12-2013

Insights Into the Beliefs and Practices of Preservice Elementary Mathematics Teacher Educators: A Naturalistic Inquiry

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To the Graduate Council:

I am submitting herewith a dissertation written by Linda Ann Arnold entitled "Insights Into the Beliefs and Practices of Preservice Elementary Mathematics Teacher Educators: A Naturalistic Inquiry." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

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Insights Into the Beliefs and Practices of Preservice Elementary Mathematics Teacher Educators: A Naturalistic Inquiry

A Dissertation Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Linda Ann Arnold

December 2013

Acknowledgments

I would like to thank the members of my committee who so generously gave of their time, and supported me through the process of writing this dissertation.

Dr. Vena Long was an amazing chair and manager for this research project. She knew what to say want what to do in order to get me to the finish line. At times, I needed her to set deadlines, and she did this without my asking, at just the right times. At time, I needed her to tell me to slow down. At the end, I even needed her to tell me, “Stop, already.” She possessed a great spectrum of knowledge, and a huge font of expertise in the world of mathematics education. What is more, she supervised my dissertation as a professor emerita, at a time when she did not need to do any more work. Dr. Long and I share an interest in multi-dimensional geometry, for itself and also, I think, as a metaphor for possible new directions for growth. I thank Dr. Long for helping me to perceive some of these new dimensions.

Dr. Amos Hatch donated many hours of his valuable time to helping me. He is a great editor and methodologist. His help was invaluable to me in the process of writing, and with the myriad of details related to qualitative methodology. I was inspired by his love of knowledge and by his wisdom. I have grown through the helpful feedback and suggestions that he took the time to share. Through his help and encouragement, I have been inspired to believe that I can both write and do research. I also thank Dr. Ji-Won Son, whose knowledge and whose encouragement to me was of immense value. She believed in me during dark times and bright, and helped me to go places I never thought I would go. Finally, thanks to Dr. Mary Ziegler, who helped me when I was first beginning to understand how to work with adults as a teacher educator, and without whose wise and thoughtful questions and valuable suggestions this dissertation would not have been the same.

Abstract

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. Until now, researchers have had little understanding of beliefs and practices of teachers in elementary mathematics methods classrooms, despite the fact that teacher educators' beliefs and practices have profound effects on what happens in thousands of classrooms every day. The qualitative research in this study offered insights with the potential to inform higher educational practice in regard to the preparation of elementary mathematics teachers and mathematics teacher educators. Participants were six professors and instructors of elementary math methods courses at six different colleges and universities. A series of 18 in-depth interviews, three with each participant, allowed context, nuance, and detail to be closely examined. Typological analysis led to the emergence of six themes. They dealt with consistency, challenge, content, methods, resources, and reasons. Details related to those themes helped to illuminate what the working experience of a preservice elementary mathematics teacher is like. Study conclusions indicated (a) general consistency, with some exceptions, between reported beliefs and reported practices; (b) some common challenges faced by most participants, particularly in regard to limited elementary classroom experience, and perceived time constraints; (c) wide variation in content and methods despite agreement on teaching philosophy; (d) perceived student needs as the main reported basis of participant teaching decisions, and; (e) some indication of the extent of influence of curricular and pedagogical reform as elementary mathematics teacher educators sought to enact generally constructivist inspired teaching visions. Recommendations for educators, researchers and policy makers stemming from these conclusions have potential benefits for elementary mathematics teacher preparation.

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Chapter 1

Introduction

> "The school mathematics experience of most Americans is and has been uninspiring at best and mentally and emotionally crushing at worst…Ironically, the most logical of the human disciplines of knowledge is transformed through a misrepresentative pedagogy into a body of precepts and facts to be remembered 'because the teacher said so.' Despite its power, rich traditions, and beauty, mathematics is too often unknown, misunderstood, and rendered inaccessible. The consequences of traditional mathematics teaching have been documented; they include lack of meaningful understanding, susceptibility to 'mathematical puffery and nonsense' (Schoenfeld, 1991), low and uneven participation, (and) personal dread" (Ball, 1988a, p. 1).

Although many researchers have examined the beliefs and practices of preservice elementary mathematics teachers (Kagan, 1992; Kennedy 2011, Wu, 2011), we know little about the beliefs and practices of the teacher educators who prepare them. Yet, the beliefs and practices of such teacher educators may have profound implications for what happens in thousands of classrooms every day. The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. The qualitative research completed sought to yield insights with the potential to help inform higher educational practice in regard to the preparation of mathematics teachers and mathematics teacher educators. Participants in the study were six professors and instructors of elementary math methods courses at six different colleges and universities.

This chapter begins with an overview of the context and background that frame the study. Following this are the problem statement, the formal statement of purpose, and the accompanying research questions. Also included in this chapter is a brief discussion about the research approach, assumptions, the researcher, and research perspective. The chapter concludes with a discussion of the rationale and significance of this study, and definitions of key terms.

Background and Context

The literature is replete with examples of those who stress that looking at preservice teacher (PST) beliefs is essential. Brownlee (2003) encourages such exploration because "these beliefs will influence their (PST's) teaching practices in the classroom" (p. 87). Bischoff and Golden (2003) describe a general disconnect between preservice elementary teacher concept knowledge and procedural knowledge. Ambrose, et.al. (2004) remark, "because of the important role beliefs play in the teaching and learning of mathematics, mathematics educators need to consider ways to assess beliefs and belief change" (p. 56). However, while dozens studies have looked at preservice elementary mathematics teachers, my extensive review of the literature did not find any research specifically addressing the beliefs and practices of those who prepare such teachers.

Researchers can and do look at the beliefs and practices of elementary school children when it comes to mathematics. They can and do look at the beliefs and practices of those who teach elementary school children. This study seeks to shed more light on the beliefs and practices of those who prepare the teachers of these children. By looking at these source beliefs and practices, insight into the experience of teaching courses for preservice elementary mathematics teachers can be gained. From this improved vantage point, recommendations can be made, in order to better inform higher educational practice.

Problem Statement

A gap exists in the literature regarding the beliefs and practices of those who prepare future elementary mathematics teachers. Research reveals little about their views of mathematics, or their perceptions on its teaching and learning. Little is known about what elementary mathematics methods teachers ask their students to do, why they choose certain task structures, and how they engender and structure classroom discourse about what students are learning and doing.

Statement of Purpose and Research Questions

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. Situated within the qualitative research paradigm, the goal of this research was to contribute to an emerging professional conversation by exploring participants' perceptions and gaining an understanding of their beliefs and practices as indicated by their experiences in the context of their classroom environments. The research questions were:

1.What are the participants' beliefs regarding preparing elementary math teachers?

(a) What do they believe elementary students need to learn?

(b) What do they believe preservice elementary teachers should learn?

(c) How to they believe elementary students should be taught?

(d) How do they believe they should use class time with their students?

(e) What do they believe are the major challenges that they face in their work?

2. How do participants describe their approaches to teaching elementary math methods course?

(a) How do they use class time with their students?

(b) What content do they teach?

(c) How do they decide what to teach?

Research Approach

With the approval of the University's Institutional Review Board, I studied the experiences and perspectives of elementary math methods instructors. The investigation was a naturalistic inquiry using qualitative research methods. In-depth, semi-structured interviews gathered over a period of three months were the primary sources of data. Each participant completed three interviews; an introductory interview, a follow-up interview, and a member check interview. Prior to conducting interviews, I sent a demographic survey instrument to 450 mathematics educators, yielding 92 responses. Information from this instrument helped to establish parameters which in turn helped to choose participants for the study. Qualitative information obtained through the individual interviews with participants chosen formed the basis for the overall findings of the study. Pseudonyms identified each interviewee, and all interviews were taped and transcribed verbatim.

A review of literature helped to shape and refine the data collection methods used. I developed coding categories and refined them on an ongoing basis, guided by the study's conceptual framework. Data were analyzed through typological analysis (Hatch, 2002). In order to establish credibility, I employed various strategies, including searching for discrepant evidence, and peer review at different stages as the study progresses.

Assumptions

I assumed that participants were professionals with a sincere interest in improving the future teaching practice of their students. I made no assumptions about the presence or absence of any other factors that might motivate participants to teach an elementary math methods course.

I assumed that teachers provided accurate demographic information when completing the email survey/questionnaire, to the best of their knowledge. Regarding interview data, I assumed teachers accurately reported their perceptions and beliefs in regard to their practice, that is, their constructed realities. In this study, set within the constructivist paradigm, I did not seek objective reports of actual actions in regard to practice. In such a study, absolute reality is not sought nor even assumed to exist; rather, constructions of reality are what is important (Lincoln & Guba, 1994). I wanted to know what professors and instructors believed in order to support them and the teacher candidates with whom they work.

The Researcher

At the time of conducting this study, I was pursuing a doctorate in education at the University of Tennessee. Additionally, I worked part time as an adjunct instructor at a small Southeastern liberal arts college, teaching three sections per year of a methods course for preservice elementary mathematics teachers each spring. I continued my adjunct work while writing the dissertation.

My desire to undertake this study emerged from my interest in teacher preparation. After several years of teaching in school mathematics classrooms, my personal journey was one of wanting to give back, to help and support new teachers coming into the profession. I sought a doctoral degree, in part, to better understand problems of teacher preparation. Therefore, this topic, which provided an opportunity to construct understandings about practices and beliefs with those teaching math methods courses, seemed to be an ideal undertaking.

During my career, I have mainly taught mathematics to students in grade levels 6-12, with about 85% of my teaching done at the grade 9-12 level. However, three years ago, I began teaching college classes for preservice teachers who plan to teach math to students in grades K-6. This is not an unusual assignment, since policy makers desire a certain degree of specialization in mathematics for those teaching such classes (Pope & Mewborn, 2008). I wanted to discover more about the phenomenon of being a preservice elementary teacher educator, in part because I might incidentally learn more about working with my students and with elementary school children through hearing participants share their beliefs and practices. Thus, I viewed and continue to view research as a two way street when it comes to learning. This is consistent with a constructivist perspective. Below, I discuss this perspective in greater detail.

Research Perspective

The challenge researchers face as they begin reflecting on and designing a study is to appropriately match the research approach to the paradigm of the researcher, the purpose, questions, issues to be addressed, and the research problem. This is methodological congruence (Morse & Richards, 2002). In keeping with my ontological and epistemological beliefs, my methodological research paradigm was constructivism. This paradigm is one which is compatible with qualitative methods (Hatch, 2002), specifically with the naturalistic inquiry undertaken.

Qualitative research is best suited to promoting a deep understanding of a social setting or activity as viewed from the perspective of the research participants. The emphasis is on exploration, discovery, and description (Bloomberg & Volpe, 2008). This study of math methods teachers will emphasize just those things: discovering, understanding and describing the perspective of the participants.

A basic tenet of constructivism is that all reality is socially, culturally and historically constructed (Lincoln & Guba, 1985). Research stemming from this paradigm helps in attempts to understand social phenomena from a context-specific perspective. Inquiry is seen as value-bound rather than objective. The researcher and context under study influence the process of inquiry (Lincoln & Guba, 1985). Individuals construct reality, and so develop subjective meanings regarding their personal experience. This gives way to multiple meanings. The researcher's role is to understand the multiple realities from the perspectives of the participants (Bloomberg & Volpe, 2008). Constructivist researchers may seek to understand the process of interaction among individuals. They may focus on specific contexts in which people live and work to understand particular cultural and historical settings (Bloomberg & Volpe, 2008). The constructivist researcher's role is essentially that of a fellow participant as well as that of the facilitator of a multivoiced reconstruction (Lincoln & Guba, 2000). Rather than starting with a theory, constructivists pose research questions and then generate or inductively develop meaning from the data collected in the field (Bloomberg & Volpe, 2008).

Speaking with mathematics methods teachers about their own experiences, beliefs and practices were my interest. In interviews, we worked together to construct mutual understanding of these factors. From these interviews, significant themes and patterns emerged related to consistency, challenges, content, methods, resources, and reasons.

Rationale and Significance

The rationale for this study emanated from my desire to learn more about the beliefs and practices of preservice elementary mathematics methods teachers. If we understand more about the thinking of those who prepare teachers, this may have significant benefits for teacher preparation.

This study was timely in light of current concerns about mathematics achievement in the United States. Although most groups have shown some improvement in mathematics achievement in the past twenty years (National Center for Education Statistics, 2012), there is more work to do. Leaders in mathematics education reform have commented:

> Mathematics education works virtually nowhere as well as it needs to if we are to prepare students for life in a world where practical, intellectual and critical quantitative competence will matter more than ever before, and for more people… (Ball & Even, 2009, p. 255).

In order to help ensure effective mathematics teaching, we need to learn more about the beliefs and practices about those who are preparing mathematics teachers.

Additional rationale for the study had its basis in the importance of pedagogical preparation for teachers. For example, recent survey data indicate that in the first year of teaching, teachers with a mathematics baccalaureate, but little or no pedagogical preparation, left teaching at twice the rate of those with the same degree, but more comprehensive pedagogical preparation. (Conference Board of the Mathematical Sciences, 2012). Yet, an American Educational Research Association (AERA) commissioned report suggests that the preparation of mathematics teachers is an under-researched topic (Cochran-Smith & Zeichner, 2005).

Regarding significance, increased understanding of instructors of math methods courses may help the mathematics education community to better prepare teachers to take on the challenge of teaching mathematics in today's changing global society. The interviews of preservice elementary mathematics teacher educators (PEMTEs) provided information that will help make it possible to; (a) examine possible support practices for enhanced classroom practice among PEMTEs; (b) provide insights that may enhance the preparation and professional development of potential and current PEMTEs, and; (c) see the emergence of themes and patterns for possible examination in larger studies.

Two decades ago, in a review of literature on teacher belief, Pajares (1992) pointed out that beliefs may be the best indicators of the decisions about practice that individual teachers make throughout their lives. Students bring beliefs to teacher education and then teachers bring them into their schools and classrooms. Pajares stated, "for these reasons, investigating the educational beliefs of teachers and teacher candidates should become a focus of current educational research, and also for these reasons teacher preparation programs can ill afford to ignore the…beliefs of preservice teachers" (p. 322). I believe research until now has neglected what could be a third side of this "triangle" involving those responsible for instruction. Teachers and teacher candidates have been foci, but also need to focus on teacher educators. This study is an attempt to address that need.

Definitions of Key Terminology Used in This Study

A list of terms follows below, defined within the context of this study:

"Teaching Practices" will generally refer to reported classroom actions and routines.

"Belief" will generally be used as Pajares (1992) used the term, including attitudes, values, judgments, opinions, ideology, perceptions, conceptions, preconceptions, dispositions, implicit theories, explicit theories, personal theories, practical principles, perspectives, and social strategies. Beliefs can include individuals' philosophical contemplations. For the teacher participants in this study, "belief" will also be used in keeping with Kagan (1990), who defined teacher beliefs as "the highly personal ways in which a teacher understands classrooms, students, the nature of learning, the teacher's role in a classroom, and the goals of education" (p. 423).

"Constructivist" will refer to either the constructivist epistemological/ontological paradigm or constructivist learning theory. These can overlap, but are not identical. Constructivist learning theory is a theory of learning, and not teaching (O'Shea & Leavy, 2013). It refers to a belief that learners construct *learning*, and may be embraced by persons identifying as "constructivist" but philosophically subscribing to any one of various paradigms, including constructivist, post-positivist, post-structuralist or critical (Hatch, 2002). A constructivist ontological/epistemological paradigm, in contrast, refers to a belief that individuals construct *reality*, including, but not limited to learning. In this study, the adjective constructivist, as well as the noun, constructivism, will normally refer to learning theory. Where the reference is to a philosophical paradigm, that will be clear from the context.

"Mathematics Methods Course" will refer to a course preparing future mathematics teachers through emphasizing pedagogy. This is in contrast to courses such as College Algebra, with pure mathematics content. This is also in contrast to mathematics content courses designed for future elementary teachers, which some universities specifically required (McCrory & Cannata, 2011) for elementary teacher certification.

“PEMTEs” will refer to “preservice elementary mathematics teacher educators.” Such teachers will be at college and university level and may be professors, lecturers, teaching fellows or instructors.

“Teacher” will refer to teachers at any level. Preservice teachers will always be identified as such, or as teacher candidates. When preservice teachers are referred to as “students” it will be clear that this is in the context of a class preparing them to become teachers.

“Preservice Elementary Mathematics Teachers” will refer to undergraduate students preparing to teach elementary school mathematics, generally to children at levels below grade 7. The teacher candidates are *not* study participants. Rather, they are the students of study participants.

“Naturalistic Inquiry” will refer to a specific research methodology, originally set forth by Lincoln and Guba (1985, 2000), which is part of the qualitative research tradition.

Organization of Dissertation

Subsequent to this introductory chapter, my study is organized into four additional chapters. In Chapter 2, I present a review of literature which concentrates upon the status of math learning in the U.S.; current and historic movements in United States mathematics education; research on elementary mathematics teaching and on preservice and inservice elementary teachers’ beliefs about mathematics. In Chapter 3, I explain the methodology of naturalistic inquiry, and the specific methods of this study are set forth. Within Chapter 4, I report findings stemming from data analyses. Finally, in Chapter 5, I close by discussing themes, patterns, conclusions and recommendations emerging from the study.

Chapter 2

Review of Literature

Overview

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. To carry out this study, it was necessary to complete a critical review of current literature. The review was ongoing throughout the data collection, data analysis, and synthesis phases of research. The present form of the review provides background information and illustrates what the literature yields in terms of informing the study at hand.

This review explored topics related to participants' classroom practices and to their beliefs about mathematics, teaching and learning. The context was one of present day changing standards and methods of evaluation in United States schools. Four areas of literature were critically reviewed: (1) University and college mathematics methods teachers and their work of teacher preparation. (2) Reform movements in United States mathematics education, including context, structure, and the standards pertaining to mathematics methods teachers' work. (3) Elementary mathematics teaching, including substantive theory on classroom tasks and discourse, on teachers' subject matter knowledge and pedagogical content knowledge. (4) Finally, beliefs, including beliefs about mathematics in general, elementary teachers' beliefs about mathematics in particular, and elementary teachers' beliefs about teaching.

The beliefs and practices of mathematics methods instructors themselves were not directly examined in this review. There is a gap in the literature on those matters. Thus, this review concerns topics related to such beliefs and practices.

Topics

The Work of Mathematics Teacher Preparation

Who becomes a mathematics teacher educator? In their report to the National Council of Teachers of Mathematics, the Task Force on Teacher Preparation, Certification and Shortage (2005), stated “For all students to learn mathematics well, they must have a highly skilled, committed, and professionally supported teacher of mathematics” (p. 2). They further stated that the quality of teachers of mathematics is, at least in part, a function of the quality of mathematics teacher educators (Task Force on Teacher Preparation, Certification and Shortage, 2005).

Such courses are often taught by faculty composed primarily of adjuncts and clinical instructors who do not hold tenure-line positions (Wolf-Wendel, Baker, Twombly, Tollefson, & Mahlios, 2006). Pope and Mewborn (2008) gave information on mathematics teacher educators in the United States in their report to the International Commission on Mathematics Instruction (ICMI). Routes to becoming a mathematics teacher educator in the United States vary considerably. Three types of professionals who train future mathematics teachers are; (a) generalists, with special training in education, but not mathematics; (b) mathematicians, who, for the most part, have no formal preparation for becoming mathematics educators; and; (c) specifically prepared mathematics educators. Generalists and mathematics educators preparing preservice mathematics teachers have most often begun their own careers as primary or secondary classroom teachers. There is sometimes interchange of levels, however. For instance, a mathematics educator may have mainly taught secondary school pupils, but work mostly in preparing primary teachers. Most mathematicians given the task of preparing preservice teachers have experience teaching at the university level only. Mathematics teacher educators may or may not hold a doctorate in mathematics education and may or may not have any formal training in mathematics education. Some mathematics teacher educators are graduate students employed to teach classes as part of an assistantship. Many are adjuncts, often retired teachers, who teach part-time. While there is growing recognition that teaching mathematics education, and therefore preparing future teachers of mathematics, requires a specialized knowledge base (Ball & Forzani, 2010, Viadero, 2004), institutions in the United States are just beginning to grapple with what this means (Pope & Mewborn, 2008).

Shortages exist, both of mathematics teachers and of mathematics teacher educators (American Association of Colleges of Teacher Education, 2010b). The teacher supply for critical shortage areas in the STEM field (science, technology, engineering, and mathematics) is insufficient to meet the needs in P-12 classrooms. Furthermore, changes in federal policy regarding teacher effectiveness may affect both beginning teachers and teachers who are more advanced. According to the Statistics for Survival, 40 of the 50 states reported mathematics teacher shortages. The shortage of P–12 mathematics teachers is, in part, a function of a shortage of mathematics teacher educators (Task Force on Teacher Preparation, Certification and Shortage, 2005).

Teacher preparation programs. Issues in mathematics teacher preparation include alternative methods of preparing teachers, balancing clinical experience with classroom settings and changes currently proposed. Such changes include changes in program requirements, accountability and evaluation. This section examines those issues**.**

Collegiate and alternative teacher preparation. Preservice elementary mathematics methods teachers prepare candidates for teaching in a social context that includes popular films, such as "Waiting for Superman"; that highly tout alternative preparation programs for teachers such as Teach for America. In this setting, questions are being asked about previously unquestioned preferences and requirements for teachers trained in collegiate schools of education rather than through alternative means.

How relevant and important is the job of mathematics methods teachers in collegiate-based programs? Numerous studies have examined alternative or non-standard teacher preparation and certification. The preponderance of evidence finds significant links between types and qualities of teacher preparation and student achievement. For example, the U.S. Department of Education commissioned a review of fifty-seven studies considered to be of rigorous design which found positive relationships between teacher effectiveness and collegiate teacher preparation programs, but not between such effectiveness and alternative preparation programs. These relationships were found across studies using different units of analysis, different measures of preparation and in studies controlling for students' socioeconomic status and for prior academic performance (Wilson, Floden, & Ferrini-Mundy, 2001). A study of New York City Teaching Fellows on the effect of teacher qualifications on 4th and 5th graders' mathematics performance in high-poverty schools concluded that teachers who graduated from collegiate preparation programs were significantly more effective than teachers lacking certification, and performed better than Teaching Fellows and Teach for America teachers (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2008). Three recent, large, longitudinal studies, using individual-level student data found that teachers who enter as temporary or emergency hires or as alternative route candidates, are less effective than fully-prepared beginning teachers with similar students (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2006; Darling-Hammond, Holtzman, Garlin, & Heilig, 2005; Kane, Rockoff, & Staiger, 2006). Finally, the positive effects of standard teacher certification on students' mathematics achievement exceeded those of content majors in mathematics with non-standard certification. This highlights the importance of the pedagogical portion of teacher training (Goldhaber & Brewer, 2000).

Classroom preparation and preparation in the field. Methods classes are only one dimension of teacher preparation. Another important component takes place through fieldwork. The literature addresses the balance between these two aspects of teacher preparation.

Some would contend that teacher preparation is now increasingly recognized as an academically taught, but clinical practice profession, such as nursing, clinical psychology and medicine (Alter & Coggshall, 2009). The American Association of Colleges for Teacher Education (AACTE) points out the importance of clinical practice, in addition to methods classroom experience (American Association of Colleges of Teacher Education, 2010a). One of their recent major policy reports states, "much of what teachers must know in order to teach effectively emerges in the context of actual practice" (p. 6). AACTE's position is that factors such as information regarding students' misconceptions, the effectiveness of certain strategies with varying student groups, or knowledge about how different learners learn best can only be fully understood in the field work component of teacher education. They do not recommend decreased time in methods classes, but they do recommend a strong component of field experience in teacher preparation in addition to methods classes. The AACTE (2010a) report goes on to state, "The skilled application of theory to benefit a student is developed through learning situated in practice, interacting with real children of various cultural backgrounds and developmental levels, under the guidance of experienced mentors" (p. 7).

Changes in teacher preparation programs. Teacher preparation programs are in a process of change and reform, and not only with regard to balance between university classroom taught methods and field experience. Various reports exist as to perceptions of mathematics teacher preparation and necessity for change. For example, Wu (2011), a mathematician who worked on the NAEP steering committee, stated that universities must improvement the way universities they prospective mathematics teachers. Directors of the Third International Mathematics and Science Study (TIMSS), in an article entitled *"Preparing Future Mathematics Teachers",* stated, "teacher quality may be the Achilles heel, as more than 40 U.S. states move to implement the mathematics Common Core State Standards (Schmidt, Houang, & Cogan, 2011, p. 1266). In contrast, the 25th-anniversary Metlife survey in 2008 reported that teachers' perceptions of their preparation had improved; 67% of teachers (compared to 46% in 1984) agreed that they were prepared to do a good job in the classroom. Additionally, 51% of principals agreed that the quality of new teachers coming into the profession was stronger than it had been in earlier years (American Association of Colleges of Teacher Education, 2010a).

A summary of current reform recommendations from the American Association of Colleges for Teacher Education, includes the following: Revise the "highly qualified teacher" definition in the Elementary and Secondary Education Act; invest in the development of a national teacher performance assessment; maintain the Teacher Quality Partnership grants with a specific clinical preparation focus; require teacher preparation accrediting bodies to develop and apply strong standards for clinical preparation; require all providers of teacher preparation to be professionally accredited; and require all providers to report on the impact of their programs, including their effect on the learning of P-12 students (American Association of Colleges of Teacher Education, 2010a). All of these suggested reforms will impinge on what it means to prepare preservice teachers in the mathematics methods classroom.

Data systems that attempt to link student outcomes to teacher preparation programs and characteristics is a recent emphasis tied to recommended reforms (American Association of

Colleges of Teacher Education, 2010a). The hope is that such data systems, including value added type evaluations, will provide opportunities to expand knowledge of what works for whom and under what conditions. The fact that future teachers' effectiveness will be subject such analysis, however, has a potentially profound impact on future teacher preparation programs, and thus on the experience of those working to prepare future teachers.

Reform Movements in United States Mathematics Education

The present day context of preparing future mathematics teachers is one of controversy, change and reform. Mathematics skills are a critical component necessary for economic progress in the United States (International Association for the Evaluation of Educational Achievement, 1996). The public debate is no longer whether to study mathematics at every level of schooling, but how to raise achievement at every level, and for all students.

Prospective elementary school mathematics teachers need opportunities to experience reform-based teaching and learning in order to reconceptualize what it means to make sense of mathematics and of teaching (Conference Board of the Mathematical Sciences [CBMS], 2001; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). Various reform movements deal with both pedagogy and content. For any reform to be successful, teacher preparation programs, including mathematics methods courses, need to embrace and incorporate the changing ideas and pedagogy associated with curriculum, assessment and equity issues. However, changing ideology is not a simple matter (Borko, Flory, & Cumbo, 1993; Hiebert, 2013; Loucks-Horsley, 1998; Woodbury & Gess-Newsome, 2002)

Reform movements in mathematics instruction pre-1980. Reform movements in education are nothing new. Horace Mann's (1845) criticism of Boston's brightest "Brag Scholars," showed that Mann was troubled that the students could recite numerous facts by rote, and yet were lacking the critical thinking skills to use their knowledge in a constructive way. John Dewey (1936a) supported a vision of education connected with real life and argued that, "Number arises in connection with the measuring of things in constructive activities; hence arithmetic should be so taught (pp. 213-214)." At Dewey's Chicago Laboratory School, mathematics experiences were integrated with other disciplines, in contrast to being taught in isolation. His contention was that "Until educators have faced the problem and made an intelligent choice between the contrasting conceptions…I see no great hope for unified progress in the reorganization of studies and methods in the schools (Dewey, 1936b, p. 396)".

Modern media attention has focused on mathematics education since the Space Race in the 1950s. Reaction to the launch of Sputnik in 1958 resulted in attempts to increase numbers of scientists and engineers. Mathematics curriculum was rewritten on a national scale, involving terms and operations new to the public. These efforts collectively became known as the New Math. However, these efforts were top-down and did not include K-12 teacher input. Additionally teachers received little, if any, training. Disappointed by the 1960s efforts at reform, sharp calls for back-to-basics began to be heard 1970s with such publications as "Why Johnny Can't Add: The Failure of the New Math" (Kline, 1973). Some suspected that the back-to-basics movement was, at least in part, a reactionary swing from the "new math" with its emphasis on abstract mathematical structures (Ball, 1992).

Standards Based Reform through the National Council of Teachers of Mathematics. The National Council of Teachers of Mathematics (NCTM) stated in *An Agenda for Action* (1980) that the so-called 'back-to-basics' movement was too inclined to place a low ceiling on mathematical competence. Life was becoming more and more permeated by both multiple and diverse uses of mathematics. Returning to classes centered on memorization of facts, formulae and algorithms without attention to conceptual problem solving ability would be detrimental to all students. *An Agenda for Action* outlined eight directions needed to improve mathematics teaching and learning. This included a focus on problem solving, viewing basic skills as more than computational facility, the use of calculators and computers at all grade levels, efforts to increase public support for mathematics instruction, wider ranges of measures than conventional testing, requiring that students take more mathematics with flexible curriculum choices, high levels of professionalism from teachers and development of standards of both effectiveness and efficiency to be applied to the teaching of mathematics (NCTM, 1980). The report was widely disseminated, but unfortunately, came to rest on many educators' shelves (Ball, 1992). A more ambitious move seemed to be needed (Crosswhite, 1990). With time, the recommendation for standards was brought to fruition by NCTM itself, and this allowed the other actions recommended in the *Agenda* to begin to take hold.

In the twenty years following the publication of *An Agenda for Action* a number of standards documents published by NCTM garnered substantial public reaction (Davidson & Mitchell, 2008): Among these were: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), *Assessment Standards for School Mathematics* (1995), and the *Principles and Standards for School Mathematics* (2000). In terms of pedagogy, the focus was non-prescriptive, but endorsed changes from a classroom model of teacher-centered, procedure-oriented, lecture-followed-by-individual-seatwork, to a model more centered on students, on building concepts, and on making connections among mathematical ideas. In terms of curriculum, the focus was on a standards-based approach to mathematics instruction. NCTM was, in fact, first among professional teaching organizations to talk in terms of standards (Romberg, 1992).

Ball (1992) believes that in some respects, from a common sense point of view, the notion of “standards” appears uncontroversial. Standards can seem worthwhile, whether for behavior, quality or measurement. However, our national relationship with standards is in conflict with values such as individuality, avoidance of “sameness” and dislike of control, particularly government control. On the whole, our nation’s relationship with the idea of standards is an ambivalent one.

NCTM standards are not prescriptions for making everyone the same. The NCTM standards represent a vision, informed by multiple perspectives such as research knowledge, philosophical orientations, and policy motives (Ball, 1992). NCTM intends the standards to direct practice and guide teaching, but not to prescribe exact methods.

In 2006, NCTM made the link between curriculum and anticipated outcomes even more specific. *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* spelled out grade level focal points for content.. In 2009, NCTM offered guidance for grades 9–12 mathematics with *Focus in High School Mathematics: Reasoning and Sense Making*. Today, many mathematics educators encourage teachers to embrace reform by establishing classrooms focused on problem solving and characterized by students’ active engagement in meaningful mathematical activities involving investigation, inquiry, conjecturing, communication and reasoning (Shilling, 2010).

The Common Core State Standards for Mathematics. The NCTM standards were not an act of government. They were developed and financed by the professional organization. In response to them, the governments of the 50 states developed their own standards. However, the standards varied markedly from state to state (McCallum, 2012). In 2010, the Council of Chief State School Officers and the National Governors Association Center for Best Practices released their own standards, known as the Common Core State Standards for Mathematics (CCSSM). Authors state that they drew upon documents from the National Council of Teachers of Mathematics, the National Research Council, reports from the National Mathematics Advisory Panel, and the College Board, as well as “standards of U. S. states and high achieving countries, particularly in East Asia” (McCallum, 2012, p. 2).

The context of the word “standards” needs to be clear. There are both NCTM standards and CCSSM standards. NCTM standards are not designed to be enacted into law, but the CCSSM standards do lend themselves to legislation by offering a grade-by-grade listing of performance expectations.

CCSSM contains two types of standards: content and practice. Mathematical Content Standards address specifics to be learned. For example, second graders should be able to read and write numerals to 1000. Mathematical Practice Standards, in contrast, describe ways that students engage in mathematics. They call on students to do such things as reason abstractly, make sense of problems and construct viable arguments. According to CCSSM (2010), the Standards for Mathematical Practice were based on the NCTM Process Standards (National Council of Teachers of Mathematics, 2000) and on the strands of mathematical proficiency described in *Adding It Up* (National Research Council, 2001).

The internationally benchmarked CCSSM were intended to engage the mathematical community at large and to move U.S. mathematics education toward a single set of standards (McCallum, 2012). The endeavor was an attempt to add rigor, focus and coherence to school mathematics (Schmidt, 2012).

Critiques of Reform. Two principal aspects of reform have resulted in the most criticism. One relates to implementation. The other is related to philosophical underpinnings

Philosophical Critiques. Both popular and scholarly critiques have examined reform. Some held that NCTM standards endorsed "fuzzy math", replacing individual accountability with group work, proficiency in basic skills with reliance on calculators, and serious attention to algorithmic processes with more pragmatic type problems (Davidson & Mitchell, 2008). These voices continued to be heard even as a new document and new facets of reform in mathematics education arose.

As an example of popular criticism, Garelick (2012), writing for *EducationNews.org*, described reform mathematics as based upon the NCTM process standards and as:

> A method of teaching math that eschews memorization, favors group work and student-centered learning, puts the teacher in the role of "guide" rather than "teacher" and insists on students being able to explain the reasons why procedures and methods work that they may not be able to perform (retrieved from http://www.educationnews.org/education-policy-and-politics/the-pedagogical-agenda-of-common-core-math-standards).

Scholarly opinions, however, differ from these critiques. A National Science Foundation (National Science Foundation, 2002) publication states that the practices recommended by the NCTM standards are "grounded in views of knowledge, learning and teaching informed by a constructivist perspective" (p. 3). In this perspective, knowledge is socially constructed, shaped by context and purposes, and is validated through a process of negotiation within a community of practice (Brooks & Brooks, 1999). Learning is a process of making personal meaning (National Science Foundation, 2002). Teaching mathematics is facilitating learning by creating an environment conducive to inquiry, setting up problem solving situations and supporting students' attempts to solve problems and make sense of mathematical concepts (Borasi & Siegel, 2000).

Turning to the Common Core State Standards, both popular and scholarly critiques are beginning to emerge that sound different from criticisms of NCTM for "fuzzy math." Here, the contention is that the CCSS encourage emphasis on direct instruction and the development of fact knowledge with less emphasis on concept development and learning to be problem solvers (Ohanian, 2012). Tienken and Zhao (2010) writing for the American Association of School Administrators Journal of Scholarship and Practice state, "When we looked at the underlying theories of mandated statewide testing from the…Common Core State Standards, and the (associated) proposals put forth in the Race To The Top program, we found them driven by behaviorism and rational choice theories" (p. 4).

The critique that behaviorism informs aspects of the Common Core State Standards for Mathematics bears closer inspection. The CCSS documents (2010) state that the Mathematical Content Standards are a "balanced combination of procedure and understanding" (p. 8). The documents advise educators that Mathematical Content Standards that begin with the word "understand" are often especially good opportunities to connect with the Mathematical Practice Standards (based upon the NCTM process standards). Examination shows, however, that the large majority of *Content* Standards in the CCSSM (2010), do not begin with the word "understand" but with words such as "describe, classify, name, and identify"; words more closely associated with behavioral, measurable objectives. In contrast, the Mathematical *Practice* Standards ask learners to do such things as make sense, reason abstractly, construct arguments, critique, and model. Thus, the content standards and the practice standards seem to differ in emphasis.

Questions arise as to whether CCSSM standards are being built to depend more on behaviorist or constructivist foundations. To help make sense of this, consider Fosnot (1996), who states,

> Constructivism is fundamentally nonpositivist and as such it stands on completely new ground,-often in direct opposition to both behaviorism and maturationism. Rather than behaviors or skills as the goal of instruction, concept development and deep understanding are the foci; rather than stages being the result of maturation, they are understood as constructions of active learner reorganization (p. 10).

Thus, since the Content Standards of the CCSSM often reference skills such as "classify, name", and "identify", and the Practice Standards of the CCSSM call upon students to "make sense, reason abstractly, construct arguments, critique, and model", there is some evidence that the CCSSM harken to blended philosophical calls: constructivism and behaviorism.

Critiques of Implementation. Regarding implementation, attempts to bring about reform often fail. The substantive nature of what happens in classrooms stays much the same (Handal & Herrington, 2003; Lamb, 2010; Stigler & Hiebert, 2009, Hiebert, 2013). Studies conducted in United States mathematics classrooms by Cuban (1984), Mewborn (2001), Romberg and Carpenter (1986), and Grubb (2010) reveal that most mathematics lessons follow a pattern of "show and tell" style of teaching and whole-class lecturing with little discussion. This is commonly followed by passive, independent seatwork. Small group work is uncommon, and student participation is not active. Teachers' questions emphasize right or wrong answers, most often at the "recall" level, rather than asking for analysis. Emphasis is on rote learning, procedures, and facts. Thus, despite reform efforts, large numbers of lessons in American classrooms can continue to be characterized as traditional in orientation.

Teachers' response to change is actually a facet of teacher beliefs. This will be dealt with separately in the section on teacher beliefs. To summarize this section, I attempted to show that ideas of philosophy, belief and practice are deeply intertwined with mathematics reform and movements encouraging reform. The next section examines knowledge related to the practice of teaching mathematics. Following that, beliefs about mathematics and mathematics teaching are examined in greater depth before closing this review.

Elementary Mathematics Teaching and Teaching Knowledge

Teacher knowledge can be thought of as a large integrated functioning system (Fennema & Franke, 1992). Teaching practices are related to content knowledge and to pedagogical content knowledge (Shulman, 1986). They involve task design and discourse about tasks. Practices may be judged to be effective or ineffective. Generally they have their foundation teacher beliefs, that is, in a teaching model consciously or unconsciously endorsed by the teacher. Teacher beliefs will be dealt with in a later section. This section on elementary mathematics teaching will look at teaching practices deemed effective and at teacher content and at pedagogical content knowledge. Attention will be given to task design and classroom discourse, since plans are to ask methods instructors about these two items, and since studies have indicated the importance of content knowledge, pedagogical content knowledge, task design, and discourse in effective teaching.

Effective Teaching. Wu (2011), a mathematician and critic of many practices in the mathematics education community, states that to help teachers teach effectively, we must provide them with a body of mathematical knowledge that:(a) is relevant to teaching, that is, does not stray far from material taught in school, and; (b) is "consistent with the fundamental principles of mathematics" (p.373).

A rigorously designed longitudinal study (Fennema, et al., 1996), with its basis in Cognitively Guided Instruction (CGI), a method that stems from a foundation in constructivist learning theory, found that certain instructional features were associated with higher student achievement in mathematics. They were (a) Providing for students to work with mathematical ideas in problem solving contexts (task design); (b) Providing opportunities for students to converse with each other about mathematical ideas (discourse); and (c) Adapting instruction to the problem solving level of students (PCK). Students in the CGI longitudinal study referenced, in many cases, improved one standard deviation on achievement tests (Fennema, et al., 1996). More recent reviews of literature bear out similar findings. Hodara (2011), in a large scale review of recent publications on reforming mathematics classroom pedagogy, found that studies which employed rigorous design suggested that structured forms of student collaboration and instructional approaches that focus on problem representation improve mathematics learning and understanding. Studies qualifying as rigorous in this review avoided common methodological flaws: student design non-equivalency, teacher design non-equivalency, possible attrition issues, and lack of a comparison group. Donovan and Bransford (2005), in a synthesis of research literature in the cognitive and developmental sciences outline best practices for K-12 mathematics teaching. They recommend that educators (1) connect with, build on and refine the mathematical understandings, intuitions and resourcefulness that students bring to the classroom, (2) use a variety of instructional methods, (3) include learner-centered activities, including metacognitive strategies and (4) provide students with feedback through the use of ongoing assessment.

Content knowledge. Regarding content knowledge, comparatively early studies state "there is consensus that teachers' knowledge is a major determinant of mathematics instruction and learning" (Fennema & Franke, 1992). More recent statements hedge this somewhat. Krauss, et al (2008) state there is wide consensus, at least that teachers' domain-specific knowledge is an essential ingredient of high quality instruction, especially in the mathematics classroom.

Shulman (1987) identified a minimum of seven knowledge bases for teaching: (1) content knowledge; (2) pedagogical knowledge; (3) curricular knowledge; (4) pedagogical content knowledge; (5) knowledge of students; (6) knowledge of context, and; (7) knowledge of educational goals. Of particular interest to this study are two areas: content knowledge, to be dealt with in this section, and pedagogical content knowledge, to be dealt with in forthcoming pages. Content knowledge is the understanding of a discipline's key facts, concepts and principles. Pedagogical content knowledge, to be discussed later, enables an instructor to present material in a way that students can understand.

Profound understanding of fundamental mathematics. Cooney (1999) and Knuth (2002) both concluded that, as undergraduates, preservice teachers do not experience mathematics instruction that enables them to build deep mathematical understanding when they go on to teach their own students. In her book *Knowing and Teaching Elementary Mathematics*, (Ma, 1999), detailed what elementary teachers know and can do mathematically, using a comparative method of analysis of elementary mathematics teaching in the United States and China. She documented issues and implications involved in having a surface understanding of mathematics content. Ma described the subject matter knowledge needed for teaching elementary mathematics as a profound understanding of fundamental mathematics (PUFM), that is “an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough” (Ma, 1999, p. 120). She found that while elementary school teachers in both countries could perform and, to some extent, explain procedures, the vast majority of Chinese teachers had a thorough conceptual understanding of such topics as place value, whereas many U.S. teachers did not.

One possible inference from Ma’s study is that university coursework may not have addressed the kinds of mathematical understanding prospective teachers need. More specifically, Schoenfeld (2002) explains that:

> Teaching for mathematics understanding is hard. It requires a deep understanding of the mathematics involved and of how to create instructional contexts that lead students to engage with mathematics in meaningful ways. The vast majority of today’s American mathematics teachers learned the traditional curriculum in the traditional way. Hence they neither have models nor experience teaching in the ways that would best facilitate their students’ development of mathematical understanding (p. 20).

The current study examines the instructional strategies reported by college mathematics methods teachers, in part, as a way to begin to better understand the learning opportunities preservice elementary teachers have in their undergraduate mathematics methods courses.

Mathematical knowledge for teaching. Inconsistent results and small effects have resulted from research attempting to tie such things as the number of mathematics courses successfully completed in college by teachers with high scores on standardized mathematics tests by the students of these teachers (Conference Board of the Mathematical Sciences, 2012). Thus, in the 1980s, scholars began to investigate “knowledge for teaching”, something earlier research had ignored. The focus was identifying the kinds of knowledge relevant for teaching. For instance, teacher candidates might be asked to respond to a classroom scenario in which a student asked why division by zero was undefined. Even mathematics majors were not always able to answer satisfactorily (Conference Board of the Mathematical Sciences, 2012). Teachers "need to know how to use pictures or diagrams to represent mathematics concepts and procedures to students, provide students with explanations for common rules and mathematical procedures, and analyze students' solutions and explanations (Hill, Rowan, & Ball, 2005, p. 372)." Knowledge for teaching has shown promise. For instance, scores of third-grade teachers in 115 elementary schools on one test of knowledge for teaching were better predictors of student achievement than other measures such as the number of mathematics courses they had taken, years of experience or their certification status (Hill, Rowan, & Ball, 2005). Current research considers knowledge for teaching a better predictor of student success than what are now considered proxy variables, such as numbers of courses taken in mathematics or degrees attained. (Hill, Rowan, & Ball, 2005).

The Conference Board of the Mathematical Sciences (CBMS) is an umbrella organization whose members are the presidents of sixteen professional associations, including the American Mathematical Society (AMS) and the Mathematical Association of America (MAA). Current recommendations are that prospective K-4 teachers take 12 hours of math content courses focused on knowledge for teaching and that middle grades math teachers (5-8) be taught be mathematics specialists with 24 semester hours in mathematics. The board also recommends that university mathematics departments work closely with mathematics education departments (Conference Board of the Mathematical Sciences, 2012)

Efforts have also been launched by individual professional organizations for mathematicians. For example, the MAA's Preparing Mathematicians to Educate Teachers (PMET) project is geared toward providing assistance to college and university mathematics teachers in finding appropriate instructional strategies for helping future teachers connect their college mathematics to the mathematics they will teach (Katz & Tucker, 2003).

Pedagogical content knowledge. At the intersection of knowledge of mathematics and knowledge of teaching is knowledge of how to teach mathematics, that is, pedagogical content knowledge. Shulman (1986), explains, "the key to distinguishing the knowledge base for teaching lies at the intersection of content and pedagogy, in the capacity of the teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variation in ability and background presented by the students" (p. 15). Further, pedagogical knowledge is described as "knowledge that helps teachers understand how students learn mathematics, to be able to use a range of different teaching techniques and instructional materials, and organize and manage the classroom" (National Council of Teachers of Mathematics, 2000, p. 17). This is consistent with Shulman's definition of pedagogical content knowledge

Research has explored how elementary and secondary teachers learn mathematical pedagogical content knowledge. For example, Ball (1991) studied the role of subject matter knowledge in teaching place value. Teachers' subject matter knowledge, she states, "interacts with their assumptions and explicit beliefs about teaching and learning, about students, and about context to shape the ways in which they teach mathematics to students" (p. 1).

Tasks. For future teachers, building conceptions of mathematically rich and cognitively and socially stimulating school mathematical activities is at the heart of the process of their professional formation (Gellert, et al., 2008). PEMTEs help their preservice teachers learn to design tasks/activities to facilitate learning. Tasks form the basis for opportunities to learn what mathematics is and how it is done; they influence learners by directing their attention to particular aspects of content; they may indicate ways to process information. The level and kind of thinking required by mathematical instructional tasks influences what pupils learn. Differences in the level and kind of thinking called for by tasks used by different teachers, schools and districts, is a major source of inequity in opportunities to learn mathematics (Briars, 2011). Higher level tasks ask for application and analysis. Learners apply, demonstrate, interpret, appraise, experiment and question. They attain a conceptual understanding of and build mental mathematical structures with which they may work.

The phrase “deep processing” was first used in mathematics education by Marton and Saljo (Marton & Saljo, 1976) to describe distinctions in how learners respond to a task. Deep learning is described by Biggs (1987) and Tagg (2003) as an approach to the learning process that focuses on meaning making. This is in contrast to a surface approach that relies on rote memorization in order to pass a test or earn a grade. Deep learning can occur when learners make connections and formulate personal meaning

Not all tasks are created equal. Different tasks will provoke different levels and kinds of thinking. Theorists have attempted to classify different levels of task in the mathematics classroom. Stein, Smith, Henningsen and Silver (2000), describe four levels of cognitive demand typically used in mathematics instruction: memorization, procedures without connections, procedures with connections and doing mathematics (problem solving) (Stein, Smith, Henningsen, & Silver, 2000). During the five years of the QUASAR project, Silver and Stein (1996) utilized these classifications and engaged teachers in developing and implementing an instructional style that engaged urban middle school students in developing meaningful understanding of mathematical ideas through problem solving. They had strikingly positive results in student achievement.

Professional groups have made recommendations involving task design at the college level. A concept paper titled "Finding Common Ground in K-12 Mathematics Education" published by the American Mathematical Society (Ball, Ferrini-Mundy, Kilpatrick, Milgram, & Scharr, 2005) has been influential in identifying some common areas of agreement about mathematics education. The authors included three mathematics educators (Deborah Ball, Joan Ferrini-Mundy and Jeremy Kilpatrick), two research mathematicians and two representatives from technology corporations. Areas of agreement in the document are (1) Students can learn effectively via a mixture of direct instruction, structured investigation, and open exploration. (2) Decisions about what is better taught through direct instruction and what might be better taught by structuring explorations for students should be made on the basis of the particular mathematics, the goals for learning, and the students' present skills and knowledge.

Discourse. The mathematics classroom, and by extension the mathematics methods classroom, should provide students with "a rich set of experiences and a discourse environment in which they negotiate meaning as they construct their knowledge of mathematics (Romberg, 1992, p. 434). From statements like the above, the importance of discourse in the mathematics classroom can be seen. Students do not need merely to have rich experiences; they need to participate in the discourse surrounding them in order to facilitate the social construction of knowledge.

Stein, Engle, Smith and Hughes (2008) have proposed a pedagogical framework to support teachers in orchestrating whole group mathematics discourse. Key practices are: (a) anticipating, envisioning how students might approach a task with correct and incorrect strategies; (b) monitoring, paying close attention to students mathematical thinking at they work on a task; (c) selecting, choosing certain students to present their thoughts and get certain topics on the table while remaining in control of what the mathematical content of the discussion will likely be; (d) sequencing, making decisions about how to order students' presentations to maximize learning; and (e) connecting, making sure that presentations build on one another and that students make relations among ideas presented in order to develop their own powerful mathematical ideas.

Additional schema classifying knowledge for teaching mathematics. As discussed above, teacher knowledge is often discussed as being comprised of three strands: content knowledge, pedagogical knowledge, and didactical (that is, pedagogical content) knowledge (Durand-Guerrier & Winslow, 2008). This is not the only way that teacher knowledge can be partitioned, however. For example, Bergsten and Grevholm (2008) speak of teacher knowledge as being comprised of disciplinary knowledge and PK. Disciplinary knowledge is the substantive knowledge of facts, procedures, concepts, and so forth, as well as knowledge of mathematics as a discipline. PK, on the other hand, is PCK and curriculum knowledge as well as knowledge of general issues in education, such as learning, developmental psychology, and socialization. Adler and Davis (2008) view the acquisition of teacher knowledge as learning to teach and learning mathematics for teaching. Rowland, Huckstep, and Thwaites (2005) have introduced of the notion of the 'Knowledge Quartet' as a tool for thinking about the ways that a teacher's subject knowledge comes into play in the classroom. This quartet, which is being increasingly cited in international literature (Millman, Iannona, & Johnston-Wilder, 2008), is comprised of foundation, transformation, connection and contingency. It can be briefly explained as follows:

> foundation (teachers' knowledge, beliefs, and understandings acquired "in the academy"), transformation (teachers' knowledge in action as demonstrated both in planning to teach and in the act of teaching itself), connection (binds together certain choices and decisions that are made for the more or less discrete parts of mathematics education), and contingency (witnessed in classroom events that are almost impossible to prepare for) (Rowland, Huckstep, & Thwaites, 2005, p. 2).

This third section of the literature review considered teaching practices deemed effective for mathematics, and at the content knowledge and pedagogical content knowledge deemed necessary for teaching mathematics. The final section will leave concrete matters of research on practice and knowledge. Since this study examines both teaching practice and beliefs, the final section will turn to beliefs.

Teacher Beliefs

In research literature, the term "beliefs", in general, has included attitudes, values, judgments, opinions, ideology, perceptions, conceptions, preconceptions, dispositions, implicit theories, explicit theories, personal theories, practical principles, perspectives, and social strategies. Beliefs can include individuals' philosophical contemplations (Pajares, 1992). Separating the term "belief" from, "teacher belief", Kagan (1990), defined teacher beliefs as "the highly personal ways in which a teacher understands classrooms, students, the nature of learning, the teacher's role in a classroom, and the goals of education" (p. 423).

Prospective elementary teachers do not come to teacher education believing that they know nothing about teaching. "Long before they enroll in their first education course or mathematics methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988b, p. 40). These ideas are more than just fleeting notions about mathematics and mathematics teaching. They most often form the foundation that teaching candidates will eventually build their own practice on as teachers of mathematics (Skott, 2001). At times, these deep-seated beliefs often run counter to research on good practice (Liljedahl, et al., 2009). As such, some hold that it is the role of teacher education program to reshape these beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971). There is evidence, however, that in some cases, teacher education programs are busy concentrating on pedagogical knowledge and give little consideration to modifying candidate's beliefs (Tillema, 1995). As a consequence, teacher education programs may not produce teachers whose beliefs are consistent with curriculum innovation and research (Handal, 2003; Kennedy, 1991)

The important role of teacher beliefs in the learning and teaching of mathematics has been largely acknowledged in the literature (Shilling, 2010). Researchers have suggested that pedagogical decisions are deeply rooted in teacher beliefs (Ernest, 1989; Schoenfeld, 2010) Teachers do not make decisions based solely on their pedagogical knowledge or curriculum guidelines. Instead, their beliefs act as a filter, through which they make decisions (Clark & Peterson, 1986).

The study of teachers' instructional beliefs and their influence on instructional practice has gained momentum since the early 1990s (Handal, 2003). Ernest (1989) goes so far as to suggest that beliefs are the primary regulators for mathematics teachers' professional behavior in the classroom. Teacher beliefs may be strong enough to either facilitate or slow proposed educational reform (Handal & Herrington, 2003). But while beliefs are often highly resistant to change, it is possible to motivate change during a university mathematics course (Shilling, 2010).

In this section, teacher beliefs about mathematics will be examined, followed by beliefs about models for teaching, with their underlying philosophies. The section closes by looking at various classification schemes for teaching beliefs related to both mathematics and teaching, and at changing teacher beliefs.

Beliefs about mathematics. The National Research Council (1989) defined mathematics as “a science of pattern and order” (p. 31). Some see mathematics as a process of inquiry that allows individuals to develop meaning or make sense of their world (Timmerman, 2003). Yet, for many people, the word “mathematics” in free association, produces strong, negative images (Fuson, Kalchman, & Bransford, 2005). Gary Larson published a cartoon entitled “Hell’s Library” that consisted of nothing but book after book of mathematics word problems.

What do children believe about mathematics? The authors of *Everybody Counts* (National Research Council, 1989) wrote:

> Virtually all young children like mathematics. They do mathematics naturally, discovering patterns and making conjectures based on observations....As children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually,

> most students leave mathematics under duress, convinced that only geniuses can learn it (pp. 43-44)

Adults seeking to teach young children may find the roots of their beliefs about mathematics in their own experiences as children. Ma (1999) found that many practicing elementary teachers in the U.S., reflecting upon their own experiences as students, held the belief that mathematics was "an arbitrary collection of facts and rules in which doing mathematics means following set procedures step-by-step to arrive at answers" (p. 123). Some future teachers of mathematics in the elementary school see mathematics as boring, and some are anxious about mathematics.

For those with high levels of math anxiety, mathematics associated with conscious or unconscious beliefs and results in feelings of fear, tension, or apprehension. Preservice elementary teachers experience higher levels of math anxiety than the general undergraduate population (Bursal & Paznokas, 2006; Harper & Daane, 1998; Hembree, 1990). One study found, "a considerable proportion of students entering primary teacher education programs with negative beliefs and attitudes towards mathematics" (Uusimaki & Nason, 2004, p. 370). Another study found that when the participants with high levels of math anxiety saw that they would be presented with a mathematics problem, brain scans showed activation in the same neural areas associated with physical threats and bodily harm (Lyons & Beilock, 2012). Studies identify many factors associated with mathematics anxiety: environmental factors such as family pressure for higher achievement, intellectual factors as learning styles and personality factors such as low self-esteem (Uusimaki & Nason, 2004; Woodard, 2004). Researchers looked at math anxiety in female elementary teachers and found that while there was no relation between a teacher's math anxiety and her students' mathematics achievement at the beginning of the school year, at year-end, the more anxious teachers were about math, the more likely girls were to adopt the belief that "boys are good at math, and girls are good at reading" and the lower the mathematics achievement of these girls (Beilock, Gunderson, Ramirez, & Levine, 2010). Mathematics anxiety is a complex and multifaceted construct with cognitive and affective dimensions. Working with students preparing to teach elementary mathematics who nonetheless are mathematics anxious complicates the role of the mathematics methods instructor.

Models of teaching and their underlying philosophies in learning theories. Consciously or unconsciously, teachers' beliefs include a teaching philosophy which will point toward a theoretical teaching model. During the past 40 years, theoretical models for teaching have undergone change. Early approaches were based on a transmission model (Blumenfeld, Krajcik, Marx, & Soloway, 1994). In this model, the focus is on clear explanations and the teacher's organization of material. Instruction is based on *telling* pupils what they need to learn. Information or content is seen as passing directly from the teacher to the pupils (Hassad, 2011). A form of transmission style learning widely used at the college level is the lecture method of teaching (Finn, 2010).

In transmission models, the focus on is on what the teacher does. Turning attention to the student's cognitive processes led to the development of the transformation model of teaching (Blumenfeld, Marx, Krajcik, & Soloway, 1997). Like the transmission model of teaching, there are many formulations of the transformation model, for example, the information processing approach, individual construction, and social constructivism (Blumenfeld, et al., 1997). Von Glasersfeld (1998), in advocating constructivist leaning theory, says that knowledge cannot be transmitted, it must be constructed. He speaks of a didactical constructivism, a learning theory of learning and teaching model built on the idea that students reconstruct knowledge. The student is required to act upon the environment to both acquire and test new knowledge.

Grubb (2010) organizes teaching models along a continuum, rather than utilizing two distinct categories. In this view, though some have divided classroom practices into two polar opposites – behaviorist and constructivist – many practices fall in between, drawing on both. Such practices have been called "balanced." Grubb, referring to Little (2006) and Lampert (2001), then goes on to develop a two-dimensional system for classifying instructional practices. Central to this system is the idea that in effect, high and low quality practices that educators and researchers agree are important. Dimensions of quality for constructivist teaching are somewhat different from dimensions of quality in behaviorist teaching, yet such dimensions can still be identified. Two brief examples may clarify. For behaviorist teaching, perceived higher quality techniques of direct instruction currently suggest a careful progression of introducing a new topic, presenting and modeling it to students, having students practice with guidance and scaffolding, and finally having students work independently. This is in contrast to older, lower quality "hear a lecture, complete a worksheet" classroom practices. In constructivist instruction, student groups, for example, can be of lower quality, with a few students dominating, or they can be of higher quality, that is, truly cooperative. The philosophical dimension from constructivist, through balanced, and then to behaviorist forms the x-axis in this system and "quality of practice" from low to high, forms the y-axis.

Hatch and Freeman (1988) found that in working with young children, educators' philosophies tend toward behaviorism, maturationism or interactionism. These philosophical beliefs could clash with reality, however, when individuals were responsible for implementing and/or working in programs that they do not philosophically believe best serve the needs of young children. For example, their work may ask them to focus on meeting objectives stated in measurable terms, a concept associated with behaviorism (Schickedanz, York, Steward, & White, 1983). In mathematics, the context of school instruction "often obliges practicing elementary teachers to teach traditional mathematics even when they may hold alternative views about mathematics and about mathematics teaching and learning" (Handal, 2003, p. 49). In situations like this, what Hatch and Freeman called the "philosophy-reality conflict" comes into play.

This review of literature shows that educators are still very much faced with programs stressing the meeting of measurable objectives, even though they may have heard, and may even have inwardly embraced, more maturationist or interactionist–type beliefs and models. Other research has found that teachers may hold inconsistent beliefs that defy classification (Handal, 2003). Sosniak, Ethington and Varelas (1991) attempted to profile 178 secondary teachers as having either a traditional or progressive orientation to the curriculum but found, using statistical analysis, that the teachers in their study lacked a consistent theoretical orientation. Andrews and Hatch (1999) working with secondary teachers in the United Kingdom and Howard, Perry and Lindsay (1997), working in Australia, stated that within teachers' belief systems there are often beliefs that appear ideologically incompatible with others. The fact that educators hold inconsistent beliefs, however, does not mean that it is always impossible to identify a major learning theory with which their beliefs are the most consistent.

Other belief classification schemes. In this section, belief classification schemes which do not overtly mention either behaviorism or constructivism as categories are reviewed. Despite the lack of these category labels, however, it will be noted that these two poles still appear to underlie much of what is being written regarding teacher beliefs about teaching.

Trigwell, Prosser and Taylor (1994), developed a matrix-like classification scheme for teaching approaches that may help to inform this study. They first identified four posited intentions for teaching and three posited strategies. Strategies were: teacher-focused, teacher/student interaction, and student-focused. Intentions were: transmit information, acquire the concepts of the discipline, develop conceptions, and change conceptions. A ‘teaching approach’ consisted of a strategy combined with an intention, for instance, “a student-focused strategy with the intention of transmitting information.” Twelve different approaches were possible. Of note here is that in continued work with this model, Trigwell and Prosser (1996) showed that certain strategies adopted by teachers often matched certain intentions they had for their teaching; for example, the intention to transmit information was often matched with teacher-focused (not student-focused) strategies. Additionally, Trigwell, Prosser and Waterhouse (1999) found that teachers who reported adopting more of a teacher -focused approach to teaching had students who themselves reported adopting a more surface approach to learning.

The Trigwell and Prosser study led to the *Approaches to Teaching Inventory-Revised*. Recently, researchers have used this inventory as a pre and post measure to assess the impact of a professional development program on faculty approaches to teaching (Light, Calkins, Luna, & Drane, 2009). Researchers “have distinguished between faculty who are concerned with teaching as essentially an organization of the content of the teacher’s knowledge for transmission to the students and those who regard teaching as facilitating their student’s personal construction of knowledge” (Light, et al., 2009).

Handal (2003) looked at three different conceptualizations of teachers’ beliefs about the nature and pedagogy of mathematics (Kuhs & Ball, 1986; Ernest, 1991; Renne, 1992). Handal states that these schemes provide analytical frameworks to discuss teachers' belief systems. Ernest (1991) hypothesized five different mathematical belief systems he believed were found among teachers: authoritarian, utilitarian, mathematics centered, progressive and socially aware. Ernest believed that the teachers' philosophy of mathematics was the most important of these categories, and that it might very from absolutism to social-constructivism. Teachers' theories of learning and teaching define perceptions of the learners' role as active or passive, dependent or autonomous, or as a receiver or creator of knowledge. Kuhs and Ball (1986) delineated three conceptions related to teaching and learning mathematics. The learner-focused view stressed the learner's construction of mathematical knowledge with the help of social interaction. The content-focused view emphasized conceptual understanding. The content-focused view emphasized performance as evidenced by mastery of rules and procedures. Renne (1992) proposed a *Purpose of Schooling/Knowledge* matrix. This matrix included two orientations: school-knowledge oriented and child-development oriented. Additionally, it included two beliefs about where to place emphasis in learning: emphasis on 'what' is going to be learned and emphasis on learning within a holistic and meaningful structure.

I close this section with a return to the theme of dichotomy vs. continua. A posited dichotomy of beliefs is referred to in some of the mathematics education literature as "Procedural vs Conceptual" orientation. One study from the early days of the present reform movement hypothesized that there are calculational and conceptual orientations in teaching mathematics (Thompson, Philip, & Thompson, 1994). Sharp contrasts in the orientations are described in several teachers classified as one of the two orientations, with no middle ground. In contrast, Woodbury and Gess-Newsome (Woodbury & Gess-Newsome, 2002), developed grounded theory that indicated teachers had a strong sense of autonomy about their work and were teaching in unique ways that each considered to be best for their students. They proposed a model with nuance and continua, rather than a dichotomy, entitled Teacher-Centered Systemic Reform (TCSR). This model highlighted teacher thinking as a central factor shaped by "the interdependent influences of (a) the general context of reform, (b) a teacher's personal profile, and (c) the structural and cultural contexts of teachers' work" (Woodbury & Gess-Newsome, 2002, p. 764).

Changing teacher beliefs. The chairperson of the commission that wrote the NCTM Standards stated, "The single most compelling issue in improving school mathematics is to change the epistemology of mathematics in schools, the sense on the part of teachers and students of what the mathematical enterprise is all about" (Romberg, 1992, p. 433). Yet Stigler and Hiebert (1999) argue that reforms aimed at educational improvement in the United States fail, and that they do so because they do not alter widely shared and deeply embedded cultural beliefs. They point to the existence of a relatively small but tacit set of core beliefs about the nature of mathematics, how students learn, and about the role of classroom teachers. In a cross-national comparison of 231 eighth-grade mathematics lessons they were struck by "how much teaching varied across cultures and how little it varied within cultures (p.11)." The American culture of teaching mathematics in general appeared to consist of a relatively limited repertoire of methods that focused on learning terms and acquiring isolated, rather than connected, skills through repeated practice. Stigler and Hiebert contrast these methods with more than one country. They found that United States students encountered a "different kind of mathematics" (p. 56) from their counterparts in other countries, that it appeared to be less advanced and to be presented in a more piecemeal and prescriptive way. In particular, they pointed out that Japanese teaching, focusing on teaching for conceptual understanding, with Japanese teachers participated in ongoing, incremental improvement of their teaching through participation in lesson study groups.

Another reason for lack of change may be paradigm conflict. Teachers know that although those in charge verbally promote reform efforts, professional assessment and student assessment is in terms of the traditional paradigm. As such, they tend to conform to the status quo. This allows them to minimize risk and disturbance in what they see as an ethical and practical way (Anderson & Piazza, 1996; Doyle & Ponder, 1977).

Recent research has brought forward ways in which teachers' involvement in reform involves personal, social and emotional responses to change (Zembylas & Barker, 2007). Woodbury and Gess-Newsome (2002) examine conditions under which we can expect new reforms to create discernible differences in educational practices, and propose a model of educational reform, the Teacher-Centered Systemic Reform (TCSR) model. In this model, reform must reflect the intent of shifting basic patterns of schooling, must address issues of change in structure and culture in interrelated components of the educational system, and, most important, must encourage and support change in teachers' work as the center of reform efforts. Four historical perspectives are taken into consideration. Teacher thinking is highlighted as the central and is seen as being shaped by three interdependent influences: the general context of reform, a teacher's personal profile, and the structural and cultural contexts of teachers' work within embedded systems.

Summary

Elementary mathematics teaching today takes place in a context of long-sounded calls for reform, but a context where there is doubt about the extent to which these reforms have taken hold. On the one hand, classroom teachers have been cast as the root of problem due to instructional practices, seen as boring, superficial or mechanical. Practices that, in fact, resemble mathematics classes the teachers learned in. On the other hand, they have been cast as key agents of improvement, because pupils could not learn new mathematics in new ways unless teachers understood mathematics in these new ways (Cohen & Ball, 1990). This dichotomy lies at the heart of understanding the challenge of preparing preservice mathematics teachers. How can teachers teach mathematics they never learned, in ways they never experienced?

In this review of literature I addressed issues related to the vital job of preparing mathematics teacher educators. Beliefs and practices were the focus throughout. I examined teaching practice, beliefs about teaching, beliefs about mathematics and knowledge of mathematics on the part of teachers. I also examined proposed reforms, coming from both professional and political organizations, as they are an important part of the context of mathematics teacher preparation.

Mathematics teacher educators are faced with a challenge of consistency in their own philosophy of teaching. Lecturing students on the futility of lecturing would not make sense. Thus, I examined varied models for teaching and philosophies related to these models. Models examined here will be relevant to findings.

The Study of the International Commission on Mathematical Instruction [ICMI] (Even & Ball, 2008) aimed to inform and develop dialogue in the research community about international programs and practices related to teacher education. It focused on selected topics in mathematics classroom teacher education, and was grounded in the belief that no efforts to improve students' opportunities to learn mathematics can succeed without parallel attention to classroom teachers' opportunities for learning. To more fully understand teachers' opportunities for learning, knowledge about the beliefs and practices who prepare teachers is needed. This study will focus on such knowledge.

Chapter 3

Methodology

Overview

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. Such exploration may lead to a more informed perspective, that in turn may help to (a) examine possible support practices for enhanced classroom practice among PEMTEs; (b) provide insights that may enhance the preparation, professional development and study of potential and current PEMTEs, and; (c) help to see the emergence of themes and patterns for possible examination in larger studies. Thus, this study hoped to offer useful information and recommendations to be considered by researchers, policy makers and practitioners. A greater understand of the thinking of those who prepare teachers may have significant benefits for preservice teacher educators themselves, for the teachers they prepare, and for the students of those teachers.

The study addressed the following research questions:

1. What are the participants' beliefs regarding preparing elementary mathematics teachers?

    (a) What do they believe elementary students need to learn?

    (b) What do they believe preservice elementary teachers should learn?

    (c) How to they believe elementary students should be taught?

    (d) How do they believe they should use class time with their students?

2. How do participants describe their approaches to teaching elementary mathematics methods courses?

    (a) How do they use class time with their students?

    (b) What content do they teach?

(c) How do they decide what to teach?

Certain sub-questions deal more with content, specifically 1a, 1b and 2b. Other deal more with method, specifically 1c, 1d and 2a. Still others deal with both, specifically, 1e and 2c. A graphic presentation of the research questions is found in Appendix A, figure 1.

I begin this chapter with a brief overview. Next, I explain the methodological approach, including the methodological paradigm and specific research approach. I then explain the methodology in detail, discussing six major aspects: preliminaries, data collection strategies, data analysis, ethical considerations, trustworthiness, and limitations of the study. I conclude with a chapter summary.

Methodological Approach

In this section, I first discuss the methodological paradigm. After that, I discuss naturalistic inquiry, the specific research method, explaining the reasons that I chose it.

Methodological Paradigm

In this study, the overarching methodological paradigm was constructivism. Constructivist epistemology and ontology underlie the paradigmatic perspective that undergirded this study. A basic tenet of constructivism is that all reality is socially, culturally and historically constructed (Lincoln & Guba, 1985). In the constructivist paradigm, knowledge is viewed as being established through the meaning attached to the phenomena being studied (Machawira, 2008). Furthermore, in constructivism "Individuals seek understanding of the world in which they live and work. Meanings are varied and multiple, leading the researcher to look for the complexity of views" (Creswell, 2007, p. 20). Qualitative researchers attempt to understand phenomena in terms of the meanings people bring to them, rather than to quantify them in terms of variables and hypotheses. Constructivist research is concerned with particular contexts and particular points in time. Looking at complexities in the sociocultural world, it examines how such complexities are experienced by participants. The intent is to explore a situation or phenomenon by allowing the researcher to enter the world of the participant, while seeking to obtain a holistic understanding of how those participants experience that world (Bloomberg & Volpe, 2008).

Qualitative research design is suited to a constructivist paradigmatic position (Hatch, 2002). Qualitative methodology makes it possible to generate descriptions, interpretations and analyses (Wolcott, 1994). The main strength of qualitative research is that it yields data that provide depth and detail to that can be used to construct an understanding of phenomena and lived experiences (Bowen, 2005). Constructivist research is generally focused on extracting and interpreting the meaning of an experience (Bloomberg & Volpe, 2008).

Qualitative methods were needed in order to obtain the rich data required to address the research questions of this study. Several features of what it means to proceed with a qualitative study fit well with this research project. These features included (a) understanding the processes by which events and actions take place; (b) developing contextual understanding; (c) facilitating interactivity between the researcher and participants; (d) adopting an interpretive stance, and; (e) maintaining design flexibility (Bloomberg & Volpe, 2008).

According to Creswell (2007), qualitative research is used when detailed understandings of an issue are the goal. Qualitative research will not measure whether or not a certain approach always makes a difference under a given set of circumstances. However, it may examine participant reports of why an approach made a difference in their situation, for example. The choice of a qualitative methodology allowed me to “study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them" (Denzin & Lincoln, 2005).

My research was designed to rely, as much as is possible, on the participants' views of the situation (Creswell, 2007). Questionnaires and interviews focused on research questions and were open-ended. The intent was to make sense of and interpret the participants' meanings regarding their beliefs and practices related both to preparing elementary mathematics teachers and to teaching elementary mathematics courses. Thus, qualitative methodology was considered appropriate for an analysis of concepts and themes that will be derived from an in-depth exploration of the beliefs and practices of elementary mathematics methods instructors.

Naturalistic Inquiry

Within the framework of a qualitative approach, this study's goals appeared to be best suited to a research methodology of naturalistic inquiry. Willems and Raush (1969) define naturalistic inquiry as "the investigation of phenomena within and in relation to their naturally occurring contexts" (p. 3), that is, not in an experimental setting with researcher induced variables. Naturalistic research includes the following characteristics: a human or humans as the primary data gathering instrument, qualitative methods, purposive sampling, inductive analysis, grounded theory, emergent design, negotiated outcomes, idiographic interpretation, tentative application of findings, focused determined boundaries and criteria of trustworthiness (Lincoln & Guba, 1985, pp. 39-43). I considered each of these characteristics in making the decision to utilize naturalistic inquiry. I was interested in the natural contexts that instructors work in and the meanings, themes and patterns that emerge for them. I sought to contextualize interview responses via the questions I asked, along with follow-up questions. Studying real-world situations as they unfold, naturalistic inquiry is non-manipulative and non-controlling and lacks predetermined constraints on findings (Patton, 2002).

In this inquiry, the most important data source was in-depth interviews. Interviews are one of the most widely used methods of qualitative data collection (Redmond & Curtis, 2009). Defined by Dexter (1970) as a “conversation with a purpose", interviews, according to Sorrell and Redmond (1995) are not “conducted,” but are participated in by both the interviewer and the interviewee. Patton (2002) states, “The purpose of interviewing is to allow (the researcher) to enter into the other person’s perspective” (p. 341).

Interviews, as described by Lincoln and Guba (1985), may be concerned with some combination of events, activities, feelings, motivations, claims, concerns, and other entities. I structured interviews around guiding questions (Hatch, 2002). Such questions were prepared before the interview and were designed to guide, but not dictate, the conversation that I anticipated would take place. They did not call for a specific recitation of facts, such as, “Where did you go to college?” or “How long have you been teaching?” Instead, they invited the participant to begin a conversation. For instance, I asked, “What is your background?” or “Tell me about a typical day in your class?” Follow-up questions could then occur naturally, depending on what the participant said, and could invite the participant to say more, for example, “Oh, so you started teaching at the college level after you had mentored several younger teachers?”

A major strength of qualitative interviews is that they allow researchers to do what may be difficult or impossible to do in any other way, that is, to “find out what is in and on someone else’s mind” (Patton, 1990, p. 278). Qualitative interviewers design interviews carefully, and ask appropriate probing questions. Participants may not always be aware of, for example, how they make sense of their worlds and mentally organize their experiences (Hatch, 2002). Qualitative interviews offer a means to bring such things to the surface.

Methodology

Methodology included three major steps. Additionally three major factors related to methodology had to be considered. The major steps followed in order to conduct the study: (a) preliminaries, including literature review and IRB approval; (b) data collection, including short answer surveys and in-depth interviews; and; (c) data analysis. Major factors considered in relation to methodology were (a) ethical considerations; (b) issues of trustworthiness and; (c) limitations. The six factors just mentioned will be the six sub-sections comprising this section on methodology.

Preliminaries

A review of selected literature examined theory and other findings that inform the study. Transmission and transformation models of teaching were examined, along with their basis in constructivist or behaviorist learning theory. Theories of teacher response to calls for change, generally coming from mathematics reform and standards-based accountability movements, were also examined. The place of theory in the literature review is in keeping with Wolcott's (1995) statement: "Theory is supposed to help researchers of any persuasion clarify what they are up to and to help them to explain to others what they are up to" (p. 189).

The review of literature took place while I was writing a prospectus for my committee explaining the research I planned to conduct. The committee approved the prospectus in December of 2012. Permission was sought from the University of Tennessee Research

Involving Human Subject Institutional Review Board (IRB) to conduct the study. It was obtained in February 2013. All IRB guidelines were followed

Data Collection

Two main forms of data were gathered. First, in March of 2013, a demographic survey was sent via email to 450 mathematics educators, from which 92 responses were received. This survey asked questions regarding academic background, teaching preparation and professional organization membership. The purpose of the survey was to help identify candidates for in-depth interviews. The second form of data consisted of in-depth interviews with volunteer participants during April, May and June of 2013. Interview questions elicited data about preparing elementary mathematics teachers and teaching elementary mathematics methods courses. By exploring beliefs and practices of elementary mathematics methods teacher educators, this study drew attention to some important dynamics that influence teaching in the mathematics methods classroom.

In this section on data collection, I discuss contexts, participants and strategies. Strategies, here, refers to the two forms of data that were gathered; survey and interview.

Contexts. Interviews were conducted with participants from six sites. Sites were selected through purposeful sampling. Purposeful, as opposed to probabilistic sampling, is "selecting information-rich cases for study in depth" (Patton, 1990, p. 169). All sites were universities and colleges where mathematics methods classes for future elementary teachers are taught. Selection criteria included manageability in terms of the number of sites, accessibility to a variety of participants, varied geographical location of the sites, and the willingness of participants to speak freely with the interviewer. Specific demographic findings on each site are given in chapter four, when participants are described in detail, along with the settings they work in.

Participants. A purposeful sampling procedure was used for selection of participants. Merriam (1998) points out that, "Purposeful sampling is based on the assumption that the investigator wants to discover, understand, and gain insight (about a particular phenomenon) and therefore must select a sample from which the most can be learned" (p. 61). All participants were current instructors of mathematics methods courses for preservice elementary teachers at colleges and universities. Six participants took part in the in-depth interview process. Demographically, they are described in detail in chapter four, the findings chapter.

The method of selecting interview participants was as follows: From 92 responses to this email survey, 23 persons stated that they were teaching classes focused on elementary mathematics teaching methods and that they would be willing to participate in interviews. Six participants were eventually chosen for the study, based mainly on the possibility of meeting face-to-face (FtF) for at least the first interview out of three. Synchronous communication in time and place characterizes face-to-face interviews. Qualitative researchers have found that no interview method other than FtF interviews can take its advantage of social cues (Opdenakker, 2006). Rapport can be built, and unique information can be gathered, through social cues such as voice, body language and intonation (Hatch, 2002). Such cues are highly assessable as part of FtF interviews. The aim of the selections was to enable the gathering of subjective participant accounts that revealed how mathematics methods teaching was experienced and constructed by a group of people with similar experiences and concerns, but diverse means of making meaning in their individual contexts.

Strategies. Two major strategies were employed for data collection: The demographic survey was conducted first. It was followed by in-depth interviews. Interviews served as the major form of data for analysis.

Demographic Survey. A demographic survey was planned. Its purpose was to help locate willing participants for the study. The survey was first piloted in order to develop a final version. Then, once the final version had been developed, it was administered via email.

Piloting the instrument. The demographic email survey was piloted with fellow graduate students. They made suggestions and commented on anything that did not seem clear. The purpose of this process was to refine questions so as to "net the most useful information" (Creswell, 2007, p. 132). Sampson (2004) emphasizes the importance of piloting instruments and recommends doing so in order to refine questions, assess observer bias, frame questions, and adapt the research procedures. Piloting the instruments also may help to improve the data collection plans and develop better and more relevant survey questions (Yin, 2003).

Demographic survey administration. After developing a satisfactory demographic survey, which may be found in Appendix B, I emailed it to persons working as teachers in mathematics education at universities and colleges. I obtained a public email list at the International Congress for Mathematical Education held in Seoul, Korea in July 2012. On this list were the names of 450 mathematics education professionals at United States colleges and universities. As previously stated, 92 responses were received, from which 23 persons were both working as elementary mathematics educators and indicated a willingness to interview.

In-depth interviews. From the 23 persons discussed above, six were eventually chosen and invited to participate in in-depth interviews. These interviews were the primary data collection method. Invitation were sent out through an email cover letter found in Appendix C.

Each signed an informed consent form before interviewing, a copy of which is found in Appendix D.

In a constructivist interview study, the data obtained are co-constructed with participants (Hatch, 2002). All participants completed a series of three in-depth, open-ended, semi-structured interview. I prepared an interview guide that I prepared, found in Appendix E. This guide contained an interview protocol, but with open-ended questions and the freedom to pursue topics of interest that arose. Merriam (1998) explains that less structured interview formats are appropriate to qualitative study. Participants define their world in unique ways, and so, the interviewer can respond to the situation of the interview, to the emerging worldview of the interviewee, and to new ideas on the topic during the course of the interview. The interview guide can help readers of the report judge the quality of the interviewing method and the protocol used (Bowen, 2005). Transcripts of Interviews are in Appendix F.

Three interviews took place with each participant. In the first interview, the participants and I got better acquainted. I began to learn something about the beliefs of the participant and his or her classroom practices. The second interview was primarily a follow-up. Here I probed for more information and checked information from the first interview. It took place after analyzing data from the first interviews. In the third interview I asked a few further follow-up questions, but this was mainly a member check interview, in which the participants and I could agree on what was being conveyed in the interviews. Stake (1995) points out that a member check takes place when participants examine data, usually when no further data are to be collected. Member checks are an opportunity for participants to react to tentative findings that I have generated (Hatch, 2002). I allowed participants to offer feedback until we agreed that I had fairly represented their voices.

I invited participants to help me become a better researcher (Wolcott, 1995) as we constructed knowledge together. In all three interviews, I asked, "Do you have any suggestions about these interviews?" (p. 115) and, "Are there any topics we should explore that I haven't talked about?" (p.116).

All but one of the initial interviews were done in person. One interviewee had a family emergency. Though she had scheduled the first interview in person, it had to be cancelled and rescheduled later via Skype. The second and third interviews with all participants were conducted via Skype, with the exception of one participant, who stated a preference for phone interviews as opposed to Skype.

The cover email inviting participants to take part in the study was developed following the guidelines of the Institutional Review Board (IRB). It explained that an appropriate incentive provided by the researcher would be sent to participants. Incentives were in the form of $20 gift certificates to Amazon. One was given after the first interview and one at the conclusion of the interview process. The incentives were intended only as tokens of appreciation.

Naturalistic inquiry requires that data collection tools be flexible and adaptive, and usually qualitative in nature; therefore, the interview questions were open-ended (Lincoln & Guba, 1985). Interviews consisted of directed conversations seeking to understand participants' reported beliefs and classroom practices. I had the freedom to adapt and ask follow-up questions from the participants as needed.

Data Analysis

After I collected data, they were analyzed. According to Creswell (2007), "data analysis in qualitative research consists of preparing and organizing the data for analysis, then reducing the data into themes through a process of coding and condensing the codes, and finally representing the data in figures, tables, or a discussion" (p. 148). All data analysis was done with the research questions continually in mind.

The email survey data served to help select participants and helped with demographic descriptions in the final report. Survey data are not part of the formal analysis. The dissertation committee concluded that formal analysis of the email survey data would constitute a separate study not needed for the purposes of this research project. Therefore, only the data from in-depth interviews were analyzed. While not part of the analysis, data from one survey question did help to support one of the conclusions in chapter five. This will be explained fully when study conclusions are discussed.

In this section, I will discuss the type of analysis chosen, typological analysis, in detail. I will then describe, in detail, the procedures that I followed when analyzing data.

Typological Analysis. The analysis of interview transcripts was based on the typological data analysis method (Hatch, 2002). This type of analysis can be especially helpful in analyzing data from interview studies. Initial typologies are identified in the data by means of thematic codes, which have been described as "dividing everything observed into groups or categories on the basis of some canon for disaggregating the whole phenomenon under study" (LeCompte & Preissle, 1993, p. 257). Data analysis began with dividing the entire data set into typological groupings. In this case, the initial groupings flowed naturally from the research questions. Eventually, themes emerged which combined and overlapped research questions. At that point, all transcripts were re-coded based on typologies associated with six emergent themes.

Procedures. In doing typological analysis, I followed specific procedures described by Hatch (2002) making only limited, legitimate modifications to suit specific study needs. The procedures I followed were (a) I identified eight typologies to be analyzed based on the eight research questions; (b) I read and re-read interview data, marking passages related to eight typologies; (c) I began recording these passages on eight separate summary sheets, one for each typology; (d) However, at this point, I realized that much data was overlapping, and it seemed that a very large amount of data addressed multiple research questions – this was especially true because if participants believed in something, they stated that they practiced it, thus questions on belief and practice were difficult to separate; (e) As I continued looking for patterns, relationships and themes within typologies, I felt that six separate themes emerged from the data.; (f) I re-read and re-coded the data according to these six themes, and created six new summary sheets, one for each theme; (g) Applying principles from constant comparison (Glaser & Strauss, 1967) all along, I searched the interview data, comparing to be sure themes constituted patterns as well as searching for any non-examples of my patterns; (h) I reorganized the summary sheets according to some sub-themes which emerged. For instance, "challenges" was one theme. I found that several specific challenges were repeatedly mentioned by a majority of participants. These became sub-themes. (i) I wrote my six themes as one to three sentence generalizations which became findings; (j) I organized data excerpts that supported my findings. After doing this, I was ready to write a report of my findings.

Ethical Considerations

In any study, ethical issues related to the participants are a vital concern (Marshall & Rossman, 2006). This study involved human participants. All IRB guidelines were followed. In order to protect the privacy of the participants, a protocol of informed consent was followed. Full disclosure of research intentions and the clear message that consent is voluntary are essential elements to ensure genuine informed consent (Hatch, 2002). Participants were informed about the process of data collection, security and storage in the informed consent form (Appendix D).

Participants' anonymity was protected through the use of pseudonyms. Each participant signed the informed consent form indicating his or her understanding that he or she would be interviewed and audiotaped. The contents of the interviews and audiotapes were for data collection purposes only. Notes taken were kept in a locked file cabinet in the researcher's home. Only the researcher had access to these data. All participants were notified of their right to withdraw from this study at any time and for any reason, without penalty.

Regarding risks and benefits, there was no known risk to those who participated in the study. Benefits to the researcher in the form of dissertation findings and potential benefits to the research community have already been discussed. Benefits to participants were a chance to reflect upon their own beliefs and practices, and support in becoming more aware of them. Symbolic interview incentives consisted of a $20 Amazon gift certificate after completion of the first and third interviews.

Issues of Trustworthiness

Issues of trustworthiness were another factor considered when designing and conducting the study. A major objective in sharing this study with others is to persuade them that the constructions of reality put forth in the findings are of value and so should be considered in understandings of mathematics methods instruction. However, qualitative studies make no claims in terms of traditional quantitative constructs such as reliability and generalizability. Instead, they explore, reflect and analyze in terms of trustworthiness as opposed to the conventional, positivistic criteria of internal and external validity, reliability, and objectivity (Denzin & Lincoln, 1994; Padgett, 1998). Since positivist criteria are not well suited to qualitative research (Denzin & Lincoln, 1994), qualitative methodologists (Guba , 1981; Guba & Lincoln, 1989; Lincoln, 1995) suggest that four other factors be considered in establishing the trustworthiness of findings from qualitative research. They are credibility, transferability, dependability, and confirmability.

Credibility. In qualitative research, credibility refers to confidence in the claims of the study's findings, and asks whether the findings are accurate and credible from the standpoint of the researcher, the participants and the reader. Various methods can help establish credibility, including prolonged engagement, the adoption of well-established research methods, persistent observation, triangulation, third party scrutiny, tactics to help ensure honesty in informants, discrepant case analysis, progressive subjectivity checks, and member checking (Lincoln & Guba, 1985; Bloomberg & Volpe, 2008; Shenton, 2004). This study established credibility through the adoption of well-established research methods and tactics that helped ensure honesty in informants, member checking, discrepant case analysis, and utilizing opportunities for scrutiny.

The adoption of well-established research methods can help to ensure that the phenomena under study have been accurately recorded and systematically analyzed (Shenton, 2004). Specific procedures used for data gathering and analysis were based on procedures that have been successful in previous, comparable projects, such as other qualitative research projects that involved interviewing teachers. For example, the work of Hatch and Freeman (1988) utilizing interviews with teachers on their beliefs and practices was be utilized in this study.

Tactics to ensure honesty in informants are referred to by several researchers (e.g., Bayih, 2011; Shenton, 2004; Pool & Reitsma, 2010). This is not to imply an assumption of dishonesty without the use of such tactics, but to say that it is possible to encourage transparency and openness in participants. Moen (2006) stresses the importance of establishing trust with participants, stating that only if they feel safe and perceive the researcher as trustworthy will they be prepared to share openly. The researcher can encourage frankness from the outset, let participants know there are no right or wrong answers, give opportunities to refuse participation and, where applicable, make his or her independent status known, so that participants will be more comfortable sharing openly. I used these strategies. In addition, I used an empathic stance (Patton, 2002). This means I sought vicarious understanding without judgment by showing openness, sensitivity, respect, awareness, and responsiveness and implies I attempted to be fully present (mindful) when listening and observing during interviews.

Member checking is considered by Guba and Lincoln (Guba & Lincoln, 1989) to be the most important provision that can be made to enhance a study's credibility. Such checks were conducted by talking with respondents to check the accuracy of the data recorded in the interviews. This is in keeping with the idea that in constructivist research, the researcher and participants co-construct knowledge (Guba & Lincoln, 1994). Members were asked if I had fairly represented their voices.

Discrepant case analysis, also called negative or divergent case analysis, enhances rigor and is used in the quest for verification (Padgett, 1998; Strauss & Corbin, 1990). This means that, after completing my study, I searched the data for anything that seemed to refute my analysis. If anything did seem to refute it, I revised my explanation and went through the cycle again until there were no more discrepant cases to account for.

Opportunities for scrutiny of the research project (Shenton, 2004) by colleagues, peers and academics were welcomed and sought after. In particular, I sought systematic review of the project by the chair and members of the committee. This process brought a fresh perspective and helped me to challenge any unwarranted assumptions I might have been holding. Individuals' questions and observations helped me to refine my methods, strengthen my research designs and clarify or correct my arguments.

Transferability. In qualitative research, transferability means that other researchers can apply the findings of a study to their own contexts. Qualitative research makes no claims about generalizability, but references transferability instead. In naturalistic inquiry, the "naturalist cannot specify the external validity (generalizability) of an inquiry; but can provide only the thick description necessary to enable someone interested in making a transfer to reach a conclusion about whether transfer can be contemplated as a possibility" (Lincoln & Guba, 1985, p. 316). Transferability is primarily the responsibility of the reader, the one doing the generalizing to his or her particular situation, deciding whether or not a particular study applies to his or her case. The researcher can enhance transferability by doing a thorough job of describing the research context, boundaries, limitations and assumptions involved. The reader who wishes to transfer the results to a different context will then have a better understanding of the study and can better make a judgment of how sensible the transfer will be (Bassey, 1981).

Dependability. The stability of the findings over time is referred to in qualitative research as dependability (Denzin & Lincoln, 1994). In quantitative research, a reliable finding will be repeatable in the same circumstances. To speak of sameness of circumstances in qualitative research is problematic, however. This is because the investigation is tied to the situation under study. As Florio-Ruane (1991) argues, "published descriptions are static and frozen in the ethnographic present" (p. 234). In order to increase dependability, study processes were reported in detail, such that others could repeat the work, though not necessarily obtaining the same result. Thick descriptions in the form of journaling, notes, and interview transcripts, (Lincoln & Guba, 1985; Padgett, 1998) were created and serve to offer readers an opportunity to assess the findings of the study through their presence in the audit trail.

Confirmability. In qualitative research, researchers speak of confirmability rather than objectivity, the latter of which is held to be neither possible nor desirable (Bloomberg & Volpe, 2008). The central concern is the source of findings. Findings should be confirmable as the result of the research and should not be the result of the characteristics and preferences of the researcher. Techniques applied to previously discussed trustworthiness criteria also apply here. I included detailed description to help readers determine the quality of data and results emerging from the study. I provided an audit trail for public scrutiny. I was reflexive and illustrated how data could be traced back to their origins. I followed the directive implied by Code's (1991) remark that "knowledge is a construct that bears the marks of its constructors, and the more explicit a researcher is about those marks, the more the readers are able to evaluate it fairly" (p. 55).

Limitations

Limitations were the third major factor I considered in designing this study. The sample of professors and instructors consisted of those who volunteered their responses. However, in qualitative research, the purpose of sampling is not to "establish a random or representative sample drawn from a population but rather to identify specific groups of people who either possess characteristics or live in circumstances relevant to the social phenomenon being studied" (Mays & Pope, 1995, p. 110). I identified informants because of their potential to help enable exploration and understanding of the phenomenon.

This study collected and analyzed data based on reported beliefs. Such beliefs may not entirely align with participants' true beliefs. While all investigations on individuals' beliefs have this shortcoming (Pajares, 1992), current literature uses the term "beliefs" rather than terms such as "reported beliefs" or "inferred beliefs" (Perrin, 2008). This study follows that convention.

Data were collected on teachers' reported classroom practices rather than collecting data on classroom practices via observations. This was a purposeful choice since the subject is PEMTEs' reported practices and beliefs. Future studies may examine observed classroom practices and how they may or may not reflect beliefs; but that is not the purpose here.

One methodological limitation of my study was its small sample size. Purposeful sampling was employed, however, so that each of the participants studied contributed to understanding the beliefs and practices of elementary mathematics methods teachers. Qualitative research is idiographic and emic (Morrow, 2005). The focus is on one or a very few individuals. Categories of meaning are derived from the individuals studied.

A limitation of phone interviews is the loss of face-to-face interaction, with the consequent reduction of social cues. Therefore, in-person interviews were be preferred; followed by Skype interviews, with phone interviews in third place. Certain advantages do exist for phone interviews (Mann & Steward, 2000). Among them are wide geographic access, ease of scheduling interviews, and possible greater ease in working with sensitive accounts. Regarding the later, some people are reluctant to discuss personal issues in face-to-face interviews, but may decide to discuss them over the phone.

A limitation which could threaten trustworthiness if it was present would have been respondents' desires to give a response they perceive as pleasing. Respondents may have said what they think I wanted to hear. For instance, they may have spoken favorably of practices endorsed by organizations, in part, because they know I am a member. To combat this, I used an empathic stance in interviewing (Patton, 2002). As previously discussed, this means I sought vicarious understanding without judgment, and attempted to show openness, sensitivity, respect, awareness, and responsiveness. These practices helped me put respondents at ease with regard to sharing their thoughts and feelings.

Data analysis depended on my thinking and interpretive choices, and so this study was limited by researcher subjectivity. Researcher effects may include assumptions, biases, interests, perceptions and needs (Bloomberg & Volpe, 2008). During the study, I was teaching as an adjunct instructor in courses preparing elementary mathematics teachers for approximately three months of the year. I bracketed my biases, acknowledging and taking them into account as a method of dealing with them. In interviews, I made sure my verbal and nonverbal language was appropriate, encouraging respondents but not expressing an opinion of their response. Some of the criteria for trustworthiness, detailed above, also helped to address any bias. This included divergent case analysis, seeking participant feedback though member checking and committee scrutiny as I participated in accountability through an audit trail.

Chapter Summary

To sum up, a detailed description of this study's research methodology was provided in this chapter. The phenomena of the beliefs and practices of elementary mathematics teacher educators were studied using naturalistic inquiry. Data collection was accomplished first through an initial email questionnaire, which helped identify persons asked to participate in interviews, and then through a series of in-depth interviews with six volunteers. Trustworthiness was accounted for through various strategies, including peer scrutiny, use of established research practices, thick, rich description, member check, discrepant case analysis and an audit trail. After collecting initial data, examination of email questionnaires facilitated the choice of persons from varied backgrounds who were invited to complete in-depth interviews. Subsequent analysis of those interviews allowed key themes from the findings to be identified. Interview transcripts were reviewed against literature and for emergent themes, using a typological strategy. Conclusions were then drawn, and recommendations were conceived for educational practice and further research. Findings will be discussed in detail in the next chapter, and conclusions and recommendations are discussed in chapter five. My intent in conducting this study was to contribute to a better understanding of the beliefs and practices of those who prepare elementary mathematics teachers.

Chapter 4

Findings

> Our preservice candidates are a result of our failed system. They went through the system when we were all about testing and answer getting and so they really don't have that deep understanding themselves. They just want to get an answer… So, I believe they first need to learn the mathematics. They need to understand it themselves. And then they need to be able to teach that to elementary children so they can understand. (Participant E)

Overview

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. This study was timely in light of reported findings about mathematics achievement in the United States. The quote above, from a mathematics teacher educator participating in this study, reflects concerns about how mathematics is taught and learned.

At present, although many researchers have examined the beliefs and practices of preservice elementary mathematics teachers, little is known about the beliefs and practices of those who prepare them, preservice elementary mathematics teacher educators (PEMTEs). In designing and conducting this study, the goal was to understand more about the thinking of PEMTEs. This understanding might have benefits for mathematics teacher educators themselves, for the preservice teachers they help to prepare, and for the children who will be the future pupils of those teachers. We can better help children learn mathematics if we understand more about those who are preparing mathematics teachers.

This study began with a demographic survey, conducted via email. This short answer survey asked questions regarding academic background, teaching preparation and professional organization membership. The survey's purpose was to help identify good candidates for in-depth interviews. It is found in Appendix B.

The survey was sent to 450 mathematics educators, from which 92 responses were received. Of the 92 respondents, 23 indicated that they were both currently teachers of classes for preservice elementary mathematics educators and would be willing to participate in in-depth interviews. From the 23 willing interviewees, six were chosen for interviews.

This chapter focuses on presenting key findings from 18 in-depth interviews, gleaned from the six elementary mathematics teacher educators. Participants, all of whom held doctoral degrees, varied according to geographic location, age, years of experience, preparation, departments in which they worked, and types of colleges in which they taught. Within this diversity, however, common themes arose, leading to the findings reported here.

After this chapter overview, participants are described, with details related to participants' sites interwoven into participant descriptions. Next research questions are re-stated. Then, findings are summarized and their relationships to research questions explained. Finally, findings are presented in detail, followed by a chapter summary.

Participant Descriptions

The emphasis throughout this work was on allowing participants to speak for themselves. Illustrative quotations taken from interview transcripts attempt to portray participant perspectives and convey some of the complexity and richness of the subject matter. Participant descriptions, are correct as of spring and summer 2013, when interviews took place.

Participant A is a Caucasian female, over 50 years of age. She is a professor in a mid-sized private liberal arts college in an urban setting in the Northeastern United States. She has taught all K-12 grades except grade two and grade five. She prepared as an elementary teacher and taught elementary school for several years. Eventually she taught middle school math, returned for a master's in mathematics education, taught high school math, returned for a doctorate in mathematics education, and began teaching mathematics education courses at the college level. She obtained her doctorate 11 years ago. After five years teaching at the college level, she became a state coordinator for mathematics education. Her state became recognized as a top performing state during these years. After some years in this position, she came to the university where she now teaches and heads up mathematics programs for her college. She has taught elementary mathematics methods courses for 14 years, and also teaches separate content classes for elementary mathematics educators. Students in her methods courses are preparing to teach in grades PreK-6.

Participant B is a Hispanic female, in the 30-34 year old age group. She is an assistant professor at a large research I public university in a suburban setting in the Midwestern United States. She possesses a bachelor's degree in mathematics, and both master's and doctoral degrees in mathematics education. She taught high school mathematics for five years. She taught elementary math methods for three years as a graduate student and has now taught for one year as an assistant professor in an education department. In her present situation, she teaches classes that combine methods and content. Her university does not offer separate methods courses. Students in her methods/content courses are preparing to teach grades 1-6.

Participant C is a Caucasian female in the 25-30 year old age group. She is an assistant professor at a small private liberal arts college in a rural area in the Southern United States. She possesses a bachelor's degree in mathematics education, though this degree had only one methods class and included all of the mathematics needed for a degree in mathematics itself.

Her doctorate is in mathematics education. She taught middle school for three years. In her present position, which she has held for three years, she teaches two courses that combine elementary math methods and math content. She also teachers a master's level elementary math methods course for students who possess bachelor's degrees in non-education related areas but wish to become elementary school teachers. Students in her methods/content classes are preparing to teach in grades PreK – 5.

Participant D is a Canadian Caucasian male, in the 30-34 year old age group. He is a teaching fellow at a large private liberal arts college in an urban setting in the Northeastern United States. He is also an adjunct instructor of mathematics at a community college. Having obtained his doctorate as the interview process for this study was unfolding, he will become an assistant professor at a small liberal arts college in the fall of 2014, continuing to teach courses for future elementary mathematics teachers in that setting. He possesses a bachelor's degree in music, a master's in mathematics education and a doctorate in curriculum and instruction with emphasis in mathematics, science and technology. He taught high school mathematics for five years, and has taught elementary mathematics methods for three years as a teaching fellow. Most students in his methods course are preparing to teach in grades K-6 and some are preparing to teach special needs students at all levels.

Participant E is a Caucasian female over 50 years of age. She is an assistant professor at a large (population around 12,000, including online students) public university in a small town located in a rural and, as she described it, somewhat isolated area of the Western United States. She taught seventh and eighth grade math in a Western U.S. city for 25 years, and possesses national teacher certification. Her bachelor's and doctoral degrees are in mathematics education. She has taught elementary mathematics methods for four years. She is the only participant teaching under the traditional curricular arrangement in which prospective elementary mathematics teachers obtain credit in College Algebra and Statistics, and take one methods course. All other participants teach math methods in situations with specialized mathematics content courses for prospective elementary teachers. Participant E teaches elementary math methods, both in an actual classroom, and as an online course. Students in her methods course plan to teach in grades K-8.

Participant F is a Caucasian male in the 30-34 year old age group. He is an assistant professor, teaching in a suburban setting in a Mid-Atlantic state. He teaches at a mid-sized public university campus that serves as a satellite of a research I public university. His bachelor's degree is in mathematics, and his Ph.D. is in mathematics education. He taught high school for five years and taught math content courses for future elementary educators for two years as a graduate assistant. He completed his first semester as an instructor of an elementary math methods course during the interview process. Students in his methods course are preparing to teach in grades K-4.

Relationship Between Research Questions and Findings

The aim of the interviews was to gather subjective accounts that revealed how mathematics methods teaching is experienced and constructed by a group of people with similar experiences and concerns, but with individual ways of making meaning in their particular classroom contexts. Research questions aimed to examine beliefs and practices of participants. They were:

1. What are the participants' beliefs regarding preparing elementary math teachers?

(a) What do they believe elementary students need to learn?

(b) What do they believe preservice elementary teachers should learn?

(c) How to they believe elementary students should be taught?

(d) How do they believe they should use class time with their students?

(e) What do they believe are the major challenges that they face in their work?

2. How do participants describe their approaches to teaching an elementary math method course?

(a) How do they use class time with their students?

(b) What content do they teach?

(c) How do they decide what to teach?

Overall, six major findings emerged. Findings paint a picture of independence. Instructors report agreement in underlying beliefs. For example: teach big ideas, constructivism is important and more math content is needed. However, they report making a variety of individual and sometimes divergent decisions regarding their teaching practices, and regarding what content they choose to teach in the day to day life of their classrooms. Findings addressed the two major research questions, along with all sub-questions. However, there was overlap. Emergent themes did not always fit one and only one research question. Findings could address both belief and practice simultaneously. They could also address more than one sub-question on belief or practice at a time. To deal with this, a cross-reference list was created, along with a graphic figure summarizing the relationships between the research questions and the findings. The figure, presenting the same material as the cross-reference list in visual form, is found in Appendix A, figure 2. The cross-reference list follows.

The six major finding emerging from the study, along with the ways in which they relate to the research questions, were as follows:

Finding one: Participants uniformly reported that they believe their practice is consistent with their beliefs.

This finding relates to Question 1, on belief, as well as to sub-questions associated with Question 1 on beliefs related to content and practice. Centrally, when answering research questions about practice, questions about belief are also being answered, since participants report practicing what they believe.

Finding two: Participants consistently reported certain limits and challenges. Specifically, they reported lack of experience in teaching elementary school mathematics classrooms, limits in the mathematics content knowledge of many of their incoming students, student field experience issues, time allotted for their course, helping student learn to plan lessons, and attitudes about mathematics on the part of many incoming students.

This finding relates mainly to Question1e, “What do (participants) believe are the major challenges they face in their work?”

Finding three: Participants possessed a largely shared overall vision for best teaching practices, both in their own classrooms and in elementary school classrooms. The vision was consistent with constructivist learning theory.

This finding relates to most of the research questions. Belief in constructivist learning theory relates to how participants approach teaching (Question 2), how they believe both elementary students and preservice teachers should be taught (Questions 1c , 1d), how they report using class time (Question 2a), how they report reaching decisions about teaching (Question 2c), and to some extent to content participants believe should be taught (Questions 1a, 1b).

Finding four: Participants reported widely varied manners of employing resources such as textbooks, manipulatives, technology, professional organizations, and standards.

This finding relates to research Question 1d. “How do they (participants) believe they should use class time with their students?” It also relates to Question 2a: “How do they use class time with their students?” Regarding textbooks, professional organizations and standards, it relates to Question 2c “How do they decide what to teach?”

Finding five: Regarding content, although curriculum remained within broad guidelines set by universities, participants showed much individual variation, and generally exercised personal autonomy in the decisions they reached about both the mathematical and pedagogical content of their classes.

This finding relates to Question 1b “What do they (participants) believe preservice elementary teachers should learn?” It also relates Question 2c: “What content do they teach?”

Finding six: Regarding teaching decisions, all participants were aware of their own journeys from tacit acceptance of the so-called traditional methods by which they were taught mathematics to their autonomous decisions to implement reform-oriented teaching methods and content because they deemed them best for their students.

This finding mainly relates to Question 2c: How do they (participants) decide what to teach? Findings are discussed in detail, and evidence is given for each finding, in the next section.

Findings

In this section, I present details to support each finding. All findings stem from interviews. In this naturalistic inquiry, interviews were done with open-ended questions. An interview protocol is provided in Appendix E. Since questions were open-ended, ideas that appear as part of the findings, such as being challenged by lack of elementary teaching experience, were originally brought up by participants themselves in answer to the questions such as, "What challenges do you believe you face in your work?" and not in response to a leading question such as, "Do you believe lack of elementary teaching experience is a problem for you?" Open-ended questions asked of each participant, included, "How do you use class time?", "How do you reach decisions about what to teach?", "What content do you believe your preservice teachers need?", "Could you tell me about a particular lesson, perhaps one that you thought went well?" and, "How do you believe elementary students should be taught?"

Finding One

Participants uniformly reported that they believe their practice is consistent with their beliefs. On the whole, reports were that what participants believe that they *should* enact is what they *do* enact, subject to time constraints.

Some participants reported being aware that, in general, teachers' practices may not always be consistent with their beliefs. They realized, in theory, that this could apply to them. However, participants remained unaware of any actual instances where their practices and beliefs were not consistent with one another. All said they strive for such consistency.

Participant E spoke of informing her beliefs with the reading of research literature, then said, "I feel like I'm doing what I should." Participant C, noting her classroom autonomy, remarked, "about the content already that I teach…I would say that in general, it's the content that I think they should learn." Asked specifically whether what they did in the classroom was what they believed they should do, Participant D said, "Upon my own reflection, I believe yes." Participant C stated, "That would be true," and Participant F replied, "Boy howdy I hope that I, for lack of better words, practice what I preach."

Participant F, quoted above, said that while he was not aware of any instances where he did not practice what he preached, he was aware that such instances might exist. Participant D expressed a similar thought, saying that his own studies of preservice teachers had revealed that they did not always teach as they believed they should. However, Participant D said he was not aware of instances where his actions were not in keeping with what he believed he should do. In fact, the assertion from all participants was that when teaching, they did what they believed they should do.

One caveat was that time constraints prevented participants from enacting *all* they believed they should. Participant C said, "I wish that they had more content, and what I've found from my students is that they actually wish that they had more." Another caveat involved university rules: Participant B referred to administrative regulations when she said, "Certain things are imposed. I do not want to give them grades," she said, yet she had to. Participant B also said that the pacing of her course was, "somewhat dictated", with common midterms and finals. However, she had been part of writing these common assessments, and indicated they were testing what she believed should be taught.

Because participants reported putting their beliefs into practice where they could, research questions regarding what participants believe *should* be done in the classroom, and those concerning participants' reports of their *actual practices* are answered together in this dissertation. Beliefs and practices did not appear to be separate for participants. They perceived that they enacted what they believed they should.

Finding Two

Participants consistently reported certain limits and challenges. Significant limitations included their own lack of experience teaching in elementary school

mathematics classrooms, and perceived limitations in the mathematics content knowledge of their incoming students. Challenges included student field experience related issues and time allotted for their courses. Attitudes about mathematics on the part of incoming students were a challenge for some, as were concerns related to teaching students about lesson plans.

Specific challenges arose time and again in speaking with participants. Six of the major challenges are detailed below.

Experience. Lack of elementary school teaching experience, along with the knowledge and training implied by such experience, was a consistent challenge, mentioned by five of the six participants. Participant A, the exception, had originally prepared as an elementary teacher, and had taught elementary grades for several years, so she did not lack such experience All of the other participants were left to wonder: How could they prepare future elementary school teachers, when they had not taught elementary school themselves?

“I've been teaching elementary methods for four years now,” said Participant B, “and it still surprises me when things come up. Being secondary trained; I think moments just come up sometimes where I've never thought about it from a preservice elementary teacher's perspective.” She added, “I didn't have a methods course for teaching methods.” Participant D also perceived his own limitations:

> It's definitely a challenge; it's definitely a departure from what I'm comfortable with. Since I have had so much of a focus in secondary math there are still a lot of the ideas… a lot of the reform uses for content, you know … there's still a lot that I need to know.

For Participant C, lack of preparation through teaching and professional development geared to elementary students was also a problem:

> I don't feel like there was a lot of preparation specifically to teach elementary methods courses. During my time in the doctoral program I never taught a methods course, in fact, I never taught at all, I was a research assistant the whole time. And so, since I got the job where I am, I've taught elementary methods courses, every semester. It's been quite a bit of on-the-job training.

Participant F noted that, "In grad school most everything I did was sort of on that secondary focus." Participant B was responsible to help mathematics education doctoral students preparing as math educators. She had possible future mathematics teacher educators sit with tables of preservice elementary educators during her classes in order to, "understand what preservice elementary teachers don't understand, and so they can get an appreciation for what elementary preservice elementary teachers know and what their concerns are, and things like that, because most of them are secondary teachers." Her belief in doing this was that secondary teachers will have difficulty when asked to address the concerns and needs of elementary teachers.

Time. Time was a challenge for all six participants. All believed there simply was not time for students to gain the knowledge and skills that they needed to gain in order to be prepared to teach mathematics to elementary school children.

"I wish I had more time," was Participant B's comment. "I think that it's impossible to teach all the content that I would hope that they know going into the classroom," said Participant E, who continued, "In my course, I don't have time to do as much as I should, unless they add hours." Participant C could not cover all that she felt students needed. "I feel that they really do need more content knowledge," she said, "but at the same time they need to have all of those good discussions about how to actually teach the mathematics." Frustration seemed evident when Participant F stated, "37 and a half hours of contact time …to impart all this knowledge. How do I do that? I don't know." Participants appeared to believe candidates were being sent out unprepared due, in part, to lack of time.

Content knowledge. "I've had so much concern about the content knowledge of my students," said Participant E. Perceived problems with students' mathematics content knowledge were recognized and voiced by all six participants. Most came back to the concern multiple times during their series of three interviews. More than one, when asked what challenges they face in their work, mentioned the math content knowledge of their students first. Participant E said, "One of the greatest challenges is the content knowledge of the preservice candidates." Participant C, when asked "What challenges are you faced with as a preservice elementary mathematics methods educator?" had this as her first response:

> What challenge? I would say certainly the type of students that seem to be drawn to elementary education. Certainly not all of them would fit into the stereotype but I would say a majority of students lack a lot of the basic math skills. Whether that's because they've forgotten it, or because they never acquired a very deep foundation to begin with. I think that in general they tend to struggle with math, and so, that affects them not only in being able to do and teach the content themselves, but also their attitude towards it…Being able to build up their confidence and helping them to have a positive attitude is, I would say, is one of my major goals in that class.

Participant B spoke of attempting to have a discussion on a video of teacher-student interaction during one lesson and discovering, “Whoa, they don't understand equivalent fractions like we thought they did. We (she and her fellow instructors) kind of just like took equivalent fractions as more of a given.” Participant D stated, “I think that there is just an overall fear and an overall lack of understanding of the basic mathematical concepts. I don't think it's just one or two small things. I think it's very much across the board.”

Participant E linked lack of content knowledge to eventual problems with learning reform teaching methods, when she said:

> Even though right up front they realize, and we talk about how they learned mathematics, and their struggles with mathematics, they still want to teach how they were taught. Even though they know that it wasn't right for them and they didn't learn that way. They still revert back to that way, and so that's one of the greatest challenges.

Participant B recently successfully advocated for a third content /methods combination course to be added to her university’s requirements, showing she is indeed concerned about content knowledge. However, she added two caveats. First, “They have what they have and I work with that. It's just like when 9th graders come to us and we say they didn't learn anything in middle school.” She continued, “They know what they know and that's your job is to work with them where they are.” A second caveat was in regard to some researchers’ concerns about lack of what might be considered very deep content knowledge (Ma, 1999). In conversations, Participant B admitted she was not sure math educators should look for or expect what she seemed to consider an unusual degree of depth from preservice elementary math educators:

> I was a math major in college and I think if I were to take an elementary math class…somebody might be like – “Oh she doesn't really have the content knowledge in certain areas.”… I just knew invert and multiply. I didn't know why. I didn't understand that you could divide (fractions) straight across. But I had a degree in math. So, I mean it's just what kind of knowledge you're looking for and things like that.

Participant E, however, directly disagreed with the specific depth of knowledge referenced above regarding invert and multiply. She spoke about student understanding, saying, “So if they really understood the why - why you invert and multiply - then I think they would be able to explain that better to children." This illustrates that though participants are generally concerned about students’ content knowledge, they may not all agree on what specific knowledge they would like students to have.

Attitude. Several participants were concerned about students’ attitudes toward mathematics when they entered the course. Participant A gave this as her first concern when asked about challenges she faces:

> I think the biggest one with preservice teachers is that many come in stating that they hate mathematics. And, I've seen through my own classes; that when a teacher doesn’t like or understand something, students pick up on it, and the teacher's perception of not liking mathematics is often translated to children. I think that's a real serious concern that we have in helping preservice teachers recognize that they have to get over their phobia and make sure that they don't bring that dislike of mathematics into their classroom.

Others agreed with these concerns. For instance, Participant D spoke of the attitudes of his students coming into his course:

> I think a lot of these students that I've been working with; very few of them, when I talk to them, are interested in math. They're almost never interested in math… I don't know why 90% of them - a large percentage of students - are still very scared of doing math. They even say, 'We're doing elementary ed. because we didn't like doing math.'

One PST even snapped at Participant A, "I'm here to learn how to teach math, not learn math." Participant C, speaking of a graduate methods course for those who have never taught, contributed that, "The long and short of it is that you've got a bunch of people that really don't like math in general, and they're having to spend three and a half hours digging up things that maybe they haven't seen in 20 years."

Participants face the difficulty of standing before a group of students knowing that, when they first come in, a majority of them dislike and even fear their subject. However, this dislike generally did not continue through the term. There were indications that with discovery oriented activities focused on the concepts behind mathematics, and the formation of a classroom learning community through discussion, students' attitudes changed. Participant C stated:

> The students' attitudes make a really big difference. I would say that in general I'm grateful for the students that I have because, even if they come in with pretty bad attitudes, it seems like they will quickly get on board with what I'm trying to do. I have very few that end the course with a bad attitude…I can see that if you had a lot of that in a classroom, it would be very difficult to continue with the type

> of activities and discussions that I have in my classrooms, because it really is student based, and so it's dependent on the students being willing to be engaged.

Field experiences. Participants reported concerns about the field experiences of students. This impacted on what they and their students were able to do in their methods courses. “We try to tie in with the field class that they're in, but that's a little more difficult because I don't have as much control over that,” said Participant B. Asked about challenges, she also said:

> Control of field is a big one. I really wish I had more control of the field experiences...I try to improve their field experiences and make more connections between what they see in the classrooms and what they see in my class…They go for a whole day, so I have no control over what they see…what grade levels or anything. They're all in different places, so I don't really know how to engage a whole class discussion around what they're seeing.

Participant E noted that, “Especially, I think a lot of them are placed in elementary classrooms that don't model best practices, and so I'm telling them one thing and yet they're not seeing it in the field.”

Field experiences did not pose a problem for one participant. Participant C was able to have some control over what was happening in the field because local schools worked directly with her. “Teachers that I've come in contact with through … field experiences for my students have given really good feedback about what it's really important that students know.” In particular, the school district required mathematics lessons using a reform-based approach. Participant C teaches her students to work with this type of lesson, which they then go and teach in the field, reporting back to the class.

There were also problems observing in the field. For most, never having been elementary teachers, this was felt to be a crucial need. However, it could be difficult to arrange. Participant D said that he would like more time in classrooms, and explained, “I love being in classrooms. That's like, half the fun, right? Like, getting to be there and getting to spend the time with the teachers and getting to spend the time with the students.” Participant F said, “My college administration is extremely supportive of my wanting to be in the classroom (observing). It was the cooperating school district that said no,” said Participant F. He expounded further:

> If we had to come up with some challenges, that is one of them…It's almost baffling to me how much of a struggle it has been to gain access to districts. If I'm observing, there’s sort of a very strong connotative meaning to what an observation is. That I'm observing the teacher in a critical sense, not in any information gathering sense, or just wanting to watch. So, that, and I feel like that's maybe one of those reoccurring problems of not being able to feel like it's as authentic to what an elementary classroom is. You know, I had a chance at the end of my grad school career to go watch my friends teach elementary school…, but that opportunity's completely non-existent here.

Lesson Plans. Four participants avoided or limited the topic of lesson plans. Two thought lesson plans were an important topic to include. Thus, the group was somewhat polarized on the topic of lesson plans.

Two participants who spoke positively of teaching PSTs about lesson plans were Participant A and Participant C, though Participant A still reported challenges. Participant C’s had only one local district to work with, and teachers there had communicated to her regarding a specific type of lesson they used and wanted teacher candidates to know. Thus, Participant C had a warrant and a specific plan for the types of lessons she taught. Participant A is in charge of mathematics programs for her college. State records show that graduates of her professional development programs have directly improved student achievement on state assessments. One thing Participant A does is insist that preservice teachers learn to plan lessons utilizing the "Understanding by Design" (UbD) method (Wiggins & McTighe, 1998). She did report that, "It's very challenging to use UbD" since students had not taught before. However, she believed it was important to do this. Participant A modeled Understanding by Design practices in her classroom and required students to write a unit lesson plan based upon them. She attributed some of the teaching success of her students to being prepared in this way.

The other four participants encountered complications and did less, or in some cases nothing, with lesson plans in their courses. Participant D said his students were "beaten over the head with that stuff in other classes," and reported that he told students, "Ok, you guys are sick of it (lesson plans), let's not talk about it." Participant E assigned students to write and teach lesson plans, but asked, "How can they effectively do the assignments that we're asking them to do?" describing placements in which students were required by local districts to use a scripted lesson, and/or a lesson without manipulatives. Participant F avoided the topic of lesson plans, stating:

> Um, you know, I don't get much into lesson planning. Probably because I feel like that's its own little beast. Partly because …structurally, there's so many different districts. I don't know how they handle lesson plans. So, rather than trying to balance 35 different district's lesson plan, all right, just, big ideas, big ideas.

Concluding the section on challenges, findings revealed one item that was not a challenge by participants. In the words of Participant F: "There are plenty of horror stories that go around about how math educators are viewed by mathematicians." He seemed to refer to a belief or stereotype he perceived to be common. However, he went on to say, "Thankfully, my fears were horribly unfounded…Those horror stories could not be further from the truth where I'm at ...they value me as a colleague, and give me the latitude and the freedom to do what I need." Participant C also reported departmental harmony, stating, "even though I work in the math department, I'm very, very closely tied to the school of education. They have invited me over and we have meetings together frequently." Participant A, heading up math programs for her college, gave, as her first answer when asked about challenges in her work, "Being recognized as knowing math content myself and being considered a mathematician by fellow professors who are not math educators." She later clarified that such lack of recognition applied only to those who were more recently employed by the college, particularly newly hired adjuncts. She stated that those who had gotten to know her gave her respect for her knowledge of mathematics. Some participants were situated in a math department, and some in an education department. Whatever their situation, however, all six felt generally respected rather than looked down upon by mathematicians were had not specialized in mathematics education.

Finding Three

Participants possessed a largely shared overall vision for best teaching practices, both in their own classrooms and in elementary school classrooms. The vision was consistent with constructivist learning theory.

Almost all participants labeled themselves constructivists. Within this substantial agreement, however, notable variations on practice emerged, again dependent upon the decisions of individuals. Thus, instructors used class time in different manners and allotted varied amounts of time to different activities.

A common vision. Teachers' beliefs themselves are complex and may be contradictory, and therefore they resist a concise classification (Mansour, 2009). However, the focus of this research was on reported beliefs. When asked, "Is there any particular school of educational philosophy you would agree with?" five of the six participants stated that they were constructivists. "I am a radical constructivist," said Participant B, "I'm definitely a constructivist," said Participant E; whereas Participant C stated, "I am a constructivist but I don't go to the extreme with that. I don't think that all knowledge has to be constructed…Some things, students can be shown, and they can understand it." Participant F said he was a constructivist, but when asked about methods and content for elementary children, he said:

> Yes, there's discovery but the other part of that discovery is, well, once you discover it, what does it take for you to have it in your memory, in your mind. There's… exploring, reasoning and memorizing. And the goal is to pick up on all three of the parts, I think, but to pick up all three parts and not pick two of the three. It's really tempting sometimes for them to pick two of the three. They're exploring and reasoning or exploring and memorizing.

Participant A, though she made many statements consistent with constructivism, and none that seemed consistent with transmission models, stated that she took an "eclectic approach" when asked whether there was any particular school of educational philosophy she agreed with. In her words:

> From my experience as a classroom teacher, there's no one right way to teach. There's no one right philosophy. We need to be savvy enough as preservice

> educators to expose our teachers to a variety of different philosophies, and to help them understand and recognize, they need to do what's in the best interest of all their students. And, sometimes it means taking from one school of thought and another one and blending it together for the population in which they are working.

Regarding her vision for all mathematics learners however, whether in the elementary classroom or higher, Participant A said, "I firmly believe that the less telling we do, and the more we spend time giving kids an opportunity to figure it out, within reason, that they have those ah-ha moments and they don't forget them."

Participant F was concerned about constructivism in the elementary classroom itself, especially when teachers are responsible to test such a wide spectrum of students:

> We sort of shuffle the deck every year …We've got students at different levels of mathematical knowledge, mathematical preparation. So how do we find those activities that can deal with the variance of abilities across the classroom? So I mean, there's (sic) all these things that the romanticism, or the idealism I have about constructivism that sort of, argh, hits this little bit of a snag with the realities of the world they are in. So, how constructivist can we be (in elementary school classrooms), given the constraints that we live in?

In their own math methods classrooms, participants reported a plethora of examples of teaching practice consistent with constructivist belief. Participant B said, "I get the students engaged in some kind of task to open. I don't do a lot of direct instruction at all." Asked to describe a typical day in her classroom, she described the learning community:

> I try to challenge them, walk around, and then usually as a whole group, I'll try to like elicit some conversation, whole class discussions around them - bring them to

> the smart board to present their ideas and then we all talk about it…maybe a couple of tasks (takes up) the whole class.

Participant B also told about a lesson that she felt had gone well. “They had a lot of fun engaging in that problem. It was really fun, really rich. They came up with alternate solutions… then they made an extension, and then they figured out that something was wrong with their extension.” Participant F said he emphasized activities, stating, “So, these ideas of doing activities in class and having an activity driven methods course - it's so that your (teacher candidates) will have an activity focused, activity driven math class (in the future).” Asked about her students, Participant E stated, “They really need to understand the mathematics, not just be about answer getting.” When asked how elementary students should learn, Participant E stated “Well, because I'm a constructivist, I believe that's the way they should be taught… that's how students learn. They need to be taught using hands-on manipulatives, projects, and real-world applications:

> They learn, I believe they learn best, by actually doing the activity or the hands-on work that they would expect their own students to do. I do expect them to read the book, read the chapter, have that background knowledge, but just me lecturing, they don't get it. I think they really learn best when they're actually doing the mathematics themselves. And I've seen some ah-ha moments when we've been working with fractions - "Oh, is that why that works?" - you know, so that's always cool too.

Participant B noted, “I think that, as a teacher, I can find things to maybe provoke disequilibrium in my students, but I don't think I could ever impart knowledge on them. I don't think that's possible.” She continued that, “My job as a teacher is to kind of like find situations that will make them think and re-evaluate how they think about things.” She shared what she was strongly enthusiastic about in teaching: "So that's like my passion right now, finding ways to trigger these things so I can like perturb their schema, move past it."

Differences in emphasis and application. In spite of a constructivist label being embraced by most participants, there is evidence that participants had some philosophical differences in their approaches, and in what they emphasized. Each participant had a different reaction when asked "What is your educational philosophy?" There were some striking contrasts. Looking at the first thing each said in response to this question as evidence of what was important to participants reveals the following: Participant A stated "I firmly believe that every student can learn, that students learn differently, so it's my responsibility to provide experiences, interactive experiences for all of my students so that they have access to whatever the concept is that I'm teaching." Participant B: "My teaching philosophy … is that I like to really focus on rigorous content because I think people that teach math should know a lot of math…my learning theories frame a lot … I'm a pretty firm believer in radical constructivism." Participant C: "My teaching philosophy… I would say that I like to strive for a balance in my teaching in that I really, I look at the different reasons that we teach math…skills…concepts." Participant D: "I think my big, overarching teaching philosophy is that I really want people to get interested and engaged in math." He continued, "That really kind of is over everything that I do, this can be really fun, this can be really interesting, you just have to give it that chance." Participants F's remark will be of particular note later in this report in the section on hands-on learning. Asked what his teaching philosophy was, he immediately replied, "I probably should have a better name for it, but to really emphasize the hands-on aspects, maybe it does go back to that constructivism, the idea that students are going to build their own knowledge by doing hands-on activities." Participant E gave a lengthier response, when initially asked what her philosophy was, separating elementary students and preservice teachers:

> OK, well, for kids, I do believe that all kids can learn mathematics, and I do believe that they can learn to a high level. For preservice teachers I believe that they can teach all students mathematics. Philosophy: my research was around attitudes toward mathematics so sometimes I really focus a lot on that in my classroom, that your attitude is going to affect you students' attitude, and so I think my philosophy is a lot based on that.

Replication. Leaving the differences in things emphasized by participants' responses to questions of philosophy, it is notable that the idea of replication arose in talking with all candidates. Methods that allow teacher candidates to learn in the way they believed elementary students should learn appeared to be a central part of participants' teaching visions. "I feel compelled to replicate … the elementary classroom environment that I want my preservice teachers to develop... I want them to realize I teach my math methods courses the same way as I expect them to teach their students" said Participant A. "My class time with them should be largely engaged in mathematical thinking, and investigating how children think about mathematics, and trying to get them to connect how children learn to how they're (my students are) learning," stated Participant B. Participant E contributed, "I just really feel like the candidates need to actually do the activities. I will ask them to think like a kindergartener or think like a second grader." She continued, "Towards about the middle of the semester I start teaching them like I would expect them to teach. So they come in and I'll have music going – let's get excited! Or we set up learning centers, and talk about why." Participant C, speaking of a successful activity where students learned about different bases in order to remember what it is like to learn about place value in base-10 said, "They've (my students) used that as an example of something that they had to go back and learn the way (an elementary) student would learn it." Several participants, in fact, specifically mentioned learning to add and subtract in a different base as a good way to help teacher candidates learn as an elementary student would, and gain insight into that experience. Overall, the participants reported that they hoped to prepare future teachers by having them learn about mathematics in the way that their future students would best learn about mathematics.

Productive struggle. A recurrent idea mentioned by participants, particularly when asked about a "typical day" in their classes, was productive struggle (National Research Council, 2001). Participants expressed their beliefs in its powers. In the words of participant D: "I want them to be able to have the chance to think about it (mathematics) as their students would think about it, and struggle with it a little." Participant B, who mentioned that students in her class sometimes got confused, was asked how she felt about such confusion. She stated, "It's good because it means that they're actually thinking. If they weren't confused then I feel like they would just be passively like trying to get though. So, I like productive struggle." Participant C shared that:

> I try to help them understand that they're going to be teaching math regardless of whether they've labeled themselves good or bad at math. So, if you struggle with it, that's OK, but it's time to figure out how you can deal with that…it helps to see real data and concrete examples. So, can you take this abstract formula and can we come up with some examples to start with and then build up to the abstract part of it? I guess being OK with struggling is one thing… It's just helping them see that that's OK.

This idea extends to the elementary classroom. Participant B explained one thing she tells her PSTs about their future classrooms:

> When I tell them, it's good to let kids struggle, sometimes they think that sounds cruel. A lot of them, I think, had difficult things in math…They see it as a bad thing. I tell them they have to think about that the struggle is your brain kind of re-evaluating what you know and trying to fit new things in or assimilate or accommodate new ideas. And so I think it's good: if you never struggled, you would never learn anything new, I think.

Participants balanced the idea of struggle with the desire to inspire a love of mathematics in students through classes that they enjoy. "A lot of what I want from teachers is I want them to treat math as a rigorous subject where children can get enjoyment from engaging in deep problem solving or critical thinking," was how Participant B expressed it. In the end the idea seems to be that students will struggle, but we want them to be problem solvers and to see that in a positive light. Their ultimate success in understanding concepts will be ensured by the classroom community, including, but not limited to, the instructor. Such success will breed deeper understanding of mathematics, and deeper enjoyment of mathematics as well.

Enjoyment. Enjoyment of mathematics was an idea a majority of the participants discussed. Participant B, describing a lesson she felt went well, prefaced her remarks with: "They had a lot of fun engaging in that problem." In describing a decision on teaching practice, Participant A said, "So I thought, "How can I make this a fun, or a puzzle problem?" Participant D stated, "I really try to have them immersed in an experience …I really want them to just have a really great experience with math …and so I just try to make it as fun as possible." He continued:

> I think my big, overarching teaching philosophy is that I really want people to get interested and engaged in math… this can be really fun, this can be really interesting, you just have to give it that chance… And so I think that's the big thing that guides everything. How can I get people just, so excited about doing math?

Reasoning and sense making. Reasoning and sense making with rich mathematics problems were reported to be very important, both for elementary school students and for elementary mathematics educators.

> Not a class goes by that I don't have my conjecture board going, that's just a common thing, that we're not doing maybe a gallery walk where they have taken their work and they've posted it on sheets, we've hung them up around the room so that they can walk around and critique the reasoning of their fellow students.

Participants attempted to choose rich, rigorous problems for reasoning. "I guess my teaching philosophy in general is that I like to really focus on rigorous content because I think people that teach math should know a lot of math, including elementary teachers," said Participant E. Participant B contributed that, "the worth of just engaging in a good mathematics problem is, I think, super great."

Regarding elementary students' reasoning, Participant C said, "As far as processes that are important at the elementary level, I think it's very important for students to be introduced early on to problem solving and to reasoning through their solutions getting some of that justification in early." Participant A suggested, "Take a look at what NCTM is promoting, which is reasoning and sense making. If our youngsters from pre-school all the way on don't have opportunity to reason through and think through problems …they're never going to get anywhere." She later added:

> So, the students do need to be able to communicate mathematically, they need to be able to reason, they need to be able to think, they need to be able to share with one another what they're thinking, but it all has to be based on rigorous mathematics, bottom line.

Thus, a central belief of participants seemed to be that, rich and challenging mathematics problems should be as part of classes preparing effective mathematics teachers.

Active Learning. Learning was centered on exploratory activities, as opposed to lectures or show-and-practice type lessons. "Um, so I mean altogether … a big part, I would say 75-80 percent of my class is engaging in some kind of activity," said Participant B. Activities allowed teacher candidates to explore mathematics: "I definitely believe that there should be time for exploration and less direct lecture during class," said Participant C. Participant F stated, "I sort of feel like my role is to say 'Here's the activity, on your mark, get set, go. Play with it. Let's talk about it, and then come back and say: What did you learn?'" Explore before explain was the approach advocated by Participant C:

> Encourage them to explore before you have to have a meaningful discussion about whatever skill or procedure or concept that you're trying to address for that day, because if students already have that experience and exposure to it then they're able to discuss it and talk about it and understand it a lot better than if you just throw everything at it and then tell them to go explore.

Variations in specific practice. There were many common themes in participants' overall visions of student-centered classrooms, but classroom methods they described still varied, sometimes widely, from instructor to instructor. I was able to ask five of the six participants to estimate time spent on specific methods. Participant A, however, opted for a shortened third interview while recovering from surgery and was not given this question. Variation was evident in the responses given during the final interview by participants when they were asked to estimate the percent of class time devoted various pursuits.

Table 1: Participants Estimates of Time Devoted to Various Pursuits

Part.	Activity	Lecture	Group Discussion	Student Presentations	Comment
B	N/A	10%	N/A	N/A	No estimate given for categories other than lecture
C	50%	15%	15%	15%	Other = 5%
D	20%	40% or more	30%	10%	
E	55% - 60%	25%	N/A	20%	Discussions were part of activities.
F	30%	25%	45%	0%	

Above, lecture included such things as power point presentations containing information for which students would be responsible. A certain amount of question, answer and discussion would probably occur during what was identified as lecture. Still, the amount of variation in reports of estimated lecture time, from 10% to 40%, seemed particularly striking. The participant who said he lectured the most was Participant D. In interviews he had actually given voice to much enthusiasm for active learning; for the importance of “doing.” In other statements, he advocated “balance”, as follows:

> So I think that there has to be some, like, knowledge drop. Even if in my doc classes, if the teacher just has you doing all the work, you're like, at some point (mentally), 'I'm here to learn from *you* as well, you know.' I think there has to be a balance. So, I do lecture…But I do like doing.

Xxx Although Participant D firmly stated, "I'm constructivist," this statement could, on the surface, be considered to have some consistencies with the transmission model. However, lecture-based teaching is not always incompatible with constructivism (Lobato, Clarke, & Ellis, 2005). Looking at his comments as a whole, it is possible that while mainly inclined toward constructivist learning theory, Participant D's approach to teaching is more congruent with "conceptual mathematics teaching" as explained in Ball's (1988) trichotomy of ordinary mathematics teaching, conceptual mathematics teaching, and mathematics pedagogy.

Participant D was one of the participants who stated that he was aware his reported practices might not line up with his beliefs, though he could not state instances where they did not. He, along with all participants, knew that the interview process would give opportunity for reflection on teaching practices and beliefs. In fact, he stated, "I know that I probably lecture more than I think that I do, so I was trying to be as honest as possible; but, no, you really have me second guessing everything with this whole beliefs thing."

To summarize this section on visions for classroom teaching, in general, participants reported they held with a constructivist approach. In common to all, rich problems calling for reasoning were important and exploratory activities were important. However, participants did exhibit a good deal of variation in what they emphasized and in the ways they implemented their visions of teacher preparation.

Finding Four

Participants reported widely varied use of resources such as textbooks, manipulatives, technology, professional organizations, and standards.

Surprisingly, all participants used the same textbook. They did so, however, to varied extents. Manipulatives and technology were also used to different extents, and in varied manners. All participants were members of professional organizations, but such organizations affected their beliefs and practices in different ways. Participants also integrated standards, a possible curricular resource, into their classrooms in individual ways, and to varied extents.

Manipulatives. Participants were not uniform in their use of manipulatives. For Participant C, manipulatives were used in a reported 95% of activities, although this possibly referred to having them available at those times. Participant B was asked to estimate what percent of activities manipulatives were used for, and said that it was about 30%, and here she referred to all students using them during those activities. Participant F, asked which *manipulatives* he had tried with his students, spoke of a *pictoria*l resource: a black line master in which students used pencils to fill dots on pictures of tens frames, rather than having students use actual tens frames. He also said that next time he taught the course, he planned to use the Library of Virtual Manipulatives online. When I probed, asking about actual manipulatives, Participant F said that he had just finished his first semester of teaching in a new program and there were no manipulatives available on his campus. "I need to ask for (them), and I'm kind of kicking myself because I haven't asked for these things yet," was his remark. He continued, "In grad school, we had whole entire rooms devoted to these manipulatives. And so to go from all of that to, 'Oh, I need to be asking for it' - I haven't made that full mindset shift yet." For

Participant F, it appeared drawings of pictures showing manipulatives, or virtual manipulatives on a computer screen, were acceptable in place of concrete manipulative materials.

Participant F spoke repeatedly of the importance of hands-on learning. One example: "… to really emphasize the hands-on aspects…The idea that students are going to build their own knowledge by doing hands-on activities." After the discussion of manipulatives with Participant F, I wanted to find out how he and other participants defined the term "hands-on." I heard a wide variety of definitions for a term that I had thought was broadly understood. I eventually asked four other participants what they believed was meant by "hands-on." Only one of the four said it definitely referred to things students could touch, although three of the four said it would either sometimes or always be about touchable items. Responses to, "What do you think we mean when we say hands-on?" were as follows: Participant E was definite, stating, "To me it means that students are actively involved, they have manipulatives. They are actually moving pieces. They are putting things together. They are taking things apart. They're manipulating sticks or snap cubes to model a problem." Participant D said, "I mean they could be talking about them as a teacher, you know, being involved and sitting down and working with students one on one. It *could* be that they're using manipulatives." Participant B felt hands-on only needed to be "brains-on":

> I mean they're actively engaged in mathematics, it doesn't have to be real-world, it could just be a really interesting problem. It doesn't really have to be hands-on, I mean, your brain could be doing most of the work. I don't know, hands-on is like a weird thing because it makes you think about crafts or like it has to be real-world or something. But I think just a really engaging task, that makes students

think and talk is kind of my idea of hands-on. I guess it's more, not, hands-on but brains-on

For Participant C, the idea of hands-on depended on the context. She explained this as follows:

It depends on the context because sometimes when I refer to hands-on, it refers to actually being in a classroom and teaching, or field experience. But then when we're talking about just in my classroom, hands-on would be where there is actually some sort of activity, I would say almost always with manipulatives, that the students are actually manipulating something to figure it out, so I don't mean they just have some problem that they're figuring out all by themselves or with a partner or with a group, but that there's actually something being manipulated.

Technology. Participants also varied in their use of technology. Participant D was very enthusiastic about technology, devoting multiple two-and-a-half hour class sessions exclusively to the topic of helping students learn how to use and evaluate classroom technology. Participant E actually taught online sections of the math methods course, which led to questions about how she dealt with manipulatives in such circumstances. Her answer was that students bought their own Hands-on Teaching Strategies Kits (H.O.T. kits) for using math manipulatives and used them along with the live or recorded version of the on campus class sessions in their homes. The other four participants made no mention of technology use in their courses, in spite of being asked for, and giving, lengthy descriptions of typical days, materials, and types of activities students were involved in.

Textbook. All participants used the textbook *Elementary and Middle School Mathematics: Teaching Developmentally* by Van de Walle, Karp, & Bay-Williams (2012) known colloquially as "the Van de Walle." For each, it was the officially adopted textbook for their course, and all but one required students to purchase it. Variation was great in its use, however. Participant B reported, “The Van de Walle book is really good, “ but still said, “We didn’t really use the textbook much last year.” She reported that students were asked to read several articles from sources, such as the professional journal *Teaching Children Mathematics* (National Council of Teachers of Mathematics). She planned to stop using the Van de Walle textbook in the next school year, and instead, use a textbook written by Sybilla Beckman that had more mathematics content. She said this might be a better fit with the Mathematical Education of Teachers II (MET2) content recommendations (Conference Board of the Mathematical Sciences, 2012) that her university is attempting to implement to some extent. Participant D had no required textbook purchase, but listed the Van De Wall as an optional purchase. He said, “We used to use the Van de Walle text. Now I think I only use one chapter from it.” He favored journal articles and book excerpts, mentioning Robert Moses and Deborah Ball. Other participants were more enthusiastic about the book. Participant A said, “Because Van de Walle is so very well aligned with the (NCTM) standards that I think all preservice teachers need to know and be able to do, I’m pretty happy with using his material.” Participant C reported, “I feel fortunate that I have a good textbook at the elementary level that I like, I have not found such a textbook at the secondary level…I like the Van de Walle textbook and I use it.” Participants E and F reported using activities from the Van de Walle text in each class. Participant F reported working through 12 chapters of the text, and when asked to describe a typical day in his classroom, said “The VandeWalle text - and so there's a little bit of prose and there's sort of a little bit of a gap and it says activity, whatever it is, we'll do that.” Participant E shared that in a typical day, “They may rotate groups and we do the activities that are in the textbook because that book is so full of good activities to use." When asked how she reached decision on what to teach, she first mentioned standards, then said:

> I can't believe (what) I'm gonna say is what I tell my students not to do , is I do kind of let the textbook guide me. I love that book, and it pretty much lays it out in a good order… I pretty much go straight through it.

Participant E did make it clear that if the text had not happened to contain what she thought preservice teachers should know, she would not have relied heavily upon it.

Standards. Textbooks carry messages about intended curricula (Son & Senk, 2010). "The mathematics that's contained within Van de Walle is all aligned to the NCTM standards," said Participant A. Standards are normally a part of mathematics methods courses because lessons are to be based upon them. Regarding standards, the Common Core State Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) were mentioned the most frequently. Participant A and Participant E, who had taught mathematics methods course the longest, showed the most enthusiasm in regard to National Council of Teachers of Mathematics (NCTM) standards (National Council of Teachers of Mathematics, 2000), although all participants stated that they liked and used the NCTM standards when asked about them. All participants were considering how NCTM standards might continue to be used now that most states are adopting the Common Core State Standards for Mathematics (CCSSM).

Utilizing CCSSM implies an emphasis on the standards of the state where teacher candidates are preparing to teach. "I use the standards as my guide. I know these teachers, when they go into a district to do their practicums, they need to know the mathematics that's being taught in the state, and so I use the state standards," said Participant A.

Participants varied widely in the extent and manner in which they utilized the CCSSM in their classrooms. While most favored standards, not all did. "Standards and one size fits all curricula are not my favored views on education," said Participant B. Though she was teaching about standards, she saw a conflict between standards and her view, "that children should be treated as individuals and their individual thoughts and ideas should be valued." The five other participants were more enthusiastic. Participant D reported that he "sets up the course…following along (with) the standards for Common Core" and explained, "What I do is take each one of those standards and I spend about two weeks on them, and kind of infuse content within that." Further discussion revealed that his reference was to the CCSSM *practice* standards, and that the CCSSM *content* standards were not something students looked at a great deal in his methods course. Participant C explained: "I …look at the Common Core Standards and the things that they're going to be teaching. Part of that comes from working very closely with the teachers in our local school districts." Participant E looked at both practice and content CCSSM. She explained, "the Common Core (content) Standards and the eight standards for mathematical practice are so critical...I keep coming back to those all semester long." In her class, she explained:

> The last few years I've focused on the Common Core Standards and I've taught the (class) content based on those standards. I believe that's what I should be teaching. (This semester) I think I probably got through, maybe, early 5th grade…I firmly believe that the teacher's attitude and how he or she works with children and…how they teach the mathematics…affects how the children learn. So that's why the Common Core Standards and the eight standards for mathematical practice are so critical.

Most of the participants spoke more about practice standards, rather than content standards As Participant C explained, “One thing in particular that I'm thinking of are the standards of mathematical practice that I have focused on more and more as the Common Core has come,” saying this was, “due to what I feel is an emphasis from some of the professional organizations for those practices to be something espoused by the preservice teachers.” Enthusiasm for the practice standards did not always carry over to the content standards. Participant C shared that even her students have some doubts about the content standards:

> There's one project that I have my classes do where they'll look at the development of a concept over five grade levels. And so they'll look at fractions for example from first grade through fifth grade. What they tend to come back with, and I do encourage this somewhat, is an evaluation of those standards. So, not only do they look at what the standards are saying, but they also say, ‘well, there's gaps here and here, and well, they're building up to it but all of a sudden there's a huge jump in third grade’ or something like that. And so it's interesting to see the students come back with some of those nitty gritty details how they don't really line up as well as they should.

The bottom line for Participant C was, “I feel like Common Core is still at a point where there's some issues that need to be fleshed out, but they're probably just new at this point.”

Regarding NCTM standards as compared to CCSSM, Participant A contributed her views:

> I firmly believe that our students do need to know the content that was outlined in Principles and Standards by NCTM. That being said, because so many states now are using the Common Core, I also think they need to understand how the

> Common Core State Standards align with the NCTM standards, and I point out where there are some differences. NCTM does not promote *the* standard algorithm for anything, whereas the Common Core does.

Participant A saw the importance of the NCTM standards. “I…take a look at what NCTM is promoting, which is reasoning and sense making. If our youngsters from preschool on don’t have opportunity to reason through and think through problems and do problem solving, they’re never going to get anywhere. Transitions could be tough, however, as explained by Participant C, who contrasted NCTM standards and CCSSM standards:

> Getting into standards discussions is always interesting because I think that there are a lot of commonalities…I feel like there’s almost a shared vision as far as how math education should be taught, should be learned, as far as it needs to be more student centered and engage the students more in their own problem solving and reasoning and things like that. However, when you get down to the details, there is a huge difference.

Participant B also expressed selected doubts when she discussed CCSSM as a successor to NCTM standards:

> I kind of wish they would have just gone with the NCTM standards, because I was pretty happy with them. In the elementary school there's some disappointing aspects of the Common Core, like they took out all the probability and statistics pretty much, and patterning. But I like the practices, and I think they align OK with the process standards.

Participant D mentioned NCTM only once, saying, “I might start off a class and say, "This is what NCTM says, this is what the Common Core says…” Participant C said, “In the past I've had them write lesson plans that they just turn in to me and so I'll pair up, for example, with the NCTM standards, one content standard and one process standard and tell them they need to write me a lesson plan that incorporates both of those things." It was not clear whether this would continue with the widespread adoption of CCSSM, however. Participant E said , "At the beginning of the semester… we did a lot… with the NCTM Principles and Standards," and also said that in addition to the textbook, "the standards do guide me, the NCTM Principles and Standards and the Common Core Standards." Such was her enthusiasm that she said, "In fact, I require my students to join NCTM as a student member. That's on their book list, to join as a student member."

This section focused on participant use of various resources: manipulatives, textbooks, technology and more broadly defined resources such as standards and professional organizations. Individuals, though they appeared to have a good deal in common regarding their vision for active, constructivist approaches to learning mathematics, nevertheless used and applied available resources in what were sometimes very different ways.

Finding Five

Regarding content, though some material remained within broad guidelines set by universities, participants showed much individual variation and generally exercised personal autonomy in the decisions they reached about both the mathematical and pedagogical content of their classes.

Data related to content mainly came from two questions: "What do you believe preservice teachers should learn?" and, "What do you believe elementary students should learn?" Follow-up questions about particular content were asked only if a participant had brought the topic up. The answers about math content often overlapped with pedagogical content. "That's the way it should be, that there's a lot of overlap (between content knowledge and pedagogical knowledge). Hopefully we're teaching them what they need," was Participant C's view on this. Participant E contributed that:

> In general, I try to look at - What do new teachers tend to struggle with, and what kind of things will be most beneficial? What kind of skills do elementary teachers need?...I think I've said it a lot of times but, focusing on children's thinking, valuing children and learning how to learn from their own teaching. And, the content is like a vehicle, that you can do all these pedagogical moves toward because I think it's hard to separate pedagogy and content.

No participant indicated that math content was absent from his or her methods class, because all perceived that their students needed deeper knowledge of such content. In particular, that their students needed it from a teacher's perspective, which is different from and broader than the perspective of a mathematics student.

Participants B and C taught courses designed as methods/content courses. Dedicated mathematics methods courses for future elementary school teachers were not offered at their universities. However, teacher candidates had to take two or three methods/content courses. Thus, it appeared that PSTs at the schools of Participant B and Participant C received about as much methods instruction as students taking a single dedicated methods course at other schools. The other four participants taught courses identified solely as mathematics teaching methods. However, all participants indicated they would not want to attempt to teach math methods without including any math content, either formally or informally.

Curriculum control. Participants reported that they could determine a large portion of both pedagogical and mathematical content for their classes themselves, subject to some overall guidelines. “I think that, in general I have a lot of control over my classes…I completely rewrote the course descriptions…I can pick the textbooks, the material, and the content that I want to teach,” said Participant C. “I’m given pretty much free reign,” was how Participant B put it. In her case, her university had an overall push to stay within the guidelines of the MET2 (Conference Board of the Mathematical Sciences, 2012), but since the amount of mathematics content recommended by this body is large (12 hours of coursework are recommended to cover it all), Participant B and her fellow instructors chose the content they wished to teach from amongst the recommended content. They also chose the activities and problems they would use. No participant other than Participant B made reference to MET2.

Referring to levels of course content control, Participant F, teaching at a satellite campus, said that the main branch of his university had listed some topics they would like to see covered. However, those happened to correspond to what Participant F wanted to cover. He was free to choose teaching methods, problems, readings and activities related to these topics, and did so. Others referred to content decisions only on the basis of personal autonomy, without guidance from such sources as MET2, a main campus branch, or even reference to guidance from such sources as NCATE, the National Council for Accreditation of Teacher Education.

Pedagogical content. Participants chose to emphasize various items in their course pedagogical content. Two aspects of pedagogy, lesson plans and teaching through standards, have already been discussed above. Pedagogical content brought up by participants, in addition to lesson plans and standards, elicited themes of student thinking, conceptual learning, connections, representations and questions.

Student thinking. “I want them to think about student thinking,” (Participant A) was a recurrent theme in regard to pedagogical content. The mathematical thinking of elementary school pupils was mentioned by all participants. They appeared to see it as an important part of pedagogical content knowledge (Shulman, 1986). Participant B said:

> So, my class time with them should be largely engaged in mathematical thinking and investigating how children think about mathematics…I might present them with a non-routine solution to a problem, so they can think about – 'Oh, there are different ways to do it. How is this child thinking?'

Participant A explained how she helped her students to learn to focus on their students' mathematical thinking:

> We talk about why somebody was choosing one method versus another, what they were thinking while they were doing the problem, how they might get evidence of what students are thinking through the different methods that are represented by the students in the class.

Participant F contributed:

> I'm going to include more video clips so that the idea of thinking like a student or acting like a student - they can see what a student is doing… There's something about watching a second grader figure out a story problem that's pretty fascinating as well, so I still think it all kind of comes together.

Concepts. In pedagogy, it appeared participants focus was concepts, not rotely learned algorithms and procedures. For instance, Participant C explained:

> So, for example, we don't want to just jump into an algorithm, and then say, all right, the first exposure to adding is going to be, you line up the place values and then you add and then you carry the one. The algorithm needs to be the last thing… whether we're talking about fractions, whether we're talking about whole

> numbers, whether we're talking about even measurement and formulas…That last step is the really efficient algorithm or formula. We want to first establish some models and some concrete examples and then work in some student reasoning and, you know, what patterns do you see? What's going on with this? And then finally getting to, maybe an algorithm, something that is a more traditional way of looking at mathematics.

Concepts, which all participants wanted to emphasize, referred to, "the importance of developing an understanding of why algorithms work," in the words of Participant A. Participant D stated, "I want them to understand it (the math), not just do the algorithms." This was not just for teacher candidates, but for elementary school pupils as well. "Well, they definitely need to learn how to teach the mathematics so that elementary students can understand, and can understand to a deeper level, not to the basic algorithms and the shortcuts," said Participant E.

Regarding alternative algorithms, participants believed preservice teachers should be familiar with various alternatives and know how to employ them. As participant F put it:

> What is there about the partial product algorithm that is better to see for someone content-wise? The last few years I've focused on the Common Core Standards and I've taught the content based on those standards. I believe that's what I should be teaching. Seeing multiplication for the first time, then the standard algorithm (referred to in CCSSM)? Or, what's helpful about the lattice method that's not able to be seen in the standard algorithm? Those kinds of things.

Connections. In pedagogy, connections were an important part of learning to think about mathematics, and were mentioned by a majority of participants when asked what they thought their students should learn. "My preservice students then have to explain the models they use, how it connected to the number and algebra if it was a geometry class," said participant A. Participant C explained:

> I've noticed that a lot of students, especially if their math knowledge is weak, it tends to be had in isolated clumps and they don't see mathematics as a broad subject area. They just see this set of rules and procedures and skills that they have for algebra and then they have a completely new set of rules and procedures and skills that they have for geometry. It might even be more isolated than that. That's kind of broad sections. But helping them make connections between those areas helps them, not only fill in the gaps, but then also see mathematics as a whole.

Participant A, weaving together multiple and connected representations, detailed how she explained to PSTs the way to help students in elementary schools write constructed response essays:

> A table, drawing, diagram or a picture. They have to do a graph. It could be a bar graph for youngsters. It could be a line plot, but some kind of a graph. They have to write a written rule, or an equation depending upon what their grade level is, and then they have to justify their thinking." This emphasis on connections is not just between math concepts.

Participant B stated, "I am trying to get them (PSTs) to connect how children learn to how they're learning." Thus, participants looked at many types of connections in the pedagogical content of their classes. They were between types of learning, between areas of math, between math concepts in a single area of math, and between ways to represent mathematical ideas.

Representations. Teaching students to use various representations as part of pedagogy was brought up by most participants. Above, Participant A stressed the connections between various types of representations. Additionally, she said, "I try to very carefully select the problem so that they can't use algebra, so that they have to think about it either using a model, a table, a graph, acting it out." In context, this was for elementary students as well as preservice teachers. Participant D said, "So, you know, measurement, interpreting graphs at an early age."

Expressing his approaches for teaching children, Participant F had realized that pictures were a very important part of his course. "When I've talked with some of my former students….they really caught onto how much I made a big deal about drawing pictures." He went on to say, "Pictures are very helpful, especially the first time a student sees an idea for the first time mathematically. If you can't explain it abstractly then the first fall back better be pictures. The idea all participants seemed to share about pictorial representations was that as teacher candidates used them, they would come to see that they were useful and would incorporate them into their future classroom pedagogy. Participant F even saw pictures as, "a visible component of my practice of teaching elementary methods…as a predominate component."

Questions. Questions were an important part of pedagogy, both in regard to using them and analyzing them. Every participant chose to address questioning strategies in describing desired pedagogical learning for teacher candidates. Participant A spoke of the importance of, "highlighting that when we're looking at that pedagogical content knowledge, we need to know what questions to ask, and when it's appropriate to ask a question." Participant E stated, "I think that's critical that the teacher has the right kinds of questions to ask." Regarding questions in the classroom, Participant B told of her class, where, "They (PSTs) laugh and say I never tell them answers. They say I just like ask hard questions and then walk away because I don't want them to look at me for like, you know, feedback and stuff." Participant C explained questions for elementary students as follows, "They should be given many opportunities to investigate and explore and discover their own conjectures or conjectures of others, their teachers raising questions and then …liberty to explore and discover on their own." Participant D was interested in a form of curriculum knowledge when he emphasized teacher candidates understanding the cognitive demand of questions: "… being able to look at maybe a question and being able to understand the level of cognitive demand that would be involved in this particular problem."

Having brought up questions that teachers ask students, Participant A was queried as to how she helps teacher candidates learn to ask good questions. She explained:

> By modeling. It's the only way that I know how to explain the importance of choosing the right questions. And so what I do is, when we're talking about questioning techniques as part of formative assessment, I often times will ask a question that only requires a one or two word answer, or maybe even one sentence. And I follow up with a differently phrased question that requires the students to really think and reflect on what it is they were doing.

Math content. All participants discussed math content. In the words of Participant A, "I start off every class with my preservice teachers by asking a math question. A math content question that would have some relevance to the concepts that we're going to be discussing." Math content was reported to be essential, but both math content and pedagogical content varied, and sometimes varied greatly, depending on the individual decisions of instructors. Data analysis, geometry, number and operations, and what they referred to as real-world problems were all mentioned by at least some participants in relation to course content.

Data analysis. Participants varied greatly the degree to which data analysis appeared in their courses. Participant D said: "I really think that at a very early age there's going to be an importance on looking at data for students. I know that I focus on that in my methods courses when I teach," said Participant D. Participant B took the opposite tack: "We used to do probability and statistics pretty heavily…and I was proposing that we… lessen the focus on it." Participant C took what was possibly a middle ground approach when she said, "Especially now with the Common Core, there is so much data analysis being packed into middle school. I think that elementary teachers can do a lot to help students understand some of those basic concepts before they reach middle school."

Geometry. Another content area that appeared to vary in importance for participants was geometry. Participant C said, "One other content area that I've become increasingly interested in at the elementary level that I think is important, but a lot of times doesn't get pushed past the most basic level is geometry." Participant A chose an example related to geometry when asked to describe a typical day in her classroom, saying, "I might be talking about the Van Hiele materials… I would have a basket of manipulatives on each of the tables… and pose a geometry problem and observe." Participant E valued geometry, but with new material related to Common Core to be learned, she could not get to it as much as she wanted, stating, "I didn't have enough time to go more in depth in geometry (this term), like I would normally go into, but I spent more time with the Common Core Standards " However, though three participants appeared to value geometry, the other three did not make any reference to including it in their classes. Instead, they talked mainly about problems dealing with number, (including especially place value and fractions) operations, and data analysis.

Real-world problems. A content topic that arose frequently was real-world problems. There was general, though not universal, agreement on their importance. Participant D stated, "I think that students are going to have to really start to focus on using real-world examples…I think being able to apply the real-world to their math is what's going to be important in terms of content." Participant F said, "We need to prepare our teachers to go out in front of a room full of young children and to show that math is alive." He continued, "There is connection between what they are doing during their math time, what they will see out in the, lack of better words, the real-world." Participant E explained, "Because I'm a constructivist, I believe that's the way they should be taught, that's the way - I believe that's how students learn. They need to be taught using hands on manipulatives, projects, real-world applications." Participant B, however, had a divergent point of view on real-world problems. She discussed this, using an example of a problem from Stein and Smith (1998):

> There's very few real-world problems I think that I've seen that are worthwhile. I don't think you have to connect the real-world necessarily to get a lot of math out of a situation. I think it could just be a really good question to being to one of them, like one of the that we were talking about today was from one of the task sorting problems, the Mary Kay Stein, Peg Smith task sort activity. It was like, you know, this rectangle is four-fifths of a candy bar, can you draw the whole candy bar. Or maybe it wasn't even a candy bar; it was a rectangle or something. Can you draw the whole one, are there other fractions that would be similar to the size of that four-fifths bar. I mean, that's a good problem for students to think about. Like because they have to partition the four-fifths part, and then they have to iterate one of those one-fifth size pieces to get the whole again. That's a super-

> rich problem, but it's not, I mean it's not necessarily real-world, or most people wouldn't call it that, but it’s really good thinking problem.

Number and operations. The content topic of number and operations, for both preservice teachers and elementary students, was important to participants. “I think there's some basic things that really need to be set up well in elementary school. For example, number sense, I think is huge… just understanding how numbers relate to each other, the size of numbers,” said Participant C. She continued, “That kind of thing, I think is really, really important for them to develop… Certainly, the operations, and being able to efficiently do the operations, but then also understand what that operation really means.” Participant F contributed that that he emphasized, “A few ideas and learn them really, really, really, really well …I felt like probably a third, 40 percent up to 50 percent of the class was devoted to place value and number.” Place value was an aspect of number sense that came up frequently. Participant C’s lengthy quote on the algorithm for place value has already been discussed, as well as her method of helping PSTs replicate how elementary students learn, in part, by using place value activities. Participant A, who had formerly taught all grades except 2 and 5, detailed her ideas, in a similar vein:

> When I was teaching sixth grade, which would have been my first year teaching, I realized that some of my students didn’t understand place value. I had some great preservice educators at (my college)…, some of whom did not understand base-10. I thought,’ I can’t keep teaching them what it is they know they don’t understand’. So I thought, ‘How can I make this a fun, or a puzzle problem?’ So I started using different bases, and brought in models that were different bases, and we would stand around and we would count. But we could only use the digits from zero to four, and in base four, and making them do all their computations in

> base four. And we did this for quite a while, I would say probably for six weeks, all the mathematics we did, we did in different bases, and it was so satisfying when those youngsters that didn't understand base-10 finally said to me one day, "That's just like what we do with base-10! Now I get it, now I know why we go from our ones to our tens to our hundreds.

The quote above is a particular and detailed illustration of a general principle brought up by participants. The interest is not in lecturing to explain a concept, but letting students construct this understanding for themselves. This perhaps summarizes the theme of this section: In an ideal situation, preservice teachers should learn to construct mathematical knowledge for themselves. They should also gain pedagogical knowledge and use it to help elementary students construct knowledge mathematical for themselves.

Finding Six

Regarding their reasons for teaching decisions, all participants were aware of their own journeys from tacit acceptance of the so-called traditional methods by which they were taught mathematics to their autonomous decisions to implement the reform-oriented teaching methods and content that they deemed best for their students.

What were participants reasons for teaching what they did, and as they did? To find out, I asked questions such as, "How do you decide what to teach?" and "Why do you believe you teach in the way that you do?" In answering, participants reported considering the research reports and recommendations of professional and policy making bodies, then taking an eclectic approach, adding their own beliefs and experiences as a large factor in decisions they reached. They wove sometimes conflicting information from various sources together into course content and a teaching approach they believed was best for the mathematics teacher candidate classroom for which they were responsible.

Again, participants utilized autonomy. Participant F was not told how to teach his class. “I get my chance to really put my own beliefs into practice, or my own interpretations of ‘the what’ into practice a lot more clearly.” Participant decisions took the form of personal constructions arrived at after reflection, in which participants formed ideas based on their own experiences, and the ideas of colleagues, researchers, professional organizations and policy makers. Participant A explained how she was guided by standards because of what she believed students needed:

> I use the standards as my guide… they (PSTSs) need to know the mathematics that’s being taught in the state, and so I use the state standards. So, what I decide to teach is based on what my teachers’ need to know deeply when they get into the classroom, to be able to teach.

Participant B’s initial response when asked how she decided what to teach was, “I try to look at what new teachers tend to struggle with, and what kind of things will be most beneficial. What kind of skills do elementary teachers need?” She also said, “I have to listen to them to see how they can bridge their knowledge to the teaching knowledge that they need.” Participant E had said she used the textbook in teaching, but this was because it had what students needed to learn, “it pretty much lays it out in a good order,” were her words. Students’ perceived needs influenced Participant D: “It really depends, year to year, it's really changed and I've found the most important thing is to figure out what the students are looking for,” he said, echoing the emphasis on perceived student needs, rather than following any one set of prescribed rules coming from a professional group, textbook, or group of policy makers.

Participant A stressed looking at desired outcomes to help students. Teachers should, "understand that they need to take a look at the outcome as they're planning their lesson … and build the outcomes." Her lesson designs are based on, "activities that have been incorporated in them are designed to really help students develop that understanding that they need to be successful." Participant B was asked about how she reached teaching decisions and said, "I think I've said it a lot of times but, focusing on children's thinking, valuing children and learning how to learn from their own teaching." Learning how to learn was important. "I need to teach them ways to like find resources or figure out how to learn things that they're gonna need," said Participant B. Participant F looked at his students' future needs, saying:

> I hope … it's something you (PSTs) can have on the ready for when you have your classroom, whatever that classroom may look like. So that was sort of how I tried to approach the sort of philosophy behind the decisions I made and the organization of a daily session in my class.

Participant C was able to use input from cooperating elementary school teachers about what her students needed to know:

> The teachers that teach in that district, they came to me and said, it really would be helpful if they (PSTs) understood the method and how to teach and how to set up this type of lesson format before they came out in to the schools because this is something that is really, really important. And so that it one thing that I've incorporated into my classes due to that feedback that I got.

Attendance at professional conferences and the recommendations of bodies advocating for standards had some influence, helping participants to meaningfully address students' needs to learn how to help children. Participant F explained:

> I mentioned earlier about the idea of going to a conference symposium sort of thing and sort of making asterisks in those sessions that were about preservice teachers and their math methods courses, and then (asking myself) what are some ways to engage those preservice teachers meaningfully… what are sort of best practices, or what are ways to get to the students more effectively? (This was) so that they (PSTs) are a little more aware of how to know math for teaching math to young students.

Asked about the basis of her decisions, Participant C replied, “I very regularly attend conferences and have discussions with other teacher educators… also the guidelines that are set forth by some… organizations. AMTE (the Association of Mathematics Teacher Educators) is another one that publishes guidelines that I feel are very helpful…certainly that does influence the content.” However, merely following recommendations could be problematic and even contradictory. Participant C said, “When you get down to the details, there is a huge difference” between NCTM standards and CCSSM

Thus, participants considered the relevant research as well as the recommendations of several bodies, but then made autonomous decisions based on perceived student needs, and upon their own experiences of trial and error in attempting to help students. Participant D, speaking of his instructional decision, wanted to see what would work for students:

> I feel like, it's kind of , ‘this seems like a cool idea, let's see how this works’…you know, I read, I follow research and from all of that. All of a sudden maybe an idea pops into my head and I let it kind of bang around in my head for a couple of days and I'm like -Oh, this could be really fun. This might be great. This might absolutely dive, but I've gotta give it a chance, I've gotta see how it works.

Candidates were aware that they had transitioned from the traditional way in which they were taught mathematics, to teaching math, and deciding to teach their students to teach math, in a reform-oriented, active manner. Participant C explained, "One of the reasons why I went into teaching was because that I felt like I had really horrible math teachers and I thought that there's got to be a better way." Participant D explained his journey as follows: "I was taught very, very traditionally and so, and as always, it's not so easy to depart from … the way that you were taught. So, I still think that I have so much that I still need to learn."

To illustrate participants' awareness how their beliefs developed, consider Participant C's story of how she made her own decision to begin teaching in the constructivist fashion. This was a fashion she believed would be best for students, but one very different from the way in which she learned to teach, and different from the way she taught in her first teaching assignment:

> The school district that I started teaching in really, really tried to push the student centered instruction, and having the students develop their knowledge. I have to say that I hated it for the first probably year or two that I taught it. Even when I was finishing (my third and final year teaching in public schools), I still struggled teaching that method because it wasn't something that I felt comfortable with, it wasn't something that I knew how to effectively implement in the classroom. Every time somebody said, "We're gonna do this the constructivist way." I just would shut down. I would say – 'No, that doesn't work, you need to tell students how to do something and then they know how to do it'… It wasn't until graduate school that I really saw good examples of how you could engage students in learning and it wasn't just a complete mess. And, that's when I think I really started believing.

Participant C came to believe that student-centered, constructivist methods would be best for her students, and her instructional decisions came to have a basis in this belief. For Participant C, as for all participants, consideration was given to research and recommendations of professional bodies, but then her own observations combined with personal reflection on what was best for students was the deciding factor in making instructional decisions.

Chapter Summary

The findings of this chapter dealt with consistency, challenge, content, methods, resources, and reasons. Details related to those themes helped to illuminate what the working experience of a preservice elementary mathematics teacher educator is like.

Regarding consistency, participants reported beliefs were consistent with their reported practices, but two exceptions noted were noted. Regarding challenge, six major concerns were reported by a majority of participants: (a) the need for more experience and professional development related to elementary classrooms; (b) concerns about lack time for teaching content; (c) students' reported attitudes; (c) students' reported content knowledge; (e) field experiences, and; (f) teaching students to plan lessons. Regarding vision, participants held to a vision consistent with constructivist learning theory. Vision-related factors that appeared important to a majority were seeking to replicate elementary student experiences in the PST classroom, productive struggle, enjoyment of mathematics, and reasoning and sense making. In spite of commonalities in vision, participants did show noticeable differences in emphasis, application and practice. Regarding resources, participants' discussed their use of manipulatives, technology, the textbook and standards. Differences were found in how participants used these resources. Regarding content, this was discussed in terms of curriculum control, pedagogical content and mathematical content. Finally, regarding reasons for teaching decisions, participants were seen to make decisions in light of what they ultimately felt was best for their students, and these decisions tended to be related to constructivist and reform teaching practices.

One idea that seems to underlie most of the findings of this study was the importance of individual opinions in teacher decision making. Participants had some guidelines from their universities, but to a large extent, chose content and especially methods for their courses based upon their own considered ideas. The participants in this study, to varying extents, had taken time to become informed about research literature and about standards set by professional and government organizations. Each had reflected upon what he or she learned. Then each made his or her own instructional decisions.

Beliefs and practices reported by participants, while similar in many ways, evidenced some noticeable differences. Some lectured a reported 40% of the time, some only 10%. Some used technology a great deal, most, in their interviews never referred to it as a topic considered in their course. Some continued to emphasize data analysis, others (attempting to follow CCSSM) were deemphasizing the topic. Some had teacher candidates study as much as 12 chapters of their common textbook, others barely used the book, with optional purchase and only one chapter to read. Differences such as these came about in spite of being drawn from the same research, policy and professional base. No participant attempted strict conformity to any one set of standards or practices. Centrally, each reported doing what he or she believed was best for his or her students.

Chapter 5

Conclusions and Recommendations

Overview

The purpose of this naturalistic inquiry was to explore and analyze aspects of the reported beliefs and practices of a group of preservice elementary mathematics teacher educators. This chapter focuses on conclusions and recommendations arising from the study. Following this overview, it is organized as follows: (a) discussion; (b) conclusions; (c) recommendations, and; (d) summary and reflection.

In the discussion section, rationale, study design and findings will first be summarized, followed by a synthesis of three themes that were found through multiple findings. Five major conclusions follow in the conclusions section. The recommendations section will present one or more recommendations stemming from each of the five conclusions. This dissertation will end with a brief concluding summary, followed by an overall reflection on the experience of the study.

Discussion

This discussion will consist of two main parts. First, I will review the study thus far, looking at rationale, procedures, and findings. Then, a synthesis of themes found to cut across one or more findings will be discussed.

Study Review

Today we face a problem in mathematics education: How do we help future teachers learn to teach in a way they were not taught? Researchers have often studied future mathematics teachers themselves (Wu, 2011; Kagan, 1992; Kennedy 2011). However, the preparation of mathematics teachers is an under-researched topic (Cochran-Smith & Zeichner, 2005) and there is little research concerning those who prepare future teachers (Sztajn, Ball & McMahon, 2006). This study sought insights into elementary mathematics teachers' preparation through a series of in-depth interviews with six preservice elementary mathematics teacher educators (PEMTEs). The quality of teachers of mathematics is, at least in part, a function of the quality of mathematics teacher educators (Task Force on Teacher Preparation, Certification and Shortage, 2005). This study sought to generate new insights into the experience of teaching courses for preservice elementary mathematics teachers. Such insights hold the potential to inform higher education in regard to how mathematics teachers are prepared.

Participants in the study were six volunteers, located through a demographic survey of 450 mathematics education professionals. From 92 responses to this survey, 23 persons stated that they were teaching classes focused on elementary mathematics teaching methods and that they would each be willing to participate in interviews. Six participants were eventually chosen for the study through a process of purposeful sampling (Merriam, 1998).

Qualitative data were collected through 18 semi-structured, in-depth interviews, using a methodology of naturalistic inquiry. Participants included professors and instructors of elementary mathematics methods courses at colleges and universities in various locations throughout the United States. The data were first collected, then coded and analyzed through a process of typological analysis (Hatch, 2002). Typologies corresponded to themes arising from the research questions of the study, which were:

1. What are the participants' beliefs regarding preparing elementary math teachers?

(a) What do they believe elementary students need to learn?

(b) What do they believe preservice elementary teachers should learn?

(c) How to they believe elementary students should be taught?

(d) How do they believe they should use class time with their students?

(e) What do they believe are the major challenges that they face in their work?

2. How do participants describe their approaches to teaching an elementary math method course?

(a) How do they use class time with their students?

(b) What content do they teach?

(c) How do they decide what to teach?

Six major findings arose from qualitative analysis of interview data. They were as follows:

Finding one. Participants uniformly reported that they believe their practice is consistent with their beliefs.

Finding two. Participants consistently reported certain limits and challenges. Specifically, they reported lack of experience in teaching in elementary school mathematics classrooms, limits in the mathematics content knowledge of many of their incoming students, student field experience issues, time allotted for their course, helping student learn to plan lessons, and attitudes about mathematics on the part of many incoming students.

Finding three. Participants possessed a largely shared overall vision for best teaching practices, both in their own classrooms and in elementary school classrooms. The vision was consistent with constructivist learning theory.

Finding four. Participants reported widely varied manners of employing resources such as textbooks, manipulatives, technology, professional organizations, and standards.

Finding five. Regarding content, although curriculum remained within broad guidelines set by universities, participants showed much individual variation, and generally exercised personal autonomy in the decisions they reached about both the mathematical and pedagogical content of their classes.

Finding six. Regarding their reasons for teaching decisions, all participants were aware of their own journeys from tacit acceptance of the so-called traditional methods by which they were taught mathematics to their autonomous decisions to implement more reform-oriented teaching methods and content that they deemed best for their students.

Synthesis

Three themes were observed to run through multiple findings. They were: (a) unity in reported experiences, beliefs and practices; (b) diversity in reported experiences, beliefs and practices and; (c) participant autonomy in forming beliefs and implementing practices. While unity and diversity are contrasting ideas, both themes ran through multiple findings.

Unity in reported experiences, beliefs and practices. To teach preservice teachers how to teach a subject that they have never taught, in a way that they were never taught it, is the common experience of many preservice elementary mathematics teacher educators. Unpacking that statement and applying it to this study, five out of six PEMTE participants had not taught mathematics in the elementary school. Instead, their experiences were in teaching middle school, high school and/or college level mathematics content courses. Although the teacher candidates in their classes had all been taught mathematics while in elementary school, few if any had been taught elementary school mathematics in a reform-oriented manner. This means that problems related to lack of mathematics teaching and learning experiences, both on their part and on the part of students in their classes, were encountered by all participants.

Another area of unity was common challenges. Certain challenges were brought up by every participant. Without exception, participants reported weak knowledge of math content on the part of many incoming students. Additionally, every participant reported that lack of time for teacher candidates to learn what was needed in order to become effective elementary mathematics teachers was a challenge. Lack of elementary school teaching experience has already been mentioned as a challenge for all participants with a secondary teaching background, which included five out of six participants. Additionally, a majority of participants reported that field experiences were problematic, either with a need for more control of what students were seeing and doing in the field, for more opportunities to observe in the field, or both. Finally, five of the six participants reported that teaching students how to plan lessons was a challenge.

A third unifying factor for participants was their expressed belief in reform-oriented, student centered practices. All participants mentioned their support for activity based learning on multiple occasions. They believed it was important for students to construct their own learning through mathematical activities calling for reasoning and sense making. Five of the six participants stated that they were constructivists, with one participant taking an eclectic approach but still using active approaches that challenged students to construct their own mathematical knowledge. Additionally, all participants firmly stated their support for the NCTM standards, and for the Common Core standards for mathematical practice.

Participants also showed unity in their reported choice of pedagogical content, that is, content that helped PSTs learn teaching practices. Content chosen was not identical, but participant agreement in this area did appear to be substantial. A majority of participants mentioned emphasis on helping teacher candidates learn to think about their own students' thinking, as well as an emphasis on concepts, connections, discussions, multiple forms of mathematical representation, and questioning strategies. Most mentioned replication, whereby students in their classes were to have a chance to learn mathematics in the manner that elementary students should ideally learn it. Participant unity in the pedagogical content of their classes was consistent with reform-oriented and constructivist visions for their classrooms.

Diversity in reported experiences, beliefs and practices. In spite of many similar beliefs, diversity was apparent in several aspects of participant's classroom practices. For instance, although all participants spoke of "active learning", "learning by doing" and students "constructing their own knowledge," the way they put such ideas into practice varied. Some participants estimated that they lectured as much as 40% of the time, others as little as 10%. Some stated they had manipulatives available for as much as 95% of activities. Others had not used concrete manipulatives at all.. Two participants emphasized helping students learn to plan lessons, believing such learning was essential even though teaching about lesson plans was challenging. Others included the topic of lesson plans in their classes very little, or not at all. One reporting that he wanted to stick to "big ideas" (Participant F), and another said, "OK, you guys are sick of this" (Participant D). This was in reference to teacher candidates having had general information about lesson planning in other classes.

Participants also diverged somewhat in the mathematics content of their classes. There was noticeable divergence in the amount of reported attention that data analysis received. Some participants used it as the centerpiece of mathematics content because of their belief that it offered more opportunities to relate mathematics to the real-world. Others choose to deemphasize the topic in favor of topics perceived to receive more emphasis in the new Common Core standards. Additionally, participants diverged in the reported emphasis they gave to geometry. Divergence was also noticeable with regard to technology, with some instructors devoting several class sessions to technology in the mathematics classroom, and some making no mention of technology as part of class content.

Participant autonomy in forming beliefs and implementing practices. A thread of autonomous decision making ran through findings three, four and five on teaching vision, content and methods. Several of the findings pointed to the fact that participants appeared to act on their own beliefs, not merely on the recommendations of others. Participants used language consistent with having made their own decisions regarding both content and methods. Participant A referred to, “what I decide to teach.” Participant C spoke of “The standards of mathematical practice that I have focused on more and more,” indicating that she had chosen this focus. Participant D could decide to change his decisions from term to term. “ *I* (emphasis mine) would change the course based on who's there,” he said, referring to the variety of students he taught. Participant B spoke in an enthusiastic tone of implementing her own ideas, saying, “ I have so many ideas for the fall, …I am just like - how am I gonna do things differently, and I have a million ideas, but I can’t enact them all at once.”

In short, although university and NCATE guidelines set boundaries for certain topics participants were required to cover, and they did broadly cover them, participants viewed such elements as teaching methods, activity choice, activity design, reading assignments and specific mathematical content as their decisions. They considered input from colleagues, professional organizations, researchers and policy makers, but in the end, decisions about how the class was conducted were their own.

Conclusions

Five major conclusions were drawn from the findings of this study. They related to (1) consistency and inconsistency; (2) challenges; (3) variation; (4) reasons for teaching decisions, and; (5) embrace of reform methods. Finding six yielded two separate conclusions. A visual representation, showing which conclusion came from which finding or findings, is found in Appendix A, figure 3, and sources of conclusions are given individually in the text, as they are discussed.

Consistency and Inconsistency

Participants uniformly reported that, to the best of their knowledge, their practice was consistent with their beliefs (finding one). While their reported practice was indeed almost always consistent with their reported beliefs, there were some inconsistencies even at this level. Therefore, participants often, but not always, acted on their stated beliefs (conclusion one).

The fact that participants' reported behavior usually appeared to be consistent with their beliefs tends to confirm other research (Haney, Czerniak & Lumpe, 1996; Pajares, 1992; Cronin, 1991) that has determined teacher beliefs are significant indicators, and even determinants of the teacher behaviors that will be present in the classroom. However, participants occasionally reported practices appearing inconsistent with their beliefs. This confirms other literature (Galton & Simon, 1980; Fang, 1992; Mansour, 2009) that indicates teachers do not always do what they say they are doing, or what they say that they believe they should be doing.

The few inconsistencies between participant beliefs and reported practices in this study may be tied to contextual constraints. In the past, researchers have attempted to explain mismatches between teachers' beliefs and practices through external and internal constraints pressuring teachers in the context of their work (Abell, 1990; Abell & Roth, 1992; Gahin, 2001).

Mansour (2009) stated that the role of contextual constraints in teaching has almost disappeared from accounts of educational research. This study will make a small contribution towards addressing that near-disappearance.

Two participants reported practices that may have been inconsistent with their stated constructivist beliefs. Participant F, in his first year of teaching math methods, said repeatedly that he believed “hands-on learning” was important. However, probing revealed that teacher candidates in his class did not use manipulatives at all. The reason Participant F gave for this was that his university had not provided any to his campus, and additionally, he intimated that in his opinion, virtual manipulatives could be sufficient for learning mathematics. . Although virtual manipulatives might simulate some selected manipulatives, they are much more abstract, since they do not allow hands-on activities (Durmus & Karakirik, 2006). Furthermore, Hunt, Nipper, & Nash,(2011), in a study of 78 aspiring middle grades mathematics teachers, found that 76% found concrete manipulatives easier to use than virtual manipulatives, and 82% found concrete manipulatives more helpful for understanding than virtual manipulatives. Participant F had also hoped that such things as drawing on blackline masters showing tens frames, on which students could draw physical counters, could suffice. He said that the time constraints involved in teaching the course for the first time were the reason he had not gotten around to ordering manipulatives. “I'm kind of kicking myself because I haven't asked for these things (manipulatives) yet,” he said, and indicated, “In grad school, we had whole entire rooms devoted to these manipulatives.” This indicates that Participant F may intend to act on his beliefs about active learning.

These data may help confirm work by Borg (1990), which indicated that a lack of resources was one external constraint affecting teachers’ actions, and work by Blasé (1986) that time constraints were another. Furthermore, lack of detailed knowledge of research indications, in this case about virtual manipulatives vs. concrete manipulatives, may have been a third constraint. In an informal conversation in fall of 2013 (Personal Communication, Participant F) following the study, Participant F said that concrete manipulatives were now commonly used in his class, and indeed, that he was even serving as a source of resource information for other teachers on this topic. Participant F had had time for study and to actually order the materials he needed. This further confirms my impression that Participant F would act on his beliefs when constraints were removed.

Participant D, who reported lecture time was "40 percent, maybe a bit more…and I know that I probably lecture more than I think I do," appeared to be constrained by his belief about accomplishing a learning goal he saw as important. He said, "I want to learn from experts, and I think that students do too, so I think that there has to be a certain amount of lecture." Participant D's reason for lecturing seemed related to teaching the class the way he would like for it to be taught if he were in it. He also appeared to believe lecture was a desirable way, and possibly the most efficient way to learn from experts. Efficiency relates back to Blasé's (1986) concern with time constraints.

Challenges

A second conclusion is related to challenges. Researchers have pointed out the need for more information about challenges faced by teacher educators. "What is challenging about their (teacher educators') work …?" (p. 150) was one of four main questions for additional research suggested by Sztajn, Ball and McMahon (2006) in their AMTE monograph on the work of mathematics teacher educators. This study will contribute to answering that question. Interview data revealed that the same challenges tended to arise again and again for participants (finding two). Therefore, if the findings of this study match the experiences of other PEMTEs, some consistent challenges, which should be addressed, are being faced by this group (conclusion two).

Need for experience and professional development. A major challenge for participating teacher educators was lack of opportunities for professional development and lack of experience teaching mathematics to elementary school students. Some participants lacked even observational experiences in elementary schools. Participant F, a former secondary teacher, typified this. Asked, "Do you have any chance to go into the elementary schools and observe?" he replied in an emphatic tone, "No, and if we had to come up with some challenges, that is one of them!" All but one of the participants had been secondary school teachers, but all were now working to prepare future elementary school teachers. Thus, they perceived that they lacked some aspects of knowledge on how to best prepare preservice elementary mathematics teachers. An indication that this problem may be widespread comes from the demographic survey conducted in the initial phases of this study, In it, 28 out of 37 PEMTE respondents (76%) indicated a background including middle school or high school mathematics teaching experience. An N of 37 is small, but this is evidence that the six participants in this study may have been typical of the broader PEMTE population in their overall lack of elementary school teaching experience. Participants also stated, based on their observations, that their lack of elementary school teaching experience was typical for PEMTEs.

Institutions have tended to assume that teacher educators, such as PEMTEs, already possess the knowledge that they need to teach their higher education classes (Murray & Male, 2005). Pedagogy is seen to be an area where the teacher is expert, notwithstanding the different demands of teaching about teaching (Loughran, 2006). This has created a situation characterized as “novices perceived as experts” (Murray & Male, 2005, p. 138). The education community in the United States does not have a specified way in which to prepare teacher educators for the work they plan to do (Murray & Male, 2005). “I didn’t have a methods course for teaching methods” was how Participant B put it. In fact, although there has been much thought about how teacher education itself should change, little attention has focused on how to prepare mathematics teacher educators who will need to provide this education (Van Zoest, Moore & Stockero, 2006). In a broad sense, participants lacked training as what Sztajn, Ball and McMahon (2006) refer to as “teacher developers,” an inclusive term for such professionals as college instructors, professional development session leaders and school based developers.

Compounding the problem, graduate students in mathematics education who go on to take positions as college and university-based teacher educators will frequently find themselves in institutions where there is little support for their continuing development as teacher educators (Zeichner, 2005). Sztajn, Ball and McMahon (2006) argue that “the improvement of mathematics teacher education depends on building the capacity of those responsible for teaching teachers” (p. 150). Most new faculty benefited by developing their knowledge, understanding and skills as teachers in higher education if they were offered induction support (Murray & Male, 2005). However, mentor support programs on the part of senior faculty for more junior faculty were mentioned infrequently in the mathematics education literature.

The need for support may be especially acute for many PEMTEs because of their lack of elementary school teaching experience. They lack experience in working with children younger than 12, and they lack knowledge of the routines in elementary classrooms. To give one illustration, in most cases, stories can be told by teacher educators about actions in previous classrooms. For instance, in one study of teacher educators, Murray and Male (2005) found that

26 out of 28 participants reported emphasizing their past school teaching experience in talking with teacher candidates during and outside of class. PEMTEs who have never taught elementary school cannot speak of such past experience.

Content Knowledge. Another challenge noted by all participants was the lack of mathematics content knowledge they perceived on the part of most teacher candidates entering their classes (finding two). This concern echoes those of broader society. The content knowledge of preservice and inservice elementary mathematics teachers in particular has long been acknowledged a matter of concern (Ball 1990; Ma, 1999). It is widely viewed as important, for "With each passing year, our economy and our quality of life depend upon higher levels of proficiency in the STEM disciplines (Science, Technology, Engineering and Mathematics) across the workforce" (Massachusetts Department of Education, 2007, p. 2). What is more, elementary teachers need not only "regular" knowledge of mathematics, but also pedagogical content knowledge about how best to help students learn this content (Shulman, 1986) and specialized content knowledge about math itself for teaching (Conference Board of the Mathematical Sciences, 2012). Researchers have emphasized the importance of solid mathematical content knowledge/fluency and specialized mathematical knowledge for teaching. Both have been found to significantly predict student achievement gains (Ball, Hill & Bass, 2005).

Time constraints. Participants in this study autonomously decided their class content (finding five), while perceiving lack of teaching time as a challenge (finding two). Short of time in which to teach content, participants were picking and choosing from amongst the math content knowledge, mathematical knowledge for teaching and pedagogical content knowledge that they believed their students most needed to study. Participants considered that there was not enough time to cover essential material. Previous research suggests that because teachers work under perceived constraints, the link between beliefs and practice is weakened (Thompson, 1984). Raymond (1997) found that time constraints were the most frequently identifiable reason for inconsistency between a beginning elementary school teacher's beliefs and teaching practice in mathematics. While previous studies have looked at K-12 mathematics teachers, this study points toward concerns about time as a constraint for mathematics teacher educators.

Variation

Conclusion three concerns variation. Findings three, four and five, were at least partially related to variation. Participants possessed a largely shared overall vision for best teaching practices, but differed in the teaching methods through which they implemented this vision (finding three). Additionally, participants reported widely varied manners of employing resources (finding four). Finally, there was noticeable variation in content that PEMTE participants choose to teach (finding five). Therefore, participants in this study showed a large amount of variation in factors related to content, practice and resources (conclusion three).

Variation would appear to be the norm. This study's findings on variation are consistent with the a three-year National Research Council (2010) study of teacher preparation programs. This study found "enormous variation in virtually all aspects of teacher preparation programs and pathways" (p. 175). The variation in the elementary mathematics teacher preparation programs for which PEMTEs in this study were teaching would appear to confirm this finding. Additionally recent recommendations from CAEP, the Council for the Accreditation of Teacher Preparation (2013) acknowledge variation in programs.

This study contributes to knowledge about variation in teacher preparation programs by giving insight into reasons for such variation. Generally, reasons given by participants fell into two categories. First, courses differences: courses were aimed at different grade levels (some for K-4 and some for K-8, for example) and sometimes included students seeking different certifications (elementary, middle and special education). A second reason was for variation was participants' differing perceptions of what their students needed.

Basis for Teaching Decisions

Conclusion four is related to the basis for teaching decisions. "Hopefully we're teaching them what they need," said Participant C about her teaching. Asked how she decided what to teach, Participant A initially responded, "I use standards as my guide." However, she then clarified that this was because, "they (PSTs) need to know the mathematics that's being taught in the state" and then continued, "So, what I decide to teach is based on *what my teachers' need* (emphasis mine) to know deeply when they get into the classroom." In answer to, "Why do you teach the way that you do?" Participant C said, "I think that it's important for students (PSTs) to understand in a very meaningful way for them… the nature of mathematics." She believed students needed to understand this. Participant D spoke of how he decided what to teach in his course: "Being able (myself) to create the courses so that everyone gets what they need out of it," were his words. Overall, participants sought "to implement reform-oriented teaching methods and content because they deemed them best for their students" (finding six*),* Therefore, participants underlying basis for their teaching decisions appeared to be mainly their perceptions of the needs of their students (conclusion four).

Participants did not always agreed about what students needed to know. However, perceived student needs were paramount in participant decisions about what to teach and how to teach. When asked why they taught as they did, Participant A first mentioned state standards, Participant C first mentioned the nature of mathematics and Participant D first mentioned live, real-world connections. The commonality, however, was that participants believed these things were important and that students needed to know them.

Schwille, et al (1982) reported that a view of elementary school mathematics teachers as mere implementers of policies imposed from above is misleading. They contended such teachers were best viewed as “policy brokers” (p. 387) rather than mere implementers, working to adapt policies to the perceived needs of their students. Content actually taught to students was likely to be a compromise between the officially adopted content and the needs of the students as the teachers saw them. PEMTEs in this study, with fewer official content policies as constraints, appeared to be even more likely than elementary school mathematics teachers to decide what to teach based on perceived student needs.

Embrace of Reform Methods

All participants were aware of their own journeys from tacit acceptance of the so-called traditional methods by which they were taught mathematics to their autonomous decisions to implement reform-oriented teaching methods and content (finding six). Therefore, it appeared that participants had embraced curricular reform and constructivist oriented pedagogy (conclusion five).

Participants did not commonly use transmissionist type language. Thus, this study does not confirm the work of Murray & Male (2005), who said that 28 U.K. teacher educators in their first year of university level teaching commonly utilized phrases that emphasized transmission orientated teaching methods such as, “‘trying to sell my knowledge,’ or, ‘trying to graft on all my years of experience on to them’ or ‘saying this what worked for me, I’m the expert’” (p. 7.) In fact, participants in this study tended to use language such as “perturb their schema” (Participant B), “have my conjecture board going” (Participant A), “a discussion just to kind of get everyone up and talking" (Participant D), "students being able to develop their own knowledge" (Participant C), and "candidates need to actually do the activities" (Participant E.) These quotes illustrate and support the conclusion PEMTEs had embraced reform-based teaching. Thus, this study give support to the notion that the extent of the influence of curricular and pedagogical reform on elementary mathematics teacher educators has been widespread.

Recommendations

This study can help to lay groundwork, provide information for future research studies and inform higher educational practice through recommendations for policy, practice and further research. A visual summary of recommendations is provided in Appendix A, figure 4, and they are presented here in text form.

Recommendations Stemming from Conclusion One

Conclusion one, on consistency and inconsistency, indicated that participants often, but not always, acted on their beliefs. Participants gave some indication of constraints to acting on beliefs, such as lack of time, lack of resources, and lack of knowledge of research. This study has contributed to knowledge of why teachers do not always act on their stated beliefs, but more information in needed. Thus, I recommend further research to learn more about how PEMTEs' beliefs are related to their practices, and about why PEMTEs may sometimes find themselves unable or unwilling to put some of their beliefs into practice.

Recommendations Stemming from Conclusion Two

Conclusion two stated that if the findings of this study match the experiences of other PEMTEs, some consistent challenges are being faced by this group. Several recommendations can be made based on this conclusion. In each, those involved should consider ways to address the challenges commonly reported by these PEMTEs.

Recommendations on experience and professional development. Participants spoke of reading journals and attending conferences, stating that these things were certainly helpful. The profession has done much to address PEMTEs' needs. However, because so many PEMTEs lack experience in elementary classrooms, continued, special, focused, and sustained efforts should be made to help with the professional development of such educators.

Yet many PEMTEs lack elementary teaching experience relevant to their positions, having taught mathematics only in secondary schools. I found no evidence through a review of literature or through participant interviews that this particular situation has been given much attention. Thus, I strongly recommend recognizing and addressing this situation. I contend that the need for additional experience and training is particularly strong for PEMTEs. School districts, schools of education and policy makers should work together to create opportunities for more observation time, professional development and experience for PEMTEs and those who will become PEMTEs.

All participants felt that because of their lack of experience in elementary schools, they needed additional experience. Thus, I recommend that administrators from schools of education and administrators of elementary schools work together to provide more extensive opportunities for PEMTEs to observe in elementary school classrooms, and where possible, to guest teach.

Additionally, universities can provide more learning experiences for graduate students who may one day be employed as PEMTEs. Not all participants had the opportunity to teach classes for future teachers in graduate school. Of those who did, some were mentored by faculty members, but most were not. They could assign mentors from among senior faculty members to those teaching the course as graduate students. They could allow more graduate students to co-teach such courses under the supervision of a faculty mentor. Additionally, there is need for universities to offer formal coursework and seminars aimed at teacher educators both for doctoral students and for those already working as teacher educators. I found only one example of a course for PEMTEs discussed in research literature. Sztajn, Ball and McMahon (2006) described a Center for Proficiency in Teaching Mathematics offering entitled, "Developing Teachers' Mathematical Knowledge for Teaching." This course attracted 120 applications for 70 places. During course sessions, participants viewed video episodes from concurrently occurring laboratory mathematics methods class. This indicates need for and interest in such a course. Thus, I recommend that such courses be developed and offered by higher education facilities.

Research in exploring the socialization and induction needs of teacher developers is needed. Few studies examine teacher educators' professional experiences and induction needs as they enter Higher Education. This omission means that new teacher educators are an occupational group that is both under-researched and poorly understood (Murray, 2003). At the least, programs can be set up for senior faculty to mentor more junior faculty, if such programs have not been put into place.

Technology can provide help in addressing professional development needs. For example, in fall of 2013, many teacher educators are among those enrolled in a 20,000 participant MOOC from Stanford University entitled, "How to Learn Math." The Teacher Education Materials Project (TE-MAT), found at ww.te-mat.org, has materials for mathematics teacher professional developers that may apply to mathematics methods classes as well. The Association of Mathematics Teacher Educators website lists some technological material, as well as two research journals available online, their own, *Mathematics Teacher Educator Journal* and

Contemporary Issues in Technology and Teacher Education. There is also the *Journal of Mathematics Teacher Education*. Online forums can also be of help, for instance *"The Mathematics Teaching Community"* is a general online forum for discussing mathematics teaching at all levels. Searching for discussion topics related to teaching elementary school mathematics on this forum can result in discussions with current and past elementary school teachers and teacher educators. Finally, K-12 educators in some large rural states, such as Kansas, are able to take advantage of programs whereby they can contact a qualified mentor online when none are available locally (Sawchuck, 2013). PEMTEs, in a similar fashion, may not find a local mentor for teaching elementary math methods courses. Thus, I recommend exploring how technology could address lack of PEMTE elementary classroom experience.

Recommendations on Time and Content. Participants were challenged by both time constraints and lack of mathematical content knowledge on the part of incoming students. Inconsistent practices and divergent course content offerings by study participants may have been, at least in part, functions of the time constraints encountered as participants attempted to address the content needs of their students. If participants have sufficient time in which to teach their courses, they may not have to pick and choose from what they believe is essential for their students to know. They may have time to address perceived content knowledge problems. Thus, I recommend additional courses, whereby preservice elementary mathematics teachers spend more time in college classrooms and where they may become better acquainted with math content and math teaching methods.

This recommendation is consistent with educational policy recommendations now being broadly discussed. Research indicates that K-12 mathematics students learn more when their teachers have a strong foundation of content knowledge for teaching (Hill, Rowan & Ball, 2005).

This is different from simply adding "content knowledge" or recommending more math courses. Courses must be oriented to mathematical knowledge for teaching. Findings of this study imply support for recommendations calling for more coursework by preservice elementary mathematics teachers by the Conference Board of Mathematical Sciences (CBMS) in the *Mathematical Education of Teachers II (MET2)* report (CBMS, 2012). The recommendation of CBMS is that preservice elementary mathematics teachers take 12 hours (four separate courses) of offerings combining math methods and math content for teaching. This is so that elementary school teacher candidates may gain the necessary knowledge for teaching mathematics effectively in the elementary school. However it should be noted that everyone wants more time for everything they do and every teacher wants better prepared students.

Findings also lend some support to recommendations that such additional courses may be taught through either a university mathematics department, a school of education, or both. This is because (a) all participants had majored in mathematics education but reported that they felt respected and valued by colleagues in math departments who had majored in mathematics itself, and; (b) a majority of participants reported that it was difficult to separate mathematics content from mathematics pedagogy.

Lesson Plans. PEMTEs participating in this study had widely divergent opinions about whether they should address the topic of lesson plans with their students. All participants considered lesson planning to be a challenging topic, but disagreed on the value of efforts to address this challenge. Yet, research gives some indications that certain forms of lesson planning based on so-called backward design, have beneficial results (Childre, Sands, & Pope, 2009). Understanding by Design is a form of backward design in lesson planning put forth by the Association for Supervision and Curriculum Development (ASCD). Participant A related that its use had been very positive for her program. The students of graduates from the particular program for which this participant teaches for have been found, using state data, to have improved their mathematics achievement results.

I would recommend that further research be done on effective lesson plans, since current research is tentative. Thus, mathematics educators may need more information, possibly through conference presentations and journal articles, about the topic of lesson plans for future teachers.

Field Experiences. Several study participants considered field experiences by students in their classes to be a challenge. They were unable to control the types of lessons the PSTs were seeing, and the types of lessons the PSTs were asked to teach. Despite the sensitivity of the topic, I recommend that university offices of field experience make more concentrated efforts to work with methods teachers in regard to students who are enrolled in their classes. Teacher educators appear to have little input into, and no control over, such field experiences. This was in spite of the fact that students were to come to class, discuss fieldwork and in some cases submit written assignments stemming from fieldwork that PEMTEs were to grade. Thus, regarding field experiences by students in their classes, I recommend that input into fieldwork done in conjunction with elementary math methods courses, ability to establish requirements and ability to determine settings should be increased for PEMTEs.

Recommendations Stemming from Conclusion Three

Conclusion three stated that participants in this study showed a large amount of variation in factors related to content, practice and resources. This gives some indication that researchers should not be surprised by variation in what PEMTEs teach, or by their methods of teaching. . Field (2012) found that amongst six new teacher educators she interviewed, most of the teaching methods they described rested on modeling good practice and re-telling anecdotes. The fact that the PEMTEs in this study had more in their teaching arsenal than these two items may be seen as a positive, even though their stock of specific teaching methods resulted in variation.

The extent to which we should attempt to standardize teacher preparation programs requires examination of many factors (National Research Council, 2010). An accepted pedagogy of teacher education, of learning to teach others how to teach, does not yet exist (Loughran, 2006; Field, 2012). At present, elementary mathematics methods courses address many different grade bands, for instance, K-3, K-4, 1-6, and K-8. This variation exists because of different university program requirements and because of different state licensure requirements. Additionally, elementary methods courses prepare candidates for various teaching licenses: future elementary school, middle school and special education teachers may or may not be in PEMTE's classrooms. It would be difficult to set a standard curriculum when such varied grade bands and licenses are involved. Thus, while it is useful to know that programs vary, I do not recommend attempt to end all variation in mathematics methods as they are currently offered.

The recommendations of the Conference Board of the Mathematical Sciences (2012) uses the CCSS as a framework for outlining, in detail, the mathematical ideas that elementary teachers, both prospective and practicing, should study and know. These recommendations should be considered seriously for adoption and might result in less variation in the mathematical content of courses for preservice elementary mathematics teachers.

Recommendations Stemming from Conclusion Four

Conclusion four stated that participants' underlying basis for their teaching decisions appeared to be the needs of their students. However, while there was agreement, participants also evidenced differences on both how to address PSTs' needs and on exactly what those needs were. All classes and students have individual characteristics, but at the same time, preservice teachers do have some proven needs. For instance, all need to know about effective teaching practices, a general item about which participants agreed. Still, participants addressed this need to different extents and at different levels. For instance, some utilized a conjecture board, a proven practice, while others did not appear to know about it. What is more, the mathematics education community has not agreed on all the needs of PSTs, so further discussion and research in this area is welcome. In short, it seemed good for participants to make autonomous teaching decisions based on the perceived needs of their students. However, they could have received more through input about what those needs are, and how to address them. Thus, I recommend more research and continued, readily available information to help PEMTEs gain an informed perspective on the needs of their students.

Recommendations Stemming from Conclusion Five

Conclusion five stated that participants had apparently made their own decisions to embrace curricular reform and constructivist oriented pedagogy. In embracing reform, they were choosing to follow the recommendations of the National Council of Teachers of Mathematics (1989, 2000) to teach according to principles and standards emphasizing children's active involvement in learning, understanding of concepts, reasoning and sense-making.

Calderhead (1996), found that teacher candidates typically start with control-oriented belief systems that emphasize such things as good discipline, the importance of maintaining order, and guiding children's activities. During teacher training, Calderhead found, these attitudes become more child-centered, that is, more constructivist. However, when teacher candidates enter full-time teaching, they revert to a teacher control-oriented belief system. I believe the present study lends support to Calderhead's work. Participant E, who held National

Teacher Certification, had 28 years of classroom experience, and had taught elementary mathematics methods for 3 years, said this when speaking about teacher candidates:

> They still want to teach how they were taught. Even though they know that it wasn't right for them and they didn't learn that way. They still revert back to that way, and so that's one of the greatest challenges…They don't have a deep understanding of the content, so then they revert back to the way they were taught.

Participant E was acting in keeping with reform practice in her classroom, but she needed more time, more knowledge, or both in order to more fully prepare teacher candidates to effectively teach mathematics. Thus, in keeping with the findings of this study, future efforts need not be on attempting to convince PEMTEs to embrace reform. Instead, research should look at promising approaches to produce effective mathematics teachers who permanently embrace reform, rather than return to how they were taught in earlier years. For instance, allowing more time in the preservice elementary mathematics classroom through having teacher candidates take more courses may help to address students reverting to traditional methods when they become classroom teachers. Additionally, fieldwork placements for students teachers with mentor teachers known to embrace reform practices should be utilized.

Future researchers should look at how PEMTEs differ from their students and from inservice teachers in adapting to reform recommendations. For instance, in this study, participants did not dislike and avoid change. Future research can look at whether or not this is generally true. Also, much work has been done on knowledge for teaching (Thames & Ball, 2010; Ma 1999). Future work also needs to look at knowledge for teaching teachers. MET2 (CBMS, 2010) has made a beginning in their recommendations for what elements courses for preservice elementary mathematics teachers should contain. Some studies have been conducted on becoming effective teachers of teachers (Ball & Forzani, 2010, Hiebert, Morris, Berk & Jansen, 2007). However, there is much work to do in this area.

Brief Summary and Reflection

In brief, study conclusions found (a) general consistency, with some exceptions, between reported beliefs and reported practices; (b) some common challenges faced by most participants, particularly in regard to limited elementary classroom experience, and perceived time constraints; (c) wide variation in content and methods despite agreement on teaching philosophy; (d) perceived student needs as the main reported basis of participant teaching decisions, and; (e) some indication of the extent of influence of curricular and pedagogical reform as elementary mathematics teacher educators sought to enact generally constructivist inspired teaching visions. Recommendations for educators, researchers and policy makers stemming from these conclusions have potential benefits for elementary mathematics teacher preparation.

In reflecting upon this endeavor, my intent in conducting this study was to contribute to a better understanding of the beliefs and practices of those who prepare elementary mathematics teachers. Stake (1995) says that qualitative research is highly personal, and that researchers are encouraged to include their own personal perspectives in the interpretation. Thus, I will reflect on how I personally value my work and the research experience. I believe my learning is not final. I will continue to think about and progress in my understanding of the issues I examined in this study.

In doing this naturalistic inquiry I set out to understand context, nuance, details and reasons that a different kind of study might not have elicited. Here, it was possible to discover, understand and describe the in-depth perspective of the participants. The themes that arose dealt with consistency, challenges, content, methods, resources, and reasons. In looking at those six themes, there is a sense in which they underlie all of teaching, and perhaps all of life. In this case, however, I hope that details related to those themes have helped to illuminate what the working experience of a preservice elementary teacher educator is like.

This was a cooperative endeavor. It was greatly enhanced by the research participants who so generously gave of their time and insight to share their experiences with me. Their feedback in this constructive endeavor helped to shape and guide results. I felt that, in keeping with the constructivist paradigm, we constructed the findings that arose from this study together, rather than the findings being “mine”. I also formed professional relationships with participants which have continued today, months after the conclusion of the study.

This study was greatly enhanced by feedback from committee members who so patiently read drafts and dialogued about research issues. They continued to challenge me about not inserting my own biases into the research. Without their encouragement and support, the study could not have been completed. The full page of acknowledgements at the beginning of this dissertation stands in testament to this.

Future research on my part stemming from this project would seem to have two possible pathways. I can choose selected topics for more in-depth exploration about mathematics teacher educators’ beliefs and practices in regard to them. For instance, I already plan to do interviews with a larger group of teacher educators asking specifically about English language learners. How do we develop equity in the mathematics education classroom? In order to answer this question, two things we need to know more about are the beliefs and practices of teacher educators when (and if) they present topics such as sheltered Mathematics instruction for English language learners to teachers and teacher candidates in their classrooms.

A second area for further research would be to try to answer one of the central questions raised in this study. Some work has been done on this question, but more is needed. It is this: If mathematics teacher educators are indeed presenting teacher candidates with a vision for reform-oriented practices, using such practices in their classrooms, and providing techniques for implementing such practices, why do some, in fact many, classroom teachers revert to the traditional way in which they were taught mathematics? Or, as Participant E put it, “They still want to teach how they were taught. Even though they know that it wasn't right for them and they didn't learn that way.” Following students in-depth after they leave mathematics education courses and enter the classroom would be needed to examine this question.

Participating in this study challenged some of my own beliefs. When I first wrote up findings for this study, I believed we should study the variation that I found was present in mathematics methods classrooms for its effects, but did not believe we should address it in any other way at this time. After all, I reasoned, my participants were responding autonomously to the perceived needs of their students. I still do not support attempting to create identical cookie cutter methods courses. However, after consideration, talking with others and attending talks by mathematics educators, I realize that the sound recommendations we currently have for what mathematics teachers need to know can contribute to more in-depth guidance on what all teacher candidates should obtain from an elementary mathematics methods course.

Another area I changed my mind about was the separation of math methods and math content courses for future teachers. Talking with my participants revealed that these two items overlap to a considerable degree and can be almost inextricable from one other. Some of my participants had been assigned to teach methods courses that were intended to combine mathematics methods and content and others had been assigned to teach mathematics methods courses knowing that students took separate courses on content. I gained insight into both approaches. While courses can work separately, I now see how courses that combine mathematics content and methods can also work well.

I was asked at my dissertation defense by a committee member which recommendation, out of many, I considered to be most essential. I replied that I would like for elementary mathematics teacher educators to have continued opportunities for professional development through receiving more opportunities to spend time in elementary school classrooms. Teacher educators need such time in classrooms if they are to adequately prepare others to do the vital work of helping children to understand mathematics as a thing which makes sense, a thing which is beautiful, an activity which is enjoyable and an endeavor at which they are all capable of succeeding.

With the help of my committee members, I have grown as a researcher. I am grateful to them and to many people who have helped along the way. It is my hope that this dissertation may help to provide information that will help in the preparation of future elementary mathematics teachers.

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Appendices

Appendix A

Figures

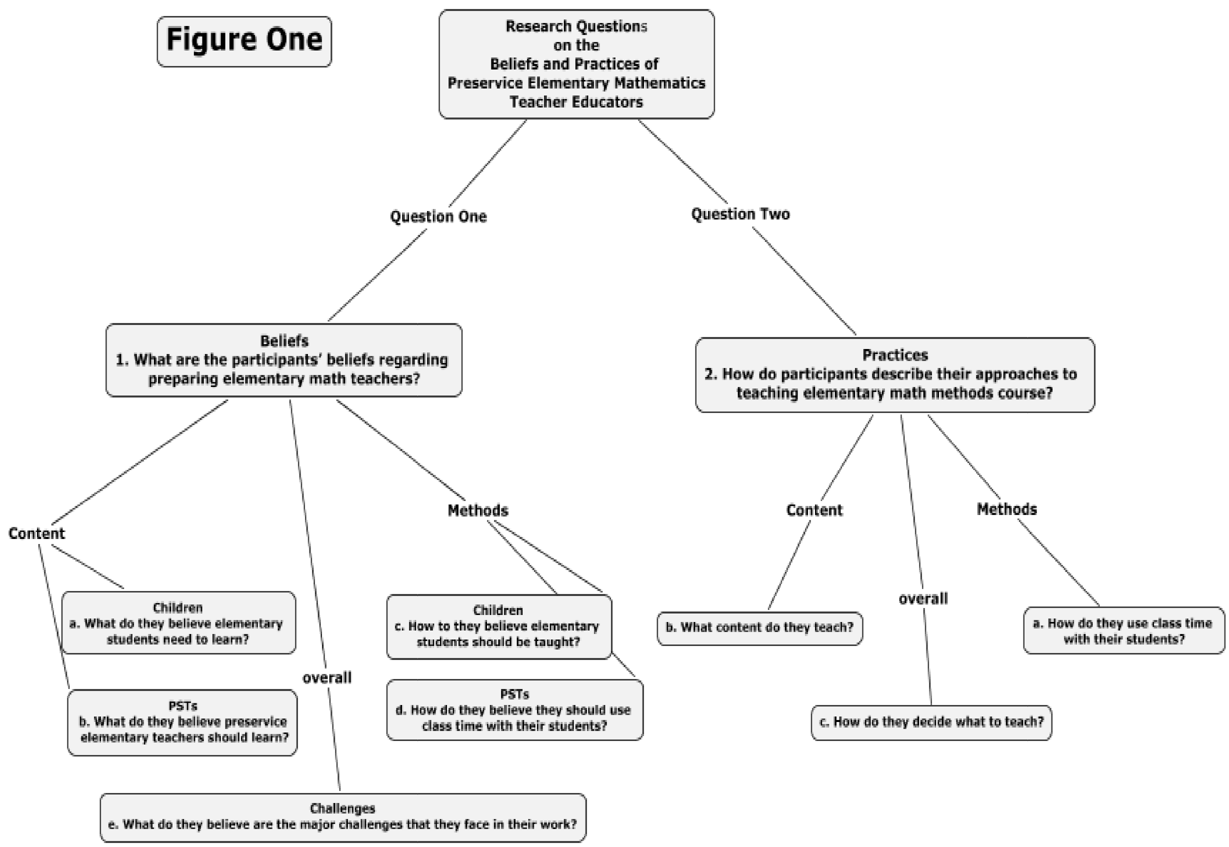

Figure 1: Research questions on the Beliefs and Practices of Preservice Elementary Mathematics Teacher Educators

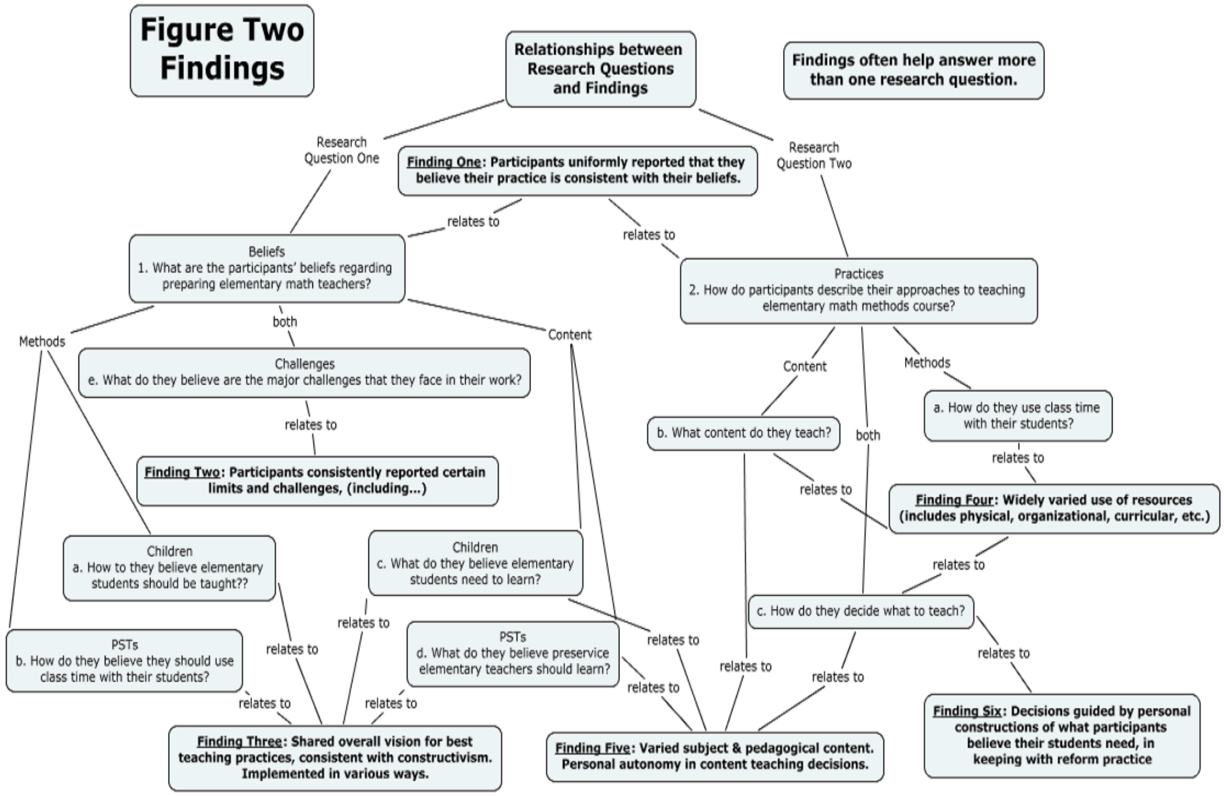

Figure 2: Relationships Between Findings and Research Questions

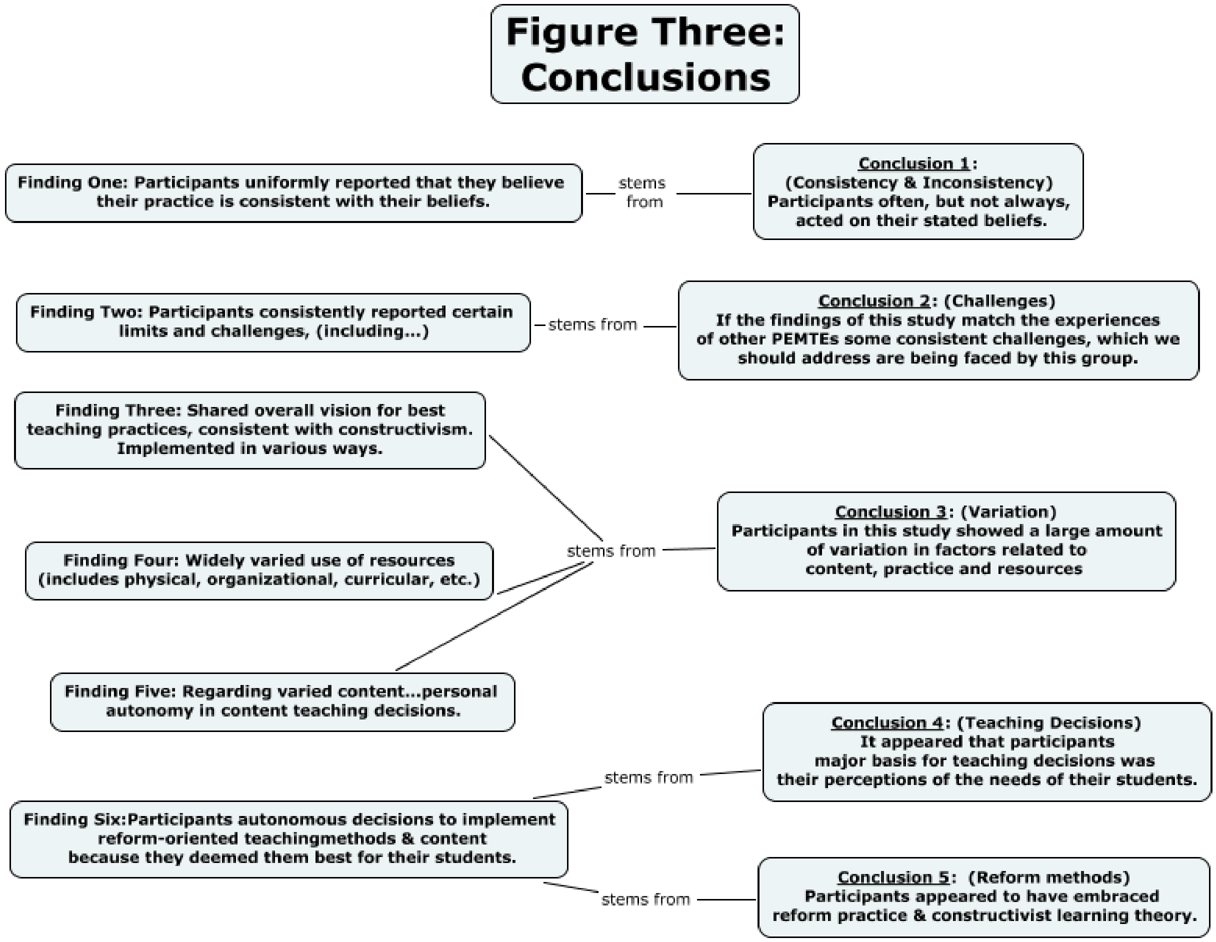

Figure 3: Conclusions

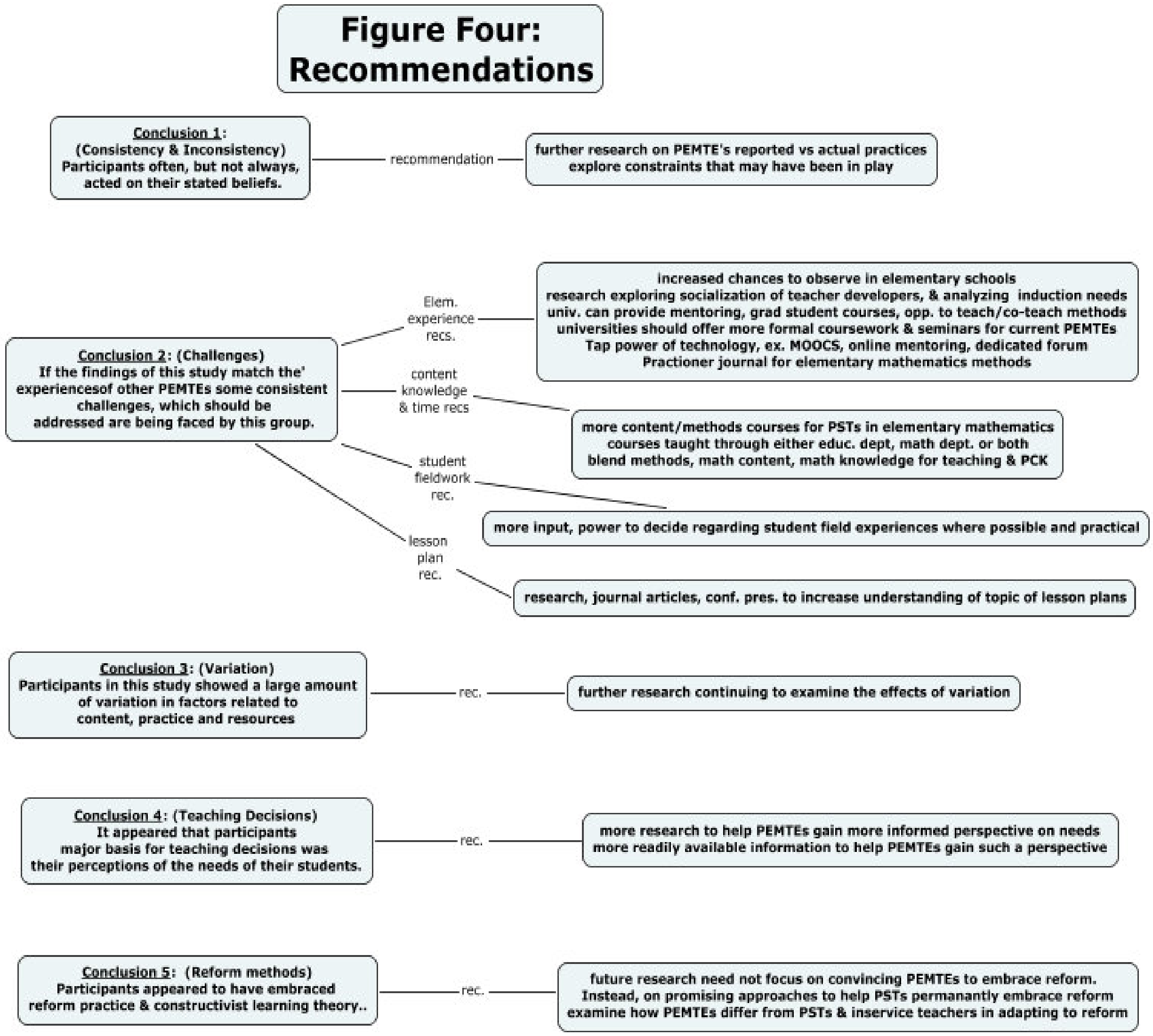

Figure 4: Recommendations

Appendix B

Demographic Survey:

In February of 2013, the following survey was emailed to 450 math education professionals.

1. Are you, currently or in the last five years, teaching a course focusing on mathematics teaching methods for preservice elementary teachers?

- ☐ Yes, currently
- ☐ In the last five years
- ☐ No

2. Which best describes your primary teaching position?

- ☐ In a Mathematics Department
- ☐ In a College of Education
- ☐ Other (please specify)

3. What is the highest degree you have received?

- ☐ Masters
- ☐ Educational Specialist
- ☐ PhD or EdD

4. Which of the following best describes the field in which you received your highest degree?

- ☐ Mathematics Education
- ☐ Education in General
- ☐ Mathematics
- ☐ Other (please specify)

5. At what levels are your students planning to teach mathematics? Indicate all that apply.

- ☐ Pre-K (or K) -6
- ☐ Grades 1 – 4
- ☐ Grades 5 – 8
- ☐ Grades 9 - 12

6. Which best describes the course(s) you teach?

- Focused almost entirely on mathematics teaching methods, with very little mathematics content.
- Primarily focused on teaching methods, with the remainder dealing with mathematics content.
- Equally divided between mathematics teaching methods and mathematics content.
- The course is almost entirely focused on mathematics content, with very little mathematics methods.

7. How would you describe your preparation for teaching elementary mathematics methods courses?

- I have not taught such a course
- Description of preparation:

8. Do you have teaching experience at the K-5 Level?

- yes
- no

9. Do you have teaching experience at the 6-8 Level?

- yes
- no

10. Do you have teaching experience at the 9-12 level?

- yes
- no

11. Below, please check the names of any professional organizations you are a member of, or have been a member of in the past three years.

- National Council of Teachers of Mathematics (NCTM)
- Mathematical Association of America (MAA)
- Association of Mathematics Teacher Educators (AMTE)
- National Council of Supervisors of Mathematics (NCSM)
- Other (please specify)

12. Please name any other organizations you are a member of related to education, or educational research (but not focusing exclusively on mathematics or mathematics education).

13. What is the average age of students in your methods classes? (Choose the best answer)

- ☐ 18-22
- ☐ 23-30
- ☐ Over 30

14. Do you have manipulatives/hands-on materials available for your use? if so, how often do you use them in your course?

- ☐ There are none available for my use
- ☐ At every class meeting, or almost every class meeting
- ☐ Occasionally
- ☐ Rarely

15. Do you use a text? If so, please give the name and primary author.

16. Would you be willing to be interviewed as I continue to gather data for my study? If you are interviewed, your identity will be protected. You will have the opportunity to review my findings or verify that I have accurately represented your views.

- ☐ yes
- ☐ no

Appendix C

Cover Email for Interview Participants

Dear Mathematics Teacher Educator:

Thank you so much for indicating your willingness to be interviewed as part of my doctoral dissertation research!

I am doing a study on the beliefs and practices of Preservice Elementary Mathematics Teacher Educators. The main interview will take place face to face. There will be two shorter follow-up phone conversations after that, which will constitute secondary interviews. As a token of appreciation you will receive two $20 Amazon gift certificates; the first upon completion of the first interview and the second after the final phone conversation. Your input is invaluable to the study and is much appreciated. If you are interviewed, your identity will be protected. You will have the opportunity to review my findings or verify that I have accurately represented your views.

I am planning to attend the NCTM national gathering in Denver, April 17 – April 20. I wonder if you will be in attendance and if so, would you be willing to get together with me there? If this will work, I can schedule the interview over coffee or lunch or at a time and place that is convenient for you in Denver.

Thank you so much and I look forward to hearing from you!

Linda Arnold

Doctoral Candidate

College of Education, Health and Human Services

The University of Tennessee, Knoxville

Appendix D

INFORMED CONSENT STATEMENT

Beliefs and Practices of Preservice Elementary Mathematics Teacher Educators

INTRODUCTION

You are invited to participate in a research study on the beliefs and practices of preservice elementary mathematics teacher educators. The purpose of this study is to describe and understand aspects of the reported teaching beliefs and practices of professors and instructors who prepare preservice elementary mathematics teachers.

INFORMATION ABOUT PARTICIPANTS' INVOLVEMENT IN THE STUDY

Participation involves being interviewed three times by a researcher from the University of Tennessee. Each interview will last approximately 30-45 minutes. Notes may be written during the interview. An audio recording of the interview and subsequent dialogue transcript will be made. Audio recordings, notes and transcripts will be stored securely in a locked file cabinet in the researcher’s home office. It is anticipated the interviews will be completed over a period of one to three months.

RISKS

There are no known risks to participants in this study.

BENEFITS

Benefits to interview participants will be a chance to reflect upon their own beliefs and practices during the interview, and an opportunity to contribute to further research knowledge in the field of mathematics education.

CONFIDENTIALITY

Information in the study records will be kept confidential. Data will be stored securely and will be made available only to persons conducting the study unless participants specifically give permission in writing to do otherwise. No reference will be made in oral or written reports which could link participants to the study.

________ Participant's initials

COMPENSATION (*If applicable to your study, add compensation information here*)

Participants will receive a token compensation given in appreciation for their participation in the study. This compensation will be (a) A $20 gift certificate for Amazon.com given upon completion of the first interview and (b) A $20 gift certificate for Amazon.com given upon completion of the final interview.

EMERGENCY MEDICAL TREATMENT

The University of Tennessee does not "automatically" reimburse subjects for medical claims or other compensation. If physical injury is suffered in the course of research, or for more information, please notify the investigator in charge, Linda Arnold 865-332-4032

CONTACT INFORMATION

If you have questions at any time about the study or the procedures, (or you experience adverse effects as a result of participating in this study,) you may contact the researcher, Linda Arnold, at 414 Olympia Drive, Maryville, TN 379804, larnold5@utk.edu and (865) 332-4032. If you have questions about your rights as a participant, contact the Office of Research Compliance Officer at (865) 974-3466.

PARTICIPATION

Your participation in this study is voluntary; you may decline to participate without penalty. If you decide to participate, you may withdraw from the study at any time without penalty and without loss of benefits to which you are otherwise entitled. If you withdraw from the study before data collection is completed your data will be returned to you or destroyed.

CONSENT

I have read the above information. I have received a copy of this form. I agree to participate in this study.

Participant's signature ______________________________ Date __________

Investigator's signature _____________________________ Date __________

Appendix E

Interview Protocols

Protocol for Interview One

This protocol is for the initial interview in a series of three. Five initial interviews (those with Participants A, B, C, D and F) took place face to face during April 2013 at the National Council of Teachers of Mathematics Annual National Conference in Denver, Colorado. The initial interview with Participant E took place in May of 2013 via Skype.

Subsequent interviews depended upon what had been said in the first interview. I asked individualized follow-up questions, and performed member checks to be sure I had understood and would fairly represent what the participant had said. All interviews were semi-structured, allowing questions to arise spontaneously, depending on context. Thus, individual interview transcripts may contain many questions that do not appear on this protocol.

- Could you tell me about your background, that is, your preparation to teach a class for preservice elementary mathematics educators?

- Please describe a typical day in your classroom.

- How would you describe your teaching philosophy?

  Probe if needed: Is there any particular school of educational thought you would agree with?

- How do you believe your students learn best?

- What do you believe preservice elementary mathematics teachers should learn?

- How do you believe you should use class time with your students?
- Could you describe a particular lesson that you feel went well.

- What content do you teach your students?

- What content do you believe you should teach your students?

- How do you decide what to teach your students?

- What do you believe elementary school students need to learn about math?

- How to you believe elementary students should be taught mathematics?

- Is there anything you would like to tell me that I have not asked about in this interview – perhaps something you thought I might ask about?

- Do you have any suggestions for how I could improve the interview process?

Protocol for Interview Two:

For the most part, candidates were asked individual follow-up and member check questions. I did add the following question for all candidates:

- Could you tell me how you approach the topic of lesson planning in your classroom?

Protocol for Interview Three:

For the most part, candidates were asked individual follow-up and member check questions. I did add the following questions for all candidates:

- (Because all candidates seemed to have indicated this.) Would I be correct to report that you do not see a conflict between what and how you believe you should teach your students and what and how you actually do teach your students?

- (Because all candidates seemed to have indicated this.) Would you like more opportunities to observe in elementary school classrooms? (If they say yes, I invite them to say more.)

- Some math methods teachers are able to observe in elementary schools. What is being done to give you experiences with K-6 children?

- How would you define the term "hands-on" in math education? (This was asked because I was surprised that one candidate had no manipulatives, since he said hands-on experiences were emphasized in his class. When pressed, he talked about drawing picture of manipulatives, etc.)

- You may pass on this question, and you may stop me if you hear the answer: What is your age group? (pause in case someone wants to pass) – 35-30; 30-34; 35-39; 40-49, 50 or over. (The two older candidates were not asked this question because their over 30 years each of experience made their age group as "50 or over" easy to determine, and it can be awkward to ask the age of a person over 50.)

- Please estimate how much of your class time you spend on the following:
    - Activities, group discussion, lecture, student presentations, other

- Agree or disagree: Real-world problems are important for students. (This question was asked because four had alluded to this, and I wondered if it was unanimous. Here, I hoped participants might expand on their remarks, but I did not probe if they did not. I now wish I had probed.)

- Agree or disagree: Elementary students should be required to make conjectures and justify. (I asked this because no participant had mentioned it for elementary students, I wanted to confirm my thought that there were things that participants believed in, but were not mentioning. The enthusiastic agreement I got confirmed my thinking that indeed, there were things that were important to participants that they had not brought up to general questions such as "What should elementary students learn about mathematic?" and "How should elementary students learn mathematics?)

- Does action research have a place in your teaching?(I asked this to try to find out more about basis for decisions, thinking that action research might be taking place, but had not been mentioned. It seemed to me that most participants did not understand the question the way it was asked, however, and I did not press.)

- I would like to ask a final question – to kind of sum up. You can take your time to think first, if you like. Why do you believe that you teach in the way that you do?

- Is there anything you would like to tell me that I have not asked about in this interview – perhaps something you thought I might ask about?

- Do you have any suggestions for how I could have improved the interview process?

Vita

Linda Ann Arnold was born in Dallas, Texas, the first child of parents James and Marjorie Duggan Arnold. She attended Anona Elementary School in Largo , Florida In neighboring Seminole, Florida, she continued to Seminole Jr. High and Seminole High School, graduating in 1975. She attended Florida State University, graduating in 1977 obtaining a bachelor's in Music Education, with a minor in mathematics. After teaching music for one year, and mathematics for three years, she returned to Texas, the place of her birth. She obtained a master's degree in Mathematics Education from TCU in Ft. Worth, Texas. This was followed by graduate studies in Phoenix, Arizona at Arizona State University. Following this she was able to travel to several cities of the world, performing street theater while circumnavigating the globe, which was an educational experience unlike any other. At age twenty-nine, she began teaching mathematics in an inner city high school. This experience challenged and inspired her, and she continued happily in this position for twenty-one years. She traveled during many of the summers, at one point taking a Spanish immersion course. Other summer travels have allowed her to volunteer with street children in Dublin, to whitewater rafting in New Zealand, to tutor in reading on the Navajo reservation, to observe school rooms in France and Germany, and to ride a camel into the Sahara desert where she camped overnight amongst Berber tribespeople. In 2007, she applied and was accepted for a teaching position in the United Kingdom, where she remained for one year, learning and traveling, until the health of her widowed mother called her to Tennessee, where she became a caregiver. At this time, she applied to the University of Tennessee and was accepted as a doctoral student in education, in the department of Theory and Practice in Teacher Education, with a specialization in mathematics education. During this time she also worked part time as an adjunct at Tusculum College, teaching three courses a year, helping to prepare future teachers to teach mathematics to elementary pupils. An important experience during her doctoral work was being selected by the National Science Foundation as a member of a team charged to report on mathematics access for additional language learners, traveling to the International Congress for Mathematical Education held in Seoul, Korea, in 2012. After completing her Ph.D., she hopes to work with teachers and future teachers as a faculty member of a college or university. She believes she has had many good things to be grateful for in this life, and it is her desire to give back.