5-2004

# Dynamic Left-turn Phase Optimization Using Fuzzy Logic Control

Zhenyang Li
*University of Tennessee - Knoxville*

To the Graduate Council:

I am submitting herewith a dissertation written by Zhenyang Li entitled "Dynamic Left-turn Phase Optimization Using Fuzzy Logic Control." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Civil Engineering.

Lee D. Han, Major Professor

We have read this dissertation and recommend its acceptance:

Arun Chatterjee, Frederick J. Wegmann, Yueh-er Kuo

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

To the Graduate Council:

I am submitting herewith a dissertation written by Zhenyang Li entitled "Dynamic Left-turn Phase Optimization Using Fuzzy Logic Control". I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Civil Engineering.

Lee D. Han
Major Professor

We have read this dissertation
and recommend its acceptance:

Arun Chatterjee

Frederick J. Wegmann

Yueh-er Kuo

Acceptance for the Council:

Anne Mayhew
Vice Chancellor and
Dean of Graduate Studies

(Original signatures are on file with official student records)

# Dynamic Left-turn Phase Optimization Using Fuzzy Logic Control

A Dissertation
Presented for the
Doctor of Philosophy Degree
The University of Tennessee, Knoxville

Zhenyang Li
May 2004

# Dedication

Dedicated to my Family
for their love and support.

## Acknowledgments

First, I would like to thank my major advisor, Dr. Lee Han, for giving me the opportunity of pursuing a Ph.D degree. He has not only inspired and encouraged me to dig deeper into the transportation field, but also guided and supported me on every single step of this endeavor. No word can convey my gratefulness toward Dr. Han for his patience, friendship, and for always been there for me.

I am very grateful to Dr. Frederick Wegmann, Dr. Arun Chatterjee for their continuous friendship and encouragement. Without their help, support, and encouragement, I would never have been able to be here, let alone finishing this work. I also give my special appreciation to Dr. Yueh-er Kuo for serving on my committee, and for her invaluable advice.

My thanks also go to my friends Fang Yuan and Zhongren Gu for interesting discussions and being fun to be with.

Last, but not the least, my greatest appreciation goes to my beloved wife Dr. Hui Wang, for always standing beside me and sharing her experience with me, and having faith in me.

## Abstract

The left-turn movement at an intersection has long been a concern of traffic engineers as it is a major capacity reduction factor. Different left-turn signal phasings have been shown to result in significant differences in delay, intersection capacity, and even safety level.

First, past studies about leading and lagging signal phases and signal control application are overviewed. Then this research gives a theoretical analysis of signal left-turn phase operations at both isolated and coordinated signalized intersections, compares the difference in delay based on leading and lagging left-turn signal phase designs, analyzes the influences of traffic control delay components for leading and lagging left-turn, identifies the main control factors, and gives a new model to guide the choosing between the leading and lagging left-turn phases.

In the third part of this research, some basic mathematical definitions and rules of fuzzy logic control are described. A four-level fuzzy logic control model is designed. To implement this control model, observed approaching traffic flows are used to estimate relative traffic intensities in the competing approaches. These traffic intensities are then used to determine whether a leading or lagging signal phase should be selected or terminated.

Finally, this research designs a dynamic traffic signal left-turn phase control system, and implements the four-level fuzzy logic control model to optimize signalized intersection operation. The performance of this dynamic traffic signal left-turn phase fuzzy logic control system compared favorably in all categories to fixed time control, actuated control, and traditional fuzzy control based on simulation using field data. The results suggest that the proposed dynamic traffic signal left-turn phase fuzzy logic control system is a superior and efficient tool for reducing intersection traffic delay. The study also demonstrated that the successful implementation of the proposed model does not rely on the installation of expensive or complicated equipment.

# Table of Contents

# List of Tables

## List of Figures

# Chapter 1 Introduction

## 1.1 Background

Traffic signals are intended to offer logical and reasonable traffic control at intersections. When properly timed, signals can improve safety and efficiency of both pedestrian and vehicular traffic. However, unjustified and ill-timed signals can cause excessive delay as well as safety concerns. Traffic engineers have long been concerned about left-turning phase at intersections, since it is one of the major reasons that reduce the traffic capacity of an intersection. Different left-turn phase arrangements in one signal cycle can result in significant differences of delay and safety level.

The two basic categories of left-turn signal phase are Leading and Lagging, determined by whether the left-turning traffic is leading the through traffic on the same approach or the other way around. Leading signal phase can be further classified into protected-leading and protected-permitted-leading designs, while lagging signal phase include protected-lagging and permitted-protected-lagging designs. Listed below are brief descriptions of these four designs:

- **Protected-leading**: a designated green phase is given to left-turn vehicles before the through traffic movements. And then left turning movements are prohibited when through traffic gets its green time.

- **Protected-lagging**: left turn is prohibited when through traffic gets its green time. And then a designated green phase is given to left-turn vehicles when the through traffic gets its red time.
- **Protected-permitted-leading**: protected left turn phase is given right before the green phase for through traffic, during which left turning movements are permitted when gaps are available in the opposing through traffic.
- **Permitted-protected-lagging**: protected left turn phase is given right after the green phase for through traffic, during which left turning movements are permitted when gaps are available in the opposing through traffic.

Under heavy through traffic flow where gaps are rare for left turn movement during the green phase for through traffic, a protected-permitted phasing becomes similar, if not equal, to a protected left-turn phasing design. Most agencies prefer the use of leading left-turn phasing since the belief is that driver expectancy weighs heavily in favor of leading left-turns. Some agencies use lagging left-turn phasing when left-turns and opposing through traffic are light in favor of permitted-protected operation.

In real world, these phases are controlled by either fixed-time (pre-timed) model and/or actuated model. The fixed-time model is based on pre-set signal timings and is, therefore, non-responsive to real-time fluctuations in traffic demand. The actuated model presents an improvement over pre-timed by detecting traffic demand and thus modifying the signal timing, but its ability of adjusting to traffic pattern fluctuation is limited by its design. For an intersection with actuated control, performance generally deteriorates with heavy traffic conditions and the proportion of stopped vehicles is generally high. Some adaptive fuzzy controllers are designed to address these deficiencies, as they have the ability to make real-time adjustments to signal settings in response to both observed and/or predicted real-time traffic demands.

## 1.2 Problem Definition

When leading left-turn phase is used, the decision to run the protected portion of the phase must be based on an educated guess of how many left turns can be accommodated during the following permissive period. In the other hand, when using lagging left-turn phase, most of this guesswork is no longer necessary. However, it still needs to determine at the end of the permissive period whether additional vehicles are waiting and will not be able to complete their turns during the change-and-clearance interval.

The decision to run the protected left turn phase depends upon more than estimating the number of vehicles that will be able to turn left permissively, it must also consider factors affecting the overall operational efficiency of the intersection. These factors include the demands of traffic for all movements and the amount of time that the signal controller allows for the available phases serving these movements. Factors reward permitted left-turn, in addition to traffic demand, include cycle length, offset, green time, the arrival time of platoons, platoon dispersion, and an individual driver's gap acceptance criteria, which can be influenced by driver frustration. With the onset of actuated signal phases and adaptive traffic control, many of these factors can vary from cycle to cycle.

To address the left-turn signal problem, some researchers began to analyze the differences between leading and lagging signal phases at isolated intersections about a dozen of years ago. Some of the results encourage leading left-turn, while the others prefer lagging. Since past studies on this matter have focused mainly on the signal phasing design aspect of leading and lagging left-turn at isolated intersections, the findings of these studies are often not transferable to more generalized situations where multiple intersections within a close proximity often interact, if not interfere, with one another.

The dynamic signal left-turn phases control process deals with a complex multi-objective and multi-constraint problem in which the optimization performed is based mainly on recent information. It relies on the fact that it must be repeated with very short time intervals. However, when the intersection is complex in terms of geometric design, channelization and types of vehicles to be handled, the control process must consider many usually mutually conflicting objectives. The control detectors sometimes cannot capture the details of prevailing conditions on the approaches (not as good as a human), so traffic conditions in the immediate future cannot be predicted. Our ability to make precise and yet significant statements about its behaviors diminishes, and significance and complexity become almost mutually exclusive characteristics. Unfortunately, all of the existing signal control models were based on operations with predetermined phase orders. The extent of the control decisions made by the various algorithms was limited to skipping, terminating, and extending certain phases in a predetermined phase sequence.

Adding, changing, and rearranging the fixed phase order in real time, which would offer more flexibility for optimization purposes, were beyond the capability of these algorithms. How to weigh and control these objectives is a big issue, which is becoming the scope of this research.

## 1.3 Research Objective

This research gives a theoretical analysis of signal left-turn phase operations at both isolated and coordinated signalized intersections, compares the difference in delay based on leading and lagging left-turn signal phase designs, identifies the main control factors, and gives a new model to guide the selection between the leading and lagging left-turn phases. Then a four-level fuzzy logic control model is designed to determine whether the leading or lagging signal phase should be selected or terminated in signal operation.

The main goals of this research include:

- To make a general theoretical analysis of signal left-turn phase operation,
- To build new mathematic models to guide the selection between the leading and lagging left-turn sequences at both isolated intersection and coordinated intersections,
- To analyze traffic control delay components and identify left-turn control factors,
- To formulate generalized fuzzy control rules for traffic signal left-turn phase control using linguistic variables,

- To validate the fuzzy control principles and to calibrate the membership functions of the linguistic variables using simulation and field data and,
- To develop a dynamic traffic signal left-turn phase fuzzy logic control system.

## 1.4 Research Approach and Layout

An introduction is given in Chapter 1, then the dissertation reviews past studies about leading and lagging signal phases and signal control applications in Chapter 2. Chapter 3 presents theoretical analysis of left-turn phase operation and compares leading and lagging left-turn at both isolated fixed-time and actuated signalized intersection. Coordinated signalized intersection left-turn phase operation is discussed in Chapter 4. Chapter 5 analyzes the influences of traffic control delay components for leading and lagging left-turn and identifies left-turn control factors from above Chapters.

In Chapter 6, fuzzy control logic is introduced, and fuzzification, defuzzification in the control process is also discussed. Based on Chapter 5 results and using linguistic variables, the fuzzy control rules and membership functions are derived. Chapter 7 develops a dynamic traffic signal left-turn phase fuzzy logic control system (Figure 1-1), and validates the fuzzy control principles and calibrates the membership functions of the linguistic variables using simulation and field trials and compares the results with other fixed-time control, actuated control and adaptive control. Chapter 8 presents a summary of the study, and then draws a number of conclusions based on the outcome of the research. Some recommendations are made for future research.

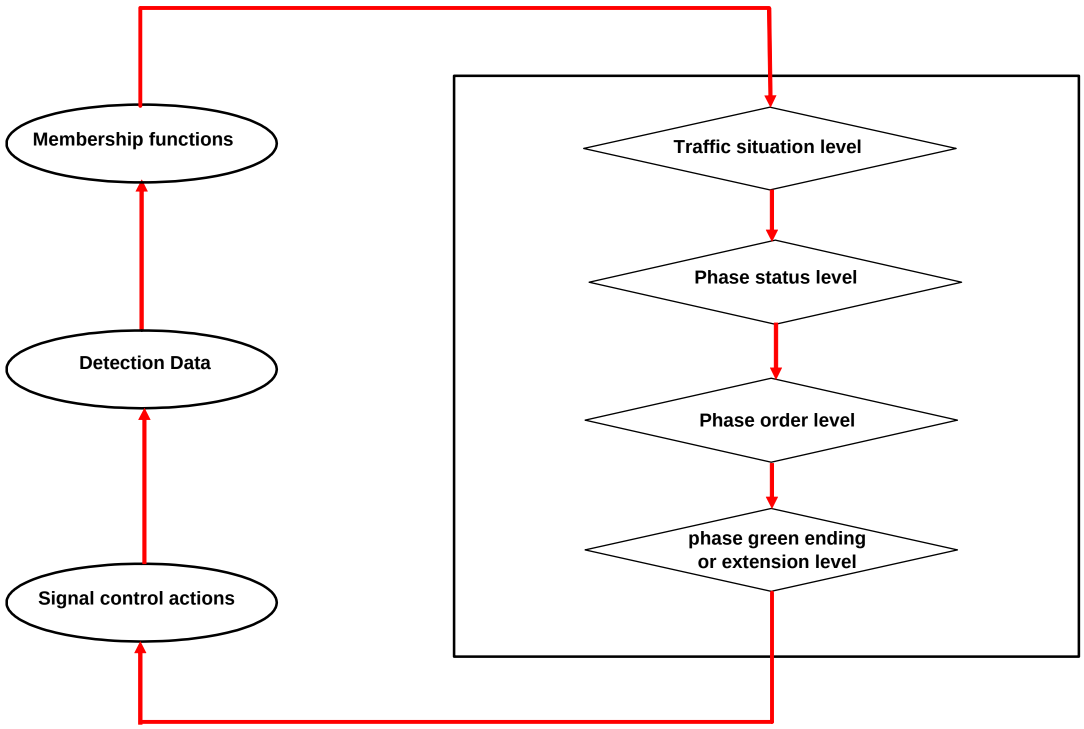



**Figure 1-1. Dynamic traffic signal left-turn phase fuzzy logic control system**

# Chapter 2 Literature Review

## 2.1 Traffic Signal Control Review

### 2.1.1 Fixed-time and actuated control

In the 1950's, F.V. Webster conducted traffic simulation studies using an early electronic computer, developed two traffic signal timing strategies that practically minimizes the resulting delay for pre-set fixed traffic isolated traffic signals. The Webster formula minimizes the total delays for the pre-known traffic volumes. Webster also demonstrated, both theoretically and experimentally, that fixed-timed signals should have their critical phases timed for equal degrees of saturation for a given cycle length to minimize the delay for that cycle time, even if it is not the minimum delay cycle. The Webster formula is as follows (Webster [1]):

$$C = \frac{1.5 * F + 5}{1 - \sum \{\max(\frac{q_i}{s_i})\}}$$

where $C$ = optimum cycle time, s
$F$ = sum of the lost times, s
$q_i$ = traffic flows of each direction (lane) in phase i, veh/h
$s_i$ = saturation flows of each direction (lane) in phase i, veh/h.

In fixed-timed operation, the red, yellow, and green indications are timed at fixed intervals. It assumes that the traffic patterns can be predicted accurately based on time of day. Fixed-timed operation does not require traffic detectors at the intersection, and is therefore much cheaper to install. Consequently, fixed-timed operation is usually used at isolated intersections only when funds do not allow actuated operation.

Intersections with actuated operation consist of actuated traffic controllers and vehicle detectors placed in or on the roadways approaching the intersection. The most important elements of actuated control are demand and extension. During actuated operation, a traffic movement will be served with a green indication. This green interval will last a user-defined minimum amount of time. As long as cars continue to cross the approach detectors frequently enough, the green interval will be extended. These extensions will continue until the cars thin out sufficiently to allow the signal to gap out, or until the interval reaches the maximum time. With actuated controllers, green intervals may terminate in one of four ways:

- Maximum green time is reached.
- Traffic flow ceases on the approach (gapping out).
- A signal system forces the termination (applying a force-off in coordinated system).
- The signal is Pre-empted. When a priority vehicle approaches the intersection, no priority green intervals may be terminated in favor of the priority movement.

The traditional actuated control of isolated intersections attempts continuously to adjust green times, and sometimes to adjust the sequence of phasing. The main disadvantage is that the control algorithm looks only at the vehicles on green while not taking into account the number of vehicles waiting at red.

### 2.1.2 Real-time adaptive control

With the introduction of microprocessor controllers it became possible to have more advanced control algorithms based on mathematical models. The optimization function can be chosen to reach a predefined goal, which usually is the minimization of vehicle delays. Miller [2] suggested a self-optimizing strategy based on the criterion of minimizing the total vehicle delay. In his strategy, the decision to extend a phase is made at regular intervals by the examination of a control function.

Adaptive signal control systems (ATCS), such as SCOOT, SCATS, OPAC and RHODES, help optimize and improve intersection signal timings by using real time traffic information to formulate and implement the appropriate signal timings [Martin et al. 3].

EPAC 300 Eagle Signal controller is an adaptive left turn control tool. Urbanik II et al. [4] indicates the basic concept of the controller is to measure the left turn volumes and monitor the gaps in the opposing traffic stream through detector actuations. The left turn phase is designed to run permissive unless there are not enough acceptable gaps in the opposing through traffic and left turning volume is high enough to justify a protected left turn phase. It is possible to omit the left turn phases by time of day in some controllers, and the significant benefit of this feature is that the left turn phases can be omitted or activated based on traffic conditions.

Mirchandani et al. [5] introduces a real-time traffic-adaptive signal control system (RHODES). The system takes as input detector data for real-time measurement of traffic flow, and “optimally’ controls the flow through the network. The prototype consists of network control logic (the network flow optimization logic and the platoon flow prediction logic) and intersection control logic (the intersection optimization logic and the link flow prediction logic).

Hernandez et al. [6] presents a general approach for real time traffic management using knowledge-based models named TRYS. TRYS is a knowledge representation environment supporting models to perform traffic management at a strategic level in urban, interurban or mixed areas. TRYS model provides traffic monitoring functions and control actions. The interface between TRYS and the control system allows the TRYS model to accept input data (i.e. speed and occupancy measurements) from the real-time data collection facilities (via the traffic control computer) and to send back control actions to the traffic control computer. Depending on the traffic control system available at the application site, control actions can range from a set of constraints limiting the selection to a library of predefined signal plans (or a library of predefined messages in VMS applications) to a set of constraints on signal setting parameters (i.e. cycle time, phase split and offsets) in a fully adaptive system.

Findler [7] described harmonization as part of our work on distributed, knowledge-based, real-time, traffic-adaptive control of street and highway ramp traffic signals. Harmonization represents the best approximation to a coordinated omni-directional progression ("green wave"). This means that the resulting control regime produces a minimum of the sum, over all intersections, of delay times due to red lights and of unused green periods, each contributing term being weighted by the respective traffic flow values.

### 2.1.3 Fuzzy logic control

When the intersection is complex in terms of geometric design, channelization and types of vehicles to be handled, the main problems of optimizing control are the fairly high number of detectors, difficulty of understanding control and its parameters, and sensitivity to detector errors (Kronborg et al. [8]). Kosko [9] indicated "as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviors diminishes, and significance and complexity become almost mutually exclusive characteristics" . So, the best solution to control complicated signalized intersection might be the fuzzy logic that is mechanism of human thinking with linguistic fuzzy values.

Pappis and Mamdani [10] considered an isolated traffic intersection control in an isolated one-way east-west/north-south signalized intersection (2+2 lanes) with random vehicle arrivals and no turning movements using fuzzy logic controller in 1977. They made a theoretical simulation study of a fuzzy logic control. In their report, they compared their fuzzy method to a delay-minimizing adaptive signal control with optimal cycle time. According to the results, the fuzzy controller was equal to, or slightly better than, the adaptive method used for comparison.

In 1984, Nakatsuyama et al [11] used fuzzy logic phase controller in two successive signalized intersections control of an arterial road under conditions such as when a fairly large number of vehicles is passing an intersection. The fuzzy logic phase controller is composed of fuzzy control statements, which determine the termination of green or amber periods. Co-operation between a fuzzy logic controller and a fuzzy logic phase controller always results in good performance, especially when the number of cars varies by a large margin as observed before or after the rush hour.

From the network point of view, Chiu et al [12] [13] applied fuzzy logic for controlling multiple two-way streets intersections with no turning movements. Chiu used fuzzy decision rules to adjust cycle time, phase split and offset parameters based on local information only. A set of 40 fuzzy decision rules was used for adjusting the signal timing parameters in a network of 3 * 3 intersections. The rules for adjusting cycle time, phase split and offset are decoupled so that these parameters are adjusted independently.

Niitymaki et al. [14] developed a fuzzy logic algorithm for controlling the timing of a pedestrian crossing signal based on the fuzzy extension principle which was used by

Pappis et al. [10]. Niitymaki indicated the algorithm can offers at least equal or better performance than conventional demand-actuated signal control.

Trabia et al [15] designed a fuzzy logic based signal controller for a four-approach isolated intersection with through and left-turning movements. The controller has the ability to make adjustments to signal timing in response to observed changes in the approach flows. Using upstream vehicle detectors, the controller measures approach flows and estimates approach queues at regular time intervals. This information is used in a two-stage fuzzy logic procedure to determine, at any given time, whether to extend or terminate the current signal phase for through movements.

Sayers et al. [16] had aimed to develop a flexible signal controller which could be configured so that it embodied the objectives appropriate to the situation in which it was to be used. They used a multi-objective genetic algorithm (MOGA) optimization technique to derive optimal solutions for fuzzy control.

Niittymaki et al. [17] tested fuzzy public transport rule at an isolated intersection. The tested intersection is a T-intersection with three phases. The traffic volume arriving from the minor street is quite small. Buses approach the intersection from both major street directions and they drive straight through.

## 2.2 Left-turn Phase Delay Comparison Review

Researches have been done to evaluate the pros and cons of leading and lagging left-turn signal design. For fixed-time signal designs, Hummer et al. [18] found that at an isolated intersection with heavy pedestrian traffic, lagging is better than leading regarding to intersection delays. The results are summarized in Table 2-1.

J. E. Hummer's research was based on the use of leading and lagging phases sequences in Indiana. The result, which favors lagging over leading phases, was narrowed by the following condition:

- Light to medium-heavy (but still unsaturated) volumes;
- Balanced flow between the directions on the street with the left-turn signals;
- Intersection angles of approximately 90 degrees;
- Narrow or nonexistent medians;
- Single left-turn lanes;
- Three-or four-leg intersections on four-lane arterials; and
- Adequate left-turn lane lengths (spillback is rear).

If the intersection has above conditions and one of the following conditions, lagging is recommended.

- at isolated signals serving heavy pedestrian traffic

**Table 2-1. Leading and lagging comparison at fixed-time isolated intersection**

| Fixed-time | Left-turn signal | Mean delay (sec/veh) |
|---|---|---|
| Four approaches | Protected-leading | 19.9 |
| | Protected-lagging | 19.4 |
| | Protected-permissive-leading | 14.7 |
| | Permissive-protected-lagging | 13.5 |

- at isolated diamond interchanges
- the signals are fixed-time or incapable of overlapping phases

For actuated signals, on the other hand, Lee et al. [19] yielded an opposite finding, in which lagging designs almost always result in more delay than leading designs. The results are summarized in Table 2-2.

Lee's research result confirmed the assumption that overlap can lightly influence intersection delay when taken into account. The result, which lagging delay is almost always more than leading delay, was narrowed by the following condition.

- All of the study locations were operation with full actuated control;
- The signals were basically isolated from other intersections;
- Light to medium-heavy (but still unsaturated) volumes; and
- Vehicle queues generally cleared during each cycle;
- The left-turn volume of opposing direction was very unbalanced so that the opportunities for phase overlap often appear in leading and was not used in lagging.

Fambro et al. [20] tested the operational efficiency of Dallas protected-permissive phasing sequence and the standard protected-permissive phasing with signal displays allowed by MUTCD. The Dallas phasing is a special type of lead-lag operation developed and implemented by traffic engineers in the cities of Dallas and Richardson,

**Table 2-2. Leading and lagging comparison at actuated isolated intersection**

| Actuated-time | Left-turn signal | Mean delay (sec/veh) |
|---|---|---|
| Four approaches | Leading | 30.3 |
| | Lagging | 40.6 |

Texas. The phasing eliminates the possibility of a left turn trap situation that was explained earlier in the case of lead-lag sequence with protected-permissive and permissive-protected left turns. The results of the study indicated that the Dallas phasing results in less delay for both the left-turning and through movements when compared to phasing with MUTCD left turn signal displays. The study also documented that at intersections along high volume coordinated arterials; the Dallas phasing offers significant operational benefits. The study, however, did not appear to measure and compare the safety impacts of the Dallas phasing versus the phasing allowed per MUTCD signal displays.

Parsonson [21] indicated that in many cases leading left turn phasing is the normal sequence of operation, which in a gap out situation caused by an early release in through movements, can potentially damage progression. It should be noted that this situation could also be caused by an early release due to a cross street gap out situation. The synthesis further discusses the applicability of lagging left turn sequences under tight storage length situations and qualifies the safety implications that may result due to left turn trap situations. The responses to a survey in the synthesis qualify that driver expectancy weighs heavily in favor of leading left turns. The respondents indicated that lagging left turns were used only when necessary and safe. One respondent indicated that the driver-expectancy problem might exist when phase sequencing is changed by time-of-day to obtain a better bandwidth. This paper includes investigation of this particular issue.

A study reported by Nassi [22] showed that changing left turn phasing from leading to lagging operation resulted in positive synchronization results. An after study of the arterial signal conversion documented decreases of 38.3% in fuel consumption, 43.1% in air pollutants, 40.0% in traffic collision rate, and 42.2% in vehicle delay.

Tian et al. [23] addresses various forms of split phasing schemes resulted from various pedestrian timing treatments. The pedestrian conflicts with each split phasing scheme are discussed based on coordinated signal system operations. The research indicates the protected/permitted phasing scheme would provide the efficiency and safety during the protected phase, and would minimize the impact of pedestrian crossing by accommodating the pedestrians in two parallel pedestrian phases. An exclusive pedestrian phase under split phasing operations can be more efficient compared to the standard protected left-turn display phasing scheme.

Buckholz, [24] indicated that one of the major pitfalls of coordinated signal timing is the reluctance by traffic engineers to use lead-lag left turn phasing to improve progression, due to possible violation of driver expectancy. He further indicated that experience has shown that where drivers become used to traffic signal phasing variations, the lead-lag left turn phasing can have positive effects on the arterial flow.

Researchers including Nandam et al. [25] then argued that choosing leading or lagging left turn phasing should not be a default decision, and dynamically changing leading and lagging phases by time-of-day may improve progression. Based on these studies, Pline

[26] indicated that phasing sequence selection should be based on analysis on a case by case basis and dependent upon acceptance of drivers using the traffic signal. However, dynamic signal design requires systematic effort at coordinated intersections since changes at one intersection may adversely affect the others. In order to find the basic difference between leading and lagging left turns at coordinated intersections, Li et al. [27] complied the signal phases, traffic patterns and delay, pointed out that the left turn and through vehicles play very different roles at coordinated intersections, and lagging (upstream) plus lagging (downstream) design for the two coordinated intersections will result in minimum delay. Li et al. [28] also gives a mathematic model to guide selecting leading and lagging between two closed intersections.

## 2.3 Left-turn Phase Safety Comparison Review

According to J. E. Hummer et al. [18], the accident number is summarized in table 2-3. This result is based on the data collected in Indiana. The more the pedestrians are, the safe the intersection is.

According to Jonathan Upchurch [29], the accident number is summarized in table 2-4. This is based on data collected in Arizona. From these results, the intersection with lagging left-turn signal phases is safer than the one with leading left-turn phases when the opposing lanes are more than 2 lanes.

Nandam et al. [25] indicate that the change in sequence of the left turns and use of

**Table 2-3. Leading and lagging left-turn safety comparison in Indiana**

| | Leading | Lagging |
|---|---|---|
| Accidents per million left turn vehicles | 0.9 | 0.8 |
| Accidents per million total vehicles | 0.09 | 0.06 |

**Table 2-4. Leading and lagging left-turn safety comparison in Arizona**

| | Leading | Lagging |
|---|---|---|
| Accidents per million left turn vehicles (2 opposing lanes) | 1.02~2.71 | 2.09~3.02 |
| Accidents per million left turn vehicles (3 opposing lanes) | 1.33~4.54 | 0.5~2.65 |

dynamic change of phase sequence by time of day did not result in change of left turn and total crash experience. The calculated t-value for the before and after left turn crash rates was 1.67. This is within the critical range of t-value at the 0.05 level. The calculated t-value for the before and after total crash rates was 0.734. This is within the critical range of t-value at the 0.05 level.

It is most often the case that a lagging protected phase is not allowable due to the possibility of opposing left turn drivers being caught in a left turn “trap,” in which they incorrectly assume that their movement is being terminated at the same time as that of traffic opposing them. This confusion is eliminated at intersections without opposing left turning traffic, such as “T” intersections or intersections with one-way streets, or it can be avoided by the use of “Dallas” left turn phasing, in which left turn drivers are shown an exclusive display of the opposing traffic’s indication.

# Chapter 3 Theoretical Analysis of Signal Left-turn Phase Operation at Isolated Intersection

## 3.1 Leading Signal Left-turn Phase Operation

Leading signal left-turn phase can be classified into protected-leading and protected-permitted-leading designs. Listed below are brief descriptions of these two designs:

- **Protected-leading**: a designated green phase is given to left-turn vehicles before the through traffic movements. And then left turning movements are prohibited when through traffic gets its green time.
- **Protected-permitted-leading**: protected left turn phase is given right before the green phase for through traffic, during which left turning movements are permitted when gaps are available in the opposing through traffic.

The following terms are defined:

$q_a$ - The arrival rate (veh/s),

$s_{pr}$ - The saturation flow rate for the protected phase (veh/s),

$s_{pm}$ - The saturation flow rate for the permitted phase (veh/s),

$t_{g_q}$ - The blocked portion of the permitted phase (s),

$t_{g_u}$ - The unblocked portion of the permitted phase (s),

$t_{g_{pr}}$ - The portion of the protected phase (s),

$t_r$ - The effective red time for the effective red time (s),

$Q_{i1}$ - The residual queue (veh) at the beginning of the red phase,

$Q_{11}$ - The queue (veh) at the end of the red phase,

$Q_{21}$ - The queue (veh) at the end of the protected green phase,

$Q_{31}$ - The queue (veh) at the end of the saturated interval of the permitted green phase,

$Q_{41}$ - The queue (veh) at the end of the unsaturated interval of the permitted green phase.

Under heavy through traffic flow where gaps are rare for left turn movement during the green phase, a protected-permissive phasing becomes similar, if not equal, to a protected left-turn phasing design. Most agencies prefer the use of leading left turn phasing since the belief is that driver expectancy weighs heavily in favor of leading left turns. The leading left-turn operations are showed in Figure 3-1 ~ 3-3.

The geometry of the triangle depends on the traffic volume, the queue discharge rate, and the length of the red and green phases. It accumulates on red time $t_r$ and blocked permitted phase $t_{g_q}$, and discharges on protected phase $t_{g_{pr}}$ and unblocked permitted phase $t_{g_u}$.

In case 1, no queue remains at the end of protected or permitted phase.

In case 2, queue remains at the end of the protected phase, but does not remain at the end

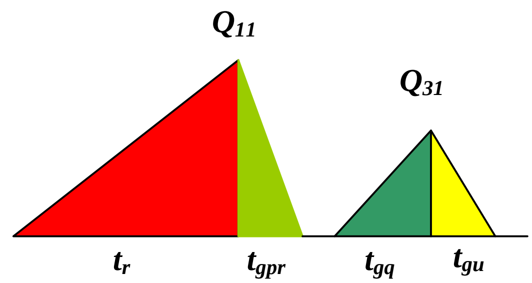



**Figure 3-1. Leading left-turn queue accumulation polygon-Case 1**

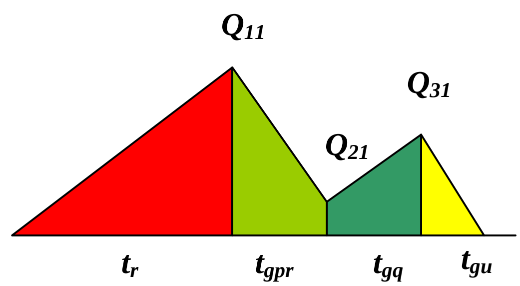



**Figure 3-2. Leading left-turn queue accumulation polygon-Case 2**

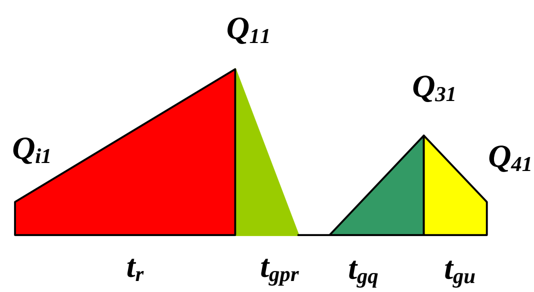



**Figure 3-3. Leading left-turn queue accumulation polygon-Case 3**

of the permitted phase.

In case 3, queue does not remain at the end of the protected phase, but remains at the end of the permitted phase. Note that it is not possible to have a queue at the end of both the protected and permitted phases if the v/c ratio is not allowed to exceed 1.0 when calculating the uniform delay term.

At the end of the protected left-turn's effective red, the left-turn queue length is:

$$Q_{11} = Q_{i1} + \int_{t_r} q_a dt_r$$

Left-turn queue begins to dissipate at $t_{gpr}$, and will completely dissipate if $Q_{11}$ departure time $t_{Q11}$ is less than $t_{gpr}$.

$$Q_{21} = Max\left[0.0, Q_{11} - \int_{t_{g_{pr}}} \left(s_{pr} - q_a\right) dt_{g_{pr}}\right]$$

Permitted left-turn capacity occurs during unblocked green time

$$t_{g_u} = Max\left(0.0, t_{g_{pm}} - t_{g_q}\right)$$

At end of the blocked green time, the left-turn queue length and time to dissipate the left-turn queue are:

$$Q_{31} = Q_{21} + \int_{t_{g_q}} q_a \, dt_{g_q}$$

Queue length at end of unblocked green time is:

$$Q_{41} = Max\left[ 0.0, Q_{31} - \int_{t_{g_u}} \left( s_{pm} - q_a \right) dt_{g_u} \right]$$

Final queue length is reduced by the average number of sneakers, $n_f$, per cycle

$$Q_{51} = Max\left[ 0.0, Q_{41} - n_f \right] = Q_{i1}$$

So, leading left-turn traffic delay $D_1$ is:

$$\begin{aligned} D_1 &= Q_{i1} \times \left( t_r + t_{Q11} \right) + Q_{21} \times \left( t_{g_q} + t_{Q_{31}} \right) \\ &+ \int_{t_r} q_a \left( t_r + t_{Q11} \right) dt_r - \int_{t_{Q11}} \left( s_{pr} - q_a \right) t_{Q11} dt_{Q11} + \int_{t_{g_q}} q_a \left( t_{g_q} + t_{Q31} \right) dt_{g_q} - \int_{t_{Q31}} \left( s_{pm} - q_a \right) t_{Q31} dt_{Q31} \end{aligned}$$

(3-1)

$$Q_{21} = \begin{cases} Q_{i1} - \int_{t_{g_{pr}}} s_{pr} dt_{g_{pr}} + \int_{t_{r+g_{pr}}} q_a dt_{r+g_{pr}} & \text{if } t_{Q11} = t_{g_{pr}} \\ 0 & \text{if } t_{Q11} < t_{g_{pr}} \end{cases}$$

The leading left-turn traffic volume which experience delay $V_{ld1}$ and total left-turn volume $V_{l1}$ are:

$$V_{ld1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{Q31}} q_a dt_{Q31} \tag{3-2}$$

$$V_{l1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{g\,pr}} q_a dt_{g_{pr}} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{g_u}} q_a dt_{g_u} \tag{3-3}$$

When left-turn is leading operation, the opposing through traffic accumulation and departure pattern is showed in Figure 3-4.

At the end of the opposing movement's effective red, opposing queue length $Q_{o1}$, and through traffic delay $D_{o1}$ are:

$$Q_{o1} = Q_{oi1} + \int_{t_{r+g\,pr}} q_o dt_{r+g_{pr}}$$

$$D_{o1} = Q_{oi1} \times t_{r+g_{pr}+g_q} + \int_{t_{r+g\,pr}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} - \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q}$$

(3-4)

The opposing through traffic volume which experience delay $V_{od1}$ and total opposing through traffic volume $V_{o1}$ for leading left-turn operation are:

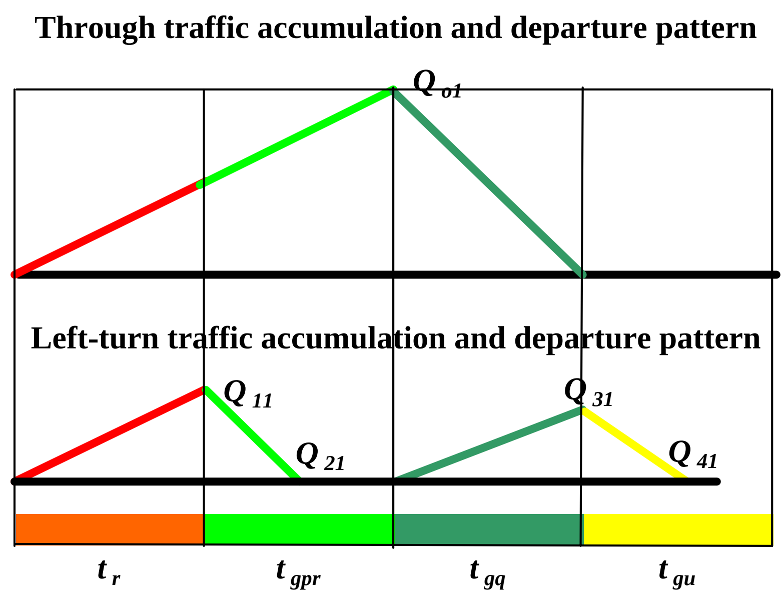



**Figure 3-4. Through traffic queue polygon with leading left-turn**

$$V_{od1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q} \tag{3-5}$$

$$V_{o1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u} \tag{3-6}$$

## 3.2 Lagging Signal Left-turn Phase Operation

Lagging signal phase may include protected-lagging and permitted-protected-lagging designs. Listed below are brief descriptions of these two designs:

- **Protected-lagging**: left turn is prohibited when through traffic gets its green time. And then a designated green phase is given to left-turn vehicles when the through traffic gets its red time.
- **Permitted-protected-lagging**: protected left turn phase is given right after the green phase for through traffic, during which left turning movements are permitted when gaps are available in the opposing through traffic.

The following terms are defined for lagging left-turn operation:

$Q_{i2}$ - The residual queue (veh) at the beginning of the red phase,

$Q_{12}$ - The queue (veh) at the end of the red phase,

$Q_{22}$ - The queue (veh) at the end of the saturated interval of the permitted green phase,

$Q_{32}$ - The queue (veh) at the end of the unsaturated interval of the permitted green phase,

$Q_{42}$ - The queue (veh) at the end of the protected green phase.

Under heavy through traffic flow where gaps are rare for left turn movement during the green phase for through traffic, a permitted-protected phasing becomes similar, if not equal, to a protected left-turn phasing design. Some agencies use lagging left turn phasing when left turns and opposing through traffic are light in cases of permissive-protected operation. The lagging left-turn operations are showed in Figure 3-5 ~ 3-6.

In case 4, no queue remains at the end of the permitted phase. Because the protected phase follows immediately after the permitted phase and will therefore accommodate all of its arrivals without further delay, so there will be no queue at the end of the protected phase either.

In case 5, queue remains at the end of the permitted phase. If the v/c ratio is kept below 1.0 as just discussed, this queue will be fully served during the protected phase.

Left-turn queue at end of effective red continues to grow during the blocked green time. Left-turn queue length and time to clear are:

$$Q_{12} = Q_{i2} + \int_{t_r} q_a dt_r$$

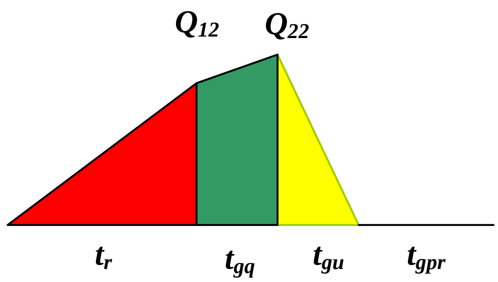



**Figure 3-5. Lagging left-turn queue accumulation polygon-Case 4**

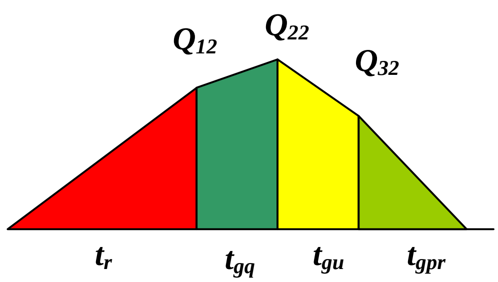



**Figure 3-6. Lagging left-turn queue accumulation polygon-Case 5**

$$Q_{22} = Q_{12} + \int_{t_{g_q}} q_a \, dt_{g_q}$$

Queue length and time to clear at end of the permitted phase (beginning of the protected phase) are:

$$Q_{32} = Max\left[ 0.0, Q_{22} - \int_{t_{g_u}} \left( s_{pm} - q_a \right) dt_{g_u} \right]$$

Queue length at end of protected green phase is:

$$Q_{42} = Max\left[ 0.0, Q_{32} - \int_{t_{g_{pr}}} \left( s_{pr} - q_a \right) dt_{g_{pr}} \right] = Q_{i2}$$

In this case, there are no sneakers.

So, lagging left-turn traffic delay $D_2$ is:

$$D_2 = Q_{i2} \times \left( t_r + t_{g_q} + t_{Q22} + t_{Q32} \right) + \int_{t_r} q_a \left( t_r + t_{g_q} + t_{Q22} + t_{Q32} \right) dt_r + \int_{t_{g_q}} q_a \left( t_{g_q} + t_{Q22} + t_{Q32} \right) dt_{g_q}$$
$$- \int_{t_{g_u}} \left( s_{pm} - q_a \right) t_{Q32} \, dt_{g_u} - \int_{t_{Q22}} \left( s_{pm} - q_a \right) t_{Q22} \, dt_{Q22} - \int_{t_{Q32}} \left( s_{pr} - q_a \right) t_{Q32} \, dt_{Q32}$$

(3-7)

The lagging left-turn traffic volume which experience delay $V_{ld2}$ and total left-turn volume $V_{l2}$ are:

$$V_{ld2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{Q22}} q_a dt_{Q22} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{Q32}} q_a dt_{Q32} \quad (3\text{-}8)$$

$$V_{l2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{g_u}} q_a dt_{g_u} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{g_{pr}}} q_a dt_{g_{pr}} \quad (3\text{-}9)$$

When left-turn is lagging operation, the opposing through traffic accumulation and departure pattern is showed in Figure 3-7.

At end of the opposing movement's effective red, opposing queue length $Q_{o2}$, and through traffic delay $D_{o2}$ are:

$$Q_{o2} = Q_{oi2} + \int_{t_{r+g_{pr}}} q_o dt_{r+g_{pr}}$$

$$D_{o2} = Q_{oi2} \times t_{r+g_{pr}+g_q} + \int_{t_{r+g_{pr}}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} - \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q}$$

(3-10)

The opposing through traffic volume which experience delay $V_{od2}$ and total opposing through traffic volume $V_{o2}$ for lagging left-turn operation are:

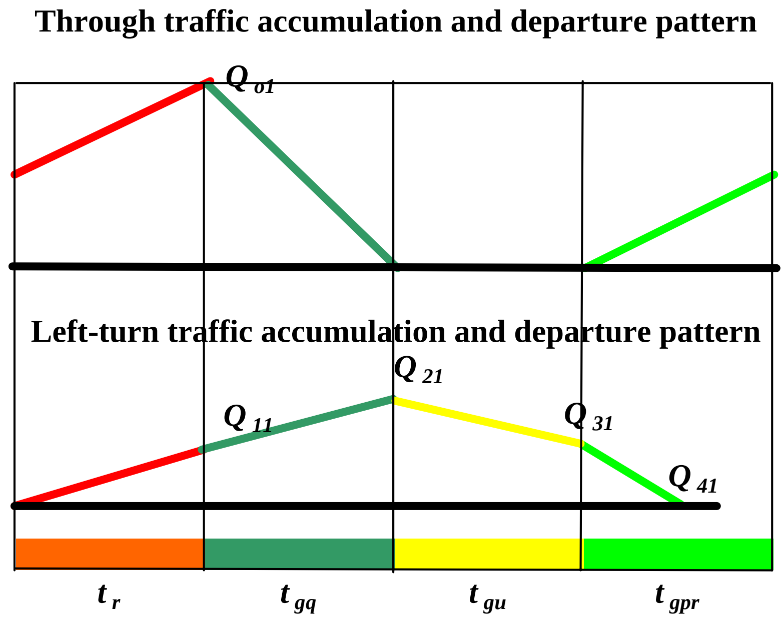



**Figure 3-7. Through traffic queue polygon with lagging left-turn**

$$V_{od2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q} \tag{3-11}$$

$$V_{o2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u} \tag{3-12}$$

## 3.3 Comparison of Left-turn Phases at Isolated Intersection

### 3.3.1 Delay comparison at fixed-time isolated intersection

From Equation 3-1 ~ 3-3, leading left-turn traffic delay and volume are:

$$D_1 = Q_{i1} \times (t_r + t_{Q11}) + Q_{21} \times (t_{g_q} + t_{Q_{31}})$$
$$+ \int_{t_r} q_a (t_r + t_{Q11}) dt_r - \int_{t_{Q11}} (s_{pr} - q_a) t_{Q11} dt_{Q11} + \int_{t_{g_q}} q_a (t_{g_q} + t_{Q31}) dt_{g_q} - \int_{t_{Q31}} (s_{pm} - q_a) t_{Q31} dt_{Q31}$$

$$Q_{21} = \begin{cases} Q_{i1} - \int_{t_{g_{pr}}} s_{pr} dt_{g_{pr}} + \int_{t_{r+g_{pr}}} q_a dt_{r+g_{pr}} & \text{if } t_{Q11} = t_{g_{pr}} \\ 0 & \text{if } t_{Q11} < t_{g_{pr}} \end{cases}$$

$$V_{ld1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{Q31}} q_a dt_{Q31}$$

$$V_{l1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{g_{pr}}} q_a dt_{g_{pr}} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{g_u}} q_a dt_{g_u}$$

From Equation 3-7 ~ 3-9, lagging left-turn traffic delay and volume are:

$$D_2 = Q_{i2} \times \left(t_r + t_{g_q} + t_{Q22} + t_{Q32}\right) + \int_{t_r} q_a \left(t_r + t_{g_q} + t_{Q22} + t_{Q32}\right) dt_r + \int_{t_{g_q}} q_a \left(t_{g_q} + t_{Q22} + t_{Q32}\right) dt_{g_q}$$

$$- \int_{t_{g_u}} \left(s_{pm} - q_a\right) t_{Q32} dt_{g_u} - \int_{t_{Q22}} \left(s_{pm} - q_a\right) t_{Q22} dt_{Q22} - \int_{t_{Q32}} \left(s_{pr} - q_a\right) t_{Q32} dt_{Q32}$$

$$V_{ld2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{Q22}} q_a dt_{Q22} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{Q32}} q_a dt_{Q32}$$

$$V_{l2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{g_u}} q_a dt_{g_u} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{g\,pr}} q_a dt_{g_{pr}}$$

So, the left-turn delay difference (leading-lagging) is:

$$D_1 - D_2 = D_i + D_r + D_{pr} + D_{pm} \tag{3-13}$$

Where:

$$D_i = \left(Q_{i1} - Q_{i2}\right) \times t_r + \left(Q_{21} - Q_{i2}\right) \times t_{g_q} + \left(Q_{i1} t_{Q11} - Q_{i2} t_{Q32}\right) + \left(Q_{21} t_{Q31} - Q_{i2} t_{Q22}\right)$$

$$D_r = \int_{t_r} \left(t_{Q11} - t_{g_q} - t_{Q22} - t_{Q32}\right) q_a dt_r$$

$$D_{pr} = \int_{t_{Q32}} \left(s_{pr} - q_a\right) t_{Q32} dt_{Q32} - \int_{t_{Q11}} \left(s_{pr} - q_a\right) t_{Q11} dt_{Q11}$$

$$D_{pm} = \int_{t_{g_u}} \left(s_{pm} - q_a\right) t_{Q32} dt_{g_u} + \int_{t_{Q22}} \left(s_{pm} - q_a\right) t_{Q22} dt_{Q22} - \int_{t_{Q31}} \left(s_{pm} - q_a\right) t_{Q31} dt_{Q31} - \int_{t_{g_q}} q_a \left(t_{Q22} + t_{Q32} - t_{Q31}\right) dt_{g_q}$$

The difference (leading-lagging) of left-turn vehicles which experience delay is:

$$V_{ld1} - V_{ld2} = Q_{i1} - Q_{i2} + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{Q31}} q_a dt_{Q31} - \int_{t_{Q22}} q_a dt_{Q22} - \int_{t_{Q32}} q_a dt_{Q32} \tag{3-14}$$

$$V_{l1} - V_{l2} = Q_{i1} - Q_{i2} \tag{3-15}$$

From Equation 3-4 ~ 3-6, leading through traffic delay and volume are:

$$D_{o1} = Q_{oi1} \times t_{r+g_{pr}+g_q} + \int_{t_{r+g_{pr}}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} - \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q}$$

$$V_{od1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q}$$

$$V_{o1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u}$$

From Equation 3-10 ~ 3-12, lagging through traffic delay and volume are:

$$D_{o2} = Q_{oi2} \times t_{r+g_{pr}+g_q} + \int_{t_{r+g_{pr}}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} - \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q}$$

$$V_{od2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q}$$

$$V_{o2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u}$$

So, the through traffic delay difference (leading-lagging) is:

$$D_{o1} - D_{o2} = \left(Q_{oi1} - Q_{oi2}\right) \times t_{r+g_{pr}+g_q} \tag{3-16}$$

$$Q_{oi1} = Max\left[0.0,\ \int_{t_{r+g_{pr}}}^{r+g_{pr}} q_o\,dt - \int_{t_{g_q}}^{g_{pm}} (s_o - q_o)\,dt\right] \tag{3-17}$$

$$Q_{oi2} = Max\left[0.0,\ \int_{t_{r+g_{pr}}}^{r+g_{pr}} q_o\,dt - \int_{t_{g_q}}^{g_{pm}} (s_o - q_o)\,dt\right] \tag{3-18}$$

From above through traffic equations 3-16 ~ 3-18, when their $q_o$ have the same distribution, $Q_{oi1}=Q_{oi2}$, $V_{od1}=V_{od2}$, $V_{o1}=V_{o2}$, so leading or lagging design does not affect through traffic delay, $D_{o1}=D_{o2}$.

When $D_1 - D_2 = D_i + D_r + D_{pr} + D_{pm} \geq 0$, leading left-turn design is selected at fixed-time isolated intersection. Otherwise lagging left-turn design is preferred. The Figure 3-8 and 3-9 show the difference between protected-permissive leading and permissive-protected lagging left-turn on fixed-time traffic condition. The study period is chosen in peak hour, cycle length is 54s, g/c for left-turn traffic is 0.59, g/c for through traffic is 0.46, through volume are 600 veh/h, the number of lanes is 4 for both directions, and pedestrians are not included. From Figure 3-8 and 3-9 comparison results, leading left-turn delay is always lower than lagging left-turn delay at isolated fixed-time signalized intersection when left-turn v/c ratio is relatively low, while leading left-turn delay is higher than lagging left-turn delay when v/c ratio is close to or exceeds 1.0. Note that leading left-turn delay will be significantly higher than lagging left-turn delay at isolated fixed-time signalized intersection when left-turn v/c exceeds 1.0.

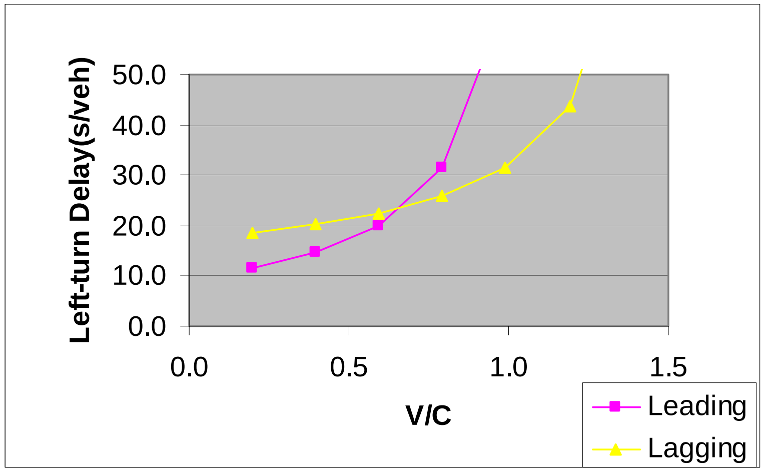



**Figure 3-8. The left-turn delay comparison for isolated fixed-time signal**

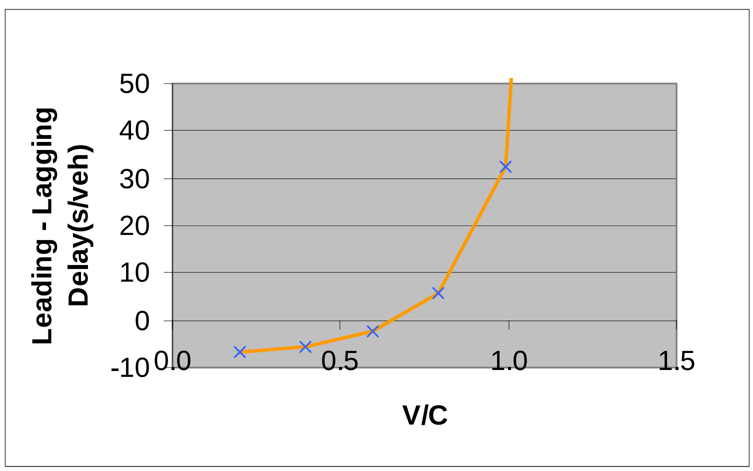



**Figure 3-9. The left-turn delay difference for isolated fixed-time signal**

3.3.2 Delay comparison at actuated isolated intersection

In actuated signal conditions, when the left-turn volumes of opposing directions were very unbalanced, the opportunities for phase overlap will often appear (Figure 3-10 Case1 + Case 2). But, the intersection with lagging phases cannot use these overlaps, because the red phase is followed next (Figure 3-10 Case 4 + Case 5). However the intersection with leading left-turn phases can reduce much through traffic delay by using overlap phases.

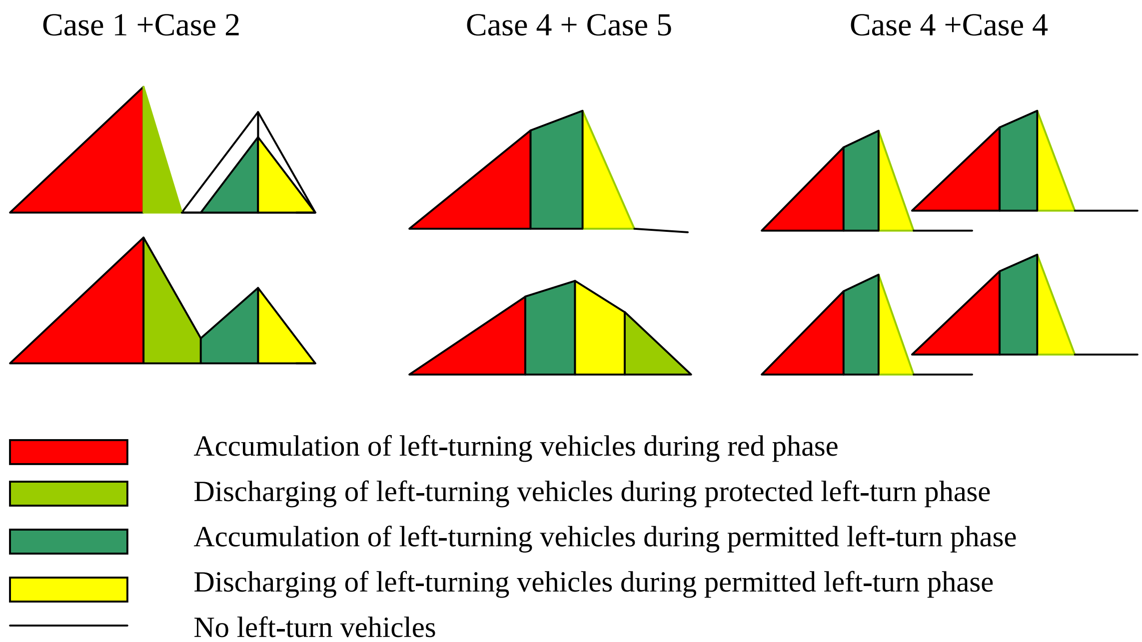
Based on Chapter 3.1 and 3.2 analysis results, leading actuated left-turn traffic delay and volume are:

$$D_1 = Q_{i1} \times (t_r + t_{Q11}) + Q_{21} \times (t_{g_q} + t_{Q_{31}}) + \int_{t_r} q_a (t_r + t_{Q11}) dt_r$$
$$- \int_{t_{Q11}} (s_{pr} - q_a) t_{Q11} dt_{Q11} + \int_{t_{g_q} + g_{pr} - Q11} q_a \, (t_{g_q} + t_{Q31}) dt_{g_q + g_{pr} - Q11} - \int_{t_{Q31}} (s_{pm} - q_a) t_{Q31} dt_{Q31}$$

(3-19)

$$Q_{21} = \begin{cases} Q_{i1} - \int_{t_{g_{pr}}} s_{pr} dt_{g_{pr}} + \int_{t_{r+g_{pr}}} q_a dt_{r+g_{pr}} & \text{if } t_{Q11} = t_{g_{pr}} \\ 0 & \text{if } t_{Q11} < t_{g_{pr}} \end{cases}$$

The leading left-turn traffic volume which experience delay $V_{ld1}$ and total left-turn volume $V_{l1}$ are:




Case 1+2: Leading design under unbalanced traffic (Phase overlapping)

Case 4+5: Lagging design under unbalanced traffic

Case 4+4: Lagging design under balanced traffic (Phase skipping)

**Figure 3-10. Queue Accumulation Diagram for Leading and Lagging Designs**

$$V_{ld1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q+g_{pr}-Q11}} q_a \, dt_{g_q+g_{pr}-Q11} + \int_{t_{Q31}} q_a dt_{Q31} \tag{3-20}$$

$$V_{l1} = Q_{i1} + \int_{t_r} q_a dt_r + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q+g_{pr}-Q11}} q_a \, dt_{g_q+g_{pr}-Q11} + \int_{t_{g_u}} q_a dt_{g_u} \tag{3-21}$$

Lagging actuated left-turn traffic delay and volume are:

$$D_2 = Q_{i2} \times \left(t_r + t_{g_q} + t_{Q22} + t_{Q32}\right) + \int_{t_r} q_a \left(t_r + t_{g_q} + t_{Q22} + t_{Q32}\right) dt_r + \int_{t_{g_q}} q_a \left(t_{g_q} + t_{Q22} + t_{Q32}\right) dt_{g_q}$$
$$- \int_{t_{g_u}} \left(s_{pm} - q_a\right) t_{Q32} dt_{g_u} - \int_{t_{Q22}} \left(s_{pm} - q_a\right) t_{Q22} dt_{Q22} - \int_{t_{Q32}} \left(s_{pr} - q_a\right) t_{Q32} dt_{Q32}$$

(3-22)

The lagging left-turn traffic volume which experience delay $V_{ld2}$ and total left-turn volume $V_{l2}$ are:

$$V_{ld2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{Q22}} q_a dt_{Q22} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{Q32}} q_a dt_{Q32} \tag{3-23}$$

$$V_{l2} = Q_{i2} + \int_{t_r} q_a dt_r + \int_{t_{g_u}} q_a dt_{g_u} + \int_{t_{g_q}} q_a dt_{g_q} + \int_{t_{g_{pr}}} q_a dt_{g_{pr}} \tag{3-24}$$

The actuated left-turn delay difference (leading-lagging) is:

$$D_1 - D_2 = D_i + D_r + D_{pr} + D_{pm} \tag{3-25}$$

Where:

$$D_i = (Q_{i1} - Q_{i2}) \times t_r + (Q_{21} - Q_{i2}) \times t_{g_q} + (Q_{i1} t_{Q11} - Q_{i2} t_{Q32}) + (Q_{21} t_{Q31} - Q_{i2} t_{Q22})$$

$$D_r = \int_{t_r} (t_{Q11} - t_{g_q} - t_{Q22} - t_{Q32}) q_a dt_r$$

$$D_{pr} = \int_{t_{Q32}} (s_{pr} - q_a) t_{Q32} dt_{Q32} - \int_{t_{Q11}} (s_{pr} - q_a) t_{Q11} dt_{Q11}$$

$$D_{pm} = \int_{t_{g_u}} (s_{pm} - q_a) t_{Q32} dt_{g_u} + \int_{t_{Q22}} (s_{pm} - q_a) t_{Q22} dt_{Q22} - \int_{t_{Q31}} (s_{pm} - q_a) t_{Q31} dt_{Q31}$$
$$- \int_{t_{g_q}} q_a (t_{g_q} + t_{Q22} + t_{Q32}) + \int_{t_{g_q} + g_{pr} - Q11} q_a (t_{g_q} + t_{Q31}) dt_{g_q + g_{pr} - Q11}$$

The difference (leading-lagging) of left-turn vehicles which experience delay is:

$$V_{ld1} - V_{ld2} = Q_{i1} - Q_{i2} + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q + g_{pr} - Q11}} q_a dt_{g_q + g_{pr} - Q11} + \int_{t_{Q31}} q_a dt_{Q31} - \int_{t_{g_q}} q_a dt_{g_q} - \int_{t_{Q22}} q_a dt_{Q22} - \int_{t_{Q32}} q_a dt_{Q32} \tag{3-26}$$

$$V_{l1} - V_{l2} = Q_{i1} - Q_{i2} + \int_{t_{Q11}} q_a dt_{Q11} + \int_{t_{g_q + g_{pr} - Q11}} q_a dt_{g_q + g_{pr} - Q11} - \int_{t_{g_q}} q_a dt_{g_q} - \int_{t_{g_{pr}}} q_a dt_{g_{pr}} \tag{3-27}$$

Leading actuated through traffic delay and volume are:

$$D_{o1} = Q_{oi1} \times t_{r + g_{pr} + g_q} + \int_{t_{r+Q11}} q_o \times (t_{r+Q11} + t_{g_q + g_{pr} - Q11}) dt_{r+Q11}$$
$$- \int_{t_{g_q + g_{pr} - Q11}} (s_o - q_o) \times t_{g_q + g_{pr} - Q11} dt_{g_q + g_{pr} - Q11} \tag{3-28}$$

$$V_{od1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q} \tag{3-29}$$

$$V_{o1} = Q_{oi1} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u} \tag{3-30}$$

Lagging actuated through traffic delay and volume are:

$$D_{o2} = Q_{oi2} \times t_{r+g_{pr}+g_q} + \int_{t_{r+g_{pr}}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} - \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q} \tag{3-31}$$

$$V_{od2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q}} q_o dt_{r+g_{pr}+g_q} \tag{3-32}$$

$$V_{o2} = Q_{oi2} + \int_{t_{r+g_{pr}+g_q+g_u}} q_o dt_{r+g_{pr}+g_q+g_u} \tag{3-33}$$

So, the actuated through traffic delay difference (leading-lagging) is:

$$\begin{aligned} D_{o1} - D_{o2} = {} & \left(Q_{oi1} - Q_{oi2}\right) \times t_{r+g_{pr}+g_q} + \int_{t_{r+Q11}} q_o \times \left(t_{r+Q11} + t_{g_q+g_{pr}-Q11}\right) dt_{r+Q11} \\ & - \int_{t_{g_q+g_{pr}-Q11}} \left(s_o - q_o\right) \times t_{g_q+g_{pr}-Q11} dt_{g_q+g_{pr}-Q11} - \int_{t_{r+g_{pr}}} q_o \times \left(t_{r+g_{pr}} + t_{g_q}\right) dt_{r+g_{pr}} \\ & + \int_{t_{g_q}} \left(s_o - q_o\right) \times t_{g_q} dt_{g_q} \end{aligned} \tag{3-34}$$

$$Q_{oi1} = Max\left[0.0, \int_{t_{r+Q11}} q_o dt_{r+Q11} - \int_{t_{g_q}+g_{pr}-Q11} (s_o - q_o) dt_{g_{pm}+g_{pr}-Q11}\right]$$

(3-35)

$$Q_{oi2} = Max\left[0.0, \int_{t_{r+g_{pr}}} q_o dt_{r+g_{pr}} - \int_{t_{g_q}} (s_o - q_o) dt_{g_{pm}}\right] \qquad (3\text{-}36)$$

From above through traffic equations, when their $q_o$ have the same distribution, $Q_{oi1}<Q_{oi2}$, $V_{od1}<V_{od2}$, leading through traffic delay $D_{o1}<D_{o2}$.

When $D_1 - D_2 + D_{o1} - D_{o2} = D_i + D_r + D_{pr} + D_{pm} + D_{o1} - D_{o2} \geq 0$ , leading left-turn design is selected at actuated isolated intersection. Otherwise lagging left-turn design is preferred.

However, on the other hand, when the left-turn volumes of opposing directions are balanced, and there are many gaps in the through traffic (Figure 3-10 Case 4 +Case 4) so that the cycle with permissive-protected lagging can skip some protected left-turn time and improve intersection's traffic capacity. In this case, lagging may be recommended.

The Figure 3-11 and 3-12 show the difference of protected-permissive leading and permissive-protected lagging left-turn on actuated traffic condition. The study period is chosen in peak hour, cycle length is 54s, g/c for left-turn traffic is 0.59, g/c for through traffic is 0.46, through volume are 600 veh/h, the number of lanes is 4 for both directions,

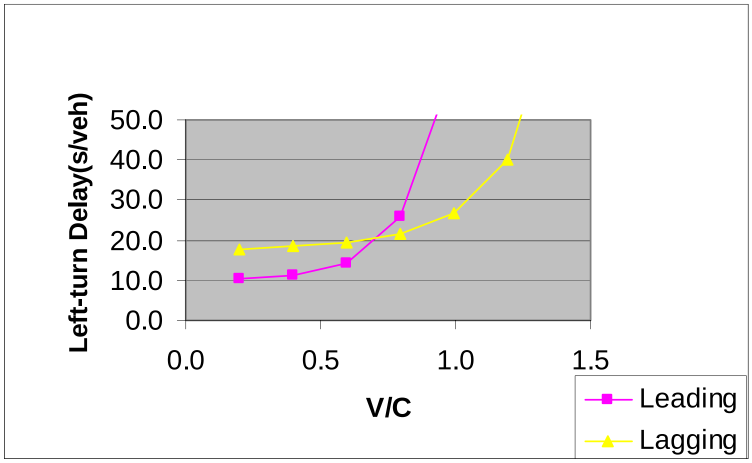



**Figure 3-11. The left-turn delay comparison for isolated actuated signal**

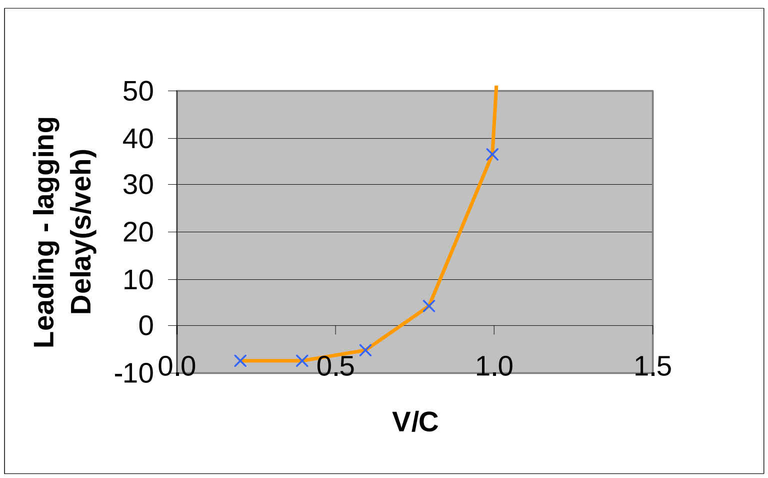



**Figure 3-12. The left-turn delay difference for isolated actuated signal**

and pedestrians are not included. From Figure 3-11 and 3-12 comparison results, leading left-turn delay is always lower than lagging left-turn delay at isolated actuated signalized intersection when left-turn v/c ratio is relatively low, while leading left-turn delay is higher than lagging left-turn delay when v/c ratio is close to or exceeds 1.0. Note that leading left-turn delay will be significantly higher than lagging left-turn delay at isolated actuated signalized intersection when left-turn v/c exceeds 1.0.

### 3.3.3 Queue length and storage time comparison at isolated intersection

From Chapter 3.1, leading left-turn maximum queue appears at $Q_{11}$ or $Q_{31}$:

At the end of the protected left-turn's effective red, the left-turn queue length and time to dissipate the left-turn queue are:

$$Q_{11} = q_a \times t_r + Q_{i1} \qquad \text{(3-37)}$$

$$t_{Q11} = Q_{11} / \left(s_{pr} - q_a\right)$$

At end of the blocked green time, the left-turn queue length and time to dissipate the leftturn queue are:

$$Q_{21} = Max\left\lfloor 0.0, Q_{11} - \left(s_{pr} - q_a\right) \times t_{g_{pr}} \right\rfloor$$

$$Q_{31} = Q_{21} + \left(q_a \times t_{g_q}\right) \qquad \text{(3-38)}$$

$$t_{Q31} = Q_{31} / (s_{pm} - q_a)$$

Lagging left-turn maximum queue appears at $Q_{22}$, left-turn queue at end of effective red continues to grow during the blocked green time. Left-turn queue length and time to clear are:

$$Q_{12} = q_a \times t_r + Q_{i2}$$

$$Q_{22} = Q_{12} + (q_a \times t_{g_q}) \tag{3-39}$$

$$t_{Q22} = Q_{22} / (s_{pm} - q_a)$$

Because lagging left-turn Q22 > leading left-turn Q11 or Q31, so lagging left-turn design needs more exclusive left-turn lane storage space, the difference length (lagging- leading) is:

$$Max\left[(q_a \times t_{g_q}), (s_{pr} - q_a) \times t_{g_{pr}}\right] \tag{3-40}$$

The difference clearing time (lagging- leading) is:

$$Max\left[(q_a \times t_r + Q_{i2} + q_a \times t_{g_q})/(s_{pm} - q_a) - (q_a \times t_r + Q_{i1})/(s_{pr} - q_a), ((s_{pr} - q_a) \times t_{g_{pr}})/(s_{pm} - q_a)\right]$$

(3-41)

Leading through traffic maximum queue is at end of the opposing movement's effective red,

$$Q_{o1} = q_o \times (t_r + t_{g_{pr}}) \tag{3-42}$$

Time to dissipate through traffic queue is:

$$T_{o1} = Q_{o1} / (s_o - q_o) = t_{g_q}$$

Lagging through traffic maximum queue is at end of the opposing movement's effective red,

$$Q_{o2} = q_o \times (t_r + t_{g_{pr}}) \qquad (3\text{-}43)$$

Time to dissipate through traffic queue is:

$$T_{o2} = Q_{o2} / (s_o - q_o) = t_{g_q}$$

From above through traffic equations, $Q_{o1}=Q_{o2}$, $T_{o1}=T_{o2}$. So, leading or lagging design does not affect through traffic operation.

# Chapter 4 Theoretical Analysis of Signal Left-turn Phase Operation at Coordinated Intersections

This research looks into the traffic flow pattern at two coordinated signalized intersections, compares the difference in delay based on leading and lagging left-turn signal phase designs, makes a general theoretical analysis of signal left-turn phase operation at both isolated and coordinated intersections, and gives a new model to guide the choosing between the leading and lagging left-turn sequences at coordinated signal intersections.

## 4.1 Traffic Flow Pattern Analysis

In Figure 4-1, the total number of vehicle arrivals is equal between the two cases. However the optimal signal timing could be significantly different. In top case, the demand occurs immediately following $t_i$, whereas there little demand immediately following $t_i$ and great demand in the future in bottom case. So, traffic flow pattern analysis is fundamentally important for signal timing and phasing design.

This study focuses on a pair of coordinated intersections on a major road and the road section between them. For coordinated intersection, the traffic pattern through the upstream intersection will largely affect the design of the downstream signal. Figure 4-2 maps the traffic pattern on the downstream section of the major road when the upstream

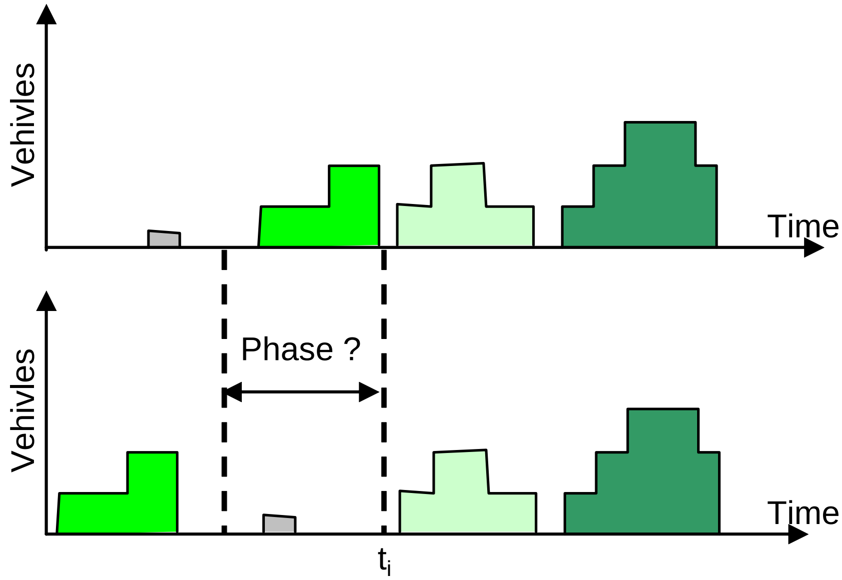



**Figure 4-1. The effect of traffic flow pattern**

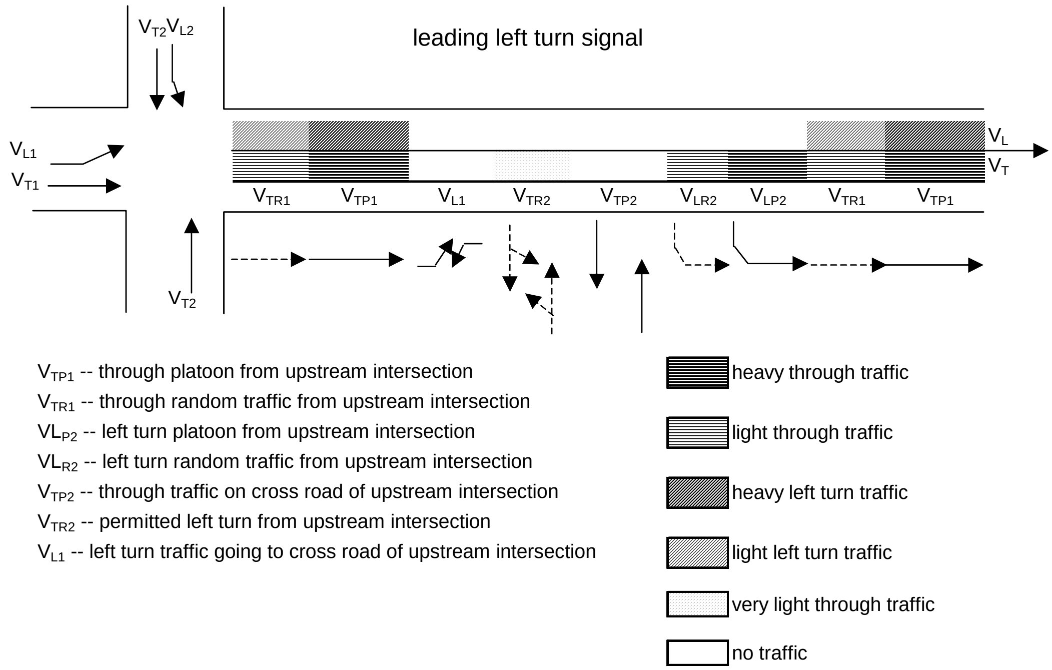



**Figure 4-2. Traffic pattern on the downstream section of the major road (A)**

intersection uses leading left turn phase design. At the start of a green phase for major road through traffic, vehicles usually depart to the downstream road section as a relatively dense platoon VTP1, which has a high flow rate. This may be partially due to the queue leftover from last signal cycle. Following this platoon, the traffic pattern becomes a less dense flow denoted as VTR1. In a leading design, the phase after this green time is the leading left turn phase for the minor road, which passes a platoon of vehicles VLP2 to the major road downstream section. Then during the green phase for minor road through traffic, a few more vehicles VLR2 may make left turn onto the major road. In a lagging design (Figure 4-3), VLP2 and VLR2 are coming before VTP1, VTR1. A percentage of the vehicles in VTP1, VTR1, VLP2 and VLR2 may make left turn at the downstream intersection. Therefore Figure 4-2 and Figure 4-3 map these left turning demand separately from the through demand traffic in order to exam their behavior in detail.

## 4.2 Traffic Flow Dispersion

Departing from the upstream intersection, these vehicle platoons will disperse on their way to the next intersection. The effect of vehicle bunching weakens as the platoon moves downstream, since vehicles in it travel at various speeds, spreading over the downstream road section. This phenomenon, known as platoon diffusion or dispersion, was modeled by Pacey [30]. He derived the travel time distribution f( $\tau$ ) along a road section assuming normally distributed speeds and unrestricted overtaking. According to Pacey's model:

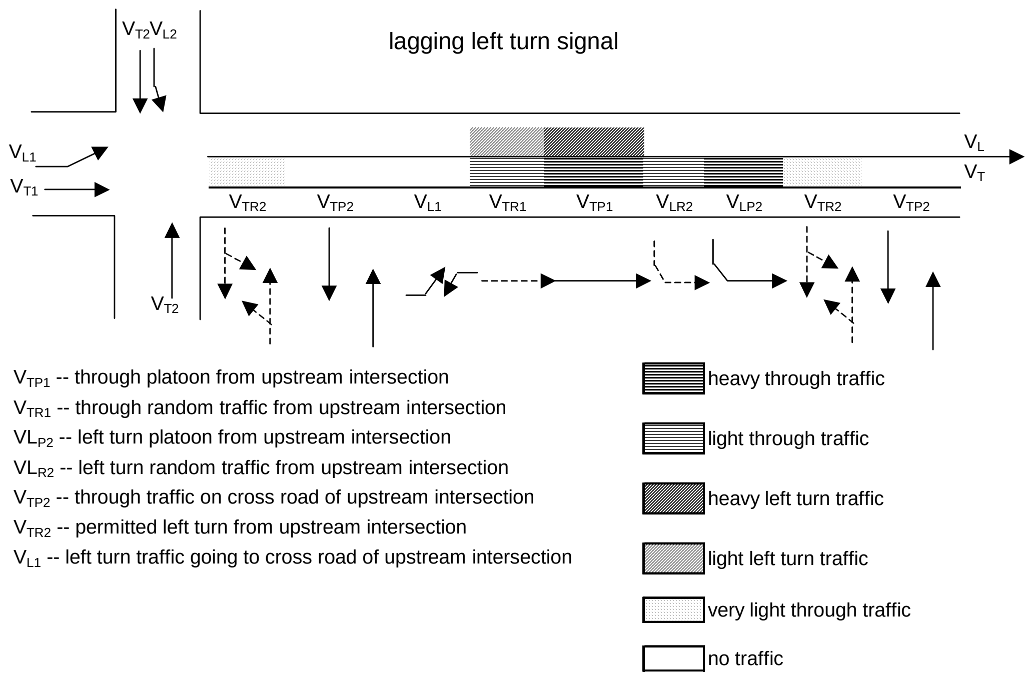



**Figure 4-3. Traffic pattern on the downstream section of the major road (B)**

$$q_2 = \int_{t_1} q_1 \times \{\frac{D}{\tau^2 \sigma \sqrt{2\pi}} \exp[-\frac{(\frac{D}{\tau} - \frac{D}{\tau'})^2}{2\sigma^2}]\} dt_1 \tag{4-1}$$

Where,

$q_2$ = The number of vehicles passing downstream intersection,

$q_1$ = The number of vehicles passing the upstream intersection,

$D$ = Distance between the upstream intersection and downstream intersection,

$\tau$ = Individual vehicle travel time along distance D,

$\tau'$ = Mean travel time, and

$\sigma$ = Standard deviation of speed.

Platoon diffusion effects were observed by Hillier and Rothery (1967) at several consecutive points located downstream of signals (Figure 4-4). They analyzed vehicle delays at fixed-timed signals using the observed traffic profiles and drew the following conclusions:

- the deterministic delay (first term in approximate delay formulae) strongly depends on the time lag between the start of the upstream and downstream green signals (offset effect);
- the minimum delay, observed at the optimal offset, increases substantially as the distance between signals increases; and

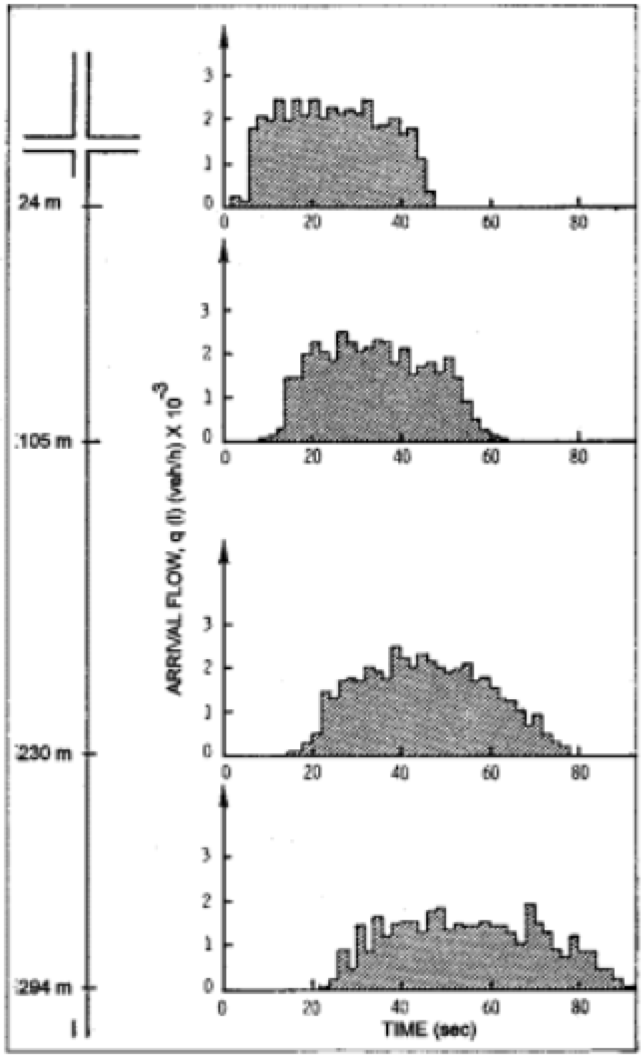



**Figure 4-4. Observed traffic flow dispersion**

- the signal offset does not appear to influence the overflow delay component.

When the distance between the upstream intersection and downstream intersection is small, which is usually the case for coordinated intersections, the diffusion would also be small so that $q_2 \Rightarrow q_1$.

## 4.3 Comparison of Leading and Lagging Signal Designs

To compare the delay of leading and lagging signals, arrival/departure diagrams are used in Figure 4-5 ~ Figure 4-8 to illustrate the delays at the downstream intersection. Signal phases are shown in green or red color while yellow time was purposely left out to simplify the diagram. It makes sense to argue that since VLP2 and VLR2 are left turned onto the major road, the likelihood of them turn to left again on the immediate downstream intersection is low. Also they usually will have fewer vehicles than VTP1 and VTR1, so that the left turn demand among them, if any, would be much less than that of VTP1 and VTR1. Therefore in order to simplify the diagrams, this portion of the left turning demand was not plotted out in these figures. This will not change the result of the comparison.

Figure 4-5 and 4-6 are for the condition when upstream intersection uses leading design. Figure 4-7 and Figure 4-8 are for conditions when the upstream intersection uses lagging

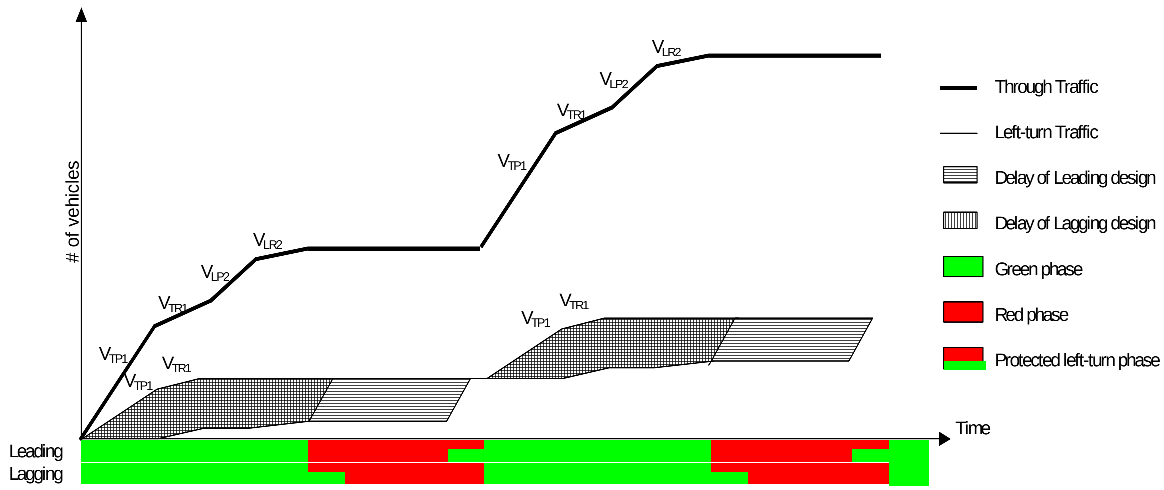



**Figure 4-5. Traffic arrival and departure diagram at the downstream intersection on major road (A) [a,b]**

a. Upstream signal using leading left turn phase

b. Downstream signal design optimize service for through traffic on the major street

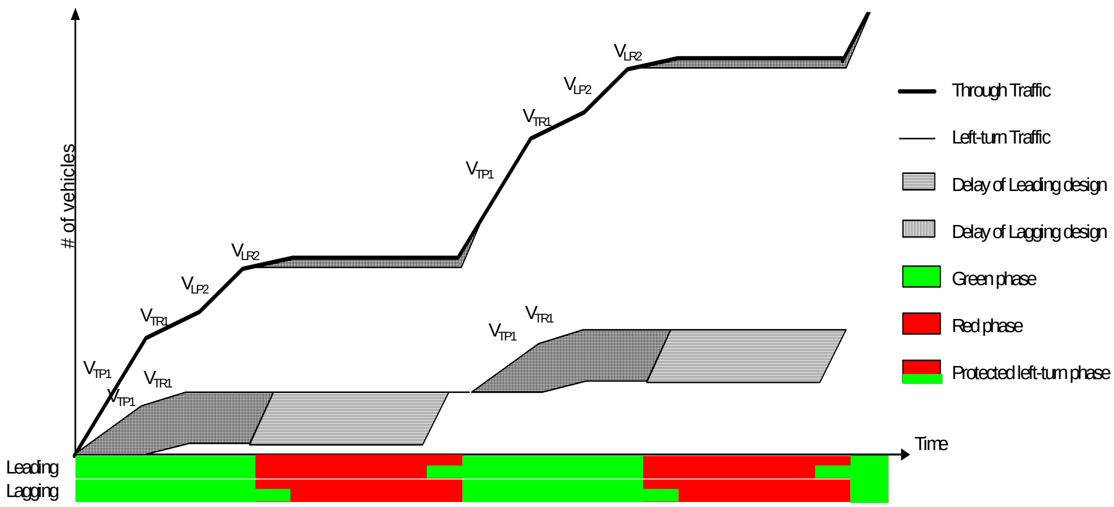



**Figure 4-6. Traffic arrival and departure diagram at the downstream intersection on major road (B)** [a,b]

a. Upstream signal using leading left turn phase

b. Downstream signal design cannot optimize service for through traffic on the major street

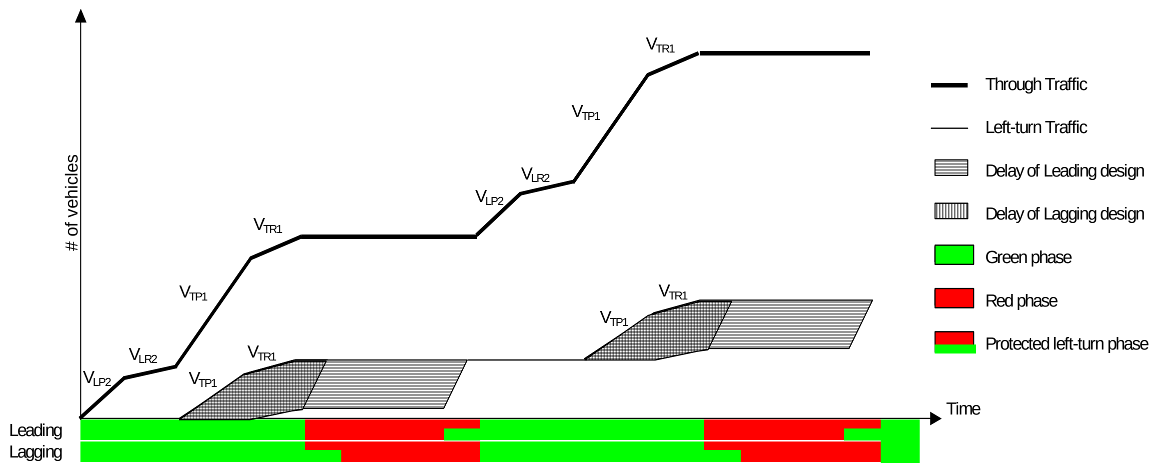



**Figure 4-7. Traffic arrival and departure diagram at the downstream intersection on major road (C) [a,b]**

a. Upstream signal using lagging left turn phase

b. Downstream signal design optimize service for through traffic on the major street

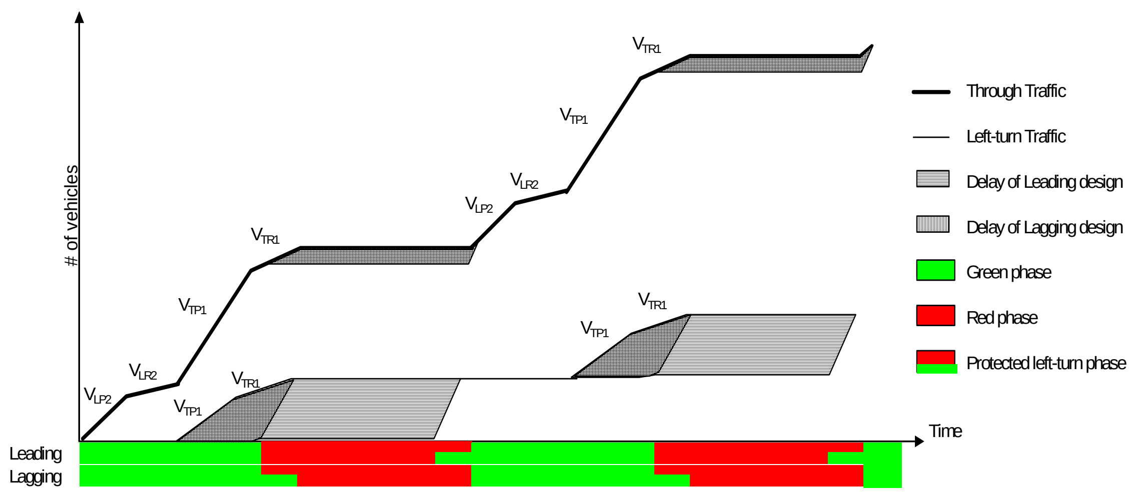



**Figure 4-8. Traffic arrival and departure diagram at the downstream intersection on major road (D) [a,b]**

a. Upstream signal using lagging left turn phase

b. Downstream signal design cannot optimize service for through traffic on the major street design.  Figure 4-5 and Figure 4-7 deal with an ideal condition, under which the signal design optimizes the service for major road through traffic at this downstream intersection.  This may happen when the intersected minor road has relatively lighter traffic.  Figure 4-6 and Figure 4-8 then assume the signal design causes unmet through demand on the major road in each cycle.  While to illustrate this situation, the red phase can be "stretched" longer towards both direction on the diagram. However, it is clear that the through traffic delay would be much less when the red phase avoid the heavy arrival "head", but tackle the light arrival "tail" instead.  Therefore, Figure 4-5 and 4-8 only plotted the latter design.

If actuated designs are used for both intersections, as discussed at the start of this research, leading design has the advantage of possible phase overlapping, while lagging design has the potential of phase skipping.  Both conditions depend on the requirements of enough gaps in the through traffic for left turning vehicles or very light left turn demand.  The difference is overlapping in leading design can happen when only one direction met this criterion, but lagging design must met it on both directions to warrant a phase skipping.  From the delay diagrams, it is reasonable to elaborate that lagging would still be better than leading if the traffic on both directions are near balanced.  For very unbalanced traffic, while gives more chance of phase overlapping in leading design than phase skipping in lagging design..

From Figure 4-5 ~ Figure 4-8 we can conclude:

1. Lagging design for the downstream signal generates less delay than leading design no matter which design was used for the upstream signal.

2. Lagging (for the upstream signal) + lagging (for the downstream signal) design for the two coordinated intersections gives the best result in terms of intersection delay.

3. Leading or lagging designs do not differ in terms of through traffic delays. Instead, their strength/weakness are due to the left turning traffic delay.

## 4.4 Left-turn Phase Selection Model

Based on above analysis, the differences between leading and lagging left turns at two coordinated intersections can be expressed as equation 4-2 and 4-3.

$$D_1 = T_R \times [\alpha \times \int_{t_{TP1}} q_{t_{TP1}} dt + (\alpha - \beta_1) \times \int_{t_{TR1}} q_{t_{TR1}} dt - \beta_2 \times \int_{t_{LR2}} q_{t_{LR2}} dt] \quad (4\text{-}2)$$

$$D_2 = T_R \times [\alpha \times \int_{t_{TP1}} q_{t_{TP1}} dt + (\alpha - \beta_1) \times \int_{t_{TR1}} q_{t_{TR1}} dt] \quad (4\text{-}3)$$

Where,

$D_1$ = delay difference between leading and lagging design at the downstream intersection when upstream intersection uses leading design,

$D_2$ = delay difference between leading and lagging design at the downstream intersection when upstream intersection uses lagging design,

$T_R$ = red phase along the major road at downstream intersection,

$\alpha$ = percentage of left turn traffic in $V_{TP1}$ and $V_{TR1}$ at downstream intersection,

$\beta_1$ = coefficient of gap availability for left turns in $V_{TR1}$,

$\beta_2$ = coefficient of gap availability for left turns in$V_{LR2}$,

$q_{t_{TP1}}$ = number of vehicles arrived at downstream intersection during $t_{TP1}$,

$q_{t_{TR1}}$ = number of vehicles passed downstream intersection during $t_{TR1}$,

$q_{t_{LR2}}$ = number of vehicles passed downstream intersection during $t_{LR2}$,

Let $Q_1 = \alpha \times (\int_{t_{TP1}} q_{t_{TP1}} dt + \int_{t_{TR1}} q_{t_{TR1}} dt)$, $Q_2 = \beta_1 \times \int_{t_{TR1}} q_{t_{TR1}} dt$, and $Q_3 = \beta_2 \times \int_{t_{LR2}} q_{t_{LR2}} dt$, Equation 4-2 and Equation 4-3 are simplified into the following formats.

$$D_1 = \begin{cases} T_R \times [Q_1 - Q_2 - Q_3] & \text{if } Q_1 > Q_2 + Q_3 \\ 0 & \text{if } Q_1 \le Q_2 + Q_3 \end{cases} \tag{4-4}$$

$$D_2 = \begin{cases} T_R \times [Q_1 - Q_2] & \text{if } Q_1 > Q_2 \\ 0 & \text{if } Q_1 \le Q_2 \end{cases} \tag{4-5}$$

When $Q_1 > Q_2 + Q_3$, $D_2 \ge D_1 > 0$. This means when gaps in trough traffic are less than the left-turning demand, lagging design for the downstream signal always generates less delay than leading design. And the best combination is lagging (upstream) + lagging (downstream).

When $Q_2 + Q_3 \geq Q_1 > Q_2$, $D_2 > D_1 = 0$. This means lagging design for the downstream signal generates less delay than leading design when the upstream signal uses lagging left turn design. But if the upstream signal uses leading left turn design, there would be no significant difference between leading and lagging design at the downstream intersection.

When $Q_2 \geq Q_1$, $D_1 = D_2 = 0$. When available gaps are more than the left-turning demand, there would be no significant difference between leading and lagging signal designs. In fact, under this situation, fixed left turn phase would not be necessary.

The following Figure 4-9 shows the difference of protected-permissive leading and permissive-protected lagging left-turn at coordinated traffic condition. The study period is chosen in peak hour, cycle length is 54s, g/c for left-turn traffic is 0.59, g/c for through traffic is 0.46, through volume are 600 veh/h, the number of lanes is 4 for both directions, and pedestrians are not included.

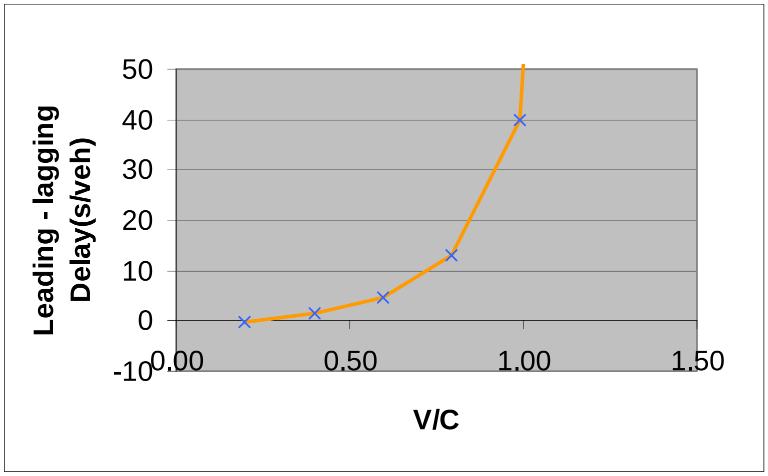



**Figure 4-9. The left-turn delay difference for coordinated signals**

# Chapter 5 Traffic Control Delay Components and Left-turn Control Factors Analysis

Chapter 5 discusses the components of intersection traffic control delay, simplifies the delay equations from above chapters, and identifies main left-turn control factors, analyzes the influences for leading and lagging left-turn.

## 5.1 Control Delay Components

The values derived from the delay calculations represent the average control delay experienced by all vehicles that arrive in the analysis period, including delays that are incurred beyond the analysis period when the lane group is oversaturated. Control delay includes movements at slower speeds and stops on intersection approaches, as vehicles move up in queue position or slow down upstream of an intersection.

For simplifying the traffic delay analysis, HCM2000 gives the following equations to estimate the average control delay per vehicle for a given lane group.

$$d = d_1 \times PF + d_2 + d_3 \tag{5-1}$$

where

$d$ = control delay per vehicle, s/veh,

$d_1$ = uniform control delay assuming uniform arrivals, s/veh,

$PF$ = uniform delay progression adjustment factor which accounts for the effects of signal progression,

$d_2$ = incremental delay to account for the effect of random and oversaturation queues, adjusted for the duration of the analysis period and the type of signal control. This delay component assumes that there is no residual demand for the lane group at the start of the analysis period, s/veh, and

$d_3$ = supplemental delay to account for oversaturation queues that may have existed prior to the analysis period, s/veh.

### 5.1.1 Estimation uniform delay $d_1$

Equation 5-2 gives an estimate of delay assuming uniform arrivals, stable flow, and no initial queue. It is based on the first term of Webster's delay formulation and is widely accepted as an accurate depiction of delay for the idealized case of uniform arrivals. This equation can be used for permitted only left-turn and through traffic delay estimation.

$$d_1 = \frac{0.5 \times C \times \left(1 - g/C\right)^2}{1 - \left[Min(1, X) \times g/C\right]} \tag{5-2}$$

where

$d_1$ = uniform control delay assuming uniform arrivals, s/veh,

$C$ = cycle length, s; cycle length used in pretimed signal control, or average cycle length for actuated control,

$g$ = effective green time for lane group, s; green time used in pretimed signal control, or average lane group effective green time for actuated control, and

$X$ = v/c ratio or degree of saturation for lane group.

However, Equation 5-2 cannot be used for protected plus permitted left-turn or permitted plus protected left-turn delay estimation. The following is a simplified method to compute uniform delay for protected plus permitted or permitted plus protected left-turn operation. When traffic flow is uniform distribution, the Equation 3-1 ~ 3-12 can be expressed as:

Leading left-turn design:

At the end of the protected left-turn's effective red, the left-turn queue length and time to dissipate the left-turn queue are:

$$Q_{11} = q_a \times t_r + Q_{i1}$$

$$t_{Q11} = Min\left[Q_{i1} / \left(s_{pr} - q_a\right), t_{g_{pr}}\right]$$

Left-turn queue begins to dissipate at, $s_{Pr}$, and will completely dissipate if $t_{Q1}$ is less than $g_{Pr}$.

$$Q_{21} = \begin{cases} q_a \times \left(t_r + t_{g_{pr}}\right) - s_{pr} \times t_{g_{pr}} + Q_{i1} & \text{if } t_{Q11} = t_{g_{pr}} \\ 0 & \text{if } t_{Q11} < t_{gpr} \end{cases}$$

Permitted left-turn capacity occurs during unblocked green time

$$t_{g_u} = Max\left(0.0, t_{g_{pm}} - t_{g_q}\right)$$

At end of the blocked green time, the left-turn queue length and time to dissipate the leftturn queue are:

$$Q_{31} = Q_{21} + \left(q_a \times t_{g_q}\right)$$

$$t_{Q31} = Min\left[Q_{31} / \left(s_{pm} - q_a\right), t_{g_u}\right]$$

Queue length at end of unblocked green time is:

$$Q_{41} = Max\left[0.0, Q_{31} - \left(s_{pm} - q_a\right) \times t_{g_u}\right]$$

Final queue length is reduced by the average number of sneakers, $n_f$, per cycle

$$Q_{51} = Max\left\lfloor 0.0, Q_{41} - n_f \right\rfloor = Q_{i1}$$

So, the leading left-turn traffic delay and volume are:

$$D_1 = Q_{i1} \times \left(t_r + t_{Q11}\right) + Q_{21} \times \left(t_{g_q} + t_{Q_{31}}\right) + \frac{1}{2} \times \begin{bmatrix} q_a \times \left(t_r{}^2 + t_{g_q}{}^2 + t_{Q11} \times t_r + t_{Q31} \times t_{g_q}\right) \\ -\left(s_{pr} - q_a\right) \times t_{Q11}{}^2 - \left(s_{pm} - q_a\right) \times t_{Q31}{}^2 \end{bmatrix}$$

(5-3)

$$Q_{21} = \begin{cases} q_a \times \left(t_r + t_{g_{pr}}\right) - s_{pr} \times t_{g_{pr}} + Q_{i1} & \text{if } t_{Q11} = t_{g_{pr}} \\ 0 & \text{if } t_{Q11} < t_{gpr} \end{cases}$$

$$V_{ld1} = Q_{i1} + q_a \times \left(t_r + t_{Q11} + t_{g_q} + t_{Q31}\right) \tag{5-4}$$

$$V_{l1} = Q_{i1} + q_a \times \left(t_r + t_{g_{pr}} + t_{g_q} + t_{g_u}\right) \tag{5-5}$$

Lagging left-turn design:

Left-turn queue at end of effective red continues to grow during the blocked green time. Left-turn queue length and time to clear are:

$$Q_{12} = q_a \times t_r + Q_{i2}$$

$$Q_{22} = Q_{12} + \left(q_a \times t_{g_q}\right)$$

$$t_{Q22} = Min\left\lfloor Q_{22} / \left(s_{pm} - q_a\right), t_{g_u} \right\rfloor$$

Queue length and time to clear at end of the permitted phase (beginning of the protected phase) are:

$$Q_{32} = Max\left\lfloor 0.0, Q_{22} - \left(s_{pm} - q_a\right) \times t_{g_u} \right\rfloor$$

$$t_{Q32} = Min\left\lfloor Q_{32} / \left(s_{pr} - q_a\right), t_{g_{pr}} \right\rfloor$$

Queue length at end of protected green phase is:

$$Q_{42} = Max\left\lfloor 0.0, Q_{32} - \left(s_{pr} - q_a\right) \times t_{g_{pr}} \right\rfloor = Q_{i2}$$

In this case, there are no sneakers.

Lagging left-turn traffic delay and volume are:

$$D_2 = Q_{i2} \times \left(t_r + t_{g_q} + t_{Q22} + t_{Q32}\right) + \frac{1}{2} \times \left[\begin{array}{l} q_a \times \left(t_r^2 + t_{g_q} \times t_r + t_{Q22} \times t_r + t_{Q32} \times t_r + t_{g_q}^{\ 2} + t_{Q22} \times t_{g_q} + t_{Q32} \times t_{g_q}\right) \\ -\left(s_{pm} - q_a\right) \times \left(t_{Q32} \times t_{g_u} + t_{Q22}^{\ 2}\right) - \left(s_{pr} - q_a\right) \times t_{Q32}^{\ 2} \end{array}\right]$$

(5-6)

$$V_{ld2} = Q_{i2} + q_a \times \left(t_r + t_{Q22} + t_{g_q} + t_{Q32}\right) \qquad (5\text{-}7)$$

$$V_{l2} = Q_{i2} + q_a \times \left(t_r + t_{g_{pr}} + t_{g_q} + t_{g_u}\right) \tag{5-8}$$

Leading through traffic delay and volume are:

$$D_{o1} = Q_{oi1} \times t_{r+g_{pr}+g_q} + \frac{1}{2} \times \left[q_o \times t_{r+g_{pr}+g_q} \times t_{r+g_{pr}} - \left(s_o - q_o\right) \times t_{g_q}{}^2\right] \tag{5-9}$$

$$V_{od1} = Q_{oi1} + q_o \times t_{r+g_{pr}+g_q} \tag{5-10}$$

$$V_{o1} = Q_{oi1} + q_o \times t_{r+g_{pr}+g_q+g_u} \tag{5-11}$$

Lagging through traffic delay is:

$$D_{o2} = Q_{oi2} \times t_{r+g_{pr}+g_q} + \frac{1}{2} \times \left[q_o \times t_{r+g_{pr}+g_q} \times t_{r+g_{pr}} - \left(s_o - q_o\right) \times t_{g_q}{}^2\right] \tag{5-12}$$

$$V_{od2} = Q_{oi2} + q_o \times t_{r+g_{pr}+g_q} \tag{5-13}$$

$$V_{o2} = Q_{oi2} + q_o \times t_{r+g_{pr}+g_q+g_u} \tag{5-14}$$

From above through traffic Equations 5-19 ~ 5-14, when fixed-time signal is designed, $Q_{oi1}$=$Q_{oi2}$, $V_{od1}$=$V_{od2}$, $V_{o1}$=$V_{o2}$, so leading or lagging design does not affect through traffic delay, When traffic signal is actuated, the difference is depended on the real traffic condition, the detail analysis is in Chapter 3.3.2.

When traffic condition is unsaturated and traffic flow is uniform distribution, the left-turn traffic delay is:

Case 1 (leading left-turn):

$$d_1 = \frac{0.5}{C}\left[t_r^{\ 2} + \frac{q_a t_r^{\ 2}}{s_{pr} - q_a} + t_{g_q}^{\ 2} + \frac{q_a t_{g_q}^{\ 2}}{s_{pm} - q_a}\right] \tag{5-15}$$

Case 2 (leading left-turn):

$$d_1 = \frac{0.5}{q_a C}\left[q_a t_r^{\ 2} + 2q_a t_r\left(t_{g_{pr}} + t_{g_q}\right) - \left(s_{pr} - q_a\right)\left(t_{g_{pr}}^{\ 2} + 2t_{g_q} t_{g_{pr}}\right) + q_a t_{g_q}^{\ 2} + \frac{\left(q_a t_r - \left(s_{pr} - q_a\right)t_{g_{pr}}\right)^2}{s_{pm} - q_a}\right] \tag{5-16}$$

Case 3 (leading left-turn):

$$d_1 = \frac{0.5}{q_a C}\left[q_a t_r^{\ 2} + 2q_a t_{g_q}\left(t_r + t_{g_u}\right) - \left(s_{pr} - q_a\right)\left(t_{g_u}^{\ 2} + 2t_{g_u} t_r\right) + q_a t_{g_q}^{\ 2} + \frac{\left(q_a\left(t_r + t_{g_q}\right) - \left(s_{pm} - q_a\right)t_{g_u}\right)^2}{s_{pr} - q_a}\right] \tag{5-17}$$

Case 4 (lagging left-turn):

$$d_1 = \frac{0.5}{C}\left[\frac{s_{pm}}{s_{pm} - q_a}\left(t_r + t_{g_q}\right)\right] \tag{5-18}$$

Case 5 (lagging left-turn):

$$d_1 = \frac{0.5}{q_a C}\left[ q_a t_r^{\,2} + 2 q_a t_r \left(t_{g_q} + t_{g_u}\right) - \left(s_{pr} - q_a\right) t_{g_u}^{\,2} + q_a t_{g_q}^{\,2} + \frac{\left(q_a \left(t_r + t_{g_q}\right) - \left(s_{pm} - q_a\right) t_{g_u}\right)^2}{s_{pr} - q_a} \right]$$

(5-19)

5.1.2 Estimation incremental delay $d_2$

Equation 5-20 is used to estimate the incremental delay due to non-uniform arrivals and temporary cycle failures (random delay) as well as delay caused by sustained periods of oversaturation (oversaturation delay). It is sensitive to the degree of saturation of the lane group (X), the duration of the analysis period (T), the capacity of the lane group (c) and the type of signal control, as reflected by the control parameter (k). The equation assumes that there is no unmet demand which causes residual queues at the start of the analysis period (T).

$$d_2 = 900T\left[ (X-1) + \sqrt{(X-1)^2 + \frac{8kIX}{cT}} \right] \qquad (5\text{-}20)$$

where

$d_2$ = incremental delay to account for the effect of random and oversaturation queues, adjusted for the duration of the analysis period and the type of signal control. This delay component assumes that there is no residual demand for the lane group at the start of the analysis period, s/veh,

$T$ = duration of analysis period, h,

$k$ = incremental delay factor that is dependent on controller settings,

$I$ = upstream filtering/metering adjustment factor,

$c$ = lane group capacity in veh/h, and,

$X$ = lane group v/c ratio, or degree of saturation.

5.1.3 Estimation supplement delay $d_3$

A generalized form of d3 appears as Equation 5-21. It provides estimation of the supplemental control delay per vehicle (in seconds) when an initial queue of size Qb is present at the start of the analysis period T.

$$d_3 = [1800 Q_b (1+u) t] / cT \tag{5-21}$$

where

$Q_b$ = initial queue at the start of period T,veh,

$c$ = adjusted lane group capacity, veh/h,

$T$ = duration of the analysis period, h,

$t$ = duration of unmet demand in T, h, and

$u$ = delay parameter.

The parameters t and u are determined according to the prevailing case. Equations 5-22 and 5-23 may be used to estimate the values for cases III, IV, and V:

if $Q_b = 0$ then $t = 0$

else $t = Min\left[T, \frac{Q_b}{c(1 - Min(1, X))}\right]$ (5-22)

if $t < T$ then $u = 0$

else $u = 1 - \frac{cT}{Q_b}[1 - Min(1, X)]$ (5-23)

In addition to the supplemental delay term, the analyst may be interested in computing the time at which the last vehicle which arrives during the analysis period clears the intersection (measured from the start of the time period T) due to the presence of an initial queue of length Qb. This time is referred to as the supplemental clearing time, Tc. In cases I, II, III, all vehicles will clear at the end of the period T (in addition to the normal delays d1 + d2). For cases IV and V, the last vehicle arriving in T will clear the intersection at time Tc > T (again, in addition to d1 + d2). Therefore, a general formula for the supplemental clearing time in the case of an initial queue, measured from the start of the analysis period, T is given as Equation 5-24:

$T_c = Max\left(T, \frac{Q_b}{c} + TX\right)$ (5-24)

Note that in order to decide whether case III (t < T) or IV (t = T) applies, the value of t must first be computed from Equation 5-22. For cases III, IV, and V, the uniform control delay component (d1) must be evaluated using X = 1.0 for the period when an oversaturation queue exists (t) and using the actual X value for the remainder of the analysis period (T-t). Therefore, in these cases, a time weighted value of d1 is to be used as shown in Equation 5-25.

$$d_1 = d_s \times \frac{t}{T} + d_u \times PF \times \frac{T - t}{T} \qquad (5\text{-}25)$$

where

$d_s$ = the saturated delay (d1 evaluated for X = 1.0), and

$d_u$ = the undersaturated delay (d1 evaluated for the actual X value).

## 5.2 Left-turn Control Factors

From Equation 5-1 ~ Equation 5-25, the factors that influence traffic delay are progression adjustment factor (PF), Incremental Delay Calibration Factor (k), traffic arriving rate ($q_{a)}$, protect departure rate ($s_{pr)}$, permitted departure rate ($s_{pm)}$, capacity (c), effective green time ($t_{gpr}$, $t_{gpm}$)and red time ($t_r$).

5.2.1 Progression adjustment factor

Good signal progression will result in a high proportion of vehicles arriving on the green. Poor signal progression will result in a low proportion of vehicles arriving on the green. Progression primarily affects uniform delay, and for this reason, the adjustment is applied only to d1. The value of PF may be determined by Equation 5-26:

$$PF = \frac{(1-P)f_{PA}}{1-g/C} \tag{5-26}$$

where

$PF$ = progression adjustment factor,

$P$ = proportion of vehicles arriving on the green,

$g/C$ = proportion of green time available, and,

$f_{PA}$ = supplemental adjustment factor for platoon arriving during the green.

An important traffic characteristic that must be quantified to complete an operational analysis of a signalized intersection is the quality of the progression. The parameter that describes this characteristic is the arrival type (AT) for each lane group. According to HCM2000, six arrival types for the dominant arrival flow are defined.

Arrival Type 1: Dense platoon, containing over 80 percent of the lane group volume, arriving at the start of the red phase. This AT is representative of network links that may experience very poor progression quality as a result of conditions such as overall network signal optimization.

Arrival Type 2: Moderately dense platoon arriving in the middle of the red phase or dispersed platoon, containing 40 to 80 percent of the lane group volume, arriving throughout the red phase. This AT is representative of unfavorable progression on two-way streets.

Arrival Type 3: Random arrivals in which the main platoon contains less than 40 percent of the lane group volume. This AT is representative of operations at isolated and non-interconnected signalized intersections characterized by highly dispersed platoons. It may also be used to represent coordinated operation in which the benefits of progression are minimal.

Arrival Type 4: Moderately dense platoon arriving in the middle of the green phase or dispersed platoon, containing 40 to 80 percent of the lane group volume, arriving throughout the green phase. This AT is representative of favorable progression on a two-way street.

Arrival Type 5: Dense to moderately dense platoon, containing over 80 percent of the lane group volume, arriving at the start of the green phase. This AT is representative of highly favorable progression quality, which may occur on routes with low to moderate side-street entries and which receive high priority treatment in the signal timing plan.

Arrival Type 6: This arrival type is reserved for exceptional progression quality on routes with near-ideal progression characteristics. It is representative of very dense platoons progressing over a number of closely spaced intersections with minimal or negligible side-street entries.

The arrival type should be determined as accurately as possible because it will have a significant impact on delay estimates and LOS determination. Although there are no definitive parameters to precisely quantify arrival type, HCM2000 recommends using the platoon ratio computed by Equation 5-27 to determine the arrival type:

$$R_P = \frac{PC}{g_i} \tag{5-27}$$

where

$R_P$ = platoon ratio,

$P$ = proportion of all vehicles in movement arriving during the green phase,

$C$ = cycle length, s, and

$g_i$ = effective green time for the movement/lane group, s.

P may be estimated or observed in the field, whereas gi and C are computed from the signal timing.  The value of P may not exceed 1.0.

When estimating delay for future situations involving coordination, it is advisable to assume Arrival Type 4 as a base condition for coordinated lane groups (except left turns). Arrival Type 3 should be assumed for all uncoordinated lane groups.  Movements made from exclusive left-turn lanes on protected phases are not usually provided with good progression.  Thus, Arrival Type 3 is usually assumed for coordinated left turns.

5.2.2 Incremental delay calibration factor

The calibration term (k) is included in Equation 5-28 to incorporate the effect of controller type on delay.

$$k = (1 - 2k_{\min})(X - 0.5) + k_{\min} \qquad (5\text{-}28)$$

For fixed time signals, a value of k = 0.50 is recommended by HCM2000.  This is based on a queuing process with random arrivals and uniform service time equivalent to the lane group capacity.  Actuated controllers, on the other hand, have the ability to tailor the green time to traffic demand, thus reducing incremental delay.  The delay reduction depends in part on the controller's unit extension (UE), and the prevailing v/c ratio. Recent research indicates that lower unit extensions (i.e., snappy intersection operation) result in lower values of k and d2. However, when v/c approaches 1.0, an actuated controller will tend to behave in a manner similar to a fixed time controller. Thus, the (k) parameter will converge to the fixed time value of 0.50 when demand equals capacity.

### 5.2.3 Arrival, departure rate and green time

In HCM2000, protected-plus-permitted phases are analyzed by separating the portions of the phase into two lane groups for the sake of analysis. Each portion of the phase is then handled as if the other were not present. The protected portion of the phase is treated as a protected phase. The permitted portion of the phase is treated as a permitted phase.

By doing this, separate saturation flow rates may be computed for each portion of the phase.

- The first portion of the phase, whether protected or permitted, is assumed to be fully utilized, that is, to have a v/c of 1.0, unless total demand is insufficient to use the capacity of that portion of the phase.

- Any remaining demand not handled by the first portion of the phase is assigned to the second portion of the phase, whether protected or permitted.

Arrival rate is determined in Equation 5-29:

$$q_a = \frac{v}{3600 \times Max(X, 1.0)} \quad (5\text{-}29)$$

where X is lane group v/c ratio, or degree of saturation.

Two departure rates are determined in Equation 5-30 and Equation 5-31:

• the protected-phase departure rate,

$$s_{pr} = \frac{s}{3600} \quad (5\text{-}30)$$

where s is saturation flow rate for the protected phase; and

• the permitted-phase departure rate,

$$s_{pm} = \frac{s\left(t_{g_q} + t_{g_u}\right)}{t_{g_u} \times 3600} \quad (5\text{-}31)$$

where s is the adjusted saturation flow rate for the permitted phase.

Since permitted departure rate has significant relationship with green time, so the green time difference between leading and lagging left-turn is discussed below. For exclusive lane operation, the leading green Figure 5-1, G1, is followed by G/Y1, a period during which the left-turn change and clearance interval is displayed, and the through movement continues with a green ball indication. G2 has a green ball indication for both the through and left-turn movements, followed by a full change and clearance interval for all north-south movements, Y2. The effective green time for the permitted phase, g*, is equal to G2 + Y2 for the NB direction. Note that there is no lost time for the NB movements, since both were initiated in the leading phase, and the lost time is assessed there. For the NB phase, gq is referenced to the beginning of the opposing (SB) effective green. Again, the value needed is the portion of the NB g* blocked by the clearance of the opposing queue. Because the NB effective green (g*) does not account for lost time, $gq^* = gq + t_L$. On the other hand, the lagging green Figure 5-2, G1, is followed by G/Y1, a period during which the left-turn change and clearance interval is displayed, and the through movement continues with a green ball indication. G2 has a green ball indication for the through traffic and a green arrow for the left-turn movement, followed by a full change and clearance interval for all north-south movements, Y2. The effective green time for the NB permitted phase, g*, is equal to $G1 + G/Y1\text{-}t_L$ for the NB direction. The gq is referenced to the beginning of the opposing (SB) effective green. The NB effective green $gq^* = gq$.

When the phases time are same between leading and lagging signal, the time $t_r$ and $tg_o$ will be same, so the through traffic will not affect left turn delay between them. However the lagging phase $tg_{pr}$ will be leading $tg_{pr}$ plus yellow and clearing time, lagging $tg_{pm}$ will be leading $tg_{pm}$ minus yellow and clearing time. Since $tg_q$ is decided by $V_{olc}$,$q_{ro}$,$tg_o$,$t_L$, the

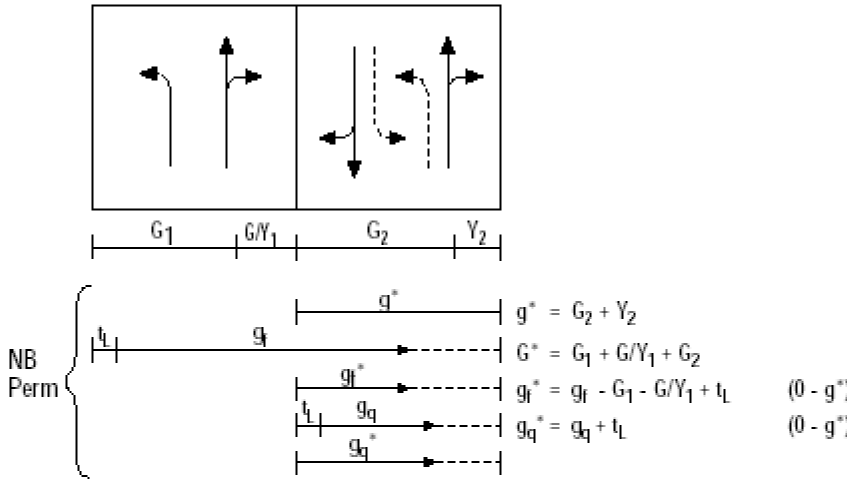



**Figure 5-1. Green time for leading green**

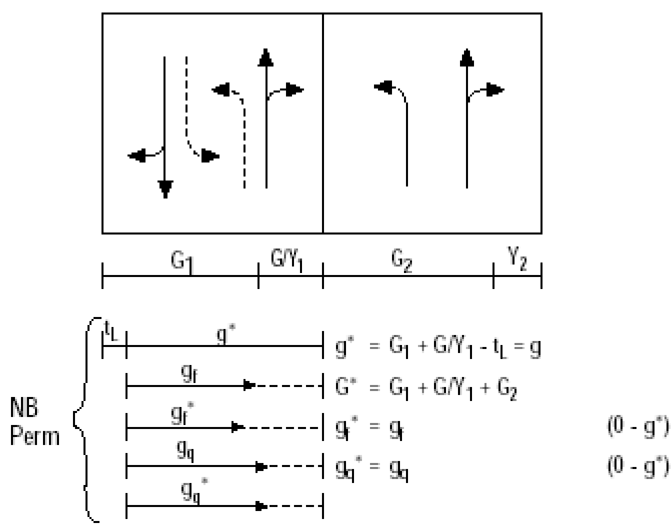



**Figure 5-2. Green time for lagging green**

leading $g_q$ will be same with lagging $g_q$. So leading $tg_u = tg_{pm} - tg_q$ is greater than lagging $tg_u$ when $tg_u > 4$ second, otherwise they have same $tg_u = 4$ seconds and leading $tg_q = tg_{pm} - tg_u$ will be greater than lagging $tg_q$.

### 5.2.4 Capacity and v/c factor

Capacity at signalized intersections is based upon the concept of saturation flow and saturation flow rate. The flow ratio for a given lane group is defined as the ratio of the actual or projected demand flow rate for the lane group (vi) and the saturation flow rate (si). The flow ratio is given the symbol (v/s)i for lane group i. The capacity of a given lane group may be stated as shown in Equation 5-32:

$$c_i = s_i \frac{g_i}{C} \qquad (5\text{-}32)$$

where

$c_i$ = capacity of lane group i, veh/h,

$s_i$ = saturation flow rate for lane group i, veh/h,

$g_i / C$ = effective green ratio for lane group i.

The ratio of flow rate to capacity (v/c), often called the volume-to-capacity ratio, is given the symbol X in intersection analysis. It is typically referred to as degree of saturation. For a given lane group i, Xi is computed using Equation 5-33.

$$X_i = (v/c)_i = \frac{v_i}{s_i\left(\frac{g_i}{C}\right)} = \frac{v_i C}{s_i g_i} \tag{5-33}$$

where

$X_i = (v/c)_i$ = ratio for lane group i,

$v_i$ = actual or projected demand flow rate for lane group i, veh/h,

$s_i$ = saturation flow rate for lane group i, veh/h,

$g_i$ = effective green time for lane group i, s, and

$C$ = cycle length, s.

Sustainable values of Xi range from 1.0 when the flow rate equals capacity to zero when the flow rate is zero. Values above 1.0 indicate an excess of demand over capacity.

## 5.3 Main Control Factor for Left-turn Comparison

Among these factors that influence traffic delay d1, d2, d3, the PF factor should be determined as accurately as possible because it will have a significant impact on delay estimates. However, there are no definitive parameters to precisely quantify arrival type right now, the PF factor is hard to be controlled. The k factor is determined when the intersection control type is selected. For fixed time signal, k=0.5; for actuated signal, k depends on the prevailing v/c ratio. When v/c approaches 1.0, an actuated controller will tend to behave in a manner similar to a fixed time controller. Thus, k will converge to the fixed time value of 0.50 when demand equals capacity. Departure rate factors are determined by green time, however green time is the control target, so these factors are not selected to be main control factor.

From Chapter 5.2 analysis, capacity and demand are relatively easy to be measured and they are also important factors for left-turn traffic control. So $X_i = (v/c)_i$ ratio is selected to compare leading and lagging left-turn delay d1, d2, d3. From equation 5-33, $v_i$ is actual or projected demand flow rate which reflects the dynamic change of upcoming traffic flow, $s_i$ is saturation flow rate which reflects the intersection real conditions including geometry, location, pedestrian, transit, policy …

Figure 5-3 ~ 5-5 show the difference of leading and lagging left-turn traffic delay. The study period is chosen in peak hour, cycle length is 54s, g/c for left-turn traffic is 0.59, g/c for through traffic is 0.46, through traffic v/c=1.0, the number of lanes is 4 for both directions, and pedestrians are not included.

From Figure5-3 and 5-4, for isolated signal, leading d1 is always lower than lagging d1 no matter fixed or actuated; leading d2 is always higher than lagging d2 no matter fixed or actuated; leading d3 is very higher than lagging d3 when v/c exceeds 1.0. Composing d1, d2, d3, the leading total delay d is lower than lagging total delay d when v/c is relatively lower; the leading total delay d is higher than lagging total delay d when v/c is relatively higher. From Figure5-5, for coordinated signals, leading d1 is almost same

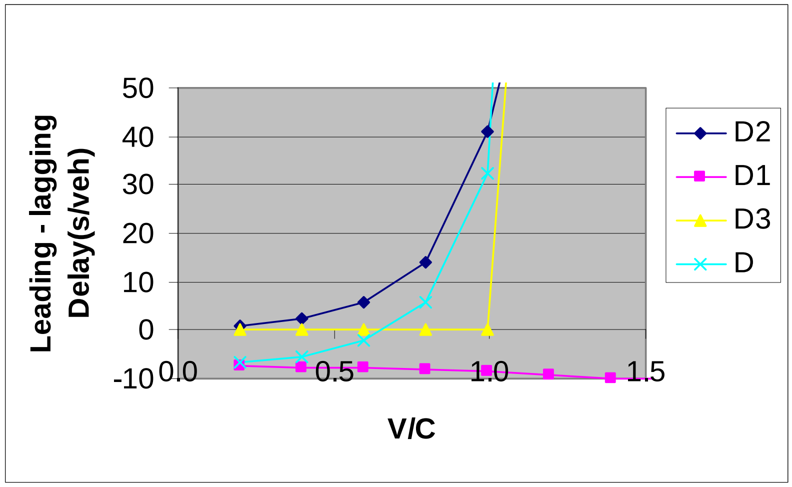



**Figure 5-3. The delay difference for isolated fixed-time signal**

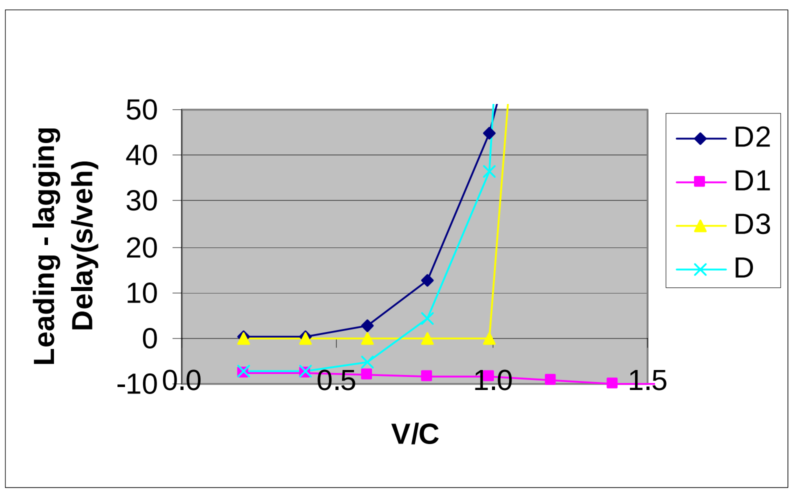



**Figure 5-4. The delay difference for isolated actuated signal**

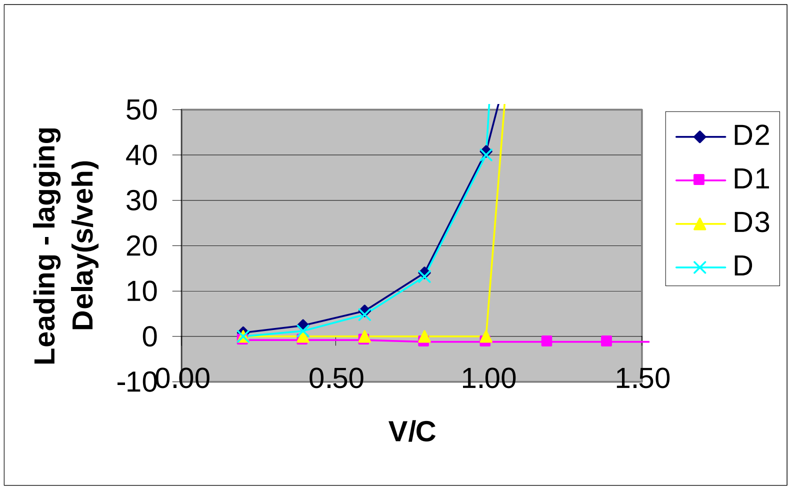



**Figure 5-5. The delay difference for coordinated signal**

with  lagging d1, the difference between them is very closed to zero; leading d2 is always higher than lagging; leading d3 is very higher than lagging d3 when v/c exceeds 1.0. Composing d1, d2, d3, the leading total delay d is higher than lagging total delay d, especially when v/c exceeds 1.0.

## 5.4 Sensitivity Results of Control Factors

Delay is relatively insensitive to demand levels until demand exceeds 90 percent of capacity, then delay is highly sensitive to not only changes in demand, but also changes in g/C.  The study period is chosen in peak hour, cycle length is 54s, g/c for left-turn traffic is 0.59, g/c for through traffic is 0.46, through traffic v/c=1.0, the number of lanes is 4 for both directions, and pedestrians are not included.

### 5.4.1 Sensitivity of delay to left-turn demand/capacity ratio

Figure 5-6 and 5-7 show the left-turn delay change based on v/c ratio.  No matter leading or lagging left-turn operation, the delay will increases as v/c ratio increases, and will increases significantly when v/c exceeds 1.0.

### 5.4.2 Sensitivity of delay to single-lane high volume through traffic g/c ratio

Figure 5-8 and 5-9 are sensitivity of left-turn traffic delay to single through lane traffic g/c ratio when through traffic volume closes to capacity and left-turn g/c is 0.59.  In such

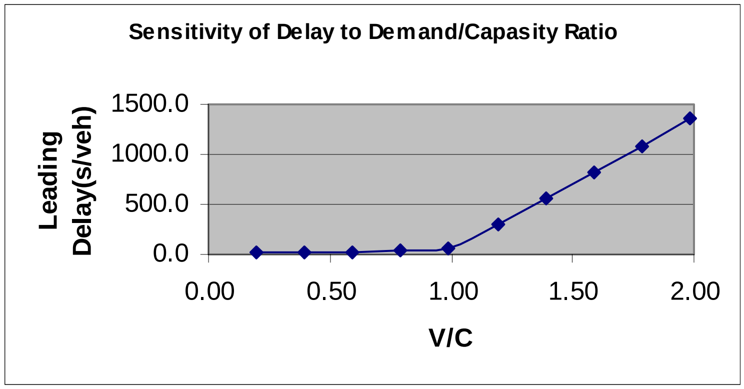



**Figure 5-6. Sensitivity of leading left-turn delay to v/c ratio**

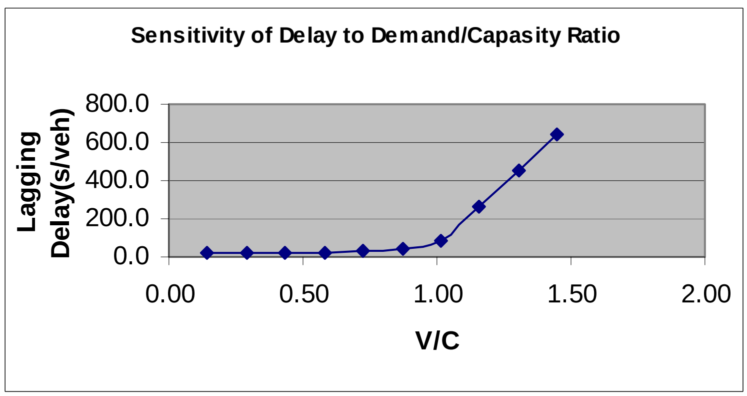



**Figure 5-7. Sensitivity of lagging left-turn delay to v/c ratio**

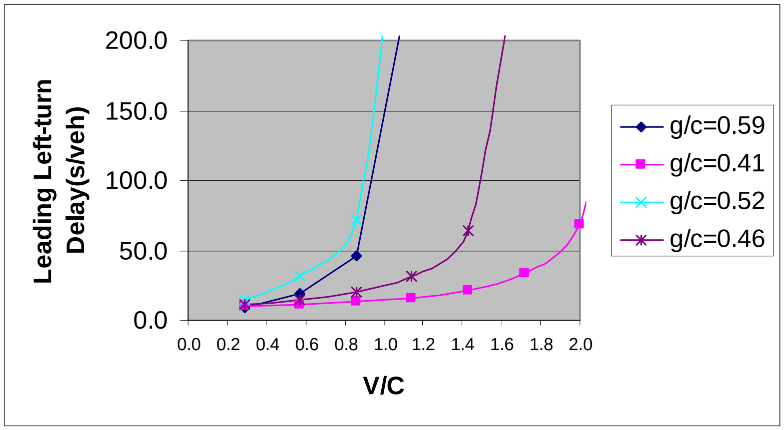



**Figure 5-8. Leading left-turn delay sensitivity to v/c for through traffic g/c ratio**

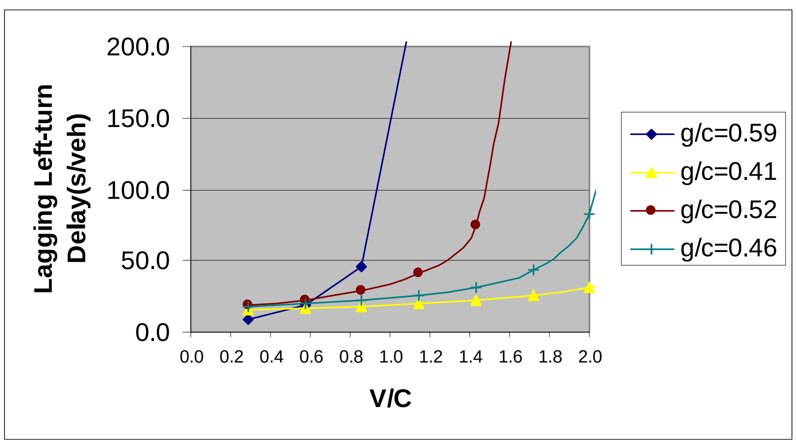



**Figure 5-9. Lagging left-turn delay sensitivity to v/c for through traffic g/c ratio**

condition, leading left-turn traffic delay will be lowest when left-turn v/c is greater than 0.3 and through traffic g/c is lowest, and it is insensitive. Lagging left-turn traffic delay will be lowest when left-turn v/c is greater than 0.5 and through traffic g/c is lowest, its delay is insensitive.

Figure 5-10 and 5-11 are sensitivity of total traffic delay to single through lane traffic g/c ratio when through traffic volume closes to capacity and left-turn g/c is 0.59. In such condition, leading total traffic delay will be lowest when left-turn v/c is lesser than 1.0 and through traffic g/c is highest, or left-turn v/c is greater than 1.0 and through traffic g/c is second lower, and it is insensitive when left-turn v/c is lower than 1.0 for highest through traffic g/c and left-turn v/c is lower than 1.4 for second lower through traffic g/c. Lagging total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9, lesser than 1.4 and through traffic g/c is second higher, or left-turn v/c is greater than 1.4 and through traffic g/c is second lower, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, left-turn v/c is greater than 0.9 and lower than 1.4 for second higher through traffic g/c, and left-turn v/c is greater than 1.4 for second lower through traffic g/c.

### 5.4.3 Sensitivity of delay to single-lane low volume through traffic g/c ratio

Figure 5-12 and Figure 5-13 are sensitivity of left-turn traffic delay to single through lane traffic g/c ratio when through traffic volume is lower and left-turn g/c is 0.59. In such condition, leading left-turn traffic delay will be lowest when left-turn v/c is greater than

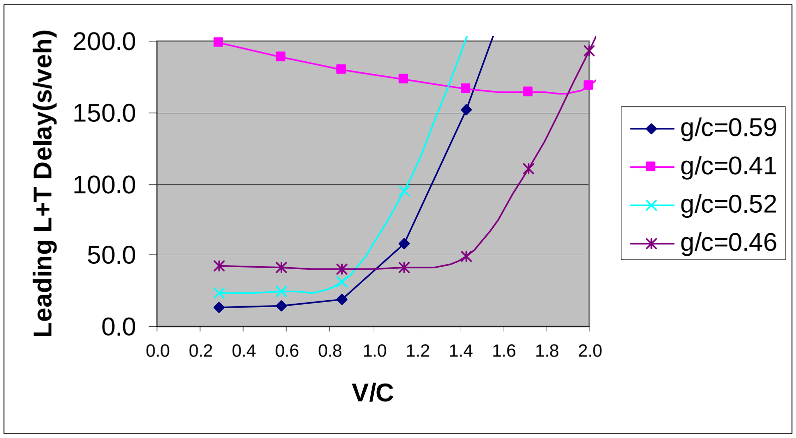



**Figure 5-10. Leading total delay sensitivity to v/c for through traffic g/c ratio**

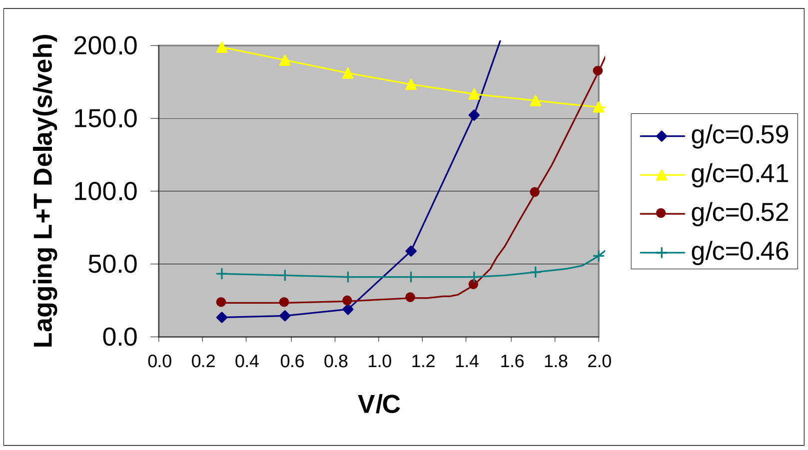



**Figure 5-11. Lagging total delay sensitivity to v/c for through traffic g/c ratio**

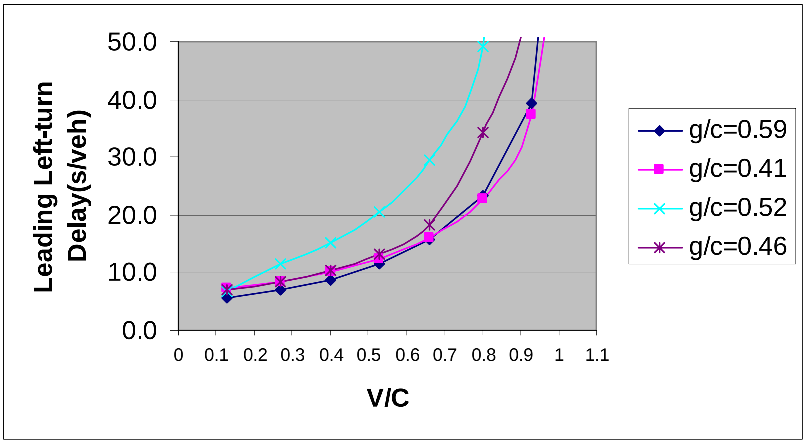



**Figure 5-12. Leading left-turn delay sensitivity to v/c for through traffic g/c ratio**

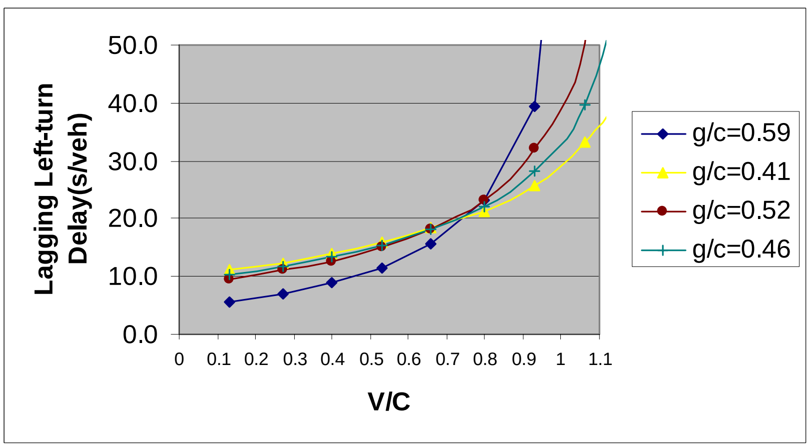



**Figure 5-13. Lagging left-turn delay sensitivity to v/c for through traffic g/c ratio**

0.55 and through traffic g/c is lowest, or left-turn v/c is lesser than 0.55 and through traffic g/c is highest, and it is a little sensitive. Lagging left-turn traffic delay will be lowest when left-turn v/c is greater than 0.75 and through traffic g/c is lowest, or left-turn v/c is lesser than 0.75 and through traffic g/c is highest, and it is a little sensitive.

Figure 5-14 and Figure 5-15 are sensitivity of total traffic delay to single through lane traffic g/c ratio when through traffic volume lower and left-turn g/c is 0.59. In such condition, leading total traffic delay will be lowest when through traffic g/c is highest, and it is insensitive when left-turn v/c is lower than 0.8 and sensitive when left-turn v/c is greater than 0.8. Lagging total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9and through traffic g/c is lowest, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, and left-turn v/c is greater than 0.9 for lowest through traffic g/c.

### 5.4.4 Sensitivity of delay to multi-lane high volume through traffic g/c ratio

Figure 5-16 and Figure 5-17 are sensitivity of left-turn traffic delay to multi through lane traffic g/c ratio when through traffic volume closes to capacity and left-turn g/c is 0.59. In such condition, either leading or lagging left-turn traffic delay will be lowest when left-turn v/c is greater than 0.4 and through traffic g/c is lowest, and it is insensitive.

Figure 5-18 and Figure 5-19 are sensitivity of total traffic delay to multi through lane traffic g/c ratio when through traffic volume closes to capacity and left-turn g/c is 0.59.

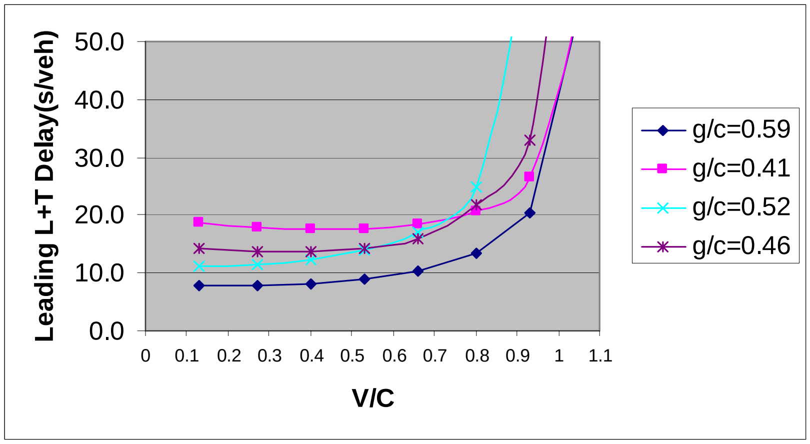



**Figure 5-14. Leading total delay sensitivity to v/c for through traffic g/c ratio**

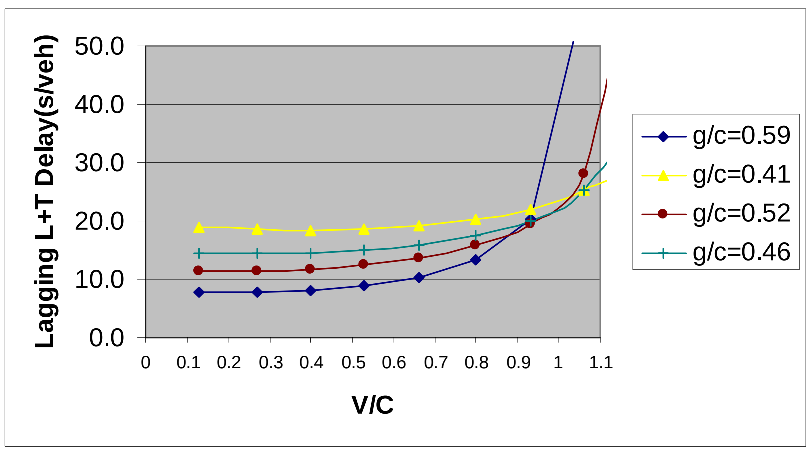



**Figure 5-15. Lagging total delay sensitivity to v/c for through traffic g/c ratio**

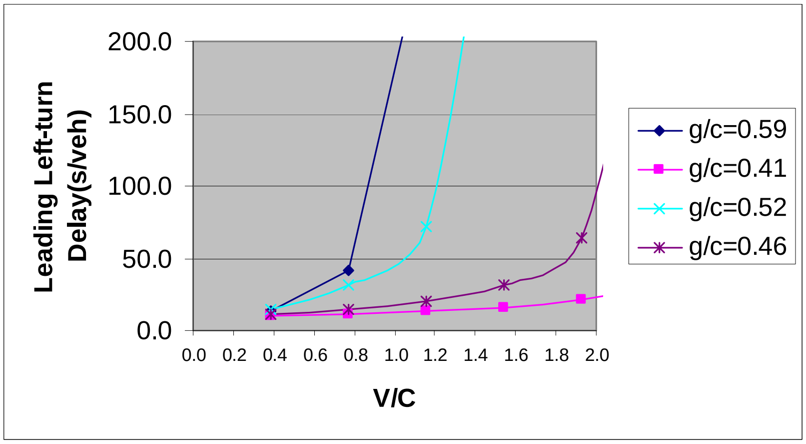



**Figure 5-16. Leading left-turn delay sensitivity to v/c for through traffic g/c ratio**

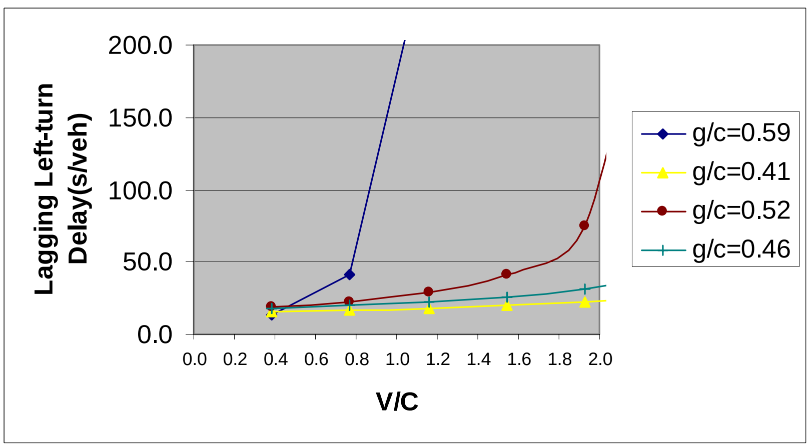



**Figure 5-17. Lagging left-turn delay sensitivity to v/c for through traffic g/c ratio**

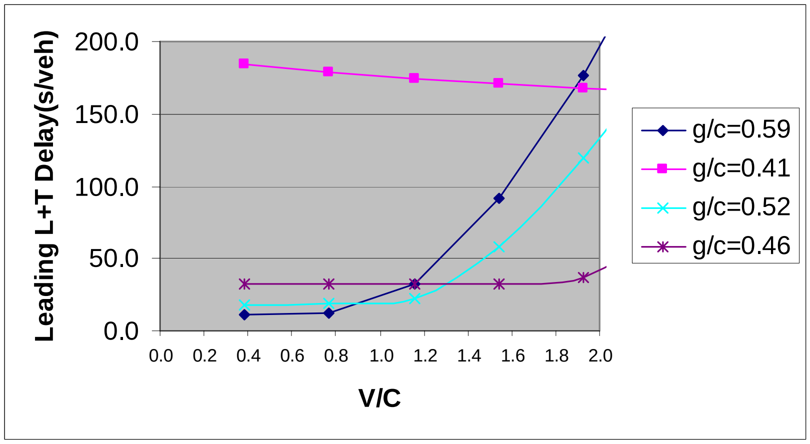



**Figure 5-18. Leading total delay sensitivity to v/c for through traffic g/c ratio**

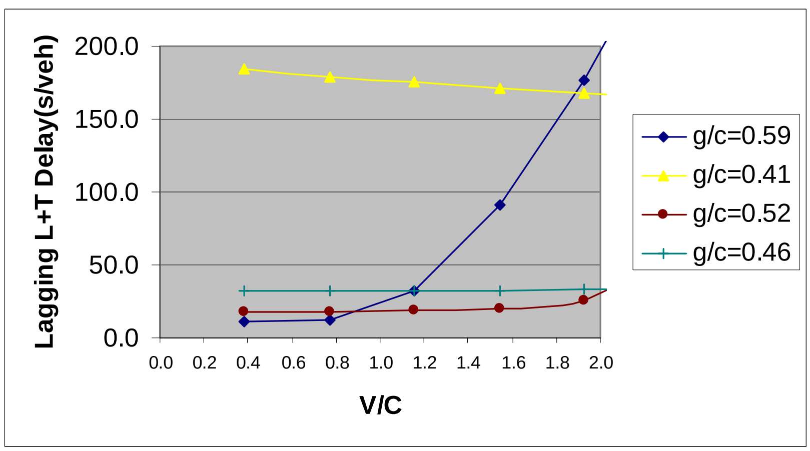



**Figure 5-19. Lagging total delay sensitivity to v/c for through traffic g/c ratio**

In such condition, leading total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9 and lesser than 1.3 for second higher through traffic g/c, or left-turn v/c is greater than 1.3 for through traffic g/c is second lower, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, left-turn v/c is greater than 0.9 and lesser than 1.3 for second higher through traffic g/c, and left-turn v/c is greater than 1.3 for second lower through traffic g/c.  Lagging total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9 and through traffic g/c is second higher, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, left-turn v/c is greater than 0.9 for second higher through traffic g/c.

### 5.4.5 Sensitivity of delay to multi-lane lower volume through traffic g/c ratio

Figure 5-20 and Figure 5-21 are sensitivity of left-turn traffic delay to multi through lane traffic g/c ratio when through traffic volume is lower and left-turn g/c is 0.59.  In such condition, leading left-turn traffic delay will be lowest when left-turn v/c is greater than 0.2 and through traffic g/c is lowest, and it is a little sensitive.  Lagging left-turn traffic delay will be lowest when left-turn v/c is greater than 0.45 and through traffic g/c is lowest, or left-turn v/c is lesser than 0.45 and through traffic g/c is highest, and it is a little sensitive.

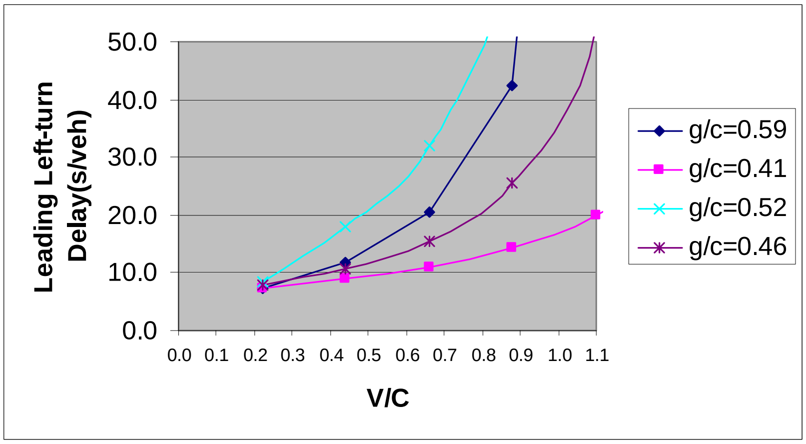



**Figure 5-20. Leading left-turn delay sensitivity to v/c for through traffic g/c ratio**

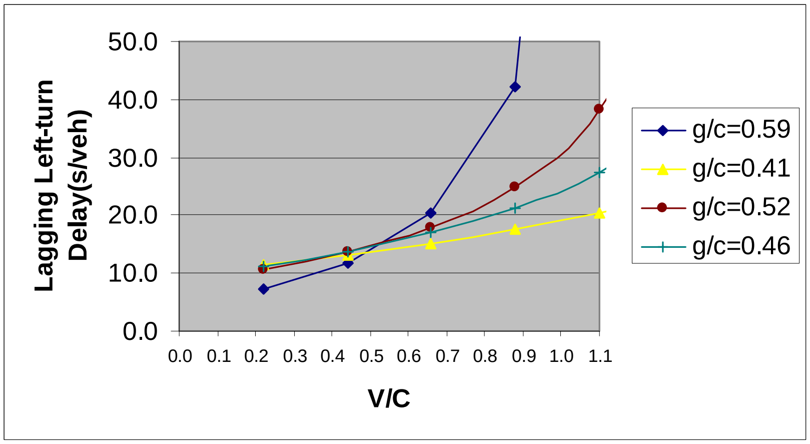



**Figure 5-21. Lagging left-turn delay sensitivity to v/c for through traffic g/c ratio**

Figure 5-22 and Figure 5-23 are sensitivity of total traffic delay to multi through lane traffic g/c ratio when through traffic volume lower and left-turn g/c is 0.59. In such condition, leading total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9 and lesser than 1.5 for second lower through traffic g/c, or left-turn v/c is greater than 1.5 for lowest through traffic g/c, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, or left-turn v/c is greater than 0.9 and lesser than 1.5 for second lower through traffic g/c, and left-turn v/c is greater than 1.5 for lowest through traffic g/c. Lagging total traffic delay will be lowest when left-turn v/c is lesser than 0.9 and through traffic g/c is highest, or left-turn v/c is greater than 0.9 and through traffic g/c is second highter, and it is insensitive when left-turn v/c is lower than 0.9 for highest through traffic g/c, and left-turn v/c is greater than 0.9 for second higher through traffic g/c.

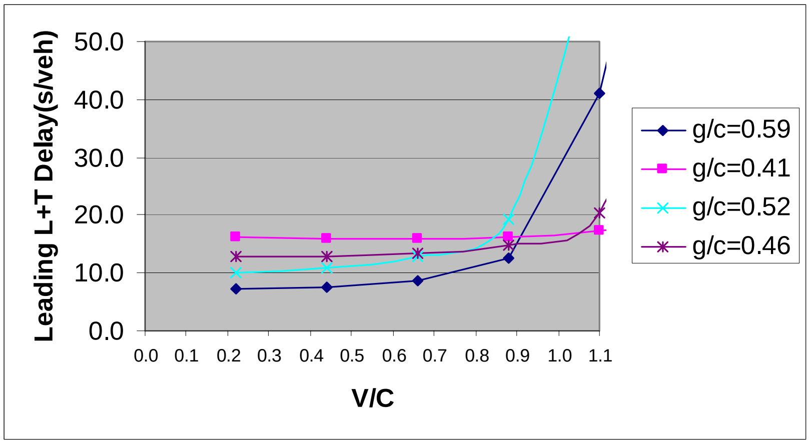



**Figure 5-22. Leading total delay sensitivity to v/c for through traffic g/c ratio**

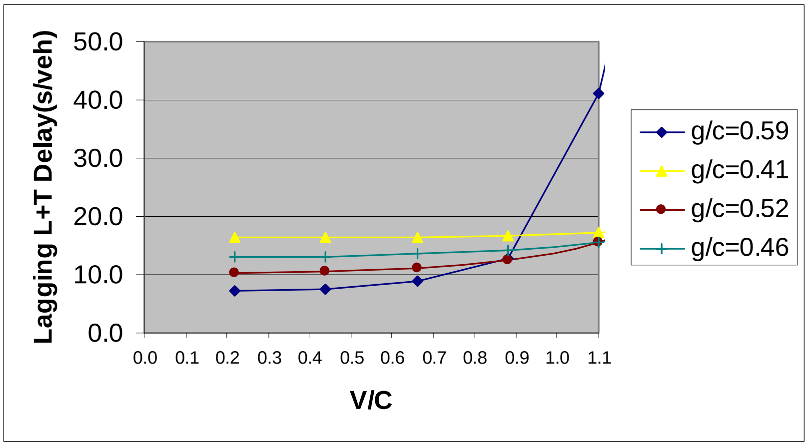



**Figure 5-23. Lagging total delay sensitivity to v/c for through traffic g/c ratio**

# Chapter 6 Formulating Left-turn Phase Fuzzy Control Rules and Membership Functions

In Chapter 6, fuzzy control logic is introduced, and fuzzification, defuzzification in the control process is also discussed. Based on Chapter 5 results and using linguistic variables, the fuzzy control rules and membership functions are derived.

## 6.1 Fuzzy Control Logic

### 6.1.1 Fuzzy logic theory

Fuzzy logic allows the implementation of real-life rules similar to the way humans would think. The beauty of fuzzy logic is that it allows fuzzy terms and conditions such as “heavy”, “less”, and “longer” to be quantized and understood by the computer.

**Fuzzy sets:** A fuzzy set s is an ordered pair (X, f), where X is a vector space (usually the real line R) and f is a set membership function mapping X onto the interval [0,1] of the real line R, $f : X \to [0,1]$.

In a fuzzy control problem, X is the signal space of a signal or a vector signal, respectively. A set $S \subset X$ is associated with the fuzzy set $s = (X, f)$ in a natural way: $S = cl\{x \in X, f(x) > 0\}$ is the closure of the set in X where f attains positive values.

Notice that the set membership function f is normalized in the sense that the value f(x) = 1 is attained for at least one element $x \in S \subset X$. However, this normalization has mainly been introduced for practical and intuitive reasons. Usually, a fuzzy set is a constant construct, a time-invariant part of a fuzzy control system.

The weight w and the centroid c of a fuzzy set $s = (X, f)$ are defined as follows:

$$w = \int f(x)dx$$

and $$c = \frac{\int xf(x)dx}{\int f(x)dx}$$

where all of the integrals are taken over the signal space X.

**Fuzzy variables:** A fuzzy variable v is an ordered pair (s,d) where s is a fuzzy set and $d \in [0,1]$ a real bounded variable.

Fuzzy variables arise in the fuzzification operation in a natural way: For the variable $x \in X$, the real variable d is the degree of membership in the fuzzy set s. In another interpretation of a fuzzy variable, the real variable d "modulates" the fuzzy set s: The scaler d and the set membership function $f : X \rightarrow [0,1]$ of the fuzzy set s define a new function $g : X \rightarrow [0,1]$. There are two modulation schemes:

“linear modulation”: $g(x) = d \bullet f(x)$

“modulation by clipping”: $g(x) = \min(f(x), d)$

Using the linear modulation scheme, the function g obtained by linear modulation typically contains more detailed information about the structure of the fuzzy variable. Notice that the linear modulation scheme results in a linear reduction of the weight of the fuzzy variable, $w_v = d \bullet w_s$, while the centroid remains unchanged, $c_v \equiv c_s$ for all $d \in [0,1]$.

However, for calculation with a fuzzy variable, it is more practical to use the “modulated” function $g$ than to keep the scalar $d$ and the set membership function $f$ of the underlying fuzzy set $s$ apart. Furthermore, the restriction $g(x) \leq 1$ for all $x \in X$ can be dropped. This is practical when sums of fuzzy variables are calculated.

**Fuzzy logic:** Fuzzy logic defines the rules governing the operators intersection and union of fuzzy sets.

Consider two fuzzy sets $s_1 = (X, f_1)$ and $s_2 = (X, f_2)$ defined on the same signal space X and their associated sets $S_1 \subset X$ and $S_2 \subset X$, respectively. An arbitrary element $x \in X$ belongs to the union $s_1 \cup s_2$ of the two fuzzy sets $s_1$ and $s_2$ with degree

$d = \max(f_1(x), f_2(x))$. An arbitrary element $x \in X$ belongs to the intersection $s_1 \cap s_2$ of the two fuzzy sets $s_1$ and $s_2$ with degree $d = \min(f_1(x), f_2(x))$.

Consequently, the union operator and the intersection operator yield the fuzzy sets $s_1 \cup s_2 = (X, \max(f_1, f_2))$ and $s_1 \cap s_2 = (X, \min(f_1, f_2))$, respectively. Notice that the intersection $s_1 \cap s_2$ is a degenerated fuzzy set in the sense that its set membership function $\min(f_1, f_2)$ does not map onto the interval[0,1] as requested by the definition of a fuzzy set. This detail is not pursued any further here because in fuzzy control, all calculation are done with fuzzy variables rather than with fuzzy sets.

### 6.1.2 Fuzzification and defuzzification in the control process

**Fuzzification:** Consider a signal space X covered by several fuzzy sets $s_i, i = 1,...,k$. The fuzzy question is: Given a vector $x \in X$, to which of the fuzzy sets $s_i$ does $x$ belong or, in which of the sets $S_i$ associated with the fuzzy sets $s_i$ does $x$ lie? In mathematical set theory, the answer for each of the sets $S_i$ is a binary one. In fuzzy set theory, set membership is “by degree”.

Consider a fuzzy set $s = (X, f)$. An arbitrary element $x \in X$ belongs to the fuzzy set $s$ with degree $d = f(x)$. Hence, the answer to the fuzzy question is : $x$ belongs to each of the fuzzy sets $s_i$ to some degree, degree $d_i = f_i(x), i = 1,...,k$.

**Defuzzification:** Defuzzification is the process of assigning a representative value to a fuzzy variable. Consider a fuzzy variable $v_u$ on the signal space $U = R$ which is represented by the modulated function $g_u$.

The defuzzification operator $D$ maps the fuzzy variable $v_u$ to the cintroid $u$ of the modulated function $g_u$,

$$u = D\{v_u\} = D\{g_u\} = \frac{\int \alpha g_u(\alpha) d\alpha}{\int g_u(\alpha) d\alpha}$$

where both of the integrals are calculated over the signal space $U = R$. The defuzzification operation $D$ is understood to accept an arbitrary representation of the fuzzy variable $v_u$ as its argument.

### 6.1.3 Fuzzy rules

Fuzzy rules are used in fuzzy control in order to define the map from the fuzzified input signals ( error signals, measured signals, or command signals) of the fuzzy controller to its fuzzy output signals (control signals).

**Fuzzy SISO rule:** The SISO rule mapping the fuzzy input variable $v_1 = (s_1, d_1)$ to the fuzzy output variable $v_u = (s_u, d_u)$ (of the fuzzy controller) is defined by $v_u = (s_u, d_1)$. If the value of the signal belongs to the fuzzy set $s_1$ to degree $d_1$ then the fuzzy set $s_u$ of the control signal is fired to degree $d_u = d_1$. The value $u(t)$ of the control signal is obtained by "defuzzification" after all of the fuzzy rules pertaining to the control signal have been processed.

**Fuzzy AND rule:** The AND rule mapping the fuzzy input variables $v_1 = (s_1, d_1)$ and $v_2 = (s_2, d_2)$ to the fuzzy output variable $v_u = (s_u, d_u)$ is defined by $v_u = (s_u, \min(d_1, d_2))$. If the value of the first signal belongs to the fuzzy set $s_1$ to degree $d_1$ and the value of the second signal belongs to the fuzzy set $s_2$ to degree $d_2$ then the fuzzy set $s_u$ of the control signal is fired to the smaller of the two degrees, $d_u = \min(d_1, d_2)$. The value $u(t)$ of the control signal is obtained by "defuzzification" after all of the fuzzy rules pertaining to the control signal have been processed. It should be obvious how the definition of the fuzzy AND rule can be extended to three or more fuzzy input variables.

**Other fuzzy rules:** In analogy to the fuzzy AND rule, fuzzy OR rule and more complicated logical combinations for fuzzy rules could be defined. This research prefers to use fuzzy AND rule exclusively because OR and AND rules together typically results in a weaker contribution to the overall fuzzy output variables and the corresponding defuzzified control variables.

### 6.1.4 Fuzzy associative memory

For a fuzzy controller, the collection of all of its fuzzy rules is called the fuzzy associative memory. For every control cycle, each of the fuzzy rules is evaluated. This can be done by massively parallel processing. The output of each fuzzy rule is a fuzzy variable. The output of the fuzzy associative memory is equal to the sum of all these fuzzy variables.

## 6.2 Deriving Membership Functions

For the traffic signal control, there are four membership functions for each of the input and output fuzzy variable of the system. Table 6-1 shows the fuzzy variables of Arrival, Queue and Extension of the system.

The graphical representation of the membership functions of the linguistic variables is presented in Figure 6-1. It can be observed that the y-axis is the degree of the membership of each of the fuzzy variable. For the input fuzzy variables the universe of discourse (the x-axis) is the quantized sensor signals which sensed the quantity of the vehicles. For the output fuzzy variable the universe of discourse is the length of time to be extended in seconds. In this control, two detectors are located per each approach lane.

**Table 6-1. Fuzzy variables for dynamic traffic signal control**

| Arrival | Queue | Extension |
|---|---|---|
| Almost zero-AZ | Almost zero-QZ | Zero-Z |
| A few-AF | A few-QF | Short-S |
| Medium-AMD | Medium-QMD | Medium-M |
| Many-AMY | Many-QMY | Long-L |

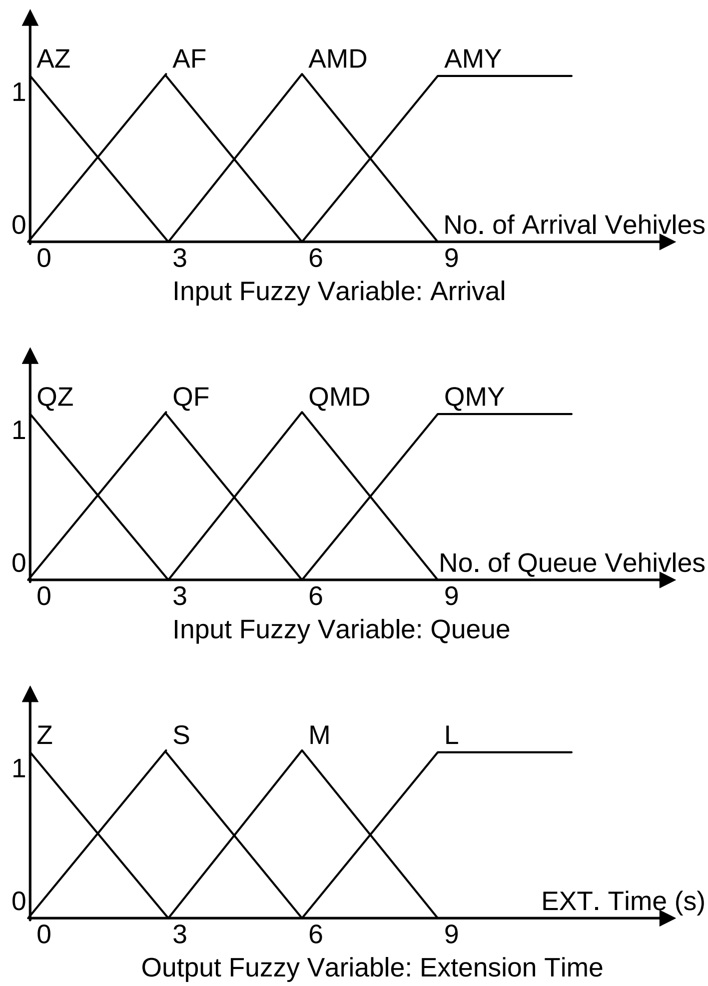



**Figure 6-1. Membership functions of the fuzzy green extender**

The location of the first one in through lane is approximate 330 ft upstream of the stop line and the second one is at the stop line. This means that we know how many vehicles are approaching the stop line within next 6-8 seconds. The location of the first one in left-turn lane is approximate 200 ft upstream of the stop line or the start end of the left-turn lane and the second one is at the stop line.

From Figure 6-1, it can be observed that nine vehicles have been assigned as “Many” fuzzy sets in this simulation which have a full membership. For “Medium” fuzzy sets, a full membership is six vehicles and so on. For the output fuzzy variable, a “long” fuzzy set with a membership of “1” would be in the region of 9 seconds, whereas a “Medium” fuzzy set would be in the region of 6 seconds, and so on. The configuration of these membership functions is done according to expert observation of the system and environment.

However, the width and center of the membership functions of these fuzzy sets can be easily changed and configured according to different traffic situations and conditions. For example, if the junction is too congested, the number of vehicles in the fuzzy sets “Many” is needed to be increased. On the other hand, for a less congested junction the width of the membership functions can be reduced. It can be observed that in fuzzy logic control the transition from one fuzzy set to another provides a smooth transition from one control action to another, thus, the need to overlap these fuzzy sets. If there is no overlapping in the fuzzy sets then the control action would resemble bivalent control. On the other hand if there is too much overlap in the fuzzy sets, there would be a lot of fuzziness and this blurs the distinction in the control action. A heuristic approach is to overlap the fuzzy sets by about 25%.

### 6.3 Formulating Left-turn Phase Fuzzy Control Rules

Signal left-turn phase control deals with a complex multi-objective and multi-constraint problem in which the optimization performed is based mainly on recent information. In other words, the complexity of a system increases, our ability to make precise and yet significant statements about its behaviors diminishes, and significance and complexity become almost mutually exclusive characteristics. How to weight and control these objectives is a big issue which is becoming the scope that this research is going to solve out.

Fuzzy logic is often used to identify and recognize certain patterns of traffic flow, allowing the most appropriate signal timings to be defined and implemented as the traffic situation change (Hoyer et al. [32]) and Zhou et al. [33]. It works in the same way as the traditional adaptive control, but the extensions are adjusted by a fuzzy selector. So, a better solution might be the mechanism of human thinking with linguistic fuzzy values rather than numbers (0/1).

However, all of the aforementioned efforts were based on operations with predetermined phase orders. The extent of the control decisions made by the various fuzzy logic algorithms was limited to skipping, terminating, and extending certain phases in a predetermined phase sequence. Adding, changing, and rearranging the fixed phase order in real time, which would afford more flexibility for optimization purposes, were beyond the capability of these algorithms.

This dissertation aims to venture beyond controlling the duration of each signal phase alone by introducing the added control of phase orders in real time. To this end, a four-level fuzzy logic control model (Author [34]) was designed to determine which phase order should be chosen per cycle, and whether the leading or lagging signal phase should be selected or terminated in signal operations. The fuzzy rules are working at following four levels: Traffic situation level, Phase status level, Phase order level, and Green ending or extension level.

### 6.3.1 Traffic situation level

The through traffic situation is divided into four different categories: oversaturated *(O)*, *normal without gap (N)*, normal with gap *(G)* and low *(L)* demand.

So the through traffic fuzzy rules are:

*If min (TOCC) is high then TS is O or*

*If max (TOCC) is zero and TVOL is low then TS is L or*

*If max (TOCC) is zero and TVOL is more than normal then TS is N or*

*If max (TOCC) is normal and TVOL is less than normal then TS is G or*

*If max (TOCC) is normal and TVOL is high then TS is O*

Where *TVOL* is the through traffic volume of the last time period, perhaps 5 min;

*TOCC* is the through traffic occupancy (in %) of the first detector during the last time period; and

*TS* is the through traffic situation.

The left-turn traffic situation is divided into three different categories: oversaturated *(O)*, normal *(N)* and low *(L)* demand.

So the left-turn traffic fuzzy rules are:

*If LVOL is any and min (LOCC) is high then LS is oversaturated (O) or*

*If LVOL is low and max (LOCC) is zero then LS is low (L) or*

*If LVOL is any and max (LOCC) is normal then LS is normal (N)*

Where *LVOL* is the left-turn traffic volume of the last time period;

*LOCC* is the left-turn traffic occupancy (in %) of the first detector during the last time period; and

*LS* is the left-turn traffic situation.

Note that in these rules there are terms such as low, normal, which are qualitative in nature and will be determined, or quantified, with membership functions to be designated by the user or traffic engineer later on when implementing the proposed framework.

### 6.3.2 Phase status level

In this level, phase status (*PS*): permitted-phase (*PP*), leading-phase (*LEP*) and lagging-phase (*LAP*) are decided by fuzzy phase status selector.

Based on chapter 3 analysis results, the phase status fuzzy rules for isolated intersection (Table 6-2) are:

*If LS is L and TS is O then PS is LEP or*

*If LS is N and TS is O then PS is LAP or*

*If LS is N and TS is N then PS is LEP or*

*If LS is O and TS is L then PS is LEP or*

*If LS is O and TS is more than L then PS is LAP*

*Otherwise the PS is PP*

Based on chapter 4 analysis results, the phase status fuzzy rules for coordinated intersections (Table 6-3) are:

*If LS is L and TS is O then PS is LAP or*

*If LS is N and TS is O then PS is LAP or*

*If LS is N and TS is N then PS is LAP or*

*If LS is O then PS is LAP or*

*Otherwise the PS is PP*

**Table 6-2. Phase status fuzzy rules for isolated traffic signal control**

| | | Through traffic | | | |
|---|---|---|---|---|---|
| | | **L** | **G** | **N** | **O** |
| **Left-turn** | **L** | *PP* | *PP* | *PP* | *LEP* |
| | **N** | *PP* | *PP* | *LEP* | *LAP* |
| | **O** | *LEP* | *LAP* | *LAP* | *LAP* |

**Table 6-3. Phase status fuzzy rules for coordinated traffic signal control**

| | | Through traffic | | | |
|---|---|---|---|---|---|
| | | **L** | **G** | **N** | **O** |
| **Left-turn** | **L** | *PP* | *PP* | *PP* | *LAP* |
| | **N** | *PP* | *PP* | *LAP* | *LAP* |
| | **O** | *LAP* | *LAP* | *LAP* | *LAP* |

It is often the case that a lagging protected phase is not allowable due to the possibility of opposing left turn drivers being caught in a left turn "trap," in which they incorrectly assume that their movement is being terminated at the same time as that of traffic opposing them. This confusion is eliminated at intersections without opposing left turning traffic, such as T-intersections or intersections with one-way streets; or it can be avoided by the use of "Dallas" left turn phasing or others, in which left turn drivers are shown an exclusive display of the opposing traffic's indication [3], [35].

### 6.3.3 Phase order level

The goal of this level is to determine the best phase order. The phase order (*PO*) will be decided based on the above fuzzy phase status results.

*If PS in all approaches is PP then PO is TT*

*If PS in approach 1 is LEP and the opposing approach 2 is PP then PO is LT1+TT*

*If PS in approach 1 is PP and the opposing approach 2 is LAP then PO is TT+LT2*

*If PS in all approaches is LEP then PO is LL+TT*

*If PS in approach 1 is LEP and the opposing approach 2 is LAP then PO is LT1+TT+LT2*

*If PS in all approaches is LAP then PO is TT+LL*

Where *TT* : through phase with permitted left-turn,

*LT1+TT*: first phase is protected left-turn and through phase in the same direction, second phase is through phase with permitted left-turn,

*TT+LT2*: first phase is through phase with permitted left-turn, second phase is protected left-turn and through phase in same direction,

*LL+TT*: first phase is protected left-turn, second phase is through phase with permitted left-turn,

*LT1+TT+LT2*: first phase is protected left-turn and through phase in the same direction, second phase is through phase with permitted left-turn, third phase is protected left-turn and through phase in other same direction,

*TT+LL*: first phase is through phase with permitted left-turn, second phase is protected left-turn.

### 6.3.4 Phase green ending or extension level

At this final level, the green duration of a phase is determined by a fuzzy green extender. The goal of this level is to adjust the cycle length, divide the cycle into appropriate durations of green phases, and maximize the capacity along the way. The input variables of the level are the numbers of arriving and queued vehicles. The output is the extension of the movement groups for the phase (EXT). All input and output variables are controlled by fuzzy sets membership functions, which are discussed in Chapter 6.2.

**Two-phase Vehicle Control**

Based on PO results in 6.3.3, if *PO* in major road and minor road both are *TT*, the fuzzy green extender uses the following two-phase control.

There are only two input variables for fuzzy rule base:

A = approaching vehicles (*AZ*, *AF*, *AMD*, or *AMY*) for the green approach at time t;

Q = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t.

Due to the membership assignment, these linguistic variables can be taken care of through fuzzy logic technology. The fuzzy rules are showed in the Table 6-4.

Based on the membership functions derived in 6.2, the output EXT is:

After minimum green (4s),

*if A is AZ then terminate immediately (0s)*

*or if A is AF and Q is more than QF then terminate immediately (0s)*

*or if A is AF and Q is less than QMD then EXT is short (3s)*

*or if A is AMD and Q is QMY then terminate immediately (0s)*

*or if A is AMD and Q is QMD then EXT is short (3s)*

*or if A is AMD and Q is less than QMD then EXT is medium (6s)*

*or if A is AMY and Q is QMY then EXT is short (3s)*

*or if A is AMY and Q is less than QMY and more than QZ then EXT is medium (6s)*

**Table 6-4. Two-phase fuzzy green extender fuzzy rules**

| | | Arrival | | | |
|---|---|---|---|---|---|
| Queue | | ***AZ*** | ***AF*** | ***AMD*** | ***AMY*** |
| | ***QZ*** | *Z* | *S* | *M* | *L* |
| | ***QF*** | *Z* | *S* | *M* | *M* |
| | ***QMD*** | *Z* | *Z* | *S* | *M* |
| | ***QMY*** | *Z* | *Z* | *Z* | *S* |

*or if A is AMY and Q is QZ then EXT is long (9s).*

After the first extension (EXT1+minimum green 4s),

*if A is AZ then terminate immediately (0s)*

*or if A is AF and Q is more than QF then terminate immediately (0s)*

*or if A is AF and Q is less than QMD then EXT is short (3s)*

*or if A is AMD and Q is QMY then terminate immediately (0s)*

*or if A is AMD and Q is QMD then EXT is short (3s)*

*or if A is AMD and Q is less than QMD then EXT is medium (6s)*

*or if A is AMY and Q is QMY then EXT is short (3s)*

*or if A is AMY and Q is less than QMY and more than QZ then EXT is medium (6s)*

*or if A is AMY and Q is QZ then EXT is long (9s).*

……

After the nth extension (EXT1+EXT2+…+EXTn+minimum green 4s),

*if A is AZ then terminate immediately (0s)*

*or if A is AF and Q is more than QF then terminate immediately (0s)*

*or if A is AF and Q is less than QMD then EXT is short (3s)*

*or if A is AMD and Q is QMY then terminate immediately (0s)*

*or if A is AMD and Q is QMD then EXT is short (3s)*

*or if A is AMD and Q is less than QMD then EXT is medium (6s)*

*or if A is AMY and Q is QMY then EXT is short (3s)* *or if A is AMY and Q is less than QMY and more than QZ then EXT is medium (6s)*

*or if A is AMY and Q is QZ then EXT is long (9s).*

**Multi-phase Vehicle Control**

If there is protected left-turn phase in the signal control, the fuzzy green extender uses the below multi-phase control Table 6-5.

The input variables for fuzzy rule base:

A = approaching vehicles (*AZ*, *AF*, *AMD*, or *AMY*) at time t for the green approach;

Q1 = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with phase 2;

Q2 = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with phase 3;

……

Qn = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with (phase 2+…+ phase n)

The output variables for fuzzy rule base:

EXT1 = extension timing for phase 1 in comparison with phase2

EXT2 = extension timing for phase 1 in comparison with phase3

……

**Table 6-5. Multi-phase fuzzy green extender fuzzy rules**

| | Arrival-phase1 | | | | |
|---|---|---|---|---|---|
| **Queue -phase 2 or 3,…,n** | | ***AZ*** | ***AF*** | ***AMD*** | ***AMY*** |
| | ***QZ*** | *Z* | *S* | *M* | *L* |
| | ***QF*** | *Z* | *S* | *M* | *M* |
| | ***QMD*** | *Z* | *Z* | *S* | *M* |
| | ***QMY*** | *Z* | *Z* | *Z* | *S* |
| **Queue –phase 2+3 or 3+4,…, (n-1)+n** | | ***AZ*** | ***AF*** | ***AMD*** | ***AMY*** |
| | ***QZ*** | *Z* | *S* | *M* | *L* |
| | ***QF*** | *Z* | *S* | *M* | *M* |
| | ***QMD*** | *Z* | *Z* | *S* | *M* |
| | ***QMY*** | *Z* | *Z* | *Z* | *S* |
| **…** | **…** | … | … | … | … |
| **Queue –phase 2+3+…+n** | | ***AZ*** | ***AF*** | ***AMD*** | ***AMY*** |
| | ***QZ*** | *Z* | *S* | *M* | *L* |
| | ***QF*** | *Z* | *S* | *M* | *M* |
| | ***QMD*** | *Z* | *Z* | *S* | *M* |
| | ***QMY*** | *Z* | *Z* | *Z* | *S* |

EXTn = extension timing for phase 1 in comparison with phase 2+…+n

EXT = min(EXT1, EXT2, …, EXTn)

The general fuzzy rules are:

If *W(p)* is many then phase *p* will be the next one,

If $W(p_i)$ is medium and $W(p_j)$ is a few then phase *(i)* will be the next one,

If $W(p_i)$ is a few and $W(p_j)$ is a zero then phase *(i)* will be the next one,

The maximum waiting time of each vehicle cannot be too long,

Otherwise the phase will be as planned.

For example, if PO results in Chapter 6.3.3 are *LL+TT* in major road and *TT* in minor road, based on above rules and membership functions in Chapter 6.2, the output *EXT* of the three phase control are:

After the minimum green (4s),

*If A is AMY then phase 1 will be extended to:*

*if Q1 is QMY then EXT1 is short (3s)*

*if Q1 is less than QMY and more than QZ then EXT1 is medium (6s)*

*if Q1 is QZ then EXT1 is long (9s).*

*if Q2 is QMY then EXT2 is short (3s)*

*if Q2 is less than QMY and more than QZ then EXT2 is medium (6s)*

*if Q2 is QZ then EXT2 is long (9s).*

*if Q3 is QMY then EXT3 is short (3s)*

*if Q3 is less than QMY and more than QZ then EXT3 is medium (6s)*

*if Q3 is QZ then EXT3 is long (9s).*

*if A is AMD then phase 1 will be extended to:*

*if Q1 is QMY then EXT1 is zero (0s)*

*if Q1 is QMD then EXT1 is short (3s)*

*if Q1 is less than QMD then EXT1 is medium (6s)*

*if Q2 is QMY then EXT2 is zero (0s)*

*if Q2 is QMD then EXT2 is short (3s)*

*if Q2 is less than QMD then EXT2 is medium (6s)*

*if Q3 is QMY then EXT3 is zero (0s)*

*if Q3 is QMD then EXT3 is short (3s)*

*if Q3 is less than QMD then EXT3 is medium (6s)*

*if A is AF then phase 1 will be extended to:*

*if Q1 is more than QF then EXT1 is zero (0s)*

*if Q1 is less than QMD then EXT1 is short (3s)*

*if Q2 is more than QF then EXT2 is zero (0s)*

*if Q2 is less than QMD then EXT2 is short (3s)*

*if Q3 is more than QF then EXT3 is zero (0s)*

*if Q3 is less than QMD then EXT3 is short (3s)*

*if A is AZ then EXT1, EXT2, EXT3 are zero (0s)*

*EXT = min (EXT1, EZT2, EXT3)*

……

After the nth extension ( $\sum_{1}^{n} EXT$ +minimum green 4s),

*If A is AMY then phase 1 will be extended to:*

*if Q1 is QMY then EXT1 is short (3s)*

*if Q1 is less than QMY and more than QZ then EXT1 is medium (6s)*

*if Q1 is QZ then EXT1 is long (9s).*

*if Q2 is QMY then EXT2 is short (3s)*

*if Q2 is less than QMY and more than QZ then EXT2 is medium (6s)*

*if Q2 is QZ then EXT2 is long (9s).*

*if Q3 is QMY then EXT3 is short (3s)*

*if Q3 is less than QMY and more than QZ then EXT3 is medium (6s)*

*if Q3 is QZ then EXT3 is long (9s).*

*if A is AMD then phase 1 will be extended to:*

*if Q1 is QMY then EXT1 is zero (0s)*

*if Q1 is QMD then EXT1 is short (3s)*

*if Q1 is less than QMD then EXT1 is medium (6s)*

*if Q2 is QMY then EXT2 is zero (0s)*

*if Q2 is QMD then EXT2 is short (3s)*

*if Q2 is less than QMD then EXT2 is medium (6s)*

*if Q3 is QMY then EXT3 is zero (0s)*

*if Q3 is QMD then EXT3 is short (3s)*

*if Q3 is less than QMD then EXT3 is medium (6s)*

*if A is AF then phase 1 will be extended to:*

*if Q1 is more than QF then EXT1 is zero (0s)*

*if Q1 is less than QMD then EXT1 is short (3s)*

*if Q2 is more than QF then EXT2 is zero (0s)*

*if Q2 is less than QMD then EXT2 is short (3s)*

*if Q3 is more than QF then EXT3 is zero (0s)*

*if Q3 is less than QMD then EXT3 is short (3s)*

*if A is AZ then EXT1, EXT2, EXT3 are zero (0s)*

*EXT = min (EXT1, EZT2, EXT3)*

The input variables of three phases (major *LL*+ major *TT*+ minor *TT*) control:

A = approaching vehicles (*AZ*, *AF*, *AMD*, or *AMY*) at time t for the green approach;

Q1 = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with phase 2;

Q2 = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with phase 3;

Q3 = queuing vehicles (*QZ*, *QF*, *QMD*, or *QMY*) for the red approach at time t for phase 1 in comparison with phase 2+ phase3;

The output variables of three phases (major *LL*+ major *TT*+ minor *TT*) control:

EXT1 = extension timing for phase 1 in comparison with phase2

EXT2 = extension timing for phase 1 in comparison with phase3

EXT3 = extension timing for phase 1 in comparison with phase 2+3

EXT = min (EXT1, EXT2, EXT3)

# Chapter 7 Dynamic Traffic Signal Left-turn Phase Fuzzy Logic Control System

Chapter 7 develops a dynamic traffic signal left-turn phase fuzzy logic control system, validates the fuzzy control principles and calibrates the membership functions of the linguistic variables using simulation and field trials.

## 7.1 Dynamic Traffic Signal Left-turn Phase Fuzzy Logic Control System

The dynamic traffic signal left-turn phase fuzzy logic control system is real dynamic control comparing fixed-time control, actuated control and traditional fuzzy control. For example, the normal phase order in one signal intersection is A-B-C-A. Some phases (B or C) can be skipped if no request was observed for them. For these “traditional” fuzzy control algorithms the output phase order may become A-B-A-C-A, A-B-A-B-A, or A-C-A-C-A when phase skipping occurs. However, the intersection can not use an otherwise undefined phase D (or E, F, …), which may provide better performance under certain circumstances. On the other hand four-level fuzzy control, which will be detailed in the ensuing section, offers the added flexibility of adding new phases and rearranging phase orders to create new feasible timing plans on a per cycle basis according to real-time traffic condition. This opens up the possibility of improving signal performance under unexpected traffic conditions with virtually unlimited number of phase orders such as A-D-C-A-B-A, A-D-E-A-F-A, etc.  The system includes the following steps:

**First step**: locating target signal intersection, and collecting data by detectors.

**Second step**: deriving fuzzy control membership function based on created fuzzy sets and fuzzy variables.

**Third step**: evaluating all input data using four-level fuzzy logic control rules, see Figure 7-1.

- In first level, traffic situation level, update traffic situation variables TS and LS per circle using last 5 minutes TOCC, TVOL, LOCC, and LVOL data.
- In second level, phase status level, update phase status variable PS per circle using updated TS and LS of first level.
- In third level, phase order level, update phase order variable PO per circle using updated PS of second level.
- In fourth level, phase green ending or extension level, evaluate phase green time and control the target signal intersection.

**Fourth step**: repeat the above steps for next cycle.

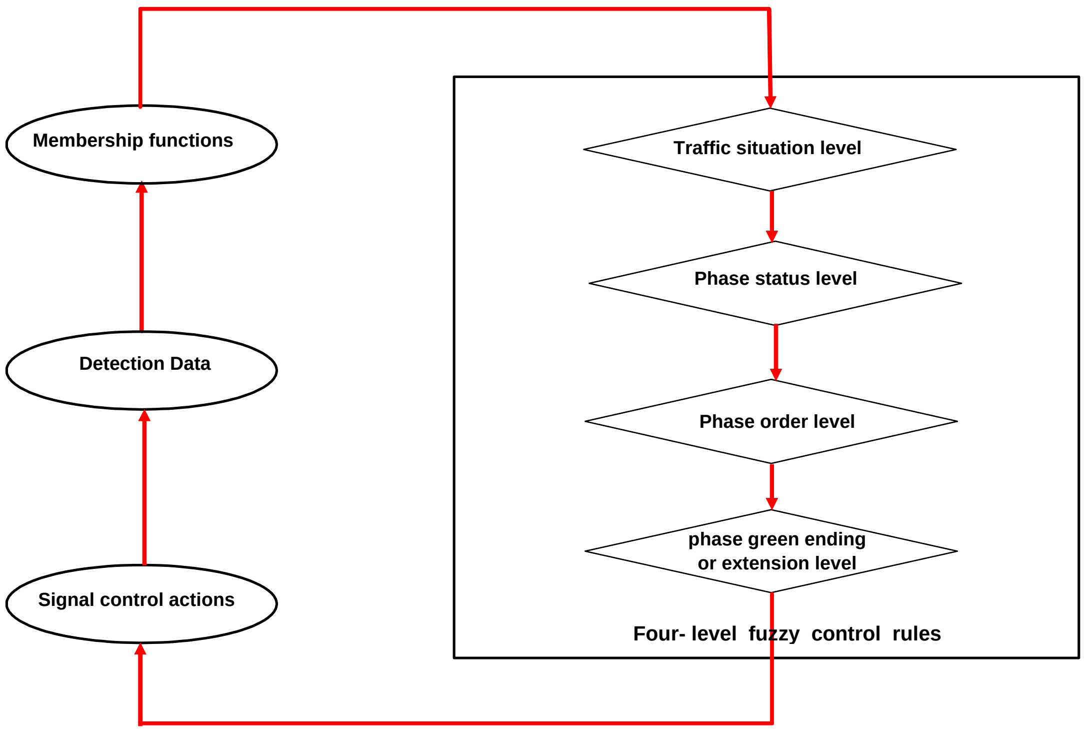



**Figure 7-1. The four-level fuzzy logic control model**

## 7.2 Simulation and Verification with Field Data

### 7.2.1 Example of simulation steps

**First step**: The test intersection is located in Herndon VA. The north-south direction is the main street – Centreville Rd, the westbound is Worldgate Dr, and Eastbound is Parcher Ave. Intersection geometry location is in Figure 7-2.

In this intersection, two detectors are located per each approach lane. The yellow line on each approach is first detector. The white one is the second detector. The location of the first one in through lane is approximate 330 ft upstream of the stop line and the second one is at the stop line. This means that we know how many vehicles are approaching the through lane stop line within next 6-8 seconds. The location of the first one in left-turn lane is approximate 200 ft upstream of the stop line or the start end of the left-turn lane and the second one is at the stop line. This means that we know how many vehicles are approaching the left-turn lane stop line within next 4-6 seconds. The data collecting date is chosen on May 5 2003, morning peak hour is 7:00~8:00, and afternoon peak hour is 17:00-18:00. The traffic peak hour volume is in Table 7-1.

**Second step**: The fuzzy control membership function is updated, the example is showed in Figure 7-3.


**Figure 7-2. The detectors location in real field test intersection**

**Table 7-1. Test intersection traffic volume**

| | | Northbound | Southbound | Eastbound | Westbound |
|---|---|---|---|---|---|
| A.M. Peak | Left-turn (v/h) | 108 | 174 | 126 | 214 |
| | Through (v/h) | 1152 | 1250 | 108 | 48 |
| | Right-turn (v/h) | 588 | 40 | 216 | 84 |
| P.M. Peak | Left-turn (v/h) | 282 | 150 | 156 | 444 |
| | Through (v/h) | 1158 | 935 | 108 | 216 |
| | Right-turn (v/h) | 222 | 25 | 206 | 150 |

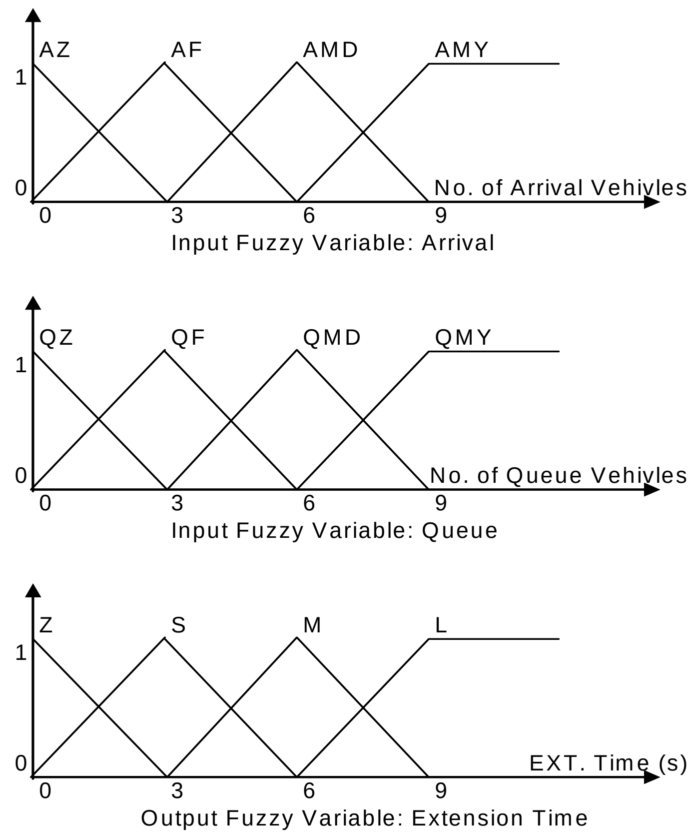



**Figure 7-3. Membership functions of the fuzzy green extender**

**Third step**: The fuzzy control variables are updated by the four-level fuzzy control rules formulated in Chapter 6.

- In first level, traffic situation level, update traffic situation variables TS and LS per circle using last 5 minutes TOCC, TVOL, LOCC, and LVOL data. The example is showed in Table 7-2.

- In second level, phase status level, update phase status variable PS per circle using updated TS and LS and fuzzy status selector. The example is showed in Table 7-3.

- In third level, phase order level, update phase order variable PO per circle using updated PS and fuzzy order selector. The example is showed in Table 7-4.

- In forth level, phase green ending or extension level, and evaluate phase green time using fuzzy green extender, controlling the target signal intersection using the output results.

**Fourth step**: repeat the above steps for next cycle.

7.2.2 Comparison results of field data test

Based on the field data collected at the study intersection four types of traffic signal control methods were selected for the purpose of signal delay comparisons. The four

**Table 7-2. Traffic fuzzy situation variables**

| | Northbound | Southbound | Eastbound | Westbound |
|---|---|---|---|---|
| TOCC | Normal | Normal | Zero | Zero |
| TVOL | Normal | Normal | Low | Low |
| LOCC | Normal | Normal | Zero | Normal |
| LVOL | Normal | High | Low | Normal |
| TS | Normal | Normal | Low | Low |
| LS | Normal | Normal | Low | Normal |

**Table 7-3. Fuzzy phase status variable**

| | Northbound | Southbound | Eastbound | Westbound |
|---|---|---|---|---|
| PS | LAP | LAP | PP | PP |

**Table 7-4. Fuzzy phase order variable**

| | N-S direction | E-W direction |
|---|---|---|
| PO | TT+LL | TT |

types of control methods include the proposed four-level fuzzy control (FFC), traditional fuzzy control (TFC), actuated control (AC), and fixed-time control (FC). The comparisons during these four types control are showed in Figure 7-4, Figure 7-5 and Figure 7-6. Based on the field data and results from numerical analysis, the proposed four-level fuzzy control has the lowest total average delay and the number of delayed vehicles in all traffic conditions. In other words, FFC is the best control methodology among the four models for the reduction of total and average traffic delay. The comparison of average delay per vehicle for the peak hour suggests that FFC outperforms TFC, AC, and FC by 10% ~ 23%, 35% ~ 36%, and 37% ~ 46% respectively. The comparison of average delay per vehicle for the non-peak hours exhibits the similar trend that FFC outperforms TFC, AC, and FC in all categories.

In AM peak hour, traffic delay comparison result is in Table 7-5 and Figure 7-7, Figure 7-8. The comparison of average delay per vehicle for the AM peak suggests that FFC outperforms TFC, AC, and FC by 23%, 35%, and 46% respectively. While at least two-third of the vehicles were delayed at the intersection for TFC, AC, and FC methods, FFC saw less than 50% of the vehicles delayed.

In PM peak hour, traffic delay comparison result is in Table 7-6 and Figure 7-9, Figure 7-10. The comparison of average delay per vehicle for the PM peak suggests that FFC once again outperforms TFC, AC, and FC by 10%, 36%, and 37% respectively. While at least three quarters of the vehicles were delayed at the intersection for TFC, AC, and FC methods, FFC saw less than 60% of the vehicle delayed.

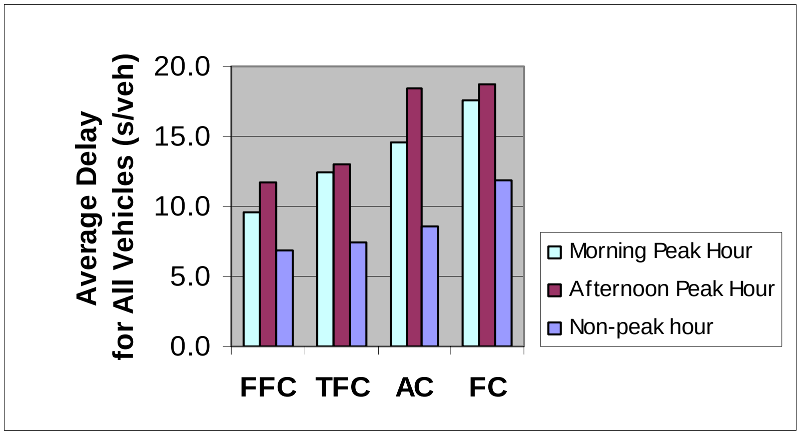



**Figure 7-4. Average delay comparison for all vehicles**

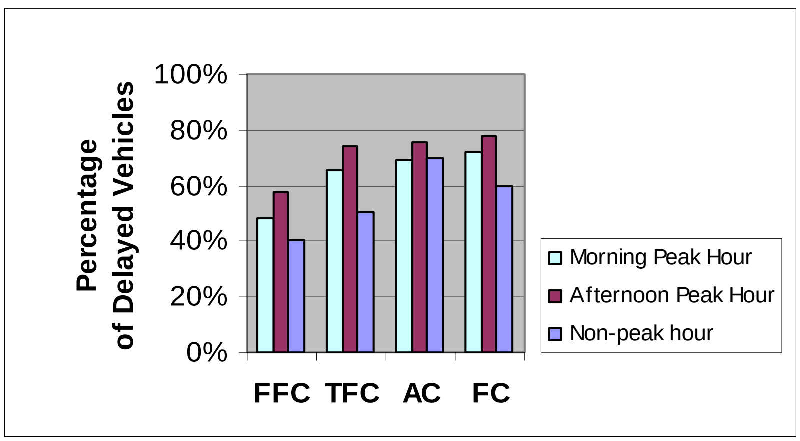



**Figure 7-5. Percentage of delayed vehicles comparison**

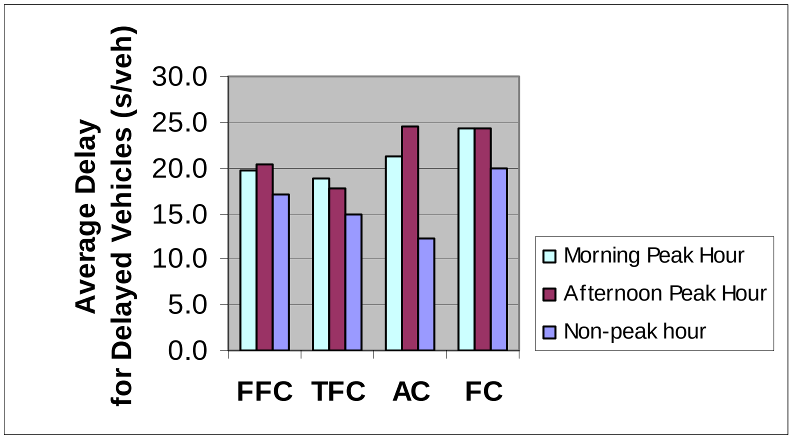



**Figure 7-6. Average delay comparison for delayed vehicles**

**Table 7-5. Traffic delay comparison for AM peak**

| | Northbound | Southbound | Eastbound | Westbound | N-S | E-W | Total |
|---|---|---|---|---|---|---|---|
| **Four-step Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 6.3 | 6.1 | 31.2 | 40.4 | 6.1 | 35.6 | 9.5 |
| Average delay of delayed vehicles | 16.3 | 12.8 | 36.8 | 44.1 | 14.2 | 40.5 | 19.7 |
| Percentage of delayed vehicles | 38% | 47% | 85% | 92% | 43% | 88% | 48% |
| **Traditional Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 8.7 | 11.6 | 26.5 | 31.8 | 10.3 | 29.1 | 12.4 |
| Average delay of delayed vehicles | 13.9 | 17.6 | 38.3 | 38.2 | 16.0 | 38.3 | 18.9 |
| Percentage of delayed vehicles | 63% | 66% | 69% | 83% | 64% | 76% | 66% |
| **Actuated Control** | | | | | | | |
| Average delay of total vehicles | 13.2 | 13.2 | 18.5 | 32.5 | 13.2 | 25.2 | 14.6 |
| Average delay of delayed vehicles | 19.3 | 19.8 | 26.8 | 35.5 | 19.6 | 31.6 | 21.2 |
| Percentage of delayed vehicles | 69% | 67% | 69% | 92% | 68% | 80% | 69% |
| **Fixed-time Control** | | | | | | | |
| Average delay of total vehicles | 17.9 | 15.7 | 21.0 | 27.4 | 16.7 | 24.1 | 17.6 |
| Average delay of delayed vehicles | 25.7 | 22.1 | 24.8 | 32.9 | 23.7 | 28.7 | 24.3 |
| Percentage of delayed vehicles | 70% | 71% | 85% | 83% | 71% | 84% | 72% |

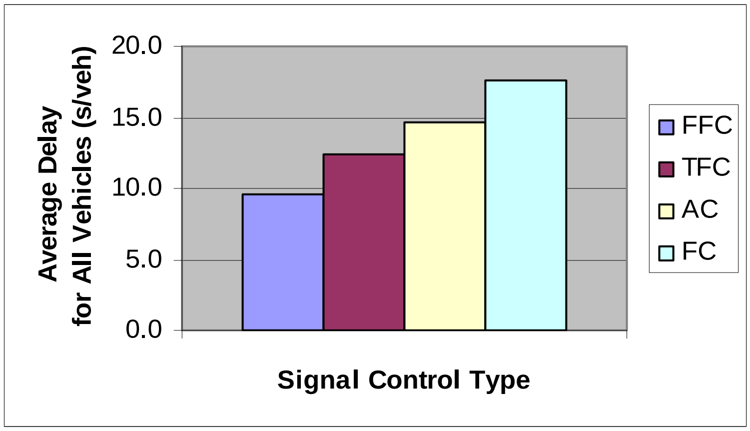



**Figure 7-7. Average delay comparison for all vehicles in AM peak**

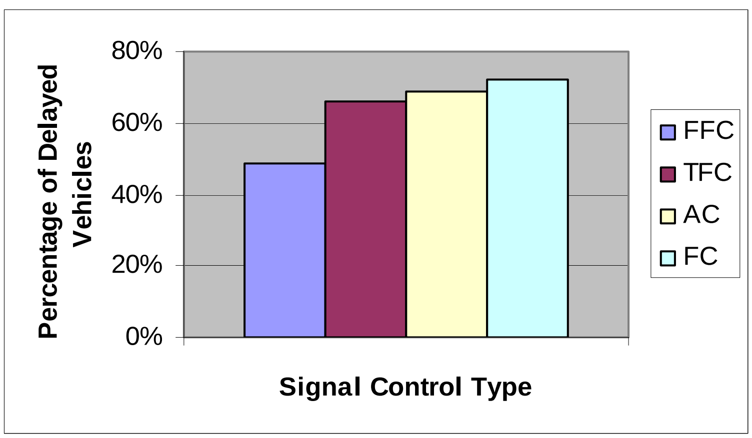



**Figure 7-8. Percentage of delayed vehicles comparison in AM peak**

### Table 7-6. Traffic delay comparison in PM peak

| | Northbound | Southbound | Eastbound | Westbound | N-S | E-W | Total |
|---|---|---|---|---|---|---|---|
| **Four-step Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 10.8 | 7.1 | 32.2 | 15.4 | 9.3 | 20.6 | 11.8 |
| Average delay of delayed vehicles | 21.3 | 15.0 | 32.2 | 18.7 | 18.8 | 23.5 | 20.3 |
| Percentage of delayed vehicles | 51% | 48% | 100% | 83% | 50% | 88% | 58% |
| **Traditional Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 14.1 | 6.0 | 29.7 | 17.5 | 10.8 | 21.3 | 13.1 |
| Average delay of delayed vehicles | 18.7 | 10.5 | 31.5 | 17.9 | 15.9 | 22.0 | 17.6 |
| Percentage of delayed vehicles | 75% | 57% | 94% | 98% | 68% | 97% | 74% |
| **Actuated Control** | | | | | | | |
| Average delay of total vehicles | 21.5 | 11.7 | 25.7 | 20.0 | 17.4 | 21.8 | 18.4 |
| Average delay of delayed vehicles | 28.2 | 19.4 | 25.7 | 21.6 | 25.0 | 23.0 | 24.5 |
| Percentage of delayed vehicles | 76% | 60% | 100% | 93% | 70% | 95% | 75% |
| **Fixed-time Control** | | | | | | | |
| Average delay of total vehicles | 21.4 | 11.5 | 23.4 | 24.1 | 17.3 | 23.9 | 18.7 |
| Average delay of delayed vehicles | 27.8 | 16.5 | 30.1 | 25.4 | 23.4 | 26.7 | 24.2 |
| Percentage of delayed vehicles | 77% | 70% | 78% | 95% | 74% | 90% | 77% |

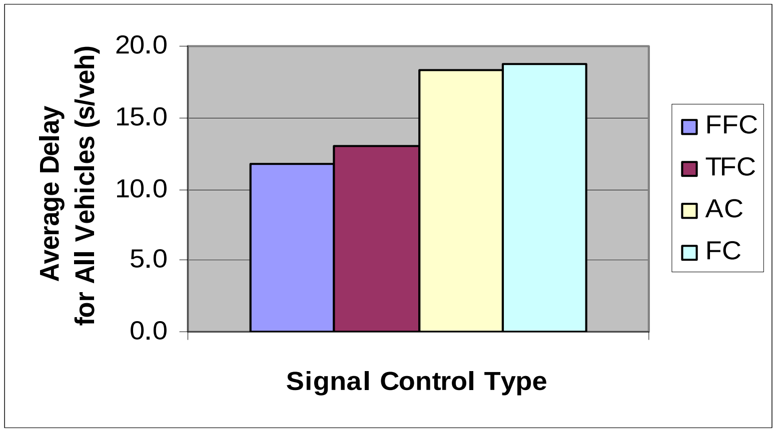



**Figure 7-9. Average delay comparison for all vehicles in PM peak**

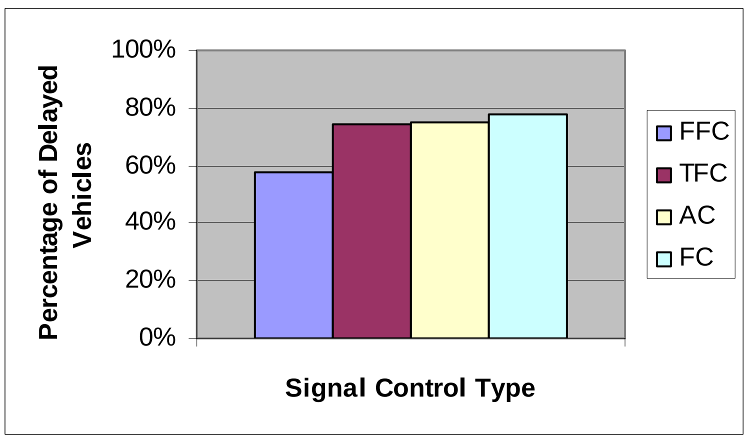



**Figure 7-10. Percentage of delayed vehicles comparison in PM peak**

In non-peak hour, traffic delay comparison result is in Table 7-7 and Figure 7-11, Figure 7-12. The comparison of average delay per vehicle for the PM peak suggests that FFC once again outperforms TFC, AC, and FC by 8%, 20%, and 42% respectively.

Based on above comparison results using field data, the proposed dynamic traffic signal left-turn phase fuzzy logic control system is a superior and efficient tool for reducing intersection traffic delay. The study also demonstrated that the successful implementation of the proposed model does not rely on the installation of expensive or complicated equipment.

## Table 7-7. Traffic delay comparison in non-peak

| | Northbound | Southbound | Eastbound | Westbound | N-S | E-W | Total |
|---|---|---|---|---|---|---|---|
| **Four-step Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 4.4 | 3.9 | 32.3 | 18.4 | 4.1 | 22.4 | 6.8 |
| Average delay of delayed vehicles | 15.2 | 12.2 | 36.9 | 19.4 | 13.3 | 24.1 | 17.1 |
| Percentage of delayed vehicles | 29% | 32% | 88% | 95% | 31% | 93% | 40% |
| **Traditional Fuzzy Control** | | | | | | | |
| Average delay of total vehicles | 4.5 | 5.1 | 24.9 | 20.6 | 4.9 | 21.8 | 7.4 |
| Average delay of delayed vehicles | 12.1 | 10.8 | 28.4 | 22.9 | 11.3 | 24.4 | 14.8 |
| Percentage of delayed vehicles | 37% | 47% | 88% | 90% | 43% | 89% | 50% |
| **Actuated Control** | | | | | | | |
| Average delay of total vehicles | 8.2 | 6.0 | 15.9 | 19.0 | 6.8 | 18.1 | 8.5 |
| Average delay of delayed vehicles | 11.5 | 9.6 | 18.1 | 20.0 | 10.4 | 19.5 | 12.2 |
| Percentage of delayed vehicles | 71% | 62% | 88% | 95% | 66% | 93% | 70% |
| **Fixed-time Control** | | | | | | | |
| Average delay of total vehicles | 13.0 | 9.4 | 10.5 | 20.5 | 10.8 | 17.6 | 11.8 |
| Average delay of delayed vehicles | 21.3 | 16.5 | 14.0 | 34.2 | 18.4 | 27.4 | 19.8 |
| Percentage of delayed vehicles | 61% | 57% | 75% | 60% | 59% | 64% | 59% |

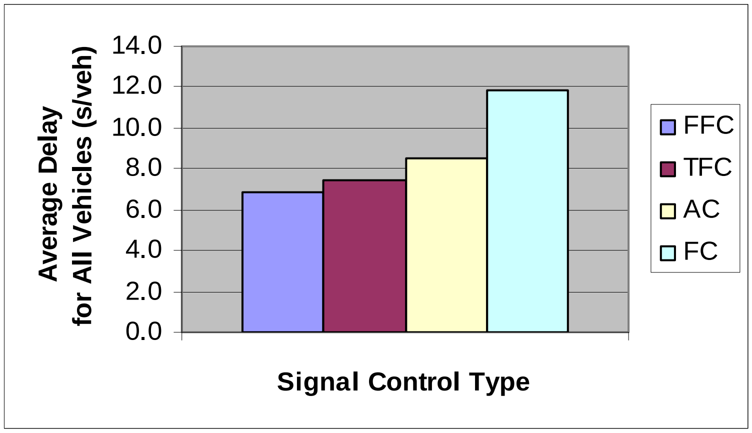



**Figure 7-11. Average delay comparison for all vehicles in non-peak**

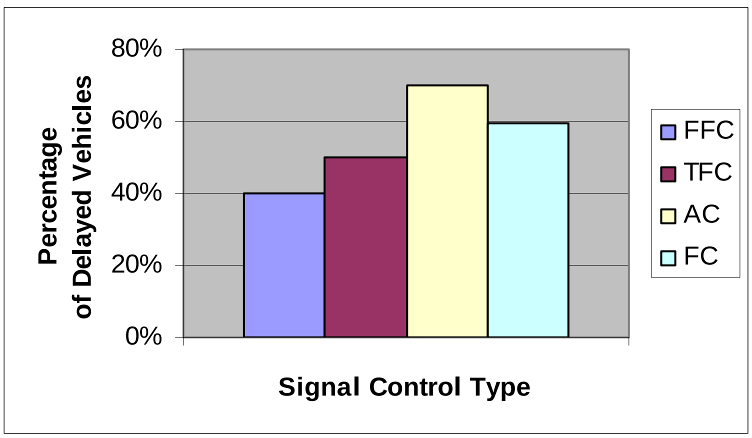



**Figure 7-12. Percentage of delayed vehicles comparison in non-peak**

# Chapter 8  Summary and Recommendation

Chapter 8 presents a summary of the study and then draws a number of conclusions based on the outcome of the research.  Some recommendations are made for future research.

## 8.1 Research Summary

Signal left-turn phase control involves a complex multi-objective and multi-constraint problem analysis in which the optimization performed is based mainly on recent information.  This research designs a dynamic traffic signal left-turn phase fuzzy logic control system.  Based on the new fuzzy phase selection model which guides the selection between the leading and lagging left-turn phases, the four-level fuzzy logic control model is used to optimize signalized intersection operation.

The four-level fuzzy logic control includes: Traffic situation level, Phase status level, Phase order level, and Green ending or extension level.  In the model, observed approach traffic flows are used to estimate relative traffic intensities in the competing approaches, then these traffic intensities are used to determine whether the leading or lagging signal phase should be selected or terminated.  For example, the normal phase order in one signal intersection is A-B-C-A.  Some phases (B or C) can be skipped if no request is observed for them.  For these “traditional” fuzzy control algorithms the output phase order may become A-B-A-C-A, A-B-A-B-A, or A-C-A-C-A when phase skipping occurs. However, the intersection can not use otherwise undefined phase D (or E, F, …), which may provide better performance under certain circumstances. On the other hand, the four-level fuzzy control model offers the added flexibility of adding new phases and rearranging phase orders to create new feasible timing plans on a per cycle basis according to real-time traffic condition. This opens up the possibility of improving signal performance under unexpected traffic conditions with virtually unlimited number of phase orders such as A-D-C-A-B-A, A-D-E-A-F-A, etc.

Based on this four-level fuzzy logic model, leading left-turn delay is always lower than lagging left-turn delay for isolated signalized intersection when left-turn v/c ratio is relatively low, while leading left-turn delay is higher than lagging left-turn delay when v/c ratio is close to or exceeds 1.0. Note that leading left-turn delay will be significantly higher than lagging left-turn delay at isolated signalized intersection when left-turn v/c exceeds 1.0. The selection model is presented by Equations 3-13, 3-16, 3-25, 3-34.

For coordinated intersections, lagging phase design for the target downstream signal generates less delay than leading phase design no matter which phase design is used for the upstream signal. Lagging (for the upstream signal) + lagging (for the downstream signal) design gives the best result in terms of the target intersection delay. Leading or lagging designs do not differ in terms of through traffic delays. Instead, their strength/weakness is due to the left turning traffic delay. The selection model is presented by Equations 4-2, 4-3, 4-4, 4-5.

The performance of this dynamic traffic signal left-turn phase fuzzy logic control system compared favorably in all categories to fixed time control, actuated control, and other traditional fuzzy control based on simulations using field data. The results suggest that the dynamic traffic signal left-turn phase fuzzy logic control system is a superior and efficient tool for reducing intersection traffic delay. The study also demonstrated that the successful implementation of the proposed control model does not rely on the installation of expensive or complicated equipment.

## 8.2 Recommendations

The dynamic traffic signal left-turn phase fuzzy logic control system is an efficient tool to reduce intersection traffic delay. In the future, based on the real operation of the system, the fuzzy control principles and the membership functions of the linguistic variables will be continuously validated and calibrated.

# List of References

# List of References


[1] Webster F., *Traffic Signal Settings*. Road Research Laboratory. Road Research Technical Paper No. 56, London, 1958.

[2] Miller A.J., *A Computer Control System for Traffic Networks*. Proceedings of the Second, International Symposium on the Theory of Traffic Flow, Paris, 1963.

[3] Peter T. Martin, Rodrigo Disegni, *TRB Signal Systems Committee: Adaptive Traffic Signals Control Systems Survey*, Final Report: UTL – (07/02) – (57), August 2002.

[4] Thomas Urbanik II, Srinivasa R. Sunkari, Kirk Barnes, Allison C. Meadors, *Adaptive Left Turn Phasing*, Institute of Transportation Engineers 2000 Annual Conference, Nashville, TN, August 2000.

[5] Pitu Mirchandani, Larry Head, A real-time traffic signal control system: architecture, algorithms, and analysis, Transportation Research Part C, Volume 9, 415-432, 2001.

[6] Cuena, J; Hernandez, J; Molina, M, Knowledge-Based Models for Adaptive Traffic Management Systems, Transportation Research. Part C: Volume: 3, Issue: 5, 1995.

[7] Nicholas V. Findler, *Harmonization for Omnidirectional Progression in Urban Traffic Control*, Computer-Aided Civil and Infrastructure Engineering. Part C: Volume: 14, Issue: 5, 1999.

[8] Kronborg P., Davidsson F. and Edholm J., SOS-Self Optimising Signal Control, Development and Field Trials of the SOS Algorithm for Self Optimising Signal Control at Isolated Intersections. TFK, report 1997.

[9] Kosko B. *Fuzzy Logic.* Art House. p.363, 1993.

[10] C. Pappis, E. Mamdani, *A fuzzy logic controller for a traffic junction*. IEEE Trans. Systems Man Cybernet, SMC-7 (10):707-717,1977.

[11] Nakatsuyama, M., Nagahashi, H., and Nishizuka, N., *Fuzzy logic phase controller for traffic junctions in the one-way arterial road*. Proceedings of the IFAC Ninth Triennial World Congress, Pergamon Press, Oxford, 2865-2870, 1984.

[12] Chiu, S., *Adaptive traffic signal control using fuzzy logic*. Proceedings of the IEEE Intelligent Vehicles Symposium, 98-107, 1992.

[13] Chiu, S. and Chang, S., *Adaptive Traffic Signal Control Using Fuzzy Logic*. Proceedings IEEE International Conference on Fuzzy Systems, 1371-1376, 1993.

[14] Niittymäki J., Kikuchi S., *Application of Fuzzy Logic to the Control of a Pedestrian Crossing Signal*. Transportation Research Record No. 1651, pp. 30-38. Washington D.C., 1998.

[15] Mohamed B. Trabia, Mohamed S. Kaseko, Murali Ande, *A two-stage fuzzy logic controller for traffic signals*, Transportation Research Part C, Volume 7, 353-367, 1999.

[16] Sayers T., Bell M.G.H., Mieden T. and Busch F. *Improving the Traffic Responsiveness of Signal Controllers Using Fuzzy Logic*. Presented at the IEE symposium on Urban Congestion Management, November 1995.

[17] Niittymäki J., Mäenpää M. *The role of fuzzy logic public transport priority in traffic signal control.* Traffic Engineering and Control, International Journal of Traffic Management and Transportation Planning, Volume. 42. No.1. pp. 22-26, January 2001.

[18] J. E. Hummer, R. E. Montgomery, and K. C. Sinha. *Guidelines for Use of Leading and Lagging Left-Turn Signal Phasing*. Transportation Research Record, 1324(3), 11-20, Washington D.C., 1991.

[19] Jim C. Lee, R. H. Wortman, D. J. P. Hook, and M. J. Poppe. *Operational Comparison of Leading and Lagging Left Turns*. Transportation Research Record, 1421(1), 1-10, Washington D.C., 1993.

[20] Daniel B. Fambro, Gilmer D. Gaston, and Christopher M. Hoff. *Comparison of Two Protected-Permitted Lead-Lag Left-Turn Phasing Arrangements.* Research Report 989-1F, Texas Transportation Institute, funded by Texas Department of Highways & Public Transportation, 1991.

[21] Peter S. Parsonson. *Signal Timing Improvement Practices.* NCHRP Synthesis of Highway Practice 172, TRB, National Research Council, Washington D.C., 1992.

[22] Nassi, R. *Tucson's Lag Left Summary.* Arizona, 1985.

[23] Tian, Z., T. Urbanik, R.E. Engelbrecht, and K. Balke. In, *Pedestrian Timing Alternatives and Impacts on Coordinated Signal Systems under Split-Phasing Operations*, Transportation Research Record, 1748, pp: 32-41, Washington, D.C., 2001.

[24] Buckholz, J.W., The Major Pitfalls of Coordinated Signal Timing, Institute of Transportation Engineers, 63(8), August 1993.

[25] L. Kanth Nandam, and T. Douglas Hess, Dynamic Change of Left Turn Phase Sequence between Time-Of-Day Patterns – Operational and Safety Impacts, Institute of Transportation Engineers 2000 Annual Conference, Nashville, Tennessee, August 2000.

[26] James L. Pline, *Left Turn Treatments at Intersections*, NCHRP Synthesis 225, Draft, TRB, National Research Council, Washington D.C., 1995.

[27] Zhenyang Li, Hui Wang, Lee D. Han, *Comparison of Leading and Lagging Left-turns for Coordinated Intersections*, Institute of Transportation Engineers 2002 Annual Conference, Philadelphia, PA, August 2002.

[28] Zhenyang Li, Hui Wang, Lee D. Han, *Selecting Leading or Lagging Left-turns Signal Phases at Coordinated Intersections*, Transportation Research Board, Washington D.C., January 2003.

[29] Jonathan Upchurch. *Comparison of Left-Turn Accident Rates for Different Types of Left-turn Phasing*. Transportation Research Record, 1324(5), 33-40, Washington D.C., 1991.

[30] Pacey, G. M. The Progress of a Bunch of Vehicles Released from a Traffic Signal. 1956, Road Research Laboratory, London (mimeo).

[31] Daniel E. Mitchell, Sean F. Skehan, *Queue Detector Design and Operation for Protected-Permissive Left Turn Phasing*, Institute of Transportation Engineers 2000 Annual Conference, Nashville, TN, August 2000.

[32] Robert Hoyer and Ulrich Jumar, *Fuzzy Control of Traffic Lights,* IEEE International Conference on Fuzzy Systems, 1526-1531,1994.

[33] Zhou, W-W., Wu, J., Lee, A., Fu, L., and Miska, E., *Fuzzy Flows*, ITS: Intelligent Transportation System, May/June 43-45, 1997.

[34] Zhenyang Li, Hui Wang, Lee D. Han, *A Proposed Four-Level Fuzzy Logic for Traffic Signal Control*, Transportation Research Board, Washington D.C., January 2004.

[35] By Kent Kacir, Christopher L. Brehmer, and David Noyce, *A Recommended Permissive Display for Protected/Permissive Left-Turn Control*, ITE Journal, Volume 73 (No. 12).

# Vita

Mr. Zhenyang Li started his academic career in Civil and Transportation Engineering at Harbin Institute of Technology in the 1987. He received his Bachelor of Science in Civil/ Highway and Urban Road Engineering in 1991 and joined the Highway Planning and Design Institute of the Ministry of Communications of China immediately after his graduation. There he involved in planning, design, and construction of the China National Expressway System, and developed an expertise in highway geometric design and project feasibility study. During this period (1991~2000), he served as assistant civil engineer, professional civil engineer and later project manager for numerous highway projects. He was also provided with opportunities to work on World Bank invested projects and therefore built his early knowledge of the international standards.

In 2001, Mr. Li began his doctoral program study at University of Tennessee in Knoxville, Tennessee. During this period, Mr. Li served as a Graduate Research Assistant in the Civil and Environmental Department. Responsible for a series of projects encompassing adaptive traffic control model development, traffic impact study, travel demand analysis, GIS application and 3D visualizations, traffic simulation, and statistical data analysis.

With his perfectly integrated knowledge and experience in transportation planning, highway geometric design, traffic engineering and simulation. Mr. Li looks forward to an even more fruitful career in the transportation field.