12-2012

Analysis of the Impact of Contagion Flow on Firm Value and Application to High Yield Bond Portfolio Optimization

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To the Graduate Council:

I am submitting herewith a dissertation written by Wendy Ann-Swenson Roth entitled "Analysis of the Impact of Contagion Flow on Firm Value and Application to High Yield Bond Portfolio Optimization." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Management Science.

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Analysis of the Impact of Contagion Flow on Firm Value and Application to High Yield Bond Portfolio Optimization

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Wendy Ann-Swenson Roth

December 2012

Dedication:

Marissa and Jason, my wonderful children. You have brought more joy and inspiration to my life than you can ever know. No, Marissa, it wasn't as hard as doing flips off a diving board and yes, Jason, I am finally done.

Steve, thank you for all the ways you helped me make it to this day and for never doubting that I would finish.

My parents, thanks for the love and support you have always given me and instilling in me the importance of education.

To numerous friends on Signal Mountain – especially Sherry, Snowe and Linda. Thanks for your friendship, support and always listening to me even when it was boring. It did take a village or at least a lot of people from a small town on a mountain.

Acknowledgements:

I would like to thank my dissertation committee, I am incredibly fortunate to have such an accomplished and supportive group of advisors. Each of you have provided me with an example of what I hope to achieve in my career.

I thank Dr. Chanaka Edirisinghe, my dissertation chair, for providing guidance and support throughout the dissertation process. I am grateful for your high standards, wealth of knowledge, constructive feedback, guidance and words of encouragement.

Dr. Aparna Gupta, I am grateful for your mutual interest in the impacts of contagion, your willingness to give your time to our numerous discussions, the guidance you provided and words of encouragement.

Dr. Ray DeGennaro, your insightful comments and valuable suggestions about the finance considerations pertaining to the development of my research are greatly appreciated.

Dr. Bogdan Bichescu, your encouraging comments, positive outlook and guidance as I progressed through the dissertation process were incredibly helpful.

Dr. Russell Zaretzki, thank you for your time and energy spent serving on my dissertation committee.

Abstract

Portfolios of financial instruments are designed to increase returns and manage risk. In high-risk investment strategies, central measures of risk must be complemented with controls on tail measures of risk. An unanticipated event that impacts securities of one firm can contagiously effect those of other firms through a contagion flow process that may occur via a set of network connections. Such connections among firms arise due to a variety of factors, such as a shared supply chain member or auditing firm. These connections spread the contagion, potentially impacting numerous other firms in the network. This can adversely affect the level of tail risk in an investment strategy, especially when many such connected and affected firms are included in a portfolio, such as a bond portfolio.

Reduced form models illustrate how connections between firms can lead to the heavy tailed default distributions seen in empirical data. Historical events help in the understanding of connections that allow contagion to spread between firms. The goal of this research is to combine the insights gained from both previous models and historical events into a structural model for flow of contagion among firms. This includes defining and calibrating contagion variables, including the network that allows the contagion to spread between firms resulting in increased defaults for a portfolio of debt instruments. The model is then utilized to assess the impact of the network structure on the portfolio of high-yield debt instruments. Additionally, simulation data from the model is a key input into a bond optimization used to provide guidance on the immunization of a portfolio of bonds from the impact of contagion flow. Two historical incidences of contagion are presented as test cases to validate the ability of the proposed firm value model including contagion, to alert investors to the impact of a contagion on a bond portfolio.

Contents

Chapter 1

Introduction and Literature Review

A portfolio of high-yield bonds has characteristics that will appeal to certain investors over other investment opportunities. Returns are usually greater than government bonds and other investment grade bonds albeit for a higher level of risk exposure. This risk is still less than equity of the same companies, and the overall volatility in returns should also be lower. Moreover, investors may gravitate to high-yield bonds for yield enhancement of their investments or the more deterministic incomes offered by bonds. For the valuation and assessment of a bond's risk, it is important to understand the probability of default of the bond, which is particularly important when considering high-yield bonds. When constructing a bond portfolio, it is not only important to assess the default risk of individual bonds, but how these default risks are correlated between bonds.

When developing a bond portfolio strategy the degree of default risk is a key determinant for defining risk-adjusted portfolio returns. Altman et al. (2000) and Altman and Bana (2004) analyzed data of defaults of high yield bonds occurring between 1971 and 2003. The results showed default rates ranged from 0.158 to 12.795% per year, with a weighted average default rate of 5.453% (Figure 1.1 shows

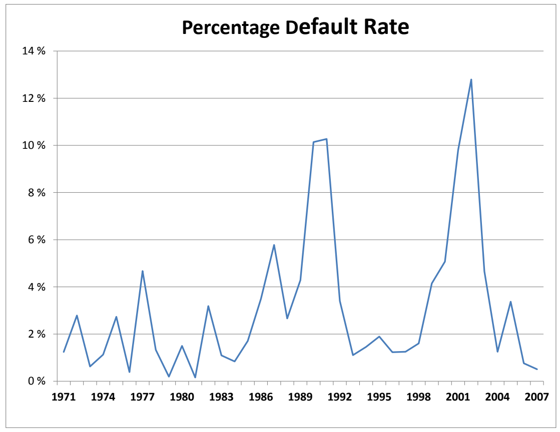


Figure 1.1: Default Rate 1970 - 2007

default rates during the 1971 to 2007 time period). In addition, during the time period covered by the study, the loss given a default increased substantially with a record par value default of 96.858 billion dollars in 2002. Due to the high default rate, it is important that selected bonds do not have higher than anticipated levels of correlated defaults. Default rates also display high yearly variability, increasing the challenge of accurate risk predictions.

The financial crisis of 2007, involving mortgage-backed securities (MBS), highlighted credit rating agencies' inabilities to timely and accurately predict risk. These agencies' ratings of bond are used to indicate a bond's credit risk and default likelihood. The issue of ratings accuracy has also been raised by 'fallen angels,' discussed by Altman and Bana (2004). Fallen angels are companies whose bond ratings are downgraded from investment to speculative grade subsequent to the actual drop in the bond's price, reinforcing that ratings often are a lagging indicator of risk.

Selection of the optimal portfolio of bonds must be based on models that accurately incorporate default risks. Macroeconomic, sector and idiosyncratic risk assessments are important factors when constructing a portfolio and have been extensively studied, see Arora et al. (2005), Leland (2006). Clustering of defaults

Figure 1.2: Stock performance of connected firms impacted by an unanticipated event

and bankruptcies, however are more substantial than macroeconomic and sector movements can explain.

Illustrative Case

Firm values of three firms in the same sector using models based on macroeconomic and sector correlations will tend to move together. The size of that movement, however, can be substantially greater than predicted by these models in some instances. An example of such a case is the stocks of Alkermes, Eli Lilly and Amylin, see Figure 1.2, for their stock returns. The drop in stock prices of these three firms was the result of a surprise announcement of the FDA denying approval of a new drug. As stated by the headline “Amylin shares crater after FDA rejects Bydureon application.” Amylin was the focus of the most negative publicity; however, this also impacted Alkermes and Eli Lilly. These three firms were connected by a joint venture, illustrating how an exogenous contagious event can spread from one firm to other firms creating an impact greater than that explained by macro economic and sector factors alone. Without an understanding of the connections between firms and how a negative impact can spread to a connected firm, a biotech portfolio containing all three firms is riskier than anticipated.

The nature of connections between firms have varied characteristics. Some are expected, such as membership in the same market sector. According to Altman and Bana (2004), in 2002, 36% of bankruptcies were telecom-related firms. In other situations, however, the firm connections were less clear. In that same year, 24% of bankruptcies were related to alleged fraud. When the initial fraud was revealed, all firms connected through the presence of a joint auditor were at increased risk. This risk was not understood until the initial event occurred and the connections between the firms became apparent.

Definition of Contagion

Historically, there are numerous examples of unanticipated events affecting an entity and spreading to similar entities for reasons not anticipated or fully understood at the time of the incident. When the entity is a firm, a bank, or even a country, these incidents can impact their stability and ability to survive as a financially-viable enterprise. The term contagion is often used to describe the spreading of a negative event due to connections among entities. Various researchers use slightly different definitions of what constitutes a contagion. Forcardi and Fabozzi (2004) define contagion as a sudden and unexplained increase in correlation levels. Egloff et al. (2007) state that through micro-structural channels, the credit deterioration of a counterparty triggers the credit deterioration of other counterparties. Our research defines contagion as the co-movement in asset values not explained by common economic fundamentals.

1.1 Statement of Hypotheses

The goal of this research is to improve understanding of the impact of contagion on a portfolio of bonds. The areas of specific focus are presented in the following research questions and related hypotheses.

The first three hypotheses focus on the understanding of the connections between firms and how they impact the flow of contagion. These connections can be represented using constructs from graph theory. Firms are represented as vertices and the connections between firms as edges. Each of these edges or connections will have an associated travel time and decay based on actual firm qualitative data. Examples of this type of data include information about business relationships and funding sources, and existing connections that allow a contagion to flow between firms.

Research Question 1: As an improvement over a stylized model used to represent connections between firms, can a framework be developed to use business-related firm information to characterize the connections in a network of firms whose corporate bonds form a portfolio? Would such a characterization provide an improved understanding of the observed (empirical) data on bond default dynamics?

H1:Firms in a portfolio with firm-pair connections, based on qualitative firm data, modeled using graph theory, result in heavy tailed default distributions seen in empirical data.

Research Question 2: Once connections between firms are established, do the structures formed by the connected firms, the strength of their connections and the size of the initial impact influence the spread of a contagion? Do both the structure of the connections among firms and the number of connections impact the overall portfolio risk to a similar degree?

Once an exogenous contagion has occurred and connections exist, three main variables control the amount of impact on other firms. One determines the strength of the initial contagion, two additional variables determine how fast it travels from one firm to another and how quickly the impact dissipates. Is the size of the initial event or what happens to the contagion as it travels equally important? These contagion variable issues will be addressed in two related hypotheses.

H2A: When firms in a portfolio of bonds experience an exogenous contagion, the structure of the connections between firms, not just the number of connections, decreases the firm value and increases lower tail risk of the overall portfolio.

H2B: For a similar percent change, a slower decay rate more heavily impacts the overall portfolio of firms than increasing the size of the initial exogenous contagion.

Research Question 3: Based on the results of the above hypotheses, a better understanding of the spreading of a contagion is ascertained. Can this knowledge be used in an optimization model to improve bond portfolio selection?

H3: Improved knowledge of the impacts and spread of contagion can be used in a bond portfolio optimization process to improve the selection process, resulting in a portfolio with less tail risk.

1.2 Summary of Conclusions

The following are the major findings of this thesis:

1. In numerous simulations, we see that firms with firm-pair connections experience increased defaults, leading to heavy tailed default distributions seen in empirical data. These results provide insight into the characteristics of these firm-pair relationships.

2. Illustrated with the stylized network connections, the structure not just the number of the connections matter. The specific firm-pair relationships impacts the portfolio's risk exposure.

3. Through the simulation study, we see that a slower decay rate has more of an impact than a similar increase in the size of the initial exogenous contagion. This points to the importance of understanding what links firm-pairs together and how to measure the strengths of these connections.

4. Results for bond portfolios show that an optimization can be formulated to reduce the exposure to bonds with increased firm-pair connection strength.

The thesis is organized as follows: The remainder of this chapter reviews prior research in the areas of financial contagion, bond defaults and portfolio optimization.

Chapter two presents our proposed firm value model incorporating contagion effects to firm value. Also covered is the proposed bond portfolio optimization model which uses the results from the above firm value model to limit portfolio's exposure to heavy tailed losses. Details of the calibration of the firm value and contagion pieces of the model and the simulation analysis are discussed in chapter three. Results of the simulation and optimization models are discussed in chapter four. Chapter five presents a case study for two contagion incidents, illustrating the types of warning the proposed model would provide to potential investors. Discussion of computational requirements for the models is also included in this chapter. The final chapter contains remarks summarizing the insights gained from this thesis on the impact of contagion on a portfolio of bonds and discussion of directions for future work in this area.

1.3 Literature Review:Firm Value

Macroeconomic, sector and idiosyncratic risks are evaluated when constructing a portfolio of corporate bonds. The impacts of these factors have been extensively studied. On the other hand, contagion has also been studied in detail, but with limited attention given to high-yield bond investment and how contagion flow can impact defaults within a portfolio. We will now consider the research in models for firm value and contagion.

Limit of Existing models

The impact of contagion is missing in many models that are used to predict defaults. When these models are tested, their results often under predict the amount of defaults seen in the the extreme loss section of the default distribution. Accounting models used to predict bankruptcy have been studied to determine their accuracy in predicting default on investment grade bonds. Marchesini et al. (2004) considers four such models; two models based on accounting ratios and two based on cash flows to determine if these could be used to predict the default of bonds. The results were barely better than flipping a coin, leading them to develop a Logistic Regression model with 24 ratios and variables, including Total Debt/Total Assets, Return on Asset, etc.. They conclude that improved results lie in the use of a variety of variables.

Models of defaults are often based on the double stochastic assumption, stating that if conditioned on risk factors, all firms default intensities are independent Poisson arrivals with conditional deterministic intensity paths. Since these models only consider observable factors, they don't allow for contagion. Das et al. (2007) explains how an understanding of corporate defaults is important when putting together a portfolio of corporate debt instruments; stating, in particular, the clustering of these defaults as an issue. If defaults cluster more than predicted by various models, risk to a portfolio's value increases. The authors analyze defaults and corporate data from 1979 to 2004 to test the double stochastic assumption. Results fail to support this assumption, leading them to conclude that better models for default correlations are needed, and speculating that contagion could be a cause of this increased clustering.

Market data shows that as default environment worsens, correlation increases. Hull and White (2008) modeled these increased default correlations using a firm hazard rate based on a deterministic process subject to periodic impulses. They use these impulses to create the increased default correlations. The larger the impulse the more firms are impacted. In order to fit the model to the term structure of CDS spreads, an increase in impulse is required. This model is a simplification in that all firms are impacted at the same time by the same size shock, and no specific network structures are involved. It does, however, show that an impulse or exogenous contagion is required for increased default correlations to model market observed data.

Previous research leads us to believe there are more intricate relationships between firms than considered by many models when considering increased default correlations observed in the market. A better understanding of what is causing this clustering of defaults is needed to prevent bond portfolios from being exposed to a greater amount of risk than expected. Network structure-based relationships among firms are often utilized as a way to explain this default clustering. The spreading of a negative impact through these networks is often referred to as contagion. The impact of contagion on other entities such as financial institutions and countries has also been studied with some of the first models of contagion in these areas.

Contagion: Banks and Countries

Contagion spreading through the international banking system can cause severe global impact. In the seminal work by Allen and Gale (2000), contagion is not viewed as a random event, but rather driven by real shocks and linkages between entities. A network of banks and the impact of a liquidity crisis are used to illustrate a contagion spreading through the banking system. The size of the liquidity shock and the structure that connects the banks determine the impact of the contagion's spread, ranging from complete dissipation to causing total network systemic failure.

Research into the impact of contagion includes spreading between countries, see Pericoli and Sbracia (2003), Corsetti et al. (2005), across markets, see Kodres and Pritsker (2002), between Sovereign Bond Markets, see Bhanot et al. (2011) and among Hedge Funds, see Boyson et al. (2011). The credit crisis of 2007-2011 continues to show how shocks spread between linked nations and banks. Recently, Stiglitz (2010) argued that benefits gained by integrating countries into the global financial system, therefore allowing easier access to capital, can be overwhelmed by the impact of contagion spreading between countries due to these connections. This systemic risk often is modeled as two components – a random shock and a network that allows the transmission, see Martinez-Jaramillo et al. (2010). Contagion's impact on banks or countries can be extreme and is well publicized, leading to model development to enhance understanding of the issues providing insight into modeling contagion among firms.

Network Structures

Bank and country connections that spread shocks are easier to recognize and understand, although it is more difficult in the case of firms. However, research has been done to develop models which show that these types of connects also exist between firms. Many of the models that have been developed are reduced form or stylized models focusing on the resulting loss distribution versus the impact on firm value. When looking at loss distributions, increases in the tail sections are often taken as an indicator of contagion. Models have been developed to show how a contagion, consisting of an exogenous shock and a network structure that allows the contagion to spread, can result in the heavy tailed loss distributions seen in empirical data.

Various stylized models have been developed to represent the connections between firms and how this allows contagion to impact the loss distribution. These show how connections between firms can result in loss distributions that more closely match the reality. They, however, often do little to understand these connections. Davis and Lo (2001) develop a model for contagion in a bond portfolio. They use Bernoulli random variables to model default distribution and also the probability of this default spreading to other bonds in the portfolio. They show how the probability of the default spreading increases the risk of the portfolio. In this model the spread of contagion is controlled by an infection probability and is limited to a given sector. No specifics are given about the structure that allows the contagion to spread.

Random and directed graphs have also been used to represent the network structure between firms that allow a contagion to spread. Forcardi and Fabozzi (2004) propose a highly idealized static model for determining the credit loss distribution based on exogenous factors and contagion. They use a Random Graph model to form a network between firms. These connections are random and not based on any particular characteristic of a given firm. They conclude that these network connections impact the loss distribution. They suggest adding a parameter to represent these connections and then the loss distributions will more closely fit empirical data, especially in the tail areas. They, however, make no attempt to understand the business relationships that can cause these linkages or the impact of the individual firm characteristics. Egloff et al. (2007) also use directed graphs, but include empirical links and more realistic assumptions. They propose a dynamic, structural, stylistic model of contagion. The model considers macro and microstructure dependencies. These microstructure dependencies represent the links between debtors, which is the method allowing the contagion to spread among the debtors. It is shown that these interdependencies have a large impact on the tails of the loss distribution.

A structural model with the network based on a queueing model was developed by Cossin and Schellhorn (2004). Random and cyclical network structure are considered. Other stylized models of contagion have been proposed by Giesecke and Weber (2006), Kraft and Steffensen (2009) and Horst (2007). These models show how a contagion, consisting of an exogenous shock and a network structure, allow the contagion to spread and result in heavy tailed loss distributions. These models, however, lack the ability to increase understanding about the characteristics that allow the contagion to spread between firms.

Firm Connections

Research shows that actual connections do exist between firms allowing contagion to flow. Hertzel et al. (2008) present data to support their hypothesis that contagion can spread through supply chain relationships from one firm to rivals and suppliers of the firm. Historical data for approximately 250 firms that have filed for bankruptcy, their customers and suppliers are used to test this hypothesis. Negative daily abnormal returns on filing and distress day indicate the spread of contagion. The impact on the suppliers is more severe if the industry of the firm has been impacted by the contagion. This contagion can also be spread beyond reliant suppliers and major customers to their industries. Huang and Li (2000) analyze the spillover effect from Enron to other firms through their connections to the accounting firm Arthur Andersen. This empirical research provides support for contagion spreading due to an auditor's industrial specialty or geographical location.

Firm characteristic information is required to understand these connections, which is a premise of this thesis. In a similar vein, Egloff et al. (2007) use firm specific knowledge to determine the firm connections, focusing on weights determined by business volume of debtors with counter parties or predefined structures. These connections between firms can be modeled as random and directed graphs. Forcardi and Fabozzi (2004) proposed a highly idealized static model for determining the credit loss distribution based on exogenous factors and contagion. A random graph model, not based on characteristics of a given firm, is used to form a network. The loss distribution based on these connections more closely fit the tail areas of empirical data. Addition of a parameter to represent these connections is suggested; however, the possible business relationships that cause these linkages or the impact of individual firm characteristics are not explored. Egloff et al. (2007) also used directed graphs, but included links based on firm specific knowledge such as the "quality of portfolio" which is determined the by ratings of the firms.

Firm Value to determine defaults

Focusing on firm value to determine defaults is similar to the model outlined by Uhrig-Homburg (2005) where firm defaults are due to either liabilities exceeding a firms assets or a firm having insufficient cash flow to meet its obligations. The author builds on Merton's approach (see Merton (1974)) and adds the impact of default and bankruptcy. The Merton model is limited only considers an individual firms and, therefore, doesn't address the spread of a contagion to other firms. The firm value is determined based on unleveraged firm asset value, value of future payments, tax benefits, bankruptcy costs and equity issuance costs.

This thesis develops a structural model providing an intuitive interpretation based on the value of a firm. Factors impacting firm value include: macroeconomic, interest rates, sector, firm specific and the addition of a contagion factor. The contagion piece of the model is created reflecting the way a contagion propagates through countries, banks and firms; an external contagious event that spreads to other firms through connections. Contagion flow is segmented into variables impacting the spread of the contagion between firms.

Since investors often purchase bonds in a portfolio, this thesis will then propose a bond portfolio optimization using the results of the firm value model. The following section will present research in the area of bond portfolio optimization.

1.4 Literature Review: Optimization

Stocks and bonds are common and easily available investment options. The methods to create portfolios of these instruments, however, have not advanced in tandem. The work of Markowitz (1959) and the creation of modern portfolio theory have long been applied to create equity portfolios. Though the bond market is several times larger than the equity market, it wasn't until much later that these concepts were applied to fixed income instruments, see for instance Altman (1996). Part of this evolution is due to the unique risk factors that impact bonds; interest rate risk and default risk.

Risk: Interest Rate and Default

When focusing on interest rate risk portfolio immunization techniques, duration and convexity have long been used. Until Merton (1974) published his seminal paper on pricing of corporate debt, most analysis focused on interest rate term structure when determining corporate debt pricing. Merton's paper changed the focus by addressing bond pricing where there is a significant probability of default. Equations are developed for the risk premium, which is a function of the variance of the firm's operations and the ratio of the present value of the payment of the bond to the current value of the firm. These measurements confirm the importance of a firm's value and variability when developing accurate measures of default risk. Default risk is the risk that a substantial amount of the principal of the bond will be lost. Though this risk may be relatively rare, the impact is severe. Bond defaults can be caused by a wide range of negative shocks; from ones with a global impact to those that impact a specific firm. In addition, impacts can spread to other firms in the same sector, country, industry or due to other less obvious connections (partners in a joint venture, supply chain members, etc.). These connections can be viewed as linkages

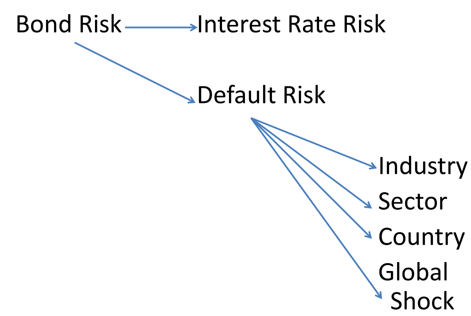


Figure 1.3: Disaggregation of Risk Factors Impacting Bonds

allowing an exogenous contagion to spread between firms. Depending on the type of shock, all firms in a connected group will see their firm values negatively impacted, resulting in increased correlations and possibly increased defaults. Figure 1.3 breaks down different categories of bond risk.

Altman and Bana (2004) analyze bond defaults from 1971 to 2003, reinforcing the view that defaults have significant and variable impacts on a portfolio. Mulvey and Zenios (1994) present data showing significant increases in variance and co-variance measures from 1981- 1991 in high-yield bond prices. These increases underscore the need to address default and correlations measures in bond portfolio optimization.

Optimization Models

Altman (1996) provides one of the earlier applications of modern portfolio theory to fixed income instruments and loans. Due to insufficient historical high yield bond data required for the classic mean-return model, an alternative risk measure is developed. The return measurement is calculated at yield to maturity minus expected loss. Yield to maturity ignores interest rate changes based on the assumption of randomness with an expected capital gain of zero. Expected loss is based on historical losses for bonds of the same ratings. The risk measurement, called unexpected loss, is determined based on the following:

1. Using information from quarterly financial statements such as (retained earnings/total assets) and (working capital/total assets), a "Z " score is assigned to each firm.

2. Comparing a firm's "Z" score to historical data representing numerous firms' Z" scores and related ratings, a equivalent rating is assigned.

3. Based on these rating equivalents, using historical data, standard deviation and correlations are determined around unexpected losses over a sample measurement period.

Heavy-tailed bond distributions are represented by unexpected loss. However, unexpected loss relies on the accuracy of bond rating data and the causes of unexpected loss are not addressed. Saunders et al. (2007) address optimal portfolio selection when considering allocation across different credit ratings. A factor model is developed based on macroeconomic and idiosyncratic factors impacting default. Large Portfolio Approximation is used to reduce computational effort. Optimization is based on minimization of CVaR and requires expected return to be above a set amount. Average risk free rate, plus the rating average credit spread, are used as proxies for expected returns. This model is, however, only considers bonds at the rating class level.

Altman (1996) and Saunders et al. (2007) also rely on bond ratings to quantify the risk inherent in an individual bond and limit the risk of a portfolio through controlling exposure to a rating's class. The quality of these ratings has been questioned most recently due to the financial crisis of 2007-2011. Additionally, these methods do not address correlations between bond defaults. With the goal of including individual risky corporate bonds in their portfolio optimization Kato and Konno (2007) use actual bond data to measure risk and return. They present both mean variance and mean absolute deviation optimization models. Bond return is based on the current forward rate associated with the planning period plus the bond spread. The bond spread is solved based on the current price of the bond and future cash flows.

Variance or Mean Absolute Deviation (MAD) compares the historical rate of return for the bond during the period minus expected returns. The results are superior when comparing integration optimization to an asset allocation portfolio. This method provides insight into return and variance measurements but doesn't address the issue of defaults.

Bond defaults are addressed for a portfolio of risk free debt, equities and risky corporate bonds in the reduced form model developed by Lui (2009). Unique business relationships between firms, including systemic, sector and close business ties, are used to impact default correlations. The model shows that different business relations between two firms determine the type of default correlation between the bonds issued by the firms. A flexible default correlation structure is presented where default intensity is a function of multiple state variables. However, since this is a reduced form model, it isn't able to consider individual firms.

Contagion Optimization

The above optimization models fail to address the clustering of defaults caused by a contagion. Consider a group of firms each having issued a bond and having business linkages, such as supply chain relationships. These linkages can impact default correlations that are not reflected in the portfolio risk assessment. Businesses with a supply chain linkage may not be in the same industry or sector. If one firm is impacted by an unanticipated event, this can pass to firm two by firm one's inability to pay its obligations to firm two due to a liquidity crisis caused by the original event. This continues throughout the network of connected firms due to cascading liquidity issues resulting in a contagion event. Additional measurements of bond risk to address defaults due to contagion would improve bond portfolio optimization.

In the following chapter, a Firm Value model is developed to include the impact of contagion flow between firms. An Optimization Model is then developed that uses simulations from the Firm Value model to optimize a portfolio with the goal of reducing the impact of heavy tailed defaults due to contagion.

Chapter 2

Model development: Firm Value and Optimization

In this chapter, a structural model for firm value is developed to explain the phenomenon of default clustering better. Various factors that impact a firm's value are included in different structural models. Moody's KMV commercially available structural model, as described by Arora et al. (2005), is based on default-risk free rate, a market risk premium, liquidity premium and expected recovery. In Uhrig-Homburg (2005) model, as previously described, the value of a firm is determined by valuing un-leveraged asset value, future payments, tax benefits, bankruptcy costs and equity issuance costs (for additional background information on structural models see Arora et al. (2005),Leland (2006), Crouhy et al. (2000), and Leland (2002)).

Our model will consider the following factors impacting firm value: macroeconomic, interest rate, sector and firm specific factors. Seeking to go beyond the common macro and micro-factors considered in many models, we include a contagion component.

The contagion model reflects an exogenous contagion impacting one firm, then spreading to other firms due to intra-firm connections. Egloff et al. (2007) uses firm specific knowledge to determine the firm connections. They focus on weights determined by business volume of debtors with counter parties or predefined structures and then selecting from 1 of 4 grouping for the "quality of portfolio" which determines the ratings of the firms.

Our model considers specific firm data, reflecting characteristics of the firm, to determine firm connections. Characteristics defining the contagion are size of an external contagion, the network through which contagion spreads to other firms and the speed and decay as it disseminates. The model represents a contagion similar to the way a contagion is actually seen, having an external event spreading to other firms through connections. Calibration of the model will be described, including the development of a framework to combine firm specific information with historical incidents of contagion to calibrate the contagion variables.

Firm value evolution begins with simulation of various factors including contagion. Evolution of firm value makes possible the analysis of events where the firm is close to default in addition to the default event. This value falling below a pre-specified threshold based on debt level is indicative of financial distress and the likelihood of a firm defaulting on its debt.

2.1 Model Development: Firm Value

Multiple factors go into the pricing of a bond; some are straightforward, such as interest payments and current market interest rates. A complication arises when the risk of the firm defaulting on the bond is considered. A common way of analyzing this risk is to consider the rating of the bond. The usefulness and accuracy of this rating has been recently called into question. Hilscher et al. (2008) point out the following flaws in credit ratings:

- All firms within a default category have the same default probability
- Time horizon is not associated with a rating
- Ratings are slow to change

Due to these reasons and concerns raised by the latest financial crisis, better ways to model the risk of default have become even more critical.

Various stochastic models for credit risk have been developed. Two main categories are reduced form and structural models. Brief summaries of these models are described by Leland (2006), Cossin and Schellhorn (2004) and Hilscher et al. (2008). Structural models are based on asset values. The two main determinants of a firm defaulting on a bond are the asset values and debt. Early structural models were developed by Black and Scholes (1973) and Merton (1974). Advantages include the intuitive nature of the model, its wide use in the industry and in financial literature. Reduced form or statistical models are not directly based on a firm's asset values. A variety of factors can lead to a firm default. Selection of these factors is based on matching their changes to the occurrence of historical defaults.

Arora et al. (2005) empirically compare two structural models and one reduced form model's ability to predict credit default swaps. They find that the more detailed structural model outperforms the reduced form model. In addition, the relatively straightforward Merton model, in certain situations, also outperforms the reduced form model.

Regardless of the type of model its ability to match the heavy tailed distribution of defaults that occurs in historical data remains a serious issue. Absence of factors for contagion in the models is often viewed as the cause for this issue. Adjustments are made to both classes of models to take contagion into account. As stated by Hilscher et al. (2008), such additional components in the model improve the predictive power of either model.

The model proposed in this thesis is developed as a structural model because asset values allow more straightforward interpretations. Reduced form models tend to use jumps to represent the impact to firms and stylized structures to represent connections between firms to create the heavy tailed distributions. These abstractions are more difficult to tie to the actual firm characteristics that influence the results. An intuitive way to match the actual default distributions, more fitting with a structural model, will be included in the model to represent the contagion process.

Structural models are a combination of numerous factors. Individual models need to be selected for each of the factors. Brigo et al. (2007) review various stochastic processes that can be used to build models for risky portfolios. Following are descriptions of some well known processes and reasons for their selection in a model.

Geometric Brownian motion (GBM) is a continuous time random walk stochastic process. It is widely used to model asset prices over time, in particular, in the Black-Scholes model. It is described by the following equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{2.1}$$

S(t) represents the asset price level which is impacted by the drift (μ) and standard deviation (σ) of the process. The third impact to the process is W which is the standard Brownian motion. This is a diffusion process that is normally distributed with a mean of zero, standard deviation equal to the square root of the time step, and independently identically distributed.

Taking the log, the following result can be derived, which is then used to simulate future values:

$$dlogS(t) = (\mu - (1/2)\sigma^2)dt + \sigma dW(t), \tag{2.2}$$

The process, however, is not mean reverting, exposing the results to a drift component, nor does it result in a heavy tailed distribution. Mean reverting equations have similar characteristics to Geometric Brownian Motion(GBM), with the addition of reversion to their mean value overtime, eliminating the drift component. Since the contagion piece will have a shock that dissipates overtime, the non-contagion factors should not contain a drift. For these reasons, the mean reverting process is a better fit than using GBM. The following is the mean reverting equation:

$$dS(t) = \gamma(\mu - S(t))dt + \sigma dW(t). \tag{2.3}$$

S(t) represents the asset price level which is impacted by the mean (μ) and standard deviation (σ) of the process. Now, in addition, we consider the rate of convergence to the mean (γ) and a standard Wiener process (W) replaces the standard Brownian motion described above.

Mean reverting equations are used for the non-contagion components of the model. This is then combined with the contagion piece of the model which addresses the issue of creating a heavy tailed distribution.

2.2 Outline of the Firm Value Model

The impact of various factors including contagion results in the evolution of firm value. When this value falls below a debt level threshold it indicates financial distress and the likelihood of a firm defaulting on its debt.

It is customary to model return on risky assets (such as portfolios or individual securities) using risk factors that disaggregate the total risk into various parts that are orthogonal to each other. The well known Fama-French (FF) Fama and French (1992) model uses three factors within a linear regression model to predict market rate of return of portfolios. The FF factors are market risk, risk of firm-size and a risk due to book-to-market value. In a similar manner, this thesis models the return on firm-asset value by decomposing the total risk into four non-contagion factors where each has a unique role to play in determining the return on asset (ROA) in excess of the risk free rate. As depicted in Figure 2.1, a contagion risk factor is also included in the model as a fifth factor. Premiums associated with each of these risk factors determine the expected return on the firm's assets.

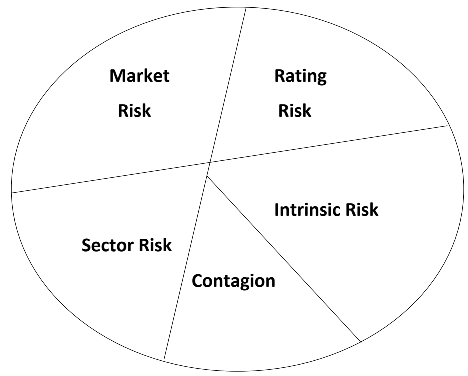


Figure 2.1: Disaggregated Risk Factors Impacting Firm Value

Risk factors impacting firm value:

1. Market Risk - Return of the market minus risk free return: Firms are exposed to risk due to changes in macroeconomic conditions. As stated in Altman and Bana (2004) there is a tie between economic performance and defaults. Common belief is that defaults increase at the beginning of a recession and peak when a recession ends, or shortly thereafter. Due to the impact of macroeconomic risk on firm value, a variable for economic outlook is included in the firm value model.

2. Sector Risk - Return of the sector minus return of the market: Sectors are groups of firms in similar businesses as identified by industries classifications. A portfolio may contain a selection of companies from one or multiple sectors based on an investors' views that this sector will outperform, either in return or stability, the market in general. Each sector has a unique risk profile. As an example, there are additional risks associated with firms in the technology sector compared to those in the more stable consumer staples sector.

3. Rating Risk - Return on Baa bond minus return on Aaa bond: Corporate borrowing costs impact a firm's value. To reflect the risk due to changes in these costs a measure of the interest-rate spread between Aaa and Baa type corporate bonds is included. As spreads widen, it presents greater risk for firm's perceived ROA.

4. Intrinsic (firm) Risk - Return of the firm's stock minus risk free return: Numerous firm-specific factors influence a firm's intrinsic(unique) risk. This factor captures the excess rate of return (long-term, such as quarterly, semi-annually or annually) that the firm's shares can command in the markets, which often is representative of the firm's fundamental managerial performance, see Fama and MacBeth (1973). Inherent company risk is the risk due to unique company factors; such as management/employee performance, intellectual property and possible litigation.

5. Contagion Risk: Consider a group of firms, each having issued a bond, and having business linkages such as common supply chain partners. If one firm is impacted by an unanticipated event, this can pass to another firm by a firm's inability to pay its obligations to the second firm. A liquidity crisis caused by the original event can continue throughout the network of connected firms due to cascading liquidity issues resulting in a contagion event. This risk is not reflected in the other four factors.

The impact of a contagion on a set of networked firms is modeled based on five parameters. The first two focus on the exogenous contagion represented by the arrival rate and amplitude of the initial impact of the contagion on one firm. The next three factors focus on how the contagion spreads to other firms in the network. The first is the network structure that exists due to relationships between firms which allows the contagion to spread to other firms. Supply chain relationships, reliance on the same bank, geographic locations (proximity to each other), same audit firm, common investment firm ownership or funding sources, alliances and common directors are possible examples. The remaining two contagion factors determine the speed and amount of decay of the exogenous contagion as it propagates through the network of firms.

The firm value model based on these five risk factors will be developed in the following two sections. The first section will address the first four risk factors. The addition of the contagion risk factor to the firm value model will be addressed in section 2.4.

2.3 Firm Value Evolution in the Absence of Contagion

The non-contagion evolution of firm value describes how firm value can evolve over time due to the impact of various risk factors on a firm's return on assets including: market, rating, sector and idiosyncratic risk, as described in section 2.2. The impact caused by these factors on firms is unique and thus separable. Additionally, contagion described in the following section propagates an exogenous impact to other firms independently of the other four risk factors. For a complete understanding of the impact of contagion at the firm and portfolio level, multiple firms will be modeled.

Definition of Terms:

V_i: Firm value of the firm issuing i^{th} Bond; i $\in [1, n]$, n: Number of firm issued bonds considered for portfolio

$r_v^i(t)$: Return on Asset (ROA) for i^{th} Firm

r_X: Return for individual factor X: m(Market index), f(Risk Free Rate), s(Industry sector), c(Firm specific), Aaa(Aaa Bond Rate) or Baa(Baa Bond Rate)

σ_x: Volatility of return for individual factor X

NC Net contagion - see section 2.4

Changes to firm value come from two sources, the rate of return on the assets of a firm and the impact of contagion. Combining the two sources, the firm value evolves as,

$$\frac{dV_i}{V_i} = r_v^i(t)dt + dNC_i(t). \tag{2.4}$$

The excess return over the risk-free rate in the firm's return on asset (ROA) is dependent on the additional risk factors an investor is exposed to through investment in the firm. These additional risk factors, as presented in Figure 2.1, describe the

ROA by the factor model:

$$r_v^i - r_f = \beta 1_i(r_m - r_f) + \beta 2_i(r_s - r_m) + \beta 3_i(r_{Baa} - r_{Aaa}) + \beta 4_i(r_{ci}), \tag{2.5}$$

where the β's represent the factor loadings.

The disaggregation of various risk factors, except for firm specific risk(r_{ci}), are straightforward as represented in equation 2.5. Firm specific risk(r_{ci}) represents only the risks associated with idiosyncratic factors related to performance. Equity returns(r_{ei}) can be used as a proxy for firm specific risk, however, as shown by the Capital Asset Pricing Model (CAPM) and the Fama-French Model (FF) (see Fama and French (1992)), equity returns are impacted by factors other than just firm specific. As stated in the FF, equity return can be disaggregated into the following factors: market risk, firm size,and book to market ratio (value) of the firm. Since market risk is already represented in equation (2.5), its effects will be eliminated from firms equity value, thus yielding a measure of idiosyncratic risk due only to specific firm characteristics. After accounting for the market risk, the residual returns leading to size premium and value premium, along with firm-specific idiosyncratic risk, are used as proxy in the factor model for ROA. Accordingly,

$$r_{ei} - r_f = \beta 5_i(r_m - r_f) + \epsilon_i. \tag{2.6}$$

Based on this equation, using historical data for return on equity values (r_{ei}), market return and risk free rate, $\beta 5$ is estimated representing the firm's sensitivity to market risk. The residuals, ϵ_i, are then the proxy for the firm specifc/idiosyncratic risk (r_{ci}), resulting in

$$r_{ci} = r_{ei} - r_f - \beta 5_i(r_m - r_f), \tag{2.7}$$

to be used as a factor in the ROA model in equation (2.5).

Since impact to firm value will be evaluated over an interval of time, the returns for the individual factor in equation (2.5) will evolve based on the mean reverting equation, see equation (2.3). The following equations reflect the evolution of returns for market, interest rates, sector and firm specific factors. Each factor with rate of convergence to the mean (γ), long term mean (μ) and a standard Wiener process (W), is represented as follows:

$$\begin{aligned} dr_m &= \gamma_m(\mu_m - r_m)dt + \sigma_m dW_m, &(2.8)\\ dr_s &= \gamma_s(\mu_s - r_s)dt + \sigma_s dW_s, &(2.9)\\ dr_{Aaa} &= \gamma_{Aaa}(\mu_{Aaa} - r_{Aaa})dt + \sigma_{Aaa} dW_{Aaa}, &(2.10)\\ dr_{Baa} &= \gamma_{Baa}(\mu_{Baa} - r_{Baa})dt + \sigma_{Baa} dW_{Baa}, &(2.11)\\ dr_f &= \gamma_f(\mu_f - r_f)dt + \sigma_f dW_f, . &(2.12) \end{aligned}$$

Evolution of Equity returns are based on

$$dr_e = \gamma_e(\mu_e - r_e)dt + \sigma_e dW_e. \quad (2.13)$$

This is combined with risk free and market returns in equation (2.7), representing the proxy for evolution of firm specific return. This results in a basic model for the evolution of a firm's value. Next we develop the contagion component of the firm value model, an often ignored aspect of firm value evolution.

2.4 Firm Value Evolution under Impact of Contagion in a Network

Unlike the non-contagion piece of the model, few examples exist in modeling contagion as part of a structural model. Similar to Martinez-Jaramillo et al. (2010), this thesis focuses on two separate components: an exogenous random event impacting an individual firm and a network-propagated contagion to other firms. The two components are described in terms of five parameters capturing the characteristics of evolution of contagion flow. Amplitude and arrival rate describe the exogenous contagion. Connections between firms define the path of contagion flow among the networked firms. The network is defined by three parts: the existence of a connection between firms, the delay in arrival and decay in the strength of the impact that the contagion experiences as it travels along the connections as presented in equations (2.14) and (2.15) below.

Definition of Terms:

R_i **:** Set of Neighbors of firm issuing i^{th} Bond; i $\in [1, n]$, n: Number of firm issued bonds considered for portfolio,

$NC_i(t)$ **:** Net Contagion at a firm (node) i at time t,

$C_i(t)$ **:** Exogenous Contagion at time t at firm (node) i at time t,

$\tau_{ji}(t)$ **:** Travel time for Contagion from firm (node) j to firm (node) i at time t,

$\lambda_{ji}(t)$ **:** Decay rate for Contagion from firm (node) j to firm (node) i at time t,

$A_i(t)$ **:** Stochastic amplitude of the exogenous contagion arriving at firm (node) i at time t,

$N_i(t)$ **:** Contagion arrival process, Poisson Process with rate $\mu_i(t)$ at time t.

For a firm i = 1,...,n , we define the evolution of Net Contagion ($NC_i(t)$) as follows,

$$NC_i(t) = C_i(t) + \sum_{j \in R_i} NC_j(t - \tau_{ji}(t)) \exp(-\lambda_{ji}(t)(\tau_{ji}(t))), \tag{2.14}$$

with

$$dC_i(t) = A_i(t) dN_i(t). \tag{2.15}$$

$NC_j(t)$ represents the contagion at all neighbors (those connected to i) that will arrive at firm (node) i in time τ_{ji} as it exponentially decays in intensity at a rate, λ_{ji}. Therefore, τ captures the fact that between two nodes, contagion takes finite time to travel and arrives at a node with a time lag since when it arrived at the neighboring nodes. λ describes the decay rate as the contagion travels. These two parameters are proposed to capture the complexity of contagion flow characteristics.

2.4.1 Contagion Factors

The following provides additional discussion on each contagion factor that appears in the contagion model described by equations (2.14) and (2.15).

Exogenous Contagion - Arrival Rate and Amplitude: Exogenously arriving contagion events are unexpected and rare events. Arrival rates reflect on average how often a contagion event is expected for a firm over a given time period. Characteristics of the triggering event will impact the strength of the contagion. The amplitude of the triggering event is used to reflect the strength of the exogenous contagion.

Contagion Spread - Connections, Decay Rate and Travel Time: Kannan and Kohler-Geib (2009) consider contagion spreading from one country to another and propose an "uncertainty channel of contagion," showing that a surprise crisis, unlike an anticipated crisis, increases the likelihood of a contagion spreading. When something unexpected happens, people trust their information less and react in ways more likely to contribute to the contagion spreading. Other contagion channels suggested by Kannan and Kohler-Geib (2009) included overexposed fund investor, trade links and common creditors. Similar ideas can be applied to firms impacted by contagion. With firm-pair connections, a negative surprise at one firm can travel to another firm due to linkages between the two firms. Just as the spread of information, in general, depends on the length and quality of the network it travels on, the spread of contagion will also depend on the strength of the linkage between the firms.

Firms can be linked in a network structure due to common relationships or characteristics. These specific firm-pair values represent the unique way that two firms are linked or connected to each other. The variables used to define the impact of these firm-pair connections are the decay rate and travel time. The decay rate represents the strength of the connection. The travel time represents the speed of transmission of contagion from one firm to another. These variables determine the speed and amount of impact an exogenous contagion will have as it spreads from one firm to other connected firms.

Connections: Business relationships are used by Liu (2009) to develop a flexible default correlation structure to optimize a portfolio of corporate bonds. The model in this thesis focuses on actual firm relationships or characteristics to determine connections between firms. The relationships can include joint venture involvement, shared directors, shared distribution channel and supply chains. It can also include financial channels, including banking relationships, investors and institutional ownership.

Similar to the random graphs used by Forcardi and Fabozzi (2004) and a directed graph model used by Egloff et al. (2007), this thesis models firms as vertices and the edges or links denote the relationship connections between firms. The edges or links are treated as directed, so the direction of flow of contagion can be captured. We provide a simple example to illustrate the essential network features to describe the contagion model.

Examples of Stylized Connections: A firm can be connected to multiple other firms with the direction determined by the nature of the connection. As illustrated in Figure 2.2, firm i is impacted by an exogenous contagion. Firm i affects firm j, which in turn spreads contagion to two other firms, as illustrated by the outgoing arrows. Connections can be represented as basic structures. In a serial network, the top example in the Figure 2.3, three firms are connected, allowing the contagion to spread from firm 1 to firm 2, and then from firm 2 to firm 3. An exogenous contagion can hit any firm and then spread downstream to other connected firms. The types of connections between firms 1 and 2 and firms 2 and 3 can be unrelated. Five firms connected allowing the contagion to spread from firm 1 to firms 2, 3, 4 and 5 in a star network is presented in the bottom of Figure 2.3. This illustrates a contagion impacting firm one with connections to multiple firms. An example scenario here

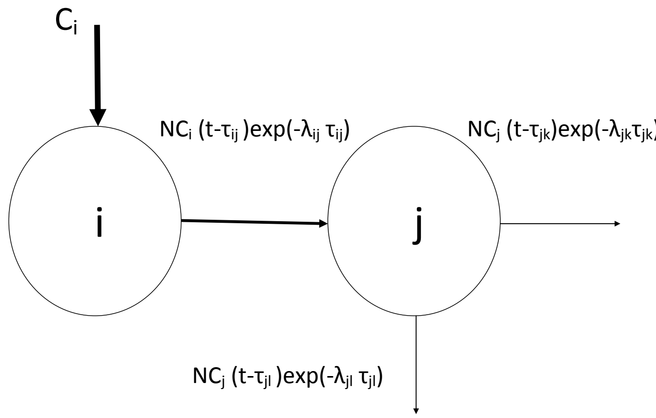


Figure 2.2: Firms i and j connected in a network

would be, firm 1 is a supplier to firms 2, 3, 4, and 5. Numerous larger structures can be created from these basic structures.

Travel Time and Decay Rate : The impact of a contagion as it travels from one firm to another is dependent on the specifics of the connection that exists between the two firms. These connections are ranked by the speed of information traveling on these channels (travel time) and the strength of the relationship between the two firms (decay rate).

Travel Time (τ) represents the delay for a contagion to travel from one firm to another connected firm. Time elapsed can be measured in various units such as days, weeks or months with $\tau \in [0, \infty)$. The larger the value of τ, the slower moving the contagion, and therefore contagion strength have a longer period to decay.

Decay Rate (λ) represents the strength of the linkage between two firms. The stronger the connection between firms the less the contagion will decay as the contagion impact spreads from one firm to another.

The amount of contagion that propagates from one firm to another connected firm is impacted together by the amplitude, travel time and decay rate. As presented in

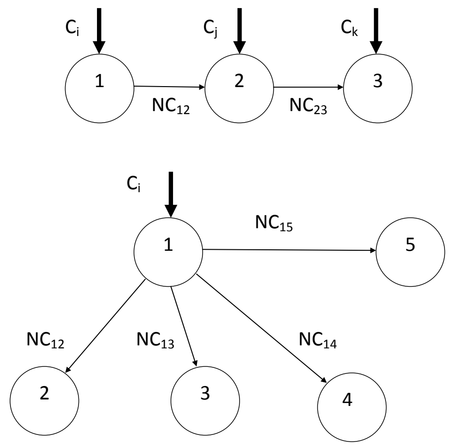


Figure 2.3: Illustrating Contagion flow: Serial and Star Network

equation (2.14), the amplitude of the exogenous contagion is multiplied by $e^{-\lambda\tau}$ to model the size of the propagated contagion. This model is selected since values of $e^{-\lambda\tau}$ range in the interval, $[0, 1]$, with 1 resulting in no decay, and 0 corresponding to complete decay. This implies when $\lambda\tau$ is greater than 4, i.e., when a contagion takes two time units to travel between firms with medium connection strength of 2, less than 2% of the exogenous contagion is experienced by a neighbor. At $\lambda\tau = 1$, a contagion that will arrive in one time unit between firms with strong connection, approximately 37% of original impact is experienced at the neighbor. The closer $\lambda\tau$ is to 0, the closer to the total propagation of the exogenous contagion, see figure 2.4.

2.5 Firm Value as a predictor of bond defaults

Firm value evolution provides insight into the financial health of a company. However, when focused on bonds, the main concern is the likelihood of default. The firm value model including contagion is combined with default boundaries to allow analysis of bond defaults. As described in Leland (2002) various methods can be used to

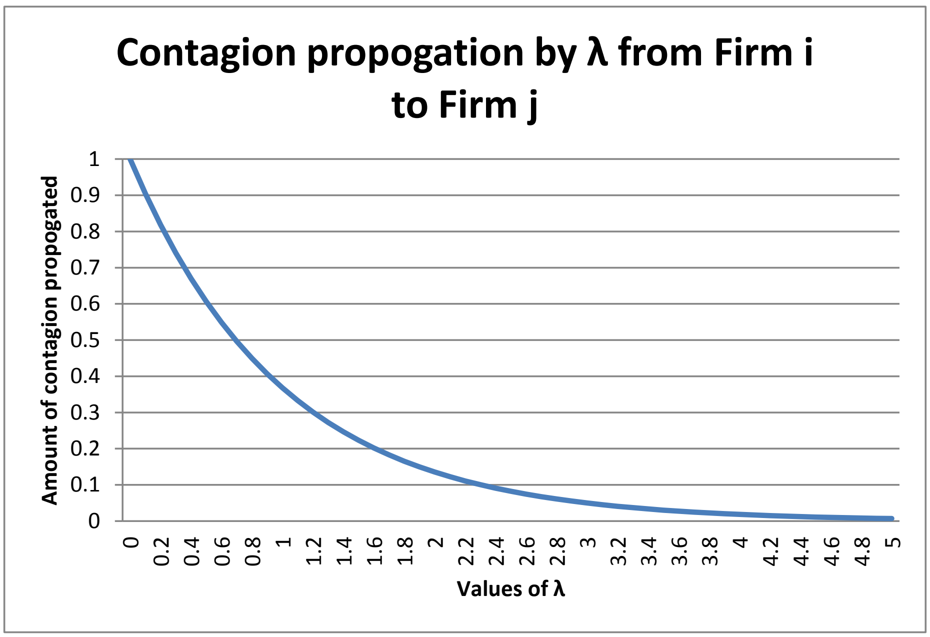


Figure 2.4: Impact of λ on Contagion Propagation

determine the default boundary. To project future debt levels(DL), an autoregressive AR(2) model

$$DL_t = c + \psi_1 DL_{t-1} + \psi_2 DL_{t-2} \tag{2.16}$$

is used. Debt level evolves over the same time interval as firm value, with default assumed to take place when firm value minus debt level becomes negative. Since the focus on the model is on defaults due to contagion, recovery amount as a percent of investment will be assumed to be similar for all firms and therefore will be ignored in the analysis. When bonds are homogeneous in terms of ratings and sectors we would expect similar recovery rates. If a more diverse pool of bonds is considered specific analysis of variation in recovery rates may be required.

2.6 Firm Value Models: Statistical Measurements of Contagion Impact

Having developed a firm value model, measurements illustrating the increased defaults, risk of default and portfolio impact, due to contagion and the network connections which allows it to spread, are necessary to quantify the impacts.

2.6.1 Firm Value Measurements

The goal of the firm value measurements is to quantify the impact of a contagion and the network on a individual firm. To have an improved understanding of a contagion's implications basic firm measurements need to be defined. These include: the impact on firm value, defaults and increased likelihood of default. To quantify how the network impacts each of these areas, measurements are calculated for the impact due to an exogenous contagion assuming no network structure ($VNC_i(t)$) versus a network structure ($VC_i(t)$). The percent increase due to the network ($PV_i(t)$) is then calculated as

$$PV_i(t) = \frac{VC_i(t) - VNC_i(t)}{VNC_i(t)}. \tag{2.17}$$

For a complete description of variables considered, see Appendix A.5.

2.6.2 Optimal Portfolio Measurements

When determining weights for bonds in a portfolio, the goal is to minimize portfolio risk for a given minimum return. The following describes the risk and return measurements used in bond portfolio selection.

Risk Measurement

The objective function reflects minimization of portfolio risk based on the Markowitz model. When the goal is controlling for default risk, standard deviation is not the best measure of risk. Therefore, a more appropriate proxy will be used for the portfolio risk equation. Addressing default risk requires a focus on variation of the lower tail, therefore, semi-standard deviation is preferred. In addition, to focus on how the variation is related to a firm's probability of default, firm value needs to be considered. Semi-standard deviation is combined with the mean firm value to calculate the coefficient of semi-variation for a firm CSV_i. This then replaces the firm standard deviation in the objective function of the Markowitz model:

Definition of Terms:

CSV_i : Coefficient of Semi-Variation

ρ_{ij} : Correlation coefficient

PR_{csv} : Portfolio Risk measure based on Coefficient of Semi-Variation

w_i : Investment weight on Firm i Bond

ρ : Correlation Coefficient

$$PR_{csv} = \sum_i \omega_i^2 CSV_i^2 + \sum_i \sum_{i \neq j} \omega_i \omega_j CSV_i CSV_j \rho_{ij}. \tag{2.18}$$

This creates a Portfolio risk measure more appropriate for controlling risk of default. The correlation coefficient (ρ) is based on stock prices for the firms in the portfolio.

Portfolio Return to Debt Holders

Bond return for an investor is based on the current yield on the day they purchase the bond. Therefore, the current yield on the day the portfolio is being determined will be the return measure for the debt holders. For a given portfolio, a set level of return will be required. Returns, however, are also impacted by defaults. This will be controlled for by setting a maximum portfolio default rate. Default rate will be determined from the results of the firm value simulation. Since the focus of the model is not on returns based on fluctuations in bond price over the simulation period, they will be ignored.

Portfolio Level Measurements

Based on the portfolios determined by the above measurements, the impact on the portfolio due to contagion will be determined based on the shift to the efficient frontier. This measurement, however, fails to address the issue of most concern in a portfolio of bonds - multiple defaults, resulting in heavy tailed default distributions. In the following section an optimization, based on the firm value model, addressing this issue will be developed.

2.7 Bond Portfolio Optimization Model Focused on Tail Risk

Investment grade bonds are an attractive investment alternative due to known and consistent income streams and higher yields than Treasuries. The goal in selecting a portfolio of bonds is to maximize wealth (return) while controlling interest rate risk and the risk of substantial declines in the value of a portfolio due to default. Various methods exist to immunize a portfolio of bonds against interest rate risk. The goal of this optimization is protection against defaults. The increased yields of high yield bonds reflects the increased likelihood of default. A singular default in a portfolio, therefore, is not totally unexpected. The most debilitating situation for an investor, however, is when multiple bonds default in a portfolio.

Risk varies by firm due to their unique characteristics. The risk and return tradeoff of the Markowitz model provides a framework to select an optimal portfolio of equities. Controlling for a given level of return, while minimizing the risk of variations in return value, are issues often addressed in portfolio optimization. Bond portfolios often control for risk by limiting the amount of investment in a specific ratings class, see Saunders et al. (2007). These methods, however, do not address a negative event at a firm contagiously spreading to other firms via firm connections resulting in an extreme event of multiple defaults in a portfolio of bonds. This section builds on the firm value with contagion model to create an optimization framework to limit the exposure to the heavy tailed section of the loss distribution seen in empirical bond default distributions, see Das et al. (2007).

2.7.1 Return Measure:

A firm specific measure of return for each bond is required. Bond yields reflect both macroeconomic interest rate risk and company specific default risk. Since the portfolio will consist of publicly traded bonds, current yields are used as the return measure. Bonds will be held selected at the beginning of the simulation period and will be held for the entire simulation. Fluctuations in yields during the simulation will be ignored.

Portfolio returns will be determined from these current yields as follows:

$$R_p = \sum_{j=1}^{n} E[R_j] X_j \tag{2.19}$$

Definition of terms:

j: Bond j $\in$ [1,n], n number of bonds in portfolio

p: Portfolio level measures

X_j: amount(percent) of portfolio invested in bond j

R_j: Bond current yield for bond j

Risk Measure I: Variability of Return

Return for the selected portfolio of Bonds is based on the current yield when the bond is purchased. Fluctuations resulting in increasing yields are not considered in this optimization. Decreasing yields however can indicate distress for the firm including the possible increased risk of default. A measure to represent this variability of the bond portfolio return,

$$Var(R_p) = \sum_{j=1}^{n} Var[R_j]_j X_j^2 + 2 \sum_{i,ji<j}^{n} Cov(R_i, R_j) X_i X_j \tag{2.20}$$

is based on Covariance(Cov) representing the correlations of firms' yields due to the impact of interest rate changes and exposure to other risk factors that impact both firms. Allowing for control of the likelihood of a price change, which in severe cases can indicate increased default risk. The highest risk scenario to a portfolio investor, which is not addressed by equation (2.20), however, is correlated defaults resulting in a heavy tailed loss distribution. Additional measures of risk which focus on defaults and correlated defaults are thus required.

2.7.2 Risk Measure II: Risk of default

Events impacting a firm can take many forms; an economic or industry event leading to substantial drop in sales, a global event that impacts lending policies or an individual firm event, resulting in decreasing firm value. When the firm value drops below a company's debt, a default is an increasingly likely occurrence.

Defaults of individual bonds result in substantial loss in value to a bond investor. In addition, these defaults tend to cluster resulting in extreme losses occurring with higher probabilities than predicted by many models. Contagion flow between firms, contained in the Firm Value model, see equation(2.4), can be used to model this clustering of defaults and needs to be considered in portfolio selection.

Based on the Firm Value model, defaults are modeled as a jump diffusion process. Contagions arrive to a firm resulting in a negative jump in firm value. Depending on the size of the jump and the health of the firm, a default may occur. A firm is considered to have defaulted when a prescribed threshold level (T), a firm's projected debt is equal to or greater than a firm's value, is met. To represent whether a default has occurred, an indicator random variables I_j is defined. Using the event that firm j defaults (D_j) and its complement ($\bar{D_j}$) when no default has occurred, i.e.

I_j=1 if D_j occurs and I_j=0 if $\bar{D}_j$occurs.

Due to connections between firms, a contagious event can spread to other firms causing similar negative jumps in their respective values resulting in multiple firm defaults in the portfolio. This can be viewed as a counting process, representing the number of defaults that have arrived in the portfolio. Due to the contagion flow, the arrival of these defaults is no longer independent but correlated due to the contagion spreading through connected firms.

From an investor's standpoint, the amount of investment loss due to defaults is of prime concern. Therefore, the random variable representing Total Investment Loss for a given level T is:

$$L(T) = \sum_{j=1}^{n} I_j(T)Z_j \tag{2.21}$$

where Z_j denotes the loss given default for a specific firm. The expected value of the random variable L is:

$$E[L] = \sum_{j=1}^{n} E[I_{Dj}]Z_j \tag{2.22}$$

Though a low probability event, due to the extreme impact on portfolio value, measures need to be considered to reduce exposures to default tail events. VaR and CVaR (see Sarykalin et al. (2008)) are additional risk measures that control for tail risk to the overall portfolio better than the use of variance alone. VaR and CVaR are used to control maximum loss that is acceptable at a given confidence level (β). VaR gives the value of loss at β whereas CVaR gives the weighted average of loss at values equal to or greater than VaR at the same β. Winker and Maringer (2007) investigate the issues and impact of using VaR for bond portfolio optimization, sighting its quadratic nature often requiring a heuristic optimizations to reach a solution. An additional drawback includes the results aren't sub additive, resulting in the possibility of the actual risk of a portfolio being greater than the sum of the individual bond's risk. CVaR, on the other hand, doesn't have these limitations and therefore is a better choice to include in an optimization. Equation (2.23), developed in Saunders et al. (2007), determines CVaR from an ordered set of simulated losses based on equations from Rockafellar and Uryasev (2000). This equation creates a weighted average of VaR (first term to right of equal sign) and CVaR(second term).

$$CVaR = \lambda z_{i*} + (1-\lambda)\Sigma z_i/(n-(i^*+1)). \tag{2.23}$$

Definition of terms:

Z_i: ordered listing of loss due to default from simulation

Z_{i*}: VaR_β

λ: $(i^*/n-\beta)/(1-\beta)$

β: Confidence level, 1- α

2.7.3 Optimization Model:

Based on Markowitz (1959), the model addresses portfolio risk while controlling for return to determine weights of instruments in a portfolio. To address default risk in a bond portfolio, the model introduces multiple risk factors including risk due to both variability in return, default and multiple default distributions. Roman et al. (2007) present an optimization model that controls for two risk factors. Their model minimizes variance while controlling for the expected value and CVaR. The proposed optimization also includes multiple risk measures.

Minimize CVaR(L)

Subject to:

(1) $E[R_p] = \Sigma_{j=1}^{n} R_j X_j \geq R_0$

(2) $E[L_p] = \Sigma_{j=1}^{n} E[I_{Dj}] X_j \leq L_0$

(3) $Var(R_p) \leq V_0$

(4) $\sum_{i=1}^{n} X_i = 1$

(5) $X_i \geq 0$

Due to the extreme impact of multiple defaults, CVaR will be minimized while controlling for: (1) Expected return for the portfolio above a required level, (2) Expected loss of the portfolio below a required value and (3) Variance of the return measure below a set value.

With the Firm Value and Optimization models defined, the next chapter addresses calibrating these models.

Chapter 3

Calibration and Simulation

Calibration of the model is critical. Variables exist in the model that need to be determined based on historical data. Empirical data used to calibrate the non-contagion piece of the model is straightforward, relying on MLE's or OLS estimation. The difficulty of calibrating the contagion piece of the model is rewarded by the additional firm-specific insight gained when compared to a reduced form model. A framework is developed in this thesis to use firm-specific information, data from historical contagious events and insight gained from previous research to assign values to the contagion variables.

This chapter addresses:

- Calibration of the no contagion factors
- Development of a framework to calibrate contagion variables. Providing a detailed 3 firm illustration and a summary of a 10 firm calibration
- Calibration of an exogenous contagion
- Data for the optimization model
- Simulation of the firm value model with contagion

3.1 Calibration of No Contagion Factors

The no contagion factors of the model are calibrated using historical data. The following lists the data used to quantify the risk factors in the firm value model. Section A.4 provides more data source information.

- Market: S&P 500
- Rating: Baa and Aaa yields
- Risk free return: 1 year Treasury
- Sector: Nasdaq Biotech (for instance)
- Firm: Stock Price

Mean Reversion Equations: Mean reversion equations are used to simulate the future values of factors that will impact the rates of return for each company. A process similar to that outlined by Smith (2010) for estimation and simulation of mean-reverting Ornstein-Uhlenbeck process is used. Historical data (July 1, 2007 until June 30, 2011), taken at 10-day intervals, to represent bi-monthly intervals, is used to calibrate the mean, standard deviation and reversion rate for the factors affecting the return on assets of the firms. Maximum Likelihood Estimation (MLE) is used to arrive at these values. Results are used to simulate one year of bimonthly values for each of the factors. Section A.2 provides the MLE estimates for each risk factor parameters.

Return on Assets Equation: Quarterly data (March 2006 - June 2011) is collected for the return on assets of each firm and for factors impacting ROA. Using Matlab, the β's in equation (2.5) are estimated. See Appendix A.3 for the results of the calibration.

3.2 Framework to Calibrate Contagion Factors

Calibration of the contagion component poses unique issues due to the challenge of extracting the impact of a contagion event. Due to this challenge, stylized structures have been used to simulate the spread of contagion, see Cossin and Schellhorn (2004) and Horst (2007). Since these structures aren't based on actual firm data, they fail to provide insight into what characteristics impact the flow of contagion. We develop and discuss a framework, consisting of four procedures, to address the challenge of calibrating the contagion model. This approach results in the ability to consider the impact of contagion at the firm level, tied to the unique characteristics of a firm, providing valuable insight in portfolio selection.

Understanding how contagion spreads between firms first requires documentation of characteristics that may define these connections broadly in the general population of firms and contagion episodes. The first two procedures of the calibration framework focus on defining these firm characteristics, along with developing an insight for their relative importance. Historical contagion episodes are examined as the next step of the framework, assigning values of τ and λ for each firm-pair in contagion episodes, translating this to contagion predictive capability of each firm characteristic. The final procedure applies the output of the first three procedures to a set of firms being analyzed for inclusion in a portfolio. Information on firms is collected based on the first two procedures and combined with the constructs of the third procedure to assign a network structure for the firms considered. The layout of the framework is presented in Figure 3.1. The top section of three boxes applies to the entire population of data, the lower box applies to each firm-pair. The procedures are stated in a specific order, however, iterative improvements may be needed as insights are gained while progressing through the four procedures.

Procedure 1: Define Categories (C) of Firm Characteristics

1. Firm linkages can occur by different functional areas of firms. We first explore and identify functional areas that may, or are known to, facilitate contagion

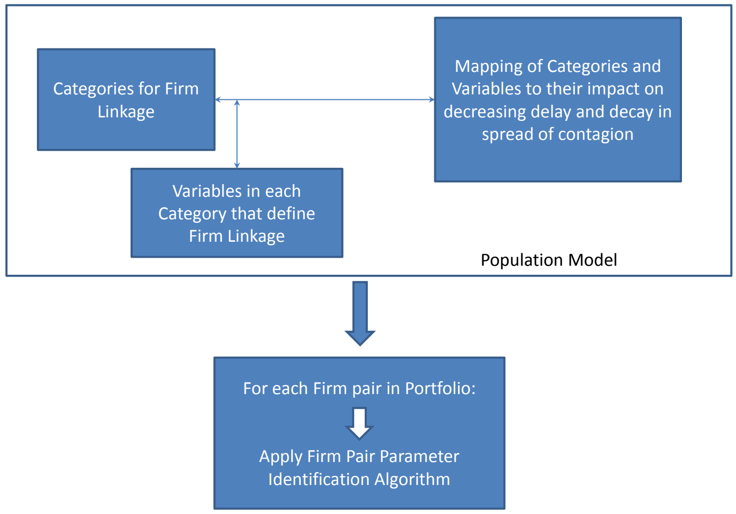


Figure 3.1: Framework to establish Contagion Spread Variables

flow. We call these the category of firm linkages. Categories may include: financial (ability to raise funds), operations (ability to deliver product or service, acquiring raw material or parts, research and development, joint ventures), marketing (firm's image) and support functions (ancillary parts of the firm).

2. Categories may have relative importance in their ability to be contagion inducing for firms. To bring attention to this aspect, categories are organized in decreasing order of business importance, such as, $C_1 \geq C_2 \geq C_3 \geq C_4$. An a "priori" insight may guide in creating this relative importance, or data on contagion events obtained in Procedure 3 may be used to validate or create this relative importance.

Procedure 2: Define Variables(V) within Categories

1. Within each category or functional area, there may be a variety of specific ways that firms may be inter-related. To increase resolution on the nature of linkage between firms, each category is further classified into variables that define firm linkages. Variables within each category are ascertained to have a unique impact

on the speed and decay of contagion flow between firms. Examples of variables within categories include:

Finance: Banking Relationships (sources of lines of credit or large loans), Institutional Investors, or other Funding Sources

Operations: Supply Chain or Joint Venture partners

Support: Audit firm, IS Support firms, Product Design partners

Marketing: Geographical Location, Competitors, Consultants

See Appendix A.1 for a more detailed listing and description of Category/Variables.

2. Drawing attention to more critical variables within a category, we organize variables in decreasing order of priority. The higher the priority, the greater the link strength anticipated by the presence of that variable in a firm's linkage: $V_1(C_i) \geq V_2(C_i) \geq V_3(C_i) \geq V_4(C_i)$.

Procedure 3: Populating Contagion Factors $\theta_{\tau cv}$ and $\theta_{\lambda cv}$

The following describes how historical incidents of contagion are used to determine values of τ and λ for a firm pair (i, j). These values are then translated to obtain the values for τ and λ for specific category/variable. We first define a few additional terms.

τ_{max} **and** λ_{max} : correspond to weakest (non-existent) link strength, implying it takes the longest time for contagion to arrive from a neighboring firm, and the decay is almost immediate.

τ_{min} **and** λ_{min} : corresponds to strongest possible link strength, where it takes the shortest amount of time for contagion to arrive at a neighboring firm with almost no decay.

$\theta_{\tau cv}$ **and** $\theta_{\lambda cv}$: contagion delay and decay contribution of cv: variable, v, in category, c.

τ_{ij} **and** λ_{ij} : delay and decay strength of a link between firm (i, j) due to contribution of all relevant categories/variables defining the firm linkage. The models for determining λ_{ij} and τ_{ij} is described in the following two equations;

$$\lambda_{ij} = \lambda_{max} - \sum_{cv \in ij} \theta_{\lambda cv}, \tag{3.1}$$

$$\tau_{ij} = \tau_{max} - \sum_{cv \in ij} \theta_{\tau cv}. \tag{3.2}$$

Based on the assumption of relative independence among categories and variables, it is an additive model.

In order to determine $\theta_{\tau cv}$ and $\theta_{\lambda cv}$ for each category/variable based on historical data, a large sample of a population of firms is required for contagion spread events between firms. Using these data, we will first determine values for τ_{ji} and λ_{ji} for each firm-pair in the data, and then after identifying relevant categories/variables for each firm-pair, convert these to values for $\theta_{\tau cv}$ and $\theta_{\lambda cv}$. The steps adopted are as follows:

1. For a large population of firms, evaluate changes in firm values for unanticipated significant impact not due to underlying market changes. Similar to Chaney and Philipich (2002) and Hertzel et al. (2008), percent abnormal equity return time series data is used in our analysis as a proxy for the impact of a contagion. Abnormal equity returns indicate an unanticipated event occurring not related to underlying market fundamentals. These changes in equity value are assumed to be indicative of changes in firm value. An event that causes firm value to decrease will be reflected in the equity value of the firm. Table 3.1 presents a three firm example; firm 1 is impacted in period 2 and firm 3 in period 3, while firm 2 is not affected.

2. Collect category and variable data for firms involved in purported contagion events to populate the category/variable structure defined in procedures 1 and 2. When firms have the same data for a category/variable, a connection is

Table 3.1: Percent abnormal return over time: 3 Firm Example

Percent Abnormal Returns

Time Interval	1	2	3	4	5	6	7
Firm 1		20					
Firm 2							
Firm 3			5				

Table 3.2: Category/Variable Matches: 3 Firm Example

Category/Variable Matches

Firm-Pair	1	2	3
Firm 1-2	C_1V_2		
Firm 1-3	C_1V_2	C_2V_3	
Firm 2-3			

suspected. Table 3.2 illustrates connections for the 3 firm example. Firm 1 has connections to both firms 2 and 3.

3. Determine if a category/variable match exists between firms with abnormal returns within an acceptable time frame. This would indicate contagion has spread from firm j to i based on connection established between the firms, as illustrated in the 3 firm example by comparing the data from Tables 3.1 and 3.2. Firms 1 and 3 both have abnormal returns within 1 time interval and match on C_1V_2, whereas firm 2 shows no impact from contagion.

4. When a contagion is indicated at firm level, determine if an exogenous contagion or a contagion,has spread from one firm to another.

Exogenous Contagion (C_j) : When abnormal returns are seen at a firm and no neighbors show abnormal returns in an earlier time period, assume an exogenous contagion impacting this firm, therefore $NC_j = C_j$.

Contagion from a connected firm, (NC_i) : Indication of a contagion spreading from firm j to a neighbor firm i are abnormal returns for firm i

preceding those for firm j. We infer the size of impact on firm i from endogenous contagion, and set NC_i equal to the impact contagion has on firm i.

Identify τ_{ji} : Set τ_{ij} equal to the time gap between abnormal returns of firm j and firm i. It is assumed this is the result of the contagion spread from neighbors and that no exogenous contagion has effected this firm in this time period.

Solve for λ_{ji} : Using τ_{ji}, NC_i, NC_j and C_i (based on equation 2.14), compute λ_{ji}.

For the 3 Firm example: firm 1 experiences a 20% abnormal return ($C_j = NC_j$ =.2). One time period later ($\tau_{ji} = 1$) firm 3, a neighbor of firm 1 (connections C_1V_2), experiences a 5% abnormal return ($NC_i = .05$) and ($C_i = 0$).

5. We next need to convert firm-pair λ_{ji}'s and τ_{ji}'s to expected values for individual category/variable, $\theta_{\lambda cv}$ and $\theta_{\tau cv}$. Since multiple historical contagion incidents can impact a specific firm pair, an additional subscript, p, is added to indicate a specific occurrence. Since a linear relationship is assumed between λ_{ji}'s, τ_{ji}'s and ($\theta_{\lambda cv}$, $\theta_{\tau cv}$), as given in equations (3.1) and (3.2), we need to simultaneously solve for all the ($\theta_{\lambda cv}$, $\theta_{\tau cv}$) from these set of equations. We can define appropriate matrices, A, to state this set of equations, as shown in Table 3.3. In the table, the first row is set to one to represent λ_{max} or τ_{max}. For the entire population of firms pairs, this represents the values to indicate a baseline negligible connection. When the listing of category/variable for firms is incomplete, this baseline may capture unaccounted for determinants for contagion. The use of one value for λ_{max} assumes the same level of connection knowledge for all contagion incidents.

6. Create a vector, b_1 and b_2, of values of λ_{ji_p} and τ_{ji_p}, respectively, for all firm-pair and all incidents. This is the connection strength for a given firm pair

Table 3.3: Matrix A: Category/Variable Matches

Firm Pairs(p)	λ_{max}	C_1V_1	C_1V_2	C_1V_3	...	C_2V_3	...	C_sV_t
1 2(1)	1	0	-1	0	0	0	0	0
1 3(2)	1	0	-1	0	0	-1	0	0
1 4(3)	1	0	0	0	0	-1	0	0
1 4(4)	1	0	0	0	0	-1	0	0
....								
n n (p)	1	0	0	-1	-1	0	0	0

Table 3.4: Vectors: X and b

X = θ_{cv}	b = λ_{ij_p}
λ_{max}	λ_{12_1}
θ_{11}	λ_{13_2}
θ_{12}	λ_{14_3}
θ_{13}	λ_{14_4}
...	...
θ_{st}	λ_{nn_p}

from one specific historical incident. See right half of Table 3.4. Based on the above values; solve for the best fit, X, for $AX = b_1$ and $AX = b_2$. This will provide the values of ($\theta_{\lambda cv}$, $\theta_{\tau cv}$), the change in connection strength due to a specific category/variable. The value of (λ_{max}, τ_{max}) will represent the weakest connection strength for this set of data collected.

Procedure 4: Populate Contagion Variables (λ_{ji} and τ_{ji}) for a Portfolio of Firms

Based on the output from the previous three procedures, contagion variables for a specific portfolio of firms will be determined as the final step of the contagion model calibration. The following describes the algorithm used to arrive at the values of τ_{ij} and λ_{ij} as an iterative process, to represent the delay and decay of contagion propagation in the network of firms in the portfolio.

Algorithm for Contagion Variables

1. Analyze firm data for each firm in the portfolio to determine the relevant firm category/variable matrix created in procedures 1 and 2.

2. Compare all firms for matches on firm category/variable matrix. For each match, collect values of $\theta_{\lambda cv}$ and $\theta_{\tau cv}$ from procedure 3.

3. For a firm pair (i, j) in the portfolio, adopt the following steps to determine the strength of each firm-pair link.

    (a) Begin by setting $\lambda_{ij} = \lambda_{max}$ and $\tau_{ij} = \tau_{max}$, at the baseline.

    (b) For all categories/variables identified as relevant for firm-pair (i, j), as per order of importance identified in procedures 1 and 2, arrange the $\theta_{\lambda cv}$ and $\theta_{\tau cv}$ in decreasing order.

    (c) For n = 1: number of category/variables relevant for firm pair (i, j);
    $\lambda_{ij} \leftarrow \lambda_{ij} - \theta_{\lambda cvn}$;
    $\tau_{ij} \leftarrow \tau_{ij} - \theta_{\tau cvn}$.

4. Pick another firm pair (i, j), and go to step 3. If all links in the firm network are calibrated, stop.

Trajectories for updating firm pair strength are shown in Figure 3.2. An illustrative example for this procedure for a three firm portfolio is provided in the following Section 3.3. Application to a larger portfolio is illustrated in Section 3.4.

3.3 Illustration of framework: Three Firm Portfolio

A portfolio of three biotechnology firms: Biogen Idec Inc.(BIIB), Gilead Sciences, Inc.(GILD) and PDL BioPharma, Inc.(PDLI) illustrates the use of the preceding

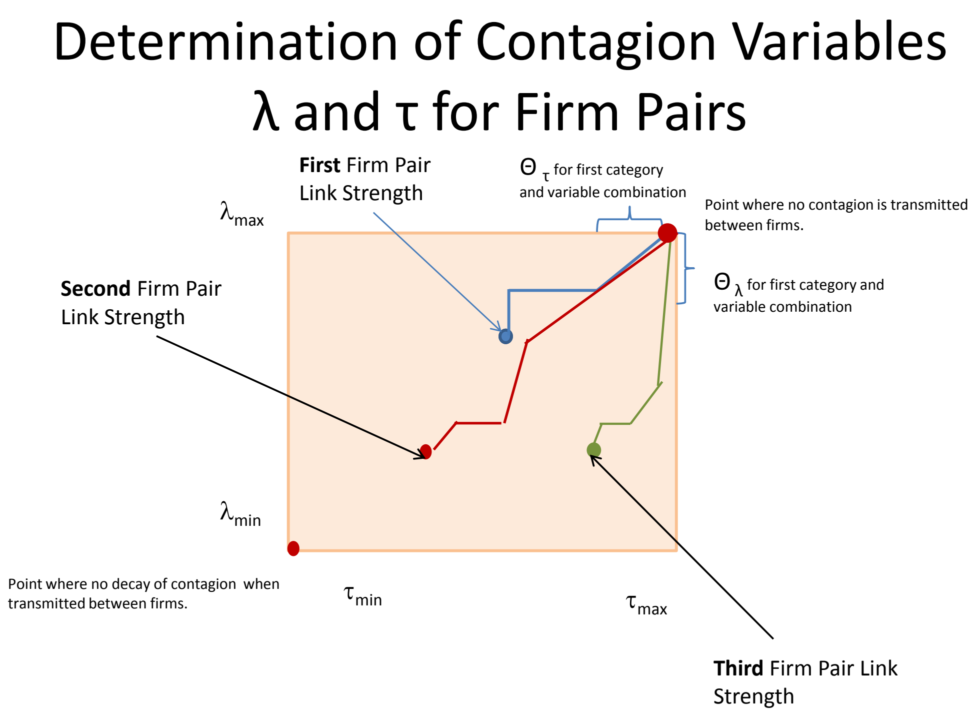


Figure 3.2: Trajectories for updated firm pair strength

framework to arrive at values for the contagion variables. As an alternative to the sizable amounts of data required to populate values of $\theta_{\tau cv}$ and $\theta_{\lambda cv}$, for this illustration, a sample of three historical contagion incidents is analyzed. To exemplify the calibration process, each event employs one major category/variable that dominates the firm-pair connection.

Procedure 1 and 2: Categories and Variables

The category/variables being considered, listed in order of decreasing priority, follows. This ordering of categories and variables will be confirmed from the results in procedure 3 and can be used to estimate values when data is limited.

1. Support: Impact to areas that provide support functions to the firm.

    - Same Auditor: As seen in the Enron case, questions about the practices of a firm's auditor or other financial irregularity result in doubt about a firm's accounting reports and will impact a firm's ability to raise funds. Other similarities between firms can result in increasing the impact.

2. Financial: Events that impact a firm's cost or access to funds.

    - Institutional Ownership: An Institutional investor with financial stakes in similar firms, such as firms in the same sector, may withdraw money from all firms if sentiment changed for one firm in the group due to an external event.

3. Marketing (firm's image): When a negative event occurs at one firm, investors will reevaluate other firms with similar images or characteristics, viewing these related firms as less valuable due to concern that they share the same issue as the original firm.

    - Competitors: When a negative event impacts a firm, this event can spread to competitors due to similarities between the firms.

4. Operations: Connections that exist between firms can impact a firm's ability to deliver a product.

    - A supply chain relationship allows a failure at a supplier to spread to other firms due to the inability to pay bills or supply parts needed to meet demand.

    - Joint ventures are close associations between firms. If the venture or one of the partners suffers a setback, the impact will affect the other firms.

Procedure 3: Determine $\theta_{\tau cv}$ and $\theta_{\lambda cv}$ based on Historical Research

First a general description of the application of procedure 3 to the sample cases will be presented. The results of this procedure will follow as applied to three incidents of contagion spreading through firm connections: Arthur Andersen (2001-2002), the US auto industry (2007-2009), and AMLYN (2010).

1. Analyze specific contagion events. Determine firms believed to be impacted and the dates of suspected significant events which could result in a contagion.

2. Collect firm equity returns (r_j), an appropriate market index return (r_m) and short term treasury risk free rate (r_f) for approximately one to two years preceding (pre-contagion) and two months subsequent to the significant event (contagion period).

3. Based on pre-contagion data, calibrate CAPM model

$$r_j = \beta_j(r_m - r_f) + r_f + \epsilon_j, \tag{3.3}$$

    determining value for β_j.

4. Based on β_j, market index returns and risk free rate, use equation (3.3) to calculate expected equity return values (r_j^{exp}) during contagion period, controlling for the impact of general market changes. Daily Actual Returns (A_j) are calculated based on equity prices during the contagion period.

5. Calculate Abnormal Returns (AR) based on

$$AR = A_j - r_j^{exp} \tag{3.4}$$

for each firm as a proxy for the percentage impact to a firm's value due to an unexpected event.

6. Calculate cumulative abnormal returns (CAR) over a 10 day window, to focus on sustained changes and avoid one day event which are canceled out by the following days result.

7. Evaluate CAR for the contagion period for each firm. Again since sustained change is desired, focus on multiple days of significant CAR (less than - 5%). From these data points, select the middle CAR value and assigning it to λ_{ij} for this firm-pair contagion incident.

8. Calculate the number of days between the contagion event and selected λ_{ij} for each firm. Convert this into τ_{ij}, number of time periods, based on the number of days in a simulations time interval.

9. Collect λ_{ij} and τ_{ij} for all firms. Based on equations 3.1 and 3.2, solve for values of $\theta_{\lambda cv}$ and $\theta_{\tau cv}$. Due to the limited data, solving these systems of equations is not straight forward. Additional constraints based on a decay rate between 0 and 100% and $\theta_{\lambda cv}$ and $\theta_{\tau cv}$ being less than λ_{max} and tau_{max} respectively, will be included in the optimization to determine an acceptable solution.

This procedure is applied to our historical incidents, as described below.

Historical Incident One: Auditor, Arthur Anderson

An exogenous event impacted Enron when questions were raised about the practices of its auditor: Arthur Andersen. This became a contagion spreading to other firms that were linked by the use of the same auditor raising concerns that they also possibly had accounting irregularities. Chaney and Philipich (2002)

Table 3.5: Abnormal Returns: Enron Events

10/22/2001 SEC inquiry into Enron accounting practices announced

Date	Firm	Cum Abnormal Return (10 days)	Time Period
10/23/2001	Enron	-0.4957	
10/25/2001	WCOM	-0.13235	2 days
11/02/2001	Waste Management (WM)	-0.1095	8 days
11/01/2001	Peregrine (PPHM)	-0.1105	7 days

11/29/2001 SEC starts investigating accounting practices

Date	Firm	Cum Abnormal Return (10 days)	Time Period
11/28/2001	Enron	-0.11058	
01/04/2002	WCOM	-0.03804	24 days
01/11/2002	Waste Management (WM)	-0.07274	29 days
01/10/2002	Peregrine (PPHM)	-0.10352	28 days

calculated abnormal returns to evaluate the impact of Arthur Andersen's audit failure at Enron on other clients of the auditor. Three days following the admission by Arthur Anderson to shredding documents, other Arthur Andersen clients experienced significant negative market reactions. Chaney and Philipich (2002) found clients of the Houston office, which was most directly linked to Enron's auditors, experienced abnormal returns of -3.96 % three days after the incident. Firms outside the Houston area linked to Arthur Andersen were also impacted, however the impact was more muted. Chaney and Philipich (2002) showed abnormal returns of -1.63% for all Andersen clients. These values give us an idea about average values. We will focus on individual firms determining values of the contagion variables based on the firm-pair connections that exist between specific firms.

Client firms of Arthur Andersen evaluated in addition to Enron include: WorldCom, Waste Management, and Peregrine Pharmaceutical. The coefficients for equation 3.3 will be determined from data spanning January 3, 2000 through Sept 7, 2001 with Nasdaq as the market index. Table 3.5 presents two events and the ten day cumulative abnormal returns ending on the date given. For further information on the Arthur Andersen event see Rauterkus and Song (2005).

Table 3.6: Abnormal Returns: GM Events

2/12/2008 GM announced $2 billion operating loss

Date	Firm	Cumulative Abnormal Return (10 days)	Time Period
2/27/2008	GM	-0.08511	
2/28/2008	Visteon	-0.07165	1 days
3/14/2008	Lear	-0.05116	12 days

3/24/2008 GM reported a cash position of $24 billion,
or $6 billion less than what was on hand September 31, 2007

Date	Firm	Cumulative Abnormal Return (10 days)	Time Period
3/18/2008	GM	- 0.16809	
4/15/2008	Visteon	-0.07518	19 days
4/02/2008	Lear	-0.06119	10 days
4/29/2008	Accuride	-0.11717	29 days

Historical Incident Two: Supply Chain, US auto industry

January 11, 2011, ClickSoftware announced its failure to make revenue expectations, resulting in a one day stock drop of 15.2%. Reasons included company hiring, which can be expected and understood. Also cited, the bankruptcy of one of ClickSoftware's customers, an exogenous event originating at another firm spreading to ClickSoftware. In this same vein, Hertzel et al. (2008) looked at abnormal returns and applied them to supply chain customers linked to distressed firms. Focus was on the filing day and a distress day, defined as the pre-bankruptcy date where a firm experienced the largest loss of shareholder wealth. The majority of firms had larger abnormal returns on distress day than on filing day. For the firms declaring bankruptcy, distress day abnormal returns average -26%. Abnormal returns of negative 1-3%, over a five day period centering on a distress day, were reported for firms linked through the supply chain.

The U.S. auto industry in 2008-2009 shows how the spread of a contagion can be particularly debilitating when the group of firms affected is already fragile. In 2009, 27 automotive suppliers filed for Chapter 11 bankruptcy, with estimated trade credit recoveries of less the two percent, see Gray (2008). While the state of the economy played a part in this, the connections between these firms increased the severity of

Table 3.7: Abnormal Returns: AMLN Events

10/20/2010 FDA Announcement			
Date	Firm	Cumulative Abnormal Return (10 days)	Time Period
10/21/2010	AMLN	-0.50113	
10/26/2010	ALKS	-0.35442	3 days
10/27/2010	Lilly	-0.07851	4 days

the situation and allowed the contagion to spread. Twelve percent of executives of industry suppliers, surveyed in November 2008 by Gray (2008), said they would likely or definitely close if General Motors declared bankruptcy. The following looks at events impacting GM and the subsequent impact on three suppliers: Lear, Accuride and Visteon. Data from the period April 3, 2006 through January 23, 2008 are used for regression with Nasdaq as the market index. Table 3.6 presents two incidents and cumulative abnormal returns over a 10 day period ending on the date given.

Historical Incident Three: Joint Venture: AMLN

Firms connected through joint ventures can have particularly strong impacts on each other when an issue arises in the product that forms the basis of their joint venture. As illustrated earlier, when an unexpected FDA announcement effected the product that linked AMLN, ALKS and LLY. Data from the period January 1, 2009, thru September 30, 2010 are used for regression with Nasdaq as the market index. Table 3.7 presents this incident and how percent change to abnormal returns evolved over a 10 day period ending on the date given.

Supplementary Research: Rivals

Yu and Leistikow (2011) and Akhigbe et al. (2005) evaluate equity returns to show contagion spreading to firms in the same industry, illustrating how contagion effects can outweigh any competitive advantage brought about by a rival's negative event. Yu and Leistikow (2011) considered the general case of an exogenous contagion event starting with a firm's stock price dropping more than 10 percent. Rivals are typically firms in the same industry. Their results showed event firms, those impacted by the exogenous events, saw equity prices drop on average of 13 %. When

Table 3.8: Values for A = Category/Variable matches

Firm Pair	λ_{max} τ_{max}	Auditor	Ind/rivals	Supply chain	Joint venture
Enron, WCOM	1	-1	0	0	0
Enron, PPHM	1	-1	0	0	0
Enron, WM	1	-1	0	0	0
Enron, WCOM	1	-1	0	0	0
Enron, PPHM	1	-1	0	0	0
Enron, WM	1	-1	0	0	0
AMLN, ALKS	1	0	-1	0	-1
AMLN, Lilly	1	0	0	0	-1
GM, Accuride	1	0	-1	-1	0
GM, Lear	1	0	-1	-1	0
GM, Visteon	1	0	-1	-1	0
GM, Accuride	1	0	-1	-1	0
GM, Lear	1	0	-1	-1	0
GM, Visteon	1	0	-1	-1	0

this spread to rivals, they saw their equity prices drop 0.4 %. The actual sizes of the impact aren't given, Akhigbe et al. (2005) show the Arthur Andersen scandal had significant negative impact on Enron's rivals. Though the primary firm-pair connections highlighted in the historical examples are not the industry/rival variable, based on Yu and Leistikow (2011) and Akhigbe et al. (2005), we see that it can impact the strength of the firm-pair connection.

Solve for $\theta_{\tau cv}$ and $\theta_{\lambda cv}$ Values

Qualitative data is used to determine category/variables matches which creates matrix A (summarized in Table 3.8) and then used to solve equations 3.2 and 3.1. For each firm-pair, the values of λ_{max} and τ_{max} are set to 1 since these values are always part of the equations. The remaining columns are -1 when a firm-pair match exists for a Category/Variable, 0 when there is no match. Based on data from the historical cases by firm pairs, Table 3.9 summarizes exogenous and propagated contagion and values of τ_{ji}, based on a weekly time period, creating vector b_2. These values are used to solve for λ_{ji} creating vector b_1. Based on equations 3.2 and 3.1, the goal is

Table 3.9: Contagion Variable values by Firm pairs

Firm Pair	C_jExog	NC_i	τ_{ji} (b_2)	λ_{ji} (b_1)	$\exp(-\lambda_{ji})$:
	Proportion drop	in firm value	1 unit =		Proportion of
			10 days		Propagation/time unit
Enron,WCOM	0.49565	0.13232	0.20	6.6032	0.0014
Enron, PPHM	0.49565	0.1105	0.70	2.1441	0.1172
Enron, WM	0.49565	0.10948	0.80	1.8877	0.1514
Enron,WCOM	0.4143	0.03804	2.40	0.9950	0.3697
Enron, PPHM	0.4143	0.10352	2.80	0.4953	0.6094
Enron, WM	0.4143	0.07274	2.90	0.5999	0.5489
AMLN, ALKS	0.50113	0.35442	0.30	1.1546	0.3152
AMLN, Lilly	0.50113	0.07851	0.40	4.6341	0.0097
GM, Lear	0.08511	0.05116	1.20	0.4242	0.6543
GM, Visteon	0.08511	0.07165	0.10	1.7215	0.1788
GM, Accuride	0.16809	0.11717	2.90	0.1244	0.8830
GM, Lear	0.16809	0.06119	1.00	1.0105	0.3640
GM, Visteon	0.16809	0.07518	1.90	0.4235	0.6548

Table 3.10: Resulting Category/Variable values

Weakest Firm pair connection

	τ_{max} 1 unit = 10 days	λ_{max}	$\exp(-\lambda_{max})$ Proportion of Propagation/time unit
Values when no connections	2.694	5.30554	0.0050

Amount Catogory/Variables Increase connection Strength

	$\theta_{\tau cv}$	$\theta_{\lambda cv}$	$\exp(-(\lambda_{max}$ - $\theta_{\lambda cv}))$
Auditor	1.0606	3.1847	0.1199
Ind/rivals	0.1001	3.4795	0.1611
Supply chain	1.1738	1.0852	0.0147
Joint venture	2.2939	0.6714	0.0097

to solve A$X_1 = b_1$ and A$X_2 = b_2$, with the resulting values of $X_2 = \theta_{\tau cv}$ and $X_1 = \theta_{\lambda cv}$. OLS regression may not provide the best solution to this problem due to the limited amount of data, the fact that not all category/variables impact a specific incident, and the limits on the acceptable solutions for X. Optimization, minimizing the total error for the solution, is therefore combined with constraints controlling for acceptable values and the error size for individual category/variables.

Constrained Regression for $\theta_{\lambda cv}$:

Minimize: Sum of the squared error for the historical values λ_{ji} and those based on equation (3.1)

$$\sum_{ij \in dataset} (\lambda_{ij} - (\lambda_{max} - \sum_{cv \in ij} \theta_{\lambda cv}))^2, \tag{3.5}$$

Constraints: Based on equation 2.14 and the assumption that decay will be between 0 and 100 %, acceptable values for $\theta_{\lambda cv}$ will fall within a set range:

$$Lowerbound \leq \theta_{\lambda cv} \leq Upperbound \tag{3.6}$$

To assure that for each incident the value of λ_{max} is greater than the associated $\theta_{\lambda cv}$,

$$\sum_{cv \in ij} \theta_{\lambda cv} \leq \lambda_{max}. \tag{3.7}$$

A similar constrained regression is performed to estimate $\theta_{\tau cv}$.

Table 3.10 summarizes the category/variable results for our limited sample. Some interesting insights can be gained by comparing the results for the different connection types. The impact between rivals/firms in same industry results in a substantial amount of the contagion traveling from one firm to another. Most of the increase in firm-pair strength comes from strengthening λ_{ij}. The increase in τ_{ij} is very small (0.1001). These values indicate that a contagion impacting firm-pair rivals, will travel slowly, however, it will not substantially decay each time interval.

At first glance, the results for Joint Venture seem unexpected. A strong connection would be expected between firms linked by a joint venture. What we see is that the impact travels quickly, but the impact isn't that strong. The logic behind these results is that joint ventures are well known between firms, therefore an impact would travel very quickly. In addition, since they are well known, there would be less of a surprise and uncertainty about the impact and therefore it is less severe.

Auditors and Supply Chains have similar impacts on the travel time, this can be expected since information about these types of connections requires similar research to obtain (listed in Compustat or in 10K reports), and are less well known when compared to Joint Ventures. The amount of the contagion that will impact the connect firm is greater for the Auditor connection. This is expected due to the critical financial information that is the responsibility of the Auditor.

Figure 3.3 presents the category/variable structure combined with values of $\theta_{\tau cv}$ and $\theta_{\lambda cv}$. Allowing for estimation of $\theta_{\tau cv}$ and $\theta_{\lambda cv}$, when historical data isn't available. Strength for a non-calibrated category/variable firm pair connection can be estimated based on a comparison to the rankings of the category/variable structure determined in Procedure 1 and 2.

Procedure 4: Populate Contagion Variables for a Portfolio of Firms

Researching firm information on Compustat and 10K reports revealed various connections as illustrated in Figure 3.4. BIIB and PDLI are connected by a joint venture. GILD and PDLI are connected by the use of the same auditor. All three firms are connected based on same Industry/Rivals. The connection strength for Industry/Rivals, however, wouldn't be the same since BIIB and PDLI develop similar types of treatments (cancer and autoimmune) versus GILD (HIV and hepatitis). Therefore the connection strength for Industry/Rivals was reduced to 25 percent for the firm pairs (BIIB, GILD) and (PDLI, GILD).

Determine Values of τ_{ij} and λ_{ij}

The coefficient results from Procedure 3 will be used to assign values to the contagion variables based on the connections revealed in procedure 4. Values of $\tau_{max} = 2.694$ and $\lambda_{max} = 5.3055$ are assigned based on no contagion spreading when

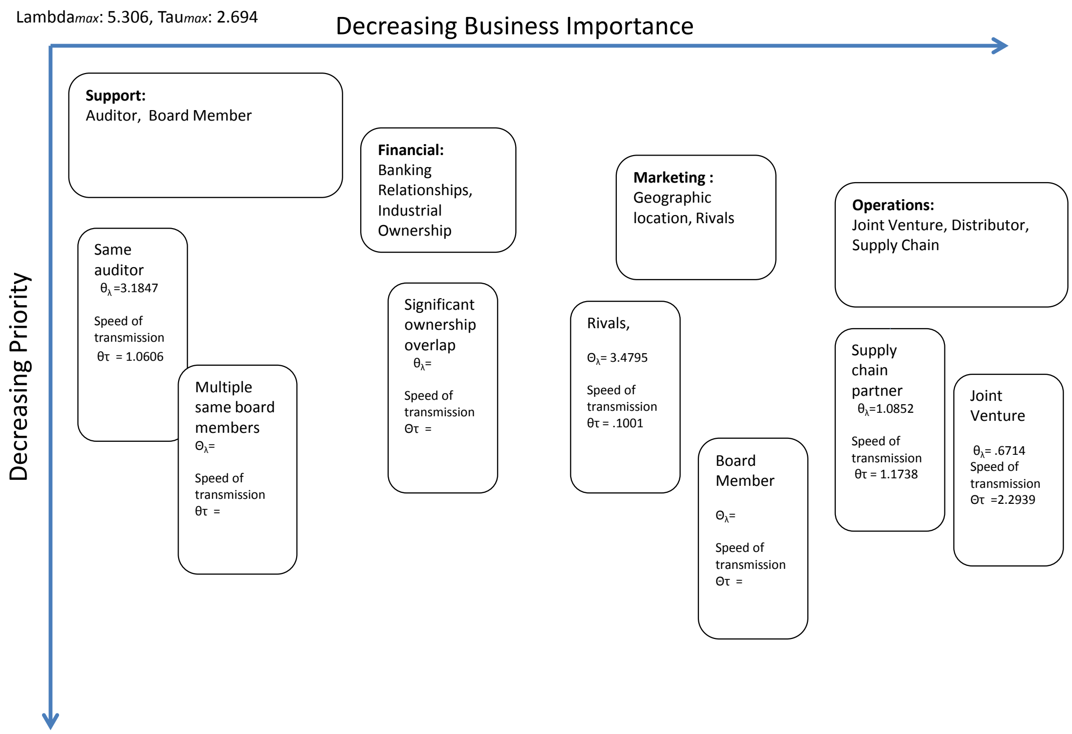


Figure 3.3: Connection strength by Category/Variable based on historical incidents

a connection doesn't exist. The connection between BIIB and PDLI is strengthened due to catogory/variables Joint Venture and Industry/Rivals resulting in $\tau_{13} = 0.3$ and $\lambda_{13} = 1.15$. The GILD and PDLI are connected by the use of the same Auditor and a weaker Industry/Rivals connection resulting in $\tau_{23} = 1.25$ and $\lambda_{23} = 1.61$. BIIB and GILD are linked by a weak Industry/Rivals resulting in values of $\tau_{12} = 2.67$ and $\lambda_{12} = 4.44$. Unit of measure for τ is 1 unit equals 10 days, exp(- λ_{max}) is Proportion of Propagation /time unit.

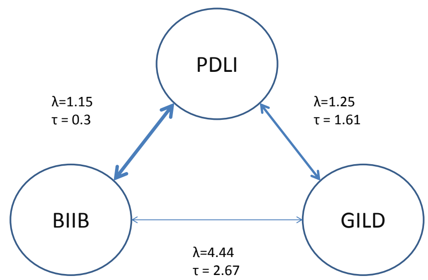


Figure 3.4: Three Firm Illustration: Connections between Firms

3.4 Illustration of framework: Larger Portfolio Implementation:

With the goals of applying the framework to a larger sample of firms and analyzing effectiveness, the following describes the process from initial bond selection to determination of values of contagion variables for 10 firms.

Bond Selection: In the current low interest rate environment and increased variability in the stock market, an investor wishes to invest in a portfolio of higher yielding bonds. Due to the investor's belief in the stability of the biotech and closely related sectors and the yields offered in this sector, the bonds are selected from this or other closely related sectors. A listing of bonds from FINRA (http://cxa.marketwatch.com/finra/BondCenter/Default.aspx) in the following categories is collected:

1. Biotech
2. Life science tools and services
3. Healthcare providers and services
4. Pharmaceuticals

Bonds, classified as senior notes, are selected based on yield and maturity date. Due to the one year time interval of the simulation, bonds with middle range maturities (4 to 10 years) are selected. This is to exclude bonds with maturity during or shortly after the simulation time period. Longer term maturities are more impacted by interest rate predictions which we seek to avoid in this simulation. It is too limiting to only select bonds with the exact same maturity, but bonds will be selected with as similar maturity dates as possible. Current yields in the 2 to 5 percent range and historical yield data for the time period of July 2010 to June 20, 2011 are desired.

Historical data required for the models:

Data will be collected based on a portfolio selection date of July 1, 2011. The required data for the Firm Value No Contagion piece of the model, from the time period July 2007 through June 2011 is:

- Market: S&P 500 (S&P website),
- Rating: Baa and Aaa (Compustat),
- Risk free return: 1 year Treasury (Compustat),
- Sector: Nasdaq Biotech (Nasdaq),
- Firm: Stock Price (Compustat and Nasdaq).
- ROA: Quarterly, July 2007 thru June 2011 (Compustat)
- Beginning Firm Value: (stock price times shares outstanding) + debt (Compustat)

Connections between firms:

Once a set of bonds/firms is selected for inclusion in a portfolio, qualitative information is required to determine the existence of firm-pair connections that provide a transmission path for a negative event's impact to spread from one firm to another. Firm specific information is collected from various sources including

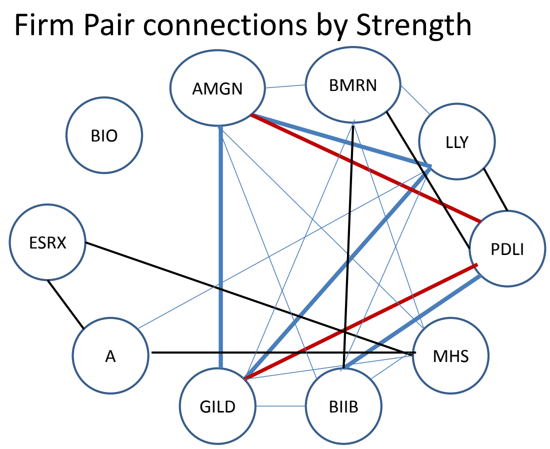


Figure 3.5: Connections that exist between 10 Firms

Compustat, Nasdaq and 10 Q reports. Figure 3.5 illustrates the connections between the firms in the portfolio. To represent the different firm pair connection strength, the thicker the lines the stronger the firm pair connections. This illustrates how complicated these relationships can become as the number of firms increases, leading to the need for an optimization program to limit the inclusion of bonds with strong connections that can spread contagion. Table 3.6 provides a description of specific firm pair connections. These connections then need to be converted to value for the contagion variables, see section 3.2 for a detailed description of procedure 4.

Using the category/variable values determined in Section 3.3, Figure 3.7 provides the values assigned to τ_{ij} and λ_{ij}. These values need to be modified due to the discrete time steps used for the evolution of the firm value model. τ_{ij}'s are set to integers, λ_{ij} are then modified so that the values of $\tau_{ij} \times \lambda_{ij}$ remain constant before and after the modification. Since we are looking at defaults at the end of a specified period, this modification should have minimal impact. To reduce the impact, the time interval can be reduced. This will, however, impact the computer memory requirements and CPU time to run the simulations.

FIRM	RIVALS	PARTIAL RIVALS	AUDITOR	JOINT VENTURE	DISTRIBUTOR
BIIB	PDLI BMRN	AMGN GILD		GENENTEC, ELAN	
GILD		AMGN BMRN PDLI BIIB	ER		AB, MCK, CH
PDLI	BIIB BMRN	AMGN GILD	ER	GENENTEC, ELAN	
A	BIOB		PWC		
BIOB	A				
AMGN		BMRN PDLI BIIB GILD	ER		AB, MCK, CH
BMRN	PDLI BIIB	AMGN GILD	KPMG		
LLY			ER		AB, MCK, CH
MHS			PWC		
ESRX			PWC		

Figure 3.6: Connections that exist between 10 Firms

Firm Pair	Connection	Tau	Lambda	Modified Tau	Modified lambda	Proportion of
		1 unit =		1 unit =		Propagation
Firm i	Firm j	10 day		10 day		/unit time
6	7	2.6690	4.4356	3.00	3.9462	0.0193
6	8	0.4596	1.0356	1.00	0.4760	0.6213
6	3	1.6084	1.2509	2.00	1.0060	0.3657
6	9	2.6940	5.3055	3.00	4.7643	0.0085
6	1	2.6690	4.4356	3.00	3.9462	0.0193
6	2	0.4346	0.1657	1.00	0.0720	0.9305
7	8	2.6940	5.3055	3.00	4.7643	0.0085
7	3	2.5939	1.8260	3.00	1.5788	0.2062
7	9	2.6940	5.3055	3.00	4.7643	0.0085
7	1	2.5939	1.8260	3.00	1.5788	0.2062
7	2	2.6690	4.4356	3.00	3.9462	0.0193
8	3	1.6334	2.1208	2.00	1.7321	0.1769
8	1	2.6940	5.3055	3.00	4.7643	0.0085
8	2	0.4596	1.0356	1.00	0.4760	0.6213
8	4	2.6940	5.3055	3.00	4.7643	0.0085
3	1	0.3000	1.1546	1.00	0.3464	0.7072
3	2	1.6084	1.2509	2.00	1.0060	0.3657
9	1	2.6940	5.3055	3.00	4.7643	0.0085
9	2	2.6940	5.3055	3.00	4.7643	0.0085
9	4	1.6334	2.1208	2.00	1.7321	0.1769
9	10	1.6334	2.1208	2.00	1.7321	0.1769
1	2	2.6690	4.4356	3.00	3.9462	0.0193
4	10	1.6334	2.1208	2.00	1.7321	0.1769

Figure 3.7: τ_{ij} and λ_{ij} values for 10 bond portfolio

3.5 Exogenous Contagion

Exogenous contagion is modeled based on two variables: arrival rate and size of impact. Historical contagion events provide guidelines for these variables. In addition, variations in these values are useful to understand the stresses on the portfolio as the number and size of exogenous contagion change. The first variable controls the arrival of an external contagion to a firm, modeled as a Poisson arrival process with the arrival rate varied based on the average arrivals projected over the simulation. As an example, arrivals with a mean of 1/(number of periods per year) will result in approximately one arrival to a firm per year. This is varied based on the amount of contagion stress one wants the firms to experience. The second variable represents the amplitude of this contagion as a percent of firm value impacted (C_i). It is modeled as an exponential random variable due to distribution characteristics that are advantageous in describing a contagion, including positive values which are close to the mean value, and few values significantly greater than the mean value.

To calibrate the mean value for C_i, abnormal returns based on equity returns were determined when calculating values for τ_{ij} and λ_{ij}, see Table 3.9. These values of historical exogenous contagion are used as a guide. Since the size of the exogenous contagion is not directly linked to the value of the firm pairs connection variables(τ_{ij} and λ_{ij}), amplitude values are varied to analyze the resulting impact variations.

3.6 Data: Bond Return, Variance, Correlations, Probability of default and CVaR

Similar to Altman (1996) and Kato and Konno (2007), bond variances and correlations will be based on historical bond yields. Due to the thinly traded nature of some bonds, on days when there are no trades the last available daily trade is used. When there are multiple trades in a day, the last trade of the day is used.

For this illustration, the portfolio selection will be made based on yields from July 1, 2011. Historical bond yields are collected from Compustat from July 1, 2010 to July 1, 2011. Depending on the issue date for the bond, data for the entire period may not be available; therefore, a shorter time period will be used.

Default probabilities will be based off of indicator data from approximately 25,000 simulations of the firm value model (described in section 2.2). This is determined by firm value minus debt with a default assumed when the value is 0 or less, resulting in the indicator being set to 1, otherwise it is set to 0. This data is also used in the CVaR calculation with, for each bond, 1 indicating total loss of the investment in that specific bond.

3.7 Simulation

Once the model is fully calibrated the simulations are run. The following describes the time evolutions and relations of the main modules of the simulation. The simulation programs are developed in Matlab. See appendix A.10 for layouts of modules. Most regressions will also be run on Matlab, some smaller regression computations will be run on Excel.

Firm Value Model: No contagion

Future firm values are determined by evolving return on assets for individual firms. Mean reverting equations for each factor that impacts firm value are simulated per time period. For interest rates, a yearly rate of return is calculated. It is then converted to a quarterly value. For all other factors, a new rolling quarterly rate of return is calculated for each 10 day period. These rates of returns are used with the betas from the calibration section to arrive at a simulated firm return on assets which is divided by 6 to convert to a bi-monthly rate.

Firm Value Model: Contagion Factors

Exogenous contagions are simulated for individual firms using the Poisson process. They then evolve using the same time interval as the no contagion piece of the model and impact on other firms in the portfolio based on the contagion variables: network structure, travel time and decay. This is combined with the results from the no contagion piece to calculate the next time period's firm value.

Default Boundary-Debt Level

Quarterly debt data (Second quarter 2006-Second Quarter 2011) will be used to calibrate the AR(2) model to project future debt level. Debt level is evolved over the same time step as the firm value. Default is considered to have occurred when firm value is less than debt level.

3.7.1 Stylized Characteristics of Contagion Propagation in Serial Network

The following provides illustrations of the simulation output. Figure 3.8 represents an exogenous contagion impacting three firms and then spreading to other firms through their connections. The charts on the left show the exogenous contagion that impacts each firm. The right side charts show the exogenous contagion plus any contagion that has spread to a firm due to the network connections. As an example, an external problem impacts firm one. Once investors are aware of the problem, they re-evaluate the prospect of other firms due to their belief that the firms are similar and could have the same problem. The contagion spreads to firm two in one time period and then from two to three in two time periods.

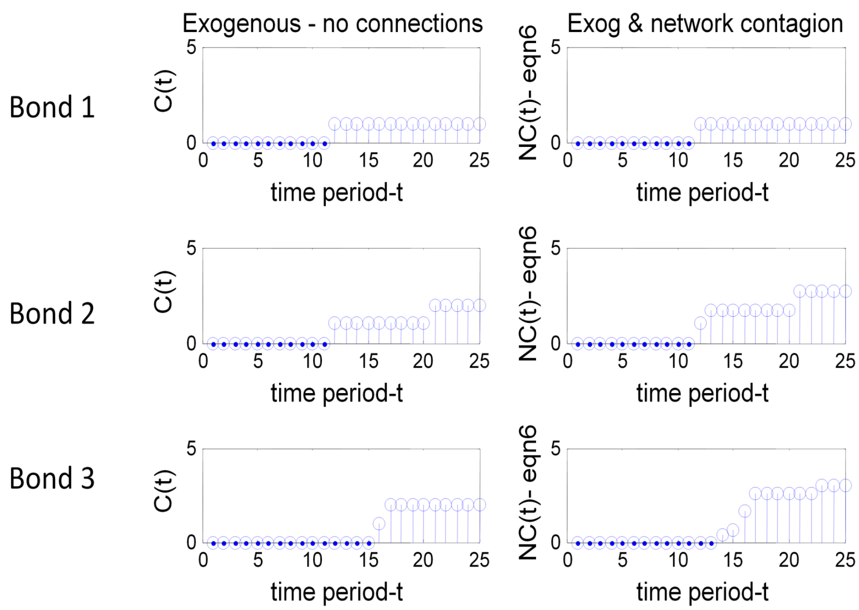


Figure 3.8: Contagion Impacting Three Bonds

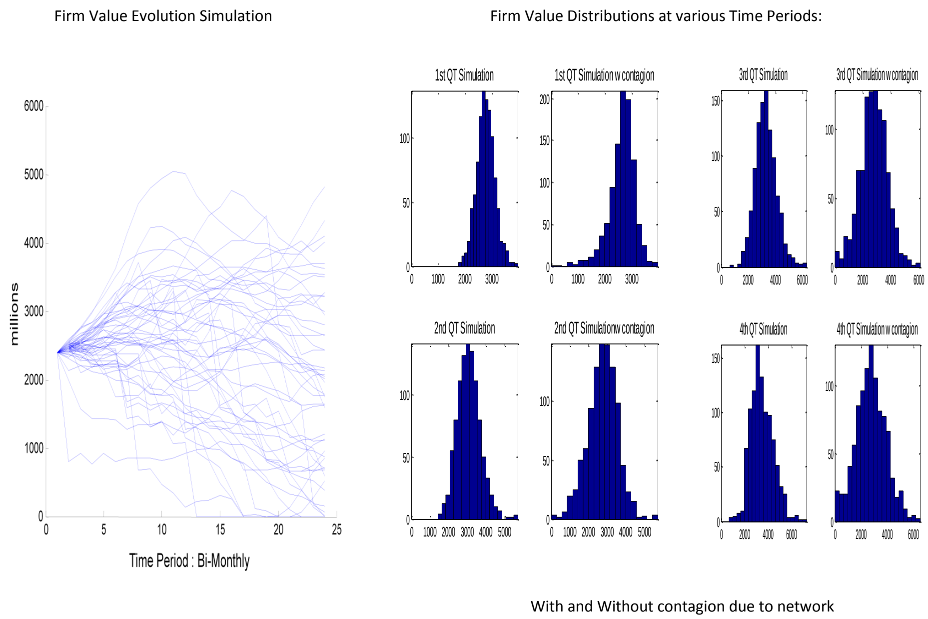


Figure 3.9: Simulation Results

3.7.2 Firm Value with Contagion: Example Simulation Results

Output for 50 firm value evolution simulations, for one firm, is presented in the Figure 3.9. The first set shows how the firm value evolves over time due to the changes in the various factors and contagion. On the right are the distributions of a specific firm's value by quarter, with and without the impact of contagion spread due to network connections. The charts including contagion, the quarterly charts on the right, show the heavy tailed default distributions we see in empirical default data.

Having developed the model and performed calibration, the following chapter discusses the results of the firm value model simulations and the bond portfolio optimization. Insights gained into the impact of the flow of contagion on individual firms and portfolios are presented.

Chapter 4

Contagion-based Firm Value and Portfolio Selection

Understanding default and lower tail risk in firm value are critical when designing a portfolio of bonds. Contagion will impact firm value, but does the diversification that results from creating a portfolio of bonds help limit this risk due to contagion? Bond portfolios of biotechnology firms are analyzed to see the importance of considering contagion during portfolio creation. Simulations are performed to gauge the impact of these variables on defaults and lower tail risk at a firm and portfolio level.

Specifically, contagion's impact on the following will be considered:

- Individual firms
- The efficient frontier of a basic risk (default) and return(yield) portfolio
- Individual firms and portfolios with stylized connections
- Portfolios of actual firms are created to control for tail risk

To gain additional insight on the portfolio size, three, five and ten bond portfolios will be analyzed created from a set of ten bonds.

Table 4.1: Average percent changes due to Network Connections

Firm	Mean Firm Value	Default
BIIB	-34.38	834.59
GILD	-8.81	43.36
PDLI	-27.16	134.55

4.1 Initial Three and Five Bond Analysis

Simulations were run with a portfolio of three bonds, described in Section 3.3, to address the following hypothesis:

H1:Firms in a portfolio with firm-pair connections, based on qualitative firm data modeled using graph theory, result in heavy tailed default distributions seen in empirical data.

A network structure that allows a contagion to spread, negatively impacts the firms by increasing defaults, and also by decreasing firm value and bringing the firm closer to the possibility of default. Defaults increased in all firms in the portfolio due to the spread of the contagion. In addition, when the contagion didn't cause a default, it negatively affected the firms through drops in the mean firm values. The measurement from the simulations of the three bond portfolio, shown in Table 4.1, includes changes to firm value and defaults. To highlight the impact of the network structure, values show the percent change occurring due to the connections. Comparisons can then be made of values from the exogenous impact to those occurring due to the exogenous impact plus the spread caused by the network structure.

The spread of the contagion has a negative impact, as seen by the drop in firm value and increased defaults. These results also show how the number and size of the connections impact the firms. The strongest connection is between BIIB and PDLI, and their firm values are the most negatively impacted. This illustrates that not all firms will be impacted to the same extent by contagion, with some more susceptible to a severe negative impact. Unless one is aware of these connections, a firm's risk can be under-estimated, leading to severe and unanticipated portfolio losses. See Section A.8 for an additional three firm example.

Impact to Efficient Frontier

Individual firms are at increased risk due to contagion. Most investors, however, invest in a portfolio to reduce risk exposure through diversification of capital. To understand how contagion impacts this desired risk reduction, the effect on the optimal portfolio is considered. Based on simulation results, the efficient frontier is plotted for portfolios, with and without the connections that allow a contagion to spread. Returns are based on current bond yields. Risk is determined by firm defaults since this is the risk factor most seriously impacted by a contagion. Additionally, the measure of portfolio risk, as described by equations 2.18, will be controlled to remain below a prescribed value. Figure 4.1 illustrates the contagion impact on the efficient frontier. The curve shifts to the right, showing an increase in risk for a given level of return when a network that allows contagion to flow is embedded in a portfolio optimization.

For further insight into the impact of contagion on a portfolio, Figure 4.2 illustrates the impact on the efficient frontier for a portfolio of five bonds, created from the previous three bonds combined with two additional bonds that have no firm-pair connections. The same impact due to contagion and the network structure is seen on the efficient frontier curve. Additionally, the composition of the portfolio changed, see Figure 4.3. With additional bonds choices (lacking any firm-pair connections), the weights of the firm-pair with the strongest connection stayed the same or decreased.

Impact to Efficient Frontier based on Stylized Connections

The impact of contagion is influenced by the specific network structure that exists between firms. This is illustrated by comparing the resulting efficient frontiers with various network structures. Firm pairs from the previous five bonds were created using stylized one directional connections; star, serial and loop networks. Figure 4.4 compares the impact on the efficient frontiers, with and without considering contagion

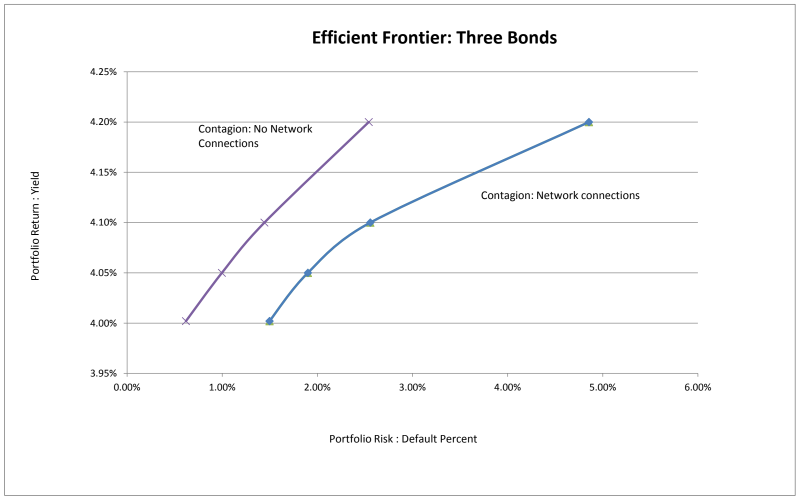


Figure 4.1: Efficient Frontier Three Bond Portfolio

for each network structure. Regardless of structure, once a network is considered that allows a contagion to spread, all portfolios are more risky than anticipated.

The specific structure of the firm connections impacts the resulting efficient frontier, as illustrated in Figure 4.5. Therefore, a group of firms hit by the same contagion will be impacted differently depending on the structure connecting them, with the greatest impact to firms, connected in a loop.

Weights of the bonds that make up the various portfolios are also impacted by contagion and firm connections. Table 4.2 presents the portfolio weights for an average return of 4 % based on a no contagion portfolio and then contagion portfolios for Star, Loop, and Serial networks. BIIB's percent increases in the serial networks and GILD in the star network. This is due to the one directional connections used in the simulation. In these networks, these firms transmit contagion to other firms, but don't receive in the opposite direction. Risk for these firms decreases relative to

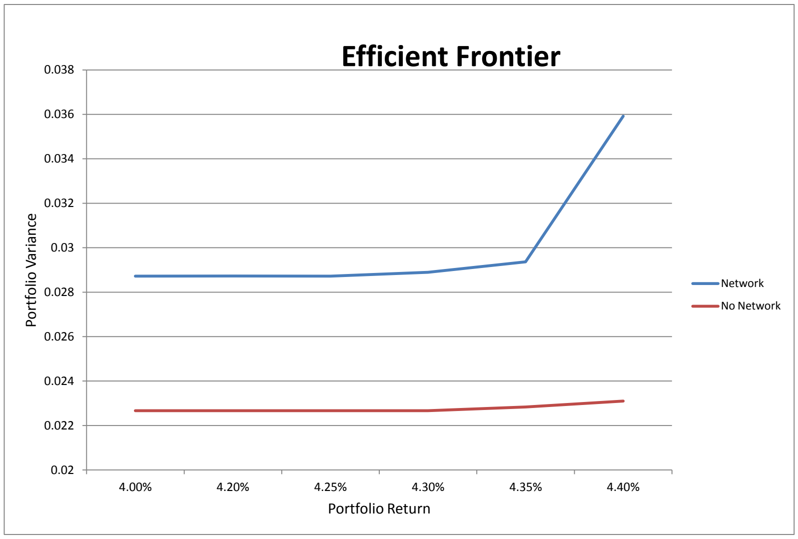


Figure 4.2: Efficient Frontier Five Bond Portfolio

Table 4.2: Five Firm Portfolio Weights (percentages): By network connections

4% Return	BIIB	GILD	PDLI	A	BIO
Aver No Contagion	22.13	47.60	5.77	0	24.50
Serial Connection	26.62	42.36	5.51	0	25.50
Loop Connection	27.32	41.30	5.88	0	25.50
Star Connection	17.25	52.70	7.03	0	23.02

the other firms in portfolio when a contagion is considered, making BIIB(serial) and GILD(star) more attractive to include in the portfolio.

It is not possible to create a portfolio which avoids completely exogenous contagions or connections to other firms. Therefore, a better understanding of the impact of the contagion variables is required to assist in the portfolio selection process. The following section applies the model to stylized firm data with focus placed on how changes in these variables impact the firm and portfolio value.

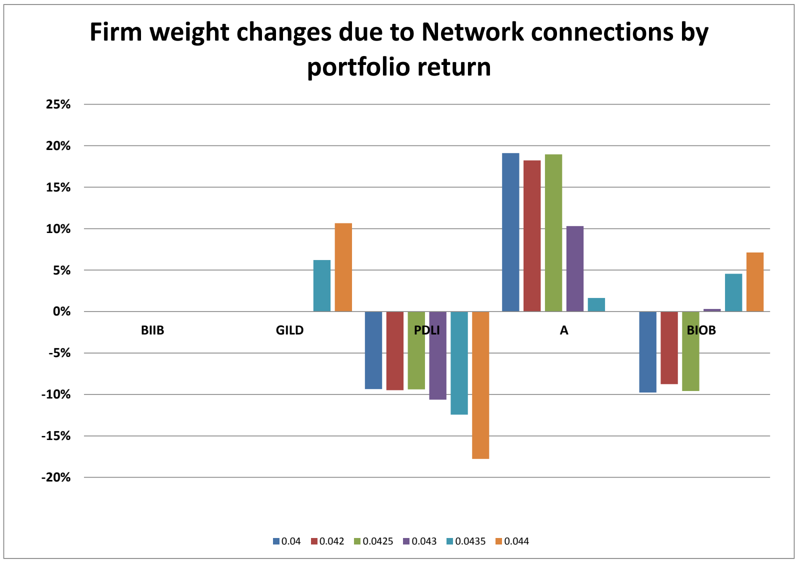


Figure 4.3: Portfolio Weight Changes due to Network Connections

4.2 Study of Impact due to Variations in Contagion Variables

The goal of these simulations is to develop a better understanding of the variables that impact the spread of a contagion by measuring the firms susceptibility to default and lower tail risk. To develop an understanding of the impact of key factors on the spread of contagion, simulations are run modifying the contagion variables. Specifically the following two hypothesis are addressed:

H2A: When firms in a portfolio of bonds experience an exogenous contagion, the structure of the connections between firms, not just the number of connections, decreases the firm value and increases lower tail risk of the overall portfolio.

H2B: For a similar percent change, a slower decay rate has more of an impact to the overall portfolio of firms than increasing the size of the initial exogenous contagion.

To better understand portfolio risk, it is important to identify the characteristics that make a portfolio more susceptible to contagion. Simulations are run modifying the contagion variables. One set of firm data is used for the mean reverting part of the model, allowing for control of the firm specific factors and placing focus on the impact of the network variables. The variables analyzed are: amplitude of exogenous contagion, decay rate times travel time, number of firms connected and shape of network connections.

The number of bonds in the portfolio are varied to study the contagion impact as the size of portfolio is increased. Various network structures are considered, representing the possible ways bonds can be connected to each other within the portfolio. All of the connections are one directional. As the name implies, in a serial network, bonds are serially-connected in a uni-directional sense. For example, firms 1 and 2 could be linked by a joint venture, as in the earlier Amylin example. Firm 2 could then be linked to another firm by the use of the same lending institution. When firm 2 experiences problems, the lender could see this as a sign of larger problems and decreases credit to firm 3, spreading the impact to this firm. A looped network connection is the same as the serial except that the first and last nodes are also connected. The star connection starts with one node and all other bonds are connected to this central node.

4.2.1 Three and Five Bond Portfolio

The connection variables between firms impact the spread of contagion. Focusing on three and five bond portfolios, the specific variables considered are network connections of serial, star or loop, an amplitude of 0.1 or 0.3, and τ x λ set to 0.25 or 0.5. Table 4.3 contains results for 3 bond simulations. Tables 4.4 and 4.5 focus on the impact of the amplitude and decay rate (τ x λ) for the various 5 bond networks. All values are averages of firms to which the contagions can spread reflecting the percent changes due to the network structure. This provides a comparison of values based

Table 4.3: Three bond Portfolio: Average percent changes due to network

	Serial	Serial	Star	Star	Loop	Loop
Amplitude	0.1	0.3	0.1	0.3	0.1	0.3
Tau*Lambda	0.25	0.25	0.25	0.25	0.25	0.25
Mean 4QT	-8.61	-23.23	-6.24	-18.31	-21.74	-43.65
Coef of Semi-Var 4QT	12.54	7.51	7.90	6.93	37.33	11.04
Defaults %	68.41	82.06	28.27	77.13	501.51	258.16
LT Tail Debt Cover 10 %	-1.97	-12.76	-1.43	-10.56	-5.52	-22.66

only on the exogenous impact and those that occur due to the exogenous impact plus the network structure, highlighting the impact of the firm-pair connections.

The type of structure influences the impact of contagion. As seen in Table 4.4, a firm in a serial network is more negatively impacted by a contagion than one in a star network. This is expected since a contagion in a serial network has a greater likelihood of impacting a firm that can spread the contagion to another firm. With a star network, the impact on all firms would require a exogenous contagion hitting the main firm. This would have an extreme impact, but the chances of this happening are rare. This increased negative impact on a serial network is alarming in portfolio selection since serial connections are harder to recognize and therefore more likely to go undetected. The network with the greatest negative impact, however, is the loop. When a contagion hits any firm in a closed network it will then dissipate within the structure, increasing the impact to the total portfolio.

As expected, results in all three tables show that the higher the amplitude of the exogenous contagion, the greater the decrease in the lower tail debt coverage. When considering Tables 4.4 and 4.5, the impact of a slower decay rate has a greater impact decreasing lower tail debt coverage on the serial network than on the star network. With a decay rate of 28% faster, an amplitude of 0.1, lower tail debt coverage dropped by 90% for a serial connection, versus 44% percent for a star connections. Similar

Table 4.4: Five Bond Portfolio: Average percent changes due to network

	Serial	Serial	Star	Star
Amplitude	0.1	0.3	0.1	0.3
Tau*Lambda	0.25	0.25	0.25	0.25
Mean 4QT	-11.68	-30.18	-6.40	-17.73
Coef of Semi-Var 4QT	15.33	7.33	6.70	6.55
Default %	104.798	119.01	43.25	75.58
Lower Tail Debt Coverage 10 %	-2.68	-16.56	-1.74	-10.30

results can be seen for the amplitude of 0.3. This indicates again that the serial connection is more susceptible to the impact of a contagion than the star connection.

In Tables 4.3, 4.4, and 4.5, the results of the coefficient of semi-variation are misleading at times. As the mean and number of firms defaulting increase, it appears that the resulting data is less dispersed. This has more to do with the mean decreasing, and therefore the range also decreasing, than the values being less dispersed.

Table 4.5 reflects the initially puzzling results that, as the amplitude increases, the percentage change in default decreases. A possible explanation is that with a higher amplitude, more failures are a result of the exogenous contagion itself, therefore seeing a smaller percent increase. When the amplitude is lower, the initial defaults will be lower from the less severe exogenous contagion. As it spreads, the percentage increase in defaults will be larger. This can be interpreted as weaker firms being more likely to default immediately due to a large initial impact, versus being further weakend due to a smaller initial impact and then defaulting due to a later impact.

At the portfolio level, the size of the network not only increases risk due to a greater number of firms being impacted, but also the risk to the individual firms. As a result, the overall portfolio can be impacted twice. When comparing the results in Tables 4.4 and 4.5, the averages of most measures for the 3 bond versus 5 bond networks show an increased risk in the case of the larger network.

Table 4.5: Five Bond Portfolio: Average percent changes due to network

	Serial	Serial	Star	Star
Amplitude	0.1	0.3	0.1	0.3
Tau*Lambda	0.5	0.5	0.5	0.5
Mean 4QT	-7.80	-21.56	-4.89	-14.09
Coef of Semi-Var 4QT	6.97	4.30	6.20	3.49
Default %	71.66	49.38	58.72	35.91
Lower Tail Debt Coverage 10 %	-1.41	-9.21	-1.21	-6.75

4.2.2 Ten Bond Portfolio: Complex Network Structures

Most bond portfolios are at risk for containing numerous firm-pair connections creating multiple different structures. The following are results for a network of ten firms with a combination of structures, including star, loop and serial. In Table 4.6 two similar structures are compared. Both have a star network of five firms. The difference in the structures is that in the first set of results the remaining five firms are connected serially off one of the spokes of the star, see Figure 4.6. In the second result these firms form a loop of four firms that are connected to one of the spokes of the star through another firms, see Figure 4.7. All the outputs are in terms of percent changes to the values due to the network structure, comparing what the values would be with only the exogenous impact and to those that occur with the exogenous impact plus the network structure.

The negative impact of the relatively minor change of going from a serial to a loop structure are shown in the first two sets of results in Table 4.6. Going from serial to loop adds only one additional connection, but results in more defaults and greater lower tail risk. To further understand the difference, Table 4.7 shows the firm averages separated by type of connection for the same amplitude and Tau times Lambda. When looking at the output for the individual firms, those in the star formation have the smallest negative impact and are very similar, regardless of how the remaining firms are connected. Firms in the serial connection are more negatively impacted for

Table 4.6: Ten Bond Portfolio, Impact for Combination Structures: Average percent change due to Network

	Star and Serial	Star and Loop	Star and Loop
Amplitude	0.1	0.1	0.3
Tau*Lambda	0.25	0.25	0.25
Mean 4QT	-11.53	-18.07	-39.44
Coef of Semi-Var 4QT	13.28	24.51	6.50
Default %	175.68	202.66	217.76
Lower Tail Debt Coverage 10 %	-2.57	-4.42	-22.01

Table 4.7: Ten Bond Portfolio, Averages by Connection Type: Average percent change due to Network

	Aver Star Firm	Aver Serial Firm	Aver Loop Firm
Mean 4QT	-6.51	-14.86	-29.07
Coef of Semi-Var 4QT	8.88	16.86	39.19
Default %	172.47	120.71	225.66
Lower Tail Debt Coverage 10 %	-1.71	-3.12	-7.27

nearly all of the measures when compared to the star formation. However, as seen in earlier results, those in the loop structure are the most severely impacted.

Understanding the impacts of variations in the contagion variables is useful. However, since a portfolio of bonds is the more common investment instrument, focus will now be on how to improve portfolio optimization by considering the impact of contagion and an underlying network structure.

4.3 Portfolio Optimization Focusing on Tail Risk

The goal of the portfolio optimization, created in Section 2.7 is to highlight the tail risk in a portfolio of bonds due to contagion spreading through a network structure, specifically addressing:

Table 4.8: Simulations where both Firms defaulted (25000 simulations); Firm-pairs: (BIIB,PDLI), (PDLI,GILD), (GILD,BIIB)

Company	BIIB	GILD	PDLI	A
GILD	1880.00%			
PDLI	6778.95%	535.71%		
A	907.14%	41.18%	158.26%	
BIO	672.73%	62.50%	128.57%	0.00%

H3: Improved knowledge of the impacts and spread of contagion can be used in a bond portfolio optimization process to improve the selection process, resulting in a portfolio with less tail risk.

The following considers the ability of this optimization on a portfolio of five and ten bonds to highlight the risks of contagion. Optimization is run using default data with and without network connections. This allows for optimization results including network connections that allow contagion to be spread to be compared to results when connections are not considered.

4.3.1 Five Bond Portfolio

An investor selects a sector specific investment strategy believing this sector will have stable returns and less risk than the market in general. Five firms' bonds are selected, three as described in section 3.3. Two additional firms, without connections to the other firms in the portfolio are also selected.

The goal of the optimization is to immunize the portfolio from the severe impact of contagion. This impact is seen in the rare but severe tail events where multiple firms default together. Table 4.8 describes the percent increase in two firms defaulting together when connections between the firms are considered. Three of the top four increases in firms defaulting together are between the three linked firms, showing the increase in multiple defaults between linked firms.

When three, four or five firms default at the same time the damage to the portfolio is debilitating. Table 4.9 presents the percent increase in multiple defaults (including

Table 4.9: Percent increase in defaulted of three, four or five firms due to network connections (25000 simulations), Firm-pairs: (BIIB,PDLI), (PDLI,GILD), (GILD,BIIB)

Percent Increase in defaults of three, four or five firms:
Due to Network connections between BIIB, GILD, PDLI

BIIB	GILD	PDLI	A	BIO
3500.00%	4275.00%	1915.00%	1369.23%	975.00%

the listed firm) due to the network connections. The percent increase is again greatest for the three connected firms.

Optimization Insights: The goal of the optimization is to decrease the exposure to the multi firm defaults illustrated above. To improve understanding, various values of β, returns, default level and return variance are used in the optimizations. β = 1 - α with α determining the percentage of the upper tail loss distributions that CVaR minimizes. The following summarizes the insights from this analysis.

Insight I: The CVaR estimate for portfolio default risk is underestimated when network connections aren't included.

Figure 4.8 illustrates values of CVaR for various levels of return and β. When connections are considered, portfolios have substantially more tail risk than when no network effects are considered. This is illustrated by the graphs with the network effects being higher (increased risk) when compared to those with no network effects. *A. Lower β failed to draw out this tail/contagion risk and therefore reduces the ability to control for it.*

Additionally, as β decreases, the curves drop and the network-based and no network curves for the same β become closer together (Figure 4.8). The ability of the optimization to represent the size of the extreme tail risk is decreased. Due to the rare and extreme nature of a contagion's impact, if a value of β is too low the severity of the contagion's impact is muted by the impact of many less extreme events. A level of β above 0.95 helps illustrate the extreme tail risk of the portfolio.

Insight II: Composition of Portfolio is Impacted, amount of impact varies by β.

Portfolio weights change when network effects are introduced. Figure 4.9 depicts the distance between the two vectors representing the optimal firm weights with and without network effects. All portfolios are substantially different when considering network connections. The portfolio with the least change is the one with the lowest β.

Additionally, in the optimal portfolio, the weights of two of the three connected firms decreased due to increased risk resulting from firm pair connections (see Figure 4.10). The total weight of the two no connection firms is often higher in the portfolio with network connections. The exception in this example is GILD. This firm has the lowest defaults in both the connected and no connection cases. In the firm connection case, it is the preferred choice of the three connected firms.

Insight III: Changes to values of other risk factors impact the ability to control for tail risk.

A. Decreasing return variance increases tail risk.

As the allowance for variance of return decreases, risk is transferred away from return variance and to tail risk (CVaR). This impact decreases for lower βs. See Figure 4.11. This indicates that creating a portfolio with lower portfolio variance of return will actually create a portfolio with greater tail risk, in the presence of network-connected underlying firms.

B. Decreasing default risk increases tail risk.

When the maximum default level of the portfolio is decreased, tail risk of the portfolio increases as seen in Figure 4.12. This is especially true with smaller $\beta's$. Optimizations were run decreasing the limit for portfolio defaults to 8% from 7%. These results indicate that controlling for default risk will not necessarily limit your tail risk and may have the unintended consequences of creating a portfolio that is more susceptible to tail risk. Separate measures of tail risk are therefore required to limit the risk of multiple defaults.

4.3.2 Ten Bond Portfolio

The preceding bond portfolio optimization is expanded to a larger set of ten bonds, described in Section 3.4. The goals of this optimization include:

1. Confirm optimization insights, developed earlier, for a larger portfolio impacted by contagion
2. Improved understanding of computational issues

The connections between the firms are described in Section 3.4. Major connections exist for firms: BIIB, GILD, PDLI, AMGN and LLY.

The CVaR estimate for portfolio default risk is under-estimated when network connections aren't included.

As seen with the 5 bond portfolio Figure 4.13 illustrates values of CVaR for various levels of return and β. Again when connections are considered, portfolios have substantially more tail risk than what is expected when no network effects are considered. The values of CVaR are lower with the 10 bond portfolio, illustrating the positive impact of having increased bonds to select,

Composition of Portfolio is Impacted.

Portfolio weights change when network effects are introduced. Figure 4.14 depicts the distance between the two vectors representing the optimal firm weights with and without network effects for β of 0.999. The weights of the firms with the strongest network connections decrease, indicating the increased risk of these bonds when connections between firms are considered.

Benefit of Optimization increased with size of Portfolio.

From the above, we see similar results in the 10 bond portfolio as in the 5 bond portfolio. Both sets of results show that the optimizations reduce the exposure to bonds with stronger firm pair connections. This would be increasingly useful as the number of bonds under consideration grows. As the bond portfolio increases, so will the firm-pair connections making it is increasingly difficult to recognize which firms are exposed to a significant number of strong connections through either visual inspection or graphing. This can be seen by comparing the complexity of figures 3.4 and 3.6. The portfolio optimization, however, is able to limit exposure to those firms with the most significant risk due to firm-pair connections.

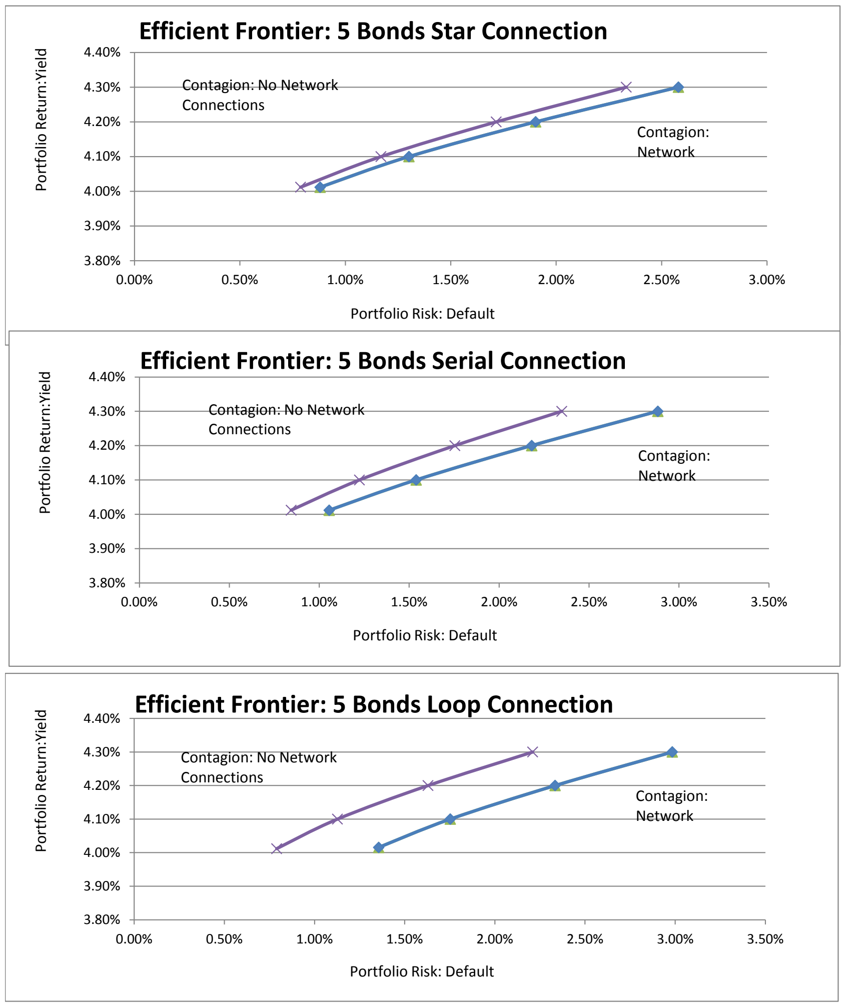


Figure 4.4: Efficient Frontier - Star, Serial, and Loop Networks

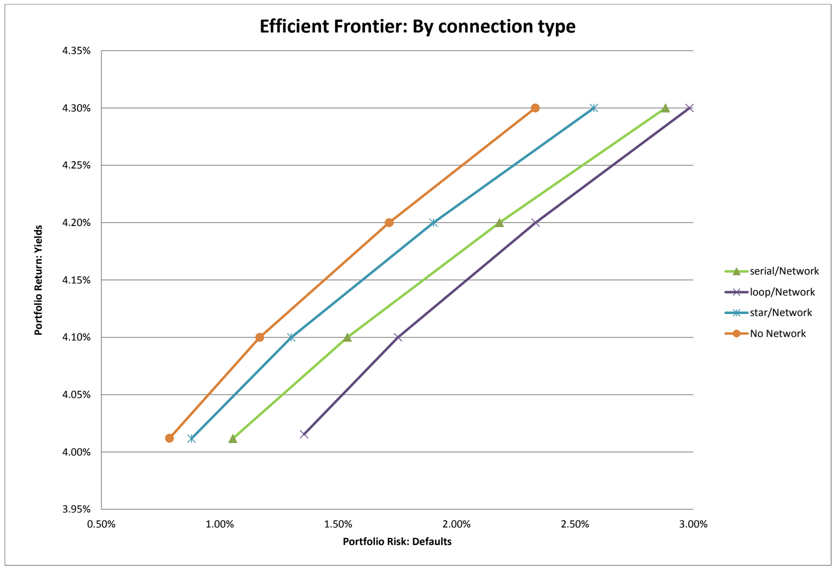


Figure 4.5: Efficient Frontier

Figure 4.6: Star and Serial Network

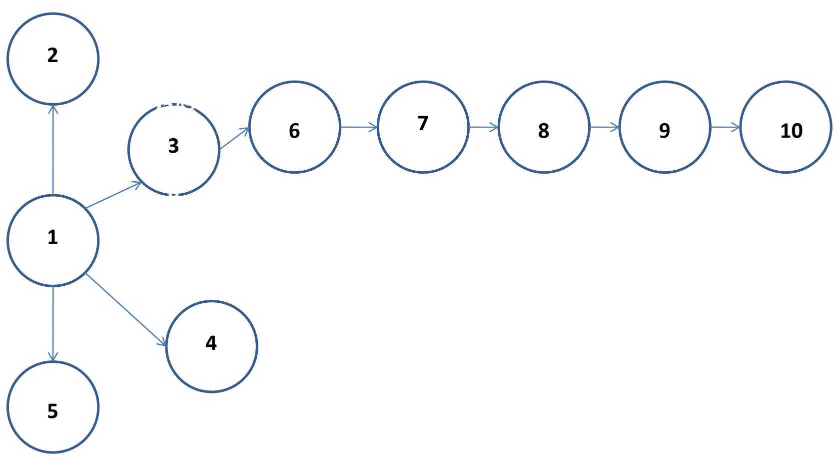
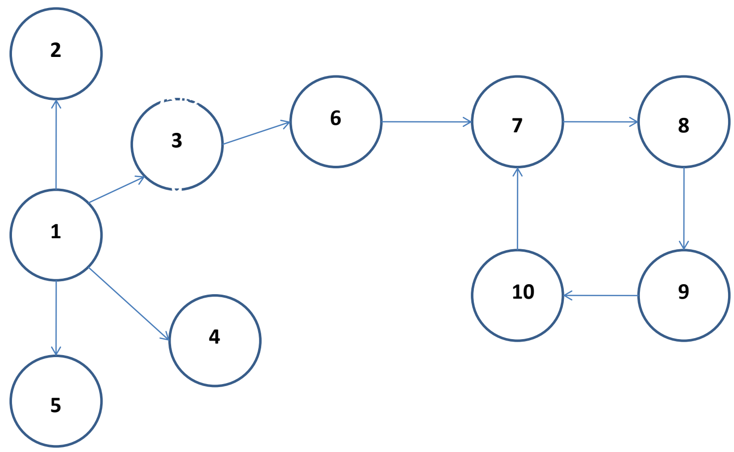


Figure 4.7: Star and Loop Network



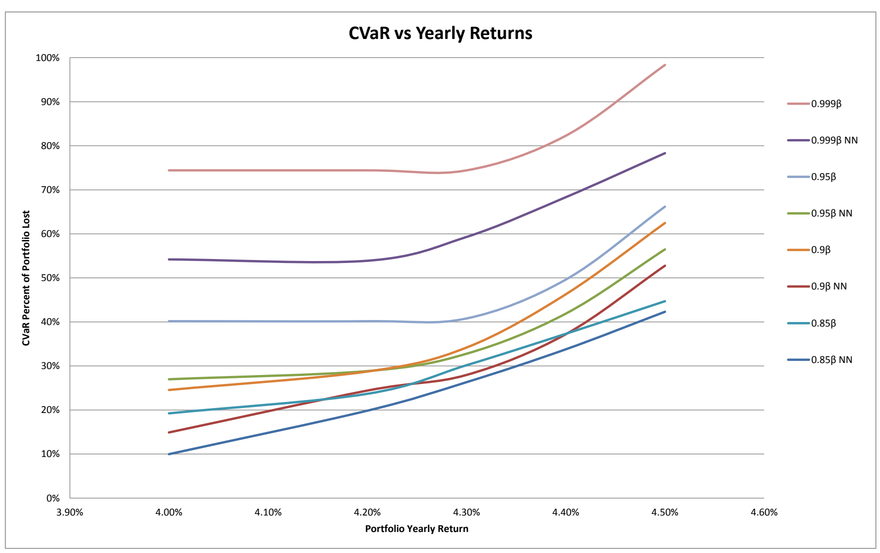


Figure 4.8: CVaR's Insight based on β

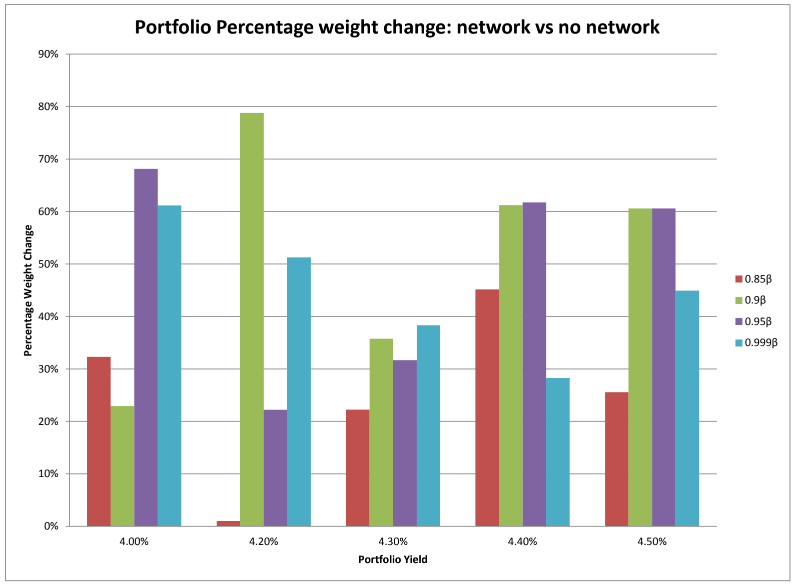


Figure 4.9: Portfolio Weight changes due to CVaR β

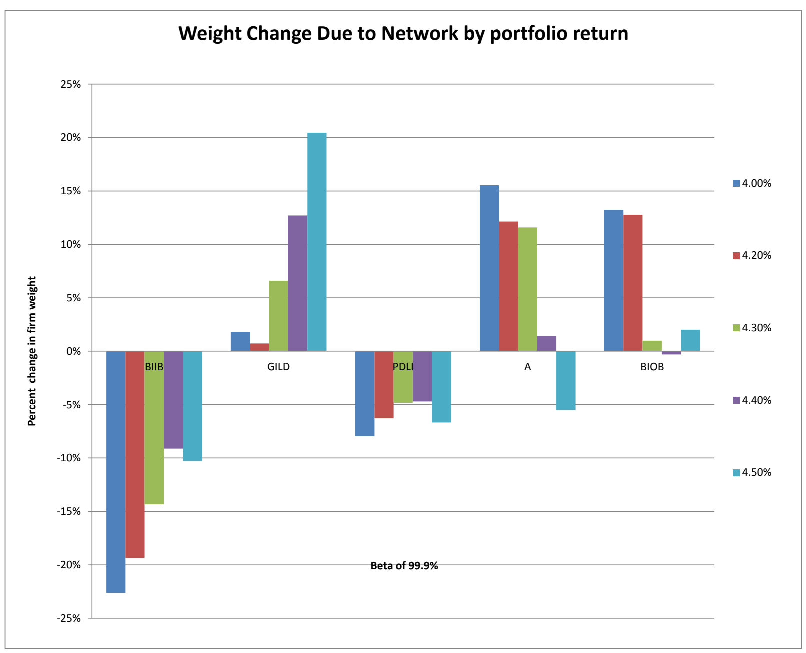


Figure 4.10: Portfolio Weight changes due to Network Connections

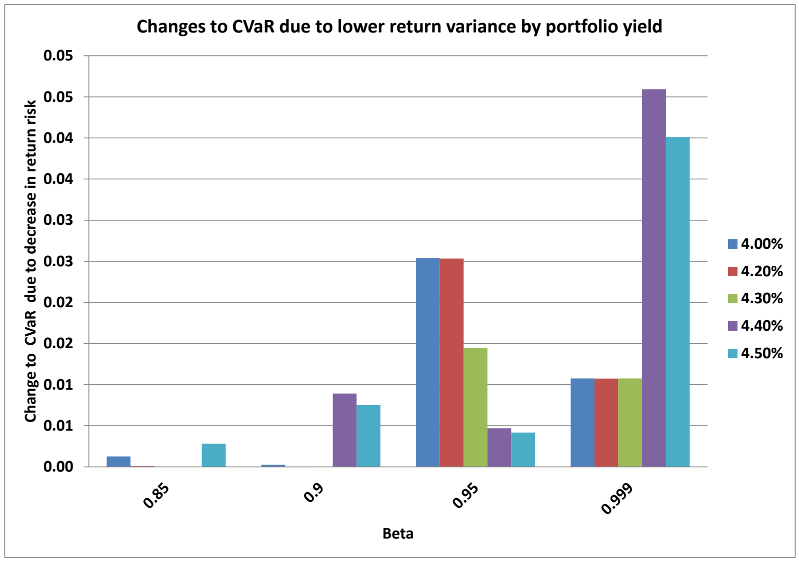


Figure 4.11: Impact to CVaR due to decrease in return variance by portfolio yield

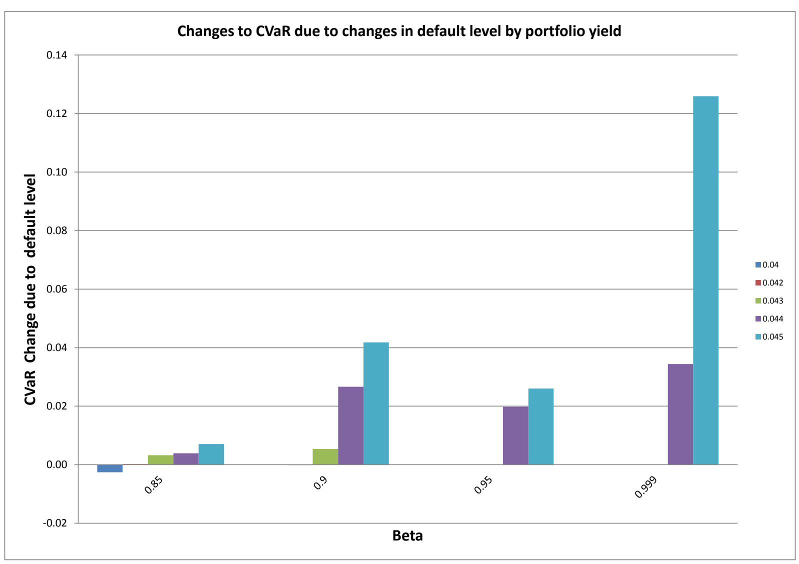


Figure 4.12: Impact to CVaR due to decrease in default level

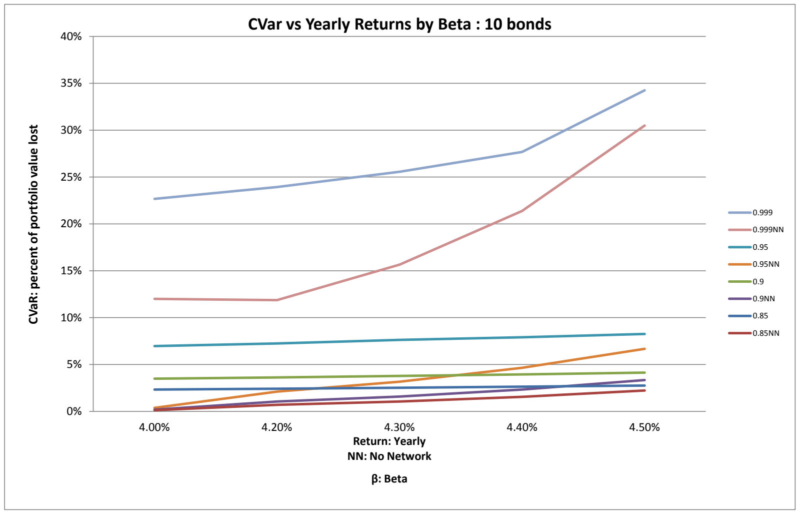


Figure 4.13: Impact to CVaR due to yield and β: 10 bond

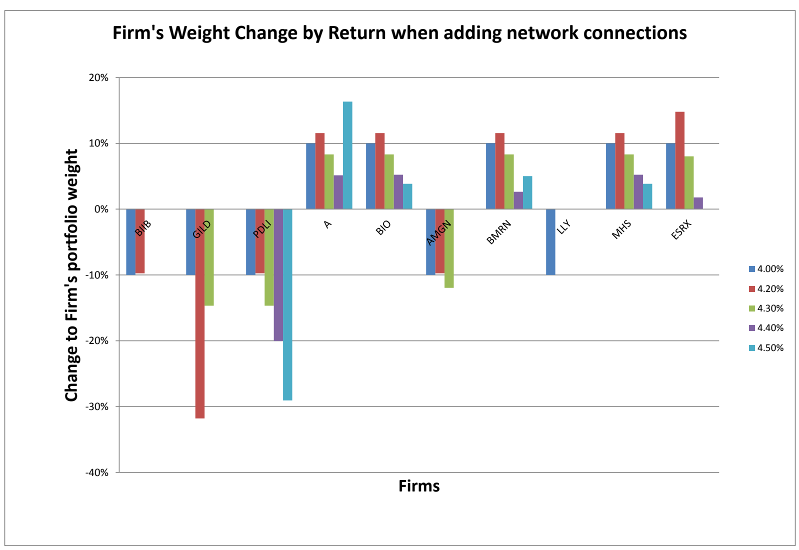


Figure 4.14: Portfolio Weight changes due to Network Connections: 10 bonds

Chapter 5

Case study and Computational Analysis

This chapter provides a case study where the Firm Value model is applied to two historical incidents of contagion to evaluate what warning the model would have provided before the incidents occurred. A section discussing computational requirements and issues will close out the chapter.

5.1 Case study of Firm Value Model

Two historical incidences of contagion are presented as test cases to validate the ability of the proposed firm value model, including contagion, to alert investors to the impact of a contagion on a bond portfolio. The primary warning about firm risk achieved via the model is "increased multiple defaults".

Hypothesis:

A firm value model including contagion helps identify individual or groups of firms that are susceptible to impact by a contagious event. Expected results are increased defaults (singularly or in tandem) for connected firms, resulting in increased tail risk for a portfolio containing these firms.

Metrics/validation:

To determine the impact on firm value and defaults, simulations are run with and without the connections that allow the impact of the contagion to spread between firms. Singular and tandem firm defaults are measured to quantify the impact.

5.1.1 Case 1: Auditor Induced:

Enron, World Com and Waste Management were linked together by the use of the same auditor, Arthur Andersen. The exogenous contagion impacted Enron when Arthur Anderson admitted to destroying evidence by shredding documents. The realization that similar unethical practices could be occurring at other firms audited by Arthur Andersen resulted in the contagion spreading to firms including World Com and Waste Management.

Generation dynamics:

Historical data for the firm value model simulation will cover the period July 1, 1997 - June 29, 2001. The contagion structure represents the spread of an exogenous contagion from any of the three firms to the other two firms, see Figure 5.1. Firm values are evolved bi-monthly over a one year period. Contagion arrival is modeled as a Poisson process with an average of one arrival per year. A relatively strong exogenous impact and strong links between the firms were modeled with amplitude of 0.6 and tau time lambda of 0.5.

Results:

Contagion allowed to spread through the network structure dramatically increased the probability of default. Figure 5.2 illustrates the probability of a firm or combination of firms defaulting. With the contagion and a network, the chances of all three firms defaulting increased substantially.

Figure 5.3 highlights the most damaging impact to a portfolio, multiple defaults. Comparing defaults due to the contagion versus the defaults from the contagion together with the network structure, allow focus on the impact of the network. All

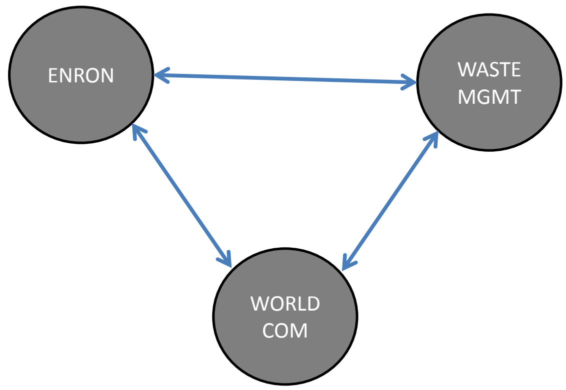


Figure 5.1: Auditor Induced Contagion

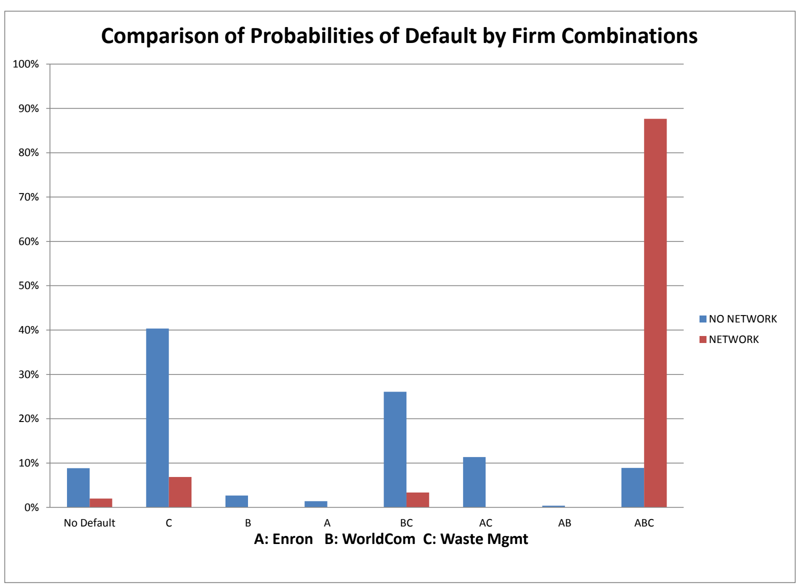


Figure 5.2: Probability of a Firm or combination of Firms Defaulting

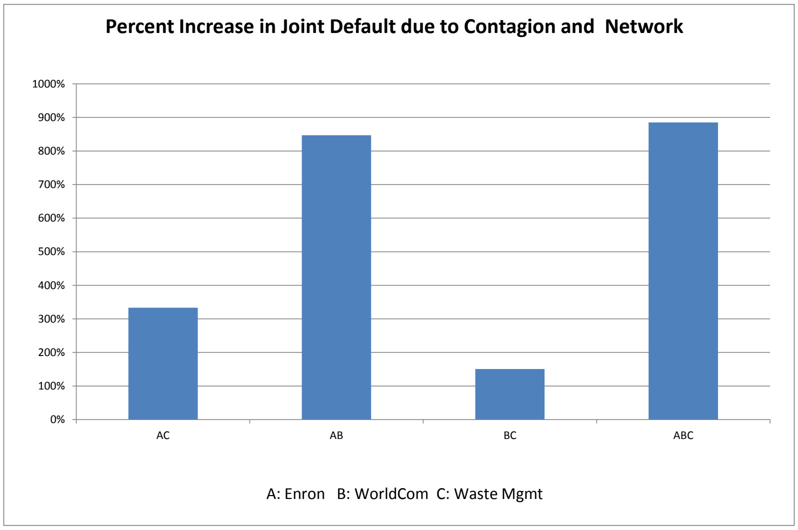


Figure 5.3: Comparison of Multiple Defaults

combinations of multiple defaults increase substantially when a contagion occurs and a network structure exists.

Conclusion:

From the preceding results we see that the model indicated that the risk of all three firms defaulting was greater due to the contagion risk. Approximately six months after the period modeled, an exogenous contagious event impacted Enron, and spread to World Com and Waste Management. Within one and a half years all three firms had declared bankruptcy. Considering contagion in portfolio selection provides an indicator of the increased risk of multiple defaults. The actual historical event had a stronger exogenous impact and slightly weaker connections than those used in the simulation.

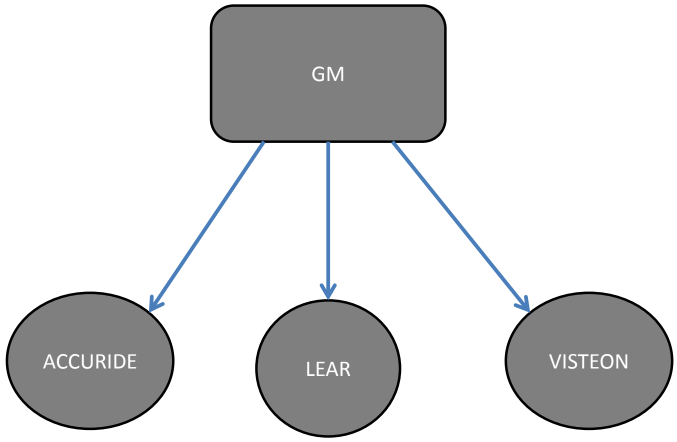


Figure 5.4: Supplier Induced Contagion

5.1.2 Case 2: Supplier Induced:

Numerous firms are linked through supplier relationships. These connections allow contagion to spread thorough lack of payments or decreases in orders.

Generation dynamics:

Accuride, Lear and Visteon were all linked to GM through supplier relationships. To illustrate the impact of problems at GM spreading to its supply partners, the firm value simulation was run based on historical data from 4th quarter 2002 to 4th quarter 2006. The structure has GM connecting to all three of its suppliers, see Figure 5.4. Firm values are evolved bi-monthly over a one year period. Contagion arrival is modeled as a Poisson process with an average of one arrival per year. A medium exogenous impact and strong links between the firms were modeled with amplitude of 0.4 and tau time lambda of 0.5. These values are in the middle of the range from the actual event.

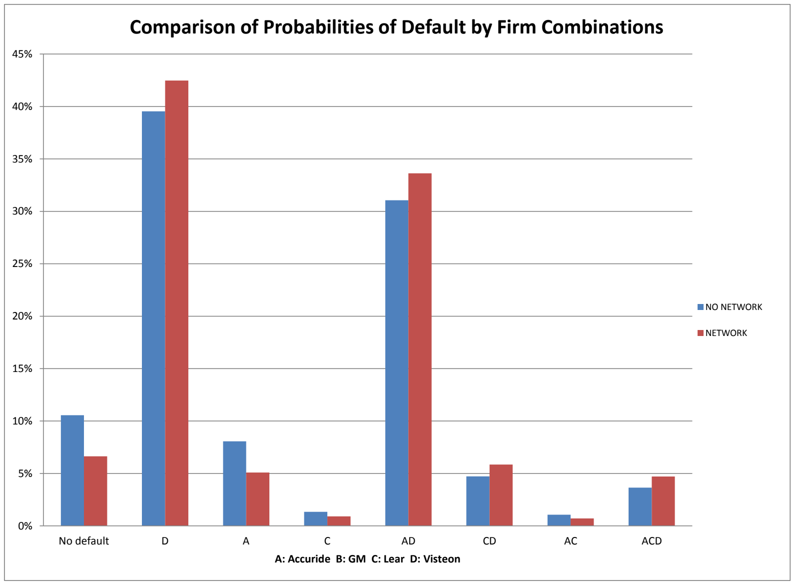


Figure 5.5: Increase in Defaults due to Contagion and Firm Network

Results:

A network structure that allows a contagion to spread results in increase single and multiple defaults as seen in Figure 5.5. Of concern is the substantial increase in multiple defaults which are particularly damaging to a portfolio as seen in Figure 5.6. GM is not included in this set of results since its financial health had already deteriorated by this point and defaulted in all simulations.

From the preceding results we see that the model indicated that the risk of multiple firms defaulting was greater due to the contagion risk. When the U.S. automotive industry began experiencing problems, all firms were negatively impacted and all eventually filed for bankruptcy. Visteon and GM filed at almost the same time, May 28, 2009 and June 8, 2009. Lear filed approximately one month later on July 2, 2009. The last to default was Accuride on October 8, 2009. The results for this case are not as decisive as for the previous example. Possible reasons include the different

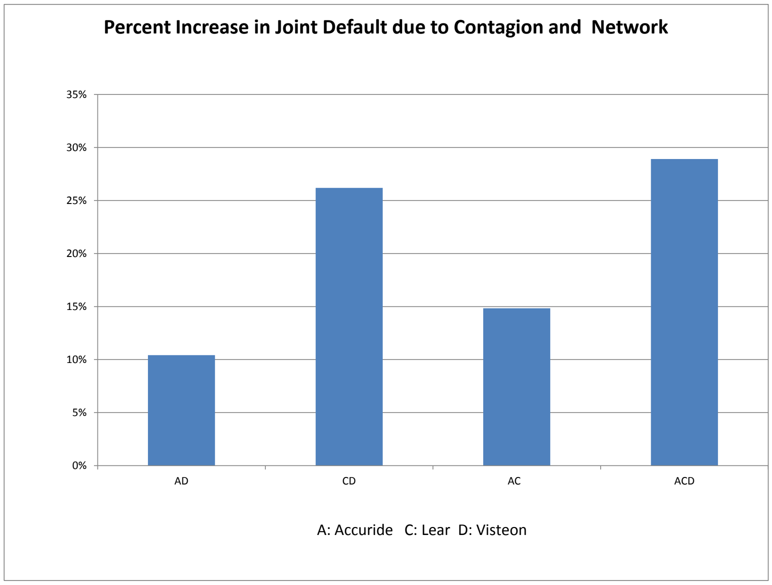


Figure 5.6: Increases in Multiple Defaults

connection structures. In addition, the time period modeled in case 2 were farther removed from the actual event time.

Hypothesis Results and Conclusions:

Based on the preceding results, the firm value model including contagion indicates increased risks for firms when connections exist and contagions are able to spread. Use of the model can provide a warning to possible tail risk if all firms are contained in a portfolio.

Summary:

The firm value model can be used to show the impact a contagion can have on a portfolio when connections exist between firms. This can provide a warning of the risks during portfolio optimization. Once connections between firms are understood, analysis can be run to see the impact of a contagion on various firms in a portfolio. Structure of the connections makes a difference. When comparing the impacts in the two cases, both experienced increases in multiple defaults. The degree of impact was much greater in the Auditor Induced case. Though other factors, such as health of the impacted firms, strength of contagion and connections could be partially causes, the shape of the structure, as seen in previous stylized models, has a considerable impact.

5.2 Computational Simulation Issues: Firm Value Model

The firm value and optimization programs are developed in Matlab. Input and output data not taken directly from Matlab are contained in Excel files. While the use of Excel to read inputs and record results is very convenient, it is very inefficient and is kept to a minimum. Though acceptable in a research environment, in a production environment these platforms would require further evaluation.

In the interest of seeing the impact of increases in the size of the bond portfolio and the number of simulations, the following describes the computational demands. Two system units were used to run the simulations, one contains an Intel i7 processor running at 3.4 GHz, with 8 GB of memory and the other running at 1.73 Ghz, with 6 GB of memory.

Memory Constraints:

The amount of memory is an issue with the firm value simulations. The system unit with 6GB would occasionally run into out of memory errors when executing 25,000 simulations. Additionally, on that system unit simulations took approximately 30 hours. Memory requirements will also increase if the length of time of each simulation is increased or if the time step is reduced.

Time Constraints:

To help understand the impact to simulation times based on the number of bonds considered and the number of simulations run, Table 5.1 provides a sample of times for various combinations. As expected, the number of simulations has a larger impact

Table 5.1: Simulation times by number of bonds

	Number of Simulations							
Bonds	5K	15K	20K	25K	30K	35K	50K	75K
5				11.6 hrs				
10	2.72 hrs	8.08 hrs		15.42 hrs			22.12 hrs	46.38 hrs
15			12 hrs	15.12 hrs	18.38 hrs	18.93 hrs		

than increasing the number of bonds. In addition, we see that neither impacts the time requirements linearly. However the significant amount of time to run the 75K simulation is of concern.

Simulation Accuracy:

A key determinant of the number of simulations is the accuracy of the default predictions for the tail default situations where multiple firms default. Figure 5.7 compares the default probabilities for multiple defaults. The probabilities converge as the number of simulations increase to 25K, 65K and 75K. This is useful because, as seen in Table 5.1, the length of time per run increases significantly when the number of simulations increases from 25K to 75K.

5.3 Computational Simulation Issues: Optimization:

The amount of time to determine an optimization solution is more difficult to quantify. The tighter the constraints the longer it can take to find a solution. The time also increases as the number of bonds increase and as the number of simulations used in the CVaR calculations increase. When a small number of bonds is being considered, each optimization takes approximately 4.5 minutes. This, however, can increase substantially as the number of bonds is increased. To limit the time, constraints are placed on the number of iterations per solution. The

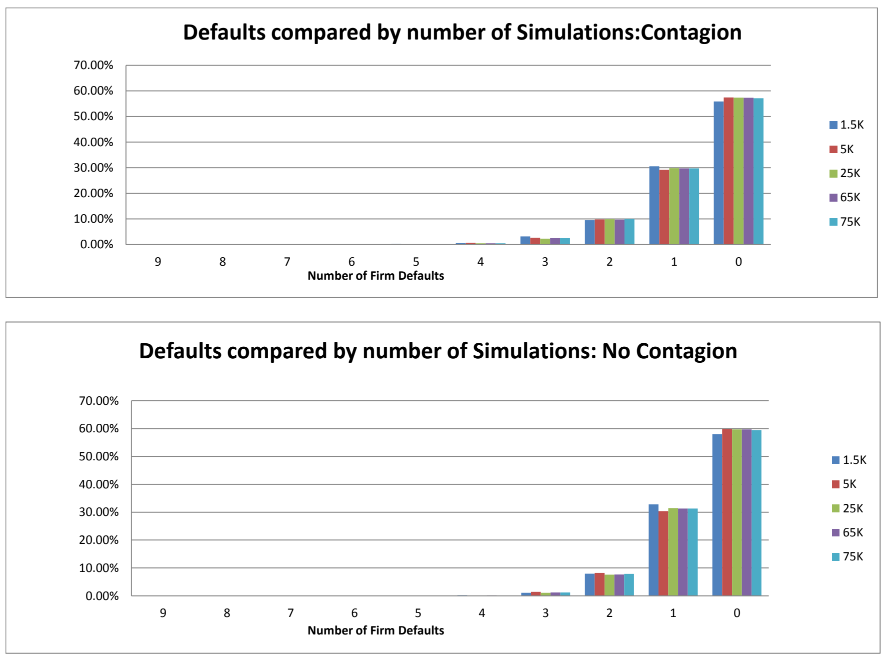


Figure 5.7: Default Probabilities Compared for Number of Simulations

CVaR optimization code is based on code developed by Manthos Vogiatzoglou (www.mathworks.com/matlabcentral/fileexchange/19907-cvar-optimization).

Chapter 6

Concluding Discussion

Much research has been done to improve understanding of the causes of increased clustering in defaults, which is of particular concern in portfolio selection. Often the research focuses on contagion. Most models, however, lack the use of actual firm data and therefore the ability to increase the understanding of how the contagion process works between firms. The aim of this thesis is to develop a model that assists in the understanding of the impact of a network structure among firms on the spread of a contagion. A structural model is developed to evolve firm value impacted by contagion in addition to commonly considered risk factors. Contagion, modeled as an exogenous event spreading from one firm to another due to a network structure, is segmented into variables increasing understanding of what characteristics impact defaults. Results from this model are then combined with other firm data to create a portfolio optimization with the goal of reducing exposure to multiple defaults.

Our research bridges the gap between stylized contagion models and the empirical data which illustrates the impact of contagion. Building on previous research, such as Egloff et al. (2007), a framework is developed to address the issue of calibrating the contagion pieces of the model through the use of firm qualitative data and historical contagion events. Thus developing a platform to analyze the impact of contagion when creating a portfolio of bonds.

The firm connections, allowing a contagion to spread, can increase the severity of an unanticipated event with negative ramifications at the firm and portfolio level. The connections can result in differing impacts for firms depending on the types of connections and their positions in the contagion flow network. This assessment can be gainfully utilized to develop a more robust investment strategy involving the debt issued by the firms.

Analysis of the proposed model's variables shows changes in impact due to varying network structures. Firms connected in a loop structure are the most susceptible to the impacts of contagion, an even greater concern due to the difficultly uncovering this type of connection in practice. The selection of an optimal portfolio is affected when adding contagion aspects to the model. The efficient frontiers are negatively impacted for all three structures considered with the type of connection determining the severity of the impact.

With a better understanding of the impact of a connections between firms on the spread of a contagion, a portfolio optimization model is developed. With this model we see that unless we consider that impact of contagion flowing between firm pairs and focus on its impact through the minimization of CVaR, the true risk of the portfolio won't be understood. When considering contagion flow between firms, portfolio weights change substantially, indicating that without considering the output of the firm value model, a portfolio will be at increased risk for multi firm defaults. As with research searching for understanding of what allows diseases to spread and result in contagion, improved understanding of how a negative event can spread and impact multiple firms, including what allows it to spread, the size of impact and the speed of transmission, is required to decrease negative impacts in the future.

6.1 Hypotheses Conclusions

The following will summarize the results of the hypothesis:

H1: Firms in a portfolio with firm-pair connections, based on qualitative firm date, modeled using graph theory, result in heavy tailed default distributions seen in empirical data.

In numerous simulations, we see that firms with firm-pair connections experience increased defaults, leading to heavy tailed default distributions, over the same firms without these connections. We can identify default distributions seen in empirical data. Unlike reduced form models that also create these default distribution, our model helps understand the causes for these heavy tailed defaults.

H2A: When firms in a portfolio of bonds experience an exogenous contagion, the structure of the connections between firms, not just the number of connections, decreases the firm value and increases lower tail risk of the overall portfolio.

As seem with the stylized network connections, the structure of the connections matter. In order of decreasing significance: Loop, Serial, Star.

H2B: For a similar percent change, a slower decay rate has more of an impact to the overall portfolio of firms than increasing the size of the initial exogenous contagion.

Through the simulation study, we see that decay rate has more of an impact than the size of the initial exogenous contagion. This points to the importance of understanding what links firm pairs together and how to measure the strengths of these connections.

H3: Improved knowledge of the impacts and spread of contagion can be used in a bond portfolio optimization process to improve the selection process, resulting in a portfolio with less tail risk.

Results for both the 5 and 10 bond portfolios show that the optimization reduces the exposure to bonds with increased firm pair connection strength. This would be increasingly useful as the number of bonds being evaluated grows. As the number of bonds in the portfolio expands, it is increasingly difficult to recognize which firms are exposed to a significant number of connections. The portfolio optimization will limit exposure to these firms with the strongest connections.

6.2 Future Work

Contagion continues to be a major issue impacting countries, banks and firms. Much remains to be studied to continue to clarify how contagion flows between firms. It is only with this understanding that methods can continue to be developed to decrease the impact of a contagion on a portfolio. In this vein, further study in this area include:

Develop a Data Analytic method to search data bases and reports such as 10K reports to determine firm-pair connections. Increasing the data available will improve understanding of the amount and type of connections that exist between firms. Searches can focus on collecting data about firms, similar to the method used in this research. Another possible method would be to look for key words, such as geographical locations, firm or product names, and then match firms based on similar references. Only by doing a substantial database search can understanding of the types and amounts of connections that exist between firms be fully quantified. This information then needs to be combined with abnormal returns data to provide additional insight into the values of the contagion variables.

The limited abnormal returns analyzed showed interesting results that require further investigation. When evaluating the Arthur Andersen and GM contagion incidents, there appeared to be multiple contagion events that flowed between these firms. Further evaluation of contagion incidents, studying speed of transmission or multi-impacts between firms, may provide insight into the possibility of interrupting the flow in severe situations where multiple events can impact over a period of time. Additionally, when multiple events exist between firms, does the decay rate change over time? A quicker decay could indicate that after an initial event, the impact decreases in subsequent events possible due to the fact that it is no longer as unexpected as before the first event.

A larger investigation of abnormal returns related to less well known incidents, such as the AMLN example, should be further studied. A study of an increased number of firms over a longer time period should be studied for abnormal returns. This then will be cross referenced with news events and firm connections to see what possible contagion flow events exist and the sizes of the impacts. This will provide much insight into values of the contagion variables.

Can the framework used to establish values for contagion variables be applied in contagion incidents outside of firms? As seen in the current economic crisis, not all countries are impacted equally. Evaluate applying the framework to understand the strength of connections to other scenarios, such as countries or banks to improve understanding of what impacts the amount of contagion that flows between entities.

When creating a portfolio a better understanding of the connections that exist between firms is required. Only by investigating what allows a negative event at one firm to impact another firm will a true measure of portfolio risk be possible.

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Appendix

Appendix A

Appendix

A.1 Categories and Variables

The following are additional examples Categories and Variables that can exist between firm pairs.

Finance

- Lending Institutions: Problems at one firm that impact its relationship at a bank can result in the bank tightening credit for other firms it views as similar. This can lead to the credit issues with firms that use the same bank.

- Same Institutional Ownership of common stock: Sharing an investment firm between firm-pairs can result in a contagion spreading, impacting firm-pairs that have the same investors.

Operations

- Joint Venture: A problem that impacts a firm can spread to another firm through a joint venture. This will impact the firm-pairs that participate in a joint venture.

- Distributors: One firm can negatively impact their distributor or a distributor can run into some unrelated issue. The distributor can spread the problem to other firms. This will Impact firm-pairs with the same distributors.

- Suppliers: A problem at a key supplier can impact all the firms that use this supplier. This will impact firm-pairs that use the same supplier.

Support

- Audit Firm: If practices of an accounting firm are called into question at one firm, the fear of a similar problem at another firm with the same auditor can cause a contagion to spread. As in the example of Enron and their auditor Arthur Anderson. This impacts firm-pairs with the same audit firm.

- Board Member: Having the same board member at multiple firms indicates a closer relationship between those firms than firms in general in that industry. In addition that provides an additional channel for information to travel between those firms.

Marketing

- Sales Region (Geographic): Due to geographically proximity, a contagion could spread among firms due to their location. This will impact firm-pairs located in the same region. As seen in the Enron case, firms with the same auditor and in the same geographical location where more negatively impacted than those outside the region.

- Competitors: Due to the linkage of being in the same industry, when a firm is impacted, this can have an impact on their rivals.

A.2 Mean Reversion Calibration

Table A.1 contains the results of the Mean Reversion Calibration based on data from July 1, 2007 to June 30, 2011

Table A.1: Results of the Mean Reversion Calibration

time length	Delta time	Factor	μ	γ	σ
4 years	10 day	Baa Interest	0.065398172	0.804797491	0.010842532
4 years	10 day	LT Interest	-0.003542363	0.483814811	0.007481217
4 years	10 day	Sector	944.6378646	0.65022903	178.8327222
4 years	10 day	S and P 500	1117.046527	1.005294181	238.8207622
4 years	10 day	Aaa Interest	0.052449111	2.862622973	0.009562894
4 years	10 day	BIIB	75.21730507	0.62652776	23.6273616
4 years	10 day	GILD	43.95502466	2.434941025	12.14348081
4 years	10 day	PDLI	6.60181284	1.701291655	4.472278019
4 years	10 day	A	38.12109338	0.325085702	10.09204842
4 years	10 day	BIOB	96.83047356	1.658390553	28.9917572
4 years	10 day	AMGN	54.2351624	4.345095196	15.48249363
4 years	10 day	BMRN	24.66514798	1.097468898	9.961966461
4 years	10 day	LLY	36.32867405	1.172299106	9.472789656
4 years	10 day	MHS	53.44859564	1.731261824	1/14/1900
4 years	10 day	ESRX	52.64359066	0.523192088	11.04699076

A.3 ROA Calibration

Table A.2 contains the results for the Beta's in the ROA minus Risk Free calibration, based on data from March 31,2006 to June 30, 2011:

A.4 Data Sources

The following lists the sources for the data collected.

Non Contagion Factors

Data sources used for calibration and time intervals available for the data are given in the Table A.3.

Contagion Factors

Firm specific data was collected from the following sources: Compustat, 10-K and 10-Q reports and the Nasdaq website.

Table A.2: Results for the Beta's

Firms	Market - Risk Free	Sector - Market	Baa - Aaa Return	Firm equity - Risk Free
BIIB	0.056648858	-0.005386656	5.2988323	0.02871769
GILD	-0.009428149	0.076940627	11.35576695	0.020887294
PDLI	0.366289784	0.070505853	27.14018172	0.010348046
A	0.072283618	0.129464046	2.299071224	-0.018926506
BIOB	0.072648109	0.049587913	2.400118152	-0.019423832
AMGN	0.074147664	-0.005424095	6.123708709	0.001036622
BMRN	0.140275825	0.212180499	0.388590245	-0.023986195
LLY	0.229434793	-0.057965084	3.906139569	-0.055704074
MHS	0.058245836	0.03427229	3.50943487	-0.00908419
ESRX	0.070629808	0.130970843	5.978442229	-0.018811023

Table A.3: Data sources used for calibration

Factor	Variable	Source	Equation	delta time
Market Return	S and P 500 Index	S and P	4	Daily
Risk Free Return	1 year treasury	Compustat	3	Daily
Spread	Return on Aaa and Baa Bonds	Compustat	3	Daily
Sector Return	NASDAQ biotech sector	NASDAQ	5	Daily
Firm	ROE - stock Price	Compustat	6	Daily

A.5 Statistical Measurements of Contagion Impact

The following describes the variables measured to quantify the contagion impact at the firm level. Each are calculated per firm per run of the simulation. The means of these values for all runs of the simulation are then calculated for use in the analysis.

1. Firm value.

2. Defaults.

   The percentage of firms where: Firm Value - Simulated Debt Level <= 0

3. Variables representing the number of firms close to default,therefore increasingly risky. Often this would be seen as a heavy tailed distribution. The following measurements quantify this exposure.

   (a) Semi-variance.

   Variance for data to the left of the mean.

   (b) Coefficient of Variation.

   Data dispersion is an important indicator or risk This measurement based on the square root of semi-variance which is then normalized for differences in the mean firm values is

   $$COV_i(t) = \frac{\sqrt{SV_i(t)}}{MC_i(t)}. \tag{A.1}$$

   (c) Lower Tail Debt Coverage.

   Measured as Firm Value/ Debt Level to indicate closeness to default. Calculated for firms in the lower tailed 1, 5, 10, and 20 percent.

The following describes the variables measured to quantify the contagion impact at the portfolio level.

1. Shift to the efficient frontier.

   Based on the above optimization, the efficient frontier for portfolios with both no contagion and contagion will be compared. In addition, various optimal portfolios in the no contagion environment will be evaluated in the contagion environment.

2. Portfolio increased risk measures due to network structure.

   To evaluate the impact of contagion and various network structures on a portfolio, the percent change to the risk factors will be calculated for the portfolio. This will show how the network between firms results in the percent change in various measurements of the risk contained in the portfolio. These will be a weighted average of the firm value measurements described by equation 2.17. The risk variables considered are the portfolio weighted average default, lower tail debt coverage and coefficient of variation. The portfolio percent change in these variable is represented by

$$PPV = \Sigma\omega_i(\frac{VC_i(t) - VNC_i(t)}{VNC_i(t)}). \tag{A.2}$$

A.6 Mean Revision Equations

A Mean reverting processes, also known as an Ornstein-Uhlenbeck process, is a stochastic process that over time will revert to the mean. They are often used in Finance, particularly to model interest rates, currency exchange rates, and commodities. The speed of the reversion will depend on the standard deviation and the reversion rate. Values are also impacted by a Weiner process, referred to as Brownian Motion. Methods used to arrive at the parameter values are least square regression and the maximum likelihood method.

An issues to consider is nothing in the formula prevents the process from going negative.

Table A.4: Three Firms Example

Average percent changes due to Network		
Firm	Mean	Default
CELG	-0.31261	1.7840
CEPH	-0.4270	1.1471
AKRX	-0.2739	0.5125

Further Reference on Mean Reverting Processes include: Smith (2010) and Brio, etc.(2007)

A.7 MLE's

Maximum-likelihood estimations (MLE's) are used to fit data to determine values of associated parameters. Based on the underlying data, the most likely values for the underlying parameters is estimated. Logarithmic likelihood functions are determined and then maximizing to determine the MLE's.

A.8 Additional Three Firm Example

The following is an additional exam of the impact on firm value and defaults due to a network structure. The three firms considered are CEPH, CELG and AKRX. Connections exist between CELG and CEPH and between CEPH and AKRX. Table A.4 summarize the results of the simulations.

The spread of the contagion has a negative impact seen by the drop in firm value and increased defaults. These results also show how the number and size of the connections impact the firms. CEPH's firm value is the most negatively impacted of the three firms, due to the connections to both CELG and AKRX. Though CELG did see a greater percent increase in defaults, CEPH had the greatest total percentage

Table A.5: Firm Yield and default percentages

Company	CELG	CEPH	AKRX	BMRN	AMLN
Yield	2.854305	1.103195	1.557236	1.936571	3.129283
Default Rate	0.02612	0.00716	0.02956	0.01024	0.44108

of defaults. Illustrating that not all firms will be impacted to the same extent by contagion, with some more susceptible to a severe negative impact. Unless one is aware of these connections, a firm's risk can be higher than anticipated.

A.9 Five Firm Portfolio: additional example

An investor selects a sector specific investment strategy believing this sector will have stable returns and less risk than the market in general. Five firms' bonds are selected. Four of the five bonds are selected for their returns and relatively low default percentage. The fifth stock has a greater return. Its risk however, is also increased (see A.5). Three of the bonds have connections that raise the concern of contagion spreading between the firms (CELG, CEPH, AKRX). The connections between the firms consist of : CELG and CEPH, CEPH and AKRX. Of the no connection bonds; one is similar to the three connected bond in terms of risk and yield (BMRN), the other is the highest risk/return bond (AMLN).

The goal of the optimization is to immunize the portfolio from the severe impact of contagion. This impact is seen in the rare but severe tail events where multiple firms default together. Table A.6 describes the likelihood of two firms defaulting together. The three linked firms are at a greater likelihood of defaulting together than to default with the similar but unlinked firm BMRN. Defaults are much higher for the risky firm ALMN, however both the linked firms and the unlink firm have similar results.

Table A.6: Simulations where both Firms defaulted (25000 simulations)

Company	CELG	CEPH	AKRX	BMRN
CELG				
CEPH	16			
AKRX	47	37		
BMRN	7	2	10	
AMLN	254	69	168	133

When three, four or five firms default at the same time the damage to the portfolio is debilitating. Table A.7 presents the number of times these multi-firm defaults occurred in the simulations. Despite the fact that the default rates for the connected firms are much lower than the riskiest firm (AMLN), they are as likely to be involved in a multiple default situation when the network connections are considered. The no connection firm (BMRN), though similar risk and return characteristics to the connected firms, is less frequently part of a multiple default. Supporting that connections which allow a contagion to spread increase a firm's likelihood of being part of a multiple default scenario. AMLN is expected to be risky, but we don't expect the three linked firms to be at a similar level of risk. The number of defaults for the same simulations, when there are no network connections between the firms allowing the contagion to spread, is presented in the second chart. Substantial drops in all firms are shown, but now the three connected firms have values much closer to the similar no connection firm BMRN.

A.10 Flow Chart of Matlab Modules

Figures A.1, A.2, and A.3 contain flow charts of the matlab modules.

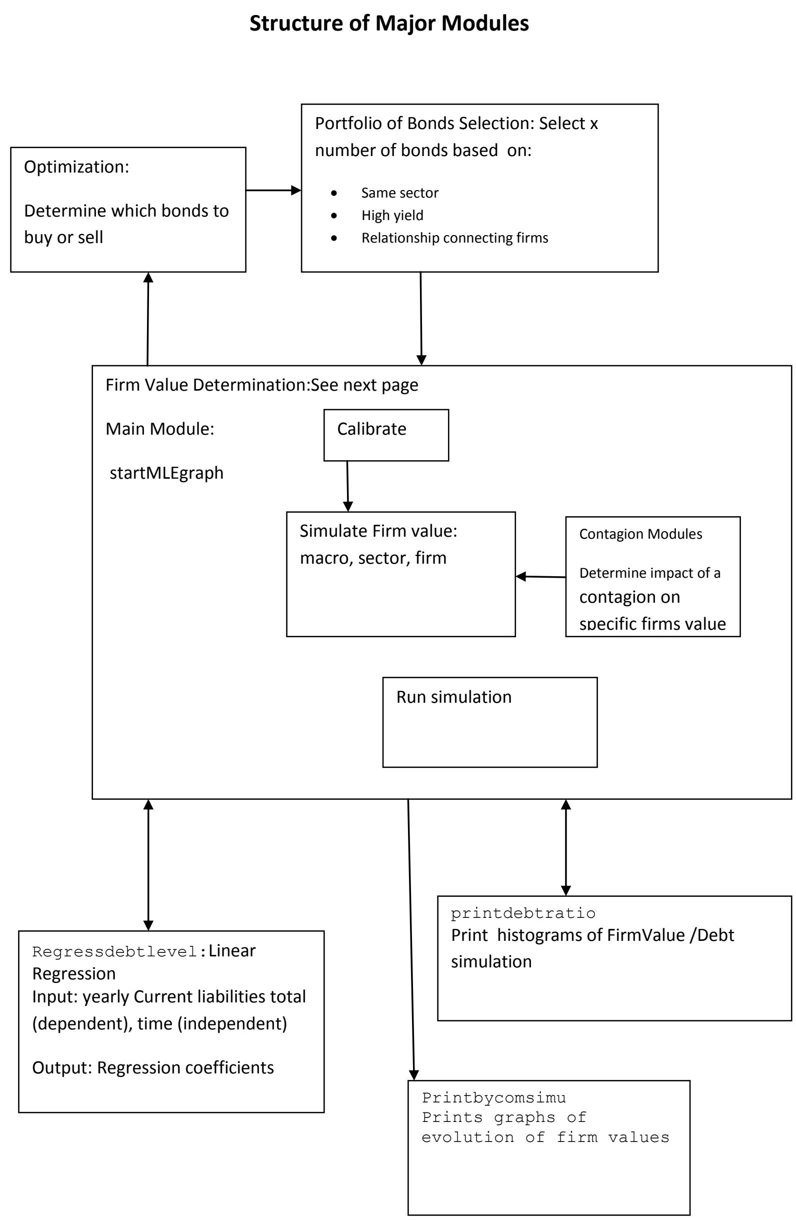

Figure A.1: Matlab Modules: Structure of Major Module

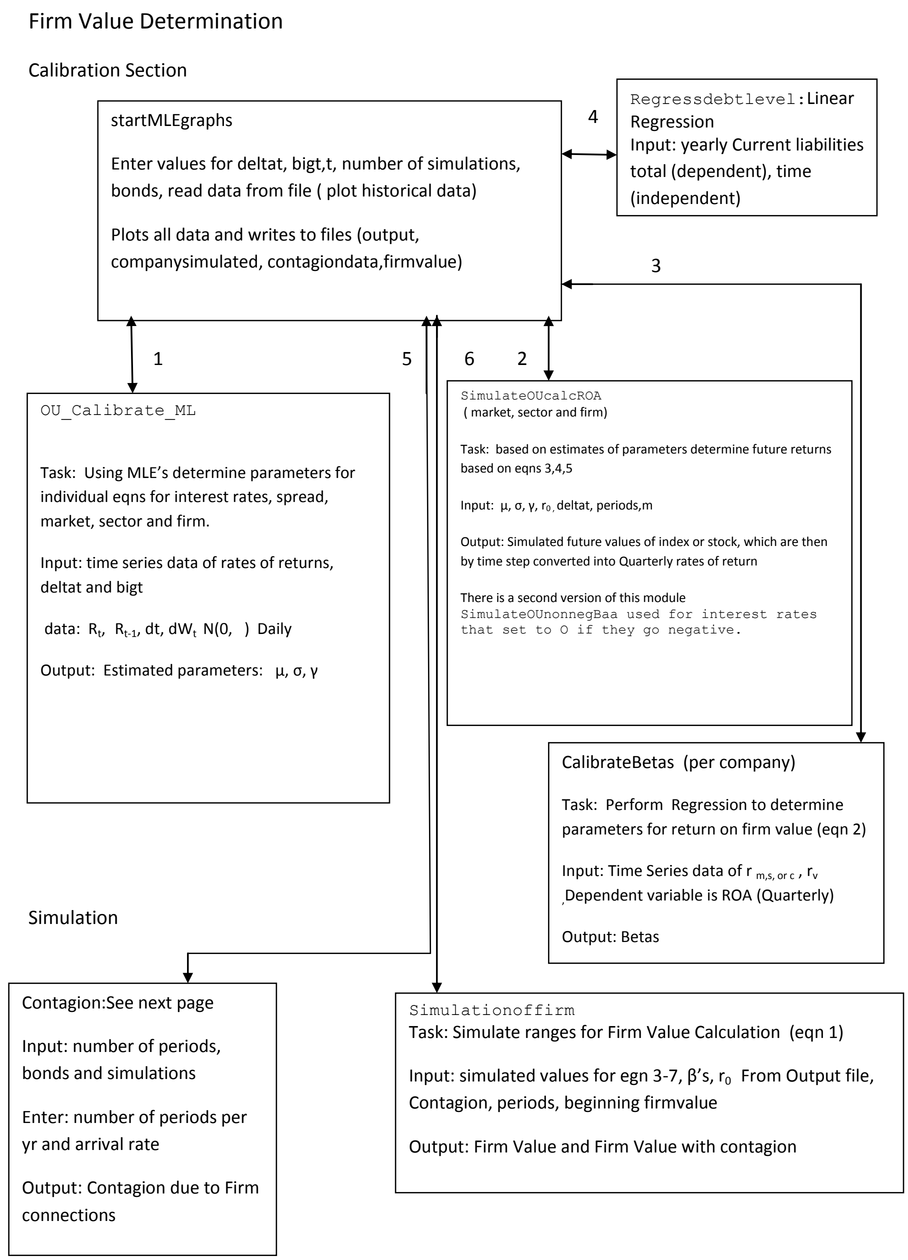

Figure A.2: Matlab Modules: Firm Value Determination

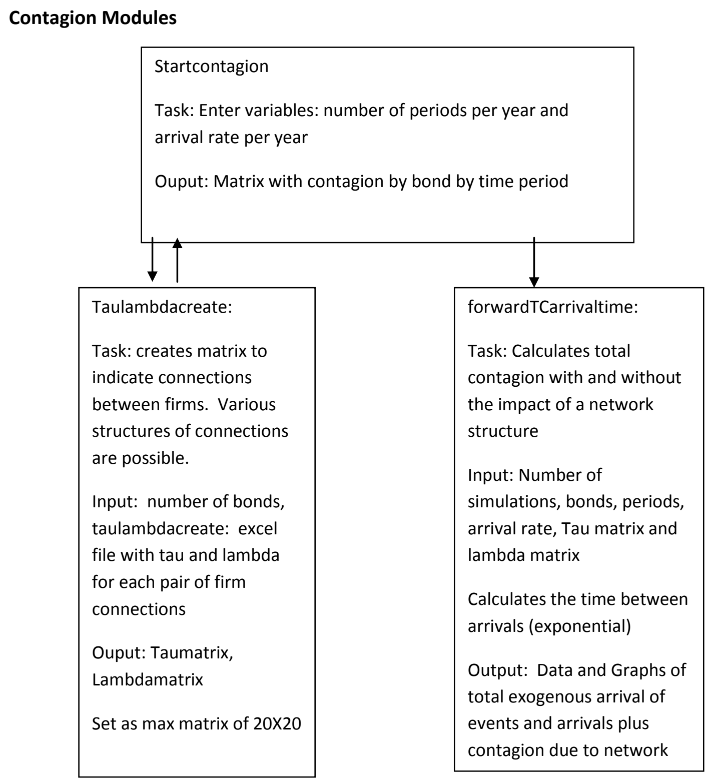

Figure A.3: Matlab Modules: Contagion Modules

Table A.7: Number of multiple firm defaulted, with and without network connections (25000 simulations)

Relative Frequency of number of simulations with defaults of three, four or five firms: (25,000 simulations) Network connections between CELG, CEPH, AKRX

CELG	CEPH	AKRX	BMRN	AMLN
34	25	28	9	31

Relative Frequency of number of simulations with defaults of three, four or five firms: (25,000 simulations)No Network connections

CELG	CEPH	AKRX	BMRN	AMLN
5	3	8	4	10

Vita

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