5-2013

Moldable Items Packing Optimization

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To the Graduate Council:

I am submitting herewith a dissertation written by Sima Maleki entitled "Moldable Items Packing Optimization." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Industrial Engineering.

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Moldable Items Packing Optimization

A Dissertation Presented for the Doctor of Philosophy Degree
The University of Tennessee, Knoxville

Sima Maleki
May 2013

DEDICATION

To my beloved parents and sisters

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my graduate advisor, Dr. Rupy Sawhney, for his guidance, support, and patience during the course of my academic study, research, and dissertation writing. Without his inspiration and encouragement, the achievements of this research and the completion of this dissertation would not have been possible.

I would also like to thank my graduate committee, Dr. Charles E. Noon, Dr. Xiaoyan Zhu, and Dr. Joseph Wilck, for their time and participation in this academic endeavor, in the classroom, in my dissertation proposal reviews, and in my dissertation defense.

ABSTRACT

This research has led to the development of two mathematical models to optimize the problem of packing a hybrid mix of rigid and moldable items within a three-dimensional volume. These two developed packing models characterize moldable items from two perspectives: (1) when limited discrete configurations represent the moldable items and (2) when all continuous configurations are available to the model. This optimization scheme is a component of a lean effort that attempts to reduce the lead-time associated with the implementation of dynamic product modifications that imply packing changes.

To test the developed models, they are applied to the dynamic packing changes of Meals, Ready-to-Eat (MREs) at two different levels: packing MRE food items in the menu bags and packing menu bags in the boxes. These models optimize the packing volume utilization and provide information for MRE assemblers, enabling them to preplan for packing changes in a short lead-time. The optimization results are validated by running the solutions multiple times to access the consistency of solutions. The solutions are visualized using Autodesk Inventor, to communicate the optimized packing solutions with the MRE assemblers for training purposes.

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CHAPTER I
Introduction

Introducing new products competitively and modifying existing products are key dimensions by which organizations grow their market shares. These dimensions become more critical because of shorter product life cycles and a greater frequency of modifications within each product's life cycle. Incorporating any of these changes requires an understanding of the consequences of those changes and the impacts on the final packing. Though the news highlights the innovations of electronic products, there are numerous examples of innovation in every industry. A lean packing modification system, which would reduce the lead-time required to model and to incorporate changes, is a key component in developing a system within an organization for the introduction of new products and the modifications of existing products.

A lean dynamic packing modification system includes four key components. The first component is the development of Quick Measure, a three dimensional laser measurement device that accurately and quickly measures product dimensions. The second component is the creation of the library that stores data. The third component of the system is an optimization scheme for packing and assembling. The final component of the system visualizes the optimized packing solutions for assembly purposes and personnel training.

Product modification also exists in the military as demonstrated by Meals, Ready-to-Eat (MRE), which are produced for military personnel. The Defense Logistics Agency (DLA) constantly provides innovations in meals, meal quality, and packing material. These innovations lead to a short life cycle for a series of MREs and a scenario in which packing assemblers are continuously modifying their assembly processes and training their personnel.

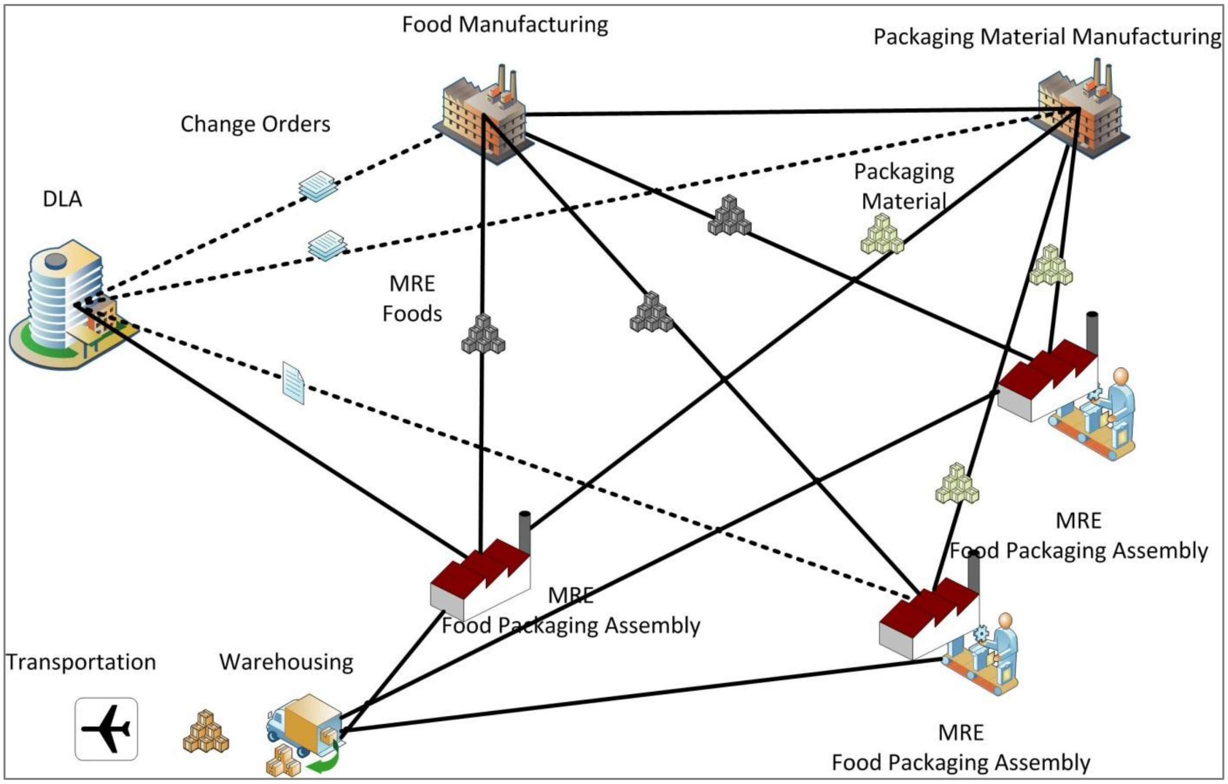

Figure 1: MRE Modifications Life Cycle

Figure 1 illustrates the flow of information and materials among entities of the MRE supply chain associated with any modification in MREs. After DLA makes decisions to introduce new food items, to change packaging material, or to modify existing products, information travels to the food and packing material producers. These facilities then modify their manufacturing processes to incorporate these changes. The updated products are transferred to packing assemblers. The assemblers then train their personnel to pack the updated MREs. Each of these processes has a long lead-time and is dependent on all previous processes. Developing a lean dynamic packing modification system for DLA will reduce the necessary lead-time to incorporate the changes throughout the MRE supply chain.

Figure 2: MRE Packaging System

Figure 2 presents the conceptual framework of the dynamic product modification system. This system receives the information, optimizes the packing, and provides access to visualized outputs for application and training purposes with a short lead-time.

1.1 Background

The idea of a product life cycle was first introduced almost 30 years ago [1, 2]. The product life cycle represents the unit sales curve for some product, extending from the time it is first placed on the market until it is removed [3]. The product life cycle is approximated with a bell shaped curve and is generally divided into four stages: introduction, growth, maturity, and decline [4]. Because industries are constantly seeking ways to grow their cash flow by maximizing revenue from the sales of products, the products' life cycles are becoming shorter and shorter, and many products in mature stage are revitalized by modifications.

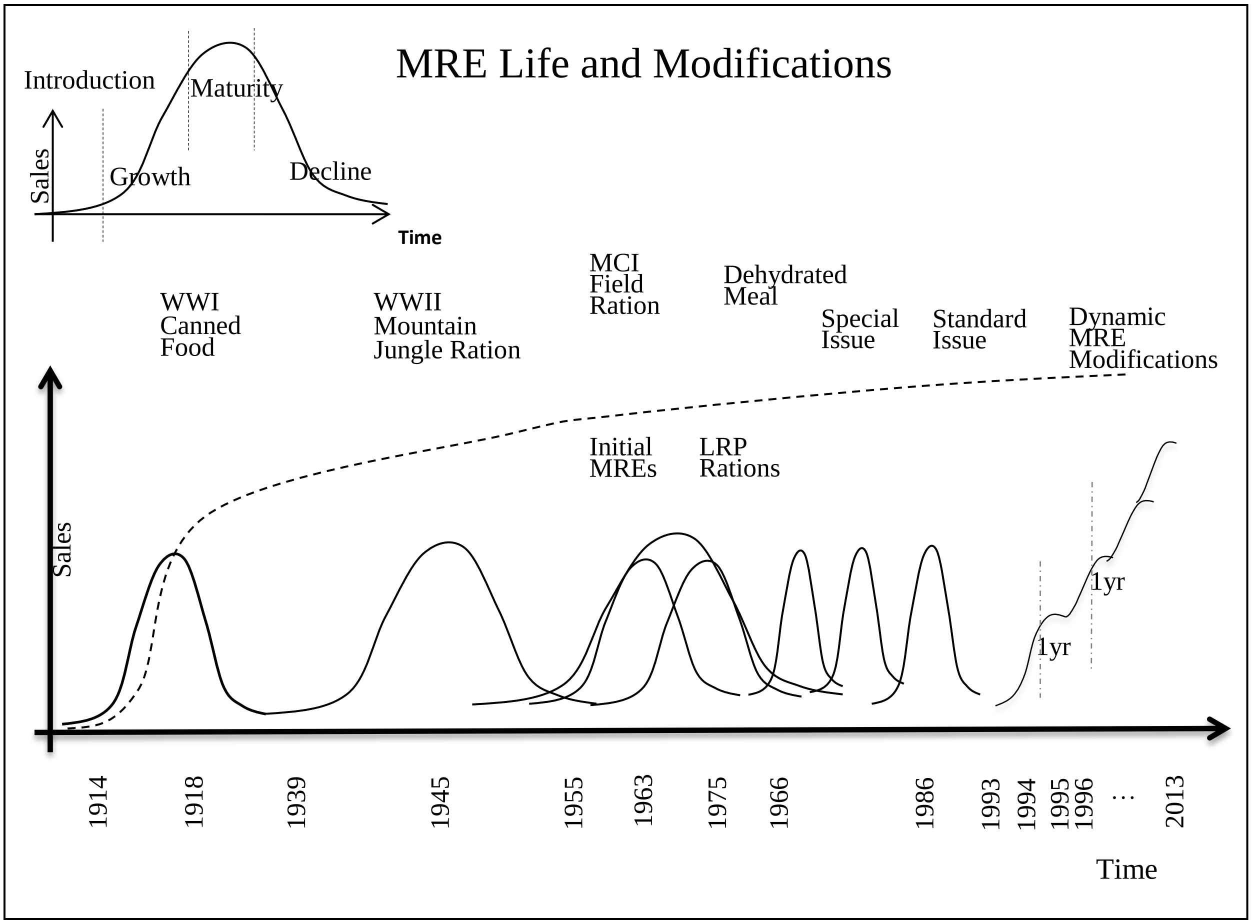

Figure 3: MRE Life and Modifications

Figure 3 presents the life cycle of various MRE products during their history. Figure 3 also shows that the life cycles of MREs have shortened dramatically and that the frequencies of changes have increased between 1914 and the present, specifically since 1993. Starting with World War I (WWI), the first rationed food for soldiers was canned food. During the beginning of World War II (WWII), the military introduced a few new field rations, including the Mountain ration and the Jungle ration. The use of non-dehydrated, canned rations continued through the Vietnam War with the improved MCI field ration [5].

After repeated experiences with rations dating from before World War II, Pentagon officials realized the necessity for palatable food over long periods of time. This palatable food needed to cater to individual tastes and preferences, to withstand extreme environments, and to act as a lighter alternative. Hence, in 1963, the Department of Defense began developing the MREs, relying on modern food preparation and packaging technology to create a lighter replacement for canned meals. In 1966, this development led to the Long Range Patrol or LRP ration, a dehydrated meal stored in a waterproof canvas pouch. However, just as with the Jungle ration, the expenses compared to non-dehydrated, canned rations and the costs of stocking and storing led to limited usage and eventual discontinuance of LRP rations.

In 1975, work began on a dehydrated meal stored in a plastic pouch. It went into special issue starting in 1981 and standard issue in 1986, using a limited menu of 12 entrees. Since 1993, the MREs have faced consistent modifications. The DLA is conducting extensive research to dynamically improve the MRE meals, meal quality, and packing material, all of which affect the final packing of MREs in their bags and boxes.

There is no standardization in packing processes due to the lack of a system that enables the manufacturers and assemblers to communicate the changes to the design in a sufficient amount of time. This lack of standardization leads to tremendous variations in the final MRE bags, bulging of boxes, and instability in pallets, all of which impacts the integrity of the food when delivered to the soldiers. The MRE boxes are often inflated and, therefore, extend the pallet size one to two inches, leading to denting during transportation and/or storage (see Figure 4).

Figure 4: Pallet for MRE Boxes

The dynamic packing modification system is a solution not only for the MRE packing problems but also for any industry that deals with extensive innovation and modification in products leading to dynamic changes in the packing. Specifically, in MRE application, this system helps the DLA and assemblers to understand the dynamic changes and their consequences, to measure the dynamic change, to communicate the change, to incorporate the change in the final packing of MREs, and finally to train personnel properly on the change. If the suggested solutions are used, assembly workers' training curves reduce significantly, and the standardized packing procedures reduce variations in MRE menu bags and boxes, increase the stability of pallets, and save the integrity of the food. This process ensures that the DLA is able to incorporate any modification effectively with a very short lead-time and to deliver the food to the soldiers with safety and integrity.

1.2 Problem Statement and Assumptions

A majority of products have three dimensions (3-D), and their content is either rigid or moldable, leading to solid or moldable rectangular dimensions. This research focuses on the optimization component of the dynamic packing modification system, investigates a unique variant of the 3-D bin packing problem [6] including a hybrid mix of rigid and moldable dimensions, and studies the implications of applying analytical models to solve packing problems.

Two 3-D mathematical models are developed to optimize the problem of packing a hybrid mix of rigid and moldable items. The developed packing models characterize moldable items from two perspectives: (1) when limited discrete configurations represent the moldable items and (2) when all continuous configurations are available to the model. Then the developed models are applied to the dynamic packing changes of Meals, Ready-to-Eat (MRE) at two different levels: packing MRE food items in the menu bags and packing menu bags in the boxes.

Current MRE food items are a hybrid set of rigid and moldable food items. Thus, the hybrid moldable model with limited discrete configurations for moldable items is used to find the optimum packing of MRE food items in the MRE menu bags. This model uses a limited number of configurations of existing moldable MRE food items. The hybrid moldable model with all continuous configurations for moldable items is used to find the optimum design for moldable MRE food items and the packing of MRE food items in the existing menu bags. The optimum design for moldable MRE food items can be used for future modifications of moldable MRE food items to pack them in the existing menu bags. The results are compared with the condition in which all the MRE food items are rigid.

The resulting optimum configurations for MRE menu bags from the optimum packing of MRE food items in the menu bags are inputs in the hybrid moldable model with limited discrete configurations used to find the optimum packing of MRE menu bags in the boxes. The hybrid moldable model with all continuous configurations for moldable items is not applicable to the existing MRE menu bag packing problem. However, the results can be used for future redesign of menu bags as well as redesign of moldable food items. The results are compared with the menu bags' optimum configuration from considering all the food items as rigid.

In various objective scenarios, the actual height, length, or width of each bin differs. The variable-sized bin packing problem consists of the classical 3-D bin packing problem, where all the bins have the same capacity and cost [6]. The 3-D bin packaging problem is an NP-hard problem in the strongest sense [7]. The flexibility of bin dimensions poses a greater challenge to providing quality solutions; in other words, unlike the traditional 3-D bin packaging problem where only the number of bins used needs to be optimized, the variable-sized 3-D bin packaging problem needs to search for all of the bins' minimum volumes overall. The flexibility of the items' orientations also significantly expands the search space and, thus, increases the difficulty of finding optimal solutions. The proposed modeling method adopts its main analytical concept for 3-D rectangular packaging from Chen (1995) and Wu et al. (2010) [6, 8].

In developing the models, the following assumptions are made:

- In this research, a bin is a rectangular container used to hold items smaller than the bin.
- Each item can have rigid rectangular dimensions or moldable rectangular dimensions.
- It is assumed that the quantity, content, and dimensions of each item type are known.
- The longest dimension of an item is referred to as its length; the shortest dimension is the height; and the middle dimension is the width.

- It is assumed that all items can be freely rotated and placed into the bin in one of six positions that keep the items' edges parallel to the bin edges. Without loss of generality, it is also assumed that all item dimension data are positive values, satisfying the constraint that each item can be placed in the bin in at least one of the six orientations.
- The items need not be packed in layers, and the so-called guillotine constraint (requiring packing patterns to be such that the items can be obtained by sequential face-to-face cuts parallel to the bin's faces) is not imposed [6].

In using the developed models for solving the MRE packing problems, the following assumptions are made:

- In packing MRE food items in the menu bags, the menu bags are bins with rigid rectangular dimensions that can hold smaller MRE food items.
- Each MRE food item can have rigid rectangular dimensions or moldable rectangular dimensions.
- In packing menu bags in the boxes, boxes are bins with rectangular dimensions that can hold MRE menu bags.

More information on definitions and terminology is provided in appendix A.

1.3 Significance of Study and Contributions

This research develops analytical models for 3-D packing design problems with a hybrid mix of rigid and moldable items and uses the findings to solve an actual packing problem of delivering food to soldiers. Modeling the 3-D packing with a hybrid mix of rigid and moldable items is unique and can be used for any product. Additionally, this dissertation contributes in the following ways:

- It introduces a valid, novel, 3-D hybrid bin packing approach to the problem of packing a combination of moldable and rigid items where limited discrete configurations for moldable items are provided for the model.
- It introduces a valid, novel, 3-D hybrid bin packing approach to the problem of packing a combination of moldable and rigid items where all continuous configurations for moldable items are available to the model, and they can be in any shape.
- It illustrates the applicability of the models to solve the MRE packing problem arising on two levels:
    - Packing MRE food items in the MRE menu bags.
    - Packing MRE menu bags in the boxes.
- It validates the findings by running the models multiple times to ensure the consistency of solutions.
- It translates the solutions by creating visualizations of the packing process of MRE food items in the menu bags as well as MRE menu bags in boxes for training purposes.

As a component of a lean effort that attempts to reduce the lead-time associated with the implementation of dynamic product modifications that imply packing changes, this dissertation contributes in the following ways:

- Assisting packing organizations by designing the optimized packing for new or modified products and enabling companies to train their personnel easily and quickly.
- The MRE application perspective enables MRE supply chain entities, including DLA, manufacturers, and assemblers, to understand the consequences of changes in MREs, to measure changes, to communicate the changes, and to incorporate the changes effectively in a short lead-time, yielding the following results:

- Reducing the dimensional variations in MRE menu bags and boxes.
- Improving the space utilization in packing MRE food items in the menu bags and packing MRE menu bags in the boxes.
- Reducing the costs associated with instability of pallets in transportation and warehouse storage.
- Saving the integrity of food when delivered to soldiers.

1.4 Approach

Figure 5 illustrates the methodology framework, including the phases and steps. The research methodology starts with reviewing the literature and explicitly defining the problems. The second step involves developing mathematical models to optimize the problem of packing a hybrid mix of rigid and moldable items. In the next steps, two scenarios for modeling moldable items are considered, and relevant mathematical models are formulated. The objectives for each problem are identified, and then the developed models are tested by solving the MRE packing problems on two levels: packing MRE food items in the bags and packing MRE menu bags in the boxes. The MREs' specific packing requirements are considered, and the developed mathematical models are used to solve the packing problems. The results are compared on three levels: a hybrid mix of rigid and limited discrete moldable configurations, a hybrid mix of rigid and continuous moldable configurations, and only a rigid rectangular configuration.

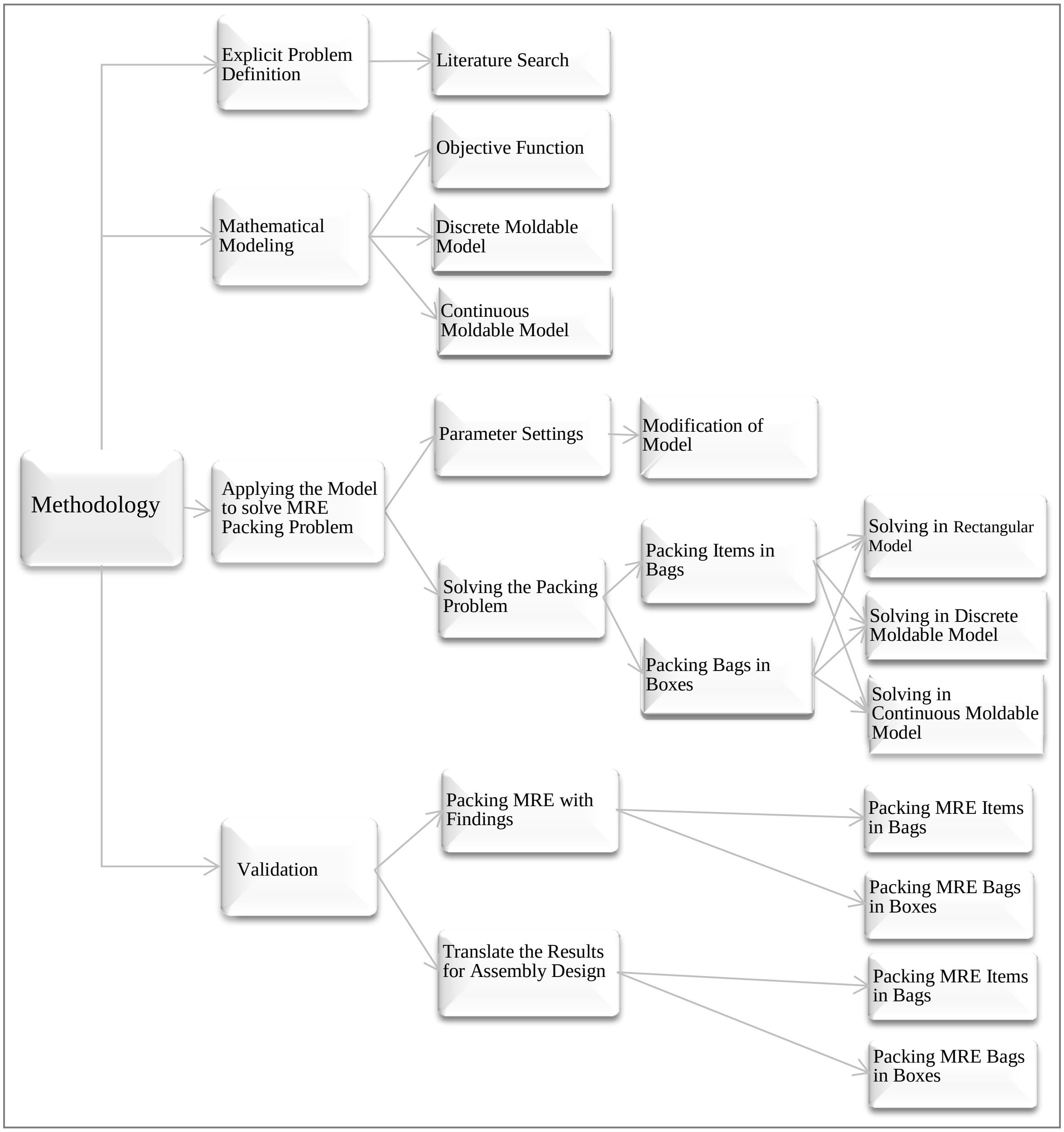

Figure 5: Research Methodology

In the last step, the optimization results are validated by running the solutions multiple times to access the consistency of solutions. Then the solutions are visualized using Autodesk Inventor to communicate the optimized packing solutions with the MRE assemblers for training purposes.

1.5 Organization

Chapter II summarizes the literature of packing problems, packing optimization, and 3-D hybrid moldable rigid packing optimization.

Chapter III presents the mathematical models for two scenarios including (1) the 3-D hybrid moldable packing problem with limited discrete configurations for moldable items and (2) the 3-D hybrid moldable packing problem with all continuous configurations for moldable items.

Chapter IV presents the results of applying the two mathematical models to solve the MRE packing problems at two different levels: packing MRE food items in the menu bags and packing MRE menu bags in the boxes. Also, Chapter IV shows the comparison between these results and conditions in which only solid rectangular configurations are present. Furthermore, this chapter presents the validation of solutions and visualizations.

Chapter V includes a summary, conclusions, and recommendations. This dissertation ends with references and appendixes. The appendixes include terminology, military backing limitation and constraints, and a list of MRE items.

CHAPTER II
Literature Review

This research investigates a unique variant of a 3-D bin packing problem arising in packing processes including a hybrid mix of rigid and moldable items. The 3-D bin packing problem is a special category of container-loading problem belonging to a larger group of packing problems. Packing problems have a structure in common which includes:

- two given sets of elements, namely a set of large objects and a set of small items;
- elements defined exhaustively in one, two, three, or an even larger number (n) of geometric dimensions (1-D, 2-D, 3-D, …, n-D);
- a select some or all small items, grouped into one or more subsets, and where each of the resulting subsets is assigned to one of the large objects, i.e. the geometric condition holds or the small items of each subset have to be laid out on the corresponding large object such that
    - all small items of the subset lie entirely within the large object and
    - the small items do not overlap,
- and a single-dimensional or multi-dimensional objective function is optimized [9-14].

The assortment of items is considered to be weakly heterogeneous if the items can be grouped into few classes. It is called strongly heterogeneous if none or only very few items are of identical shape and size.

As Figure 6 illustrates, the packing problems can be mapped into three main categories: knapsack packing, cutting problems, and container loading problems [15]. In knapsack packing problems, a container with fixed dimensions and a set of items each with a profit value are given;

the goal is to find a subset of items with maximum profit which may be packed within the container [16-33].

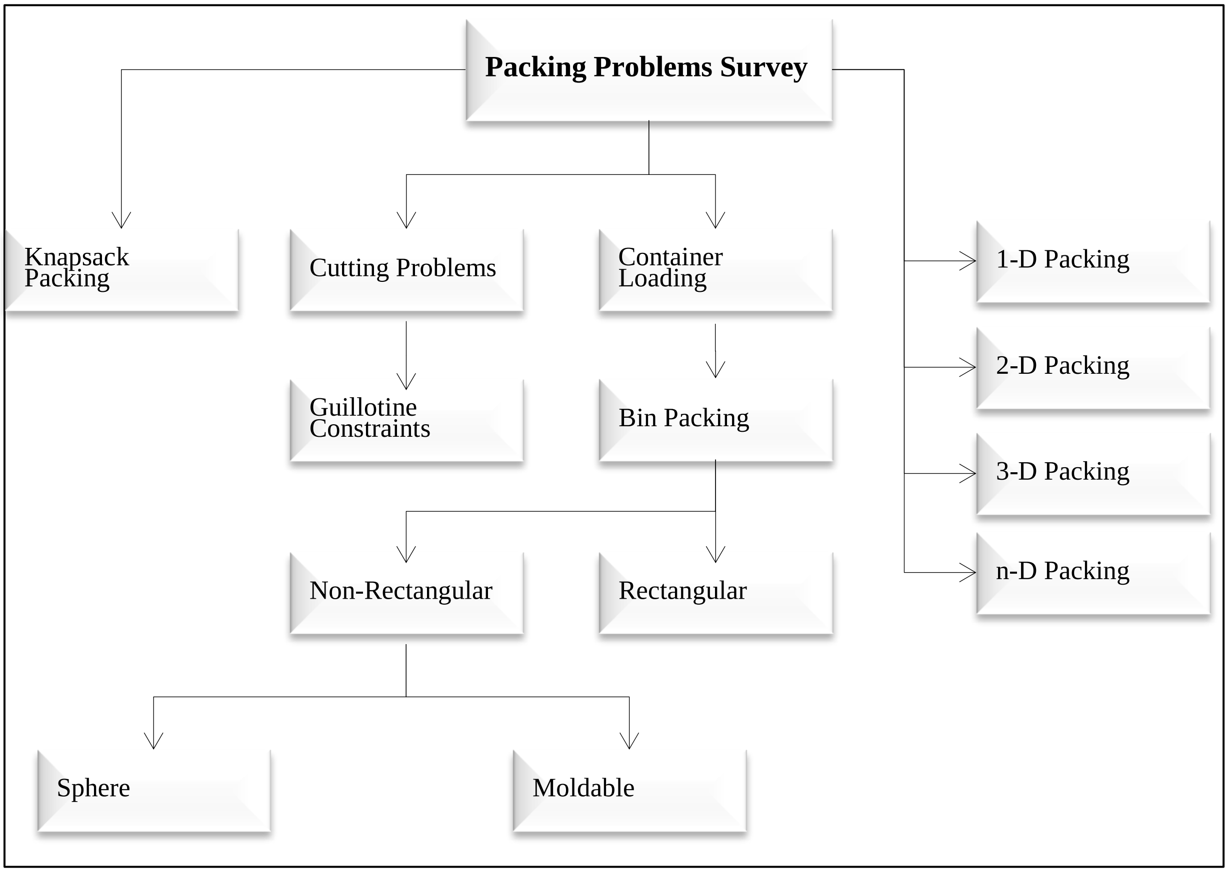

Figure 6: Packing Problems Surveys

A comprehensive survey of knapsack problems is presented in Martello and Toth (1990) [34]. Cutting problems look for cutting large objects to produce smaller objects [35-44]. Cheng et al. [45] presents surveys about the cutting stock problem, also known as the trim-loss problem, with methodologies and the practical aspects.

Three-dimensional container loading problems can be defined as geometric assignment problems, in which 3-D small items have to be packed in 3-D, rectangular, large containers [36-44, 46]. Numerous articles present possible applications and solutions for container loading problems. [5, 39-44, 47, 48].

All packing problems can be divided into two types: minimization and maximization. Wäscher, Haußner, and Schumann (2007) and Bortfeldt and Wäscher (2013) categorize minimization problem in the following ways:

- Single Stock-Size Cutting Stock Problem: packing a weakly heterogeneous set of items into a minimum number of identical containers;
- Multiple Stock-Size Cutting Stock Problem: packing a weakly heterogeneous set of items into a weakly heterogeneous assortment of containers such that the value of the used containers is minimized;
- Residual Cutting Stock Problem: packing a weakly heterogeneous set of items into a strongly heterogeneous assortment of containers such that the value of the used containers is minimized;
- Single Bin-Size Bin Packing Problem: packing a strongly heterogeneous set of items into a minimum number of identical containers;
- Multiple Bin-Size Bin Packing Problem: packing a strongly heterogeneous set of items into a weakly heterogeneous assortment of containers such that the value of the used containers is minimized;
- Residual Bin Packing Problem: packing a strongly heterogeneous set of items into a strongly heterogeneous assortment of containers such that the value of the used containers is minimized;
- Open Dimension Problem: packing a set of items into a single container with one or more variable dimensions such that the container's volume is minimized.

The maximization problem is mainly equivalent to the maximization of the container volume utilization if the value of the small items is proportional to their volumes. The following maximization problem types can be distinguished based on the literature review presented in Wäscher, Haußner, and Schumann (2007) and Bortfeldt and Wäscher (2013):

- Identical Item Packing Problem: loading a single container with a maximum number of identical small items;
- Single Large Object Placement Problem: loading a single container with a selection from a weakly heterogeneous set of items such that the value of the loaded items is maximized;
- Multiple Identical Large Object Placement Problem: loading a set of identical containers with a selection from a weakly heterogeneous set of items such that the value of the loaded items is maximized;
- Multiple Heterogeneous Large Object Placement Problem: loading a (weakly or strongly) heterogeneous set of containers with a selection from a weakly heterogeneous set of items such that the value of the loaded items is maximized;
- Single Knapsack Problem: loading a single container with a selection from a strongly heterogeneous set of items such that the value of the loaded items is maximized;
- Multiple Identical Knapsack Problem: loading a set of identical containers with a selection from a strongly heterogeneous set of items such that the value of the loaded items is maximized;
- Multiple Heterogeneous Knapsack Problem: loading a set of (weakly or strongly) heterogeneous containers with a selection from a strongly heterogeneous set of items such that the value of the loaded items is maximized [36].

Based on these categories, the research problem is an open dimension problem (to pack a set of items into a single container with one variable dimension such that the container's volume is minimized). What makes this problem different from previous studies is the hybrid mix of moldable and rigid items.

In the following discussion, section 2.1 presents a review of packing optimization studies. Section 2.2 then presents the literature related to 3-D moldable packing optimization problems.

2.1 Packing Optimization

In 1992, Dowsland et al. [49] publish a survey paper on the application of operational research techniques, giving a solution for packing problems like pallet loading, container stuffing, and placement problems. In particular, this paper reviews a number of exact and heuristic approaches and focuses on the modeling of and solution for packing problems in two and three dimensions. The authors also mention that most variants of packaging problems are NP-complete; therefore, heuristic methods are required to obtain a practical solution in a reasonable amount of computational time.

Tsai's article (1987) presents the earliest modeling approach to solve a packing problem [50]. The author provides a model for arranging boxes of different sizes in three dimensions on a pallet without overlapping, but does not note any further constraints. Chen, Lee, and Shen (1995) propose a linear, mixed-integer model which includes all the above-listed intermediate problem types as special cases [5].

Padberg (2000) introduces a mixed-integer model for constraint-free 3-D packing problems. He estimates that problem instances with 10 to 20 boxes may be solved in reasonable computing times by means of standard branch and bound algorithms with no results from numerical experiments being reported [51].

Junqueira, Morabito, and Yamashita (2012) present 0-1 linear programming models which include orientation constraints, (vertical and horizontal) stability constraints, and stacking constraints. The authors use the numerical experiments to validate the proposed methods. The authors also mention that only problem instances of moderate size can be handled by the standard problem solver (GAMS/CPLEX) used in these experiments [46].

Dowsland [52] presents practical solutions to real industrial packing problems. This paper uses the simulated annealing approach to packing problems and describes a series of experiments carried out to ascertain the effectiveness of the method. His paper also suggests that annealing does seem capable of producing “near” feasible solutions which can be transformed into feasible solutions by hand and by adding a few optimization phases to the annealing process. These optimization phases are costly in terms of computation time, and they require skill in their development and programming and require users to judge the appropriate moments to call them from the annealing process.

From Dowsland’s research, two new ideas emerge: the use of a “cooling” schedule, which also heats up when moves are not accepted, and the use of a relaxed objective either in a two-stage approach or in a parallel to help move the solution to the optimum point.

Lodi et al.’s article [53] shows recent advances obtained for the two-dimensional bin packing problem with special emphasis on exact algorithms and elective heuristic and metaheuristic approaches.

Epstein and Levy [54] consider dynamic packing of squares and rectangles into unit squares and dynamic packing of three-dimensional cubes and boxes into unit cubes. They also study dynamic d-dimensional hypercube and hyper box packing. For dynamic d-dimensional box packing, they define and analyze an algorithm (NFDH) for the offline problem and present a dynamic version.

Leung et al. [55] aims to find the containers' dimensions for distributing various kinds of towel products to a large number of retail outlets. The objective of the carton box design problem is to lower the overall future distribution costs by improving the box space utilization and reducing the number of carton types required. A Multi-Objective Genetic Algorithm (MOGA) typically generates the carton designs for the sales forecast of the upcoming first week as well as for the upcoming 53 weeks. The clustering technique has then been used to investigate the towel product order pattern so as to validate the MOGA generated results. The results demonstrate that MOGA effectively searches the best carton box spatial design to reduce unfilled space and the number of required carton types.

In applications such as the loading of tractor trailer trucks, airplanes, and ships, where a balanced load provides better fuel efficiency and a safer ride, there are often conflicting criteria to be satisfied (to minimize the bins used and to balance the load of each bin) that are subject to a number of practical constraints. Unlike existing studies that only consider the issue of the minimum number of bins, a multi objective two-dimensional mathematical model for bin packing problems with multiple constraints (MOBPP-2D) is formulated in Liu's book [56]. To solve MOBPP-2D problems, a multi objective evolutionary particle swarm optimization algorithm (MOEPSO) is proposed.

Ioannou [57] presents an integer programming formulation that integrates decisions concerning the layout of the resource groups on the shop floor with the design of the material handling system. The proposed model reflects critical practical concerns, including the capacity of the material flow network and of the handling transporters as well as the tradeoff between fixed (construction and acquisition) and variable (operational) costs. An integrated solution method, guided by a simulated annealing scheme, solves the global shop design problem. The algorithm takes advantage of the proposed decomposition and converges to a final design which is feasible with respect to all problem constraints. The method is applied to redesign the facility of a large manufacturer of radar antennas. The resulting shop configuration exhibits substantially decreased material handling effort, and requires significantly smaller investment costs compared to the existing facility.

Vassiliadis [58] presents a hierarchical, binary tree representation for bounding boxes (rectangles) comprised of shapes that are to be cut from a two-dimensional sheet of material. Vassiliadis's paper concludes that the binary tree representation is capable of capturing any configuration of such rectangular shapes in two-dimensional space, such as rotations through right angles and translations. The construction is such that these boxes must be bound together in pairs having a common edge and can be extended to contain constraints regarding vertical or horizontal space between the actual objects and the special types of cuts, e.g. guillotine cuts used in the glass cutting industry. Finally, the paper discusses a simple continuous sheet application as a demonstration of the capabilities of the algorithm in connection with a local search method, specifically threshold acceptance.

Jakobs [59] proposes a genetic algorithm for placing polygons on a rectangular board. The Orthogonal Packing Problem (OPP) is a generalization of the one-dimensional bin-packing problem. If all rectangles have the same height, then the two problems agree. Jakobs also mentions in the paper that both the problems dealing with same height and same width are considered to be NP-Complete. In his paper, the combination of genetic algorithms and deterministic methods are adopted to avoid the convergence to a local minimum. In order to effectively reduce the number of possible orthogonal packing patterns, the Bottom-Left-condition (BL-condition) algorithm is introduced. The orthogonal packing pattern fulfills the BL-condition if no rectangle can be shifted further to the bottom or to the left. Jakobs also suggests that any deterministic packaging algorithm based on permutation can be improved by using the Bottom-Left-algorithm (BL-algorithm). The disadvantage of the BL-algorithm is that groups of rectangles exist for which the BL-algorithm cannot generate the optimal packing pattern; therefore, a more expensive deterministic algorithm is required to transform a permutation into a packing pattern.

Cagan et al. [60] propose a simulated annealing-based algorithm using hierarchical models for a general three-dimensional component layout. The objective functions in their paper are (1) maximization of packing density, (2) minimization of routing costs, (3) maximization of assimilability, and (4) minimization of configuration costs. The algorithm uses simulated annealing to search for optimal layouts and hierarchical models of components to approximate efficiently the intersections of complex geometric shapes. The algorithm is able to generate consistently good quality layouts for arbitrary geometries. Also, these layouts not only optimize one or more objectives functions such as packing density, center of gravity, etc., but also they satisfy spatial constraints between the components and between the components and the container.

Hopper and Turton [61] propose two hybrid genetic algorithms that use genetic algorithms in conjunction with BL routine and the placement method (to overcome the disadvantages of BL routine) in order to solve 2-D packaging problems frequently encountered in the wood, glass, and paper industries. The two hybrid genetic algorithms are compared with random search and heuristic placement algorithms to evaluate their performance. Results show that the approach with placement method produces better results than the BL routine, but the difference between the hybrid genetic algorithm (with the BL routine) and the heuristic placement method is smaller than expected. Also, the results indicate that the performance of the two hybrid genetic algorithms is strongly dependent on the nature of the placement routine.

Feng et al. [62] discuss graph theory and group theory approaches. They also propose a global optimization algorithm for solving the layout problem of the artificial satellite module to find an optimal strategy for installing a finite number of apparatus (graph elements or pieces) with different shapes, sizes, and mass quantities on a base board (cycloid domain).

Bazargan [63] presents a multi-objective model to generate the layout designs for machines and cells in a cellular manufacturing environment for a food manufacturing and packaging company in Australia. This paper shows the process of developing the final inter-cell layout designs by providing the management with multiple layout configurations and showing the impact of each design on the material handling cost at each stage. This paper also shows that the travelling cost is not the sole criterion for generating the layout designs.

Faina [9] proposes a geometrical model which reduces the general three-dimensional packing problem to a finite enumeration scheme. Further, a statistical algorithm called "zone 3-D" based on simulated annealing is described to solve a container loading problem for low density cartons in which the cartons are loaded only on volume restrictions. Numerical results (an estimate of the asymptotic performance bound) show that the algorithm provides excellent placement for a relatively fewer numbers of boxes (less than 128) and an acceptable placement for larger numbers of boxes.

Teng et al. [64] propose a mathematical model, a solution strategy, and a heuristic algorithm for a three-dimensional packing problem which is often encountered in many advanced technologies, such as satellites, spacecrafts, rotating vessels, underwater suspended engineering, etc. In particular, their paper studies the layout optimization problem of allocating objects inside the conic satellite vessel which moves spirally; this problem is considered to be more difficult to solve than the classical packaging problem.

In 2002, Cagan et al. [65] conduct a survey of all the computational approaches to solve the three-dimensional packaging problem. In particular, their paper reviews the optimization algorithms and geometric representations used in layout problems, the advantages and limitations of each method, and application areas. The authors also suggest that heuristic and traditional optimization techniques are not suitable to solve problems involving nonlinear, non-differentiable objective and constraint functions.

Dyckhoff's article [66] demonstrates the development of a systematic approach for a comprehensive typology that integrates various kinds of cutting and packaging problems and notations. The main intention of his paper is to form a basis for unifying the different use of notations in literature and for concentrating future research on special types of problems.

Chernov et al. [67] propose mathematical models and practical algorithms for solving the cutting and packing problem. Some objects are a general shape (called "phi objects"); their layouts are characterized by means of special functions (called "phi-functions"), and their construction involves a certain degree of flexibility. The phi functions can be used to represent both 2-D and 3-D objects. The authors review and enhance the phi functions by reducing the cutting and packaging problem to a constrained minimization problem and then discuss various approaches to solve the constrained minimization problem.

Gomes and Oliveira [68] propose a hybrid heuristic approach consisting of simulated annealing and linear programming techniques to solve Irregular Strip Packing problems. Simulated annealing is used to guide the search over the solution space while linear programming models are used to generate neighborhoods during the search process. The use of a neighborhood structure based on the exchange of pieces on the layout is shown to be an extremely effective approach when used in conjunction with a simulated annealing algorithm to guide the search over the solution space. Gomes and Oliveira use this hybrid approach to improve the results of the best algorithm that had been used until then to solve the Irregular Strip Packing problems, but the computational time for large problems instances is relatively high compared to previous approaches.

Birgin et al. [69] propose an approach based on smooth nonlinear programming concepts for orthogonal packing of rectangular items within arbitrary convex regions. The authors first solve the problem of packing a given set of rectangles without rotations. Then, the possibility of orthogonal rotations is considered, and a procedure for identical rectangles is devised. Finally, the problem of packing as many identical rectangles as possible, allowing orthogonal rotations, is addressed. Computation results on test problems were able to prove the efficiency and effectiveness of this method.

Westerlund et al. [70] present an extension of previous formulations applied on two-dimensional facility layout and two- and three-dimensional process plant layout problems to solve spatial allocation problems of fitting mixed-sized boxes into unchanging dimensional containers. The most important characteristic of this formulation from its predecessors is its ability to use a set of different container types instead of one fixed pallet size in the packing pattern. The formulation is a generalized version and can be applied to solve a variety of container loading problems.

2.2 3-D Moldable Packaging Optimization

The 3-D bin packaging problem is an NP-hard problem in a strongest sense [7]. The items in the bin packing problem can be rectangular or non-rectangular. The non-rectangular items can be divided further into irregular items (spheres) or moldable categories as presented in Figure 6. Because of the increased complexity of non-rectangular problems, a large proportion of the published studies to date are limited to the packing of rectangles or cuboids. Also, there are various research studies which focus on sphere packing problems.

Stoyan (1983) tries to overcome non-rectangular problems by using a probabilistic technique. He suggests that the pieces be ordered randomly and placed with a given placement policy in a number of trials. A distribution type for the objective is assumed, and the resulting distribution is obtained from the experimental mean and standard deviation. The probability of obtaining a better solution than the best solution to date can then be determined. By defining a metric on the permutations of n objects, orderings less than a given distance from the maximum can be determined, and the process is repeated in this range until the probability of improvement is small. Stoyan reports that this method has been used successfully in both two-dimensions and three-dimensions, but Stoyan does not suggest how the distribution type or the metric be obtained. This work is reminiscent of more general investigations into a heuristic performance using the Weibull distribution. It is possible that this form of analysis may provide valuable insight into the performance of any of the heuristics described in the previous sections.

Irregular objects packing problems are more complex than regular ones. In Wong et al.'s article [71], a methodology that hybridizes a two-stage packing approach based on grid approximation with an integer representation based Genetic Algorithm (GA) is proposed to obtain an efficient allocation of irregular objects in a stock sheet of infinite length and fixed width without overlap. The experiments in the apparel industry validate the effectiveness of the proposed methodology. The results demonstrate that the proposed method outperforms the commonly used BL placement strategy in combination with random search (RS).

Various researchers conduct sphere-packing studies [72-88]. However, the problem of the unequal sphere packing in a 3-D polytope is analyzed in Sutou and Dai’s article [89]. Given a set of unequal spheres and a polytope, the double goals are to assemble the spheres in such a way that (1) they do not overlap with each other and (2) the sum of the volumes of the spheres packed in the polytope is maximized.

Leung and Huang (2008) address the problem of loading a subset of -3-D rectangular items into a 3-D rectangular container, such that the total volume of the packed items is maximized or such that the container’s wasted volume is minimized. This problem is an NP-hard problem, whose 1-D degradation, the 0–1 knapsack problem, is still NP-hard. This 3-D rectangular packing problem is also called “the container loading problem,” because the most common and important application of this problem is to load rectangular cargoes into containers, vehicles, or ships in the transportation industry [55].

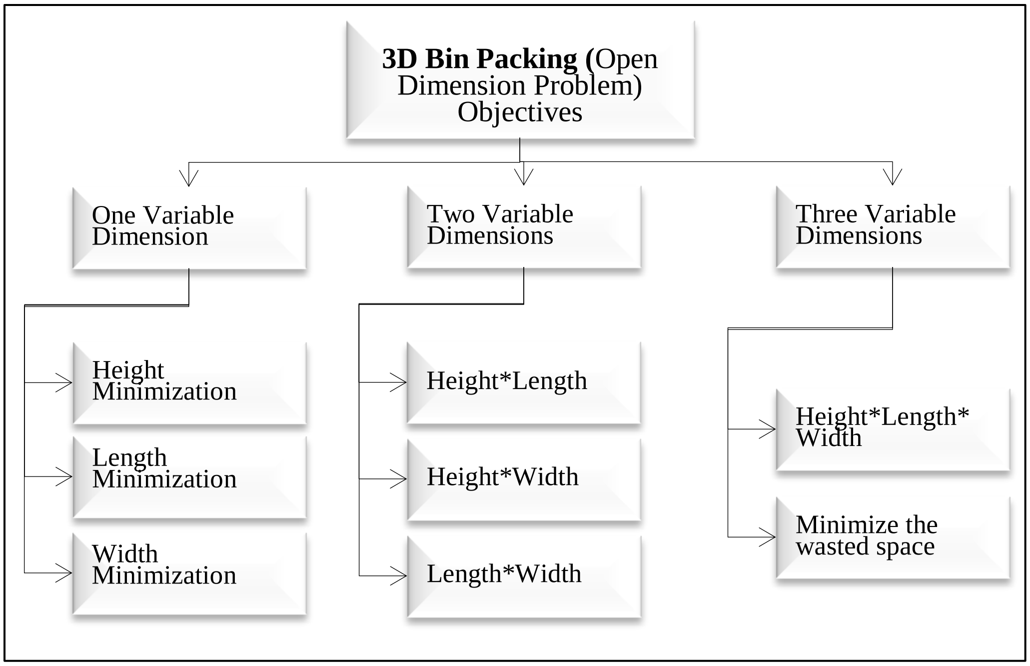

Figure 7: 3-D Packing Problems Objective Functions

The literature review reveals that there are no analytical models considering the moldable items packing. Hybrid moldable packing problems arise in many industries, and this research aims to address this problem from an application perspective. Similar problems with only rigid rectangular items are presented in the literature with the objectives of minimizing the packing volume with three variable dimensions (or reducing the wasted space), minimizing the packing volume with two variable dimensions, or minimizing the packing volume with one variable dimension. Figure 7 depicts the various objective functions. In this research, minimizing the packing volume with one variable dimension is considered.

CHAPTER III
Mathematical Models

This chapter aims to present the developed mathematical models of the 3-D hybrid moldable packing problems. Section 3.1 presents the logic of the modeling. Section 3.2 demonstrates the notations that are used throughout this chapter. Section 3.3 presents the 3-D hybrid moldable packing model with limited discrete configurations for moldable items, assumptions, objective functions, and the constraints. Section 3.4 presents the 3-D hybrid moldable packing model with all continuous configurations for moldable items, objective functions, and the related constraints.

3.1 Logic

This section clarifies the logic of modeling which is used in developing the models. In order to locate items in a bin, it is necessary to identify three sets of variables. These variables include:

- variables indicating the placement of all the items in the bin relative to the other items (see Figure 8),

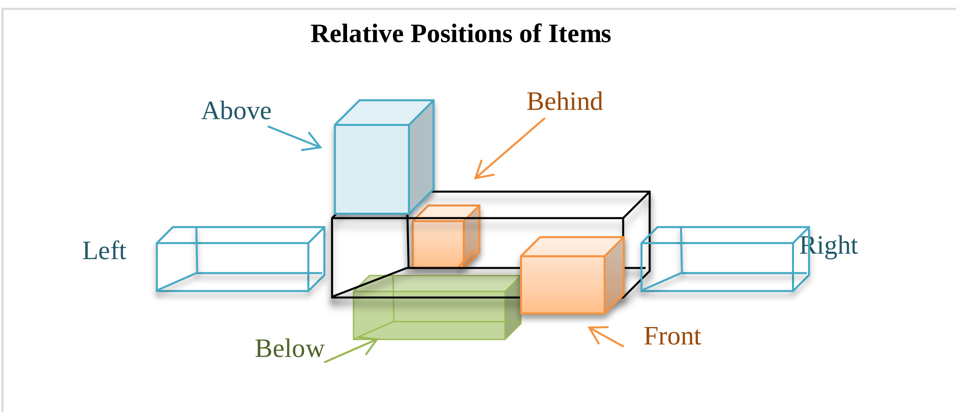

Figure 8: Relative Positions Modeling

- variables indicating the coordinates of the front-left bottom (FLB) corner of each item (see Figure 9),

- variables indicating whether the length of the item is parallel to the $X-$, $Y-$, or $Z-axis$; width is parallel to $X-$, $Y-$, or $Z-axis$; or height is parallel to $X-$, $Y-$, or $Z-axis$ (see Figure 9) [5, 6, 46].

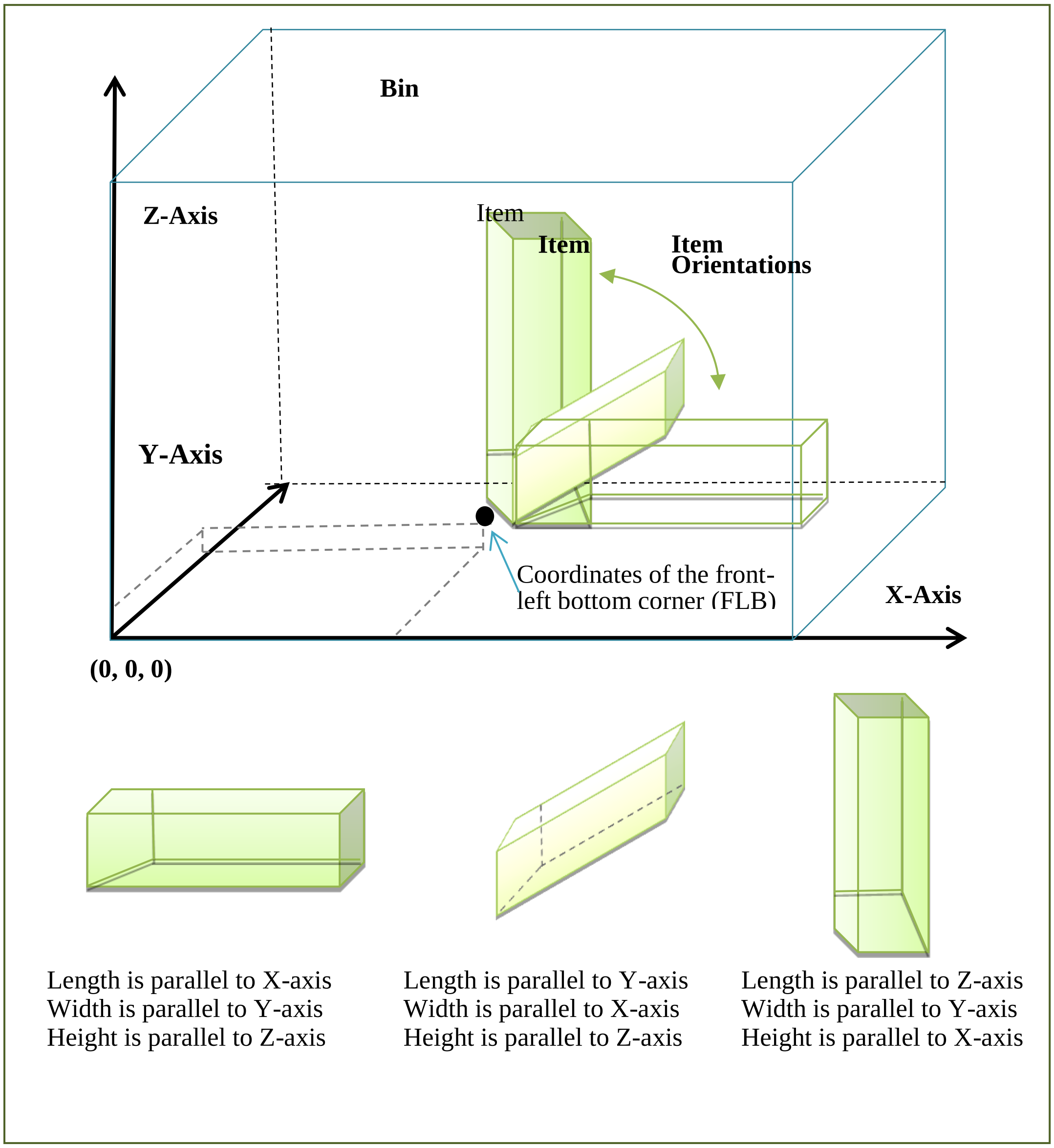

Figure 9: Modeling Logic

3.2 Notations

Table 1 presents the notations that are used in modeling the discrete moldable packing and the continuous moldable packing problems. Total items to be packed in the box are defined with the index i which belongs to set I. This set is divided into two sets of items: namely, moldable items, $\hat{I}$, and rigid rectangular items, I'. It is assumed that a bin is placed with its length along the $X - axis$ and its width along the $Y - axis$. The FLB corner of the bin is fixed at the origin.

Table 1: Moldable Items Packing Model Notation

Symbol	Description
$\boldsymbol{I}$	Total number of items to be packed
$\boldsymbol{i}, \boldsymbol{k} \in \boldsymbol{I}$	Index for each item
$\boldsymbol{D^i}$	Total number of configurations for ith moldable item
$\boldsymbol{d} \in \boldsymbol{D^i}$	Index for each configuration of ith moldable item, $i \in \hat{I}$
$\boldsymbol{\hat{I}}$	Set including all moldable items, where $\hat{I} \subseteq I$
$\boldsymbol{\acute{I}}$	Set including all rigid rectangular items, where $\acute{I} \subseteq I$, $\hat{I} \cap \acute{I} = \emptyset$, $\hat{I} \cup \acute{I} = I$
$\boldsymbol{M}$	An arbitrarily large number
$\boldsymbol{L}\ (\boldsymbol{l})$	Parameters (variable) indicating the length of bin
$\boldsymbol{W}(\boldsymbol{w})$	Parameters (variable) indicating the width of bin
$\boldsymbol{H}(\boldsymbol{h})$	Parameters (variable) indicating the height of bin
$(\boldsymbol{p_i}, \boldsymbol{q_i}, \boldsymbol{r_i})$	Parameters indicating the length, width, and height of item $i \in \acute{I}$ (rigid rectangular)
$(\boldsymbol{P_i}, \boldsymbol{Q_i}, \boldsymbol{R_i})$	Variables indicating the maximum length, width, and height of item $i \in \hat{I}$ (moldable)
$(\boldsymbol{\beta_i}^{ld}, \boldsymbol{\beta_i}^{wd}, \boldsymbol{\beta_i}^{hd})$	Parameters indicating the maximum length, width, and height of the dth configuration for the moldable item $i \in \hat{I}$
$\boldsymbol{\alpha_i}^{l}$	A lower bound (minimum) value for length of a moldable item, $i \in \hat{I}$
$\boldsymbol{\beta_i}^{l}$	An upper bound (maximum) value for length of a moldable item, $i \in \hat{I}$
$\boldsymbol{\alpha_i}^{w}$	A lower bound (minimum) value for width of a moldable item, $i \in \hat{I}$
$\boldsymbol{\beta_i}^{w}$	An upper bound (maximum) value for width of a moldable item, $i \in \hat{I}$
$\boldsymbol{\alpha_i}^{h}$	A lower bound (minimum) value for height of a moldable item, $i \in \hat{I}$
$\boldsymbol{\beta_i}^{h}$	An upper bound (maximum) value for height of a moldable item, $i \in \hat{I}$
$\boldsymbol{V_i}$	Parameter representing the volume of a moldable item, $i \in \hat{I}$
$(\boldsymbol{x_i}, \boldsymbol{y_i}, \boldsymbol{z_i})$	Continuous variables (for location) indicating the coordinates of the front-left bottom corner (FLB) of item i

Table 1. Continued.

Symbol	Description
$(\boldsymbol{l_{xi}}, \boldsymbol{l_{yi}}, \boldsymbol{l_{zi}})$	Binary variables indicating whether the length of item i is parallel to the $X-$, $Y-$, or $Z-axis$. For example, the value of l_{xi} is equal to 1 if the length of item i is parallel to the $X-axis$; otherwise it is equal to 0.
$(\boldsymbol{w_{xi}}, \boldsymbol{w_{yi}}, \boldsymbol{w_{zi}})$	Binary variables indicating whether the width of item i is parallel to the $X-$, $Y-$, or $Z-axis$. For example, the value of w_{xi} is equal to 1 if the width of item i is parallel to the $X-axis$; otherwise it is equal to 0.
$(\boldsymbol{h_{xi}}, \boldsymbol{h_{yi}}, \boldsymbol{h_{zi}})$	Binary variables indicating whether the height of item i is parallel to the$X-$, $Y-$, or $Z-axis$. For example, the value of h_{xi} is equal to 1 if the height of item i is parallel to the $X-axis$; otherwise it is equal to 0.
$\boldsymbol{a_{ik}}$, $\boldsymbol{a_{ki}}$, $\boldsymbol{c_{ik}}$, $\boldsymbol{c_{ki}}$, $\boldsymbol{e_{ik}}$,	Binary variables a_{ik} , a_{ki} , c_{ik} , c_{ki} , e_{ik}, and e_{ki} are defined to indicate the placement of items relative to each other. The a_{ik} is equal to 1 if item i is on the left side of item *k*. Similarly, the variables a_{ki} , c_{ik} , c_{ki} , e_{ik}, and e_{ki} represent whether item i is on the right of, behind, in front of, below, or above item k, respectively. These variables are needed and defined only when $i < k$.
$\boldsymbol{t_{id}}$	Binary variables t_{id} are defined to create a link between moldable item configuration constraints.

Figure 10 illustrates the interpretation of all variables. The bin in the figure 10 is loaded with two items, i and k. Since the item i is located behind and on the left-hand side of the item k, a_{ik} and d_{ik} are equal to 1. Other indicators for the relative locations of the items i and k are set to 0 in this instance. The item k is located with its length along the $Z-axis$ and its width parallel to the $X-axis$. Therefore, the orientation indicators for item k, l_{zk}, w_{zk}, and h_{zk}, are equal to 1. The binary variables, $l_{xi}, l_{yi}, l_{zi}, w_{xi}, w_{yi}, w_{zi}, h_{xi}, h_{yi},$ and h_{zi}, are dependent, and the following relationships exist among them:

$$l_{xi} + l_{yi} + l_{zi} = 1$$
$$w_{xi} + w_{yi} + w_{zi} = 1$$
$$h_{xi} + h_{yi} + h_{zi} = 1$$
$$l_{xi} + w_{xi} + h_{xi} = 1$$
$$l_{yi} + w_{yi} + h_{yi} = 1$$
$$l_{zi} + w_{zi} + h_{zi} = 1$$

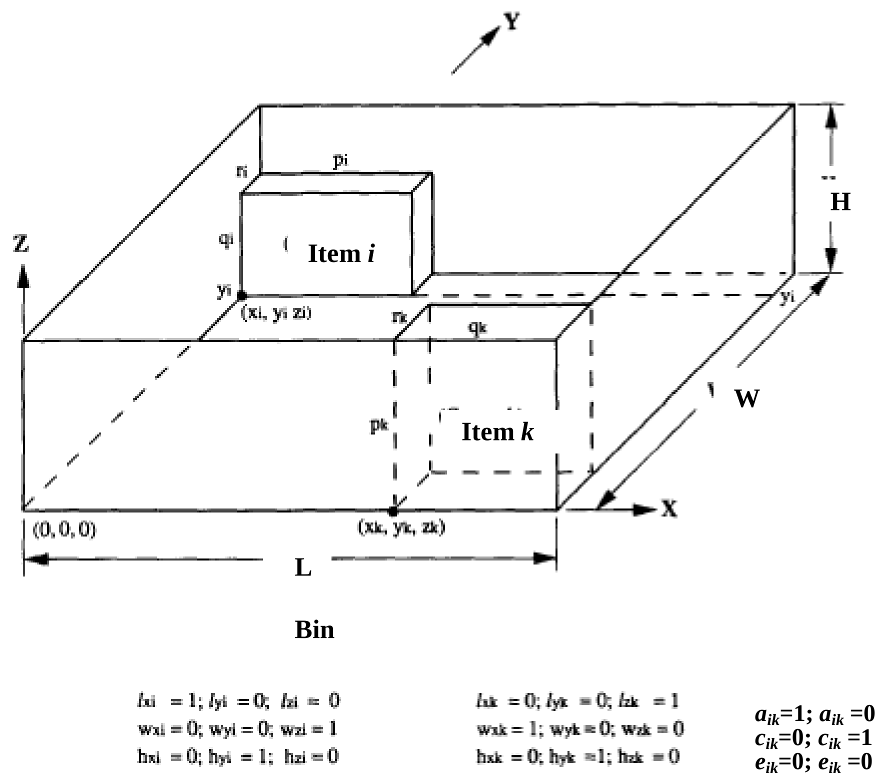

Figure 10: Variable definition (source [8])

Wu et al. (2010) has shown that by using the above relationships, the five variables l_{yi}, w_{xi}, w_{zi}, h_{xi}, and h_{yi} can be eliminated from the model resulting in a significant reduction in the model size [6].

3.3 3-D Hybrid Moldable Packing Model with Limited Discrete Configurations for Moldable Items

This section presents the model for the 3-D hybrid moldable packing problem with limited discrete configurations for moldable items. A moldable item is a 3-D item with a discrete number of recognizable configurations. The problem is how to pack a hybrid mix of rigid and moldable items into a bin. Specifically, for given alternative values for dimensions of moldable items, the following questions are addressed: what are the relative positions of all items in the bin; what are the configurations selected for each of the moldable item; and what is the packing volume required to pack all the items?

3.3.1 Assumptions

Set $I = \{1, 2, \ldots, i\}$ includes all the items that need to be packed in the bin. As mentioned in section 3.2, set I is comprised of flexible items, $\hat{I}$, and rectangular items, $\acute{I}$. Variables P_i, Q_i, and R_i represent the dimensional variables for the moldable items, where they can take any of the following recognizable discrete combination of the dimensions:

$$(P_i, Q_i, R_i) \in \{(\beta_i{}^{l1}, \beta_i{}^{w1}, \beta_i{}^{h1}), (\beta_i{}^{l2}, \beta_i{}^{w2}, \beta_i{}^{h2}), (\beta_i{}^{l3}, \beta_i{}^{w3}, \beta_i{}^{h3}), \ldots, (\beta_i{}^{ld}, \beta_i{}^{w\mathrm{d}}, \beta_i{}^{h\mathrm{d}})\},$$

where $(\beta_i{}^{ld}, \beta_i{}^{w\mathrm{d}}, \beta_i{}^{h\mathrm{d}})$ is the dth identified configuration for the ith item such that $d \in D^i$. In this configuration, $\beta_i{}^{ld}$represents the length, $\beta_i{}^{w\mathrm{d}}$ represents the width, and $\beta_i{}^{h\mathrm{d}}$ represents the height of the ith item's d^{th} configuration.

3.3.2 Objective Function

The objective function considered for this model is packing volume minimization with only one variable dimension. Accordingly, the objective may be any of the following:

- Minimize *h*
- Minimize *l*
- Minimize *w*

3.3.3 Constraints

The constraints of the 3-D hybrid moldable packing model with discrete configuration are shown below:

Subject to		
$x_i + p_i.l_{xi} + q_i.\left(l_{zi} - w_{yi} + h_{zi}\right) + r_i.\left(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\right) \leq x_k + (1 - a_{ik})M\ ;\forall\ i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	1́
$x_i + P_i.l_{xi} + Q_i.\left(l_{zi} - w_{yi} + h_{zi}\right) + R_i.\left(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\right) \leq x_k + (1 - a_{ik})M\ ;\forall\ i \in \hat{I}, k \in I; i \neq k$	overlap control for moldable items	
$y_i + q_i.w_{yi} + p_i.(1 - l_{xi} - l_{zi}) + r_i.\left(l_{xi} + l_{zi} - w_{yi}\right) \leq y_k + (1 - c_{ik})M\ ;\forall i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	2́
$y_i + Q_i.w_{yi} + P_i.(1 - l_{xi} - l_{zi}) + R_i.\left(l_{xi} + l_{zi} - w_{yi}\right) \leq y_k + (1 - c_{ik})M\ ;\forall i \in \hat{I}, k \in I; i \neq k$	overlap control for moldable items	
$z_i + r_i.h_{zi} + q_i.(1 - l_{zi} - h_{zi}) + p_i.l_{zi} \leq z_k + (1 - e_{ik})M; \forall i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	3́
$z_i + R_i.h_{zi} + Q_i.(1 - l_{zi} - h_{zi}) + P_i.l_{zi} \leq z_k + (1 - e_{ik})M; \forall i \in \hat{I}, k \in I; i \neq k$	overlap control for moldable items	
$a_{ik} + a_{ki} + c_{ik} + c_{ki} + e_{ik} + e_{ki} \geq 1; \forall\ i, k \in I; i \neq k$	relative positions of all items	4́
$x_i + p_i.l_{xi} + q_i.\left(l_{zi} - w_{yi} + h_{zi}\right) + r_i.\left(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\right) \leq L; \forall i \in \acute{I}$	bin length constraint	5́
$x_i + P_i.l_{xi} + Q_i.\left(l_{zi} - w_{yi} + h_{zi}\right) + R_i.\left(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\right) \leq L; \forall i \in \hat{I}$		
$y_i + q_i.w_{yi} + p_i.(1 - l_{xi} - l_{zi}) + r_i.\left(l_{xi} + l_{zi} - w_{yi}\right) \leq W; \forall i \in \acute{I}$	bin width constraint	6́
$y_i + Q_i.w_{yi} + P_i.(1 - l_{xi} - l_{zi}) + R_i.\left(l_{xi} + l_{zi} - w_{yi}\right) \leq W; \forall i \in \hat{I}$		
$z_i + r_i.h_{zi} + q_i.(1 - l_{zi} - h_{zi}) + p_i.l_{zi} \leq H; \forall i \in \acute{I}$	bin height constraint	7́
$z_i + R_i.h_{zi} + Q_i.(1 - l_{zi} - h_{zi}) + P_i.l_{zi} \leq H; \forall i \in \hat{I}$		
$l_{xi} + l_{zi} \leq 1; \forall\ i \in I$	orientation control	8́
$l_{zi} + h_{zi} \leq 1;\ \forall i \in I$		9́
$l_{zi} - w_{yi} + h_{zi} \leq 1;\ \forall i \in I$		1́0
$l_{zi} - w_{yi} + h_{zi} \geq 0;\ \forall i \in I$		1́1
$1 - l_{xi} - l_{zi} + w_{yi} - h_{zi} \leq 1;\ \forall i \in I$		1́2
$1 - l_{xi} - l_{zi} + w_{yi} - h_{zi} \geq 0;\ \forall i \in I$		1́3
$l_{xi} + l_{zi} - w_{yi} \leq 1\ ; \forall i \in I$		1́4
$l_{xi} + l_{zi} - w_{yi} \geq 0;\ \forall i \in I$		1́5
$P_i \geq {\beta_i}^{ld} - M(1 - t_{id});\ \forall i \in \hat{I}, d \in D^i$	moldable length alternatives	1́6
$P_i \leq {\beta_i}^{ld} + M(1 - t_{id});\ \forall i \in \hat{I}, d \in D^i$		1́7
$Q_i \geq {\beta_i}^{wd} - M(1 - t_{id});\ \forall i \in \hat{I}, d \in D^i$	moldable width	1́8

$Q_i \leq \beta_i{}^{wd} + M(1 - t_{id})$; $\forall i \in \hat{I}, d \in D^i$	alternatives	1́9
$R_i \geq \beta_i{}^{hd} - M(1 - t_{id})$; $\forall i \in \hat{I}, d \in D^i$	moldable height alternatives	2́0
$R_i \leq \beta_i{}^{hd} + M(1 - t_{id})$; $\forall i \in \hat{I}, d \in D^i$		2́1
$\sum_{d=1}^{D^i} t_{id} = 1; \forall i \in \hat{I}$	limiting only one alternative for moldable items	2́2
$t_{id} \in \{0,1\}$; $\forall i \in \hat{I}, d \in D^i$ $l_{xi}, l_{zi}, w_{yi}, h_{zi}, a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik} = 0$ or 1		2́3
$x_i, y_i, z_i, H \geq 0$ $P_i, Q_i, R_i \geq 0$; $\forall$ $i \in \hat{I}$		2́4

The constraints (1́)-(3́) ensure that any two items i and k do not overlap each other. This check for overlap is necessary since a pair of items is placed in the same bin. The variables l_{xi}, l_{zi}, w_{yi}, and h_{zi} are used to calculate the respective mappings of the item i's length, width, and height to the corresponding bin's $X-$, $Y-$ and $Z-$ *axes*. The constraint functions in (1́)-(3́) for moldable items are nonlinear because P_i, Q_i, and R_i are positive variables and l_{xi}, l_{zi}, w_{yi}, and h_{zi} are binary variable. Constraints (4́) limit the relative position of any two items i and k. Constraints (5́)-(7́) keep all items within the bin's dimensions. The constraint functions in (5́)-(7́) for moldable items are nonlinear. Constraints (8́)-(1́5) ensure that the binary variables which determine the items' positions are properly controlled to reflect practical positions. Constraints (1́6)-(1́7) ensure that length of a moldable item takes the discrete alternative values that are provided to the model. Similarly, constraints (1́8)-(1́9) and (2́0)-(2́1) ensure that the width and height of a moldable item uses the discrete alternative values that are provided to the model. Constraints (2́2) ensure that only one of the alternative configurations is selected for each of the moldable items. Constraints (2́3) ensure that the binary variables are properly controlled. Constraints (2́4) ensure that location variables yield positive values.

The 3-D hybrid moldable packing problem with limited discrete configurations for moldable items is formulated as a mixed integer nonlinear programming (MINLP) model. This mathematical model is coded in GAMS modeling language. The Branch-And-Reduce Optimization Navigator (BARON) solver is used to solve the test problems [44, 90]. Having a total of I items, including N moldable items with d different configurations, the model contains $4I^2 + 7I + 6Nd + N + 1$ constraints (1 is added by solver for the objective function) and $3I^2 + 4I + N(3 + d) + 2$ variables. If the objective function is the minimization of bin height, height is one variable, and the solver considers another variable for the objective function equation. Among the variables, there are $3I + 3N + 2$ continuous and $3I^2 + I + Nd$ binary and discrete variables.

3.4 3-D Hybrid Moldable Packing Model with All Continuous Configurations for Moldable Items

This section presents the model for the 3-D hybrid moldable packing problem with continuous configurations. A moldable item is a 3-D item with flexible dimensions and a fixed volume which can be in any shape due to the relocation of its content. Therefore, all the 3-D continuous configurations are available. The problem is how to pack a hybrid mix of rigid and moldable items into a bin. Specifically, for given values for volume and lower and upper bounds for each dimension of moldable items, the following questions are addressed: what are the relative positions of items in the bin; what are the best designs for the moldable items; and what is the packing volume required to pack all of the items?

3.4.1 Assumptions

The volume of each moldable item i is provided to the model and is equal to V_i. Each moldable item dimension can take continuous values within a certain boundary. Similar to section 3.3, set I is comprised of set $\hat{I}$ and $\acute{I}$ including the moldable and rectangular items respectively. Variables P_i, Q_i, and R_i represent the dimensional variables for the moldable items, where $\alpha_i{}^l \leq P_i \leq \beta_i{}^l, \alpha_i{}^w \leq Q_i \leq \beta_i{}^w$, and $\alpha_i{}^h \leq R_i \leq \beta_i{}^h$. Parameter $\alpha_i{}^l$ is a lower bound of length of a moldable item provided for the model. Parameter $\beta_i{}^l$ is an upper bound of length of a moldable item provided for the model. Respectively, $\alpha_i{}^w$ and $\beta_i{}^w$ are the lower and upper bounds for moldable item i's width, and $\alpha_i{}^h$and $\beta_i{}^h$ are the lower and upper bounds for the height of the moldable item i.

3.4.2 Objective Function

The objective function considered for this model is packing volume minimization with only one variable dimension. Accordingly, the objective may be any of the following:

- Minimize *h*
- Minimize *l*
- Minimize *w*

3.4.3 Constraints

The constraints of the 3-D hybrid moldable packing with continuous configurations model are presented below.

Subject to		
$x_i + p_i.l_{xi} + q_i.(l_{zi} - w_{yi} + h_{zi}) + r_i.(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}) \leq x_k + (1 - a_{ik})M \; ; \forall\, i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	1́
$x_i + P_i.l_{xi} + Q_i.(l_{zi} - w_{yi} + h_{zi}) + R_i.(1 - l_{xi} - l_{zi} + w_{yi} -$	overlap control for moldable items	

$h_{zi}\big) \leq x_k + (1 - a_{ik})M \ ; \forall\, i \in \hat{I}, k \in I; i \neq k$		
$y_i + q_i.w_{yi} + p_i.(1 - l_{xi} - l_{zi}) + r_i.\big(l_{xi} + l_{zi} - w_{yi}\big) \leq y_k + (1 - c_{ik})M; \forall i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	2́
$y_i + Q_i.w_{yi} + P_i.(1 - l_{xi} - l_{zi}) + R_i.\big(l_{xi} + l_{zi} - w_{yi}\big) \leq y_k + (1 - c_{ik})M; \forall i \in \hat{I}, k \in I; i \neq k$	overlap control for moldable items	
$z_i + r_i.h_{zi} + q_i.(1 - l_{zi} - h_{zi}) + p_i.l_{zi} \leq z_k + (1 - e_{ik})M; \forall i \in \acute{I}, k \in I; i \neq k$	overlap control for rectangular items	3́
$z_i + R_i.h_{zi} + Q_i.(1 - l_{zi} - h_{zi}) + P_i.l_{zi} \leq z_k + (1 - e_{ik})M; \forall i \in \hat{I}, k \in I; i \neq k$	overlap control for moldable items	
$a_{ik} + a_{ki} + c_{ik} + c_{ki} + e_{ik} + e_{ki} \geq 1;\ i, k \in I; i \neq k$	relative positions of all items	4́
$x_i + p_i.l_{xi} + q_i.\big(l_{zi} - w_{yi} + h_{zi}\big) + r_i.\big(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\big) \leq L; \forall i \in \acute{I}$	bin length constraint	5́
$x_i + P_i.l_{xi} + Q_i.\big(l_{zi} - w_{yi} + h_{zi}\big) + R_i.\big(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}\big) \leq L; \forall i \in \hat{I}$		
$y_i + q_i.w_{yi} + p_i.(1 - l_{xi} - l_{zi}) + r_i.\big(l_{xi} + l_{zi} - w_{yi}\big) \leq W; \forall i \in \acute{I}$	bin width constraint	6́
$y_i + Q_i.w_{yi} + P_i.(1 - l_{xi} - l_{zi}) + R_i.\big(l_{xi} + l_{zi} - w_{yi}\big) \leq W; \forall i \in \hat{I}$		
$z_i + r_i.h_{zi} + q_i.(1 - l_{zi} - h_{zi}) + p_i.l_{zi} \leq H; \forall i \in \acute{I}$	bin height constraint	7́
$z_i + R_i.h_{zi} + Q_i.(1 - l_{zi} - h_{zi}) + P_i.l_{zi} \leq H; \forall i \in \hat{I}$		
$l_{xi} + l_{zi} \leq 1; \forall\, i \in I$	orientation control	8́
$l_{zi} + h_{zi} \leq 1;\ \forall i \in I$		9́
$l_{zi} - w_{yi} + h_{zi} \leq 1;\ \forall i \in I$		10́
$l_{zi} - w_{yi} + h_{zi} \geq 0;\ \forall i \in I$		11́
$1 - l_{xi} - l_{zi} + w_{yi} - h_{zi} \leq 1;\ \forall i \in I$		12́
$1 - l_{xi} - l_{zi} + w_{yi} - h_{zi} \geq 0;\ \forall i \in I$		13́
$l_{xi} + l_{zi} - w_{yi} \leq 1\ ; \forall i \in I$		14́
$l_{xi} + l_{zi} - w_{yi} \geq 0;\ \forall i \in I$		15́
${\alpha_i}^l \leq P_i \leq {\beta_i}^l;\ \forall\, i \in \hat{I}$	moldable items length lower and upper bound	16́
${\alpha_i}^w \leq Q_i \leq {\beta_i}^w; \forall\, i \in \hat{I}$	moldable items width lower and upper bound	17́
${\alpha_i}^h \leq R_i \leq {\beta_i}^h;\ \forall\, i \in \hat{I}$	moldable items height lower and upper bound	18́
$P_i.Q_i.R_i = V_i;\ \forall\, i \in \hat{I}$	moldable volume	19́
$l_{xi}, l_{zi}, w_{yi}, h_{zi}, a_{ik}\ , b_{ik}\ , c_{ik}\ , d_{ik}\ , e_{ik}\ , f_{ik} = 0 \ or\ 1$		20́
$x_i, y_i, z_i \geq 0;\ \forall\, i \in \hat{I}$		21́

$P_i, Q_i, R_i \geq 0;\ \forall\ i \in \hat{I}$	2́2

Constraints (1́)-(1́5), (2́0), and (2́1) are similar to the previous model with only one difference that P_i, Q_i, and R_i in this model are continues variables with lower and upper bounds; in the previous model, they are continuous variables, but they can only yield any of the given discrete values. Constraints (1́6)-(1́8) ensure that the moldable items' dimensions are within the specified boundaries. Constraints (1́9) ensure that the moldable items' volumes remain fixed. Constraints (2́2) ensure that the moldable items' dimensions yield positive values.

Constraints (1́6)-(1́8) can be replaced with the following constraints in the model:

$\mathrm{P_i} \leq \beta_\mathrm{i}{}^{\mathrm{l}};\ \forall\ i \in \hat{I}$	1́6
$\mathrm{P_i} \geq \alpha_\mathrm{i}{}^{\mathrm{l}};\ \forall\ i \in \hat{I}$	
$\mathrm{Q_i} \leq \beta_\mathrm{i}{}^{\mathrm{w}};\ \forall\ i \in \hat{I}$	1́7
$\mathrm{Q_i} \geq \alpha_\mathrm{i}{}^{\mathrm{w}};\ \forall\ i \in \hat{I}$	
$R_i \leq \beta_i{}^h;\ \forall\ i \in \hat{I}$	1́8
$R_i \geq \alpha_i{}^h;\ \forall\ i \in \hat{I}$	

The 3-D hybrid moldable packing problem with all continuous configurations for moldable items is formulated as a Mixed-Integer Nonlinear Programming (MINLP) model. Similar to the previous model, this model is coded in GAMS language, and BARON solver is used to solve the problem instances [44, 90]. Having a total of I items including N moldable items, with lower and upper bounds defined for each dimension and a fixed volume, the model contains $4I^2 + 7\ (I + N)$ constraints and $3I^2 + 4I + 3N + 2$ variables. If the objective function is the minimization of the bin height, height is one variable, and the solver considers another variable for the objective function equation. Among the variables, there are $3(I + N) + 2$ continuous and $3I^2 + I$ binary and discrete variables.

CHAPTER IV
Experiments and Validation

This chapter aims to test the application of the developed mathematical models for solving 3-D bin packing problems with a hybrid mix of rigid and moldable items. To achieve this goal, the developed mathematical models are applied to the MRE packing problems on two levels: packing MRE food items in the MRE menu bags and packing MRE menu bags in the boxes. Each of these problems is addressed with the limited discrete and all continuous moldable configurations approaches presented in sections 3.3 and 3.4, respectively. These results are compared with the condition in which all the items are considered as rigid.

In the following discussions, section 4.1 presents the results of solving the problem of packing MRE food items in the meal bags. Section 4.1.1 presents the method of collecting MRE data. Section 4.1.2 presents the solution methodology. Section 4.1.3 explains the results of applying the 3-D hybrid moldable packing model with discrete configurations to the MRE food items packing problem. Section 4.1.4 presents the results of applying the 3-D hybrid moldable packing model with continuous configurations to the MRE food item packing problem. Section 4.1.5 compares the results of hybrid moldable packing of MRE food items in the menu bags with the condition in which only rigid packing is considered.

Section 4.2 illustrates the results of solving the problem of packing MRE menu bags in the boxes. Accordingly, section 4.2.1 explains the results of applying the hybrid moldable packing model with discrete configurations to the problem of packing MRE menu bags in the boxes. Section 4.2.2 presents the results of applying the hybrid moldable packing model with continuous configurations to the problem of packing MRE menu bags in the boxes. Section 4.2.3 compares the results of the hybrid moldable packing of MRE menu bags in the boxes with the condition that only rigid configurations are present. Section 4.3 presents the results of validation. Section 4.4 shows how the visualization of the packing process of MREs in menu bags and in boxes can translate the mathematical findings for training purposes.

4.1 Packing MRE Food Items in the Menu Bags

Each MRE menu bag includes a hybrid mix of rigid and moldable food items. Thus, both of the discrete and continuous approaches for moldable packing problems must be used to solve the MRE food items packing problems.

4.1.1 MREs Data Collection

The MRE samples are provided to the department of Industrial and Systems Engineering at the University of Tennessee for the purpose of solving MRE packing problems in the following format:

- MRE menu samples are provided from three assemblers. In these samples, menu items and menu bags from different assemblers are subject to variation.
- Box A includes menu bags 1-12. Box B includes menu bags 13-24. The list of food items are provided in appendix C.

Dimensional information about MRE food items in each menu bag are collected with a caliper and a ruler. All the items are categorized as rigid or moldable based on their content. Any food items with liquid, powder, or flexible contents are considered to be moldable.

4.1.2 Solution Methodology

As discussed in section 3.3.4, the mathematical formulation for the hybrid moldable packing model with discrete configurations is in the form of MINLP. Thus, BARON solver is used to solve the test problems [44]. The solver was installed on a server with 2.3 GHz Intel ® Core i5-

2410M CPU processor and 6 GB of RAM. In the experiments that follow, the computational time spent to solve each model was limited to 2000 seconds (sec.) and the optimality (relative) gaps were computed as follows:

$$Gap = \frac{(best\ bound\ obtained - best\ solution\ obtained)}{(best\ bound\ obtained\ )} \times 100\%$$

The relative gap is set to 0.0 percent. Absolute gap is the upper bound for the distance between the best integer and the optimal solution. Therefore, four possible cases, with respect to the quality of the solution obtained by BARON, can occur: (1) an optimal solution with a gap equal to zero; (2) integer solution, with a gap greater than 0.0 percent (with BARON exceeding the 2000 sec. time limit); (3) no solution, without a gap and with BARON exceeding the time limit; and (4) an insufficient computer memory to compile the model in GAMS, yielding no gap and no relevant information concerning the computational time. If the last two cases occur, they are represented by the symbols ‘‘-’’ in the tables.

4.1.3 Solving MRE Food Items Packing Problem with the Hybrid Moldable Packing Model with Limited Discrete Configurations

In this section, the 3-D hybrid moldable packing model with discrete configurations is applied to solve the problem of packing MRE food items in the menu bags. In the following description, the method of collecting MRE data and running the computational experiments, an example of the problem, and the subsequent results are presented.

4.1.3.1 Assumptions

The objective function is to reduce the packing volume with only one variable dimension. The following assumptions are considered in generating the test problems:

- Each test problem is comprised of a set of MRE food items to be packed in a menu bag belonging to the box A or B. Thus, 24 test problems are conducted to pack all the food items in menu bags 1-24.
- For the objective of minimizing the heights of various menu bags, the lengths and widths of each menu bag are provided for the model. The lengths and widths of the menu bags are considered equal to the length and width values, respectively, of the largest food item. Thus, in the experiments, the lengths of menu bags vary between 7 to 9 inches, and the width values vary between 4 to 6 inches.
- For the objective of minimizing the lengths of various menu bags, the heights and widths of each menu bag are provided for the model. The heights and widths of the menu bags are considered equal to the height and width values, respectively, of the largest food item. Thus, in the experiments, the height values vary between 0.5 to 2 inches, and the width values vary between 4 to 6 inches.
- For the objective of minimizing the widths of various menu bags, the lengths and heights of each menu bag are provided for the model. The lengths and heights of the menu bags are considered equal to the length and height values, respectively, of the largest food item. Thus, in the experiments, the length values vary between 7 to 9 inches and height values vary between 0.5 to 2 inches.
- In order to recognize the configurations of moldable items, they are placed in various positions, and the resulting dimensions are collected.

4.1.3.2 Example

An example of solving the problem of packing MRE food items in a menu bag with variable height is presented below. Table 2 shows that 7 items are selected to be packed in a menu bag including 2 moldable items and 5 rigid items. For moldable items, three configurations are recognized. These configurations' dimensions are presented in Table 2. The length of the menu bag is set to 8.25 inches and the width of the menu bag is set to 4.9 inches. The objective function value is 3.69 inches which results in 149.17 cubic inches packing volume to pack all the food items. The total volume of all the items is 114 cubic inches. Thus, the unutilized space is 35.17 cubic inches (149.17-114=35.17).

Table 2: List of Food Items and Dimensions

#	Food Items	L1	W1	H1	L2	W2	H2	L3	W3	H3
1	Cheese Spread (moldable)	5	2	0.25	4	2.1	0.3	3.75	1.25	0.5
2	Dairy shake Powder (moldable)	7	4.2	0.3	6	4.1	0.4	5	4	0.5
3	Mexican Style Corn	7.4375	4.815	0.68						
4	Accessories	6.25	4.4	0.65						
5	Tortillas	8.125	4.9	0.36						
6	Skittles	6.25	2.6	0.9						
7	Chili With Beans	8.25	4.81	0.8						

The results are presented in Table 3 and Table 4 including all the variables and their values. Variable x, y, and z illustrate the front-left bottom (FLB) corner of each item. The FLB corner of item 6 is at the origin. Variables P, Q, and R present the dimensional values selected for the moldable items. Accordingly, binary variables t_{id} (for *i*=1 and 2; *d*=1, 2, and 3) show the configuration which is selected for each moldable item. Binary variables *lx*, *lz*, *wy*, and *hz* show the orientation of each item. The total volume of items is 114 cubic inches. Table 4 illustrates the binary variables, indicating the placement of items *i* and *k* (*i.k*) relative to each other such that i, k∈I.

Table 3: Results of Solving the Hybrid Moldable Packing Problem

Item (i)	**x**	**y**	**z**	**P**	**Q**	**R**	**Volume**	**Lx**	**Lz**	**Wy**	**Hz**	**ti1(tid, d=1)**	**ti2(tid, d=2)**	**ti3(tid, d=3)**
1	6.25	0	2.06	3.75	1.25	0.5	2.3	0	0	0	1	0	0	1
2	0	0	2.71	7	4.2	0.3	8.8	1	0	1	1	1	0	0
3	0	0	3.01				24.4	1	0	1	1			
4	0	0	2.06				17.9	1	0	1	1			
5	0	0	1.7				14.3	1	0	1	1			
6	0	0	0				14.6	1	0	1	1			
7	0	0	0.9				31.7	1	0	1	1			
Total volume of the items							114							

Table 4: Results of Solving the Hybrid Moldable Packing Problem – Continued

Items (*i.k*)	**a**	**c**	**e**	**Items (*i.k*)**	**a**	**c**	**e**	**Items (*i.k*)**	**a**	**c**	**e**	**Items (*i.k*)**	**a**	**c**	**e**
1.2	0	0	1	3.1	0	0	0	5.1	0	0	1	7.1	0	0	1
1.3	0	0	1	3.2	0	0	0	5.2	0	0	1	7.2	0	0	1
1.4	0	0	0	3.4	0	0	0	5.3	0	0	1	7.3	0	0	1
1.5	0	0	0	3.5	0	0	0	5.4	0	0	1	7.4	0	0	1
1.6	0	0	0	3.6	0	0	0	5.6	0	0	0	7.5	0	0	1
1.7	0	0	0	3.7	0	0	0	5.7	0	0	0	7.6	0	0	0
2.1	0	0	0	4.1	1	0	0	6.1	1	0	0				
2.3	0	0	1	4.2	0	0	1	6.2	0	0	1				
2.4	0	0	0	4.3	0	0	1	6.3	0	0	1				
2.5	0	0	0	4.5	0	0	0	6.4	0	0	1				
2.6	0	0	0	4.6	0	0	0	6.5	0	0	1				
2.7	0	0	0	4.7	0	0	0	6.7	0	0	1				

4.1.3.3 Results

Table 5 presents the results of packing MRE food items in menu bags 1 to 24 with the objective of variable menu bag heights. Table 5 includes four sections: the MRE test problem specifications, results, computational outputs, and percent of unused space. The MRE test problem specifications include the box numbers, menu bag numbers, total number of food items in the menu bags, number of moldable food items in the menu bags, number of rigid food items in the menu bags, and the menu bags' fixed values for lengths and widths.

The results section in the table 5 presents the optimum height of the menu bag and the required volume to pack all the food items. The computational output shows the running time in seconds, the number of equations, variables, and discrete variables, relative gap, and solution status. The percent of unused space is the percent difference between the optimum volume of the menu bag and the total volume of existing food items.

The required volume to pack the MRE food items in the menu bags with the varying heights is between 83.27 to 138.96 cubic inches. The average volume that is used to pack current samples of MRE menu bags is 220 cubic inches.

The average unused space in a MRE menu bag is 19.6 percent. In 13 menu bags, the percent of unused space is less than 20 percent, and in the remaining 11 menu bags, it is more than 20 percent. The unused space is always less than 30 percent.

Table 5: Computation Results of Discrete Moldable Packing of MRE Food Items in the Menu Bags with Variable Menu Bag Height

Test Problems	MRE Test Problem Specifications								Discrete Packing Results		Computational Outputs						Percent of Unused Space
	Box	Menu Bag Number	Total Number of Food items in the Menu Bag	Number of Moldable Food items in the Menu Bag	Number of Rigid Food items in the Menu Bag	Number of Configurations for Moldable Items	Bag Length	Bag Width	Objective Function Height Min	Total Packing Volume	Computation in Sec.	# Equations	#Variables	# Discrete Variables	Relative Gap	Solution Status	
1	A	1	7	2	5	3	8.25	4.9	**2.94**	**118.85**	31	284	189	160	0	Opt	24.3%
2	A	2	9	4	5	3	7.25	4.9	**3.25**	**115.46**	150	351	238	206	0	Opt	15.1%
3	A	3	9	3	6	3	8	4.75	**3.00**	**114.00**	1800	445	299	261	0	Opt	28.1%
4	A	4	9	2	7	3	7.25	4.9	**3.25**	**115.46**	166	426	293	258	0	Opt	14.3%
5	A	5	9	2	7	3	8.25	4.9	**3.25**	**131.38**	1523	426	293	258	0	Opt	22.4%
6	A	6	7	2	5	3	8.25	4.9	**2.75**	**111.17**	200	284	189	160	0	Opt	19.0%
7	A	7	10	4	6	3	8	4.9	**3.25**	**127.40**	619	547	366	322	0	Opt	19.9%
8	A	8	8	3	5	3	8.25	4.75	**2.75**	**107.77**	340	370	244	209	0	Opt	12.8%
9	A	9	7	2	5	3	8.25	4.9	**3.00**	**121.28**	85	284	189	160	0	Opt	25.8%
10	A	10	8	2	6	3	8.25	4.9	**3.00**	**121.28**	9	351	238	206	0	Opt	21.7%
11	A	11	10	3	7	3	8.25	4.9	**3.00**	**121.28**	217	528	360	319	0	Opt	15.1%
12	A	12	8	3	5	3	8.25	4.9	**3.44**	**138.96**	105	370	244	209	0	Opt	22.3%
13	B	13	9	3	6	3	8.25	4.9	**2.75**	**111.17**	9	445	299	261	0	Opt	22.6%
14	B	14	8	3	5	3	8.18	4.75	**3.05**	**118.51**	170	370	244	209	0	Opt	18.1%
15	B	15	8	3	5	3	8.25	4.75	**2.88**	**112.66**	97	370	244	209	0	Opt	18.3%
16	B	16	7	2	5	3	8.25	4.75	**2.13**	**83.27**	128	284	189	160	0	Opt	15.9%

Table 5. Continued.

Test Problems	MRE Test Problem Specifications								Discrete Packing Results		Computational Outputs						Percent of Unused Space
	Box	Menu Bag Number	Total Number of Food items in the Menu Bag	Number of Moldable Food items in the Menu Bag	Number of Rigid Food items in the Menu Bag	Number of Configurations for Moldable Items	Bag Length	Bag Width	Objective Function Height Min	Total Packing Volume	Computation in Sec.	# Equations	#Variables	# Discrete Variables	Relative Gap	Solution Status	
17	B	17	7	2	5	3	8.25	4.75	**2.56**	**100.42**	171	284	189	160	0	Opt	14.4%
18	B	18	8	3	5	3	8.18	4.75	**3.50**	**135.99**	899	370	244	209	0	Opt	20.6%
19	B	19	8	4	4	3	8.25	4.75	**3.25**	**127.36**	12	389	250	212	0	Opt	23.1%
20	B	20	7	3	4	3	8.25	4.75	**3.00**	**117.56**	54	303	195	163	0	Opt	23.4%
21	B	21	8	4	4	3	8	4.75	**2.40**	**91.20**	30	389	250	212	0	Opt	25.4%
22	B	22	8	3	5	3	8	4.9	**3.19**	**124.95**	235	370	244	209	0	Opt	15.2%
23	B	23	9	3	6	3	8.25	4.9	**3.00**	**121.28**	94	445	299	261	0	Opt	15.9%
24	B	24	8	4	4	3	8.18	4.75	**3.34**	**129.68**	171	389	250	212	0	Opt	16.7%

Table 6 presents a summary of numerical results of packing MRE food items in the menu bags with the objective of packing volume minimization with a variable menu bag length. Various test problems are conducted to find the minimum volume to pack MRE food items in the menu bags with variable lengths. The height and width of the menu bags are fixed inputs to the models. In test problem 1 for packing food items in menu 1, the width of the menu bag is set to 3, and the height is set to 0.5. The problem is infeasible because the heights of the items in the bag are larger than 0.5. In test problem 2, the height is increased from 0.5 to 3 inches, and the width is increase to 4. The optimum objective function is 13.15, and the total required volume to pack all the items is 157.80. In the results presented in table 5 with variable height objective, the required space is 118.85. The reason for the increase in the total space requirement is the width input. The widest item in menu bag 1 is 4.9 inches. Therefore, this item cannot fit in a menu bag with a width of 4 inches. If the input width of the menu bag is increased to 4.9 inches, the length of menu bag decreases to 8 inches which is the length of the longest item. The volume required to pack all the items in test problem 3 is 121.28.

Similarly, test problems 4, 7, 11, and 16 in Table 6 illustrate that if the menu bag height input is smaller than the largest height value among the food items and if the width value is smaller than the largest width value among the food items, the problem is infeasible. The input setting does not fit any of the large items.

Accordingly, if the value of the menu bag height is set to the largest height value among the food items and if the value of width is set to the largest item’s width among the food items, the menu bag yields a narrow, long configuration. Some of the items are packed next to each other increasing the menu bag length value. This design is not applicable to the current MRE packing problem because it requires a change in the current menu bags’ design.

Table 6: Computation Results of Discrete Moldable Packing of MRE Food Items in the Menu Bags with Variable Menu Bag Length

Test Problems	MRE Test Problem Specifications								Results		Computational Outputs					
	Box	Menu Bag Number	# of Food items in the Menu Bag	# of Moldable Food items in the Menu Bag	# of Rigid Food items in the Menu Bag	Number of Configurations for Moldable Items	Fixed Menu Bag Height	Fixed Menu Bag Width	Objective Function Length Min	Total Packing Volume	Computation in Sec.	# Equations	#Variables	# Discrete Variables	Relative Gap	Model Status
1	A	1	7	2	5	3	0.5	3	-	**-**	-	284	189	160	-	Inf
2	A	1	7	2	5	3	3	4	13.15	**157.80**	16	284	189	160	0	Opt
3	A	1	7	2	5	3	3	4.9	8	**121.28**	35	284	189	160	0	Opt
4	A	2	8	2	6	3	0.5	3	-	**-**	-	351	238	206	-	Inf
5	A	2	8	2	6	3	4	5	14	**280.00**	66	351	238	206	0	Opt
6	A	2	8	2	6	3	3.8	5.5	8.25	**151.53**	71	351	238	206	0	Opt
7	A	3	9	3	6	3	0.5	4	-	**-**	-	445	299	261	-	Inf
8	A	3	9	3	6	3	6	6	8.25	**288.00**	70	445	299	261	0	Opt
9	A	3	9	3	6	3	6	4.75	8.25	**228.00**	95	445	299	261	0	Opt
10	A	3	9	3	6	3	3	4.75	8.25	**114.00**	665	445	299	261	0	Opt
11	A	4	9	2	7	3	0.5	4	-	**-**	-	426	293	258	-	Inf
12	A	4	9	2	7	3	5	5	8.25	**181.25**	88	426	293	258	0	Opt
13	A	4	9	2	7	3	4.9	5	8.25	**177.63**	110	426	293	258	0	Opt
14	A	4	9	2	7	3	5	4.9	8.18	**177.63**	150	426	293	258	0	Opt
15	A	4	9	2	7	3	4.9	3.25	8.25	**115.46**	700	426	293	258	0	Opt
16	A	5	9	2	7	3	0.5	3	-	**-**	-	426	293	258	-	Inf
17	A	5	9	2	7	3	4	5	8.25	**165.00**	233	426	293	258	0	Opt
18	A	5	9	2	7	3	3.25	4.9	8.18	**131.38125**	1133	426	293	258	0	Opt

Test problems 10, 18, and 15 clarify that if the optimum results from the variable height objective function are used as inputs to this problem, the same outputs are achieved. Test problems 13 and 14 show that replacing the height and width values results in the same objective function value. This is because this change allows room for rotation. Comparable results hold for other menu items.

Based on the results, the height objective is able to find better solutions in short time while many problem instances to reach the same results for the length objective are required. The majority of the menu bags' designs with the length objective results in a narrow, long configuration that is not applicable with the existing menu bags' dimensions. However, such designs can be used to find a better layout of menu bags in the boxes with less packing volume.

With the length objective, the menu bag's length is at least equal to the food item with the longest length. This manual adjusting of the inputs (height and width) are required to reach the minimum packing volume.

Various test problems are conducted to find the minimum volume to pack MRE food items in the menu bags with variable widths. The results of conducted test problems are presented in Table 7. Test problem 1 shows that if the fixed height of the menu bag is smaller than the largest height value among of the food items' height, the problem is infeasible. When the height and length of the menu bags are set to the largest values of the items' height and length, the resulting value for the menu bag width is not optimum. The resulting menu bag is long and narrow. If the heights of the menu bags are set to the width value of the widest items, then the problem performs similarly to the height minimization objective because of allowing rotation. In this instance, the menu bag's width and height are replaced with each other. The quality of solution is equal with the height objective.

Table 7: Computation Results of Discrete Moldable Packing of MRE Food Items in the Menu Bags with Variable Menu Bag Width

Test Problems	MRE Test Problem Specifications								Results		Computational Outputs					
	Box	Menu Bag Number	Total Number of Food items in the Menu Bag	Number of Moldable Food items in the Menu Bag	Number of Rigid Food items in the Menu Bag	Number of Configurations for Moldable Items	Bag Length	Bag Height	Objective Function Width Min	Total Packing Volume	Computation in Sec.	# Equations	#Variables	# Discrete Variables	Relative Gap	Solution Status
1	A	1	7	2	5	3	8.25	0.5	-	-	-	284	189	160	-	Inf
2	A	1	7	2	5	3	8.25	2.94	4.9	**118.85**	368	284	189	160	0	Opt
3	A	1	7	2	5	3	8.25	4.9	2.94	**118.85**	46	284	189	160	0	Opt
4	A	2	9	4	5	3	7.25	4.9	3.25	**115.46**	120	351	238	206	0	Opt
5	A	3	9	3	6	3	8	4.75	3.00	**114.00**	170	445	299	261	0	Opt
6	A	4	9	2	7	3	7.25	4.9	3.25	**115.46**	166	426	293	258	0	Opt
7	A	5	9	2	7	3	8.25	4.9	3.25	**131.38**	231	426	293	258	0	Opt
8	A	6	7	2	5	3	8.25	4.9	2.75	**111.17**	50	284	189	160	0	Opt
9	A	7	10	4	6	3	8	4.9	3.25	**127.40**	238	547	366	322	0	Opt
10	A	8	8	3	5	3	8.25	4.75	2.75	**107.77**	121	370	244	209	0	Opt
11	A	9	7	2	5	3	8.25	4.9	3.00	**121.28**	98	284	189	160	0	Opt
12	A	10	8	2	6	3	8.25	4.9	3.00	**121.28**	147	351	238	206	0	Opt
13	A	11	10	3	7	3	8.25	4.9	3.00	**121.28**	390	528	360	319	0	Opt
14	A	12	8	3	5	3	8.25	4.9	3.44	**138.96**	109	370	244	209	0	Opt
15	B	13	9	3	6	3	8.25	4.9	2.75	**111.17**	23	445	299	261	0	Opt
16	B	14	8	3	5	3	8.18	4.75	3.05	**118.51**	111	370	244	209	0	Opt
17	B	15	8	3	5	3	8.25	4.75	2.88	**112.66**	99	370	244	209	0	Opt
18	B	16	7	2	5	3	8.25	4.75	2.13	**83.27**	231	284	189	160	0	Opt

Table 7. Continued.

Test Problems	MRE Test Problem Specifications								Results		Computational Outputs					
	Box	Menu Bag Number	Total Number of Food items in the Menu Bag	Number of Moldable Food items in the Menu Bag	Number of Rigid Food items in the Menu Bag	Number of Configurations for Moldable Items	Bag Length	Bag Height	Objective Function Width Min	Total Packing Volume	Computation in Sec.	# Equations	#Variables	# Discrete Variables	Relative Gap	Solution Status
19	B	17	7	2	5	3	8.25	4.75	2.56	**100.42**	204	284	189	160	0	Opt
20	B	18	8	3	5	3	8.18	4.75	3.50	**135.99**	432	370	244	209	0	Opt
21	B	19	8	4	4	3	8.25	4.75	3.25	**127.36**	440	389	250	212	0	Opt
22	B	20	7	3	4	3	8.25	4.75	3.00	**117.56**	87	303	195	163	0	Opt
23	B	21	8	4	4	3	8	4.75	2.40	**91.20**	190	389	250	212	0	Opt
24	B	22	8	3	5	3	8	4.9	3.19	**124.95**	201	370	244	209	0	Opt
25	B	23	9	3	6	3	8.25	4.9	3.00	**121.28**	365	445	299	261	0	Opt
26	B	24	8	4	4	3	8.18	4.75	3.34	**129.68**	298	389	250	212	0	Opt

Test problem results reveal that the height objective provides the optimum solution in a timely manner and results of bin height can be used as inputs in length and width objectives.

4.1.4 Solving MRE Food Items Packing Problem with Hybrid Moldable Packing Model with Continuous Dimensions

In order to test this model, various test problems are generated. As discussed in section 4.1.3.3, the best objective function is the variable menu bag height. While rigid items' dimensions are based on the MRE food items, approximate volume and upper and lower bounds for moldable items are considered. In running this set of test problems, boundaries on the variables are applied. The gap is set to 0, and the time is limited to 2000 seconds.

4.1.4.1 Assumptions

The objective function is to reduce the packing volume with one variable dimension. The following assumptions are considered in generating the test problems:

- Each test problem is comprised of a set of MRE food items to be packed in a menu bag belonging to the box A or B. Thus, 24 test problems are conducted to pack all the food items in menu bags 1-24.
- As discussed in section 4.1.3.3, the best objective function is the variable menu bag height. Also, with the best settings, all three objective functions (length, width, and height) reach the same results. Hence, only the objective of minimizing the menu bag's height is considered.
- Upper and lower values for moldable items' dimensions are based on the largest and smallest discrete recognized configurations dimensions.

- The volumes for moldable items are randomly generated between the maximum and minimum volumes of configurations which are considered as inputs to the discrete model.

4.1.4.2 Results

Table 8 illustrates the results of running 24 sets of test problems for solving the problem of packing hybrid MRE food items in the menu bags when continuous configurations for moldable items are considered. The number of equations, total variables, and binary variables are reported in Table 8. The unused space is presented in the last column of Table 8.

The optimum height value for menu bag 1 is reduced from 2.94 in discrete moldable model to 2.60 in the continuous model. This results in a 11.56 percent reduction in the packing volume. This test problem contains 260 equations (constraints), 183 variables, and 154 binary variables. The solver took 95 second to find this problem's optimum level. The unused space in menu bag 1 is 14.4 percent. The unused space in menu bag 1 when using discrete configurations for the moldable items is 24.3 percent. The space utilization in menu bag 1 improves more than 40 percent.

The optimum height value for menu bag 2 reduces from 3.25 in the discrete moldable model to 3 in the continuous model. This test problem contains 327 equations (constraints), 232 variables, and 200 binary variables. The solver took 34 second to find this problem's optimum level. The unused space in menu bag 2 is 8.0 percent. The unused space in menu bag 2 when using discrete configurations for the moldable items is 15.1 percent. The space utilization in menu bag 2 improves more than 45 percent.

Table 8: Results of MRE Food Items Moldable Packing with Discrete Packing

MRE Test Problem Specifications								Continuous Packing Results		Computational Outputs						Percent of Unused Space
Test Prob.	Box	Menu Bag #	Total # of Food items	# Moldable Food items	# Rigid Food items in the Menu Bag	Bag Length	Bag Width	Objective Function Height Min.	Total Packing Volume	Computational time in Sec.	Equations	Variables	Discrete Variables	Relative Gap	Solution Status	
1	A	1	7	2	5	8.25	4.9	2.60	105.11	95	260	183	154	0	Opt	14.4%
2	A	2	8	2	6	7.25	4.9	3.00	106.58	34	327	232	200	0	Opt	8.0%
3	A	3	9	3	6	8	4.75	2.25	85.50	120	409	290	252	0	Opt	4.1%
4	A	4	9	2	7	7.25	4.9	2.90	103.02	195	402	287	252	0	Opt	3.9%
5	A	5	10	3	7	8.25	4.9	3.00	121.28	249	522	351	310	0	Opt	15.9%
6	A	6	7	2	5	8.25	4.9	2.50	101.06	10	260	183	154	0	Opt	10.9%
7	A	7	10	4	6	8	4.9	2.75	107.80	515	499	354	310	0	Opt	5.4%
8	A	8	8	3	5	8.25	4.75	2.50	97.97	133	334	235	200	0	Opt	4.1%
9	A	9	7	2	5	8.25	4.9	2.50	101.06	11	260	183	154	0	Opt	10.9%
10	A	10	8	2	6	8.25	4.9	2.50	101.06	50	327	232	200	0	Opt	6.0%
11	A	11	10	3	7	8.25	4.9	2.60	105.11	515	492	351	310	0	Opt	2.0%
12	A	12	8	3	5	8.25	4.9	2.90	117.23	73	334	235	200	0	Opt	7.9%
13	B	13	9	3	6	8.25	4.9	2.25	90.96	51	409	290	252	0	Opt	5.4%
14	B	14	8	3	5	8.18	4.75	2.80	108.79	57	334	235	200	0	Opt	10.8%
15	B	15	8	3	5	8.25	4.75	2.63	102.87	98	334	235	200	0	Opt	10.6%
16	B	16	7	2	5	8.25	4.75	2.00	78.38	19	260	183	154	0	Opt	10.7%
17	B	17	7	2	5	8.25	4.75	2.50	97.97	17	260	183	154	0	Opt	12.2%

Table 8. Continued.

MRE Test Problem Specifications								Continuous Packing Results		Computational Outputs						Percent of Unused Space
Test Prob.	Box	Menu Bag #	Total # of Food items	# Moldable Food items	# Rigid Food items in the Menu Bag	Bag Length	Bag Width	Objective Function Height Min.	Total Packing Volume	Computational time in Sec.	Equations	Variables	Discrete Variables	Relative Gap	Solution Status	
18	B	18	8	3	5	8.18	4.75	3.25	126.28	25	341	238	200	0	Opt	14.5%
19	B	19	8	4	4	8.25	4.75	3.00	117.56	41	341	238	200	0	Opt	16.6%
20	B	20	7	3	4	8.25	4.75	2.50	97.97	16	267	186	154	0	Opt	8.1%
21	B	21	8	4	4	8	4.75	2.00	76.00	129	341	238	200	0	Opt	10.5%
22	B	22	8	3	5	8	4.9	2.88	112.70	203	334	235	200	0	Opt	5.9%
23	B	23	9	3	6	8.25	4.9	2.75	111.17	517	409	290	252	0	Opt	8.2%
24	B	24	8	4	4	8.18	4.75	3.00	116.57	150	341	238	200	0	Opt	7.3%

The optimum height value for menu bag 3 reduces from 3 in the discrete moldable model to 2.25 in the continuous model. This test problem contains 409 equations (constraints), 290 variables, and 252 binary variables. The solver took 120 second to find this problem's optimum level. The unused space in menu bag 3 is 4.1 percent. The unused space in menu bag 3 when using discrete configurations for the moldable items is 28.1 percent, which shows a significant improvement in space utilization.

Similar improvements in the optimum height and space utilization value for menu bags 4-24 are observed. The average unused space reduces from19.6 percent to 8.9 percent. The solver is able to reach the optimum solution for all the problem instances in less than 520 seconds.

The results reveal that the packing volume necessary to pack MRE food items in the menu bags significantly reduces. The resulting unused space in the menu bags when using the continuous configurations for moldable items model is less than the resulting unused space when using the discrete configuration for moldable items model; the continuous moldable modeling takes advantage of the flexibility in moldable items to fill the empty spaces, thus reducing the required packing volume.

The results of the test problems of the limited discrete configurations model for packing MRE food items in menu bags 1 through 24 can be used to improve current packing of menu bags. The continuous configurations model test problem results can be used when MREs are modified and/or when new moldable MRE items are introduced.

4.1.5 Comparison of Results for MRE Food Items with the Hybrid Moldable Packing with the Rigid Packing

In this section, the results for the hybrid moldable packing with continuous configurations and the hybrid moldable packing with discrete configurations for moldable items are compared with the condition in which only one configuration for moldable items are considered. Thus, all the food items are considered with rigid dimensions.

4.1.5.1 Assumptions

- One rigid rectangular configuration is considered for each of the items including the moldable items. All the equations related to the moldable items are removed. The discrete model is reformulated as a mixed integer linear programming model [6]. The model is coded in GAMS modeling language, and the CPLEX solver is used to solve the problem.
- The solver was installed on a server with 2.3 GHz Intel ® Core i5-2410M CPU processor and 6 GB of RAM. Other assumptions are similar to section 4.1.2.

4.1.5.2 Results

Table 9 presents the results of comparison between all the considered scenarios for menu bags 1-12. The results are compared in terms of total packing volume for each menu bag, the percentage of packing volume reduction based on each modeling assumption, and the percentage of unutilized space in each menu bag. For menu bags 1-12, the total required space for all the menu bags, considering all the food items as rigid, is 1614.74 cubic inches. Total required space for all the menu bags containing hybrid rigid and moldable food items with discrete configurations for moldable items is 1444.26.

Table 9: Comparison of Packing Volume Results for MRE Food Items Moldable Packing Solutions-Menu Bags 1-12

Test Problems Specifications			**Tested Models**			**Percent Improvement in the Packing Volume Results (%)**						**Percent Unused Space**		
Test Prob.	**MRE Box**	**Menu**	**Min- Packing Volume With Rigid Modeling**	**Min- Packing Volume With Discrete Moldable Modeling**	**Min- Packing Volume With Continuous Moldable modeling**	**Rigid Model Compared with Average Space (131.16) for MRE Menu Bags**	**Discrete Model Compared with Rigid**	**Discrete Model Compared with Average Space for MRE Menu Bags**	**Moldable Model Compared with Discrete**	**Moldable Model Compared with Rigid**	**Moldable Model Compared with Average Space for MRE Menu Bags**	**Unused Space in Rigid Model**	**Unused Space in Discrete Moldable Model**	**Unused Space in Continuous Moldable model**
1	A	1	118.85	118.85	105.11	9.38	0.00	9.38	11.56	11.56	19.86	24.3%	24.3%	14.4%
2	A	2	124.34	115.46	106.58	5.20	7.14	11.97	7.69	14.29	18.74	21.2%	15.1%	8.0%
3	A	3	123.50	114.00	85.50	5.84	7.69	13.08	25.00	30.77	34.81	33.6%	28.1%	4.1%
4	A	4	115.46	115.46	103.02	11.97	0.00	11.97	10.77	10.77	21.45	14.3%	14.3%	3.9%
5	A	5	141.49	131.38	121.28	-7.88	7.14	-0.17	7.69	14.29	7.54	27.9%	22.4%	15.9%
6	A	6	121.28	111.17	101.06	7.54	8.33	15.24	9.09	16.67	22.95	25.8%	19.0%	10.9%
7	A	7	147.00	127.40	107.80	-12.08	13.33	2.87	15.38	26.67	17.81	30.6%	19.9%	5.4%
8	A	8	137.16	107.77	97.97	-4.57	21.43	17.84	9.09	28.57	25.30	31.5%	12.8%	4.1%
9	A	9	131.38	121.28	101.06	-0.17	7.69	7.54	16.67	23.08	22.95	31.5%	25.8%	10.9%
10	A	10	141.49	121.28	101.06	-7.88	14.29	7.54	16.67	28.57	22.95	32.9%	21.7%	6.0%
11	A	11	141.49	121.28	105.11	-7.88	14.29	7.54	13.33	25.71	19.86	27.2%	15.1%	2.0%
12	A	12	171.33	138.96	117.23	-30.62	18.89	-5.95	15.64	31.57	10.62	37.0%	22.3%	7.9%
Available Space	**1573.9**	**Average Space 131.16**	**1614.74**	**1444.26**	**1252.77**	**-2.60%**	**10.56%**	**8.24%**	**13.26%**	**22.42%**	**20.40%**	**28.1%**	**20.1%**	**7.8%**

The total required space for all the menu bags containing hybrid rigid and moldable food items with continuous configurations for moldable food items is 1252.77. Thus, by comparing the discrete configurations for moldable items with moldable items as rigid items, it is apparent that the total packing volume reduces by 10.56 percent. By examining the continuous configurations for moldable items side-by-side with the limited discrete configurations for moldable items, it is apparent that the total packing volume decreases by 13.26 percent. By juxtaposing the continuous configurations for moldable items with the continuous configurations with rigid items, it is apparent that the total packing volume decreases by 22.42 percent.

Current MRE box A has 1573.9 cubic inches of space for packing MRE menu bags 1-12. Therefore, comparing the discrete configurations for moldable items with box A's available space shows an 8.24 percent space requirement reduction. Observing the continuous configurations for moldable items compared with current box A's available space shows a 20.40 percent space requirement reduction. Considering the rigid configuration for all the food items in the menu bag increases the space requirement by 2.6 percent.

The average available space for each MRE menu bag in box A is 131.16 cubic inches. By comparing the results of using moldable items as rigid items with the average available space, negative values for menus 5 through 12 appear. This change occurs because the current packing of MREs benefits from moldable items in arbitrary packing of the food items in the bags. For example, considering rigid dimension for menu 12 has increased the required space by 30.62 percent. One of the moldable items in menu 12 is $9 \times 5.5 \times 0.0625$ inches. Considering rigidity for this item increases the length and width requirements of the menu bag to 9 and 5.5 inches, while another alternative for this item's dimensions are 5.5, 4.5, and 0.125. In the current packing of menu 12, this item is bent, and the second alternative configuration is used. By considering discrete configurations for all moldable items in menu 12 as opposed to the rigid considerations improves the require space by 18.89 percent. By comparing continuous configurations for moldable items in menu 12 with the rigid considerations, it is clear that the required space improves by 31.57 percent.

Table 10 presents the results of comparison between all the considered scenarios for menu bags 13-24. For menu bags 13-24, the total required space for all the menu bags with all the food items as rigid is 1545.74 cubic inches. The required space is reduced to 1545.74 cubic inches by solving the problem of packing MRE food items in the menu bags, considering discrete configurations for moldable items. Additionally, the total required space for all the menu bags with all the food items as rigid is 1374.05 cubic inches by solving the problem of packing MRE food items in the menu bags, considering continuous configurations for moldable items. The percent reductions in the space requirements are 11.11 percent, 9.96 percent, and 19.96 percent, respectively.

Current MRE box B has 1573.9 cubic inches of space for packing 12 MRE menu bags. The results reveal that using discrete configurations for moldable items compared with box B's available space shows a 12.70 percent space requirement reduction. Considering the continuous configurations for moldable items compared with the current box B are available space shows a 21.39 percent space requirement reduction. Implementing the rigid configuration for all the food items reduces the total space requirement by 1.79 percent. The results of this comparison for each individual menu bags are presented in Table 10. The results of comparisons among menu bags 1, 4, and 22 shows no improvement when the results of using discrete configurations for moldable items are compared with considering moldable items as rigid items. This outcome occurs because the optimum selected configuration in the moldable items in the hybrid moldable packing with discrete configurations for moldable items is equal to the rigid dimensions considered for those moldable items.

A comparison between developed hybrid moldable packing approaches and the current available space coupled with the condition in which only rigid packing is considered shows that the unutilized space with the hybrid moldable modeling approach is reduced. Specifically, by comparing the moldable discrete and moldable continuous models, it is apparent that the continuous model can use the packing volume more effectively and reduce the unutilized packing volume even further.

A comparison between the developed hybrid moldable packing approaches and the current available volume coupled with the condition in which only rigid packing is considered reveals that the hybrid moldable modeling takes advantage of the flexibility in moldable items to fill the empty spaces, thus reducing the require packing volume. Specifically, comparing the moldable discrete and moldable continuous models shows that the continuous model can find a better design of the dimensions and reduce the packing volume even further.

Table 10: Comparison of Packing Volume Results for MRE Food Items Moldable Packing Solutions-Menu Bags 13-24

Test Problems Specifications			Tested Models			Percent Improvement in the Packing Volume Results (%)						Percent Unused Space		
Test Prob.	MRE Box	Menu	Min- Packing Volume With Rigid Modeling	Min- Packing Volume With Discrete Moldable Modeling	Min- Packing Volume With Continuous Moldable modeling	Rigid Model Compared with Average Space for MRE Menu Bags	Discrete Model Compared with Rigid	Discrete Model Compared with Average Space for MRE Menu Bags	Moldable Model Compared with Discrete	Moldable Model Compared with Rigid	Moldable Model Compared with Average Space for MRE Menu Bags	Unused Space in Rigid Model	Unused Space in Discrete Moldable Model	Unused Space in Continuous Moldable model
13	B	13	121.28	111.17	90.96	7.54	8.33	15.24	18.18	25.00	30.65	29.1%	22.6%	5.4%
14	B	14	137.94	118.51	108.79	-5.17	14.08	9.65	8.20	21.13	17.05	29.7%	18.1%	10.8%
15	B	15	115.11	112.66	102.87	12.23	2.13	14.10	8.70	10.64	21.57	20.1%	18.3%	10.6%
16	B	16	88.17	83.27	78.38	32.77	5.56	36.51	5.88	11.11	40.24	20.6%	15.9%	10.7%
17	B	17	110.21	100.42	97.97	15.97	8.89	23.44	2.44	11.11	25.30	22.0%	14.4%	12.2%
18	B	18	145.71	135.99	126.28	-11.09	6.67	-3.69	7.14	13.33	3.72	25.9%	20.6%	14.5%
19	B	19	149.40	127.36	117.56	-13.91	14.75	2.90	7.69	21.31	10.37	34.4%	23.1%	16.6%
20	B	20	127.36	117.56	97.97	2.90	7.69	10.37	16.67	23.08	25.30	29.3%	23.4%	8.1%
21	B	21	114.00	91.20	76.00	13.08	20.00	30.47	16.67	33.33	42.05	40.4%	25.4%	10.5%
22	B	22	124.95	124.95	112.70	4.73	0.00	4.73	9.80	9.80	14.07	15.2%	15.2%	5.9%
23	B	23	138.36	121.28	111.17	-5.49	12.35	7.54	8.33	19.65	15.24	26.3%	15.9%	8.2%
24	B	24	173.25	129.68	116.57	-32.09	25.15	1.13	10.11	32.72	11.13	37.7%	16.7%	7.3%
Available Space	**1573.9**	**131.16**	**1545.74**	**1374.05**	**1237.20**	**1.79%**	**11.11%**	**12.70%**	**9.96%**	**19.96%**	**21.39%**	**27.5%**	**19.1%**	**10.1%**

4.2 Packing MRE Menu Bags in the Boxes

In this section, all the packing models, including the model with limited discrete configurations for moldable items, the model with all continuous configurations for moldable items, and the model in which all items are considered as rigid, are used to solve the problem of packing the MRE menu bags in the boxes. The continuous molding approach is not applicable for solving the current issue of packing MRE menu bags in the boxes. However, the results can be used for future designs.

4.2.1 Solving MRE Menu Bags Packing Problem with the Moldable Packing Model with Discrete Configurations

In this section, the 3-D hybrid moldable packing model with discrete configuration is applied to solve the problem of packing MRE menu bags in the boxes.

4.2.1.1 Assumptions

The objective function is to reduce the packing volume with only one variable dimension. The following assumptions are considered in generating the test problems:

- Each test problem is comprised of a set of MRE menu bags to be packed in a box. Thus, two sets of test problems are conducted to pack all the menu bags in the boxes. Menu bags 1-12 are packed in one box and menu bags 13- 24 are packed in another box.
- The length of the current boxes is equal to 16.625 inches. The width of the current boxes is equal to 9.125 inches. The height of the current boxes is equal to 10.375 inches.

- The resulting dimensions for menu bags 1-24 presented in sections 4.1.3 and 4.1.4 are used as inputs for the model. Two discrete configurations are considered for each menu bag.

4.2.1.2 Results

Table 11 presents the results of packing MRE menu bags in the boxes with three objective functions. In the best scenario, 1483.78 cubic inches for packing menu bags 1-12 is required. In the best scenario, 1443.88 cubic inches for packing meal bags 12-24 is required.

Table 11: Packing Menu Items in the Boxes with Discrete Configurations for MRE Menu Bags

Menu 1-12	Objective	Box Length	Box Width	Box Height	Packing Volume Requirement
Box A Results	Variable Height Min.	16.625	9.125	10.50	1592.88
	Variable Width Min.	16.625	8.50	10.50	1483.78
	Variable Length Min.	18.44	8.50	10.50	1645.77

Menu 13-24	Objective	Box Length	Box Width	Box Height	Packing Volume Requirement
Box B Results	Variable Height Min.	16.625	9.125	9.65	1463.94
	Variable Width Min.	16.625	9.00	9.65	1443.88
	Variable Length Min.	17.59	9.00	9.65	1527.69

4.2.2 Solving MRE Menu Bags Packing Problem with the Hybrid Moldable Model with Continuous Dimensions

The continuous molding approach is not applicable for packing MRE menu bags in the boxes because of the assumption that continuous dimensions for the menu bags can result in configurations that conflict with the possible packing of food items. However, Table 12 presents the result of packing MRE menu bags in the boxes with variable heights. The results of the height objective function are presented. In the best scenario, 1289.48 cubic inches for packing meal bags 1-12 is required. In the best scenario, 1250.03 cubic inches for packing meal bags 13-24 is required.

Table 12: Packing Menu Items in the Boxes with Continuous Configurations for MRE Menu Bags

Box	Box Length	Box Width	Box Height	Packing Volume Requirements
Box A Results	16.625	9.125	8.50	1289.48
Box B Results	16.625	9.125	8.24	1250.03

4.2.3 Comparison of Results of MRE Menu Bags Packing with Rigid Configurations

In this section, the hybrid moldable packing approaches are compared with the menu bag dimension results when only rigid configurations for all the food items are considered (presented in section 4.1.5). Table 13 presents the results. The results are compared with the current box volume and show that the discrete model can improve the required packing volume by 5.73 percent in box A and 8.26 percent in box B.

Table 13: Packing Menu Items in the Boxes-Comparison of All Scenarios

Box	Scenarios	Results	Percent Improvement
Box A	Current Volume	1573.90	-
	All items Considered Rigid	1780.87	-13.15
	Discrete Configurations Model	1483.78	5.73
	Continuous Configurations Model	1289.48	18.07
Box B	Current Volume	1573.90	-
	All items Considered Rigid	1789.30	-13.69
	Discrete Configurations Model	1443.88	8.26
	Continuous Configurations Model	1250.03	20.58

4.3 Validation

This research is a component of a lean effort that attempts to reduce the lead-time associated with the implementation of dynamic product modifications that imply packing changes. The results of this lean system include measuring changes, communicating changes, and incorporating changes effectively in a short lead-time. The implementation of this system will result in the reduced dimensional variations in MRE menu bags and boxes.

Based on the observation of MRE assembly lines, the sources of variation in MRE packing processes can be categorized into measurement systems and packing processes. In this section, variation in measurement and packing are studied to validate the possible implications of the lean packing modification system. In order to evaluate the variation in the measurements, two measurement methods are selected, and the results are compared with each other. In method 1, all the measurements are collected manually with a caliper and a ruler. In method 2, the measurements are collected with a 3-D scanner. For method 1 measurements, human errors are more prevalent in the results. Therefore, multiple samples are required to increase the precision. Ten measurement samples are collected from each dimension. The average value, the standard deviation, variation, and maximum and minimum values are reported in Table 13. Method 2 results are presented in Table 13 as well and can be compared with the average and maximum and minimum values. In the proposed lean packing modification system, the 3-D scanner will reduce the potential error of measurement method.

Table 14: Measurement and Data Analysis

	Method 1 Measurement Samples						Method 2
Items	Inside Item	Avg	Std. Dev.	Var.	Max	Min	
Coffee	Length	0.7725	0.080983537	0.006558333	0.89	0.71	0.8478
	Width	4.585	0.019148542	0.000366667	4.61	4.57	4.64184
Peanut butter	Length	5.6875	0.072168784	0.005208333	5.75	5.625	5.5315
	Width	2.3275	0.091787799	0.008425	2.43	2.25	2.3732
Potato Cheddar Soup with Bacon	Length	4.74275	0.0095	9.025E-05	4.75	4.73	4.68279
	Width	7.37375	0.001892969	3.58333E-06	7.375	7.371	7.28413
Tabasco	Length	0.5965	0.124906632	0.015601667	0.703	0.46	0.66665
	Width	2.3275	0.002886751	8.33333E-06	2.33	2.325	2.26782
Beverage Base Powder Tropical Punch	Length	4.285	0.025166115	0.000633333	4.32	4.26	4.29144
	Width	4.91	0.059441848	0.003533333	4.96	4.83	4.92251

The variation due to the measurement system is either because of human error or because of the device of measurement. The packing process variation is because of the difference in the layout of items in the menu bags and the difference in the final layout of menu bags in the boxes. Identified sources of these variations are depicted in Figure 11.

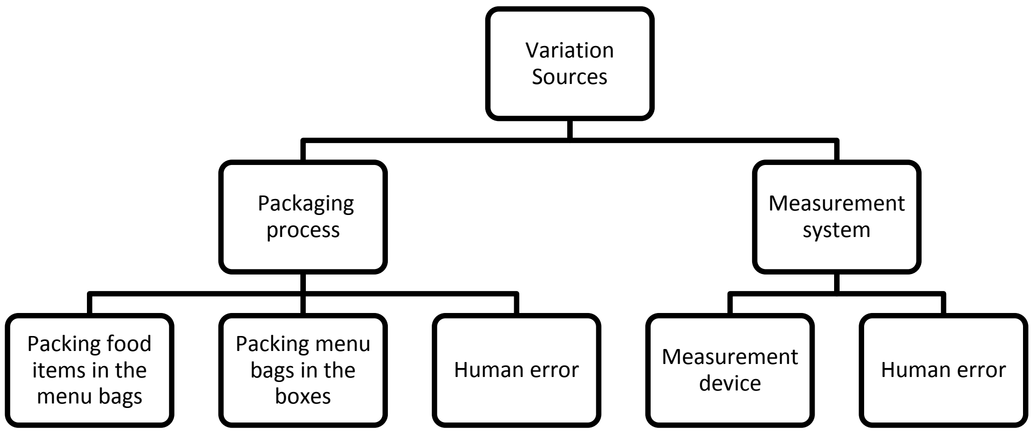

Figure 11: Variation Sources

The Quick Measure component of the lean dynamic packing modification system reduces the variation caused by the measurement device and eliminates human error. The optimization scheme for packing and assembling provides the assemblers with a standard packing methodology. Visualization of standard packing allows the assembly personnel to follow a similar packing process which reduces the variations cause by arbitrary packing in both packing levels.

To identify the effect of the lean dynamic packing modification optimization scheme, the optimization models are run multiple times to achieve a consistency of solutions. Then the results are used to pack the MRE food items and menu bags. The resulting dimensional variation is compared with 10 MRE package samples from one of the assemblers. This specific assembler preplans the process of packing before the assembly line and also designs the assembly in a way that the order of items is fixed for all the packing personnel. Each personnel has a fixed location in the assembly line and picks the items in front of him/her and puts them in a bin which later will be replaced with the menu bag. This assembly line is depicted in Figure 12.

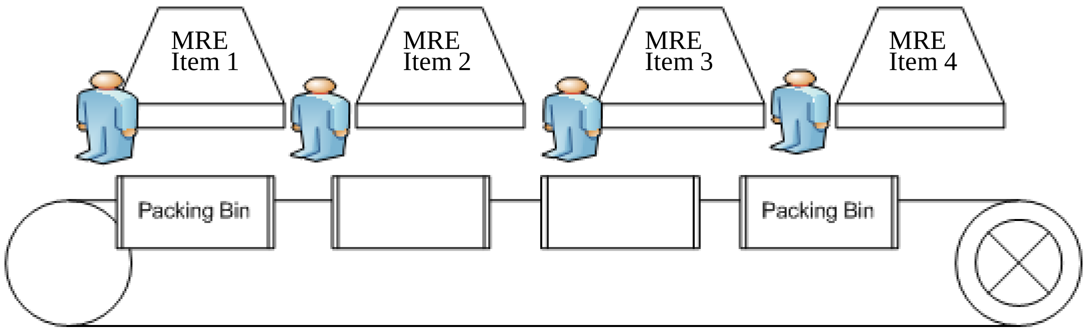

Figure 12: MRE Packing Assembly Line

The standard deviation in height of menu 1 in the 10 samples is 1.059 inches. The standard deviation for other menu bags varies from 0.51 to 1.6 inches.

The packing results from section 4.1 are used to pack food items in menu bags 1-24. The standard deviation for the height of menu bag 1 is reduced to 0.09. Similarly, for the other 23 menu bags, the standard deviation of height varies from 0 to 0.3.

This assembly line can use the optimization results to change the order of packing items and to improve the final layout. In this MRE assembly plant, no planning for packing menu bags in the boxes exists. Therefore, the layouts of the menu bags in the boxes are arbitrary. These layouts cause variations which serve as one of the sources of bulging boxes and unbalanced pallets, causing transportation risks. Ten samples of boxes are evaluated by focusing on the height of the box. The standard deviation of box A height is 0.36. The packing results from section 4.2 are used to pack menu bags in the boxes. The standard deviation of box A height is reduced to 0.11. This reduction in variation improves the packing processes, reduces the costs associated with pallet instability in transportation and warehousing, and saves the integrity of food when delivered to soldiers.

As discussed earlier, the optimization is a component of a packing modification system, including a precise measurement tool called Quick Measure which was developed in the Natural Interactions Lab[1]. Quick Measure collects the lengths, widths, and heights of items in seconds without any human post-processing requirements. In this research, the dimensional data for input to the optimization model is collected manually. If, in the future, the input information is collected with the Quick Measure, the variation will be reduced even further.

4.4 Visualization

Translation of the mathematical outputs to a visualized format allows MRE assemblers to use the results easily and to train their personnel to pack the food items and menu bags quickly. The visualization also helps to standardize the packing processes and to reduce the packing variation on two levels: packing food items in the menu bags and packing menu bags in the boxes. Autodesk Inventor is used to visualize the outputs. Food items and menu bags are simulated individually. Then, based on the optimization outputs, the layout of food items within each bag and the layout of menu bags in the final box are defined.

Figure 13 illustrates the process for packing various food items in a menu bag, including the relative position of food items to each other (1-5) and the final menu bag (6). Figure 14 shows a typical translation of the mathematical results to a visual format for training purposes. In this way, the workers can easily follow the instructions and pack the items. Figure 15 illustrates the visualization of the menu bags' packing in a typical box. Based on the input values for the menu bags, the objective function can result in alternative layouts in the boxes.

[1] Dr. Gewei Zhang in the Department of Industrial and Systems Engineering, Natural Interactions Lab

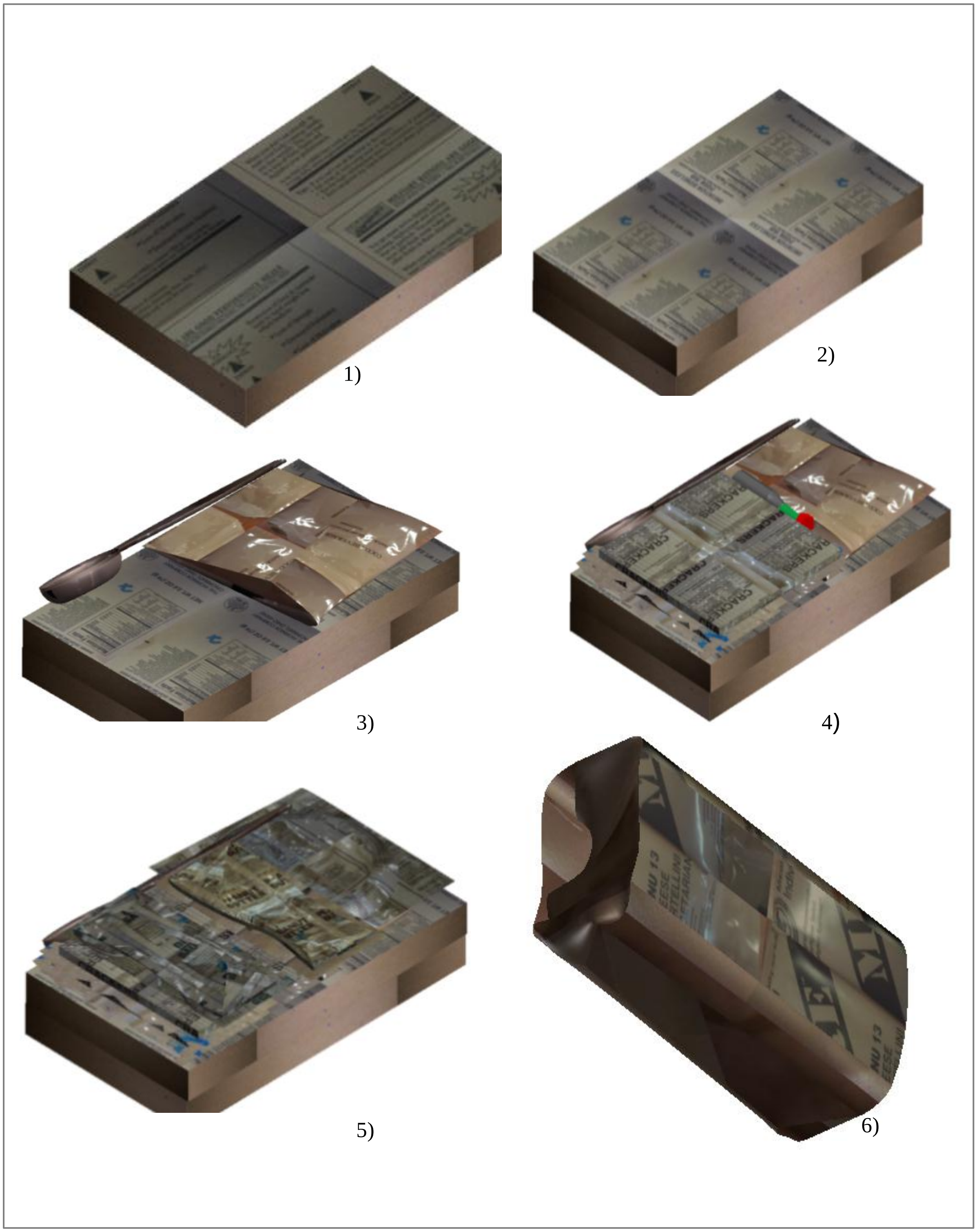

Figure 13: Visualization of Packing Items in a Menu Bag

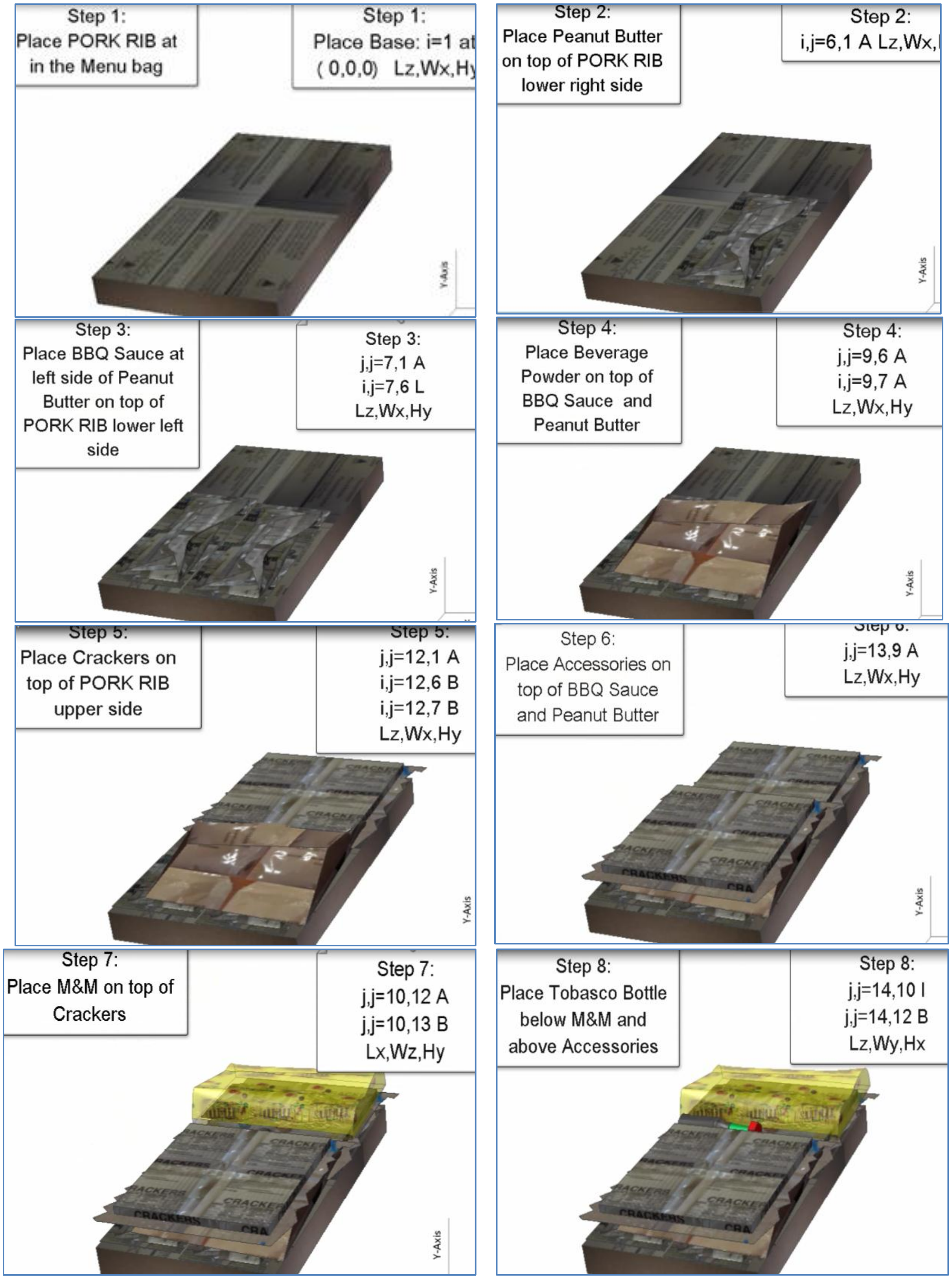

Figure 14: Translating the Outputs for Training Purposes

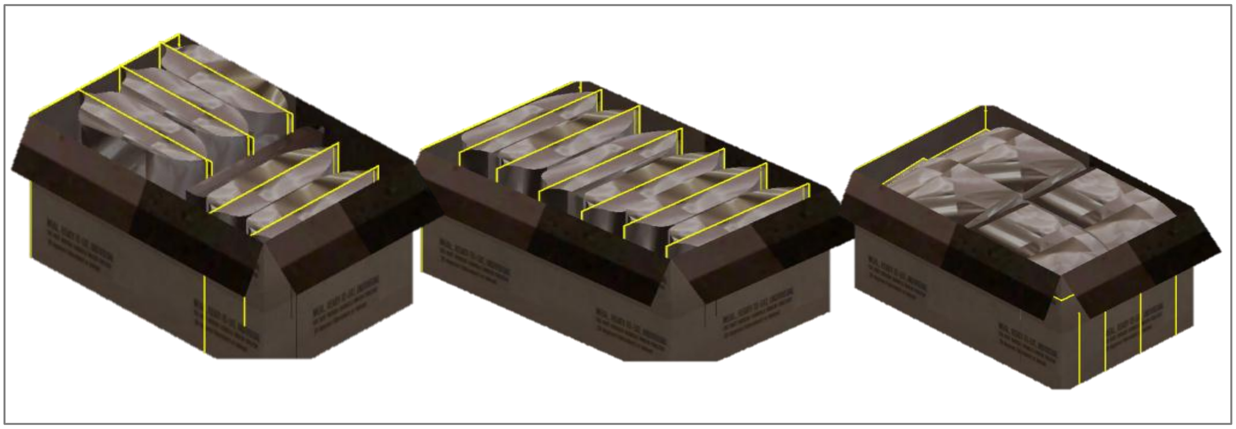

Figure 15: Visualization of Packing Menu Bags in a Box

Figure 16 illustrates another example of packing fours items. If the proper cover for each item is used, the relative position of items toward each other can be easily understood by the assembly personnel.

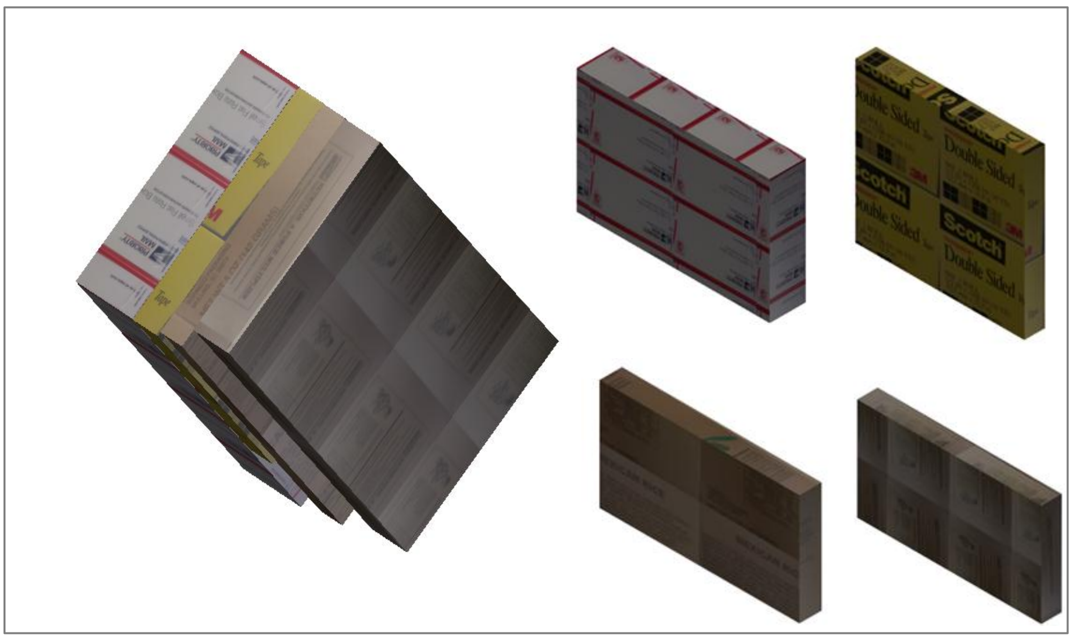

Figure 16: Visualization of Packing

CHAPTER V
Summary, Conclusions, and Recommendations

Chapter V includes a summary of this dissertation, the study's conclusions, and the recommendations for future research.

5.1 Summary

This study's literature review includes the following: the packing optimization problem and previous studies of 3-D hybrid moldable packing problems. First, the most important contributions, as well as their authors, for this problem are highlighted. The literature review reveals that there are no analytical models considering the moldable items packing problem. Two mathematical models to solve the problem of packing a hybrid mix of rigid and moldable items are developed. Theses packing models scrutinize moldable items from two perspectives: (1) when limited discrete configurations are provided for the model and (2) when all continuous configurations are available to the model. This optimization scheme is a component of a lean effort that attempts to reduce the lead-time associated with the implementation of dynamic product modifications that imply packing changes.

The models were tested based on dynamic packing changes of MRE on two different levels: packing MRE food items in the menu bags and packing the menu bags in the boxes. These models optimize the packing volume and provide information for MRE assemblers enabling them to preplan for packing changes. The results are validated by running the solutions multiple times to access consistency of solutions. The solutions are visualized using Autodesk Inventor to communicate the optimized packing solutions with the MRE assemblers for training purposes.

5.2 Conclusions

The two developed models solve the packing problems of a hybrid mix of rigid and moldable items. Computational tests for 24 MRE menu bags (which are based on the available samples of MRE food items) reveal that the developed models are able to solve the packing problem of a hybrid mix of rectangular and moldable items.

After comparing the developed hybrid moldable packing approaches with the approach that only considers packing rigid items, it is clear that the moldable modeling takes advantage of the flexibility of moldable items to fill empty spaces. This flexibility within the hybrid moldable packing approach, therefore, reduces the required packing volume. More specifically, by comparing the moldable discrete model with the moldable continuous model, it is apparent that the continuous model can find a better design of the dimensions ultimately reduce the packing volume even further.

Results show that the developed models are able to accomplish three main goals: to standardize the packing processes, to help achieve a consistency of packing design, and to reduce the variations in the packing solutions. By applying these models to the packing process of MREs, MRE supply chain entities, such as the DLA, manufacturers, and assemblers, can better understand the consequences of changes in MREs. These entities can also better measure those changes, communicate the changes, and incorporate the changes effectively in a short lead-time. The ability to understand, measure, communicate, and incorporate these changes ultimately achieves four main effects: it reduces the dimensional variations in MRE menu bags and boxes, improves the space utilization in packing MRE food items in the menu bags and packing MRE menu bags in the boxes, reduces the costs associated with pallet instability in transportation and warehousing, and saves the integrity of food when delivered to soldiers.

Furthermore, the visualizations illustrate how a translation of results enables packing assemblers to train their personnel easily and quickly.

5.3 Recommendations

The proposed models can be useful in motivating future research about decomposition methods, relaxation methods, and heuristics, among others, to solve bigger and more complex moldable packing problems.

The methodology used in this research is a decomposition approach to the problem of packing MREs in boxes. In this approach, the packing problems are solved by breaking them up into two smaller problems and solving each of the smaller ones sequentially. Each MRE packing level is considered to be a separate packing problem. The interaction between the two problems was controlled by using the results of packing the first level (food items in the menu bags) to solve the second level (packing menu bags in the boxes). Solving the packing problems simultaneously is another approach which can be applied in future research.

In 3-D packing problems, each dimension of an item can serve as height, giving rise to three vertical orientations. By setting a particular dimension to be an item's height, the item can be aligned horizontally to the bin's walls by means of two horizontal orientations. Therefore, six orientations exist in which a (rectangular) item can be placed orthogonally into a bin. In application fields, however, the number of orientations of an item may be restricted both in vertical and in horizontal directions.

Also, the load-bearing strength of an item depends on its vertical orientation. Subsequently, not all possible vertical orientations can be used when a large bin is being loaded. It may even be possible that a particular orientation is possible on an upper loading layer which is not permitted on a lower one. Vertical orientation constraints are introduced either to prevent goods and packaging from being damaged or to ensure the stability of the load [36]. On the contrary, relaxing constraints, which control the rotations of items, allow any orientation for each item in the packing space. To control the overlap of items and to fit the items within the bin's dimensions, the partial rotation items should be properly controlled. One way to control the items is to allow the length, width, or height of each item to take an angle of 30, 45, or 60 degrees. This approach increases the complexity of modeling and difficulty of packing the items manually; however, it may result in improved packing space utilization.

Load bearing or stacking constraints restrict how boxes can be placed on top of each other. They arise from the limited loadbearing strength of the boxes [90]. The load bearing capability of each box depends on the amount of weight or pressure a box can tolerate. Additionally, this capability depends on the strength of the box case, which is determined by the construction of the case and the material used.

Moreover, positioning constraints can be used to restrict the location of items within the bin. Such constraints can be imposed when certain items are to be placed or not to be placed within the bin or when items are to be positioned or not to be positioned relative to each other. Such constraints are typically set by the size, weight, or content of an item. Identifying subsets of cargo to be delivered to specific customers [91] during loading/unloading operations [92] is among a positioning constraint. The problem can be extended to a case where a subset of items cannot be placed adjacent to other items or within a close proximity to each other. Including these real world constraints in the model, especially when the model is used for solving large packing problems, such as loading boxes on pallets, achieves more stable practical results [93]. Spreading the items' weight in the bin as evenly as possible results in balanced loads and reduces the risk that the cargo shifts while the bin is moving. (The following constraints, for example, can be added to the model to control the weight imbalance along the *X-axis*:

$-c_x \leq \sum_{i=1}^{N} \frac{w_i}{2} \left[\sum_{j=1}^{m} L_j . n_j - 2x_i - p_i . l_{xi} - q_i \left(l_{zi} - w_{yi} + h_{zi} \right) - r_i . \left(1 - l_{xi} - l_{zi} + w_{yi} - h_{zi} \right) \right] \leq c_x,$ where w_i is the weight of item i and c_x is the weight imbalance limit along the *X-axis* [8]). The literature concerning packing problems claims that there is a gap between the practical value of the results and the packing approaches that are considered [36]. Thus, future research can address this gap and consider practical constraints in large, real-world packing problems.

Numerical experiments indicated that for the 3-D bin packing problem in moderate size (MRE packing problem) application, BARON can be used. However, BARON needs a significant amount of computational time to generate a reasonably good result for large problems; this time requirement makes it unsuitable for computing on-site operations in larger applications. Hence, heuristics are necessary if quality and fast solutions are required. For example, a heuristic can be used to incorporate heuristics strategies with a handling method for remaining spaces to generate optimal loading arrangements of boxes with stability considered [6].

Other interesting topics for future research include exploring multidisciplinary applications such as integrating models in coupled vehicle routing, container loading models with multi-drop constraints [46], and circuit board design. In dynamically reconfigurable Field-Programmable Gate Arrays (FPGAs), each task is assigned to the computation resources of a rectangular region on the FPGA for a certain time period [94]. This problem is defined as a 3-D rectangular packing problem of a 2-D plane and time axis [95, 96]. The 3-D packing problem can be applied to solve very large scale 3-D integrations, which consist of many layers of active devices [97].

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APPENDIX

A) Terminology

Packing: the assembly of items into a unit, intermediate, or exterior pack with necessary blocking, bracing, cushioning, weatherproofing, reinforcement, and marking [39].

Packaging: the processes and procedures used to protect material from deterioration, damage, or both.

MRE: Meals, Ready-to-Eat is currently the main individual operational ration for the U.S. military [2, 22].

MRE Menu: a bag filled with various menu items and accessories (Figure 17).

MRE Box: 12 MRE menu bags packed in a box for delivery purposes (see Figure 18).

Box A/B: Menu bags 1-12 are in box A; menu bags 13-24 are in box B.

Figure 17: MRE Food Items

Figure 18: MRE Menu Bags

B) Military Packing Limitations and Constraints

The application of the developed 3-D packing models shall be in accordance with the requirements specified in the MIL-STD-2073 document. The MIL-STD-2073 document outlines the standard processes for the development and documentation of military packaging, as distinct from commercial packaging [98]. This standard covers the methods of preservation to protect material against environmentally induced corrosion and deterioration, physical and mechanical damage, and other forms of degradation during storage, multiple handling, and shipment of materiel in situations where commercial packaging cannot meet known distribution and environmental requirements. There are protection levels of military preservation and packing to ensure that a given item is not damaged during shipment and storage. Level A is the protection required to meet the most severe worldwide shipment, handling, and storage conditions. Level B is the protection required to meet moderate worldwide shipment, handling, and storage conditions [98].

C) List of MREs

Tables 15, 16 and 17 illustrate the list of MRE food items in box A, in box B, and content categories. Each menu bag has an accessory bag.

Table 15: List of MRE Food Items in Box A

Case A			
Bag No.	List of food items	Bag No.	List of food items
1	Mexican Style Corn	7	Crackers
	Crackers		Apple Jelly
	Skittles		Peanut Butter
	Dairy Shake Powder Strawberry Banana		BBQ Sauce
	Cheese Spread		Cocoa Beverage Powder
	Chili With Beans		Biscuit
2	Pork Rib		Cookies with Pan Coated Chocolate Discs
	Potato Cheddar		Candy, Toffee Rolls
	Cookies	8	Nacho Cheese Pretzels
	Peanut Butter		Smoked Almonds
	Strawberry Jam		Cheese Spread
	BBQ Sauce		Tropical Punch Flavored Fruit juice
	Crackers		Wheat Snack Bread (a)
	Lemon- Lime Powder		Wheat Snack Bread (b)
3	Beef Ravioli		Marinara Sauce & Meatballs
	Toaster Pastry Frosted Brown Sugar Cinnamon	9	Lemon Poppy Seed Pound Cake
	Creamsicle Cookies		Crackers
	Beef Snack Strip		Chocolate Dairy Shake Powder
	Vegetable Crackers		Cheese Spread
	Carbohydrate Electrolyte Beverage Powder		Mashed potato
	Cheese Spread		Beef Stew
4	Chocolate Chip Toaster Pastry	10	Mango Peach Applesauce
	Cinnamon Scone		Twizzlers
	Granola		Lemon Lime Beverage Powder
	Apple Butter		
	Crackers		Cornbread
	Salsa Verde		Cheese Spread Jalapenos
	French Vanilla Cappuccino Instant Powder		Chili and Macaroni

Table 15. Continued

Bag No.	List of food items	Bag No.	List of food items
5	Chicken Breast	11	Accessories
	Corn Bread Stuffing		Mango Peach Apple Sauce
	Chocolate Chip Chocolate Covered Ranger Bar		Wheat Snack Bread
	M&Ms		Marble Pound Cake
	Cheese Spread with jalapeno		Chocolate Hazelnut Cocoa Beverage Powder
	Wheat Snack Bread		Peanut Butter
	Beverage Based Powder Lemon - Lime		Vegetable Lasagna
	Accessories		Cranberries, Sliced
	Grilled Chicken Fillet	12	Chocolate Banana Nut Top Muffins
	Cocoa Beverage Powder		Carbohydrates Electrolyte Beverage Powder
6	Cranberry Apple Ranger Bar		Wheat Snack Bread
	M&Ms		Veggie Burger in BBQ
	Crackers		
	Cheese Spread		
	Chicken, Vegetables and Noodles in Sauce		

Table 16: List of MRE Food Items in Box B

Case B			
Bag No.	List of food items	Bag No.	List of food items
13	Crackers, plain	19	Crackers, plain
	Accessory Packet, C & Seasoning		Steak Sauce
	Peanut Butter		Beverage Base Powder
	Beverage base Raspberry		Chocolate Peanut Butter
	M&Ms		Cranberries, Fruit, Dried
	FSR Bar		Carrot Pound Cake
	Cheese Tortellini		Roast Beef with Vegetables
	Apples Pieces in Spiced Sauce	20	Chipotle Snack Bread
14	Crackers		Cheese Spread
	Peanut Butter		Beverage Base Raspberry
	Beverage Base Powder Orange		Tabasco
	Cranberries		Hot & Spicy Baked Snack Cracker
	Marble Pound Cake		Spaghetti with Meat and Sauce
	Penne with Vegetable Sausage		Cherry Blueberry Cobbler
15	Cheese Spread with Jalapenos	21	Tortillas
	Beverage Base Power Tropical Punch		Dairy Shake
	Picante Sauce		Mayonnaise, Fat Free
	Cookies with Pan Coated Chocolate Disc		Chip Cookies
	Beef Enchilada in Tomato Sauce		Pretzels
	Refined Beans		Chocolates
	Crackers, Vegetable		White Tuna
16	Tortilla	22	Wheat Snack Bread
	Cheese Spread, Plain		Grape Jelly
	Coffee, Irish Cream		Chunky Peanut Butter
	Baked Snack Cracker		Beverage Base Powder
	Chicken Fajita		Candy I M&Ms
	Mexican Rice		Sterling Foods
17	Wheat Snack Bread		Chicken with Dumplings
	Cheese Spread with Jalapenos		Tabasco
	Spoon	23	Wheat Snack Bread
	Fruit Punch		Beverage Base Powder
	Nut Raisin Mix		Cheese Spread with Bacon
	Fudge Brownie		Pudding, Vanilla
	Sloppy Joe Filling		Chicken Pesto & Pasta

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Table 16. Continued

Bag No.	List of food items	Bag No.	List of food items
18	Wheat Snack Bread 2		Pineapple
	AC	24	Wheat Snack Bread
	BBQ Sauce		Cheese Spread with Jalapenos
	Beverage Base Powder Orange		Mocha Cappuccino Instant Powder
	Cheese Spread		Reese Pieces
	Pepperoni Pizza Crackers		Patriotic Cookies
	Beef Patty, Grilled		Chicken, Pulled with Buffalo Style Sauce
	Macaroni and Cheese Mexican style		Fried Rice

Table 17: List of Moldable Items in Each Menu Bag

Type	Item	Menu Bag
Moldable (Powder)	Cheese Spread	1, 3, 8, 13, 16, 18, 20
	Cheese Spread with Bacon	23
	Cheese Spread with Jalapenos	5, 10, 15, 17, 24
	Picante Sauce	15
	Peanut Butter	7, 2, 11, 13, 14, 19, 22
	Grape Jelly	22
	Salsa Verde	4
	Mayonnaise	21
	Mocha Cappuccino Instant Powder	24
	Dairy Shake Powder	1,21
	Vanilla Pudding Powder	23
	Beverage Base Powder	3, 5, 6, 7, 10, 11, 12, 13, 14, 18, 19, 20, 23
	Tropical Punch	8, 15, 17, 22

Vita

Born in Iran, Sima Maleki is the daughter of Morteza and Fatemeh. Her father is a Persian Miniaturist, and her mother is a Persian literature teacher. A watercolor artist, Sima participated in an exhibition at the Knoxville Museum of Art in 2011. She has a Bachelor's degree in applied mathematics and a Master's in industrial engineering. Immigrating to the U.S.A. in 2009, she joined Dr. Sawhney's group to pursue her Ph.D.