were you feeling as the session began?
2. After you first read this problem, how did you feel? What did you think? Did you understand the problem after you read it the first time?
3. How many times would you say you reread this problem? When you reread it, did you read the entire problem or part of it? Which part? Why?
4. Had you seen this problem or a similar problem before? After reading it the first time, did you have any thoughts about the difficulty of the problem? What made you think that? After working it did you have any thoughts concerning the difficulty of the problem?
5. Was enough or too much information given in the problem? Was there anything else you needed to know besides math to understand and work this problem?
6. When (if at all) did you select a strategy? What did you choose to do? Why? Did you change this strategy as you worked the problem? Why or why not? Did your chosen strategy work? How do you know? What math skills did you use in this problem?
7. During the time that you were sitting and not saying anything, what were you thinking? What part(s) of the problem were you focusing on? How did you feel during this time?
8. What did you do at this particular point? Why? What were you thinking? Feeling?
9. Were you sure you had the right answer? If so, at what point were you certain you had the correct answer?
10. Did you check your work? When? How?
11. How did you feel when this session was over? About the problem? About your work? About yourself?

APPENDIX D

FORM FOR PROBLEM SOLVING SESSION TIME CHART

STUDENT: MINUTE:	STUDENT ACTION	STUDENT VERBAL	FOLLOW-UP INTERVIEW	RESEARCHER COMMENTS
ONE				
TWO				
THREE				
FOUR				
FIVE				
SIX				
SEVEN				
EIGHT				
NINE				
TEN				

STUDENT:

PAGE 2 MINUTE:	STUDENT ACTION	STUDENT VERBAL	FOLLOW-UP INTERVIEW	RESEARCHER COMMENTS
ELEVEN				
TWELVE				
THIRTEEN				
FOURTEEN				
FIFTEEN				
SIXTEEN				
SEVENTEEN				
EIGHTEEN				
NINETEEN				
TWENTY				

COMPLETED PROBLEM SOLVING SESSION TIME CHART

STUDENT: #1 MINUTE:	STUDENT ACTION	STUDENT VERBAL	FOLLOW-UP INTERVIEW	RESEARCHER COMMENTS
ONE	Picks up problem. Reads.	Reads aloud. Reads silent. Reads aloud. Reads 3 to 2	Says he forgot what his teacher	Seems calm.
TWO	Scratches head. Writes 3/2=4/11	Reads silently. Repeats 3 to 2	taught him about ratio. Was try-	
THREE	Stares at prob-lem.	" I think it's a proportion. There's more	ing to remember. Remembered they used	
FOUR	Stares at problem.	boys than girls. I'm gonna go on.	proportions but could not remember how to set	
FIVE	Reads new problem.	Reads problem out loud. "The first	up. Thought prob-lem was easy.	
SIX	Writes 13,15	one's even, the next two are odd. The next two are	Was certain of answer.	Did not hesi-tate.
SEVEN	Writes 18.	even. The next two are odd, so the last one is even.		
EIGHT NINE	Reads new problem. Stares at problem. Writes 42x60	Reads problem out loud. Reads out loud again. "42 cars, 60' 42 pass by" divide.	Did not understand when first read. Decided to multiply, then	

STUDENT: #1

PAGE 2 MINUTE:	STUDENT ACTION	STUDENT VERBAL	FOLLOW-UP INTERVIEW	RESEARCHER COMMENTS
TEN	=2520/60 =42 Reads again.	Reads prob- lem out loud again.	But saw he was where he started.	
ELEVEN	Writes 60.	Reads problem again.	Decided to make an "educated" guess.	Guess based on how fast trains travel
TWELVE	Reads new problem. Writes L 11	Reads out loud. Reads again	Stated he did not think a bat and ball	
THIRTEEN	A 7 Looks at writing. Looks at	$5 for 1 $5 for another. $1 for	would cost so little.	Monitor- ing. Reason- by real
FOURTEEN	problem. Writes 2 443 7 334	a ball 2 bats OK it has to be the	Didn't think his answer was right. Did	life. Only student
FIFTEEN	Taps pencil Looks at problem.	same amt. But the bat bat will be	not cost enough.	who has used "real
SIXTEEN	Points at nos. Looks at problem. Writes 5 1	more. Counts 5,6,7. I'd say bat $5 ball $1.		life" to help solve problems
SEVENTEEN	Reads new problem. Reads again.	Reads out loud. Reads out loud again.	Says he knew there could be several	Looking for the "right" way to
EIGHTEEN	Writes 2- 3-1. Looks at problem. Writes	15=$1 OK. I'd say he has... Let's see...	Says he was just trying dif- ferent com-	work it.

STUDENT: #1

PAGE 3 MINUTE:	STUDENT ACTION	STUDENT VERBAL	FOLLOW-UP INTERVIEW	RESEARCHER COMMENTS
NINETEEN	10-5 20-10 Writes 100	Let's say he has 10 nick-els and 20 dimes. No.	binations until he found one that worked.	Trial and error.
TWENTY	Writes 5 dimes, 10 nickels 50 and 50	5 dimes & 10 nickels. That's right. It equals $1.		

APPENDIX E

COMPLETED GRID FORM

CATEGORIES IDENTIFIED BY STUDENT #2

CONSTRUCTS	THINGS I FEEL	THINGS I DO TO GET ANSWERS	READING THE PROBLEM	THINKING ABOUT THE QUESTION	WAYS TO COME UP WITH THE ANSWERS
DON'T UNDERSTAND IT	3	1	3	3	1
ADD	1	2	1	1	3
READ	1	1	3	1	1
ASK TEACHER	1	2	3	3	1
WRITE ANSWER	1	3	1	1	3
THINK ABOUT QUESTION	1	2	3	3	1
REREAD IT	1	1	3	3	1
UNDERSTAND	2	2	3	3	3
MULTIPLY	1	1	1	1	3
DIVIDE	1	1	1	1	3
CHECK	1	3	1	2	2
FIND A PATTERN	1	1	1	1	3
SUBTRACT	1	1	1	1	3
SKIP IT	3	1	1	1	1
COUNT	1	2	1	2	3
SET PROBLEM UP	1	3	2	2	2

3 = Definitely Related
2 = Sometimes Related
1 = Not Related

GRID CATEGORIES IDENTIFIED BY THE SIX STUDENTS

STUDENT #1

1. Math skills used to work the problem.
2. Feelings or thoughts.
3. What you do while you work the problem.
4. What happens when you can't think of what to do next.

STUDENT #2

1. Things I feel.
2. Things I do to get answers.
3. Reading the problem.
4. Thinking about the question.
5. Ways to come up with the answers.

STUDENT #3

1. Reading.
2. Things I do when I get aggravated.
3. Things I do when I work the problem.
4. The way I feel.
5. The kind of math I use.
6. What I do when I'm finished with the answer.
7. What I do when I'm finished working it out.
8. Getting stuck.

STUDENT #4

1. Strategies
2. Ways to get help from others.
3. Thoughts.
4. Ways to help you solve the problem.
5. Steps for solving problems.

STUDENT #5

1. Action (things I did).
2. Decide which operation to do first.
3. Things that have to do with strategies.
4. Things related to checking the problem.
5. Feelings and Concerns.
6. Similar Problems.
7. Thought I have when I'm working the problem.
8. What I do when I get stuck.
9. Things I think are important in the problem.

STUDENT #6

1. Things I do when I understand the problem.
2. Things I do when I don't understand it.
3. Things I feel.
4. Things I do when I think I've seen the problem before.

EIGHTY-THREE CONSTRUCTS IDENTIFIED BY THE STUDENTS

COGNITIVE CONSTRUCTS:

Addition, subtraction, multiplication, and division
Read the problem
Reread the problem
Reread parts of the problem
Start to work
Carry out strategy
Fractions
Measurement
Divide problem into pieces
Scrap attempts to work
Draw a picture or a diagram
Use graphs
Use scales
Label/Write answer
Writing information or numbers from the problem
Think about the question, the numbers, or the problem
Find a pattern
Count
Work in head
Estimation or rounding
Exponents
Ratios and Proportions
Not sure what part of the problem means
Use a formula
Determine if I've seen the problem before
Do the opposite operation
Key words
Writing
Concentrate
Trial and Error
Do what's in parentheses first
Use a calculator
Think of a useful memory device
Convert fractions to decimals
Draw Mental Pictures

METACOGNITIVE CONSTRUCTS:

Understand the problem
Get stuck
Skip it
Look back at book
Unrelated doodling
Guess how to work
Correct myself
Answer/problem doesn't work out
Problem doesn't look right
Check answer
Ask teacher/parents/another student
Decide if the problem is hard or easy
Picking out important parts
Trying different solutions in head
Reread after answering
Look back at work
Don't understand
Setting problem up
Change strategy
Common sense
Wonder if I'm doing the right thing
Guess answer
Change numbers and try the same strategy
Figure out why something doesn't work
Not sure of answer
Educated guess
Go back to old strategy
Think: I've got the answer
Stuff pops into my mind
Instinct
How did I work a similar problem?
Think about getting answer right
Think about how much time is left
Missing/Extra information
Think of quickest way to solve
Don't get frustrated
Slow yourself down

AFFECTIVE CONSTRUCTS:

Fidget/play
Feel delayed or behind
Frustrated
Feel good about answer
Feel like I don't know the answer
Feel like I'm doing the right thing
Feel lost or confused
Feel relieved
Nervous
Tired
Quiet

VITA

Terry Denise Rose was born on March 14, 1958 in Knoxville, Tennessee. Her parents are William R. and Barbara D. Rose. She received her Bachelor of Science degree in Secondary Mathematics Education and Elementary Education from The University of Tennessee in June of 1981. She began teaching eighth grade mathematics for the Knoxville City school system in August of 1981. In August of 1987, she received her Master of Science degree in Mathematics Education from The University of Tennessee.

She entered The University of Tennessee to pursue the Doctor of Education degree in the fall of 1988. Taking an educational leave of absence from Knox County Schools, she was employed by The University of Tennessee as a graduate assistant for two years involved in the Needs Assessment project with Dr. Jerry Bellon for the Tennessee State Department of Education.

Her major field of study was Curriculum and Instruction with collaterals in Mathematics, Research, and Evaluation. She received the Doctor of Education degree in May of 1991.