8-2006

Specialized Understanding of Mathematics: A Study of Prospective Elementary Teachers

Margaret Viola Moss
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Margaret Viola Moss entitled "Specialized Understanding of Mathematics: A Study of Prospective Elementary Teachers." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

P. Mark Taylor, Major Professor

We have read this dissertation and recommend its acceptance:

Vena Long, Ramon Leon, Thomas Turner

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

To the Graduate Council:

I am submitting herewith a dissertation written by Margaret Viola Moss entitled "Specialized Understanding of Mathematics: A Study of Prospective Elementary Teachers." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

P. Mark Taylor

Major Professor

We have read this dissertation
and recommend its acceptance:

Vena Long

Ramon Leon

Thomas Turner

Accepted for the Council:

Anne Mayhew

Vice Chancellor and
Dean of Graduate Studies

(Original signatures are on file with official student records.)

Specialized Understanding of Mathematics:
A Study of Prospective Elementary Teachers

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Margaret Viola Moss
August 2006

Acknowledgements

If I were to thank by name all of the people who had helped and inspired me along the way, then my acknowledgements would be longer than my dissertation. Let me begin by thanking my two wonderful sons, Nathan and Adam, who have inspired me, supported me, and for surrounding me with love. Thank you to Nathan for his wisdom and encouragement, and to Adam for his knowledge and always reminding me to take time to play. Also thank you to my parents, Fred and Kay Moss, who have always believed in me. Thank you to Dad for reminding me to persevere, to be a beautiful person, and to maintain the proper attitude. Thank you to Mom for teaching me mathematics and being such a great role model as a teacher, mother, and person.

I would like to thank my committee chair, Dr. P. Mark Taylor, for his patience, his analogies, and his wisdom. Thank you to my other committee members, Dr. Vena Long, Dr. Ramon Leon and Dr. Thomas Turner, as well as Dr. Kristin Rearden and Dr. Don Dessart for serving on my committee earlier in the process. The guidance, expertise and time that each of these professors shared with me is greatly appreciated.

Thank you also to the Appalachian Math Science Partnership (AMSP), a National Science Foundation project, for not only their incredible financial support that made this dissertation possible, but also to the project directors for their encouragement and advice. Thank you to Deborah Ball and Heather Hill and their colleagues at the Learning Mathematics for Teaching project for developing the Content Knowledge for Teaching Mathematics instruments and for their

advice. Thank you to all of the mathematics methods professors who allowed me into their classes to study their students and for providing me with information and help. Also thank you goes to Mike Newman in the statistical consulting center at UT for his advice and help.

There are so many friends, family members, and colleagues that have helped me along this journey. Thank you to my colleagues at Treasure Valley Community College as they helped me in my early career. Thank you to my colleagues at Pellissippi State Technical Community College who encouraged me as I went through the PhD program and to colleagues from other colleges and universities.

Thank you to friends who provided writing retreats in the redwoods, friends who helped me to ask the right questions, friends who reminded me to have fun, friends who drank caparinhas with me, friends who took care of my children, friends who let me cry on their shoulders, friends who laughed with me, friends who promised me celebratory parties at the end, friends who played music for me, friends who talked to me late at night from the hot tub, friends who brought me coffee in bed, friends who traveled with me, and friends who supported and encouraged me.

“What a long, strange trip it’s been!” Robert Hunter

Abstract

This dissertation study informs the field on how, when and where a specialized understanding of math (SUM) might be developed within a teacher education program by focusing on the three following research questions and related methodology.

1) What are the strengths and weaknesses in prospective elementary teacher's specialized understanding of mathematics as they enter their mathematics methods course?

The Number and Operation and Geometry items from the Content Knowledge for Teaching Mathematics instruments, which have been developed at The University of Michigan's Learning Mathematics for Teaching Project, were administered to 244 prospective elementary teachers at four universities during the first two weeks of the mathematics methods course. An item analysis sheds light on areas of strengths and weaknesses, and a statistical analysis was conducted to see any relationships between content understanding and quantity and type of content courses. A relationship was found between participants who took specialized content courses and the pretest scores. Another interesting finding was that simply taking more mathematics content courses is not related to higher scores.

2) Does the specialized understanding of mathematics change as they take the mathematics methods course?

The CKTM items were administered as a post test during the last two weeks of the methods course and compared with the pre test to look at changes,

both as a paired samples t test and an item analysis. Growth in SUM was found between the pretest and posttest.

3) What learning opportunities during the methods course may improve the specialized understanding of mathematics of prospective elementary teachers?

Interviews were conducted with mathematics methods instructors who saw significant growth on specific items. The general philosophy of the course, as well as specific learning opportunities that may have helped understanding in the specific items that saw growth were explored, and a framework was created of learning opportunities that may impact understanding of mathematics. The learning opportunities that seem to add to improved SUM include readings, communication, experiencing children's mathematical thinking, mathematics activities, manipulatives, and field experiences.

Table of Contents

List of Tables

List of Figures

CHAPTER I

INTRODUCTION

Success in a technologically advanced world is possible only with mathematical power. To read the newspaper, achieve higher paying jobs, or understand the effects of clear-cutting the old growth forests, one must be able to think mathematically in a powerful, conceptual way. Mathematics has become as critical a civil rights issue today as the right to vote was in the 1960's (Moses, 2002). The current slogan of the National Council of Teachers of Mathematics (NCTM) is "Do math and you can do anything." The key to providing the general population with an equal opportunity to acquire this mathematical power is to help teachers acquire and be able to use this mathematical knowledge (Ma, 1999).

Changing the way that mathematics is taught and learned in schools requires a large paradigm shift in both the knowledge and the beliefs of the teachers (Cooney, 2001). This change or reform can be thought of as "a form of liberation rather than as a movement toward something perceived to be better…[Let's consider] teacher development as a personal journey from a static world to one in which exploration and reflection are the norm" (Cooney, p. 10). A classroom where the teacher is teaching with reform methods is more in line with a democratic society. Imparting information maintains the status quo, whereas leading students to be able to think mathematically empowers them. For this to actually happen, the student's mathematical thinking must be valued and the teacher must possess the necessary beliefs and knowledge to foster that (Cooney).

From the “New Math” of the 1960’s and 1970’s, to the “Back to Basics” movement of the 1980’s, the pendulum has swung between many ideas of how mathematics should be taught. In 1989, the NCTM published a document with a holistic vision of mathematics teaching and learning. This was followed in 2000 with the *Principles and Standards of School Mathematics* document which not only included content standards, but also principles of mathematics education such as the equity principle and the technology principle. This vision requires that a teacher have a specialized understanding of mathematics.

The current image of effective mathematics teaching and learning creates a dynamic and connected image of school mathematics. This vision requires that teachers have very different kinds of mathematical understanding and experiences than in the past (Conference Board of the Mathematical Sciences [CBMS], 2001; National Council of Teachers of Mathematics [NCTM], 2000). Teachers must have a specialized understanding of mathematics in order to teach in ways that reflect the standards. This type of understanding was not necessary for the arithmetic algorithm curriculum of the past (Lappan & Even, 1989). The current recommendations require that teachers experience and understand mathematics differently to transform the cycle of teachers teaching the way they were taught (National Center for Research on Teacher Learning [NCRTL], 1992).

In order to better define and understand this “specialized understanding of mathematics (SUM)”, consider the following question in figure 1.1 from the released items of the Content Knowledge for Teaching Mathematics instruments. When asked to perform 35 times 25, most adults can obtain a correct answer, however simply being able to get a correct answer is not sufficient for a teacher who will likely encounter something similar to the following situation in her classroom.

Teachers must be able to use their knowledge to explain concepts, algorithms, and connections (NCRTL, 1992). A specialized understanding of the

---

Imagine that you are working with your class on multiplying large numbers. Among your students’ papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 X25 175 +700 875	35 X25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

Figure 1.1. Example of Specialized Understanding of Mathematics.

mathematics and the curriculum is necessary for teachers to perform the intricate tasks of teaching such as selecting worthwhile activities, asking good questions, and understanding what students know, need to know, and how to guide their learning (Ball & McDiarmid, 1990). Natural curiosity will inspire elementary students to question their teachers about why things work. Teachers who only possess an algorithmic set of memorized facts are unable to help students understand concepts such as why division by zero can not be defined (Ball & Wilson, 1990). Evidence exists that teachers can usually follow an algorithm correctly but they often can not express the deeper concepts that explain why and how the procedures work (Leinhardt & Smith, 1985).

Teachers must draw on a SUM to effectively respond to this type of situation that arises in the classroom. However, responding to mathematical classroom situations seems to draw from a special kind of understanding of mathematics that the average person, or even the mathematician, may not possess (Hill, Schilling, & Ball, 2004). Therefore, teachers must have a specialized understanding of the mathematics they teach in order to ask good questions, choose proper activities, and decide how and where to guide a discussion (Ball, 1988a).

Several attempts have been made recently to better define and name this specialized understanding of mathematics that teachers must have. Within the related literature, many words, such as deep, conceptual, connected, flexible, and profound, are used to describe this specialized type of mathematical understanding. Ma (1999) uses the phrase “a profound understanding of fundamental mathematics (PUFM)" to identify the deep understanding of mathematics that teachers need. Pedagogical content knowledge is a term that has been popular in the literature since it was coined by Shulman (1986, 1987). This type of knowledge is the intersection of mathematics content and mathematics pedagogy and addresses the special type of mathematical knowledge necessary for teachers. Ball and Bass (2005) suggest that content knowledge for teaching includes the domains that Shulman suggested of subject matter knowledge and pedagogical content knowledge. Ball further divides the subject matter knowledge into common content knowledge and specialized content knowledge. Common content knowledge is the mathematical knowledge that any educated adult has, for example being able to get a correct answer when multiplying 25 times 35. Specialized content knowledge is an important concept in that it helps to give credit to the teaching field as a profession in signifying that the knowledge needed for teaching is specialized. However, this researcher would like to propose a combination of some of these terms. Knowledge is not a strong enough word, understanding is stronger. Webster's New World Dictionary defines knowledge as a collection of facts or information. However, "understanding" is comprehension and the power to think and learn. The premise here is that teachers may have knowledge, but understanding is the critical piece. Therefore, throughout this paper, the term "specialized understanding of mathematics (SUM)" will be used to signify this specialized type of mathematical knowledge that teachers must possess and be able to use in order to encourage their student's mathematical thinking and to implement the reform vision of mathematics teaching and learning. "Understanding" is deeper than "knowledge" and signifies that they can use this knowledge. Looking and thinking closely about what types of knowledge teachers need has furthered the field greatly in the past two decades, but much work still needs to be done in learning how to help teachers gain this specialized understanding (Mewborn, 2000; Rand Mathematics Study Panel, 2003).

Elementary school mathematics has been plagued by a rule memorization curriculum which is now being criticized. A perception exists that elementary school mathematics is easy, since most adults can perform the basic operations. However, a deep, connected understanding of elementary school mathematics is not something that most adults possess (Ball, 1988b). While being able to perform basic computations is important in elementary school mathematics, truly understanding the computations and their meanings is a much more powerful understanding than simply memorizing the algorithms.

Teachers need to understand the mathematics that came before and the mathematics that will come after the grade level they are teaching. By having this knowledge, teachers can better make connections to what already has been learned as well as what will lay a better foundation for the future. Seeing the bigger picture is important in understanding where each topic fits into the nature of mathematics. Having a specialized understanding helps teachers teach mathematics in a coherent and connected way that links concepts together (CBMS, 2001; NCTM, 2000).

Statement of the Problem

Many descriptive and comparative studies exist that consider the mathematical understanding that both pre-service and in-service teachers possess. These studies present an image of content understanding in teachers that is not enough to support the current vision of elementary school mathematics. Most of the descriptive studies focus on a particular content area and delve into a small number of teachers' understanding of a particular area through interviews and surveys. These studies provide evidence that although most teachers possess a procedural knowledge of mathematics (for example they can get a correct answer when multiplying 25 times 35), very few teachers possess a SUM that allows them to explain why the procedure works. They possess a fragmented set of memorized rules, but do not understand the connections or the underlying meanings (Ball, 1988a; Baturo & Nason, 1996; Even, 1993; Fuller, 1997; Lappan & Even, 1989; Ma, 1999; Tirosh, Fischbein, Graeber, & Wilson, 1999; Tirosh & Graeber, 1991). Comparative studies have also provided evidence that in the United States, in-service elementary teachers do not have significantly more mathematical understanding than prospective elementary teachers. These findings cast doubt on the idea that teachers will learn mathematics more deeply while teaching it, at least within the current contextual constraints that exist in the American educational structure (Ma, 1999). Evidence also exists that the mathematical knowledge of secondary teachers is not significantly deeper or more conceptual than elementary teachers, which casts doubt on the idea that taking more mathematics courses will solve the problem (Even, 1993; Ball, 1990).

Most prospective elementary teachers are not gaining a SUM from their content coursework (NCTM, 2000). Teacher education programs need more empirical evidence of what learning opportunities most contribute to more knowledgeable and confident teachers in order to make more informed changes to their programs (Mewborn, 2000). The National Science Foundation has supported many reforms in mathematics teacher education (both pre-service and in-service) through programs such as the Teacher Professional Continuum and the Advanced Technological Education Articulation programs, and recently they have moved towards requiring more research into which efforts are most effective. The field is in agreement that most elementary teachers do not have the mathematical understandings necessary to teach effectively, however very little evidence on how to solve this problem exists.

Not much is known about what and how teachers are learning mathematical content from their college courses (Ball & McDiarmid, 1990). More of the research literature focuses on other aspects of teaching such as beliefs and pedagogy. Content knowledge is an important area to focus research on so that educators can learn more about how to help teachers gain content knowledge so that they have an understanding of the mathematics they are teaching.

Purpose of the Study

The purpose of this study was to identify the strengths and the weaknesses of the mathematical understanding of a selected sample of prospective elementary teachers as they entered and exited their mathematics teaching methods course. This identification then enabled the researcher to determine whether this understanding grew as this sample of prospective elementary teachers took their methods course. A corollary purpose was to determine what learning opportunities existed within the methods course which might have contributed to growth in the specialized understanding of mathematics (SUM) necessary for effective teaching. The following three research questions served to focus this endeavor.

1) What are the areas of strengths and what are the areas of weaknesses in the SUM, as measured by the Content Knowledge for Teaching Mathematics instruments, of prospective elementary teachers as they enter their mathematics methods course?

2) Does a SUM change as prospective elementary teachers take their methods course?

3) What learning opportunities during the methods course may contribute to growth in SUM?

Need for the Study

Many mathematics teacher educators are putting much time and effort into reforming their teacher preparation programs. Seemingly great ideas are being implemented, such as changes in the content course requirements, more conversations between colleges of education and colleges of arts and sciences and field experiences in connection with the content and/or methods courses. While these all seem like promising ideas, the field needs more empirical evidence of which learning opportunities and reform efforts are most worthwhile. As mathematics teacher educators reform their programs, they must carefully consider where and how teachers will acquire a SUM within the program (Floden, McDiarmid, & Wiemers, 1990). The RAND Mathematics Study Panel (2003) suggests the field needs to consider “What learning opportunities enable teachers to develop the mathematical knowledge … needed for teaching?” (p. 24)

This study will add to the knowledge base of what specific areas of mathematics content are lacking, and, therefore, need to be improved during the mathematics content courses. It will also add to the knowledge base of how, when and where prospective elementary teachers might improve upon their SUM. Large scale quantitative studies in this area are critically needed to further the field and improve teacher education (Adler, Ball, Krainer, Lin, Novotna, 2004; Mewborn, 2000). This study will provide evidence on how to improve content knowledge in teachers and will help mathematics teacher educators to make more informed changes to their programs.

Many mathematics teacher educators, including this researcher, have spent many years trying to improve the SUM of prospective elementary teachers. This task is often overwhelming, and sometimes discouraging. A need exists to understand more deeply and fully how to create experiences that help teachers to develop their mathematical understanding (Ball, 1988b). This study will help provide insight into what learning opportunities may be more effective, and what areas of mathematics are most lacking in current structures.

Organization of the Study

After an introduction and exploration of the problem and how this study sheds light on the problem in chapter one, a complete review of the related literature is the focus of chapter two. Chapter three contains a complete description of the design of the study and the procedures used, including the sample, the measurements, the data analysis techniques, and a description of the research sites. Chapter four contains the statistical analysis of the data, including the item analysis of the pretest and the analysis of changes in scores reflected in the pre and post tests. Finally, chapter five reports the conclusions that can be drawn from the statistical analysis, the implications for the field of mathematics teacher education, and recommendations for further study.

Definitions of Terms

Procedural understanding of mathematics is an algorithmic understanding of mathematics. A person has a procedural understanding of mathematics if she can follow an algorithm (procedure) to get the right answer to computational problems.

Specialized understanding of mathematics (SUM) is a conceptual, connected understanding of mathematics that allows a person to know why the procedures work, how the concepts are related, provide explanations and understand multiple representations and algorithms. This is the type of understanding of mathematics that those in the teaching profession need in order to encourage and guide a student's mathematical understanding.

Direct instruction teaching method is the "teaching by telling" method where the teacher tells the student a process, and the student practices the procedure. In this classroom, the teacher does most of the talking.

Reform teaching methods are those that support the process standards of The Principles and Standards of School Mathematics (2000). This type of teaching evolves from a constructivist theory of learning and includes methods such as collaborative group work, problem solving, discussions, and manipulative use.

Prospective elementary teachers refers to students enrolled in a four or five year teacher education program on the pathway to becoming licensed elementary teachers.

Teacher is used in this study to refer to both prospective and in-service teachers, with the viewpoint that from the time they begin a teacher education pathway and throughout their teaching career, they are on the teacher professional continuum.

In-service teacher is used in this study to reference current classroom teachers.

Mathematics methods course refers to the course usually taken in the junior or senior year of college in which the prospective teacher learns about teaching techniques and theory of teaching mathematics in the elementary schools. This is usually taken after the content courses.

Theoretical Framework

Elementary school teachers must have a SUM in order to teach effectively. If they are to teach in the reform vision of the NCTM Principles and Standards for School Mathematics document, then they must understand mathematics in this way. A SUM includes being able to see and appreciate the connections between mathematical ideas and between mathematics and other subjects. It includes both a conceptual and a procedural knowledge, although these must be connected. Teachers must know and be able to use many representations and provide explanations. A wider variety of mathematics topics must be understood such as geometry, data analysis, probability, number and operations.

The global theoretical framework is depicted in figure 1.2. Many factors may affect a teacher's SUM. SUM may be influenced in a teacher's own K-12 mathematics experiences. Aspects of their college level mathematics content courses; such as the number and type of courses, the professor's philosophies, and the learning opportunities may affect their mathematical understanding. The number and type of mathematics methods courses, the professor's philosophy, and the learning opportunities in the course may impact their SUM. Also their own teaching practice may have an effect on their understanding of mathematics. During each of these phases, their beliefs and attitudes may be impacted as well, but the focus of this research is on content knowledge so that is depicted in figure 1.2.

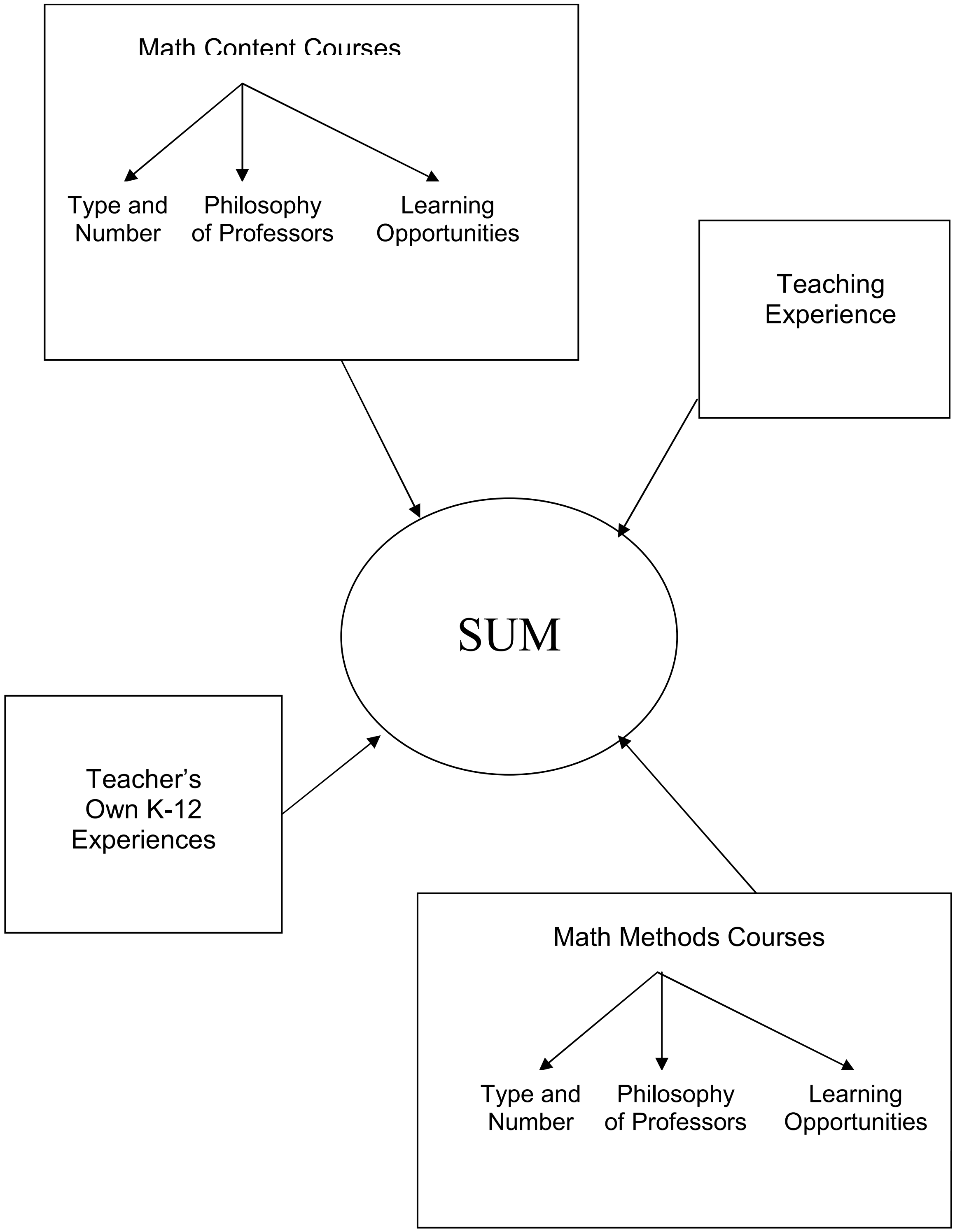


Figure 1.2. Factors Affecting Teacher's SUM.

Since the focus of this research is on the mathematics methods course, the next theory to identify is what learning opportunities within a methods course may affect a SUM. Figure 1.3 depicts this researcher's theory, based on the literature review and experience, on learning opportunities within a mathematics methods course that impact the SUM of prospective teachers. This theory includes five categories of learning opportunities.

Readings and discussions may include journal articles, textbooks, or mathematics curriculum materials and classroom discussions stemming from that. Activities and problem solving include specific problems that the students engage in and mathematical discovery activities and explorations. Experiences with children's mathematical thinking may include looking at student's work samples, watching video clips of children talking through their mathematical thinking, or experiences talking with children about their mathematical thinking. The tactile and visual experiences of using manipulatives to think about mathematics may be another factor that impacts the mathematical understanding of prospective teachers. Lastly, field experiences in elementary classrooms during the mathematics lesson may not only give prospective teachers more experiences with children's mathematical thinking, but also provide other experiences such as lesson development and observing the teacher that influence mathematics understanding. While beliefs and attitudes are important to consider in the model as they are intertwined with content knowledge, these opportunities are put in a rectangle as they are different from learning opportunities that may impact SUM.

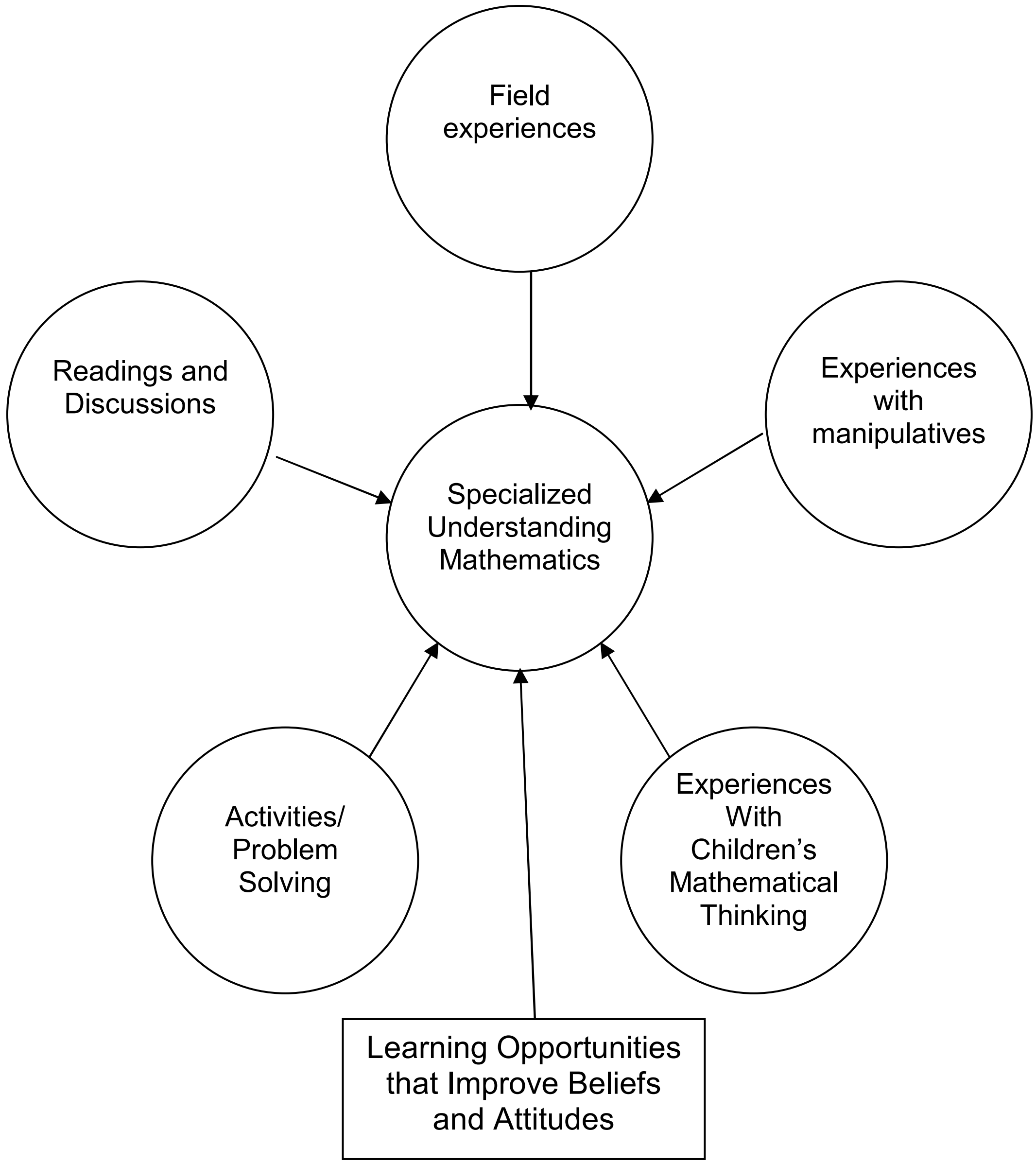


Figure 1.3. Learning Opportunities in a Mathematics Methods Course Affecting SUM.

CHAPTER II

REVIEW OF THE LITERATURE

"Mathematics is a dynamic cultural invention that grows and changes as the needs and interests of society evolve. In the modern world this evolution of mathematical knowledge and society's dependence on mathematical ideas has become a revolution" (Lappan & Even, 1989, p. 20). Research has shown a strong correlation between the mathematics content knowledge of teachers, the quality of teaching, and the mathematical achievements of K-12 students. Evidence exists that good teaching matters and that content knowledge of teachers is critical to effective teaching (National Research Council [NRC], 2001b). The related research and literature clusters around the following themes: mathematical beliefs and attitudes, types of mathematical knowledge, why teachers need a SUM, how teachers acquire a SUM and improved beliefs and attitudes, and where teachers gain SUM and improved beliefs and attitudes. While the focus of this study is on content knowledge, beliefs and attitudes are included in this literature review since content knowledge is so intertwined with beliefs and attitudes. Mathematics teacher educators need to consider both as they are not mutually exclusive.

Mathematical Beliefs and Attitudes

A productive disposition towards mathematics and learning mathematics must be intertwined with deep knowledge in order to teach effectively (NRC, 2001a). Having understanding alone does not guarantee that the teacher will teach with reform methods (Mewborn, 2000; Lubinski, Otto, Rich, & Jaberg, 1998). This suggests that other factors besides content knowledge influence teaching, such as beliefs, attitudes and contexts. Gaining a better understanding of mathematics alone is not enough to change the limiting beliefs that many prospective elementary teachers have about the needs and abilities of their future students to learn mathematics as well as methods to help them learn it (Wilcox, Lanier, Schram, & Lappan, 1992). The beliefs and attitudes about mathematics have been deeply engrained into future teachers during fourteen years of mathematics classes, which may be one of the largest challenges to changing the teaching of mathematics (Lappan & Even, 1989). The five dimensions of beliefs identified by Ball (1987) can be useful in organizing the research literature about beliefs: beliefs about mathematics, beliefs about learning mathematics, beliefs about pupils as learners and doers of mathematics, beliefs about teaching mathematics, and beliefs about learning to teach mathematics.

Beliefs About Mathematics

Teachers beliefs about what mathematics is - its origin, its uses, and its stability – appear to affect how they portray mathematics to their students. They tend to believe that mathematics is not connected to other disciplines or daily life except for simple computations (NCRTL, 1992). Many prospective elementary teachers have low self confidence in their own abilities in mathematics and often admit to not liking mathematics (NCRTL, 1992). However, because they believe mathematics at the elementary level to be very basic, they are confident in their abilities to teach mathematics at this level (Bobis & Cusworth, 1995). Many prospective elementary teachers believe mathematics to be a static set of rules and algorithms to be memorized and that for most problems one correct method exists to find the one right answer (Benbow, 1993; Philipp, Clement, Thanheiser, Schappelle, & Sowder, 2003).

Beliefs About Learning Mathematics

A teacher's perceptions about learning mathematics may interfere with her teaching of mathematics (Ball, 1988a). Prospective elementary teachers have long held beliefs about how people learn mathematics that often come in conflict with the more conceptual ways of teaching and learning that mathematics educators and the NCTM (2000) are suggesting (Philipp et al., 2003). Teachers who believe that learning mathematics is stressful and are afraid of the subject will often convey and transmit these anxieties to their students (Gellert, 2000).

Beliefs About Pupils as Learners and Doers of Mathematics

A common belief among many prospective elementary teachers is that learning mathematics is a natural ability that some students have and some do not. This belief can greatly influence how a teacher approaches teaching mathematics to a class of thirty students, some of whom they believe just do not have a mathematical mind. In this case, teachers may believe that teaching mathematics to some, especially in a conceptual way, is not worth the effort (Featherstone, Smith, Beasley, Corbin, & Shank, 1995). If prospective elementary teachers believe that only some of their students have the ability to learn mathematics, then they believe that what they do as a teacher has little effect. Therefore no reason exists to put much effort into teaching challenging mathematics (Foss & Kleinsassser, 1996).

Because of beliefs about what mathematics is and about students as doers of mathematics, there appears to be a plethora of activities in the elementary classroom linked to computational real world applications such as making change or adding up a shopping list, but for the activities to involve deep mathematical thinking, communications, or imagination is unusual (Foss & Kleinsasser, 1996). Gellert’s (2000) findings suggest that prospective elementary teachers plan to use games and fun to shelter their students from the mathematics, which they perceive as a difficult and scary subject. However, these fun and games chosen to protect the kids from frustration often involve trivial mathematics that do not challenge the students.

Beliefs About Teaching Mathematics

Prospective elementary teachers need to believe that teaching and learning mathematics in conceptual ways is important if they are going to value and therefore attempt to teach with methods that seek to develop a connected understanding of mathematics (Hill, 1997). Teacher educators must challenge prospective elementary teachers’ beliefs about teaching mathematics so that they can let go of “teaching the way they were taught” (Wilson, 1990). Because teachers care so much for children, they want to create a “safe space” for their students, which may not challenge the students. These teachers avoid problem solving explorations where the students may feel uncomfortable (Gellert, 2000). This belief reduces the role of the teacher from “nurturing” to simply “caring” and perpetuates mathematic anxieties (Gellert, 2000). Upon completion of teacher education programs, teachers may state their belief in the use of teaching techniques such as manipulatives and discussions, but often are unable to translate these beliefs into their teaching partly because of their weak knowledge (NCRTL, 1992). Once again, an interwoven need exists for both knowledge and productive beliefs.

Beliefs About Learning to Teach Mathematics

Many prospective elementary teachers believe that only a basic understanding of mathematics is necessary to teach elementary students. They can add, subtract, multiply and divide, so they believe they do not need to learn much more. However, the vision of the NCTM (2000) document includes other areas of mathematics, such as geometry and statistics that the teachers may have never experienced but must now understand. Because this vision also calls for a more connected understanding of mathematics for all students, prospective elementary teachers need to understand that their rote memorization of the facts is not sufficient to teach children effectively. They need to become discontented with their current understanding of mathematics, and realize that their lack of understanding is a result of the way they were taught in school (Hill, 1997).

Types of Mathematical Knowledge

Teachers need a SUM that people in other professions do not. Teachers use mathematics every day, but in very different ways from others. They need to understand more connections and concepts (NCRTL, 1992). While teachers must understand the subjects they are teaching, defining this knowledge for teaching has been a source of discussion and debate in the field (Ball & McDiarmid, 1990).

Shulman started much discussion on the types of knowledge that teachers need when he suggested three categories of teacher knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (1986). Subject matter content knowledge refers to the facts and procedures of a discipline, as well as justifications of these facts and why these ideas are important. Pedagogical content knowledge is an intersection of content, teaching, and learning and has been discussed broadly in the field since Shulman first coined the phrase. It refers to an understanding of representations and examples that can be used to illustrate a given idea, as well as an understanding of what ideas may be more difficult for students, why these ideas are more difficult, and examples and representations that can best be used to clarify these ideas for learners. Curricular knowledge refers to the understanding of curriculum materials available and the ability to decide which of these materials is most appropriate in different situations and the ability to utilize these materials in different contexts effectively (Shulman, 1986).

Another term in the discussion of mathematics knowledge for teaching is "a profound understanding of fundamental mathematics (PUFM)" which was coined by Ma in 1999. This type of understanding refers to a deeper, more conceptual, knowledge of the mathematics taught in elementary school. Ma discusses the need for elementary teachers to understand connections within mathematics and between mathematics and other subjects, to understand and be able to explain standard algorithms, and to use multiple representations of a fundamental mathematical idea (Ma, 1999).

Another construct of the types of knowledge needed by teachers is organized into interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition (National Research Council [NRC], 2001a). This vision of mathematics proficiency for teachers is seen as an intertwined weaving of these strands. Conceptual understanding in this model refers to a connected and useable understanding of mathematical ideas. Procedural fluency is defined as the ability to perform mathematical procedures and algorithms effectively. Strategic competence is what has often been referred to in the literature as the ability to solve mathematics problems encountered in every day life. Adaptive reasoning is being able to think logically about mathematics and to be able to justify ideas and mathematical facts. In this model a productive disposition is intertwined into the types of knowledge mentioned above. Teachers and students must see a reason to learn mathematics, believe that mathematics is valuable and that an understanding of mathematics is attainable and worthwhile (NRC, 2001a).

Evidence exists that teachers generally do not have enough mathematics knowledge. Ma's (1999) study of in-service elementary teachers from both the United States and China provides evidence that the mathematics knowledge of the U.S. teachers tends to be superficial, disconnected, and procedural. Many studies that focus on the content knowledge of teachers focus on a particular content area such as rational numbers (Tirosh, Fischbein, Graeber, & Wilson, 1999), perimeter/area (Baturo & Nason, 1996; Fuller, 1997), or division concepts (Ball, 1988a; Ma, 1999; Tirosh & Graeber, 1991). All of these studies suggest that the elementary teachers' knowledge of these mathematical concepts is procedural and fragmented. The teachers lack the ability to use appropriate representations of mathematical concepts or to use justifications to explain mathematical truths.

Some Closing Ideas About Knowledge and Beliefs

Tasks of teaching, such as facilitation of a discussion and choosing activities, are greatly influenced by the mathematical beliefs of the teacher (Ball & McDiarmid, 1990). Prospective elementary teachers tend to believe that computational skills are the primary goal of elementary school mathematics, learning mathematics is memorizing a fragmented set of rules, and telling is teaching (Lappan & Even, 1989). This narrow view of mathematics knowledge will most likely be transferred to students and will limit the teacher's ability to teach in ways that help students to think mathematically (Ball & McDiarmid, 1990).

However, a productive belief system alone is not sufficient, and neither is mathematical understanding. These must be intertwined. Prospective elementary teachers have strong beliefs about mathematics teaching and learning before they enter teacher education programs, and these beliefs must be challenged in teacher education just as a stronger mathematics understanding must be fostered (Wilcox et al., 1992). More research is needed on how to improve the beliefs and attitudes, how to improve content knowledge, and how these domains intertwine.

Why Teachers Need a SUM

A SUM is critical in being able to effectively listen to students. A teacher must possess content understanding to be able to hear what the student understands and to allow the teacher to expand on student's thinking (Ball & Bass, 2000). For example, suppose a student solves the following problem this way: $53-28 = 50 - 20 + 3 - 8 = 30 + -5 = 25$. Only a teacher who has a specialized understanding of concepts such as place value and number properties will be able to understand if and how this works and be able to nurture this student's discovery (CBMS, 2001). "If anything is to be regarded as a specific preparation for teaching, priority must be given to a thorough grounding in something to teach" (Peters, 1977, p. 151). Content knowledge is essential to effective teaching, and more needs to be known about how teachers might gain this understanding.

The vision of effective mathematics teaching and learning suggested in recent documents requires that teachers have very different kinds of mathematical understanding and experiences than in the past (CBMS, 2001; NCTM, 2000). The arithmetic algorithm curriculum of the past required very little of the teacher as far as a specialized understanding, but that has changed (Lappan & Even, 1989). Elementary school mathematics is not rule memorization under the reform vision, and most adults do not understand mathematics conceptually (Ball, 1988b). The current reform recommendations require that teachers must understand mathematics differently to break the cycle of teachers teaching the way they were taught (NCRTL, 1992). Elementary teachers are often those most uncomfortable with mathematics. They must have different experiences with mathematics to break this cycle, with mathematics courses focusing on developing concepts of elementary school mathematics and taught with effective pedagogical methods (Cipra, 1991).

Simply being able to get a correct answer is sufficient for many professions, but is not sufficient for teachers who must draw on their knowledge to explain concepts, algorithms, and connections (NCRTL, 1992). A specialized understanding of elementary school mathematics is necessary for teachers to effectively teach and perform the intricate tasks of teaching such as selecting appropriate tasks, asking effective questions, hearing what students know and understanding what they need to know (Ball & McDiarmid, 1990). Young students have a natural curiosity and will wonder why things work. Teachers who only possess an algorithmic set of memorized facts are unable to deal with inquisitive questions such as why “invert and multiply” gets the correct answer when dividing fractions (Ball & Wilson, 1990). Evidence exists that students can get through mathematics classes with only a procedural understanding of mathematics, and this lack of conceptual understanding creates barriers when these students become teachers trying to teach mathematics in meaningful ways (Ball & McDiarmid, 1990). Both elementary and secondary prospective teachers do not possess this conceptual understanding of the content they have memorized (NCRTL, 1992).

Teachers must have a connected understanding of the mathematics they teach if they are to teach using reform methods. Only a teacher who possesses a SUM will be able to ask effective questions, choose appropriate activities, or decide how and where to lead a discussion (Ball, 1988a). Therefore it is not a question of why they need this understanding, but how and where mathematics teacher educators can help them to gain this understanding.

How Teachers Acquire a SUM

"Teachers need experiences that enable them to revisit the content that they will teach in order to revise and develop deeper understandings of the underlying principles and connection among ideas inherent in school mathematics" (NCTM, 1989, p. 74). Research suggests that simply taking more mathematics courses does not necessarily provide opportunities for the learners to unpack their knowledge in order to examine and understand mathematical meanings. Majoring in mathematics or taking more mathematics courses also does not guarantee that the students will experience different models of teaching (Ball & Wilson, 1990). In fact, research suggests that the students of teachers who major or minor in mathematics do not achieve at higher rates than students of teachers who do not major or minor in mathematics (Begle, 1979). Among other things, this raises the question of whether the number of mathematics courses is an accurate measure of mathematics knowledge for teaching (Ball, 1988a; Mewborn, 2000). McDiarmid's (1989) research suggests that liberal arts mathematics courses may not provide experiences that help students learn mathematics in a connected and flexible way. These findings question the assumption that getting rid of education degrees in favor of liberal arts degrees will improve the subject matter knowledge of teachers.

Prospective teachers must experience a wide array of mathematical content areas such as statistics and probability that they are now expected to teach, but may not have studied before college (NCTM, 1991). Strong content knowledge must be intertwined with learning opportunities that consider the learner's mind, interests, and experiences. Without a good model of integrating curriculum and without teachers who make connections between and beyond mathematics concepts, the integrated connected mathematics experience often does not happen (Ball & Bass, 2000). Teacher education professors have the additional challenge of making appropriate connections between the mathematics that the students are learning and the mathematics that they will teach (CBMS, 2001). In Judson and Sawada's study of mathematics content courses for prospective elementary teachers using flexible, interactive and innovative methods, the course and faculty development were part of the Arizona Collaborative for Excellence in the Preparation of Teachers. Their findings report evidence that teachers who take these reform courses are much more likely to teach with these types of methods than those who take more traditional mathematics courses. Although their research does not make any claims about the knowledge of these teachers, it does suggest that experiencing new ways of learning mathematics may positively affect the ability of teachers to use similar methods (Judson & Sawada, 2001). Research is lacking on specific learning opportunities that have this positive impact.

Fostering collegial interactions and collaboration among all the participants shows promise in creating a SUM. Ma reports that teachers in China have time built into their daily schedule to reflect, work with colleagues, and learn mathematics. Professional development is built into the schedule and expected for teachers in China. In fact, evidence exists that Chinese teachers gain most of their conceptual understanding of mathematics through this collegial interaction and professional support. Veteran teachers in the United States, where collegial interactions are not built into a day, do not appear to have more mathematics knowledge than their novice colleagues, further suggesting that ongoing collegial interactions focused on mathematics can help teachers gain mathematics knowledge (Ma, 1999). Results from The Third International Math and Science Study showed that in higher performing countries the teachers are given time to learn and collaborate (CBMS, 2001). Collegial interactions can support content and pedagogical growth in teachers, but must be supported in the school setting, must align colleagues with compatible philosophies, and must ensure teacher ownership of the interactions (Taylor, 2004).

Barriers between faculty in colleges of education and colleges of arts and sciences need to be overcome so that the content and methods preparation are more connected (Ball & Bass, 2000). John Dewey writes of this tension in teacher preparation programs of finding a balanced relationship of subject matter and method (1916). Dewey argues that method and content must be so closely intertwined that differentiating between them is difficult. He writes: "Scholastic knowledge is sometimes regarded as if it were something quite irrelevant to method. When this attitude is even unconsciously assumed, method becomes an external attachment to knowledge of subject matter" (Dewey, 1916, p. 160). This requires that mathematics faculty and mathematics education faculty work together to create and implement the mathematical preparation of future teachers (CBMS, 2001).

Creating strong collaborations between K-12 teachers and mathematics teacher educators shows promise. Observing elementary teaching can help mathematics education faculty to better understand the knowledge needed by elementary teachers as well as to better understand the challenges faced by these teachers. Evidence exists that having a successful K-12 teacher develop and team teach mathematics courses for prospective teachers can improve the courses as well as the teaching and understanding of the faculty members and K-12 teacher (Roth McDuffie, Mather, & Reynolds, 2004).

Hill (1997) asked a sample of students finishing their methods course about what they perceive as important to making mathematics more attainable. Many of the respondents answered that manipulative use and real life mathematics problems were the most important. Hill also writes that creating a supportive collegial atmosphere in the class, providing experiences where the students are successful at mathematics, and concrete learning experiences are significant influences on the student's beliefs about mathematics and their ability to learn mathematics. However, Gellert (1999) cautions that providing experiences that they are successful with must not be trivial games with no mathematical content. If a professor can help students walk out of the course liking mathematics instead of hating it, then a huge hurdle has been overcome. Once teachers have positive mathematics experiences and learn that they are capable of having interesting mathematical thoughts and can “do math,” then their anxieties are transformed into enthusiasm for learning (CBMS, 2001).

Where Teachers Gain SUM

Teachers are in school for 13 years before they enter college. Therefore their beliefs and attitudes about mathematics and learning mathematics are so deeply ingrained by the time they get to college that the relatively short time they are in teacher education may not be enough time to truly change these beliefs. Prospective elementary teachers reveal that their instructional strategies are largely based on their earlier experiences with mathematics and as mathematics students. They still see mathematics as computational driven and only see superficial real life uses such as money (Foss & Kleinsasser, 1996). These deep rooted beliefs about what teaching mathematics is do not seem to change significantly as a result of their teacher education (Ball & Wilson, 1990). These limiting beliefs may negatively impact their openness to learning mathematics in the specialized way needed for teaching.

As for content knowledge, during the K-12 mathematics experiences, prospective elementary teachers probably have not learned mathematics in meaningful ways that enable them to teach mathematics effectively (Foss & Kleinsasser, 1996). Beginning college students have frail understandings of the mathematics procedures that they have mostly memorized in K-12 mathematics (NCRTL, 1992).

So, if they are not developing mathematics understanding or a productive disposition in the K-12 years, then it needs to happen in teacher education, but where? The Teacher Education and Learning to Teach (TELT) study suggests that graduating teacher education students have weak understandings of mathematics. The only exception to this is noted in one program with a collaborative, focused effort on developing these ideas through an integrated four semester content/methods experience (NCRTL, 1992). This suggests that teacher education can have an impact but only if much effort and cooperation is expended on this goal (NCRTL, 1992).

Some claim that colleges of education do not provide teachers with the content knowledge they need. These critics fail to realize however, that prospective teachers take their content in colleges of arts and sciences. Ideally these colleges work together towards the goal of preparing teachers but that is not a reality in many universities. Also as mentioned previously, liberal arts mathematics courses do not necessarily provide teachers with the specialized understanding they need. This is further supported by the TELT study which suggests that prospective secondary mathematics teachers do not have significantly more mathematical understanding than the prospective elementary teachers. Even though they are required to take substantially more mathematics courses, their knowledge is still algorithmic and procedural, with little understanding of the underlying meanings and connections (Even, 1993; Ball, 1990). A need exists to train the teachers of teachers, meaning the college mathematics faculty responsible for helping prospective elementary teachers to understand mathematics (Cipra, 1991). Currently, many organizations in the field, such as the Mathematical Association of America, the Center for Proficiency in Teaching Mathematics, and the American Mathematical Association of Two Year Colleges, are focusing efforts in helping mathematics faculty to have a better understanding of the type of mathematics knowledge that teachers need and methods that content faculty can employ to help prospective teachers to gain this knowledge. These organizations are providing summer institutes and collegial support with this goal, but no research results about what impact these may be having exist yet.

Mathematics methods instructors often assume a large role in trying to develop mathematical understanding. Strawhecker (2004) found more gains in content knowledge during a methods course than in a content course. Prospective elementary teachers usually take only one or two college level mathematics courses, and often these are general liberal arts courses that do not address the SUM or pedagogical content knowledge (Floden, McDiarmid, & Wiemers, 1990). Evidence exists that methods instructors often have very different views about their role in developing content knowledge. Many methods instructors tend to believe that their primary goal is to create a productive disposition, not improve content knowledge. While the main goal of the methods course is supposed to be to provide methods for teaching mathematics (Floden et al., 1990), the main objective for many methods instructors is to relieve mathematics anxieties and to provide a bag of creative and fun teaching tricks (NCRTL, 1992). Some methods instructors report that they believe that the best way to improve pedagogical content knowledge is to provide teaching experiences, either to peers in the class or in a field experience setting (Floden et al., 1990). The methods instructors often want to portray mathematics as fun and creative in order to alleviate anxieties, and are hesitant to engage the prospective elementary teachers with challenging mathematics (Gellert, 2000; NCRTL, 1992). If content is taught in a methods course it usually consists of topics such as statistics that are now included in the reform elementary curriculum but have not been traditionally (Ball, 1988b).

Whether the methods course can actually have an impact on beliefs and attitudes that have developed over many years of mathematics classes is unclear. Benbow (1993) suggests that beliefs about what mathematics is and about the teachers ability to impact mathematics learning can be enhanced through an integrated content and methods experience that includes innovative teaching and an in-depth field experience. However, other evidence exists that even if prospective elementary teachers do have innovative experiences in their methods courses, they may not change their beliefs that procedural teaching is still the best method (Foss & Kleinsasser, 1996). These beliefs may inhibit the prospective teachers ability to be open to learning mathematics in a more conceptual way.

Even if prospective elementary teachers do improve their beliefs about mathematics and the learning and teaching of mathematics, their teaching practice may not be impacted. They may not be convinced that these methods are realistic when faced with 30 students from very different backgrounds.

Beginning teachers may have difficulty in translating these nontraditional teaching methods into their practice. Contextual constraints are also a factor. A beginning teacher is likely to slide back into direct instruction teaching methods if the principal, parents and fellow teachers are not supportive of reform teaching methods or have narrow views of the nature of mathematics and mathematics teaching (Taylor, 2000). Therefore, support and guidance during the early years of teaching is important to create real change in the teaching and learning of mathematics (Wilcox et al., 1992).

Conclusion

A SUM and a productive disposition towards mathematics must be intertwined, and are very related in the research literature. Which comes first? Does a productive disposition lead to more specialized understanding? Or does more specialized understanding lead to a more productive disposition? Perhaps as prospective teachers transition from student to teacher through experiences such as a mathematics methods course, they become more open to learning and thinking about mathematics in different ways, both from a beliefs and attitudes viewpoint as well as in mathematical understanding.

Prospective teachers want to understand mathematics better because they care about the kids that they will teach and they want to be effective teachers for them (Hill, 1997). Much remains to be learned about how, when and where prospective and current teachers can gain this understanding. Teachers must understand the content they are going to teach in a specialized way. They must also become open to learning mathematics this way through improved beliefs and attitudes towards mathematics. Perhaps the methods course is a possible place to do this as they are becoming more aware of the realities of teaching mathematics in the twenty first century.

Mewborn (2000), in considering research in the field over the last 45 years, writes of several major movements in the field. During the 1960's and 1970's, most of the studies were quantitative and tried to link teacher knowledge with student achievement. However, these studies could not find correlations, and have been criticized because their measures of teacher knowledge such as number of mathematics courses taken and grade point average in mathematics may not have been accurate measures of mathematical knowledge. Also during the 1960's and 1970's, and on into the 1980's, there were many studies describing what teachers do know, which showed that they have a procedural knowledge but not a conceptual knowledge. These frightening findings spurred a flurry of studies comparing the content knowledge of elementary teachers to secondary teachers, or United States teachers to other nationalities, or pre-service to in-service teachers.

Recently, the International Congress on Mathematics Education commissioned an in-depth study on the recent mathematics teacher education research that has been conducted to see where the research in the field is and to make recommendations about what research needs to be done to further the field of mathematics teacher education. This analysis of two international mathematics teacher education journals reports a preponderance of qualitative studies looking at a small number of teachers, often conducted by researchers studying their own programs. While these studies add to the knowledge base, this study group calls for larger scale and longitudinal quantitative studies (Adler et al., 2004).

Anyone who works with prospective elementary teachers knows the incredible challenges of not only improving their content knowledge, but also helping them to create a productive disposition towards mathematics and the learning and teaching of mathematics. While research documenting elementary teachers' lack of conceptual mathematical understanding is helpful, the field does not need more studies documenting the status quo (Mewborn, 2000).

A significant need exists to study what learning opportunities most contribute to gains in mathematics knowledge within teacher education programs, as well as to study teachers over time as they engage in these opportunities (Mewborn, 2000; RAND Mathematics Study Panel, 2003). Learning about how to effectively and sustainably improve the mathematics knowledge of teachers is important to the field. Focused attention needs to be given to when, where, and how teachers gain a deeper knowledge of mathematics that is necessary for effective teaching (Mewborn, 2000). The improvement of the mathematics knowledge for teaching within teacher education has not been a main focus of research in the field. Other aspects of effective teaching, such as teacher's beliefs and attitudes about mathematics and themselves as learners and teachers of mathematics, as well as method and curricular knowledge have been studied more extensively (Ball & McDiarmid, 1990). While these studies help provide a bigger picture of the skills that a teacher must acquire, a need exists to study mathematics content knowledge of prospective elementary teachers. Therefore, the following research questions are proposed to shed light on this area.

1) What are the areas of strengths and what are the areas of weaknesses in the SUM, as measured by the Content Knowledge for Teaching Mathematics instruments, of prospective elementary teachers as they enter their mathematics methods course?

2) Does a SUM change as prospective elementary teachers take their methods course?

3) What learning opportunities during the methods course may contribute to growth in SUM?

CHAPTER III

METHODOLOGY

Description of Sample

The sample consists of 244 students enrolled in a mathematics methods course at four public universities in the Appalachian region of the United States of America during the fall and/or spring of 2005-2006. This sample therefore is pulled from the population of students enrolled in these courses at the universities overall. This sample is a snapshot in time. Table 3.1 illustrates the sample size at each site.

Data Collection Procedures

During the first two weeks of the elementary mathematics methods course at each institution, the Content Knowledge for Teaching Mathematics (CKTM)

Table 3.1
Sample Size by Site

Research Site	Number Taking Pre-test	Number Taking Post-test
A	25	22
B	69	68
C	26	25
D – main campus	50	44
Da – University D on community college-a campus	33	30
Db – University D on community college-b campus	30	22
Dc – University D on community college-c campus	11	10
Total sample size	244	221

multiple choice instruments were administered to all enrolled students. This administration was conducted by the researcher when possible. However because of time constraints, the administration was done at two of the sites by another graduate student and at another site by the methods professor. All of the people administrating the measures had clear written instructions and conversations were held between the researcher and the other administrators to ensure conformity of techniques. Data on which mathematics content courses were taken and where they were taken was collected with the pretest. At all but one of the sites, the pre-test measures were administered during class time. At the other site, the researcher visited the classes to ask for volunteers, and administered them at four different times during the following week. During the last two weeks of the semester, the CKTM measures were administered again as a post test to the students who were still enrolled in the methods course. All of the post tests were given during class time.

A coding system was used for anonymity of the participant responses. A $25 Amazon gift certificate was given to each participant who took the pre and post test measures as an incentive to participate seriously. No time limit was imposed, except by the length of the class time which was at least 60 minutes at each site. The CKTM measures took the students at most 45 minutes. Calculators were allowed on the measures in accordance with the specifications of CKTM, although not necessary due to the nature of the questions. These measures were given in a paper and pencil format. Two well trusted people were hired to input the data into SPSS. They checked each entry for accuracy and the researcher randomly checked a subset for accuracy.

Data was also collected about the learning opportunities of the methods courses through an interview with each of the methods instructors where significant growth was found on particular items. The interview protocol in appendix B first asked general information about the course format and then asked what happened in the course that they believe may have helped them to understand each concept better. At least one week in advance, each instructor received via email the individual items that their students showed gains on so they could reflect on the individual items and their class. These semi formal interviews were conducted face to face during the last two weeks in May, and the methods instructors received a $25 Amazon gift card for their time. Notes were taken during the interview by the researcher, no audio recording was made. The notes were destroyed after the analysis and all methods instructors were kept anonymous and reviewed the reporting of the interview for accuracy

Measures and Variables

The Content Knowledge for Teaching Mathematics (CKTM) Instrument was used to measure content knowledge in the areas of number and operation and geometry. These items were developed to measure the knowledge necessary to teach mathematics, not just do mathematics. Many sources were used to guide the development of these items including research literature, classroom observations, and elementary curriculum materials. As part of the validation work, a content mapping to the NCTM PSSM document was conducted. These measures were developed through the Learning Mathematics for Teaching (LMT) project at the University of Michigan. Released items to these measures are attached in Appendix A. Because of the costs associated with developing items, the actual items used can not be published. Other quantitative measures of content knowledge, such as the Praxis, measure more general knowledge, and do not focus on the specialized understanding of mathematics needed for teaching. Each item is placed in context of a classroom situation where a teacher might need to explain why a process works, determine the validity of a non-traditional algorithm, or analyze definitions or mathematical representations and relationships. The items used in this study involve a SUM in the content areas of number and operations and geometry.

The instruments and their measurement items have been extensively studied and validated. Piloting each item with over 600 elementary teachers has provided extensive information about item difficulties and overall scale reliabilities. Scale reliabilities typically average in the high .70s to low .80s for 25-item assessments (Hill, Schilling, & Ball, 2004). A link between teachers who do well on the CKTM measures and students who achieve well on Terra Nova tests has been found (Hill, Rowan, & Ball, 2005).

Scientifically-based, quantitative, large-scale research is now greatly enabled with the development of the CKTM measures. Large scale studies of content knowledge for teaching mathematics were previously complex because qualitative measures are difficult to score for large numbers of teachers. Multiple choice measures allow researchers to know the statistical qualities of items such as difficulty and reliability. Many of these items grew out of qualitative measures and the distracters were chosen from years of qualitative research which allows mapping of the most common wrong answers (Hill, Rowan, & Ball, 2005). Each of the items have an “I’m not sure” option to reduce the lucky guess problem.

The CKTM measures are available free of charge to researchers after they have attended a training session held at The University of Michigan in Ann Arbor. This researcher has participated in this training twice, once in March, 2004 and again in August, 2004.

Data Analysis Procedures and Relation to Research Questions

1) What are the areas of strengths and what are the areas of weaknesses in the SUM, as measured by the Content Knowledge for Teaching Mathematics measures, of prospective elementary teachers as they enter their mathematics methods course?

This question was answered by conducting an item analysis to determine which questions were the least challenging and which were the most challenging for the prospective elementary teachers as they entered the mathematics methods courses.

In considering this question, frequency tables were created for each item showing the frequency and percentages of each answer option. The frequency tables included how many subjects answered each option, not merely whether they got the wrong or right answer, as what they answered wrongly seemed to provide some insight into their misconceptions and understandings. The frequency of correct answers, as well as percentages of correct answers and z scores for each item were input into a data table in SPSS and sorted into descending order. The eleven items with the highest number of correct responses (and with a z score greater than 1.0) and the eleven items with the highest number of incorrect responses (and with a z score less than -1.0) were then analyzed for content to determine the areas of strengths and weaknesses in their SUM.

The NCTM (2000) publication, *Principles and Standards for School Mathematics (PSSM)*, was used as a framework for this item analysis. This document makes sense to use as a framework for two reasons. First, in order to teach in this vision, the teachers must understand the content standards of this document. Analyzing the items using this framework will shed light on which areas of the NCTM PSSM document the students are strong in and which areas need improving. Also, the validation work of the CKTM items involved content mapping to the NCTM standards, so they fit easily together. The only two content areas investigated were number and operation content knowledge (NOCK) and geometry. Under the NOCK content area, the test developers identified common content knowledge (CCK), which would be questions that any educated adult should be able to answer. They also developed items that fell under a specialized content knowledge (SCK), which would be items requiring a specialized understanding in order to be able to represent mathematical ideas and operations, provide mathematical explanations, and interpret non-standard computation algorithms (Hill, Schilling, & Ball, 2004). The items from the NOCK content area were also analyzed through this lens.

Previous content courses were analyzed to determine any correlation between number and type of content courses and mathematical knowledge by conducting a univariate analysis of variance comparing quantity of mathematics courses and score on the pretest. Also, independent samples t tests were conducted to determine any relationships between students who took mathematics content courses specifically designed for teachers and the pretest scores. College catalog course descriptions (see Appendix C) of these specialized content courses were analyzed. However, no claims about the methodology used to teach these courses can be made.

2) Does a SUM change as prospective elementary teachers take their methods course?

A paired samples t-test on the pre and post administrations of the CKTM measures was used to determine if knowledge growth occurred during the methods course. An item analysis, including a McNemar test on marginal homogeneity, was also conducted to determine which individual items saw significant gains, and if any showed significant loss.

3) What learning opportunities during the methods course may contribute to growth in SUM?

Interviews with the methods instructors were analyzed to determine what learning opportunities may have led to these gains in particular areas. These interviews were analyzed through the lens of figure 1.2: learning opportunities in a mathematics methods course affecting a SUM. The interviews were first analyzed individually, as the particular items that each site saw significant growth in were different across sites. Then the interviews were analyzed together looking for themes that emerged across sites. Table 3.2 shows the timeline for this study.

Limitations and Assumptions

A limitation of this study is that only a snapshot of the prospective teacher's experiences will be studied since the focus is on the methods course only. Longitudinal studies in this area are needed, but are beyond the scope of this study. Participants are providing information about what mathematics content courses they took, but listing courses does not guarantee a particular type of learning opportunity.

Table 3.2
Time Line of Study

Time	Activity
August, 2005 First two weeks of fall semester at each institution	Administer the CKTM measures in the elementary methods courses at four campuses of research site D
December, 2005 Last two weeks of fall semester at each institution	Administer the CKTM measures in the elementary methods courses at four campuses of research site D
January, 2006 First two weeks of spring semester at each institution	Administer the CKTM measures in the elementary methods courses at all research sites except the community college site c of university D
April, 2006 Last two weeks of spring semester at each institution	Administer the CKTM measures in the elementary methods courses at al research sites except the community college site c of university D
May, 2006	Interview mathematics methods instructors

The results of the CKTM measures are related directly to how motivated the participants are to do well on them. This limitation was addressed in two ways. First, when the participants received the instruments, they were urged to take them seriously in order to help the profession and help improve teacher preparation. Also, each participant received a $25 gift certificate for their efforts in the hope that this will further motivate them to take the measures seriously.

At six of the seven research sites, the surveys were given during class time. One of the sites, the pretest was given outside of class time, during times that met the needs of the students. At this one site, the post test was given during class time.

Each of these methods courses are semester long courses, but they do have different structures and differences exist in the background and methodologies of the instructors. They each include field experiences, and each have similar goals. However, the face to face instruction time varies from 75 minutes per week at one institution to four hours per week at another institution.

To improve generalizability to other prospective teachers, four different universities are involved in this study. However, each of the universities is located in the Appalachian area which may limit the generalizability. Keeping within this region makes the study manageable for the researcher, so this limitation must exist. However, there is no reason to believe that these prospective teachers are different than at other institutions.

This research is based on the assumption that the participants will take the CKTM measures seriously so that the results accurately represent their content knowledge. Another assumption is that the participants accurately reported which mathematics content courses they had successfully taken.

Delimitations

The researcher either traveled to the universities to administer the measures personally, or provided written and verbal instructions to the people who administered them at the other sites. This ensured that each group received the same instructions and that the measures were administered in exactly the same ways. Therefore, the administration of the measures was uniform across all sites.

Although not being a longitudinal study is a limitation, efforts were made to make it a larger scale study than what has typically been done in the field. The measures were administered both fall and spring at one university to increase the sample size even more.

CHAPTER IV

DATA ANALYSIS

The data was entered into SPSS statistical software by a team of two data entry people. They double checked each entry, and the researcher checked for accuracy by inspecting a sample of the surveys. The following data analysis will be organized by way of each research question.

Question 1

What are the areas of strengths and what are the areas of weaknesses in the mathematical knowledge for teaching, as measured by the Content Knowledge for Teaching Mathematics measures, of prospective elementary teachers as they enter their mathematics methods course?

Areas of Strength

The eleven items with the highest number of correct responses (and with a z score greater than 1.0) are shown in Table 4.1. The first column in the table gives the item number and whether it is from the geometry or number and operation content knowledge (NOCK). If the item is from the NOCK construct, then it can be further analyzed into the common content knowledge (CCK) domain and the specialized content knowledge (SCK) domain. The SCK domain can be further subdivided into representing mathematical ideas and operations, interpreting non-standards computational methods, and providing mathematical explanations (Hill, Schilling, & Ball, 2004). Column two of table 4.1 shows to which NCTM content standard the item maps. Column three shows the z score

Table 4.1
Items with Z Scores Greater than 1

Item	NCTM content standard and grade level	Z score
Q9c NOCK CCK	Grades 3-5 Expectations: 1) understand and use properties of operations, such as the distributivity of multiplication over addition 2) develop fluency in adding, subtracting, multiplying, and dividing whole numbers	1.76734
Q19b2 Geometry	Grades Pre-K-2 Expectations: Recognize, name, build, draw, compare, and sort two- and three- dimensional shapes Grades 3-5 Expectations: Classify two- and three- dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids	1.56687
Q7a NOCK CCK	Grades 3-5 Expectations: Explore numbers less than 0 by extending the number line and through familiar applications	1.36640
Q12c NOCK SCK Representing math ideas and operations	Grades 3-5 Expectations: Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to student's experiences	1.13252
Q19c1 Geometry	Grades Pre-K-2 Expectations: Recognize, name, build, draw, compare, and sort two- and three- dimensional shapes Grades 3-5 Expectations: Classify two- and three- dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids	1.13252
Q7b NOCK CCK	Grades 3-5 Expectations: Explore numbers less than 0 by extending the number line and through familiar applications	1.08240

Table 4.1
Continued

Item	NCTM content standard and grade level	Z score
Q9a NOCK CCK	Grades 3-5 Expectations: 1) understand and use properties of operations, such as the distributivity of multiplication over addition 2) develop fluency in adding, subtracting, multiplying, and dividing whole numbers	1.08240
Q17c Geometry	Grades 3-5 Expectations: Identify, compare, and analyze attributes of two- and three- dimensional shapes and develop vocabulary to describe attributes	1.04899
Q19a1 Geometry	Grades Pre-K-2 Expectations: Recognize, name, build, draw, compare, and sort two- and three- dimensional shapes Grades 3-5 Expectations: Classify two- and three- dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids	1.03228
Q9f NOCK CCK	Grades 3-5 Expectations: 1) understand and use properties of operations, such as the distributivity of multiplication over addition 2) develop fluency in adding, subtracting, multiplying, and dividing whole numbers	1.03228
Q17d Geometry	Grades 3-5 Expectations: Identify, compare, and analyze attributes of two- and three- dimensional shapes and develop vocabulary to describe attributes	1.01557

of the item. These z scores equal the number of participants who answered correctly minus the average number answering each item correctly divided by the standard deviation.

Geometry. On page 164 of the NCTM PSSM document, one of the geometry standards for grades 3-5 states that students should be able to "Analyze characteristics and properties of two- and three-dimensional geometric shapes." The geometric items that the participants found easiest relate to this standard. A majority of them were able to identify properties of different types of quadrilaterals. Interestingly, even though two of the most correct items involved identifying properties of quadrilaterals, the 13th most difficult question involved identifying properties of a less common quadrilateral. Four of the items that they scored highest on involved interpreting definitions of three dimensional geometric shapes. In these items, they were also able to analyze and apply mathematical language.

Number and operation. Participants did well in certain areas of the number and operation content domain, particularly in the comment content knowledge items. The six NOCK items that were answered correctly more often included the content standards of "understand meanings of operations" (NCTM, 2000, p. 148), especially in terms of whole numbers and with subtraction resulting in negative integers. The participants also appear to be able to "compute fluently" (p. 148) including being able to compute numerical expressions involving order of operations, especially with knowing that multiplication is performed before addition. However an exception to this appears to be in knowing when a numerical expression of the form $-x^y$ produces a positive or negative answer.

The meaning of subtraction as a "what is left" operation seems to be understood, although whether a deeper understanding of operations exists is unclear. Item 12c involves evaluating a representation of fraction subtraction, where the question is "what is left" so many of the participants chose this as a correct representation of fraction subtraction. However, item 12a was one of the most frequently missed items, which also involves evaluating a representation of fraction subtraction and a "what is left" question. The difference with this item is that the unit whole is not the same for the two fractions being subtracted.

Areas of Weakness

Eleven items had a z score below -1, nine of which are from the domain of number and operation and two from the geometry content area. The z scores were calculated by subtracting the number who answered the item correctly minus the average number answering correctly divided by the standard deviation. Four of the most missed questions involve fraction concepts. Specialized content knowledge seems to be found in many of the most missed NOCK items. Table 4.2 shows the items with z scores below -1, which content area the item is from, as well as the NCTM standard to which the item can be mapped.

Geometry. As for the two geometry questions with a z score below -1, they both require understanding relationships between different measurements of figures (length, width, area, volume, etc.) as well as the meanings behind the

Table 4.2
Items with Z Scores Less than -1

Item	NCTM content standard and grade level	Z score of item
Q8 NOCK SCK Interpreting non-standard computational methods	Grades Pre-K-2 Expectations: develop and use strategies for whole number computations, with a focus on addition and subtraction Grades 3-5 Expectations: identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.	-1.00583
Q6 NOCK SCK Providing mathematical explanations	Grades 3-5 Expectations: 1) Understand the place value structure of the base-ten number system and be able to represent and compare whole numbers and decimals.	-1.15618
Q16d Geometry	Grades 3-5 Expectations: Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life. Grades 6-8 Expectations: Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes	-1.18959
Q3 NOCK SCK Providing mathematical explanations	Grades 3-5 Expectations: 1) understand various meanings of multiplication and division 2) understand the effects of multiplying and dividing whole numbers	-1.23971
Q4 NOCK SCK Providing mathematical explanations	Grades 6-8 Expectations: Develop, analyze and explain methods for solving problems involving proportions such as scaling and finding equivalent ratios	-1.23971

Table 4.2
Continued

Item	NCTM content standard and grade level	Z score of item
Q13 NOCK SCK Interpreting non-standard computation methods	Grades 3-5 Expectations: Recognize equivalent representations for the same number and generate them by decomposing and composing numbers Grades 6-8 Expectations: Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use	-1.25642
Q14 NOCK SCK Interpreting non-standard computation methods	Grades 6-8 Expectations: Develop and analyze algorithms for computing fractions, decimals, and integers and develop fluency in their use	-1.35665
Q9e NOCK CCK	Grades 3-5 Expectations: 1) understand and use properties of operations, such as the distributivity of multiplication over addition Grades 6-8 Expectations: 1) Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation	-1.39006
Q12a NOCK SCK Representing mathematical ideas and operations	Grades 3-5 Expectations: Developing understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers Grades 6-8 Expectations: Understand the meaning and effects of arithmetic operations with fractions, decimals and integers	-1.49030
Q18 Geometry	Grades 6-8 Expectations: Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects	-1.50700

Table 4.2
Continued

Item	NCTM content standard and grade level	Z score of item
Q10 NOCK SCK Representing mathematical ideas and operations	Grades 3-5 Expectations: Developing understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers	-1.77430

formulas they have memorized. Students should "understand the relationship between the measurement of an object and the succinct formula that produces the measurement" (NCTM, p. 175, 2000). Understanding the meaning behind the formulas for area and volume seem to be missing for many of the prospective teachers. Another concept needing improvement is in understanding how changing one dimension of a figure affects areas and volumes. Also the relationship between the area of a circle and the number pi seems to be a weakness in their understanding.

Number and operation. One of the items with low z scores in the number and operation content area are from the common content knowledge domain. Several related items had very high z scores but this item required that participants understand exponential notation, especially whether $-x^y$ yields a positive or negative answer when y is even.

The other eight NOCK items with low z scores were of a specialized content knowledge. One area of specialized content knowledge was in representing mathematical ideas and operations. One of these items involves representing fraction subtraction. The misconception here seems to be in understanding that the unit whole must be the same for both fractions when subtracting them. For example, taking one third of a cake and eating one sixth of what is left is not a valid representation for 1/3 -1/6 since the unit whole changes.

Representing fractions in general seems to be another problem. While the responses indicate that the participants are comfortable with representations of fractions as an area model in parts of a unit whole, and representations of fractions as a set, few participants were able to understand a representation of fractions as divisions of whole numbers.

Another area of specialized content knowledge as defined by the developers of CKTM items was in providing mathematical explanations. Providing illustrations as to why division by zero can not be defined was difficult for the participants, and the most common choices indicate a lack of understanding of the meaning of the operation of division. Many participants simply restated the rule when choosing answers to explain why the standard method for simplifying fractions works without changing the value of the fraction.

The third area of specialized content knowledge that appeared in the items with z scores below -1 was in interpreting non standard algorithms. Two involved whole number subtraction, while the other involved division of fractions. The participants were often unable to evaluate whether the non-traditional methods were valid or not. Interestingly, these three questions involving non-traditional algorithms, had high responses of "I'm not sure" (35.2%, 18.9% and

18%, which in two of those cases was higher than the percentage answering correctly).

Indicators. Previous mathematics courses were considered to determine if the type or number of content courses they took were indicators of how they scored on the CKTM items. A Shapiro-Wilk's test and Levine's test assured that the two assumptions of sampling from a normal distribution and of equality of variances were valid assumptions for each of the t tests conducted.

All of the universities in the study, and the associated community colleges, offer two semesters of specialized content courses for elementary teachers. The catalog course descriptions (Appendix C) for these specialized content courses were analyzed. Although the course titles and course numbers were not consistent across all sites, similarities between the course descriptions were found. Both of these courses were three credit hours at each site and were specifically designed for prospective elementary teachers. The Math for Teachers I course at each site includes number and operation in the course description. The Math for Teachers II course at each site includes two and three dimensional geometry as well as measurement. No claims can be made about the methodology of the instruction of these classes, only that students who took these classes were exposed to number and operation and geometry content with a focus on mathematical understanding that elementary teachers need.

An independent samples t-test was conducted to test the null hypothesis of equality of means of the NOCK portion of the pretest between those who had taken math for teachers I and those who had not. The z scores were used in this analysis rather than the raw scores, but the same significance is found using either. The z score of the pretest for each participant equals the number of items correct minus the average number correct divided by the standard deviation. This test showed that while people who have taken math for teachers I, had a slightly higher mean on the number and operation portion of the pretest, it was not significantly higher. The results of this analysis are shown in tables 4.3 and 4.4.

An independent samples t-test was conducted to test a relationship between students who took math for teachers II and the score on the geometry items on the pretest. A positive and significant relationship exists between these two variables as shown in tables 4.5 and 4.6. The p value is .017 and the effect size is .38

An independent samples t test was then run to test the null hypothesis of equality of means between participants who had taken both math for teachers I and II and the total z score on the pretest. This shows a positive and significant relationship between participants who took both specialized content courses and their overall pretest score. The p value is .008 and the effect size is .40. These results are shown in tables 4.7 and 4.8.

The total number of mathematics courses taken and the pretest score were analyzed in a univariate analysis of variance test to see if taking more mathematics classes is related to higher pretest scores. The data was banded into thirds, with the lower third being less than or equal to two mathematics

Table 4.3

Group Statistics for Math for Teachers I and NOCK Score

	Math for Teachers I	N	Mean	Std. Deviation	Std. Error Mean
Zscore(scoreNOCK)	Yes	197	.0418017	.96957948	.06907968
	No	47	-.1752112	1.11273633	.16230927

Table 4.4

Results of Independent Samples t test for NOCK Score

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Zscore(scoreNOCK)	Equal variances assumed	.629	.429	1.339	242	.182	.21701288	.16207106	-.10223715	.53626290
	Equal variances not assumed			1.230	63.684	.223	.21701288	.17639814	-.13541661	.56944237

Table 4.5
Group Statistics for Math for Teachers II and Geometry Score

	Math for Teachers II	N	Mean	Std. Deviation	Std. Error Mean
Zscore(scoregeo)	Yes	196	.0751715	.98262659	.07018761
	No	48	-.3069501	1.02195829	.14750697

Table 4.6
Results of Independent Samples t test for Geometry Score

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Zscore(scoregeo)	Equal variances assumed	.160	.690	2.396	242	.017	.38212159	.15949661	.06794275	.69630043
	Equal variances not assumed			2.339	69.829	.022	.38212159	.16335424	.05630781	.70793538

Table 4.7

Group Statistics for Math for Teachers I and II and Pretest Score

	Both Math for Teachers I and II	N	Mean	Std. Deviation	Std. Error Mean
Zscore(scorepre)	Yes	186	.0940932	.97198252	.07126922
	Yes	58	-.3017472	1.03697867	.13616197

Table 4.8

Results of Independent Samples t test for Pretest Score

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Zscore(scorepre)	Equal variances assumed	.102	.750	2.665	242	.008	.39584039	.14853857	.10324686	.68843391
	Equal variances not assumed			2.576	90.419	.012	.39584039	.15368599	.09053562	.70114515

content courses, the middle third was three mathematics classes and the upper third was four or more mathematics classes. There was one participant with eleven mathematics classes, which was much more than others, but this person was left in the data. There was not a statistically significant difference in the means of these three groups, as shown in tables 4.9, 4.10, and 4.11.

This analysis provides evidence that if they took two semesters of mathematics for teachers courses, then they scored significantly higher on the CKTM survey at the point they enter their methods course. Analyzing this more deeply shows that students who took the mathematics for teachers second semester which includes geometry, scored significantly higher on the geometry items than those students who had not taken this course. Students who took the mathematics for teachers first semester, which includes number and operation, did not score significantly higher on the number and operation items of the CKTM survey. The total number of mathematics courses that students take does not appear to be an indicator of their score on the CKTM survey. Students who took more mathematics classes did not score significantly higher on the test.

Table 4.9
Between-Subjects Factors

		Value Label	N
totalmath (Banded)	1	<= 2	99
	2	3 - 3	87
	3	4+	58

Table 4.10
Descriptive Statistics

totalmath (Banded)	Mean	Std. Deviation	N
<= 2	-.0909223	1.03805914	99
3 - 3	-.0468434	.91190588	87
4+	.2254600	1.04231224	58
Total	.0000000	1.00000000	244

Table 4.11
Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3.958(a)	2	1.979	1.995	.138
Intercept	.198	1	.198	.200	.655
bandtotmath	3.958	2	1.979	1.995	.138
Error	239.042	241	.992		
Total	243.000	244			
Corrected Total	243.000	243			

Question 2

Does a SUM change as prospective elementary teachers take their methods course?

To investigate this question, a paired samples t-test was conducted (after verifying the assumptions of sampling from a normal distribution) between the pre and post tests. The scores were standardized to the pretest by creating a z score to raw score conversion table for the pretest scores, and then scoring the post test scores using this standardization table. This standardization of z scores is equivalent to z score post = (raw score post – mean of pretest raw scores)/ standard deviation of pretest raw scores. The Cronbach's alpha reliability measure was found to be .837, which is well in the acceptable range. Tables 4.12, 4.13, and 4.14 show the results of this test.

Table 4.12
Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Zscore: pre	.0000000	221	1.00000000	.06726728
	Standzpost	.1261064	221	1.04741198	.07045655

Table 4.13
Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Zscore: score pre & standzpost	221	.721	.000

Table 4.14
Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Zscore: score pre - standzpost	-.12610642	.76641272	.05155450	-.22771032	-.02450253	-2.446	220	.015

With a p value of .015, which is less than .05, we reject the null hypothesis that the means are equal for the pre and post tests. Evidence exists that their content understanding is growing as they take their methods course. The average improvement roughly translates to improving one question out of the 48. This is a statistically significant improvement, although intuitively it is not a huge improvement, and the effect size is small at .123. The pre and post test scores were highly and significantly correlated with a coefficient of .721, which is to be expected with the paired sample.

An item analysis was conducted to consider which questions saw the most gains, and if any saw a decrease. This analysis is descriptive in nature, no causal relationship is claimed. The McNemar Test was conducted on individual items with a large increase (or decrease) to test the null hypothesis of marginal homogeneity on each item. This test is used to test for proportional change. For example, is a 30% gain enough to be significant, or 20%? The McNemar Test is used to determine this. Overall, there were eight items where there was a significant gain. Only one item had a significant loss. While not all items showed a positive gain, there were eight items that showed a significant gain, and more items showed a marginal gain than loss, so the balance made a significant gain overall that was found in the paired samples t test. Table 4.15 shows the items and content that showed a significant proportional gain as well as the pre and post z scores of the item and the McNemar p-value. These z scores are equivalent to the number correct on that item minus the average number of correct answers on all items divided by the standard deviation.

Table 4.15
Items with Significant Improvement

Item	NCTM Content Standard	Z score of item on pretest	Z score of item on posttest	McNemar p- value
5 NOCK SCK Interpreting non-standard computation methods	Grades 6-8 Expectations: understand the meaning and effects of arithmetic operations with fractions, decimals, and integers	-.989	-.569	.002
11 NOCK SCK Interpreting non-standard computation methods	Grades 3-5 Expectations: 1) Recognize equivalent representations for the same number and generate them by decomposing and composing numbers; 2) Develop fluency in adding, subtracting, multiplying and dividing whole numbers	-.087	.312	.005
16d Geometry	Grades 3-5 Expectations: Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life. Grades 6-8 Expectations: Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes	-1.190	-.850	.004

Table 4.15
Continued

Item	NCTM Content Standard	Z score of item on pretest	Z score of item on posttest	McNemar p- value
22c Geometry	Grades 6-8 Expectations: Select and apply tools and techniques to accurately find length, area, volume, and angle measures to appropriate levels of precision	-.789	-.475	.021
21 Geometry	Grades 3-5 Expectations: Predict and describe the results of sliding, flipping, and turning two-dimensional shapes	.414	.687	.048
4 NOCK SCK Providing Mathematical Explanations	Grades 6-8 Expectations: Develop, analyze and explain methods for solving problems involving proportions such as scaling and finding equivalent ratios	-1.240	-1.019	.008
16a Geometry	Grades 3-5 Expectations: Identify, compare, and analyze attributes of two- and three- dimensional shapes and develop vocabulary to describe attributes	-.204	.031	.036
9f NOCK CCK	Grades 3-5 Expectations: 1) understand and use properties of operations, such as the distributivity of multiplication over addition 2) develop fluency in adding, subtracting, multiplying, and dividing whole numbers	1.032	1.212	.012

Four of the items that showed the most improvement were from the geometry content area, four were from the number and operation content area. Of the four number and operation items that showed the most improvement, three of those were from the specialized content knowledge domain.

The one item that showed significant loss went from a z score of -1.357 on the pretest to a z score of -1.750 on the posttest, with a McNemar p-value of .021. This one item is a NOCK item, specialized content knowledge in interpreting non-standard computation methods, specifically in fraction division. The positive gains on so many other items still produced a significant gain overall.

Question 3

What learning opportunities during the methods course may contribute to growth in SUM?

This sample of 221 prospective teachers came from seven different campuses, and therefore seven different methods experiences. While this is not a comparative study, at this point the data was disaggregated to conduct an item analysis for each site to determine whether each site may have seen growth in certain areas. The analysis on individual items is descriptive in nature and no causal relationship can be claimed. The McNemar test was conducted on each item with at least a 10% increase in percentage of participants getting a correct answer at each site to see where significant change occurred. Four out of the seven sites saw significant change on particular items. Interviews were then conducted with the methods instructors to focus in on learning opportunities that may have contributed to the students having a better understanding of certain concepts after the methods course. The background and individual pedagogy of the methods instructors are other variables that affect what is learned within a methods class, but these variables are beyond the scope of this study. Table 4.16 summarizes the areas of growth at each site.

Each of these methods instructors agreed to talk with me about their methods courses and about what learning opportunities may have contributed to these improvements. During each interview, the instructors shared their general philosophy with the course, the layout of the course, and then focused on learning opportunities related to the items that saw growth. While this analysis certainly can not claim a causal relationship, or perhaps even a correlational relationship, the methods instructors are the experts on what happened in their classes that may have helped understanding, and therefore are in the best position to theorize on what learning opportunities help understanding within a methods course.

Methods Instructor A

This instructor believes that an important goal in the methods course is to help prospective teachers learn to see mathematics in a variety of interconnected ways so they can better understand how their students are seeing mathematics. This enables the teacher to support the students' ability to build on their own knowledge develop deeper understanding.

Table 4.16
Areas of Growth at Each Site

Instructor	Number of questions with significant growth	Concepts addressed in items
A	2	• Why simplification of fractions works • Alternate algorithms for division of whole numbers
B	5	• Comparing fractions • Alternate algorithm for decimal multiplication • Relationship between area of circle and pi • Interpreting geometric definition • Area of a figure on a rectangular grid
C	4	• Alternate algorithms for decimal multiplication, subtracting whole numbers, dividing whole numbers • Properties of parallelograms
D	3	• Order of Operations • Relationship between area of circle and pi

A method to help the prospective teacher develop this interconnected understanding of mathematics, and perhaps a contributing factor to the improvement that this instructor's methods students had in analyzing non-traditional algorithms, is working in a non-base ten numeration system. The methods students work with this system using concrete materials and they think about why the traditional algorithms work. These activities may also help them to realize that they would come up with alternate algorithms on their own if they had not been told the traditional methods.

Perhaps another technique that helped this group of methods students to understand alternate algorithms for division of whole numbers may be in helping them to think about the relationships between division and multiplication. Also viewing division as repeated subtraction may help. To help develop this concept, the methods instructor asks the methods students to do a division problem on the calculator, but they can not use the division key because it is supposedly broken.

The methods students also model the division of whole numbers with base ten blocks. They go through the traditional algorithm, as they model it and discuss the model and record the model. They also do each step on paper and then model it with the blocks. These activities break down the conventional algorithm into partial steps that can be seen, thus producing visual images of the concepts.

Precise language that reflects the model rather than language that refers to the abstract is a goal that this instructor tries to foster. Concrete meaning must be associated with mathematical language. An example of this, and an area where this instructor saw improvement on the CKTM items, is in the phrases “reducing fractions” and “improper fractions.” This instructor stresses the idea, through questions such as “Reducing something, what does that word mean?” Questions like this helps the methods students to realize that reducing means to make smaller and improper is bad, neither of which reflect the concrete ideas that these phrases are supposed to represent.

Throughout the class, many opportunities to examine children’s mathematical thinking exist. The format of many of the items on the CKTM measures are similar in format to questions on the exams in this course in that they are often situated in a classroom and require analyzing children’s mathematical thinking as well as providing explanations. Another opportunity to analyze children’s mathematical thinking is through watching video clips of children thinking and communicating mathematically.

The methods students are also required to conduct two interviews with children, one focused on place value and the other on number sense. Through this activity, the prospective teachers not only have an opportunity to analyze the children’s mathematical thinking but also to analyze the questions that they ask and what might have been a better question to ask. The instructor and prospective teacher look for improvement in the questioning technique between the first and second interview.

Methods Instructor B

This instructor has two major goals for students in the methods course. The first of these goals is that they be able to make sense of mathematics. They need multiple ways of making sense of concepts, not just one way. The second goal is the "affective objective." To this instructor, this includes several aspects of beliefs and attitudes. Prospective teachers often come to a methods course not only afraid of mathematics, but afraid that they will harm their future students by trying to teach them mathematics. This instructor wants them to look forward to teaching mathematics and finds it exciting when the prospective teachers, after experiencing the opportunities in the methods course, will confess that "I didn't think I would like teaching mathematics, but I do!" Other aspects of the affective objectives are to become more comfortable with mathematical thinking and to gain intrinsic reasons to learn.

This methods instructor creates many opportunities for conversations about mathematics and about the teaching and learning of mathematics, which this instructor believes is very important. Conversations about mathematics help the prospective teachers to make sense of mathematics by talking it through with each other. Conversations about mathematics with colleagues are perhaps the most important professional development for teachers, so fostering this in the methods courses is essential. They need to be comfortable and feel safe in verbalizing their mathematical thinking.

One of the items that this instructor's students improved on involved interpreting definitions of three dimensional objects. An activity that may have helped this involves precise communication. The instructor hands around models of a variety of three dimensional objects and the students have conversations about the characteristics of the objects. They must use very precise language and communicate this thoroughly with each other. After this, they talk about the names of the objects, and can then see why a triangular prism and a hexagonal prism are both prisms.

Questioning is an important part of these mathematical conversations. The instructor often answers a question with another question that guides the prospective teacher's thinking. Questions that relate what they are trying to figure out to what they already know are a means of helping them to construct their own knowledge as well as to see connections in mathematical concepts. For example, when the students are thinking about division of fractions, the instructor helps them to see connections by asking something like "Is division of fractions different than division of whole numbers?" The students also learn to ask each other questions, and will ask each other to explain a concept differently. The instructor wrote on the board a division of fractions problem where he first found an answer by inverting the first number and multiplying and then found a different answer by inverting the second number and multiplying. All the students knew that the second answer was correct, but when the instructor asked them to explain why, they struggled. Throughout the semester, as students brought in explanations of this concept, they had to explain it to fellow classmates and make sure that they all understood.

The readings in this course are focused on the textbook, which focuses on methods for K-4 mathematics teaching and learning. The philosophy of the instructor is that mainly reading the text helps the students to focus on the content, instead of being overwhelmed by activities. This also addresses literacy issues in helping the prospective teachers to read a text, and decide how to teach the content based on that information.

This instructor also believes in the importance of the students having visual images to help them to understand and remember mathematical concepts. One of the questions that the students grew significantly in answering correctly involves a deeper understanding of area and pi. The instructor talks about providing visuals for area and how it is measured. The relationship between the area of a circle and the number pi seems to be a visual concept that the students need to be able to see.

The structure of the course helps to foster the conversations, questions, visual images, and explanations through three pedagogical techniques: a modified jigsaw technique, learning communities, and lesson study. Each of these interrelated opportunities helps the students to take responsibility for their own learning, as well as the learning of members of their community. They foster collegial interactions, and promote conversations with colleagues about mathematics and mathematics teaching.

The modified jigsaw component entails each member of the learning community taking responsibility for reading, analyzing, and teaching some topic from the text. The prospective teachers create a lesson plan on the topic which is mainly based on the text materials, as well as their own knowledge and experiences. The lesson plan includes why the topic is important, procedures for communicating the ideas, materials to be used, and assessment procedures.

This helps the prospective teachers to see that they can figure things out for themselves.

The modified jigsaw component lives within a community of learners. The learning community piece of this structure is a group of students who have the common goal of teaching and learning mathematics content to each other. They have a shared responsibility for this and they depend on each other to learn.

These methods are also connected by a lesson study model where they together analyze the lesson that each member of the community has taught via a jigsaw method of each student being responsible for teaching a certain topic to the others. Each person in the community plays the role of the leader when they are teaching a concept, and the role of the learner when they are experiencing a lesson when another member of the community is the leader. After the lesson is taught, both the leader and the learner discuss the lesson and how it could be improved. Each student is also required to teach three mathematics lessons in their field experience during the semester, and often the learning community will discuss these lessons as well.

Methods Instructor C

Modeling best practices in teaching is a main goal of this instructor, so that the prospective teachers can experience a different way of learning mathematics. This instructor may focus on a specific technique each class period, such as questioning and wait time or centers. At the end of class the instructor makes explicit what technique was being modeled and they discuss the technique.

Another major goal is to help the prospective teachers to understand that if you allow children opportunities to solve problems on their own, then they can figure out the mathematics. The methods students get excited when they figure out a concept on their own. This instructor helps the methods students to learn how to think by encouraging them to think. One of the methods students commented near the end of the semester that if she had been taught mathematics this way in K-12 school, then perhaps she would not be so scared of it now.

This instructor's students saw growth in analyzing alternate algorithms on three different items involving multiplication, subtraction, and division. The instructor spends a lot of time on place value concepts and alternate algorithms which may have impacted the growth in these areas. Interestingly, while the instructor spends a lot of time on place value and alternate algorithms for addition and subtraction, no time is spent on alternate algorithms for multiplication and division. However, a better understanding of place value and being open to alternate algorithms for addition and subtraction seemed to translate into a better understanding of alternate algorithms for multiplication and division.

A specific activity that is done to develop place value concepts involves using different symbols and names to develop a base five system. The scenario given to the students is that they only have five symbols, different from Hindu Arabic numerals, available for their numeration system. They are asked to count and develop a chart similar to a hundreds chart with this notation. The instructor asks questions to help the students to make connections between the chart, the manipulatives, and the written numerals. The class discusses what is important about a place value system and connects this back to a base ten system.

Another activity that may help the methods students to better understand and be open to alternate algorithms involves viewing video clips of children's mathematical thinking. The instructor first gives the methods students an addition problem with two three-digit numbers. The methods students find the answer mentally and then share the methods of computation they used. After putting the different methods on the board, the class watches a video clip where children are sharing their invented algorithms, which usually match the mental algorithms the methods students used. A similar activity is done with subtraction algorithms. The methods students are amazed that the children invented the different strategies and at how well the children communicate their thinking.

While this instructor's students saw growth on a geometry question involving analyzing characteristics of a two dimensional figure, no specific activities could be pinpointed that might have impacted this. Just as the students were able to figure out alternate algorithms for division and multiplication although the class had not specifically done this, perhaps the students improve their mathematical thinking and ability to figure things out through problem solving and collaborations during class.

Methods Instructor D

This instructor, who saw a large increase in students understanding of order of operations, reported that each semester of this study (as well as in many previous semesters) questions arise from prospective teachers about order of operations. These questions are always rooted in field experiences where the prospective teachers ask for a review of order of operations because they have experienced needing to know it during their field experiences. So, field experiences may guide a need to know feeling and therefore make the prospective teachers have a more productive disposition to learning the material.

This instructor's students also had an increase in an understanding of pi. This instructor does two activities related to pi that may have led to this increased understanding. First, they do an activity with circular objects of many sizes, coffee cans, coins, hula hoops, etc. They measure the circumference of the circle with a tape measurer, and then lay the tape measurer on the table, holding their finger on the circumference mark. Then, laying the diameter of the object across the tape measurer, they see that the circumference is three of the diameters, plus a little. This activity may help them to see pi and to truly understand that it is a little more than three.

The second related activity is reading the children's book *Sir Cumference and the Dragon of Pi* (Neuschwander & Geehan, 1999). The prospective teachers are asked to pull out the mathematical concepts and think about how the children's literature could be used to teach mathematics concepts. The Sir Cumference series of books are very clever, and include visually stimulating illustrations.

Both the measuring activity and the children's literature provide "visual imagery" which this methods instructor believes is very important. Because her students have seen pi as a little more than 3 in the measuring activity, and because they have seen it developed in the children's book, they should have a clear visual image of what pi is. This instructor talked about the importance of hands-on activities to help them see the mathematics.

Analysis of Interviews

All of the learning opportunities in the theoretical framework and accompanying figure 1.2 from chapter one emerged in the interviews. However, after analyzing the interviews some adjustments in this framework are important. These learning opportunities that may increase a SUM in prospective teachers are not mutually exclusive. They are all interrelated. For example, a field experience may be a good opportunity for a prospective teacher to gain experiences with children's mathematical thinking. However there are other ways to gain experiences with mathematical thinking and there are other things that may happen in a field experience that may lead to increased SUM. Each opportunity is important and interrelated. Each of the methods instructors commented on how much time all of these takes, and commented that they need more time with the prospective teachers.

Readings

In the original framework, readings and discussions included journal articles, textbooks, and mathematics curriculum materials, all of which showed up in the interviews. Relevant children's literature may be another reading to be added to this list. Children's literature may provide visual images of the mathematical concepts, as well as help with the prospective teacher's attitudes towards learning mathematics. Reading and analyzing NCTM and/or state standards was also part of each of these courses. Analyzing these documents may contribute to the affective goals of helping prospective teachers understand why they need more content understanding.

Activities and Problem Solving

Activities that encourage and help prospective teachers to construct their own knowledge and gain visual images are important. Specific hands on activities such as finding pi by measuring circles of different sizes, analyzing three dimensional geometric models, and using base ten blocks and non-base ten models to illustrate numbers and operations were brought up in the interviews. Another factor important to the activities was that they are either situated in a classroom setting, or the idea is related to children's thinking and pedagogical issues.

Experiences with Children's Mathematical Thinking

Several opportunities for experiences with children's mathematical thinking are created within these methods classes. Video clips of mathematics interviews with children as well as video clips of classroom interactions are used. Having prospective teachers interview children not only provides opportunities to listen to children talk and think about mathematics, but also provides experiences in forming good questions to encourage and better understand their thinking. A well designed field experience also provides experiences with children's mathematical thinking. All of these experiences help prospective teachers to unpack and better understand mathematics themselves. These experiences also seem to develop productive beliefs about the depth of children's understanding as well as attitudes about teaching and learning mathematics.

Manipulatives

Manipulatives are an integral part of these methods courses. Manipulatives provide "visual images" of the mathematics that help prospective teachers to make sense of the mathematics as well as to hopefully remember the mathematics better through those images. Many of the activities in the methods courses involve hands on manipulative use. Modeling number operations with concrete materials helps the prospective teachers to make sense of the algorithms. One instructor reported that towards the end of the semester, the students do not pull the manipulatives off the cart as often as they are able to visualize them. They are still thinking with the visual images of the manipulatives but no longer feel as much of a need to actually use them once they understand the mathematics in that way. Hands on materials help them to make sense of the mathematics and to construct visual images of the concepts.

Field Experiences

Field experiences connected to a methods course can provide opportunities to increase SUM, as well as opportunities to improve beliefs and attitudes about mathematics. When the prospective teachers see a mathematics topic being taught in the elementary classroom that they do not remember, then this can lead to a discussion in the methods course that refreshes the topic for them and perhaps gives them a new way of looking at the concept. Field experiences can lead to opportunities to experience children's mathematical thinking and communication. These opportunities can help prospective teachers to see the depth of the mathematical thinking that the children are capable of and therefore help the prospective teachers to understand the need to learn mathematics more deeply themselves.

Communication

Opportunities for discussion were originally included with the readings, but a broader category of communication needs to be included in the learning opportunities list. Communication includes using precise language about mathematics. Opportunities to ask appropriate questions to guide and understand other's mathematical thinking help to develop understanding. Opportunities to listen to children's and colleague's mathematical communications may impact SUM. Finally, providing explanations of mathematical concepts in ways that both colleagues and children can understand may help prospective teachers to make sense of mathematics. Therefore, opportunities for communication are another learning opportunity that may impact content knowledge and is therefore being added to the original model.

Beliefs and Attitudes

Affective goals are intertwined with content goals in these methods courses. Although this study is focused on content understanding, beliefs and attitudes are so intertwined that they can not be left out of the model. Prospective elementary teachers are often very afraid of mathematics and of teaching mathematics to children. Improving beliefs and attitudes helps content knowledge, and improving content knowledge helps beliefs and attitudes. They need to become more comfortable verbalizing their knowledge, as well as confronting their unproductive beliefs. They need to feel safe talking about mathematics within the methods course. Prospective teachers must become comfortable constructing their own mathematical knowledge and allowing their future students to do the same. While this study makes no claims about what learning opportunities improve beliefs and attitudes, this researcher suspects that the six opportunities in this model would be a good theory to be tested.

Considering all of this, figure 4.1 illustrates the learning opportunities that may impact SUM during the mathematics methods course. All of these are interrelated with each other, as well as with beliefs and attitudes. The circles contain learning opportunities that impact SUM, while the rectangle of learning opportunities that impact beliefs and attitudes can not be left out of the model.

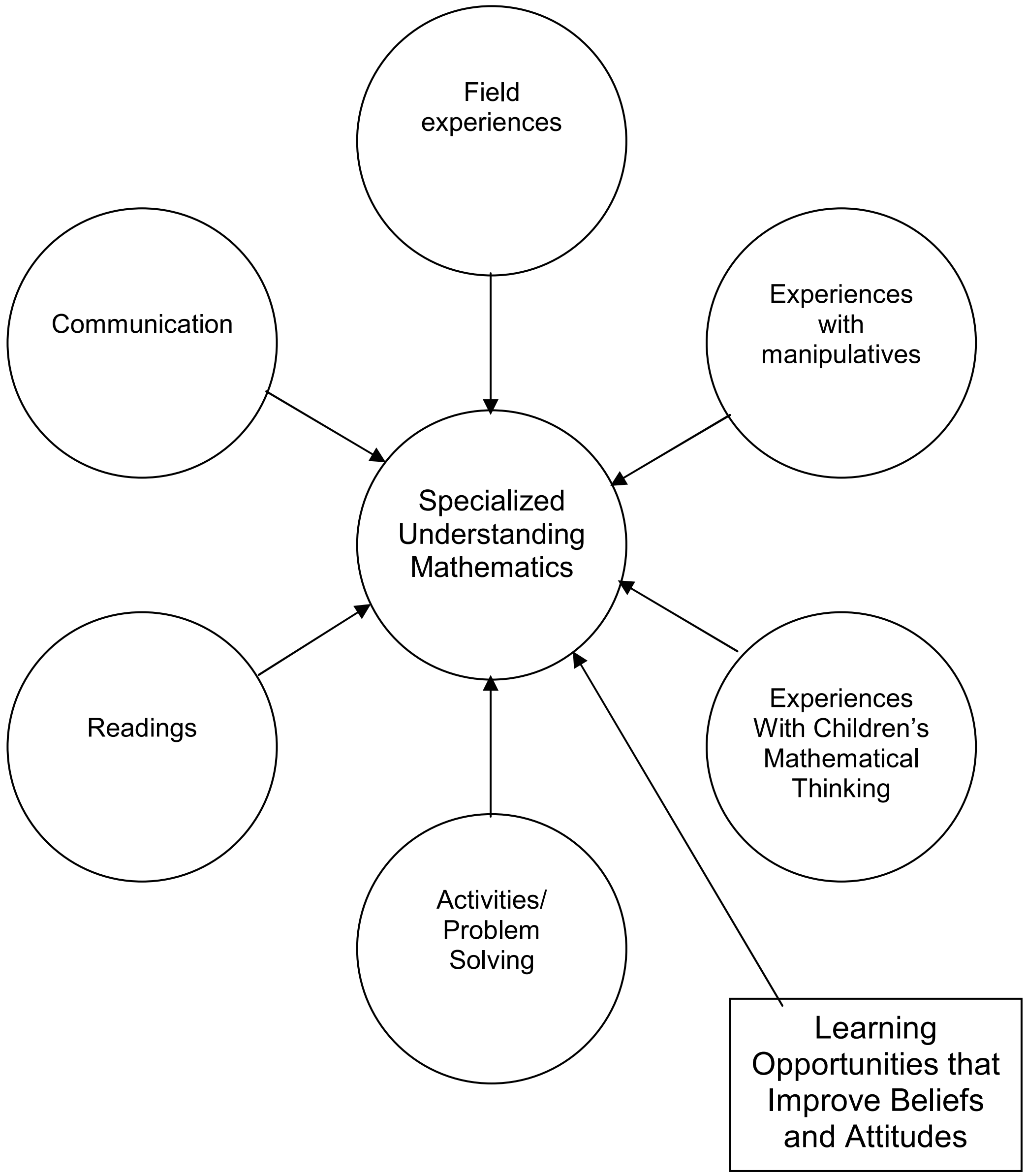


Figure 4.1. Learning Opportunities Impacting SUM.

CHAPTER V

CONCLUSIONS AND IMPLICATIONS

This study has provided insight into areas of mathematical understanding that specifically need improving in prospective elementary teachers and where and how this understanding is perhaps gained. Helping prospective teachers understand mathematics better is imperative to create a mathematically literate population necessary for a healthy economy, environment, and society. Mathematics educators need to reflectively analyze and research their practice, so that we can learn what works and what does not work in helping to improve the SUM that teachers possess and are able to use.

The reform vision of the NCTM PSSM document requires that teachers have a conceptual and connected understanding of mathematics so that they can guide discussions, ask appropriate questions, and implement effective activities. A teacher must understand place value and operations flexibly if they are to analyze alternate algorithms, and encourage alternate ways of thinking about mathematics. Multiplying 25 times 35 can be done in many ways, and teachers need to understand this and be able to understand alternate ways besides the process they memorized when they were young. How mathematics educators can help teachers to gain this knowledge is important to understand.

Summary of the Study

The Content Knowledge for Teaching Mathematics items from the number and operation and geometry constructs were administered during the first two weeks of the semester to 244 prospective elementary teachers enrolled in a mathematics teaching methods course at seven sites to determine what areas of strengths and what areas of weaknesses exist in the prospective teachers' mathematical understanding at this point in their teacher training program. Information about previous content courses taken was also collected at this time to determine if there was a relationship between quantity and type of content courses and content understanding. The same form of the CKTM instrument was given as a post test during the last two weeks of the semester. Some students were either absent or had withdrawn, so 221 of the original sample of 244 students took the post test.

An item analysis was conducted on the pretest items that were missed the most often and the items that were answered correctly most often to better understand the areas of strengths and weaknesses of their knowledge. Statistical tests were also run to look for relationships between number and type of content courses taken and scores on the pretests.

A paired samples t test was run on the pretest and posttest scores to consider whether the content knowledge had changed during the methods course. An item analysis was conducted on the overall posttest results to determine how their understanding compared on the pretest and the posttest. To determine whether growth in a particular area grew at a particular research site, an item analysis was conducted by site. Any growth on a particular item was followed up by an interview with the methods instructor to ask them to reflect on learning opportunities in their course that may have impacted this better understanding.

Findings

Question 1

What are the areas of strengths and what are the areas of weaknesses in the SUM, as measured by the Content Knowledge for Teaching Mathematics instruments, of prospective elementary teachers as they enter their mathematics methods course?

Prospective teachers in this sample showed knowledge in being able to perform computations, interpret definitions, and seemed to understand geometry more than specialized number and operation concepts. They were able to identify properties of two and three dimensional shapes and were able to use the order of operations. Many of them were able to interpret and apply a geometric definition. Six of the easiest questions were from the number and operation content area and five were from the geometry content area.

In geometry, the participants struggled with questions where they needed to understand the meanings of the formulas, and how changing dimensions affects volume, areas, and perimeter. What it means to say that the area of a circle equals pi times radius squared and understanding this as a relationship between area and pi is a concept that the participants seemed to not have clarity on. Irregular shapes such as non-isosceles trapezoids, as well as rotated figures such as parallelograms that are not parallel with the top of the paper, are not as familiar to the students.

Multiple representations and alternate algorithms were areas of weakness in the participant's understanding. Several questions that were missed the most frequently required that they evaluate the validity of a student's different way of doing an operation. These questions were answered "I'm not sure" at a high rate, in fact more people answered this on two of these questions than got the correct answer. Also, understanding that fractions can be modeled with many representations besides one circle or a set of objects, particular in understanding a division model was found to be an area of weakness.

Students who took the Math for Teachers I and II courses scored significantly higher on the pretest, meaning there was a correlation between students who took both of these courses and the SUM they had as they entered their methods course. In examining this finding more closely, a significant correlation was found between students who took the Math for Teachers II course, which includes geometry concepts, and the geometry items on the pretest. However, there was not a significant correlation between the students who took the Math for Teachers I course, which includes number and operation concepts, and the number and operation items on the pretest. Quantity of mathematics courses did not have a significant relationship with the score on the pretest.

Question 2

Does a SUM change as prospective elementary teachers take their methods course?

The students did exhibit a statistically significant growth ($p = .015$) in content understanding as they took their methods course. This was analyzed using a paired samples t test. The growth translated to approximately one question higher. A very high correlation was found between which items the participants found easiest and hardest on the pretest and the posttest.

Question 3

If differences in growth in mathematical knowledge are found, what learning opportunities during the methods course may have contributed to any growth in knowledge?

The item analysis of each site's responses showed that four of the sites had significant increases on particular items. Each of the methods instructors at these sites were interviewed to try to pinpoint what learning opportunities might have helped the students to understand these concepts better. These were analyzed through the theoretical framework in figure 1.2.

While all of the opportunities in the theoretical framework emerged within the interviews, adjustments were made to this theory based on the interviews. Figure 4.1 illustrates the new theory. Opportunities for communication, including using precise language, listening, questioning, and explaining was added as another category that may help in developing a SUM in prospective teachers. Readings, such as textbooks, curriculum materials, standards, and children's literature are important learning opportunities. Manipulatives help prospective teachers to make sense of mathematics and create visual images of the concepts. Experiences with children's mathematical thinking, through video clips, analyzing student work, interviews, and field experiences, impact both mathematical understanding as well as beliefs about mathematics teaching and learning. Mathematical activities and problem solving help prospective teachers to unpack knowledge and explore the mathematics. Connected field experiences can also provide prospective teachers with a better understanding of mathematics, as well as the teaching and learning of mathematics.

Conclusions

Prospective teachers need more and better opportunities to increase their SUM. While their SUM does significantly increase from a statistical perspective during their methods course, and while students who take specialized content courses do have statistically significantly more specialized understanding than those who did not, much still needs to be done in improving their understanding of mathematics even more significantly. By way of example, teachers who are unable to answer why division by zero can not be defined, are highly unlikely to be able to help their students understand this “why.”

Improvements are needed in the areas of understanding multiple representations and the explanations behind the mathematics. With a better SUM teachers are better able to teach their students to understand mathematics deeply and conceptually. Perhaps an understanding of number and operation is so much more difficult for them to “relearn” because they already know one method of multiplying 25 times 35, so it is very difficult for them to open up to multiple representations and algorithms, as evidenced by the most frequently missed items on both the pretest and posttest in this study. They have memorized rules and processes, but need experiences that help them to understand why these rules and processes are valid. Interestingly, three questions involving non-traditional algorithms, had high responses of “I’m not sure" (35.2%, 18.9% and 18%, which in two of those cases was higher than the percentage answering correctly). So, perhaps when confronted with this in the classroom, they may answer, "I am not sure" and investigate it further with the students instead of just saying the method is wrong.

As an example in geometry, meaning needs to be associated with the formulas for perimeter, area and volume. Simply memorizing the formulas is not sufficient for them to apply the formulas flexibly. Relationships between different measurements (such as the diameter and circumference of a circle) must be explored and understood. Also, it is critical that students explore different forms of geometric figures, such as non-isosceles trapezoids and non-regular polygons to be able to recognize these figures as trapezoids and polygons. Different rotations of the figures need to be seen often. If a square is always looked at with its sides parallel to the edges of the paper, then a square rotated 45 degrees may not be recognized as a square.

Experiences with multiple representations and non-traditional algorithms are important for prospective teachers. In France, the name for a fraction is "camembert", a round cheese. In the United States, this circular area model is used so often that prospective teachers have difficulty modeling fractions in other ways. Not only are other area models important, such as a square or rectangle or pattern blocks, but also measurement, ratio, division and set models need to be explored and understood.

Prospective teachers need to believe that it is important to understand alternate algorithms before they can be open to learning them. Mathematics educators need to help these students to relearn and unpack knowledge they believe they already have. More time and focus on the math for teachers I course are important.

While the Math for Teachers II course does seem to be associated with more specialized understanding of geometry, improvements in it need to be made in giving meaning to formulas and in looking at non-traditional forms of figures. Quality, not quantity, of mathematics courses seems to be the key.

While the SUM did show a significant increase in prospective teachers during the mathematics methods course, more needs to be done so that a larger increase can be accomplished. Several learning opportunities may help to develop an increased SUM. These learning opportunities include field experiences, manipulatives, experiencing children's mathematical thinking, good activities, readings, and opportunities for communication. Mathematics educators might find each of these components helpful in developing a SUM in their students, whether in a methods or a content course.

Implications for Practice and Further Research

How do we help teachers to acquire a SUM? That is the overarching question that many mathematics educators struggle with. This study can not answer this huge question, but it does shed some light.

More time and focus in improving the mathematics content courses for prospective teachers is important. Instructors of these content courses should have opportunities to share activities and problems that they believe to help improve student's understandings in the areas that this study showed they had weaknesses in. Research should then be done on implementing these activities to determine which of the activities lead to more growth in a SUM.

Quality, not quantity, of mathematics courses seems to be the key. The number of mathematics courses taken was not found to be an indicator of how well they scored on the pretest in this study. Simply having the students take more content courses does not seem to be the answer. Content courses must provide opportunities for students to truly understand concepts that they have memorized processes for. They must have experiences with multiple representations and reasoning why things work the way they do in mathematics.

Exploring alternate algorithms should be encountered often within a teacher education program. However, beliefs and attitudes about mathematics and teaching and learning mathematics are very intertwined with learning alternate algorithms. Prospective teachers first need the belief that it is important to learn alternate algorithms. Many prospective teachers are very comfortable with their memorized procedures and it may be difficult for them to be open to other methods. It is important that they have opportunities to explore different methods and different ways of looking at mathematics, as they will encounter students in their classrooms who will think about mathematics differently than the way they themselves were taught. Watching video clips of children's mathematical thinking, more structured field experiences or examination of student's mathematical work samples may be helpful for prospective teachers to understand the depth of children's mathematical thinking. Research is recommended to see whether these types of activities improve both the beliefs about mathematics teaching and learning as well as the SUM. In order to encourage their students' mathematical thinking, teachers must be able to appreciate and evaluate the reasonableness of their thinking. However to be able to do this, they must have for themselves a deeper understanding of mathematics.

A large percentage of participants answered "I'm Not Sure" for the alternate algorithm questions on the CKTM survey. Further investigation into this would be interesting. What will they do in practice when faced with this type of situation? Will they answer I am not sure in the classroom and investigate with the students? Or will they fall back into traditional algorithmic methods of teaching? On a measure currently under development in the content area of earth sciences, a follow up question to each content item is "How sure are you of your answer?" (Leslie, Dockers, & Wavering, 2006). In science, people often have misconceptions that they believe to be true, and therefore it is difficult to help them to let go of these misconceptions. This assessment is also followed up by questions of how they might teach a certain topic that is often filled with misconceptions, such as a solar eclipse. This type of questioning brings up misconceptions, as well as links it to pedagogical content knowledge. Similar items in the mathematics content area would be interesting to help mathematics educators to understand what beliefs about mathematics students hold on to, what areas they are really unsure of, and how their content understanding is linked to pedagogical content understanding.

Multiple representations and multiple uses of manipulatives are important for prospective teachers to encounter. The circle is used so often for a representation of a fraction that prospective teachers find it difficult to think of fractions in any other way. Learning mathematics through the use of manipulatives for both prospective teachers and their future students helps learners of mathematics to see and touch the mathematics. However, prospective teachers need to be able to use these flexibly. Base ten blocks are a great tool for understanding whole numbers, place value, and operations. However, prospective teachers after using the blocks this way, need to also think about how the unit would change if these same manipulatives were being used for decimal concepts.

Mathematics methods instructors should consider ways to incorporate all six learning opportunities depicted in figure 4.1 into their classes and research how each one works. Sharing of ideas of how to provide these opportunities into a methods, or content, course should be encouraged in the field. Observing mathematics methods courses to document these opportunities from an outside prospective is an area of research worth doing. For example, the techniques of incorporating the use of manipulatives may be very different. Perhaps the instructor is doing it, or the students are using the manipulatives individually, or in groups. Perhaps the activity is guided in detail, or completely open discovery. Observation of this and analysis of which specific techniques seem most fruitful is important.

Some methods and content courses have mathematics focused field experiences in connection with the courses. One of the methods courses involved in this study normally has a focused field experience during the semester but was unable to this semester due to a variety of circumstances. An interesting and important area of study is to examine the effects of such field experiences on the SUM of prospective teachers. Does having a connected field experience in a methods course affect the SUM of the participants? Does a connected field experience in a methods course affect the beliefs and attitudes about mathematics and the teaching and learning of mathematics?

Sankey (2006) found that prospective elementary teachers involved in a focused mathematics and science field experience connected with their mathematics and science content courses improved their attitudes about the content course and why learning the mathematics and science in specialized ways was so important. Further investigation into these types of experiences is recommended to understand how they affect beliefs and attitudes as well as content understanding. Does a connected field experience in a content course affect the SUM of the participants? Does a connected field experience in a content course affect the beliefs and attitudes about mathematics teaching and learning?

Because mathematics content knowledge and beliefs and attitudes are intertwined (NRC, 2001a), studies comparing prospective and in-service teachers beliefs and attitudes to their content understanding are recommended. Is there a relationship between the beliefs and attitudes about mathematics teaching and learning and the understanding of mathematics in prospective teachers? Does this relationship exist in in-service teachers?

Do more specialized mathematics content courses improve mathematics understanding? All of the sites involved in this study offer only two specialized content courses specifically designed for future elementary school teachers. Recent policy documents such as those published by the National Science Foundation and the Conference Board of Mathematical Sciences recommend three special content courses for prospective elementary teachers. Research is recommended to see whether the content knowledge of students who do take three specialized content courses that are beginning to be offered at some schools around the country, improve the SUM that prospective teachers possess.

Replication of this study at other institutions around the country, and around the world, is recommended to see if similar results are found elsewhere. Multi country studies of this type would be wonderful in providing information on whether other countries have similar findings as in the United States. Do prospective teachers at other institutions, both within the United States and in other countries, have similar strengths and weaknesses coming into their methods experiences? If programs are found where they have a stronger understanding when entering their methods courses, what experiences before then may have led to this stronger understanding? If their knowledge improves during the methods course, what learning opportunities may have led to this improved knowledge?

Longitudinal studies in this area are much needed in the field. Following a cohort of prospective students as they enter their content courses, as they exit their content courses, as they enter their methods courses, as they exit their methods courses, as they graduate with their teaching license, and as they enter the field of teaching is important, although a difficult prospect to track students this long. One of the challenges to this is documenting what the learning opportunities are within the teacher education programs. Perhaps asking students to journal opportunities that they believe have contributed to increased content knowledge, as well as have professors journal the learning opportunities that they have provided could offer insight. This requires compensation for the professors, as well as professors who are willing to open their practice to this type of investigation. Understandably, opening one's practice to analysis by others can be intimidating, but it seems important in understanding what we are doing well and what needs improvement. Doing this at several different institutions with different models of mathematics teacher preparation would help to define what learning opportunities at which points in the teacher training program lead to improved understanding of mathematics, as well as improved beliefs and attitudes.

Following the teachers into their practice would inform the field as to whether more content understanding and reform oriented beliefs and attitudes are evident in their teaching practice and whether these change as they gain teaching experience. Hill, Rowan, & Ball (2005) have been able to provide evidence that teachers who do better on the CKTM items have students who do better on the Terra Nova tests. Do teachers who do better on CKTM teach in more reform oriented methods?

For this researcher, more questions have been raised than answered. This study has provided a clearer picture of the strengths and weaknesses in the understanding of mathematics as prospective teachers enter their methods course. This study has provided evidence that content understanding does grow as the prospective teachers take their methods course and insight into learning opportunities that may affect SUM. Much needs to be learned about how to help prospective teachers gain more SUM as well as improved beliefs and attitudes. Mathematics educators need time and financial resources to continue to learn about this important issue.

References

References

Adler, J. Ball, D. L., Krainer, K., Lin, F. L., & Novotna, J. (2004, July). *Research on mathematics teacher education: Mirror images of an emerging field*. Plenary session at the International Congress on Math Education, Copenhagen, Denmark. Slides retrieved June 20, 2005, from http://www-personal.umich.edu/~dball/ST3_ICM10.pdf

Ball, D. L. (1987, April). Dimensions of teacher beliefs. Paper presented at the National Council of Teachers of Mathematics Research Presession, Anaheim, CA.

Ball, D. L. (1988a). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Vol. 2. Teachers' subject matter knowledge and classroom instruction* (pp. 1-48). Greenwich, CT: JAI Press.

Ball, D. L. (1988b). *The subject matter preparation of prospective mathematics teachers: Challenging the myths* (Research Report No. 88-3). East Lansing: Michigan State University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., & Bass, H. (2005, September*). Learning the mathematical work of teaching*. Paper presented at the meeting of the Appalachian Association of Mathematics Teacher Educators, Lexington, KY.

Ball, D. L., & McDiarmid, G. W. (1990). The subject matter preparation of teachers. In W. Houston, M. Haberman, & J. Sikula (Eds.), *Handbook of research on teacher education* (pp. 437-449). New York: Macmillan Publishing.

Ball, D. L., & Wilson, S. W. (1990). *Knowing the subject and learning to teach it: Examining assumptions about becoming a mathematics teacher* (Research Report No. 90-7). East Lansing: Michigan State University, National Center for Research on Teacher Education. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268.

Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of empirical literature*. Washington, DC: Mathematics Association of America and the National Council of Teachers of Mathematics.

Benbow, R. M. (1993, April). *Tracing mathematical beliefs of preservice teachers through integrated content-methods courses.* Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

Bobis, J., & Cusworth, R. (1995). Attitudinal shifts towards mathematics of preservice teachers. In B. Atweh & S. Flavel (Eds.), *Proceedings of the 18th annual conference of the Mathematics Education Research Group of Australasia* (pp. 109-144). Darwin, Australia: Mathematics Education Research Group of Australasia.

Cipra, B. (1991). *On the mathematical preparation of elementary school teachers* (Report on a Conference held at University of Chicago). Chicago, IL: University of Chicago Press.

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Washington, DC: Mathematical Association of America.

Cooney, T. J. (2001). Considering the paradoxes, perils and purposes of conceptualizing teacher development. In Fou-Lai Lin & T. J. Conney (Eds.), *Making sense of mathematics teacher education* (pp. 9-32). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Dewey, J. (1916). *Democracy and education*. New York: The Free Press.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94-116.

Featherstone, H., Smith, S.P., Beasley, K., Corbin, D., & Shank, C. (1995). *Expanding the equation: Learning mathematics through teaching in new ways* (Research Report No. 95-1). East Lansing: Michigan State

University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Floden, R. E., McDiarmid, G. W., & Wiemers, N. (1990). *Learning about mathematics in elementary methods courses* (Research Report No. 90-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Foss, D. H., & Kleinsasser, R. C. (1996). Preservice elementary teachers' views of pedagogical and mathematical content knowledge. *Teaching and Teacher Education, 12*(4), 429-442.

Fuller, R. A. (1997). Elementary teachers' pedagogical content knowledge of mathematics. *Mid-Western Educational Researcher, 10*(2), 9-16.

Gellert, U. (1999). Prospective elementary teachers' comprehension of mathematics instruction. *Educational Studies in Mathematics, 37,* 23-43.

Gellert, U. (2000). Mathematics instruction in safe space: Prospective elementary teachers' views of mathematics education. *Journal of Mathematics Teacher Education, 3,* 251-270.

Guralnick, D. B. (Ed.) (1987). *Webster's New World Dictionary*. New York: Warner.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H. C., Schilling, S., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Hill, L. (1997). Just tell us the rule: Learning to teach elementary mathematics. *Journal of Teacher Education, 48*(3), 211-222.

Judson, E., & Sawada, D. (2001, April). *Tracking transfer reform: Tracking transfer of reform methodology from science and math college courses to the teaching style of beginning teachers of grades 5-12*. Paper presented at the meeting of the American Educational Research Association, Seattle, WA.

Lappan, G., & Even, R. (1989). *Learning to teach: Constructing meaningful understanding of mathematical content* (Research Report No. CP 89-3). East Lansing: Michigan State University, National Center for Research on

Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

Leslie, K. M., Dockers, J.E., & Wavering, M.J. (2006, January). *The development and validation of a two-tiered instrument to identify alternative conceptions in earth science.* Paper presented at the meeting of the Association of Science Teacher Educators International, Portland, OR.

Lubinski, C. A., Otto, A. D., Rich, B. S., & Jaberg, P. A. (1998). An analysis of tow novice K-8 teachers using a model of teaching-in-context. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the twentieth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 704-709). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. (ERIC Document Reproduction Service No. ED430776)

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum.

McDiarmid, G. W. (1989). *What do prospective teachers learn in their liberal arts courses?* (Issue Paper No. 89-8). East Lansing: Michigan State University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Mewborn, D. S. (2000, April). *An analysis of the research on K-8 teachers' mathematical knowledge*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Moses, R. P., & Cobb, C. E., Jr. (2002). *Radical equations: Math literacy and civil rights.* Boston, MA: Beacon Press.

National Center for Research on Teacher Learning. (1992). *Findings from the Teacher Education and Learning to Teach study* (Final report). East Lansing: Michigan State University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: The National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000*). Principles and standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics.

National Research Council. (2001a). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.). *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education.* Washington, DC: National Academy Press.

National Research Council. (2001b). *Educating teachers of science, mathematics, and technology: New practices for the new millenium*. Washington, DC: National Academy Press.

Neuschwander, C., & Geehan, W. (1999). *Sir Cumference and the dragon of Pi.* Watertown, MA: Charlesbridge.

Peters, R. S. (1977). *Education and the education of teachers.* London: Routledge and Kegan Paul.

Philipp, R. A., Clement, L. L., Thanheiser, E., Schappelle, B. P., & Sowder, J. T. (2003, April). *Integrating mathematics and pedagogy: An investigation of the effects on elementary preservice teachers' beliefs and learning of mathematics.* Paper presented at the National Council of Teachers of Mathematics Research Presession, San Antonio, TX.

RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND Publications.

Roth McDuffie, A., Mather, M., & Reynolds, K. (2004). Professional development of teacher and teacher educator through sustained collaboration. In T. Watanabe & D. Thompson (Eds.). *The work on mathematics teacher educators: Exchanging ideas for effective practice.* San Diego, CA: Association of Mathematics Teacher Educators.

Sankey, Y. K. (2006, January). *The 5 E's as a tool for planning a math and science learning community.* Paper presented at the international meeting of the Association of Science Teacher Educators, Portland, OR.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Strawhecker, J. E. (2004). *Preparing elementary teachers to teach mathematics: How pedagogy, content, and field experience impact content knowledge and beliefs.* Unpublished doctoral dissertation, University of Nebraska.

Taylor, P. M. (2000). When are we ever going to use this? Lessons from a mathematics methods course. *School Science and Mathematics, 100*(5), 252-255.

Taylor, P. M. (2004). Encouraging professional growth and mathematics reform through collegial interaction. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the Teaching of Mathematics: Sixty-sixth yearbook* (pp. 204-221). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Tirosh, D., Fischbein, E., Graeber, A. O., & Wilson, J. W. (1999). *Prospective elementary teachers' conceptions of rational numbers.* Retrieved July 12, 2005 from http://jwilson.coe.uga.edu/Texts.Folder/tirosh/Pros.El.Tchrs.html

Tirosh, D., & Graeber, A. O. (1991). The effect of problem type and common misconceptions on preservice elementary teachers' thinking about division. *School Science and Mathematics, 91*(4), 157-163.

Wilcox, S. K., Lanier, P., Schram, P., & Lappan, G. (1992*). Influencing beginning teachers' practice in mathematics education: Confronting constraints of knowledge, beliefs, and context* (Research Report No. 99-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning. Retrieved June 10, 2005, from http://ncrtl.msu.edu/full.htm

Wilson, S. M. (1990). The Secret garden of teacher education. *Phi Delta Kappan, 72*(3), 204-209.

Appendices

Appendix A

Study of Instructional Improvement/Learning Mathematics for Teaching

Content Knowledge for Teaching Mathematics Measures (CKTM measures)
Elementary Mathematics Release Items
2002
CONTENT KNOWLEDGE ITEMS[1]

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

 Measures development supported by NSF grants REC-9979873, REC-0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
x 25	x25	x 25
125	175	25
+75	+700	150
875	875	100
		+600
		875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all Whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

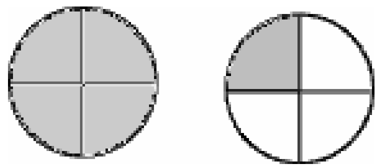
a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

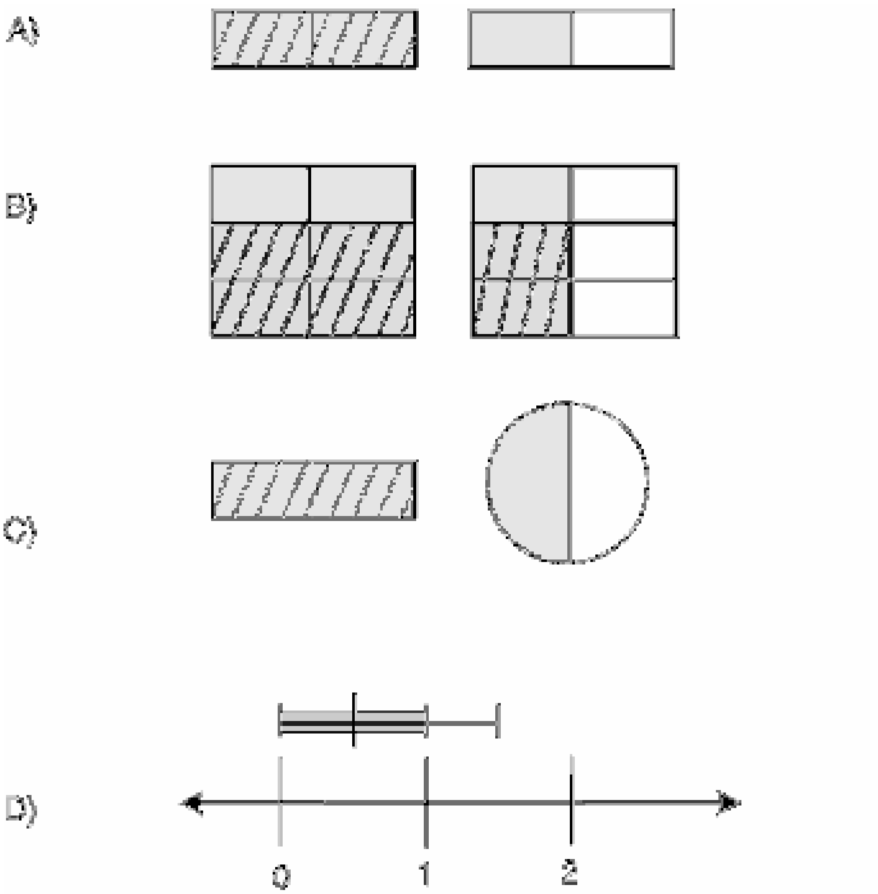
b) 5/3

c) 5/8

d) 1/4

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)



7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

Appendix B

Interview Protocol for Mathematics Methods Instructors Interviews

1) How many hours a week does your mathematics methods course meet?

2) Is there a field experience component with your course? If so, how many hours?

3) What are your major goals for the mathematics methods course?

4) What happened in your class that may have impacted the content understanding on each of the items that your students showed a significant gain on?

Appendix C

Catalog Course Descriptions of Math for Teachers at Each Research Site

A. Catalog descriptions of math for teachers I at each site

Math 201 Structure of the Number System 3 credits

Problem solving, sets and relations, numeration systems, integers, elementary number theory, rational numbers and decimals. Prereq: Two years of algebra and one year of geometry in high school and satisfactory placement test score.

MATH 1410 The Structure of the Number System 3 Credits

Recommended for prospective elementary education teachers. Topics include problem solving, sets and relations, numeration systems, integers, elementary number theory, rational numbers, decimals and algebraic applications. Prerequisite(s): High school algebra I and algebra II and geometry and ACT math score of at least 19; or DSPM 0850 or equivalent math placement score

MATH 1410 Survey of Elementary Mathematics I 3 Credits

Introduction to sets and operations on sets, properties and operations on whole numbers, integers, rational and real numbers. Prerequisite: Admission is restricted to students majoring in Elementary Education.

MATH 1410 Number Concepts/Algebra Structures 3 Credits

This course includes symbolic logic, logical reasoning, history of early numeration systems, set theory with rules of operations and Venn diagrams, relations and functions, the systems of whole numbers, of integers, and of rational numbers. Any student would profit from this course, but it is especially targeted to the education major (elementary and non-math secondary).

(Prerequisite: Two years of high school algebra and one year of geometry or appropriate developmental math.)

MATH 1410 Number Concepts for Elementary Education 3 Credits

This course is a conceptual approach to the study of the properties of number sets within the real number system. Topics include tools for problem solving, sets, functions, logic, numeration systems, properties of and operations with whole numbers, integers, rational numbers and real numbers. Successful completion of an Arithmetic Proficiency Test is required. Prerequisites: Documented eligibility for collegiate mathematics; one high school credit each in algebra I, algebra II, and geometry.

MA 201 Mathematics for Elementary Teachers 3 Credits

Sets, numbers and operations, problem solving and number theory. Recommended only for majors in elementary and middle school education. Prereq: MA 109, 111.

MATH 231 Mathematics for the Elementary Teacher I 3 Credits

Number systems, primes, and divisibility; fractions; decimals; real numbers; algebraic sentences. Successful completion of a basic skills exam in mathematics is required for credit in this course. Designed for preservice teachers P-9. Prerequisite: completion of a general education required core course in mathematics.

B. Catalog descriptions of math for teachers II at each site

Math 202 Probability, Statistics, and Euclidean Geometry 3 Credits

Probabilities in simple experiments, measures of central tendency and variation. Basic plane and three-space geometry, congruence and similarity, constructions with compass and straightedge, transformations, area and volume measurement. Turtle graphs. Prereq: Two years of algebra and one year of geometry in high school and satisfactory placement test score.

MATH 1420 Geometry/Statistics 3 Credits

Recommended for prospective elementary education teachers. Topics include elementary probability and statistics, basic plane and 3-space geometry, congruence and similarity, constructions, transformations, area, volume, surface area and measurements. Prerequisite(s): High school algebra I and algebra II and geometry and ACT math score of at least 19; or DSPM 0850 or equivalent math placement score

MATH 1420 Survey of Elementary Mathematics II 3 Credits

Admission is restricted to students majoring in Elementary Education. Introduction elements of probability and statistics, basic concepts of Euclidean geometry including congruence, similarity, measurements, areas and volumes. Prerequisite: "C" or better in MATH 1410.

MATH 1420 Problem Solving/Geometry 3 Credits

A continuation of MATH 1410, this course includes elementary number theory, irrational number, basic algebra, interest (simple and compound), elements of plane and solid geometry (especially working with measurements and formulas), the metric system, and basic statistics. (Prerequisites: _MATH 1410 or consent of instructor).

MATH 1420 Geometry for Elementary Education 3 Credits

Topics include measurement, congruence, similarity, and graphing; constructions, theorems, and proofs in both non-coordinate and Cartesian settings; historical development of geometry as a tool. Activities will include creating models and manipulatives. Prerequisites: Documented eligibility for collegiate mathematics; one high school credit each in algebra I, algebra II, and geometry. Students who are subject to A89 admission requirements who do not have a high school credit in geometry must successfully complete MATH 0990 prior to enrollment in MATH 1420. (Formerly MAT 1240)

MA 202 Mathematics for Elementary Education 3 Credits

Algebraic reasoning, introduction to statistics and probability, geometry, and measurement. Prereq: A grade of "C" or better in MA 201. Also recommended: a course in logic (e.g. PHI 120) or a course in calculus (e.g. MA 123).

MATH 232 Mathematics for the Elementary Teacher II 3 Credits

Introduction to probability and statistics; geometric shapes; geometry of measurement; congruence and similarity. This course satisfies the area studies-natural and mathematical sciences for general education. Designed for preservice teachers P-9. Prerequisite: MATH 231.

Vita

Margaret Viola (Meg) Moss was born in Gastonia, North Carolina where she lived until going to college at the University of North Carolina, Chapel Hill. There she earned a bachelors degree in Secondary Mathematics Education in 1989. In 1992, she earned a Masters degree in Mathematics Education at Appalachian State University in Boone, North Carolina.

Meg Moss then moved to Ontario, Oregon where she was employed as a Mathematics instructor at Treasure Valley Community College. Her first year there, she was assigned to teach Math for Elementary Teachers courses, where she found her professional love of helping teachers to understand mathematics better so that they can teach mathematics better. She was a faculty fellow with the Oregon Collaborative for Excellence in the Preparation of Teachers (OCEPT), taught a variety of math courses and coordinated the teacher education program during her eight years at Treasure Valley.

It was then time to move closer to family and earn a doctorate degree, so Ms. Moss moved to Knoxville, Tennessee. Ms. Moss began her doctoral pursuit at the University of Tennessee, Knoxville in August, 2001 and has been a full time faculty member at Pellissippi State Technical Community College since August, 2000. At Pellissippi State, Meg Moss is an Associate Professor of Mathematics, Teacher Education Coordinator, and Principal Investigator of a National Science Foundation grant (#0302907).

Meg Moss has two wonderful sons, Nathan age ten and Adam age seven. She loves to travel, hike, listen to live music, and enjoy time with her sons. Upon graduation, Meg plans to relax, celebrate and consider the possibilities.