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Some Instances of Ic1/Ic5 Interaction in Post-Tonal Music (and Their *Tonnetz* Representations)

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SOME INSTANCES OF IC1/IC5 INTERACTION IN POST-TONAL MUSIC
(AND THEIR *TONNETZ* REPRESENTATIONS)

STEPHEN C. BROWN

In a prior article, I explored the role of interval classes 1 and 5 in Shostakovich's music, showing how they work together in a number of his works to shape both melodic and harmonic design.[1] Of late, other scholars have investigated the same phenomenon in the music of Copland and Ruggles.[2] Though composers have certainly worked with other interval-class pairings,[3] ic1 and ic5 do make an especially potent combination. For one thing, they are the only two interval classes capable of generating the aggregate (via the chromatic scale and the circle of fifths/fourths), and therefore can join to provide a uniquely effective means for traversing pitch space. On a related level, ic1 and ic5 play special roles as the basic intervallic generators of, respectively, chromaticism and diatonicism. In addition, ic1 and ic5 are the most sharply differentiated interval classes—representing the opposite poles of consonance and dissonance— and thus can foster the strongest of musical contrasts. Perhaps for some combination of these reasons, the pairing of ic1 and ic5 has appealed to many composers—far more than those mentioned above, including several central figures in twentieth-century music.

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[1]   Stephen C. Brown, "Ic1/Ic5 Interaction in the Music of Shostakovich," *Music Analysis* 28/2–3 (2009): 185–220.

[2]   David Heetderks explores ic1/ic5 interaction in Copland's *Quiet City* (see Heetderks, "A Tonal Revolution in Fifths and Semitones: Aaron Copland's *Quiet City*," *Music Theory Online* 17/2 (2011), accessible at <http://www. mtosmt.org/issues/mto.11.17.2/mto.11.17.2.heetderks.html>). Stephen P. Slottow and I have both independently discussed ic1/ic5 structuring in the music of Ruggles (see Brown, "Dual Interval Space in Twentieth-Century Music" [Ph.D. diss., Yale Univ., 1999], 143–161; Slottow, "A Vast Simplicity: Pitch Organization in the Music of Carl Ruggles" [Ph.D. diss., City Univ. of New York, 2001], 136–148; and Slottow, *A Vast Simplicity: The Music of Carl Ruggles* [Hillsdale, NY: Pendragon Press, 2009], 118–131).

[3]   For example, I have discussed the combination of ic1 and ic4 in Webern, as well as ic3 and ic4 in Schoenberg, in Brown, "Dual Interval Space in Twentieth-Century Music," *Music Theory Spectrum* 25/1 (2003): 35–57.

In the hope of further understanding and appreciating this recurring post-tonal practice, the following discussion offers a series of analytical examples drawn from the music of Stravinsky, Webern, and Bartók. One might add that all three composers are ones in whom Allen Forte has taken an abiding interest over the decades, having published more than once on each, and particularly extensively in the case of Stravinsky and Webern.[4] In fact, some of these passages considered below are ones that Forte himself has illuminated in his writings.

STRAVINSKY: THREE PIECES FOR STRING QUARTET

The second of Stravinsky's Three Pieces for String Quartet furnishes a clear example of ic1 and ic5 combining to underlie a passage of music. Quoted in Figure 1a, the entire opening passage dwells solely on the six pitch classes E♭, E, F, A♭, A, and B♭, among which a clear hierarchy emerges as the music progresses. Pitch class A functions as pitch center, owing largely to its reiteration as a pedal in the cello part; E plays a secondary role as "supporting fifth" to the A, a relationship cemented by the E–A–E–A ("V–I–V–I") melodic gesture in mm. 4–5 and 9–10. A and E also serve as the sole common tones linking the repeated, upbeat-downbeat chords: A remains constant in the cello part, while E moves back and forth between the viola and first violin. In sum, A and E serve as the "ic5 backbone" of the passage, relative to which we hear the remaining pitches as "ic1 auxiliaries": the first violin's F and viola's E♭ flank each other about E, while the second violin's B♭ and A♭ relate in the same manner to A.[5]

These relationships emerge more clearly with the help of the ic1/ic5 *Tonnetz* diagram shown in Figure 1b, a visual representation of what I term *ic1/ic5 space*. For ease of comparison, this and all subsequent ic1/ic5 diagrams follow the convention that moving one space to the right corresponds to an ascending half step or descending major seventh (i.e., T_1), while moving one space upward corresponds to an ascending perfect fifth or descending perfect fourth (i.e., T_7). As the diagram shows, the passage establishes a symmetrical, 2x3 region in ic1/ic5 space, with the cello and second violin parts inhabiting one ic1 layer, the first violin and viola parts another.[6] In addition, the alternating upbeat and downbeat tetrachords—each a member of set class 4-8 (0156)—occupy 2x2 squares in the space, intersecting symmetrically about the central A/E dyad (i.e., the ic5 backbone of the music).

At the end of the movement, the opening music returns in a shortened version, followed after a pause by a separate, two-measure concluding gesture, as shown in Figure 2a. This final gesture conveys a version of 4-6 (0127) that conspicuously omits both A and E. Thus the movement's ending initially suggests a tonal return (to the original hexachord with its underpinning A and E), but then subverts it in the final two measures. Figure 2b depicts these events in ic1/ic5 space, showing the total pitch-class content of the movement's last six measures and laying bare the ambivalence of Stravinsky's ending: although the final two measures undermine the A/E

Structure of Atonal Music (New Haven: Yale Univ. Press, 1973), 30. Forte succinctly shows that the second chord of each upbeat-downbeat pair is T_{11} of the first, and that the two chords share A and E as common tones, which are then isolated and highlighted in the melodic gesture of mm. 4–5. Rainier Sievers also notes that the pitch structure here can be understood in terms of the fifth A/E combined with chromatic adjacencies (see Sievers, *Igor Strawinsky, Trois pieces pour quator à cordes: Analyse und Deutung* [Wiesbaden: Breitkopf und Härtel, 1996], 79–81).

[6] Whenever a given set takes the shape of rectangle in a *Tonnetz*, its pitch content can be generated through the transpositional combination of two smaller sets, each formed by a partial interval cycle corresponding to one of the axes of the space. In this case, for example, the hexachord underlying the passage results from the operation (012) * 5—in other words, a three-note segment of a 1-cycle (i.e., three notes of a chromatic scale) transpositionally combined with a two-note segment of a 5-cycle (i.e., an ic5 dyad). For more on transpositional combination, see Richard Cohn, "Inversional Symmetry and Transpositional Combination in Bartók," *Music Theory Spectrum* 10 (1988): 19–42. For more on the connection between the *Tonnetz* and transpositional combination, see Brown, "Dual Interval Space in Twentieth-Century Music," 68–73.

FIGURE 1. Stravinsky, Three Pieces for String Quartet, No. 2, mm. 1–12

(a) score
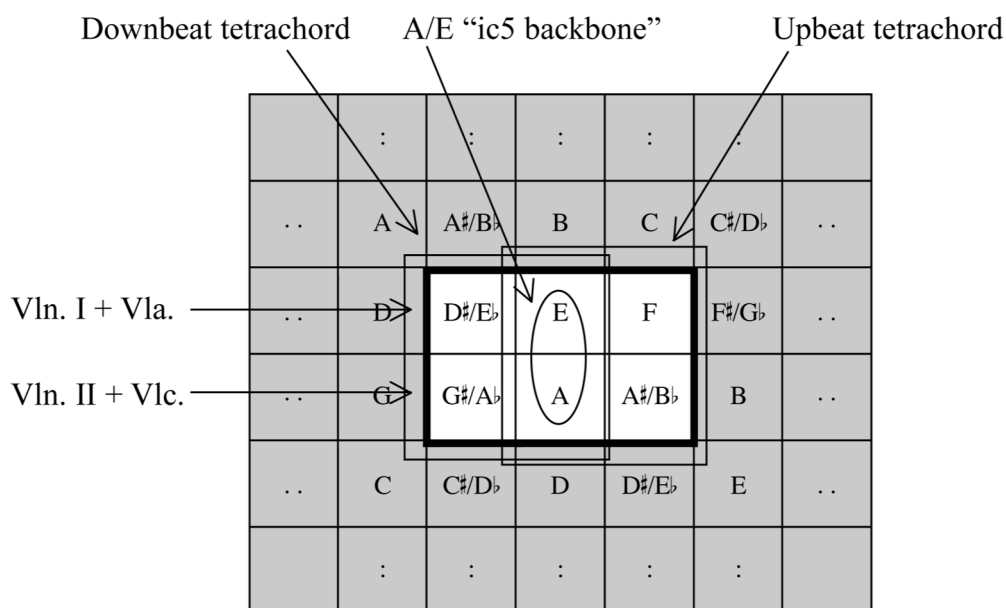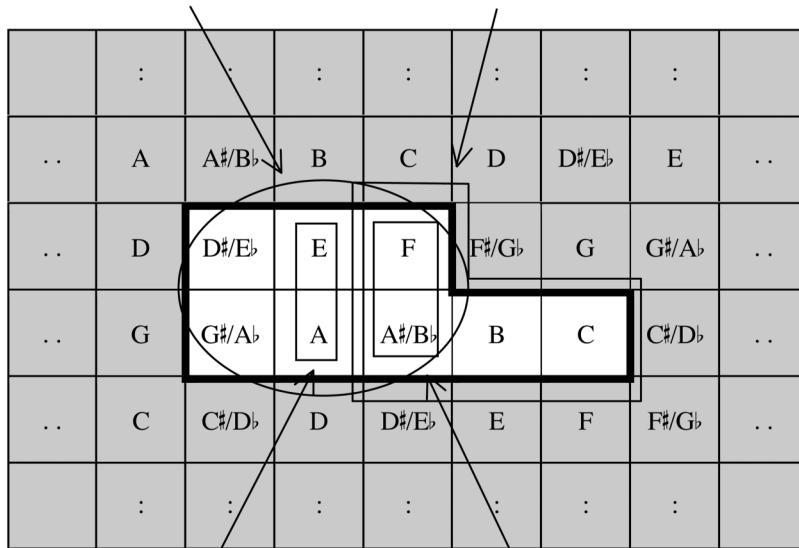

(b) depiction in ic1/ic5 space

FIGURE 2. Stravinsky, Three Pieces for String Quartet, No. 2, m. 56–end

(a) score

(b) depiction in ic1/ic5 space

Opening hexachord (restated in mm. 56–58) Final tetrachord (last two measures)

Initial A/E "ic5 backbone"   Final underpinning ic5 dyad B♭/F

backbone (and the symmetry about it), they nonetheless arise as a natural outgrowth of the preceding music. Pitch classes B♭ and F, which take over as the final ic5 underpinning for the movement, occupy the right-hand portion of the movement's initial 2x3 region and relate to A and E as ic1 adjacencies. Meanwhile, C and C♭ arise as a pair of T_1 extensions, trailing off

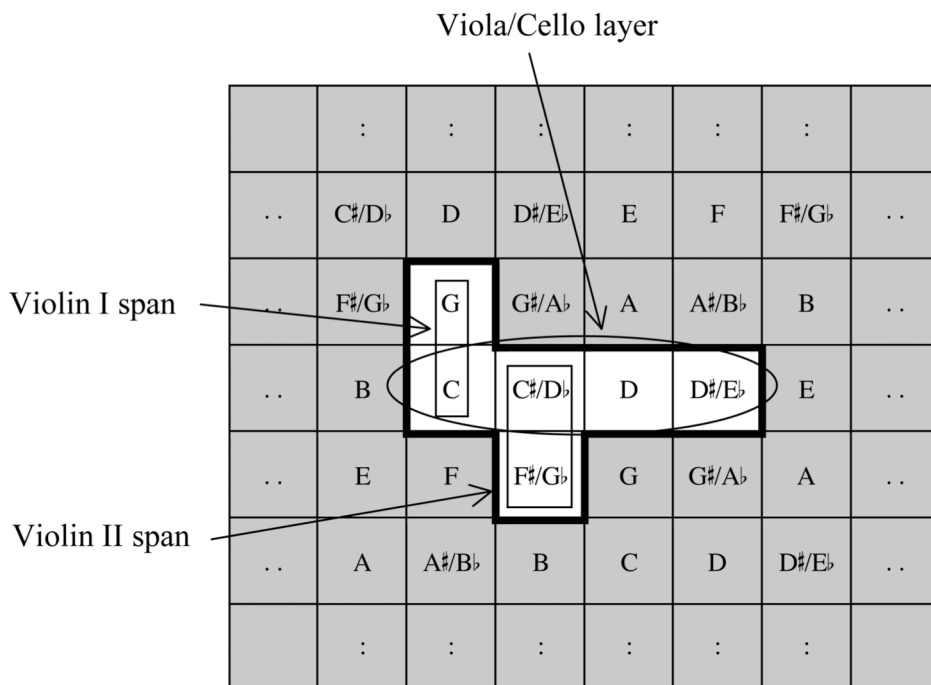**FIGURE 3.** Stravinsky, Three Pieces for String Quartet, No. 1, mm. 1–8

"rightward" in the space. Thus the movement's ending feels both unresolved—even imbalanced—and yet organically connected.

Ic1/ic5 structuring occurs in the other two movements as well. Consider the opening of the first movement, quoted in Figure 3. (Given the repetitive nature of the movement, this excerpt provides a good sense of its entirety and in fact captures its entire pitch content.) Together, the viola and cello express the chromatic tetrachord C–C♯/D♭–D–E♭, while the two violins each play *diatonic* tetrachords: G–A–B–C in the first, C♯–D♯–E–F♯ in the second. Each of the violins' tetrachords is therefore framed by an ic5 dyad, one of whose notes intersects with the viola/cello tetrachord. The pitch structure anchoring the first movement can therefore be depicted as the hexachordal region shown in Figure 4a.

Owing to the particular topography of ic1/ic5 space—specifically, the way the same pitch classes recur at different locations—this same hexachord can also be portrayed as the connected region shown in Figure 4b. This latter rendering has two virtues. First, the viola's D, present from the outset and sustained throughout the movement as an inner-voice pedal, now lies along the vertical center line of a nearly-symmetrical structure, while the cello's D♭ and E♭—which

FIGURE 4. Stravinsky, Three Pieces for String Quartet, No. 1

(a) underlying structure interpreted in ic1/ic5 space
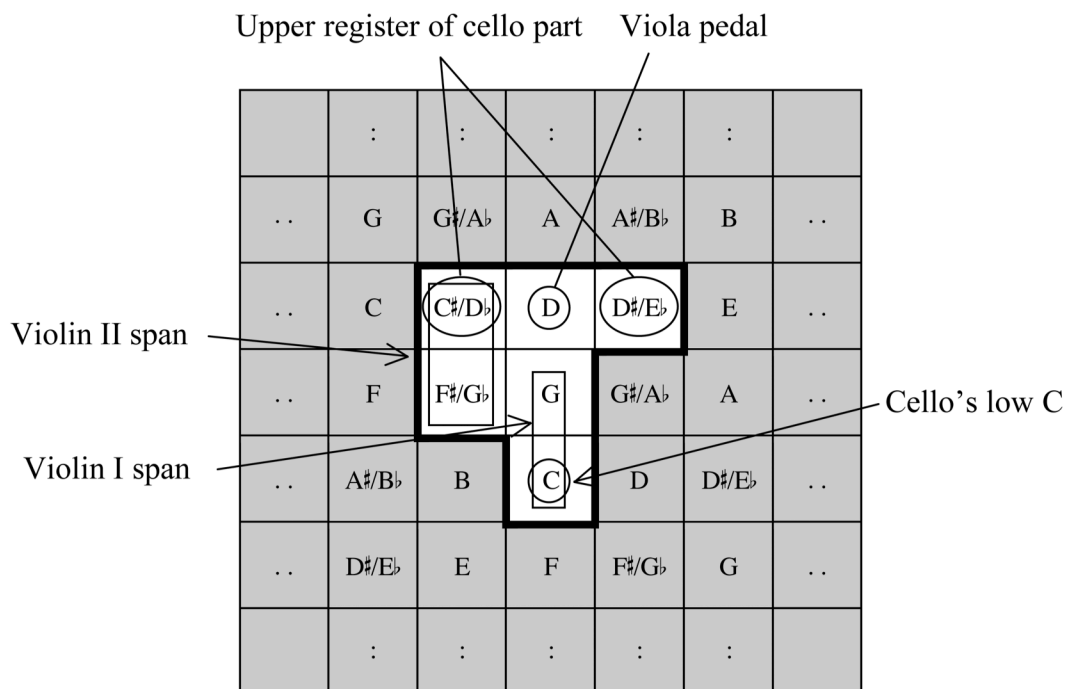

(b) another interpretation

FIGURE 5. Stravinsky, Three Pieces for String Quartet, No. 3, mm. 1–9

always accompany the viola's *pizzicato* Ds—symmetrically flank it on either side.[7] Second, this revisualization reveals the close relationship between this hexachord and the one underlying the beginning of the second movement: sliding the C in Figure 4b diagonally up to the right (i.e., to the A♭) produces the same rectangular region as in Figure 2b (and hence the same hexachord type, albeit at a different transposition level).[8]

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[7] The relationship here between the viola's D and the cello's D♭ and E♭ returns in a different guise at the outset of the second movement, where the second violin's A♭ and B♭ symmetrically flank the cello's A (likewise in a different register).

[8] For an alternative interpretation of this movement, see Pieter C. van den Toorn, *The Music of Igor Stravinsky* (New Haven: Yale Univ. Press, 1983), 147–154. Whereas my reading emphasizes the interplay of chromatic and

FIGURE 6. Stravinsky, Three Pieces for String Quartet, No. 3:
ic1/ic5 components of the chords in mm. 3–9

The final, chorale-like movement features ic1/ic5 activity in a more veiled form: most of its chords comprise an underlying ic1/ic5 "skeleton" that is "fleshed out" by one additional pitch class unrelated by ic1 or ic5. The first nine measures can suffice to illustrate; see Figure 5. Note first that the opening harmony fuses C♯-major and D-minor triads, with F (=E♯) serving as common third. The chord thus comprises two ic1-related ic5 dyads (C♯/G♯ and D/A) joined by E♯/F as a non-ic1/ic5-related "color note." The ic5 dyads together form a version of set class 4-8 (0156), the same ic1/ic5-based construction that figures so prominently in the beginning of the second movement.

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diatonic elements (and of course on a more basic level the interaction of ic1 and ic5, which can be understood as tokens or generators of chromaticism and diatonicism), van den Toorn contends that the movement involves a combination of diatonicism and octatonicism.

The ensuing music in mm. 3–9 presents ten distinct chords (several of which are repeated), numbered on the figure. Figure 6 diagrams these chords, using labeled brackets to highlight their ic1 and ic5 components. With the exception of Chord 9, each one consists minimally of an ic5 dyad played by the violins (in the form of a perfect fourth) linked by ic1 to one of the pitches in the lower two string parts. In the case of Chord 9, the lower strings play an ic5 dyad (in the form of a perfect fifth) linked by ic1 to one of the violins' pitches.[9] As a result, each chord either belongs to set class 3-4 (015) or 3-5 (016)—the two "trichordal tokens" of ic1/ic5 interaction—or includes 3-4 or 3-5 as a subset. These trichord types, in turn, comprise the only possible three-note subets of tetrachord 4-8, which forms the ic1/ic5 foundation of the movement's opening sonority (as just noted). Ic1/ic5 structuring thus provides a thread of inner continuity uniting all three of Stravinsky's outwardly contrasting movements.[10]

WEBERN: FIVE PIECES FOR STRING QUARTET, OP. 5, NO. 4

As a way into this oft-analyzed piece, let us begin with the boxed portion of the music shown in Figure 7 (extending from the last eighth note of m. 4 through the downbeat of m. 6). During this span, interval classes 1 and 5 clearly predominate.[11] The first violin melody begins

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[9] Another intervallic feature is almost as consistent: Chords 2 through 7 all contain ic4 in the lower strings, spelled as a major third or diminished fourth. For a network analysis of this passage predicated on the ic4/ic5 structure of the chords, see Shaugn O'Donnell, "Transformational Voice Leading in Atonal Music" (Ph.D. diss., City Univ. of New York, 1997), 85–94.

[10] Ic1/ic5 structuring also plays a role in some of Stravinsky's later, serial music. See Massimiliano Locanto, "'Composing with Intervals': Intervallic Syntax and Serial Technique in Late Stravinsky," *Music Analysis* 28/2–3 (2009), especially 230–234 and 242.

[11] In a discussion of this same passage, Richard Parks reaches a similar conclusion, but from a significantly different perspective (see Parks, "Pitch-Class Set Genera: My Theory, Forte's Theory," *Music Analysis* 17/2 [1998], 219–224). Interpreting the music with his version of pitch-class set genera, he argues that a complex genus generated by set classes 5-7, 6-5, and 7-7 best fits the set structure of the music, and that the sets in this genus "display high concentrations of ics 1 and 5" (224). Benjamin Boretz analyzes the piece through the lens of a "syntactical model" based on a chromatic chain of ic5 dyads (i.e., <{C, G}, {C#, G#}, {D, A} . . . >), thus obliquely acknowledging the primacy of ic1 and ic5 (see Boretz, "Meta-Variations, Part IV: Analytic Fallout (I)," *Perspectives of New Music* 11/1 [1972], 218–223).

FIGURE 7. Webern, Five Pieces for String Quartet, Op. 5, No. 4: beginning to middle of m. 6

with an ic5 dyad (C–F), followed by an ic1 dyad (D♭–C), followed by another ic1 dyad (D♮–C♯), this one ic1-related to previous one. The cello follows in canon with the first violin (two octaves lower and an eighth note later) and therefore conveys the same intervals. Meanwhile, the viola states the ic1 gesture F♯–G–F♯. As for the harmonic dimension, each chord in the passage belongs to either set class 3-4 (015) or 3-5 (016)—barring a single instance of 3-8 (026)—and

therefore contains *both* interval classes 1 and 5.[12] Moreover, we can interpret that errant 3-8 as resulting from a suspended D in the first violin, a sort of "post-tonal non-harmonic tone," and thereby regard it as *not* a full-fledged harmony within the context of the passage.[13]

The intervallic profile of this span clearly warrants an interpretation in ic1/ic5 space. As Figure 8 shows, the violin-and-cello melody forms a contiguous, L-shaped region in that space. This spatial rendering of the passage suggests a particular logic to the viola part: its F♯ and G fill in the "empty space" of this L shape, completing a symmetrical 2x3 region. This passage thus conveys the same hexachord type that we observed above in the opening of Stravinsky's second piece for string quartet (compare Figures 1b and 8)—and also by means of ic1/ic5 interaction. In this case, however, Webern presents the set in a more elaborate manner, and without a strong sense of pitch center.

Probing further into the harmonic structure of the passage, we note that because each verticality contains both ic1 and ic5 (barring the exceptional 3-8), each takes the form of a triangle in ic1/ic5 space. These ic1/ic5 trichords are numbered 1 through 6 within the boxed span of Figure 7. As Figure 9 details, a combination of flipping, rotation, and translation in the space can account for the progression from each trichord to the next.[14]

Turning now to the beginning of the movement, we observe that the initial, *tremolo* tetrachords in the violins (boxed in Figure 7) can be taken as "source harmonies" for the later

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[12] Forte noted the centrality of set classes 3-4 and 3-5 in this piece as early as 1964 (see Forte, "A Theory of Set-Complexes for Music," *Journal of Music Theory* 8/2 [1964], 173–179).

[13] The violin's D♮ can in fact be heard as a very traditional suspension. It enters during the middle of beat 2, where it forms part of a 3-4 trichord. Given the prevalence of 3-4 and 3-5 in the passage, these sonorities can be considered contextually consonant. The D therefore begins as a metrically weak "consonant preparation." It then holds into beat 3, where it forms part of the contextually dissonant 3-8 trichord, and finally "resolves" down by step, into a contextually consonant 3-5.

[14] Rotation can also be achieved by combining a diagonal flip with a horizontal or vertical one. For more on these flipping operations and their effects on pitch-class sets, see Brown, "Dual Interval Space in Twentieth-Century Music" (2003).

FIGURE 8. Webern, Five Pieces for String Quartet, Op. 5, No. 4: end of m. 4 to the beginning of m. 6 (boxed portion of Figure 6), interpreted in ic1/ic5 space
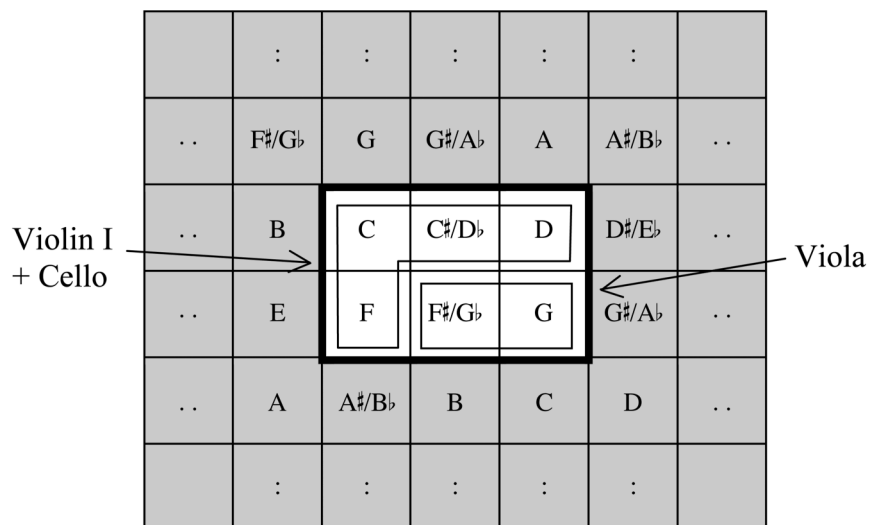
FIGURE 9. Webern, Five Pieces for String Quartet, Op. 5, No. 4: m. 4, beat 4 to the downbeat of m. 6 (boxed portion of Figure 7), with trichordal relations interpreted as flips, rotations, and translations in ic1/ic5 space

1 to 2: flip about a vertical axis running between F and F♯
2 to 3: rotate 90° clockwise about D♭
3 to 4: rotate 180° about D♭
4 to 5: translate one space to the left (= T_{11})
5 to 6: flip about a vertical axis running through F♯ and C♯
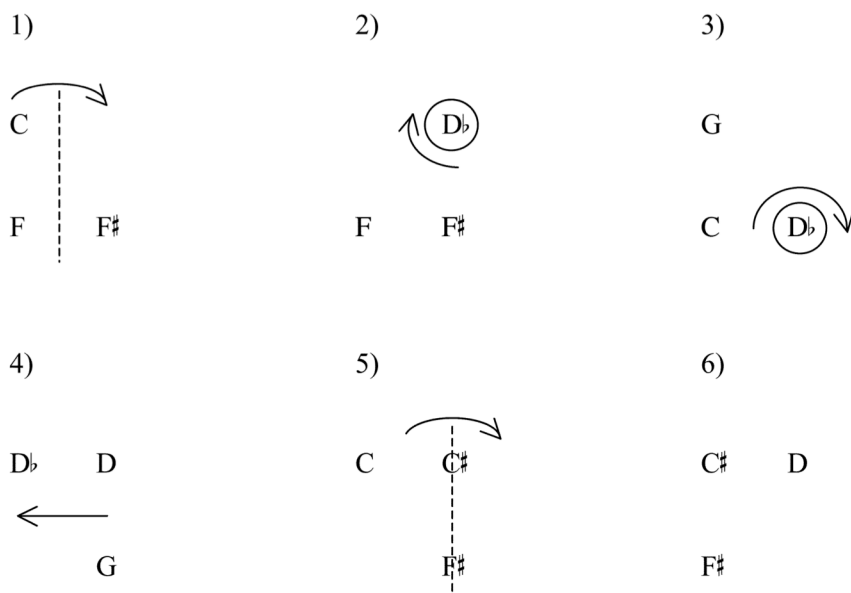
FIGURE 10. Webern, Five Pieces for String Quartet, Op. 5, No. 4:
violin tetrachords in mm. 1–2, depicted in ic1/ic5 space

(a) m. 1 (set class 4-8)

(b) m. 2 (set class 4-9)

trichords. The first tetrachord, set class 4-8 (0156), contains among its four trichordal subsets two members each of 3-4 (015) and 3-5 (016), while the second, set class 4-9 (0167), comprises four instances of 3-5. On a more basic level, however, ic1/ic5 interaction serves as a deeper source for both these tetrachords and the subsequent trichords; whereas each trichord unites single instances of ic1 and ic5, the tetrachords combine two instances of each.

As Figure 10 shows, both tetrachords can be interpreted as contiguous regions in ic1/ic5 space. The initial tetrachord, 4-8, occupies a square region and thus opens the piece by expressing symmetry with respect to ic1 and ic5. The following tetrachord, 4-9, results from the E of the first chord ascending to F♯, which skews the symmetry of the 4-8 into a zigzag region. (The E–F♯ motion occurs in the top register of the first violin part and then is echoed by the viola at the end of m. 2.) This spatial interpretation suggests a rationale for the cello's E♭ (the only other pitch class in the opening two measures). As Figure 11 illustrates, the E♭ not only augments the connected area established by the opening tetrachords but also symmetrically offsets the F♯, forming a larger region that is balanced about the same vertical axis as is the opening 4-8.

Comparing Figures 8 and 11, we discover that both musical spans occupy hexachordal ic1/ic5 regions confined to two adjacent ic1 layers. In fact, as Figure 12 shows, all of the music in mm. 1–6 can be understood as staking out a rectangular region in ic1/ic5 space, measuring two units "tall" and five "wide," with the music mostly conveying a gradual "rightward drift" within the space.[15] The measures thus complete a larger, symmetrical ic1/ic5 structure, generated by a five-note chromatic segment transpositionally combined with an ic5 dyad, or (01234) * 5.

Ic1/ic5 structuring also underlies the music of mm. 11–12 (which rounds out the movement with a modified reprise of the boxed music in Figure 7). But the two remaining portions of the movement relate more tangentially to ic1/ic5 design. First is the seven-note ascending

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[15] The 2x5 region depicted Figure 12 is essentially a more compact—and arguably more flexible and readily apprehensible—version of Boretz's model for the same excerpt. See note 11 above and Boretz, "Meta-Variations, Part IV," 221. In the extensive analytical literature on this piece, the sets identified in this section are typically ones that occupy compact, connected subregions of this 2x5 area. More specifically, each of these sets tends to occupy a rectangular zone, or a rectangle combined with one or two semitonal adjacencies. In addition to Forte's "A Theory of Set-Complexes for Music" and "Aspects of Rhythm in Webern's Atonal Music," see for example David Beach, "Pitch Structure and the Analytic Process in Atonal Music: An Interpretation of the Theory of Sets," *Music Theory Spectrum* 1 (1979): 7–22; Patricia Hall, "Letter to the Editor," *Music Theory Spectrum* 4 (1982): 163–167; Richard A. Kaplan, "Transpositionally-Invariant Subsets: A New Set-Subcomplex," *Intégral* 4 (1990): 37–66; and David Clampitt, "Ramsey Theory, Unary Transformations, and Webern's Op. 5, No. 4," *Intégral* 13 (1999): 63–93.

FIGURE 11. Webern, Five Pieces for String Quartet, Op. 5, No. 4:
pitch content of mm. 1–2 interpreted in ic1/ic5 space

Opening tetrachord

E to F♯ motion in viola and
upper register of violin I

Balancing E♭ in the cello

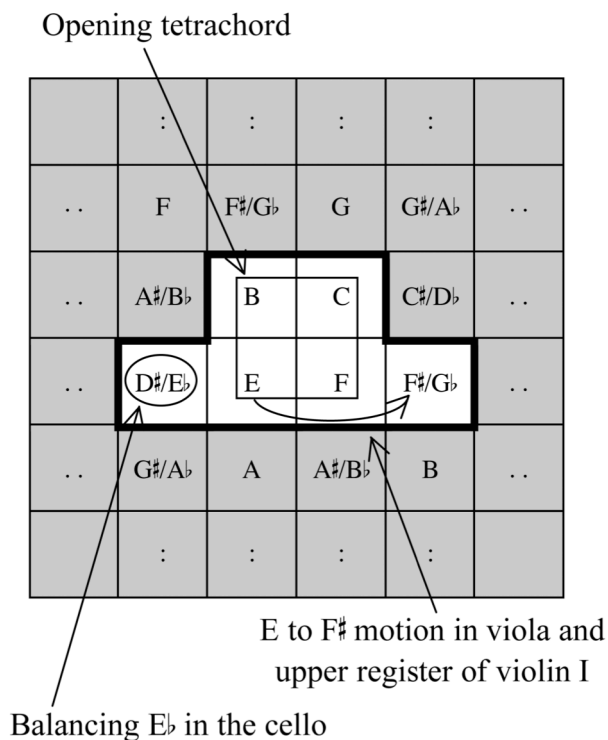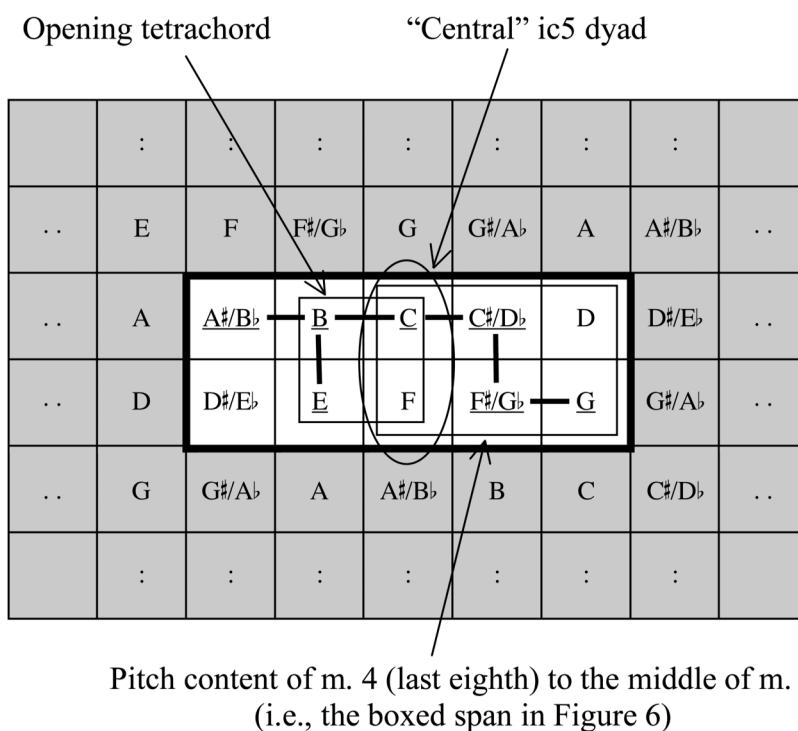**FIGURE 12.** Webern, Five Pieces for String Quartet, Op. 5, No. 4:
entire pitch content of mm. 1–6 interpreted in ic1/ic5 space
NB: The pitch classes of the second violin's ascending seven-note gesture at the end of m. 6
are underlined and form a connected subregion of the 2x5 area

Opening tetrachord   "Central" ic5 dyad

Pitch content of m. 4 (last eighth) to the middle of m. 6
(i.e., the boxed span in Figure 6)

melody played by the second violin at the end of m. 6 (see Figure 7), which returns later at two different transposition levels. Interval classes 1 and 5 do not stand out strongly within this fleeting gesture: its intervallic adjacencies feature a single instance of ic5 (F♯–B) and no instances of ic1. Yet as Figure 12 shows, the melody's pitch classes stem entirely from the 2x5 region of ic1/ic5 space established in the preceding measures, and even form a connected subregion of that area. Thus in at least a limited sense, the melody could be understood to emerge as a byproduct of ic1/ic5 activity.

The middle section of the movement, shown in Figure 13, also downplays interval classes 1 and 5: the viola locks into an augmented-triad ostinato and therefore plays ic4 exclusively, while the first violin's melody almost entirely expresses ic3 and ic4. The passage nonetheless can be interpreted from the standpoint of ic1/ic5 interaction. For example, the viola's constant ic4 motion can be taken as resulting from a combination of ic1 and ic5 (5–1=4); thus the viola's trichord assumes the form of a diagonal in ic1/ic5 space, as depicted in Figure 14. The remaining pitch content of the passage constitutes a five-note subset of a hexatonic collection (set class 6-20 (014589)), which can be depicted as an adjacent zigzag area forming a "crooked parallel" to the viola's region. Altogether, the pitch content of the middle section inhabits a symmetrical ic1/ic5 zone, but in this case the symmetry takes place not about a horizontal or vertical axis, but rather about *diagonal*, tritone axis, as marked on the figure: it is as if Webern creates intervallic contrast in this section by rotating his initial, ic1/ic5-oriented framework by 45 degrees.[16] The ic1/ic5 structuring that pervades most of the piece can thus be understood to exert at least an indirect influence in those sections where ic1 and ic5 are less prominent.

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[16] The idea of diagonals in ic1/ic5 space, and their relevance for this passage, suggests a connection with Forte's early discussion of this piece. He notes that the difference between the crucial trichords 3-4 and 3-5, as well as between tetrachords 4-8 and 4-9, boils down to a swapping of interval classes 4 and 6—none other than the intervals corresponding to the diagonals in ic1/ic5 space. See Forte, "A Theory of Set-Complexes for Music," 174–176.

FIGURE 13. Webern, Five Pieces for String Quartet, Op. 5, No. 4: mm. 7–9

FIGURE 14. Webern, Five Pieces for String Quartet, Op. 5, No. 4: mm. 7–9 depicted in ic1/ic5 space
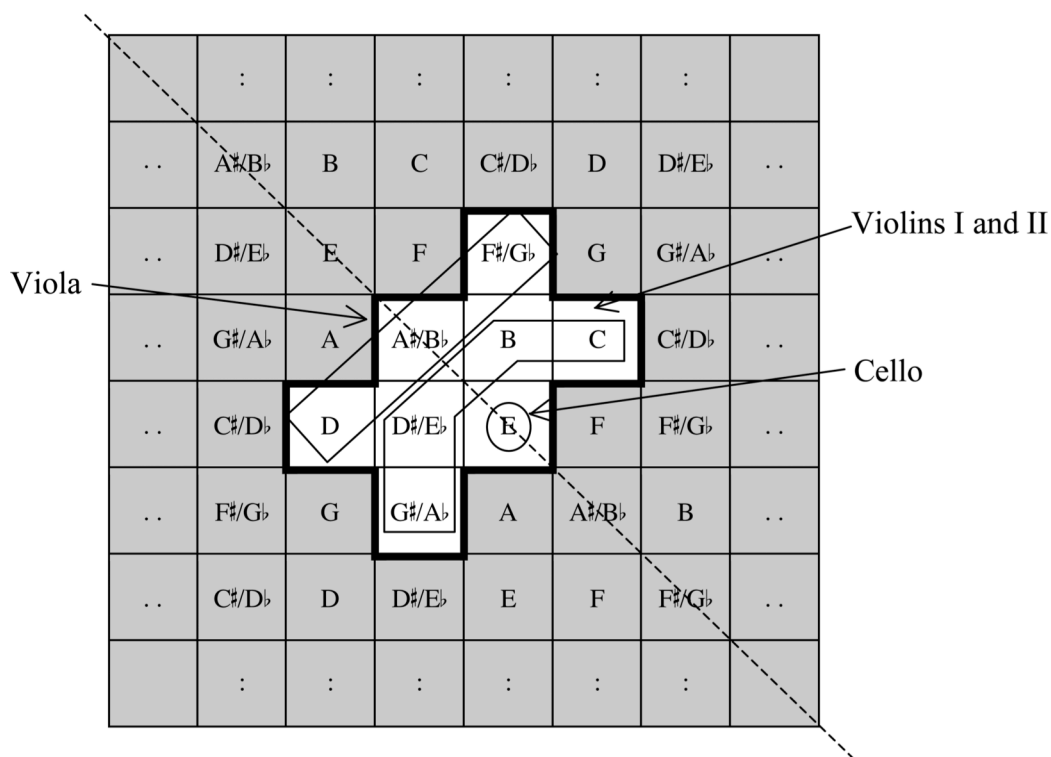
BARTÓK: "MINOR SECONDS, MAJOR SEVENTHS" (*MIKROKOSMOS* NO. 144)

For a final example, let us turn to a work by Bartók: "Minor Seconds, Major Sevenths," from *Mikrokosmos* (No. 144). Although the title of this piece obviously signals the importance of ic1, ic5 also plays a major role throughout—in fact, often equal to that of ic1. As a result, the piece presents a veritable treasure trove of ic1/ic5 relationships, which we can explore by considering four spans of music that together are highly representative of the piece as a whole.[17]

Figure 15 quotes the first span, extending from the beginning to the downbeat of m. 8. Within this span, the first two measures introduce the primary generative idea of the piece—its *Grundgestalt*. As illustrated in Figure 16, this *Grundgestalt* is solidly based on ic1/ic5 inter-action. In the first measure, the two hands wedge outward three times, chromatically saturating a perfect-fourth span from F♯ up to B. Ic5 thus serves as the initial frame, while ic1 provides the means for filling it in. Both interval classes are also conspicuous on the musical surface, in the form of the repeated melodic gestures B♭–B♮ (right hand) and G–F♯ (left hand), together with the vertical F♯/B stated three times as the long part of the short-long rhythmic figure.

In m. 2, the wedge recedes by one semitone on each side, shedding F♯ and B to form a four-note chromatic span from G up to B♭.[18] Meanwhile, G♯ and A, which form the inner core of this span, each "sprout" ic5-related notes, E♭ and D, which *themselves* form an ic1 dyad, expressed as a major seventh (which is then isolated for emphasis in the second half of the measure). The right-hand part of Figure 16 depicts this web of ic1/ic5 relationships. Crucial to

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[17] Early recognition of the significance of ic1/ic5 combinations in Bartók came in the form of Ernő Lendvai's "1:5 distance model." See Lendvai, *Béla Bartók: An Analysis of His Music* (London: Kahn and Arverill, 1971).

[18] Among Bartók analysts, a four-note chromatic span such as this is often referred to as "set X" or an "X cell," a designation originating with George Perle. See Perle, "Symmetrical Formations in the String Quartets of Béla Bartók," *Music Review* 16 (1955): 300–312; Leo Treitler, "Harmonic Procedure in the Fourth Quartet of Béla Bartók," *Journal of Music Theory* 3/2 (1959): 292–298; and Elliot Antokoletz, *The Music of Béla Bartók: A Study of Tonality and Progression in Twentieth-Century Music* (Berkeley: Univ. of California Press, 1984).

FIGURE 15. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): mm. 1–8

FIGURE 16. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): ic1/ic5 structuring in mm. 1 and 2
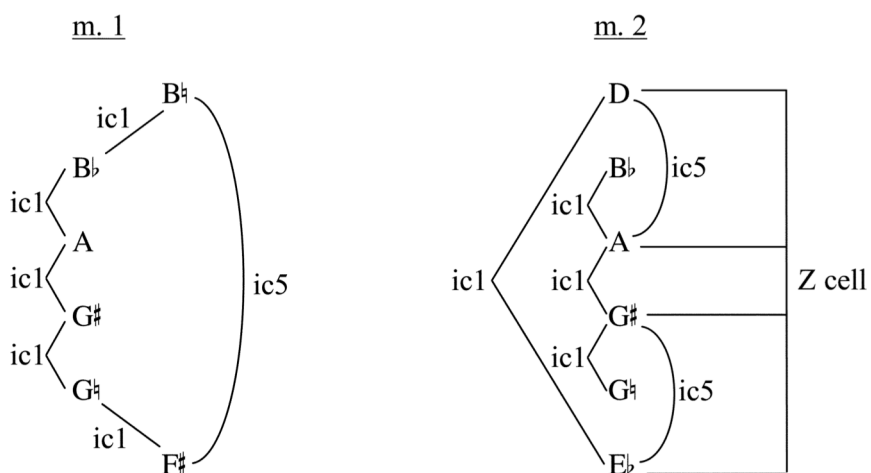
note here is that the inner and outer ic1 dyads combine to form an instance of 4-9 (0167), a Bartókian "Z cell" in its characteristic formation of two perfect fourths linked by a half step in the middle.[19] The entire chord of m. 2 thus comprises the ic1/ic5-generated Z cell E♭–G♯–A–D, whose inner ic1 dyad (G♯/A) is augmented by two chromatic adjacencies (G♮ and B♭).[20]

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[19] The term "Z cell" was coined by Treitler (1959) and figures prominently in Antokoletz's work on Bartók (Antokoletz 1984).

[20] Or to put it another way, the entire chord could be interpreted as a fusion of X and Z cells.

FIGURE 17. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144):
two excerpts from the last section: mm. 61–63 and final two measures

FIGURE 18. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144):
examples of ic1/ic5 structuring in the final section
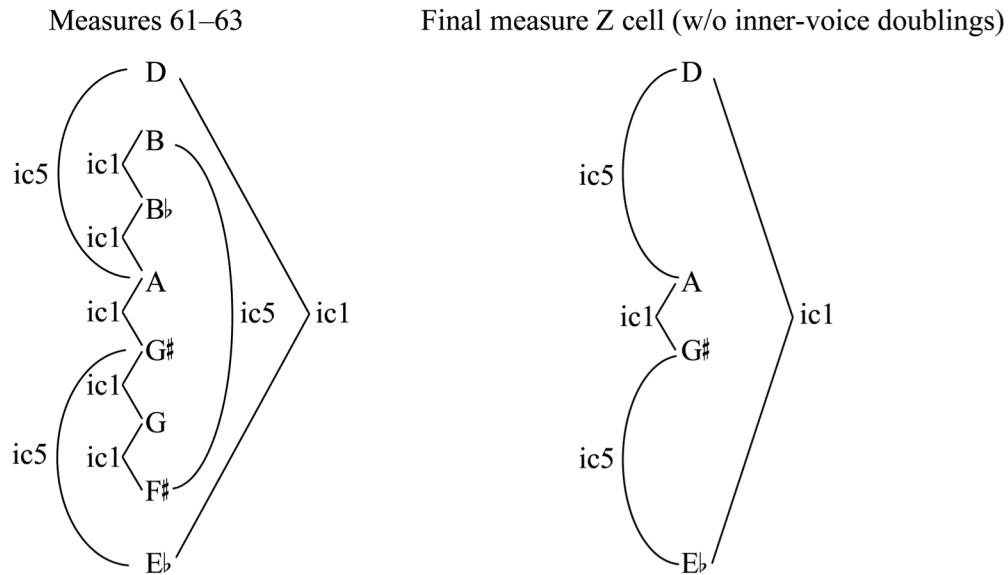
Before considering the rest of mm. 1–8, let us briefly skip to the end of the piece, which is entirely based on the *Grundgestalt*. Figure 17 quotes two representative portions of this section: m. 61 to m. 63, beat 3, and the final two measures. Measures 61 and 62 feature a varied reprise of m. 1, in a version that clarifies and emphasizes its wedge aspect. Measure 63 then

FIGURE 19. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144), mm. 1–8:
the major-seventh dyads gradually stake out a 2x5 region in ic1/ic5 space

presents a modified and expanded version of m. 2. The chord here is the same as in m. 2, except

that it is now an octave higher, arpeggiated, and its inner chromatic stretch is lengthened by a

semitone above and below (i.e., F♯ and B are not omitted as they are in m. 2). The result is the

expanded network of ic1/ic5 relationships shown in Figure 18. In the last two measures of the

piece, Bartók pares away the inner embellishing chromatic notes, ending the piece with a

revoiced version of the Z cell, as diagrammed on the right-hand side of Figure 18.

Returning to the opening of the piece, we observe that mm. 3–8 are pervaded by modified

repetitions of the initial *Grundgestalt* (in mm. 3–4, 5, and 6). Emerging as an outgrowth of this

Grundgestalt is a progression of major-seventh dyads played by the right and left hands. Figure

19 isolates and interprets these dyads. We first hear D/E♭, which is paired with G/A♭ in mm. 4–5.

FIGURE 20. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144), mm. 18–24

(a) score

(b) underlying Z cell and its ic1/ic5 basis
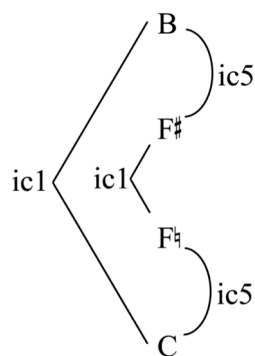
Since these dyads are ic5-related, they occupy a 2x2 region in ic1/ic5 space, as depicted in the lower half of the figure. In mm. 5–6, C/D♭ enters the mix. Since this dyad in turn relates by ic5 to G/A♭, the three ic1 dyads conveyed thus far—all present in mm. 5–6—combine to form a 2x3 region. The progression culminates in mm. 6–8, where the lower line of the right-hand part now plays a complete black-key pentatonic collection, each of whose pitches is paired with the white-key pitch a major seventh above. (The left hand, meanwhile, plays a subset of these dyads.) Since the pentatonic collection derives from a five-note span of the circle of fifths, mm. 6–8

therefore complete a 2x5 region in ic1/ic5 space: a natural fulfillment of the intervallic processes leading up to it.

Shown in Figure 20, our second span also develops the ic1/ic5 relations of the *Grundgestalt,* but now in a different way. Consider the first portion of this span, from m. 18 to the downbeat of m. 21. Whereas the original *Grundgestalt* begins with the two hands together filling in the perfect fourth from F♯ up to B, during these measures the right hand completes the same span by itself, while the left hand follows suit, a tritone lower. The passage therefore chromatically elaborates a Z cell comprising the notes C–F♮–F♯–B (spelling upward), which is then highlighted as a "cadential harmony" on the downbeat of m. 21. Thus, while m. 1 presents a single ic5 span filled in by means of ic1, this passage simultaneously presents two ic5 spans completed in this manner. Moreover, because these ic5 spans combine to form a Z cell in its characteristic formation, they are linked both internally and externally by ic1 (in the form of the interior F/F♯ and the exterior C/B). After Bartók lays bare the Z cell on the downbeat of m. 21, he then elaborates it via two upward runs: the first stretches from F♯ up to F♮, the second from C up to B; thus the terminal notes combine to express the Z cell. After this brief interruption, the right- and left-hand melodies remain within the confines of their respective perfect-fourth zones, and thus continue to operate within the bounds of the underpinning Z cell.

Our third span follows directly on the heels of the second, and features the most overt ic1/ic5 structuring in the piece; see Figure 21. On the downbeat of each measure (and the end of the $\frac{3}{8}$ measure), the two hands each play ic5 dyads in the form of perfect fifths, while in the middle of each measure, they play ic1 dyads in the form of major sevenths. In each case but one, the paired dyads involve different pitch classes. (The one exception occurs in the latter half of m. 29, where the two hands both play D♯/E.) As a result, the two hands combine to form a series of seven distinct tetrachords, numbered on Figure 21. Figure 22 provides ic1/ic5 network diagrams

FIGURE 21. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): mm. 25–33

FIGURE 22. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): connected ic1/ic5 networks for each of the vertical tetrachords in mm. 25–33

for each of these tetrachords. As it shows, each tetrachord is connected (in the graph-theoretical sense) through some combination of ic1 and/or ic5 moves. For these particular tetrachords, this property does not obtain with any other pair of interval classes.

Given their intervallic basis, interpreting the tetrachords in ic1/ic5 space can reveal significant relationships. For example, consider the pivotal harmonic motion connecting the previous span (mm. 18–24) to this one. As noted above, the earlier section is underpinned by a Z

FIGURE 23. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144):
partial inversion in ic1/ic5 space—in this case modeled by flipping about a horizontal axis—
relates the tetrachord on the left (which underpins mm. 18–24) to the one on the right
(which initiates the next section, starting in m. 25)
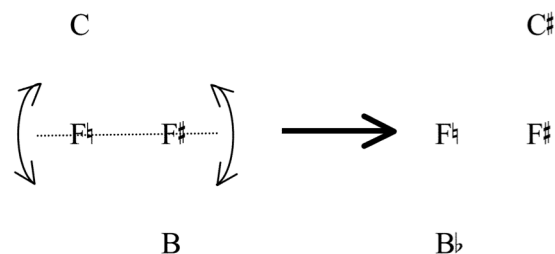
cell comprising the pitch classes C–F♮–F♯–B. This tetrachord can be interpreted as the "zigzag"

shape in ic1/ic5 space shown on the left side of Figure 23. Now compare this tetrachord with its

successor: the prominent downbeat chord in m. 25 inaugurating the next section of the piece (i.e.,

Chord 1 in Figures 21 and 22). The new chord likewise comprises two ic5 dyads, one in each

hand, linked by the same internal ic1 dyad (F/F♯). Thus in moving from the first tetrachord to the

second the F/F♯ dyad remains invariant, while the ic5 dyads invert from perfect fourths to perfect

fifths. In other words, the first chord partially inverts into the second, an operation I call *partial*

inversion (logically enough), and which in this case can be modeled by flipping about a hori-

zontal axis in ic1/ic5 space running through the invariant F/F♯, as shown in Figure 23.

The ic1/ic5 design of the resulting tetrachord (i.e., Chord 1) suggests the potential for

hexatonicism: spelling upward, it features the beginnings of a 7/1 combination cycle (B♭–F♮–F♯–

C♯), which if extended to two more pitch classes, would generate the complete hexatonic collec-

tion B♭–F♮–F♯–C♯–D–A. This incipient, ic1/ic5-derived hexatonicism does in fact come to

fruition as the music progresses. In the second half of the section, the downbeat tetrachord

repeats several more times, now a minor third lower (see Chord 3 in Figure 21, mm. 29–32).

Meanwhile, the major-seventh dyads in the two hands wedge inward, converging on the chord

G–F♯–B–A♯, spelling upward (Chord 7 in Figure 21). Figure 24 interprets the entire culminating

FIGURE 24. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144):
the entire culminating sonority achieved in m. 32 completes an ic1/ic5-generated hexatonic collection,
which can be depicted as a zigzag region in ic1/ic5 space
NB: The dotted line on the figure shows the axis of interval-exchange symmetry
about which the two tetrachords relate
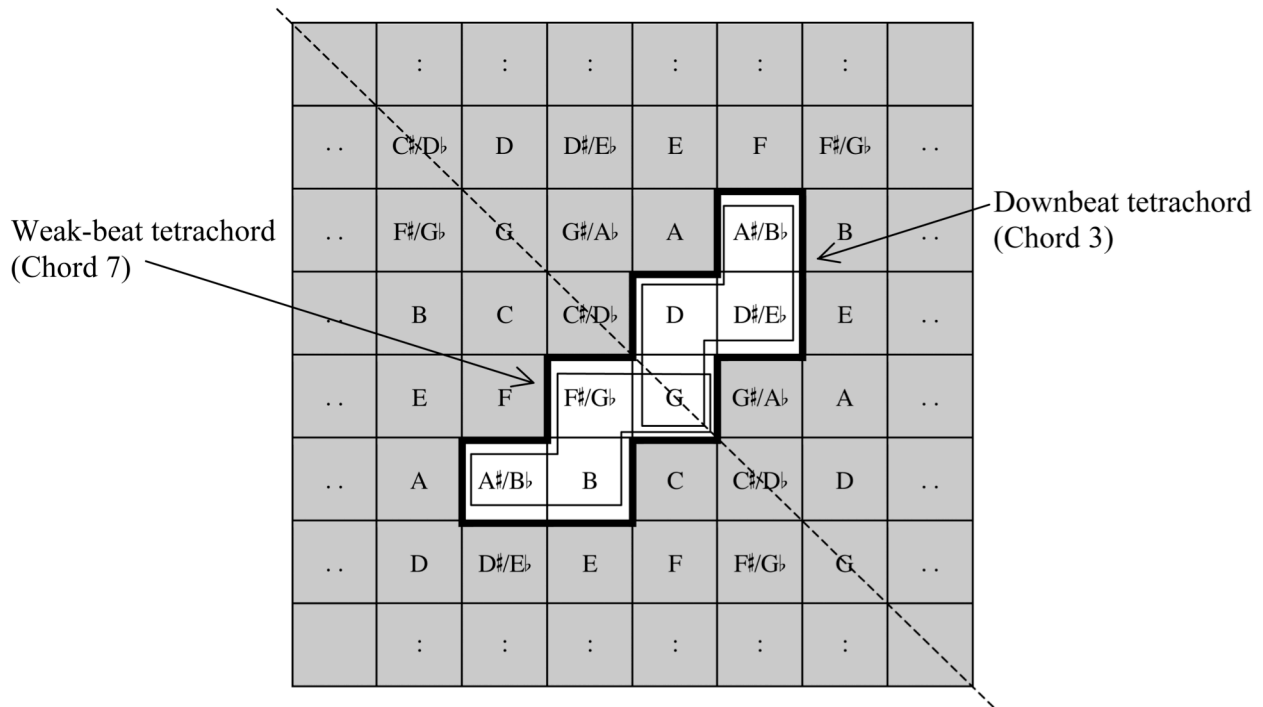
sonority reached in m. 32, combining the downbeat and weak-beat chords (i.e., Chord 3 plus Chord 7). As it shows, the zigzag pattern extends to encompass all six pitch classes of a hexatonic collection. The figure also highlights a crucial relationship between the two tetrachords: the downbeat tetrachord (i.e., Chord 3) comprises two ic5 dyads linked by ic1, whereas the weak-beat tetrachord (Chord 7) consists of two ic1 dyads linked by ic5. The two chords therefore relate by the operation of *interval exchange* (literally, an exchange of intervals), which can be modeled by flipping not about a horizontal or vertical axis, but rather about a *diagonal* axis. In this case, the diagonal axis runs NW–SE through the central G, which serves as the common bass note of both chords.

The ic1/ic5-generated hexatonicism we witness here reaches its culmination a few measures later, in our fourth and final span, shown in Figure 25. Starting in the middle of m. 40, the left hand plays a four-note, descending gesture (derived from the lower half of the *Grundgestalt*), which it then sequences downward twice in a row, as shown on the figure. The right hand answers each of these left-hand statements with an upward gesture, which comprises an inversion of the left-hand part, followed by a fifth, "extra" note (marked with parentheses). Each of these "extra" notes can be interpreted as anticipating the right-hand's following note (see the dotted arrows); owing to the pattern's cyclical nature, the right-hand's final D would also serve as an anticipation, were the sequence to continue (hence the parenthesized, backward-pointing arrow on the figure).

Figure 26 reduces the passage by isolating the descending perfect fourths framing each of the three left-hand gestures, together with the corresponding, ascending perfect fourths in the right hand. As the figure shows, the music is underpinned by a series of three Z cells, each new one a major third lower than the previous one. Together these three Z cells complete the aggregate; thus the passage expresses a deeper and more systematic level of chromaticism (thanks to the aggregate completion of its underlying framework), in addition to the more superficial chromaticism conveyed by the local (012) segments in the right and left hands.[21]

But consider finally the intervallic processes driving the passage. For instance, the pitches of the left-hand's underpinning perfect fourths together convey an entire 7/1 interval cycle, as shown on the first line beneath the staff in Figure 26. (If the pattern were to continue, it would

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[21] In terms of transpositional combination, this aggregate completion results from the operation 4-9 (0167) * 3-12 (048). Bartók's completion of the aggregate in this passage—involving as it does multiple transpositions of a single set type—resonates with Forte's 1960 discussion of set complementation in the third movement of the String Quartet No. 4, which engages similar procedures but with different set types (see Forte, "Bartók's 'Serial' Composition").

FIGURE 25. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): mm. 40–41

FIGURE 26. Bartók, "Minor Seconds, Major Sevenths" (*Mikrokosmos* No. 144): reduction and interpretation of the pattern underpinning mm. 40–41

return to its initial C♯, as shown on the figure.) The left hand thus features an ic1/ic5-based pattern that completes a hexatonic collection, specifically $HEX_{0,1}$. (The perfect fourths of the right hand meanwhile complete the complementary collection $HEX_{2,3}$.) Observe also the pattern formed by the initial pitches in the alternating left- and right-hand gestures (C♯, D, A, etc., as shown at the bottom of Figure 26). These pitches convey essentially the same interval cycle, but now starting with T_1 rather than T_7. Thus on another, simultaneous level, the music features an underlying ic1/ic5-based pattern completing a different hexatonic collection, in this case $HEX_{1,2}$. In sum, this brief passage represents a true culmination of the processes we have traced in this

discussion: a fusion of hexatonicism, aggregate completion, and Z-cell usage, all stemming from and united by the ic1/ic5 interaction pervading the piece.[22]

—◆—

Comparing the analyses presented here reveals some interesting connections along with some notable differences. On a general level, of course, all three pieces feature fundamental musical processes involving the interaction of two specific interval classes, namely ic1 and ic5; these processes take place on a basic level of musical structure and transcend set class and cardinality. The pieces differ, however, in the extent to which they are driven by these processes. In the Stravinsky, all three pieces display significant ic1/ic5 structuring, and yet the work as a whole also contains stretches that do not involve conspicuous ic1/ic5 organization (especially some parts of the second movement) or stretches where that organization comprises just a facet of a more elaborate structure (as in the first movement, where an ic1/ic5 design furnishes the underpinning that links the diatonic scale segments in the two violin parts). In the Webern, the outer sections of the movement are saturated with ic1/ic5 relationships, but the middle section largely revolves around other interval classes—though in this case the intervallic relationships can to an extent be interpreted as *derived* from ic1/ic5 interaction (through a 45 degree rotation of ic1/ic5 space). Bartók's piece, finally, is most suffused with ic1/ic5 relationships, to the point where it could truly be dubbed a "study in ic1 and ic5."

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[22] For an interesting alternative reading of the piece, see Edward Gollin, "Some Unusual Transformations in Bartók's 'Minor Seconds, Major Sevenths,'" *Intégral* 12 (1998): 25–51. Gollin engages a number of relationships in the music—especially ones involving exchanges of ic1 and ic5—using the machinery of Klumpenhouwer networks along with the operations "Q" and "X" introduced in David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale Univ. Press, 1987), 251–253. Notably, Q and X involve transformations of pitch classes within *octatonic* collections. Gollin's interpretation thus contrasts with the present reading, which focuses on ic1/ic5 relationships unmediated by octatonic transformations, and indeed highlights certain *hexatonic* patterns rather than octatonic ones.

More specific comparisons reveal other interesting correspondences. As noted earlier, the passage centering around m. 5 of the Webern and the opening measures of the second Stravinsky piece both present set class 6-z6 (012567) by means of ic1 and ic5, such that they can both be understood as staking out 2x3 regions in ic1/ic5 space (compare Figures 1b and 8); yet the Webern does so more elaborately and non-centrically. For another example, compare mm. 1–6 of the Webern (Figure 12) with mm. 6–8 of the Bartók (Figure 19). Both excerpts inhabit 2x5 regions of ic1/ic5 space, but here the regions are 90-degree rotations of each other (or flipped about a diagonal axis relative to each other); in other words, the musical processes relate via interval exchange.

Also significant is that all three pieces feature different kinds of symmetries involving ic1 and ic5—not just traditional transpositional and inversional symmetry, but also symmetry with respect to the operations of partial inversion and interval exchange, which can be modeled via horizontal, vertical, or diagonal flips in ic1/ic5 space. And finally, all three pieces involve processes and relationships that do not necessarily involve voice-leading parsimony. In much Neo-Riemannian and transformational music theory, the *Tonnetz* is upheld largely, if not primarily, as way to model relationships involving smooth voice leading. Yet as the analyses here have sought to demonstrate, invoking the *Tonnetz* free of any "parsimony agenda" can provide a useful means for interpreting a significant subset of the post-tonal repertoire.

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ABSTRACT

Drawing on the author's Dual Interval Space methodology, this essay uses a *Tonnetz* model to explore the interaction of interval classes 1 and 5 in pieces by Bartók, Stravinsky, and Webern. The analyses attest to the importance of ic1/ic5 pairing for a significant range of post-tonal music, while illuminating processes and relationships that transcend the standard pitch-class operations. In addition, the analyses demonstrate that a generalized *Tonnetz* model, freed from its customary associations with voice-leading smoothness, can serve as a compelling means for interpreting post-tonal music.

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