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Data-Enabled Cognitive Modeling: Validating student engineers' fuzzy design-based decision-making in a virtual design problem

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Abstract

The ability of future engineering professionals to solve complex real-world problems depends on their design education and training. Because engineers engage with open-ended problems in which there are unknown parameters and multiple competing objectives, they engage in *fuzzy decision-making*, a method of making decisions that takes into account inherent imprecisions and uncertainties in the real world. In the design-based decision-making field, few studies have applied fuzzy decision-making models to actual decision-making process data. Thus, in this study, we use datasets on student decision-making processes to validate approximate fuzzy models of student decision-making, which we call *data-enabled cognitive modeling*. The results of this study (1) show that simulated design problems provide rich datasets that enable analysis of student design decision-making and (2) validate models of student design cognition that can inform future design curricula and help educators understand how students think about design problems.

Keywords: design cognition, decision-making, fuzzy numbers, cognitive modeling

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1. Introduction

Always a central aspect of the engineering field, design has become a focus in engineering education in recent decades. Throughout engineering design education, decision-making is an important element in defining the problem, deciding which elements of a project to work on first, and choosing designs for testing. Thus, a key skill for 21st century engineering students is the ability to make design decisions and understand the complex social, environmental, and economic consequences associated with each decision.

Because engineers engage with open-ended problems in which there are unknown parameters and multiple competing objectives, they engage in *fuzzy decision-making*, a method of making decisions that takes into account inherent imprecisions and uncertainties in the real world. Fuzzy decision-making uses numbers whose values do not refer to one single, deterministic, or “crisp” value, but instead refer to a range of possible values. However, very few studies that report on fuzzy decision-making analyze actual engineering students’ decision-making processes.

In this work, we examined how undergraduate students make engineering design decisions through the lens of fuzzy engineering design decision-making and developed a mathematical framework to quantify their decision-making processes. In particular, we first collected discourse data from a small sample of students (n = 13 students) who individually solved a simulated engineering design problem. We then analyzed this data to inform and develop two sets of decision-making models: crisp and fuzzy. Finally, we applied these models to data from a virtual internship program in which students (n = 175 students) worked in teams (n = 35 teams) on the same simulated design problem in the context of an internship simulation. Since we used large sets of student discourse data to test these proposed models, which to our knowledge is the first report of data-enabled validation of a decision-making model, we call this approach *data-enabled cognitive modeling*.

Understanding and modeling student decision-making based on collected student process data has several important applications for engineering design education. First, when creating design problems or scenarios for students, educators can use the appropriate cognitive models to guide their curriculum development and create opportunities for students to practice appropriate decision-making in the context of design. Such cognitive models are especially useful in digital and computer applications of design problems because the models drive the development and design of virtual design problems.

Model-based virtual design problems offer several advantages for engineering educators. First, all students are given the same simulated design problem to solve and the same resources with which to solve it. This approach gives all students an equal starting point and provides a basis for standardized assessment. Instructors can collect student process work in the form of digital engineering notebooks, conversations, or reports and collect student product work in the form of final design specifications. Using this collection of student work, instructors can make valid comparisons among different students' design thinking and assess students' design thinking against engineering learning standards.

Second, model-based simulated design problems facilitate scaling up and allow for a sophisticated yet accessible examination of design learning with a large number of participants. In turn, such digital learning environments can also offer a way for educators to scale up their instruction and provide more students access to quality design instruction which simulate real-world uncertainties.

Thus, developing and testing design-based decision-making models help educators and researchers better understand how students are thinking about design problems and guide students through critical decision-making processes that better align with how engineers design under realistically imprecise and uncertain conditions.

2. Theory

2.1 Engineering Design Education

Design is a critical part of the engineering profession (Dym, 1994a; Simon, 1996). As a result, design is a central focus of engineering education in terms of teaching, learning, and assessment (Atman, Eris, McDonnell, Cardella, & Borgford-Parnell, 2014; Dym, Agogino, Eris, Frey, & Leifer, 2005). In a recent study, Sheppard and others (Sheppard, Macatangay, Colby, & Sullivan, 2009) interviewed faculty and students about the field of engineering and concluded that design is the most critical component of engineering education. One faculty member asserted that “guiding students to learn ‘design thinking’ and the design process, so central to professional practice, is **the** responsibility of engineering education” (p. 98; emphasis ours).

Two decades ago, ABET, the accreditation board for engineering programs, developed criteria that included opportunities for design learning. ABET (ABET, 2014) defined engineering design as the “process of devising a system, component, or process to meet desired needs. **It is a decision-making process** (often iterative), in which the basic sciences, mathematics, and the engineering sciences are applied to convert resources optimally to meet these stated needs” (p. 4; emphasis ours). According to ABET and others (Dym, Wood, & Scott, 2006; Thurston, 2001), decision-making is integral to engineering and occurs in nearly every phase of the design process. Engineers make critical design decisions during concept exploration, research, model selection, feasibility analysis, prototype testing, and final design documentation (Cross, 2011; Ulrich & Eppinger, 2011).

In an authentic engineering design process, engineers most often deal with ill-structured problems that do not have specific procedures, possess conflicting goals, and unexpectedly develop complications (Dym, 1994b; Jonassen, 2000). In these types of open-ended and complex scenarios, design-based decision-making requires the engineer to consider the possibilities that may result from imaginable choices and make reasonable estimates based on limited information. Using their experiences and content knowledge, "designers construct and impose a coherence of their own… Their designing is a web of projected moves and discovered consequences and implications, sometimes leading to a reconstruction of the initial coherence—a reflective conversation with the materials of a situation" (Schön, 1987). Engineering design problem have many feasible solutions, but such solutions fall within a reasonable set of designs that have been chosen to satisfy particular design goals as best as possible. Therefore, one of the key processes in engineering design, and thus a critical aspect in design education, is the way in which engineers make design decisions.

2.2 Engineering Design-Based Decision-Making

2.2.1 Theoretical and Mathematical Models

In the last several decades, engineering design decision-making has been studied, parameterized, and represented mathematically. In the late 1990s, Hazelrigg (1998; 1999) argued for an engineering design decision-making framework based on elements from classical decision theories. These decision theories make claims about how people make decisions in real world situations. Originating in the field of economics, the traditional point of view is that the preferred decision is the option whose expectation has the highest value (von Neumann & Morgenstern, 1944). Connecting to research in decision-making in economics, Hazelrigg claimed that decision theory can be applied to engineering design because design involves decision-making under uncertainty and risk. He defined uncertainty as a lack of precise knowledge regarding the inputs to a model (which may include manufacturing variations, unanticipated wear and tear, or future cost of maintenance) and risk as the result of uncertainty on the outcome of decisions.

At its simplest, Hazelrigg's framework claims that engineering involves two steps: (1) determine all possible design options, and then (2) choose the best one. This assumes that the engineer is a rational decision-maker who will choose the option with the highest expected value. Using this framework, Hazelrigg began to quantify engineering design decision-making and identified the *design vector*, x, which is the set of variables that an engineer can choose, such as dimensions or materials. His framework also included the logical process of decision-making. He constructed a set of axioms for designing and formulated two theorems that could be applied to statistical models that roughly account for uncertainty, risk, information, preferences, and external factors. The expected utility theorem states that given a pair of designs, each with a range of possible outcomes and associated probabilities of occurrence, the preferred choice is the design that has the highest expected *utility*, a scalar that indicates the value of the choice. Thus, the preferred design is the design with the highest utility value, U.

In order to calculate the total utility of a device, engineers consider multiple performance criteria (Sen & Yang, 2012; Thurston, 1991, 2001; Zeleny & Cochrane, 1973). Engineers consider not only the required functionality, but several other criteria, including the cost, reliability, manufacturability, and aesthetics of a product. Because there are multiple criteria, some criteria may contradict each other and require tradeoffs to be made when making choices. To analyze these tradeoffs, engineers may rank design choices using various methods (Ashby, Shercliff, & Cebon, 2014; Dym & Little, 2003; Ulrich & Eppinger, 2011). A *design decision matrix* allows engineers to assign a ranking value to every design choice for every performance parameter (Pugh, 1991). For example, if an engineer is examining materials for a bicycle frame design, they may assign rankings to several different materials—stainless steel, aluminum, or a composite—for strength. The engineer may also rank these three materials for compression strength and brittleness. The rankings that are assigned could be unweighted (i.e., equally weighted) or they can be weighted according to the importance of the particular performance parameter. To determine the importance of the performance parameter and thus the weight of the ranking, a commonly used mathematical method is the Analytic Hierarchy Process (Saaty, 1990). In this method, the engineer conducts comparisons of the performance parameters or attributes and assigns a value to each attribute that represents how much more important one attribute is over the other. These values are then assembled into a matrix and the eigenvalues are calculated. Thus, each attribute has a respective weighting and the weights across all attributes sum to one.

To obtain one scalar utility for each design, engineers combine the rankings for the various performance attributes. If the rankings are on the same scale, then a weighted-sum model in which the weighted rankings are summed across all attributes for each design can be used (Dlesk & Liebman, 1983; French, 1988). If the rankings are not on the same scale, the engineer may prefer to use a weighted product model that multiplies the ratios of one design to another design for each attribute. In either case, the goal is to obtain one total utility value for each design such that a comparison can be made among possible designs. The total utility function then is a function of the design vector, $U(x)$, that ranks the multiple attributes of a design to obtain one total utility.

In this classical decision-making model, once the engineer has obtained a total utility for each device, they can compare the utilities of various devices to determine an optimum design. In design, one common method for comparing devices (or performance attributes) is by conducting pairwise comparisons (Dym et al., 2006; See, Gurnani, & Kemper, 2004). The pairwise comparisons can be organized in a chart or matrix that lists all the options as rows and columns and then assesses each pair of options. When comparing pairs of designs, a pairwise comparison matrix allows the engineer to take two options at a time, x_1 and x_2, and compare the options using utility values. The engineer can then evaluate which design is preferred based on its utility value and assign a *preference, p*. This preference value can be a simple statement (1 = x_1 is preferred over x_2, -1 = x_1 is not preferred over x_2, or 0 = neither $x_1$or $x_2$is preferred) or a more complex function. For example, the pairwise comparison matrix in table 1 compares three designs: x_1, x_2, and x_3. This matrix values indicate how much the design in the row is preferred over the design in the column. In this example, $x_1$is preferred two times more than x_2 but , $x_1$is preferred four times less than x_3. This pairwise comparison matrix is assembled as an antisymmetric matrix, where the diagonal consists of zeroes and the bottom triangle of the matrix has opposite signs.

To simplify the decision-making process when deciding among several designs, an engineer can opt to sum the rows of the pairwise comparison matrix to obtain a *preference score*. Formally, the preference score between two designs, $x_1$and each of the other possible designs is

$$\phi_{x_1,x_k} = \sum_{i=1}^{k} p_i$$

where the preference values, p, are being summed for one row, i, that compares $x_1$to each possible devices, x_k (Table 1). More generally, the preference score is a function of the utility,

$\phi(U)$. Thus, the preference score allows for each device to have one value associated with it that the engineer can use to represent a quantifiable preference for that design.

2.2.2 Fuzzy Decision-Making

When applying utility functions and conducting pairwise comparisons, Hazelrigg[5] claims that engineers make deterministic and precise decisions. However, this is not always the case. Particularly in the preliminary stages of design, engineers often work with a vague, incomplete description and make design decisions to reduce the ill-formed nature of the problem (Antonsson & Otto, 1995; Wood & Antonsson, 1990). Even in the later stages of design, when the design problem is likely to have been defined more clearly, deterministic and precise decision-making may not be possible for the engineer to use in producing a final product. In many cases, engineers design completely innovative products and do not know the product's exact performance parameters until the design is finalized and manufactured (Cross, 2007; Dorst, 2011). Thus, engineering design decision-making involves not only uncertainty and risk, but also *imprecision*, which is uncertainty when choosing *among* designs. Because there is imprecision in the decision-making process, there is not always one clear choice in a design situation and engineers therefore make choices based on their preference for one device over another. This preference could be based on objective evidence or subjective past experience. Antonsson and Otto (1995, p.27) explain,

> An imprecise variable in preliminary design is a variable that may potentially assume any value within a possible range because the designer does not know, *a priori*, the final value that will emerge from the design process. The nominal value of a length dimension

[5] Hazelrigg briefly mentions the possibility of using fuzzy logic to determine utilities of alternatives in a footnote in 1998, but doesn't elaborate on this concept.

> is an example of an imprecise variable. Even though the designer is uncertain about what length to specify, they usually have a preference for certain values over others. This preference, which may arise objectively (e.g., cost or availability of components or materials) or subjectively (e.g., from experience), is used to quantify the imprecision with which design variables are known.

In other words, engineers can be precise with calculations about known situations, but during the creative design process they routinely face unknown situations, and in turn, develop preferences and use imprecise variables. Thus, engineers engage in an approximate way of thinking or what is referred to as *fuzzy decision-making* (Bellman & Zadeh, 1970; Rao & Lafayette, 1992; Triantaphyllou & Chi-Tun, 1996; Zadeh, 1997). This framework for decision-making uses the mathematics of *fuzzy numbers*, which are approximations of numbers. A single fuzzy number does not refer to one single value but rather can be thought of as a function or a range of possible values. The function that determines a fuzzy number is known as a *membership function* which determines the degree of membership of values with the range of the fuzzy number (Klir & Yuan, 1995; Zadeh, 1971). In contrast, a *crisp* number (Klir & Yuan, 1995) has a single, deterministic value. Antonsson and Otto (1995) provide an example of fuzzy vs. crisp decision-making in engineering design (Figure 1):

> Specifications and requirements also embody design imprecision, even though most are written as if they were crisp, e.g., “This device must have a range of at least 250 km.” Such a requirement implies that given two designs arbitrarily close together, one with a range of 250 km and one just below, the first would be acceptable but not the second, as shown by the dashed line.

In Antonsson and Otto's design scenario, there is a preference function, μ_p, that ranges from zero (not preferred) to one (preferred). In the crisp decision-making scenario, shown by the dashed line, if a design meets or exceeds the 250 km functional requirement, then the design is preferred ($\mu_p = 1$). If the design falls anywhere below the 250 km functional requirement, then the design is not preferred ($\mu_p = 0$). On the other hand, in the fuzzy decision-making scenario, shown by the solid line, the preference function becomes a *range of possible values* from zero to one. As the performance of the device approaches 250 km, the preference increases. As the performance of the device exceeds 250 km, the preference continues to increase until it plateaus at a maximum value of one. Therefore, the crisp decision making function only allows for a binary decision-making process (either zero or one) whereas the fuzzy decision-making function allows for a range of values between zero and one.

Similarly, if an engineer is conducting a pairwise comparison and compares two utilities that are close in value, then they might determine that these two designs are the same, even if the utilities are not exactly equal. From the engineer's perspective, the functional requirements or customer requests are not crisp, but have some fuzziness associated with them.

The fuzziness of the number may be represented with a membership function that is linear, quadratic, triangular, or some other mathematical relationship (Klir & Yuan, 1995; Zadeh, 1971). If the membership function of the fuzzy number is complex and difficult to interpret, fuzzy numbers can be *approximated* using various methods. One approximation method is using a piece-wise linear function (Coroianu, Gagolewski, Grzegorzewski, Firozja, & Houlari, 2014; Dombi & Gera, 2005; Inuiguchi, Ichihashi, & Kume, 1990; Yang & Ignizio, 1991). The piece-wise linear function models linear intervals that exist in close proximity to the fuzzy number and thus approximates a continuous membership function (Figure 2). Approximating fuzzy numbers using piece-wise linear functions have several benefits. Approximations may help reduce calculations errors, reduce the amount of computational resources required, and provide more interpretable results than complicated membership functions (Rohani, Mazaheri, & Jesmani, 2014). Approximations are not only useful for more complicated membership functions, but are also used when the membership function is unknown. In these unknown cases, the membership function can be approximated from a set of samples and an approximate fuzzy membership function can be abstracted from the samples (Klir & Yuan, 1995). Taken together, both approximations and actual membership functions represent fuzzy numbers that account for the fact that in real world engineering design decision-making there is some degree of inherent imprecision.

Although there have been several proposed fuzzy engineering decision-making models with worked examples (Mikhailov, 2003; Möller & Beer, 2008; Rao & Lafayette, 1992; Wang, 2001; Xia, Chan, & Yeung, 2011; Zadeh, 1971), there have been few studies that examine authentic fuzzy decision-making processes of either actual engineers or student engineers (See Tian, Thurston, & Carnahan, 1994 for one example). In our work, we examined how undergraduate students make engineering design decisions through the lens of fuzzy engineering design decision-making and developed a mathematical framework to quantify their decision-making processes.

2.3 Simulated Engineering Design Problems

To examine authentic decision-making processes, we studied how students solve *simulated* engineering design problems in which real students design virtual devices. The simulated design problem that we analyzed is posed within a virtual internship that our group has previously developed for first-year introduction to engineering design courses, *Nephrotex*. As has been described in detail elsewhere (Chesler et al., 2015; Chesler, Arastoopour, D'Angelo, Bagley, & Shaffer, 2013) students in *Nephrotex* role-play as interns to design a filtration membrane for a hemodialysis machine. Individually they conduct background research and summarize customer requests and technical constraints. Then, in teams, they design and test several devices before deciding on a final prototype. When deciding on a final prototype, students consider conflicting stakeholder requests and choose a design that best meets all of the stated thresholds. For example, the clinical engineer is concerned about the blood cell reactivity and flux of the filtration membrane, and the manufacturing engineer values reliability and cost. At the end of the course, student teams select and present their final hemodialysis machine filtration membrane design to their colleagues and instructor.

The simulation collects a rich dataset on students' decision-making processes, which includes data on students' choices for testing designs, students' final prototypes, and digital notebook entries in which students justify their design decisions. Based on this collection of data, to examine student decision-making processes we consider the simulated problem used within *Nephrotex* as a representation of a real engineering design problem. An advantage of this approach is that the numbers of input choices and performance parameters, as well as the information provided, are fixed. Thus, multiple students can solve identical design problems. This allows for a standard comparison among students' design decisions and decision-making processes.

To understand students' cognitive processes when solving a simulated design problem, we used data from a previous study in which we conducted several c*ognitive clinical interviews* (Arastoopour, Collier, Chesler, Linderoth, & Shaffer, 2015). Such interviews are semi-structured, task-focused events developed to investigate student thinking and understanding of a task or event. This method has been used extensively by researchers in education (diSessa, 2007; Posner & Gertzog, 1982; Russ, Lee, & Sherin, 2012). Interviews begin typically by providing the participant with a prompt for tasks and relevant tools. As the participant completes the task, the researcher encourages the participant to discuss their thinking and asks the participant questions along the way. The researcher may explore different ways of framing the task or may ask impromptu questions after receiving a particular response from a participant. The goal of the interview is to "allow the interviewee to expose his/her 'natural' ways of thinking about the situation at hand" (diSessa, 2007). In other words, the researcher is collecting evidence that explicitly reveals how a participant is thinking about a situation or task.

In our prior work, we conducted cognitive interviews with undergraduate students (Arastoopour et al., 2015). We asked participants to individually solve the simulated design problem in *Nephrotex* and explain their strategies and choices. We then analyzed the results of the cognitive interviews for details about students' design decision making when solving a simulated design problem. In our previous work, we discovered that students were incorporating fuzzy utility functions when comparing multiple attributes, as well as incorporating fuzzy pairwise comparisons when comparing among designs. For this current study, we used the analyses from these prior cognitive interviews to inform the design of our mathematical models. Then, we tested these models on digital data collected from students who solved the simulated design problem in teams by participating in the *Nephrotex* virtual internship (Figure 3).

We call this approach to design-based decision-making *data-enabled cognitive modeling*. This framework mathematically models cognitive decision-making processes and validates decision-making models using process data from participants in authentic settings. In other words, our approach uses real-world data to validate mathematical decision-making models.

2.4 Research Questions

To validate the decision-making models, we collected data from a previous study with cognitive interviews in which students engaged in the simulated design problem from *Nephrotex*. We validated two models: (1) a crisp model, which included a crisp utility model, and a (2) fuzzy model, which included a fuzzy utility model, fuzzy preference score model (based on pairwise comparisons), and a fuzzy percent change preference score model (which was a modified version of the fuzzy preference score model). Our fuzzy models used piecewise functions to approximate a fuzzy membership function. Table 2 summarizes the two categories of models. Then, we correlated each of these models with the devices that students chose as their final design to determine which of these models was most useful for representing student engineers' design-based decision-making.

Our research questions were:

1) Does the distribution of devices that students selected as their final device provide evidence for design-based decision-making in *Nephrotex*?

2) Are **crisp utilities** a significant predictor of students' final design selections?

3) Are **fuzzy utilities** a significant predictor of students' final design selections and how does this model compare to the previous model?

4) Are **fuzzy preference scores** a significant predictor of students' final design selections and how does this model compare to the previous models?

5) Are **fuzzy percent change preference scores** a significant predictor of students' final design selections and how does this model compare to the previous models?

3. Methods

3.1 Virtual Internship Description

In *Nephrotex*, students role-play as interns at a fictional medical device design company, where they work in teams to design dialyzer membranes for ultrafiltration. Research and design activities and team interactions all take place through the web platform, which includes an email and chat interface. Acting as interns, they send and receive emails to and from their supervisor and use the chat window for instant messaging with other team members and their assigned design advisor.

After collecting and summarizing research data, interns begin the actual design process using the simulated engineering drawing tool. First individually and then in teams, students develop hypotheses based on their research, test these hypotheses in the provided design space, and analyze the results provided. The design space contains four inputs and five outputs (Figure 4). Interns also become knowledgeable about internal consultants within the company who have a stake in the outcome of their designed prototype. These consultants value different outputs, which are essentially performance attribute. Each of the five internal consultants in *Nephrotex* prioritizes two output parameters (i.e., performance attributes) and identifies specific threshold values for each output. For example, the clinical engineer would like a high degree of biocompatibility and high flux, and the manufacturing engineer would like a device with high reliability but low cost. The stakeholders' concerns are often in conflict with one another (e.g., as flux increases, cost also increases), reflecting the conflicting demands common in professional engineering design.

During the second half of the internship, students switch teams and inform their new team members of the research they have conducted thus far in the internship. In the new teams, students test more devices, analyze the second iteration of results, and make a choice for a final prototype. During the final days of the internship, students present their final device design and justify their design decisions to the class and instructor.

3.2 Virtual Internship Design Space

All design problems, including the simulated problem in *Nephrotex*, have a set of inputs (design choices) and outputs (functions or performance parameters). Mathematically, the set of inputs can be described as a set $I = \{i_1, i_2, \dots, i_p\}$ where each i represents an input category. In turn, each input category i is composed of a set of choices, $C_i = \{c_1^i, c_2^i, \dots, c_r^i\}$ where each element c^i within C_i represents a choice within category i.

For example, in the virtual internship, *Nephrotex*, the set of inputs is described as

$I = \{material, surfactant, process, carbon\ nanotube(cnt)\}$.

The choices for the first three categories material, surfactant, and process are categorical where

$C_{material} = \{PSF, PRNLT, PMMA, PESPVP, PAM\}$

$C_{surfactant} = \{hydrophillic, negative\ charge, steric\ hindrance, biological, none\}$

$C_{process} = \{phase\ inversion, dryjet\ wet, vapor\ deposition\}$.

The choices for the last category, cnt, are numeric and ordered where

$C_{cnt} = \{0, 0.5, 1, 1.5, 2, 4, 6, 10, 15, 20\}$.

A potential solution for a design problem, i.e., a set of design choices, is within a design space, X, where $X = \times_{i \in I} C_i$. A set of design choices can be described as a vector $x = [x_{i_1}, x_{i_2}, \dots, x_{i_p}]$ for which there is one choice for every input category, i. There are 4 input categories in *Nephrotex*, so x is always a vector with 4 elements, $[x_{material}, x_{surfactant}, x_{process}, x_{cnt}]$. One example of a solution (or possible device design) in *Nephrotex* is

$x = [PAM, Hydrophilic, Phase\ Inversion, 2\%]$.

Within the design space, X, there is a space, D, that consists of the devices that students have selected as their optimum, final design. The elements within D are represented as solution vectors, d.

In design problems, there is also a set of outputs, where $O = \{o_1, o_2, \dots, o_n\}$, in which each element of o is an aspect of design function or performance. The performance of every solution to the design problem, which we call y, must reside within the output space, that is $y \in \mathbb{R}^O$. In general, the performance is a n dimensional vector $[y_1, y_2, \dots, y_n]$ for which there is one real number value for every output category. In *Nephrotex*, there are five aspects of performance for which the student is designing, so the vector O has five components:

$O = \{marketability, cost, reliability, flux, bcr\}$.

A representative solution to *Nephrotex* is a device with marketability 600,000, cost \$120, reliability 8 hours, flux 23 m^2/day and blood cell reactivity 43.3 nanograms/mL. Thus, the performance of this device can be described as:

$y = [600000, 120, 8, 23, 43.3]$

where

$y_{marketability} = 600000, y_{cost} = 120, y_{reliability} = 8, y_{flux} = 23, y_{bcr} = 43.3$

In all design problems, the selection of inputs affects the performance of the device. Thus, the design function can be represented as a mapping from the solution space, X, to the performance space, Y, which is a subset of real values in the output space

$$F: X \rightarrow Y \subseteq \mathbb{R}^O$$

Or

$$f(x) = y$$

Where x is a design vector and y is a performance vector. For example, in *Nephrotex*,

$f([PAM, Hydrophilic, Phase\ Inversion, 2\%])$= [600000, 120, 8, 23, 43.4].

3.3 Virtual Internship Participants

We implemented the *Nephrotex* virtual internship into two first-year, introductory, cornerstone engineering design courses at two large institutions. The first implementation occurred in fall 2013 at a university and contained 24 students in 5 student teams. The second implementation occurred in spring 2014 at a different university and contained 152 students in 30 student teams. In total, we collected final device specifications from 35 student teams in *Nephrotex* and thus collected the design specifications of 35 devices. We used this entire sample of 35 student teams final device specifications to test all four models.

3.4 Crisp Methods

3.4.1 Crisp Utility Model

In *Nephrotex*, the stakeholders collectively request that the device performance parameters meet a total of 20 thresholds. Each stakeholder identifies two outputs and then identifies two thresholds values for each output—a required threshold and a preferred threshold (Table 3).

Design advisors in the virtual internship advise students to meet as many of the thresholds as possible, thus implying that one of the main objectives of the design problem is to design a device with the highest utility value possible. One of the ways to model utility is by using crisp numbers. The output-specific crisp utility function, $u_{crisp}^{j}(y)$, consists of assigning one point for every threshold, τ, that a device satisfied for a given output, j. This method is considered a crisp method because there is no flexibility when determining the utility of a device specification—the device either meets the threshold or the device fails to do so (Figure 5).

The output-specific crisp utility function, is represented as

$$u_{crisp}^{j}(y) = \begin{cases} 0\ if\ y < \tau_1^j \\ 1\ if\ \tau_1^j \leq y < \tau_2^j \\ 2\ if\ \tau_2^j \leq y < \tau_3^j \\ 3\ if\ \tau_3^j \leq y < \tau_4^j \\ 4\ if\ y \geq \tau_4^j \end{cases}$$

That is, the crisp utility function indicates how many thresholds a device meets for one output. The result of $u_{crisp}(y)$ then is either 0, 1, 2, 3, or 4.

Based on data collected from previous (Arastoopour, Shaffer, Swiecki, Ruis, & Chesler, 2016), our first hypothesis was that students calculated the utility of a device by summing the utilities, or number of thresholds that a device satisfies, across the outputs to obtain a total utility, U_{crisp}.

$$U_{crisp}(y) = \sum_{j \in O} u_{crisp}^{j}(y)$$

For example, if a device meets all of the marketability thresholds, 1 of the cost thresholds, all of the reliability thresholds, 3 of the flux thresholds, and 3 of the BCR thresholds, then its crisp utility is calculated as 15 (Figure 6).

3.5 Fuzzy Methods

3.5.1 Approximate Fuzzy Utility Model

Based on data collected from cognitive interviews (Arastoopour et al., 2015), our second hypothesis was that students used fuzzy numbers to calculate utility. To create fuzzy models, we used approximations of fuzzy numbers that were more nuanced than the crisp numbers. As an initial approximation of a fuzzy utility model, we revised the crisp utility model so that it used additional values that were not given as explicit thresholds in *Nephrotex*. These additional values were derived from data collected in the cognitive interviews which showed that students made design decisions based on two self-constructed, additional intermediate thresholds. If the device met their constructed thresholds, the student was more likely to choose the device than if it did not meet their constructed thresholds. This type of decision-making process can be interpreted as a form of fuzzy decision-making.

As an approximation of a fuzzy number membership function, we used a piecewise linear function. To construct this model, we added additional utility values in the fuzzy model that are *between* the crisp utility values (from 1 to 4) (Figure 7), which allowed for values intermediate to the ones in the crisp model.

As an example, the lowest threshold to meet for flux in *Nephrotex* is 10 $\frac{m^3}{m^2 s}$ (meters cubed per second times meters squared). In the crisp utility model, a device that has a flux of 10 yields a crisp utility of 1. The next stakeholder threshold is 12, so devices that have a flux of 11.5, 11, and 10.5 would all be assigned a utility score of 2 in the crisp model. However, the fuzzy model assigns a device with a flux of 11 and 11.5 an intermediate utility score of 1.5. In other words, we assign different utilities to intermediate values between stakeholder requests, which we call *fuzzy utilities* (Table 4).

Then, for each device we summed the fuzzy utilities to obtain a total fuzzy utility, U_{fuzzy}.

$$U_{fuzzy}(y) = \sum_{j \in O} u^j_{fuzzy}(y)$$

For example, the same device, used as an example for calculating crisp utility had a crisp utility of 15, but has a fuzzy utility of 16.5 (Figure 8).

3.5.2 Fuzzy Preference Score Model

A fuzzy pairwise comparison compares two devices or choices and assigns a fuzzy comparison value instead of a crisp value. Because fuzzy numbers can be thought of as a family of nested intervals, in this study, we approximated a fuzzy comparison by employing a piecewise function. In this function, there exists some epsilon value when comparing two devices which allows for a range of numbers rather than one fixed number (Figure 9). The epsilon values takes into account some flexibility, or tolerance, in determining whether one device performs better or worse than another. For example, if one device has a utility of 13 and another has a utility of 14, an engineer may determine that the two devices have equal utility if he considers epsilon to be greater than 1.0.

Thus, the fuzzy pairwise comparison matrix (PCM) is defined by the function PCM_{fuzzy}^{ϵ} where

$$PCM_{fuzzy}^{\epsilon}(x_1, x_2) = \begin{cases} 1 \; if \; U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) > \epsilon \\ 0 \; if \; -\epsilon < U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) < \epsilon \\ -1 \; if \; U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) < -\epsilon \end{cases}$$

That is, the pairwise comparison matrix indicates, for each pair of designs, whether one is preferred over the other, given some epsilon value. If the total utility of device $x_1$is **greater than** the total utility of device x_2, then the result of the function is 1. If the total utility of device $x_1$is **equal to** the total utility of device x_2, then the result of the function is 0. If the total utility of device $x_1$is **less than** the total utility of device x_2, then the result of the function is -1. This comparison is repeated for all possible devices, x, in X. The complete result of PCM_{fuzzy} is a square matrix with values -1, 0, and 1.

For this study, we constructed a pairwise comparison matrix of all 570 devices using the fuzzy utilities and an epsilon value of 2.5. We determined the epsilon value of 2.5 empirically from the results of the student cognitive interviews. According to the interviews, on average, students considered the utilities of two devices equal when the differences were less than 2.5.

Lastly, to obtain one value for every device, which we call the *fuzzy preference score*, we summed each row of the PCM_{fuzzy}^{ϵ} matrix. The fuzzy preference score, ϕ_{fuzzy}, is calculated as

$$\phi_{fuzzy}(x) = \sum_{x \in X} PCM_{fuzzy}^{\epsilon}(x)$$

For example, if device 1 has a fuzzy utility of 8.2, device 2 has a fuzzy utility of 10.6, and device 3 has a fuzzy utility of 16.4, and device 4 has a fuzzy utility of 18.8, then device 3 and device 4 would be tied for the best device because they both have a preference score of 2 (table 5).

3.5.3 Fuzzy Percent Change Preference Score Model

In this model, we incorporated our fuzzy pairwise comparison function in addition to a percent change function to account for devices that had fuzzy utilities within 2.5 units. According to the cognitive interview results, if two devices were within 2.5 utility units, then students calculated the percent difference. This updated PCM piecewise function now contains a non-linear component (Figure 10).

This function is defined by the function $PCM_{fuzzy,\%\Delta}$ where

$$PCM_{fuzzy,\%\Delta}^{\epsilon}(x_1, x_2) = \begin{cases} 1 \; if \; U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) > \epsilon \\ \boldsymbol{P(x_1, x_2)} \; if \; -\epsilon < U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) < \epsilon \\ -1 \; if \; U_{fuzzy}\big(f(x_1)\big) - U_{fuzzy}\big(f(x_2)\big) < -\epsilon \end{cases}$$

where

$$P(x_1, x_2) = \sum_{j \in O} \frac{y_1^j - y_2^j}{mean(y_1^j, y_2^j)}$$

All $P(x_1, x_2)$ values were rescaled such that they ranged from -1 to 1 and were on the same scale as the other pairwise comparison scores. The result of $PCM^{\epsilon}_{fuzzy,\%\Delta}$ then is a square matrix in which the rows and consist of -1, 1, and, instead of 0, these values are now replaced with values ranging from -1 to 1. Finally, we summed each row so that there was one fuzzy preference score, $\phi_{fuzzy,\%\Delta}(x)$, for each device (See Table 6 for an example).

3.6 Poisson Regression Models with Student Data

We used the data collected on the number of times students chose a device for their final prototype to evaluate the applicability of the crisp and fuzzy utilities to model engineering student design decision-making. In particular, we calculated the crisp utility, fuzzy utility, fuzzy preference scores, and fuzzy percent change preference scores of each of the devices selected by students in *Nephrotex*. For each model, we first examined the summary statistics and because our outcome variable distribution was skewed (see result 1) and we are using count data, we conducted a Poisson regression as it takes into account the non-normal distribution of the outcome. We used this method to predict the frequency of final devices selected based on the values from each model. The regression was used to determine how closely each model predicted student data on their final device selection. For each model, we computed a Wald test to determine if the predictor was statistically significant ($p < .05$), and then we calculated the deviance G^2 and the AIC (Akaike, 1987) to determine the goodness of fit. We compared these statistics among all four models.

4. Results

4.1 Design-Based Decision-making

RQ 1: Does the distribution of devices that students selected as their final prototype provide evidence for design-based decision-making in *Nephrotex*?

Each team of students chose a final device at the end of the virtual internship. Out of a possible 570 devices, student teams chose 24 different devices. Because there were 35 student teams, several teams chose the same device. The number of times a device was selected by student teams is shown in Figure 11.

We denote the most popular device as device 1, which was chosen by 8 student teams. Devices 2 through 9 were also chosen by more than one student team in decreasing frequency of selection and the remaining 15 devices were each chosen by one team. The distribution of choices shows a skewed distribution, suggesting that students use similar methods for choosing final designs, but because this is a design problem with no one correct answer, there is some variety in their decision-making methods. Thus, we conclude that the simulated problem in *Nephrotex* provides student design-based decision making data with which we can test our models.

4.2 Summary of Results

All three models had significant predictors. The model with the best fit according to AIC, deviance, and Wald statistic was Model 4, the Fuzzy Percent Change Preference Score Model (Table 7).

4.3 Crisp Utility

RQ 2: Are crisp utilities a significant predictor of students' final design selections?

We constructed an initial model to predict student teams' final device choice in *Nephrotex*. Our predictor for this model was the utility of the device using the u_{crisp} function. The Poisson regression model was significant β = .36, Wald $\chi^2(1,22) = 2.71$, p < .05. This model had an AIC of 76.8 and a G^2 of 15.88.

4.3 Fuzzy Utility

RQ 3: Are fuzzy utilities a significant predictor of students' final design selections and how does this model compare to the previous model?

Using students' fuzzy utilities as a predictor, the Poisson regression model was significant β = .55, Wald $\chi^2(1,22) = 2.88$, p < .05. This model had an AIC of 75.2 and a G^2 of 14.28 and was a better fit than the model with a crisp utility function.

4.4 Fuzzy Preference Score

RQ 4: Are fuzzy preference scores a significant predictor of students' final design selections and how does this model compare to the previous models?

Using students' fuzzy preference scores as a predictor, the Poisson regression model was significant β = .005, Wald $\chi^2(1,22) = 3.37$, p < .05. This model had an AIC of 73.4 and a G^2 of 12.45 and was a better fit than the model with a fuzzy utility function.

4.5 Percent Change Preference Score

RQ 5: Are fuzzy percent change preference scores a significant predictor of students' final design selections and how does this model compare to the previous models?

Using students' fuzzy percent change preference score, the Poisson regression model was significant β = .006, Wald $\chi^2(1,22) = 3.57$, p < .05. This model had an AIC of 72.5 and a G^2 of 11.58 and was a better fit than the model with fuzzy preference score function.

5. Discussion

The development of this study was based on the key idea that imprecision occurs in real world, design-based decision-making. As a result, engineers tend to make fuzzy decisions based on a range of possibilities rather than a series of binary or crisp decisions. In this study, we examined students' decision-making processes while solving a simulated design problem. We used results from a previous study in which we conducted clinical cognitive interviews. Based on these results, in this current study, we developed four models: crisp utility, fuzzy utility, fuzzy preference score, and fuzzy percent change preference score. We used simple, piece-wise linear functions to approximate fuzzy membership functions for this analysis. When these models were applied these models to student decision-making data from a virtual internship, there were differences in terms of the goodness of fit of these models. The regression analysis showed that fuzzy models more accurately represented students' decision-making processes than crisp models. The fuzzy model with the best goodness of fit statistics was the fuzzy percent change preference score model, in which students used a fuzzy utility function and then conducted two different types of pairwise comparisons between devices. These results align with previous studies in which researchers developed fuzzy models for multi-criteria decision-making and showed that such fuzzy models are suitable for particular decision- making scenarios (Wang, 2001; Xia et al., 2011). For example, Triantaphyllou and Lin (1996) investigated five fuzzy multiattribute decision-making methods based on the original and revised analytic hierarchy model, the weighted-sum model, the weighted-product model, and the TOPSIS method (Hwang & Yoon, 1981). The authors concluded that the fuzzy revised analytic hierarchy model was better than the others in terms of the evaluation criteria for that particular decision-making scenario.

Our work builds on previous findings in fuzzy decision-making in design, but is unique in analyzing process data from undergraduate students who participated in a simulated design problem. Thus, this study can be viewed as a validation study that uses data-enabled cognitive modeling to provide evidence to support fuzzy decision-making models as representations of student engineers' decision-making processes.

The results in this validation study also have several implications for engineering educators. First, the results suggest that models of fuzzy decision-making are more aligned with how students make decisions in simulated design environments than crisp models and that a percent change model preference model is the best aligned with this particular simulated design problem. This implies that simulated design problems which are developed based on fuzzy decision-making models are useful approaches for allowing students to engage in realistic design scenarios. Second, in this study all students were given the same model-based simulated design problems to solve and the same resources with which to solve it. This approach gives all students an equal starting point and provides a basis for standardized assessment. Instructors can collect student process work in the form of digital engineering notebooks, conversations, or reports and collect student product work in the form of their final design specifications. They can then use this work to make valid comparisons among different students' design thinking and assess students' design thinking against engineering learning standards. Third, although the model analyzed 35 teams of students, the total number of students in this analysis was 175 because each team consisted of 5 students. As such, the results suggest that model-based simulated design problems may allow educators to scale up traditional instruction and provide more students access to quality design instruction which simulate real-world uncertainties. More generally, when creating design problems or scenarios for students, educators can use the fuzzy-decision making framework to create opportunities for students to practice fuzzy decision-making in the context of design and better understand how students are solving design problems.

Notwithstanding, this study has several limitations. First, as stated above, the models used in this study were developed using data from a previous study in which students solved a design problem individually. The models were then tested on data in which students made design decisions in teams, not as individuals. However, as shown in the results, the models significantly predicted decision-making when applied to team decision-making settings.

In addition, the models were tested with 35 data points (the number of devices that students chose in teams) from the virtual internship, which may not account for a sufficient amount of variability. In future studies, we will fit our model to a larger number of teams' design choices to test the robustness of the model.

Finally, we examined one particular simulated design problem, the design of a filtration membrane for a dialysis machine and how one population, first-year undergraduate students, solved simulated design problems. For future studies, we will examine simulated design problems with students and engineers at a variety of levels and also simulated problems in various engineering disciplines and potentially in design scenarios outside of the engineering domain.

In conclusion, this study presents data-enabled cognitive modeling, an approach to developing decision-making models that are validated with real-world process data. This work is beneficial for informing engineering design education curricula and for training future generations of design engineers.

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