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REAL-TIME ORDER TRACKING FOR SUPPLY SYSTEMS WITH MULTIPLE TRANSPORTATION STAGES

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I am submitting herewith a dissertation written by Nana Bryan entitled "REAL-TIME ORDER TRACKING FOR SUPPLY SYSTEMS WITH MULTIPLE TRANSPORTATION STAGES." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Management Science.

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REAL-TIME ORDER TRACKING FOR SUPPLY SYSTEMS WITH MULTIPLE TRANSPORTATION STAGES

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DEDICATION

To Natalie who had to share her “mommy and me” time with this dissertation and still thinks I’m the best mommy ever.

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ABSTRACT

This dissertation studies a supply system consisting of a retailer, a manufacturer, and multiple transportation stages. The manufacturer fulfills the demand from the retailer for a single product. The replenishment process is not instantaneous. Orders may take more than one time period to be shipped from the manufacturer's location, and shipped orders pass through multiple transportation stages until they reach the retailer. Each stage may represent a physical location or a step in the delivery process. Shipments are not allowed to cross over in time. The movement of each shipment depends on the congestion and movements of shipments ahead of it.

A stochastic model is developed to evaluate the long-run average cost incurred by the retailer. The cost is modeled for a myopic order-up-to-level policy. Depending on the availability of real-time order tracking information, the cost function can have different expressions. The behavior of the cost functions with or without real-time tracking information and the difference between the two are studied for different parameters.

The first main section studies a model with the manufacturer's delays in the shipping process. Orders may take several time periods to leave the manufacturer's site. Numerical examples for various transportation congestion scenarios and for different shipping policies show which settings guarantee the lowest long-run average cost. The model also helps to draw some insights on how and when the retailer should place orders with the manufacturer.

The second section studies a model with no manufacturer's delay but with a limited number of tracking devices. The model calculates the long-run average cost using information collected from the tracking devices. The numerical examples help to determine the optimal placement of a given number of tracking devices to minimize the long-run average cost. The model also suggests the optimal number of tracking devices that brings the long-run average cost as close as possible to the long-run average cost with full real-time tracking information.

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CHAPTER 1
INTRODUCTION

Techniques such as just in time and lean are very effective approaches to reduce investment in inventory and, thereby, increase free cash flow. While the implementation of these techniques varies widely based on the business model, structure, domain, and the market served, they all require knowledge of the inventory status at different stages in the supply system.

Advances in information technology make it possible to obtain data that can be used to track order status in real time through all stages in the supply chain. This data can potentially be used to track partial orders and shipments even at the item level (Angeles, 2005; Kohn *et al.,* 2005; and Murphy-Hoye *et al.,* 2005). Such real-time tracking data can enable better supply chain management (Grahovac and Chakravarty, 2001; Karkkainen *et al.,* 2004). However, the value of this tracking information is not adequately understood (Zhang *et al.,* 2006). The lack of a firm understanding can lead to incorrect decisions on when, where, and to what extent these technologies should be deployed. It presents a challenge and an opportunity for quantitative methods to model and analyze the value of real-time information.

This thesis develops a stochastic model to evaluate the value of real-time shipment tracking information in a supply system. The supply system consists of a retailer, a manufacturer, and multiple stages of transportation. The retailer aggregates demand for a single product from his end customers and places orders on the manufacturer. The replenishment process between the manufacturer and the retailer is not instantaneous. Delays can occur at the manufacturer's site and during transportation. The manufacturer's lead time, the time it takes for a given order to leave the manufacturer's site after it is placed by the retailer, is a random variable that is also affected by the number of orders outstanding at the manufacturer's site.

Products shipped from the manufacturer also incur a transportation lead time; they pass through a series of transportation stages before the retailer receives them. Each transportation stage can represent a physical location for shipments or a step in the delivery process. During a given time period, each shipment moves through a random number of stages in the supply chain. The number of stages a shipment moves in one time period also depends on the movement of shipments ahead of it. Shipments are not allowed to cross over in time. That is, they are not allowed to cross shipments ahead of them. The lead time for a given shipment is thus a random variable that depends on the distribution of shipments at various stages. The total lead time is the sum of the manufacturing and the transportation lead times.

The stochastic model analyzes the long-run average cost for the retailer. The model is applied to quantify the value of real-time order tracking information along with the associated cost savings and to draw insights on how and when the retailer should place orders with the manufacturer. When real-time tracking information is not available from all transportation stages, beacons are placed to monitor one or several stages and collect aggregated information. The model presents two types of beacons. The type I beacon can detect the presence or absence of shipments only at the stages it is monitoring. The type II beacon can give information about the number of occupied stages. The model calculates the long-run average cost using information collected from the beacons. From the retailer's point of view, it is important to decide where to place the available beacons to achieve the minimum average cost. The other important point of interest is to determine the number of beacons that brings the average cost as close as possible to the average cost using full real-time tracking information.

CHAPTER 2
LITERATURE REVIEW

Supply chain models with information sharing are well studied. The literature on supply chain information sharing is growing rapidly (see, for instance, Lee and Padmanabhan (1997), Gavirneni et al. (1999), Cachon and Fisher (2000), Chen et al. (2000), Lee et al. (2000), Raghunathan (2001), Karaesmen et al. (2002), Dejonckheere et al. (2004), Gaur et al. (2005) and Kim et al. (2006)). Most work has focused on the value of demand information. Only a small body of work studies the value of upstream information, such as information on the supplier's inventory status and order lead times (Whitt, 1999; Chen and Yu, 2005; and Li *et al.,* 2006). Furthermore, these models focus on uncertainties caused by the production or inventory condition at the supplier and assume no uncertainties due to the shipping or manufacturing processes that can affect lead times under review.

Some of the models analyze the benefits of providing the retailer with access to the inventory status at a manufacturer's warehouse (Jain and Moinzadeh, 2005; Zhang, 2006; Croson and Donohue, 2006; Zhang *et al.,* 2006). Zhang (2006) studies the effect of horizontal information sharing on the inventory status between suppliers in a two-echelon assembly system. Dobson and Pinker (2006) use an M/M/1 queuing model to analyze the factors that determine whether or not sharing state-dependent lead-time information can benefit a firm. Croson and Donohue (2006) investigate the "bullwhip" effect when inventory information is shared across a supply chain. Zhang et al. (2006) analyze the impact of sharing information on uncertain shipment quantity in a simple linear supply chain with stochastic demand for a single product.

Kaplan (1970) achieves a major breakthrough in the study of stochastic lead-time supply systems. His paper studies the supply chain with stochastic lead times and random demands. The model assumes no crossover for shipments, linear ordering costs and fixed non-negative setup costs. The parameters in his model are not related to the lead-time distribution in a simple manner. Sufficient conditions for the optimality of myopic ordering policies are also not specified. Ehrhardt (1984) extends Kaplan's work by establishing conditions for the optimality of myopic base-stock policies and for the optimality of (*s, S*) policies for both finite and infinite planning horizons. Both Kaplan and Ehrhardt assume that the orders (shipments) cannot cross over but do not present details about how shipment congestions happen and affect the order lead times.

Song and Zipkin (1996) model the supply chain as a Markov chain. An exogenous random variable that models the Markov chain is assumed to be independent of the demand and of outstanding orders. A state-dependent, base-stock inventory policy is shown to be optimal for the inventory model. The optimal policy has the same structure as in standard models, but its parameters depend on the supply conditions.

The papers by Kaplan, Ehrhardt, and Song and Zipkin do not take into account the locations of outstanding orders and assume that the location does not affect order lead times. Eppen *et al.* (1988), Ray *et al.* (2004), and Krever *et al.* (2005) make a similar assumption. They come to the conclusion that the inventory position information is adequate to make efficient ordering decisions. While this assumption simplifies problems, some valuable information is omitted.

For example, Song and Zipkin (1996) assume that order lead times are dependent on supply conditions. While they show that observing supply status information is valuable, they also assume that supply conditions are independent of the real-time outstanding order status and conclude that real-time information on the location of outstanding orders is unnecessary for generating efficient decisions. Chen and Yu (2005) analyze the value of lead time information in a single-location inventory system consisting of a supplier and a retailer. Assuming that the lead time is modeled as a Markov chain, they quantify the value of lead-time to the retailer under two information scenarios: whether or not the supplier shares lead-time information with the retailer. Song and Zipkin (1996) and Chen and Yu (2005) also assume that shipments do not cross over in time; that is, shipment congestion can occur wherein some shipments may be blocked by other shipments against orders placed earlier in time. However, they only model shipment congestion implicitly by simply regulating the characteristics of lead-time distributions. For example, Chen and Yu (2005) assume that the congestion happens at the supplier's queue and not in transit. The transition matrix of the Markov chain that models the lead-time is restricted to a semi-upper triangular form to ensure no shipment crosses over. This restriction leads to the invariance of the lead time distribution with or without shipment congestions.

Liu *et al.* (2009) model a supply chain with multiple stages between a manufacturer and a retailer. A stochastic model is used to evaluate the value of real-time shipment tracking information in a supply system with a manufacturer that fulfills demand from a retailer for a single product using a periodic review, order-up-to-level inventory control policy. Shipment congestions are modeled explicitly so that the lead time has different distributions depending on whether or not shipment congestion is present. The order-up-to levels explicitly depend on the number and position of the outstanding orders. Liu shows that when the supply status depends on the location of outstanding orders, real-time information on the location of outstanding orders is valuable and he quantifies this value under different information scenarios. However, it is assumed that the manufacturer has unlimited capacity and is able to fulfill any order immediately, thus eliminating any delays (congestion) from the manufacturer's site. In addition, it is assumed that the retailer orders every time period. Furthermore, Liu's approach is limited to supply chains with up to only 4 transportation stages.

This thesis studies a more generalized supply system where the manufacturer has limited ability to ship orders immediately and the retailer does not have to order every time period. There are multiple stages in the supply chain between the manufacturer and the retailer. Shipment congestion in the supply chain and the number of unshipped orders at the manufacturer's site are modeled explicitly so that the lead time has different distributions depending on transportation congestion and outstanding orders.

The model shows how real-time tracking information affects the retailer's ordering decision. The new approach to using transitions gives the model the ability to study supply chains with 8, 9, 10 or more transportation stages. Clearly the cost of operating the system is reduced when the retailer has information on the supply status at the manufacturer's site and has access to real-time information on shipments in transit as well as on unshipped orders. When the retailer has limited access to real-time tracking information but can choose a stage to collect partial information, the model can choose the location of one or more beacons. It can also determine the number of beacons necessary to give a desired level of closeness to the long-run average cost of the full-information case.

CHAPTER 3
THE SUPPLY SYSTEM WITH MANUFACTURER'S LAG

3.1 The Supply System

Consider a supply system consisting of a manufacturer that supplies goods to a retailer through a transportation channel. The retailer aggregates demand for a single product and places orders with the manufacturer using a periodic review, state-dependent, order-up-to-level inventory control policy. The retailer demand is assumed to be independent and identically distributed (i.i.d.) over time. Products shipped by the manufacturer pass through a number of transportation stages before the retailer receives them.

The supply status in this system is determined by the orders outstanding at the manufacturer's site and by the status of orders in transit that have not yet reached the retailer's site. The lead time for the retailer is the sum of two components:

a) The manufacturing lead time, which is the time period from the instant an order is placed with the manufacturer until it is shipped from the manufacturer's site. This lead time is a random variable that can be affected by the number of orders outstanding at the manufacturer's site at the time the order is placed.

b) The transportation lead time, which is the time period from the instant an order is shipped from the manufacturer until the shipment is received by the retailer. This lead time is a random variable that also depends on the locations of shipments already in transit at the time the order is shipped.

The following sequence of events takes place during each time period.

- At the start of each time period, the retailer receives zero or more shipments from the supply system against orders placed in earlier periods.

- The retailer fulfills customer demand to the extent possible with inventory on hand and updates his inventory level. Any unsatisfied part of customer demand is backlogged.
- The retailer next reviews his inventory on hand and the total supply chain status and decides whether to order, and if so, what the order quantity should be. It is assumed that retailer's orders are conveyed immediately to the manufacturer, but the manufacturer can take multiple time periods to process the order.
- The manufacturer ships any orders that have completed processing. Clearly, if there are no ready-to-ship orders, no shipments take place, but all completed orders ready to ship are aggregated and dispatched in one shipment.
- Shipments against orders move downstream in the same sequence as the order in which the retailer places the orders, through multiple transportation stages. These shipments do not cross each other while in transit. In other words, they are not allowed to overtake other shipments already in transit.

The supply system described above is analyzed with a stochastic model to determine the optimal cost of operating the system. The retailer incurs two types of costs: the holding cost for any positive inventory on hand at the end of each time period, and the shortage cost whenever a demand is not met. In addition to analyzing the cost of system operation, one item of interest is to determine the value of tracking information that provides real-time data on the status of orders at various stages in the supply system.

It is assumed that the unit holding cost and the unit shortage cost are both constant over time. It is also assumed that the retailer has full knowledge of the mechanics of the supply system and the distribution of customer demand. Liu *et al.* (2009) study a special case of this model in which the retailer orders every time period and where the manufacturer is able to fulfill each order immediately.

3.2 The Stochastic Model

Let $m(t)$ denote the number of orders outstanding at the manufacturer's site at the end of time period t - 1. Note that $m(t)$ can take values from 0 up to N. If the retailer places a new order at the beginning of the time period t, the number of orders outstanding at the manufacturer's site prior to any shipment increases to $m(t) + 1$.

The number of orders shipped out of the manufacturer's site each time period is governed by a random variable, Y_m, that is independent of customer demand and of time t. There is, however, a limit, N, on the number of outstanding orders at the manufacturer's site at the end of each time period. Thus, if the manufacturer starts with N outstanding orders at time t, and if the retailer places a new order during that time period, the number of outstanding orders will increase to $N + 1$, implying that the manufacturer must ship at least 1 order by the end of that time period. The number N models either the physical size of the manufacturer's facility (there's no room to store more than N orders) or the maximum lead time agreed between the retailer and the manufacturer (the order cannot take more than N time periods to ship out of the manufacturer's site).

Let q_{m_1,m_2} denote the probability that the manufacturer has m_1 orders before a shipment occurs but after the retailer places a new order (if any) that time period, and there are m_2 orders left after the shipment of any orders during that same time period. Thus, Y_m has the following distribution:

$$\Pr[Y_m = y] \overset{def}{=} q_{m,y},$$

with (1)

$$\sum_{y=0}^{m} q_{m,y} = 1, \text{ for } m < N+1, \qquad \text{with } \sum_{y=0}^{N} q_{N+1,y} = 1.$$

After an order ships from the manufacturer, it goes through a series of transportation stages before it reaches the retailer. The transportation process through the supply system follows a similar process to the one studied by Liu *et al.* (2009). This process takes up to K stages, labeled $1, 2, \ldots, K$, as shown in Figure 3.1. Each stage can represent a physical location or a step in the process. Orders shipped by the manufacturer (stage 0 in the process) progress through zero or more stages during each time period and are finally received by the retailer (at stage $K + 1$).

Shipments against individual orders are not allowed to cross over in time and, therefore, a shipment at a downstream stage can block the movement of an order from upstream stages. More specifically, a shipment against an order can only move forward as far as the next downstream shipment's location. When order shipments from upstream stages move to the same downstream stage, they merge into a larger shipment and continue to move as one shipment from then onwards until the shipment reaches the retailer.

The number of stages a shipment at stage k moves during time period t is governed by the status of other shipments in the system, and an exogenous i.i.d. random variable, X_k, that is independent of customer demand and of t. The new location of this shipment is updated at the start of time period $t + 1$. As noted earlier, shipments are not allowed to cross over in time. Axsater (2000) notes that this assumption simply reflects common practice. The movement of the shipment at stage k is independent of the location of shipments at stages upstream from it, but is dependent on the status of shipments at downstream stages due to possible shipment congestions.

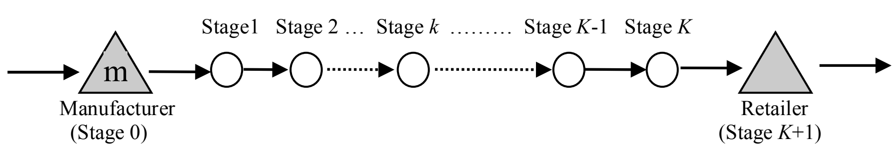


Figure 3.1. The K transportation stages between the manufacturer and the retailer, with m orders outstanding at the manufacturer's site.

Let $p_{k,x}$ represent the probability that the shipment at stage k will move to stage x in a single time period, if shipment congestions are absent. Thus, X_k has the following distribution:

$$\Pr[X_k = x] \overset{def}{=} \begin{cases} p_{k,x}, & \text{if } 0 \le k \le K,\ k \le x \le K+1, \\ 0, & \text{otherwise}, \end{cases}$$

with (2)

$$\sum_{x=k}^{K+1} p_{k,x} = 1, \qquad 1 \le k \le K, \qquad \text{and} \qquad \sum_{x=1}^{K+1} p_{0,x} = 1 .$$

Equation (2) implies that shipments do not move backward.

Let the binary variable, $s_k(t)$, denote the presence or absence of a shipment at stage k at time period t. That is,

$$s_k(t) \overset{def}{=} \begin{cases} 1, & \text{if there is a shipment present at stage } k \text{ at time } t, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Let $\vec{s}(t) = [s_1(t), s_2(t), ..., s_K(t)]$ denote the supply status in the transportation process between the manufacturer and the retailer at the end of time period t. Note that $\vec{s}(t)$ can take on one of 2^K possible states denoted by $\vec{s}_j, j = 1,2,...,2^K$. Let $\mathbf{s} = \{\vec{s}(t), t = 1,2,...\}$ and let $\Theta 1 = \{\vec{s}_1, \vec{s}_2, ..., \vec{s}_{2^K}\}$ denote the state space of **s**. Figure 3.2 shows an example of the supply status vector. The shaded circles indicate occupied stages.

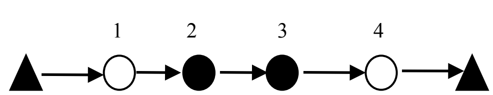


Figure 3.2. Example for K=4, $\vec{s}(t) = [0,1,1,0]$.

As noted earlier, shipments are not allowed to cross over and a shipment at a downstream stage can block movement of shipments at upstream stages. As in Liu *et al.* (2009), for each shipment located at stage k at the start of time period t, let M_k, $0 \le k \le K$, represent the location of this shipment at the start of time period t+1. The movement of a shipment is regulated by this function, defined recursively as follows:

$$M_k \stackrel{def}{=} \begin{cases} X_k & \text{if } \; s_{\hat{k}}(t) = 0 \quad \text{for any} \quad k < \hat{k} \le K, \\ \min\{X_k, M_{\bar{k}}\} & \text{otherwise,} \end{cases} \tag{4}$$

where $\bar{k} \stackrel{def}{=} \min\{\hat{k} : k < \hat{k} \le K, \quad s_{\hat{k}}(t) = 1\}$.

The shipment movement function, M_k, models shipment congestions explicitly. If shipments at downstream stages progress slowly, shipments at upstream stages may be blocked and shipment congestion is present. If shipments at downstream stages move fast enough or there are no shipments at upstream stages, there is no shipment congestion and X_k solely determines the movement of a shipment.

Since there may not be a new shipment at the beginning of a time period, the transitions from one state vector to another are modeled in two different ways. If a new shipment is released at the beginning of time period, the transition from state $\vec{s}(t)$ to $\vec{s}(t+1)$ follows the same logic as in Liu *et al*. (2009), as illustrated in Figure 3.3.

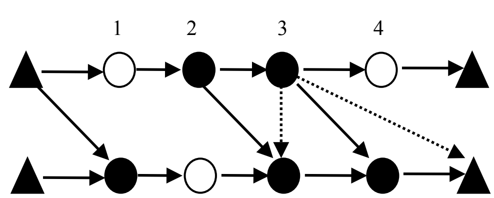
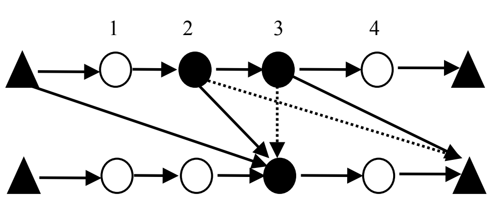


Figure 3.3. Possible transitions from $\vec{s}(t) = [0,1,1,0]$ to $\vec{s}(t+1) = [1,0,1,1]$ with a new shipment.

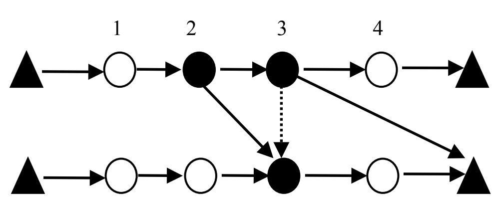
Note that if a new shipment is not released, the transition from $\vec{s}(t) = [0,1,1,0]$ to $\vec{s}(t+1) = [1,1,1,0]$ is not possible. Figure 3.4 shows possible transitions with and without a new shipment. Figure 3.4.a shows the transitions with a new shipment while Figure 3.4.b shows the transitions without a new shipment.

The retailer's decision to place a new order when the on-hand inventory level is lower than the specified order-up-to-level depends on the supply status $\vec{s}(t)$ and the number of orders outstanding at the manufacturer's site $m(t)$. Define the retailer's supply status as a pair $\vec{Rs}(t) = (m(t); \vec{s}(t))$. Note that since $m(t)$ can take any one of N+1 values, and $\vec{s}(t)$ can take any of 2^K values, $\vec{Rs}(t)$ takes on one of $(N+1)2^K$ possible values, $\vec{Rs}_j$, $j = 1,2,...,(N+1)2^K$. Let $\mathbf{Rs} = \{\vec{Rs}(t), t = 1,2,...\}$ and let $\Theta 2 = \{\vec{Rs}_1, \vec{Rs}_2, ..., \vec{Rs}_{(N+1)2^K}\}$ denote the state space of **Rs**.

It is assumed that for each state $\vec{Rs}_j$, $j = 1,2,...,(N+1)2^K$ the retailer places an order with pre-specified probability ρ_i. After fulfilling the customer's demand, the retailer observes the supply status vector, $\vec{Rs}_j$, at the end of each time period and places a new order with probability ρ_i. If the decision is to place an order but the on-hand inventory is greater than the pre-specified order-up-to-level, a pseudo order with an infinitesimally small amount is placed. Pseudo orders are necessary for mathematical tractability, and as noted in Liu *et al.* (2009) they have a low effect on the system's performance. To ensure that the state (0,0,…,0) is not absorbing, it is assumed that there is a non-zero probability of an order being placed while in this state.



a) Transition with a new shipment



b) Transition without a new shipment

Figure 3.4. Possible transitions from $\vec{s}(t) = [0,1,1,0]$ to $\vec{s}(t+1) = [0,0,1,0]$.

Let the binary variable $o(t)$ denote the presence or absence of a new order from a retailer. That is,

$$o(t) \overset{def}{=} \begin{cases} 1, & \text{if the retailer placed a new order at time } t, \\ 0, & \text{otherwise.} \end{cases} \qquad (5)$$

With this definition, the complete supply status vector is $\vec{T}s(t) = [o(t); m(t); \vec{s}(t)]$. Note that $\vec{T}s(t)$ takes on one of $(N+1)2^{K+1}$ possible states, $\vec{T}s_j, j = 1,2,...,(N+1)2^{K+1}$. Let $\mathbf{Ts} = \{\vec{T}s(t), t = 1,2,...\}$ and let $\Theta 3 = \{\vec{T}s_1, \vec{T}s_2, ..., \vec{T}s_{(N+1)2^{K+1}}\}$ denote the state space of **Ts**.

Property 3.1: Process **Rs** is a time-homogenous Markov chain.

Proof: Since process **s** is independent of the number of orders at the manufacturer's site,

$$\begin{aligned} &\Pr\{(m(t);\vec{s}(t)) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\} \\ &= \Pr\{m(t) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\} \\ &\ \text{x} \Pr\{\vec{s}(t) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\}. \end{aligned} \qquad (6)$$

By definition, the number of orders at the manufacturer's site at time t does not depend on transportation state vectors; moreover, it is completely determined by the number of orders at the manufacturer's place at time t-1 and the realization of the exogenous random variable Y_m in time period t. That is,

$$\begin{aligned} &\Pr\{m(t) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\} \\ &= \Pr\{m(t) \mid m(t-1), m(t-2),...,m(0)\} \\ &= \Pr\{m(t) \mid m(t-1)\}. \end{aligned} \qquad (7)$$

Also by definition, $\vec{s}(t)$, the supply status vector at time t, is a K-length binary string with 2^K possible states. The status of stage k in time period t+1 is completely determined by $\vec{s}(t)$, the realization of the exogenous random variable X_k in time period t, and the presence/absence of a new shipment at the beginning of time period t+1, which can be determined as the difference between $m(t$-1) and $m(t)$. And since $m(t)$ is completely determined by $m(t\text{-}1)$ and the realization of the exogenous random variable Y_m in time period t, it follows that

$$\Pr\{\vec{s}(t) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\} \\ = \Pr\{\vec{s}(t) \mid (m(t-1);\vec{s}(t-1))\}. \tag{8}$$

Thus

$$\Pr\{(m(t);\vec{s}(t)) \mid (m(t-1);\vec{s}(t-1)),(m(t-2);\vec{s}(t-2)),...,(m(0);\vec{s}(0))\} \\ = \Pr\{m(t) \mid m(t-1)\}\Pr\{\vec{s}(t) \mid (m(t-1);\vec{s}(t-1))\} \\ = \Pr\{(m(t);\vec{s}(t)) \mid (m(t-1);\vec{s}(t-1))\}, \tag{9}$$

or

$$\Pr\{\vec{R}s(t) \mid \vec{R}s(t-1),\vec{R}s(t-2),...,\vec{R}s(0)\} = \Pr\{\vec{R}s(t) \mid \vec{R}s(t-1)\}. \tag{10}$$ ■

Thus **Rs** is Markovian. As X_k and Y_m are i.i.d. over time, $\Pr\{\vec{R}s(t) \mid \vec{R}s(t-1)\}$ is independent of time period t.

Property 3.2: Process **Ts** is a time-homogenous Markov chain.

Proof: By definition, the retailer's ordering probability for time period t depends only on the retailer's supply status vector at time t. As noted earlier, the retailer's supply status vector at time t is determined only by the retailer's supply status vector at time t-1, and so it follows that

$$\Pr\{\vec{T}s(t) \mid \vec{T}s(t-1),\vec{T}s(t-2),...,\vec{T}s(0)\} = \Pr\{\vec{T}s(t) \mid \vec{T}s(t-1)\}, \tag{11}$$

and so **Ts** follows a Markov chain. ■

As the retailer's ordering probabilities are time independent and process **Rs** is time-homogenous, process **Ts** is also time-homogenous.

Let **PTs** denote the one-step transition matrix for process **Ts**. The algorithm to evaluate **PTs** is presented in the appendix.

It is assumed that **Rs** and **Ts** are ergodic. This is not a restrictive assumption, since ergodicity is satisfied whenever stage K+1 is reachable from stage 0 and $\rho_1 > 0$, where $\vec{R}s_1 = (0, 0, ..., 0)$.

Since **Ts** is ergodic, it has a unique limiting distribution. Let $\vec{\pi} = [\pi_j]$ denote the limiting distribution for **Ts**. That is,

$$\pi_j \stackrel{def}{=} \lim_{t\to\infty} \Pr[\vec{T}s(t) = \vec{T}s_j], \qquad 1 \le j \le (N+1)2^{K+1}. \tag{12}$$

3.3 Order Lead Time

Let the random variable, $L(t)$, denote the lead time for the order placed in time period t. The retailer is interested in the conditional lead time for each pair of $(m(t);\vec{s}(t))$ as this is the information the retailer needs to make his decision about the order-up-to level. This information, paired with the ordering probabilities, either results in a new order or the absence of an order during a time period. The conditional lead times are found for each $\vec{T}s(t) = [o(t); m(t); \vec{s}(t)]$ with $o(t) = 1$, in other words for each supply status vector with a new order. Note that if $o(t) = 0$, there is no new order and the lead time is not defined for that time period. The calculation of the conditional distribution of $L(t)$ is given in the appendix.

Note that the order placed at the end of time period t is still outstanding in time period t+$L(t)$ but arrives (is accounted for) at stage K + 1 at the beginning of time period t+$L(t)$+1.

Let Γ be the set of all indices in $\vec{T}s$ that correspond to vectors $\vec{T}s(t) = [o(t); m(t); \vec{s}(t)]$ with $o(t) = 1$. That is,

$$\Gamma = \{i : \textit{such that } \vec{T}s_i = (1; m; s_j) \text{ for any } 0 \le \text{m} \le \text{N and any } 1 \le \text{j} \le 2^{\text{K}}\}. \tag{13}$$

The limiting probability distribution for lead times is

$$\nu_l \overset{def}{=} \lim_{t\to\infty} \Pr[L(t) = l] = \sum_{i\in\Gamma} \pi_i \Pr[L(t) = l \mid \vec{T}s_i]. \qquad (14)$$

The following section presents the analysis for the long-run average cost under different information scenarios.

3.4 Ordering Decisions and Long-Run Average Cost

It is assumed that the retailer adopts a state-dependent, myopic order-up-to-level policy. That is, the optimal order-up-to level for each time period is obtained by minimizing the one-period expected cost. It is also assumed that the retailer has predetermined ordering probabilities for each state. For the linear cost structure assumed in this paper, a state-dependent, order-up-to-level policy is optimal; and the optimal policy tends to follow the same qualitative pattern as the simple state-dependent, myopic, base-stock policy (Song and Zipkin, 1996). For ease of exposition, Table 3.1 summarizes the important notations presented so far, as well as the notation used in the ensuing analysis.

Table 3.1. Notations.

Symbol	Definition
N	The maximum number of orders pending shipment at the manufacturer's site
K	The number of transportation stages
Y_m	Random variable that governs # orders shipped by manufacturer each time period
q_{m_1,m_2}	Pr{m_2 out of m_1 unshipped orders are left at manufacturer's site after a shipment}
X_m	Random variable that governs # transportation stages moved each time period
$p_{k,x}$	Pr{shipment at stage k will move to stage x in a single time period}
$o(t)$	The presence or absence of a new order from a retailer at time period t
$s_k(t)$	The presence or absence of a shipment at stage k during time period t
$m(t)$	The number of orders outstanding at the manufacturer's site at time period t
$\vec{T}s(t)$	The supply status vector: $\vec{T}s(t) = [o(t); m(t); \vec{s}(t)]$
L(t)	The lead time for an order placed at time period t
ρ_j	Pr{the retailer places an order while the system is in state $(m(t);\vec{s}(t))_j$ }
$d(t)$	External demand during time period t
μ_d	Average demand per time period
σ_d	Standard deviation of the demand per time period
h	Unit holding cost per period
r	Unit shortage penalty cost per period
C^*	The minimum long-run average cost with real-time tracking information
C_s^*	The minimum long-run average cost without real-time tracking information

3.4.1 Long-Run Average Cost with Real-Time Tracking Information

Let $D(t, \eta)$, $\eta \geq 0$ denote the sum of all external demands made during $(t, t+\eta+1]$. That is,

$$D(t,\eta) \overset{def}{=} \sum_{i=0}^{\eta} d(t+i+1). \qquad (15)$$

Let's assume that the retailer placed an order at the end of some time period t. As he may not order all the time, the next order may not be placed at the end of the time period $t+1$. Let's define the time period the next order is placed by $t + \tau$. Let $IP(t)$ be the inventory position at the end of time period t after the current order is placed and $IL(t)$ be the inventory level at the end of time period t. Since orders cannot cross over, the following relationship between demand, the inventory position and the inventory level holds:

$$IL(t+\eta+1) = IP(t) - D(t,\eta), \quad L(t) \leq \eta \leq L(t+\tau) + \tau\text{-}1. \qquad (16)$$

Therefore, the distributions of the inventory levels, $IL(t+\eta+1)$, for $L(t) \leq \eta \leq L(t+\tau)+\tau$-1, are determined by the inventory position $IP(t)$. Given $L(t)$ and $L(t+\tau)$, it is proper to assign the cost incurred in periods $t+L(t)+1$, $t+L(t)+2$, …, $t+L(t+\tau)$, to the order placed in period t. That is, the order placed in period t will cover the periods from when it is received until the time the next order is received. If $L(t+\tau) = L(t) - \tau$, i.e., the orders placed in periods t and $t + \tau$ arrive in the same period, zero cost is charged to the order placed in period t. For any $y \geq 0$, define

$$g(l,y) \overset{def}{=} E[\ h \quad \max(0, y - D(t,l)) - \ r \quad \min(0, y - D(t,l))], \qquad l \geq 0. \qquad (17)$$

Let $G(\vec{T}s_j, y), j \in \Gamma$, denote the one-period expected cost, i.e., the expected cost charged to the order placed in period t, given $\vec{T}s(t) = \vec{T}s_j$ and $IP(t) = y$. Then,

$$G(\vec{T}s_j, y) = \sum_{l \geq 0} \sum_{\tau \geq 1} \Pr[L(t) \leq l \leq L(t+\tau) + \tau - 1 \,|\, \vec{T}s(t) = \vec{T}s_j] g(l,y), \qquad (18)$$

where

$$\Pr[L(t) \le l \le L(t+\tau)+\tau-1 \mid \vec{T}s(t)=\vec{T}s_j]$$

$$= \Pr[L(t) \le l \mid \vec{T}s(t)=\vec{T}s_j] - \Pr[L(t) \le l, L(t+\tau)+\tau-1 < l \mid \vec{T}s(t)=\vec{T}s_j]$$

$$= \Pr[L(t) \le l \mid \vec{T}s(t)=\vec{T}s_j] - \Pr[L(t+\tau)+\tau-1 < l \mid \vec{T}s(t)=\vec{T}s_j] \tag{19}$$

$$= \Pr[L(t) \le l \mid \vec{T}s(t)=\vec{T}s_j] - \sum_{k\in\Gamma} \Pr[L(t+\tau)+\tau-1 < l \mid \vec{T}s(t+\tau)=\vec{T}s_k] NO(j,k,\tau)$$

and $NO(j,i,\tau)$ is the probability that the system transitions from state $\vec{T}s_i$ to state $\vec{T}s_j$ in τ steps with all intermediate states $\vec{T}s_k$ satisfying the condition that $k \notin \Gamma$ (i.e., there are no new orders placed until the time period $t + \tau$). The algorithm to compute $NO(j,k,\tau)$ is given in the appendix.

Let IP_j^* be the optimal order-up-to level minimizing the one-period expected cost, given that the real-time complete supply status vector is $\vec{T}s(t)=\vec{T}s_j$ when an order is placed. That is,

$$IP_j^* \stackrel{def}{=} \arg\min_y G(\vec{T}s_j, y). \tag{20}$$

Then, by the Ergodic theorem for Markov chains (Norris, 1997),

$$C^* \stackrel{def}{=} \lim_{n\to\infty} E[\frac{1}{n}\sum_{i=1}^{n} G(\vec{T}s(t+i), IP(t+i))] = \sum_{j\in\Gamma} \pi_j G(\vec{T}s_j, IP_j^*), \tag{21}$$

where C^* is the minimum long-run average cost under the myopic policy.

3.4.2 Long-Run Average Cost without Real-Time Tracking Information

Suppose the retailer does not have any information on the supply status when the current order is placed, and suppose that the retailer uses the limiting distribution of **Ts** to determine the optimal order-up-to level, which is a constant.

Let $C_s(y)$ denote long-run average cost for the retailer when the order-up-to level is y, under the assumption that no information is available on the supply status. Thus

$$C_s(y) = \sum_{j\in\Gamma} \pi_j G(\vec{T}s_j, y), \tag{22}$$

where y is the constant order-up-to level. Let C_s^* denote the minimum long-run average cost for the retailer without real-time tracking information. That is,

$$C_s^* = \min_y \sum_{j\in\Gamma} \pi_j G(\vec{T}s_j, y). \tag{23}$$

Note that the optimal ordering policy and the myopic policy are identical in this case.

It can be argued that the retailer always knows if there are no outstanding orders. In other words, he does not need real-time tracking information to identify that the complete supply status vector is either $\vec{T}s_1 = (0,0,...,0)$ or any other $\vec{T}s_j \neq (0,0,...,0)$, $1 \leq j \leq (N+1)2^{K+1}$. Thus one may say that the minimum long-run average cost for the retailer without real-time tracking information can be defined as

$$\tilde{C}_s^* = \pi_1 G(\vec{T}s_1, IP_1^*) + \min_y \sum_{\substack{j\in\Gamma \\ j\neq 1}} \pi_j G(\vec{T}s_j, y). \tag{24}$$

But to calculate $\tilde{C}_s^*$ the retailer needs to know π_j limiting distribution as well as the conditional lead-time distribution. If this information is not available, the long-run average cost should be calculated as in formula 23. The numerical investigation in the next section assumes that "no information" means that the retailer does not have real-time tracking information nor the information about the complete supply status vectors' limiting probabilities.

3.5 Numerical Investigation of Performance

This section presents the results of a systematic investigation to determine scenarios under which the retailer benefits from real-time tracking information, using a wide range of parameters and settings. Note first that the relative advantage provided by real-time tracking information is fairly small when shipments progress in a rather deterministic manner. This conclusion is based on the following property.

Property 3.3: If the retailer orders every time period with probability 1, the exogenous variable X_k has the distribution $p_{k,k+1} = 1$ for all $0 \leq k \leq K$, and the exogenous variable Y_k has the distribution $q_{n,n-1} = 1$, for all $n > 0$ and $q_{0,0} = 1$, then $C^* = C_s^*$.

Proof. When $q_{n,n-1} = 1$, the manufacturer ships one order every time period. Also with $p_{k,k+1} = 1$ for all $0 \leq k \leq K$, shipments progress from the manufacturer to the retailer in a completely deterministic manner. With such a progression of orders and shipments, it can be readily observed that, in steady state, there are no orders at the manufacturer's site besides the one that is received at the beginning of the time period and that every transportation stage is occupied by a shipment. Thus, the limiting distribution of **Ts** has only one state, $\vec{T}s_a = (1;0;1,\ldots,1)$ for which $\pi_a = 1$.

From equation (21)

$$C^* = \sum_{j \in \Gamma} \hat{\pi}_j G(\vec{T}s_j, IP_j^*) = \pi_a G(\vec{T}s_a, IP_a^*) = G(\vec{T}s_a, IP_a^*) .$$

Similarly, from equation (23)

$$C_s^* = \min_y \sum_{j \in \Gamma} \hat{\pi}_j G(\vec{T}s_j, y) = \min_y G(\vec{T}s_a, y).$$

Since $IP_a^* = \min_y G(\vec{T}s_a, y)$, the result follows. ■

Property 3.3 implies that there is perfect information available and in such a situation no further cost savings are possible.

3.5.1 Parameter Settings

For orders at the manufacturer's site, two shipping scenarios are considered:

- An expedited shipment scenario: Under this scenario, there is a high probability that all unshipped orders will leave the manufacturer's site in a single shipment. There is only a small probability that a shipment will not take a place. If the number of unshipped orders reaches the maximum, at least one order is shipped with a non-zero probability. For this scenario, the values of $q_{i,j}$ are set as follows: $q_{n,0} = 0.9$ and $q_{n,n} = 0.1$, for $0<n\leq N$ and $q_{N+1,0} = 0.9$ and $q_{N+1,N} = 0.1$.

- A batch shipment scenario: Under this scenario, most of the shipments contain two orders. If there are *n* unshipped orders at the manufacturer's site, there is a high probability that two of them are shipped at a time, leaving behind *n-2* unshipped orders. There is a non-zero but small probability that a shipment contains only one order. For this scenario, the values of $q_{i,j}$ are set as follows: $q_{1,0} = 0.1$, $q_{1,1} = 0.9$, and $q_{n,n-2} = 0.9$, and $q_{n,n-1} = 0.1$ for $n > 1$.

For both shipment scenarios, $q_{0,0} = 1$.

Three delivery modes are considered for shipments from the manufacturer's site: premium, priority, and economy. Under these modes of delivery, the shipper attempts to dispatch orders leaving the manufacturer's site directly to the retailer's site with a high, medium, or low probability, respectively. If none of the transportation stages are occupied, there is no congestion in the transportation channel, and a shipment dispatched directly to the retailer's site gets delivered in the next time period. However, if there is congestion in the transportation channel, this shipment merges with the shipment at the first occupied stage downstream since orders are not allowed to cross over. Orders that are not dispatched directly to the retailer's site proceed to the first transportation stage. The parameters for the 3 scenarios are set as follows:

- Premium delivery mode: $p_{0,K+1} = 0.9$ and $p_{0,1} = 0.1$.
- Priority delivery mode: $p_{0,K+1} = 0.7$ and $p_{0,1} = 0.3$.
- Economy delivery mode: $p_{0,K+1} = 0.3$ and $p_{0,1} = 0.7$.

The congestion at transportation stages is modeled using two settings: low and high.

- For the low-congestion transportation setting, a shipment in location $0 < k < K$ either stays in the same location during the next time period with probability $p_{k,k}$ = 0.1, moves to the next stage with probability $p_{k,k+1}$ = 0.8, or moves two stages downstream with probability $p_{k,k+2}$ = 0.1. As a shipment at the stage K cannot move two stages downstream, the probabilities are as follows: $p_{K,K}$ = 0.1 and $p_{K,K+1}$ = 0.9.
- For the high-congestion transportation setting, $p_{k,k}$ = 0.5, $p_{k,k+1}$ = 0.4 and $p_{k,k+2}$ = 0.1 for $1 \le k < K$. And for the shipments at stage K the probabilities are as follows: $p_{K,K}$ = 0.5 and $p_{K,K+1}$ = 0.5.

It is possible to set non-zero values for $p_{i,j}$ for $j>i+1$. However, in practice it is less likely for a shipment to move across multiple stages during a single time period unless there is some expediting taking place.

For the low-congestion transportation setting, the expected time a shipment stays at stage k is 1.11, while for the high-congestion transportation setting a shipment at location $k < K$ stays in that location for an average of two time periods.

In addition, a scenario with uniformly distributed probabilities for both unshipped orders and for orders at the transportation stages is studied. For this scenario, $q_{n,y} = 1/(n+1)$ for any $y = 0, 1, \ldots, n$, and $p_{k,x} = 1/(K-k+2)$ for $x = k,\ldots,K+1$ and $k > 0$, with $p_{0,x} = 1/(K+1)$ for $x = 1, \ldots, K+1$.

Using these parameter settings for the manufacturer's shipping process and the transportation process, a series of experiments was carried out for values of K, ranging from 1 to 8. The effect of shortage cost was investigated by varying the shortage cost while keeping the holding cost fixed. Demand was generated from a normal distribution with different values of μ_d and σ_d. The effect of premium delivery and the effect of varying ρ_j, the retailer's ordering probabilities, were also studied. For each case, the difference in cost between C^* and C_s^*, referred to as the cost savings, is also computed.

3.5.2 Case Where the Retailer Orders Every Time Period

The following set of examples assumes that the retailer orders every time period. That is,

$$\rho_i = 1, \quad \text{for} \quad \text{any} \qquad i = 1,\ldots,(N+1)2^K.$$

3.5.2.1 The Long-Run Average Cost

To investigate the effect of each above-mentioned parameter on the long-run average costs, a full factorial design was carried out. Some results of the experiment were intuitive and uniform, but some of them were not as anticipated. Starting from the intuitive results, the long-run average costs (with real-time tracking information as well as with no information) increase whenever

- the number of transportation stages (K) increases,
- the shortage cost increases,
- the demand variation increases, or
- the transportation setting is at high-congestion.

All these results are easy to explain. It is harder to manage on-hand inventory with high variation in demand. With high shortage costs the retailer tries to order more every time period. The high-congestion and higher values of K tend to increase the lead times. All this complicates inventory management and contributes to higher long-run average costs.

Some results were less intuitive. For example, an increase in N, the maximum number of pending shipments at the manufacturer's site, does not affect much the long-run average cost values. The values are very close for different values of N with the expedited shipment scenario as well as with the batch shipments scenario. This result can be explained by observing that the model considers costs only from the retailer's point of view. Hence, the maximum number of unshipped orders at the manufacturer's site has relatively little influence on the retailer's cost.

3.5.2.1.1 Expedited Shipment vs. Batch Shipment Scenario

While the expedited shipment scenario does not always give the lower long-run average costs (with full real-time tracking information and with no information) compared to the batch shipment scenario, analyzing which shipment scenario provides the lower long-run average cost revealed an interesting trend. For some values of K there is a forward/backward switch. The shipment scenario that provides the lower long-run average cost switches from the expedited to the batch as the shortage cost increases (forward switch) or from the batch to the expedited (backward switch). The results vary mostly according to the delivery modes.

- With the economy delivery mode the expedited shipment scenario always gives the lower long-run average costs compared to the batch shipment scenario.
- With the priority delivery mode the expedited shipment scenario gives the lower long-run average costs when the transportation congestion is low and, in most cases, when the transportation congestion is high. For the high-congestion transportation setting there are some instances with low values of K (K = 1, 2) with forward/backward switch. A forward switch is observed with K = 1, and a backward switch is observed with K = 2.

- With the premium delivery mode the expedited shipment scenario gives the lower long-run average costs for small values of K. As K increases, forward switches are observed. For even greater values of K the lower long-run average costs are always achieved with the batch shipment scenario. And for the highest values of K, backward switches are observed. The corresponding values of K are higher with the high-congestion transportation setting.

Table 3.2 shows some of the results for the priority and premium delivery modes. It shows which shipment scenario gives the lower long-run average cost. The letter “E” stands for the expedited shipment scenario, and the letter “B” stands for the batch shipment scenario. The forward/backward switches can be observed from the table. For example, with premium delivery mode and high-congestion transportation setting the forward switches can be observed for K=1 and 2. For K=3, 4 and 5 the batch shipment scenario gives the lower long-run average costs. Backward switches can be observed for K=6, 7 and 8.

In trying to explain the results, it should be noted that the batch shipment affects the congestion. In this scenario a new shipment finds the system “less congested” compared to the expedited shipment scenario as on average there are two time periods between two consecutive shipments and shipments already on their way have two time periods to move forward and clear upstream stages. Thus the transportation part has less impact on the whole system, making it a bit more predictable. Also it is more likely that the new shipment does not find any congestion in the transportation channel and gets delivered directly to the retailer’s site. It can be noted, though, that when the value of K is small and congestion is low or the value of K is high and congestion is high the batch shipment scenario has less influence on the system.

Table 3.2. Shipment scenarios providing the minimum long-run average cost with real-time tracking information (E=expedited shipment scenario; B=batch shipment scenario).

Shortage Cost, r	Demand Variation, σ_d	Transportation Congestion Setting	Delivery Mode	K=1	K=2	K=3	K=4	K=5	K=6	K=7	K=8
5	10	Low	Priority	E	E	E	E	E	E	E	E
30	10	Low	Priority	E	E	E	E	E	E	E	E
5	10	Low	Premium	E	E	E	E	E	E	B	B
30	10	Low	Premium	E	B	B	B	B	B	E	E
5	10	High	Priority	E	B	B	E	E	E	E	E
30	10	High	Priority	B	E	E	E	E	E	E	E
5	10	High	Premium	E	E	B	B	B	B	B	B
30	10	High	Premium	B	B	B	B	B	E	E	E
5	30	Low	Priority	E	E	E	E	E	E	E	E
30	30	Low	Priority	E	E	E	E	E	E	E	E
5	30	Low	Premium	E	E	E	E	E	B	B	B
30	30	Low	Premium	E	E	B	B	B	B	B	E
5	30	High	Priority	E	B	E	E	E	E	E	E
30	30	High	Priority	B	E	E	E	E	E	E	E
5	30	High	Premium	E	E	B	B	B	B	B	B
30	30	High	Premium	B	B	B	B	B	E	E	E

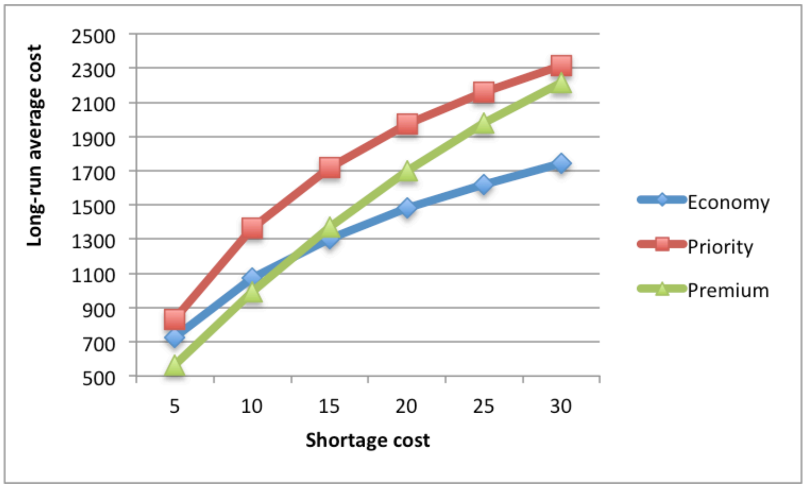
When the batch shipment scenario is combined with high delivery modes, there is a high probability that a new shipment will proceed directly to the retailer's site. This translates into lower long-run average costs compared to the expedited shipment scenario and the lower delivery modes. This phenomenon combined with the impact of the shortage cost explains the results.

In general, it appears that the expedited shipment scenario is more beneficial for the retailer than the batch shipment scenario, but there are situations (the premium delivery mode with only some values of K) when the batch shipment scenario can guarantee lower long-run average cost for the retailer.

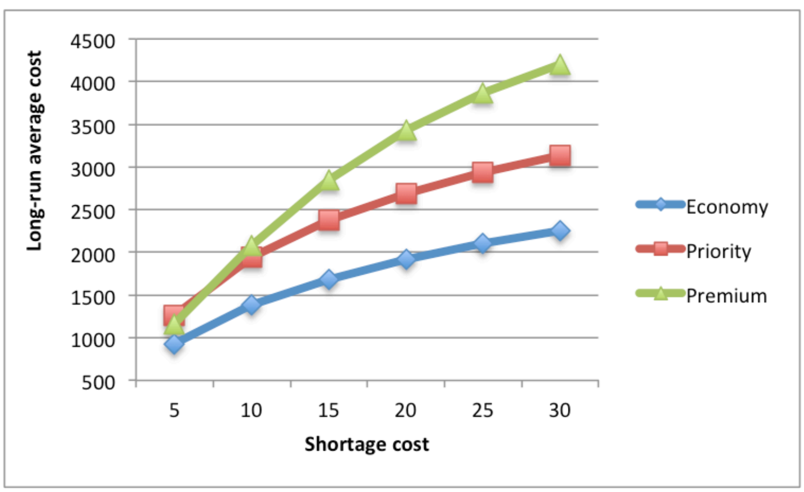
3.5.2.1.2 Economy, Priority and Premium Delivery Modes

Since freight costs are not included in the long-run average cost of inventory, differences in delivery costs are entirely irrelevant to this analysis. For this reason, intuition would say that long-run average costs with the premium delivery mode are always lower than with the economy and the priority delivery modes, but the results show that this is true only for small values of K (K = 1, 2). As the number of transportation stages increases, the transportation channel has more impact on the system and managing it becomes more complicated. As K increases, the delivery mode that gives the lowest long-run average cost is the economy delivery mode. The first transition happens for the high shortage cost (r=30) with the low-congestion transportation setting and the expedited shipment scenario with demand variation σ_d = 10. As K increases, the economy delivery mode gives the lowest long-run average costs with even smaller values of the shortage cost and high demand variation. With the batch shipment scenario the transition happens with higher values of K, and the batch shipment with high-congestion and the low shortage cost is the last one to change. Figure 3.5 and Figure 3.6 show the long-run average costs with real-time tracking information for different shortage costs and the three delivery scenarios. Figure 3.5 shows the results for expedited shipment scenario with low-congestion transportation setting and σ_d=30. Figure 3.6 shows the results for batch shipment scenario with low-congestion transportation setting and σ_d=10.

From both figures it can be seen that the premium delivery mode gives the lowest long-run average cost for smaller values of K and the shortage cost. While premium delivery is most beneficial with small values of K and lower values of shortage cost, with higher values of K and higher values of shortage cost, economy delivery mode yields the lowest long-run average cost. As K increases, order lead-times increase and managing inventory becomes more difficult. This is especially true in the case of high shortage cost where further variation in the system is particularly disadvantageous for the retailer. The premium delivery mode has more variation in the system than the economy delivery mode and therefore the shipment movement in the transportation channel is less predictable. It is intuitive for the retailer to protect against the high shortage cost by ordering more, thus driving up the average cost.

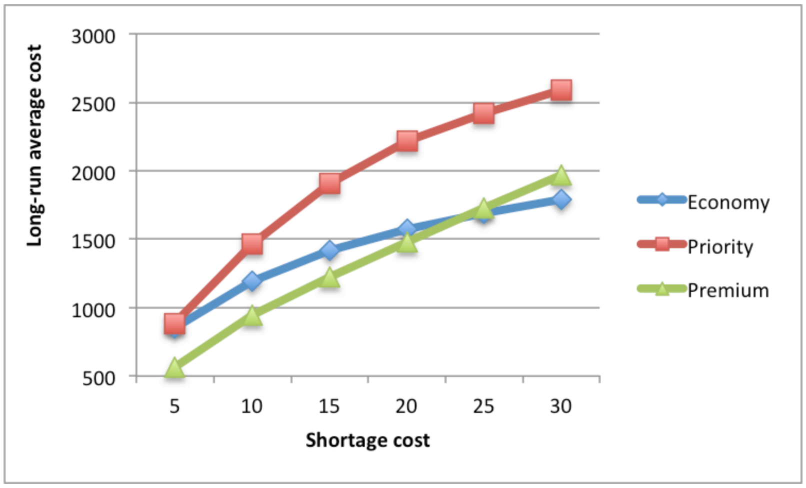


a) K=4

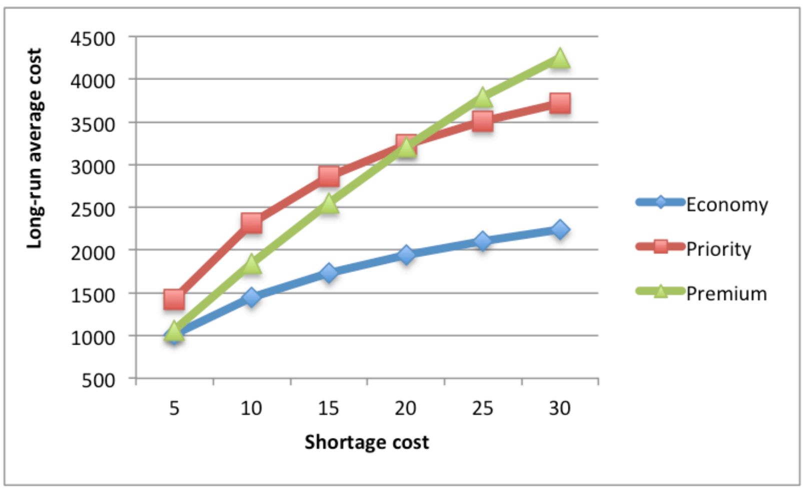


b) K=8

Figure 3.5. Long-run average costs with real-time tracking information with economy, priority and premium delivery modes; expedited shipment scenario with low-congestion transportation setting; σ_d=30.



a) K=4



b) K=8

Figure 3.6. Long-run average costs with real-time tracking information with economy, priority and premium delivery modes; batch shipment scenario with low-congestion transportation setting; σ_d=10.

Table 3.3 presents more detailed results. It shows which delivery mode provides the lowest long-run average cost. The letter "P" stands for the premium delivery mode, and the letter "E" stands for the economy delivery mode. It can be seen that for *K*=1 the Premium delivery mode gives the minimum long-run average cost for all settings and for *K*=9 the economy delivery mode gives the minimum long-run average cost for all settings. In can be observed that the first factor that contributes to the switch from the premium to the economy delivery mode is the high shortage cost. The second factor is the low-congestion and the last one is the expedited shipment scenario.

Table 3.3. Minimum long-run average cost with real-time tracking information by delivery modes (P=premium, E=economy).

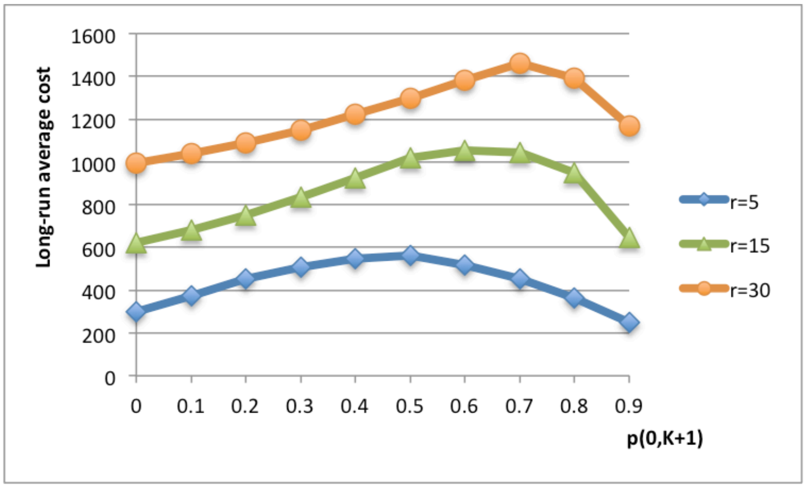
Shortage Cost, r	Demand Variation, σ_d	Transportation Congestion Setting	Shipment Scenario	K=1	K=2	K=3	K=4	K=5	K=6	K=7	K=8	K=9
5	10	Low	Expedited	P	P	P	P	P	E	E	E	E
30	10	Low	Expedited	P	E	E	E	E	E	E	E	E
5	10	High	Expedited	P	P	P	P	P	P	E	E	E
30	10	High	Expedited	P	P	P	E	E	E	E	E	E
5	30	Low	Expedited	P	P	P	P	P	E	E	E	E
30	30	Low	Expedited	P	P	E	E	E	E	E	E	E
5	30	High	Expedited	P	P	P	P	P	P	E	E	E
30	30	High	Expedited	P	P	P	E	E	E	E	E	E
5	10	Low	Batch	P	P	P	P	P	P	P	E	E
30	10	Low	Batch	P	P	P	E	E	E	E	E	E
5	10	High	Batch	P	P	P	P	P	P	P	P	E
30	10	High	Batch	P	P	P	P	E	E	E	E	E
5	30	Low	Batch	P	P	P	P	P	P	P	P	E
30	30	Low	Batch	P	P	P	E	E	E	E	E	E
5	30	High	Batch	P	P	P	P	P	P	P	P	E
30	30	High	Batch	P	P	P	P	E	E	E	E	E

As noted above, batch shipment affects congestion and makes the premium delivery mode more efficient. This combination is especially beneficial with small values of K. When there are many transportation stages, the premium delivery mode loses its benefit as there is a higher chance that a new shipment will be blocked somewhere in the transportation channel. The results indicate that with higher values of K the economy delivery mode guarantees more steady passage of the shipments through the transportation channel and results in lower long-run average costs.

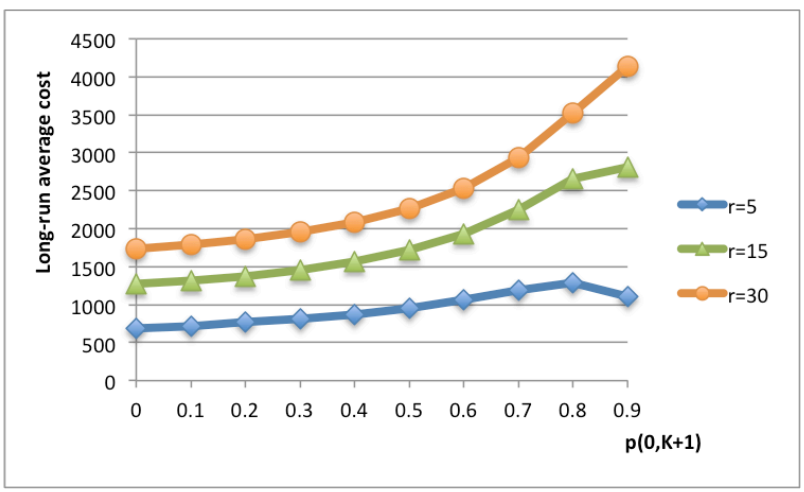
To summarize, it can be said that the premium delivery mode is most beneficial with the batch scenario. It is also beneficial with the high-congestion transportation setting and low shortage cost and small values of K. But when the transportation channel has many stages the economy delivery has more advantage.

3.5.2.1.3 Additional Investigation of the Effect of the Delivery Mode

To further investigate the effect of different delivery modes the following experiment was run. The congestion scenario and shipping scenario are fixed, and the delivery mode is varied by setting $p_{0,K+1}=0.1*i$ (and $p_{0,1}=1 - p_{0,K+1}$) for i=0,1,2,…,9. The results show that the long-run average cost functions (with full real-time tracking information and with no information) are convex for smaller values of K and they are attaining their minimums at a border point. As K increases, the cost functions stay convex with r = 5 but become increasing functions of $p_{0,K+1}$ for higher values of the shortage cost. The minimums are attained at $p_{0,K+1}=0.1$. Figure 3.7 shows the long-run average costs with real-time tracking information for different delivery modes with expedited shipment scenario and low-congestion transportation setting. It can be said that, when transportation congestion is present and downstream shipments block new shipments, the long-run average costs are smaller when there is more steady movement from the manufacturer's site. Only for smaller values of K do high delivery modes guarantee lower long-run average costs. With high values of K any parameter setting that steadies the movement of shipments through the system appears to result in lower cost. It would not be beneficial to adopt high (relatively more premium) delivery modes with high values of *K,* especially if the retailer incurs fixed costs with the high delivery mode.



a) K=2



b) K=8

Figure 3.7. Long-run average costs with real-time tracking information for different delivery modes; expedited shipment scenario with low-congestion transportation setting.

3.5.2.2 Cost Savings

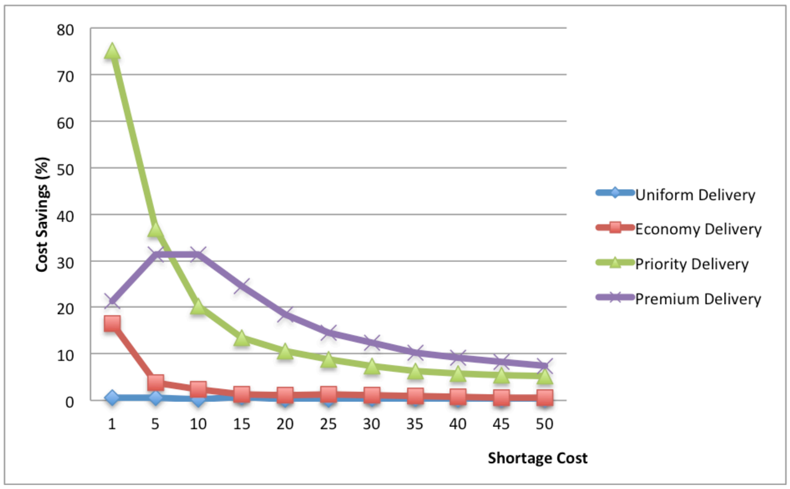
This section compares the long-run average cost with full real-time tracking information to the long-run average cost without any information. Figure 3.8, Figure 3.9, Figure 3.10 and Figure 3.11 show the percentage difference between these two costs (cost savings) for $K = 8$ and $N = 2$ for a fixed holding cost $h = 10$ and varying shortage cost r. In these figures, the demand follows a normal distribution with $\mu_d = 100$. Figure 3.8 and Figure 3.9 show cost savings for the expedited shipment scenario, and Figure 3.10 and Figure 3.11 show cost savings for the batch shipment scenario. Table 3.4 and Table 3.5 present some of these results in a tabular form.

The results for both shipment scenarios follow a similar pattern. It can be seen from these figures that the uniform delivery mode does not have significant cost savings. Moreover, even the economy delivery mode does not have any significant cost savings for shortage costs more than 5.

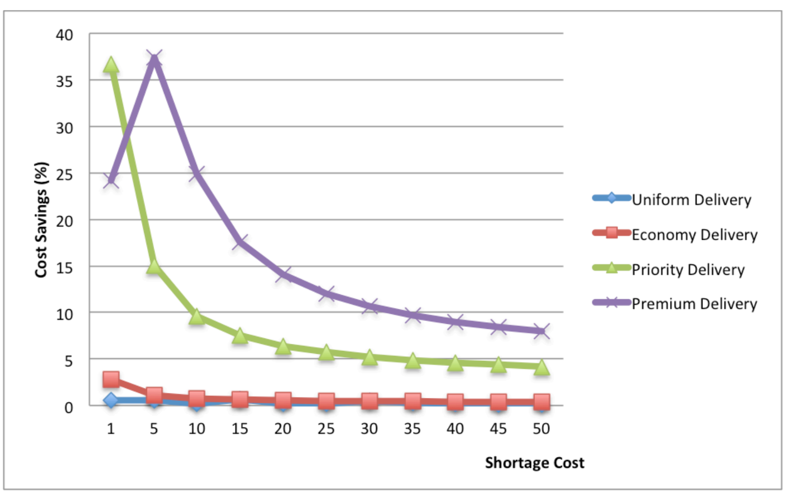
The results show that the priority delivery mode gives the best cost savings for smaller values of the shortage cost. When the shortage cost is higher than the holding cost, the best cost savings are obtained with the premium delivery mode. It can be observed from Figure 3.8, Figure 3.9, Figure 3.10 and Figure 3.11 that for all delivery modes except premium the cost savings decrease as the shortage cost increases. With the premium delivery mode the cost savings increase for smaller values of the shortage cost (r = 1, 5, 10) and start decreasing after the shortage cost reaches 15.

It can be seen from Figure 3.8 that the cost savings can be as high as 75% (the low-congestion transportation setting under the priority delivery mode, with $\sigma_d = 10$) for r=1. This result is more of an analytical value as in practice it is not usual to have such a low shortage cost. Cost savings are substantial for higher, more realistic values of r. The premium delivery mode with low-congestion gives cost savings of 31% for both shortage costs $r = 5$ and $r = 10$ ($\sigma_d = 10$). The priority delivery mode with low-congestion gives cost savings of 36% for shortage cost $r = 5$ ($\sigma_d = 10$).

It can be observed from Figure 3.10 and Figure 3.11 and from Table 3.5 that the batch shipment scenario gives substantial savings too. For example, the low-congestion transportation setting with the priority delivery mode gives 38% for $r = 5$ ($\sigma_d = 10$), and the high-congestion transportation setting with the premium delivery mode gives 26% for $r = 10$ ($\sigma_d = 10$).

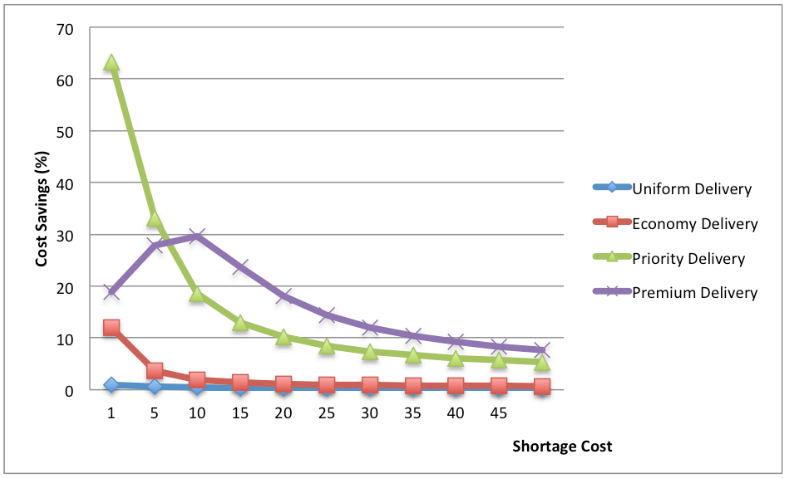


a) low-congestion

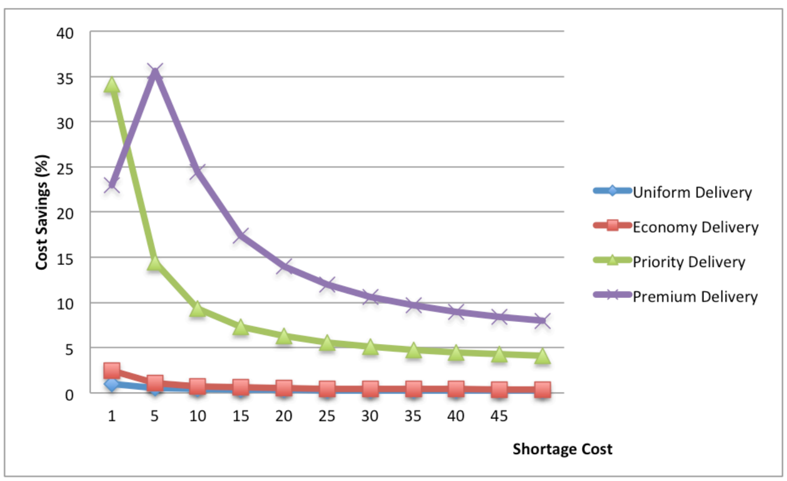


b) high-congestion

Figure 3.8. Cost savings with $h = 10$, $K = 8$, N=2; $\mu_d = 100$ and $\sigma_d = 10$; expedited shipment scenario.

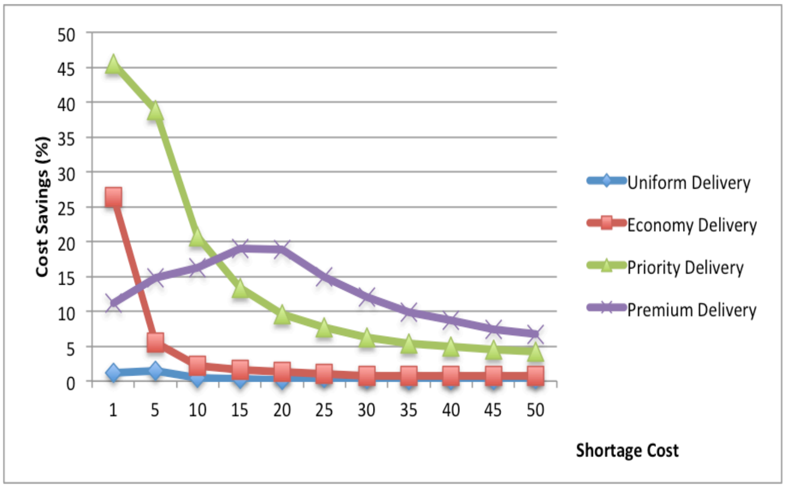


a) low-congestion

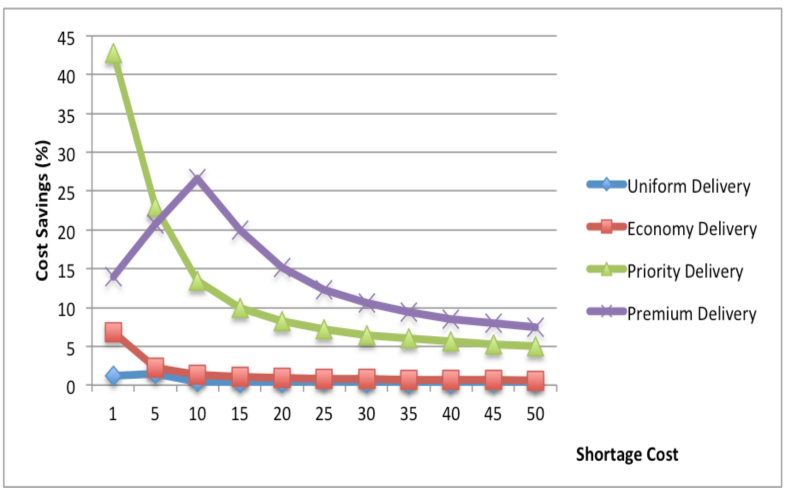


b) high-congestion

Figure 3.9. Cost savings with $h = 10$, $K = 8$, N=2; $\mu_d = 100$ and $\sigma_d = 30$; expedited shipment scenario.

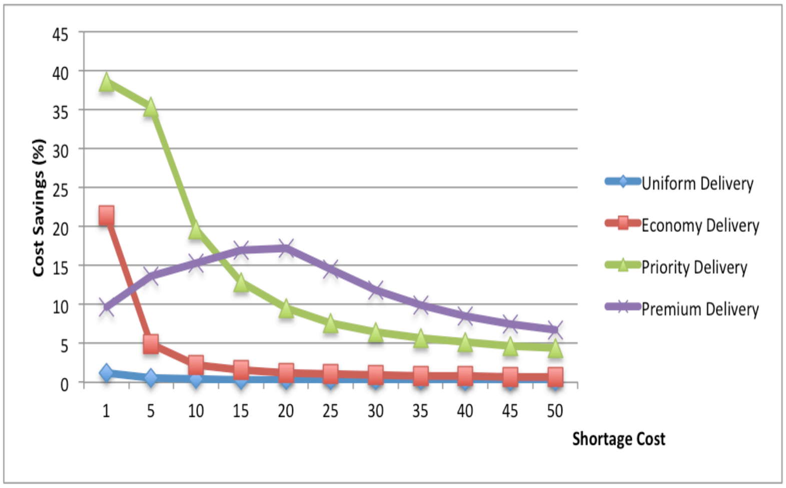


a) low-congestion

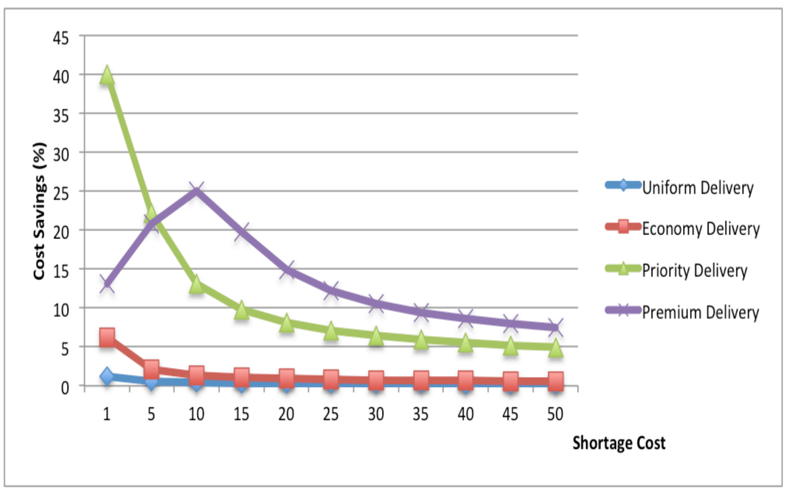


b) high-congestion

Figure 3.10. Cost savings with h = 10, K = 8, N=2; μ_d = 100 and σ_d = 10; batch shipment scenario.



a) low-congestion



b) high-congestion

Figure 3.11. Cost savings with h = 10, K = 8, N=2; μ_d = 100 and σ_d = 30; batch shipment scenario.

Table 3.4. Cost savings with normal distribution for demand with h = 10, K = 8 and expedited shipment scenario with N = 2.

Congestion Scenario	Delivery Scenario	r	$\mu_d = 100, \sigma_d = 10$			$\mu_d = 100, \sigma_d = 30$		
			C^*	C_s^*	$100\% (C_s^* - C^*)/C^*$	C^*	C_s^*	$100\% (C_s^* - C^*)/C^*$
-	Uniform	5	966.2	972.0	0.6%	1011.4	1017.1	0.6%
-	Uniform	10	1521.3	1524.8	0.2%	1596.9	1603.7	0.4%
-	Uniform	15	1940.1	1953.5	0.7%	2021.3	2028.4	0.4%
Low	Economy	5	814.9	846.6	3.9%	929.8	963.5	3.6%
Low	Economy	10	1198.6	1227.1	2.4%	1380.1	1407.3	2.0%
Low	Economy	15	1459.5	1478.0	1.3%	1684.7	1708.7	1.4%
Low	Priority	5	1193.1	1632.9	36.9%	1257.1	1673.9	33.2%
Low	Priority	10	1839.8	2211.4	20.2%	1939.3	2299.1	18.6%
Low	Priority	15	2248.2	2552.0	13.5%	2377.8	2686.2	13.0%
Low	Premium	5	1101.0	1446.4	31.4%	1159.9	1482.3	27.8%
Low	Premium	10	2050.7	2693.9	31.4%	2086.3	2704.9	29.7%
Low	Premium	15	2806.3	3496.5	24.6%	2848.3	3522.2	23.7%
High	Economy	5	2061.1	2083.7	1.1%	2134.1	2156.5	1.0%
High	Economy	10	3161.6	3185.8	0.8%	3271.4	3295.4	0.7%
High	Economy	15	3932.0	3956.6	0.6%	4066.5	4091.2	0.6%
High	Priority	5	2266.4	2608.1	15.1%	2322.0	2657.7	14.5%
High	Priority	10	3488.3	3823.4	9.6%	3574.6	3908.0	9.3%
High	Priority	15	4331.1	4657.0	7.5%	4439.4	4765.3	7.3%
High	Premium	5	2319.9	3188.1	37.4%	2356.2	3196.0	35.6%
High	Premium	10	4050.9	5059.0	24.9%	4087.3	5086.8	24.5%
High	Premium	15	5252.5	6176.3	17.6%	5301.9	6225.7	17.4%

Table 3.5. Cost savings with normal distribution for demand with h = 10, K = 8 and batch shipment scenario with $N = 2$.

Congestion Scenario	Delivery Scenario	r	$\mu_d = 100,\ \sigma_d = 10$			$\mu_d = 100,\ \sigma_d = 30$		
			C^*	C_s^*	$100\%\ (C_s^* - C^*)/C^*$	C^*	C_s^*	$100\%\ (C_s^* - C^*)/C^*$
-	Uniform	5	1008.8	1023.3	1.4%	1053.0	1059.6	0.6%
-	Uniform	10	1579.1	1585.5	0.4%	1651.5	1659.0	0.5%
-	Uniform	15	1996.6	2003.0	0.3%	2083.5	2091.3	0.4%
Low	Economy	5	1011.2	1067.2	5.5%	1105.9	1160.6	4.9%
Low	Economy	10	1449.4	1480.4	2.1%	1609.4	1646.2	2.3%
Low	Economy	15	1736.3	1765.3	1.7%	1938.2	1968.4	1.6%
Low	Priority	5	1426.2	1980.5	38.9%	1475.9	1999.0	35.4%
Low	Priority	10	2314.3	2793.8	20.7%	2379.8	2847.9	19.7%
Low	Priority	15	2862.9	3245.9	13.4%	2954.2	3334.3	12.9%
Low	Premium	5	1063.4	1220.6	14.8%	1095.5	1245.4	13.7%
Low	Premium	10	1849.5	2149.7	16.2%	1925.9	2221.6	15.4%
Low	Premium	15	2552.9	3037.7	19.0%	2634.8	3081.9	17.0%
High	Economy	5	2135.2	2183.1	2.2%	2206.4	2253.0	2.1%
High	Economy	10	3258.2	3302.8	1.4%	3365.6	3410.8	1.3%
High	Economy	15	4039.2	4082.5	1.1%	4171.8	4215.9	1.1%
High	Priority	5	2501.4	3077.0	23.0%	2544.9	3111.7	22.3%
High	Priority	10	3897.2	4422.0	13.5%	3967.7	4489.9	13.2%
High	Priority	15	4831.7	5314.4	10.0%	4923.9	5408.3	9.8%
High	Premium	5	2109.4	2548.4	20.8%	2142.7	2589.0	20.8%
High	Premium	10	3762.5	4764.3	26.6%	3815.5	4771.9	25.1%
High	Premium	15	5206.6	6245.9	20.0%	5236.0	6269.6	19.7%

Comparison of the two transportation settings shows that the high-congestion transportation setting gives lower cost savings than the low-congestion transportation setting for

- the economy delivery mode,
- the priority delivery mode,
- the premium delivery mode with the expedited shipment scenario for $K>5$ and $r\geq 10$.

The low-congestion transportation setting gives lower cost savings than the high-congestion transportation setting for

- the premium delivery mode with $K\leq 5$,
- the premium delivery mode with the batch shipment scenario,
- the premium delivery mode with the expedited shipment scenario for $K>5$ and $r\leq 5$.

It is intuitive that the high-congestion transportation setting gives the lower cost savings, as it is harder to manage high congestion. As for the premium delivery mode, when it is combined with the batch shipment scenario, congestion has less impact on the system and the results are reversed. Obtaining real-time tacking information has greater value for the low-congestion transportation setting for the economy and priority deliveries. With the premium delivery mode, real-time tacking information gives more benefits for small values of *K* and for the batch shipment scenario.

The next step is to compare cost savings with the expedited and batch shipment scenarios. The results show that with the low-congestion transportation setting the cost savings are always higher with the expedited shipment scenario than with the batch shipment scenario. Since the batch shipment scenario clears up transportation congestion, the probability that a new shipment will get delivered the next time period is high, especially with the low-congestion transportation setting. Thus the long-run average cost with full real-time tracking information is very close to the "no-information" cost.

With the high-congestion transportation setting, cost savings are higher with the batch shipment scenario than with the expedited scenario for the economy and priority delivery modes. The same result holds for the premium delivery mode but only with certain values of *K*. In case of the high-congestion transportation setting, probabilities of immediate delivery with batch shipment scenario are lower. Thus managing the system is getting harder and real-time tracking information gains more value. The batch shipment and the premium delivery combination can still overcome high congestion for small values of K, making cost savings with the batch shipment scenario less than with the expedited shipment scenario.

In general, the cost savings obtained with real-time tracking information are more pronounced with $\sigma_d = 10$ than with $\sigma_d = 30$. This is intuitive as it is more difficult to deal with high variation.

As discussed in section 3.5.2.1, increasing *N*, the maximum allowable number of orders at the manufacturer's site, does not really change the long-run average cost values. As a result the cost savings stay the same too.

As *K* increases, the cost savings are almost identical for the economy delivery mode. For the priority delivery mode with both low- and high-congestion transportation settings, the difference in cost savings is observed only for the low shortage cost $r = 1$ or 5. The cost savings with premium delivery mode increase as *K* increases with shortage cost value less than 20 (for the expedited shipment scenario) or 35 (for the batch shipment scenario), and they are almost identical for the higher values of *r*. The initial difference in cost savings is more pronounced with the low-congestion transportation setting. The results follow the same pattern regardless of the demand variation size. Figure 3.12 shows the cost savings for the expedited shipment scenario, and Figure 3.13 shows the cost savings for the batch shipment scenario for different values of *K* with $N = 2$. In Figure 3.12 σ_d=10, and in Figure 3.13 σ_d=30. In both figures the low-congestion transportation setting and the premium delivery mode were used.

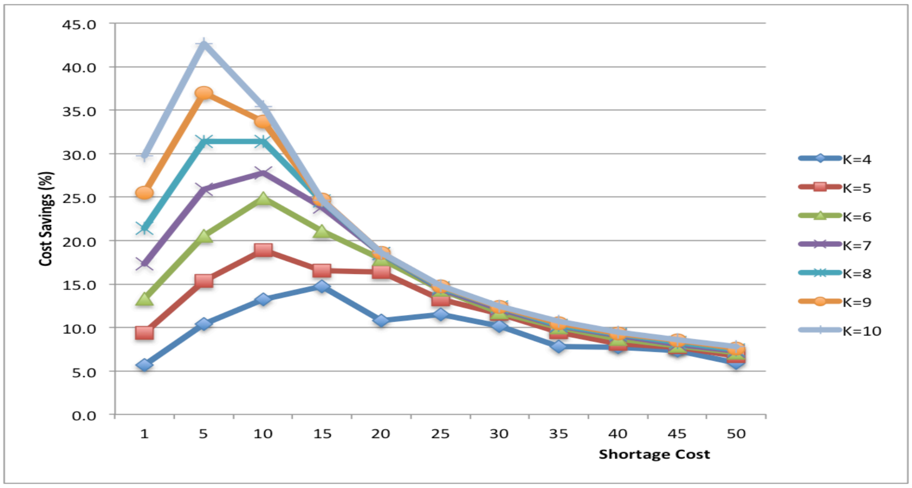


Figure 3.12. Cost savings with h=10, μ_d=100 and σ_d=10; expedited shipment scenario with low-congestion transportation setting and premium delivery mode with N=2.

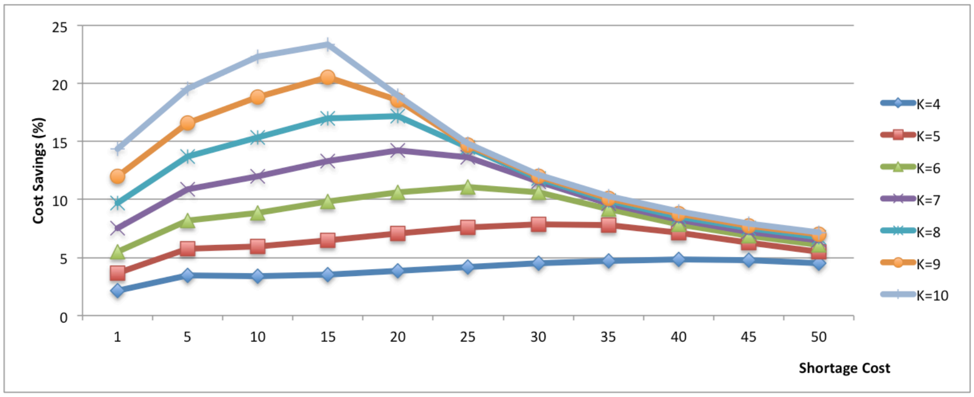


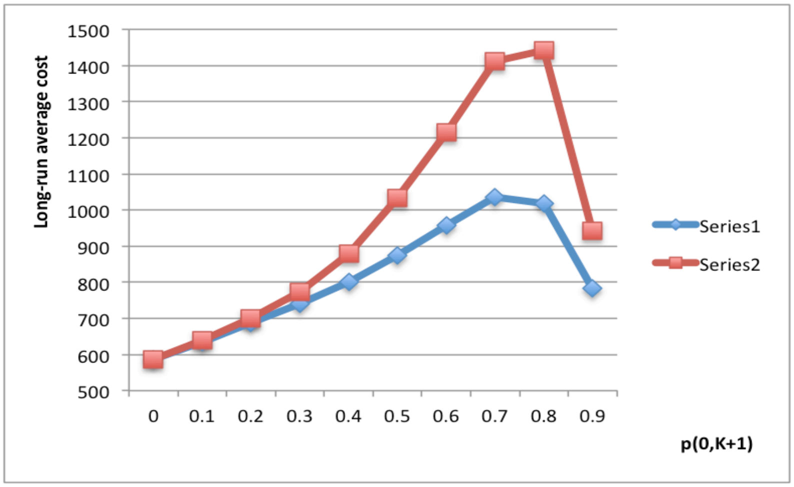
Figure 3.13. Cost savings with h = 10, μ_d = 100 and σ_d = 30; batch shipment scenario with low-congestion transportation setting and premium delivery mode with N=2.

With the premium delivery mode, real-time tracking information has a greater value for small values of the shortage cost. As K increases, the long-run average costs with and without information are getting farther from each other; thus, the value of real-time tracking information increases.

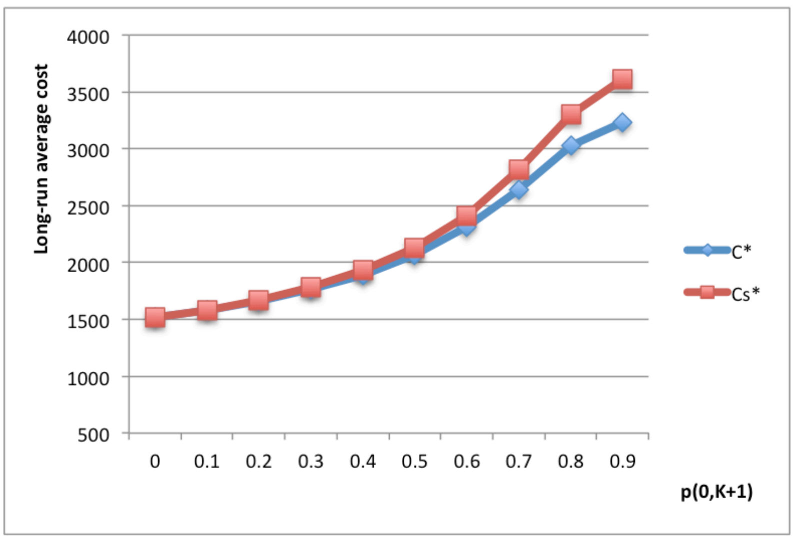
3.5.2.2.1 Additional Investigation of the Effect of the Delivery Mode

The priority delivery mode gives the best cost savings for smaller values of the shortage cost while the premium delivery mode gives the best cost savings for higher shortage costs. To further investigate the effect of different delivery modes the results of the experiment from section 3.5.2.1.3 were used. Both long-run average cost with real-time tracking information and the long-run average cost with no information are convex for smaller values of K and for high values of K with the small shortage cost $r = 5$. For high values of K with high shortage cost, the long-run average costs become increasing functions of $p_{0,K+1}$=*0.1*. The results show that the two long-run average costs are very close to each other for small values of $p_{0,K+1}$ and the difference becomes more pronounced for the higher values of $p_{0,K+1}$. Figure 3.14 shows the long-run average costs for the low-congestion transportation setting and expedited shipment scenario with $N = 2$ and K=6. It can be seen from Figure 3.14 that the difference between the long-run average cost with real-time tracking information and the long-run average cost with no information is more pronounced for the range of $p_{0,K+1} \in [0.5;0.9]$.

The results show that the maximum difference between the long-run average cost with real-time tracking information and the long-run average cost with no information are attained at $p_{0,K+1} = 0.9$ for high values of K and the shortage cost. For smaller values of K and the smaller values of the shortage cost, the maximum difference is attained at $p_{0,K+1} <$ 0.9. Figure 3.15 shows the cost savings for the expedited shipment scenario and the low-congestion scenario. For $K = 2$ and $r = 5$ the maximum cost savings are attained at $p_{0,K+1}$ = 0.6 while for the higher values of r the maximum cost savings are attained at $p_{0,K+1}$ = 0.8. For $K = 8$ and $r = 5$ the maximum cost savings are attained at $p_{0,K+1} = 0.8$ while for the higher values of r the maximum cost savings are attained at $p_{0,K+1} = 0.9$. With the high congestion transportation setting the maximum cost savings are attained at the higher values of $p_{0,K+1}$. The results are similar for the batch shipment scenario with the difference that the maximums are attained at one step lower $p_{0,K+1}$.

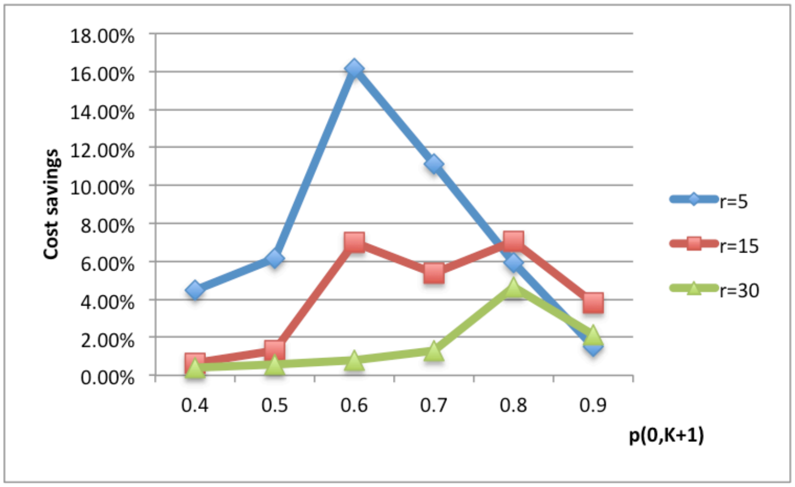


a) $r = 5$

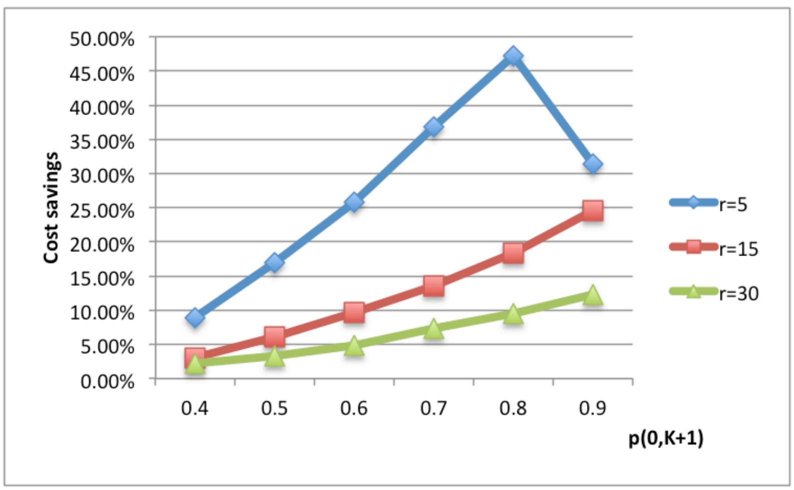


b) $r = 30$

Figure 3.14. Expected costs with real-time tracking information (C^*) and with no information (C_s^*) with $h = 10$, $\mu_d = 100$ and $\sigma_d = 10$; low-congestion transportation setting; expedited shipment scenario with $N = 2$ and K=6.



a) K=2



b) K=8

Figure 3.15. Cost savings for different delivery modes; expedited shipment scenario with low-congestion transportation setting.

To sum up, congestion has less impact when shipments don't occur every time period. There are settings when obtaining real-time tracking information can bring substantial benefits to the retailer.

3.5.3 The Lowest Long-Run Average Cost as a Function of the Retailer's Ordering Probabilities

The model allows the retailer to vary his ordering probabilities based on the total state vectors. This section investigates the expected system cost as a function of these ordering probabilities. The investigation shows that it makes sense for the retailer to order every time period if he does not have full information on the supply status.

3.5.3.1 The Retailer's Ordering Process with Partial or No Information

The following cases regarding the availability of information were considered:

i. The retailer knows how many orders are at the manufacturer's site and how many stages are occupied. But he has no information about how orders are being handled at the manufacturer's site and no information on how orders shipped from the manufacturer's site move downstream. In other words, he has no information about the behavior of the random variables X and Y. Lacking such information, it is assumed that the retailer proceeds on the assumption that the number of orders shipped during a given time period follows a uniform distribution, and that shipments are equally likely to move to any of the downstream stages. In other words, the p and q matrices are the same as in the uniform scenario in section 3.5.1.

ii. The retailer has information on the number of orders awaiting shipment at the manufacturer's site and on how shipped orders move downstream (information on the behavior of X). However, he does not have any information on how orders are shipped from the manufacturer's site (no information about the behavior of Y). He therefore proceeds on the assumption that the number of orders shipped during a given time period follows a uniform distribution.

iii. The retailer has information on how orders at the manufacturer's site are handled (the random variable Y), and he has knowledge on which stages are occupied.

However, he has no information about the shipment movement (no information on the behavior of X). Lacking information on the behavior of X, he assumes that shipments are equally likely to move to any of the downstream stages.

The baseline expected cost is the long-run average cost that results when the retailer orders every time period. Two ordering approaches were considered:

- Order according to the number of unshipped orders: set the retailer's ordering probabilities to 0 when there is a pre-specified number, i, of unshipped orders at the manufacturer's site, for $i = 0, 1, \ldots, N$. This provides $N + 1$ different ordering scenarios. This approach was used for cases i and ii.

- Order according to the number of occupied stages: set the retailer's ordering probabilities to 0 when there is a pre-specified number, j, of occupied stages, for $j = 1, \ldots, K$. This provides K different ordering scenarios. This approach was used for cases i and iii.

The goal is to find out what probabilities give the retailer the lowest long-run average cost. Different settings for the number of transportation stages (K) and the maximum number of unshipped orders at the manufacturer's place (N) were studied. To model the manufacturer's known shipping policies, the expedited and batch shipment scenarios were used (see section 3.5.1). To model the known transportation policies, the premium, priority and economy delivery modes were used combined with the high- and low-congestion transportation settings (see section 3.5.1). The retailer's ordering probabilities assume values 0 (order) or 1 (do not order). The parameters are $\rho_1 = 1,\ \rho_i = 0$ or 1, for $i = 2, \ldots, (N+1)2^K$. Values other than 0 or 1 are hard to implement in practice and were not considered.

For all cases, the results show that the lowest expected cost is achieved when the retailer orders every time period. The results are in agreement with intuition. When there is a lack of information, it is better for the retailer to protect himself by ordering every time period. Some of these results are shown in Table 3.6 for the case when demand is normally distributed with $\mu_d = 100$, $\sigma_d = 30$, and K=6, N=3, $h = 10$, and $r=5$.

Case i in Table 3.6 shows the results when the retailer has no information on either the shipping policy or the shipment movement. The results show that the retailer should order every time period regardless of the number of occupied stages or the number of unshipped orders. The lowest expected cost is C^{*}=1090.2. Note that the static policy gives the expected cost C_s^{*}=1390.2, which is higher than C^{*}, further underscoring the observation that a little information is better than no information at all.

Case ii in Table 3.6 shows results for the premium delivery and low-congestion transportation setting with no information from the manufacturer's side. It is observed that the expected cost gradually increases as the retailer orders less and less frequently.

Case iii in Table 3.6 shows the results for the expedited shipment scenario with no information on the movement of shipments. Once again, the lowest expected cost is achieved when orders are placed every time period.

Table 3.6. Ordering with partial or no information.

Ordering policy	Expected cost		
	Case i	**Case ii**	**Case iii**
Order every time period	1090.2	994.0	974.5
Order only if at most 1 stage is empty	1102.2		974.6
Order only if at most 2 stages are empty	1102.2		974.6
Order only if at most 3 stages are empty	1102.2		974.7
Order only if at most 4 stages are empty	1104.5		979.7
Order only if at most 5 stages are empty	1150.3		1035.3
Order only if all stages are empty	1446.6		1324.9
Order only if # unshipped orders is ≤ 2	1153.1	1047.3	
Order only if # unshipped orders is ≤ 1	1221.1	1102.8	
Order only if there are no unshipped orders	1424.9	1288.0	

To summarize, when there is a lack of information on the supply status, it is better for the retailer to order every time period. This result holds for all values of K and for the expedited shipment scenario as well as the batch shipment scenario.

3.5.3.2 The Retailer's Ordering Process with Full Information

A series of experiments was run for different scenarios and different values of *K* and *N* for the case where the retailer has complete information on the behavior of the *p* and *q* matrices.

The results show that the retailer should order every time period for the batch shipment scenario. For the expedited shipment scenario, the results follow the same pattern with the exception of the premium delivery mode with the high-congestion transportation setting. In this special case the results suggest that the retailer should order only when all the stages are empty if the shortage cost r=5. If the shortage cost is high (r=15), the retailer should order more often, specifically when there are less than K or K-1 occupied stages. This result is in agreement with intuition. When the shortage cost is low, the retailer can wait until the congestion clears out. As with the premium delivery mode $p_{0,K+1}$=0.9, no congestion in the transportation channel guarantees that the new shipments will reach the retailer's site basically in one time period. When the shortage cost is higher, the retailer tends to order more often to protect himself from the expected high shortage cost. The results for the high-congestion scenario and the premium delivery mode also suggest that the retailer should order only when there are no unshipped orders. Table 3.7 shows some results for the expedited shipment scenario with the premium delivery mode and the high-congestion transportation setting with K=6, N=3, holding cost h=10, shortage cost r=5 and 15 and the normally distributed demand with μ_d = 100, σ_d = 30.

Table 3.7. Ordering for the premium delivery mode with the high-congestion transportation setting.

Ordering policy	Expected Cost	
	r=5	*r*=15
Order every time period	1767.6	4232.3
Order only if at most 1 stage is empty	1767.2	4231.7
Order only if at most 2 stages are empty	1767.2	4231.7
Order only if at most 3 stages are empty	1767.2	4231.8
Order only if at most 4 stages are empty	1767.2	4235.9
Order only if at most 5 stages are empty	1767.5	4288.4
Order only if all stages are empty	1753.3	4509.7
Order only if # unshipped orders is ≤ 2	1764.1	4226.5
Order only if # unshipped orders is ≤ 1	1737.1	4181.8
Order only if there are no unshipped orders	1553.1	3795.4

CHAPTER 4
MANAGING THE SUPPLY CHAIN WITH PARTIAL INFORMATION

In today's globalized business environment, many companies ship their products, subassemblies, parts and raw materials over great distances. The destination point may be in the same country, in a neighboring country, or overseas. Transportation of these products and parts requires sophisticated methods and varying means of travel (by rail, sea, air, and road, for example) by a single carrier or by a multimodal transport operators (MTO). Multimodal transport is performed under a single contract, using at least two different means of transport. The carrier is liable for the entire carriage, even though the carrier does not necessarily possess all the means of transport. In practice, sub-carriers often perform at least a portion of the carriage.

When transportation involves many sub-carriers, especially when the transportation channel goes through several countries, the process for tracking a shipment can become complex and full real-time visibility of shipments may not be available at all times during the transportation process; however, real-time information about which sub-carrier is handling the shipment is generally available. A similar situation may occur in a production line. Information on which machine is working on a part may not be tracked, but information on which department is working on this part usually is.

For some situations tracking shipments may be challenging or not possible. Some carriers use passive RFID (radio frequency identification devices), i.e. devices that only emit signal queried by an outside source. In order to track shipments that are equipped with passive RFIDs, readers (either hand-held or fixed) need to be installed to collect information. Installation of these readers adds additional costs and may not be always possible. In addition, to work properly, specific frequencies will need to be designated for RFID use only. Government assistance and cooperation is needed in order to prevent interference with other existing devices and applications.

In some situations collecting information is very challenging. Prater *et al.* (2001) describe the following case study with VAI, a large international producer of steel products. Seeking an expansion of its production capabilities, VAI set up a joint venture with steel mills in the Ural Mountains of Russia. This operation is coordinated from VAI's offices in Austria. The joint venture allows VAI to deal with increased demand in steel while keeping costs fairly low. However, VAI has to deal with the transportation difficulties. The steel is first transported by rail from the Ural in Russia to Odessa, Ukraine on the Black Sea, then by ship to Southeast Asia.

VAI works with both Russian and Ukrainian freight forwarders. The main problem is the flow of information and reliability of transportation times. A three-week lead time is required for the first sequence of the main transport plan. The first sequence includes the following steps:

- The mills order railway wagons through the Moscow railway mission.
- Odessa is informed that VAI wants rail capacity for 10,000 tons of pallets.
- Odessa informs Ukrainian railway ministry of rail needs.
- Ukrainian railway tells Russian railway ministry of its needs.

The next step is to get railway confirmation from the freight forwarders and set up the sea transportation. All this must be done using telegrams since email is non-existent and phone service is unreliable. To track the progress of shipments, VAI hires people to observe various points of the rail line. As each train passes by, the observer notes the apparent loads of the rail cars (in order to check for theft) and sends a telegram to VAI giving the train's location. This is the "information system."

These situations where real-time tracking information is not available throughout all stages of transportation and processing create an interesting supply chain management problem.

This chapter develops a methodology to evaluate the value of partial real-time order tracking information in a supply system through a stochastic model. The modeled supply system consists of a retailer, a manufacturer, and multiple stages of transportation. The retailer aggregates demand for a single product from his end customers and places orders to the manufacturer. The replenishment process between the manufacturer and the retailer is instantaneous but the transportation process may incur some amount of delay. Shipments pass through a series of transportation stages before the retailer receives them. Each transportation stage represents either a physical location for shipments or a step in the delivery process. Shipments are not allowed to cross over in time and real-time tracking information is available only for some of the transportation stages.

The stochastic model computes the retailer's long-run average cost when only partial real-time tracking information is available. The model also compares the partial information long-run average cost to the full information long-run average cost, i.e. the cost with real-time tracking information from all the transportation stages. The calculations demonstrate the relationship between full, partial and no information long-run average costs and draw insights as to the optimum locations of tracking devices at different stages.

4.1 The Supply System

The supply system discussed in this chapter is similar to the supply system discussed in the previous chapter (when *N*=0 and $\rho_i = 1$, for $i = 1,...,(N+1)2^K$) with the following exceptions:

- The retailer orders every time period. During those time periods when the order quantity is zero, it is assumed that the retailer places a pseudo order. Pseudo orders are necessary for mathematical tractability. As noted in Liu *et al.* (2009) the effect of pseudo orders is very low on the system's performance.

- It is assumed that the manufacturer is able to fulfill all orders placed by the retailer completely and ship them in a single shipment.

- Products shipped from the manufacturer's site pass through K transportation stages before they reach the retailer's site. Even when the shipment movement behavior across all the transportation stages is known, the real-time tracking information about a shipment's presence/absence can be collected from less than K transportation stages.

4.2 The Stochastic Model

To track shipments, tracking devices called beacons are placed at $M < K$ unique stages. Each tracking device monitors the progress of shipments from its location to the location of the next tracking device.

For instance, consider a system with $K = 5$ stages and $M = 2$ tracking devices, and suppose these devices are located at stages 1 and 4. In this example, the tracking device at stage 1 monitors stages 1, 2 and 3. The tracking device at stage 4 monitors stages 4 and 5, as illustrated in Figure 4.1. To account for situations when the first tracking device is located at a stage downstream from stage 1, information on the status of shipments at the stages upstream from it is monitored by a zero-beacon that is placed at the manufacturer's location.

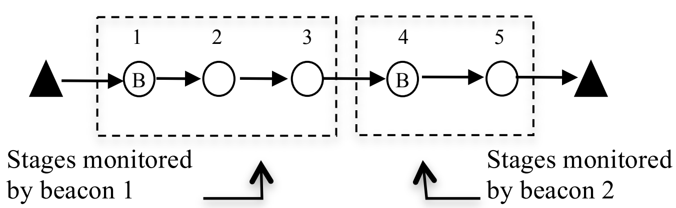


Figure 4.1. Example of two beacons for K = 5.

The monitoring can be configured in two different ways according to the information that can be collected from the beacons.

- Type I beacon: This beacon can detect the presence or absence of shipments at monitored stages. In other words, the beacon can provide data as to whether a shipment or shipments passed the beacon but did not reach the next beacon.

- Type II beacon: This beacon can identify how many stages are occupied among monitored stages.

Define a tracking device vector $\vec{b}(t)=[b_0(t),b_1(t),b_2(t),...,b_M(t)]$, where the variable $b_i(t),\, i=0,...,M$ is binary and indicates the presence or absence of shipments among the monitored stages for type I beacons and the number of occupied stages among stages monitored by tracking device *i* at time *t* for type II beacons. Let l_i denote the location of tracking device *i*, for $i=1,...,M$, and set $l_0=0$ and $l_{M+1}=K+1$. Thus, for $i=0,\dots,M$,

$$b_i(t)=\begin{cases} 1, \text{ if there is an occupied stage among the stages } l_i,...,l_{i+1}-1 \\ 0, \text{ otherwise} \end{cases}, \quad \text{for type I beacon}$$

and

$$b_i(t)=\text{number of occupied stages among the stages } l_i,...,l_{i+1}-1, \quad \text{for type II beacon.}$$

If a tracking device is placed at the first stage, the variable $b_0(t)$ can be omitted from a tracking device vector $\vec{b}(t)$ as it does not carry any information in such a situation.

For type I beacons there are 2^M possible values for the tracking device vector if the first beacon is placed at the first stage, and there are 2^{M+1} possible values if the zero-beacon is used. For type II beacons, each $b_i(t)$, for $i=1,...,M$ variable can assume values from 0 to $l_{i+1}-l_i$ and if $l_1>1$ the variable $b_0(t)$ can assume values from 0 to l_1-1. Thus the number of all possible values for the tracking device vector is

$$bn \overset{def}{=} \begin{cases} 2^M \begin{cases} 2 & if\ l_1 > 1 \\ 1 & f\ l_1 = 1 \end{cases} & \text{for Type I beacons} \\ \prod_1^M (l_{i+1} - l_i + 1) \begin{cases} l_1 & if\ l_1 > 1 \\ 1 & f\ l_1 = 1 \end{cases} & \text{for Type II beacons.} \end{cases}$$

Let $\vec{b}_i, i=1,\ldots,bn$ denote a tracking device vector. Let **b**=$\{\vec{b}(t), t=1,2,\ldots\}$ and let Θ4= $\{\vec{b}_1,\vec{b}_2,\ldots,\vec{b}_{bn}\}$ denote the state space of **b**. To illustrate the relationship between the tracking device vector and the supply status in the transportation process vector, suppose there are K = 4 stages and M = 2 tracking devices located at stages $l_1 = 1$ and $l_2 = 3$. The tracking device vector values and the values of the corresponding supply status vector in the transportation process values are shown below.

Each tracking device vector, $\vec{b}_i, i=0,\ldots,bn,$ thus accounts for ("covers") a set of supply status vectors. Let $B_i = \{j\}$ denote the set of supply status vector indices covered by tracking device vector $\vec{b}_i$. Given this, the following equation holds:

$$\bigcup_{i=1}^{bn} B_i = \{1,2,\ldots,2^K\}.$$

Note that since tracking devices are placed at unique locations, we have

$$B_i \cap B_j = \varnothing \text{ for any } 1 \le i, j \le b_n$$

Table 4.1 and Table 4.2 illustrate the relationship between tracking device vectors and supply status vectors. In these examples K=4 and beacons are placed at stages 1 and 3.

Table 4.1. Type I beacons and the total supply status vectors.

Tracking device vector ($\vec{b}$)	Supply status vectors in the transportation process ($\vec{s}$)
[0,0]	(0,0,0,0)
[0,1]	(0,0,0,1) (0,0,1,0) (0,0,1,1)
[1,0]	(0,1,0,0) (1,0,0,0) (1,1,0,0)
[1,1]	(0,1,0,1) (0,1,1,0) (1,0,0,1) (1,0,1,0) (0,1,1,1) (1,0,1,1) (1,1,0,1) (1,1,1,0) (1,1,1,1)

Table 4.2. Type II beacons and the supply status vectors.

Tracking device vector ($\vec{b}$) (Type II beacons)	Supply status vectors in the transportation process ($\vec{s}$)
[0,0]	(0,0,0,0)
[0,1]	(0,0,0,1) (0,0,1,0)
[0,2]	(0,0,1,1)
[1,0]	(0,1,0,0) (1,0,0,0)
[1,1]	(0,1,0,1) (0,1,1,0) (1,0,0,1) (1,0,1,0)
[1,2]	(0,1,1,1) (1,0,1,1)
[2,0]	(1,1,0,0)
[2,1]	(1,1,0,1) (1,1,1,0)
[2,2]	(1,1,1,1)

Property 4.1: Process **b** is a time-homogenous Markov chain.

Proof: First, note that when the retailer places orders every time period, process **s** is a time-homogenous Markov chain (see Liu *et al.,* 2009). The value of $\vec{b}(t)$ is determined only by the value of $\vec{s}(t)$, and the value of $\vec{b}(t-1)$ is determined only by the value of $\vec{s}(t-1)$. $\Pr\{\vec{s}(t)\mid\vec{s}(t-1),\vec{s}(t-2),...,\vec{s}(0)\}=\Pr\{\vec{s}(t)\mid\vec{s}(t-1)\}$ as process **s** is Markovian. Thus, it can be concluded that

$$\Pr\{\vec{b}(t)\mid\vec{b}(t-1),\vec{b}(t-2),...,\vec{b}(0)\}=\Pr\{\vec{b}(t)\mid\vec{b}(t-1)\} \qquad \blacksquare$$

Note that the limiting probability that the system is in state $\vec{b}_i$ is $\sum_{l\in B_i}\omega_l$, where ω_i is the limiting distribution for process **s**.

All the characteristics of process **s** can be obtained using the same algorithm and methods as described in Chapter 3 by setting all the retailer's ordering probabilities to 1 ($\rho_j=1$) and the maximum number of orders pending shipment at the manufacturer's site to 0 (N=0).

4.3 The Long-Run Average Cost

When the system is in state $\vec{s}_i$, the one-period expected cost (the expected cost charged to the order placed in period t, given $\vec{s}(t)=\vec{s}_i$ and $IP(t)=y$) is as follows:

$$G(\vec{s}_j,y)=\sum_{l\geq 1}\Pr[L(t)\leq l\leq L(t+1)\mid\vec{s}(t)=\vec{s}_j]g(l,y),$$

where L(t) is the conditional lead-time for an order placed at time period t and $g(l,y)$ is defined as

$$g(l,y) \overset{def}{=} E[\ h \quad \max(0, y - D(t,l)) - \ r \quad \min(0, y - D(t,l))], \qquad l \geq 0.$$

Then when the system is in state $\vec{b}_i$ it is appropriate to define the one-period expected cost (the expected cost charged to the order placed in period t, given $\vec{b}(t) = \vec{b}_i$ and $IP(t) = y$) as follows:

$$GB(\vec{b}_i, y) = \sum_{j \in B_i} \left[\frac{\omega_j}{\sum_{l \in B_i} \omega_l} G(\vec{s}_j, y) \right]$$

where ω_i-s are limiting probabilities for the process **s**.

Let IP_i^* be the optimal order-up-to level that minimizes the one-period expected cost, given that the system is in state $\vec{b}(t) = \vec{b}_i$ when an order is placed. That is,

$$IP_i^* \overset{def}{=} \arg\min_y GB(\vec{b}_i, y).$$

Thus the optimal (myopic) long-run average cost is

$$C_B^* = \sum_{i=1}^{bn} \left(\sum_{l \in B_i} \omega_l \right) GB(\vec{b}_i, IP_i^*) .$$

Property 4.2: $C^* \leq C_B^* \leq C_s^*$.

Proof:

1. Relationship between C^* and C_B^*

$$C^* = \sum_{j=1}^{2^K} \omega_j G(\vec{s}_j, IP_j^*) = \sum_{j=1}^{2^K} \omega_j \min_y G(\vec{s}_j, y) = \sum_{j=1}^{2^K} \min_y \omega_j G(\vec{s}_j, y)$$

$$= \sum_{i=1}^{bn} \sum_{j \in B_i} \min_y \omega_j G(\vec{s}_j, y) \le \sum_{i=1}^{bn} \min_y \sum_{j \in B_i} \omega_j G(\vec{s}_j, y)$$

$$= \sum_{i=1}^{bn} \min_y \left(\sum_{l \in B_i} \omega_l \right) \sum_{j \in B_i} \frac{\omega_j}{\left(\sum_{l \in B_i} \omega_l \right)} G(\vec{s}_j, y)$$

$$= \sum_{i=1}^{bn} \min_y \left(\sum_{l \in B_i} \omega_l \right) GB(\vec{b}_i, y) = \sum_{i=1}^{bn} \left(\sum_{l \in B_i} \omega_l \right) \min_y GB(\vec{b}_i, y)$$

$$= \sum_{i=1}^{bn} \left(\sum_{l \in B_i} \omega_l \right) GB(\vec{b}_i, IP_i^*) = C_B^*$$

2. Relationship between C_s^* and C_B^*

$$C_s^* = \min_y \sum_{j=1}^{2^K} \omega_j G(\vec{s}_j, y) = \min_y \sum_{i=1}^{bn} \sum_{j \in B_i} \omega_j G(\vec{s}_j, y)$$

$$\ge \sum_{i=1}^{bn} \min_y \sum_{j \in B_i} \omega_j G(\vec{s}_j, y) = \sum_{i=1}^{bn} \min_y \left(\sum_{l \in B_i} \omega_l \right) \sum_{j \in B_i} \frac{\omega_j}{\left(\sum_{l \in B_i} \omega_l \right)} G(\vec{s}_j, y)$$

$$= \sum_{i=1}^{bn} \left(\sum_{l \in B_i} \omega_l \right) \min_y GB(\vec{b}_i, y) = \sum_{i=1}^{bn} \left(\sum_{l \in B_i} \omega_l \right) GB(\vec{b}_i, IP_i^*) = C_B^*$$

Thus we have $C^* \le C_B^* \le C_s^*$. ■

Type II beacons provide more information than type I beacons. In addition to the presence or absence of the shipments, type II beacons also supply information about the number of occupied stages. Because of the capability to provide this additional information, the expected long-run average cost with type II beacons is less than the corresponding cost with type I beacons.

To illustrate the relationship between the two types of beacons, suppose a beacon is covering 3 stages. Let superscript *I* denote the variables for type I beacons and the superscript *II* denote the corresponding variables for type II beacons. If the beacon value $b^I(t)$ is 0 then for both types of beacons the stages are empty or, in other words, the representation of the monitored 3 stages is (0,0,0). If the type I beacon value $b^I(t)$ is 1 (there is at least 1 shipment among the 3 monitored stages), then the corresponding value of type II beacon $b^{II}(t)$ is 1, 2, or 3 (there is only 1 stage occupied, or there are 2 or 3 stages occupied). Thus for each set of supply status vector indices covered by each type II tracking device vector, B_j^{II}, there is a set of supply status vector indices covered by a type I tracking device vector, B_i^I for which $B_j^{II} \in B_i^I$. Each B_j^{II} completely belongs to only one of B_i^I. Let Ω_i be the set of all B_j^{II} that are subset of B_i^I.

Let $C_{B^I}^*$ be the long-run average cost obtained using type I beacons and $C_{B^{II}}^*$ be the corresponding long-run average cost obtained using type II beacons. An important assumption is that the beacons are placed at the same positions and only the type of information provided is different.

Property 4.3: $C_{B^{II}}^* \le C_{B^I}^*$.

$$C_{B^{II}}^* = \sum_{i=1}^{bn^{II}} \left(\sum_{l \in B_i^{II}} \omega_l \right) GB(\vec{b}_i^{II}, IP_i^*) = \sum_{i=1}^{bn^{II}} \min_y \sum_{l \in B_i^{II}} \omega_l G(\vec{s}_l, y) =$$

$$= \sum_{i=1}^{bn^{II}} \min_y \left(\sum_{j \in \Omega_i} \left(\sum_{l \in B_j^I} \omega_l G(\vec{s}_l, y) \right) \right) \le \sum_{i=1}^{bn^{II}} \left(\sum_{j \in \Omega_i} \min_y \left(\sum_{l \in B_j^I} \omega_l G(\vec{s}_l, y) \right) \right) =$$

$$= \sum_{j=1}^{bn^I} \min_y \left(\sum_{l \in B_j^I} \omega_l G(\vec{s}_l, y) \right) = \sum_{j=1}^{bn^I} \left(\sum_{l \in B_j^I} \omega_l \right) GB(\vec{b}_j, IP_j^*) = C_{B^I}^*.$$

■

Combining property 4.2 and 4.3 gives

$$C^* \le C_{B^{II}}^* \le C_{B^I}^* \le C_s^*.$$

4.4 Numerical Investigations

A wide range of parameters and settings was tested to investigate the value of long-run average costs and cost savings with a different number of beacons as well as to determine the optimal number of beacons.

4.4.1 Transition Matrices from Chapter 3

The same congestion transportation settings and delivery modes as described in paragraph 3.5.1 were used. It is assumed that the retailer knows the one-step transition matrix *p*; in other words, he has theoretical knowledge about the movements of shipments through the transportation stages. But the real-time information can be collected from less than *K* stages if any.

As anticipated, all the calculation results are in agreement with Property 4.2 and 4.3: $C^* \le C_B^* \le C_s^*$ and $C_{B^{II}}^* \le C_{B^I}^*$. The next step was to investigate how the number and the placement of beacons impact the long-run average cost. If there are *K* transportation stages but only a small number of beacons can be placed, what would be the better position to lower the long-run average cost as much as possible? Implementation of this decision can be as wide as just placing beacons to choosing different carriers.

4.4.1.1 "Baseline" Expected Cost

It is appropriate to assume that the retailer knows whether there are any outstanding orders. In other words, he knows whether the supply system is in state s_1=(0,0,…,0) or in any other state even without collecting information from the transportation stages. Mathematically this corresponds to situation when there is only one zero-beacon placed at the manufacturer's location and the tracking device vector is $\vec{b}(t) = [b_0(t)]$. Let $C_{B_0}^*$ be the corresponding long-run average cost.

The first experiment investigates cost difference between the full real-time tracking information long-run average cost (C^*) and the “baseline” cost ($C^*_{B_0}$). The difference between the full real-time tracking information long-run average cost (C^*) and the long-run average cost with no information (C^*_s) is also calculated.

If the retailer does not have real-time information about occupied stages but has information about the shipment's movement and the transition probabilities, it is better to use the “baseline” cost function to determine order-up-to levels. Still this cost will be higher than the long-run average cost with real-time tracking information. Table 4.3 shows the long-run average cost values and cost savings for $K = 8$ with the holding cost of 10, normally distributed demand with $\mu_d = 100$ and $\sigma_d = 10$.

It can be seen that the cost savings are more pronounced with the shortage cost of 5. The low-congestion transportation setting always gives better cost savings (difference between C^* and $C^*_{B_0}$) than the high-congestion transportation setting. Using the “baseline” cost gives the better approximation to the real-time information cost, but the difference between $C^*_{B_0}$ and C^* still can be as high as 19.9% (low-congestion transportation setting with the priority delivery mode), thus emphasizing the importance of real-time tracking information.

The difference between the long-run average costs with and without full information is not high for the economy delivery mode. The maximum difference is achieved for a low-congestion transportation setting with $K = 4$, the shortage cost $r = 5$, and the standard deviation $\sigma_d = 30$.

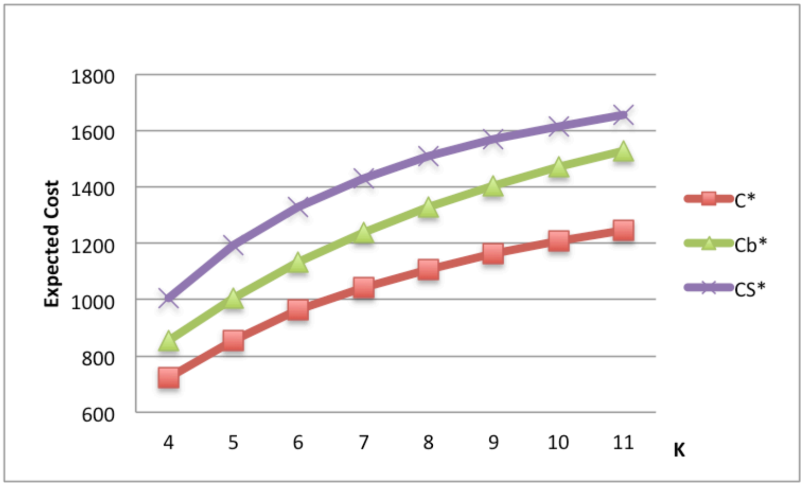
$$\max\left(\frac{C^*_s - C^*}{C^*} 100\%\right) \approx 4\% .$$

And the difference can get as small as 0.4%. In these situations the “baseline” cost stays closer to the long-run average cost with no information.

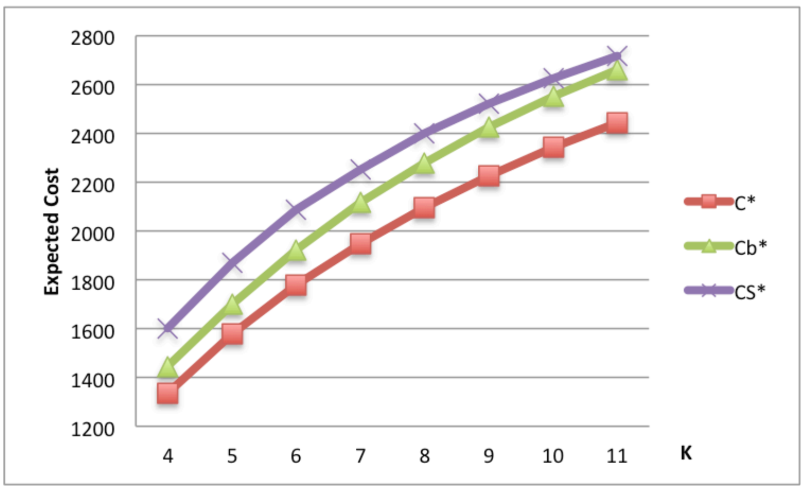
Table 4.3. The long-run average cost values and cost savings for $K = 8$ with $h = 10$.

			$\mu_d = 100, \sigma_d = 10$				
Congestion Setting	**Delivery Mode**	r	C^*	$C^*_{B_0}$	C^*_s	100% $(C^*_{B_0} - C^*)/C^*$	100% $(C_s^* - C^*)/C^*$
Low	Economy	5	741.9	764.3	764.7	3.0%	3.1%
Low	Economy	15	1326.5	1341.4	1341.6	1.1%	1.1%
Low	Priority	5	1107.4	1328.1	1509.3	19.9%	36.3%
Low	Priority	15	2065.7	2215.0	2340.4	7.2%	13.3%
Low	Premium	5	1075.6	1268.7	1473.1	17.9%	36.9%
Low	Premium	15	2775.7	2935.0	3479.1	5.7%	25.3%
High	Economy	5	1922.9	1940.1	1940.2	0.9%	0.9%
High	Economy	15	3698.4	3716.0	3716.1	0.5%	0.5%
High	Priority	5	2096.6	2280.0	2399.0	8.7%	14.4%
High	Priority	15	4031.0	4219.8	4319.7	4.7%	7.2%
High	Premium	5	2137.7	2362.5	3044.0	10.5%	42.4%
High	Premium	15	4894.0	5092.4	5776.1	4.1%	18.0%

Figure 4.2 shows long-run average costs C^*, $C^*_{B_0}$, and C^*_s for the priority delivery mode with $h = 10$, $r = 5$ for different values of K. Figure 4.2.a shows the results for the low-congestion transportation setting and $\sigma_d = 30$ and Figure 4.2.b shows the results for the high-congestion transportation setting and $\sigma_d = 10$. Figure 4.3 shows the corresponding cost savings. It can be observed that as K increases the difference between C^* and C^*_s and the difference between C^* and $C^*_{B_0}$ are getting closer to each other. In other words, as K increases $C^*_{B_0}$ gets closer to C^*_s.

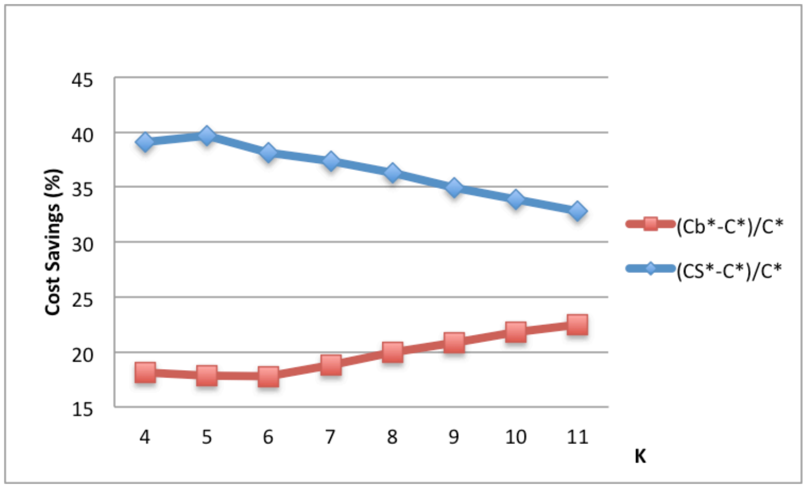


a) low-congestion transportation setting; $\sigma_d = 30$

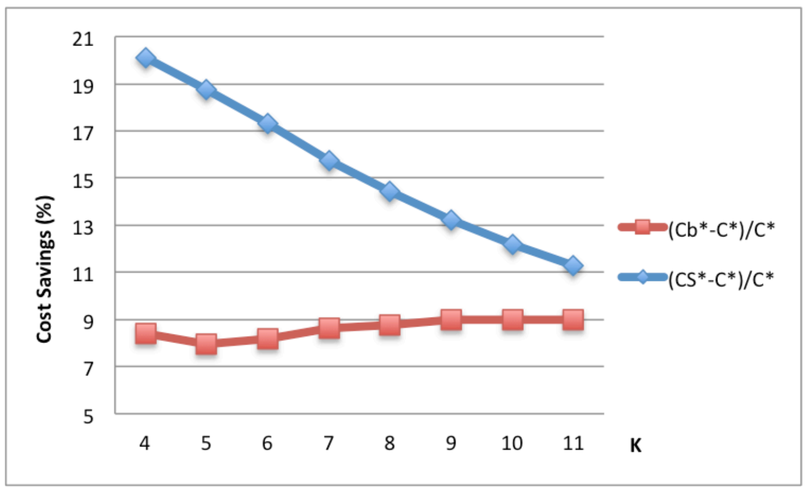


b) high-congestion transportation setting; $\sigma_d = 10$

Figure 4.2. Long-run average costs for and the priority delivery mode with $h = 10$, $r = 5$.



a) low-congestion transportation setting; $\sigma_d = 30$



b) high-congestion transportation setting; $\sigma_d = 10$

Figure 4.3. Cost savings for the priority delivery mode with $h = 10$, $r = 5$.

4.4.1.2 One Beacon

The next experiment investigates the impact of only one beacon on the long-run average cost. The beacon monitors stages downstream from its location. It is assumed that the stages upstream from the only beacon are monitored by the zero-beacon. The tests were run for the normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 10 or 30. The holding cost is 10 and the shortage cost varies.

In general the results depend mostly on the delivery scenario, as summarized in Table 4.4. The value of the shortage cost and the variation in demand have little effect on the placement of the only beacon. With the economy deliver mode the beacon is always placed at the stage 3 regardless of the size of *K* (*K* = 4, 5, …, 11) and the congestion transportation setting. The economy delivery mode does not give initial high cost savings and that can be the reason why the beacon placement does not depend on any other parameter.

With the priority delivery mode the only beacon is placed at stage 4 for smaller values of *K* and at stage 5 for higher values of *K*. The size of *K* when the beacon moves from stage 4 to the stage 5 depends on the shortage cost and the congestion transportation setting. High shortage cost and the high-congestion transportation setting tend to keep the beacon at stage 4 for higher values of *K*.

With the premium delivery mode the beacon is placed further downstream. With the low-congestion transportation setting placement is at the *K* - 1 stage (with lower shortage cost) or at stage *K* (with higher shortage cost). The results with the premium delivery mode and the high-congestion transportation setting are a little different. The beacon is placed at the last stage with *K* = 4, 5, 6, 7 and sometimes 8, but as *K* increases the beacon is kept at stage 7 or 8. The demand variation does not affect the results. The value of the shortage cost has a little effect for higher values of *K*. The beacon is mostly placed at stage 8 for lower shortage cost and at stage 7 for the higher value of shortage cost.

Table 4.4. The only beacon position for the shortage cost r = 15 and σ_d = 30.

	Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
6	3	3	4	4	6	6
7	3	3	4	4	7	7
8	3	3	4	4	8	7
9	3	3	5	5	9	7
10	3	3	5	6	10	7
11	3	3	5	5	11	7

The above results are the same for both types of beacons. Table 4.4 shows the beacon position for the shortage cost r = 15 and the standard deviation σ_d =30.

4.4.1.3 Two or Three Beacons

The next experiment investigates the impact of two or more beacons. As mentioned earlier, it is assumed that the stages upstream from the first beacon are monitored by the zero-beacon. The tests were run for the normally distributed demand with μ_d = 100 and σ_d = 10 or 30. The holding cost is 10, the shortage cost varies, and K = 6 and up.

As for the one beacon case, with the economy delivery mode, congestion transportation settings and demand variation have little effect on the beacon placements with two beacons. Two beacons are mostly placed at stages 2 and 4 or at stages 2 and 3. With three beacons the first two beacons are always placed at stages 2 and 3 and the third one is placed at stage 5 (with the lower shortage cost) or at stage 4 (with the higher shortage cost).

With the priority delivery mode beacons are placed a little further downstream than with the economy delivery mode. With the low-congestion transportation setting two beacons are placed at stages 3 and 6 (for lower values of K) or at stages 4 and 7 (for higher values of K). The switch depends on the shortage cost value. With higher shortage costs, the beacons are kept at stages 3 and 6 for higher values of K. The high-congestion transportation setting keeps the beacons at stages 3 and 6 for higher values of K. With three beacons the first two beacons are placed almost always at stages 3 and 5 and the third one is placed at the last stage 7 (for $K \leq 8$) or at stage 8 (for $K \geq 9$). The demand variation, the shortage cost and the congestion transportation settings do not affect this result.

With the premium delivery mode beacons are placed closer to the last stage. The demand variation, congestion transportation settings and the shortage cost do not affect the placement of beacons. The only factor seems to be the size of K. With two beacons the second beacon is always placed at stage K - 1 and the placement of the first beacon changes as K increases. The first beacon is mostly placed at stage 4 for K = 6, at stage 5 for K = 7, 8, 9 and at stage 6 for K = 10, 11. Similar to the two beacon case, with three beacons the third one is always placed at the last stage. The first two beacons are placed at stages 4 and 6 (or 4 and 7) for lower values of K and at stages 5 and 8 for higher values of K.

The above results are the same for both types of beacons with very little difference, which is an occasional shift by one stage. Table 4.5 shows positions of two beacons and Table 4.6 shows positions of three beacons. Both results are for the shortage cost r = 15 and the standard deviation σ_d = 30.

Table 4.5. Positions of two beacons for the shortage cost r = 15 and σ_d = 30.

	Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
6	2, 3	2, 3	3, 6	3, 6	4, 6	4, 6
7	2, 4	2, 3	3, 6	3, 6	4, 7	4, 7
8	2, 4	2, 3	3, 6	3, 6	5, 8	5, 8
9	2, 4	2, 3	3, 6	3, 6	5, 9	5, 9
10	2, 4	2, 3	4, 7	3, 6	5, 10	6, 10
11	2, 4	2, 4	4, 7	4, 7	6, 11	6, 11

Table 4.6. Positions of three beacons for the shortage cost r = 15 and σ_d =30.

	Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
6	2, 3, 4	2, 3, 4	2, 4, 6	3, 4, 6	3, 5, 6	3, 5, 6
7	2, 3, 4	2, 3, 4	3, 5, 7	3, 5, 7	3, 6, 7	4, 6, 7
8	2, 3, 4	2, 3, 4	3, 5, 7	3, 5, 7	4, 7, 8	4, 6, 8
9	2, 3, 4	2, 3, 4	3, 5, 8	3, 5, 7	4, 7, 9	4, 7, 9
10	2, 3, 4	2, 3, 4	3, 5, 8	3, 5, 8	4, 7, 10	4, 7, 10
11	2, 3, 4	2, 3, 4	3, 5, 8	3, 5, 8	4, 8, 11	5, 8, 11

4.4.1.4 Comparison of $C, C_{b0}, C_{b1}, C_{b2}, C_{b3}, \ldots, C_s$

When K is large enough for more than three beacons, the natural question is to ask what the optimum number of beacons is. At some point placing more than that number does not give much benefit. It is more appropriate to investigate this problem for those settings that give large initial cost savings.

As noted above, the economy delivery mode does not give very large cost savings. The maximum cost saving achieved for this delivery mode is 4% with the low-congestion transportation setting, the shortage cost $r = 5$, and the standard deviation $\sigma_d = 30$. The "baseline" cost is as close to the real-time tracking information cost as 3.4%. This result is for $K = 4$. Placement of one beacon gives the cost savings of 1.98% and with two beacons the savings are 0.72%. Moreover, it is of no importance to investigate placement of more than three beacons for $K = 4$.

The priority delivery mode for the shortage cost $r = 5$ and the premium delivery mode for the shortage cost $r = 15$ give high initial cost savings (between C^* and C_s^*). The next experiment investigates these settings.

Let's define the tolerance level to be 1%. In other words, the experiment starts with the zero-beacon; then one beacon is added at a time and the corresponding long-run average cost is calculated. There will be no additional beacons if the cost difference between the long-run average cost using beacons and the long-run average cost with full real-time tracking information is less than the pre-specified tolerance level, in this case 1%.

The results show that in most cases four beacons are enough to achieve less than 1% difference between the long-run average costs. For the premium delivery mode with the shortage cost of 15 it is enough to have only three beacons. The priority mode with the shortage cost of 5 requires five beacons to achieve the desired level for $K = 10$ and 11. The results are basically the same for both types of beacons with an occasional difference of one less beacon for the type II beacons. Table 4.7 shows the long-run average costs and corresponding cost savings for the low-congestion transportation setting with the priority delivery mode for $K = 11$, the shortage cost $r = 5$, the holding cost $h = 10$ and normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 10. Note that the cost difference drops significantly from three beacons to four beacons and three beacons can be considered enough if the tolerance level does not have to be strict. Table 4.8 illustrates a small difference between type I and II beacons. The results are for the low-congestion transportation setting with the premium delivery mode for K = 11, the shortage cost r = 15, the holding cost h = 10 and normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 30.

In such cases when obtaining real-time tracking information is difficult or involves more investments, a carrier may argue that providing real-time tracking information from only four beacons should be sufficient for the retailer as he can lower his long-run average cost and make it only 1% off from the full real-time tracking information long-run average cost.

Table 4.7. The long-run average cost values and cost savings for K = 11; low-congestion transportation setting with priority delivery mode; h = 10, r = 5; μ_d = 100, σ_d =10.

	Type I beacons		Type II beacons	
	Expected Cost	Difference from C^*	Expected Cost	Difference from C^*
No information (C_s^*)	1655.6	32.8%	1655.6	32.8%
"Baseline" cost ($C_{B_0}^*$)	1527.3	22.5%	1527.3	22.5%
Cost with 1 beacon	1476.8	18.5%	1387.1	11.3%
Cost with 2 beacons	1440.7	15.6%	1366.8	9.7%
Cost with 3 beacons	1404.7	12.7%	1346.8	8.0%
Cost with 4 beacons	1262.5	1.3%	1260.6	1.1%
Cost with 5 beacons	1255.6	0.7%	1254.7	0.7%
Full information (C^*)	1246.5		1246.5	

Table 4.8. The long-run average cost values and cost savings for K = 11; low-congestion transportation setting with premium delivery mode; h = 10, r = 15; μ_d = 100, σ_d =30.

	Type I beacons		Type II beacons	
	Expected Cost	Difference from C^*	Expected Cost	Difference from C^*
No information (C_s^*)	4622.2	23.8%	4622.2	23.8%
"Baseline" cost ($C^*_{B_0}$)	3938.1	5.4%	3938.1	5.4%
Cost with 1 beacon	3819.7	2.3%	3804.3	1.9%
Cost with 2 beacons	3794.7	1.6%	3784.5	1.3%
Cost with 3 beacons	3778.9	1.2%	3772.0	0.99%
Cost with 4 beacons	3741.2	0.2%		
Full information (C^*)	3735.0		3735.0	

In summary,

- Economy delivery mode requires beacons closer to the manufacturer's site.
- Priority delivery mode requires beacons a little further downstream than the economy delivery mode.
- Premium delivery mode requires beacons closer to the end of the transportation channel.
- In most cases placing four beacons is enough to be only 1% off from the full real-time tracking information long-run average cost.

4.4.2 Sling Matrices

For the following set of tests the transition matrices discussed above are modified. It is assumed that there is a special "sling" stage S from where shipments can be dispatched directly to the retailer; in other words, the stage has the same "sling effect" as stage 0 (manufacturer's site). These special stages can be warehouses or some sort of distribution centers. For shipments that leave the manufacturer's site, the same three delivery modes discussed earlier are considered: premium, priority, and economy. Shipments from the sling stage follow the same delivery modes. Under these modes of delivery, the shipper (the manufacturer as well as the sling stage) strives to dispatch orders leaving the stage directly to the retailer's site with a high, medium, or low probability, respectively. If there is no congestion in the transportation channel shipments arrive directly to the retailer's site the next time period. However, if there is congestion in the transportation channel, this shipment will merge with the shipment at the first occupied stage downstream. The congestion is modeled with the two settings discussed above, low-congestion and high-congestion.

The parameters for the three delivery modes are set as follows:

- Premium delivery mode: $p_{0,K+1} = 0.9$ and $p_{0,1} = 0.1$, $p_{S,K+1} = 0.9$ and $p_{S,S+1} = 0.1$
- Priority delivery mode: $p_{0,K+1} = 0.7$ and $p_{0,1} = 0.3$, $p_{S,K+1} = 0.7$ and $p_{S,S+1} = 0.3$
- Economy delivery mode: $p_{0,K+1} = 0.3$ and $p_{0,1} = 0.7$, $p_{S,K+1} = 0.3$ and $p_{S,S+1} = 0.7$

The parameters for the two congestion transportation settings are the same as in Chapter 3. Note that there still is a positive probability of 0.1 (for both congestion transportation settings) that a shipment at stage S - 1 will move to stage S + 1, thus avoiding the warehouse.

The tests were run for the normally distributed demand, varying number of transportation stages (K) and shortage cost (r). The sling stage was set for S = 4, 5, … K-2. As before all the results are in agreement with Property 4.2 and 4.3: $C^* \leq C_B^* \leq C_s^*$ and $C_{B^{II}}^* \leq C_{B^I}^*$.

4.4.2.1 "Baseline" Expected Cost and Cost Savings

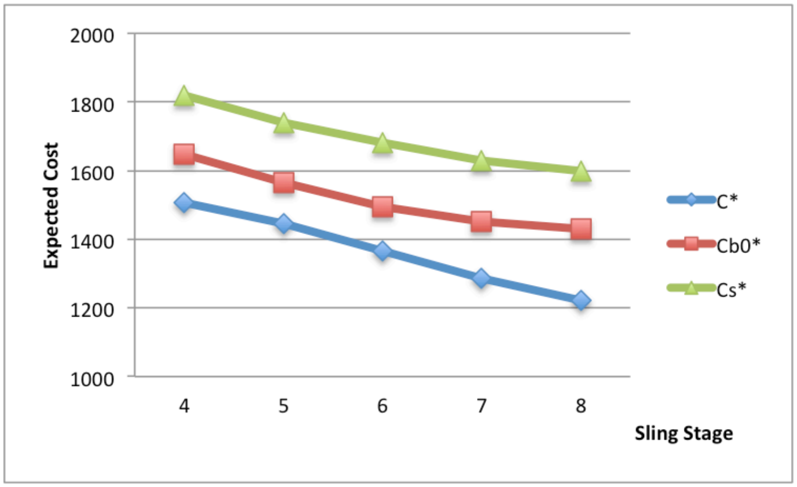
The results show that when a sling stage is present all long-run average costs, C^*, $C_{B_0}^*$, and C_s^*, are lower than corresponding long-run average costs without a sling stage. This result holds for all delivery modes. Orders at the sling stage move forward faster than from a regular stage. The retailer benefits from having a sling stage as order lead times are getting shorter and managing inventory is becoming easier. The results also show that in the cases of the economy and priority delivery modes all long-run average costs, C^*, $C^*_{B_0}$, and C^*_s, are decreasing as the sling stage is moved further downstream. With the premium delivery mode the long-run average costs are increasing as the sling value increases. This result is intuitive as it is less important to use premium delivery when the sling stage gets closer to the end of the transportation channel. Figure 4.4 shows the long-run average costs for K = 10, holding cost h = 10, shortage cost r = 5 for the normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 10.

Table 4.9 shows the long-run average cost values and cost savings for K = 10 with the shortage cost r = 5, normally distributed demand with the mean μ_d = 100 and standard deviation σ_d = 10 for different values of the sling stage and low-congestion transportation setting with priority delivery mode.

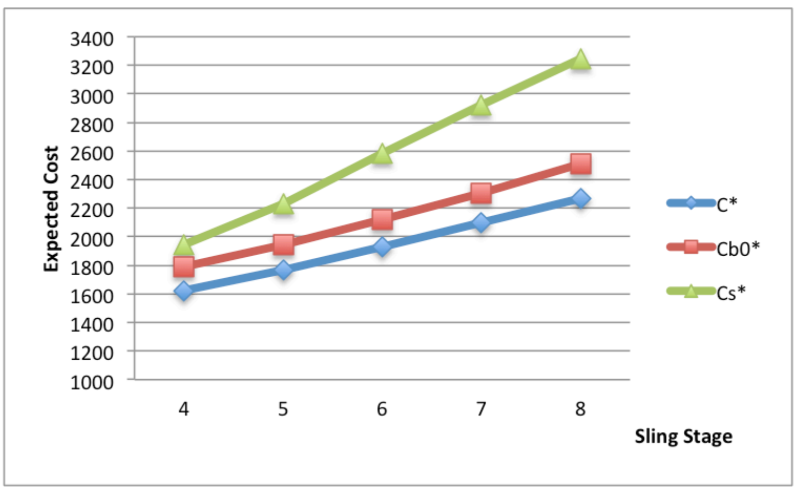
Table 4.10 shows the long-run average cost values and cost savings for K = 10 with the shortage cost r = 5, normally distributed demand with the mean μ_d = 100 and the standard deviation σ_d = 10 for different values of the sling stage and high-congestion transportation setting with premium delivery mode.

It can be seen that the difference between the long-run average cost with real-time tacking information and no information increases as S increases for both delivery modes. The difference between the long-run average cost with real-time tacking information and the "baseline" cost keeps steady for the premium delivery mode and increases as S increases for the priority delivery mode.

The difference from the real-time tracking information long-run average cost can be as high as 42% for long-run average cost with no information and 16% for the "baseline" cost. Once again the results emphasize the importance of real-time tracking information.



a) low-congestion transportation setting with priority delivery mode



b) high-congestion transportation setting with premium delivery mode

Figure 4.4. The long-run average costs for $K = 10$, with $h = 10$, $r = 5$ and $\sigma_d = 10$.

As before the economy delivery mode does not give high cost savings. The cost savings are almost non-existent with the high-congestion transportation setting. The maximum difference of around 3.7% is achieved for the low-congestion transportation setting with $K = 8$, the shortage cost $r = 5$, and the standard deviation σ_d =10 when the sling stage is at stage 6. The difference can get as small as 0.4% with the high-congestion transportation setting.

Table 4.9. The long-run average cost and cost savings for K=10; low-congestion transportation setting with priority delivery mode; h=10, r=5; μ_d=100, σ_d =10.

Sling Stage	C^*	$C^*_{B_0}$	C^*_s	100% $(C^*_{B_0} - C^*)/ C^*$	100% $(C_s^* - C^*)/ C^*$
No sling stage	1208	1471	1617	21.8%	33.9%
4	1508	1648	1818	9.3%	20.6%
5	1446	1566	1741	8.3%	20.4%
6	1366	1494	1681	9.4%	23.1%
7	1286	1453	1628	13.0%	26.6%
8	1222	1429	1599	16.9%	30.9%

Table 4.10. The long-run average cost and cost savings for K=10; high-congestion transportation setting with premium delivery mode; h=10, r=5; μ_d=100, σ_d=10.

Sling Stage	C^*	$C^*_{B_0}$	C^*_s	100% $(C^*_{B_0} - C^*)/ C^*$	100% $(C_s^* - C^*)/ C^*$
4	1624	1787	1944	10.0%	19.7%
5	1771	1944	2234	9.8%	26.1%
6	1931	2119	2582	9.7%	33.7%
7	2097	2308	2926	10.1%	39.5%
8	2270	2510	3244	10.6%	42.9%
No sling stage	2671	2956	3836	10.7%	43.6%

In summary, having a sling stage is beneficial for the retailer for all delivery modes. For the economy and priority delivery modes, the further a sling stage is placed from the manufacturer's site the lower the retailer's long-run average cost. For the priority delivery mode it is more beneficial for the retailer if a sling stage is placed closer to the manufacturer's site.

4.4.2.2 One Beacon

The following set of experiments investigates the placement of only one beacon. As mentioned above, it is assumed that the stages upstream from the only beacon are monitored by the zero-beacon. The results of the experiment reveal that the placement of the only beacon depends mostly on the delivery scenario.

With the economy delivery mode the only beacon is placed mostly at stage 3. There are occasionally cases when the beacon is placed at stage 2. For example with K = 8 the experiments with lower demand variation (σ_d = 10) place the beacon at the stage 2. The results are same regardless the value of the sling stage. It has to be noted that even with the lowest value of the sling stage (S = 4), the beacon is placed upstream from it. The results are similar for both type I and type II beacons.

With the priority delivery mode the beacon is placed somewhere halfway between the manufacturer and the retailer and the placement does not depend on the sling stage value. The beacon is always placed at stage 4 or 5. In most cases if the beacon is placed at stage 5 it corresponds to the smaller value of the shortage cost (r = 5). The congestion transportation setting does not have a big effect on beacon placement. The results are in general the same for both types of beacons. The difference between beacon placements is never more than one stage. The beacons are always placed either at stage 4 or 5.

With the premium delivery mode the only beacon is always placed at the sling stage. Results are the same regardless of the size of variation in demand, shortage cost, value of K or congestion transportation setting. Table 4.11 shows the only beacon position for the shortage cost r = 5 and the standard deviation σ_d =30 with the sling stage at stage 6.

Table 4.11. The only beacon position for the shortage cost r = 5 and σ_d = 30; S = 6.

	Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
8	3	3	5	4	6	6
9	3	3	5	4	6	6
10	3	3	4	4	6	6

4.4.2.3 Two or Three Beacons

To investigate the impact of two or more beacons the tests were run for the normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 10 or 30. The holding cost was 10 and the shortage cost varied. The tests were run for K = 8 and up and the sling value of S = 4 up to S = K - 2.

With the economy delivery mode the further downstream the sling stage (S = 6 and up) is the more likely it is to place the two beacons at the beginning of the transportation channel at stages 2 and 4. With smaller sling values (S = 4 and 5) combined with higher values of K, beacons are placed a little further downstream than stages 2 and 4. Sometimes the beacons are placed at both sides of the sling stage. For example with K = 9 and S = 4, the beacons are placed at stages 3 and 6. This result is more prominent with the high-congestion transportation setting. The same pattern is observed with three beacons. For larger values of K and S the three beacons are mostly placed at stages 2, 3, and 4 or 2, 3, and 5. With higher values of K and smaller values of S and mostly for the high-congestion transportation setting the third beacon is placed after the sling stage.

With the priority delivery mode beacons are placed mostly upstream from the sling stage. The placement changes as K and S increase. For the sling value equal to 4 the first beacon is placed at stage 3 and the second beacon is placed at stage 4 or the sling stage. As the sling value increases, beacons are placed upstream from the sling stage. The second beacon is mostly placed at the sling stage and the first one is placed two or three stages upstream. As *K* increases both beacons are placed upstream from the sling stage. The second is placed at stage *S* - 1 or *S* - 2 and the first one is placed at stage *S* - 2 or *S* - 3. The placement of three beacons shifts as the sling stage moves towards the end of the transportation channel. For small values of *S* = 4, the three beacons are placed at stages *S*, *S* + 1, and *S* + 3 or *S* + 4. For *S* = 5, the three beacons are placed at stages *S* - *2*, *S*, and *S* + 1 or *S* + 2. For higher values of *S*, the third beacon is placed at stage *S* and the first two beacons are placed upstream from the third one with one or two stages apart.

With the premium delivery mode results are slightly different for different congestion transportation settings. For the high-congestion transportation setting two beacons are placed downstream from the sling stage at stages *S* and *S* + 1 unless the sling stage is too close to the retailer's site. When *K* – *S* = 2 or sometimes even 3, the beacons are placed at stages 4 and *S*, or 5 and *S*. The low-congestion transportation setting follows the same pattern, except for smaller values of *S* with the small shortage value *r* = 5 the beacons are placed a couple of stages closer to the manufacturer. With three beacons the third beacon is placed at stage *K* for smaller values of *S* (*S* = 4, 5, 6). When the sling stage is close to the retailer's site, the third beacon is placed at stage *S* and the first two beacons are placed upstream from the third one with one or two stages apart.

The above results are the same for both types of beacons with very little difference, which is a small occasional shift by one stage. Table 4.12 shows positions of two beacons and Table 4.13 shows positions of three beacons. Both results are for the shortage cost *r* = 15 and the standard deviation σ_d = 30.

Table 4.12. Positions of two beacons for the shortage cost r = 15 and σ_d = 30.

		Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	S	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
8	5	2, 3	2, 3	3, 5	3, 5	5, 6	5, 6
8	6	2, 3	2, 3	3, 6	3, 5	3, 6	4, 6
9	5	2, 4	2, 4	3, 5	3, 5	5, 6	5, 6
9	7	2, 4	2, 3	3, 6	3, 5	4, 7	4, 7
10	5	2, 4	2, 4	3, 5	3, 5	5, 6	5, 6
10	8	2, 4	2, 4	3, 6	3, 6	4, 8	5, 8

Table 4.13. Positions of three beacons for the shortage cost r = 15 and σ_d = 30.

		Economy Delivery Mode		Priority Delivery Mode		Premium Delivery Mode	
K	S	Low Congestion	High Congestion	Low Congestion	High Congestion	Low Congestion	High Congestion
8	5	2, 3, 4	2, 3, 4	3, 5, 8	3, 5, 6	5, 6, 8	5, 6, 8
8	6	2, 3, 4	2, 3, 4	2, 4, 6	3, 4, 6	3, 6, 7	4, 6, 7
9	5	2, 3, 4	2, 3, 5	3, 5, 9	3, 5, 6	5, 6, 9	5, 6, 9
9	7	2, 3, 4	2, 3, 4	3, 5, 7	3, 5, 7	4, 7, 8	4, 7, 8
10	5	2, 3, 5	2, 3, 5	3, 5, 6	3, 5, 6	5, 6, 10	5, 6, 10
10	8	2, 3, 4	2, 3, 4	3, 5, 8	3, 6, 7	3, 5, 8	4, 6, 8

4.4.2.4 Comparison of $C, C_{b0}, C_{b1}, C_{b2}, C_{b3}, \ldots, C_s$

This section investigates how many beacons are needed to be as close to the real-time tracking information long-run average cost as some predetermined tolerance level. As noted earlier the economy delivery mode does not give high cost savings. All the cost savings fall under 3% and in most cases adding only one beacon gives the desired tolerance level. Thus this section concentrates on the priority and premium delivery modes.

As before let's assume that the tolerance level is 1%. The goal of the experiment is to investigate how many beacons are needed to achieve less than 1% difference between the long-run average cost with full real-time tracking information and the long-run average cost with the information from beacons.

In most cases placing three beacons is enough to achieve less than 1% difference in long-run average costs. In rare cases four beacons are needed for the same result. The number of beacons does not depend on where the sling stage is.

It is interesting to note that there are settings when the difference in costs falls from around 13% to 1% or less from three beacons to four beacons. The difference decreases slowly from the "baseline" to the three-beacon cost and then falls sharply when the fourth beacon is added, as illustrated earlier in Table 4.7. The results are basically the same for both types of beacons with an occasional difference of one less beacon for the type II beacons.

Since the shortage cost r=5 gives high initial cost savings, the following tables present results for this setting. Table 4.14 shows the long-run average costs and corresponding cost savings for the low-congestion transportation setting with the priority delivery mode for $K = 10$ and the sling stage $S = 4$, the shortage cost $r = 5$, the holding cost $h = 10$ and the normally distributed demand with mean $\mu_d = 100$ and standard deviation $\sigma_d = 10$. In this table the number of type I beacons needed is one greater than the number of type II beacons needed to achieve the tolerance level.

Table 4.14. The long-run average cost values and cost savings for K = 10 and S = 4 with holding cost h = 10, shortage cost r=5 and standard deviation σ_d =10.

	Type I beacons		Type II beacons	
	Expected Cost	Difference from C^*	Expected Cost	Difference from C^*
No information (C_s^*)	1818	20.6%	1818	20.6%
"Baseline" cost ($C^*_{B_0}$)	1648	9.3%	1648	9.3%
Cost with 1 beacon	1625	7.8%	1590	5.4%
Cost with 2 beacons	1620	7.4%	1587	5.2%
Cost with 3 beacons	1619	7.4%	1584	5.0%
Cost with 4 beacons	1525	1.1%	1521	0.9%
Cost with 5 beacons	1518	0.7%		
Full information (C^*)	1508		1508	

Table 4.15 shows the results for the high-congestion transportation setting with the premium delivery mode for K = 10 and the sling stage S = 8, the shortage cost r = 5, the holding cost h = 10 and the normally distributed demand with mean μ_d = 100 and standard deviation σ_d = 10.

Table 4.15. The long-run average cost values and cost savings for K = 10 and S = 8 with holding cost h = 10, shortage cost r = 5 and standard deviation σ_d = 10.

	Type I beacons		Type II beacons	
	Expected Cost	Difference from C^*	Expected Cost	Difference from C^*
No information (C_s^*)	3244	42.9%	3244	42.9%
"Baseline" cost ($C^*_{B_0}$)	2510	10.6%	2510	10.6%
Cost with 1 beacon	2481	9.3%	2447	7.8%
Cost with 2 beacons	2473	8.9%	2443	7.6%
Cost with 3 beacons	2336	2.9%	2321	2.2%
Cost with 4 beacons	2277	0.7%	2276	0.6%
Full information (C^*)	2270		2270	

In summary,

- The beacon placement depends on the delivery mode. The economy delivery mode requires beacons closer to the manufacturer's site. The priority delivery mode requires beacons a little further downstream and the premium delivery mode requires beacons closer to the end of the transportation channel. If there is only one beacon with the premium delivery mode, it is placed at the sling stage.
- Four or sometimes five beacons are enough to be only 1% off from the full real-time tracking information long-run average cost regardless of the sling stage value.

Also it must be noted that

- Long-run average costs for type I beacons are greater than the long-run average costs for type II beacons. The difference is achieved by setting different order-up-to levels, which need to be a little higher for type II beacons than for the type I beacons.

- Beacon placement and the number of beacons needed to be close to the long-run average cost with full real-time tracking information in general do not depend on the type of beacons.

CHAPTER 5
CONCLUSIONS

A stochastic model was used to evaluate the value of real-time information on supply status on both the manufacturer's process and on the transportation process when the order lead time is dependent on the distribution of the locations of outstanding orders and the number of unshipped orders at the manufacturer's site. Shipment congestions were explicitly modeled.

With the optimal myopic ordering policy, the long-run average cost for the retailer is higher when he does not have any information available. The numerical examples indicate that the cost savings are especially significant when real-time tracking information and the manufacturer's shipping policies are available. When the retailer has no or partial information, it is better to order every time period. When full information is available, there are situations when the retailer does not need to order every time period to lower his long-run average total cost. The retailer may adjust his ordering policy according to the information he has available.

When the retailer knows the supply chain behavior, it is better to use the knowledge of whether the supply system is empty or not and calculate the long-run average cost to define two order-up-to levels, one when the system is empty (no congestion present) and another one when there is at least one outstanding order. The cost incurred by the retailer using two order-up-to levels is much less than the long-run average cost with no information or the cost when using just one order-up-to level. Moreover, using beacons lowers the long-run average cost even further. There is an optimum number of beacons that gives a good approximation to the full information real-time tracking long-run average cost.

The thesis assumes that orders do not cross in time. Order crossover cases are somehow less resolved. Among the limited literature on order crossover there are Robinson *et al.* (2001), Bradley and Robinson (2005) and Hayya *et al*. (2008). As noted in Robinson *et al.* (2001), in order crossover cases it is better to use shortfall distribution instead of the distribution of demand during the lead time. The model in Robinson *et al.* (2001) studies the order-up-to level approach where the level does not depend on the supply chain status.

The attempt to model the crossover case with the same approach as the no-crossover case was not successful. The basic idea for the current model is that for each complete supply status vector the conditional lead times can be calculated. As the orders arrive in the same sequence as they were shipped, it is mathematically possible to track the on-hand inventory and calculate the long-run average costs charged to each complete supply status vector. When crossover is allowed, any order can move downstream freely and congestion is no obstacle. The lead times for any order are the same regardless of the congestion, thus regardless of the complete supply status vector at the time when the order left the manufacturer's site. Moreover, as orders can cross over it is impossible to track the value of on-hand inventory, thus making it impossible to calculate the long-run average cost. One approach to tackle the problem would be instead to model the supply chain by only the number of outstanding orders to implicitly track the quantities of outstanding orders. This approach would make the model cumbersome and, moreover, the only way to find the order-up-to levels would be the linear programming approach. That is basically the same approach taken by Robinson *et al.* (2001) in the paper's supplement, which describes the linear programing formulation for a crossover case with order-up-to-levels that depend on the supply status. Robinson is able to solve the problem only for a simple policy of two possible replenishment lead times. Even with the simple example the linear programing is too big and requires placing bounds to be manageable. As noted by Robinson *et al.* (2001), the problem is too big to have any practical use and it is more of theoretical interest.

The thesis assumes a myopic ordering policy for the retailer. For future studies it would be of interest to investigate the value of information in the case of lost sales. Although the supply system is modeled with multiple stages, the model is still only considering a two-echelon supply system consisting of a manufacturer and a retailer. It would be of interest to identify how the value of real-time information changes as the number of levels in the supply system increases.

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APPENDICES

A. The Transition Probabilities for Process s

Define the variable *z(t)* to be the first non-zero stage in the vector $\vec{s}(t)$. For example, if $\vec{s}(t) = (0,1,0,1,1,0)$ then $z(t) = 2$.

Define $\vec{s}^{\,0}(t)$ to be a vector $\vec{s}(t)$ but with its first non-zero element set to 0. For example if $\vec{s}(t) = (0,1,0,1,1,0)$ then $\vec{s}^{\,0}(t) = (0,0,0,1,1,0)$. Thus for any $1 \le i \le 2^K$

$$\vec{s}_i^{\,0}(j) = \begin{cases} \vec{s}_i(j), & if \quad j \neq z(t) \\ 0, & if \quad j = z(t) \end{cases}$$

It is assumed that states are ordered lexicographically; i.e., each state vector $\vec{s}(t) = \vec{s}_i$ from the first non-zero state to the last state is the binary representation of index *i*-1. For example when *i*=7, $\vec{s}_7 = (0,...,0,1,1,0)$.

For modeling convenience it is assumed that a shipment received by the retailer has reached its final destination and thus remains in stage K+1 thereafter. Also for convenience, the time period index for a variable is omitted, and the superscript "+" is used to indicate the next period, whenever such an omission does not obscure clarity. For example, x represents x(t) and x+ represents x(t + 1).

Let $P_s^0(s_j \mid s_i) = \Pr[\vec{s}^{\,+} = s_j \mid \vec{s} = s_i, \text{ no new shipment}]$ denote the one-step transition from $\vec{s}(t) = \vec{s}_i$ to $\vec{s}(t+1) = \vec{s}_j$ without a new shipment for any $1 \le$ i, j $\le 2^K$, and let $P_s^1(s_j \mid s_i) = \Pr[\vec{s}^{\,+} = s_j \mid \vec{s} = s_i, \text{ new shipment}]$ be the corresponding transition with a new shipment.

A.1. Transitions for Process s without a New Shipment

The state s_1=(0,0,…,0) transitions to the state s_1=(0,0,…,0) without a new shipment with probability of 1; i.e., $P_s^0(s_1 \mid s_1) = 1$ and $P_s^0(s_j \mid s_1) = 0$ for any $j > 1$. For any i, such as $1<i\leq 2^K$, state $\vec{s}_i = (s_i(1), s_i(2), ..., s_i(K))$ transitions into the state s_1=(0,0,…,0) if all the shipments at all the occupied stages move to the $K+1$ stage; thus the transition probability is $P_s^0(s_1 \mid s_i) = \prod_{\substack{\text{for all } k \text{ such as} \\ s_i(k)=1}} p(k, K+1)$

To calculate transition probability from state $\vec{s}_i = (s_i(1), s_i(2), ..., s_i(K))$ to state $\vec{s}_j = (s_j(1), s_j(2), ..., s_j(K))$, for any i and j, such as $1< i \leq 2^K$ and $1< j \leq 2^K$, three different situations are possible.

- *z(i)>z(j)*. The transition is not possible, as shipments do not go backward, so the transition probability equals zero.
- *z(i)=z(j)*. The transition is possible only when the shipment at stage *z(i)* stays at *z(i)* and state $\vec{s}_i^{\,0}$ transitions to state $\vec{s}_j^{\,0}$. For example, state (0,1,1,0) transitions to state (0,1,0,1) when the shipment at stage 2 stays at the same stage and state vector (0,0,1,0) transitions to state (0,0,0,1). Figure A.1 illustrates this example.
- *z(i)<z(j)*. The shipment at stage *z(i)* moves to stage *z(j)* and the shipments downstream from stage *z(i)* move downstream from stage *z(j)* ($\vec{s}_i^{\,0}$ transitions to the state $\vec{s}_j^{\,0}$), or all the shipments downstream from *z(i)* move to the stages from *z(j)* to K+1 ($\vec{s}_i^{\,0}$ transitions to the state $\vec{s}_j$) and the transition from *z(i)* is blocked by the shipment at the stage *z(j)*.

Figure A.2 illustrates an example with *z(i)<z(j)*. The state (0,1,1,0) transitions to the state (0,0,1,0) when the shipment at stage 2 transitions to stage 3. The state vector (0,0,1,0) transitions to the state (0,0,0,0) (Figure A.2, left) or the state vector (0,0,1,0) transitions to the state (0,0,0,0) and the movement of the shipment from stage 2 is blocked by the shipment at stage 3 (Figure A.2, right).

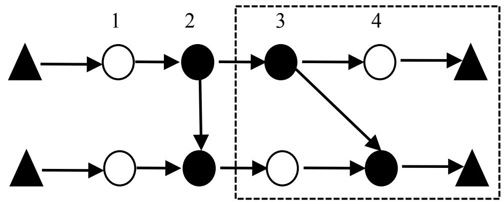

Figure A.1. Transition from the state (0,1,1,0) to the state (0,1,0,1) without a new shipment.

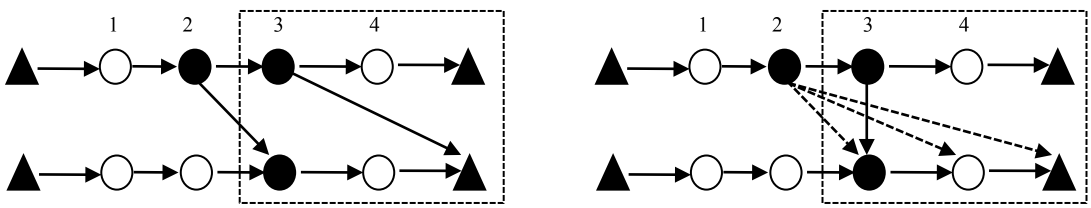

Figure A.2. Two possible transitions from (0,1,1,0) to (0,0,1,0) without a new shipment.

Thus the transitions without a new shipment are as follows:

$P_s^0(s_1 \mid s_1) = 1$ and $P_s^0(s_j \mid s_1) = 0$ for any $j > 1$

$$P_s^0(s_1 \mid s_i) = \prod_{\substack{\text{for all } k \text{ such as} \\ s_i(k)=1}} p(k, K+1), \text{ for any } i > 1$$

$$P_s^0(s_j \mid s_i) = \begin{cases} 0, & \text{if } z(i) > z(j) \\ p(z(i), z(j)) P_s^0(s_j^0 \mid s_i^0), & \text{if } z(i) = z(j) \\ p(z(i), z(j)) P_s^0(s_j^0 \mid s_i^0) + \sum_{k=z(j)}^{K+1} p(z(i), k) P_s^0(s_j \mid s_i^0), & \text{if } z(i) < z(j) \end{cases}, \text{ for any } i, j > 1.$$

A.2. Transitions for Process s with a New Shipment

The logic to calculate transitions for the process **s** with a new shipment is similar to the one without a new shipment.

Transition from s_1=(0,0,…,0) state to s_1=(0,0,…,0) state with a new shipment happens when the new shipments move directly to the retailer's site, i.e. $P_s^0(s_1 \mid s_1) = p(0, K+1)$.

The state vector s_1=(0,0,…,0) can transition into a state with only one stage occupied with the probability $P_s^0(s_j \mid s_1) = p(0, z(j))$ with $s_j(k) = 0$ for all k such as $k \neq z(j)$.

Transition from any state to the state s_1=(0,0,…,0) can happen when all outstanding shipments get delivered in a single time period; in other words, the given state transitions to the state s_1=(0,0,…,0) without a new shipment and the new shipment moves to the retailers site too.

For any i and j, such as $1 < i, j \leq 2^K$, the logic is similar to the no new shipment case assuming that $z(i)$=0. Thus the transitions without a new shipment are as follows:

$$P_s^1(s_1 \mid s_1) = p(0, K+1)$$

$P_s^1(s_j \mid s_1) = p(0, z(j))$ for all j such as s_j has only one occupied stage

$$P_s^1(s_1 \mid s_i) = p(0, K+1) P_s^0(s_1 \mid s_i), \text{ for any } i > 1$$

$$P_s^1(s_j \mid s_i) = p(0, z(j)) P_s^0(s_j^0 \mid s_i) + \sum_{k=z(j)}^{K+1} p(0,k) P_s^0(s_j \mid s_i), \text{ for all } i, j > 1.$$

A.3. Transitions for Process Ts

Using transitions of process **s** it is possible to find transition probabilities for process **Ts.** Let $\mathrm{P}(\vec{T}s^{+} \mid \vec{T}s)$ denote the probability that the complete supply status vector $\vec{T}s$ transitions into the complete supply status vector $\vec{T}s^{+}$ in one time period; then

$$\Pr(\vec{T}s^{+} \mid \vec{T}s) = q_{o+m,m^{+}} \left(\begin{cases} \rho_i & if \quad o^{+} = 1 \\ 1-\rho_i & if \quad o^{+} = 0 \end{cases} \right) \left(\begin{cases} \mathrm{P}_s^1(\vec{s}^{+} \mid \vec{s}), & if \quad o+m < m^{+} \\ \mathrm{P}_s^0(\vec{s}^{+} \mid \vec{s}), & otherwise \end{cases} \right)$$

where

$$\begin{aligned}
&o = \vec{T}s(1) \\
&m = \vec{T}s(2) \\
&\vec{s} = \vec{T}s(3:K+2) \\
&o^{+} = \vec{T}s^{+}(1) \\
&m^{+} = \vec{T}s^{+}(2) \\
&\vec{s}^{+} = \vec{T}s^{+}(3:K+2) \\
&i = \quad index \quad such \quad that \quad \vec{T}s(2:K+2) \;= (m,\vec{s}) = \vec{R}s_i.
\end{aligned}$$

B. Conditional Lead Times

B.1. Conditional Lead Time for Process s

To find the distribution of transportation lead times, a "backward" approach is used. For each state vector $\vec{s}_i$, the conditional lead time for the shipment at the first non-zero stage (stage *z(i)*) is calculated starting from the state with *z(i)=K* and proceeding to the states with *z(i)=K*-1,*K*-2,…,1. Each step uses conditional lead times for shipments that are situated downstream from the current shipment of interest. The last step is to calculate conditional lead times for new shipments.

Let's consider a shipment at stage *k*. First let's note that only shipments downstream from it (shipments at stages *k*+1, *k*+2,…, *K*) can influence its movement and thus its lead time. Shipments at stages 1, 2,…, *k-1* do not interfere with its movement.

Let $LT_k(i,l)$ denote the probability that given state $\vec{s}_i$, such as $k = z(i)$, the shipment situated at stage k will reach the retailer in l time periods.

If $k = K$, there is only one state with z(i)=K. This is state $\vec{s}_2 = (0,0,...,0,1)$. It is easy to see that

$$LT_K(2,l) = p(K,K+1)p(K,K)^{l-1}.$$

For $1 \leq k < K$, the conditional lead time is 1 only if $\vec{s}_i$ transitions to $\vec{s}_1 = (0,0,...,0)$ without a new shipment. The conditional lead time is l, $l>1$, if the current state $\vec{s}_i$ transitions to a new state, $\vec{s}_j$, $j>1$, without a new shipment and then the shipment takes $l-1$ time periods to reach the retailer's site. Thus the conditional lead times are calculated as follows:

$$LT_k(i,1) = \Pr_s^0(s_1 \mid s_i)$$
$$LT_k(i,l) = \sum_{j=2}^{2^K} \Pr_s^0(s_j \mid s_i) LT_{z(j)}(j,l-1), \text{ for } K-1 \geq k \geq 1 \text{ and } l > 1$$

The logic for k =0 or for the new shipments (shipments coming from the manufacturer's site) is similar. Let $LT_0(i,l)$ be the transportation lead time and denote the probability that given state $\vec{s}_i$, the new shipment will reach the retailer in l time periods. Then

$$LT_0(i,1) = \Pr_s^1(s_1 \mid s_i)$$
$$LT_0(i,l) = \sum_{j=2}^{2^K} \Pr_s^1(s_j \mid s_i) LT_{z(j)}(j,l-1), \text{ for } l > 1$$

B.2. Conditional Lead Time for Process Ts

The lead time of the new order is the sum of two components, the time it takes for a new order to leave the manufacturer's site plus the time it takes a new shipment to reach the retailer's site (transportation lead time).

Consider a tagged unshipped order, Z, placed by the retailer that is still at the manufacturer's site. Let $n(t)$ denote the position of the tagged order in the queue at the manufacturer's site before the retailer made a decision about placing a new order before the transition takes place. Assume that $n(t)$=0 to tag a new order just placed.

If $n(t) > 0$, the implicit assumption is that there are at least $n(t)$ orders outstanding at the manufacturer's location before the retailer made his ordering decision. Let's recall that the retailer's decision about placing a new order is defined by $o(t)$, where $o(t)$ =1 if the new order was placed and $o(t)$ = 0 otherwise. Thus after the decision was made there are $m(t)+o(t)$ orders at the manufacturer's site. The position $n(t)>0$ implies that there are $m(t)-n(t)$ orders ahead of the tagged order at the manufacturer's site at time period t.

Let $LM(n,j,l)$ be the conditional lead time and denote the probability that an order at the n^{th} position in the queue at the manufacturer's site will reach the retailer's site in l time periods given that the total supply vector is $\vec{T}s(t) = Ts_j$.

If $n=M$ and a new order is placed or, in other words, the number of unshipped orders at the manufacturer's site reached the maximum after the retailer's ordering decision and the tagged order is at the M^{th} position, at least one order will be shipped including the tagged order and the time it reaches the retailer is equal to the new shipment lead time for process **s**. Thus

$$LM(M,j,l) = L_0(i,l), \text{ for all } Ts_j = (1,M,s_i). \tag{A.1}$$

If $0<n<M$, or $n=M$ but the new order was not placed (o(t) = 0), the tagged order either stays at the manufacturer's site or gets shipped. If the shipment takes place, then the lead time is calculated as in formula A.1. If the tagged order was not shipped, its position becomes $n^+ = n + o$ and proceeds further from this position. If the original conditional lead time equals l, for the tagged order at position n, given the current state is Ts_j, this means that the state transitions to a new state Ts_k with the tagged order at the position $n^+ = n + o$ and from there takes $l-1$ time periods to reach the retailer's site. Thus the conditional lead times are calculated as follows:

$$LM(n,j,l) = L_0(i,l)\sum_{m^+=0}^{n-(1-o)} q_{o+m,m^+} + \sum_{m^+=n+o}^{M} q_{o+m,m^+} \sum_k LM(n+o,k,l-1)$$

for any j such as $Ts = \vec{T}s_j = (o,m,s_i)$ with $o+m < M+1$,
for any k such as all $Ts^+ = \vec{T}s_k = (o^+,m^+,s^+)$ with $m^+ \geq n+o$,
and for any $l > 0$

It is assumed that $LM(n,j,0) = 0$ for any $n > 1$ and any j.

If $n=0$, the tagged order has been just placed by the retailer. The conditional lead time can be calculated only for the total supply vectors $Ts_j = (o,m,s)$ with $\text{o} = 1$. The logic is the same as above. Note that, if a newly placed order ships out the same time period then there are no unshipped orders left at the manufacturer's site. Thus

$$LM(0,j,l) = L_0(i,l)q_{o+m,0} + (1-q_{o+m,0})\sum_k LM(1,k,l-1)$$

for any j such as $Ts = \vec{T}s_j = (1,m,s_i)$,
for any k such as $Ts^+ = \vec{T}s_k = (o^+,m^+,s^+)$ with $m^+ \geq 1$.

The conditional probability distribution of lead times for the process **Ts** is given as

$$\Pr[L(t) = l \mid \vec{T}s(t)] = LM(0,j,l), \text{ for all } j \text{ such as } \vec{T}s(t) = Ts_j = (1,m,s).$$

C. The Probabilities NO(i,j,τ)

For any i and j, let $NO(i,j,\tau)$ be the probability that a complete supply status vector $\vec{T}s_i$ transitions into a complete supply status vector $\vec{T}s_j$ in τ steps with all intermediate complete supply status vectors $\vec{T}s_k$ satisfying the condition $k \notin \Gamma$; in other words, $\vec{T}s_k$ is a complete supply status vector without a new order. It's the probability that if the supply chain status is any complete supply status vector $\vec{T}s_i$, the next ordering happens at time t+ τ and the complete supply status vector at that time is $\vec{T}s_j = (1,m,s)$.

If τ = 1 the calculation is straightforward:

$$NO(i,j,1) = \Pr(\vec{T}s_j \mid \vec{T}s_i) \text{ for any } i \text{ and any } j \in \Gamma.$$

If $\tau > 1$, the probability that the next ordering happens in τ periods is equal to the probability that a complete supply status vector $\vec{T}s_i$ will transition to some complete supply status vector $\vec{T}s_k = (0,m,s)$ and from there the next ordering happens in τ – 1 periods. That is,

$$NO(i,j,\tau) = \sum_{k \notin \Gamma} \Pr(\vec{T}s_k \mid \vec{T}s_i) NO(k,j,\tau-1) \text{ for any } i \text{ and any } j \in \Gamma.$$

VITA

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