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Self-Organized Criticality Studies in Carbon Fiber Reinforced Polymer Matrices

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Self-Organized Criticality Studies in Carbon Fiber

Reinforced Polymer Matrices

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 $\bar{\zeta}$

Abstract

Self-Organized Criticality Studies in Carbon Fiber Reinforced Composite Matrices. Ben Rogers (University of Tennessee, Knoxville, Tennessee 37919) S. Simunovic (Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831).

Self-organized critical (SOC) behavior is exhibited by systems ranging from earthquakes to fluctuations in the stock market and is characterized by critical events occurring on all time and length scales after some critical state has been established. Classifying a phenomenon as SOC gives scientists a better foundation for describing and understanding many of the underlying principles of the process. The field of Self-Organized Criticality theory has led to insights in many areas of research. Based on the original theories of Bak, Tang, and Wiesenfeld, and expanded by numerous other studies, SOC models have given researchers valuable tools for exploring the behavior of complex systems. The work presented here explores the SOC behavior of the fracture properties of carbon fiber reinforced composite materials. Materials with randomly oriented fibers and materials with braided carbon fibers were subjected to laboratory tests (crushing) and the results were analyzed for possible SOC behavior patterns. In both types of materials evidence has been found to suggest that the progressive fracture follow SOC patterns. Establishing an SOC pattern of behavior in material fracture is an important step toward our goal of developing predictive stochastic finite element models.

Introduction:

This project grew out of my time as an undergraduate researcher at the Oak Ridge National Laboratory in Oak Ridge Tennessee. Working under the supervision of Dr. Srdan Simunovic I was asked to study self-organized criticality (SOC) and determine if such behavior was present in the fracture of composite materials. The goal of this research was to determine if carbon fiber reinforced polymer matrices possessed a characteristic behavior pattern termed selforganized critical behavior. It was our belief that the evidence of such a characteristic would appear in the patterns of various measurable quantities tracked during the fracture of the material. Those quantities include force with respect to time, force with respect to displacement, and the normalized amount of energy released during fracture. To measure these quantities several tests

were run under controlled conditions. Three main testing conditions were explored, dynamic drop testing, static load testing, and constant velocity load testing. The resulting experimental data was analyzed by computer programs developed specifically for this purpose. Results from the analysis of the experimental data were studied and their relationship to SOC systems was explored.

The evidence collected shows that there is very good reason to believe that carbon fiber composite materials do exhibit SOC behavior when subjected to dynamic, static, and constant velocity loading under the testing conditions. This discovery will help with the eventual goal of developing a stochastic, predictive model of composite material behavior.

Background:

In August of 1987, Per Sak, Choa Tang, and Kurt Wiesenfeld submitted a paper to the journal, PhYSical Review A, detailing an idea they felt explained an unexplored principle of dynamic, dissipative systems. They termed the idea selforganized criticality. The paper argued that the appearance of scale invariant (fractal) structure is one of the fingerprints of self-organized criticality. Additionally it described a sand-pile model that simply and elegantly illustrates the complex behavior of a dynamic system reaching its critical state. [1]

A self-organized critical system is one that has attained a critical state without specific initial conditions that promote the formation of such a state. The critical state is one such that small imbalances in the system can cause large

disruptions or rearrangements. Critical systems are not in balance, but are metastable. However, they possess multiple metastable states. They can be arranged in infinite ways that are closer or further away from true stability. As one part of the system changes it can trigger changes in far distant parts of the system. The sand-pile model helps to visualize this state of metastability and change.

Imagine picking up a handful of sand and letting it slip one grain at a time through your fingers onto a flat table. Initially there will be no long-range connection of grains of sand. When one grain is dropped it stays in the same area it entered. As more and more sand piles up, the slope of the sand-pile reaches a critical state. When the critical state is reached adding more sand causes the already present sand to shift or slide down the sloping sides. This is the critical state, the point at which any changes to the system begin to force rearrangements of the surrounding portions of the system. Now that the sandpile is high enough to have reached its critical state continue dropping one grain at a time onto the pile and picture what is going on where the grain of sand lands.

As the grain of sand lands on the slope of the pile it can either be caught and stay where it landed, or it can cause a rearrangement or avalanche. The key image for understanding the mechanism for rearrangement is the avalanche. If only a few grains of sand shift when the new one has been added it caused a small avalanche. If it causes the entire sand-pile to shift it created a very large avalanche. The size of the avalanche created depends on the proximity of the metastable state to true stability. Avalanches in other systems can take on many forms. In plate tectonics an avalanche would be analogous to an earthquake. In materials fracture it would be crack formation and propagation.

Another important feature of the sand-pile model is the stationary state that is developed in the critical region. The average slope of the sand-pile and the average amount of sand in the system (on the table) are fixed, yet the slope and amount of sand are continually changing. This is the stationary state within the metastable system. The model is robust enough to incorporate changes in the type of input to the system. If a different type of sand is used it may cause a change in the slope of the pile but not in the long-term dynamics. This ability to adapt to changes in input was one of the keys to applying the self-organization idea to real world systems. [2]

For application to the materials science field it was clear that more specific criteria could be necessary to have a system develop into a self-organized state than were present in the sand-pile situation. There are, in fact, four distinct criteria for self-organized criticality in materials, the first three of which apply to all systems. First, like in the sand-pile model a system must possess a stationary state with global conservation laws. Second, there must be long-range correlations present in the system, meaning that changes in one part of the system can affect other distant parts of the system. Third, there must be local rigidity in the system, meaning that energy or mass can accumulate locally in the system before being dissipated. [3] The final criterion for SOC behavior in materials is the presence of annealed disorder. [4] Annealed disorder is a condition that shows bond weakening due to nearby fracture. Essentially, it

means that if a bond breaks at a nearest neighbor site, damage is done to the bonds around it. This damage weakens the bonds and can cause them to break as well. After determining the necessary criteria for SOC behavior, we began testing to find if our chosen materials possessed the proper criteria and if they would exhibit SOC behavior in the crush testing experiments.

Experimental Method:

The testing of our samples of carbon fiber composite materials consisted of three types of test, dynamic drop testing, static loading, and constant velocity crush testing. There are two different machines for testing, one being the drop tower and the other being the intermediate strain rate device. The drop tower is a device with rail guides to control the fall of a large weight. The carbon fiber composite tubes are attached to the bottom of the weight and are released. The large weight crushes the tube onto a pressure plate that records the forces exerted against it during impact. The tower has stops so that the tower drop mass does not impact the pressure plate. This device is for constant energy testing.

Constitutive Modeling of Composite Materials for Impact Analysis

Figure 1: Computer generated image of a drop tower test 1

The intermediate strain rate device is a more complex in that is capable of generating variable forces on the composite tubes. This device is computer controlled to produce the static loading and constant velocity loading data. There is a feedback loop that informs the computer when more force is required and it is applied to the tube. For static loading the tube is placed in the machine and subjected to a force capable of crushing the tube. For constant velocity testing the tube is attached to the machine and is then crushed against the pressure plate at constant speed. This device is for constant displacement testing.

The results of each test are recorded as force versus time. The data files are then edited to remove portions of the curve that do not involve the fracturing

¹ Image from Dr. Srdan Simunovic Oak Ridge National Laboratory

of the material. Generally this involves removing the initial loading portion of the curve and the final unloading portion. The edited data is then transformed from the US unit system to the metric unit system. The force vs. time data is then manipulated in conjunction with other data from the testing to produce force vs. displacement data. From the force vs. displacement data calculations are performed by Matlab© programs developed for the purpose to generate energy release data and statistical measures to explore the fractal and self-organized critical nature of the data sets. The Matlab© code can be found in Appendix A.

The statistical measures determined by the Matlab© program include the number of drops of size x where x varies from the smallest drop to the largest drop. This measure is taken for the data sets of force-displacement, energy dissipated and in some cases for energy-time as well. Once the drop size and frequency have been found the fractal dimension is calculated. The fractal dimension is the slope of the log-log plot of drop size vs. drop frequency. The final measure taken is of the normalized energy drops or the percentage of energy dissipated by fracture during testing. The statistical significance of these measures is that if the system obeys the SOC rules it should have characteristic "fingerprints" in these measured quantities.

Results:

Drop Tower Testing

The results are separated into three sets by the testing procedure. First, the drop tower testing results. The drop tower testing used materials of several **configurations, braided fibers or random fibers, thick or thin tubes, and square or circular tubes. The composites tested were all combinations of each set of possibilities.**

Figure 2: Unedited force vs. time data from drop tower crush test of a braided circular thin tube

Figure 3: Edited force vs. time data from same drop test

The fractal dimension of the tested materials is an indication of the fractality of the force vs. displacement data. If the data has a fractal dimension higher than one it possesses some self-similar characteristics. Comparisons of the fractal dimensions for the various tests and the appearance of SOC behavior in the normalized energy calculations suggest that the magnitude of the fractal dimension is a good indication of SOC behavior. The average fractal dimension of the tests that appear to exhibit SOC behavior is 1.55 with a maximum of 1.84 and a minimum of 1.23. The average fractal dimension of those tests that do not appear to have SOC behavior is 1.42 with a maximum of 1.57 and a minimum of 1.18. Based on these results values of the fractal dimension under 1.2 would indicate non-SOC behavior and fractal dimensions greater than 1.6 would indicate SOC behavior. The numbers inside that range would not clearly indicate either.

Figure 4: Fractal Dimensions of Force vs. Displacement from Drop Tower **Testing**

Figure 4 above shows the calculated values for the force vs. displacement data from the drop tower testing. Figure 5 below is a typical normalized energy drop result for the drop tower method of testing. Of the eight samples tested in this

manner six showed SOC patterns and two did not. Figure 5 is an example of one of the tests that shows SOC patterns while Figure 6 is an example of a test that does not exhibit SOC behavior patterns. The rest of the normalized energy calculations for drop tower testing can be found in Appendix B. In Figure 5 the large number of drops that dissipated less that 10% of the energy in the system and the smaller and smaller amounts of drops that dissipate larger amounts of energy is indicative of SOC behavior. You can immediately see the difference in Figures 5 and 6. Figure 5 shows a steady decrease in the number of drops as energy dissipated increases. Figure 6 shows a large drop from small drop sizes but then levels off as size increases. The appearance of very large drops such as the <90% drop in Figure 5 is most likely an artifact of the initial fracture of the material where it is not clear if the critical state has been attained. The progressive fracture is almost certainly a critical phenomenon, but the first fracture may not be part of the critical regime as it is very often the largest fracture occurring in the system.

Figure 5: Normalized Energy Drop Graph for Circular Thick tube in drop tower test

Figure 6: Normalized Energy Drop Graph from Random Square Thick tube in Drop Tower Test

Static Testing

The results from the static testing are similar to those from the drop tower testing. Of the four tests conducted under static loading conditions three exhibit SOC behavior and one does not. The fractal dimensions of the static testing are shown in Figure 7. Figure 8 is a typical force vs. displacement data set from the static testing. Figure 9 is a typical normalized energy data set from the static testing.

The observations that were made with regards to the drop tower testing apply to the static testing as well. The two testing configurations produce very similar data, especially for those samples that have braided carbon fibers. The braided carbon fiber composites all exhibit SOC behavior where as half of the randomly oriented carbon fiber materials do not show the SOC patterns.

Thin Random Composite 1.78279 . . . **Figure 7: Fractal DimenSions of Force** vs. **Displacement data from Static Loading Testing**

Figure 9: Normalized Energy Drop Graph for Braided Thin Composite from Static **Testing**

Dynamic Constant Velocity Testing:

The constant velocity testing did not yield the same type of results as the drop tower and static testing. Where as the other two testing methods produced better than two thirds of the results with SOC behavior, only half of the constant velocity testing showed SOC characteristics. There is no data on what shape and composition materials were used for the constant velocity tests and therefore it is impossible to make any reasonable conclusions based on those tests. The data will be included in Appendix C.

Conclusions and Recommendations:

There is good reason to believe that the progressive fracture of carbon fiber reinforced polymer matrices is a self-organized critical system. The data collected throughout this research project shows that in over 66% of the tests run the process exhibited self-organized critical characteristics. This is a major step toward developing a stochastic, predictive model for the fracturing process. However, it is also clear that more testing must be done in order to be certain that the results of this research are not a statistical aberration. Running several trials on each individual material configuration would be very useful as you would be able to develop an average fractal dimension number for circular, square, thick, thin, random, and braided materials. Having this average fractal dimension for comparison to any future tests could be invaluable. The further testing would also be very important in quantifying how important and accurate

and indicator of SOC behavior the fractal dimension would be. As is stands now the fractal dimension of the force vs. displacement data for each test is at best a clue as to the SOC behavior of the material.

As regards the constant velocity testing data, without any indication as to the type of material used in the testing it is impossible to make any guess as to why there was a much lower rate of occurrence of SOC behavior. If further testing shows the same pattern as that done to date, it would suggest that the mechanism of fracture is different in the constant velocity situation than in either the drop tower or static testing. This is possible, but it seems more likely to me that the pattern will be linked to a separate variation in the testing. It is my belief based on my observations and study of the results from all tests, that the main cause is in the carbon fiber deposition. Every confirmed braided sample that was tested showed SOC behavior where as the randomly oriented sample had only 50% with SOC behavior. Based on this observation it is my belief that the pattern of fiber deposition in the material will have a great impact on the appearance of SOC behavior in the materials fracture. It is my belief that materials with randomly oriented fibers have a good chance of showing "quenched" disorder whereas the braided materials will almost certainly have "annealed" disorder. As was stated earlier "annealed" disorder is a prerequisite to SOC behavior in materials fracture. Since the orientation of the fibers inside the material can have a drastic impact on the materials mechanical properties, it is reasonable to assume that the orientation of the fibers inside the material will have an effect on the type of disorder observed. Further testing is necessary to examine the validity of my hypothesis based upon the research conducted so far.

References

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Bak, Per How Nature Works, Springer-Verlag New York 1996

"Local Rigidity and Self-Organized Criticality for Avalanches" Cafiero, Loreto, Vespignani, Zapperi, EuroPhysics Letters January 1995

"Self-Organization and Annealed Disorder in a Fracturing Process" Caldarelli, Tolla, Petri, Physical Review Letters September 1996

Appendix A

Matlab_© Code

%Ben Rogers %ORNL

% This program is designed to determine the fractal dimension of composite fiber tube %crush test data. The data analyzed is to be presented in a text file of two columns. % The first column is to be displacement in meters and the second column is to be %force in Newtons. $\frac{0}{0}$ $\frac{6}{6}$ % The data file is to be saved as a .m file in Matlab. The file name should be

%of the form testtestnumber.m (example test00000.m)

%Clearing all variables and closing all plots

clear all close all

%Initializing counters

 $j=0;$ $q=0;$

% The first step is to load the data into Matlab memory so that it can be manipulated %It is important to remember to change the loaded variable name every time you run the %program

fprintf('Before running this program check to be sure that the file is properly named\n') fprintf('and contains the appropriate data. The file should have two columns.\n') fprintf('First displacement in meters, second force in Newtons. Filename should be of the form\n') fprintf('test00000.m If a file name is not of this form it will not work. $\ln \ln \ln$ ') name=input('Please input the name of the .m file to analyze. Be sure to include .m\n>>','s');

```
%%%%%%%%%%%%% DO NOT FORGET TO CHECK THIS POINT %%%%%%%%%%%%%%%
```
load (name);

data=input('Please retype the file name without the .m at the end\n\n>>');

%This will generate a plot of the raw data.

 $figure(1)$ $plot(data(:, 1), data(:, 2), 'b-');$ xlabel=('Displacement (m)'); ylabel=('Force (N)');

% This will determine the slope at every point in the data set

```
for i = [1:1:length(data) - 1]slope(i) = (data(i+1,2) - data(i,2)) / (data(i+1,1) - data(i,1));end
```

```
% This will deterimine the local minima and maxima in the data set 
lastslope=O; 
r=0;
 for r=[1:1:length(slope)-1]if slope(r)\geq0
      lastslope=l;
```

```
elseif slope(r)<O 
    lastslope=-l; 
 else 
    lastslope=lastslope; 
 end 
  if slope(r+1) > 0currentslope=1;
  elseif slope(r+1) < 0
    currentslope=-l; 
  else 
    currentslope=O; 
  end 
  if lastslope== I & currentslope==-l 
    locmax(1+j)=r+1;j=j+1;elseif lastslope==-l & currentslope== 1 
    locomin(1+q)=r+1;q=q+1;else 
     lastslope=lastslope; 
  end 
end
```
% This will find the smallest drop so that it may be used as the initial measure of size %with a minimum size of one

```
for h = [1:1:length(locmin)]difference=data(locmax(h),2)-data(locmin(h),2); 
 smalldrop(h)=difference; 
end
```

```
sizestart=min(smalldrop);
if sizestart <= 1 
  sizestart=1;
else 
  sizestart=round(sizestart);
end
```
%This portion of the code is intended to find the average slope for the buildup in stored energy %Ieading to fracture in the crush testing. The average slope is then used as the basis for %calculations of the energy dissipated during fracture.

```
%Need to find way to make this parameter automatically enterable 
elasticmax00=1;
elasticmax=5;
```

```
upslope(1)=(data(locmax(1),2)-0)/(data(locmax(1),1)-0);
for k=[2:1:elasticmax]upslope(k)=(data(locmax(k),2)-data(locmin(k-1),2))/(data(locmax(k),1)-data(locmin(k-1),1));end
```

```
%Calculating the average upslope 
avslope=sum(upslope)/length(upslope);
slopefactor(1)=upslope(1)*(data(locmax(1), 1));
for i=[2:1:length(upslope)]slopefactor(i)=upslope(i)*(data(locmax(i), 1)-data(locmin(i-1), 1));slope2(i)=slopefactor(i)/(data(locmax(5),1));end 
avslope2=sum(slope2);
```

```
alpha=tan(avslope)*(180/pi);
beta=90-alpha;
```
%Calculating the angle for energy calculations

```
% Taking two points from the data set, a local maxima and the following local minima two triangular areas 
%are calculated using the two points as heights and a slope equal to the average slope calculated above. 
% The two areas are subtracted smaller from larger and the difference is the energy dissipated in fracture 
%by the crush test 
for p=[ 1:1: length(locmin)] 
  downslope = (data(locmin(p), 2) - data(locmax(p), 2))/(data(locmin(p), 1) - data(locmax(p), 1));rat=(l/downslope); 
  angle(p)=atan(rat)*(180/pi);
end 
energy(1)=0;
for i=[1:1:length(locmin)]height!=data(locmax(j),2);
   height2=data(locmin(j),2); 
   base I =height I /avslope; 
   base2=height2/avslope; 
   area 1=(1/2)*base 1*height 1;area2=(1/2)*base2*height2;energy(j+1)=area1-area2;areatotal(j)=areal; 
end 
energy=energy'; 
areatotal=areatotal';
```
%Getting fractal dimension of energy drops

```
%Will need to adjust this number to correspond to the greatest height drop 
sizemax=max(energy);
x=[1:10:sizemax];
for i = [1:1:length(x)]sizec=x(i);sizesumc=0;
  for h=[1:1:length(energy)]difference=energy(h);numc=difference/sizec; 
   numc=floor(numc);
    sizesumc=sizesumc+numc; 
    drop(h)=difference;
  end 
  totalc(i)=sizesumc; 
end
```

```
drop=sort(drop);
basis=3; 
i=basis; 
ylast=l; 
y=1;
z=0;
num=O; 
power=[basis basis^2 basis^3 basis^4 basis^5 basis^6 basis^7 basis^8 basis^9 basis^10 basis^11 basis 12
basis^13 basis^14 basis^15 basis^16 basis^17 basis^18 basis^19 basis^20 basis^21 basis^22 basis^23
basis/\24 basis/\25 basis"26 basis/\27 basis/\28 basis"29 basis/\30 basis/\31 basis/\32 basis/\33 basis/\34 
basis"35 basis"36 basis/\37 basis/\38 basis/\39 basis/\40 basis/\41 basis/\42 basis/\43 basis/\44 basis"45 
basis\sqrt{46} basis\sqrt{47} basis\sqrt{48} basis\sqrt{49} basis\sqrt{50} basis\sqrt{51} basis\sqrt{54} basis\sqrt{55};
while i<=sizemax
  if y>length(drop)break 
  end 
  while y>=ylast & y<=length(drop)
    if drop(y) \leq inum=num+l; 
       y=y+1;
       continue 
     else 
       ylast=y; 
       i=i*basis; 
       z=z+1;
       count(z)=num;num=O; 
       continue 
     end 
  end 
end 
if y>=length(drop)
   z=z+1;
   count(z) = num;end 
logx = log(x);logy=log(totalc); 
 figure(2) 
 D=polyfit(logx,logy,1);
D=-D(1,1);loglog(x,totalc,'k')
 title('test00010 Energy Drops')
 fprintf('Energy Drops Fractal Dimension - D= %g at %g maximum step size\n',D,sizemax) 
 for i=[1:1:length(count)]counter(i, 1) = power(i);counter(i,2)=count(i);end 
 counter 
 figure(3) 
 loglog( counter(:, 1), counter(:, 2), 'b*-')title('test00010 Drop Count Plot')
```
%Getting a Fractal Dimension for the Force vs. Displacement Graph %Clearing Variables that will be used again clear drop clear counter clear power clear count clear logy clear logx % This number will need to be adjusted to correspond to the largest drop in the data set sizemax=max(smalldrop); $x=[1:50:sizemax];$ % These numbers define a search region to find the best set of maximum/minimum lrangemax= 100; Irangemin=40; groupsize=IO; %Calculating the height difference between maxima and minima for $i=[1:1:length(x)]$ sizea=x(i); sizesuma=O; for $h=[1:1:length(locmin)]$ differencea=data(locmax(h),2)-data(locmin(h),2); Irange=0.5*differencea; lrange=floor(lrange); if lrange>lrangemax lrange=lrangemax; elseif lrange<lrangemin lrange=lrangemin; end p=h; for u=[1: 1 :groupsize] $if (h+u)>\nleq h (locmin)$ continue end diffa=data(locmax(h),2)-data(locmin(h+u),2); if $locomin(h)-locmin(h+u)$ lrange diffa=O; end if diffa > differencea differencea=diffa; $p=p+1$; end end numa=differencea/sizea; numa=floor(numa); sizesuma=sizesuma+numa; drop(h)=differencea; end totala(i)=sizesuma; end

%Graphing the Log of the drop size versus the log of the number of drops

 $logx = log(x);$ logy=log(totala); figure (4) loglog(x,totala) title('testOOOIO Force Displacment Fractal Dimension') D=polyfit(logx,logy,1); $D=-D(1,1);$ fprintf('Force Displacement Fractal Dimension - D= %g at %g maximum step size\n',D,sizemax)

```
%%%%%%%%%%%%%%%%%%%% Normalized Energy %%%%%%%%%%%%%%%%%%%%% 
     %%%%%%%%%%%%%%%% Total Area %%%%%%%%%%%%%%%%%%%%
```

```
clear totalareacounter 
clear count 
for h=[2:1:length(energy)]normE2=energy(h)/(areatotal(h-1));
  normenergy2(h-1)=normE2;end 
normenergy2=sort(normenergy2'); 
basis=O.l; 
i=basis; 
x=[basis:basis: I]; 
y=l; 
z=0;
ylast=O; 
num=O; 
while i<=1if y>length(normenergy2) 
     break 
  end 
  while y>=ylast & y<=length(normenergy2)
    if normenergy2(y)=1normenergy2(y)=.999999;
    end 
    if normenergy2(y) \leq inum=num+l; 
       y=y+1;
       continue 
    else 
       ylast=y; 
       i=i+basis; 
       z=z+1;
       count(z)=num; 
       num=O; 
       continue 
    end 
  end 
end 
if y>= length( normenergy2) 
   z=z+1;
   count(z)=num;end
```

```
for i=[1:1:length(count)]totalareacounter(i, l) = x(i);totalareacounter(i,2)=count(i);end 
totalareacounter
```
Appendix B:

Test Results from Drop Tower and Static Testing

Appendix C:

Testing Results from Dynamic Constant Velocity Testing

