5-2011

# Three Macroeconomic Essays: Budget Stabilization Funds, Terms of Trade, Durability and the Small Open Economy Business Cycle

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To the Graduate Council:

I am submitting herewith a dissertation written by Ali Mohammad Al-Nadi entitled "Three Macroeconomic Essays: Budget Stabilization Funds, Terms of Trade, Durability and the Small Open Economy Business Cycle." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Economics.

Mohammed Mohsin, Major Professor

We have read this dissertation and recommend its acceptance:

Phillip Daves, Matthew N. Murray, William S. Neilson

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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# Three Macroeconomic Essays: Budget Stabilization Funds, Terms of Trade, Durability and the Small Open Economy Business Cycle

A Dissertation
Presented for the
Doctor of Philosophy Degree
The University of Tennessee, Knoxville

Ali Mohammad Al-Nadi
May 2011

# Dedication

To Sitah, Fadwa, Jawad, and Ibrahim

# Acknowledgments

I wish to express my gratitude to my advisor, Mohammed Mohsin for his diligence, assistance, patience and dedication. I am so grateful to him. I also thank my committee members, Phillip Daves, Matthew Murray and William Neilson for their assistance and willingness to serve. In addition, I like to thank those who helped me through the course of my PhD program particularly Robert Bohm, the head of the economics department,  Donald Bruce, the director of graduate program and the Central Bank of Jordan. This research project couldn’t be completed without their kind support. Finally, I like to thank my family members for their patience, support and sacrifices that they had to undertake due to my program. Particularly, I thank my mother Sitah, my wife Fadwa and my two sons, Jawad and Ibrahim.

# Abstract

In this dissertation we use Dynamic Stochastic General Equilibrium (DSGE) models to explain empirical regularities and policy implications related to (1) durable goods, interest rates and small open economy business cycles, (2) Terms-of-Trade (ToT) and economic fluctuations in small open economies and (3) Budget Stabilization Funds (BSFs) and States' business cycles. In the first essay, we document that durable spending in developed small open economies constitutes a large share of their total income. Their spending is highly procyclical, sensitive to interest rates, and leads the business cycle. We address these regularities with a RBC model with durable goods. The model successfully replicates the observed business cycle regularities and explains many anomalies not explained in the existing literature. It also emphasizes the role of interest rates uncertainty in explaining the dynamics of the small open economies. The second essay addresses the impacts of the ToT fluctuation on the business cycles of various small open economies. We argue that differences in the degree of durability in domestic production and imports may make these economies more or less sensitive to an identical ToT shock. We found that economies with higher durability usually enjoy more stable business cycle comparing with economies with lower degree of durability. Differences in the persistence of the ToT do affect the dynamic of the external accounts but it cannot explain the observed differences business cycles across small open economies. In the last essay, we evaluate the economic impacts of the Budget Stabilization Funds (BSF) on State-level business cycles. We lay out a State economy RBC model in which a State's government applies a designated saving rule consistent with households' optimization. Given the suggested rule we find that the BDFs become a significant automatic stabilizer. It is not only mitigates the procyclicality of the government spending but it also smooth the State's business cycle.

# Preface

Following the pioneering work of Kydland and Prescott (1982), dynamic stochastic general equilibrium (DSGE) framework has become the standard tool in analyzing various issues in macroeconomics. This framework suggests constructing an artificial economy that is capable of replicating an actual economy. The main attractive features of these models are their flexibility in specifying the objectives and the constraints faced by different economic units and the institutional framework under which they interact. These models indeed have solid micro foundations. Apart from the basic aggregate supply or 'technology' shocks, these models are also able to address the effects of various exogenous policy shocks (fiscal, monetary and commercial for example). Despite the significant success of these models in explaining macroeconomic fluctuations and dynamics, they still face important challenges in explaining some of the empirical regularities observed in the actual real business cycle data. In this dissertation we analyze three distinct macroeconomic issues, the first two are specific to small open economies business cycles while the third one is specific to State economy business cycles. In the first essay we document that households, in developed small open economies, in general spend a relatively higher share of their income on purchasing durable consumption goods. Their spending on durables is strongly procyclical, strongly correlated with all national accounts, sensitive to interest rates and leads the business cycle. We construct a business cycle model with durable and nondurable goods, habit formation and variable capital utilization. Our model is subject to two exogenous shocks to technology and country premium. We calibrate to the model to Canadian data (1980:Q1-2009:Q4) and find that the model economy closely matches the observed business cycle and replicates well the comovement between durable spending and the rest of the national accounts. It also predicts that durable spending leads the cycle. Moreover, we find that interest-rate uncertainty improves the model predictions and plays an important role in the dynamics of the external debt of a small open economy.

In the second essay we extend the framework established by the first one to investigate the effects of adverse ToT shock on the business cycles of small open economies. We argue that differences in the degree of durability in domestic production and imports may make these economies more or less sensitive to ToT fluctuations. To achieve our stated goal we develop a small open economy model with two goods – domestic and foreign. We assume both home and foreign goods are two aggregate consumption goods with certain degree (s) of durability. The model economy is able to replicate the different moments of the national accounts in developed small open economies. For the first time, this model replicates the comovement between ToT and national accounts. Further, the model predicts that fluctuations in ToT are responsible for about one fifth of the aggregate fluctuations of developed economies and one third of the aggregate fluctuations in emerging economies. We also find that the differences in the persistence of the ToT can affect the dynamic of the external accounts in response to ToT fluctuations but it cannot be enough reason for the differences in the small open economies business cycle. Our model provides very reasonable explanation to the differences between developed and developing economies in terms of their business cycle characteristics.

The main objective of the third essay is to develop a State-level real business cycle (RBC) model with fiscal policy to evaluate the State's BSF in terms of its impact on State's government budget and State's business cycle. Almost all American States have legal provisions mandating that the budget should be balanced on a yearly basis. It is relatively easy to comply with the rule in good times as revenue will be abundant. Keeping a balanced budget in bad economic times is challenging. It calls for procyclical tax increases and/or expenditure cuts, unless significant surpluses are run in the upturn. To circumvent these problems state governments across the country adopted BSFs. Recently European Monetary Union countries also show interest in BSFs as the Stability and Growth Pact (SGP) limits the amount of the budget deficit in these countries. Despite the evidence on the role of on stabilizing States government expenditures, there are no empirical studies investigating the effect of the BSF on the business cycle in a systematic way. We consider two theoretical State economy models. In the first model economy, the State government collects revenues using distortionary income and consumption taxes and uses the revenues to provide consumption services, public productive investment and income transfer to households. The second model economy is almost identical to the first economy except that State government has a budget stabilization fund BSF. In the second model economy, the government allocates part of its total revenues to accumulate deposit in the BSF to be used to finance the government expenditure during the downturn of the economy. We derive an optimal dynamic rule for government saving that is consistent with households' optimization. Given that rule the State decision on savings and spending from its BSF become directly linked to the State economic fundamentals rather than the discretion of the policy makers. Given the rule, the State BSF becomes an important automatic stabilizer to the aggregate State macroeconomic fluctuations. In particular we find that government's tax revenues become less volatile and more persistence with BSF. More importantly, we find that the BSF reduces the aggregate fluctuations in percapita income, employment and consumption and government revenues and improves the persistence of these important macro variables. Household welfare becomes less correlated with income and hence becomes smoother.

The dissertation is organized as follows. In the first essay we analyze the open economy real business cycle with durables and interest rates. In the second essay we address the business cycle impacts of the ToT fluctuations in small open economies. In the third essay, we evaluate the impacts of BSFs on the State's government budget and the State's business cycle.

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# Essay 1

## Durable Goods, Interest Rates and the Small Open Economy Business Cycle

### 1.1 Introduction

Households' spending on consumption of durable goods in a typical advanced small open economy like Canada constitutes almost 15 percent of GDP. Their spending on durables is strongly procyclical, highly correlated with trade balance, sensitive to interest rates and leads the business cycle. Motivated by these facts, we constructed a real business cycle model (RBC) with durable and nondurable consumption and stochastic country premium. Additional features of our model include habit persistence in durable and nondurable consumption so as well in labor supply which mainly helps to match with the degree of persistence in the actual business cycle data. The model also relies on variable capital utilization as an additional channel to control over the aggregate volatility of the theoretical model. The model economy is subjected to two sources of uncertainty related to technology and county risk premium.

We calibrate our model to Canadian data (1980:Q1-2009:Q4) and find that the model economy closely matches the observed business cycle and replicates well the comovement between durable spending and the rest of the national accounts. Further, we find that country premium plays an important role in explaining the dynamic of external debt and investment and helps explaining the observed comovement between durable spending and national accounts. This result is intuitive since the stock of durables constitutes an important share of the aggregate portfolio of the advanced open economy. Our model is also consistent with the empirical observation that durable spending leads the business cycle.[1] The main feature of our model is that it takes into consideration the adjustments in durable goods in explaining the actual business cycle of the small open economy. However, the existing RBC models of small open economies have pursued different approaches. We highlight the main contributions to the small open economy RBC analysis and its main features and limitations comparing to our proposed small open economy RBC model.

It is well known that most of the difficulties in matching a theoretical RBC model with actual data have initial raised in the initial work of Mendoza (1991). In his first small open economy RBC model, Mendoza (1991) successfully explains main stylized facts particular to these economies.[2] His model ignited a series of important research papers. Investigating on sources for these short outcomes of the open economy RBC models one can sort them in three categories (i) the vulnerability of these models to technological progress estimators, (ii) the choice of the specific preference of households, and (iii) the inability of these models to capture effects of the fluctuations in the international interest rates.

[1] In a counterfactual version of the model, without interest rate uncertainty, the model predicts durable spending to lag the business cycle, which can be considered as an additional argument on the importance of interest rates uncertainty in the explaining the business cycle behavior of the underlined small open economy.

[2] Mendoza (1991) successfully explains the positive correlation between domestic savings and investment, and accounts for the counter cyclicality of the current account and balance of trade in the economy documented by Backus and Kehoe (1989, 1992).

Mendoza (1991) finds that when technical progress estimators are derived from Solow residuals, his model overstates the observed volatility and the persistence of annual frequency business cycle data. To circumvent this problem, he departed away from the conventional wisdom of the RBC literature. Instead of using Solow residuals to obtain the estimators of the technical progress driving force estimators, he calibrated these impotent estimators just to bring the simulated volatility and persistence in the aggregate output fluctuations closer to the actual data.[3] Unlike Mendoza, Baxter and Farr (2001), Letendre, and Gau and Janko utilize the variable capital utilization to improve the predictions in their open economy RBC models as we intend to do in our model. [4] Their approach with variable capital utilization indeed improved the performance of the international RBC model in many aspects. The same approach is also adopted by Letendre (2004) and Guo and Janko (2009) in their open economy RBC analysis. Aguiar and Gopinath (2007) alternatively decompose the Solow residuals into two components: a transitory productivity shock and a random walk or trend shock. The later component is parameterized with GMM method to minimize the difference between observed business cycle moments and its parallel moments obtained from the model. Of particular interest, their model predicts that emerging countries are driven mainly by shocks to the trend while developed economies are driven mainly by transitory shocks to technology. They interpret the stochasticity of the trend as evidence to unobserved regime switching, friction and/or domestic policies shifts.

[3] Schmitt-Grohe and Uribe (2003) and many others used the same methodology.

[4] In a close economy RBC analysis Burnside, Eichenbaum and Rebelo (1993), Burnside and Eichenbaum (1996), and King and Rebelo (1999), find that factory utilization reduces the variance of the innovation in productivity shocks necessary to match the observed volatility of output, and reduces the likelihood of a technological regress.

In this context, Garcia-Cicco, Pancrazi, and Uribe (2010) argue that the trend estimators of the trend process should steam directly from data. By so doing, they find that the standard open economy model with stochastic trend is incapable of predicting the main stylized facts of the business cycle of emerging economies. Otherwise Garcia-Cicco et al find that a model with financial friction combined with exogenous preference and domestic spending shocks performs better in explaining the Argentinian economic business cycle.

Additional limitations arise from the choice of a particular household preference, commonly known as GHH preferences according to Greenwood, Hercowitz, and Huffman (1988).[5] Although this class of preference performs better comparing to other common specifications, it yields high correlations between output and other national accounts, particularly employment and consumption. In addition, it yields very low persistence in the generated macro data comparing to the actual business cycles. To overcome these problems, many authors augment this preference with habit formation. For example Uribe (2002), Letendre (2004) and Uribe and Yue (2006) among others use habit formation in consumption to improve the outcomes of the model with GHH preference. Interestingly, Guo and Janko (2009) introduced habit formation in labor supply and improved additional characteristics of the model.

Additional critic on the performance of the small open economy RBC models is due to its inability to capture the effect of interest rates. In particular, when interest rates uncertainty is

---

[5] In an open economy RBC framework such preference is often recommended since the work of Mendoza (1991). Under this specification, the labor supply becomes independent of consumption, exclusively dependent on the marginal product of labor. This eliminates the income from households' labor supply decision making consumption and labor supply more sensitive to technology shock. Detailed discussion on the prosperities of of the GHH preference is carried by Correia, Neves, and Rebelo (1995).

added to these models its ability to replicate main business cycle features deteriorates.[6] In this later issue recently Neumeyer and Perri (2005) document that the responses of various small open economies to interest rates significantly differ. Emerging economies are more sensitive to international capital flows and interest rates. Hence, modeling interest rates requires other structural modifications that can better describe the unique structure of an underlying economy. They proposed a working capital constraint as an additional channel for interest rates in the case of emerging economies. With these modifications, their model produced countercyclical interest rates as observed in a group of emerging economies. In addition their model also replicates various stylized facts specific to these economies.[7] Unfortunately, their business cycle model is limited to emerging economies only. Neumeyer and Perri, Garcia-Cicco et al, and Aguiar and Gopinath all realize that small open economies are distinct in many structural structures issues and for a RBC models must account for distinguished characteristics of the underlining.

We note that adjustments of durables stock are particularly important for advanced small open economies. Spending on durables in advanced small open economies constitutes a larger share of households' income and closely commove with all national accounts. At the same time, in closed economy RBC models, the user cost of durable is considered a natural channel for analyzing interest rates policies. Hence, we postulate a two goods (durable and nondurable) model and incorporate interest rates uncertainty into the model in the form of a stochastic country specific risk premium. Including durable goods in a small open economy RBC model significantly improves their ability to capturing the effect of interest rates and improves our

[6] See, for example, Mendoza and Correia, Neves and Rebelo (1995).

[7] Neumeyer and Perri correctly predict that that emerging economies are highly volatile, consumption volatility exceeds output volatility, trade balance is strongly countercyclical and interest rates are countercyclical.

understanding of the distinct features of the dynamics of the small open economies. In a two-goods economy, the substitution between durables and nondurables arises from changes in real interest rates, given the nonseparability of preferences in durable and nondurable goods. As pointed out by Ogaki and Reinhart (1998), excluding durables from a RBC model, understates the estimated intertemporal elasticity of substitution, unless the two goods are completely independent. Since durable goods constitute a natural channel for interest rates, one can conclude that the absence of durable may also affect the macroeconomic response to changes in interest rates. Bernanke (1985), for example, shows that durable stock adjustments may substantially affect the time series properties of durable and nondurable consumption. Iscan (2002) finds empirical evidence that the Canadian current account reflects to some extent adjustments in the stock of durables. Uribe (2002) recommends time nonseparablity in preference (with durability in consumption or habit persistence) to improve the prediction of the international RBC model in terms of the observed "Price-Consumption Puzzle of Currency Pegs." Engel and Wang (2010) show that a standard international business cycle model with durability in traded goods can explain the behavior of imports and exports over the business cycle. However we find that a two goods model needs further modifications to be able to overcome the traditional limitations of the small open economy RBC models and be able to mimic various features of the observed advanced economies business cycles. Hence, we add additional features to our model including: habit persistence, variable capital utilization.

Durability in consumption and habit formation intensifies the time nonseparability of household's preference. The combination of these two features significantly improves the performance of our model in many aspects, particularly with respect to the dynamics of aggregate portfolio of the open economy. Habit forming household requires a higher premium to forsake consumption for holding a risky asset than a household with time-separable preference.[8] Accordingly, it is imperative to take into consideration habit persistence when saving dynamics, in response to interest rates and other shocks, plays a crucial role. In our model we account for deep habit formation in the consumption of both durable and nondurable goods, and in labor supply.

The performance of the model, in terms of the second moments, provides a remarkable match with the various features of the Canadian quarterly business cycle. The model closely predicts both the volatility and the persistence of all national accounts and their cyclical behavior. Nondurable consumption, durable consumption and employment are strongly procyclical with high persistence levels. Trade balance is moderately countercyclical and persistent. Spending on durable consumption is more volatile than output, moderately persistent, highly procyclical, and leading the business cycle. We find that interest rate uncertainty explains about 1.3 percent of the aggregate output fluctuations. It also plays an important role in improving the model's predictions, particularly in terms of the volatility of the trade balance to output ratio and the procyclicality of durables and investment. Interest rates uncertainty also explains why durable spending leads the business cycle in small open economies. While recent literature explains the countercyclicality of interest rates in emerging economies, it does not address the procyclicality of interest rates in advanced open economies, as we do in this paper.

[8] This aspect of time non-separable preferences is exploited by Abel (1990), Constantinides (1990), Gali (1994), and Campbell and Cochrane (1995) and empirically tested to study the current account dynamics in Gruber (2004), Uribe (2002) and Letender (2004). Further, empirical evidence in favor of habit persistence has been provided by Heaton (1993), Ferson and Constantinides (1991), Fuhrer and Klein (1998), and Naik and Moore (1996) as well.

We show that the sensitivity of the open economies to interest rates depends on the degree of the intraperiod elasticity of substitution between durables nondurables and the degree of elasticity of the labor supply.

The rest of the paper is organized as follows. In Section 2 we provide some empirics involving consumer durables and business cycles. The detailed structure of the model is outlined in Section 3. Parameter values used in our calibration are provided in Section 4. Section 5 provides a detailed outcome of our calibration exercise. In Section 6 we capture the role of durable spending and interest rates. The importance of habit formation and capacity utilization are evaluated in Section 7, followed by some concluding remarks in Section 8.

## 1.2 Consumer Durables: Empirics

In 2009 Canadian spending on durable consumption constitutes 24.1 percent of total consumption spending on goods and services or equivalently 15.4 percent of GDP. These shares are relatively consistent in many advanced small open economies and significantly higher compared to a larger economy like the USA.[9] In addition to constituting a large share of total income, spending on durable goods significantly influences every aspect of the national accounts in small open economies. Table A.1 reports the pairwise correlation coefficients of different national accounts using the Canadian quarterly data spanning 1981:Q1 to 2009:Q4. We observe that durable spending $(d)$ is positively correlated with output $y$, employment $(h)$, and investment $(i)$ but negatively correlated with trade balance-output ratio $(tby)$ and current account-output

[9] In the US economy spending on durable goods consumption is 12.1 percent of total personal spending on goods and services (8.6 percent of GDP) in 2009.

ratio $(cay)$. Figures B.1.a through B.1.c display a strong comovement over the business cycle between spending on durable goods and other macro aggregates in the national accounts. Figure B.2 plots the dynamic crosscorrelation between durables spending and output. The shape of the cross correlation between of durable spending and output, in the actual data, indicates that durable spending is positively leading the cycle for almost four quarters.

The strong comovement between durable spending and employment indicates that durable spending has a strong effect on labor market outcomes by influencing the households' labor supply decision. Given that, one could reasonably argue that households' demand for durables plays an important role in the underlined business cycle. Another important characteristic in our reference economy is the negative comovement between trade balance and durable spending. We interpret this comovement as part of the economy wide portfolio adjustment.[10] Hence, any aggregate portfolio adjustments that may result due to interest rates changes should result in a correlated durable spending and trade balance. Additional opserved feature is the strong comovement between spending on durable and nondurable goods which we attripute it to the substitute for between durable and nondurable consumption.

## 1.3 The Model

We consider a small open economy populated by infinitely lived households. Each household is endowed with one unit of time and has identical preference over an index of

[10] Recall that Durable spending reflects the adjustments of durable stock, and the trade balance reflects the adjustments in the external debt.

composite good and working hours. The representative household maximizes its expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t,\ h_t - \varphi_h h_{t-1}\right) , \tag{1.1}$$

where the expectation operator $E_t$ is conditional on the information available at time $t$; $\beta \in (0,1)$ is the subjective discount factor; $C_t$ represents a CES consumption index in nondurable consumption $c_t$ and durable service which is proportional to the stock of durables $D_t$. We denote the fraction of time devoted to work at time $t$ by $h_t$ and measure the habit intensity in labor supply by $\varphi_h \in (0,1)$.

We assume deep habits in both consumption goods, thus the CES utility index, take the following form:

$$C_t = \left[ \gamma \left(c_t - \varphi_c c_{t-1}\right)^{\frac{\rho-1}{\rho}} + (1-\gamma)\left(D_t - \varphi_D D_{t-1}\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} . \tag{1.2}$$

The share parameter $\gamma \in (0,1)$ determines the weight attached to the nondurable consumption in the period utility. The intraperiod elasticity of substitution between the two goods $(\rho > 0)$ reflects the households taste for diversity. The habit intensity in nondurable consumption is measured by $\varphi_c \in (0,1)$ and the habit intensity durable service is measured by $\varphi_D \in (0,1)$. A larger habit intensity parameter implies a stronger internal habit formation, in each specific good. The specific period utility takes the following form:

$$u(C_t, h_t) = \frac{\left[ C_t - \xi \left( h_t - \varphi_h h_{t-1} \right)^{\omega} \right]^{1-\sigma} - 1}{1-\sigma}. \tag{1.3}$$

The parameter $\sigma > 0$ is the intratemporal elasticity of substitution between the two goods.[11] The labor supply elasticity parameter is assumed to be positive $(\omega > 0)$ and $\xi$ is a scaling parameter. The utility function is assumed to be increasing in the current period consumption of nondurables, and durables but decreasing in current period labor supply. At the same time, the utility is decreasing in previous period consumption of nondurables and durables, and increasing in previous period labor supply. Our model differs from the existing one good RBC models in that the labor supply depends on the substitution between durable and nondurable consumption, which is supported by the actual data. In one-good model interest rates affect dynamics of the model through its effect on the cost of international borrowing only. In our model interest rates has an additional channel through the substitution between durable and nondurable consumption. The household portfolio includes three types of assets: non-traded capital, non-traded durable stock and internationally traded bond. The household's dynamic budget constraint is given by:

$$b_{t+1} = R_t \left( b_t - y_t + c_t + p_t d_t + i_t \right), \tag{1.4}$$

[11] When $\gamma = 1$ and $\varphi_c = \varphi_D = \varphi_h = 0,$ the preference in (3) collapses to what is commonly known as the GHH preferences according to Greenwood, Hercowitz, and Huffman (1988). We have also experimented with Cobb-Douglas and the standard additively separable preference. The simulations from these two experiments smooth produces too smooth consumption on the nondurable good. The results of both experiments are not reported in this study but can be available under request from the authors.

where $b_t$ denotes the household's debt position at the beginning of the period $t$ while the gross interest rate at which the domestic residents can borrow in period $t$ is represented by $R_t$. Domestic output is represents by $y_t$, expenditure on nondurable consumption by $c_t$. Household's expenditure on durables is represented by $p_t d_t$. We take the nondurable good as a numeraire, hence $p_t$ is defined as the relative price of the durable good in terms of the nondurable good. For simplicity, we normalize the relative price to one $(p_t = 1)$. Expenditure on domestic investment is denoted by $i_t$. The trade surplus is defined as the difference between domestic production and domestic absorption is used entirely to reduce the outstanding stock of external debt plus the interest cost on that debt. Similarly, the trade deficit is financed by issuing more debt. Thus, the trade balance is given by $tb_t = -(q_t b_{t+1} - b_t)$ where $q_t = 1/R_t$ is the price of one unit of international bond. The current account in tern measures the changes in the international investment position, i.e. $ca_t = -(b_{t+1} - b_t)$. Since spending on durables is also considers as investment in consumption goods from the household perspective, the forward looking nature of of the consumption decision should be controlled to avoid the excess volatility in consumption of such goods. Hence, it is common to incorporate adjustment cost on durable stock. Accordingly, the transition equation for the durable stock is given by

$$D_t = (1-\tau)D_{t-1} + d_t - \frac{\phi_D}{2}\left(\frac{D_t}{D_{t-1}} - 1\right)^2 D_{t-1} \ , \tag{1.5}$$

where $\tau$ represents the rate of depreciation of durables and $\phi_D$ controls the speed of adjustments in the stock of durables. As in Uribe and Yue (2006), the gross interest rate faced by the small open economy consists of the long run average gross interest rate $R^*$ plus a country specific premium. The premium in our model is assumed to reflect the domestic fundamentals of the open economy as indicated by its external percapita debt and income.[12] We assume that the country's premium is positively driven by the percapita debt and negatively on the percapita output. Specifically:

$$R_t = R^* + \psi\left(e^{\bar{b}_{t+1}-b} - 1\right) + \zeta\left(e^{-\bar{y}_t + y} - 1\right) + e^{\eta_t - 1} - 1, \qquad (1.6)$$

where $\left(R_t - R^*\right)$ is the country risk premium and $\bar{b}_{t+1}$ is the debt outstanding of the open economy measured at the end of the period $t$ (beginning of the period $t+1$). The percapita output is $\bar{y}_t$ while $b$ and $y$ represents the percapita steady state levels of output and debt respectively. The elasticity of the premium to debt is $\psi > 0$ and to output is $\zeta > 0$.[13] The exogenous country premium is denoted by $\eta$, assumed to reflect the exogenous fluctuations in the international capital markets. Hence we express these fluctuations in the international cost of borrowing in the following AR (1) process:

$$\ln \eta_t = \rho_\eta \ln \eta_{t-1} + \varepsilon_t^\eta \quad , \quad \varepsilon_t^\eta \sim i.i.d.\ N\left(0, \sigma_\eta^2\right). \qquad (1.7)$$

[12] Uribe and Yue attribute about two thirds of the volatility in the country premium in emerging economies to exogenous disturbances and only one third to its domestic fundamentals as indicated by external debt only.

[13] Note that we assume debt elastic interest rate. This is necessary to "close the open economy." Other equivalent alternatives are discussed in Schmitt-Groh and Uribe (2003).

It is also common in the literature to incorporate a convex adjustment cost function to control for the speed of capital adjustment. Hence, the transition equation for capital stock is

$$k_{t+1} = (1-\delta_t)k_t + i_t - \frac{\phi_k}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t, \tag{1.8}$$

where the parameter $\phi_k$ controls the speed of adjustment in capital stock. The time varying depreciation rate $\delta_t \in [0,1]$ is assumed to be increasing function in the utilization rate, specifically:

$$\delta_t = \upsilon \vartheta_t^{\mu}, \tag{1.9}$$

where $\vartheta_t \in [0,1)$ is the utilization rate at time $t$; $\upsilon > 0$ is a scaling parameter to ensure that the depreciation rate $\delta_t$ in equilibrium will equal its steady state value $(\delta)$ .The elasticity of the marginal depreciation of capital with respect to the utilization rate is defined by $\mu - 1 \in (0,1)$. As intensive capital utilization accelerates the depreciation rate, firms select the optimal rate of utilization by weighing the benefits of greater output against the costs of greater depreciation. The production function is assumed to be a Cobb-Douglas of the following form:

$$y_t = e^{z_t}\left(\vartheta_t k_t\right)^{\alpha} h_t^{1-\alpha}, \tag{1.10}$$

where $z_t$ is the technological progress and assumed to follow a stationary AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad , \quad \varepsilon_t^z \sim i.i.d.\ N\left(0, \sigma_\varepsilon^2\right). \tag{1.11}$$

Household's objective is to maximize the its life time expected utility (1.3) subject to the budget constraint (1.4), the transition equation for durable stock (1.5), the transition equation for capital stock (1.8), and the no-Ponzi constraint given by:

$$\lim_{j\to\infty} E_t \frac{b_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \le 0 . \tag{1.12}$$

The Households optimal conditions are:

$$\begin{aligned}\lambda_t = & \left[C_t - \xi\left(h_t - \varphi_h h_{t-1}\right)^{\omega}\right]^{-\sigma} C_t^{\frac{1}{\rho}} \gamma\left(c_t - \varphi_c c_{t-1}\right)^{\frac{-1}{\rho}} \\ & - E_t \beta \varphi_c \left[C_{t+1} - \xi\left(h_{t+1} - \varphi_h h_t\right)^{\omega}\right]^{-\sigma} C_{t+1}^{\frac{1}{\rho}} \gamma\left(c_{t+1} - \varphi_c c_t\right)^{\frac{-1}{\rho}} ,\end{aligned} \tag{1.13}$$

$$\begin{aligned}\lambda_t (1-\alpha)\frac{y_t}{h_t} = & \left[C_t - \xi\left(h_t - \varphi_h h_{t-1}\right)^{\omega}\right]^{-\sigma} \xi\omega\left(h_t - \varphi_h h_{t-1}\right)^{\omega-1} \\ & - E_t \beta \varphi_h \left[C_{t+1} - \xi\left(h_{t+1} - \varphi_h h_t\right)^{\omega}\right]^{-\sigma} \xi\omega\left(h_{t+1} - \varphi_h h_t\right)^{\omega-1},\end{aligned} \tag{1.14}$$

$$\begin{aligned}& \lambda_t \left\{1 + \phi_D\left(\frac{D_t}{D_{t-1}} - 1\right)\right\} + \beta E_t \lambda_{t+1}\left\{(1-\tau) + \frac{\phi_D}{2}\left(\frac{D_{t+1}}{D_t}\right)^2 - \frac{\phi_D}{2}\right\} = \\ & \left[C_t - \xi\left(h_t - \varphi_h h_{t-1}\right)^{\omega}\right]^{-\sigma} C_t^{\frac{1}{\rho}} (1-\gamma)\left(D_t - \varphi_D D_{t-1}\right)^{\frac{-1}{\rho}} + E_t \beta \varphi_D \left[C_{t+1} - \xi\left(h_{t+1} - \varphi_h h_t\right)^{\omega}\right]^{-\sigma} C_{t+1}^{\frac{1}{\rho}} (1-\gamma)\left(D_{t+1} - \varphi_D D_t\right)^{\frac{-1}{\rho}},\end{aligned} \tag{1.15}$$

$$\lambda_t + \varphi_k \lambda_t \left(\frac{k_{t+1}}{k_t} - 1\right) = \beta E_t \lambda_{t+1}\left\{\alpha \frac{y_{t+1}}{k_{t+1}} + \left(1 - \delta_{t+1}\right) + \frac{\varphi_k}{2}\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - \frac{\varphi_k}{2}\right\} , \tag{1.16}$$

$$\lambda_t = (1 + r_t)\beta E_t \lambda_{t+1} , \tag{1.17}$$

$$\delta_t = \frac{\alpha}{\mu}\frac{y_t}{k_t} , \tag{1.18}$$

where $\lambda$ denotes the Lagrangian multiplier associated with the dynamic budget constraint. The solution to the model is a set of stochastic processes for the endogenous variables that satisfies the budget constraint (1.4), equation (1.5), and the first-order conditions (1.13)–(1.18), and the consistency conditions (that values of individual per capita debt and output equal are equal to its cross sectional levels $\left(y_t = \bar{y}_t, b_t = \bar{b}\right)$ and the no-Ponzi-scheme (1.12) given the initial conditions $\{k_0, b_0, D_{-1}, c_{-1}, h_{-1}\}$. The model is solved for the long run steady state values around which the log linearized version of the model is approximated, as in Campbell (1994). The approximated log-linearized system is solved by using the method of the undetermined coefficient as in Uhlig (1999). The detailed linearized form of the model and its solutions are appended in the Technical Note C.1.

## 1.4 Calibration

We calibrate our model to the Canadian economy. Each period is taken to be a quarter. The capital share $(\alpha)$ is set to 0.32 and the steady state depreciation rate of capital $(\delta)$ is set to 0.02. The steady state employment $(h)$ is normalized to 20 percent of the total time endowment. Since there is no specific estimate for the elasticity of the intratemporal substitution between durables and nondurables $(\rho)$ for the Canadian economy, we borrow the value of this parameter (1.1) from Engel and Wang. We set the risk aversion parameter $\sigma = 0.5$ as in Neumeyer and Perri among others.

We assign low value for labor elasticity $\omega = 1.2$ comparing to the similar parameter value in Mendoza (1991) and many others. We find that this level of employment elasticity improves the predictions of the model given interest rate uncertainty.[14] The value of the labor parameter $\xi$ is set at $1.85$ to ensure that the steady state level of employment is $20\%$ of the time endowment, given other parameters' values. The long run real interest rate is $R^{*} = 1.04^{1/4}$. Given this interest rate and the assumption of a zero long run growth rates, the consistent value of the subjective discount factor $\beta$ is $1.04^{-1/4}$. We set the elasticity of the country risk premium to external debt as equal to the elasticity of the premium to domestic output $(\psi = \zeta = 0.80)$. The elasticity the country premium to debt is calibrated to match the correlation between output and trade balance. The elasticity of the premium to output is calibrated to produce a negative correlation between investment and interest rates. We explain the importance of the elasticity of the risk premium to output in details in section 6. According to Basu and Kimball (1997), the elasticity of depreciation to utilization in US manufacturing is in the range [1, 2]. Given the observed cycle, we set $\mu$ at 1.5. The steady state rate of capital depreciation $\delta$ is set at 0.02, and the historical average utilization rate $\vartheta$ is estimated to be 0.817. Accordingly, the consistent value of $\upsilon$ becomes 0.0271. Table A.2 summarizes the baseline parameter values.

## 1.5 The Model Fit With the Canadian Data

To assess the model's fit with the Canadian business cycle, it requires comparing the second moments generated from the actual business cycle data, presented in Table A.3 (column 2), with its counterpart moments generated from the theoretical model using the baseline parameter values, presented in Table A.2 (column3). The volatility of all national accounts in the

[14] In Section 1.6 we analyze the macroeconomic sensitivity of using alternative feasible values of employment elasticity and intraperiod elasticity of substitution.

baseline model closely matches with the corresponding observed volatility of the actual national accounts, though it slightly understates the volatility of trade balance to output ratio and slightly overstate the volatility of the current account to output ratio.

The model economy replicates the procyclicality of all national accounts as well as the countercyclicality of trade balance and current accounts. It also replicates the persistence levels of output, employment, durable spending, trade balance and current account. The model however slightly overstates the procyclicality of investment and slightly understates the persistence in investment and durable spending.[15] The results in Table A.3 show that the model successfully predicts the main futures of households' observed spending on durable goods as documented in Section 1.2. In figure B.2 we compare the observed crosscorrelation between output and durable spending against the baseline model predictions. The shape of the observed dynamic crosscorrelation suggests that durable spending strongly leads the cycle by up to four quarters; a similar result can be reached from the predicted crosscorrelations. The model slightly overstates the positive correlation between current output and durable spending at higher lags.[16] The success of the model of replicating the crosscorrelation is attributed in part to the effect of interest rates uncertainty. In a version of the baseline model without interest rate uncertainty, the

[15] The reason behind the observed high persistence in investment and durable data could be due to the actual national accounts classification of the components of investment and durable goods. A large component of aggregate investment data is the households' spending on residential structure, which may have high persistence level due to the household incentive to smooth the utility generated from these assets. Similarly the actual data on durable spending include consumption spending on semi-durables that has higher direct utility than other durables, given utility smoothing semi durables expected to be more persistence than other types of durables, thus the aggregate durable spending is more persistence than only pure durables.

[16] Including semi-durables in the aggregate durable spending may overstate the persistence of the durable spending in the actual data and also bias the shape of the actual crosscorrelation.

crosscorrelation diagram shows that durable is lagging the business cycle. When unexpected interest rate shock occurs, given the technology, this creates a direct adjustment in the aggregate portfolio of the open economy including durable stock. The business cycle fluctuations will follow the redistribution of the resources with some time lag hence we observe durable spending to lead the cycle.

To assess further the rule of interest rates in our model we compare the performance of the baseline model with interest rate uncertainty against a version of the model without interest rates uncertainty.[17] We shut off the interest rate shock by setting $\sigma_{\mu,t} = 0 \;\forall\; t$, and keeping all other parameter values unchanged, as in the baseline model. The results reported in the Table A.2 (columns 3 and 4) show that interest rate uncertainty explains only 1.3 percent of the aggregate fluctuations in output. At the same time the quality of the model prediction with interest rate uncertainty is matching better the actual data, particularly in terms of the volatility of the trade balance to output ratio and the procyclicality of durables and investment expenditures.

As it is documented in Neumeyer and Perri, Figure B.3 shows that the interest rate is moderately procyclical as our baseline model which is calibrated to advanced small open economies. The same figure shows other important features of the baseline model. In particular, a positive interest rate shock leads to a trade surplus associated and lowers the level of external debt comparing to the reference steady state levels of the economy. Further, it leads to a significant decrease in investment and durable spending as the household substitutes away from durable and capital stocks to compensate for the decrease in the external financing of the wide

[17] Recall that the predictions of the standard open economy model relative to the open economy stylized facts deteriorates when interest rates uncertainty included in the model.

economy. Given that, as well as household's incentive for consumption smoothing, the household temporarily forsakes investments and durable spending and work more hours to compensate for the costly debt until the shock vanishes.[18] In the following, we explain why the baseline model is able to replicate the Canadian business cycle well comparing to earlier efforts. We do so analytically by highlighting the effects of durables and interest rate uncertainty on the dynamics of the model economy.

## 1.6 Durables and the Interest Rates

The best way to see the effect of the interest rates is by eliminating all kinds of friction from the model (including habits and adjustment costs). Combining the household's first two first order conditions and evaluating the outcomes at the steady state, we obtain

$$\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{D}{c}\right)^{\frac{-1}{\rho}} = p\left[1-\frac{(1-\tau)}{R^*}\right]. \tag{1.19}$$

The intratemporal marginal rate of substitution between durable and nondurable goods on the left side of this equation is equal to the user cost of durables on the right side. It is obvious that an increase in the user cost of durables due to higher interest rates will result in a decrease in the durable to nondurable ratio, given $\rho > 1$. The higher the value of $\rho$, the more intensive is the substitution between durable and nondurable goods. Similarly, steady state employment can be described by the following equation

[18] Our results are significantly different from Neumeyer and Perri. In their working capital model of emerging countries output, employment, consumption and investment response negatively to positive external interest rates. Trade balance, however, response positively to interest rates (see Figure 7, pp. 367).

$$\xi\omega h^{\omega-1} = (1-\alpha)\frac{y}{h}\left(1+\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{D}{c}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}}\gamma^{\frac{\rho}{\rho-1}}. \qquad (1.20)$$

The equilibrium level of employment implied by (1.20) is a positive function of the durable to nondurable ratio (given $\omega > 1$ and $\rho > 1$). Therefore, an increase in the international cost of borrowing not only decreases the durable to nondurable ratio, it also implies a higher level of employment and output in the long run. Higher values of the elasticity of substitution between durable and nondurable goods increase the sensitivity of employment and accordingly output to interest rates. Moreover, the above equation also implies that the higher labor elasticity $\omega$ reduces the sensitivity of the model to interest rates. Thus, the cyclicality of interest rates depends on the combined effects of the elasticity of labor supply and the intratemporal elasticity of substitution between the two consumption goods.

To examine the effects the elasticity of substitution between durables and nondurables and the elasticity of labor supply on the sensitivity of the macroeconomy to interest rates we do the following experiment. We subject the baseline model to a one standard deviation interest rate shock and calculate the impulse response functions of the main variables. We recalculate the impulse response functions with higher labor elasticity $\omega = 1.7$, keeping all other parameters as in the baseline model. We do another calculation for the impulses but with higher intratemporal elasticity of substitution $\rho = 5.0$, while other parameters are the same as in the baseline model. Figures B.4 compares the responses of each individual national account relative to the baseline model.

The impulses in Figure B.4.a show that aggregate output in economies with high intratemporal substitution between durables and nondurables are more sensitive to interest rates. On the other hand, small open economies with higher labor elasticity are less sensitive to interest rates fluctuations. Inspecting on the responses of all other national accounts, we fine that the main reason behind the sensitivity to interest rates is the response of employment and nondurable consumption. Employment is more sensitive to interest rates in economies with high intraperiod elasticity of substitution between durable and nondurable, Figure B.4.b. In the same figure we note employment is less sensitive to interest rates in economies with high labor elasticity. Consumption of nondurables becomes more sensitive to interest rates with higher intratemporal elasticity of substitution between durable and nondurable consumption, Figure B.4.c .Interestingly with high labor elasticity consumption response negatively to interest rates. It is usual that investment responds negatively to positive interest rates shock. In our model, such negative effects get more pronounced with higher elasticity of substitution between durables and nondurables, (see Figure B.4.e). In the same figure we note that investment becomes less sensitive to interest rates with higher labor supply elasticity. Interestingly, different values of the labor supply elasticities and or the intratemporal elasticities substitution do not significantly alter the response of durable spending, investment, and the trade-balance-to-output ratio or the current-account-to-output ratio (Figures B.4.d –B.4.g).

We also note here that the country premium must be negative on output in order to obtain a countercyclical trade balance. For the trade balance to be countercyclical, output growth must be slower than the domestic absorption. Given our controls over the speed of the adjustment of durable spending and investment, the domestic absorption becomes slower than output resulting in procyclical trade balance. To avoid such a possibility, our model requires the country specific risk premium to depend on percapita output. Accordingly, output growth reduces the country premium and speed up investment and expenditure on durables to the extent that makes the trade balance countercyclical.

## 1.7 Habits and Variable Utilization

The baseline model incorporates three types of habits – habits in nondurable consumption, in durable consumption and in the labor supply. We numerically examine the role of each type of habit formation on the overall performance of the baseline model. We so by constructing four versions of the baseline model and compare the performance of each to the baseline model. In the first version we exclude all types of habits, by setting $\varphi_c = \varphi_D = \varphi_h = 0$, while in the other three versions we exclude only one type of habits at a time. The results from these experiments are reported in the Table A.4. Eliminating all types of habits leads to a significantly over volatile business cycle comparing to the baseline model or the actual data. All national accounts become more volatile except durable expenditure spending, which become less volatile without habits in the model. Further, in the absence of habits, all national accounts become strongly procyclicality particularly employment, which become perfectly correlated with output. Another important conclusion is in terms of the data persistence. Without habits, the persistence level of the simulated data become very low comparing to the baseline model or the actual data. The model that excludes habit in nondurable consumption $(\varphi_c = 0)$ yields an over volatile business cycle. Interestingly, the volatility of the durable expenditures so as well investment decreases when we eliminate the nondurable habit volatility. The model without habit in nondurable consumption overstates the procyclicality in national accounts relative to the baseline model and the actual data. Ignoring habit persistence in nondurable consumption also reduces the persistence of the consumption of the nondurable good and the trade balance. Excluding habit in the consumption of durable services $(\varphi_D = 0)$ yields an under volatile business cycle, and makes durable spending as well as nondurable consumption, employment and investment less procyclical. Interestingly, when the labor supply is not subjected to habit formation $(\varphi_h = 0)$, the model yields a significantly over smoothed business cycle and a cyclical trade balance to output ratio and a cyclical current account to output ration. The persistence of the simulated national accounts data also decreases, except for consumption of nondurables.

Finally, we evaluate the role of endogenous capital utilization on the results of the baseline model. We do so by assuming time invariant utilization $(\delta_t = \delta \forall t)$ and rewriting the production function as $y_t = e^{z_t} k_t^{\alpha} h_t^{1-\alpha}$. This eliminates the first order condition (1.18). Given the assumption of constant utilization, the consistent estimators of the technology progress are $\rho_z = 0.944$ and $\sigma_z = 0.006$ as in Letendre. The last column in the Table A.4 reports the business cycle features of an economy with fixed capital utilization. The business cycle in a fixed utilization economy is over smooth, yet the cyclicality of the national accounts and their persistence seem to be unaffected comparing to the variable utilization economy.

## 1.8 Conclusion

Aggregate household spending on durable goods constitutes a large share of GDP in advanced small open economies. It also significantly influences other national accounts. We observe that durable spending is positively correlated with output, employment and investment and negatively correlated with the trade balance to output ratio. Interestingly, it also leads the cycle. Motivated by these observations, we construct a real business cycle model with both durable and nondurable goods. We also account for the presence of habit persistence, variable capital utilization and interest rates uncertainty. Calibrating the model to the Canadian business cycles, we find that our model does an excellent job in matching the moments. In particular, the model predicts the volatility of all national accounts, their cyclicality as well as their persistence, very well. This is significant especially when we compare our results for advanced small open economies to those of other models in the existing literature.

Our model captures the procyclicality of interest rate and replicates the observed crosscorrelation between output and durable spending. Interest rate uncertainty explains 1.3 percent of aggregate fluctuations and plays an important role in improving the model performance. We highlight that durable spending and its substitution with nondurable spending constitute the main channel for interest rates in the macroeconomy. We show that the composition of the economy wide portfolio is also sensitive to interest rates. With higher interest rates the external debt of the economy decreases below its steady state level leading to a trade surplus. At the same time, investment and durable consumption decreases. Consistent with our empirical observation, we also observe that durable spending leads the business cycle.

Though the model does an excellent job overall, it slightly overstates the procyclicality of consumption and slightly understates the persistence of durable and investment expenditures. We believe that these problems are due to actual data definition. Finally, future research should incorporate monetary and fiscal dimensions of the economy to analyze the effects of various government policies. Studying various commercial policies within this framework will also be fruitful.

## 1.9 Data Sources

Data are obtained from the Canadian Socio-economic Information and Management (CANSIM) database. CANSIM labels are in parentheses:

*Population*: Quarterly estimates of population for Canada (D1).

*Output*: real gross domestic product (D100126).

*Nondurable Consumption*: personal expenditure on non-durable goods (D100106) and services (D100107).

*Durable spending*: personal expenditure on durable goods (D100104) plus personal expenditure on semi-durables (D100105).

*Investment*: investment in machinery and equipment (D100115), non-residential structures (D100114) and residential structures (D100112).

*Exports*: exports of goods and services (D100119).

*Imports*: imports of goods and services (D100122).

*GDP deflator*: ratio of nominal GDP (D14816) and real GDP (D100126).

*Employment*: employment age 15+ (D980595).

*Current account*: total nominal current account balance (D59832) deflated using the GDP deflator.

# Essay 2

## Terms of Trade, Durability, and Small Open Economies Business Cycles

### 2.1 Introduction

A characteristic feature of small open economies is their openness to international trade which has become increasingly important in the face of globalization. But developing economies have shown concerns about the evenness of the globalization process and its impact on their macroeconomic stability. One such concern is due to the fact that many developing economies are vulnerable to adverse Terms-of-Trade (ToT) shocks.[19] Understanding the true effects of ToT

[19] There exist some empirical studies on the effects of terms of trade shocks. Otto (2003) tests the relationship between the terms of trade and the trade balance for fifty-five small open economies. Using a structural vector autoregressive model, he finds that a positive terms of trade shock leads to an initial improvement in the trade balance. The finding is consistent for developing countries and small OECD countries. In another study, Cashin and McDermott (2002) also use a structural VAR model to show that terms of trade shocks have significant impacts on the current account balance in Australia and New Zealand. However, in Canada, the United Kingdom, and the United States, they find that terms of trade movements are not important. Furthermore, using panel data for non-oil commodity exporters in sub-Saharan Africa, Agénor and Aizenman (2004) find that terms of trade increases have a positive effect on private savings. Using a panel VAR model and data from 75 developing countries, Broda (2004) finds that terms of trade shocks differ systematically across exchange rate regimes. He observes that they explain 30% of real GDP fluctuations in fixed regimes and about 30% of real exchange rate fluctuations in countries with flexible regimes. In an earlier panel data study, Spatafora and Warner (1999) find that permanent terms of trade shocks have significant positive effects on consumption, investment and output, no effect on saving, and an adverse effect on the trade balance.

shocks is thus an important issue in open economy macroeconomics, where many alternative models have provided conflicting evidences about the effect of ToT shocks on the macroeconomic outcomes. The main objective of this study is to explore this issue in detail and offer some insights.

Typically small open economies specialize in the production of a few products. Developed economies usually specialize in the production of relatively highly durable commodities such as transportation vehicles, telecommunication products, electronics, etc. As the level of income in the advanced economies usually is higher comparing to emerging economies, they import relatively more durable consumption goods. As a result, the degree of durability in consumption goods in advanced economies is significantly higher than the consumption goods in emerging economies. We show that differences in the degree of durability of domestic output and imports across small open economies significantly affect their sensitivity to ToT fluctuations. The reason behind our argument is due to the forward looking nature of the demand for durables which enables the economies with higher degree of durability to better mitigate the external fluctuations in their ToT. Accordingly, the observed business cycles in developed economies are more stable comparing to the business cycles in the emerging economies.

On the issue of ToT, two lines of research have been pursued. One group of studies investigates on how consumption, employment, investment, and external accounts (such as current account and trade balance) are affected by fluctuations to ToT. Often these models are deterministic in nature. In the other line of inquiry, stochastic optimizing models have been adopted within a small open economy RBC framework. The main focus in this group of studies is to understand the role of terms of trade in shaping the business cycles of these economies; for example, how much of the aggregate fluctuations of the open economy can be attributed to ToT fluctuations that open economies are subject to. Our study contributes to both lines of research mentioned above.

Another issue that has surfaced recently at the top of the research agenda in small open economy macroeconomics is to explain the differences between developed and developing economies in terms of their business cycle characteristics. In emerging economies, business cycles are more volatile than in developed economies. Consumption volatility in emerging economies exceeds the volatility of output. Net exports tend to be strongly countercyclical in emerging economies but weakly countercyclical in developed economies. One leading explanation of such empirical regularity advocated by Neumeyer and Perri (2005) and Garcia-Cicco, Pancrazi, and Uribe (2010), attributes these differences to the relatively high sensitivity of emerging economies to international interest rates due to financial frictions the relatively high dependent on international capital flows, comparing to developed economies. Another leading point of view, supported by Aguiar and Gita Gopinath (2007), agrees that small open economies vary significantly in many structural aspects including the importance of international capital flows, financial frictions and domestic economic policies. However, Aguiar and Gita Gopinath claim that these differences across open economies can be modeled as shocks to the growth trend rather than by simple frictions of different forms. This study provides an alternative explanation that underscores the natural differences among these economies in terms of their production and demand patterns. Developed open economies usually produce and consume more durable goods than many developing economies. Not only domestic production but the trade classification of imports also differs significantly among open economies. Developed economies usually enjoy relatively higher percapita income hence the share of durables in their imports is significantly higher compared to developing economies. The reason behind the ability of durability in explaining the differences in the business cycles among small open economies is due to the forward looking nature of the demand for durables. Durability in households' consumption goods enhances the forward looking nature in their economic decisions, hence their ability to avoid fluctuations in the international prices in general and ToT in particuler.

We justify this argument with a two goods (domestic and foreign ) small open economy RBC model. This model is an extended version of Obstfeld's (1982) and Eicher, Schubert and Turnovsky's (2008). We modify these models in various ways. We incorporate endogenous labor/leisure choice, habit persistence and optimal capacity utilization. Instead of dealing with completely nondurable consumption, we assumed that home and foreign goods are both aggregate consumption goods with a certain degree(s) of durability. Habit formation helps the model to to be consistent with the observed persistence in the actual business cycle data. Optimal capacity utilization improves the ability of the model to match with the aggregate volatility of the underlined business cycles. We incorporate ToT uncertainty as a stationary stochastic process. ToT fluctuations assumed to enter the model economy through two distinct channels. The regular channel is the relative price of domestic and foreign goods, while the new channel that we incorporate here is the country specific risk premium channel. We assume that deterioration in ToT is likely to increase the country specific risk premium.

There are many compelling reasons for us to propose such a model. Mendoza (1991) was the first to offer a small open economy RBC model to explain various stylized facts particular to these economies. His model ignited a series of important research papers. As outlined in the previous essay, these models have important common limitations that can be attributed to (i) the vulnerability of the models to technological progress estimators, (ii) the vulnerability of the models to household preferences, and (iii) the inability of the models to capture the interest rate uncertainty. To overcome these limitations we have developed (in the previous essay) a model economy with two goods - durables and nondurables. This is particularly important since households' spending on the consumption of durable goods in a typical advanced small open economy like Canada constitutes almost 15 percent of GDP. Spending on durables, in such economies, is strongly procyclical, highly correlated with trade balance, sensitive to interest rates and leads the business cycle.[20] Upon calibrating that model to Canadian data (1980:Q1-2009:Q4) we found that the model economy closely matches the observed business cycle and replicates well the comovement between durable spending and the rest of the national accounts. Further, interest rates play an important role in the allocation of the aggregate portfolio and particularly affect the external debt of the economy. Along with durable and nondurable consumption, the performance of that model improved significantly when we incorporated habit persistence, and variable capital utilization.

[20] According to Ogaki and Reinhart (1998) durable consumption improves the performance of the general equilibrium models. Uribe (2002), show that introducing time nonseparablity of the preference by assuming durability can explain the "The Price-Consumption Puzzle of Currency Pegs". Engel and Wang (2010) show that when traded goods are durable in nature, the standard international business cycle model perform better in explaining the stylized facts of the international business cycle.

In the present work we extend the framework developed in the previous essay to investigate the effects of adverse ToT shock on the dynamics of small open economy and their business cycles. To be completely consistent with the previous model we need to consider two domestic goods - home durables and home nondurables - and two imported goods, foreign durables and foreign nondurables. So doing would be extremely complicated and possibly the model would intractable. As a compromise, we have assumed that households consume both home and foreign goods with a certain degree of durability. See Mohsin (2006) and Gregorio et al (1998) for details.

The results of the model are tested against the developed Canadian economy. The model economy is able to replicate the different moments of the national accounts. Moreover, for the first time, this model replicates the comovement between ToT and national accounts. The model correctly predicts moderately countercyclical ToT; the negative correlation between ToT and consumption, employment and investment; and the positive correlation between the ToT and the trade balance as observed in the Canadian data. The model replicates all these comovements well. The model predicts that ToT explains less than one fifth (17 percent) of the aggregate fluctuations.

We lower the degree of durability to calibrate to developing economies, given everything else is equal. Consistent with the observed business cycle in developing economies, the model predicts that consumption is more volatile than output and trade balance-output ratio become strongly countercyclical. In this case the model predicts that ToT explains around one third of the aggregate fluctuations in developing economies. The contribution of the ToT in aggregate economic fluctuations in our experiment is relatively lower than the predictions found in Mendoza (1995) and Kose (2002). According to these two papers ToT shocks account for at least half of output volatility in developing economies. Of course Mendoza and Kose did not account for durable consumption.

The effect of adverse ToT on a small open economies has been a subject of controversy since the early 1950s, when Laursen and Mezler (1950) and Harberger (1950) developed what has become known as the the Harberger-Laursen-Mezler (HLM) effect.[21] With endogenous time preferences, Obstfeld (1982) was first to test this proposition in an optimizing framework. He showed that a permanent deterioration in a small country's terms of trade leads it to save more, to consume more in the future and to run a current account surplus. In a related study, Svensson and Razin (1983) found that the effects of ToT shocks on trade balance depend crucially on the perceived persistence of the terms of trade. In their model, the HLM effect weakens as the terms of trade become more persistent and may even be overturned if the ToT shock is a permanent nature. This view is known as the Obstfeld-Razin-Svensson (ORS) effect. Persson and Svensson (1985) used an overlapping generation (OLG) framework. They showed that the results depend on the duration of the shock, temporary or permanent, and whether it is anticipated or unanticipated. Moreover, in all these models, the external ToT shocks are deterministic in nature. An important aspect of our study is its ability to contribute substantially to that debate as well. In

[21] Using a static framework, they argue that an adverse ToT leads to a decline in real income and aggregate savings, resulting in a deterioration of the current account balance. Based on static Keynesian assumptions, Laursen and Mezler show that an exogenous rise in the tot of a small open economy leads to an improvement in its trade balance. The reason is that an improvement in a country’s ToT raises its current income, and, given a marginal propensity to consume less than unity, current consumption increases less than current income, causing private savings to increase.

reality, small open economies are subject to stochastic shocks. Our model, as a result, is more suitable to comment on these two hypotheses. In this study we firmly reject the HLM perspective and confirm the OSR premise. However, we find that simple differences in ToT persistence cannot account for the differences in the business cycles across small open economies.

The rest of the paper is organized as follows. In Section 2 we provide some empirics involving ToT and Canadian business cycles. The detailed structure of the model is outlined in Section 3. Parameter values used in our calibration are provided in Section 4. Section 5 provides a detailed outcome of our calibration exercise and compares the results of our model economy to those of the actual Canadian economy. In Section 6 we estimate the contribution of ToT shocks to the business cycles of different theoretical small open economies. Sections 7 and 8 evaluate the importance of durability and the role of interest rates. In Section 9 we investigate the validity of the HLM and OSR hypotheses. We conclude in Section 10.

## 2.2 Empirical Regularities

To provide some empirical documentation, we used the Canadian quarterly data spanning from 1981:Q1 to 2009:Q4. In this benchmark economy, we found a significant comovement between ToT (defined as the relative price of imports to exports) and all the national accounts.[22] The results, as reported in Table A.5, are summarized as follows:

[22] Note that the standard text book definition of the ToT is the relative price of exports to imports. Using this definition in a macro model requires that all national accounts be expressed in foreign or international prices. Since we are going to match with domestic currency data we define ToT as the relative price of imports to exports. Obstfeld (1982) and Eicher, Schubert and Turnovsky (2008) adopt similar ToT definition.

1. The ToT is weakly countercyclical. An adverse ToT is associated with economic downturn with an estimated correlation coefficient of 0.27.
2. ToT deterioration is negatively correlated with aggregate consumption, with an estimated correlation of 0.35.
3. Deterioration in ToT is negatively correlated with employment, with a correlation coefficient of 0.37.
4. ToT deterioration is negatively correlated with investment, with correlation coefficient of 0.44.
5. ToT deterioration is positively correlated with the trade balance, with a correlation coefficient of 0.41.

ToT cyclical fluctuations are plotted against the main national accounts in Figure B.5.a. This figure shows clearly that ToT is more volatile than output with moderate negative comovement between the two series. However, during the large swings of the cycles the negative correlation between the TOT and the aggregate fluctuations is more pronounced.

In figure B.5.b, one can observe that ToT is less volatile than investment and more volatile than total consumption spending (including durable and nondurable consumption) and employment. Figure B.5.b also shows moderate negative comovement between these variables. The strongest comovement of the ToT is with investment and trade balance to output ration in Figure B.5.c. This figure also shows that ToT is more volatile than the trade balance, with a positive relation between the two series.

The Canadian spending on durable consumption constitutes a high share of total consumption spending on goods and services (24.1 percent) or equivalently 15.4 percent of GDP in 2009. These shares are relatively consistent with those of many advanced small open economies and significantly higher compared developing small open economies. Unfortunately, there is no readily available economic classification dealing with the degree of durability in aggregate consumption. However, it is well known that production in advanced economies includes a high share of durable products such as electronics, transportation vehicles, telecommunication equipment and other durables. On the other hand, it is also commonly known that intra-trade constitutes the majority of the trade volume among the advanced economies. [23] Accordingly, it is reasonable to assume that developed economies have a higher degree of durability in total consumption compared to developing or emerging economies.

## 2.3 The Model

The model developed here is a modified version of the small open economy models introduced by Obstfeld (1982) and Eicher, Schubert and Turnovsky (2008). We incorporate various significant changes. First, our model adopts a discrete time framework and investigates the effects of stochastic ToT shocks. Instead of dealing with completely nondurable consumption, we assumes that home and foreign goods are both aggregate consumption goods with a certain degrees of durability. Our model incorporates endogenous labor/leisure choice and capacity utilization. In addition, it accounts for households' habit formation. The model

[23] The intra-trade phenomena, among advanced economies, are explained by the similarity in the taste and product differentiation among developed countries.

economy is populated by identical and infinitely lived households. The representative household is endowed with one unit of time and has preference over an index of composite goods and working hours. The representative household maximizes the following expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, h_t - \upsilon h_{t-1}) \ , \tag{2.1}$$

where the expectation operator $E_t$ is conditional on the information available at time $t$; $\beta \in (0,1)$ is the subjective discount factor; $C_t$ represents a CES consumption index in the stock of domestic (home) goods $D_h$ and the stock of imported (foreign) goods $D_f$. We denote the fraction of time devoted to work at time $t$ by $h_t$ and measure the habit intensity of the labor supply by $\upsilon \in (0,1)$. We assume deep internal habits in each good, thus the CES utility index takes the following form:

$$C_t = \left[ \gamma \left( D_{h,t} - \varphi_h D_{h,t-1} \right)^{\frac{\rho-1}{\rho}} + (1-\gamma) \left( D_{f,t} - \varphi_f D_{f,t-1} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \ , \tag{2.2}$$

the share of home goods in the consumption index is denoted by $\gamma \in (0,1)$. The parameters $\varphi_h \in (0,1)$ and $\varphi_f \in (0,1)$ measure the habit intensity in home and foreign goods respectively. The intratemporal elasticity of substitution between the two consumption goods is measured by

$\rho > 0$ and reflects the household's taste for diversification. The specific period utility takes the following form:[24]

$$u(C_t, h_t) = \frac{1}{1-\sigma}\left\{\left[C_t - \xi\left(h_t - \upsilon h_{t-1}\right)^{\omega}\right]^{1-\sigma} - 1\right\}, \tag{2.3}$$

where $\sigma > 0$ is the intertemporal elasticity of substitution between the two goods. The labor supply elasticity parameter is assumed to be positive ($\omega > 0$).

We denote the ToT at time $t$ by $p_t$ and express it as the price of the foreign good $p_{f,t}$ relative to the price of the domestically produced goods $p_{h,t}$, in particular $\left(p_t = p_{t,f} / p_{t,h}\right)$. Given, the household's total consumption expenditure evaluated at domestic good prices $(z_t)$ can be defined as

$$z_t = d_{h,t} + p_t d_{f,t} \quad , \tag{2.4}$$

where $d_{h,t}$ represents the household's spending on domestic good in period $t$ evaluated in domestic currency prices; and $d_{f,t}$ represents the household's spending on imported foreign good in period $t$.[25] For a small open economy, ToT is determined in the international markets

[24] It should be noted that when $\varphi_h = \varphi_f = \upsilon = 0,$ and $\gamma = 1$ the preference in (2.3) collapses to what is commonly known as the GHH preferences due to Greenwood, Hercowitz, and Huffman (1988). In open economy RBC models such preference is often recommended.

[25] Although it is it is possible for household to make negative purchases of either or both durable goods, their consumption of durables will continue to be positive as long as they carry some stock of durables from previous periods.

and completely exogenous. We assume the ToT follow a first order autoregressive process AR (1) of the following form:

$$\ln p_t = \left(1-\rho_p\right) p + \left(1-\rho_p\right) \ln p_{t-1} + \varepsilon_t^p \ , \quad \varepsilon_t^p \sim i.i.d.\ N\left(0, \sigma_p^2\right). \tag{2.5}$$

Where $\rho_p$ represents the persistence level in the observed ToT historical data, and $\varepsilon^p$ is the unexpected shock to the ToT which assumed to be independent and identically distributed with zero mean and $\sigma_p^2$ variance.

Following conventional practices, we assume that the accumulation of the stock of durables is subject to a convex adjustment cost function. Since both goods have a certain degree of durability, we must account for their rate of depreciation. With the rate of depreciation of domestic and imported goods $\tau_h \in (0,1)$ and $\tau_f \in (0,1)$ respectively, the accumulation of stock of home and foreign goods can be expressed as

$$d_{h,t} = D_{h,t} - (1-\tau_h) D_{h,t-1} + 0.5\phi_h \left(D_{h,t} / D_{h,t-1} - 1\right)^2 D_{h,t-1} \ , \tag{2.6}$$

$$d_{f,t} = D_{f,t} - (1-\tau_f) D_{f,t-1} + 0.5\phi_f \left(D_{f,t} / D_{f,t-1} - 1\right)^2 D_{f,t-1}, \tag{2.7}$$

where the adjustment cost parameters $\phi_h$ and $\phi_f$ control the volatility of the flow of consumption spending on domestic and foreign goods respectively.

The household has access to four types of assets - domestic capital, stock of home durables, stock of imported durables and to a one period internationally traded bond. The household’s dynamic budget constraint is given by

$$p_t b_{t+1} = R_t\left(p_t b_t - y_t + z_t + i_t + 0.5 p_t \phi_b \left(b_{t+1} - b\right)^2\right) , \qquad (2.8)$$

where $b_t$ denotes the household's debt position at the beginning of the period t; $R_t$ denotes the gross interest rate at which the domestic residents borrow in period $t$ ; $y_t$ represents domestic output; and $i_t$ represents domestic investment spending. We add to the expenditure side a debt adjustment cost which is assumed to be convex in the deviation of the external debt from its desired the long-run equilibrium level. The debt adjustment cost as discussed in Schmitt-Grohe' Uribe (2003) is one of several techniques for "closing" the open economy. The surplus in the trade balance is used to repay the existing debt, while the trade deficit is financed by issuing more debt. Thus, the trade expressed in domestic currency prices is defined as $tb_t = -p_t\left(q_t b_{t+1} - b_t\right)$ where $q_t = 1/R_t$ is the price of one unit of the international bond.

Following Uribe and Yue (2006), we assumed that the gross interest rate faced by the domestic economy consists of the long-run average gross interest rate $R^w$ plus a country-specific risk premium that depends on domestic fundamentals and on other exogenous factors. Accordingly, we assume that the country premium is increasing in terms of trade and decreasing in percapita output. Hence, the international cost of borrowing can be described by the following equation:

$$R_t = R^w + \psi\left(e^{p_t - p} - 1\right) + \varsigma\left(e^{-\bar{y}_t + y} - 1\right) , \qquad (2.9)$$

where the country premium $\left(R_t - R^w\right)$ is increasing in ToT and decreasing in the wide economy percapita output $\bar{y}_t$ . The elasticity of the risk premium to ToT is $\psi > 0$ and to output is $\varsigma > 0$ .

It is also common to incorporate a convex adjustment cost function to control the speed of capital adjustment. Hence, investment spending is assumed to evolve as follows:

$$i_t = k_{t+1} - (1-\delta_t)k_t + 0.5\phi_k \left(k_{t+1}/k_t - 1\right)^2 k_t, \tag{2.10}$$

where $k$ is the stock of domestic capital and $\phi_k$ controls the speed of adjustment of the capital stock. Following Baxter and Dorsey (2001), we use variable capacity utilization to improve our control over the volatility of the artificial business cycle data. Accordingly, the time varying depreciation rate of capital $\delta_t \in [0,1]$ is an increasing function of the utilization rate:

$$\delta_t = \vartheta w_t^{\mu}, \tag{2.11}$$

where $w \in [0,1)$ is the utilization rate, $\mu > 1$ is the elasticity of the depreciation to utilization rate and $\vartheta > 0$ is a scaling parameter to guarantee that the steady state rate of depreciation equals the equilibrium rate. $\mu - 1 \in (0,1)$ is the elasticity of marginal depreciation with respect to the utilization rate. As intensive capital utilization accelerates the rate of depreciation, firms select a rate of utilization by weighing the benefits of greater output against the costs of greater depreciation. The Cobb-Douglas production function has the following form:

$$y_t = a_t \left(w_t k_t\right)^{\alpha} h_t^{1-\alpha}, \tag{2.12}$$

where $\alpha \in (0,1)$ is the capital share in the production process. Technological progress denoted by $a_t$, is assumed to follow a stationary AR (1) process:

$$\ln a_t = (1-\rho_a)\ln a + (1-\rho_a)\ln a_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim iid \quad N\left(0, \sigma_a^2\right). \tag{2.13}$$

In addition, households are subject to No-Ponzi-scheme constraint of the form

$$\lim_{j\to\infty} E_t \frac{b_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \le 0. \tag{2.14}$$

Household's objective is to maximize its life-time expected utility (2.3) subject to the budget constraint (2.8), the transition equation for the stocks of home durables (2.6), the transition equation for foreign durables (2.7) and the transition equation for capital (2.10). The dynamic programming yields the following first-order conditions:

$$\lambda_t\left(1+\phi_h\left(\frac{D_{h,t}}{D_{h,t-1}}-1\right)\right)-\beta_t E_t\lambda_{t+1}\left\{(1-\tau_h)+0.5\phi_h\left(\frac{D_{h,t+1}}{D_{h,t}}\right)^2-0.5\phi_h\right\}=$$
$$\left\{C_t-\xi\left(h_t-\upsilon h_{t-1}\right)^{\omega}\right\}^{-\sigma}C_t^{\frac{1}{\rho}}\gamma\left(D_{h,t}-\varphi_h D_{h,t-1}\right)^{\frac{-1}{\rho}}-\beta\varphi_h E_t\left\{C_{t+1}-\xi\left(h_{t+1}-\upsilon h_t\right)^{\omega}\right\}^{-\sigma}C_{t+1}^{\frac{1}{\rho}}\gamma\left(D_{h,t+1}-\varphi_h D_{h,t}\right)^{\frac{-1}{\rho}}, \tag{2.15}$$

$$p_t\lambda_t\left(1+\phi_f\left(\frac{D_{f,t}}{D_{f,t-1}}-1\right)\right)-\left\{C_t-\xi\left(h_t-\upsilon h_{t-1}\right)^{\omega}\right\}^{-\sigma}C_t^{\frac{1}{\rho}}(1-\gamma)\left(D_{f,t}-\varphi_f D_{f,t-1}\right)^{\frac{-1}{\rho}}=$$
$$\beta_t E_t p_{t+1}\lambda_{t+1}\left\{(1-\tau_f)+0.5\phi_f\left(\frac{D_{f,t+1}}{D_{f,t}}\right)^2-0.5\phi_f\right\}-\beta\varphi_h E_t\left\{C_{t+1}-\xi\left(h_{t+1}-\upsilon h_t\right)^{\omega}\right\}^{-\sigma}C_{t+1}^{\frac{1}{\rho}}(1-\gamma)\left(D_{f,t+1}-\varphi_f D_{f,t}\right)^{\frac{-1}{\rho}}, \tag{2.16}$$

$$\lambda_t(1-\alpha)\frac{y_t}{h_t}=\left\{C_t-\xi\left(h_t-\upsilon h_{t-1}\right)^{\omega}\right\}^{-\sigma}\xi\omega\left(h_t-\upsilon h_{t-1}\right)^{\omega-1}-\beta\upsilon E_t\left\{C_{t+1}-\xi\left(h_{t+1}-\upsilon h_t\right)^{\omega}\right\}^{-\sigma}\xi\omega\left(h_{t+1}-\upsilon h_t\right)^{\omega-1}, \tag{2.17}$$

$$\lambda_t\left\{1+\phi_k\left(\frac{k_{t+1}}{k_t}-1\right)\right\}=\beta E_t\lambda_{t+1}\left\{\alpha\frac{y_{t+1}}{k_{t+1}}+(1-\delta_{t+1})+0.5\phi_k\left(\frac{k_{t+2}}{k_{t+1}}\right)^2-0.5\phi_k\right\}, \tag{2.18}$$

$$\delta_t=\frac{\alpha}{\mu}\frac{y_t}{k_t}, \tag{2.19}$$

$$\lambda_t p_t \left[1 - \phi_b \left(\bar{b}_{t+1} - b\right)\right] = \beta E_t R_t \lambda_{t+1} p_{t+1} \ , \qquad (2.20)$$

where $\lambda$ is the Lagrangian multiplier associated with the budget constraint. The solution to the model is a set of stochastic processes for the endogenous variables that satisfies the budget constraint (2.9), equation (2.10), the first-order conditions (2.15)–(2.20), the No-Ponzi-scheme constraint (2.14) and the initial conditions $\left\{k_0, b_0, D_{h,-1}, D_{f,-1}, h_{-1}\right\}$. The model is solved for the long run steady state values around which the log-linearized version of the model is approximated, as in Campbell (1994). The linearized version of the model is solved with the method of undetermined coefficient as in Uhlig (1999). The detailed linearized form of the model and its solutions are appended in the Technical Note C.2.

## 2.4 Calibration

The baseline model is calibrated to the Canadian economy. Versions of the model with relatively low degree of durability, given everything else as in the baseline model, are calibrated to capture emerging economies' business cycles. Each period is taken to be a quarter. For our benchmark model economy, we assume that domestic and imported goods have an identical degree of durability. Accordingly, we set the depreciation rates on domestic and foreign goods to be thirty percent $(\tau_h = \tau_h = 0.3)$. The adjustment cost parameters are set to match the volatility of the aggregate consumption expenditure. For simplicity, we assume equal values for domestic and foreign durable adjustment parameters $\phi_h = \phi_f = 2.0$. Habit parameters in consumption of domestic and foreign goods consecutively are $\varphi_h = \varphi_f = 0.7$, while habit intensity in labor is set at $\upsilon = 0.8$.

Since there is no specific estimate for the elasticity of substitution between foreign and home goods we used $\rho = 2.0$ in all our simulations. The curvature parameter $\sigma$ is set to 0.5. The parameter that defines labor elasticity $(\omega)$ is set at 1.4 as in Mendoza (1991). The long run international real interest rate is approximately 1 percent per quarter. This is consistent with the assumption of 4 percent at the annual basis as in Mendoza (1991). Given the interest rate and the assumption of zero real percapita growth rates in the long run, the implied value of the subjective discount $\beta$ becomes $0.99$. We set the portfolio adjustment cost parameter $\phi_b$ equal to 0.03. This value is chosen to match the correlation between output and trade balance with the observed Canadian data. The value of the labor parameter $\xi = 6.745$ is chosen to fix the steady state value of labor at $0.2$ as in Mendoza (1991). We assume that the ratio of foreign to domestic consumption equals one. Accordingly, and given other parameter values, the share of the domestic consumption good in the utility is set to $\gamma = 0.495$. We calibrate $\vartheta$ such that the steady-state rates of capital depreciation and utilization $\delta = 0.02$. The consistent value of the elasticity of depreciation to utilization $(\mu)$ is 1.4. This is reasonable as Basu and Kimball (1997) estimate the elasticity of depreciation to utilization to be between one and two in the case of the US manufacturing data over the business cycle.

As in Letendre (2004), we set the persistence parameter for the technology shock at $\rho_z = 0.93012$ and $\sigma_z = 0.00509$. Using the ToT data as reported in the International Financial Statistics (IFS) for the period (1981:Q1-2009:Q4), we estimate the AR (1) process in (2.5) and find $\rho_p = 0.93012$ and $\sigma_p = 0.01677$. It is also important to note that we assume the two exogenous shocks to be independent. Hence we have $\text{cov}(\varepsilon_p, \varepsilon_z) = 0$. The baseline parameters for our calibrations of the model are summarized in Table A.6.

## 2.5 The Model Fit With the Canadian Economy

The quality of the model performance is judged based on the proximity of the simulated business cycle obtained from the theoretical model to the underlined actual developed economy (Canadian business cycle in our case). In particular, we compare the second moments obtained from the simulated model economy with its observed counterpart obtained from the Canadian data. We report the observed second moments of the Canadian economy in Table A.7 (column 2). The simulation results are reported in Table A.7 (column 3). It should be noted that total consumption expenditure $z_t$ includes total spending on domestic and foreign goods including durables and nondurables as we explained earlier in the previous section. The volatilities of the different national accounts closely match its counterpart volatilities in the Canadian national accounts. The model economy replicates the procyclicality of all national accounts and the countercyclicality of trade balance. In addition, the model correctly predicts a moderate countercyclical ToT, negative correlations between ToT and consumption, employment and investment and a positive correlation between ToT and the trade balance as observed in the Canadian data. However, the model slightly overstates the volatility and the procyclicality of total consumption expenditure, slightly understates the persistence level of output, and noticeably understates the persistence level in the investment data.[26] In addition, we find that

[26] It is very challenging to account for the high level of persistence in investment data. It seems that the reason for the observed high persistence in investment data is due to actual national accounts

ToT is moderately countercyclical, negatively correlated with consumption, employment and investment and positively correlated with trade balance. The model replicates all these comovements well. It however slightly overstates the negative comovement between consumption expenditure and ToT and overstates more the positive comovement between trade balance and ToT alone.

## 2.6 How Important Are the ToT Shocks?

Following Kose (2002) and Mendoza (1995) we compare the second moments generated from the full theoretical model with ToT uncertainty and a version of the model without such uncertainty. We do so by shutting off the ToT shock, by setting $\sigma_t^p = 0 \; \forall \; t$, and keeping all other parameter values unchanged as in the model economy. We also carry out this exercise for two hypothetical economies – one with a higher degree of durability in consumption and the other with a relatively lower degree of durability in consumption. It is reasonable to assume that advanced open economies have a higher share of durable consumption compared to many developing open economies. The results are reported in Table A.8. In an economy with a higher degree of durability, if we compare the results with and without ToT uncertainty, we find that ToT explains 17.2% of the aggregate business cycle fluctuations. On the other hand, such effects are significantly higher in an economy with relatively lower degrees of durability. In our sample calibration, the ToT uncertainty in such an economy explains around one third of the aggregate output fluctuations. It is also worthwhile to compare our results with those of Kose and Mendoza

classification of the components of investment and durable goods. A large component of aggregate investment data is households' spending on residential structure, which may have high persistence level due to household's incentives for consumption smoothing.

(1995). They find ToT shocks account for at least half of the aggregate fluctuations. This is much higher than our model predictions. It is reasonable to argue that a model open economy without durables (like the model economy of Kose or Mendoza) is more sensitive to external ToT shocks compared to an open economy with durable consumption goods. We elucidate this issue further in the following section.

## 2.7 The Role of Durability

To capture the role of durability in our model economy, we investigate how the model without durability in both consumption goods performs comparing to the baseline model. To do so, we construct a version of the model where home and imported goods are completely perishable goods with $\tau_h = \tau_f = 1$. In addition, when the goods are completely perishable then we will have no reason to assume adjustment cost in durable stock, hence we set $\phi_h = \phi_f = 0$. The business cycle characteristics of such an economy are reported in Table A.9 (column 3). It is very important to note that under this scenario aggregate consumption becomes more volatile than output and the trade balance becomes strongly countercyclical. Such business cycle characteristics are typical in emerging economies. This comparison further confirms our conjecture that emerging economies are characterized by a low degree of durability in consumption. Moreover, the volatility of the business cycle is about 47 percent higher than in our benchmark model economy. This indicates that the absence of durability increases the volatility of the business cycles.

We also experimented with a few alternative situations. The business cycle of a hypothetical economy with durability in domestic production and nondurable imports is obtained by setting $\tau_f = 1$ and $\phi_f = 0$. The results of such a model economy are reported in Table A.9 (column 4). This particular economy is about 12 percent more volatile than the baseline economy but much less volatile than an economy with durability in production and imports at the same time. When imports are durable while domestic production is not, i.e. when $\tau_h = 1$ and $\phi_h = 0$, the business cycles becomes about 21 percent more volatile than in the baseline business cycle model, as shown in Table A.9 (column 5).

The critical question is why the economies with a high degree of durability are more stable economies than economies with lower degree of durability. The answer to this key question becomes obvious if we compare the household's optimal consumption-decision rule for these alternative economies. The decisions rules on consumption of durables in the baseline model economy are represented by conditions (2.15) and (2.16). Without durability in consumption, these conditions reduce to:

$$\lambda_t = \left\{C_t - \xi\left(h_t - \upsilon h_{t-1}\right)^{\omega}\right\}^{-\sigma} C_t^{\frac{1}{\rho}} \gamma\left(D_{h,t} - \varphi_h D_{h,t-1}\right)^{\frac{-1}{\rho}} - \beta\varphi_h \left\{C_{t+1} - \xi\left(h_{t+1} - \upsilon h_t\right)^{\omega}\right\}^{-\sigma} C_{t+1}^{\frac{1}{\rho}} \gamma\left(D_{h,t+1} - \varphi_h D_{h,t}\right)^{\frac{-1}{\rho}}, \tag{2.21}$$

$$p_t\lambda_t = \left\{C_t - \xi\left(h_t - \upsilon h_{t-1}\right)^{\omega}\right\}^{-\sigma} C_t^{\frac{1}{\rho}} (1-\gamma)\left(D_{f,t} - \varphi_f D_{f,t-1}\right)^{\frac{-1}{\rho}} - \beta\varphi_h \left\{C_{t+1} - \xi\left(h_{t+1} - \upsilon h_t\right)^{\omega}\right\}^{-\sigma} C_{t+1}^{\frac{1}{\rho}} (1-\gamma)\left(D_{f,t+1} - \varphi_f D_{f,t}\right)^{\frac{-1}{\rho}}. \tag{2.22}$$

The main difference between (2.21) and (2.22) and equations (2.14) and (2.15) is that in the nondurable model the expected ToT has no effect in household's consumption decision. In equation 2.15, one can note that the higher the degree of durability on the foreign good $(\tau_f \rightarrow 0)$, the greater is the effect of the ToT expectation on household's decision. Without durability, household has no ability to delay or postpone its current purchases of consumption goods; hence a shock to ToT will lead to substitution between home and foreign goods as the only way to avoid more expenses due to higher import prices. With some degree of durability in home and foreign consumption goods, household response to a transitory increase in ToT by reducing their current purchases of imported goods and at the same time they also substitute away some imported consumption for home good consumption. As a result, an economy with higher degree of durability will have a reduced effect of ToT fluctuations on aggregate consumption, output, and other national accounts.

## 2.8 The Role of Interest Rates

The country risk premium in (2.9) is shaping the interest rates volatility in our model. Based on this specification, the country premium constitutes an important channel for the ToT. To highlight the importance of the country premium on the dynamics of the model, we simply shut down this channel by setting $\psi = \varsigma = 0$ in (2.9).[27] Figure B.6 shows the dynamic responses of the national accounts to a one standard deviation shock in ToT. An increase in relative price (adverse ToT) leads to a decrease in total consumption spending $z_t$ along with the substitution of leisure for consumption, which reduces employment and output. As it become more expensive to

[27] Most of the RBC literature with ToT is based on the assumption of constant interest rates.

hold and consume foreign durables, household reacts by cutting its investment in consumption durables in favor of holding more domestic capital. Thus, this portfolio re-allocation decreases in aggregate consumption expenditure. While household is decreasing its aggregate consumption it consumes more leisure which reduces the labor supply. As a result output fall below its steady state so as well aggregate consumption while investment increases above its steady state level. Since output decreases while the total effect on domestic absorption is positive, the trade balance deteriorates. This can be seen in the response of the trade balance.[28] As the trade balance deteriorates, the external debt of this economy increases. Unfortunately, this dynamics does not fit the benchmark economy. We recall that in the benchmark economy, the TOT deterioration must be associated with a decrease in investment and an increase in the trade surplus. For the trade balance to be in surplus, the domestic absorption of the economy should decrease faster than the decrease in output. With variable interest rates, Figure B.7, the increase in the ToT results in a higher country premium. This increase in interest rates forces household to reduce its investment spending and holding of external debt. The logic behind the negative relation between investment spending and interest rates is related to a household's portfolio allocation. With higher interest rates the cost of borrowing relative to the return from investment becomes higher. Accordingly, household is better off reducing its liability of debt at the cost of other assets, particularly investment. In conclusion, investment decreases, the trade balance improves while external debt subsides.

[28] Domestic absorption consists of household's spending and on consumption and household's aggregate spending investment.

## 2.9 Implications of the Persistence of the ToT

The relationship between the ToT and the trade balance is dominated by two competing hypotheses. The HLM hypothesis predicts that any ToT deterioration is associated with a trade balance deficit. On the other hand, the OSR hypothesis indicates that only transitory deterioration to the ToT may lead to a trade balance deficit. Contrary to the HLM, OSR predicts that a permanent deterioration of the ToT leads to a trade balance surplus. Accordingly, one may suspect that the differences in the characteristics of open economies' business cycles are due to differences in the persistence of the ToT that they face. In this section we investigate the possibility that differences in the persistence of the ToT could be the reason for the distinct features of the business cycles of small open economies. In the following, we have two objectives. First, we analyzed the effect of the persistence of the ToT on the trade balance in our baseline model economy. This also directly tests the general hypothesis of the OSR against the HLM in the baseline economy environment. Second, we investigate the effects of the persistence of the shock on the business cycle's characteristics. This could provide a reasonable explanation as to why different open economies often exhibit distinct real business cycle characteristics.

### 2.9.1 HLM versus OSR in the Model Economy

To test these hypotheses within our framework, we need only to compare the response of the trade balance to tot deterioration with different levels of persistence in the ToT. The response of the trade balance to tot in the baseline model is depicted in Figure B.7. As explained in the previous section, the deterioration of the ToT in the baseline model is associated with a trade balance surplus. We found this to be consistent in our benchmark economy. The direct conclusion that we can draw is that tour baseline model economy rejects the HLM hypothesis, which requires that trade balance be in deficit after deterioration in ToT.

The question that remains to be answered is whether the rejection of the HLM implies any support in favor of the OSR hypothesis or not. To provide a precise answer, we need to investigate and examine the dynamics of the trade balance with alternative ToT shocks with different degrees of persistence. For a given ToT shock, the OSR hypothesis requires the trade balance to be more in surplus the higher the persistent level of the shock.

Figure B.8 plots the responses of the model economy to ToT under the assumption of a highly persistent shock. In particular, we assign a higher parameter value for $\rho_p$ than in the baseline model and keep all other parameters unchanged. Consistent with our baseline model predictions, a more persistent ToT shock leads to a surplus in the trade balance as well. Of course, now the surplus of the trade balance is more profound than in the baseline economy - consistent with the OSR hypothesis. As can be observed in Figure B.8, the decrease in investment is larger and the decrease in output is smaller, relative to the baseline model, which translates to relatively more surplus in the trade balance. We also investigate the effects of a ToT shock with lower parameter value for $\rho_p$ comparing to the baseline model. Repeating the same analysis with lower degree of durability we find similar results involving the relationship between trade balance and the persistence of the ToT (see Figure B.9).[29] In conclusion, these experiments reject the HLM hypothesis and confirm the OSR hypothesis in this study.

[29] The IRFs for these experiments are available upon request.

### 2.9.2 Persistence of the ToT and the Business Cycle

Finally, we investigate whether differences in the business cycles among different economies are due to differences in persistence of the ToT shocks faced by those economies rather than differences in the degree of durability. To do so, we first use the nondurable version of the model and then assign different persistence levels to the ToT and compare the resulting business cycles. Column 2 in Table A.10 includes the moments of the model given the estimated persistence for the benchmark economy data. In column 3 of the same table, we obtain different results by assuming higher persistence level. We find that with higher persistence the volatility of the business cycle is slightly lower than in the baseline model without durability. The main effects of higher ToT persistence are on investment volatility, and on the correlation between investment and ToT. In column 4 (Table A.10), we report the results for shorter persistence. In this version of the model, the correlation between the ToT and the trade balance becomes negative, as mentioned earlier. However, the aggregate fluctuations of the resulting business cycle are slightly less than in the baseline model without durability. Overall, the differences in the ToT persistence in various economies cannot explain the observed differences in their business cycles.

## 2.10 Conclusion

Analyzing the observed differences in the business cycles of emerging small open economies compared to advanced small open economies is an important issue in the recent literature. One point of view attributes the differences to the sensitivity of open economies to international capital markets and interest rates. Another point of view claims that emerging

economies vary from developed economies with regard to domestic policy shocks and regime switching. We provide an alternative explanation. We argue that the two groups of economies are distinct in terms of their production and demand patterns. Developed open economies usually produce and consume more durable goods than many developing economics. Economies with a higher level of income also import more durable goods than poor economies as well. We construct a small open economy real business cycle model with two goods - domestic and foreign and examine the effects of adverse terms of trade. In our model, both composite goods exhibit a certain degree of durability. We also calibrate our model to the Canadian quarterly data. The model is able to replicate the different moments of the national accounts. The model is also the first to replicate the comovement between ToT and national accounts and to decipher the dynamics behind this observed comovement. We find that ToT helps explain around one fifth of the aggregate business cycle fluctuations of the developed economy and one third of the aggregate fluctuation in the developing economy. We show that differences in the demand and production structure among different small economies affect the degree of sensitivity of these countries to the ToT due to the forward looking nature of the demand for durables.
Traditionally, the relationship between the ToT and the current account (or trade balance) is debatable and commonly explained based on either the HLM and OSR hypotheses. Our model rejects the HLM premise and confirms the OSR premise. We underscore that terms of trade persistence affect the outcomes of the trade balance but cannot justify the observed differences in the actual business cycles of the small open economies. Since our two goods model with terms of trade can replicate Canadian business cycle and explain various comovements remarkably well, it will be useful to employ this framework to study various government policies.

## 2.11 Data Sources

Data are obtained from the Canadian Socio-economic Information and Management (CANSIM) database. CANSIM labels are in parentheses.

*Population*: Quarterly estimates of population for Canada (D1)

*Output*: real gross domestic product (D100126)

Unite price of imports: International Financial Statistics, International Monetary Fund (IMF).

Unite price of exports: International Financial Statistics, International Monetary Fund (IMF).

*Nondurable Consumption*: personal expenditure on non-durable goods (D100106) and services (D100107)

*Durable spending*: personal expenditure on durable goods (D100104) plus personal expenditure on semi-durables (D100105)

*Investment*: investment in machinery and equipment (D100115), non-residential structures (D100114) and residential structures (D100112)

*Exports*: exports of goods and services (D100119)

*Imports*: imports of goods and services (D100122)

*GDP deflator*: ratio of nominal GDP (D14816) and real GDP (D100126)

*Employment*: employment age 15+ (D980595)

*Current account*: total nominal current account balance (D59832) deflated using the GDP deflator.

# Essay 3

## Optimal Budget Stabilization Funds, State Budgets and the Business Cycle

### 3.1 Introduction

Almost all American States have legal provisions mandating that their budgets should be balanced on a yearly basis. In good times, States find it relatively easy to comply with this rule, as revenues are abundant. However, in bad economic times, keeping a balanced budget is challenging. It requires for procyclical tax increases and/or expenditure cuts, unless significant surpluses are run in the upturn. In the years following the back-to-back recessions of the early 1980s States governments across the country adopted Budget Stabilization Funds (BSFs) or Rainy Day Funds. At the time, these recessions together had the most dramatic impact on state budgets of any downturn since the Great Depression. There were sever contraction in public service, some States were forced to raise taxes, and some witnessed the exhaustion of unemployment-insurance trust-fund balances. The introduction of BSFs was intended to provide insurance in the face of future recessions, in thereby to smooth the flow of public services.

Today most states have a formal BSF.[30] The main objective of this study is to develop a State-level real business cycle (RBC) model with fiscal policy to examine the effects of BSFs on the stability of government expenditures and the State level business cycles.

While State BSFs are intended to provide insurance to the consumers of state-provided public services, they may impart a countercyclical externality on the macroeconomy. Generally, BSF balances are accumulated during periods of economic expansion, dampening economic growth, and are then expended during contraction, stimulating the economy. Annual increments to reserve funds are admittedly modest in size, but cumulatively they can be substantial. For example, in 2006 BSF balances, along with idle general fund balances, approached 0.5 percent of gross domestic product. Some have suggested that the states develop even much larger reserve balances, which would increase the potential impact of this State policy on the macroeconomy.

If state BSF policies prove to have consequences for macroeconomic performance, this may call into question the view of Musgrave (1959) and others who have concluded that the stabilization function of government is best placed in the hands of the central government. Moreover, there may be implications for federal monetary and fiscal policy. If federal policymakers fail to take state fiscal behavior into account, they may overshoot policy targets with aggressive stimulus policies in downturns and policies to dampen growth during expansions.

[30] Balassone, Franco, and Zotteri (2007) among others, argues that under some conditions European Monetary Union countries can benefit from BSFs as the Stability and Growth Pact (SGP) limits the amount of the budget deficit in these countries.

Empirical evidence has shown that BSFs are an effective means to reduce fiscal stress in lean years and dampen the political business cycle. Relevant studies include Russell and Holcombe (1996), Douglas and Gaddie (2002), Yilin (2003), Fatas (2006) and Rose (2006). Unfortunately there are no empirical studies investigating the effect of the BSF on the business cycle in a systematic way.[31] In addition, there are no studies that evaluate the role of BSFs on business cycle volatility. The latter issue has many important implications for state economies, the national economy in general and the monetary union member countries. Issues related to procyclicality of governments have received tremendous attention among researchers recently due to the limited ability of the monetary authority to counter the economic downturn, caused by the massive onslaught of the financial crises all over the world (see Baunsgaard, and Symansky (2009)).

In this paper, the real business cycle model with fiscal policy is extended to incorporate a BSF. We aim to address many important concerns. One of the main purposes of this study is to evaluate the effects of BSFs on the government budgets in terms of government procyclicality and persistence of spending. Second, we evaluate the effect of a BSF on the business cycle characteristics of a typical state economy. Finally, we provide some welfare analysis and evaluate the costs of establishing the BSF. Our model benefits from the studies by Malley, Philippopoulos and Woitek (2009) and Scott and Glomm (2000) in terms of a few modeling features. In terms of tax policies, our model adopts distortionary tax policies and fixed allocation

[31] Moreover, Levinson (1998, 2007), Krol and Svorny (2007) find that states with strict balanced budget rules encounter volatile business cycles. In a recent study on BSF management, Rose (2008) finds that U.S. state politicians manipulate rainy day funds for political purposes. Given these studies, one may suspect that the balanced budget role of, even with BSF, may increase the volatility of business due to political business cycles.

of government revenues. These two assumptions are necessary to highlight the effect of adopting the BSF as an additional government instrument.

Upon calibration, our model economy captures the effects of BSFs significantly well. The overall findings are as follows. First, the stabilization fund allows the government mitigates its current expenditures procyclicality. The volatility of current spending with a BSF is 3.1 percentage points less than current spending in an identical economy without a BSF. The persistence of current expenditure is also improved by the BSF by 1.2 percentage points. Total spending in an economy with a BSF compared to an economy without a BSF is 11.4 percentage points more volatile. The second group of findings is related to the effect of BSFs on the State business cycle. Here we find a significant impact of the BSF on smoothing the business cycle. In particular, we find that the aggregate volatility in real income with a BSF decreases by 3.9 percentage points, and aggregate income persistence increases by 1.2 percentage points. In a state economy with a BSF, employment gains more stability and persistence. The BSF also reduces the volatility of private consumption by 1.2 percentage points but has no effect on the procyclicality and persistence of private consumption. Last but not least, we find that a BSF yields smoother household utility over the business cycle and reduces the dependence of household utility on the aggregate income fluctuations by a small amount.

The rest of the paper is organized as follows. A brief background along with some anecdotal evidences on BSFs in the context of the American states is outlined in Section 2. A complete real business cycle model of a state economy along with an optimal BSF rule is developed in Section 3. In Section 4, we discuss the parameter values and solution technique used for our stochastic calibration of the model economy. We report our important results in Section 5 followed by some concluding remarks in Section 6.

## 3.2 Background

The conventional view is that subnational governments should not pursue countercyclical stabilization policies. Musgrave (1959) made this case based on the practical issues of policy coordination that would arise across the potentially large set of subnational government units. Moreover, it could be argued that states and localities cannot be effective in pursuing countercyclical stabilization policies, because they generally face balanced budget restrictions that limit the scope of deficit finance and subnational budget multipliers are small relative to the multipliers for a national economy.

Despite the conventional logic, the fact is that State and local fiscal policies may have consequences for macroeconomic performance. In the midst of the Great Depression, State and local governments contributed to the downturn in economic activity through their use of procyclical tax and expenditure policies. Of course this was not the goal of their fiscal policy, but an unintended consequence. Hansen and Perloff (1944) have examined subnational fiscal policies during the Great Depression and periods of economic expansion, together refer to this procyclical behavior as “Fiscal Perversity in Boom and Depression.” A similar pattern has emerged over the course of the recent recession began in December 2007, offering some support for providing federal fiscal assistance to the states.

On the other hand, some programs, like State unemployment insurance systems, impart a countercyclical influence on the State’s business cycle, as trust fund balances accumulate during periods of expansion and are then drawn down when workers become unemployed during a contraction. This again is an unintended consequence since the goal of the unemployment insurance is to provide social insurance to individuals, not to stimulate the macro economy.

Anecdotal evidence like that presented by Hansen and Perloff supports the perversity hypothesis. For example, Lav and Berube (1999) point to $27 billion in tax increases and a wide range of service cuts in the 1989-92 window, which included a slowdown and the recession of 1991. Fox and Murray (1997) note $1.6 billion in procyclical state tax cuts during the strong expansion year of 1994. A small and more rigorous body of empirical research has emerged since the 1960s to examine the extent to which State fiscal policy has had a procyclical or countercyclical influence on the macro economy. Rafuse (1965) examined both revenue and spending and concluded that they each have a stabilizing influence on the macroeconomy. Matoon and Testa (1992) concluded that State policy was countercyclical in economic downturns. Recent evidence from the National Association of State Budget Officers and the National League of Cities points to contractionary policies over the course of the Great Recession.

The more specific question addressed here is the extent to which State BSFs have a countercyclical influence on the State's macroeconomy business cycle. Public sector entities have likely maintained budget reserve funds in various forms since the early formation of governments, but it wasn't until the 1980s that American States developed formal BSFs.[32] In the early years of fund accumulation (i.e. the 1980s) it was not clear what guided the States to their choice of a fund balance. Joyce (2001) discusses the so-called "5 percent target" which calls for

[32] Wagner and Sobel (2006) examine empirically the BSF adoption decision of the states.

BSF balances to equal 5 percent of general fund expenditures. But Joyce cannot trace this target back to an original source.

Joyce (2001) and others have sought to identify the optimal size of a BSF based on the unique characteristics of the States by using different objective functions. As one would expect, the optimal balances that emerge from this work differ widely. For example, Wagner and Elder (2007) conclude that the optimal saving rate is on average in the range of 2.5-2.8 percent. Lav and Berube (1999), on the other hand, indicate that several states would need to have reserve funds in excess of 25 percent to smooth spending; this conclusion was reached well before the most recent recession and subsequent crisis in subnational government finances in the U.S. Bond rating agencies have argued that the states should have balances in the 5-10 percent range, but this too pre-dates the recession. Given the recent recession and speed at which BSF balances were brought down, it is likely that they will grow significantly in size in the years ahead, potentially increasing their effects on macroeconomic performance.

## 3.3 The Model

Bellow we describe the model that we intend to develop for a State economy. We design a RBC type model to capture the dynamic effects of BSFs in terms of state business cycles. To accomplish this we need first to develop a baseline model without a BSF and then extend the baseline model to include a government managing BSF. In the baseline model economy, as we show later, households have access to private managed financial funds and their decisions for saving is derived optimally to guarantee household's consumption smoothing. In the construction of the BSF the assumed benevolent State government needs to set an optimal rule that govern the

BSF policies. Hence, it should be consistent with households incentives for consumption smoothing.

### 3.3.1 The Model Economy without BSF (Baseline Economy)

The baseline model economy is populated by a unit measure of infinitely lived households. Each household is endowed with one unit of time and has identical preferences over a private consumption good, a publicly provided consumption service and working hours. The government levies distorting income and consumption taxes and uses the revenues to finance its spending on public investment, consumption services, and transfer payments. The state government implements a period-by-period balanced-budget rule and at the same time administrates a BSF. The stabilization fund is being financed from the government current revenues. During the expansion of the macroeconomy, the government allocates part of its tax revenues to accumulate reserves in the BSF while during contraction times the government uses the BSF to offset the shortfall in its tax revenues. Thus, the BSF not only reduces the need for spending cuts or tax increases during an economic downturns it also limit the government size during the expansions of the business cycle. However, the accumulation and disbursement of the BSF needs to follow a specified rule by which we prevent discretionary behavior of the government and guarantee an efficient use of these funds. We suggest that the government applies the households saving rule which guarantees that savings in the BSF will be used for the benefit of households' consumption smoothing in addition to the positive effect of the BSF on reducing the procyclicality of its spending.

#### 3.3.1.1 Households

Private agents (households and firms) are assumed to take fiscal policies as given when making optimal decisions. The representative household maximizes the expected present value of lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t,\ h_t - \varphi_h h_{t-1}\right), \tag{3.1}$$

where $E_t$ is the expectation operator conditional on the information available at time $t$, $\beta \in (0,1)$ is the subjective discount factor. The household consumes $C_t$ (a CES index of private good $c_t$ and publicly provided services $s_t$) and works $h_t$ hours of its total time endowment. For consistency with macroeconomic data in terms of observed persistence, household's decisions on working hours $h_t$ and private consumption $c_t$ are assumed to be subject to habit formation. We refer to habit intensity in labor by $\varphi_h \in (0,1)$. The CES index of consumption $C_t$ is written as follows:

$$C_t = \left( \left(c_t - \varphi_c c_{t-1}\right)^{\frac{\rho-1}{\rho}} + \gamma s_t^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{3.2}$$

where $\varphi_c \in (0,1)$ is the habit intensity in private consumption, $\gamma > 0$ is a scaling parameter that maintains the ratio of private consumption to public service consumption in the model economy equal to its observed counterpart in the actual state economy. The parameter $\rho > 0$ reflects the intraperiod elasticity of substitution between private consumption and the consumption of public services.[33] In this study we use the following preference of the household:[34]

$$u(C_t,h_t)=\frac{1}{1-\sigma}\left\{\left(C_t-\xi\left(h_t-\varphi_h h_{t-1}\right)^{\omega}\right)^{1-\sigma}-1\right\}. \tag{3.3}$$

The parameter $\sigma>0$ measures the intertemporal elasticity of substitution and $\frac{1}{1-\omega}$ measures the elasticity of labor supply. The utility function is increasing in current period consumption of private good and public services and decreasing in current period employment level. As a result of habit formation, the utility is decreasing in the previous period's consumption of private good and increasing in previous period hours worked. The household allocates its disposable income into private good consumption, investment and purchases of financial assets. Accordingly, the household dynamic budget constraint is defined as follows:

$$\left(1-\tau^y\right)y_t+T_t=\left(1+\tau^c\right)c_t+I_t+H_t. \tag{3.4}$$

The left hand side of this equation measures the disposable income of the household, where $\left(1-\tau^y\right)y_t$ is the after tax aggregate income, and $\tau^y$ rate is income tax rate which we assume to be fixed over the business cycle. In addition household receives positive lump sum government transfers $T_t$. The right is the household aggregate expenditure on of the private good

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[33] When $\rho\rightarrow\infty$ the two goods become perfect substitutes, and when $\rho\rightarrow 1$ utility function takes a standard Cobb-Douglas form.

[34] It should be noted that with $\varphi_c=\varphi_h=0$ and $\gamma$=0 the preference in (3.3) collapses to what is commonly known as the GHH preferences due to Greenwood, Hercowitz, and Huffman (1988). In open economy RBC models such preferences are often recommended. Under this specification, the labor supply which is independent of consumption depends only on the marginal product of labor. This eliminates the income effect and makes consumption and labor supply more sensitive to a productivity shock.

consumption $c_t$, which is taxed at a rate $\tau^c$, investment $I_t$ , and household's net purchases of financial $H_t$. Following various RBC models, we assume that investment follows the following process:

$$I_t = k_{t+1} - (1-\delta_t)k_t + 0.5\phi_k \left(k_{t+1}/k_t - 1\right)^2 k_t, \tag{3.5}$$

where $k_t$ is the total holding of firm capital and $\delta \in (0,1)$ is the average per period depreciation rate of capital. The last term in the above equation is the adjustment cost, assumed to be convex in the rate of change in capital stock with an adjustment parameter of $\phi_k$.[35] Similarly, the model also assumes portfolio adjustment costs. Hence, net total spending on financial assets $H_t$ is expressed as follows:

$$H_t = q_t D_{t+1} - D_t + 0.5\phi_D \left(D_t - D\right)^2, \tag{3.6}$$

where $D_t$ is the net balance of one period financial assets at the beginning of the period. The financial asset that is being purchased at the beginning of the period is due at the end of the period at a price $q_t = 1/R_t$ Here $R_t$ is the gross interest rate that is being determined outside the State. The State does not have any monetary policy to conduct and interest rates are assumed to be given. The term $\phi_D > 0$ is the portfolio adjustment parameter.[36]

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[35] It is common in the literature to incorporate adjustment costs to control the volatility of investment in the model economy.

[36] Adding convex portfolio adjustment cost is necessary to guarantee stationarity in the model.

The firm uses three inputs: private capital, public capital and labor, to produce one unit of output according to the following Cobb-Douglas technology:

$$y_t = \exp(z_t) K_t^{\alpha} h_t^{1-\alpha} , \tag{3.7}$$

where $z_t$ represents technical progress and is assumed to follow a first order autoregressive process:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \ , \quad \varepsilon_t^z \sim i.i.d.\ N\left(0, \sigma_\varepsilon^2\right). \tag{3.8}$$

The persistence of technology is measured by $\rho_z$, and the innovation to the technology is measured by $\varepsilon_t^z$. The total capital in the production process $K_t$ is a CES index of private capital $k_t$ and public capital $G_t$ and is assumed to follow this function form:

$$K_t = \left( k_t^{\frac{\mu-1}{\mu}} + \phi G_t^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} , \tag{3.9}$$

where $\mu > 0$ is the intra-period elasticity of substitution between the two inputs.[37] The parameter $\phi > 0$ is a scaling parameter that takes its value from the actual ratio of public capital to private capital which can be estimated from the historical data.

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[37] When the elasticity of substitution tends to infinity, the two goods become perfect substitutes, and when it approaches one, the production function becomes a standard Cobb-Douglas: $y_t = A\exp(z_t)(k_t G_t)^{\alpha} h_t^{1-\alpha}$, where A is some constant parameter.

### 3.3.1.2 Government

The role of the government is to collect taxes and use the tax revenues for transfer payments, infrastructure investment, and the provision of utility enhancing services. Like Barro (1990), Lucas (1990) and Glomm and Ravikumar (1994), we assume the State government maintains a balanced budget period by period, that is

$$\tau^y y_t + \tau^c c_t = s_t + T_t + g_t \ . \qquad (3.10)$$

The left side of (3.10) is the total tax revenue of the State government that equals the total government expenditure to right side. Total revenues of the government assumed to be allocated into three expenditure categories. First is the government expenditure on utility enhancing public services, denoted by $s_t$. This type of expenditure provides households with direct utility. As specified in (3.3) above we assume that private consumption good and public consumption service are two substitute goods. Hence, smoothing government provision of public consumption services will also enhance households consumption smoothing. Second, it is assumed that the State government is interested in income equality among its constituents. Hence, the government spends $T_t$ in form of positive private transfer to households. The last expenditure category is the government investment in public capital $g_t$ that provides the private sector with substitute productive capital as in (3.9). The transition of public capital can be described by

$$g_t = G_{t+1} - (1-\delta_g)G_t , \qquad (3.11)$$

where $G_t$ is the public capital stock at the beginning of the period and $\delta_g$ is the per period rate of depreciation on that stock.

We assume that the government allocates its expenditures in fixed proportion. For example, $0 < m_1 < 1$ is the proportion of government revenue used for the economic stabilization function similarly $0 < m_2 < 1$ is the proportion of government revenue used for the welfare enhancing function and $(1 - m1 - m2)$ is the proportion that is allocated for income redistribution. In particular

$$g_t = m_1\left(\tau^y y_t + \tau^c c_t\right), \tag{3.12}$$

$$s_t = m_2\left(\tau^y y_t + \tau^c c_t\right), \tag{3.13}$$

$$T_t = (1 - m_1 - m_2)\left(\tau^y y_t + \tau^c c_t\right). \tag{3.14}$$

The allocation of government spending among the three types of spending is assumed to reflect the taste or priority for each objective. To keep our analysis focused on the importance of the BSF, we shall assume that all government instruments including the tax rates ($\tau^y$ and $\tau^c$) and the expenditures' allocations ($m_1$ and $m_2$) are time invariant parameters. In other words the only instrument available for the government is the BSF which becomes an automatic stabilizer once the State government adopts the optimal rule that we derive bellow.

#### 3.3.1.3 The Optimal Conditions

Household's objective is to maximize its lifetime expected utility (3.1) subject to its budget constraint (3.4). In each period household decides on the amount of consumption of private good, hours worked, the next period holding of fiscal capital and financial assets. Solving the household's problem we obtain the following optimal conditions:

$$\left(1+\tau^{c}\right)\lambda_t=\left(C_t-\xi\left(h_t-\varphi_h h_{t-1}\right)^{\omega}\right)^{-\sigma}C_t^{\frac{1}{\rho}}\left(c_t-\varphi_c c_{t-1}\right)^{\frac{-1}{\rho}}-\beta\varphi_c\left(C_{t+1}-\xi\left(h_{t+1}-\varphi_h h_t\right)^{\omega}\right)^{-\sigma}C_{t+1}^{\frac{1}{\rho}}\left(c_{t+1}-\varphi_c c_t\right)^{\frac{-1}{\rho}}, \tag{3.15}$$

$$\lambda_t\left(1-\tau^{y}\right)\frac{\partial y_t}{\partial h_t}=-\left(C_t-\xi\left(h_t-\varphi_h h_{t-1}\right)^{\omega}\right)^{-\sigma}\omega\xi s_{h,t}^{\omega-1}+\beta\varphi_h u_{ht}\left(C_{t+1}-\xi\left(h_{t+1}-\varphi_h h_t\right)^{\omega}\right)^{-\sigma}\omega\xi s\left(h_{t+1}-\varphi_h h_t\right)^{\omega-1}, \tag{3.16}$$

$$\lambda_t\left[1+\varphi_k\frac{k_{t+1}}{k_t}-\varphi_k\right]=\beta\lambda_{t+1}\left[\left(1-\tau^{y}\right)\frac{\partial y_{t+1}}{\partial k_{t+1}}+1-\delta_k+0.5\varphi_k\frac{k_{t+2}^2}{k_{t+1}^2}-0.5\varphi_k\right], \tag{3.17}$$

$$\lambda_t\left[q^{*}+\phi_D\left(D_{t+1}-D\right)\right]=\beta E_t\lambda_{t+1}, \tag{3.18}$$

where $\lambda$ is the Lagrangian multiplier associated with the budget constraint . The Euler equations (3.15)-(3.18) equate the marginal benefits and costs due to household's optimal choices in terms of consumption, hours, capital and financial asset accumulation. A detailed discussion is avoided for the sake of brevity.

### 3.3.2 The Model Economy with BSF

In this section we establish a stabilization fund as part of the government budget. During the expansion of the state economy, the state government is required to allocate part of its total tax revenues to the stabilization fund, which will be used during the contraction. The BSF is intended to stabilize government expenditure over the business cycle. Sustaining government expenditure in the downturn may compensate households by providing them with public consumption services that is substitute to the private good. It also helps the firm by providing it with additional productive public capital to substitute for the fall in their private capital purchases which also sustain the marginal product of labor or labor demand of the firms. Hence, the BSF is designed to provide a kind of insurance to all agents in the economy during bad economic times.

#### 3.3.2.1 Optimal BSF Rule

The State government is managing the BSF in terms of accumulation and disbursement. The optimal savings rule should be consistent with households' behavior. The guiding rule for government saving requires the government to follow rules that are consistent with household's objective to smooth its consumption over the business cycle. Otherwise, the government saving decision could be subject to the discretion of the budget planner, which might generate negative effects on the state economy.[38] For example, overstating the need for more BSF results in current spending cuts that in turn reduce household's welfare, slow down production and widen

[38] Empirical evidences show that the existing stabilization funds without optimal rules suffer from lack of efficiency in reaching their goals, and leaving the BSF under the discretion of the States may lead to negative effect on the stability of government expenditures.

the income gap among households. Similarly, overusing disbursements from the stabilization fund depletes the fund before it reaches its goals and leads to more volatile government spending and welfare loss. It also amplifies the political business cycle at that point. Since households are making optimal decisions, given all the information available to them, we suggest the government applies household's saving rules when it makes its budget allocation decisions. In particular, the government sector faces the following optimal condition while making other spending decisions:

$$\lambda_t \left[ q^* + \phi_D \left( D_{t+1} - D \right) \right] = \beta E_t \lambda_{t+1} . \tag{3.19}$$

Given (19), we implicitly assume that the government is voluntarily acting to maximize household's expected utility.

### 3.3.2.2 Government Current and Capital Spending

The government sector with BSF is governed by the following modified budget constraint along with (3.19) which regulates the disbursement or accumulation of resources from/in the BSF:

$$g_t + s_t + T_t = \tau^y y_t + \tau^c c_t - H_t , \tag{3.20}$$

where $H_t$ is still defined as by equation (3.6). The government revenues on the right side of equation (3.20) consist of current income or tax revenues $\left( \tau^y y_t + \tau^c c_t \right)$ in addition to capital income or net disbursement from the stabilization fund $H_t$.

In the baseline model we have already explained how the government's current-plus-capital revenues are used to finance its three functions. With BSF the equations that determine the allocation of government are modified as follow:

$$g_t = m_1\left(\tau^y y_t + \tau^c c_t - H_t\right), \tag{3.21}$$

$$s_t = m_2\left(\tau^y y_t + \tau^c c_t - H_t\right), \tag{3.22}$$

$$T_t = \left(1 - m1 - m2\right)\left(\tau^y y_t + \tau^c c_t - H_t\right). \tag{3.23}$$

### 3.3.2.3 Aggregate Resource Constraint

By combining the budget constraints of the households and the government we must always have $y_t = c_t + s_t + g_t + I_t$ with or without the stabilization fund. This is a standard market clearing condition. Having identical aggregate resource constraints is important to validate the comparison of the two macro economies, i.e. with and without BSF. Accordingly the modified household's budget constraint is given by:

$$\left(1-\tau^y\right) y_t + T_t = \left(1+\tau^c\right) c_t + I_t. \tag{3.24}$$

It should be noted that in this modified model, government savings (in the form of the BSF) is replacing a part of private sector savings. Hence, even when the government is behaving optimally regarding its stabilization fund policies, the two models will yield different results. This is very crucial in our model. In the baseline model, savings are spent based on households' priorities while in the economy with BSF, the savings are spent based on the state government priorities. This completes the basic description of our models with and without BSF.

## 3.4 Calibration and Solutions

Calibration of the State economy requires the selection of various parameters' values. It is important to note that some of these parameters are unavailable at State level. We overcome this problem by making a few simplifying assumptions. First important assumption is that State's government expenditures and revenues include all in-State, both State government plus and national government. This assumption not only helps simplifying the model, by avoiding the distinction between State level and national level, it also underscores the rule of the BSF in reducing the cost of the business cycle at the national level. Since BSF is offsetting the shortage of government revenues resulted in from the downturn of the cycle, then the national level intervention will be to offset the shortage of the aggregate revenues after it accounts for the withdraws from the BSF. Second, we assume that our State economy is having identical business cycle features like the one observed at the national level. Hence we calibrate our model in such a way as to replicate the national level business cycle stylized features.

Each period is taken to be a quarter. Following the important RBC studies, we set the capital share $\alpha$ to be $0.34$ and the steady state depreciation rate of capital $\delta_k$ to be 0.015. The steady state employment $h$ is normalized to 30 percent of the total time endowment. Since there is no specific estimate for the elasticity of substitution between private consumption and public services, we set $\rho = 1.1$. This implies that the two types of consumption are moderately substitutes. The elasticity of substitution between private and public capital is assumed to be 1.12

as in Baier and Glomm (2000). We set the risk aversion parameter $\sigma$ as equals to 0.5. The labor elasticity parameter $\omega = 1.2$ is chosen to match the observed labor volatility in the US business cycle. Habit parameters in consumption and hours are set to match with the observed persistence in these variables in US business cycle data. It also helps matching with the corresponding correlation coefficient between output and these variables in the actual data.[39] The value of the labor parameter $\xi = 0.860$ is set to ensure that the steady state level of employment is 30 percent of the time endowment, given other parameters values. The long run real quarterly interest rate is set to $R^* = (1.04)^{1/4}$. Given the interest rate and the assumption of zero growth rate in equilibrium (in per-capita terms), the consistent value of the subjective discount factor $\beta$ is $1.04^{-1/4}$. Government size measured relative to output measured by $(s_y + g_y)$ is set to 20 percent, where $(s_y)$ is 15 percent. The size of the BSF at steady state is 10 percent of total government size. The rest of the parameters used in the model are summarized in Table A.11.

Both models are solved for identical long-run steady-state values, using similar parameter values. Following Campbell (1994), we log linearized and approximated each model around their identical steady state values. However, the two models provide different off-steady state dynamics. Needless to say, we are particularly interested in understanding these differences to gauge the role of the BSF in the state economy. As we mentioned earlier, the aggregate level resource constraints are identical in both models. However, the budget constraints of the households and the government are different in the two models. For example, in the model

[39] Habit persistence also improves the persistence of output and leads to persistence in government revenues as well.

economy without BSF consumption smoothing requires household to choose their saving and investment plan to insure itself against future income fluctuations and the fluctuations in the government provision of public goods.

The solution to the model without the BSF is a set of stochastic processes for endogenous variables $\{y_t, c_t, i_t, k_{t+1}, D_{t+1}, G_{t+1}, g_t, s_t, T_t\}_{t=0}^{\infty}$ that satisfies the household's budget constraint (equation 3.4), private investment (equation 3.5), the transition equation for financial asset holdings (equation 3.6), the production technology (as defined in equations 3.7, 3.8 and 3.9), the government's budget constraint (equation 3.10) , the transition equation defining public capital (equation 3.11), the government's spending allocation rules (as defined in equations 3.12, 3.13 and 3.14), the household's optimal conditions (expressed in equations 3.15-3.18), given the government's instruments $\left\{\tau^y, \tau^c, m_1, m_2\right\}$, the initial conditions $\left\{K_0, G_0, D_0, c_{-1}, h_{-1}, \lambda_{-1}\right\}$, and the economy-wide cross sectional output $\bar{y}_t$.

With the BSF, the solution is a set of stochastic processes for endogenous variables $\{y'_t, c'_t, i'_t, k'_{t+1}, D'_{t+1}, G'_{t+1}, g'_t, s'_t, T'_t\}_{t=0}^{\infty}$ that satisfies the household's modified budget constraint (equation 24), the transition equation for private capital (equation 3.5), the transition equation for financial asset holding (equation 3.6), the production technology (as defined in equations 3.7, 3.8 and 3.9), the modified government's budget constraint (equation 3.20), the transition equation for public capital (equation 3.11), the government's modified spending allocation rules (equations 3.21, 3.22 and 3.23), the first order conditions obtained from the households optimization problem (equations 3.15-3.18), given the government's instruments $\left\{\tau^{y},\tau^{c},m_1,m_2\right\}$, the initial conditions $\{K_0,G_0,D_0,c_{-1},h_{-1},\lambda_{-1}\}$ , and economy-wide cross sectional output $\bar{y}_t$.

The approximated log-linearized system consists of rational difference equations that can be solved using the method of the undetermined coefficient (see Uhlig (1999) for details). In the following, we use the solutions for both models to simulate and calculate the impulse response functions to exogenous technology shocks. The detailed linearized form of the model and its solutions are appended in the Technical Note C.3.

## 3.5 Results

In this section, we report the main simulation results of the log- linearized version of the model. In addition we calculate the impulse response functions of the main macroeconomic variables to one-time technology shocks. By doing so, we can clearly compare the business cycle characteristics and the dynamics of the two models. Characterizing the dynamics of each model is important for many reasons. First, it helps budget planners to make dynamic decisions about the usage of BSFs given the observed economic fluctuations. Second, it fosters an understanding of the dynamic co-movement between the different macro aggregates. We simulate both models using identical parameter values as reported in Table A.11 and obtain the second moments that describe the main business cycle features of each economy. The importance of the BSF is also evaluated from three different perspectives. First, we examine its effect on the government budget in terms of stability, volatility, procyclicality and persistence.

Second, we evaluate the effects of the BSF on the business cycle characteristics of the state economy. Finally, we also examine the effect of the BSF on household welfare.

Figure B.9 depicts the impulse response function to a technology shock with the parameter values described in the previous section. The solid line represents the response of government expenditure in the baseline economy while the dashed line represents the economy with the BSF. On impact, the shock has no effect on equilibrium government spending. Government expenditures increase due to the positive effect of the shock on income and consumption and gradually slowdown to reach its steady state level as the shock vanishes. The response of total government expenditure in the economy with a BSF is much slower compared to the baseline economy. When a positive shock is realized the BSF administration automatically withholds part of the increase in the total government revenues in the BSF account. As a result, the increase in government spends will be proportional to the increase in the tax revenues. As the shock vanishes, the amount allocated to the BSF decreases faster than the slowdown in total revenues. This dynamic adjustment in the BSF will lead to improve the persistence of total government expenditures. However we note here that the volatility of total expenditure with a BSF is higher compared to the baseline model even though the government's current revenues with BSF are smoother than the baseline model. This is clearly evident in Figure B.10. The reason for the lower amplitude of current expenditure with the BSF is the smoother income tax revenues and the smoother consumption tax revenues due to smoother output response (see Figure B.11) and smoother consumption adjustments (see Figure B.12), given that the tax rates are unchanged.

Since the budget stabilization fund has the effect of smoothing the relative amount spent on public infrastructure during good times, we expect that output will be smoother too. However, since private capital is a substitute for public capital, the positive effect of the productivity shock on private capital is offset by the loss in public capital. This is why investment becomes more volatile with the BSF compared to the baseline model (Figure B.14). Note that the increase in private capital after the productivity shock cannot fully offset the slowdown in public investment. This is why the immediate response of the output in the economy with the BSF is slightly less than the output response in the baseline model. Figure B.13 shows that the employment adjustment is smoother in an economy with BSF. The reason behind the stability gain in employment is the improved stability in the labor market with BSF. Since household is substituting leisure for consumption (of private and public goods), the enhanced stability in consumption necessarily stabilizes the demand for leisure (the labor supply). The effect of the BSF on stabilizing labor demand is uncertain since private capital becomes more volatile (as noted above) while public capital becomes more stable. Hence, the net effect on the marginal product of labor is uncertain. However, the simulation results in Table A.12 show that the equilibrium employment becomes more stable with BSF than it is otherwise.

Table A.12 also shows that the volatility of the current government spending in an economy with a BSF is 3.1 percentage points less than in its counterpart economy without BSF. The persistence of current expenditure also improves with the BSF by 1.2 percent. Total spending in an economy with a BSF is 11.4 percentage points more volatile than current spending without a BSF.

The BSF has a significant effect on smoothing the business cycle. In particular, we find that the aggregate real income fluctuations with the BSF decrease by 2.7 percent and the persistence of output increases by 1.2 percent. We also find that the BSF yields a more stable and persistent employment process and a more volatile and less persistent private investment process. The BSF also reduces the volatility of private consumption by 1.2 percent but has no effect on the procyclicality and persistence of private consumption.

The government with a BSF decides on spending these funds according to its pre-determined priorities. However, in the baseline model, with private saving funds, household uses these funds to switch its purchasing power across time. Accordingly, deposits or the withdrawals from these funds have only income effects on household's demand. In the alternative model with the existing BSF, savings will have a substitution effect due to the changes it causes in the provision of public consumption services and public capital. The income effect in the later case will be limited by the change in the transfer payments of the State government. The reduced income effect of savings will also stabilize the labor supply. With these discrepancies between the two macroeconomies one may have concerns about the wealth effect of the BSF. However, as in Table A.12, we find interesting results. The BSF yields smoother household utility over the business cycle and reduces slightly the dependence of household's utility on the aggregate income fluctuations. In other world, the BSF results in limiting the expansion of household's welfare during the expansion period.

## 3.6 Conclusion

The recent global financial and fiscal crises have led to more interest in countercyclical fiscal policies to mitigate the macroeconomic business cycle. This is especially important in light of the perceived weaknesses of the U.S. fiscal stimulus program. One important fiscal instrument available to state governments is the BSF, which may help States' governments reducing the procyclicality in their total revenues and at the same time enhancing the countercyclicality of their aggregate expenditure and reduces the cost of the national level intervention to mitigate the aggregate business cycle of the US economy. The central question that we ask here is whether or not BSFs can mitigate the business cycles in the State economies. To answer this question, we have developed a real business cycle model in which government spending is important for three reasons; direct welfare enhancing, income distribution and productivity enhancing. Based on an optimal rule that we derived for government savings in the BSF, we find that the BSF stabilizes the government's current spending (financed by tax revenues) and improves its persistence. Total government spending becomes significantly less procyclical. More importantly, we find that the States' BSFs reduces business cycle fluctuations in general, smoothes utility and reduces its dependence on current income. Our model should serve as a micro foundation for an activist government and strongly recommends the establishment of BSFs. Given this foundation, our future research aims at examining the optimal size of the BSF for different States.

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# Appendices

# A. Tables

Table A.1: Pairwise Correlation Coefficients

| | *y* | *c* | *d* | *h* | *i* | *tby* |
|---|---|---|---|---|---|---|
| *c* | 0.82 | | | | | |
| | (0.00) | | | | | |
| *d* | **0.83** | **0.70** | | | | |
| | **(0.00)** | **(0.00)** | | | | |
| *h* | 0.88 | 0.85 | **0.79** | | | |
| | (0.00) | (0.00) | **(0.00)** | | | |
| *i* | 0.77 | 0.80 | **0.71** | 0.77 | | |
| | (0.00) | (0.00) | **(0.00)** | (0.00) | | |
| *tby* | -0.29 | -0.49 | **-0.44** | -0.41 | -0.65 | |
| | (0.00) | (0.00) | **(0.00)** | (0.00) | (0.00) | |
| *cay* | -0.07 | -0.08 | **-0.28** | -0.17 | -0.17 | 0.38 |
| | (0.49) | (0.42) | **(0.00)** | (0.06) | (0.06) | (0.00) |

Note: The sample period covers from 1981:Q1 to 2009:Q4. The data used are in real and per capita terms. They are logged (except for trade-balance-output and current-account-output ratios) and HP filtered with smoothing parameter 1600. Standard errors are reported inside the brackets.

Table A.2: Parameters Values in the Baseline Model

| Parameter | Value | Description |
|---|---|---|
| $\alpha$ | 0.32 0 | Capital share in the production |
| $\delta$ | 0.020 | Depreciation rate of capital stock |
| $\tau$ | 0.050 | Depreciation rate of durable stock |
| $\vartheta$ | 0.817 | Average utilization rate |
| $\mu-1$ | 1.500 | Depreciation elasticity to utilization |
| $\upsilon$ | 0.027 | Utilization scaling parameter |
| $\sigma$ | 0.500 | Risk aversion parameter |
| $R^*$ | $1.04^{1/4}$ | Average rate of return on world capital markets |
| $\beta$ | $1.04^{-1/4}$ | Subjective discount factor |
| $\rho$ | 1.100 | Elasticity of substitution between durable and nondurables |
| $\gamma$ | 0.803 | Nondurable share in the utility index |
| $\omega$ | 1.200 | Labor supply elasticity |
| $\xi$ | 1.849 | Labor scaling parameter |
| $\varphi_c$ | 0.820 | Habit intensity of nondurable expenditure |
| $\varphi_D$ | 0.850 | Habit intensity of durable expenditure |
| $\phi_k$ | 0.700 | Habit intensity of labor supply |
| $\beta$ | 42.00 | Capital adjustment cost parameter |
| $\phi_D$ | 17.00 | Durable stock adjustment cost parameter |
| $tby$ | 0.008 | Historical average of trade balance to output ratio |
| $dc$ | 0.237 | Historical average of durable to nondurable expenditures |
| $\psi$ | 0.800 | Elasticity of risk premium to external debt |
| $\zeta$ | 0.800 | Elasticity of risk premium to GDP |
| $p$ | 1.000 | Relative price of durable to nondurable |
| $\rho_z$ | 0.930 | AR (1) coefficient of technical progress |
| $\sigma_z$ | 0.005 | Standard deviation of technology shock |
| $\rho_\eta$ | 0.531 | AR(1) coefficient of interest rate exogenous shock |
| $\sigma_\eta$ | 0.006 | Standard deviation of interest rate shock |

Table A.3: Moments (Canadian Data and the Baseline Model)

| | Data | Baseline model | Without interest rate uncertainty |
|---|---|---|---|
| | | *Volatilities* | |
| $std(y_t)$ | 1.63 | 1.56 | 1.54 |
| $std(h_t)$ | 1.16 | 1.05 | 1.01 |
| $std(c_t)$ | 0.90 | 0.91 | 0.90 |
| $std(d_t)$ | 2.98 | 2.99 | 2.89 |
| $std(i_t)$ | 5.16 | 5.23 | 5.07 |
| $std(tby_t)$ | 0.93 | 0.68 | 0.45 |
| $std(cay_t)$ | 0.32 | 0.56 | 0.38 |
| | | *Contemporaneous Correlations with Output* | |
| $corr(h_t, y_t)$ | 0.88 | 0.90 | 0.90 |
| $corr(c_t, y_t)$ | 0.82 | 0.81 | 0.81 |
| $corr(d_t, y_t)$ | 0.83 | 0.80 | 0.87 |
| $corr(i_t, y_t)$ | 0.77 | 0.84 | 0.90 |
| $corr(tby_t, y_t)$ | -0.29 | -0.32 | -0.63 |
| $corr(cay_t, y_t)$ | -0.07 | -0.31 | -0.60 |
| | | *Serial Correlations* | |
| $corr(y_t, y_{t-1})$ | 0.91 | 0.84 | 0.84 |
| $corr(h_t, h_{t-1})$ | 0.91 | 0.93 | 0.93 |
| $corr(c_t, c_{t-1})$ | 0.84 | 0.95 | 0.95 |
| $corr(d_t, d_{t-1})$ | 0.80 | 0.69 | 0.70 |
| $corr(i_t, i_{t-1})$ | 0.89 | 0.71 | 0.73 |
| $corr(tby_t, tby_{t-1})$ | 0.67 | 0.69 | 0.61 |
| $corr(cay_t, cay_{t-1})$ | 0.45 | 0.58 | 0.59 |

Note: Data in the second column are in per capita. They were logged (except for trade-balance-output and current-account-output ratios) and HP filtered with smoothing parameter 1600 before computing the moments. Moments from all models are averages of 1,000 replications of length 200. They were computed using HP filtered % deviations from steady state. For symmetry with Canadian data, artificial data on the trade balance-output and current account to output ratios are not expressed in % deviation from steady state.

Table A.4: Moments (Model with Different Specifications)

| | Baseline model | Without habits | Without habit in nondurable | Without habit in durable | Without habit in labor supply | With fixed utilization |
|---|---|---|---|---|---|---|
| | | | *Volatilities* | | | |
| $std(y_t)$ | 1.56 | 3.12 | 1.95 | 1.48 | 1.34 | 1.23 |
| $std(h_t)$ | 1.05 | 2.63 | 1.47 | 0.98 | 0.88 | 0.83 |
| $std(c_t)$ | 0.91 | 3.73 | 2.88 | 0.87 | 0.54 | 0.76 |
| $std(d_t)$ | 2.99 | 2.64 | 1.37 | 2.94 | 2.48 | 2.65 |
| $std(i_t)$ | 5.23 | 6.03 | 2.85 | 4.96 | 4.85 | 4.20 |
| $std(tby_t)$ | 0.68 | 1.32 | 1.23 | 0.67 | 0.63 | 0.63 |
| $std(cay_t)$ | 0.56 | 1.14 | 1.20 | 0.55 | 0.57 | 0.51 |
| | | | *Contemporaneous Correlations with Output* | | | |
| $corr(h_t, y_t)$ | 0.90 | 1.00 | 0.95 | 0.88 | 0.82 | 0.88 |
| $corr(c_t, y_t)$ | 0.81 | 0.98 | 0.85 | 0.80 | 0.53 | 0.79 |
| $corr(d_t, y_t)$ | 0.80 | 0.91 | 0.90 | 0.78 | 0.82 | 0.81 |
| $corr(i_t, y_t)$ | 0.84 | 0.95 | 0.97 | 0.84 | 0.85 | 0.79 |
| $corr(tby_t, y_t)$ | -0.32 | -0.59 | -0.31 | -0.30 | -0.06 | -0.30 |
| $corr(cay_t, y_t)$ | -0.31 | -0.59 | -0.33 | -0.29 | 0.01 | -0.29 |
| | | | *Serial Correlations* | | | |
| $corr(y_t, y_{t-1})$ | 0.84 | 0.72 | 0.85 | 0.83 | 0.75 | 0.82 |
| $corr(h_t, h_{t-1})$ | 0.93 | 0.72 | 0.91 | 0.93 | 0.74 | 0.92 |
| $corr(c_t, c_{t-1})$ | 0.95 | 0.64 | 0.62 | 0.95 | 0.96 | 0.95 |
| $corr(d_t, d_{t-1})$ | 0.69 | 0.52 | 0.67 | 0.68 | 0.70 | 0.67 |
| $corr(i_t, i_{t-1})$ | 0.71 | 0.60 | 0.81 | 0.70 | 0.71 | 0.67 |
| $corr(tby_t, tby_{t-1})$ | 0.69 | 0.48 | 0.42 | 0.58 | 0.52 | 0.57 |
| $corr(cay_t, cay_{t-1})$ | 0.58 | 0.48 | 0.42 | 0.59 | 0.51 | 0.57 |

Note: See the note on Table 3.

Table A.5: Contemporaneous Correlation between ToT and Macro Variables

| | Output | Employment | Total consumption | Investment | Trade balance to output ratio |
|---|---|---|---|---|---|
| Correlation | -0.27 | -0.37 | -0.35 | -0.44 | 0.41 |
| Standard errors | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Note: The sample period covers from 1981:Q1 to 2009:Q4. The data used are in real and per capita terms. They are logged (except for trade-balance-output ratio) and HP filtered with smoothing parameter 1600. Standard errors are reported inside the brackets.

Table A.6: The Parameters' Values Used in the Baseline Model

| Parameter | value | Description |
|---|---|---|
| $\alpha$ | 0.320 | Capital Share in the Production |
| $\delta$ | 0.020 | Depreciation Rate of Capital Stock |
| $\tau_h$ | 0. 300 | Depreciation rate of Domestic goods |
| $\tau_f$ | 0.300 | Depreciation rate of Foreign goods |
| $u$ | 0.817 | Capital Utilization Rate |
| $\mu$ | 1.400 | Capita Utilization Elasticity |
| $\sigma$ | 0.200 | Curvature Parameter |
| $\beta$ | $1.040^{-1/4}$ | Subjective Discount Factor |
| $R^w$ | $1.040^{1/4}$ | Gross World Interest Rate |
| $\rho$ | 2.000 | Elasticity of Substitution Between Home and Foreign Goods |
| $\gamma$ | 0.495 | Share of Domestic good in the Consumption Index consumption |
| $\omega$ | 1.400 | Labor Supply Elasticity Parameter |
| $\xi$ | 6.745 | Labor parameter |
| $\varphi_h$ | 0.700 | Habit Intensity in Domestic Consumption |
| $\varphi_f$ | 0.700 | Habit Intensity in Domestic Consumption |
| $\upsilon$ | 0.800 | Habit Intensity/Persistence in Labor |
| $\phi_k$ | 42.00 | Capital Adjustment Cost |
| $\phi_h$ | 2.000 | Domestic Durable Stock Adjustment Cost |
| $\phi_f$ | 2.000 | Imported Durable Stock Adjustment Cost |
| $\phi_b$ | 0.030 | Portfolio Adjustment Parameter |
| $by$ | 0.350 | Debt Output Ratio |
| $d_h / d_f$ | 1.000 | Spending on Home Relative to Foreign Goods |
| $\psi$ | 0.045 | Interest Rate Elasticity to ToT |
| $\varsigma$ | 0.020 | Interest Rate Elasticity to Domestic Income |
| $\rho_z$ | 0.930 | AR(1) Coefficient of Technology Shock |
| $\sigma_z$ | 0.005 | Standard Deviation of Technology Shock |
| $\rho_p$ | 0.531 | AR(1) Coefficient of ToT Shock |
| $\sigma_p$ | 0.01677 | Standard Deviation of ToT Shock |

Table A.7: Canadian Economy versus the Model Economy

| | Canadian Economy | Model Economy |
|---|---|---|
| | *Volatilities* | |
| $std(y_t)$ | 1.63 | 1.57 |
| $std(h_t)$ | 1.22 | 1.18 |
| $std(z_t)$ | 1.16 | 1.37 |
| $std(i_t)$ | 5.16 | 5.11 |
| $std(tby_t)$ | 0.93 | 0.92 |
| $std(p_t)$ | 2.54 | 2.10 |
| | *Contemporaneous Correlations with Output* | |
| $corr(h_t, y_t)$ | 0.89 | 0.94 |
| $corr(z_t, y_t)$ | 0.88 | 0.86 |
| $corr(i_t, y_t)$ | 0.77 | 0.76 |
| $corr(tby_t, y_t)$ | -0.29 | -0.29 |
| | *Contemporaneous Correlations with ToT* | |
| $corr(y_t, p_t)$ | -0.27 | -0.27 |
| $corr(h_t, p_t)$ | -0.35 | -0.58 |
| $corr(z_t, p_t)$ | -0.37 | -0.40 |
| $corr(i_t, p_t)$ | -0.44 | -0.43 |
| $corr(tby_t, p_t)$ | 0.41 | 0.74 |
| | *Serial Correlations* | |
| $corr(y_t, y_{t-1})$ | 0.91 | 0.86 |
| $corr(h_t, h_{t-1})$ | 0.86 | 0.85 |
| $corr(z_t, z_{t-1})$ | 0.91 | 0.95 |
| $corr(i_t, i_{t-1})$ | 0.89 | 0.67 |
| $corr(tby_t, tby_{t-1})$ | 0.67 | 0.63 |
| $corr(p_t, p_{t-1})$ | 0.81 | 0.72 |

Note:

1. Data in the second column are in per capita. They were logged (except for trade-balance-output and current-account-output ratios) and HP filtered with smoothing parameter 1600 before computing the moments. Moments from all models are averages of 1,000 replications of length 200. They were computed using HP filtered % deviations from steady state. For symmetry with Canadian data, artificial data on the trade balance-output and current account to output ratios are not expressed in % deviation from steady state.

2. For the sake of symmetry $\tilde{z}_t$ in the bench mark model represents the percentage deviation from the hp trend of total consumption expenditure on goods and services including: 1 spending on durables, 2 spending on nondurables and 3 spending on semi durables.

Table A.8: The Role of ToT Uncertainty

| | Economy with Lower Durability | | Economy with Higher Durability | |
|---|---|---|---|---|
| | With ToT | Without ToT | With ToT | Without ToT |
| | | *Volatilities* | | |
| $std(y_t)$ | 2.64 | 1.98 | 1.57 | 1.34 |
| $std(h_t)$ | 2.41 | 1.40 | 1.18 | 0.74 |
| $std(z_t)$ | 3.21 | 2.02 | 1.37 | 0.94 |
| $std(i_t)$ | 6.40 | 4.85 | 5.11 | 4.56 |
| $std(tby_t)$ | 1.31 | 0.74 | 0.92 | 0.50 |
| $std(p_t)$ | 2.10 | | 2.10 | |
| | | *Contemporaneous Correlations with Output* | | |
| $corr(h_t, y_t)$ | 0.96 | 0.96 | 0.89 | 1.00 |
| $corr(z_t, y_t)$ | 0.97 | 0.99 | 0.95 | 0.85 |
| $corr(i_t, y_t)$ | 0.68 | 0.85 | 0.69 | 0.90 |
| $corr(tby_t, y_t)$ | -0.55 | -0.63 | -0.44 | -0.57 |
| | | *Contemporaneous Correlations with ToT* | | |
| $corr(y_t, p_t)$ | -0.53 | | -0.39 | |
| $corr(h_t, p_t)$ | -0.65 | | -0.54 | |
| $corr(z_t, p_t)$ | -0.60 | | -0.47 | |
| $corr(i_t, p_t)$ | -0.58 | | -0.56 | |
| $corr(tby_t, p_t)$ | 0.71 | | 0.75 | |
| | | *Serial Correlations* | | |
| $corr(y_t, y_{t-1})$ | 0.88 | 0.86 | 0.87 | 0.82 |
| $corr(h_t, h_{t-1})$ | 0.92 | 0.92 | 0.94 | 0.81 |
| $corr(z_t, z_{t-1})$ | 0.91 | 0.89 | 0.91 | 0.94 |
| $corr(i_t, i_{t-1})$ | 0.65 | 0.67 | 0.67 | 0.69 |
| $corr(tby_t, tby_{t-1})$ | 0.83 | 0.79 | 0.74 | 0.60 |
| $corr(p_t, p_{t-1})$ | 0.72 | | 0.72 | |

Note: Moments from all models are averages of 1,000 replications of length 200. They were computed using HP filtered % deviations from steady state.

Table A.9: The Role of Durability

| | Higher Durability in Both Goods | No Durability in Both Goods | Durability in Domestic Goods | Durab in Foreign Goods |
|---|---|---|---|---|
| | *Volatilities* | | | |
| $std(y_t)$ | 1.57 | 2.64 | 1.77 | 1.90 |
| $std(h_t)$ | 1.18 | 2.41 | 1.43 | 1.50 |
| $std(z_t)$ | 1.37 | 3.21 | 1.69 | 1.84 |
| $std(i_t)$ | 5.11 | 6.40 | 6.05 | 5.41 |
| $std(tby_t)$ | 0.92 | 1.31 | 0.88 | 0.98 |
| $std(p_t)$ | 2.10 | 2.10 | 2.10 | 2.10 |
| | *Contemporaneous Correlations with Output* | | | |
| $corr(h_t, y_t)$ | 0.86 | 0.96 | 0.89 | 0.92 |
| $corr(z_t, y_t)$ | 0.94 | 0.97 | 0.95 | 0.97 |
| $corr(i_t, y_t)$ | 0.76 | 0.68 | 0.69 | 0.73 |
| $corr(tby_t, y_t)$ | -0.29 | -0.55 | -0.44 | -0.36 |
| | *Contemporaneous Correlations with ToT* | | | |
| $corr(y_t, p_t)$ | -0.27 | -0.53 | -0.39 | -0.27 |
| $corr(h_t, p_t)$ | -0.40 | -0.65 | -0.54 | -0.38 |
| $corr(z_t, p_t)$ | -0.58 | -0.60 | -0.47 | -0.47 |
| $corr(i_t, p_t)$ | -0.43 | -0.58 | -0.56 | -0.46 |
| $corr(tby_t, p_t)$ | 0.74 | 0.71 | 0.75 | 0.72 |
| | *Serial Correlations* | | | |
| $corr(y_t, y_{t-1})$ | 0.86 | 0.88 | 0.87 | 0.87 |
| $corr(h_t, h_{t-1})$ | 0.95 | 0.92 | 0.94 | 0.94 |
| $corr(z_t, z_{t-1})$ | 0.85 | 0.91 | 0.91 | 0.89 |
| $corr(i_t, i_{t-1})$ | 0.67 | 0.65 | 0.67 | 0.67 |
| $corr(tby_t, tby_{t-1})$ | 0.63 | 0.83 | 0.74 | 0.69 |
| $corr(p_t, p_{t-1})$ | 0.72 | 0.72 | 0.72 | 0.72 |

Note: See the comments on Table 3

Table A.10: Persistence of the ToT and the Business Cycle

| | $\rho_p = 0.963$ | $\rho_p = 0.98$ | $\rho_p = 0.90$ |
|---|---|---|---|
| | *Volatilities* | | |
| $std(y_t)$ | 2.64 | 2.61 | 2.52 |
| $std(h_t)$ | 2.41 | 2.35 | 2.27 |
| $std(z_t)$ | 3.21 | 2.94 | 3.25 |
| $std(i_t)$ | 6.40 | 8.52 | 6.04 |
| $std(tby_t)$ | 1.31 | 1.71 | 1.29 |
| $std(p_t)$ | 2.10 | 2.10 | 2.07 |
| | *Contemporaneous Correlations with Output* | | |
| $corr(h_t, y_t)$ | 0.96 | 0.96 | 0.95 |
| $corr(z_t, y_t)$ | 0.97 | 0.97 | 0.96 |
| $corr(i_t, y_t)$ | 0.68 | 0.61 | 0.46 |
| $corr(tby_t, y_t)$ | -0.55 | -0.44 | -0.4 |
| | *Contemporaneous Correlations with ToT* | | |
| $corr(y_t, p_t)$ | -0.53 | -0.52 | -0.48 |
| $corr(h_t, p_t)$ | -0.65 | -0.64 | -0.6 |
| $corr(z_t, p_t)$ | -0.60 | -0.56 | -0.54 |
| $corr(i_t, p_t)$ | -0.58 | -0.75 | 0.46 |
| $corr(tby_t, p_t)$ | 0.71 | 0.76 | -0.31 |
| | *Serial Correlations* | | |
| $corr(y_t, y_{t-1})$ | 0.88 | 0.88 | 0.88 |
| $corr(h_t, h_{t-1})$ | 0.92 | 0.92 | 0.92 |
| $corr(z_t, z_{t-1})$ | 0.91 | 0.9 | 0.91 |
| $corr(i_t, i_{t-1})$ | 0.65 | 0.63 | 0.59 |
| $corr(tby_t, tby_{t-1})$ | 0.83 | 0.67 | 0.50 |
| $corr(p_t, p_{t-1})$ | 0.72 | 0.72 | 0.69 |

Note :

1. See the comments on Table 3.

2. In all model economies the degree of durability is zero in production and imported goods. Accordingly differences in the business cycles across the theoretical economies are due to different degrees of persistence of ToT persistence.

Table A.11: Parameters Values in the Baseline Model

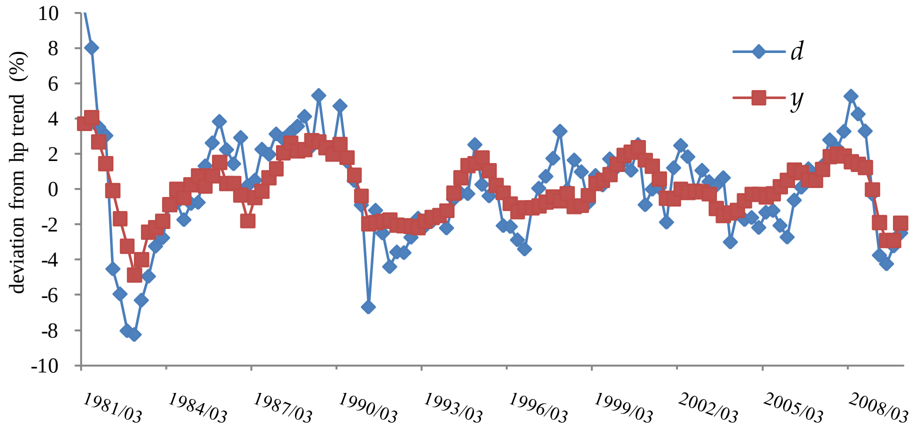
| Parameter | Value | Description |
|---|---|---|
| $\alpha$ | 0.340 | Capital share in the production |
| $\delta_k$ | 0.015 | Depreciation rate of private capital stock |
| $\delta_q$ | 0.025 | Depreciation rate of public capital stock |
| $\rho$ | 1.100 | Elasticity of substitution between private and public consumption |
| $\varepsilon$ | 1.120 | Elasticity of substitution between private and public capital |
| $\sigma$ | 0.500 | Risk aversion parameter |
| $h$ | 0.300 | Steady state hours worked |
| $\omega$ | 1.200 | Labor supply elasticity |
| $\xi$ | 0.860 | Labor scaling parameter |
| $R^*$ | $1.04^{1/4}$ | Long run average gross interest rate |
| $\beta$ | $1.04^{-1/4}$ | Subjective discount factor |
| $\varphi_c$ | 0.840 | Habit intensity of nondurable expenditure |
| $\varphi_h$ | 0.700 | Habit intensity of labor supply |
| $\gamma$ | 0.100 | Share parameter in the utility |
| $\phi$ | 0.193 | Share parameter in the production function |
| $\phi_k$ | 25.00 | Capital adjustment cost parameter |
| $\phi_D$ | 0.350 | Portfolio adjustment cost parameter |
| $\zeta$ | 0. 010 | Elasticity of risk premium to GDP |
| $\tau^y$ | 0.180 | Average income tax rate |
| $\tau^c$ | 0.090 | Average consumption tax rate |
| $m_1$ | 0.209 | Share of gvt productive spending in total gvt revenue |
| $m_2$ | 0.628 | Share of gvt spending on consumption services in total gvt revenue |
| $s_y$ | 0.150 | Gvt consumption services to output ratio |
| $g_y$ | 0.050 | Gvt investment in public capital to output ratio |
| $T_y$ | 0.039 | Gvt transfer payment to output ratio |
| $D_y$ | $0.1*(gy+sy)$ | BSF size is 10 percent of government size |
| $\rho_z$ | 0.950 | AR (1) coefficient of technical progress |
| $\sigma_z$ | 0.007 | Standard deviation of technology shock |

Table A.12: Moments Comparison (With and Without BSF)

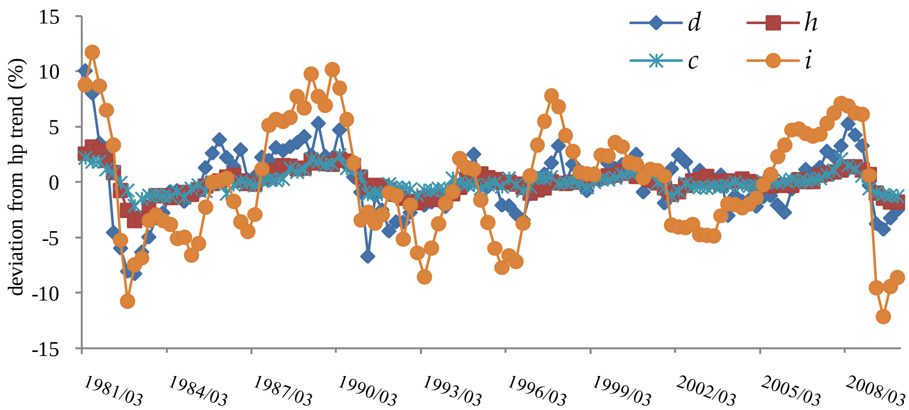
| | Baseline Economy | BSF Economy |
|---|---|---|
| | *Volatilities* | |
| $std(y_t)$ | 1.431 | 1.392 |
| $std(h_t)$ | 0.894 | 0.877 |
| $std(c_t)$ | 0.870 | 0.860 |
| $std(texp_t)$ | 1.263 | 1.377 |
| $std(cexp_t)$ | 1.263 | 1.232 |
| $std(i_t)$ | 4.773 | 5.012 |
| $std(utility_t)$ | 1.128 | 1.114 |
| | *Contemporaneous Correlations with Output* | |
| $corr(h_t, y_t)$ | 0.93 | 0.90 |
| $corr(c_t, y_t)$ | 0.84 | 0.84 |
| $corr(texp_t, y_t)$ | 1.00 | 0.78 |
| $corr(cexp_t, y_t)$ | 1.00 | 1.00 |
| $corr(i_t, y_t)$ | 0.94 | 0.91 |
| $corr(utility_t, y_t)$ | 0.92 | 0.91 |
| | *Serial Correlations* | |
| $corr(y_t, y_{t-1})$ | 0.81 | 0.82 |
| $corr(h_t, h_{t-1})$ | 0.91 | 0.93 |
| $corr(c_t, c_{t-1})$ | 0.94 | 0.94 |
| $corr(texp_t, texp_{t-1})$ | 0.83 | 0.90 |
| $corr(cexp_t, cexp_{t-1})$ | 0.83 | 0.84 |
| $corr(i_t, i_{t-1})$ | 0.74 | 0.72 |
| $corr(utility_t, utility_{t-1})$ | 0.72 | 0.72 |

Note: See the comments on Table 3.

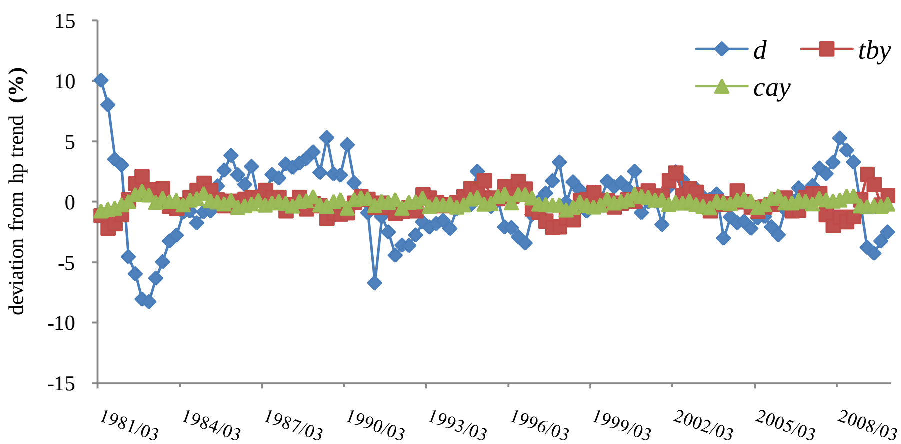
# B. Figures




a. Durable spending and output




b. Durable spending, labor, nondurable consumption and investment




c. Durable spending and external accounts

Figure B.1: Comovement between durable spending and national accounts (actual data)

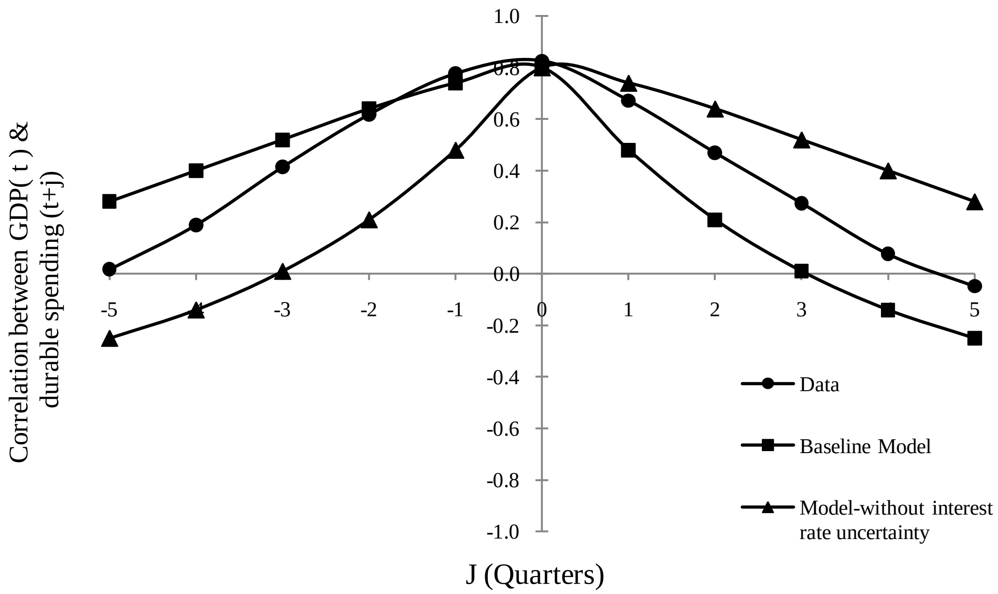



Figure B.2: Cross correlation between GDP and durable spending

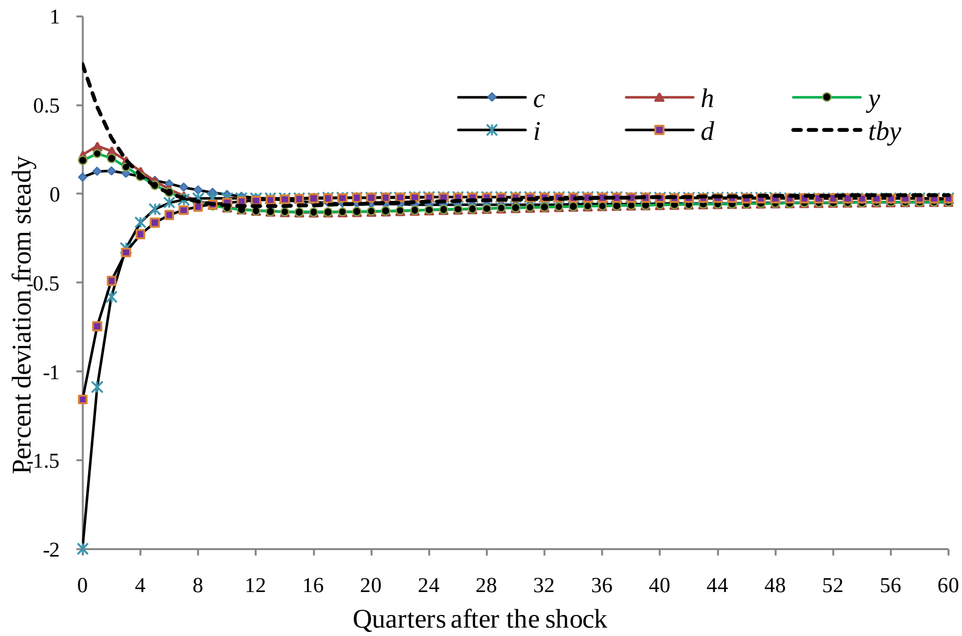
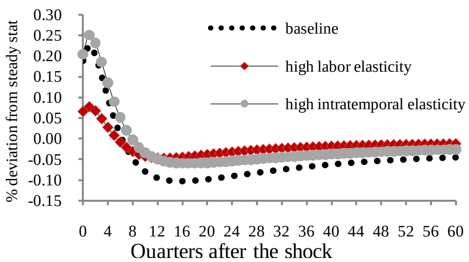
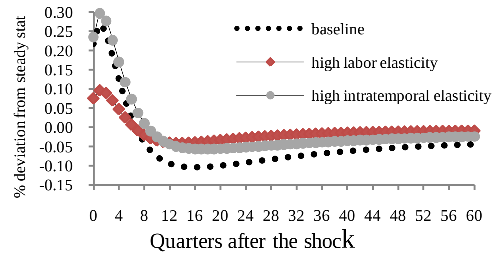


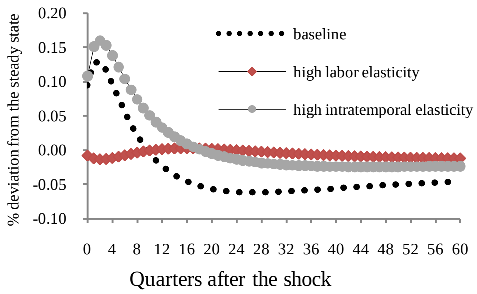

Figure B.3: Impulse responses of national accounts to an interest rate shock (baseline model)



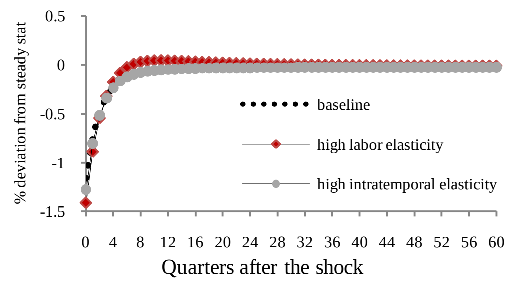

a. Output



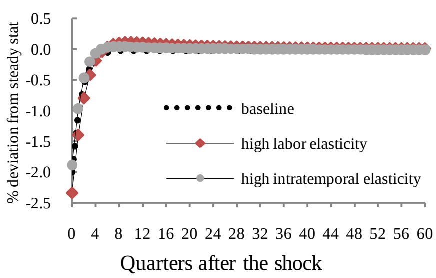

b. Employment



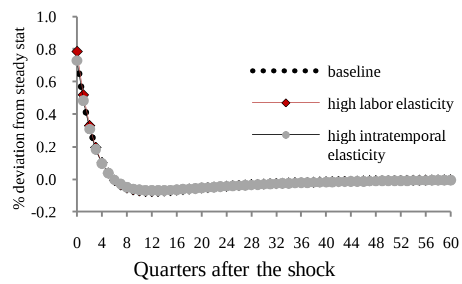

c. Nondurable consumption



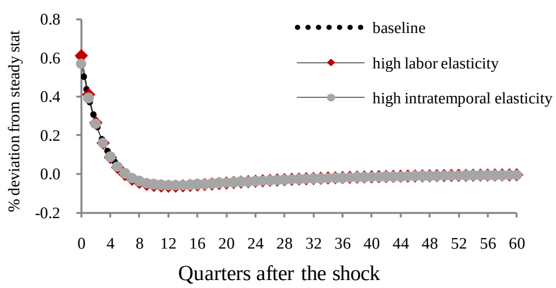

d. Expenditure on durables




e. Investment spending

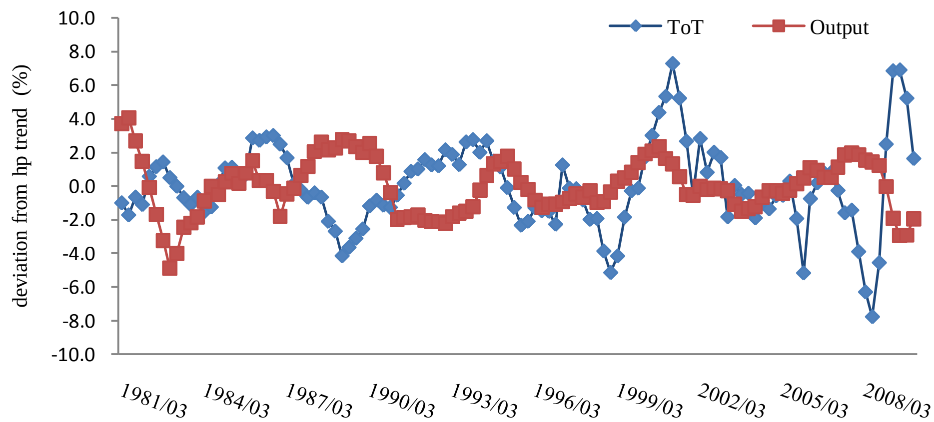



f. Trade balance to output ratio

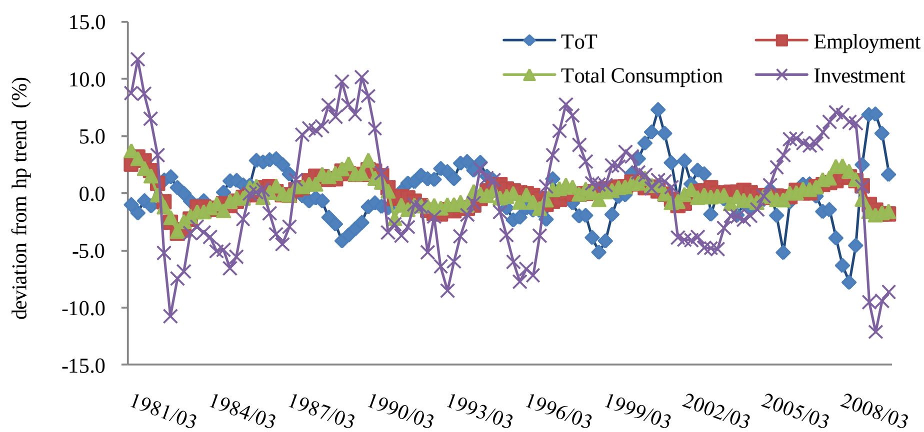



g. Current account to output ratio

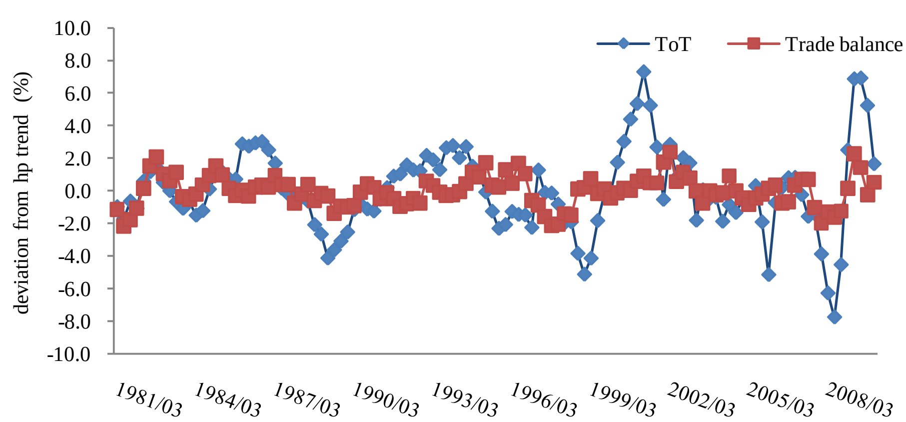
Figure B.4: Impulse responces of national accounts to an interest rate shock (alternative scenarios)




a: ToT and output




b: ToT, employment, aggregate consumption and investment




c: ToT and the trade balance

Figure B.5: Comovement between terms of trade and national accounts (actual data)

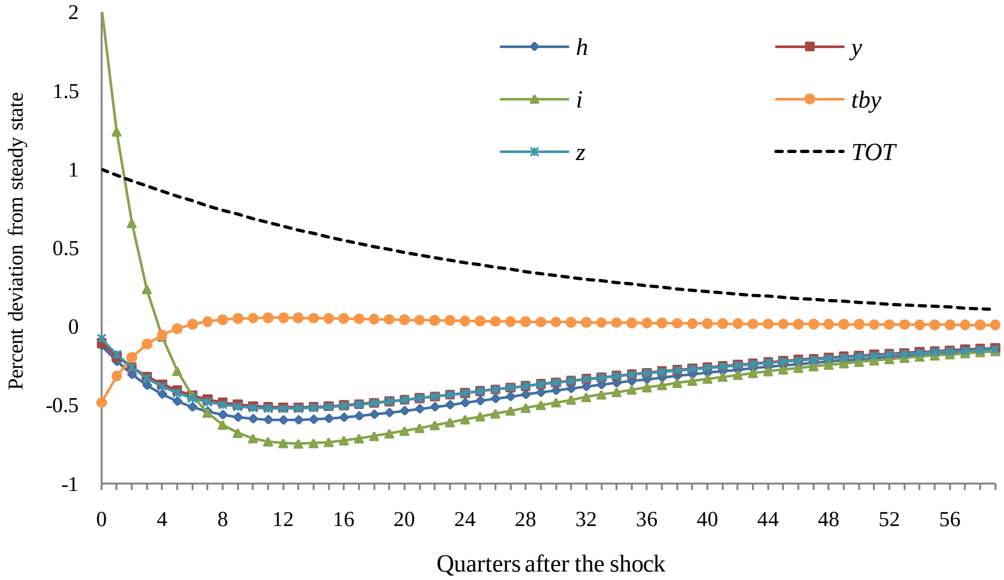



Figure B.6: Impulse responses due to a ToT Shock with constant interest rate ($\psi = \varsigma = 0$)

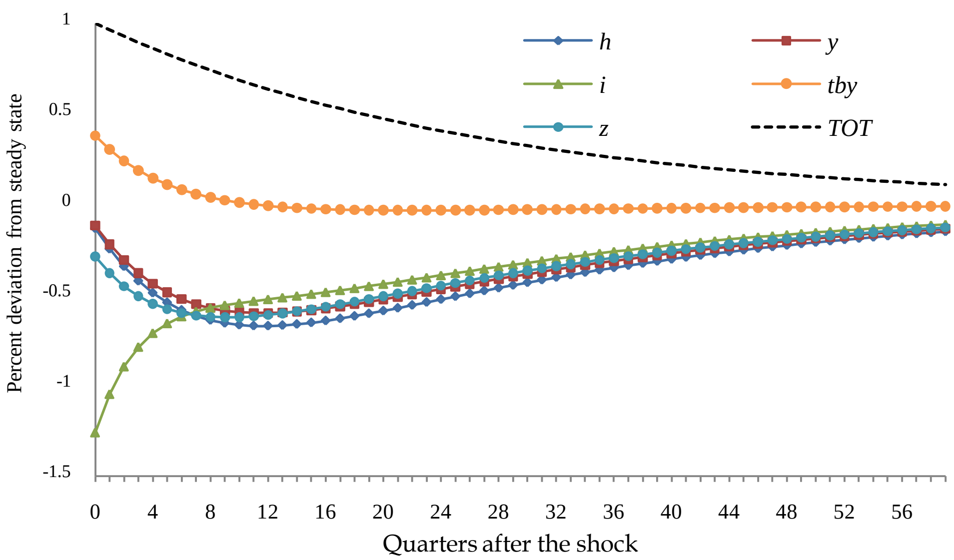



Figure B.7: Impulse responses due to a ToT shock (Baseline Model)

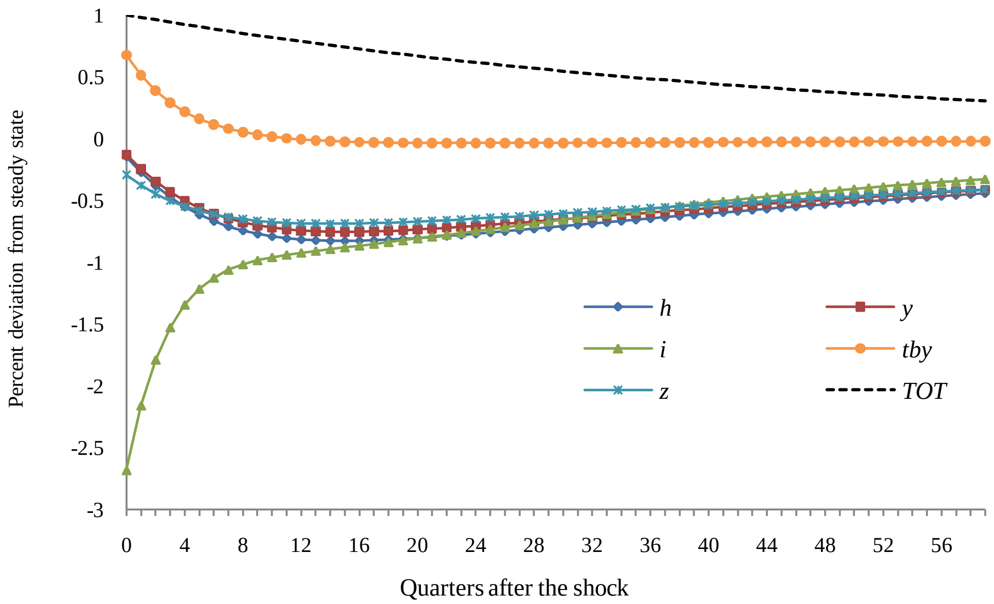



Figure B.8: Impulse responses due to a high-persistent ToT Shock $\left(\rho_p = 0.98\right)$

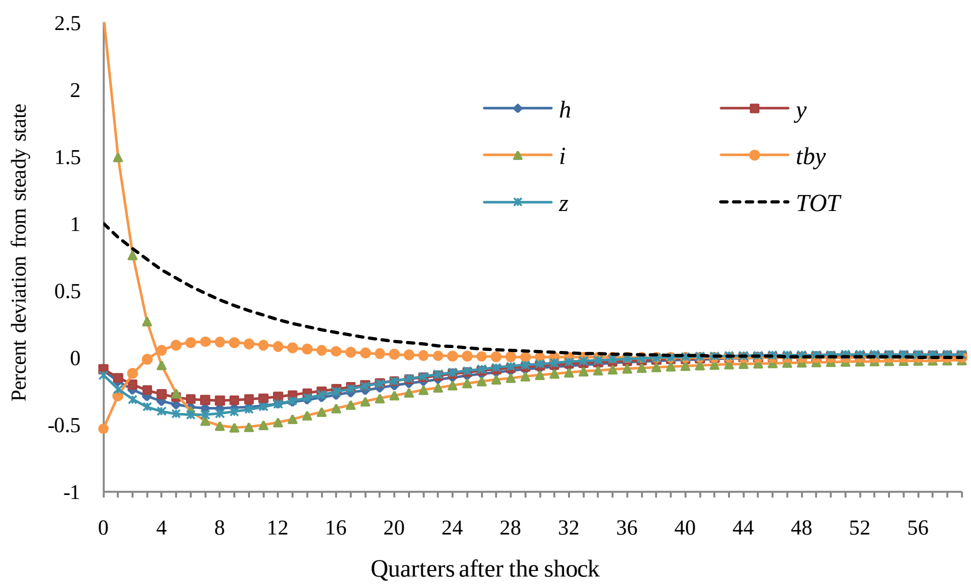



Figure B.9: Impulse responses due to a moderately-persistent ToT Shock $\left(\rho_p = 0.90\right)$

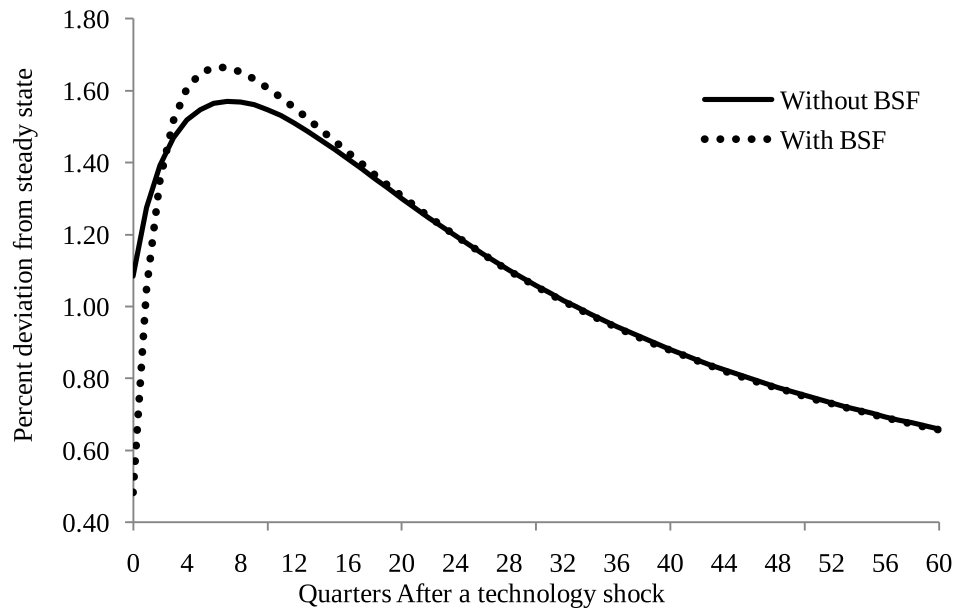



Figure B.10: Impulse responses of total government expenditure (expenditures) to a technology shock

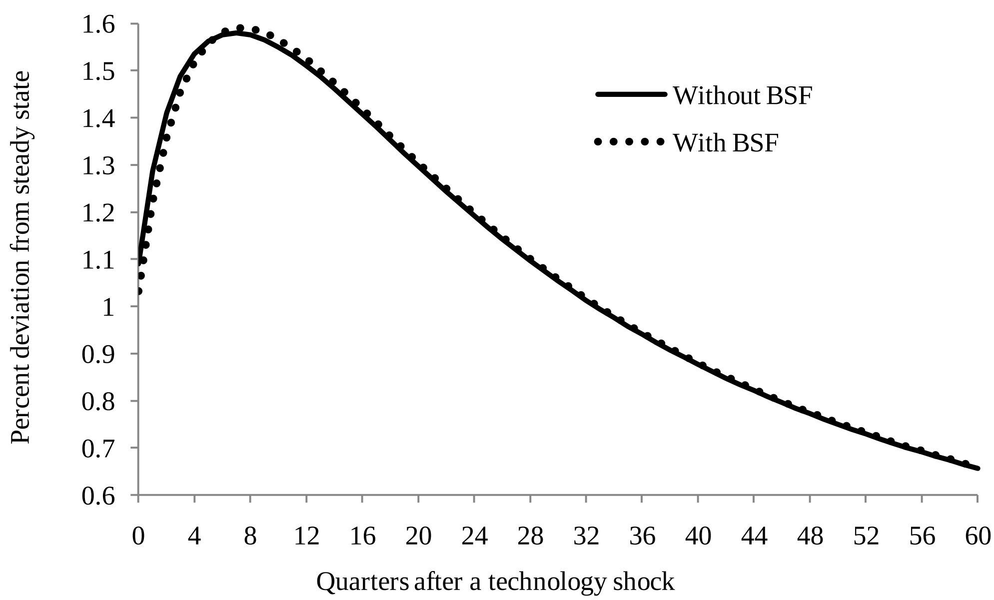



Figure B.11: Impulse responses of current government expenditure to a technology shock

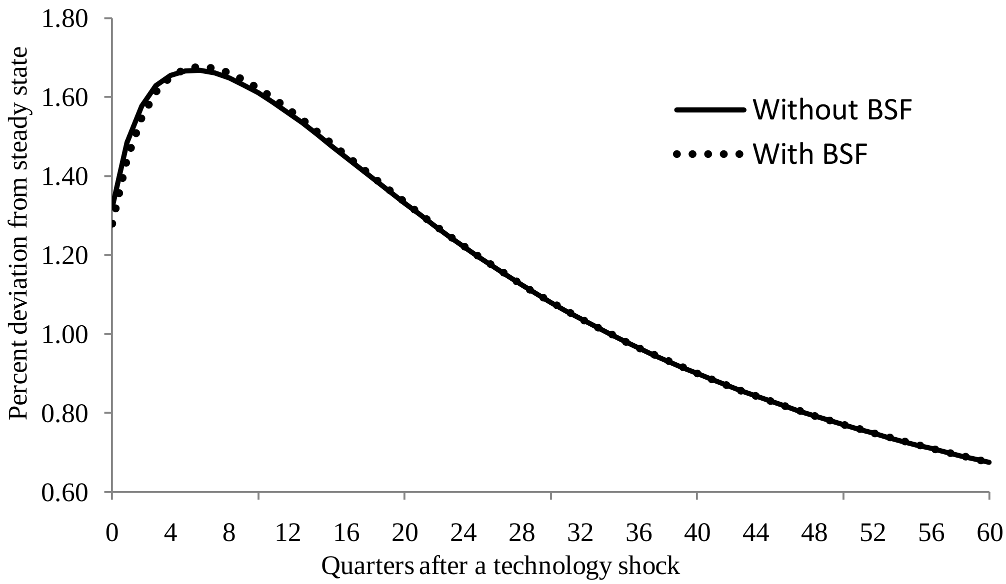



Figure B.12: Impulse responses of output to a technology shock

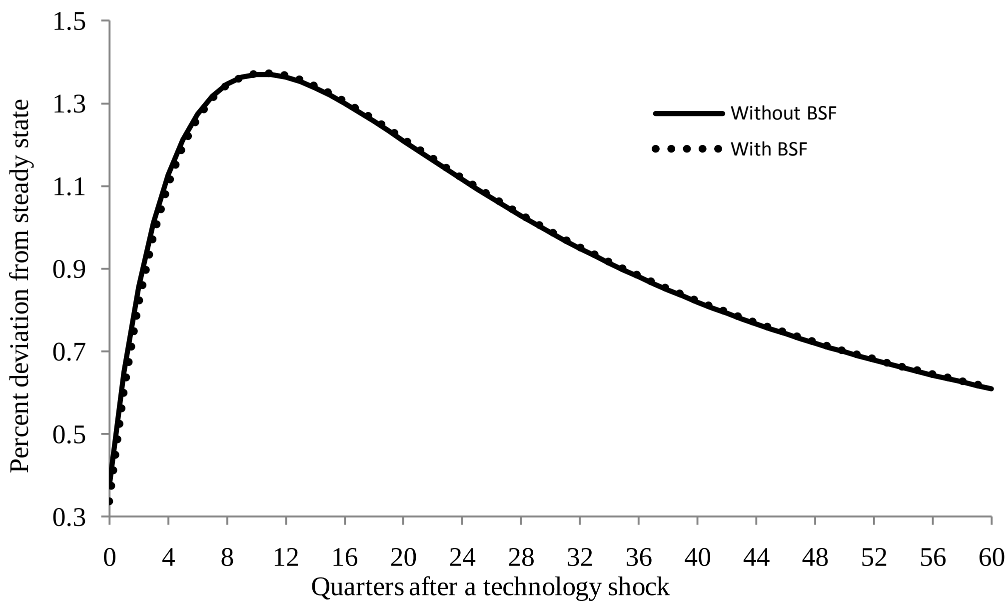



Figure B.13: Impulse responses of private consumption to a technology shock

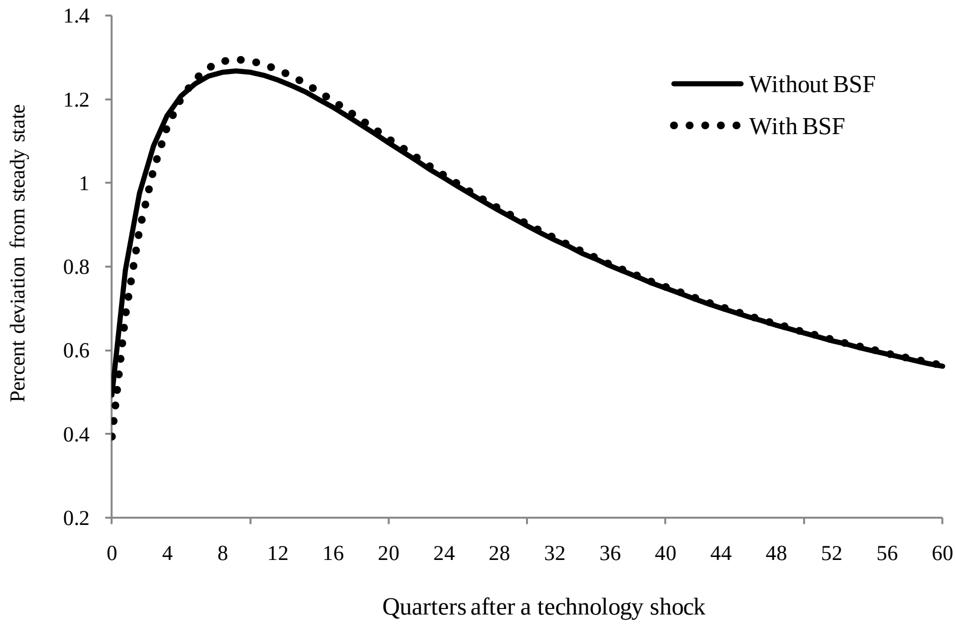



Figure B.14: Impulse responses of employment to a technology shock

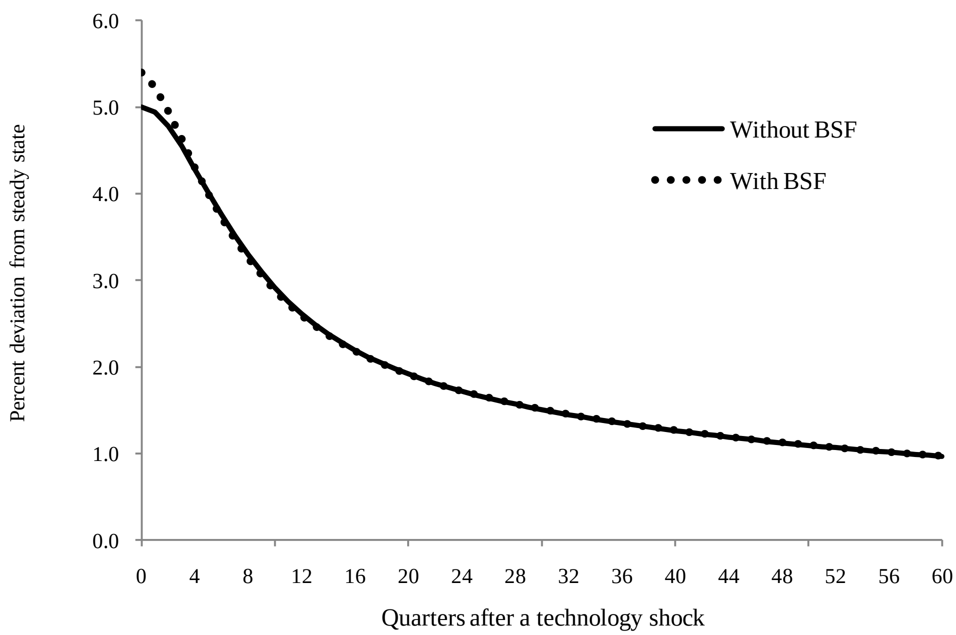



Figure B.15: Impulse responses of private investment to a technology shock

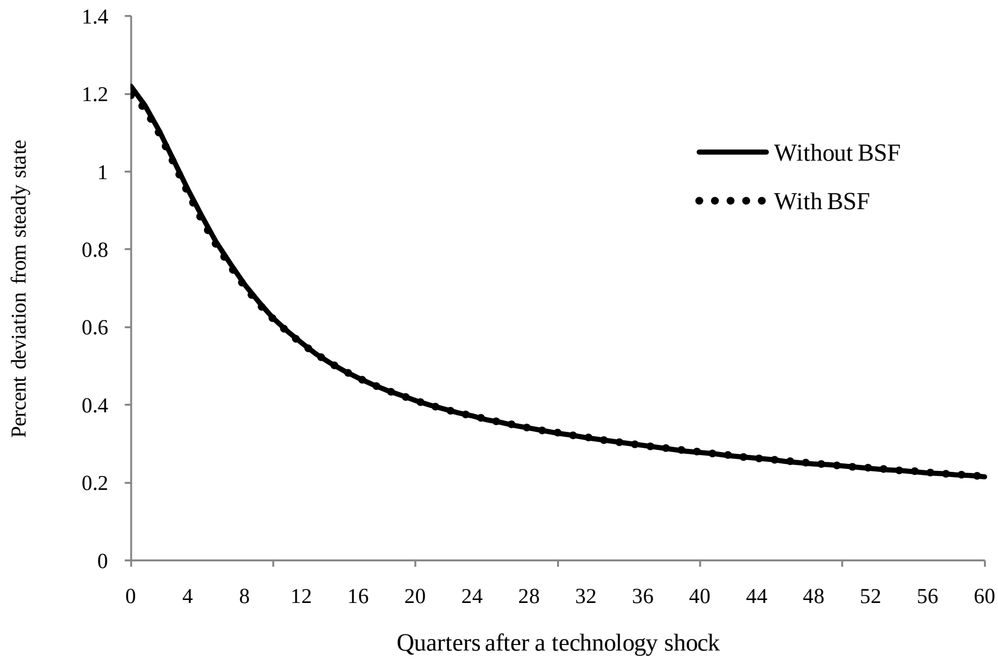



Figure B.16: Impulse responses of household welfare to a technology shock

# C. Technical Notes

# C.1: Solutions of the Dynamic Model in Essay 1

To study the dynamic effects of external shocks we log-linearize the model around the initial steady state. The full dynamic model is, thus, expressed in terms of percentage deviations from the steady state values. In particular, a variable $x_t$ with steady state value $\bar{x}$ is expressed as $\tilde{x}$ where $\tilde{x}_t = \log(x_t) - \log(\bar{x})$. However, as trade balance and current account may take negative values we employ simple linearization for these two variables: $\tilde{x}_t = x_t - \bar{x}$. This is a very standard approach in the literature. For details see Campbell (1994).

There are 16 equations in our model that control the dynamics of the economy. These are numbered as equations (1.4-1.11), (1.13-1.18) in the main text along with two equations defining trade and current accounts (not numbered). We linearized them one by one. Accordingly, we rewrite the linearized version of the model

$$-\frac{b}{R^f}\tilde{b}_{t+1} + \frac{tb}{R^f}\tilde{r}_t + R^f b\tilde{b}_t - y\tilde{y}_t + I.\tilde{I}_t + c\tilde{c}_t + d\tilde{d}_t = 0 \tag{C.1.4}$$

$$\tilde{D}_t - (1-\tau)\tilde{D}_{t-1} - \tau\tilde{d}_t = 0 \tag{C.1.5}$$

$$-\psi y\tilde{y}_t + \psi b\tilde{b}_{t+1} + \tilde{\mu}_t - r\tilde{r}_t = 0 \tag{C.1.6}$$

$$\tilde{\eta}_t = \rho_\eta \tilde{\eta}_t + \varepsilon_t^\eta \tag{C.1.7}$$

$$\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t + \delta.\tilde{\delta}_t - \delta.\tilde{I}_t = 0 \tag{C.1.8}$$

$$\mu.\tilde{u}_t - \tilde{\delta}_t = 0 \tag{C.1.9}$$

$$z_t + \alpha.\tilde{u}_t + \alpha.\tilde{k}_t + (1-\alpha)\tilde{h}_t - \tilde{y}_t = 0 \tag{C.1.10}$$

$$z_t = \rho_z z_t + \varepsilon_t^z \tag{C.1.11}$$

$$E_t\Big[BC_1\tilde{c}_t + BC_2\tilde{c}_{t-1} + BC_3\tilde{c}_{t+1} + BC_4\tilde{D}_t + BC_5\tilde{D}_{t-1} + BC_6\tilde{D}_{t+1} + BC_7\tilde{h}_t + BC_8\tilde{h}_{t-1} + BC_9\tilde{h}_{t+1} - \beta\varphi_c\tilde{\beta}_t - (1-\beta\varphi_c)\tilde{\lambda}_t\Big] = 0 \tag{C.1.13}$$

$$E_t\Big[BE_1\tilde{c}_t + BE_2\tilde{c}_{t-1} + BE_3\tilde{c}_{t+1} + BE_4\tilde{D}_t + BE_5\tilde{D}_{t-1} + BE_6\tilde{D}_{t+1} + BE_7\tilde{h}_t + BE_8\tilde{h}_{t-1} + BE_9\tilde{h}_{t+1} + BE_{11}\tilde{y}_t + BE_{11}\tilde{\lambda}_t\Big] = 0 \tag{C.1.14}$$

$$E_t\Big[BD_1\tilde{c}_t + BD_2\tilde{c}_{t-1} + BD_3\tilde{c}_{t+1} + BD_4\tilde{D}_t + BD_5\tilde{D}_{t-1} + BD_6\tilde{D}_{t+1} + BD_7\tilde{h}_t + BD_8\tilde{h}_{t-1} + BD_9\tilde{h}_{t+1} + BD_{11}\tilde{\lambda}_{t+1} - E_1\tilde{\lambda}_t\Big] = 0 \tag{C.1.15}$$

$$E_t\Big[(\alpha\beta y/k)\tilde{y}_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \beta\delta\tilde{\delta}_{t+1} + \varphi_k\beta\tilde{k}_{t+2} + E_2\tilde{k}_{t+1} + \varphi_k\tilde{k}_t\Big] = 0 \tag{C.1.16}$$

$$E_t\Big[(1-1/R)\tilde{r}_t - \tilde{\lambda}_t + E_t\tilde{\lambda}_{t+1}\Big] = 0 \tag{C.1.17}$$

$$\tilde{y}_t - \tilde{k}_t - \tilde{\delta}_t = 0 \tag{C.1.18}$$

Equations involving the trade balance and the current account (defined in the main text without numbers) are

$$-tby_t + (1-tby)\tilde{y}_t - \frac{c}{y}\tilde{c}_t - \frac{I}{y}\tilde{I}_t - \frac{d}{y}\tilde{d}_t = 0 \tag{C.1.19}$$

$$-cay_t + \tilde{y}_t - \frac{c}{y}\tilde{c}_t - \frac{I}{y}\tilde{I}_t - P\frac{d}{y}\tilde{d}_t - \frac{tby}{1+r}\tilde{r}_t - tby\tilde{b}_{t+1} = 0 \tag{C.1.20}$$

This is a 16X16 system of equations. Some of the coefficients used above are defined in terms of model parameters as follows:

$$BC1 = -\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\,\sigma\, x^{\frac{\rho}{\rho-1}}\,(1+\beta\varphi c^2)}{x\left(x^{\frac{\rho}{\rho-1}} - \xi\,(h-\varphi h\,h)^{\omega}\right)(1-\varphi c)}$$

$$BC2 = \frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\,\sigma\, x^{\frac{\rho}{\rho-1}}\,\varphi c}{x\left(x^{\frac{\rho}{\rho-1}} - \xi\,(h-\varphi h\,h)^{\omega}\right)(1-\varphi c)}$$

$$BC3 := \frac{\beta\,\varphi c\,\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\,\sigma\, x^{\frac{\rho}{\rho-1}}}{x\left(x^{\frac{\rho}{\rho-1}} - \xi\,(h-\varphi h\,h)^{\omega}\right)(1-\varphi c)}$$

$$BC4 = \frac{1}{\rho\,(1-\varphi D)}\left(\left(1-\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)(1+\beta\,\varphi c\,\varphi D)\right)$$

$$BC5 = -\frac{1}{\rho\,(1-\varphi D)}\left(\left(1-\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)\varphi D\right)$$

$$BD1 = \frac{1}{x\,\rho\,(1-\varphi c)}\left(\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)(1+\beta\,\varphi c\,\varphi D)\right)$$

$$BD2 = -\frac{\gamma\,((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)\varphi c}{x\,\rho\,(1-\varphi c)}$$

$$BD3 = -\frac{\beta\,\varphi D\,\gamma\,((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)}{x\,\rho\,(1-\varphi c)}$$

$$BD4 = \frac{1}{\rho\,(1-\varphi h)}\left(\left(\left(1-\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)-1\right)(1+\beta\,\varphi D^{2})\right)-\frac{(1-\beta\,\varphi D)\,\varphi D\,(1+\beta)}{1-\beta\,(1-\tau)}$$

$$BD5 := \frac{(1-\beta\,\varphi D)\,\varphi D}{1-\beta\,(1-\tau)}-\frac{1}{\rho\,(1-\varphi h)}\left(\left(\left(1-\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)-1\right)\varphi D\right)$$

$$BD6 = \beta\left(\frac{(1-\beta\,\varphi D)\,\varphi c}{1-\beta\,(1-\tau)}-\frac{1}{\rho\,(1-\varphi h)}\left(\left(\left(1-\frac{\gamma((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)-1\right)\varphi D\right)\right)$$

$$BD7 = -\frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)\omega\,(1+\beta\,\varphi D\,\varphi h)}{1-\varphi h}$$

$$BD8 = \frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)\omega\,\varphi h}{1-\varphi h}$$

$$BD9 = \frac{\beta\,\varphi D\,\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\,(h-\varphi h\,h)^{\omega}}\right)\omega}{1-\varphi h}$$

$$BD11 = \frac{\beta\left(1-\beta\varphi D\right)\left(1-\tau\right)}{1-\beta\left(1-\tau\right)}$$

$$BE1 = -\frac{\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}\left(\beta\varphi c\,\varphi h+1\right)}{x\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi c\right)}$$

$$BE2 = \frac{\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}\varphi c}{x\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi c\right)}$$

$$BE3 = \frac{\beta\varphi h\,\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}}{x\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi c\right)}$$

$$BE4 = -\frac{\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1+\beta\varphi D\,\varphi h\right)}{\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi D\right)}$$

$$BE5 = \frac{\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}}{x}\right)\varphi D}{\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi D\right)}$$

$$BE6 = \frac{\beta\varphi h\,\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma\left(\left(1-\varphi c\right)c\right)^{\frac{\rho-1}{\rho}}}{x}\right)}{\left(x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}\right)\left(1-\varphi D\right)}$$

$$BE8 = -\frac{\left(\left(1-\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}}\right)\right)\omega-1\right)\varphi h}{1-\varphi h}$$

$$BE9 = -\frac{\beta\varphi h\left(\left(1-\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\varphi h\,h\right)^{\omega}}\right)\right)\omega-1\right)}{1-\varphi h}$$

$$BE11 = -1+\beta\varphi h$$

$$E1 = \frac{1-\beta\varphi D}{1-\beta\left(1-\tau\right)}$$

$$E2 = -\beta\alpha yk-\phi k\left(1+\beta\right)$$

Where $x=\gamma\left(1-\varphi c\right)^{\frac{\rho-1}{\rho}}c^{\frac{\rho-1}{\rho}}+\left(1-\gamma\right)\left(1-\varphi D\right)^{\frac{\rho-1}{\rho}}\mathrm{D}^{\frac{\rho-1}{\rho}}$.

Note that the above system consists of three types of variables:

The state variables: $X_t \equiv \left[\tilde{k}_{t+1} \quad b_{t+1} \quad \tilde{D}_{t+1} \quad \tilde{c}_t \quad \tilde{h}_t \quad \tilde{\lambda}_t\right]'$

Other endogenous variables: $Y_t \equiv \left[\tilde{y}_t \quad \tilde{i}_t \quad \tilde{d}_t \quad tby_t \quad cay_t \quad \tilde{\delta}_t \quad \tilde{u}_t \quad \tilde{r}_t\right]'$

The stochastic variables: $Z_t \equiv \left[z_t \quad \eta_t\right]'$

Moreover, for simplification, we rearrange the system of equations into three distinct blocks:

**Block 1: Forward looking rational equations**

$$E_t\left[BC_1\tilde{c}_t + BC_2\tilde{c}_{t-1} + BC_3\tilde{c}_{t+1} + BC_4\tilde{D}_t + BC_5\tilde{D}_{t-1} + BC_6\tilde{D}_{t+1} + BC_7\tilde{h}_t + BC_8\tilde{h}_{t-1} + BC_9\tilde{h}_{t+1} - \beta\varphi_c\tilde{\beta}_t - (1-\beta\varphi_c)\tilde{\lambda}_t\right] = 0$$

$$E_t\left[BE_1\tilde{c}_t + BE_2\tilde{c}_{t-1} + BE_3\tilde{c}_{t+1} + BE_4\tilde{D}_t + BE_5\tilde{D}_{t-1} + BE_6\tilde{D}_{t+1} + BE_7\tilde{h}_t + BE_8\tilde{h}_{t-1} + BE_9\tilde{h}_{t+1} + BE_{11}\tilde{y}_t + BE_{11}\tilde{\lambda}_t\right] = 0$$

$$E_t\left[BD_1\tilde{c}_t + BD_2\tilde{c}_{t-1} + BD_3\tilde{c}_{t+1} + BD_4\tilde{D}_t + BD_5\tilde{D}_{t-1} + BD_6\tilde{D}_{t+1} + BD_7\tilde{h}_t + BD_8\tilde{h}_{t-1} + BD_9\tilde{h}_{t+1} + BD_{11}\tilde{\lambda}_{t+1} - E_1\tilde{\lambda}_t\right] = 0$$

$$E_t\left[(\alpha\beta y/k)\tilde{y}_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \beta\delta\tilde{\delta}_{t+1} + \varphi_k\beta\tilde{k}_{t+2} + E_2\tilde{k}_{t+1} + \varphi_k\tilde{k}_t\right] = 0$$

$$E_t\left[(1-1/R)\tilde{r}_t - \tilde{\lambda}_t + E_t\tilde{\lambda}_{t+1}\right] = 0$$

**Block 2: Static equations**

$$-\frac{b}{R^f}\tilde{b}_{t+1} + \frac{tb}{R^f}\tilde{r}_t + R^f b\tilde{b}_t - y\tilde{y}_t + I.\tilde{I}_t + c\tilde{c}_t + d\tilde{d}_t = 0$$

$$\tilde{D}_t - (1-\tau)\tilde{D}_{t-1} - \tau\tilde{d}_t = 0$$

$$-\psi y\tilde{y}_t + \psi b\tilde{b}_{t+1} + \tilde{\mu}_t - r\tilde{r}_t = 0$$

$$\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t + \delta.\tilde{\delta}_t - \delta.\tilde{I}_t = 0$$

$$\mu.\tilde{u}_t - \tilde{\delta}_t = 0$$

$$z_t + \alpha.\tilde{u}_t + \alpha.\tilde{k}_t + (1-\alpha)\tilde{h}_t - \tilde{y}_t = 0$$

$$\tilde{y}_t - \tilde{k}_t - \tilde{\delta}_t = 0$$

$$-tby_t + (1-tby)\tilde{y}_t - \frac{c}{y}\tilde{c}_t - \frac{I}{y}\tilde{I}_t - \frac{d}{y}\tilde{d}_t = 0$$

$$-cay_t + \tilde{y}_t - \frac{c}{y}\tilde{c}_t - \frac{I}{y}\tilde{I}_t - P\frac{d}{y}\tilde{d}_t - \frac{tby}{1+r}\tilde{r}_t - tby\tilde{b}_{t+1} = 0$$

**Block 3: Stochastic equations**

$$\tilde{\eta}_t = \rho_\eta\tilde{\eta}_t + \varepsilon_t^\eta$$

$$z_t = \rho_z z_t + \varepsilon_t^z$$

These three blocks can be re-written as:

1. $E_t[F.X_{t+1} + G.X_t + H.X_{t-1} + J.Y_{t+1} + K.Y_t + L.Z_{t+1} + M.Z_t] = 0$

2. $A.X_t + B.X_{t-1} + C.Y_t + D.Z_t = 0$

3. $Z_{t+1} = N.Z_t + \varepsilon_{t+1}$ With $E_t(\varepsilon_{t+1}) = 0$

*A, B, …M, N* are coefficient matrices with the elements descried in above three blocks. We solve the system following the method of undetermined coefficients. The systems **1-3** is a set of difference equations with rational expectation feature. The solution for such difference equations consist of policy function of the form "suggested solution"

$$Y_t = R.X_{t-1} + SZ_t$$
$$X_t = P.X_{t-1} + QZ_t$$

If we plug this solution into 1-3 above, the coefficient matrices that solve the system $(P, Q, R, S)$ must satisfy

$$R = -CC^{-1}(A.P + BB)$$
$$(F - J.C^{-1}.A)P^2 - (J.C^{-1}.B - G + K.C^{-1}.A)P - K.C^{-1}.B + H = 0$$
$$N' \otimes (F - J.C^{-1}.A) + I_k \otimes (J.R + F.P + G - K.C^{-1}.A)vec(Q) = vec\left(\left(J.C^{-1}.D - L\right)N + KC^{-1}D - M\right)$$
$$S = -C^{-1}(A.Q + D)$$

The coefficient matrices that solve the "suggested solution" are:

$$P = \begin{pmatrix} 1.0070 & -0.0071 & 0.0077 & 0 & -0.0244 & 0.0300 \\ 0.2378 & 0.7325 & 0.2741 & 0 & 0.4468 & -0.2339 \\ 0.0440 & -0.0096 & 0.9709 & 0 & -0.0335 & 0.0368 \\ -0.5387 & 0.2045 & -0.1428 & 0 & 0.7885 & -0.8650 \\ 0.0801 & 0.0126 & 0.0735 & 0 & 0.8963 & -0.0816 \\ 0.0883 & 0.0337 & 0.1493 & 0 & 0.1071 & 0.5481 \end{pmatrix} \qquad Q = \begin{pmatrix} 0.1129 & -0.0438 \\ 1.8330 & -1.6265 \\ 0.2213 & -0.0579 \\ -2.8310 & 1.2056 \\ 0.4553 & 0.0942 \\ 0.6200 & 0.2180 \end{pmatrix}$$

$$R=\begin{pmatrix} 0.2119 & 0.0291 & 0.1290 & 0 & 0.0926 & 0.4738 \\ 0.5633 & -0.3269 & 0.5122 & 0 & -1.1278 & 1.9742 \\ 0.8801 & -0.1920 & 0.4176 & 0 & -0.6705 & 0.7369 \\ -0.0919 & 0.1199 & -0.0902 & 0 & -0.1326 & -0.0097 \\ -0.0832 & 0.0936 & -0.0959 & 0 & -0.1564 & 0.0819 \\ -0.5254 & 0.0194 & 0.0860 & 0 & 0.0617 & 0.3158 \\ -0.7881 & 0.0291 & 0.1290 & 0 & 0.0926 & 0.4738 \\ -2.5966 & 7.0445 & 1.5445 & 0 & 6.6730 & -26.3730 \end{pmatrix} \qquad S=\begin{pmatrix} 1.8071 & 0.1885 \\ 7.4506 & -1.9996 \\ 4.4253 & -1.1583 \\ -0.7465 & 0.7302 \\ -0.6415 & 0.5693 \\ 1.2047 & 0.1257 \\ 1.8071 & 0.1885 \\ -31.0631 & 49.4297 \end{pmatrix}$$

Plugging the solution in the policy functions we obtain the solution to our model as defined by the following processes:

$$\begin{pmatrix} \tilde{k}_{t+1} \\ b_{t+1} \\ \tilde{D}_{t+1} \\ \tilde{c}_t \\ \tilde{h}_t \\ \tilde{\lambda}_t \end{pmatrix} = \begin{pmatrix} 1.0070 & -0.0071 & 0.0077 & 0 & -0.0244 & 0.0300 \\ 0.2378 & 0.7325 & 0.2741 & 0 & 0.4468 & -0.2339 \\ 0.0440 & -0.0096 & 0.9709 & 0 & -0.0335 & 0.0368 \\ -0.5387 & 0.2045 & -0.1428 & 0 & 0.7885 & -0.8650 \\ 0.0801 & 0.0126 & 0.0735 & 0 & 0.8963 & -0.0816 \\ 0.0883 & 0.0337 & 0.1493 & 0 & 0.1071 & 0.5481 \end{pmatrix} \begin{pmatrix} \tilde{k}_t \\ b_t \\ \tilde{D}_t \\ \tilde{c}_{t-1} \\ \tilde{h}_{t-1} \\ \tilde{\lambda}_{t-1} \end{pmatrix} + \begin{pmatrix} 0.1129 & -0.0438 \\ 1.8330 & -1.6265 \\ 0.2213 & -0.0579 \\ -2.8310 & 1.2056 \\ 0.4553 & 0.0942 \\ 0.6200 & 0.2180 \end{pmatrix} \begin{pmatrix} z_t \\ \tilde{\eta}_t \end{pmatrix}$$

$$\begin{pmatrix} \tilde{y}_t \\ \tilde{i}_t \\ \tilde{d}_t \\ tby_t \\ cay_t \\ \tilde{\delta}_t \\ \tilde{u}_t \\ \tilde{r}_t \end{pmatrix} = \begin{pmatrix} 0.2119 & 0.0291 & 0.1290 & 0 & 0.0926 & 0.4738 \\ 0.5633 & -0.3269 & 0.5122 & 0 & -1.1278 & 1.9742 \\ 0.8801 & -0.1920 & 0.4176 & 0 & -0.6705 & 0.7369 \\ -0.0919 & 0.1199 & -0.0902 & 0 & -0.1326 & -0.0097 \\ -0.0832 & 0.0936 & -0.0959 & 0 & -0.1564 & 0.0819 \\ -0.5254 & 0.0194 & 0.0860 & 0 & 0.0617 & 0.3158 \\ -0.7881 & 0.0291 & 0.1290 & 0 & 0.0926 & 0.4738 \\ -2.5966 & 7.0445 & 1.5445 & 0 & 6.6730 & -26.3730 \end{pmatrix} \begin{pmatrix} \tilde{k}_t \\ b_t \\ \tilde{D}_t \\ \tilde{c}_{t-1} \\ \tilde{h}_{t-1} \\ \tilde{\lambda}_{t-1} \end{pmatrix} + \begin{pmatrix} 1.8071 & 0.1885 \\ 7.4506 & -1.9996 \\ 4.4253 & -1.1583 \\ -0.7465 & 0.7302 \\ -0.6415 & 0.5693 \\ 1.2047 & 0.1257 \\ 1.8071 & 0.1885 \\ -31.0631 & 49.4297 \end{pmatrix} \begin{pmatrix} z_t \\ \tilde{\eta}_t \end{pmatrix}$$

Now using the above solutions we obtain the impulse response functions (IRFs) and the simulated data for our baseline model economy in Essay 1. For more details see Uhlig (1999).

# C.2: Solutions of the Dynamic Model in Essay 2

There are 17 equations in our model that control the dynamics of the economy. These are numbered as equations (2.1-2.2), (2.5), (2.9-2.20) in the main text along with one equation defining the trade balance. We linearized them one by one. Accordingly we rewrite the linearized version of the model

$$\tilde{D}_{h,t} - (1-\tau_h)\tilde{D}_{h,t-1} - \tau_h \tilde{d}_{h,t} = 0 \tag{C.2.1}$$

$$\tilde{D}_{f,t} - (1-\tau_f)\tilde{D}_{f,t-1} - \tau_f \tilde{d}_{f,t} = 0 \tag{C.2.2}$$

$$\tilde{p}_t = \rho_p \log \tilde{p}_{t-1} + \varepsilon_t^p \tag{C.2.5}$$

$$-p\frac{b}{R}\tilde{b}_{t+1} + pb\tilde{b}_t + \frac{tb}{R}\tilde{r}_t + BC.\tilde{p}_t - y\tilde{y}_t + d_h\tilde{d}_{h,t} + pd_f\tilde{d}_{f,t} + I\tilde{I}_t = 0 \tag{C.2.9}$$

$$\psi p.\tilde{p}_t - \varsigma y\tilde{y}_t - r\tilde{r}_t = 0 \tag{C.2.10}$$

$$\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t + \delta.\tilde{\delta}_t - \delta.\tilde{I}_t = 0 \tag{C.2.11}$$

$$\mu.\tilde{u}_t - \tilde{\delta}_t = 0 \tag{C.2.12}$$

$$z_t + \alpha.\tilde{u}_t + \alpha.\tilde{k}_t + (1-\alpha)\tilde{h}_t - \tilde{y}_t = 0 \tag{C.2.13}$$

$$\tilde{a}_t = \rho_a \log \tilde{a}_{t-1} + \varepsilon_t^a \tag{C.2.14}$$

$$\begin{aligned} &E_{13}\tilde{D}_{h,t+1} + E_{11}\tilde{D}_{h,t} + E_{12}\tilde{D}_{h,t-1} + E_{16}\tilde{D}_{f,t+1} + E_{14}\tilde{D}_{f,t} + E_{15}\tilde{D}_{f,t-1} + E_{17}\tilde{h}_t + \\ &\qquad E_{18}\tilde{h}_{t-1} + E_{19}\tilde{h}_{t+1} + E_{10}\tilde{\lambda}_{t+1} - R_1\tilde{\lambda}_t = 0 \end{aligned} \tag{C.2.15}$$

$$\begin{aligned} &E_{21}\tilde{D}_{h,t} + E_{22}\tilde{D}_{h,t-1} + E_{23}\tilde{D}_{h,t+1} + E_{24}\tilde{D}_{f,t} + E_{25}\tilde{D}_{f,t-1} + E_{26}\tilde{D}_{f,t+1} + E_{27}\tilde{h}_t + E_{28}\tilde{h}_{t-1} + E_{29}\tilde{h}_{t+1} + \\ &\qquad E_{20}\tilde{\lambda}_{t+1} - R_2\tilde{\lambda}_t + R_2\left[\beta\left(1-\tau_f\right)\tilde{p}_{t+1} - \tilde{p}_t\right] = 0 \end{aligned} \tag{C.2.16}$$

$$LM_1\tilde{D}_{h,t} + LM_2\tilde{D}_{h,t-1} + LM_3\tilde{D}_{h,t+1} + LM_4\tilde{D}_{f,t} + LM_5\tilde{D}_{f,t-1} + LM_6\tilde{D}_{f,t+1} + LM_7\tilde{h}_t + LM_8\tilde{h}_{t-1} + LM_9\tilde{h}_{t+1} + LM_{10}\tilde{y}_t + LM_{10}\tilde{\lambda}_t = 0 \tag{C.2.17}$$

$$\beta\alpha\frac{y}{k}\tilde{y}_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \beta\delta\tilde{\delta}_{t+1} + \phi_k\tilde{k}_t + E_3\tilde{k}_{t+1} + \beta\phi_k\tilde{k}_{t+2} = 0 \tag{C.2.18}$$

$$\tilde{y}_t - \tilde{k}_t - \tilde{\delta}_t = 0 \tag{C.2.19}$$

$$\tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \tilde{p}_{t+1} - \tilde{p}_t + R\phi_b b\tilde{b}_{t+1} + (1 - 1/R)\tilde{r}_t = 0 \tag{C.2.20}$$

The trade balance

$$-tby_t - \frac{d_h}{y}\tilde{d}_{h,t} - \frac{pd_f}{y}\tilde{d}_{f,t} - \frac{pd_f}{y}\tilde{p}_t - \frac{i}{y}\tilde{i}_t + (1 - tby)\tilde{y}_t = 0 \tag{C.2.25}$$

Plus extra equation that

$$\tilde{p}_t = ToT_t \tag{C.2.26}$$

This is a 17X17 system of equations. Some of the coefficients used above are defined in terms of model parameters as follows:

$$E3 = -\beta\alpha yk - \phi k(\beta + 1)$$

$$E10 = \frac{(1 - \beta\varphi h)\beta(1 - \tau h)}{1 - \beta(1 - \tau h)}$$

$$E11 = -\frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}(1 + \beta\varphi h^2)}{x\left(x^{\frac{\rho}{\rho-1}} - \xi(h - \upsilon h)^{\omega}\right)(1 - \varphi h)} - \frac{(1 - \beta\varphi h)\phi h(\beta + 1)}{1 - \beta(1 - \tau h)}$$

$$E12 = \frac{(1 - \beta\varphi h)\phi f}{1 - \beta(1 - \tau h)} + \frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}\varphi h}{x\left(x^{\frac{\rho}{\rho-1}} - \xi(h - \upsilon h)^{\omega}\right)(1 - \varphi h)}$$

$$E13 = \beta\left(\frac{(1 - \beta\varphi h)\phi h}{1 - \beta(1 - \tau h)} + \frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}\varphi h}{x\left(x^{\frac{\rho}{\rho-1}} - \xi(h - \upsilon h)^{\omega}\right)(1 - \varphi h)}\right)$$

$$E14 = \frac{1}{\rho(1 - \varphi f)}\left(\left(1 - \frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1 - \frac{\rho\sigma x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}} - \xi(h - \upsilon h)^{\omega}}\right)(1 + \beta\varphi h\varphi f)\right)$$

$$E15=-\frac{1}{\rho\left(1-\varphi f\right)}\left(\left(1-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\varphi h\right)$$

$$E16=-\frac{1}{\rho\left(1-\varphi f\right)}\left(\beta\varphi h\left(1-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\right)$$

$$E17=-\frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\omega\left(\beta\varphi h\upsilon+1\right)}{1-\upsilon}$$

$$E18=\frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\omega\upsilon}{1-\upsilon}$$

$$E19=\frac{\beta\varphi h\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\omega}{1-\upsilon}$$

$$E20=\frac{\left(1-\beta\varphi f\right)\beta\left(1-\tau f\right)}{1-\beta\left(1-\tau f\right)}$$

$$E21=\frac{1}{x\rho\left(1-\varphi h\right)}\left(\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\left(1+\beta\varphi h\varphi f\right)\right)$$

$$E22=-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\varphi h}{x\rho\left(1-\varphi h\right)}$$

$$E23=-\frac{1}{x\rho\left(1-\varphi h\right)}\left(\beta\varphi f\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)\right)$$

$$E24=\frac{1}{\rho\left(1-\varphi f\right)}\left(\left(\left(1-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)-1\right)\left(1+\beta\varphi f^{2}\right)\right)-\frac{\left(1-\beta\varphi f\right)\phi f\left(\beta+1\right)}{1-\beta\left(1-\tau f\right)}$$

$$E25=\frac{\left(1-\beta\varphi f\right)\phi f}{1-\beta\left(1-\tau f\right)}-\frac{1}{\rho\left(1-\varphi f\right)}\left(\left(\left(1-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)-1\right)\varphi f\right)$$

$$E26=\beta\left(\frac{\left(1-\beta\varphi f\right)\phi f}{1-\beta\left(1-\tau f\right)}-\frac{1}{\rho\left(1-\varphi f\right)}\left(\left(\left(1-\frac{\gamma\left(\left(1-\varphi h\right)Dh\right)^{\frac{\rho-1}{\rho}}}{x}\right)\left(1-\frac{\rho\,\sigma\,x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi\left(h-\upsilon h\right)^{\omega}}\right)-1\right)\varphi f\right)\right)$$

$$E27 = -\frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\omega(1+\beta\varphi f\upsilon)}{1-\upsilon}$$

$$E28 = \frac{\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\omega\upsilon}{1-\upsilon}$$

$$E29 = \frac{\beta\varphi f\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\omega}{1-\upsilon}$$

$$L1 = -1+\beta\upsilon$$

$$LM1 = -\frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}(\beta\varphi h\upsilon+1)}{x\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi h)}$$

$$LM2 = \frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}\varphi h}{x\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi h)}$$

$$LM3 = \frac{\beta\upsilon\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}\sigma x^{\frac{\rho}{\rho-1}}}{x\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi h)}$$

$$LM4 = -\frac{\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}}{x}\right)(1+\beta\varphi f\upsilon)}{\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi f)}$$

$$LM5 = \frac{\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}}{x}\right)\varphi 2}{\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi f)}$$

$$LM6 = \frac{\beta\upsilon\sigma x^{\frac{\rho}{\rho-1}}\left(1-\frac{\gamma((1-\varphi h)Dh)^{\frac{\rho-1}{\rho}}}{x}\right)}{\left(x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}\right)(1-\varphi f)}$$

$$LM7 = \frac{1}{1-\upsilon}\left(\left(\left(\left(1-\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\right)\omega-1\right)(1+\beta\upsilon^{2})\right)+1-\beta\upsilon\right)$$

$$LM8 = -\frac{\left(\left(1-\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\right)\omega-1\right)\upsilon}{1-\upsilon}$$

$$LM9 = -\frac{\beta\upsilon\left(\left(1-\sigma\left(1-\frac{x^{\frac{\rho}{\rho-1}}}{x^{\frac{\rho}{\rho-1}}-\xi(h-\upsilon h)^{\omega}}\right)\right)\omega-1\right)}{1-\upsilon}$$

$$LM10 = -1+\beta\upsilon$$

Where $x=\gamma(1-\varphi h)^{\frac{\rho-1}{\rho}}Dh^{\frac{\rho-1}{\rho}}+(1-\gamma)(1-\varphi f)^{\frac{\rho-1}{\rho}}Df^{\frac{\rho-1}{\rho}}$

Note that the above system consists of three types of variables:

The state variables: $X_t \equiv \left[\tilde{k}_{t+1} \quad \tilde{b}_{t+1} \quad \tilde{D}_{h,t+1} \quad \tilde{D}_{f,t+1} \quad \tilde{\lambda}_t \quad \tilde{h}_t\right]'$

Other endogenous variables: $Y_t \equiv \left[\tilde{y}_t \quad \tilde{d}_{h,t} \quad \tilde{d}_{f,t} \quad \tilde{i}_t \quad tby_t \quad \tilde{u}_t \quad \tilde{\delta}_t \quad \tilde{r}_t \quad ToT_t\right]'$

The stochastic variables: $Z_t \equiv [\tilde{a}_t \quad \tilde{p}_t]'$

Moreover, for simplification, we rearrange the system of equations into three distinct blocks:

**Block 1: Forward looking rational equations**

$$E_{13}\tilde{D}_{h,t+1} + E_{11}\tilde{D}_{h,t} + E_{12}\tilde{D}_{h,t-1} + E_{16}\tilde{D}_{f,t+1} + E_{14}\tilde{D}_{f,t} + E_{15}\tilde{D}_{f,t-1} + E_{17}\tilde{h}_t + E_{18}\tilde{h}_{t-1} + E_{19}\tilde{h}_{t+1} + E_{10}\tilde{\lambda}_{t+1} - R_1\tilde{\lambda}_t = 0$$

$$E_{21}\tilde{D}_{h,t} + E_{22}\tilde{D}_{h,t-1} + E_{23}\tilde{D}_{h,t+1} + E_{24}\tilde{D}_{f,t} + E_{25}\tilde{D}_{f,t-1} + E_{26}\tilde{D}_{f,t+1} + E_{27}\tilde{h}_t + E_{28}\tilde{h}_{t-1} + E_{29}\tilde{h}_{t+1} + E_{20}\tilde{\lambda}_{t+1} - R_2\tilde{\lambda}_t + R_2\left[\beta\left(1-\tau_f\right)\tilde{p}_{t+1} - \tilde{p}_t\right] = 0$$

$$LM_1\tilde{D}_{h,t} + LM_2\tilde{D}_{h,t-1} + LM_3\tilde{D}_{h,t+1} + LM_4\tilde{D}_{f,t} + LM_5\tilde{D}_{f,t-1} + LM_6\tilde{D}_{f,t+1} + LM_7\tilde{h}_t + LM_8\tilde{h}_{t-1} + LM_9\tilde{h}_{t+1} + LM_{10}\tilde{y}_t + LM_{10}\tilde{\lambda}_t = 0$$

$$\beta\alpha\frac{y}{k}\tilde{y}_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \beta\delta\tilde{\delta}_{t+1} + \phi_k\tilde{k}_t + E_3\tilde{k}_{t+1} + \beta\phi_k\tilde{k}_{t+2} = 0$$

$$\tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \tilde{p}_{t+1} - \tilde{p}_t + R\phi_b b\tilde{b}_{t+1} + \left(1-1/R\right)\tilde{r}_t = 0$$

**Block 2: Static equations**

$$\tilde{D}_{h,t} - (1-\tau_h)\tilde{D}_{h,t-1} - \tau_h\tilde{d}_{h,t} = 0$$

$$\tilde{D}_{f,t} - (1-\tau_f)\tilde{D}_{f,t-1} - \tau_f\tilde{d}_{f,t} = 0$$

$$\tilde{p}_t = \rho_p \log\tilde{p}_{t-1} + \varepsilon_t^p$$

$$-p\frac{b}{R}\tilde{b}_{t+1} + pb\tilde{b}_t + \frac{tb}{R}\tilde{r}_t + BC.\tilde{p}_t - y\tilde{y}_t + d_h\tilde{d}_{h,t} + pd_f\tilde{d}_{f,t} + I\tilde{I}_t = 0$$

$$\psi.\tilde{p}_t - \varsigma y\tilde{y}_t - r\tilde{r}_t = 0$$

$$\tilde{k}_{t+1} - \left(1-\delta\right)\tilde{k}_t + \delta.\tilde{\delta}_t - \delta.\tilde{I}_t = 0$$

$$\mu.\tilde{u}_t - \tilde{\delta}_t = 0$$

$$z_t + \alpha.\tilde{u}_t + \alpha.\tilde{k}_t + (1-\alpha)\tilde{h}_t - \tilde{y}_t = 0$$

$$\tilde{y}_t - \tilde{k}_t - \tilde{\delta}_t = 0$$

$$-tby_t - \frac{d_h}{y}\tilde{d}_{h,t} - \frac{pd_f}{y}\tilde{d}_{f,t} - \frac{pd_f}{y}\tilde{p}_t - \frac{i}{y}\tilde{i}_t + (1-tby)\tilde{y}_t = 0$$

**Block 3: Stochastic equations**

$$\tilde{p}_t = \rho_p \log \tilde{p}_{t-1} + \varepsilon_t^p$$

$$\tilde{a}_t = \rho_a \log \tilde{a}_{t-1} + \varepsilon_t^a$$

These three blocks can be re-written as:

**1.** $0 = E_t[F.X_{t+1} + G.X_t + H.X_{t-1} + J.Y_{t+1} + K.Y_t + L.Z_{t+1} + M.Z_t]$

**2.** $0 = A.X_t + B.X_{t-1} + C.Y_t + D.Z_t$

**3.** $Z_{t+1} = N.Z_t + \varepsilon_{t+1}$ With $E_t(\varepsilon_{t+1}) = 0$

where $(A, B...M, N)$ are coefficient matrices with the elements descried in above three blocks.

We solve the system following the method of undetermined coefficients. The system **(1-3)** is a set of difference equations with rational expectation feature. The solution for such difference equations consist of policy function of the form "suggested solution"

$$X_t = P.X_{t-1} + QZ_t$$

$$Y_t = R.X_{t-1} + SZ_t$$

The coefficient matrices that solve that “suggested solution”:

$$P = \begin{pmatrix} 0.9776 & -0.0075 & -0.0079 & -0.0056 & 0 & 0.0408 \\ -0.1960 & 0.7240 & 0.3340 & 0.2609 & 0 & -0.1088 \\ 0.1392 & -0.0099 & 0.8651 & 0.0148 & 0 & 0.0344 \\ 0.0860 & -0.0067 & 0.0140 & 0.8173 & 0 & 0.0216 \\ -0.1304 & 0.0174 & 0.0386 & 0.0246 & 0 & -0.0820 \\ 0.0403 & -0.0013 & 0.0410 & 0.0299 & 0 & 0.7872 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.1081 & -0.0229 \\ 2.2754 & -1.0083 \\ 0.4438 & -0.0658 \\ 0.2848 & -0.4068 \\ 0.0546 & -0.1779 \\ 0.3932 & -0.1326 \end{pmatrix},$$

$$R=\begin{pmatrix} 0.1540 & -0.0011 & 0.0361 & 0.0264 & 0 & 0.6939 \\ 0.4640 & -0.0330 & 0.5502 & 0.0492 & 0 & 0.1146 \\ 0.2868 & -0.0224 & 0.0466 & 0.3910 & 0 & 0.0719 \\ -0.9673 & -0.3764 & -0.3577 & -0.2550 & 0 & 2.7364 \\ 0.0680 & 0.1012 & -0.1186 & -0.0926 & 0 & 0.0321 \\ -0.6043 & -0.0008 & 0.0258 & 0.0188 & 0 & 0.4956 \\ -0.8460 & -0.0011 & 0.0361 & 0.0264 & 0 & 0.6939 \\ -0.2603 & 0.0019 & -0.0610 & -0.0446 & 0 & -1.1724 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0 & -0.0000 \\ 0.3745 & -0.0277 & 0.2957 & 0.2220 & 0 & 0.0931 \end{pmatrix}, \quad S=\begin{pmatrix} 1.6429 & -0.1168 \\ 1.4795 & -0.2192 \\ 0.9492 & -1.3560 \\ 7.0468 & -1.2617 \\ -0.8211 & 0.3795 \\ 1.1735 & -0.0835 \\ 1.6429 & -0.1168 \\ -2.7759 & 5.3820 \\ 0.0 & 1.0000 \\ 1.2115 & -0.2883 \end{pmatrix}.$$

Plugging the solution in the policy functions we obtain the solution to our model as defined by the following processes:

$$\begin{pmatrix} \tilde{k}_{t+1} \\ \tilde{b}_{t+1} \\ \tilde{D}_{h,t+1} \\ \tilde{D}_{f,t+1} \\ \tilde{\lambda}_{t} \\ \tilde{h}_{t} \end{pmatrix} = \begin{pmatrix} 0.9776 & -0.0075 & -0.0079 & -0.0056 & 0 & 0.0408 \\ -0.1960 & 0.7240 & 0.3340 & 0.2609 & 0 & -0.1088 \\ 0.1392 & -0.0099 & 0.8651 & 0.0148 & 0 & 0.0344 \\ 0.0860 & -0.0067 & 0.0140 & 0.8173 & 0 & 0.0216 \\ -0.1304 & 0.0174 & 0.0386 & 0.0246 & 0 & -0.0820 \\ 0.0403 & -0.0013 & 0.0410 & 0.0299 & 0 & 0.7872 \end{pmatrix} \begin{pmatrix} \tilde{k}_{t} \\ \tilde{b}_{t} \\ \tilde{D}_{h,t} \\ \tilde{D}_{f,t} \\ \tilde{\lambda}_{t-1} \\ \tilde{h}_{t-1} \end{pmatrix} + \begin{pmatrix} 0.1081 & -0.0229 \\ 2.2754 & -1.0083 \\ 0.4438 & -0.0658 \\ 0.2848 & -0.4068 \\ 0.0546 & -0.1779 \\ 0.3932 & -0.1326 \end{pmatrix} \begin{pmatrix} \tilde{a}_{t} \\ \tilde{p}_{t} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{y}_{t} \\ \tilde{d}_{h,t} \\ \tilde{d}_{f,t} \\ \tilde{i}_{t} \\ tby_{t} \\ \tilde{u}_{t} \\ \tilde{\delta}_{t} \\ \tilde{r}_{t} \\ ToT_{t} \\ \tilde{c}_{t} \end{pmatrix} = \begin{pmatrix} 0.1540 & -0.0011 & 0.0361 & 0.0264 & 0 & 0.6939 \\ 0.4640 & -0.0330 & 0.5502 & 0.0492 & 0 & 0.1146 \\ 0.2868 & -0.0224 & 0.0466 & 0.3910 & 0 & 0.0719 \\ -0.9673 & -0.3764 & -0.3577 & -0.2550 & 0 & 2.7364 \\ 0.0680 & 0.1012 & -0.1186 & -0.0926 & 0 & 0.0321 \\ -0.6043 & -0.0008 & 0.0258 & 0.0188 & 0 & 0.4956 \\ -0.8460 & -0.0011 & 0.0361 & 0.0264 & 0 & 0.6939 \\ -0.2603 & 0.0019 & -0.0610 & -0.0446 & 0 & -1.1724 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0 & -0.0000 \\ 0.3745 & -0.0277 & 0.2957 & 0.2220 & 0 & 0.0931 \end{pmatrix} \begin{pmatrix} \tilde{k}_{t} \\ \tilde{b}_{t} \\ \tilde{D}_{h,t} \\ \tilde{D}_{f,t} \\ \tilde{\lambda}_{t-1} \\ \tilde{h}_{t-1} \end{pmatrix} + \begin{pmatrix} 1.6429 & -0.1168 \\ 1.4795 & -0.2192 \\ 0.9492 & -1.3560 \\ 7.0468 & -1.2617 \\ -0.8211 & 0.3795 \\ 1.1735 & -0.0835 \\ 1.6429 & -0.1168 \\ -2.7759 & 5.3820 \\ 0.0 & 1.0000 \\ 1.2115 & -0.2883 \end{pmatrix} \begin{pmatrix} \tilde{a}_{t} \\ \tilde{p}_{t} \end{pmatrix}$$

Now using the above solutions we obtain the impulse response functions (IRFs) and the simulated data for our baseline model in Essay 2.

# C.3: Solutions of the Dynamic Model in Essay 3

There are 17 equations in our model that control the dynamics of the economy. These are numbered as equations (3.3), (3.5-3.8), (3.11), (3.15-3.17), (3.19-3.21), (3.24-3.26) in the main text along with two other definitions. We linearized them one by one and obtained the following equations:

$$U_1\left(\tilde{c}_t-\varphi_c\tilde{c}_{t-1}\right)+U_2\tilde{s}_t+U_3\left(\tilde{h}_t-\varphi_h\tilde{h}_{t-1}\right)-\tilde{u}_t=0 \tag{C.3.3}$$

$$\tilde{k}_{t+1}-\left(1-\delta_k\right)\tilde{k}_t-\delta_k.\tilde{I}_t=0 \tag{C.3.5}$$

$$-\frac{D}{R}\tilde{D}_{t+1}+D\tilde{D}_t-RDF_t=0 \tag{C.3.6}$$

$$z_t+\alpha x_1\tilde{k}_t+\alpha\left(1-x_1\right)\tilde{G}_t+\left(1-\alpha\right)\tilde{h}_t-\tilde{y}_t=0 \tag{C.3.7}$$

$$z_t=\rho_z z_{t\text{-}1}+\varepsilon_t^z \tag{C.3.8}$$

$$\tilde{G}_{t+1}-\left(1-\delta_g\right)\tilde{G}_t-\delta_g.\tilde{g}_t=0 \tag{C.3.11}$$

$$MU_1\tilde{c}_t+MU_2\tilde{c}_{t-1}+MU_3\tilde{c}_{t+1}+MU_4\tilde{s}_t+MU_5\tilde{s}_{t+1}+MU_6\tilde{h}_t+MU_7\tilde{h}_{t-1}+MU_8\tilde{h}_{t+1}+MU_9\tilde{\lambda}_t=0 \tag{C.3.15}$$

$$\begin{aligned}&LM_1\tilde{c}_t+LM_2\tilde{c}_{t-1}+LM_3\tilde{c}_{t+1}+LM_4\tilde{s}_t+LM_5\tilde{s}_{t+1}+LM_6\tilde{h}_t+LM_7\tilde{h}_{t-1}+LM_8\tilde{h}_{t+1}+LM_9 z_t+\\&\qquad LM_{10}\tilde{k}_t+LM_{11}\tilde{G}_t+LM_9\tilde{\lambda}_t=0\end{aligned} \tag{C.3.16}$$

$$EK.z_{t+1}+EK.D_2\tilde{G}_{t+1}+EK.\left(1-\alpha\right)\tilde{h}_{t+1}+\tilde{\lambda}_{t+1}-\tilde{\lambda}_t+\varphi_k\beta\tilde{k}_{t+2}+EUK.\tilde{k}_{t+1}+\varphi_k\tilde{k}_t=0 \tag{C.3.17}$$

$$\tilde{\lambda}_{t+1}-\tilde{\lambda}_t-\phi_D D\tilde{D}_{t+1}=0 \tag{C.3.19}$$

$$t^c c.\tilde{c}_t+t^y y.\tilde{y}_t-id_2.\frac{D}{R}\tilde{D}_{t+1}+id_2.D\tilde{D}_t-TGR.TGR_t=0 \tag{C.3.20}$$

$$TGR_t - \tilde{g}_t = 0 \tag{C.3.21}$$

$$TGR_t - \tilde{s}_t = 0 \tag{C.3.22}$$

$$TGR_t - \tilde{T}_t = 0 \tag{C.3.23}$$

The aggregate budget constraint (combination of (20) and (24)) is linearized as

$$c\tilde{c}_t + g\tilde{g}_t + s\tilde{s}_t + I\tilde{I}_t + \frac{D}{R}\tilde{D}_{t+1} - D\tilde{D}_t - y\tilde{y}_t = 0 \tag{C.3.24}$$

The definition of government current revenues

$$t^y.y.\tilde{y}_t + t^c.c.\tilde{c}_t - TCR.TCR_t = 0 \tag{C.3.25}$$

Plus extra equation that

$$z_t = sh_t \tag{C.3.26}$$

This is a 17X17 system of equations. Some of the coefficients used above are defined in terms of model parameters as follows:

$$MU1 = \frac{1}{\rho(1-\varphi c)}\left(\left(\frac{1}{H1}\left(\left(1 - \frac{\rho\,\sigma\,H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}} - \xi((1-\varphi h)\,h)^{\omega}}\right)((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\right) - 1\right)(1+\beta\varphi c^2)\right)$$

$$MU2 = -\frac{1}{\rho(1-\varphi c)}\left(\left(\frac{1}{H1}\left(\left(1 - \frac{\rho\,\sigma\,H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}} - \xi((1-\varphi h)\,h)^{\omega}}\right)((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\right) - 1\right)\varphi c\right)$$

$$MU3 = -\frac{1}{\rho(1-\varphi c)}\left(\beta\varphi c\left(\frac{1}{H1}\left(\left(1 - \frac{\rho\,\sigma\,H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}} - \xi((1-\varphi h)\,h)^{\omega}}\right)((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}\right) - 1\right)\right)$$

$$MU4 = \frac{\left(1 - \frac{\rho\,\sigma\,H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}} - \xi\,sh^{\omega}}\right)\left(1 - \frac{sc^{\frac{\rho-1}{\rho}}}{H1}\right)}{\rho}$$

$$MU5 = -\frac{1}{\rho}\left(\beta\,\varphi c\left(1-\frac{\rho\,\sigma\,H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\left(1-\frac{\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}}{H1}\right)\right)$$

$$MU6 = -\frac{\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega\left(1+\beta\,\varphi c\,\varphi h\right)}{1-\varphi h}$$

$$MU7 = \frac{\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega\,\varphi h}{1-\varphi h}$$

$$MU8 = \frac{\beta\,\varphi c\,\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega}{1-\varphi h}$$

$$LM1 = -\frac{\sigma\,H1^{\frac{\rho}{\rho-1}}\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}\left(1+\beta\,\varphi c\,\eta 2\right)}{\left(H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}\right)H1\,(1-\varphi c)}$$

$$LM2 = \frac{\sigma\,H1^{\frac{\rho}{\rho-1}}\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}\varphi c}{\left(H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}\right)H1\,(1-\varphi c)}$$

$$LM3 = \frac{\beta\,\varphi h\,\sigma\,H1^{\frac{\rho}{\rho-1}}\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}}{\left(H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}\right)H1\,(1-\varphi c)}$$

$$LM4 = -\frac{\sigma\,H1^{\frac{\rho}{\rho-1}}\left(1-\frac{\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}}{H1}\right)}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}$$

$$LM5 = \frac{\beta\,\varphi h\,\sigma\,H1^{\frac{\rho}{\rho-1}}\left(1-\frac{\left((1-\varphi c)\,c\right)^{\frac{\rho-1}{\rho}}}{H1}\right)}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}$$

$$M6 = \frac{1}{1-\varphi h}\left(\left(\omega-1-\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega\right)\left(1+\beta\,\varphi h^{2}\right)+\left(1-\beta\,\varphi h\right)\alpha\right)$$

$$LM7 = -\frac{\left(\omega-1-\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega\right)\varphi h}{1-\varphi h}$$

$$LM8 = -\frac{\beta\,\varphi h\left(\omega-1-\sigma\left(1-\frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}}-\xi\left((1-\varphi h)\,h\right)^{\omega}}\right)\omega\right)}{1-\varphi h}$$

$$LM9 = -1 + \beta \varphi h \qquad LM10 = -\frac{(1-\beta\varphi h)\,\alpha\, k^{\frac{\varepsilon-1}{\varepsilon}}}{x}$$

$$LM11 = -(1-\beta\varphi h)\,\alpha \left(1 - \frac{k^{\frac{\varepsilon-1}{\varepsilon}}}{x}\right) \qquad EK = 1 - \beta\,(1-\delta 1)$$

$$EUK = (1-\beta(1-\delta 1)) \left( \frac{k^{\frac{\varepsilon-1}{\varepsilon}}(\alpha-1)}{x} + \frac{\frac{k^{\frac{\varepsilon-1}{\varepsilon}}}{x} - 1}{\varepsilon} \right) - \varphi c\,(\beta+1)$$

$$D1 = \frac{k^{\frac{\varepsilon-1}{\varepsilon}}(\alpha-1)}{x} + \frac{\frac{k^{\frac{\varepsilon-1}{\varepsilon}}}{x} - 1}{\varepsilon} \qquad D2 = \left(1 - \frac{k^{\frac{\varepsilon-1}{\varepsilon}}}{x}\right)\left(\alpha - 1 + \frac{1}{\varepsilon}\right)$$

$$U1 = \frac{(1-\sigma)\, H1^{\frac{\rho}{\rho-1}}\,((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{\left(H1^{\frac{\rho}{\rho-1}} - \xi\,((1-\varphi h)\,h)^{\omega}\right) H1\,(1-\varphi c)} \qquad U2 = \frac{(1-\sigma)\, H1^{\frac{\rho}{\rho-1}} \left(1 - \frac{((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}}}{H1}\right)}{H1^{\frac{\rho}{\rho-1}} - \xi\,((1-\varphi h)\,h)^{\omega}}$$

$$U3 = \frac{(1-\sigma)\left(1 - \frac{H1^{\frac{\rho}{\rho-1}}}{H1^{\frac{\rho}{\rho-1}} - \xi\,((1-\varphi h)\,h)^{\omega}}\right)\omega}{1-\varphi h}$$

Where

$x = k^{\frac{\varepsilon-1}{\varepsilon}} + \phi\, G^{\frac{\varepsilon-1}{\varepsilon}}$ and $H1 = ((1-\varphi c)\,c)^{\frac{\rho-1}{\rho}} + \gamma s^{\frac{\rho-1}{\rho}}$

Note that the above system consists of three types of variables:

The state variables: $X_t \equiv \left[\tilde{k}_{t+1} \quad \tilde{G}_{t+1} \quad \tilde{D}_{t+1} \quad \tilde{c}_t \quad \tilde{h}_t \quad \tilde{\lambda}_t \quad \tilde{u}_t\right]'$

Other endogenous variables: $Y_t \equiv \left[\tilde{y}_t \quad \tilde{s}_t \quad \tilde{g}_t \quad i_t \quad TGR_t \quad TCR_t \quad \tilde{T}_t \quad RDF_t \quad sh_t\right]'$

The stochastic variables: $Z_t \equiv z_t$

For simplification we rearrange the system of equations into three distinct blocks:

**Block 1: Forward looking rational equations**

$$MU_1\tilde{c}_t + MU_2\tilde{c}_{t-1} + MU_3\tilde{c}_{t+1} + MU_4\tilde{s}_t + MU_5\tilde{s}_{t+1} + MU_6\tilde{h}_t + MU_7\tilde{h}_{t-1} + MU_8\tilde{h}_{t+1} + MU_9\tilde{\lambda}_t = 0$$

$$LM_1\tilde{c}_t + LM_2\tilde{c}_{t-1} + LM_3\tilde{c}_{t+1} + LM_4\tilde{s}_t + LM_5\tilde{s}_{t+1} + LM_6\tilde{h}_t + LM_7\tilde{h}_{t-1} + LM_8\tilde{h}_{t+1} +$$
$$LM_9 z_t + LM_{10}\tilde{k}_t + LM_{11}\tilde{G}_t + LM_9\tilde{\lambda}_t = 0$$

$$EK.z_{t+1} + EK.D_2\tilde{G}_{t+1} + EK.(1-\alpha)\tilde{h}_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \varphi_k\beta\tilde{k}_{t+2} + EUK.\tilde{k}_{t+1} + \varphi_k\tilde{k}_t = 0$$

$$\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \phi_D D\tilde{D}_{t+1} = 0$$

**Block 2: Static equations**

$$U_1\left(\tilde{c}_t - \varphi_c\tilde{c}_{t-1}\right) + U_2\tilde{s}_t + U_3\left(\tilde{h}_t - \varphi_h\tilde{h}_{t-1}\right) - \tilde{u}_t = 0$$

$$\tilde{k}_{t+1} - (1-\delta_k)\tilde{k}_t - \delta_k.\tilde{I}_t = 0$$

$$-\frac{D}{R}\tilde{D}_{t+1} + D\tilde{D}_t - RDF_t = 0$$

$$z_t + \alpha x_1\tilde{k}_t + \alpha(1-x_1)\tilde{G}_t + (1-\alpha)\tilde{h}_t - \tilde{y}_t = 0$$

$$z_t = \rho_z z_{t\text{-}1} + \varepsilon_t^z$$

$$\tilde{G}_{t+1} - (1-\delta_g)\tilde{G}_t - \delta_g.\tilde{g}_t = 0$$

$$t^c c.\tilde{c}_t + t^y y.\tilde{y}_t - id_2.\frac{D}{R}\tilde{D}_{t+1} + id_2.D\tilde{D}_t - TGR.TGR_t = 0$$

$$TGR_t - \tilde{g}_t = 0$$

$$TGR_t - \tilde{s}_t = 0$$

$$TGR_t - \tilde{T}_t = 0$$

$$t^y.y.\tilde{y}_t + t^c.c.\tilde{c}_t - TCR.TCR_t = 0$$

**Block 3: Stochastic equations**

$$z_t = sh_t$$

These three blocks can be re-written as:

**1.** $0 = E_t[F.X_{t+1} + G.X_t + H.X_{t-1} + J.Y_{t+1} + K.Y_t + L.Z_{t+1} + M.Z_t]$

**2.** $0 = A.X_t + B.X_{t-1} + C.Y_t + D.Z_t$

**3.** $Z_{t+1} = N.Z_t + \varepsilon_{t+1}$ With $E_t(\varepsilon_{t+1}) = 0$

Where $(A, B...M, N)$ are coefficient matrices with the elements descried in above three blocks (1-3).

We solve the system following the method of undetermined coefficients. The systems **1-3** is a set of difference equations with rational expectation feature. The solution for such difference equations consist of policy function of the form "suggested solution":

$$X_t = P.X_{t-1} + QZ_t$$

$$Y_t = R.X_{t-1} + SZ_t$$

The coefficient matrices that solve the "suggested solution":

$$P = \begin{pmatrix} 0.9979 & 0.0037 & 0.0006 & -0.0323 & 0.0190 & 0 & 0 \\ 0.0082 & 0.9763 & -0.0000 & 0.0069 & 0.0066 & 0 & 0 \\ 1.4421 & 0.2391 & 0.6792 & -9.6121 & 7.8697 & 0 & 0 \\ 0.1185 & 0.0194 & -0.0004 & 0.8359 & -0.0472 & 0 & 0 \\ 0.1573 & 0.0256 & -0.0023 & 0.1437 & 0.5511 & 0 & 0 \\ -0.5107 & -0.0840 & -0.0154 & 0.9989 & -0.5393 & 0 & 0 \\ 0.3830 & 0.0630 & 0.0057 & -0.5955 & 0.3356 & 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.0751 \\ 0.0273 \\ 6.4765 \\ 0.3727 \\ 0.4931 \\ -1.7246 \\ 1.2206 \end{pmatrix},$$

$$R = \begin{pmatrix} 0.3961 & 0.0646 & -0.0015 & 0.0949 & 0.3637 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 1.5282 & 0.2485 & 0.0379 & -2.1506 & 1.2662 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ -0.0575 & -0.0095 & 0.0132 & 0.3830 & -0.3136 & 0 & 0 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1.3255 \\ 1.0909 \\ 1.0909 \\ 5.0034 \\ 1.0909 \\ 1.0909 \\ 1.0909 \\ -0.2581 \\ 1.0000 \end{pmatrix}$$

Plugging the solution in the policy functions we obtain the solution to our model as defined by the following processes:

$$\begin{pmatrix} \tilde{k}_{t+1} \\ \tilde{G}_{t+1} \\ \tilde{D}_{t+1} \\ \tilde{c}_t \\ \tilde{\lambda}_t \\ \tilde{h}_t \\ \tilde{u}_t \end{pmatrix} = \begin{pmatrix} 0.9979 & 0.0037 & 0.0006 & -0.0323 & 0.0190 & 0 & 0 \\ 0.0082 & 0.9763 & -0.0000 & 0.0069 & 0.0066 & 0 & 0 \\ 1.4421 & 0.2391 & 0.6792 & -9.6121 & 7.8697 & 0 & 0 \\ 0.1185 & 0.0194 & -0.0004 & 0.8359 & -0.0472 & 0 & 0 \\ 0.1573 & 0.0256 & -0.0023 & 0.1437 & 0.5511 & 0 & 0 \\ -0.5107 & -0.0840 & -0.0154 & 0.9989 & -0.5393 & 0 & 0 \\ 0.3830 & 0.0630 & 0.0057 & -0.5955 & 0.3356 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{k}_t \\ \tilde{G}_t \\ \tilde{D}_t \\ \tilde{c}_{t-1} \\ \tilde{\lambda}_{t-1} \\ \tilde{h}_{t-1} \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{pmatrix} 0.0751 \\ 0.0273 \\ 6.4765 \\ 0.3727 \\ 0.4931 \\ -1.7246 \\ 1.2206 \end{pmatrix} z_t$$

$$\begin{pmatrix} \tilde{y}_t \\ \tilde{s}_t \\ \tilde{g}_t \\ \tilde{i}_t \\ TGR_t \\ TCR_t \\ \tilde{T}_t \\ RDF_t \\ sh_t \end{pmatrix} = \begin{pmatrix} 0.3961 & 0.0646 & -0.0015 & 0.0949 & 0.3637 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 1.5282 & 0.2485 & 0.0379 & -2.1506 & 1.2662 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ 0.3278 & 0.0535 & -0.0012 & 0.2773 & 0.2626 & 0 & 0 \\ -0.0575 & -0.0095 & 0.0132 & 0.3830 & -0.3136 & 0 & 0 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{k}_t \\ \tilde{G}_t \\ \tilde{D}_t \\ \tilde{c}_{t-1} \\ \tilde{\lambda}_{t-1} \\ \tilde{h}_{t-1} \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{pmatrix} 1.3255 \\ 1.0909 \\ 1.0909 \\ 5.0034 \\ 1.0909 \\ 1.0909 \\ 1.0909 \\ -0.2581 \\ 1.0000 \end{pmatrix} z_t$$

Now using the above solutions we obtain the impulse response functions (IRFs) and the simulated data for our model economy with BSF in Essay 3.

# Vita

Ali Mohammad Al-Nadi was born in Jordan on September 5, 1970. He received his Bachelor in Economics from the Yarmouk University in 1992. In February 1993, Ali joined the Central Bank of Jordan as a research economist. He received his Master degree in Economics from the University of Jordan in 1997. In 2005, he promoted to a Senior Economist at the Central Bank of Jordan. Ali enrolled at the University of Tennessee in August, 2005. His doctoral degree would be received in May 2011. After the accomplishment of his PhD, the author will be joining the Central Bank of Jordan. The author is a member of the American Economic Association and the Sothern Economics Association. Ali is married to Fadwa Khwaileh and has two sons, Jawad and Ibrahim.