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An Experiment in Difference Thresholds for the Visual Perception of Flexible Objects

Christopher H. Holloman
University of Tennessee - Knoxville

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SENIOR PROJECT - APPROVAL

Name: Christopher Holloman

College: Arts and Sciences Department: Psychology

Faculty Mentor: Dr. Stephen Handel

PROJECT TITLE: An Experiment in Difference Thresholds for the Visual Perception of Flexible Objects

I have reviewed this completed senior honors thesis with this student and certify that it is a project commensurate with honors level undergraduate research in this field.

Signed: Stephen Handel, Faculty Mentor

Date: 10 May 1999

Comments (Optional):

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Christopher H. Holloman

Faculty Advisor: Dr. Stephen Handel

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Abstract

Subjects manipulated straight-line representations of flexible objects presented on a computer screen to determine the point at which differences in flexibility become perceptible. The ratio of the spring constant to the mass of the endpoints could be used more effectively for determining difference thresholds than a traditional approach using only the flexibility difference to induce different perceptions. Over a small range of spring constant to mass ratios, subjects showed very consistent results revealing one threshold at approximately 2.5 times the spring constant to mass ratio of the standard. Searches for a second threshold, theoretically located between 0 and 1 were unsuccessful. Investigations into subjects' different methods and styles of manipulating objects yielded inconclusive evidence.

With the advent of rapid telecommunications that can broadcast information across the country and around the world in a matter of seconds, it is becoming increasingly possible for people to perform tasks from locations quite distant from the actual work being performed. Mostly, these sorts of tele-operations have been performed because the working environment is either too dangerous (such as working in extreme temperatures or radioactive areas) or too costly (such as flying a surgeon across the country to perform specialized operations) to make actual human participation feasible. One notable tele-operated system that has commanded recent attention is the Mars Pathfinder rover controlled from a station on Earth. Controlled by a human, the rover explored the surface of the planet and sent back pictures to be interpreted by scientists.

The human controllers of the Pathfinder received feedback that was only visual in nature. As a consequence, it is important to understand how visual information about the mechanical properties of objects obtained by manipulating those objects by any means (e.g. pushing, pulling) is interpreted. Up to this point, nearly all research has been conducted on visual perception of rigid objects; the visual perception of flexible objects has yet to be adequately studied.

Bingham (1995) demonstrated that subjects are able to discriminate between mechanical and biological movements in patch light displays quite effectively. This ability indicates that subjects should be able to interpret the visual movement of a flexible object to discover the physical properties of that object. This interpretation is based on several implicit rules about the behavior of the object, and certain expectations about its reactivity to movement. For example, the subject can expect that the object will not gain or lose any kinetic or potential energy except the energy that they supply by moving the object. Also, the subject should expect that the object's properties would not change from one movement to the next. In the present experiment, flexible objects are represented by straight-line representations as in Figure 1. Assuming that these objects will make mechanical motions does not make the interaction of even simple two-endpoint objects simplistic. A full discussion of the movement of a 2-point point light display for a rigid object across a visual field (including curvilinear paths) can be found in Johansson, 1976. The vector relationships are somewhat complex, and become only more complex when the object is flexible along the line between the two points.

In the present experiment, a simple flexible object composed of two equal masses connected by a flexible spring was simulated by an interactive computer model. The object was drawn as a straight line on a computer screen (Figure 1) that represented a configuration of masses and springs similar to that shown in Figure 3. Figure 1 shows a schematic of one screen shot. This straight-line representation of a flexible object could compress and extend only along its longest axis, so the spring could be extended and compressed but not bent. Since the presentation of the object is a uniform line, it is not possible for the subject to gain any information from the thickness or change of thickness of the line. Only the distance between the endpoints can give information about the flexibility and mass properties of the object. Hence, it is possible to think of the object as being represented by two points, as in a point light display, connected by a line.

All line segments presented in this experiment were scaled to respond to manipulation as would objects in the real world. The actual values that the variables take is arbitrary, but they scale correctly to any metric basis (e.g., m·k·s or c·g·s). Hence, we have chosen to use the m·k·s system such that spring constants are measured in N/m (kg/s^2), weight in kg, and distance in meters. In all cases, the friction of the weights against the surface was set to 1000 (kg/s if considered in similar metric units). One caution should be noted: the springs did not respond exactly as would a spring in a theoretical mechanical system. The lines in the experiment (especially the more flexible ones) were easily deformed – if the object is considered to be a spring connecting two masses, that spring did not maintain a constant natural length, but responded to stresses. The result of this flexibility (more like plasticity than springiness) was that objects could change in size quite dramatically as a result of manipulation.

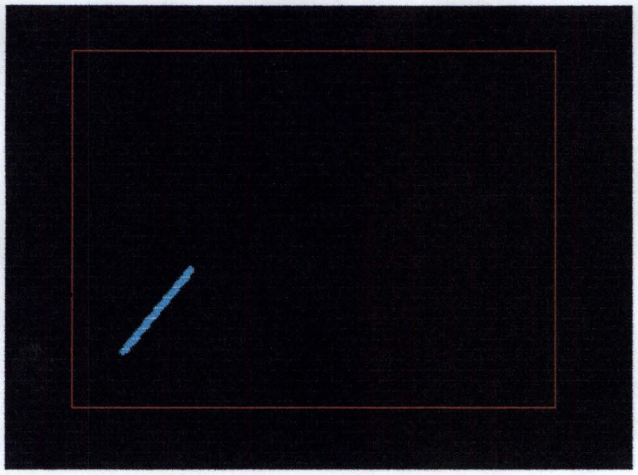


Figure 1: Schematic of a screen shot. The blue line is a straight-line representation of a flexible object with a mass-spring configuration like the one shown in Figure 3.

When the subject moves one endpoint, that endpoint provides no information – it is programmed to simply move in whatever direction the subject forces it to move and does not provide any weight feedback. The only information available to the subject is the movement of the opposite endpoint due to the movement of the grasped endpoint. Of course, the movement of the line as a whole is available, but the line is only a function of the two endpoints.

When a subject moves an endpoint, the vector in the direction of movement can be broken into two component vectors (see Figure 2). One of these vectors runs perpendicular to the object and serves only to rotate it. This perpendicular movement does not compress the spring and provides no feedback about flexibility. The other vector runs along the length of the object and serves to compress or expand it. It is this latter movement that makes the other endpoint react and thus gives information to the subject. On this basis, we predict that "inline" movements will provide the best information about the properties of the simulated object.

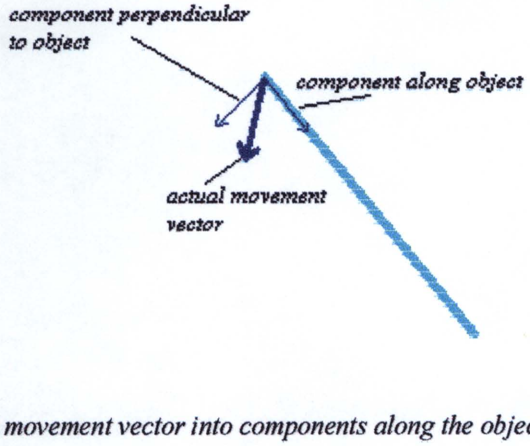


Figure 2: Decomposition of a movement vector into components along the object and perpendicular to the object.

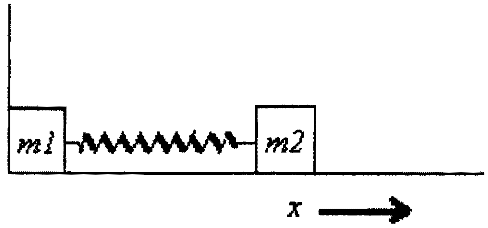


Figure 3: Simplification of flexible object.

Figure 3 shows a system of two masses and a spring similar to the objects studied in this experiment. If we consider a force applied to the first mass (m1), it is possible to model the acceleration of the second mass as follows:

$$\frac{d^2x_2}{dt^2} = \frac{k(l-(x_2-x_1))-F_f}{m_2} = \frac{k}{m_2}(l-(x_2-x_1)) - \frac{F_f}{m_2}$$

In this equation, x_1 and x_2 model the location of mass 1 (m1) and mass 2 (m2) respectively along the x axis. l is the natural length of the spring, k is the spring constant, F_f represents the force of friction (neglected in this discussion), and m_2 represents the actual mass of mass 2. Higher values of k represent stiffer springs. According to this equation, the acceleration of mass 2, represented by the second derivative of its location on along the x-axis, is proportional to the spring constant and inversely proportional to the actual mass of mass 2 (neglecting friction). The expression $(l - (x_2 - x_1))$ represents compression (or expansion) in the spring from its natural length. Overall, the expression indicates that when one endpoint is moved the acceleration of the opposite endpoint is a function of the ratio of the spring constant to the mass of that endpoint. In this simplification of the system, flexibility cannot be perceived separately from mass, and the interactions between flexibility and mass might play some significant role in the degree to which people can perceive flexibility.

Since it is difficult to uncouple the effects of the flexibility of the spring from the effects of the weight of the second mass in predicting how the second mass moves, the ratio *k/m* is an important value. Theoretically, changes in this value should create different percepts and equal ratios should yield the same percept regardless of the magnitudes of k and m. Thus, the purpose of the present experiment is to determine the ability of people to judge differences in the *k/m* ratio by observing the movement of simulated objects that vary in *k/m*.

In addition to exploring the difference threshold for flexibility as represented by the objects, it would be interesting to find out if that threshold could be affected by different strategies for discerning flexibility. For instance, it is possible that quick moves that make the spring react more violently could produce more accurate flexibility perceptions than slower movements. Lederman and Klatzky experimented with perceptual differences created by different exploratory procedures (1987), and found that different stereotyped movement patterns are used to discern different qualities of a stimulus. As an example, we would expect a subject to interact with a frying pan differently when trying to determine its size than they would in trying to determine its texture. By the same token, subjects might interact with the flexible objects differently based on how they expect to perceive flexibility. In turn, these different styles of interaction could lead to different discrimination sensitivities for different subjects.

Subjects:

Subjects were 14 undergraduate students from the University of Tennessee.

Methods:

Stimuli:

There were four standard stimuli and eight comparisons for each standard generating a total of 36 stimuli. Each stimulus, representing an object with a different flexibility/weight ratio (k/m), was portrayed on the computer screen as a line segment that initially was 3 cm long and .25 cm wide. Each stimulus was flexible only along the length of the line; that is, each line could not bend but could be stretched and compressed along its longest axis. The line segments varied in the weight of their endpoints (always equal for both endpoints of one object) and the flexibility of the segment itself. The standards consisted of four line segments generated by forming all possible combinations between two values of flexibility (k = 25 and 75) and two values of endpoint weight (m = 20 and 50). These four combinations are therefore k25/m20, k75/m20, k25/m50, and k75/m50. The comparisons consisted of 32 line segments chosen to find the difference threshold for each object. For each standard, 4 comparison segments were created to determine the difference threshold for flexibility and 4 comparison segments were created to determine the difference threshold for weight. These objects are summarized in Table 1. Included are the ratios of flexibility to weight (*k/m*) for each standard and comparison. Since higher values of k represent stiffer springs, objects with a lower *k/m* are more flexible and objects with a higher *k/m* are stiffer.

Stimulus Presentation:

All stimuli for this experiment were presented on a Silicon Graphics computer with a high-resolution 19" SVGA monitor. Each line segment shown was presented in a separate 15.8 cm wide by 11.4 cm tall window on a computer screen. The window containing the line segment contained a red border that delineated the limits within which the object could be manipulated. The red border measured 9.5 cm wide by 6.9 cm tall, and all segments were presented in the lower left corner of the box. The subjects manipulated all objects presented on the screen with a regular PC mouse. Manipulations were executed by using the mouse to grip an endpoint (by pressing the mouse button) and moving that endpoint around within the restricted area delineated by the red line. Line segments could only be manipulated by the endpoints. In each move made by the subjects, the program assumed that sufficient force was applied to the gripped endpoint to move it in whatever manner the subject desired. Parts of the line segment could be pushed outside of the red borders, but parts outside could not be directly manipulated unless they were brought back inside the red border by moving the object from the opposite endpoint.

In each trial of the experiment, subjects were presented with three line segments in three separate windows. One of the line segments in each trial was one of the standards. A second line segment was one of the comparison segments generated from that standard. The third line segment was a replicate of either the standard or the comparison, so each trial contained two line segments that were exactly the same and one dissimilar line segment. In each trial, the standard or comparison segment was chosen randomly to be replicated. With four standards compared to eight comparison segments each, there were 32 possible combinations of standards with respective comparison segments. Since each of these 32 combinations could be presented in two ways (either with the standard replicated or the comparison replicated), there were 64 total presentation scenarios possible. Each subject saw each standard-comparison combination once (with either two standards or two comparisons for each combination) creating 32 trials per subject. Over all 14 subjects, approximately equal numbers of both types of replication were shown for each standard-comparison combination.

Table 1: Standards and Comparison Objects. Each of the four standards was compared to eight comparison segments, four that differed in flexibility and four that differed in endpoint weight.

Standard #1			**Standard #2**			**Standard #3**			**Standard #4**		
wt (m)	**flex (k)**	***k/m***	**wt (m)**	**flex (k)**	***k/m***	**wt (m)**	**flex (k)**	***k/m***	**wt (m)**	**flex (k)**	***k/m***
20	25	1.25	20	75	3.75	50	25	.5	50	75	1.5

To compare flexibility:

	wt (m)	**flex (k)**	***k/m***	**wt (m)**	**flex (k)**	***k/m***	**wt (m)**	**flex (k)**	***k/m***	**wt (w)**	**flex (k)**	***k/m***
Comparison 1	20	30 (+5)	1.5	20	100 (+25)	5	50	30 (+5)	.6	50	100 (+25)	2
Comparison 2	20	45 (+20)	2.25	20	150 (+75)	7.5	50	45 (+20)	.9	50	150 (+75)	3
Comparison 3	20	60 (+35)	3	20	200 (+125)	10	50	60 (+35)	1.2	50	200 (+125)	4
Comparison 4	20	75 (+50)	3.75	20	250 (+175)	12.5	50	75 (+50)	1.5	50	250 (+175)	5

To compare weight:

	wt (w)	**flex (k)**	***k/m***	**wt (w)**	**flex (k)**	***k/m***	**wt (w)**	**flex (k)**	***k/m***	**wt (w)**	**flex (k)**	***k/m***
Comparison 1	100 (+80)	25	.25	200 (+180)	75	.375	100 (+80)	25	.25	200 (+180)	75	.375
Comparison 2	300 (+280)	25	.0833	400 (+380)	75	.1875	300 (+280)	25	.0833	400 (+380)	75	.1875
Comparison 3	500 (+480)	25	.05	600 (+580)	75	.125	500 (+480)	25	.05	600 (+580)	75	.125
Comparison 4	700 (+680)	25	.0357	800 (+780)	75	.0938	700 (+680)	25	.0357	800 (+780)	75	.0938

Procedure:

After some general instructions on the use of the computer terminal, subjects were presented with a practice line segment, k125/m35, and allowed to familiarize themselves with the computer interface for a maximum of two minutes. After this practice time, the line segment was removed from the computer screen.

Before the first trial, each subject was told the following: "I am going to present you with a series of flexible objects very similar to the one you just saw. In each trial, you will be shown three objects. When I say, 'Go,' you will have one minute to move the three objects within their windows however you wish. At the end of one minute I will say, 'Stop,' and you are to stop moving the objects and tell me which two of the three objects you think share the same characteristics. All of the objects look exactly the same; they only vary in their flexibility or the weight of their endpoints. You may spend as much or as little time on any one object as you wish, but you must give an answer at the end of one minute. Do you have any questions?" If the subject had no questions, the 32 trials were presented in random order. Subjects were given no feedback during the experiment regarding whether their answers were correct or not.

After reporting his answer on the final trial, each subject was asked to describe any strategies he used to determine the characteristics of the line segments. The experimenter recorded these answers for later review. Also, during the experiment the experimenter recorded general characteristics about the subjects' line segment movement styles including speed of movement and type of movement (curved or straight-line movements). No objective measure was used to determine the subject's style, the experimenter simply evaluated the subject visually.

Results and Discussion:

Since the variable of interest is the ratio of flexibility to weight (*k/m*), one way of approaching the difference threshold is to determine the difference in *k/m* between the standard and comparison that creates a different percept. In other words, the difference threshold could be considered the difference between *k/m* for the standard (hereafter referred to as j_1) and *k/m* for the comparison (hereafter referred to as j_2). Instead of using differences, a ratio of these two ratios is used (j_2/j_1) – this value is the proportional increase in *k/m* from the standard to the comparison. This proportion, j_2/j_1 should actually yield two different thresholds. One threshold will be the proportional increase in *k/m* necessary to create a different percept (i.e., a more rigid object), and the other will be the proportional decrease in *k/m* necessary to create a different percept (i.e., a more flexible object). Obviously, $j_2/j_1 = 1$ when there is no difference between the objects (or when one object has proportional increases in both flexibility and weight from the standard to the comparison). In searching for flexibility thresholds (using the first set of comparison objects), the comparisons have the same mass as the standard, but higher values of flexibility (as shown in Table 1). Hence, j_2 will have a higher value than j_1 and the threshold will be larger than 1. Varying the flexibility upward in the experiment while holding weight constant provides a way to search for the more rigid threshold. In searching for weight thresholds (using the second set of comparison objects), the comparisons have the same flexibility as the standard, but higher values for mass. In this case, j_2 will be smaller than j_1, and the threshold for the ratio should be smaller than 1. Varying the mass upward in the experiment while holding flexibility constant provides a way to search for the lower, more flexible, threshold – the proportional decrease necessary in *k/m* to create a difference in perception.

Flexibility Thresholds:

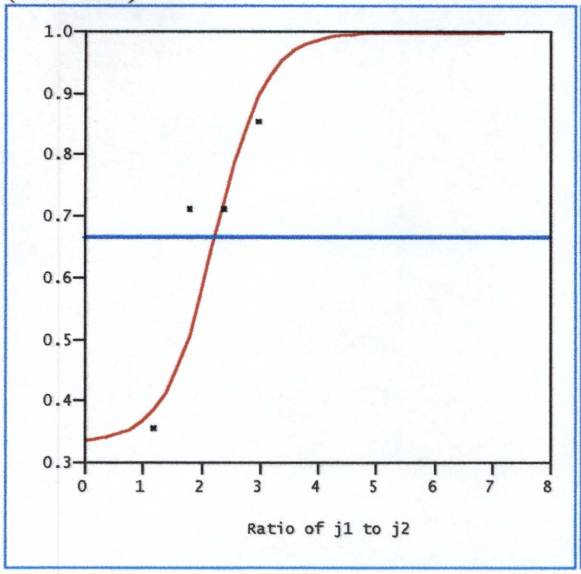
Thresholds for flexibility were determined using logistic regression to fit a smooth curve to the data. The method of analysis applied required some adjustments due to the design of the experiment, but the thresholds are relatively clear for all four standards (see Figure 4). In all cases, the probability of guessing the correct answer was 1/3. To correct the logistic function to allow for guessing, a linear transformation was made. This transformation altered the curve to be asymptotic to 1/3 and 1 instead of 0 and 1. A more complete discussion of the function and adjustments made to certain values can be found in the appendix. The threshold was adjusted accordingly from the traditional .5 to 2/3 (the location of the threshold line in each chart).

Difference thresholds for flexibility:

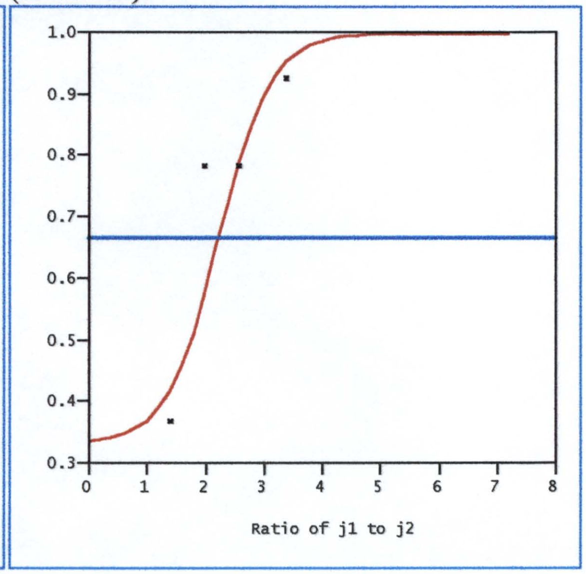
(k25/m20): $k/m = 2.25$ (k75/m20): $k/m = 2.25$

(k25/m50): $k/m = 3.73$ (k75/m50): $k/m = 2.38$

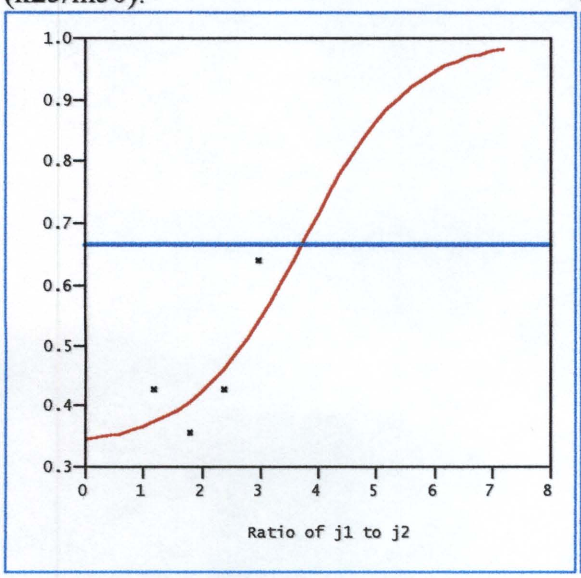
(k25/m20):



(k75/m20):



(k25/m50):



(k75/m50):

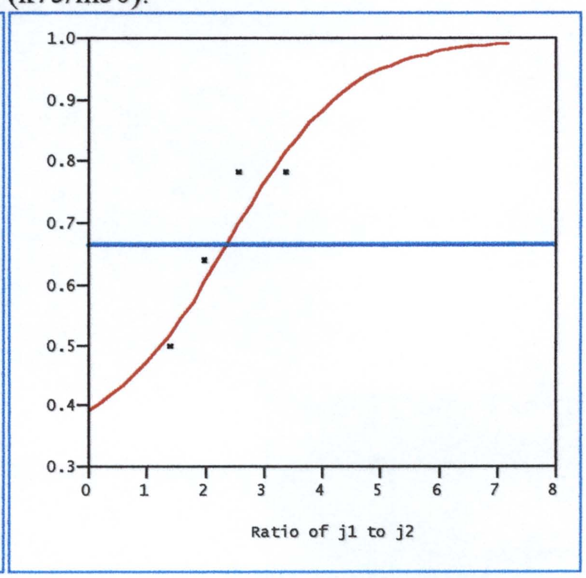


Figure 4: Flexibility Difference Thresholds. The logistic curve runs from a minimum of 1/3 to a maximum of 1 and denotes the probability of a correct answer at different levels of j_2/j_1. The threshold is defined at the midpoint of the curve (probability of a correct answer = 2/3) and is denoted in each graph by a line.

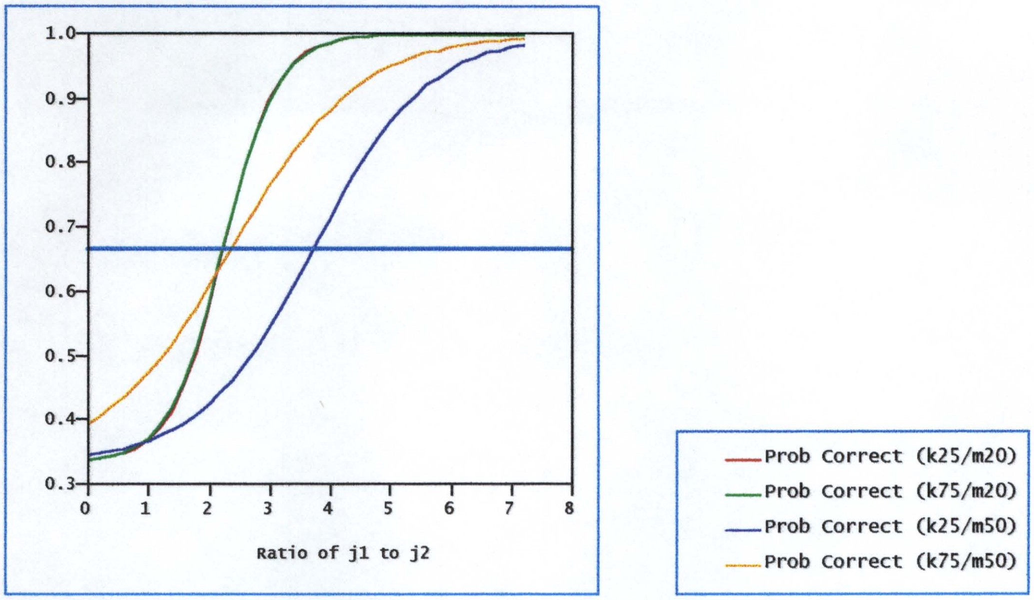


Figure 5: Comparison of Flexibility Difference Thresholds. Same plots as in Figure 4, overlain.

There is no simple statistical approach to testing whether the thresholds are different for the four objects, and the small number of subjects available for this study does not provide sufficient power to make those tests worth performing. However, it does appear that three of the four objects have approximately the same threshold, and the remaining object (k25/m50) is close. Assuming that no significant differences exist between the different objects (this evidence is supported by F-tests on the intercepts and slopes of the 4 lines), all four objects can be combined into one regression to find the overall difference threshold. Figure 6 shows the overall approximation yielding an overall difference threshold of $k/m \approx 2.5$. The regression model is relatively good with an R^2 of .5793 and no lack of fit.

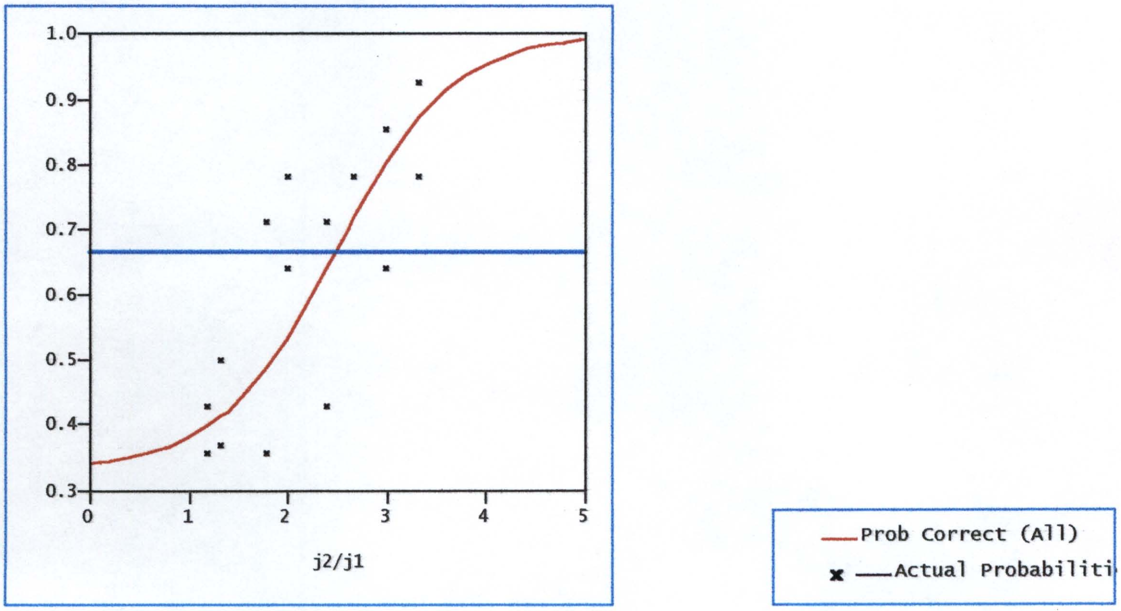


Figure 6: Overall difference threshold for k/m.

According to Weber's Law, the difference threshold should increase proportionately with higher levels of the standard stimulus. The standards have *k/m* values of 1.25, 3.75, .5, and 1.5, and plotting the difference threshold for each object against its k/m value yields pattern in figure 7. The dotted line shows the regression created using all four points. In this regression, it seems that one point, the object with *k/m* = .5, is having undue influence. From Figure 5, it is apparent that this object has a relatively different threshold from the other three objects. Removing the object with *k/m* = .5 produces the solid threshold line. In this case, the threshold seems to be about the same proportional increase in *k/m* for each of the three remaining objects as predicted by Weber's Law. It seems that a proportional increase in the *k/m* value of approximately 2.5 is a valid threshold over the tested range of standard *k/m* values.

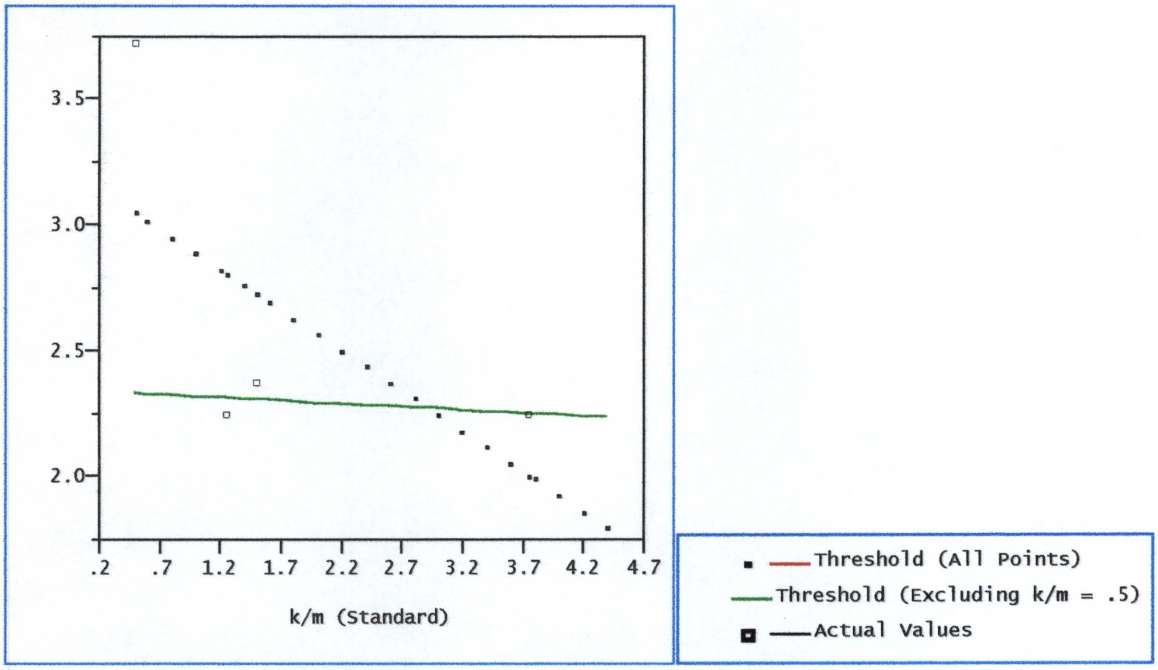


Figure 7: Weber's ratio for k/m differences in standards. The value for the standard with k/m = .5 could be considered an outlier, and it seems more plausible that the actual threshold is more uniform over a range of k/m values (as shown by the solid line).

Weight Thresholds:

The weight thresholds are included in this report merely for completeness. Figure 8 shows the probability for answering correctly for different proportional decreases in *k/m* from the standard to the comparison. The actual points are plotted in figure 8 showing that the majority of the function is extrapolated beyond the data points. Unfortunately, the graph shows that the thresholds are all below zero (an impossible condition). Apparently, the weights were not varied enough in the experiment to locate the value at which the difference in *k/m* is perceptible. It is also possible that such low values of *k/m* were used for the standards that locating a lower difference threshold is not feasible. Nonetheless, the regressions are running in the correct direction – it becomes more difficult to discern the difference as the ratio of j_2 to j_1 gets closer to one. Unfortunately, there is no way to explore the lower hypothesized threshold further without more investigation with smaller comparison *k/m* ratios or larger standard *k/m* ratios than those used in this experiment.

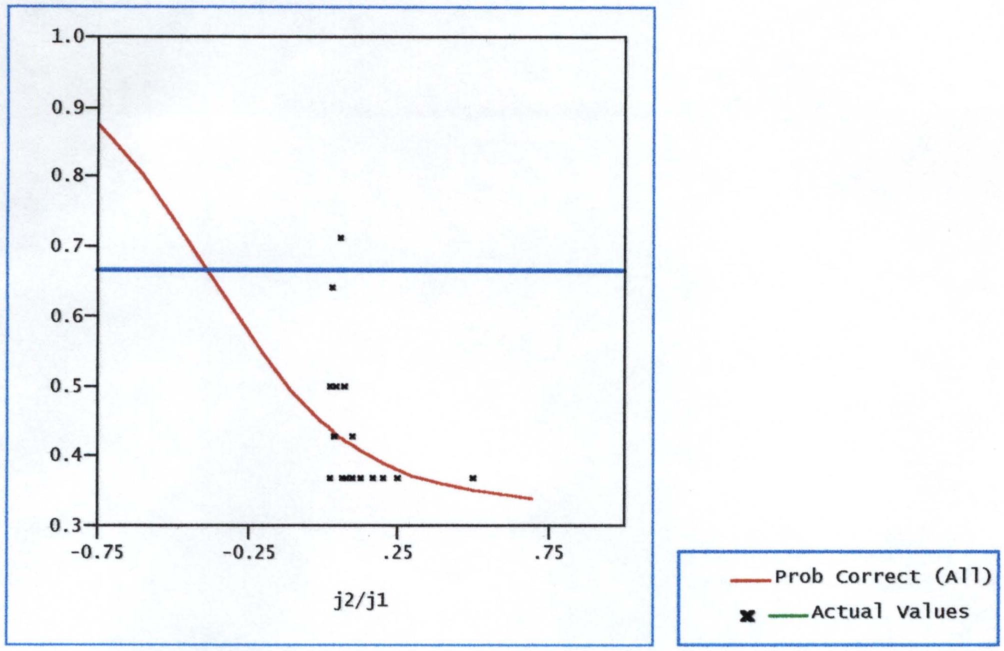


Figure 8: Compilation of All Weight Difference Thresholds.

Mediators in Flexibility Difference Thresholds:

In this section, we compare the discrimination performance of subjects using different movement strategies. Only the flexibility thresholds were explored since all of the weight thresholds were beyond the limits of the data and could not be considered significant. Based on the observations of subjects' movements used to explore the simulated objects, three dichotomies emerged. In each case, data were divided into classes, and thresholds were calculated for each class as before. First, subjects who used low-speed movements were compared to subjects who used high-speed movements. Next, subjects who made mostly straight-line movements were compared to those who used curved and straight-line movements. Finally, subjects who used a kinetic approach to perception were compared to those who used a more static approach.

Speed as a mediator in flexibility perception:

Speed of movement was recorded as being slow or fast. Subjects that used a variety of speeds or changed styles often were not considered in the analyses considering movement speed as a factor. Five subjects used low-speed movements and seven subjects used high-speed movements. Figure 9 shows the thresholds for low- and high-speed movements. The thresholds are very close and the curves are almost identical. F-tests performed on differences in intercept and slope for each line revealed no significant difference between the lines.

Move type as a mediator in flexibility perception:

The subject's type of move was recorded as being either straight-line movement only or including curved movements. Straight-line movements tended to be back and forth along the length of the line segment with very little rotation of the line itself. Curved movements resulted in more rotation of the line and less expansion or contraction. Eight subjects were classified as using straight moves, and five subjects used a mix of curved and straight moves. Figure 10 shows the threshold curves for straight-line and mixed (including a combination of curved and straight moves) move types. Again, there is no significant difference between the lines. However, it does appear that straight-line moves produce slightly more accurate perceptions of flexibility.

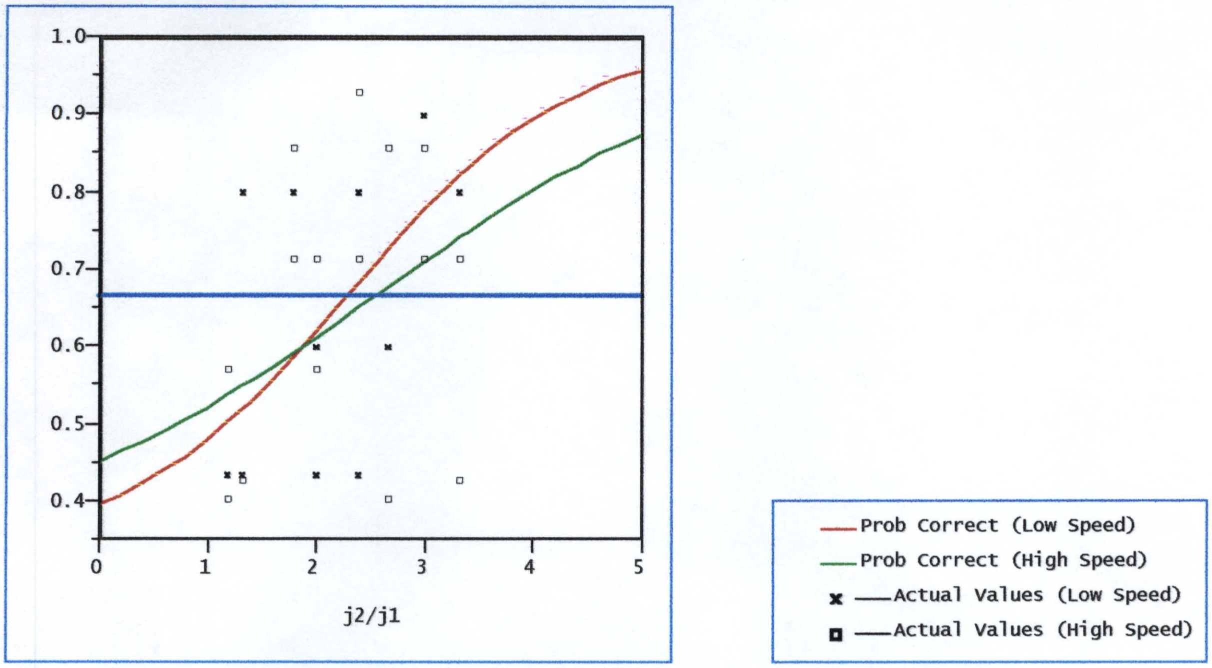


Figure 9: Effects of speed on k/m difference thresholds for flexibility.

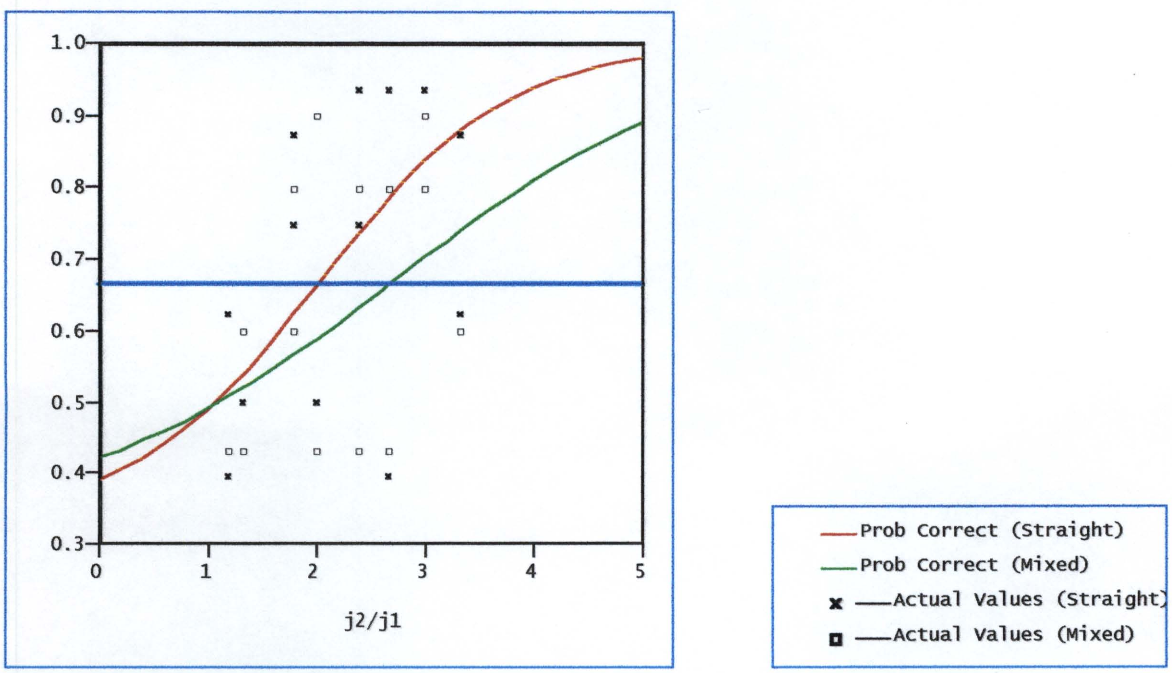


Figure 10: Effects of move type on k/m difference thresholds for flexibility.

Static and kinetic approaches as mediators in flexibility perception:

The final classification was static vs. kinetic assessment of line properties. Subjects tended to use one of two strategies to learn about the properties of the line segments. Either the subject would attempt to move the three lines in a similar fashion and evaluate the similarity of the final, still shapes of those lines (static), or the subject would move the lines and simply observe their kinetic properties. This second class of subjects tended to evaluate the "feel" of the line segments rather than using a systematic approach to determining the answer. Seven subjects used the static approach, and six subjects used the kinetic approach. Figure 11 shows the results of difference threshold analyses using static and kinetic learning approaches as mediators to flexibility perception. Again, the lines are very similar. Although these lines show large differences in slope, the actual thresholds are almost identical. No evidence exists to claim that either learning approach is more effective.

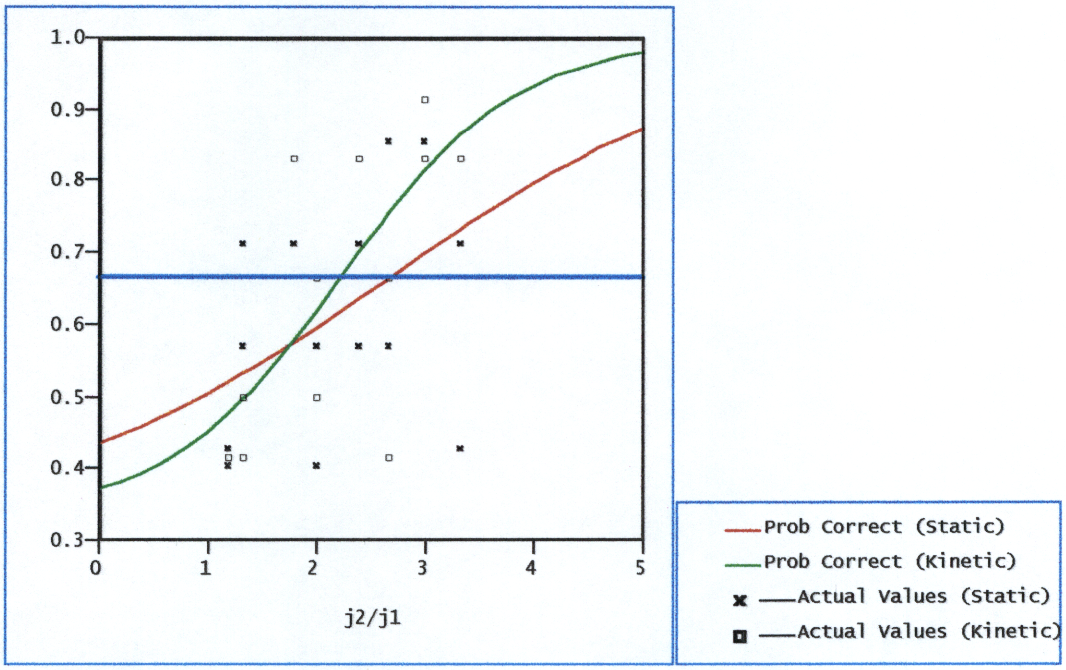


Figure 11: Effects of static vs. kinetic learning approaches on k/m difference thresholds for flexibility.

Conclusions:

The experiment conducted in this study was effective in finding one of two theoretical flexibility perception thresholds. It appears that the upper flexibility threshold for ***k/m*** is approximately 2.5 for objects over the range tested. In other words, an increase in the spring constant to mass ratio by a factor of 2.5 is perceptible. Over the range of values tested, this approximation of Weber's ratio was found. Assuming that the one standard with ***k/m*** = .5 is an outlier, it appears that the threshold might be approximately the same over a wide range of standard ***k/m*** values.

Attempts to find the lower theoretical threshold (the proportional decrease in ***k/m*** necessary in a stimulus to produce a different flexibility perception) were not successful, and the threshold appears to be much lower than originally expected. In some cases, the decrease in ***k/m*** was as much as 1/40, yet the difference was still not perceptible for many subjects. It is possible that the values of *k/m* for the standards were too low to be useful in the search for the lower threshold.

Analysis of the different approaches taken by subjects to learn about the objects yielded inconclusive evidence. There was no clear-cut evidence that movement speed, movement type, or learning strategy played an important role in improving perceptions. The biggest difference in the three strategies observed is that between straight-line and mixed moves. Although not statistically significant, this effect may be worthy of more investigation, especially with larger values of *k/m* for the standards.

Appendix: Statistical Methods

This experiment makes a departure from the traditional threshold experiment by creating a more objective measure of difference threshold evaluation. Traditionally, the experimenter or the subject adjusts the stimulus until the subject reports that a difference is just barely noticeable. In the present experiment, nothing is left to the subjective assessment of the subject. Stimuli are presented and the subject attempts to discriminate between different objects. Since three objects are presented during each trial, two of which are similar, the probability that the subject will make a correct choice by chance is 1/3. For this reason, a probability distribution that places its lower bound at 0 (as does traditional logistic regression) is not appropriate. Instead, a linear transformation of the traditional logistic function is used:

$p = \frac{\left(.5 + 1.5e^{\beta_0+\beta_1 x}\right)}{\left(1.5 + 1.5e^{\beta_0+\beta_1 x}\right)}$ which is modeled as $\ln\left(\frac{1.5p - .5}{1-(1.5p-.5)}\right) = \beta_0 + \beta_1 x$. In both cases, x is the difference between the standard and its comparison. Estimates of probability for the regression were obtained by calculating the probability across all subjects for any one trial. For example, the standard k25/m20, at a proportional increase in *k/m* of 2.4, was discerned correctly by 10 out of 14 subjects. Thus, the probability of a correct response was approximated by 10/14 = .7142. Some data had to be adjusted to give valid responses. If the probability of a correct response was lower than 1/3, an adjustment had to be made to so that the logarithm would produce a result. The approach adopted was that suggested by Freund and Wilson with a slight modification. Probabilities that fell below 1/3 were estimated as:

$p_i = \frac{1}{3} + \frac{1}{2n_i}$ where n_i is the number of observations used to calculate that probability. For example, if only 3 out of 14 subjects answered correctly, p_i was estimated as $\frac{1}{3} + \frac{1}{2 \times 14} \approx .3690$.

Other values that fell below this calculated value were also adjusted up for consistency. It should be noted that the logistic regression function can be adjusted for any base probability by a linear transformation. In this case, the transformation took into account a base probability of 1/3. However, it is possible to adjust the function for any base probability α as follows:

$p = \frac{e^{\beta_0+\beta_1 x}\left(1 - \frac{\alpha}{\alpha-1}\right) - \left(\frac{\alpha}{\alpha-1}\right)}{\left(\frac{1}{\alpha-1}\right)\left(1 + e^{\beta_0+\beta_1 x}\right)}$ which is modeled as $\ln\left(\frac{\left(\frac{1}{1-\alpha}\right)p + \left(\frac{\alpha}{\alpha-1}\right)}{1 - \left[\left(\frac{1}{1-\alpha}\right)p + \left(\frac{\alpha}{\alpha-1}\right)\right]}\right) = \beta_0 + \beta_1 x$.

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