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Energy Consumption Modeling and Analyses in Automotive Manufacturing Plant

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CUTTING FORCE CONTROL IN MACHINING: BAYESIAN UPDATE OF MECHANISTIC FORCE MODEL

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ABSTRACT

For closed loop control of machining forces in the turning process, it is well established that identification of the mechanistic force model is necessary to ensure stable operation of the process. This work proposes a novel approach to update the mechanistic force model by incorporating uncertainty in the deterministic framework. Force coefficient values reported in literature are based on wide spectrum of machining conditions and so cause difficulty in predicting the machining force using the mechanistic force model. This variability stems from variation in material workpiece input quality variation. This work proposes to treat force coefficient and process variables (shear stress and friction angles) as random variables and use Bayesian Statistical techniques to infer true distribution of force coefficients via observing cutting force and feed force values and updating shear stress and friction angle joint probability distribution. A numerical analysis is performed for calculating force coefficients for Titanium alloy (Ti6-Al4V) Markov Chain Monte Carlo (MCMC) simulation is performed to sample from the posterior distribution of the force coefficient. A single update cycle shows high reduction in the variability of the force coefficient. Numerical simulations presented indicate that it is possible to implement Bayesian update scheme in a closed loop control of cutting force for online identification of force coefficients and shear stress and friction angle distributions with few required update cycles and efficiently rejects the disturbance caused by changing machining parameters.

MACHINING FORCES MONITORING AND CONTROL: A REVIEW

Machining process force monitoring is valuable for tool wear state, chatter detection and overall process health. Though accurate means of measuring force with piezoelectric based dynamometers exist, they cannot be deployed in industrial environment mostly because of the inhibitive cost. Strain gauge based force sensors are relatively inexpensive, but suffer from low bandwidth because of slower response. There have been attempts to estimate the feed force by measuring feed axis motor current [1][2][3][4]. However, this method requires sweeping regions that machine will be operating in and generating a reliable model that will produce satisfactory estimation.

Machining force control problem has been investigated by variety of researchers over 4 decades by now. The pioneering work in this area is done by Ulsoy, Koren and Mesory [5][6][7] which discuss about Adaptive control, variable gain control, online estimation of the parameters. Some of the control structures are discussed in this work, mainly to give idea about the approaches already taken, and what can be done to improve them.

Integrator based controller based Adaptive Control Constraint (ACC) system

This approach was proposed first by [6], where the feed servo dynamics are represented by a second order dynamic system. The cutting force dynamic is represented as a first order dynamic system with the time constant solely dependent upon the spindle speed. They make an important observation about the stability of the ACC system stating the stabilizing gain has dependence on the spindle speed and depth of cut (Figure 1).

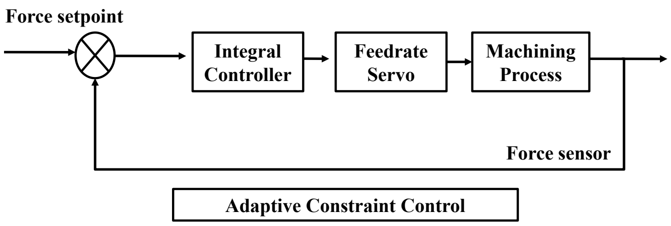

FIGURE 1: ADAPTIVE CONSTRAINT CONTROL

Variable gain Adaptive Control system:

Building on their earlier work, [7], proposed a way to update the gain of the control system in a way that it will not lead it to instability. It was accomplished by in-process estimation of the stabilizing control gain; the controller input is given to both the plant and the estimated model of the plant. The output of the plant and the model are compared, and the gains are so adjusted that the error between plant output and model output is driven to zero. The idea is presented in Figure 2.

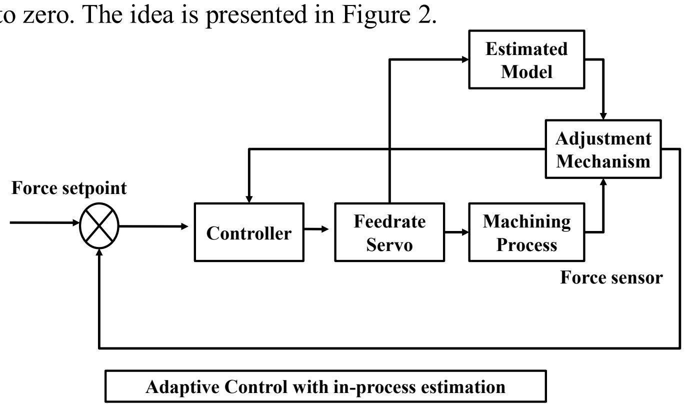

FIGURE 2: ACC WITH IN-PROCESS ESTIMATION

Other control strategies:

Apart from the control strategies mentioned earlier, there are other techniques reported which have been implemented. These include variable structure control [8][9] , intelligent sliding mode control [10] fuzzy logic control[11], robust control[12] and model predictive control [13]. Important point to note in all the work mentioned here is that the force is based on mechanistic model, the force coefficient is assumed to be known a priori and is constant. This coefficient is known to vary with tool wear, material flow stress and tool-chip friction conditions. Also, a first order force- feed dynamic model is chosen in all the references indicating that it is sufficient to describe the dynamics of the force.

To summarize, most of the prior art in this area assumes a specific static or dynamic feed-force model and attempts to drive the variable error to zero. They produce excellent results, but lack to reveal any insight in the physical nature of machining. The aim of this work is to control the machining force during the machining process, at the same time, get information about the cutting force coefficients during various conditions, estimating the shear stress and tool-chip interface friction which remains unobserved during the machining process. To that end, the current draft of the paper discusses how variability in force coefficient can be included in the deterministic model.

MODEL DEVELOPMENT AND PROPOSED APPROACH

As discussed in the review section, the most fundamental model proposed for the force control problem is the first order dynamics of force with feed as an input. In our paper, we shall be considering the same model since it describes the force dynamics up to a reasonable accuracy and it is easy to identify. The first order force dynamics model is given as follows.

$$\dot{F}_c + \frac{1}{\tau} F_c = K_c a f(t) \quad (1)$$

In equation (1), F_c and $\dot{F}_c$ are the cutting force and first order time derivative of cutting force respectively. The only identification required in this model is the force coefficient. τ is the time constant for the first order system and is given by $\frac{30}{N}$. a is depth of cut in this case. As discussed in the section before, most of the work done in this area deals with the assumption that the force coefficient K_c constant and identified a priori. Current work proposes update in value of K_c once the cutting and feed force values have been obtained. Also, gain insight in the variation of shear stress, tool-chip interface friction and shear plane angle through the process. This is achieved with Bayesian estimation of force coefficients, the control loop is shown in Figure 3.

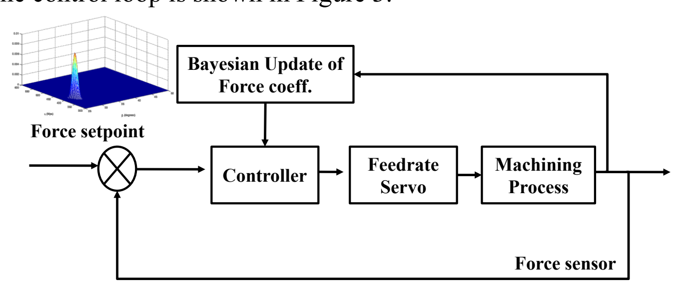

FIGURE 3: CONTROL LOOP WITH BAYESIAN UPDATE OF FORCE COEFFICIENT

It is noted [7], that the open loop gain of the system affects the stability of the closed loop system. In case of the force control in machining, this problem is addressed by estimating the open loop system gain [14], and performing variable gain tuning control. It is important to note that such approaches aim towards the control system performance (tracking) rather than model improvement. Bayesian update is proposed not only as a way to ensure system stability by updating the open loop gain, but also a way to estimate shear stress and friction angles-otherwise unobservable variables in the machining process.

Since the force coefficients are influenced by both shear stress and friction angle, update is only reliable with knowledge of both cutting force and feed force. This can be considered as one of the limitations of the method, since it can lead to more expensive instrumentation. Next few sections discuss the numerical recipe necessary to understand the nature of Bayesian update.

BAYESIAN UPDATE: A BRIEF INTRODUCTION

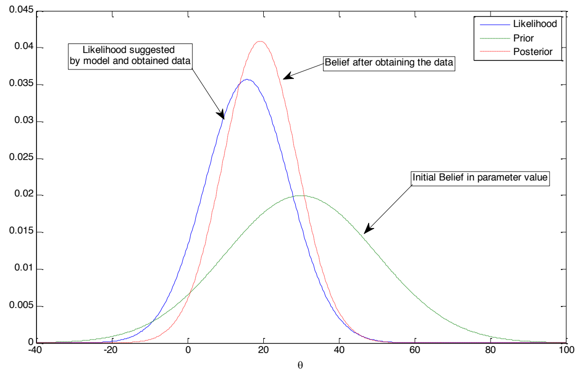

FIGURE 4: CENTRAL IDEA OF BAYESIAN UPDATE

In the simplest sense, Bayesian view of probability indicates the state of knowledge or belief in a certain hypothesis. It originates from Thomas Bayes, for proving the Bayes theorem. In context of parameter identification, let ω, be the parameter of interest, K be initial state of knowledge, D be the data point, the Bayes' theorem can be written as follows,

$$p(\omega|D,K)=\frac{p(D|\omega)p(\omega|K)}{\int p(D|\omega)p(\omega|K)d\omega} \quad (2)$$

In equation(2), $p(\omega|K)$ is read as "probability distribution of value of parameter ω, given initial state of knowledge K" often referred to as a "*prior*". $p(D|\omega,K)$ is read as "probability that the data point observed would relate to the parameter value", called the "*likelihood*". Likelihood often relates the data point to the parameter of interest via a model; it is a very important part of the solution as we shall observe in the later sections. And finally, $p(\omega|D,K)$ is "probability distribution of value of parameter ω, given initial state of knowledge K *and* having observed the data point D, called a "*posterior*". The denominator is a normalization factor, since the probability distribution must sum to unity.

Figure 4 shows this process graphically. Few points are worth noting, the posterior distribution has much less spread as compared to prior. Also, the definitiveness of both prior and likelihood dictate the variance of the posterior. Furthermore, the prior can be uninformative (uniform distribution), thus showing complete ignorance about the value of the parameter, in that case, it is the likelihood that dominates the posterior behavior.

NUMERICAL TOOLS: MONTE CARLO SIMULATIONS AND MARKOV CHAIN MONTE CARLO (MCMC) METHODS

As discussed in the previous section, the calculation of probability distributions includes integrals of different distributions. In an applied Bayesian inference scheme, this may not be feasible to do because of unavailability of analytical expression for probability distribution or the integral itself is tedious to perform. Thus, Markov Chain Monte Carlo (MCMC) methods provide a means to perform these integrals numerically. This technique is widely used in biostatistics; image and video processing, voice recognition and machine learning fields. Major applied work in MCMC area is reported by [15]. MCMC is useful when one wants to generate samples from a distribution that is analytically intractable. This is achieved by strategically constructing Markov Chains whose stationary distribution converges to the desired distribution. To deploy this in practice, there are various algorithms which include Gibbs Sampling, Metropolis algorithm and Metropolis Hastings Algorithm.

In this work, Metropolis Hastings algorithm is used to generate samples from posterior distribution. The specifics will be described in the next section, but in this section, the key points of algorithm are explained [16].As described earlier, Markov Chain needs to be generated whose stationary distribution is the target distribution we want to sample from $p(\bullet)$. At each iteration step k, the next state X_{k+1} is generated by sampling a candidate point Y from a proposal distribution $q(\bullet|X_k)$

In cases where the candidate generating distributions are symmetric, $q(Y|X_k)=q(X_k|Y)$, yielding ,

$$\alpha=\min\left\{1,\frac{p(Y)}{p(X_k)}\right\} \quad (3)$$

This is the algorithm that was proposed by [17]. In our work, we use the Random Walk Metropolis sampler, initially introduced by [18]. In the following pseudo-code, the algorithm is described, please refer to Figure 5.

Initialize X_0; set k=0;
set n=large number;
for k=1:n
{
Y=X_k+ normal random(0,Σ);
Sample a Uniform(0,1) random variable u

Calculate $\alpha(X_k, Y) = \left\{1, \frac{p(Y)}{p(X_k)}\right\}$

If u<=$\alpha(X_k, Y)$ set X_{k+1}= Y
Else set X_{k+1}= X_k
}

FIGURE 5: PSEUDO-CODE FOR RANDOM WALK METROPOLIS ALGORITHM

In the next section it will be described how MCMC methods can be used to sample from posterior distribution, which is almost intractable analytically.

NUMERICAL ANALYSIS

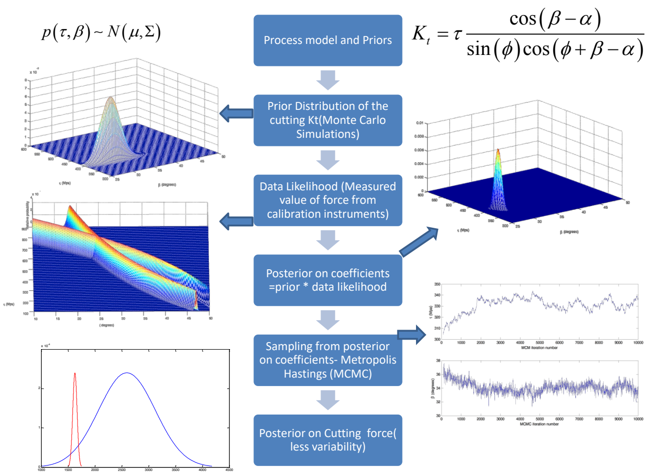

FIGURE 6: ALGORITHMIC VIEW OF APPROACH TAKEN IN THIS WORK

Figure 6 depicts algorithmically one update cycle in “belief” of the value of force coefficients. The steps involved are

- Establishment of priors
- Data likelihood generation
- Posterior distribution calculation & sampling

Establishment of priors:

The mechanistic force model given as follows

$$F_c = K_c bh + \varepsilon$$
$$F_t = K_t bh + \psi \tag{4}$$

Where b is depth of cut and h is feed per revolution. ε and ψ represent uncertainty in measurement of the cutting force because of variation in force coefficients.
The force coefficients are given as[19],

$$K_c = \frac{\tau\cos(\beta-\alpha)}{\sin(\phi)\cos(\phi+\beta-\alpha)}$$
$$K_t = \frac{\tau\sin(\beta-\alpha)}{\sin(\phi)\cos(\phi+\beta-\alpha)} \tag{5}$$

Where τ is the shear stress during the cutting (assuming orthogonal machining model), β is the friction angle, ϕ is the shear plane angle. Now the variability in force is directly proportional to variability in force coefficient since depth of cut and feed are machine parameters usually known and controlled.

$$p(K) \propto p(\tau,\beta,\phi) \tag{6}$$

Where $p(K)$ indicates the probability distribution of force coefficient and $p(\tau,\beta,\phi)$ is joint probability distribution of shear stress, friction angle and shear plane angle. Shear plane angle is independent of shear stress and friction angle and usually known. Thus equation (6) can be reduced to,

$$p(K) \propto p(\tau,\beta)p(\phi)$$
$$\propto p(\tau,\beta) \tag{7}$$

Thus variability in force coefficient is directly proportional to variability in shear stress and friction angle. Therefore, for the estimation of the forces, it is necessary to observe the joint variability (or joint probability distribution) of τ and β.
It is important to note here that for the accurate update of the force coefficient, it is necessary to have values of τ, β and ϕ. However, the measureable quantities here are only F_c and F_f (cutting and feed force). For the update of the shear plane angle, with the knowledge of the chip thickness, following relation can be used.

$$\phi = \frac{r_c\cos\alpha}{1-r_c\sin\alpha}; r_c = \frac{t_c}{t_{un}} \tag{8}$$

With which the initial belief in the shear plane angle can be updated after every cut. In the scenario where the dynamic update of force coefficients has to be made, one needs to resort to the empirical relationships, one of the popular ones given as follows[20],

$$\phi = 45^\circ - \frac{\beta}{2} + \frac{\alpha}{2} \tag{9}$$

Based on some primary literature search [21] [22] [23], for alloy Ti6-Al4V, shear stress and friction angle joint distribution can be represented by,

$$p(\tau,\beta) \sim N\left(\begin{bmatrix}500\\30\end{bmatrix},\begin{bmatrix}200 & 0\\0 & 5\end{bmatrix}\right) \tag{10}$$

This is a Bivariate Gaussian distribution with no cross variance. Figure 7 shows the 2-dimensional probability distribution of coefficients.

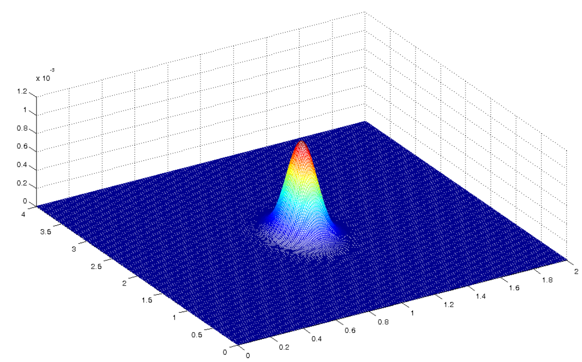

FIGURE 7: PRIOR ESTABLISHMENT FOR COEFFICIENTS

It is important to mention that the convergence to true force coefficient values depend upon the selection of prior distribution. That is, if the prior is chosen close to actual value of force coefficient, the convergence will be faster. Though this demonstration assumes a Gaussian prior centered around the literature reported values, the scheme is also valid for a uniform distribution (non-informative prior).

Data Likelihood for the Force

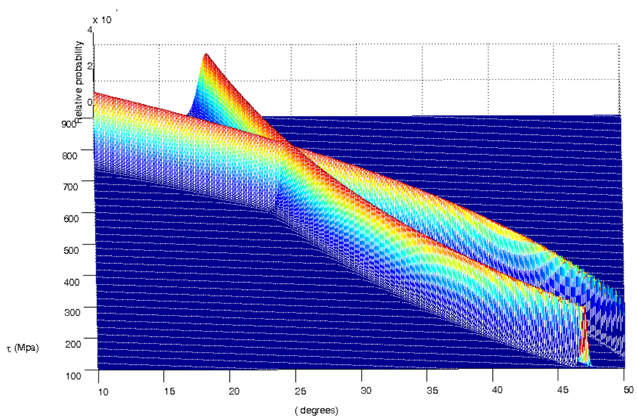

FIGURE 8: DATA LIKELIHOOD FOR FORCES

Update in force coefficient is made whenever a new data point is made available. Since shear stress and friction angle contribute to cutting and feed forces, both cutting and feed forces help update the force coefficient value. This is done by using equation(5). The method deployed here is called discrete grid method [19]. First, the shear stress and friction angle values are divided in a finite grid, and then with the measured force value, probability of all possible values of shear stress and friction angles are calculated that will produce that force. To introduce uncertainty, the measured value of torque is assumed to have some measurement noise (2-5%). This way, we get the likelihood function which solves the inverse problem of "given the data point and my model, what is the probability that estimated coefficients (parameters) produce the observed data". And that selected value of shear stress and angle will give the measured value of force using a deterministic model in the presence of uncertainty. The data likelihood is shown in Figure 8. The calculation of the posterior follows from point to point multiplication of the prior density with the data likelihood.

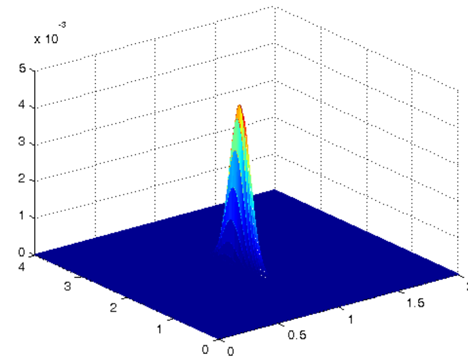

FIGURE 9: POSTERIOR DISTRIBUTION OF COEFFICIENTS

Sampling from posterior: MCMC scheme

Once the data likelihood is established, the posterior distribution of the shear stress and friction angle is generated by point by point multiplication of the prior distribution and the likelihood function (Figure 9). Since at this point, we do not have the analytical expression that represents posterior distribution; we use MCMC methods discussed in earlier sections to generate samples that represent the posterior distribution.

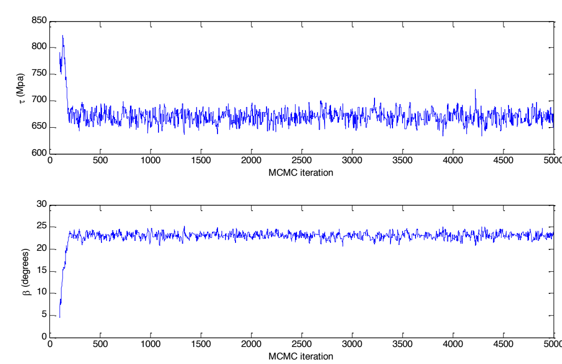

FIGURE 10: MARKOV CHAIN MONTE CARLO SIMULATIONS TO GENERATE SAMPLES FROM POSTERIOR DISTRIBUTION OF COEFFICIENTS

While using the Random Walk Metropolis algorithm, it is important to have the increment size generating a new sample (the Σ in Figure 5) smaller than the distribution one is sampling from. If this is not the case, then the convergence will not be observed [24]. Also, [25] discuss about the selection of the random walk increment matrix and acceptance ratio. In the presented work, first the values for the coefficients that produce some minimal probability were calculated. Then the Σ matrix was multiplied with a gain factor that produced the acceptance ratio between 40-50%. Additionally, there is some “burn-in” time required for the candidate samples to get converge. This evolution is shown in Figure 10. In the, Figure 11 the samples produced from the MCMC scheme are compared with the posterior distribution, indicating that MCMC scheme does produce the samples that represent posterior distribution.

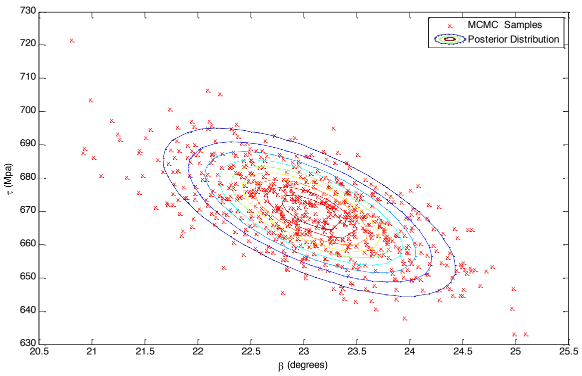

FIGURE 11: MCMC SAMPLES COMPARED WITH POSTERIOR

The mean of the posterior distribution indicates the updated shear stress and friction angle values. These values are then used in equation (5) to generate updated force coefficient values.

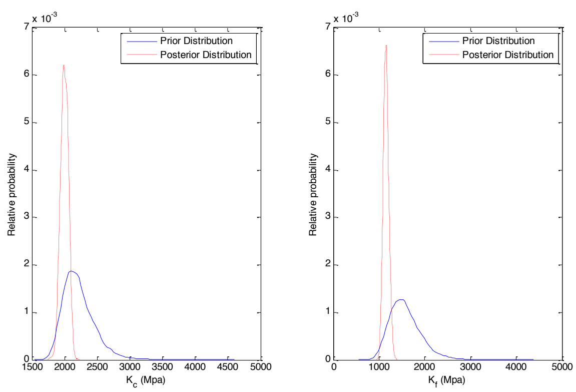

FIGURE 12: FORCE COEFFICIENT UPDATE - REDUCED VARIABILITY

As shown in Figure 12, reconstruction for the force coefficient distribution reveals much reduced variability before and after the update. The prior distribution is indicated with blue solid line and posterior distribution is indicated with red dotted line. This validates the numerical scheme accuracy and stability.

It is worth mentioning how this method is novel from the other non-model based (purely feedback based) methods. Though the force coefficient values are known to be constants, they often vary for different speed and feed regimes. If a deterministic mechanistic model is chosen, it is quite possible that the prediction of forces might be accurate in a particular regime, but not across the entire range. This method provides a means not only for a prediction of forces from mechanistic point of view, but also provides understanding in distribution of friction values, shear stress and shear plane angles, and how it varies across different cutting load and speed regimes. From the control theory point of view, it provides an automatic tuning feature. In the continuing work, authors are investigating treatment of outliers and in process identification of shear plane angles.

NUMERICAL SIMULATION: CLOSED LOOP IDENTIFICATION OF FORCE COEFFICIENT

To evaluate functionality of the Bayesian update approach proposed in this work, numerical simulation of cutting force control was performed in MATLAB Simulink® package. Since Bayesian update scheme requires variety of calculations not included in standard Simulink blocks, Embedded Matlab Function was written to execute the Bayesian update part. Following modifications were made to the control loop for the simulation purposes:

- The force setpoint is converted to feedrate set-point using machining parameters (spindle RPM and depth of cut) and initial belief of force coefficient.
- PI controller achieves desired feedrate by generating control signal and receiving feedback from servo position.
- The output feedrate is converted back in to force value using machining parameters and actual value of force coefficient. This is the part which gets replaced by plant in experimental implementation. Gaussian noise is added to induce uncertainty in force value. This is the value of force coefficient we want to estimate.
- Input to Bayesian update scheme are machining parameters, initial beliefs on shear stress, friction and cutting force.
- Output of Bayesian update scheme are cutting force coefficients and updated distributions of shear stress and friction angles. These distributions are used as initial beliefs in next time-step.
- Variance estimation by MCMC scheme is omitted because of limit of memory allocation in computational equipment used (when used in closed loop simulation).

It is important to observe the response of the controller in light of step changes in depth of cut. In the simulation presented, initial depth of cut was 1.5 mm and then stepped up to 2 mm at time 2 seconds. Here, simulation was performed for Al6061-T6 alloy cutting force control. The Bayesian scheme starts after spindle has completed one full revolution.

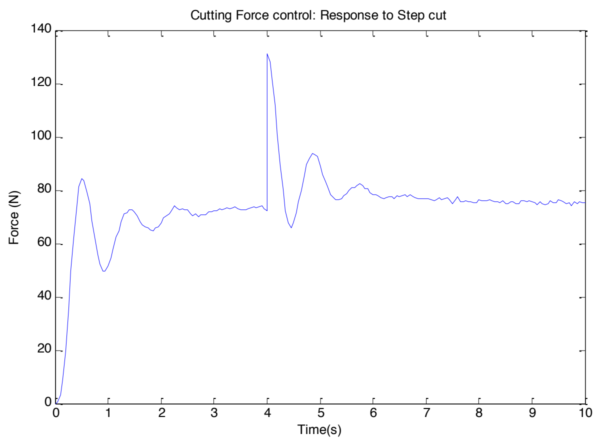

FIGURE 13: CUTTING FORCE VALUE (SET POINT 75 N) CHANGE IN DEPTH OF CUT AT 2 SEC.

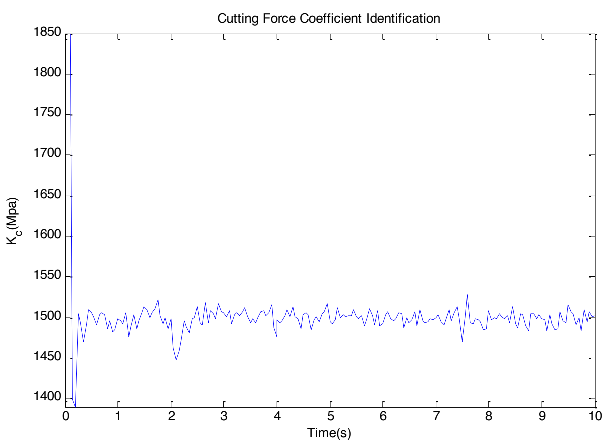

FIGURE 14: ONLINE FORCE COEFFICIENT IDENTIFICATION

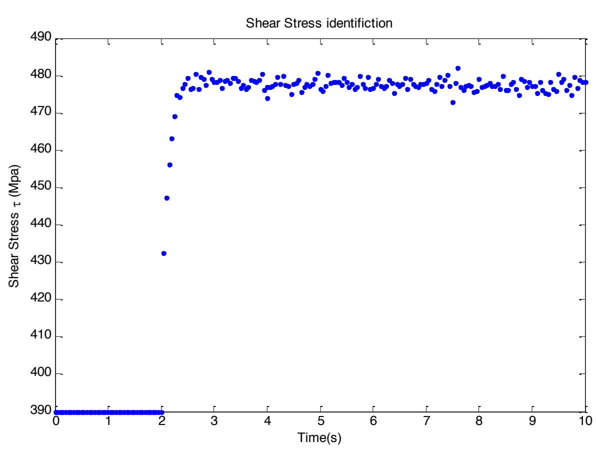

FIGURE 15: ONLINE SHEAR STRESS IDENTIFICATION

Figure 13 shows the cutting force value when closed loop control is deployed. The force set point is 75 N and at 2 seconds, depth of cut changes from 1.5 mm to 2.5 mm. After some transience, the force value settles to the desired set point. Now, our interest is also in identification of force coefficients and shear stress values. Figure 14 shows the estimated force coefficient value. It can be observed that the estimated value of force coefficient is very close to the actual value (1500 MPa), the minor oscillations in this value is because of Gaussian noise in force. Also note that force coefficient value is not much affected by change in depth of cut. Figure 15 shows the identified shear stress values, as it can be observed, only few updates are needed to identify the shear stress values and it shows very small variations for the rest of the duration of cut.

EXPERIMENTAL VALIDATION

This section describes the experimental set up to validate the numerical scheme. The tests will be taken on Okuma Lb4000 EX CNC lathe. The lathe is instrumented with a commercially available current transducer based power monitoring unit along with custom made strain gage based force sensor. The schematic of the experimental set up is shown in Figure 16.

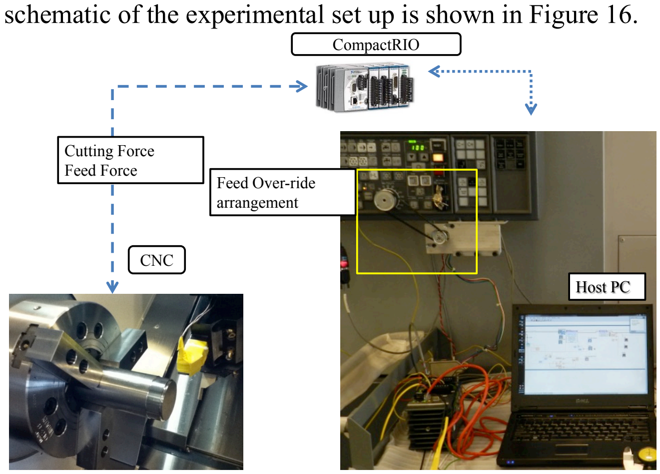

FIGURE 16: EXPERIMENTAL SET UP OF CUTTING FORCE CONTROL IN TURNING

The output from the current transducer is an analog signal (0-10V) which represents the power measured in HP. This signal is acquired with NI –CompactRIO (cRIO-9023) control prototyping module for signal processing and data storage. In the same set up, there are additional sensors measuring cutting and feed force and temperatures near cutting edge.

Since the goal of this work is the control the cutting force in real time, it is important to be able to change the machining parameters that enable one to achieve this target. Since it is not possible to access the internal signal of the feed servo signal, authors plan to do this via attaching a DC servo motor to the feed override knob. This way, though discrete, but an external means is available to control the feed input and thereby controlling the force. A separate publication (under preparation) will discuss the deployment and results of the experimental study.

CONCLUSION AND CONTINUING WORK:

In this work, a novel way of incorporating uncertainty in the mechanistic force machining model was introduced. Depending upon the machining regime, the shear stress, friction angle and shear angle influence the cutting forces. It is important to update the mechanistic force model since it affects the open loop gain. Here, based on the cutting process feedback (cutting and thrust force), the belief in shear stress and friction angles were updated, ultimately reducing variability in the force coefficient. This was also deployed in closed loop numerical simulations.

As the next step, authors wish to integrate the Bayesian update scheme in a closed loop control. The framework will be similar to one that involves model learning using Recursive Least Square (RLS) methods of parameter estimation in case of linear Gaussian dynamic models for cutting force control or power control in turning or milling process. Later on, more complex and non-linear mechanistic force models will be incorporated.

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