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Manufacturing System and Supply Chain Analyses Related to Product Complexity and Sequenced Parts Delivery

Hui Sun
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Hui Sun entitled "Manufacturing System and Supply Chain Analyses Related to Product Complexity and Sequenced Parts Delivery." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Industrial Engineering.

Fong-Yuen Ding, Major Professor

We have read this dissertation and recommend its acceptance:

Denise F. Jackson, Dukwon Kim, Melissa R. Bowers

Accepted for the Council:

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Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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(Original signatures are on file with official student records.)

MANUFACTURING SYSTEM AND SUPPLY CHAIN ANALYSES RELATED TO PRODUCT COMPLEXITY AND SEQUENCED PARTS DELIVERY

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Hui Sun
December 2005

DEDICATION

This dissertation is dedicated to my parents, Yubao Sun and Jun Xu.

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I would like to express heartfelt thanks to my advisor, Professor Fong-Yuen Ding, who provides precious guidance and encouragement for research, and continuous support during this study. Special thanks are due to my advisory committee Professor Denise F. Jackson, Professor Dukwon Kim, and Professor Melissa R. Bowers for serving on my committee.

ABSTRACT

Mixed model assembly has been widely used in many industries. It is applied in order to effectively deal with increasing product complexity. Sequencing and resequencing on a mixed-model assembly line is also complicated by high product complexity. To improve the performance of a mixed-model assembly system and the supply chain, one can develop efficient sequencing rules to address sequencing problems, and manage product complexity to reduce its negative impact on the production system. This research addresses aspects of sequence alteration and restoration on a mixed-model assembly line for the purpose of improving the performance of a manufacturing system and its supply chain, and addresses product complexity analysis. This dissertation is organized into Parts 1, 2, and 3 based on three submitted journal papers.

Part 1. On a mixed-model assembly line, sequence alteration is generally used to intentionally change the sequence to the one desired by the downstream department; and sequence restoration is generally applied to achieve sequence compliance by restoring to the original sequence that has been unintentionally changed due to unexpected reasons such as rework. Rules and methods for sequence alteration using shuffling lines or sorting lines were developed to accommodate the sequence considerations of the downstream department. A spare units system based on queuing analysis was proposed to restore the unintentionally altered sequence in order to facilitate sequenced parts delivery. A queuing model for the repairs of defective units in the spare units system was developed to estimate the number of spare units needed in this system.

Part 2. Research was conducted on product complexity analysis. Data envelopment analysis (DEA) was first applied to compare product complexity related to product

variety among similar products in the same market, two DEA models including their respective illustrative models considering various product complexity factors and different comparison objectives were developed. One of these models compared the product complexity factors in conjunction with sales volume. The third DEA model was developed to identify product complexity reduction opportunities by ranking various product attributes. A further incremental economic analysis considering the changes in costs and market impact by an intended complexity change was presented in order to justify a product complexity reduction opportunity identified by the DEA model.

Part 3. Two extended DEA models were developed to compare the relative complexity levels of similar products specifically in automobile manufacturing companies. Some automobile product attributes that have significant cost impact on manufacturing and the supply chain were considered as inputs in the two extended DEA models. An incremental cost estimation approach was developed to estimate the specific cost change in various categories of production activities associated with a product complexity change. A computational tool was developed to accomplish the cost estimation.

In each of the above stated parts, a case study was included to demonstrate how these developed rules, models, or methods could be applied at an automobile assembly plant. These case studies showed that the methodologies developed in this research were useful for better managing mixed-model assembly and product complexity in an automobile manufacturing system and supply chain.

TABLE OF CONTENTS

Part **Page**

INTRODUCTION **1**

PART 1

SEQUENCE ALTERATION AND RESTORATION RELATED TO SEQUENCED PARTS DELIVERY ON AN AUTOMOBILE MIXED-MODEL ASSEMBLY LINE **5**

1. ABSTRACT 6
2. INTRODUCTION 7
3. SEQUENCE ALTERATION 10
   3.1. Alteration on the line 10
   3.2. Resequencing to batch a single attribute by using sorting lines 16
   3.3. Resequencing by substitution 21
4. SEQUENCE RESTORATION 22
   4.1. Reservoir system 23
   4.2. Spare units system 24
5. A CASE EXAMPLE 31
6. CONCLUSIONS 34

REFERENCES 35

APPENDIX A. COMPUTATIONAL RESULTS OF THE HEURISTIC SORTING AND RELEASING RULES 37

APPENDIX B. AN INTEGER PROGRAMMING FORMULATION OF THE SORTING PROBLEM TO BATCH A SINGLE ATTRIBUTE 38

PART 2

ANALYZING PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY IN A MANUFACTURING FIRM **40**

1. ABSTRACT 41
2. INTRODUCTION 42
   2.1. Relevant literature 43
   2.2. Specific objectives and outline of this paper 46
3. REPRESENTATION AND MEASURES OF PRODUCT VARIETY 47
   3.1. Basic elements in describing product variety 47
   3.2. Measures of product variety 47
4. ANALYZING PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY 48
   4.1. Data envelopment analysis for analyzing product complexity 48
   4.2. Applying DEA in comparing product complexities related to product variety 50
   4.3. Ranking various attributes for product complexity reduction considerations.. 54
   4.4. Economic analysis for a product complexity reduction 57
5. A CASE STUDY 58

5.1. Comparison of product complexities related to product variety among seven similar products in the U.S. market ... *59*
5.2. Attribute ranking within a company ... *62*
6. CONCLUSIONS ... 63
REFERENCES ... 64
APPENDIX A. REPRESENTATION OF A PRODUCT VARIETY STRUCTURE AS STRUCTURED ON A WEBSITE ... 66
APPENDIX B. PRODUCT VARIETY STRUCTURES OF SEVEN PICKUP TRUCKS IN THE U.S. MARKET BASED ON THE CUSTOMER SELECTION PROCESS ... 69
APPENDIX C. RESULTS FOR RANKING VARIOUS ATTRIBUTES IN THE CASE STUDY ... 70
APPENDIX D. THE ECONOMIC ANALYSIS SPREADSHEET FOR A COMPLEXITY REDUCTION EFFORT IN THE CASE STUDY ... 71
APPENDIX E. DATA ENVELOPMENT ANALYSIS (DEA) BCC MODEL FORMULATION ... 71

PART 3

EXTENDED MODELS AND A PRACTICAL TOOL TO ANALYZE PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY FOR AN AUTOMOBILE ASSEMBLY PLANT ... 73

1. ABSTRACT ... 74
2. INTRODUCTION ... 75
3. RELEVANT LITERATURE ... 76
4. COMPARISON OF PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY AMONG VARIOUS AUTOMOBILE MANUFACTURERS ... 78
5. ESTIMATING COST-CHANGE ASSOCIATED WITH A COMPLEXITY CHANGE ... 82
6. A CASE STUDY ... 85
7. SUMMARY AND CONCLUSIONS ... 90
REFERENCES ... 91
APPENDIX A. VISUAL BASIC FORMS FOR THE CASE STUDY ... 93

SUMMARY AND CONCLUSIONS ... 96

VITA ... 99

LIST OF TABLES

Table **Page**

Part 1

Table 1. Number of units needed for the reservoir and spare units system under different defective rates 30
Table 2. Computational results of the heuristic sorting and releasing rules 37

Part 2

Table 1. The complexity comparison scores by using DEA models 1 and 2 61
Table 2. Improvement targets and reference sets by using DEA models 1 and 2 for inefficient companies 62
Table 3. Notation for the description of a product variety structure 68
Table 4. Product structures represented by the proposed notation 69
Table 5. Correspondence of attributes at level 1 of each product 69
Table 6. Results of ranking attributes in the case study 70
Table 7. A sample economic analysis spreadsheet for a product complexity reduction effort 71

Part 3

Table 1. Cost base for various cost areas or production activities at an automobile plant 83
Table 2. Data envelopment analysis for comparison of product complexity of full size cars in the U.S. market 87

LIST OF FIGURES

Figure **Page**

Part 1
Figure 1. A simple configuration for sequence alteration on the line……………..……10
Figure 2. Placement of 52 units in the example…………………………………..……..16
Figure 3. A sorting area 17
Figure 4. Enlarged paint blocks using 6 sorting lines in the example 19
Figure 5. Assembly plant in the case study... 31

Part 2
Figure 1. Relationship between product complexity and product variety 43
Figure 2. Model 1 – (a) A conceptual DEA model, and (b) an illustrative model for complexity comparison.. 51
Figure 3. Model 2 – (a) A conceptual DEA model, and (b) an illustrative model for complexity comparison... 53
Figure 4. Model 3 – (a) A conceptual DEA model, and (b) an illustrative model for ranking attributes.. 55

Part 3
Figure 1. Model A for complexity comparison 81
Figure 2. Model B for complexity comparison……………………………………….82
Figure 3. Computation of cost change associated with a product complexity change…..84
Figure 4. Visual Basic form 1 93
Figure 5. Visual Basic form 2 94
Figure 6. Visual Basic form 3 95

NOMENCLATURE

Part 1

Notation in 3.2. and Appendix A.

L	the number of sorting lines
b	the number of spaces in each sorting line
c	the number of attribute codes
B_1	the average block size before sorting
B_2	the average block size after release
B_3	the ideal average block size
r_1	the percentage increase in the average block size after release
r_2	the average percentage difference between the average block size after release and the ideal average block size

Notation in 4.1.

W	the average system time to repair a defective unit in a M/M/1 queuing system

Notation in 4.2.

s_i	the number of spare units for vehicle model i
p	the average percentage of defective units
m_i	the number of units of model i produced in a day
α	the service level or rate of sequence compliance
h	the number of work hours per day
λ_i	the arrival rate of defective units of model i
N	the number of units of all vehicle models produced per day
λ	the arrival rate of all defective vehicles arriving at the inspection point
μ	the repair rate in a shared repair facility
ρ	λ/μ
ω_i	m_i/N
d_i	$\rho(1-\omega_i)$
μ_i	$\mu(1-d_i)$
P_n	the probability of having n vehicles of any model in the queuing system for a spare units system
$p_i(n)$	the probability of having n units of other models in front of a unit of model i joining the queue
$p_i'(n)$	the probability of n units of other models in the queue of n units
$p_i''(n)$	the probability of having n consecutive units of other models after an existing unit of model i for the last n+1 units in the queue of more than n units of any models
S_{n+1}	the time to repair a joining unit of model i and n units of the other models that are already in the queue
T_n	the service time of unit n
V_i	the total virtual service time due to servicing the other models prior to a unit of model i and a unit of model i

$g_i(t)$ the probability density function of the virtual service time of a unit of model i due to its service requirement and serving units of other models before an existing unit of model i

$G_i(t)$ the distribution function of virtual service time V_i

P_n^i the probability of n defective units of model i in the queuing system for a spare units system

r_i the regularly produced units of model i in a spare units system

R Σr_i, the total number of units produced regularly

Notation in Appendix B.

X_{ij} binary variable, 1 if unit at position i before sorting is assigned to position j after sorting; otherwise 0

Y_{1j} one of the two binary variables to indicate whether the attribute codes of two adjacent units are different in positions j and j+1 or not

Y_{2j} one of the two binary variables to indicate whether the attribute codes of two adjacent units are different in positions j and j+1 or not

C_i the attribute code of unit i in the sequence before sorting

M a very large positive number

Part 2

Notation in 3.2.

N_v the number of product variants

N_A the number of product attributes

A_i the collection of the values of attribute i

$|A_i|$ the number of values of product attribute i

Notation in 4.1.

h_0 the efficiency score for DMU j_0

y_{rj} the amount of output r of DMU j

x_{ij} the amount of input i of DMU j

u_r the weight assigned to output r

v_i the weight assigned to input i

ε a very small positive number

m the number of inputs

t the number of outputs

n the number of DMUs

Notation in 4.2. and 5.1.

N_v the number of product variants

N_A the number of product attributes

$\overline{N}_a$ the weighted average number of attribute values

MS market share

Notation in 4.3. and 5.2.

N_a the number of attribute values

MC impact of a product attribute on manufacturing costs

$\overline{N}_p$ the weighted average number of unique parts for producing an attribute

SD the standard deviation of the percentages of demands of the various values of a product attribute

Notation in Appendix A.

L the number of levels of product variety structures

N_A^i the number of attributes in level i of a product variety structure

$[L:(N_A^1,...,N_A^L),N_A,N_V]$

the numeric index of a basic product variety structure information, where N_A the total number of attributes, and N_V the number of variants

Part 3

Notation in 4.

θ the efficiency score for DMU j_0

x_{ij} the amount of input i from DMU j

y_{rj} the amount of output r from DMU j

λ_j the multiplier for DMU j

ε a very small positive number

s_i^+ the slack variable with respect to input i

s_r^- the slack variable with respect to output r

INTRODUCTION

This research is aimed at developing production systems and analysis for improving the performance of a manufacturing system and the supply chain of an automobile assembly plant by addressing two aspects: 1) resequencing mixed-model assembly lines for downstream considerations and restoration of sequences for sequenced parts delivery, 2) product-complexity analysis and product-complexity related cost estimation. Product complexity is a significant contributor to assembly line complexity. Sequencing attempts to optimize the effect of mixed models resulted from product complexity. This research has been motivated by some major U.S. automobile manufacturers, who are interested in improving their performances by better addressing sequencing on mixed-model assembly lines and analyzing product complexity.

Sequencing mixed-model assembly lines with different goals such as workload balancing and part-usage leveling has been studied by many researchers (e.g., Miltenburg, 1989; Inman and Bulfin, 1991; Monden, 1998; Zhu and Ding, 2000; see references in Part 1), while accommodating different sequence considerations of various departments on a mixed-model assembly line is addressed very little in the literature. On the other hand, some practice of intentional sequence alteration considering different production requirements of the downstream department and sequence restoration to deal with unintentional sequence alterations to facilitate sequenced parts delivery can be seen in industry. In this research, first, rolling sequencing is used in sequence alteration. To perform sequencing, shuffling lines are considered to physically change the sequence of vehicles and release vehicles in the sequence desired by the downstream department. Thus, effective rules to facilitate the vehicle placement and release operations on the shuffling lines need to be studied. Second, increasing the block size of vehicles of the same attribute is also desired in some operations, e.g., a large block size of same color vehicles for painting. Sorting lines can be used to increase the block sizes. Models and rules with an objective of obtaining the largest block size are needed. Third, to accomplish sequence restoration, the existing practice include using substitution or a reservoir system, which generally will lead to delay in the delivery of vehicles later in the sequence or a high inventory. A more efficient way that requires a lower inventory, for example, using spare units to replace defective units, can be developed.

In Part 1 of this dissertation, research will be conducted in the following aspects: 1) Developing effective rules to address the placement and releasing of vehicle units on the shuffling lines to enable achieving the altered sequence using a rolling sequencing method. 2) Modeling the resequencing process using sorting lines in order to obtain the optimal block size. Mathematical programming will be considered in this modeling. Heuristic rules will also be developed to allow solutions for sorting. 3) Developing a spare units system that can be used to effectively restore the unintentionally altered sequence due to defects to the original sequence to keep sequence compliance using a low inventory. Queuing analysis will be used to model the repair process of defective units of various vehicle models. A case study will be conducted based on the practice at an automobile assembly plant. Recommendations to address different requirements by applying the developed rules and methods to improve the performance of this plant will be presented.

The complexity encountered by sequencing can be better understood by a product complexity analysis. Product complexity generally has a negative impact on the performance of a manufacturing system and the supply chain (MacDuffie, 1996; Fisher and Ittner, 1999; see references in Part 2). Good product complexity management can help reduce the negative impact and improve the overall performance of a production system. A benchmarking effort can help better understanding the relative complexity level of a product and provide insight in decision making on product complexity. It is also desirable to prioritize complexity reduction opportunities. Some manufacturers would also desire to know the cost impact of product complexity on their manufacturing systems and supply chains.

Part 2 of the dissertation is mainly focused on analyzing product complexity related to product variety. To perform a comparison of product complexity of similar products in the same market, data envelopment analysis (DEA) will be applied to multiple factors (Ulrich et al., 1998; MacDuffie et al., 1996; Fisher and Ittner, 1999; see references in Part 2) in measuring the product complexity related to product variety within a single product. DEA is a linear-programming-based technique that has been developed to compare the relative efficiencies of multiple homogenous decision making units. DEA models will be developed according to different comparison objectives to compare the product complexities of similar products in the same market. DEA can also be applied to prioritize various product-complexity reduction opportunities related to product attributes. Further economic analysis will be presented attempting to justify a product-complexity reduction opportunity identified from DEA ranking. The applications of these developed DEA models and an economic analysis will be illustrated in a case study.

Part 3 of the dissertation presents two DEA models extended from the DEA models 1 and 2 in Part 2 to compare product complexities of similar products specifically in automobile industry. Considering that some product attributes can have significant impact on automobile manufacturing (MacDuffie et al., 1996; see references in Part 3), in these two extended DEA models, the numbers of attribute values of these significant product attributes will be included as inputs. Also, an incremental cost estimation approach will be proposed to calculate the cost impact of a product complexity change in various categories of production activities. A case study will be given to compare product complexity levels of similar vehicles in some major U.S. automobile manufacturing companies by applying the two extended DEA models. A computation tool will be included to implement the incremental cost estimation approach in an automobile company.

PART 1
SEQUENCE ALTERATION AND RESTORATION RELATED TO SEQUENCED PARTS DELIVERY ON AN AUTOMOBILE MIXED-MODEL ASSEMBLY LINE

This part is a paper published in the journal *International Journal of Production Research* in 2004 by Fong-Yuen Ding and Hui Sun:

Ding, F. and Sun, H. (2004) Sequence alteration and restoration related to sequenced parts delivery on an automobile mixed-model assembly line with multiple departments. *International Journal of Production Research*, 42(8), 1525-1543.

My primary contributions to this paper include (1) part of the gathering and interpretation of literature, (2) part of development of heuristic sorting and releasing rules, (3) computer programming for the computational experimentation to test the heuristic sorting and releasing rules, (4) part of development of the integer programming model, (5) calculation for the number of units needed for the reservoir and spare units system under different defective rates, (6) part of development of the calculation procedure used to determine the number of units as produced on the assembly line regularly or as spare units for a spare units system, and (7) part of draft writing and editing.

1. Abstract

A mixed-model assembly line is commonly used in the automobile industry. When there are multiple departments on an assembly line, there are usually different sequencing considerations from various departments. Intentional sequence alteration to accommodate a different sequencing consideration can be needed for a downstream department. Unintentional sequence alteration may also take place due to rework or equipment breakdowns. There is also an increasingly common practice in automobile assembly to have parts sequenced before delivering to the final assembly line. To achieve sequenced parts delivery, the sequence needs to be known in advance. Thus, addressing sequence alteration and restoration becomes more relevant for an automobile mixed-model assembly line. In this paper, a number of sequence alteration methods to accommodate a downstream department's sequencing considerations are presented. One of these methods easily supports sequence restoration of the sequence altered by the method. Two sequence restoration methods for restoring the sequence altered by unintentional reasons are discussed; and the proper sizing of the two restoration methods

are addressed. These sequence alteration and restoration approaches mainly address the design and control aspects of the mixed-model assembly line. A case study based on an automobile assembly plant is presented to demonstrate the use of these methods.

2. Introduction

The automobile industry involves a large supply chain. Automobile manufacturing includes a wide variety of manufacturing activities including casting, stamping, part manufacturing, welding, painting, and assembly. A typical automobile assembly plant consists of a body, a painting, and a final assembly department. A final assembly department usually has several subassembly lines for items such as engine, frame, and instrument panel, and a main assembly line for trim and finish assembly.

Mixed-model assembly is commonly used in automobile manufacturing. It has the benefit of reducing facility and inventory costs, and a potential of achieving a better balance in workload and part usage (Monden, 1998). When mixed-model assembly is applied in an automobile assembly plant, a linear flow can pass through the whole assembly system, and the model sequence can thus, affect the production efficiency of various departments. However, production considerations for the sequence in various departments are generally different. For example, the body department may need to follow a repetitive pattern of models due to the machine setup consideration, while the painting department may need to have larger paint color blocks.

The sequence of the linear flow of models on a mixed-model assembly line can often be altered intentionally or unintentionally in an automobile assembly plant. In some cases, the sequence can be altered intentionally to achieve a better efficiency in a downstream department such as having larger paint blocks for the painting department.

The sequence can also be altered unintentionally due to unavoidable reasons such as equipment breakdowns and defective products.

When the initial model sequence is intentionally or unintentionally revised before reaching the final assembly department, it becomes difficult for the final assembly department to anticipate the exact sequence. At the final assembly department, where a significant number of parts are used, both operators and suppliers can benefit from knowing the model sequence in advance. One important reason for needing to know the sequence in advance is due to the manufacturer's desire to have parts delivered to the final assembly line according to the sequence. "Sequenced parts delivery" has become an increasingly popular practice in automotive assembly operations (Bukey and Davies, 1991), such as with Toyota (Monden, 1998), Ford (Sawyer, 1994; Vasilash, 1996; Voller and Kistler, 1997), and BMW (Automotive News, 2001). By sequenced parts delivery, parts needed at many assembly stations on the line can be organized and delivered according to the mixed-model sequence. The benefits of this practice include reduced inventory level, reduced space requirement, and ease of material retrieval for assembly operations.

There is a rich body of literature dealing with sequencing mixed-model assembly lines (e.g., Milturburg, 1989; Inman and Bulfin, 1991; Hindi and Ploszajski, 1994; Duplaga et al., 1996; Monden, 1998; Zhu and Ding, 2000; Yan et al., 2003). These various sequencing procedures are aimed at leveling work loads on the stations, smoothing part usage on the line, or minimizing the variation of production rates of the finished products for a mixed-model assembly line. Various sequencing considerations of different departments on a mixed-model assembly line are generally not addressed.

Many rescheduling methods were developed to deal with dynamic production environments (e.g., Jain and Elmaraghy, 1997; Wu and Li, 1995; Hohzaki et al., 1995). These methods are mainly concerned with rescheduling in a manufacturing system; various assembly line considerations are not considered.

Lahmar et al. (2003) noted that different departments usually do not share one optimal sequence on a moving assembly line, which raises the need to resequence jobs upon leaving a department and before entering the next one. They developed an integrated model to solve the resequencing problem for the downstream department with the objective of minimizing changeover costs incurred whenever two consecutive jobs do not have the same feature such as vehicle color. Limited offline buffers, or pull-off tables were suggested for the proposed vehicle resequencing operation. Sequence restoration was not explicitly considered. Choi and Shin (1997) presented a sequence control algorithm to perform resequencing at a multi-line buffer for the downstream assembly department considerations, i.e., spacing constraints of various options. Inman (2003) presented a sizing methodology for an AS/RS in performing sequence restoration in an automobile assembly system. An exact method and approximation method were presented under the cases with and without decoupling orders.

This paper presents methodologies in sequence alteration and restoration related to gaining advance knowledge of the model sequence so that sequenced parts delivery can be achieved. The methods presented in this paper address the design and control aspects of resequencing and sequence restoration on an automobile mixed-model assembly line. Two problem scenarios are discussed. First, considering that two consecutive departments on a mixed-model assembly line usually do not share the same sequencing consideration, the case of altering the sequence to satisfy the special consideration of the downstream department is presented. Secondly, considering that the sequence desired by the downstream department is the original sequence, the case of restoring a revised sequence to its initial model sequence is discussed. A case example based on an existing automobile assembly operation is presented to demonstrate the use of these methodologies.

3. Sequence Alteration

3.1. Alteration on the line

When two consecutive departments require different mixed-model sequences, the model sequence after leaving the upstream department can be intentionally altered so that it can be more suitable for the downstream department. This kind of sequence alteration can be done on the line followed by a shuffling area to ensure that the altered sequence is followed later in the sequence. A common configuration for a shuffling area consists of several lines as depicted in Figure 1. This kind of sequence alteration generally does not consider restoration at a later point. However, the altered sequence by the following resequencing and shuffling methods can be restored later with a configuration similar to

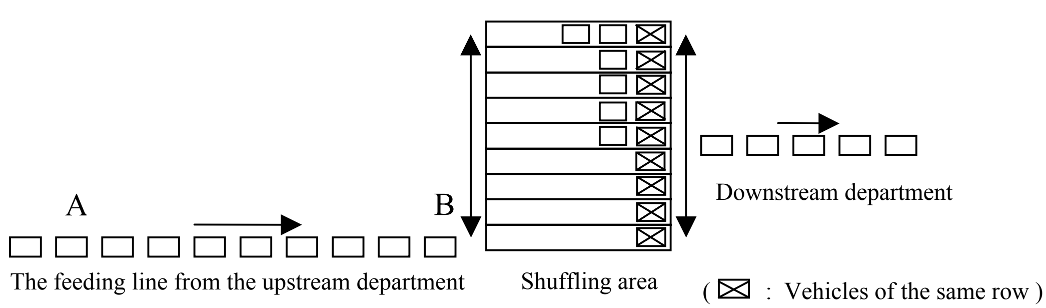

Figure 1. A simple configuration for sequence alteration on the line

the initial shuffling lines. The following sequence alteration method can also be performed at a further upstream location to allow for a longer lead time for the advance knowledge of the sequence. This sequence alteration can be performed in three steps: 1) resequencing on the line, 2) placing vehicles in the shuffling area, and 3) releasing vehicles.

1) Resequencing on the line

In this step, each vehicle coming out of the upstream department is reassigned a new "rotation number" (the order of a vehicle in the sequence) before entering the shuffling area in order to accommodate the production consideration of the downstream department. Based on the number of vehicles to be resequenced each time, there can be two categories of resequencing methods for sequence alteration on the line. In a *block sequencing method*, all units within a block of vehicles are resequenced in the same iteration. In a *rolling sequencing method*, the range of units to be selected for resequencing is kept constant, but not all units in the range have to be selected in each iteration.

For each iteration of resequencing in block sequencing, k consecutive rotation numbers are assigned to the units within a block of k vehicles according to the sequencing consideration of the downstream department. Once all vehicles are assigned the new rotation numbers, the sequence of the whole block of units can be broadcast to the downstream department.

The rolling sequencing method will select the next suitable unit, based on the sequencing consideration of the downstream department, within a given selection range of k units (from point A to B, as depicted in Figure 1). New rotation numbers are consecutively assigned to selected vehicles until the unit at point B is assigned a new rotation number. Point B is the point of the last assigned vehicle within the current selection range before the assembly line moves one unit forward. Once the vehicle number at Point B is broadcast to the downstream departments, the assembly line moves forward by one unit; and the selection range is kept constant by adding the new unit at point A into the range. Thus, rolling sequencing increases sequencing flexibility as compared to block sequencing.

2) Placing vehicles in the shuffling area

Each vehicle with a new rotation number is placed on a line in the shuffling area to ensure that the vehicles can be released according to the order of the new rotation numbers. When a unit joins the shuffling lines, the unit must be able to find a line with at least one open space, and the last unit of the line must have a smaller rotation number than that of the joining unit; otherwise, there can be "blocking" with which a unit can not be released sequentially according to the rotation number. Another consideration of placement is to achieve "even release," that is, to release units by turns from various lines. Even release has a benefit of keeping roughly an equal number of open spaces in various lines. Thus, when a unit is joining the shuffling lines, there is a widest choice for lines, and the chance of blocking is reduced.

When block sequencing of a block size k is applied, there needs to be *k shuffling lines* to avoid blocking. This is because a sequence of k consecutive rotation numbers in backward order needs k shuffling lines to avoid blocking. To achieve even release, each unit in a block of k consecutive rotation numbers is to be placed on a different line in the shuffling area; and a vehicle of a certain block can be placed behind a unit of the previous block in such a way that the difference between the rotation numbers of two consecutive units in each line is k.

While applying rolling sequencing of a selection range k, the following placing and stacking rules for joining the shuffling lines will give the most even release and will not have blocking. These placing and stacking rules attempt to place vehicles of a group of k consecutive rotation numbers in the same row (see Figure 1 regarding the term "row"), and stack the vehicles of the next group of k rotation numbers. The term "row" refers to the spaces of the same position in different shuffling lines. It can also be shown, as stated in "Result," that, with *k shuffling lines,* there will be no blocking under these placing and stacking rules.

<u>Placing rule</u>. Place the current group of k vehicles of consecutive rotation numbers in the k available positions in the "first open row" of the shuffling area as the units arrive. The "current group of k vehicles" is the first group of k consecutive rotation numbers that has not been completely placed in the shuffling area. A row is "closed" if all k units of the associated rotation-number group have been placed.

<u>Stacking rule</u>. Stack any unit that is not one of the current group of k consecutive rotation numbers in the row next to the first open row. When a unit is stacked, it will be stacked behind a unit so that the difference between their rotation numbers is the closest to k.

It can be shown that all the units of a k-rotation-number group would have arrived and be placed in the shuffling area before units of the third group of consecutive numbers start to arrive. Moreover, as will be shown in Result, there are always less vehicles of the next rotation-number group than those of a given rotation-number group before any point in the resultant sequence from rolling sequencing. This guarantees stacking to be always feasible. Therefore, the above placing and stacking rules will place units of various groups row by row without blocking.

Result The rolling sequenced vehicles based on a selection range of k can be placed on k shuffling lines without blocking using the above stated placing and stacking rules.

(Proof)

Assume that the sequence from rolling sequencing consists of multiple k-unit sections of which units are assigned numbers from multiple rotation-number groups. Due to the nature of the rolling sequencing method, a rotation number of a group of k consecutive rotation numbers (say, group q) is assigned to the next section (i.e., section q+1) of k vehicles only if a smaller rotation number within this group has been assigned to the unit at point "B" (see Figure 1) in the last iteration of the assignment. Thus, for each rotation number of the current rotation-number group (q) assigned to the next section (q+1), there must be a smaller rotation number of the current rotation-number group (q) assigned before the position of a rotation number of the next rotation-number group (q+1) assigned to the current section (q). Therefore, before any point in the resultant sequence from rolling sequence, there are always no less units of the current rotation-number group than those of the next rotation-number group. This guarantees the above placing and stacking rules to work effectively without causing blocking. (It can also be shown that there will be at least $\lfloor k/2 \rfloor$ rotation numbers of the current rotation-number group in the current and previous sections, where $\lfloor k/2 \rfloor$ represents the smallest integer that is greater than or equal to k/2.)

3) Releasing Vehicles

Vehicles placed in the shuffling area are released according to the newly formed sequence to the downstream assembly department. Since a block of units is evenly distributed among the k shuffling lines, even release is achieved. Also, the vehicles coming out of the shuffling area follow the new sequence for the downstream department's production considerations.

Based on the above discussion, it can be seen that the selection range of k vehicles to which resequencing is applied can be shifted further upstream as long as the selection range (k) remains to be equal to the number of shuffling lines. To allow for a longer lead time of having the advance knowledge of the downstream sequence, the selection range can therefore, be shifted upstream if the conditions of the manufacturing system permit.

Based on the above discussion, it can also be seen that, given that the input and output rates at the shuffling area are equal, only 2 rows of k shuffling lines are needed for releasing an altered sequence from rolling sequencing. Moreover, it can be shown that, given that the input and output rates are equal, such an altered sequence can be restored by 2 rows of k shuffling lines at a later point of the assembly line system. This can be accomplished by using the same pattern placed in the earlier shuffling area.

An example

Consider a system with a feeding line into a shuffling area of 13 lines with 4 spaces on each line. Thus, the selection range for the rolling sequencing approach is 13 vehicles. Assume 52 vehicles coming from the feeding line are resequenced in a rolling manner based on a certain downstream assembly line consideration, and the resultant sequence of rotation numbers is as follows: 48-52-49-41-51-47-50-40-43-44-37-36-34-33-45-38-39-42-46-35-31-23-29-27-22-26-21-30-24-28-20-25-32-18-17-12-11-8-9-7-3-

15-13-1-16-19-2-14-4-10-6-5 (moving in the forward direction). These units can be placed on the 13 shuffling lines according to the above placing and stacking rules. The placement result is shown in Figure 2. These vehicles can be released evenly according to the new rotation numbers without causing blocking.

3.2. Resequencing to batch a single attribute by using sorting lines

Resequencing can be performed by using sorting lines in order to accumulate vehicles with the same attribute code to a larger block. For example, this can be applied for a painting department to accumulate units of the same color before the painting operation. It is noted that physically there is no difference between the "shuffling lines" stated earlier and "sorting lines" stated here, but different names are used to highlight the difference in their objectives. Two cases can be addressed in this resequencing approach; one is to require restoring the revised sequence at a later point on the line, and the other is not to require restoration.

Case 1. Resequencing to batch a single attribute intended for later restoration

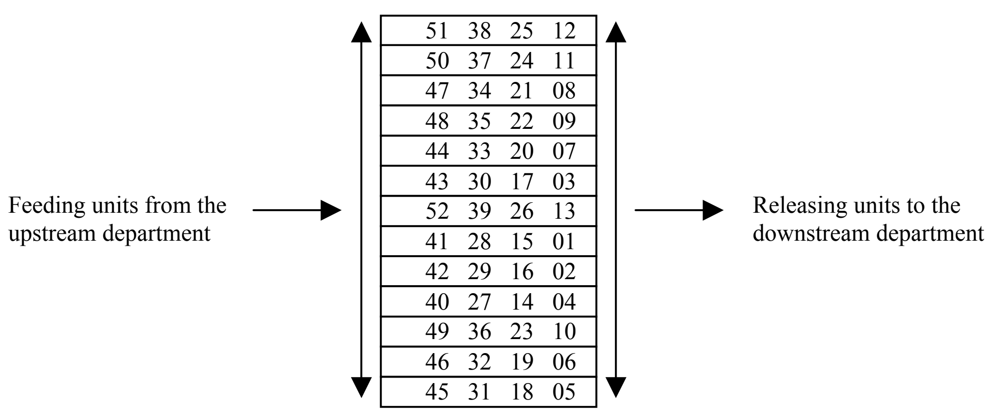

Figure 2. Placement of 52 units in the example

The accumulation of units of a certain attribute, such as color, can be performed by using a sorting area of multiple lines. This can be illustrated in Figure 3.

The above figure shows a sorting area with 4 sorting lines and 6 spaces on each line. In general, the number of sorting lines, L, and the number of spaces in each line, b, can be determined based on considerations in the available floor space, inventory cost, and desired sorting capability. Each L·b consecutive vehicles will be considered as a "section." As the units of a section enter the sorting lines, each unit will join a certain line for possible connection with another unit of the same attribute code. In this way, the downstream department can benefit from having vehicles of a common attribute code grouped into a longer string (a larger block). The same configuration of L lines of b spaces in each line at a later point on the assembly line can also ensure complete restoration to the sequence before the resequencing.

A 0-1 integer programming formulation as given in Appendix A can be developed for determining the selection of the sorting lines for a block of vehicles. Since real-time decision making is needed for the system, solving the 0-1 integer program repeatedly will generally not be practical. To allow quicker decision making, the following heuristic sorting rules are developed for sorting each section of vehicles:

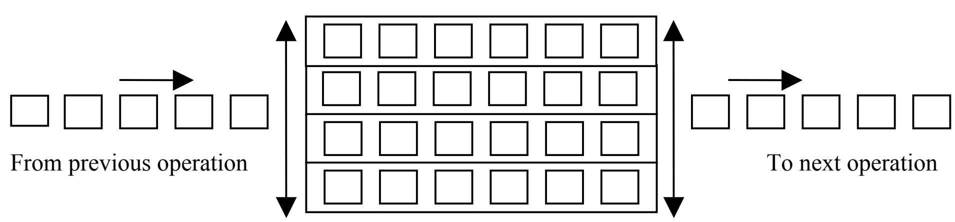

Figure 3. A sorting area

Rule 1. Place an incoming vehicle on a sorting line of which the last unit has the same attribute code as the incoming vehicle; otherwise place the unit on an empty line. If an empty line is not available, apply Rule 2 if an inactive line exists; otherwise apply Rule 3. (An "inactive" line is a sorting line of which the last unit does not have another unit of its attribute code among the unsorted units.)

Rule 2. Place the incoming unit on the inactive line that has the closest number of spaces (v) to the number of unsorted units (u) of the same attribute code as the incoming unit. In this step, the sorting line with a positive difference (v-u) of these numbers has a priority to be selected over ones with a negative difference.

Rule 3. Place the vehicle on a line that is waiting for the minimal number of unsorted units of the same attribute code as that of the last unit on this line. If several lines have the same minimal number, choose the line that has the closest number of spaces (v) to that of unsorted units (u) of the same attribute code as the incoming unit; and the line with a positive difference (v-u) is given a priority over ones with a negative difference. If there is still a tie, choose the line so that the next unit of the same attribute code as the last unit on that line will arrive the latest at the sorting area.

After all vehicles are placed, the sorted vehicles can then be released from the sorting area line by line according to the following releasing rule:

Releasing rule. First, release a pair of sorting lines with the longest positive "connecting length." The "connecting length" between a pair of lines is the total number of the units with the same attribute code at the front end of one line and the rear end of another line. (If a pair with a positive connecting length does not exist, use any order for these lines.) Then, release the next line that has the longest positive connecting length with the last released line. However, if a line with a positive connecting length does not exist, release a pair of lines that have the longest positive connecting length. If there still isn't a pair of lines with a positive connecting length, use any order for the remaining lines. Continue releasing a line or a pair of lines according to the above steps.

An example

Consider an unsorted section of 24 vehicles as follows: C_{24} A_{23} C_{22} A_{21} C_{20} A_{19} D_{18} A_{17} D_{16} A_{15} D_{14} A_{13} B_{12} A_{11} B_{10} A_9 C_8 A_7 B_6 A_5 D_4 A_3 C_2 A_1, where A, B, C, and D represent 4 different colors, and the subscripts are positions in the section before sorting. The initial average block size is 1. To obtain larger paint color blocks, a sorting area of 6 lines with 4 spaces on each line is assumed. By applying the heuristic rules to sort these vehicles, the result on the 6 lines is depicted in Figure 4.

The units can be released line by line in the order of lines 1, 5, 6, 4, 2, and 3. In this way, the average size of the same color blocks is 24/4 = 6 with 4 blocks. In order to restore the changed sequence in the downstream department, a shuffling area of 6 lines each with 4 spaces will be needed.

(line 1): A_7 A_5 A_3 A_1
(line 2): C_{22} C_{20} C_8 C_2
(line 3): D_{18} D_{16} D_{14} D_4
From previous operation → (line 4): C_{24} B_{12} B_{10} B_6 → To next operation
(line 5): A_{15} A_{13} A_{11} A_9
(line 6): A_{23} A_{21} A_{19} A_{17}

Figure 4. Enlarged paint blocks using 6 sorting lines in the example

Computational experimentation

In order to test the above heuristic sorting and releasing rules, various sorting line configurations with different number (L) of lines and line length (b), and number (c) of attribute codes are assumed in a computational experimentation. A Basic program is coded for the experiment and run on a PC. In this experimentation, the attribute under consideration will be the color of a vehicle. Assume that the ratios of the numbers of units of vehicles of the high, medium, and low quantities are roughly 3:2:1. Also assume the number of colors of the vehicles of high, medium, or low quantities accounts for approximately one third of the total number (c) of colors. For each L, b, c configuration (with 12, 24, or 36 vehicles), ten sequences based on the above ratios are randomly generated. The sorting and releasing rules are applied to these sequences and the average paint block size before sorting (B_1) and that after release (B_2) are calculated. The results are given in Table 2 in Appendix A.

It can be seen in Table 2, that the average block size increases from applying the sorting and releasing rules can be quite significant. It is noticed that for configurations with the same line number and line length, the percentage increase (r_1) in the average block size after release generally increases as the number of color decreases. This is likely due to the fact that, with a smaller number of colors, paint blocks can be accumulated more easily. In Table 2, the average block size after release is also compared to the ideal average block size (B_3), which represents a sequence using the same number of blocks as the number of colors. The average percentage differences (r_2) between the average block size after release and the ideal average block size are under 40%. Since the ideal average block size is larger than or equal to the optimal average block size for randomly generated initial sequences, the heuristic solutions could be closer to the optimal solutions than indicated by these percentage differences (r_2). All CPU times of different runs are well under 1 second and thus, negligible.

Case 2. Resequencing to batch a single attribute not intended for later restoration

If the original sequence doesn't need to be restored at a later point, the sorting lines can be used to accumulate units until a desirable block size of the same attribute code is formed, while units of a unique attribute code can bypass the sorting area.

3.3. Resequencing by substitution

Resequencing by substitution for an attribute can be performed on the line by substituting a vehicle later in the sequence for a unit with a different attribute code earlier in the sequence. These two units usually are two identical units in terms of the other attributes. Substitution doesn't physically interchange the two vehicles on the line, instead it switches their vehicle identification numbers. By switching the vehicle identification numbers, the desired attribute code of the unit later in the sequence is moved up in the sequence, while the attribute code of the unit earlier in the sequence is placed in the back. An example of this practice can take place in the painting operation. A vehicle of a desirable color later in the sequence can substitute for a unit earlier in the sequence in order to obtain larger paint blocks. A drawback of this practice is to cause delay in the final delivery of the unit placed in the back during the substitution if the distance between the two vehicles on the line is long. Another drawback is the difficulty to restore the changed sequence to the original sequence at a later point on the assembly line.

4. Sequence Restoration

When a downstream department on the assembly line needs to know the sequence much earlier in advance, and if the upstream sequence is altered during its process for intentional or unintentional reasons, the sequence may need to be restored to the original one before vehicles enter the downstream department. Advance knowledge of sequence makes it possible for suppliers to deliver parts to the downstream department accordingly. Another possible reason for needing to restore the sequence is that the original sequence may have been intended for downstream production considerations. Knowing the incoming sequence to a department in advance may also enable the prediction of the resultant sequence possibly further altered by the department.

There are various possible ways to restore the sequence after intentional or unintentional sequence alterations. A way to restore the sequence is by using sorting lines after intentional sequence alteration by sorting lines as described in the previous section. For restoring the sequence with unintentional sequence alterations, one way is to use a "reservoir system" to hold sufficient units in a bank in order to release the next needed unit to the assembly line according to the original sequence. A plant can also use "spare units" to restore unintentional sequence alterations. Since the sorting line approach has been discussed, the following sections will address the reservoir and spare units systems considering defective units that occur randomly.

It is noted that a reservoir system or spare units system needs to use a storage and retrieval system. The configuration of the storage and retrieval system depends on the required storage capacity. If the required storage capacity is high, an automated storage/retrieval system (AS/RS) may be required with a significant investment. When the required storage capacity is low, a smaller AS/RS is needed or a less expensive sorting line system may suffice.

4.1. Reservoir system

A reservoir system (Sawyer, 1994; Voller and Kistler, 1997; Inman, 2003) is used to hold a stream of vehicles in order to restore the altered sequence before releasing these vehicles. By holding a stream of vehicles in the bank, an out-of-sequence unit can join the bank and be released according to the originally intended sequence. The critical parameter of such a system is the needed holding capacity, or the maximum number of vehicles needing to be stored in the reservoir. Inman (2003) presented an AS/RS sizing methodology for sequence restoration based on a given service level and an input sequence altered by various reasons. When AS/RS sizing is determined in this paper, only repairs for the painting operation are considered as the cause for sequence alteration. For a system that considers sequence alteration only due to defective units, the holding capacity depends on the distribution of the repair time for a defective unit, production rate during a day, defective rate, and required service level. That is, the holding capacity needs to be large enough to give a sufficient chance (service level) for a repaired unit to join the reservoir before the units behind the repaired unit in the original sequence are released.

Assume that there is one repair facility to repair all defective units, arrivals of defective units follow a Poisson process, and the repair time for each defective unit is exponentially distributed, an M/M/1 queue can be used to calculate the average system time (W) to repair a defective unit. Since W is also exponentially distributed, the capacity of the reservoir system can be calculated accordingly to achieve a certain service level for the defective units to rejoin the AS/RS, so that the composite service rate of the defective and nondefective units exceeds a certain required level.

4.2. Spare units system

When a defective unit is identified for needing to be repaired off line, it is removed from the assembly line, repaired, and then inserted back to the line. In order to restore the sequence from unpredictable sequence alterations caused by rework, using spare units to replace the defective vehicles can be a viable method. At the beginning of each shift or day, spare units of various models can be produced first based on calculated quantities. These vehicles will act as spare units to replace the defective units during the shift or day, and a repaired unit will become a spare unit. All spare units will rejoin the line at the end of the shift or day according to the production schedule. To successfully run a spare units system, there needs to be an adequate storage space for storing the spare units, and equipment to retrieve the needed units from the spare units bank. To estimate the needed quantities of the spare units, a queuing model is presented here.

A queuing model

The system assumes s_i spare vehicles for model i of which m_i units are produced in a day; and the parameter s_i is determined so that a certain service level (α) is achieved. It is assumed that each defective vehicle needs to be repaired only once. Defective units are repaired in the order of arrival. The interarrival times of defective units of model i are assumed to be exponentially distributed with a rate $\lambda_i = m_i p/h$, where p is the average percentage of defective units, and h the number of work hours per day. It follows that the interarrival times of all defective vehicles arriving at the inspection point are also exponentially distributed with a rate λ = Np/h, where N = $\sum m_i$. The repair time of each unit in a *shared* repair facility is exponentially distributed with a repair rate μ. It can be seen that, if units of various models are not differentiated, the vehicle rework process is an M/M/1 queue, and the average system time of each unit in the repair system is thus, W = $\frac{1}{\mu - \lambda}$.

With respect to a certain model i, it can be shown that the system of repairing defective units of model i behaves exactly as an M/M/1 queue with a "virtual repair rate" μ_i that is lower than μ. This is due to the fact that a defective unit of model i needs to wait for units of other models to be repaired; that is, it is as if that the repair of a unit of model i takes longer time. The fact that the repair system of model i behaves exactly as an M/M/1 queue can be shown below.

Theorem

With an exponential overall service rate μ for any model, the repair process of the defective units of each model i that has an exponential arrival rate λ_i behaves exactly as an M/M/1 queue with a service rate of $\mu_i = \mu(1-d_i)$, where $d_i = \rho(1-\omega_i)$, $\rho = \lambda/\mu$, and $\omega_i = m_i/N$.

(Proof)

Note that the probability, P_n, of having n vehicles of any model in the queuing system is $\rho^n(1-\rho)$. Also the probability, P(X > n), of having more than n units of any model in the queuing system, is:

$$P(X > n) = 1 - (P_0 + P_1 + \ldots + P_n)$$

$$= 1 - [(1-\rho) + \rho(1-\rho) + \rho^2(1-\rho) + \ldots + \rho^n(1-\rho)] = \rho^{n+1}.$$

When a unit of model i joins the queue, either it finds that all units in the queue are of the other models (Case 1), or a number of units of the other models are after an existing unit of model i (Case 2). The probability, $p_i(n)$, of having n units of other models in front of a unit of model i joining the line can be determined as follows.

(Case 1) The probability that n units of other models are in the queue of n units = $P_n \cdot P$ (all units are of the other models | there are n units of any models in queue) is $p_i'(n) = \rho^n(1-\rho)(1-\omega_i)^n$.

(Case 2) The probability of having n consecutive units of other models after an existing unit of model i for the last n+1 units in the queue of more than n units of any models = $P(X > n) \cdot P$(n units of other models are after an existing unit of model i | X > n) is $p_i''(n) = \rho^{n+1}\omega_i(1-\omega_i)^n$.

Therefore, the probability that n consecutive units of other models in the queue are in front of an arriving unit of model i is:

$$p_i(n) = p_i'(n) + p_i''(n) = \rho^n(1-\omega_i)^n[1-\rho+\rho\omega_i] = (d_i)^n[1-\rho+\rho\omega_i)] = (d_i)^n[(1-\rho(1-\omega_i)]$$

$$= (d_i)^n(1-d_i), \tag{1}$$

where $d_i = \rho(1-\omega_i)$.

Let S_{n+1} denote the time to repair a joining unit of model i and n units of the other models that are already in the queue. Thus, the total repair time of the n+1 units is the sum of T_1, T_2, …, T_n, and T_{n+1} which are independent service times following an exponential distribution with a service rate μ. Thus,

$$S_{n+1} = T_1 + T_2 + \ldots + T_{n+1} \tag{2}$$

follows an Erlang distribution with parameters $\mu/(n+1)$ and $n+1$. From (1), $p_i(n)$, the probability of having n consecutive units of other models in front of an arriving unit of model i, is $(1-d_i)(d_i)^n$. The probability of having the total "virtual service time," V_i, due to servicing the other models prior to a unit of model i and a unit of model i greater than t is thus,

$$P\{V_i>t\} = \sum_{n=0}^{\infty} p_i(n) P\{S_{n+1} > t\}$$

$$=\sum_{n=0}^{\infty}(1-d_i)(d_i)^n\left[\int_t^{\infty}\frac{\mu^{n+1}x^n e^{-\mu x}}{n!}dx\right]$$

$$=\sum_{n=0}^{\infty}(1-d_i)(d_i)^n\left[1-\int_0^{t}\frac{\mu^{n+1}x^n e^{-\mu x}}{n!}dx\right]. \tag{3}$$

Differentiating (3) (representing $1-G_i(t)$, where $G_i(t)$ is the distribution function) with respect to t gives $-g_i(t)$, where $g_i(t)$ is the probability density function of the virtual service time of a unit of model i due to its service requirement and servicing units of other models before an existing unit of model i. Thus,

$$g_i(t) = \sum_{n=0}^{\infty}(1-d_i)(d_i)^n\left(\frac{\mu^{n+1}t^n e^{-\mu t}}{n!}\right) = \mu(1-d_i)e^{-\mu t}\sum_{n=0}^{\infty}\frac{\left((d_i)^n\mu^n t^n\right)}{n!}$$

$$=\mu(1-d_i)e^{-\mu t}\sum_{n=0}^{\infty}\frac{\left(\lambda(1-\omega_i)t\right)^n}{n!} = \mu(1-d_i)e^{-\mu t}\ e^{\lambda(1-\omega_i)t} = \mu(1-d_i)\ e^{-\mu(1-d_i)t}. \tag{4}$$

Therefore, V_i follows an exponential distribution with a parameter $\mu(1-d_i)$; that is, the average "virtual service rate" μ_i is equal to $\mu(1-d_i)$.

(Q.E.D.)

Based on the above discussion, the probability of n defective units of model i in the system can be calculated as $P_n^i = (\rho_i)^n(1-\rho_i)$, where $\rho_i = \lambda_i/\mu_i$. Thus, s_i can be determined to have $\sum_{n=0}^{s_i} P_n^i$ greater than a certain service level α, so that the system achieves a satisfactory composite service rate (say, 99%) that is equal to the percentage (p%) of defective units times α plus the percentage ((100-p)%) of nondefective units times 1. Since the spare vehicles will rejoin the line at the end of the shift or day, it should be noted that actually there are r_i+s_i units of model i produced totally, where r_i denotes the regularly produced units of model i one the line. The calculation procedure must ensure that r_i+s_i equals the total production requirement, m_i.

A calculation procedure can be applied by initially setting $r_i = m_i$ and calculating s_i, for all models. The (r_i, s_i) values for all models are then readjusted in each iteration. In each iteration, for all models, reset $r_i = m_i-s_i$; then for all models, recalculate s_i values based on the updated λ_i and μ_i values, that is $\lambda_i= \omega_i\lambda$, $\mu_i = \mu(1-d_i)$, where $\omega_i = r_i/R$, $R = \sum r_i$, and $d_i = \rho(1-\omega_i)$ (note that λ and μ are constant). The procedure is repeated until $r_i + s_i = m_i$ holds for all models. In some rare cases, before and after an adjustment of r_i by 1 unit, $r_i+s_i>m_i$ but $r_i'+s_i'<m_i$, or vice versa; to obtain a correct (r_i, s_i) pair in this case, a higher-than-necessary s_i value can usually be adopted to ensure $r_i+s_i=m_i$. By following this calculation procedure, the total number of units produced can be determined whether produced on the assembly line regularly or as spare units.

Two-stage repair operation

Based on the above repair system, another stage of the repair operation can be included in the model. In a painting department, for example, the second stage can represent painting and paint curing, while the first stage can be a repair operation. It is assumed here that the second stage operation is a fixed-time-interval operation. To deal with such a two-stage repair system, "additional spare units" are needed. Based on the assumptions for the first-stage operation as described in the previous section, the arrival process to the second stage is also an exponential (Poisson) one. Thus, the additional number of spare units for a model is equal to the number of units of the model present in the second stage operation based on a service level (i.e., at a given probability value), such that the multiple of the service levels of the two stages will exceed a specified service level for the defective units.

An example

Assume an automobile plant with 16 work hours per day and a daily schedule of 960 units of 50 models. Also assume 10%, 20%, 30%, and 40% of the 50 models have, on the average, 40%, 30%, 20%, and 10% of the 960 units, respectively. The number of units of each model of each case is then randomly generated following a uniform distribution within a range of ±50% of the mean number of units as stated above; however, the total number of all models will be equal to 960. At the inspection point the average percentage of defective units is 10%. The repair time for each defective vehicle is exponentially distributed with a mean of 5 minutes plus a second-stage operation of 1 hour.

In the case of applying a reservoir system, the average system time (W) to repair a defective unit is 10 minutes according to an M/M/1 queue. Thus, according to the exponential distribution of W, a defective unit can be repaired in 23 minutes plus 60 minutes from painting at a service level of 0.9. Notice that the service level of nondefective units is 1.0. This gives a composite service rate ((10%)(0.9) + (90%)(1.0)) of 99%. Therefore, approximately 83 vehicles need to be held in the reservoir system. While a spare units system requires 24 spare vehicles to achieve a composite service level of 99% based on the above queuing model and the second stage requirement. Further comparison for the defective rates ranging from 1-10% is made; and Table 1 gives the number of units needed to achieve a 99% composite service level.

In Table 1, it can be seen at a defective rate of 1%, that the number of needed units for either a reservoir system or spare units system is 0. This is because the 99% service level is achieved without either system. However, when the defective rate is higher than 1%, the number of units for the reservoir system is at least 60 because at least 1 hour of repainting time is required for a defective unit in the repair process. It also can be seen that under the given system parameters, the spare units system uses a much smaller inventory to address sequence restoration, and the inventory size of the spare units system reduces quite noticeably as the defective rate decreases. Such an analysis also helps motivate continuous improvement to reduce the defective rates.

Table 1. Number of units needed for the reservoir and spare units system under different defective rates

Defective Rate	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
Reservoir System	0	64	66	69	71	73	75	77	80	83
Spare Units System	0	0	0	0	3	5	10	13	19	24

5. A Case Example

A case example is presented here to demonstrate the concepts and methods of the sequence alteration and restoration methods presented in this paper. An automobile assembly plant in North America is used as the case example. Recommendations made to the plant have not been implemented, but are given here as examples to help clarify the concepts presented. The assembly plant produces various models of pickup trucks at a rate of about a thousand vehicles each day. The assembly system consists of a Body Department, a Painting Department, and a Final Assembly Department. The plant applies mixed-model assembly throughout the plant, and a linear flow of vehicles starts from the Body Department and moves through the plant. Figure 5 shows the linear flow of vehicles in this plant with points A through F representing the entry or departure points in various departments, and G, H representing the transit sections between departments. The sequence considerations for mixed models are different from department to department. For the Body Department, the main consideration is to follow a predetermined production pattern to have minimum setups for the welding equipment. For the Painting Department, the main consideration is to have larger paint blocks of vehicles of the same color in order to lower cost. The Final Assembly Department has a main sequencing consideration of keeping balanced workloads in several assembly areas.

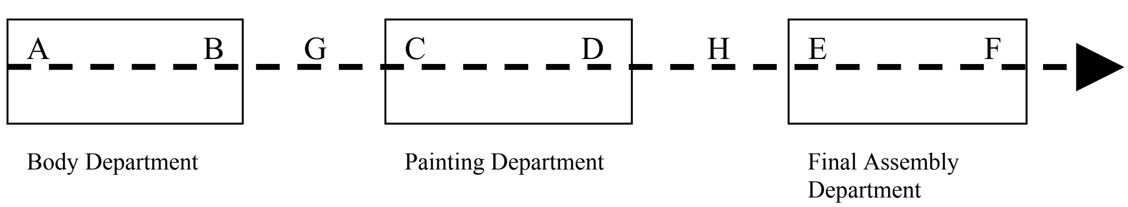

Figure 5. Assembly plant in the case study

Due to the presence of the linear flow, the consideration of the Body Department is currently given the highest priority when deciding the initial sequence. Having large painting block sizes as desired by the Painting Department is thus, not fully considered in the original sequence. Presently, the Painting Department uses the substitution method (at Point C) to obtain larger paint blocks, and this significantly changes the initial sequence. Moreover, vehicles with defects in the Body and Painting Departments usually need to be repaired (reworked) off line. After the repair is completed, the vehicles rejoin the assembly line. In this way, repair causes significant sequence alteration.

As the initial model sequence is changed in the upstream production process, the final sequence does not guarantee adequate workload balance in the Final Assembly Department. To meet the workload balance requirement for some downstream work areas, there is a "Selectivity Bank" area (at Point H, after the Painting Department) of 13 shuffling lines to resequence vehicles for the Final Assembly Department. The plant applies sequenced parts delivery for a number of parts. These parts are organized in the warehouse and delivered to the final assembly line according to the sequence broadcast from the Selectivity Bank area. The automobile assembly plant desires to know in advance the sequence in the Final Assembly Department to enable sequenced parts delivery directly from suppliers.

Based on the results presented in this paper, recommended improvements are given here to demonstrate the use of concepts and methods presented in this paper.

<u>1. To prolong the time of knowing the final sequence in advance</u>

Resequencing and broadcasting of the resequenced vehicles are currently performed at point H, the Selectivity Bank. This can be started about an hour earlier at the point right after vehicles are inspected in the Painting Department (Point D). This will result in an earlier time in knowing the downstream sequence, and consequently, give the warehouse and nearby suppliers more time to arrange parts in sequence accordingly. The rolling sequencing, placing, and stacking rules for resequencing presented in Section 2.1 can be applied using a selection range of 13 lines which is equal to the number of shuffling lines in the system.

2. <u>To restore the final sequence to the original sequence for final assembly</u>

To allow suppliers to directly deliver parts in sequence, the assembly plant can restore the final sequence to the originally intended one. Three efforts can be proposed to restore the final sequence to the original one:

i. Increase the paint block sizes by using sorting lines which allows further restoration as stated in Section 2.2. In this way, the average paint block size can be increased. Such a revised sequence can also be easily restored to the original sequence owing to the nature of the sorting approach as stated in Section 2.2. The current substitution practice can then be discontinued.

ii. Based on the assembly system parameters (not stated here due to confidentiality), 56 models (from 4 colors) per day are assumed. The number of units of each model was randomly generated based on the assignment of 40/30/20/10% of all units to 10/20/30/40% of the models. It is computed that 76 and 10 vehicles for the reservoir and spare units systems, respectively, are needed to restore the altered sequence due to rework at a desired composite service level of 98%. Similarly, when 112 models (from 8 colors) of vehicles per day are assumed, only 6 units are needed for the spare units system while the needed capacity of the reservoir system remains to be 76.

Therefore, it is suggested to use spare units to address the sequence alteration due to rework in the painting process. The procedure presented in Section 3.2 was used to calculate the quantities of the needed spare units for a spare units system. A shuffling system or small AS/RS may be used. Further reduction in the defective rate would also reduce the required capacity.

iii. Include the sequencing consideration for the Final Assembly Department in the original sequence. This will make the completely restored sequence to be suitable for the Final Assembly Department, and resequencing will not be needed at its entry point. An alternative approach to address the sequencing consideration of Final Assembly is to perform resequencing using a rolling or block sequencing method at a point before the Final Assembly Department with a prediction of the resequencing outcome based on an input sequence that is restored to the original assembly line sequence.

6. Conclusions

On an automobile assembly line, different sequencing requirements in various departments can lead to the need for intentional sequence alterations. There are also unintentional sequence alterations due to reasons such as rework and equipment breakdowns. An increasingly popular practice in sequenced parts delivery makes sequence alteration and restoration relevant topics in order to know the sequence in advance on a mixed-model assembly line.

This paper addressed sequence alteration and sequence restoration on a mixed-assembly line related to sequenced parts delivery in an automobile assembly environment primarily from its control and design aspects. Sequence alteration methods for the purposes of meeting downstream production considerations were presented. The presented methods included alteration on the line, using sorting lines, and by substitution. For the method of using sorting lines, it considered the case of either needing further restoration or not. Sequence restoration by using a reservoir system or spare units was also presented. A queuing model was presented to estimate the number of spare units in the spare units system. A spare units system performed noticeably better than a reservoir system under the considered parameters.

A case example based on an automobile assembly plant was used to demonstrate the use of these sequence alteration and restoration approaches. The methods presented in sequence alteration and sequence restoration in this paper can be useful to an assembly plant where different sequencing considerations are required in different departments, and advance knowledge of the final sequence is needed.

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Appendix A. Computational Results of the Heuristic Sorting and Releasing Rules

Table 2. Computational results of the heuristic sorting and releasing rules

Prob. Set	Configuration (L×b×c)	Avg. block size before sorting (B_1)	Avg. block size after release (B_2)	% Increase in avg. block size $r_1=(B_2-B_1)/B_1$	Ideal average block size (B_3)	% Difference btw. B_2 and B_3 $r_2=(B_3-B_2)/B_3$
1	6×4×8	1.102	2.720	147%	3.0	9%
2	6×4×6	1.166	3.476	198%	4.0	13%
3	6×4×4	1.356	4.880	260%	6.0	19%
4	4×6×8	1.174	1.962	67%	3.0	35%
5	4×6×6	1.164	2.892	148%	4.0	28%
6	4×6×4	1.264	3.909	209%	6.0	35%
7	8×3×8	1.088	2.720	150%	3.0	9%
8	8×3×6	1.141	3.340	193%	4.0	17%
9	8×3×4	1.275	4.643	264%	6.0	23%
10	12×2×8	1.134	2.415	113%	3.0	20%
11	12×2×6	1.154	4.000	247%	4.0	0%
12	12×2×4	1.314	4.600	250%	6.0	23%
13	9×4×8	1.137	3.381	197%	4.5	25%
14	9×4×6	1.197	4.154	247%	6.0	31%
15	9×4×4	1.337	9.000	573%	9.0	0%
16	6×6×8	1.116	2.737	145%	4.5	39%
17	6×6×6	1.203	4.392	265%	6.0	27%
18	6×6×4	1.333	5.629	322%	9.0	37%
19	12×3×8	1.203	3.542	194%	4.5	21%
20	12×3×6	1.181	6.000	408%	6.0	0%
21	12×3×4	1.361	7.380	442%	9.0	18%
22	6×2×8	1.084	1.437	33%	1.5	4%
23	6×2×6	1.142	1.893	66%	2.0	5%
24	6×2×4	1.200	2.580	115%	3.0	14%
25	4×3×8	1.200	1.420	18%	1.5	5%
26	4×3×6	1.130	1.807	60%	2.0	10%
27	4×3×4	1.202	2.520	110%	3.0	16%
28	3×4×8	1.072	1.366	27%	1.5	9%
29	3×4×6	1.099	1.571	43%	2.0	21%
30	3×4×4	1.322	2.640	100%	3.0	12%
31	2×6×8	1.076	1.232	14%	1.5	18%
32	2×6×6	1.075	1.207	12%	2.0	40%
33	2×6×4	1.295	1.916	48%	3.0	36%

Note:
L: the number of sorting lines
b: line length
c: the number of attribute codes
B_1: average block size before sorting
B_2: average block size after release
B_3: ideal average block size
r_1: percentage increase in average block size after release
r_2: percentage difference between average block size after release and ideal average block size

Appendix B. An Integer Programming Formulation of the Sorting Problem to Batch a Single Attribute

For each section of L·b units, there are L·b positions indicating the order of these units before entering the sorting area. Similarly, there are L·b positions after the units leave the sorting lines. Let X_{ij} be 0 or 1 to indicate whether or not the unit at position i before sorting is assigned to position j after sorting, the sum of Y_{1j} and Y_{2j} denote whether the attribute codes of the two units are different in positions j and j+1 after sorting. Also let the parameter C_i denote the attribute code of unit i in the sequence before sorting. This integer program is as follows:

$$\text{Minimize} \quad \sum_{j=1}^{L\bullet b-1}(Y_{1j} + Y_{2j}) \tag{5}$$

$$\text{Subject to} \quad \sum_{j=1}^{L\bullet b} X_{ij} = 1, \quad \forall\, i \tag{6}$$

$$\sum_{i=1}^{L\bullet b} X_{ij} = 1, \quad \forall j \tag{7}$$

$$\sum_{i=1}^{L\bullet b} i \bullet X_{i,\,j+1} \geq \sum_{i=1}^{L\bullet b} i \bullet X_{ij}, \quad \forall j,\ j \neq b, 2b, 3b, \ldots, L\cdot b \tag{8}$$

$$\sum_{i=1}^{L\bullet b} C_i \bullet X_{ij} - \sum_{i=1}^{L\bullet b} C_i \bullet X_{i,\,j+1} \leq MY_{1j}, \quad \forall j,\ j \neq L\cdot b \tag{9}$$

$$\sum_{i=1}^{L\bullet b} C_i \bullet X_{i,\,j+1} - \sum_{i=1}^{L\bullet b} C_i \bullet X_{ij} \leq MY_{2j}, \quad \forall j,\ j \neq L\cdot b \tag{10}$$

$$X_{ij} = \{0, 1\}, \qquad \forall i, j$$

$$Y_{1j} = \{0, 1\},\ Y_{2j} = \{0, 1\}, \qquad \forall j,\ j \neq L\cdot b$$

The first two constraints (6) and (7) ensure that the vehicle occupying position i before sorting is assigned to a position j after sorting. Constraint (8) guarantees that in each sorting line of length b, vehicles are placed in ascending order of their original position indexes. Constraints (9) and (10) use two 0-1variables, Y_{1j} and Y_{2j}, for each position j to ensure one of them equal to 1 if the adjacent units j and j+1 have different attribute codes. The objective function (5) seeks to minimize the number of attribute-code changes in the sequence.

PART 2

ANALYZING PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY IN A MANUFACTURING FIRM

This part is a paper submitted to the journal *International Journal of Logistics Systems and Management* in 2005 by Fong-Yuen Ding, Hui Sun, and John Kallaus:

Ding, F., Sun, H., and Kallaus., J. (2005) Analyzing product complexity related to product variety in a manufacturing firm with a case study at an automobile assembly plant. *International Journal of Logistics Systems and Management*.

My primary contribution to this paper include (1) most of gathering and interpretation of literature, (2) part of development of data envelopment analysis models, (3) data collection and calculation for the case study, and (4) part of draft writing and editing.

1. Abstract

Due to technological advances and consumer interests, product variety can become significantly high. In this paper, product variety refers to product complexity due to various customer choices within a product, while product complexity involves all factors that make a product complex. Product variety can be a major contributing factor to product complexity of a manufactured product. High product complexity can have a significant impact on many cost areas including material, manufacturing, inventory, and distribution. Motivated by a desire to better understand its product complexity and to identify product complexity reduction opportunities in a U.S. automobile plant, a number of tools are applied in this paper to analyze product complexity related to product variety and identify product complexity reduction opportunities associated with product attributes. Measures of product variety are discussed. Two data envelopment analysis (DEA) models for comparing the relative product complexities related to product variety among similar products, and a DEA model for ranking various attributes of a product for complexity reduction consideration are proposed. An economic analysis template is suggested. A case study based on the considerations of a U.S. automobile plant is also presented to illustrate the applications of these tools.

2. Introduction

Due to increasing technological sophistication and market competition, product variety and complexity of various manufactured products can become significantly high. Some companies view that high product variety gives customers more choices and consequently, a chance for the company to gain market share. For an industry that depends on economies of scale to reduce production costs, however, product variety can have a negative impact. High product complexity can have a significant impact on costs of product design, manufacturing, and distribution. This paper attempts to apply a number of analytical tools for better understanding and managing product variety.

The term "product variety" in this paper refers to product variations due to choices by customers within a single product. "Product complexity" refers to the level of product sophistication from all factors. Product variety can be a major contributing factor to product complexity, but there are usually other non-customer-choice factors contributing to product complexity. For example, an automobile manufacturer can use many kinds of wire harnesses or bolts on a car; and this is an aspect of product complexity that is not product-variety related. The relationship between product complexity and product variety factors can be depicted in a simple Venn diagram in Figure 1. The scope of this paper will be limited to dealing with product complexity related to product variety. Although how much product variety contributes to product complexity is not easily quantifiable, product variety's cost impact on the production system can be seen in automobile (MacDuffie et al., 1996; Fisher and Ittner, 1999; Kim and Chhajed, 2000), computer (Swaminathan and Tayur, 1999), and other industries (Martin et al., 1998; Randall and Ulrich, 2001).

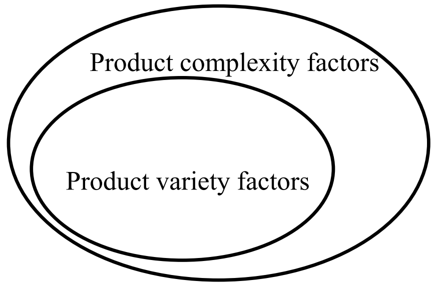

Figure 1. Relationship between product complexity and product variety

2.1. Relevant literature

Many factors such as quality and price (Marquez, 2004) contribute to gaining market share of a product. Product variety also plays an important role in gaining market share. Kekre and Srinivasan (1990) concluded positive relationships among product line breadth, market share, and profit margin based on empirical studies. It is generally viewed that both the market influence and customer behavior lead to the growth of product variety; and the ultimate feasibility constraint on product variety is technological (Lancaster, 1998; Kahn, 1998). Fader and Hardie (1996) presented a statistical choice model to show that customer choice is often made on the basis of a set of attributes of stock keeping units (SKUs). Chong et al. (1998) presented empirical models to examine how different brand width measures affect the brand share based on measures of brand width in terms of the number of SKUs, the number of “distinct SKUs” (after similar products of the same salient product attributes are combined into one SKU), and the number of distinct sizes and flavors (considered as salient attributes) of ice cream. It was stated that a company must assess the level of product complexity to keep both the company’s low costs and high product attraction to consumers (Child et al., 1991; Rommel et al., 1995; Desai et al., 2001).

Many manufacturing firms view offering more variety as an advantage in gaining market share but as a disadvantage to economies of scale. Some researchers attempted to model the impact of product complexity on manufacturing. Banker et al. (1990) directly traced overhead costs for areas including supervision, quality control, and tool maintenance, in the presence of product and process complexity of individual automobile lamps, for complexity factors such as the number of moving parts in a mold and number of functions. They developed several regression models which identified the impact of several complexity factors on overhead costs; and it was found that certain product complexity factors had a significant impact on the studied cost areas. Child et al. (1991) observed that the costs of complexity constitute 10 to 40 percent of the total costs including material, manufacturing, logistics, and inventory in some manufacturing companies. Benjaafar et al. (2004) presented a model to study the effect of product variety on inventory costs in a production-inventory system and showed that inventory costs increased linearly with the number of products, and the rate of increase was sensitive to system parameters such as the demand rate and setup time. Kekre and Srinivasan (1990) concluded that there was no negative impact on production costs from an increase in product line breadth based on self-reported survey data from a sample of U.S. manufacturers. They conjectured that this was due to companies' adopting managing strategies such as just-in-time practices or flexible manufacturing technologies to lessen any possible adverse impact. In this paper, we assume that there is no immediate improvement on the manufacturing system along with changes in product variety.

MacDuffie et al. (1996) examined the effects of four measures (model mix complexity, parts complexity, option content, and option variability) of product complexity on productivity and product quality, and discovered that parts complexity has a persistent negative impact on productivity. Fisher and Ittner (1999) performed an empirical study and simulation analysis to investigate the impact of product variety in terms of option content and option variability on automobile assembly operations. They showed that option variability has a more negative impact on productivity than option content in automobile assembly. In addition, they showed that option variability causes increases in overhead hours, rework, inventory, and the excess labor capacity assigned to a work station to buffer against variability. Regarding managing product variety for reducing manufacturing costs, Martin et al. (1998) developed three indices to indicate the levels of the manufacturing costs for providing variety, and a process sequence graph to assist in reducing manufacturing cost associated with variety on a production line.

Research efforts were made in the area of product-line design for a company considering the market, price, and costs with an intent to optimize profit (e.g., Chen et al., 1998; Yano and Dobson, 1998; Chen and Hausman, 2000; Kim and Chhajed, 2002). Morgan et al. (2001) proposed a mathematical programming model for product-line selection with the objective of maximizing profits by considering marketing implications and manufacturing costs. Ramdas and Sawhney (2001) presented a mixed-integer programming model to evaluate multiple new product lines of assembled products by considering both incremental revenues and life-cycle costs. Ulrich et al. (1998) presented five criteria including the competitive distinctness of variety dimensions, cost effectiveness of product architecture and production/distribution system choices, and design/operations capabilities to support the dimensions of variety, for the selection of a variety strategy based on the performance of four bicycle manufacturers with different product variety. Assuming that customer demand for different product variants can be represented by a Bayesian logit model, Hopp and Xu (2005) showed that an increasing degree of modularity would result in the increase of optimal product-line length and optimal market share. These research efforts developed product variety models, procedures, and strategies related to determining the variety level for a company. In general, such models and procedures depend on having estimates for product variety's impact on the market.

2.2. Specific objectives and outline of this paper

Motivated by a desire to better understand its product complexity and to identify product complexity reduction opportunities in a U.S. automobile plant, a number of tools are applied in this paper to analyze product complexity related to product variety and identify product complexity reduction opportunities associated with product attributes. The first part is a horizontal comparison of similar products for complexity. The second part is a horizontal comparison among various attributes within a product. Performing an economic analysis for a complexity reduction action will also be addressed.

This paper is organized as follows. In Section 2, the notation for describing product variety is briefly introduced, and measures of product variety are discussed. In Section 3, two data envelopment analysis (DEA) models for comparing the relative complexity levels related to product variety among similar products in the same market are proposed. A DEA model for ranking various attributes for the purpose of identifying areas of complexity reduction is also proposed. An economic analysis model is presented to review the economic impact of a change in product complexity. In Section 4, a case study based on the considerations of a U.S. automobile plant is presented to illustrate the applications of the proposed analytical tools and models.

3. Representation and Measures of Product Variety

3.1. Basic elements in describing product variety

Product variety can be thought of as variation in many "*attributes*" from which product differentiation is made. Various terms including dimension and characteristic (e.g., Yano et al., 1998; Ulrich et al., 1998) have been used for the same meaning as attribute. Each attribute has multiple selections, which will be termed as "*values*" of an attribute. An attribute of only one value will not be considered as an attribute throughout the paper. Appendix A is included to use notation to represent a product-variety structure of the customization process shown on a company's website to facilitate data collection regarding product variety.

3.2. Measures of product variety

Measures of product variety may be considered by terms including N_v, the number of product variants, N_A, the number of attributes, $|A_i|$, the numbers of values of various attributes or certain significant product attributes, where A_i is the set of all values of attribute i. Lancaster (1990) used the term product variety to refer to the number of variants within a specific product group. Chong et al. (1998) introduced three measures of brand width for products within a brand: the number of SKUs, the number of distinct SKUs, the number of distinct attributes, and the numbers of values corresponding to these attributes. Furthermore, certain product attributes have a stronger cost impact on manufacturing and the supply chain, and may be considered as product variety measures (e.g., MacDuffie et al., 1996). It can be seen that a single measure is usually not sufficient for comparing product variety. Choice of measures of product complexity depends on the objectives of the analysis, the product and production system, and in some cases, accessibility of data.

4. Analyzing Product Complexity Related to Product Variety

4.1. Data envelopment analysis for analyzing product complexity

Higher complexity generally results in a higher system cost; and it is desirable to reduce complexity whenever the benefit of complexity reduction surpasses cost. In a complexity reduction effort, quantitative tools may be needed to analyze and compare product complexity, and identify complexity reduction opportunities. To have a horizontal comparison of the product complexity related to product variety, noting that multiple factors exist, the data envelopment analysis (DEA) can be applied to products or attributes that can be considered homogenous. A comparison of product complexities related to product variety among similar products in the market gives an opportunity to benchmark similar products and motivate improvement within a company. An emphasis in this paper is to show that DEA is an adequate method for the proposed comparison.

Data envelopment analysis

Data envelopment analysis is a linear programming based methodology that has been widely used in evaluating and comparing the relative efficiencies of decision making units (DMUs) with multiple inputs and outputs. It allows use of the most favorable weights for inputs and outputs in assigning an efficiency score for each DMU. This avoids the need to determine weights for multiple factors. The initial fractional form of a DEA mathematical programming model (known as CCR model by Charnes, Cooper, and Rhodes, 1978) for determining the efficiency score of DMU j_0 is as follows:

(P1) Maximize $$h_0 = \frac{\sum_{r=1}^{t} u_r y_{rj_o}}{\sum_{i=1}^{m} v_i x_{ij_o}} \qquad (1)$$

Subject to

$$\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad j = 1, 2, \ldots, n \qquad (2)$$

$$u_r \geq \varepsilon, \qquad r = 1, 2, \ldots, t \qquad (3)$$

$$v_i \geq \varepsilon, \qquad i = 1, 2, \ldots, m \qquad (4)$$

where y_{rj} represents the amount of output r of DMU j, and x_{ij} the amount of input i of DMU j; u_r is the decision variable for the weight assigned to output r, and v_i the weight assigned to input i; ε is a very small positive number to ensure that u_r and v_i hold positive values. It is assumed that there are t outputs, m inputs, and n DMUs.

The above model (P1) can be easily transformed to a linear programming model by setting $\sum_{i=1}^{m} v_i x_{ijo}$ to 1 in (1) and converting the set of constraints (2) to linear inequalities by moving the denominator to the right hand side. The linear program can also be transformed to a dual formulation. A DMU is said to be efficient if and only if, the optimal solution value of the primal or dual equals 1. Solving the dual formulation has the important benefit of giving the targets (based on the dual solution) for adjusting the inputs and outputs of an inefficient DMU to become efficient (Boussofiane et al., 1991). The hyperplanes (in a multi-dimensional case) through the efficient DMUs in the feasible region form efficient frontiers in the CCR model. A reference set is the set of efficient DMUs that form a composite DMU on an efficient frontier to represent the target for improvement for an inefficient DMU. The reference sets can be identified from the DEA results. An inefficient DMU can move toward this target composite DMU by a proportional decrease (or increase) of inputs (or outputs).

An extension of the dual model is known as the BCC model (Banker, Charnes, and Cooper, 1984) which is formulated by adding a convexity constraint to the dual formulation of the CCR model. The convexity constraint ensures that a BCC model gives an attainable composite unit of similar scale size as that of unit j_0. The BCC model (see Appendix E) will be applied in Case Study in Section 4.

A rationale for using DEA in this paper: By including all DMUs on efficient frontiers as efficient without using preset weights, DEA gives the most favorable evaluation to each DMU and therefore, enhances the classification of the inefficient units and results in a stronger rationale for improvement.

4.2. Applying DEA in comparing product complexities related to product variety

Two DEA models are proposed here to compare the complexity levels related to product variety among similar products of different companies. Model 1 attempts to compare the complexity levels of various products related to the variety measures, while model 2 attempts to compare the efficiencies of offering product complexities in contrast to economic outputs. An illustrative model is presented for each of models 1 and 2. Although such models would vary depending on system conditions and comparison objectives, the two illustrative models can be considered when data regarding product varieties of various products of different companies are attainable only through publicly accessible information sources such as company websites.

Homogeneous DMUs consideration: Care must be given to ensure that the compared products are homogenous considering similarities in products, processes, resources, and environments (Dyson et al., 2001). For example, only products of similar vehicle lines produced in the same geographical area and sold in the same market should be compared.

Model 1

The main idea of model 1 is as follows: From the viewpoint of product complexity related to product variety, a product having smaller values in key product variety measures would be more efficient. In this proposed DEA model, each product in the same market is considered as a DMU. Key product variety measures are included as inputs and outputs. Figure 2 depicts the conceptual model 1 and an illustrative model. In the illustrative model 1, each DMU has two inputs, the number of attributes (ones with more than one code), N_A, and the weighted average number of attribute values, $\overline{N}_a$, while weights can be based on estimated levels of cost impact on manufacturing of various attributes in the same industry. For example, it is typical that painting is a major operation in the automobile industry, and color has a pervasive impact on parts variety; therefore, the color attribute would be assigned a high weight. Both inputs of illustrative model 1 attempt to capture impact on the manufacturing system. Another indicative product variety measure regarding the number of configurations of the finished products,

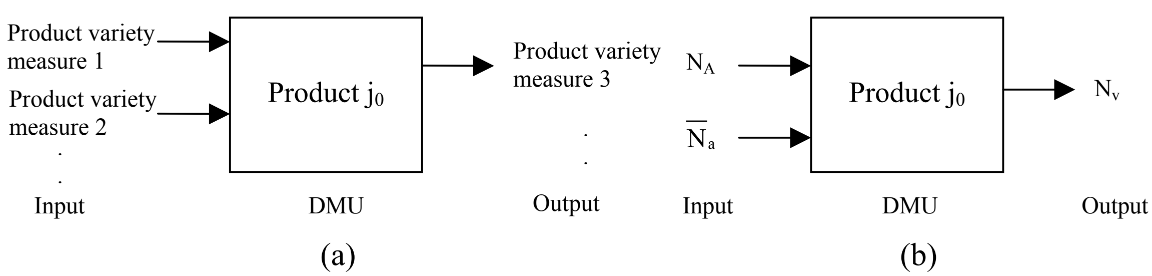

Figure 2. Model 1 – (a) A conceptual DEA model, and (b) an illustrative model for complexity comparison

the number of product variants, N_v, which is affected by N_A and $\overline{N}_a$, is considered as an output in the illustrative model 1. These input and output factors are intended to represent important measures of product complexity related to product variety. It is noted that in this illustrative model, N_v is an "undesirable" output, which is more desirable when it is smaller; and thus, there is a need to specially handle an undesirable output in its DEA computation.

Through the DEA comparison, a benchmarking of the product variety structures may be achieved and insight gained for improvement. For example, an automobile company may decide to reduce its relative complexity level by incorporating more option packages. The illustrative model 1 attempts to compare the extents of complexity of various products in the same market without considering potential revenues associated with product complexity (see model 2 for improved consideration). However, cost impact can be implicitly considered in selecting the product complexity measures for input factors.

Undesirable output or input consideration: Scheel (2001) compared various methods for treating an undesirable output in DEA, and introduced a new radial measure which assumes that any change of the output level will involve both undesirable and desirable outputs. Scheel showed that the "additive inverse" method for treating an undesirable output, which multiplies the undesirable output values by –1, was among the methods that generate a larger and more inclusive efficient DMU set. The additive inverse method will be adopted in Case Study to treat the undesirable variable so that the compared products are more likely to be classified efficient in order to motivate the improvement of a product classified as inefficient.

Model 2

The main idea of model 2 is as follows: The lower the values in product variety measures, and the higher the economic output, the more "efficient" the product is; that is, a small variety that has a large economic output is preferred. This is intended to be from the economic viewpoint of the manufacturer instead of customers. The conceptual model 2 and an illustrative model are presented for this purpose as depicted in Figure 3. In the illustrative model 2, the market share is included as an output of each product, and the three inputs are the number of attributes, N_A, the weighted average number of attribute values, $\overline{N}_a$, and the total number of product variants, N_v; these inputs represent three aspects of product-variety measures of a product. N_v is considered as an input in this model to present a metric in product complexity related to product variety. $\overline{N}_a$ is weighted because not all attributes have equal impact on production costs; and weights can be set to represent different levels of impact to the production system in the same industry. This illustrative model attempts to compare complexity levels related to

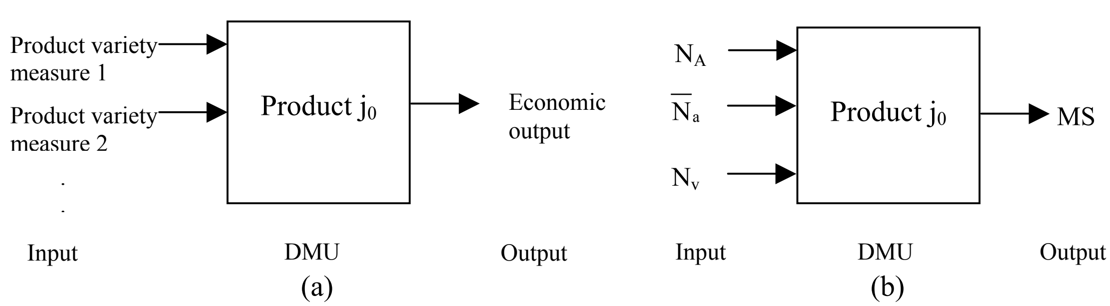

Figure 3. Model 2 – (a) A conceptual DEA model, and (b) an illustrative model for complexity comparison

product variety in contrast to sales, which represent economic outputs of products in the same market. Moreover, it is generally perceived that an increase or decrease in product variety would affect market share (closely related to sales volume). The intent of model 2 is to include important complexity measures related to variety as inputs while economic justification as output, so that a simplest product that captures the highest market share is considered the most successful product.

Post-analysis consideration: It is noted that the inputs and output in the proposed models do not necessarily have a cause-effect relationship, and that this analysis is intended for making a ranked-score comparison of various products instead of identifying the reasons for a high or low ranking. An identified product (and company) in the reference set can be further studied to determine the reasons of its success in achieving a high ranking.

Correlation consideration: It is also necessary that within each model, correlations between any variable pairs be evaluated to see whether extremely high correlation exists and whether elimination of a variable is justifiable (Nunamaker, 1985).

Sufficient DMUs consideration: Care also needs to be given to ensure that a sufficient number of DMUs are compared based on the numbers of outputs and inputs (m and t). A rule of thumb for the number of DMUs is at least [2m × t] DMUs (Dyson et al., 2001) where m and t are the number of inputs and number of outputs, respectively.

4.3. Ranking various attributes for product complexity reduction considerations

If a company is interested in reducing the complexity of a certain product, many options, selections, parts, and various aspects of the manufacturing system can make it difficult to select certain areas of focus for this effort. Since each attribute causes increased complexity by providing various selections, DEA may be performed to provide a prioritized list of attributes for focusing the product complexity reduction effort on. Possible ways to reduce complexity (Child et al, 1991) can include reducing the number of values of an attribute, offering option packages, eliminating some available attributes, and increasing the number of common parts. To rank various attributes, multiple factors can be considered in the proposed conceptual model 3 and its illustrative model as depicted in Figure 4. The main idea of model 3 is as follows: The higher the cost impacts associated with attribute-related cost impact factors, and the lower the market impact, the more "efficient" the attribute is; that is, an attribute with higher cost impact and a lower market impact is preferred as a candidate for complexity reduction consideration.

The following factors are proposed for the illustrative model 3:

Number of attribute values (N_a). An attribute that has a high number of attribute values will generally have a significant impact on the manufacturing system in inventory, scheduling, and production costs.

Impact on manufacturing costs (MC). Various attributes have different impacts on the manufacturing costs. Since the exact manufacturing cost impact from a certain attribute is generally difficult to quantify, a ranking based on a scale of say, 1 to 4 with 1

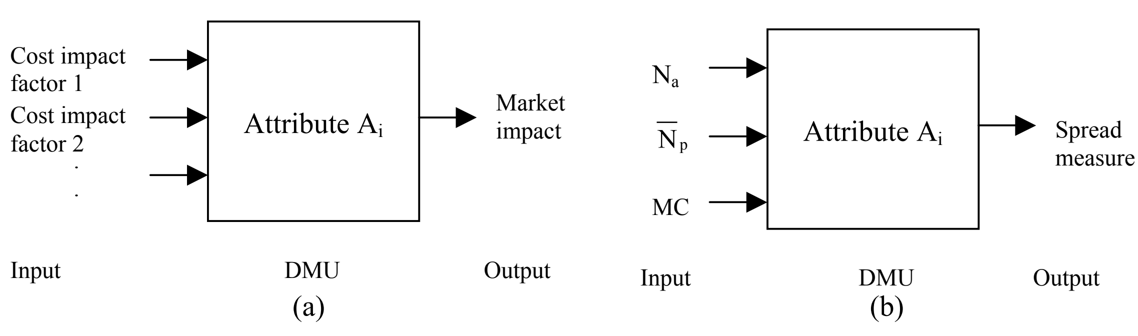

Figure 4. Model 3 – (a) A conceptual DEA model, and (b) an illustrative model for ranking attributes

representing the highest impact can be used. Care needs to be given to properly deal with this categorical variable (Banker and Morey, 1986).

A spread measure of the percentages of demands of various attribute values. Various attribute values of an attribute would have different percentages of customer demand. In conjunction with N_a, a small spread of these percentages can be an indicator for the negative effect of the *market impact* if there is a reduction of the number of attribute values. The standard deviation (SD) of the percentages of demands of the various attribute values is proposed here as a spread measure for the percentages of demands of various values of an attribute; and a higher SD favors the attribute to be selected for improvement consideration by eliminating an attribute value that has the smallest demand to have a smaller market impact. For example, assuming that there are two attributes, of which each has two attribute values, and the demand percentages of the two values of the two attributes are 80%:20% and 50%:50%, respectively. The former has an SD score of 0.424 ($\sqrt{(80\% - 50\%)^2 + (20\% - 50\%)^2}$) and the latter has an SD of 0.

In this illustrative DEA model, each attribute (A_i) of the considered product is assumed to be a DMU. The three inputs for each DMU are assumed to be the number of attribute values (N_a), the weighted average number of unique parts ($\overline{N}_p$), and impact on manufacturing costs (MC). The output of each DMU is the spread measure of percentages of sales volumes associated with various attribute values. In this model, an attribute with a higher spread measure resulted from more attribute values, more unique parts, and higher impact on manufacturing would suggest a higher priority for complexity reduction consideration. It is noted that the inputs N_a and $\overline{N}_p$ are undesirable inputs, and the impact on manufacturing cost (MC) is a desirable but uncontrollable categorical input.

Categorical input consideration: Categorical variables generally cannot form a composite unit (a convex combination of some efficient DMUs) to represent the target of an inefficient DMU due to the fact that such a composite unit may not have a meaningful interpretation (Banker and Morey, 1986). However, Banker and Morey also proposed that, if the DMUs (peer group) that form the composite DMU only consisted of DMUs of the same or lower values on the categorical input, the assessment could be considered fair (Banker and Morey, 1986; Boussofiane, 1991).

4.4. Economic analysis for a product complexity reduction

A further economic analysis is needed after a product complexity reduction area is identified. A reduction in complexity regarding a certain product aspect can lead to cost changes (increase or decrease), and an impact on sales volume and/or price. Thus, the cash flows associated with this product complexity reduction can be classified into two categories: *cost changes* and "*nominal profit changes*." Here "nominal profit changes" refers to a change in profit without considering the concurrent cost changes in order to apply an incremental comparison in the economic analysis.

Regarding *cost changes*, the product complexity reduction may reduce (or increase) its associated costs including materials, manufacturing, and distribution costs. The cost changes related to materials costs can include those for warehouse storage, inventory carrying, and material handling. The changes in manufacturing costs can include those in labor, equipment, and facility rearrangement. Similarly, the changes in distribution costs (for finished goods) can be estimated. A complexity reduction may also lead to an increase (or decrease) in the part unit cost in some cases; for example, by using more versatile parts, the part unit cost may increase while product complexity reduces.

Regarding *nominal profit changes*, two areas need to be included. One is the lost (gained) sales associated with a loss (gain) in market share after the complexity reduction. This can be calculated by multiplying the "nominal profit per unit," which is the estimated profit per unit without considering the concurrent cost changes, by the estimated number of units of the lost (or gained) sales. The other area is the nominal profit change associated with a price change with the retained sales.

The overall cash flows after a complexity reduction effort can be the basis for making a decision regarding a change in product complexity. A common spreadsheet format may be used to perform an economic analysis for various complexity reduction areas. (Although this paper addresses product complexity reduction related to product variety, the above economic analysis framework is applicable to general product complexity reduction considerations.) Such an economic analysis also helps the firm develop a better understanding of the costs and benefits involved in offering a certain aspect of complexity. It should be noted that deriving accurate estimates for various costs can be very tedious. (An example is provided in Appendix D.)

5. A Case Study

A case study was conducted for providing an analysis in product complexity for a U.S. automobile assembly plant based on the methodology presented in this paper. The automobile assembly plant produces about a thousand pickup trucks daily. Due to market considerations the plant offers many vehicle options that can be selected by dealers and customers. It has been estimated by the company that there are more than 180,000 vehicle variants as a result of the possible selections of various options. The assembly plant has experienced a high level of complexity in sequencing and scheduling, a high number of inventory items, a high level of manufacturing complexity, and a noticeable level of undesirable manufacturing conditions including misbuilt assemblies. The company is therefore, interested in being able to reduce their product complexity. The first analysis conducted was to compare similar products in the U.S. market.

5.1. Comparison of product complexities related to product variety among seven similar products in the U.S. market

In the U.S. market, there are six other vehicles of the similar size as compared to the one manufactured by the studied plant. These products can be considered generally *homogeneous* due to the same market (mainly sold in the U.S.), similar product lines (same size trucks), and similar manufacturing environments (all U.S. plants). Due to a data accessibility consideration, product complexity data were collected from the company websites. The "product variety structure" of each product offered to a common geographical location according to the customer selection process provided on each company's website was first represented using the notation described in Appendix A. The results are given in Table 4 in Appendix B. Due to data collection for the models is from the company websites, illustrative DEA models 1 and 2 described in Section 3.2 were applied. The values of N_A and the numbers of attribute values are obtained from the product feature list. Specifically, $\overline{N}_a$ is a weighted average (weights are based on estimated impact on manufacturing costs, ranging from 1 to 4, as estimated by the company conducting the analysis) of the numbers of values of all attributes for each product. The total number (N_v) of vehicle variants of each product is calculated considering available combinations of the values of various attributes. It represents the total number of vehicles that are available to customers. The sales volumes of the seven companies in the first season of the year were collected to represent the market shares.

For a small set of DMUs such as in this case problem, it might be possible for one to identify an efficient DMU by observation. However, DEA ensures that all efficient units can be identified, a ranking by scores representing the proportions of reduction (or increase) of input (or output) in order to become efficient can be obtained for all DMUs, and a reference set and improvement targets for each inefficient DMU can be obtained.

The *correlation coefficients* are 0.22 between N_v and $\overline{N}_a$, and 0.55 between N_v and N_A, respectively, based on the data of the seven companies. These relatively low correlations suggest that none of these factors (Nunamaker, 1985) should be omitted. Because the only output is an *undesirable* one in model 1, the undesirable output is first multiplied by –1 (termed as "*additive inverse transformation*"); and the *undesirable-output-oriented efficiency measure* introduced by Scheel (2001) based on the output-oriented BCC model is employed. For model 2, the *input-oriented BCC formulation* is used considering the fact that improvement can be more easily made for the input than the output (market share). The ε value used in the experiment is 10^{-8}. In general, a higher score indicates a more desirable (lower) product complexity related to product variety in model 1, and a better overall efficiency considering the market share in relation to product complexity related to product variety in model 2. Table 1 gives the results from DEA illustrative models 1 and 2. The identities of the companies are not shown.

Table 1. The complexity comparison scores by using DEA models 1 and 2

Company	N_A	$\overline{N}_a$	N_v	MS	Model 1 DEA Score	Model 2 DEA Score	Alternative Model 1 DEA Score
1	49	2.567	4,609,440	31,527	0.00199	0.963	0.948
2	59	2.735	43,417,360	27,980	0.00019	0.841	0.829
3	46	2.883	908,468	55,832	0.01316	1.000	0.857
4	43	2.533	3,571,488	7,307	0.00412	0.975	0.975
5	41	2.701	75,174	3,742	0.22034	0.953	0.953
6	50	2.443	8,266	13,794	1.00000	1.000	1.000
7	33	2.563	23,940	34,388	1.00000	1.000	1.000

Based on the model 1 scores, It can be seen that Companies 6 and 7 have the best (lowest) complexity level, followed by Companies 5, 3, 4, 1, and 2. It is noted that Company 6 has the smallest N_v and $\overline{N}_a$ and Company 7 has the smallest N_A. These two companies provide benchmarks regarding product simplification for other companies according to model 1. It is noted that Companies 6 and 7 use a significant number of "option packages" to reduce N_v. Model 2, which considers the additional factor of market share, gives products of Companies 3, 6, and 7 a score of 1. An alternative score for model 1 is also given in the last column of Table 1. These alternative DEA scores are based on a *"multiplicative inverse" transformation*, i.e., by using the inverse of the undesirable output N_v. It can be seen that the set of efficient DMUs turns out to be the same as that based on the additive inverse transformation.

In order to improve inputs or outputs to become an efficient DMU in model 1, the *output-oriented improvements* (based on the additive inverse transformation of the undesirable output) are calculated; this is because it is generally easier to reduce N_v (through using option packages, for example) than to reduce N_A and $\overline{N}_a$. With model 2, the *input-oriented improvements* are calculated since it is generally more difficult to change the market share. The sets of improvement targets are given in Table 2. The reference set for each inefficient product is also given; and the products and their companies in the reference set of an inefficient product can be studied for improvement purposes.

5.2. Attribute ranking within a company

The assembly plant under study has a total of 43 attributes that are practical to change. The plant is interested in prioritizing the complexity reduction opportunities. One way of choosing areas to focus their complexity reduction effort on is to rank attributes so that a plan of addressing product complexity reduction can be further developed accordingly. The attributes that have the most impact on manufacturing and inventory costs, and the least market impact prospect from a reduction in complexity may be considered the most favorable attributes for complexity reduction considerations. An *input-oriented BCC formulation* for the DEA illustrative model 3 for attribute ranking as stated in Section 3.3 is applied based on the data from the company. The 43 attributes are considered as 43 DMUs. Each time a DMU is computed for the efficiency score, only

Table 2. Improvement targets and reference sets from models 1 and 2 for inefficient companies

Company	Original Values				Target from model 1 (Output-oriented)			Reference set	Target from model 2 (Input-oriented)				Reference set
	N_A	$\overline{N}_a$	N_v	MS	N_A	$\overline{N}_a$	N_v		N_A	$\overline{N}_a$	N_v	MS	
1	49	2.567	4,609,440	31,527	49	2.481	9,173	6,7	35	2.472	23,578	31,527	6,7
2	59	2.735	43,417,360	27,980	50	2.477	8,249	6	38	2.300	20,896	27,980	6,7
3	46	2.883	908,468	55,832	46	2.491	11,955	6,7	(Not associated)				
4	43	2.533	3,571,488	7,307	43	2.501	14,714	6,7	42	2.504	14,084	23,553	6,7
5	41	2.701	75,174	3,742	41	2.508	16,563	6,7	39	2.514	18,304	27,015	6,7
6	50	2.443	8,266	13,794	(Not associated)				(Not associated)				
7	33	2.563	23,940	34,388	(Not associated)				(Not associated)				

the DMUs with the same or lower MC values (a *categorical input*) are compared to. The data and efficiency score based on *undesirable-input-oriented efficiency measure* proposed by Scheel (2001) for each attribute from this DEA computation are included in Appendix C. It is noted that, due to the *additive inverse transformation* and use of the efficiency measure by Scheel, DEA scores are all ≥ 1 with a score of 1 indicating an efficient DMU.

Some attributes having high DEA efficiency scores were considered for complexity reduction. One of these considered attributes is the option for the length of the cargo box. An economic analysis was conducted to compare the cost savings and nominal profit reduction from a complexity reduction of 2 values to 1 for the lengths of the cargo box. The results show that there is a slight overall cost increase. The company is carefully considering this complexity reduction option. The spreadsheet used to calculate this product complexity cost impact is included in Appendix D.

6. Conclusions

Offering more product variety can help a company gain market share; however, it can also result in higher costs in manufacturing, inventory, and distribution. Product variety can be a major contributing factor to product complexity. Properly analyzing product complexity related to product variety can develop a better understanding of the product complexity of a firm, and help develop a product complexity reduction plan. In this paper, measures of product variety considering multiple attributes were discussed. Three data envelopment analysis models and their illustrative models were proposed to compare relative complexity levels related to product variety among similar products and to prioritize attributes for complexity reduction considerations. Economic analysis was proposed to determine whether a product complexity reduction effort is economically justifiable.

A case study based on the presented analytical tools can be performed to analyze the product complexity of an automobile assembly plant. Seven similar products were compared based on two proposed DEA models. It was interesting that a product that was considered inefficient by considering only product variety related factors, can be considered efficient when the market share was included in the comparison. DEA results also suggested improvement targets for complexity reduction related to product variety and a reference set for an inefficient product. The product attributes were ranked for complexity reduction considerations. An economic analysis was applied to evaluate complexity reduction opportunities. The case study showed that applying the proposed tools to analyze product complexity can help a company compare product complexity related to product variety, identify product complexity reduction areas, and justify complexity reduction actions. While a tool such as DEA was shown to be useful in analyzing complexity related to product variety in this paper, other tools, such as AHP, may also be instrumental for such analysis.

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Appendix A. Representation of a product variety structure as structured on a website

In this paper, a "product variety structure" refers to the organization of various attributes associated with the product variety of a certain product for customer selection on a company website. The description of product variety by notation based on the structure provided on a company website becomes useful when product varieties of various products are compared for benchmarking purposes.

First, a product variety structure *based on the customer selection process* can be described in multi *levels*. Each level has one or more attributes that can be customized mutually independently possibly with some conditions. An attribute belongs to a lower level if the value selection of this attribute depends on the value selection of a higher level attribute. Among the same level of attributes defined independently of each other, there can be certain *preclusion conditions* to preclude some *combinations of attribute values* due to the nature of the product. An *override condition* may also replace the selected attribute value of an attribute of another level. For example, for an automobile, a specific kind of radio selected at a level can take the place of the radio selected at an earlier level. An "option package" used often in the automobile industry can be represented as a *combination of attribute values from a set of attributes*. The product variety structure of a certain product can be more easily compared using notation as given in Table 3.

A numeric index can also be used to include the basic structure information for a quantitative comparison of various product variety structures; for example, a numeric index can be $[L:(N_A^1,...,N_A^L), N_A, N_V]$, where L is the number of levels, $N_A^1,...,N_A^L$ are the numbers of attributes in levels 1, …, L; N_A the total number of attributes, and N_V the number of variants.

Example

An example of the variety structure is as follows:

$A_1 \otimes A_2 \otimes (A_3 \rightarrow \{ A_4^1 \otimes A_5^1,\ A_4^2 \otimes A_5^2 \} - [C_d, C_e]) - [C_a, C_b, C_c]$

This expression represents a product variety structure with three attributes (A_1, A_2, A_3) and three combinations [C_a, C_b, C_c] precluded at the first level, two attributes (A_4, A_5) whose value combinations are determined by the two value selections of attribute A_3, with two combinations [C_d, C_e] precluded at the second level.

Table 3. Notation for the description of a product variety structure

Notation	**Explanation**
{a, b, c, …}	A set of items a, b, c, … of which one is selected.
[x, y, z, …]	A collection of items x, y, z, … of which all are selected.
$A_1 \otimes A_2 \otimes \ldots \otimes A_a$	The set of all possible combinations of values from attributes A_1, A_2 , …,A_a, respectively, where $A_i=\{c_{i1}, c_{i2}, \ldots\}$, i.e., the set of all values of attribute i.
A_i^k	$A_i^k \subset A_i$, that is, A_i^k is the subset k of A_i.
$A_i \rightarrow \{ A_j^1, A_j^2, A_j^3, \ldots \}$	Branching; that is, the choice $\{ A_j^1, A_j^2, A_j^3, \ldots \}$ of the lower level depends on the value selection of attribute A_i of the higher level in a corresponding manner.
C_a	A combination of attribute values from a set of attributes.
$B_k = \{C_a, C_b, \ldots, \phi\}$	An attribute that consists of various combinations (packages) of attribute values from attributes i, j, …, where $C_a = [c_{ia}, c_{ja}, \ldots]$, $C_b = [c_{ib}, c_{jb}, \ldots]$, etc.
$-[C_k, C_e, \ldots]$	Preclusion of attribute value combinations C_k, C_e, …

Appendix B. Product variety structures of seven pickup trucks in the U.S. market based on the customer selection process

Table 4. Product structures represented by the proposed notation

Product	Basic structure information	Level 1	Level 2
1	[2: (4, 45), 49, 4609440]	$\bigotimes_{i=1}^{4} A_i - [C_{11}^1, C_{12}^1, \ldots]$	$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=5}^{6} A_i^1, \bigotimes_{i=5}^{6} A_i^2, \ldots, \bigotimes_{i=5}^{6} A_i^c\}$
			$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=7}^{49} A_i^1, \bigotimes_{i=7}^{49} A_i^2, \ldots, \bigotimes_{i=7}^{49} A_i^c\} - [C_{21}^1, C_{22}^1, \ldots]$
2	[2: (3, 56), 59, 43417360]	$\bigotimes_{i=1}^{3} A_i - [C_{11}^2, C_{12}^2, \ldots]$	$\bigotimes_{i=1}^{3} A_i \rightarrow \{\bigotimes_{i=4}^{5} A_i^1, \bigotimes_{i=4}^{5} A_i^2, \ldots, \bigotimes_{i=4}^{5} A_i^c\}$
			$\bigotimes_{i=1}^{3} A_i \rightarrow \{\bigotimes_{i=6}^{59} A_i^1, \bigotimes_{i=6}^{59} A_i^2, \ldots, \bigotimes_{i=6}^{59} A_i^c\} - [C_{31}^1, C_{32}^1, \ldots]$
3	[2: (6, 40), 46, 908468]	$\bigotimes_{i=1}^{6} A_i - [C_{11}^3, C_{12}^3, \ldots]$	$A_i \rightarrow \{\bigotimes_{i=7}^{8} A_i^1, \bigotimes_{i=7}^{8} A_i^2, \ldots, \bigotimes_{i=7}^{8} A_i^c\} - [C_{21}^3, C_{22}^3, \ldots]$
			$\bigotimes_{i=1}^{6} A_i \rightarrow \{\bigotimes_{i=9}^{46} A_i^1, \bigotimes_{i=9}^{46} A_i^2, \ldots, \bigotimes_{i=9}^{46} A_i^c\} - [C_{31}^3, C_{32}^3, \ldots]$
4	[2: (4, 39), 43, 3571488]	$\bigotimes_{i=1}^{4} A_i - (C_{11}^4, C_{12}^4, \ldots)$	$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=5}^{6} A_i^1, \bigotimes_{i=5}^{6} A_i^2, \ldots, \bigotimes_{i=5}^{6} A_i^c\}$
			$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=7}^{43} A_i^1, \bigotimes_{i=7}^{43} A_i^2, \ldots, \bigotimes_{i=7}^{43} A_i^c\} - [C_{21}^4, C_{22}^4, \ldots]$
5	[2: (6,35), 41, 75174]	$\bigotimes_{i=1}^{6} A_i - [C_{11}^5, C_{12}^5, \ldots]$	$\bigotimes_{i=1}^{2} A_i \rightarrow \{\bigotimes_{i=3}^{4} A_i^1, \bigotimes_{i=3}^{4} A_i^2, \ldots, \bigotimes_{i=3}^{4} A_i^c\} - [C_{21}^5, C_{22}^5, \ldots]$
			$\bigotimes_{i=1}^{6} A_i \rightarrow \{\bigotimes_{i=7}^{41} A_i^1, \bigotimes_{i=7}^{41} A_i^2, \ldots, \bigotimes_{i=7}^{41} A_i^c\} - [C_{31}^5, C_{32}^5, \ldots]$
6	[1: (50), 50, 8266]	$\bigotimes_{i=1}^{2} A_i - [C_{11}^6, C_{12}^6, \ldots]$	
		$\bigotimes_{i=3}^{4} A_i - [C_{21}^6, C_{22}^6, \ldots]$	
		$\bigotimes_{i=5}^{50} A_i - [C_{31}^6, C_{32}^6, \ldots]$	
7	[2: (4, 29), 33, 23940]	$\bigotimes_{i=1}^{4} A_i - [C_{11}^7, C_{12}^7, \ldots]$	$\bigotimes_{i=1}^{4} A_i \rightarrow \{A_5\}$
			$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=6}^{7} A_i^1, \bigotimes_{i=6}^{7} A_i^2, \ldots, \bigotimes_{i=6}^{7} A_i^c\} - [C_{21}^7, C_{22}^7, \ldots]$
			$\bigotimes_{i=1}^{4} A_i \rightarrow \{\bigotimes_{i=7}^{33} A_i^1, \bigotimes_{i=7}^{33} A_i^2, \ldots, \bigotimes_{i=7}^{33} A_i^c\} - [C_{31}^7, C_{32}^7, \ldots]$

Note: 1. In basic structure information, $[L : (N_A^1, \ldots, N_A^L), N_A, N_V]$ is used.

2. $\bigotimes_{i=1}^{n} A_i$ is equivalent to $A_1 \otimes A_2 \ldots \otimes A_n$.

3. c is the number of corresponding attribute value combinations at level 1 in branching to level 2.

4. Some A_i's are actually packages in some cases.

Table 5. Correspondence of attributes at level 1 of each product

No.	Attribute	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6	Product 7
1	A_1	Cab	Cab	Box	Cab	Series	Cab	Drive
2	A_2	Drive	Drive	Drive	Box	Trim level	Series	Cab
3	A_3	Box	Box	Cab	Series	Transmission	Engine	Engine
4	A_4	Series	n/a	Doors	Drive	Engine	Transmission	Transmission
5	A_5	n/a		Engine	n/a	Drive	Drive	n/a
6	A_6			Series		Cab	etc.	

Appendix C. Results for ranking various attributes in the case study

Table 6. Results of ranking attributes in the case study

Attribute No.	1. Number of Attribute Values (N_a)	2. Number of Weighted Unique Parts ($\overline{N_p}$)	3. Impact on Manufacturing Cost (MC)	4. Standard Deviation of the Percentages of Sales of Various Attribute Values	DEA Efficiency Score
1	2	5.574	2	61.943	1.047
2	2	37.617	2	29.840	1.000
3	6	25.180	3	8.965	1.000
4	2	8.262	2	25.032	2.198
5	2	8.459	2	3.253	2.386
6	9	3.607	3	7.237	1.000
7	3	22.951	3	38.727	1.000
8	4	2.623	3	33.578	1.448
9	6	12.459	2	11.959	1.000
10	2	13.377	2	38.042	1.557
11	2	1.311	3	65.337	1.108
12	3	0.656	4	38.727	1.797
13	3	2.623	2	16.848	2.062
14	3	5.410	2	12.065	2.035
15	3	0.328	3	25.586	2.296
16	2	0.033	4	6.081	4.500
17	2	10.557	2	45.679	1.435
18	2	0.033	3	45.396	2.303
19	2	0.328	4	33.658	2.986
20	7	0.885	2	9.592	1.000
21	7	0.164	2	11.230	1.000
22	3	8.852	3	25.173	2.375
23	2	0.492	3	68.024	1.000
24	2	5.475	2	56.710	1.261
25	2	0.656	4	18.526	3.853
26	2	0.295	3	29.698	3.207
27	4	7.607	2	44.406	1.000
28	2	0.164	4	68.307	1.000
29	4	1.443	3	49.311	1.037
30	2	1.639	3	62.933	1.222
31	2	0.016	4	68.024	1.016
32	2	0.984	1	22.486	1.463
33	3	0.656	4	26.212	2.275
34	2	0.164	4	8.485	4.428
35	2	1.967	1	3.536	1.500
36	2	0.656	2	55.013	1.555
37	3	1.967	2	19.785	2.027
38	2	0.164	2	67.599	1.000
39	2	8.623	1	48.083	1.000
40	2	4.787	2	33.375	2.258
41	3	10.656	1	20.438	1.000
42	2	0.328	2	58.690	1.394
43	2	0.656	2	18.385	3.165

*Attributes in alphabetical order are:

Air conditioning	Assist handle	Axle ratio	Bed extender	Bed liner
Body side	Box	Bright appearance Package	Bumper	Door trim
Doors	Drive	Engine size	Entertainment system	Exterior color
Floor covering	Floor mats	Fog lamp	Grill	Instrument panel
Interior color	Leather wrapped steering wheel	Limited slip rear axle	Model	Payload package
Power package	Rear sliding window	Rear stabilizer bar	Seat	Serial 1 package
Serial 2 appearance package	Skid plate	Spare tire	Step bar	Storage tray
Tilt steering wheel	Tire	Tonneau cover	Tow hook	Trailer tow
Transmission	Wheel	Wheel lip molding		

Appendix D. The economic analysis spreadsheet for a complexity reduction effort in the case study

Table 7. A sample economic analysis spreadsheet for a product complexity reduction effort

Item	Current	Future	Difference	Unit cost per year	Total Cost Per year
Cost Changes					
1. Material cost					
1) Materials handling			0		
2) Inventory carrying	$5,433.66	0	$5,433.66		$5,433.66
3) Warehouse storage space	11,140 ft^2	0	11,140 ft^2	$12 per ft^2	$1,604,160
			0		
2. Manufacturing cost	(Values not shown here)				$79,280
3. Distribution cost	(Values not shown here)		9,126 units	$8 per unit	$73,008
4. Part cost			0		0
Other cost changes			0		0
Subtotal 1					$1,761,882
Nominal-Profit Changes					
1. From estimated lost market share	(Values not shown here)				-$1,785,000
2. From a price change in the retained sales			0		0
Other nominal profit change			0		0
Subtotal 2					-$1,785,000
Net Profit Changes (Subtotal 1 + Subtotal 2)					-$23,118

Note: A positive cost change indicates a cost reduction.

Appendix E. Data envelopment analysis (DEA) BCC model formulation

The BCC model (Banker, Charnes, and Cooper, 1984) is an extension of the dual model from the initial fractional form of the CCR model (P1). By adding a convexity constraint (8) to the dual formulation of the CCR model, the BCC model takes account of the input excesses and output shortfalls of the DMUs around the efficient DMUs of the CCR model. The output-oriented BCC model is as follows.

(P2) Minimize $\theta - \varepsilon\left[\sum_{i=1}^{m} s_i^+ + \sum_{r=1}^{t} s_r^-\right]$ (5)

Subject to

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^+ = x_{ij_0}, \quad i = 1, 2, \ldots, m, \tag{6}$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^- = \theta y_{rj_0}, \quad r = 1, 2, \ldots, t, \tag{7}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{8}$$

$$\lambda_j, s_i^+, s_r^- \geq 0. \tag{9}$$

In the above formulation, θ represents the efficiency score of DMU j_0; s_i^+ and s_r^- represent the slack variable of input i and output r, respectively. The convexity constraint (8) requires the sum of multipliers λ_j to be 1; and this ensures that this BCC model gives an attainable composite unit of similar scale size as that of unit j_0.

PART 3

EXTENDED MODELS AND A PRACTICAL TOOL TO ANALYZE PRODUCT COMPLEXITY RELATED TO PRODUCT VARIETY FOR AN AUTOMOBILE ASSEMBLY PLANT

This part is a paper submitted to the journal *Supply Chain Management: An International Journal* in 2005 by Hui Sun and Fong-Yuen Ding:

Sun H., and Ding, F. (2005) Extended Models and a Practical Tool to Analyze Product Complexity Related to Product Variety for an Automobile Assembly Plant. *Supply Chain Management: An International Journal*.

My primary contribution to this paper include (1) most of gathering and interpretation of literature, (2) development of extended data envelopment analysis models, (3) development of a computational tool using Visual Basic and Microsoft Access for the cost estimation associated with a product complexity change, (4) data collection and calculation for the case study, and (5) part of draft writing and editing.

1. Abstract

In this paper, two extended data envelopment analysis models are developed to compare product complexity levels of similar products of multiple automobile manufacturing firms. The numbers of attribute values of some major product attributes that have significant cost impact on the manufacturing system and its supply chain are included as inputs in the proposed models. This benchmarking effort can help automobile manufacturers evaluate their product complexity levels in comparison to competitors and motivate improvement in managing product complexity. Furthermore, an incremental approach is presented to estimate the cost change associated with a certain product complexity change. A computational tool can be developed to apply this approach. By applying the above approach and tool, a firm can estimate the cost impact associated with a certain product complexity change to aid decision making in this area by considering costs and market impact. A case study that applies the extended models and cost estimating tool at a U.S. automobile assembly plant is also presented in this paper.

2. Introduction

Product complexity is mainly resulted from product design and marketing influence, and can have a significant impact on the manufacturing system and supply chain. Providing product complexity within a product can lead to higher costs in manufacturing and the supply chain. A certain level of product complexity within a product is necessary in a competitive market to meet customer demand and win a market share. Nevertheless, increasingly more automobile manufacturing companies are considering the cost and market impact aspects in setting product complexity. It is important but generally difficult to evaluate the tradeoffs between winnning the market share and having high costs from product complexity. The purpose of this paper is to provide some useful benchmarking models and a cost estimating tool for analyzing product complexity in the automobile industry.

In this paper, product complexity refers to the extent of complexity associated with a single product. It involves all factors that make a product complex, whether directly selectable or not by customers. A closely-related term "*product variety*" used in this paper refers to the product complexity factors that are selectable to customers. For example, the product complexity factors of a car include exterior color, body style, and engine size, that are selectable by customers; and the product complexity factors also include factors such as different kinds of wire harnesses, bolts and nuts, that are not customer selectable. The scope of this paper is limited to product complexity related to product variety.

3. Relevant Literature

A product can be specified by a collection of various attributes with multiple selections that can be termed as *attribute values* (Ding et al., 2005); for example, a car that allows the selection of various attribute values of different engines; a computer that allows selection of various attribute values of CPUs. Product complexity related to product variety of a manufacturing firm can be attributed to many factors. Ramdas (2003) pointed out that product variety stems from differences in both physical product features, and augmenting product features such as brands, packaging, and marketing channels. One or more measures have been considered in measuring product variety within a company; e.g., the number of product variants within a specific product group (Lancaster, 1990), the number of attributes, the number of attribute values and the number of end items (Ulrich et al., 1998). The impact of product variety on the production system in the automobile industry was studied by MacDuffie et al. (1996) and Fisher and Ittner (1999). Regression analyses were performed to show that different measures of product variety could have significantly negative impact on automobile assembly operations or total labor productivity. In these analyses, product-complexity measures including model-mix complexity, parts complexity, option content, and option variability were considered.

Some approaches of estimating the cost impact associated with product complexity or product variety can be found in the literature. Considering the cost impact of product complexity on manufacturing overhead costs, Banker et al. (1990) developed linear regression models that can be used to estimate the absolute unit-overhead costs in three categories: supervision costs, indirect quality control and inspection costs, and tool maintenance overhead costs. In addition to direct labor and machine hours that are included in the conventional overhead-cost allocation method, six other product- and process-complexity factors are identified and also included in the regression models as independent variables. The regression results indicated that these product-complexity and process-complexity factors are indispensable in explaining the overhead costs.

To select new product lines that can result in maximal incremental profits, Ramdas and Sawhney (2001) developed a linear mixed-integer programming model considering the incremental revenues and costs from introducing new product lines. It was stated that activity-based costing (ABC) was applied to estimate the life-cycle costs of new products or components in terms of new product development costs and life-cycle support costs. Only the development cost of the product with the highest development intensity needs to be computed while the development costs of other products can be estimated by scaling down the calculated cost using their intensity levels.

This research is motivated by a major automobile manufacturer in the U.S. The company is interested in better understanding its product complexity. They desire to know the complexity levels of their products in comparison to similar products of competitors in the U.S. market as a benchmarking effort can provide useful insight in this area. They also desire to know the cost impact of product complexity. Based on these interests of this automobile manufacturer and an existing methodology proposed by Ding et al. (2005), two extended models for an automobile plant and a practical tool are developed and presented in this paper. Data envelopment analysis (DEA) is applied to compare levels of product complexity related to product variety among similar vehicles in the same market. A cost estimation approach for a product complexity change is presented. A case study conducted at an assembly plant of this automobile manufacturer for applying these extended models and tool is presented.

4. Comparison of Product Complexity Related to Product Variety among Various Automobile Manufacturers

While making decisions on product complexity related to product variety in a competitive environment, a manufacturing company may be interested in knowing its relative product complexity level in comparison to their competitors. To accomplish benchmarking on the complexity levels, a viable analytical tool is the data envelopment analysis (DEA), a linear programming based tool that can be applied to measure relative efficiencies of a set of homogenous *decision making units* (DMUs) with multiple inputs and outputs. In a DEA analysis, each product in the similar market sector can be considered as a DMU. The most favorable weights are assigned to the input and output factors by solving the corresponding linear programming model so that a most favorable evaluation, in the form of efficiency score, can be given to each DMU. By performing the data envelopment analysis, a decision making unit can be classified into efficient or inefficient. Furthermore, improvement targets for an inefficient DMU can be computed based on the solutions of the DEA models. A review of DEA was presented by Boussofiane et al. (1991). A data envelopment analysis model known as the BCC model (Banker, Charnes, and Cooper, 1984), which was extended from the dual formulation of the original CCR model (Charnes et al., 1978), is one of the applicable formulations. This model formulation (output-oriented) is as follows:

$$\text{Minimize} \qquad \theta - \varepsilon\left[\sum_{i=1}^{m} s_i^{+} + \sum_{r=1}^{t} s_r^{-}\right] \tag{1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^+ = x_{ij_0}, \quad \text{i} = 1, 2, \ldots, \text{m}, \tag{2}$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^- = \theta y_{rj_0}, \quad \text{r} = 1, 2, \ldots, \text{t}, \tag{3}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{4}$$

$$\lambda_j, s_i^+, s_r^- \geq 0, \tag{5}$$

where x_{ij} is a parameter for the amount of input i from DMU j, y_{rj} a parameter for the amount of output r from DMU j; θ is a decision variable representing the efficiency score for DMU j_0; λ_j is a decision variable representing the multiplier for DMU j; ε is a constant of a very small positive number; and s_i^+ and s_r^- are slack variables with respect to input i and output r, respectively.

Ding et al. (2005) applied the data envelopment analysis in comparing the relative product complexity levels of similar products in the same market and in attempting to identify product complexity reduction opportunities. Two DEA models including their respective illustrative models were proposed for a comparison of product complexity related to product variety. These illustrative models have been developed for a product of any industry in general. One of the input factors is the weighted average number of attribute values. In this paper, two DEA models extended from these illustrative DEA models are proposed for automobile manufacturing applications to include numbers of attribute values of some attributes that have a significant cost impact in lieu of the weighted average number of attribute values.

In order to compare product complexity among various automobile manufacturing firms or evaluate its impact on automobile manufacturing and its supply chain, numbers of attribute values of certain significant product attributes such as numbers of engines, transmissions, and trim levels and exterior colors have been used (MacDuffie et al., 1996; Pil and Holweg, 2004) in representing product complexity. A DEA model can be constructed to include as input factors the numbers of attribute values of such significant product attributes. Other important product complexity factors can also be considered unless such information is unattainable; for example, the total number of parts and modules on a vehicle is usually not easily attainable.

Two extended DEA models, Models A and B, that include as input factors the numbers of attribute values of significant automobile product attributes can be used in comparing the product complexity levels of similar products among automobile manufacturing companies in the same market. Model A (see Figure 1) attempts to compare the complexity levels of multiple vehicles. Specifically, the inputs of Model A consist of the numbers of bodies, power trains, paint-and-trim combinations, which are commonly-considered major attributes in describing automobile product complexity, and the number of options. The number of product variants is considered as the output since it is affected by the input factors. In this model, a vehicle with low numbers of variants, bodies, power trains, paint-and-trim combinations, and options is considered as an efficient product from the viewpoint of product complexity related to product variety. Since a high number of product variants is undesirable by an efficient DMU in Model A, this output factor is an *undesirable* output. To solve this DEA model with an undesirable output, an appropriate transformation (Scheel, 2001) of the output factor is needed.

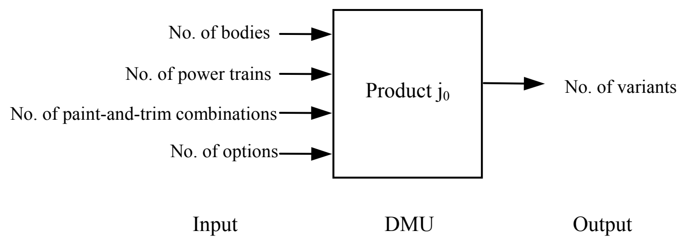

Figure 1. Model A for complexity comparison

In comparison to the DEA illustrative model 1 proposed by Ding et al. (2005), the numbers of significant product-complexity factors including the numbers of bodies, power trains, paint-and-trim combinations, and options take the place of the weighted number of attribute values used in the illustrative Model 1 by Ding et al. This eliminates the requirement of assigning weights to calculate the weighted average number of attribute values. Another advantage of Model A is to have specific improvement targets for the number of attribute values for the considered major attributes when calculating the improvement targets in DEA. It is noted that a different set of product attributes other than body, power train, and color-trim combination may be used depending on system considerations.

Model B depicted in Figure 2 attempts to compare the product complexity levels in conjunction with the sales volume. In Model B, inputs include the number of bodies, number of power trains, number of paint-and-trim combinations, number of options, and number of product variants, and the output is *sales volume*, which can be considered as the economic output of the system. In this model, a vehicle with high sales volume, low numbers of bodies, power trains, paint-and-trim combinations, options, and product variants is thus considered as efficient from the viewpoint of effectively offering a certain level of product complexity related to product variety in the market. Depending on the

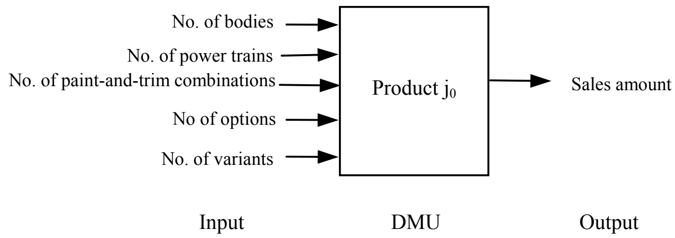

Figure 2. Model B for complexity comparison

comparison objective, Model A or Model B can be applied accordingly. When the comparison is focused on product complexity factors related to product variety, Model A can be applied; when the comparison considering the economic effect of a product with a certain complexity level, Model B can be applied.

It is noted that although benchmarking with other similar products in the same market using DEA provides a comparison of the relative product complexity and improvement directions associated with the considered complexity factors, it does not provide a cause-effect relationship, e.g., in how to increase sales or in economic justification for a certain action to be taken. Thus, a cost analysis associated with a product complexity action can be further applied by estimating the cost impact in order to facilitate decision making in a complexity reduction action. This is addressed in the following section.

5. Estimating Cost-change Associated with a Complexity Change

It is generally not possible to calculate the absolute cost related to a certain product complexity factor, e.g., the cost of offering 20 wire harnesses, or the cost of offering 7 exterior colors. This is due to the fact that many cost items associated with multiple attributes are blended together and not practically separable for individual attribute values. In this paper, we propose to estimate the *cost change* from increasing or decreasing the complexity level of a certain complexity factor to provide input for managing product complexity. To calculate the cost change associated with a certain complexity change, an approach is to estimate the cost impact on various departments, or various categories of production activities. Moreover, a change in product complexity related to product variety usually affects the associated costs for a set of *parts* in various departments, or categories of production activities. At a U.S. automobile manufacturing plant, for example, most of cost areas related to manufacturing and the supply chain management are affected by product complexity change in terms of parts as shown in Table 1. Thus, to calculate the total cost difference associated with a complexity change, one can usually consider the cost changes of the affected parts for each department or each category of production activities. However, if a cost area is not affected directly by parts, it can also be calculated according to the appropriate factor.

A cost-estimation procedure can be depicted in Figure 3. The cost changes in most cost areas or activity categories can be calculated by considering every cost element of each affected *part* due to a complexity change, and then the total cost change in all

Table 1. Cost base for various cost areas or production activities at an automobile plant

Cost Area or Production Activity Category	**Cost base**
Inventory control	Parts
Material storage	Parts
Material handling	Parts
Parts ordering	Parts
BOM maintenance	Number of part numbers
Misbuilt parts repair	Parts
Inbound transportation	Parts
Production scheduling	Number of buildable vehicles

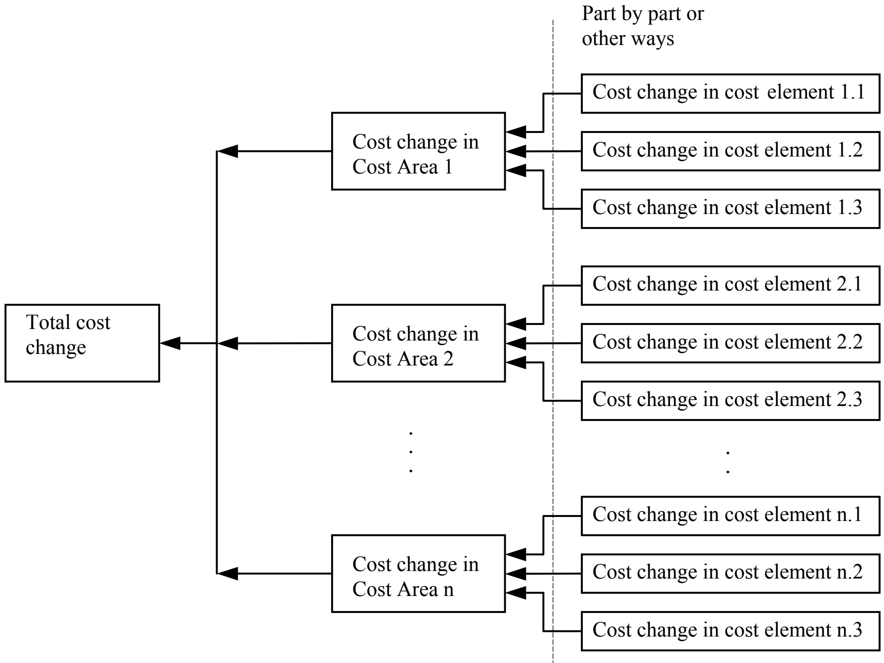

Figure 3. Computation of cost change associated with a product complexity change

cost areas can be obtained. For example, the cost items of a certain part affected by a certain complexity change can include the inventory-control cost area that has the cost elements of inventory carrying, cycle count, obsolescence, and part-loss costs. As another example, due to much part variety, a wire harness can be misassembled to incur a retrofit cost after inspection or a warranty-claim cost after sales as various possible cost elements of misbuilt parts; and each of these cost elements could be estimated for each affected part. Since a great deal of part data are needed in such calculation and a significant amount of part data can be retrieved from the company database, a computational tool can be developed to make use of available data and to automate the calculation.

The associated cost areas or parameters in cost calculation would generally be different from part to part and from case to case. This necessitates calculation for various categories of production activities for the affected parts. In some cases, the total demands before and after the complexity change may be different, and affect the cost-change calculation. In making a product-complexity related decision, using the cost-change estimates in conjunction with the market impact factors including a potential profit change and demand change can be carefully considered.

In comparison, Banker et al. (1990) used regression models to estimate overhead costs that were dependent on various factors for measuring the extent of complexity of multiple products instead of the product variety within a product as considered in this research. The overhead costs in three categories (supervision, indirect quality control and inspection, and tool maintenance) were allocated directly to individual products by interviewing with supervisors and inspectors or collecting labor costs of repairing each tool associated with each part. Using an incremental cost estimation approach in selecting production lines was used by Ramdas and Sawhney (2001). The activity-based costing (ABC) was applied for the cost calculation of the product (or component) with the highest development intensity. The costs of other products (or components) were estimated by scaling down the calculated cost by their intensity levels.

6. A Case Study

A major automobile assembly plant in the U.S. is interested in conducting a complexity study for benchmarking and for estimating the complexity costs. This plant has two assembly lines for the daily production of multiple vehicle lines. The plant consists of a stamping, body, painting department, and a final assembly department, which has a trim and chassis line. To support these production departments, there are various departments including inventory control, transportation, material handling, parts ordering, and scheduling to provide needed support in various production activity areas. The plant has many domestic and overseas suppliers for parts used on the assembly line. Parts from domestic suppliers are delivered to the plant as frequent as 4 times a day. Parts are delivered to the assembly stations in three different ways: some parts are sequenced by suppliers in a predetermined sequence of vehicle assembly order and delivered directly to the assembly line; some parts are sequenced in the plant warehouse and then delivered to the assembly line; and other parts are delivered to the assembly line in batches from the plant warehouse.

The plant is interested in knowing its relative product complexity level in comparison to its competitors. The product complexity levels of the best practice competitor may become a good reference for this plant in product complexity. To perform a benchmarking study, the DEA Models A and B for an automobile manufacturer presented in Section 3 are applied. Twelve full-size cars manufactured by 12 companies and sold in North America are considered as 12 decision making units. The values for product-complexity input and output factors are collected or calculated from the website information of these companies. The DEA scores and ranking results from the two DEA models are given in Table 2. The total number of variants is the number of buildable cars, which is calculated based on the number of options and exclusions shown on the websites. The sales volumes of a recent month of these cars were used as the sales volumes used in DEA Model B. The number of options in the computation is the number of individually selectable options other than bodies, power

Table 2. Data envelopment analysis for comparison of product complexity of full size cars in the U.S. market

Car	No. of bodies	No. of power trains	No. of paint-and-trim combinations	No. of options	Total number of variants	Sales volume	**DEA Model A score**	**DEA Model B score**
1	**1**	**6**	**91**	**36**	**28,200**	**18,819**	**0.0010**	**0.977**
2	3	4	128	23	202,880	35,887	0.0001	1.000
3	1	1	92	16	19,136	2,637	0.0014	1.000
4	2	5	75	18	192,768	27,489	0.0001	0.892
5	2	7	68	31	33,344	7,941	0.0008	0.484
6	1	3	51	17	9,232	22,939	0.0029	1.000
7	2	2	30	14	14,696	23,030	0.0018	1.000
8	2	3	38	20	91,932	16,710	0.0003	0.716
9	1	2	43	10	1,464	3,102	1.000	1.000
10	2	4	56	16	645,120	5,940	0.00004	0.242
11	1	1	18	11	27	1,681	1.0000	1.000
12	1	3	33	21	264	8,589	0.1023	1.000

trains, and paint-and-trim. Also, individual options within a package are included in the counting of the number of options. In Table 2, the identities of the compared automobile companies are not shown.

For Model A, an output-oriented BCC model is employed after applying an additive transformation on the undesirable output, the number of variants, and the efficiency scores are represented by output-oriented efficiency measures proposed by Scheel (2001); for Model B, due to the fact that it is relatively difficult to increase the sales volume, the input-oriented BCC model is applied and the efficiency scores calculated.

According to the results from DEA Model A, 2 cars are classified as efficient; while in Model B, 7 cars are classified as efficient. This difference in ranking and efficiency classification is due to the different models and objectives in Models A and B. Model A focuses on a comparison of product complexity factors related to product variety, while sales volume is incorporated in the comparison of product complexity in

Model B. It can be seen that the efficiency score of the car (car no. 1 in bold face) of the automobile company that initiated this study is 0.001 in Model A, in which only product complexity factors are considered; while its efficiency score is 0.997 in Model B, in which product complexity factors are considered in contrast to the sales volume. Even though car no. 1 is ranked to be relatively complex in Model A as compared to other cars, it is ranked close to being efficient in Model B due to its relatively high sales volume. In Model B, car no. 6 is the only vehicle included in the reference set of car no. 1, and it provides improvement targets from the input-oriented DEA Model B for car no. 1. Specifically, the improvement targets are to decrease the number of options to 17, the number of paint-and-trim combinations to 51, the number of power trains to 3, the number of variants to 9,232, and increase the sales volume to 22,939. Furthermore, the company practice of car no. 6 in manufacturing and the supply chain, and managing product complexity can be further studied.

By performing this complexity analysis, the company that initiated the study gained a better understanding of its relative product complexity. Based on the analysis, even though the company has a relatively complex product, with a relatively high sales volume, it can be considered economically close to being efficient; moreover, the analysis enabled the company to identify that car no. 6 and its company as a benchmark for improvement purposes. An interesting practice of the manufacturer of car no. 4 came to the company's attention; that is, the manufacturer of car no. 4 had most of its options installed at dealers instead of on its assembly line. Car no. 4 had a good sales volume and would have been ranked as efficient in both Models A and B if the number of options installed at the dealers were excluded in the DEA models.

The plant also desires to know the cost impact of a certain product complexity factor on its manufacturing system and the supply chain. This understanding can be used in managing product complexity in conjunction with market considerations. The cost areas related to product-complexity at this plant include categories of production activities in inventory control, material handling, material storage, inbound transportation, misassembled-part repair, bill of materials (BOM) maintenance, parts ordering, and production scheduling.

Regarding the cost-change estimation associated with a product complexity change, a computational tool using Visual Basic (VB) 6.0 and Microsoft Access was developed. The Access database contains part information downloaded from the plant database for use in the cost-change estimation. VB uses three major forms for the cost change computation process. To launch a new project, a user enters on the first VB form basic project information including the part numbers used before and after the complexity change. Each time a part number is entered, the data associated with this part are extracted from the Access database and placed in the cost-change computation tables of each cost area considered in this study, and the data can be further modified by the user on the second form. After the costs associated with each cost area are calculated in the program, the user can view the cost-change calculation results and other related information on the third VB form. These three major VB forms are included in the Appendix.

Using the VB computation tool, the company conducted complexity studies including one that estimates the cost impact of a complexity reduction effort for engine wire harnesses. In this study, 29 and 23 parts are used before and after the complexity change, respectively. All parts belong to the same commodity, and are delivered in sequence from the supplier. The cost-change estimation results indicated that this product complexity reduction effort would incur an annual cost saving. The company is considering this product complexity change option. There is no market impact in this product complexity change option.

In this case study, using the computational tool is analogous to a simulation run in this regard. Through the cost estimation using this computational tool, it can be seen that with more part numbers, there are generally more safety stocks, more floor space, cycle counts, part losses, and misassembled parts if the total demand remains unchanged. This in turn results in a higher cost in cost areas including inventory control, material storage, and misassembled-part repair. In the cost areas of parts ordering and BOM maintenance, more part numbers also lead to a higher cost for these areas roughly proportionally. Such observations would likely be different from plant to plant and from case to case; and a cost-change estimation needs to be carefully compared to the market impact.

7. Summary and Conclusions

In this paper, two extended DEA models and a tool for analyzing product complexity related to product variety are proposed to evaluate relative product complexity and to consider cost impact on a manufacturing system and the supply chain. The two extended DEA models are for the comparison of product complexity levels among similar automobiles in the same market. The numbers of several attribute values of product attributes that have a significant impact on automobile manufacturing and the supply chain are included as inputs in these two DEA models. An incremental cost estimation approach is proposed to calculate the cost change associated with a product complexity change. This approach attempts to estimate the cost impact of a product complexity change on various departments or categories of production activities. The estimated total cost impact of the affected parts by a product complexity change can be used as valuable input in understanding tradeoffs in product complexity.

A case study at a U.S. automobile assembly plant using the proposed models and tool is presented. The case study demonstrated that applying these extended models and cost estimation tool can provide insight in better understanding product complexity in a company and help make better decisions regarding product complexity related to product variety. A decision process in product complexity for a firm should incorporate considerations in complexity costs and market impact, while being aided by an understanding of its relative product complexity and related practice in the industry.

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Appendix A. Visual Basic forms for the case study

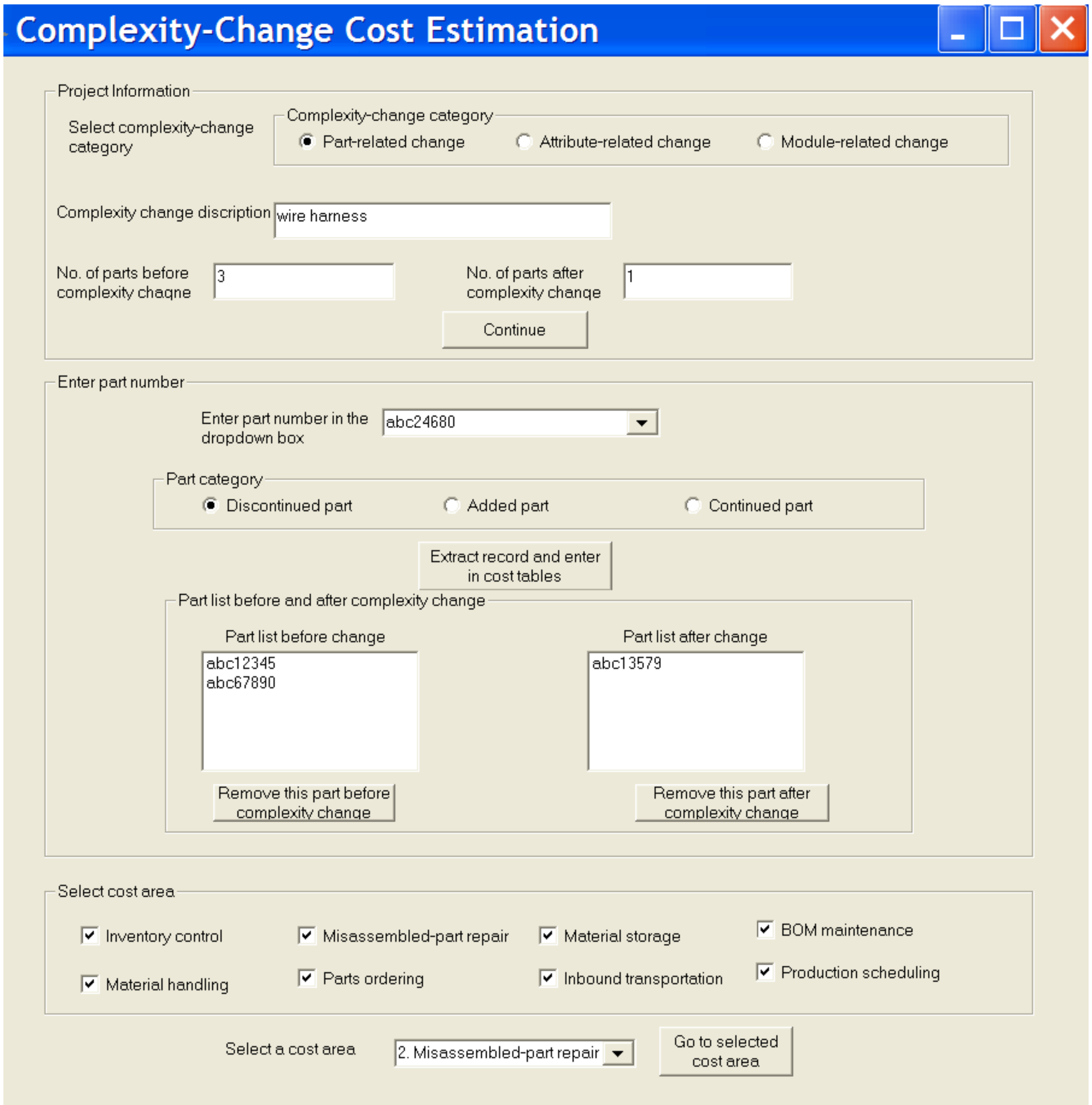

Figure 4. Visual Basic form 1

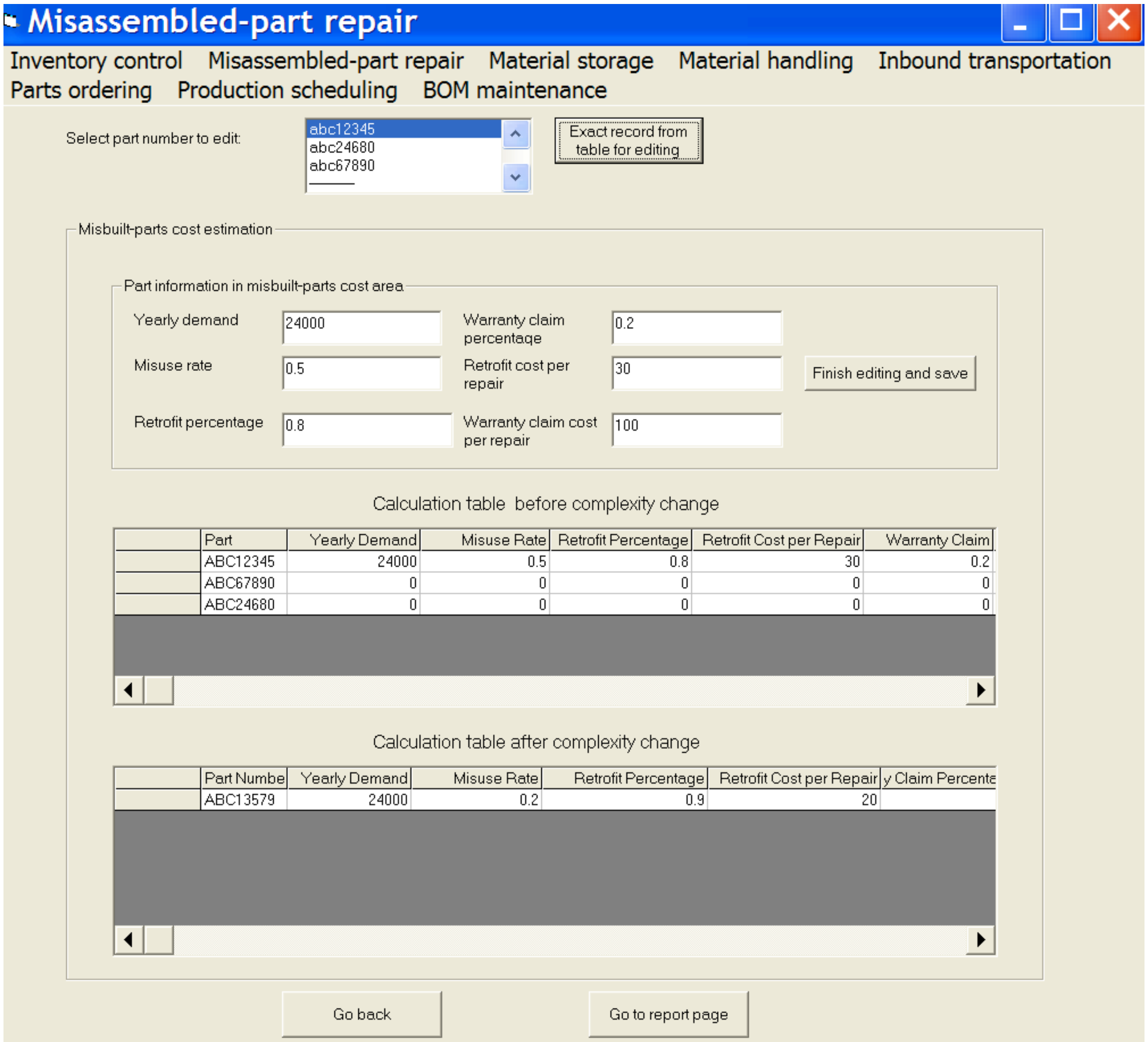

Figure 5. Visual Basic form 2

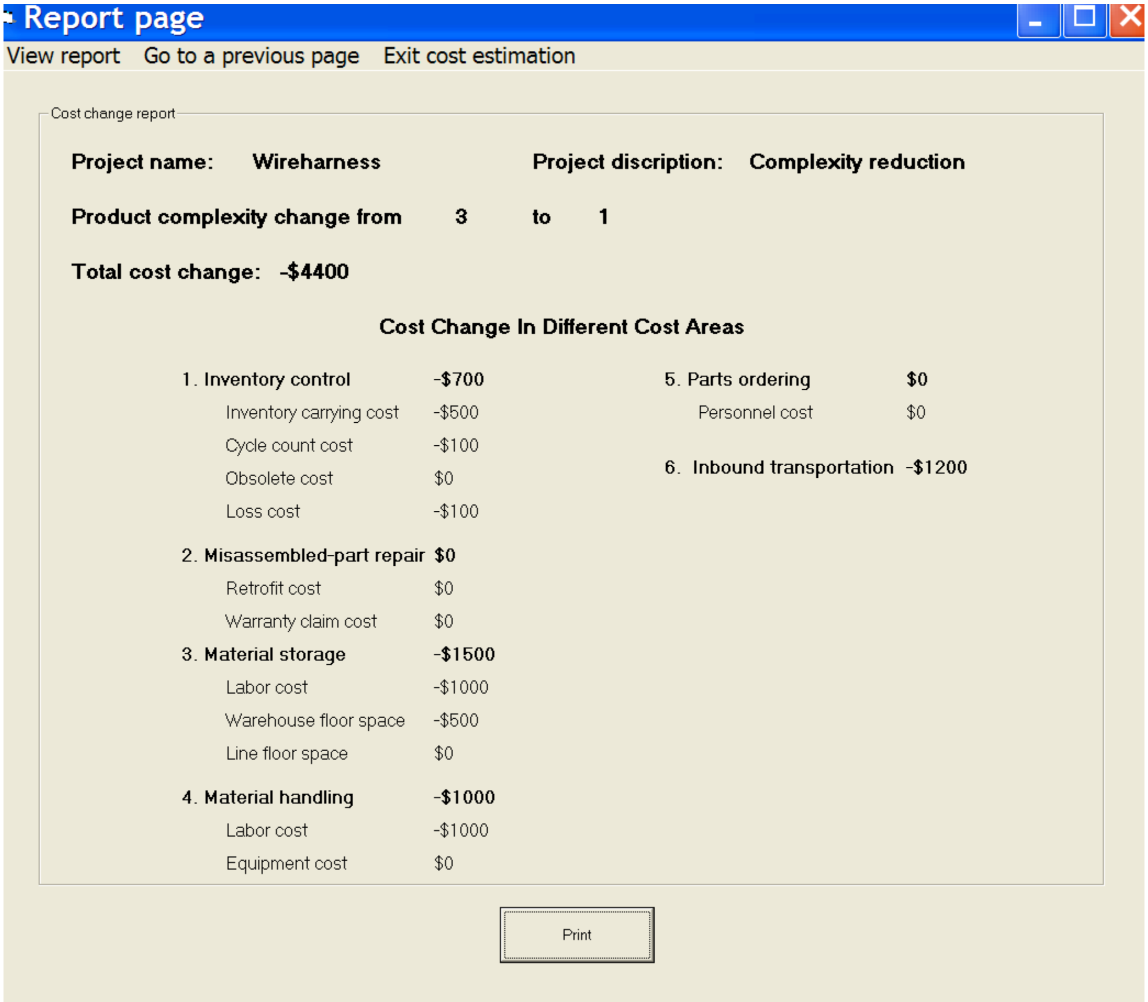

Figure 6. Visual Basic form 3

SUMMARY AND CONCLUSIONS

Sequencing mixed-model assembly lines is complicated by significant product complexity in the automobile industry. In this regard, this research presented some rules and methods to deal with intentional and unintentional sequence alterations considering production requirements of the downstream department and sequenced parts delivery. A rolling sequencing method is applied in conjunction with the developed placing and stacking rules to enable placing and releasing vehicles evenly for shuffling lines to prevent blocking. Further developed heuristic rules can increase the average block size using sorting lines when a larger block size is desirable by a downstream department. The spare units system developed in this research can be applied to restore unintentionally altered sequence due to defective units to the original order to facilitate sequenced parts delivery. The number of spare units needed by this spare units system can be estimated using the developed queuing model for the repair process of defective units in the system. Compared to a reservoir system, a spare units system generally needs a smaller inventory at a given sequence-consistency level.

High product complexity generally has a negative impact on a manufacturing system and the supply chain. Effectively managing product complexity can help improve the production system performance. This research also focuses on studying product complexity related to product variety regarding its impact on manufacturing and the supply chain system. To compare product complexity levels of similar products in the same market, data envelopment analysis (DEA) is applied by considering multiple factors related to product complexity to classify similar products treated as multiple decision making units (DMUs). The DEA analysis can provide a better understanding of the relative product complexity level of a manufacturing firm in relation to its competitors. Improvement targets for an inefficient DMU can also be obtained from its reference set from the DEA results. DEA can also be applied to identify product complexity reduction opportunities as product attributes are considered for product complexity reduction. A further incremental economic analysis considering the changes in costs, sales volume, and/or price affected by a complexity change can be performed to justify the identified opportunities identified by the third DEA model.

Two extended DEA models were presented specifically for the comparison of the product complexity levels of similar automobiles. Significant product features that have significant cost impact on automobile manufacturing and the supply chain are included in the extended DEA models. The cost impact of product complexity on an automobile manufacturing system and supply chain can be estimated by applying an incremental cost estimation approach by estimating various categories of production activities. A computational tool can aide in conducting repetitive cost estimations.

The models and methods presented in this research were developed for improving the assembly operations and the supply chain system of a manufacturing firm while the major focus has been on applications in automobile assembly plants. A case study was included in each part of the dissertation to provide some empirical examples regarding how these methods and models could be applied to real production systems.

Possible future research can be conducted in the following areas: 1) Developing a mixed-model assembly sequencing algorithm which uses various sequences to address different requirements by various departments on the assembly line. This may eliminate the requirement of intentional sequence alterations. 2) Improving the heuristic sorting rules. This may further improve color blocking since there exists an improvement opportunity between the average block sizes using the heuristic rules and the optimal block size. 3) Considering other significant product complexity factors in DEA. For example, the number of parts was not used due to lack of such data, but could be used in DEA. 4) Analyzing the cost impact of a product complexity change associated with multiple commodities of parts. Currently the computational tool developed in Part 3 implicitly assumes that the parts of the same commodity are considered even though multiple commodities of parts are allowed.

VITA

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