# Software Development and Platform Adoption as Successive Games of Real Options Investment and Valuation 

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# Software Development and Platform Adoption as Successive Games of Real Options Investment and Valuation 

\(\left.\begin{array}{c}A Dissertation <br>
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the Graduate School of <br>
Clemson University <br>
In Partial Fulfillment <br>
of the Requirements for the Degree <br>
Doctor of Philosophy <br>

Computer Science\end{array}\right]\)| by |
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## Abstract

Platform based software engineering is at the heart of a new mode software product development in the context of software ecosystems. In this setting, an organization develops a software platform with the intention of providing that platform for use and extension by software-producing organizations. Multiple benefits arise from engaging in platform-based software engineering from both the perspective of the platform developer and the software product developer, including decreased time to market, defrayed cost of development and increased software quality. Organizations have been engaging in platform-based software engineering for years, exemplified by cases such as Eclipse, Android and SAP.

However, the body of research that studies software ecosystems and platform-based software engineering is still growing, with many areas still requiring further investigation. One such area is decision-making support for software platform adoption. Platform adoption, more strategically significant than simple acquisition and use of third party libraries, represents a reciprocal relationship between the software platform developer and the product developer. This relationship, and the products developed from the platform, may be long-lived, necessitating a close relationship between the platform developer and the product developer. Thus, platform adoption is strategic, rather than tactical, in nature.

However little research exists that investigates decision making in the context of software platform adoption. While the research community is cognizant of prominent decision support criteria for software platform adoption, including licensing, hardware and operating systems compatibility, little research attempts to quantify the benefits afforded to the software platform developer, and even less that investigates the benefits realized by adopting organizations who produce software products based on a software platform.

This work is the first stage in a long term research plan for quantifying the cost and earned
value of engaging in platform-based software engineering from the perspective of a software product developer adopting a software platform. We have illustrated the adoption decision through two scenarios that exemplify strategic concerns raised in software platform adoption. The central assumption of this work is that software platform adoption reduces the cost of software development while increasing the earned value of the software product being built. Using this central theory, we propose a model for quantifying the cost and earned value of a platform-based software development. This model views software development as a series of decisions, or rather options, concerned with the decision of whether to engage or halt software development. Our model utilizes the Black-Scholes model for options evaluation. The research illustrates utilization of stochastic Monte Carlo simulation in order to perform experimentation on our underlying model as applied to our scenarios. From this research, we intend to develop theory from our simulation results that helps support strategic decision making in the context of the software ecosystems surrounding the platform and products.

## Dedication

This work is dedicated to my mother, whose loving support and editorial expertise helped me finish.

## Acknowledgements

There are far too many people to acknowledge for their contribution, direct or indirect, to the completion of this work. First and foremost, I would like to thank adviser, Dr. John D. McGregor, for his tireless mentoring of me throughout my graduate studies. I would also like to thank my committee for reading and providing valuable feedback on my work, as well as the School of Computing for its continued funding in my graduate endeavors. From the School of Computing, particularly Dr. Mark Smotherman for giving my a chance back in 2008 when I was but a timid undergraduate searching for higher-higher education.

I would like to acknowledge every teacher and professor I have had in my long and storied academic career. There are far too many of you to list by name, but each and every one of you had a profound impact on me as a student, on my view of education and on the path that got me to this point today.

I would like to thank my family for their countless questioning of defense and graduation dates. While I groaned at the same question being asked at every family gathering, the constant reminder that I had to defend and graduate helped keep me on track.

## Table of Contents

[^0]Abstract ..... ii
Dedication ..... iv
Acknowledgements ..... v
List of Tables ..... viii
List of Figures ..... x
1 Introduction ..... 1
1.1 Problem Statement ..... 2
1.2 Research Contribution ..... 4
1.3 Dissertation Organization ..... 4
2 Background ..... 6
2.1 Software Ecosystems ..... 6
2.2 Options and Their Value ..... 11
2.3 Formalizing Options Valuation and Black-Scholes Option Value ..... 15
2.4 Black-Scholes Formula: Call Options ..... 16
2.5 Parameterization of Black-Scholes ..... 17
3 Model Development ..... 19
3.1 Period-Based Software Development ..... 19
3.2 Modeling Cost and Value ..... 24
3.3 Experimental Approach ..... 30
3.4 Sources of Stochasticity ..... 32
3.5 Summary of Existing Work ..... 32
4 Experimental Design and Execution ..... 34
4.1 Period Length and Project Duration ..... 34
4.2 Software Development Cost ..... 35
4.3 Software Development Cost Reduction ..... 36
4.4 Software Development Value ..... 37
4.5 Value Addition ..... 38
4.6 Riskless Discount Rate ..... 38
4.7 Volatility ..... 39
4.8 Probability of Successful Implementation ..... 40
4.9 Discussion ..... 40
5 Experimental Results ..... 41
5.1 Expected Results and Outputs ..... 41
5.2 Simulation Structure and Computation ..... 42
5.3 Scenario 1: Choice of Two Platforms ..... 45
5.4 Scenario 2: Switching Platforms ..... 58
5.5 Summary of Results ..... 76
6 Verification and Validation ..... 78
6.1 Sensitivity Analysis ..... 79
6.2 Value Addition ..... 84
6.3 Internal Validity ..... 94
6.4 Extreme Conditions Validity ..... 99
7 Related Work ..... 105
8 Future work ..... 109
8.1 Probability of Successful Completion ..... 109
8.2 Failure Model ..... 110
8.3 Cost and Value Carry Over Model ..... 110
8.4 Cost Modeling ..... 111
8.5 Cost and Value Theoretic ..... 111
8.6 Scope of Model ..... 111
9 Conclusion ..... 113
Appendices ..... 114
A Expanded Problem Statement Scenarios ..... 115
B Software Product Lines: A Quantified Model for Valuing Variation Points ..... 118
C Rational Unified Process ..... 125
D Experimental Data ..... 128
E Verification and Validation Data ..... 144
Bibliography ..... 160

## List of Tables

5.1 4-Bucket Stratified Failure Model ..... 43
5.2 Scenario 1 Project Constants ..... 46
5.3 Scenario 1 RUP Project Timeline ..... 46
5.4 Scenario 2 Project Constants ..... 60
5.5 Scenario 2 Construction Parameters ..... 60
5.6 Scenario 2 Cost Reduction and Added Value Coefficients ..... 61
5.7 Scenario 2 Parameters: Native Web Platform ..... 62
5.8 Scenario 2 Parameters: Open Source ..... 64
5.9 Scenario 2 Parameters: Proprietary Platform ..... 66
5.10 Scenario 2 Parameters: Open Source Before Proprietary ..... 68
5.11 Scenario 2 Parameters: Proprietary Before Open Source ..... 70
6.1 Baseline Scenario for Sensitivity Analysis ..... 80
6.2 Cost Reduction Regression Analysis for Net Value ..... 82
6.3 Cost Reduction Regression Analysis for Sum Net Value ..... 83
6.4 Value Addition Regression Analysis for Net Value ..... 86
6.5 Value Addition Regression Analysis for Sum Net Value ..... 87
6.6 Probability Regression Analysis for Net Value with Stratified Failure Model ..... 90
6.7 Probability Regression Analysis for Sum Net Value with Stratified Failure Model ..... 91
6.8 Probability Regression Analysis for Sum Net Value with Binary Failure Model ..... 95
6.9 Probability Regression Analysis for Sum Net Value with Binary Failure Model ..... 96
6.10 Riskless Discount Coefficient for T-t $=[0 \ldots 10]$ ..... 101
1 Asset-Product Mapping ..... 122
2 Asset Development Cost Per Period ..... 123
3 Revenue per Product ..... 123
4 Simulation Results ..... 124
5 Scenario 1 Cost and Value Structure ..... 128
$6 \quad$ Scenario 1 Bespoke Platform Net Value per Period ..... 129
$7 \quad$ Scenario 1 Bespoke Platform Sum Net Value Per Period ..... 130
8 Scenario 1 Code-Centric Platform Net Value Per Period ..... 131
$9 \quad$ Scenario 1 Code-Centric Platform Net Value Per Period ..... 132
10 Scenario 1 Architecture-Centric Platform Net Value Per Period ..... 133
11 Scenario 1 Architecture-Centric Platform Sum Net Value Per Period ..... 134
12 Scenario 2 Native Web Platform Cost and Value ..... 135
13 Scenario 2 Native Web Platform Net Value Per Period ..... 135
14 Scenario 2 Native Web Platform Sum Net Value Per Period ..... 135
15 Scenario 2 Open Source Platform Cost and Value ..... 136
16 Scenario 2 Open Source Platform Net Value Per Period ..... 136
17 Scenario 2 Open Source Platform Sum Net Value Per Period ..... 136
18 Scenario 2 Proprietary Platform Cost and Value ..... 137
19 Scenario 2 Proprietary Platform Net Value Per Period ..... 137
20 Scenario 2 Proprietary Platform Sum Net Value Per Period ..... 137
21 Scenario 2 Open Source Before Proprietary Platform Cost and Value ..... 138
22 Scenario 2 Open Source Before Proprietary Platform Net Value Per Period ..... 139
23 Scenario 2 Open Source Before Proprietary Platform Sum Net Value Per Period ..... 140
24 Scenario 2 Proprietary Before Open Source Platform Cost and Value ..... 141
25 Scenario 2 Proprietary Before Open Source Platform Net Value Per Period ..... 142
26 Scenario 2 Proprietary Before Open Source Platform Sum Net Value Per Period ..... 143
27 Period Net Value for Cost Reduction Coefficient Sensitivity Test ..... 144
28 Period Net Value for Cost Reduction Coefficient Sensitivity Test ..... 145
29 Period Net Value for Value Addition Coefficient Sensitivity Test ..... 146
30 Sum Net Value of Final Period for Value Addition Coefficient Sensitivity Test ..... 147
31 Period Net Value for Probability Sensitivity Test with Stratified Failure Model ..... 148
32 Sum Net Value for Probability Sensitivity Test with Stratified Failure Model ..... 149
33 Period Net Value for Probability Sensitivity Test with Binary Value Model ..... 150
34 Sum Net Value for Probability Sensitivity Test with Binary Value Model ..... 151
35 Internal Validity: Experiment 1 Net Value Period Period ..... 152
36 Internal Validity: Experiment 1 Net Value Period Period Standard Deviation ..... 153
37 Internal Validity: Experiment 2 Net Value Period Period ..... 154
38 Internal Validity: Experiment 2 Net Value Period Period Standard Deviation ..... 155
39 Internal Validity: Experiment 2 - Experiment 1 Absolute Value ..... 156
40 Extreme Value Test: Net Value Per Period with r $=0.05$ ..... 157
41 Extreme Value Test: Net Value Per Period with $\mathrm{r}=0.5$ ..... 158
42 Extreme Value Test: Net Value Per Period with $\mathrm{r}=1.0$ ..... 159

## List of Figures

3.1 Waterfall Method of Development ..... 22
3.2 Iterative-Incremental Method of Development ..... 23
5.1 Experimental Process ..... 42
5.2 Bespoke Development: Net Value per Period ..... 48
5.3 Bespoke Development: Sum Net Value per Period ..... 49
5.4 Bespoke Development: Cost Carry Out Per Period ..... 50
5.5 Bespoke Development: Value Carry Out Per Period ..... 51
5.6 Architecture Centric Platform: Net Value per Period ..... 52
5.7 Architecture Centric Platform: Sum Net Value per Period ..... 53
5.8 Architecture Centric Platform: Cost Carry Out Per Period ..... 54
5.9 Architecture Centric Platform: Value Carry Out Per Period ..... 54
5.10 Code Centric Platform: Net Value per Period ..... 55
5.11 Code Centric Platform: Sum Net Value per Period ..... 56
5.12 Code Centric Platform: Cost Carry Out Per Period ..... 57
5.13 Code Centric Platform: Value Carry Out Per Period ..... 57
5.14 Native Web Platform: Net Value Per Period ..... 63
5.15 Native Web Platform: Sum Net Value Per Period ..... 64
5.16 Open Source Platform: Net Value Per Period ..... 65
5.17 Open Source Platform: Sum Net Value Per Period ..... 66
5.18 Proprietary Platform: Net Value Per Period ..... 67
5.19 Proprietary Platform: Sum Net Value Per Period ..... 68
5.20 Open Source Before Proprietary: Net Value Per Period ..... 69
5.21 Open Source Before Proprietary: Sum Net Value Per Period ..... 70
5.22 Proprietary Before Open Source: Net Value Per Period ..... 71
5.23 Proprietary Before Open Source: Sum Net Value Per Period ..... 72
5.24 Open Source Before Proprietary: Fiscal Solvency Outlay ..... 75
5.25 Proprietary Before Open Source: Fiscal Solvency Outlay ..... 76
6.1 Sensitivity Analysis: Cost Reduction vs. Net Value vs. Elapsed Time ..... 81
6.2 Sensitivity Analysis: Cost Reduction vs. Sum Net Value vs. Elapsed Time ..... 84
6.3 Sensitivity Analysis: Value Addition vs. Net Value vs. Elapsed Time ..... 85
6.4 Sensitivity Analysis: Value Addition vs. Sum Net Value vs. Elapsed Time ..... 88
6.5 Probability vs. Net Value vs. Elapsed Time with Stratified Failure ..... 89
6.6 Probability vs. Sum Net Value vs. Elapsed Time with Stratified Failure ..... 92
6.7 Probability vs. Net Value vs. Elapsed Time with Binary Failure ..... 93
6.8 Probability vs. Sum Net Value vs. Elapsed Time with Binary Failure ..... 97
6.9 Internal Validity: Development Period vs. Net Value Difference vs. Elapsed Time ..... 98
6.10 Development Period vs. Period Net Value vs. Elapsed Time with r $=0.05$ ..... 102
6.11 Development Period vs. Period Net Value vs. Elapsed Time with r $=0.5$ ..... 103
6.12 Development Period vs. Period Net Value vs. Elapsed Time with r $=1.0$ ..... 103

## Chapter 1

## Introduction

Development of a software intensive product has evolved from an isolated activity, conducted totally within a single organization, to a global activity among a set of collaborating organizations. The recent recognition of software ecosystems, a community of collaborating organizations, highlights a new approach to software development, distributed across multiple organizations, in which developers and other stakeholders participate in software development outside of cultural, geographical and domain boundaries. Both organizations and developers engage in competitive and cooperative behavior in order to reap the rewards of this globally diverse development and innovation.

At the heart of the software ecosystem approach is the notion of a software platform that attracts users to come together and participate in a diverse community of developers and users. This platform serves as a unifying asset that is developed, shared and used together by an often heterogeneous group of stakeholders. This platform and the community of stakeholders defrays the cost of development, while a software platform confers benefits to organizations adopting the platform as a basis for their product. These benefits include increased sales, customer lock-in and faster time to market, as well as the formation of strategic relationships with other organizations, relationships that mimic the relationships found in the biological ecosystems from which software ecosystems take their name.

A software ecosystem and its software platform reside within a market segment complete with competitors and rival products. Consequently, software platform and software ecosystem adoption is not a simple choice, but rather a strategic business decision that positions the adopting organization within the market segment and defines its relationships with the other organizations
in the larger context of the software ecosystem. The decision to adopt a software platform and ecosystem is not a choice to be taken lightly on the whims of technical functionality, business insight or social networking alone, but rather a strategic decision with impacts that reach into the future.

### 1.1 Problem Statement

Just as an organization within an ecosystem co-exists cooperatively and competitively with its co-inhabitants, the ecosystem which surrounds the software platform exists in a larger context of competing software platforms. The decision to adopt a software platform and its surrounding ecosystem is a complex one requiring technical and business analyses; however, this decision is further complicated by the existence of multiple, sometimes many, platforms within a particular market segment. While the community engaging in software ecosystems research agrees that software platform adoption is a viable strategy for defraying the cost of software and reducing time to market, among other benefits, little research currently exists in providing a quantified and analytical view of the strategy behind adopting a software ecosystem. Even less research exists analyzing the strategy for choosing between multiple platforms from the perspective of a software producing organization. Furthermore, there exists a tension between the costs of developing software from scratch versus using a compositional approach that may obscure hidden learning costs and additional implicit and explicit factors that affect the total cost of ownership.

### 1.1.1 Scenarios for Consideration

In this section we present two different situations that demonstrate the need for methods and models for the analysis of software ecosystem. Basing our research context on these scenarios provides a context for the computation, modeling and simulation that is performed in this research. Compelling research scenarios highlight situations where the methods explored in this research are applicable to real world settings and guide the parameterization and application of the models defined in this research. Expanded versions of these scenarios are contained in Appendix A. Consider the following scenarios:

Scenario 1 An organization wishes to create a tool to aid in software development. Creating a new workbench from scratch is reinventing the wheel, and the project manager would like to explore open source platforms to serve as the basis for their tool. After evaluating the platform
landscape, the project manager identifies two candidate platforms which meet the needs of their project: Eclipse and NetBeans. Considering that this tool may undergo extensive modification and extension, the project manager wishes to evaluate both platforms to determine which is the better fit for the project, given its perceived evolution over time. However, the project manager is unsure how to compare each of these platforms for use. Given that profit margins are low, saving money on engineering costs are a priority.

Scenario 2 An organization has developed a software application on an existing open source platform. As the application, and subsequently the platform, has aged, it is no longer clear that this platform is the best platform for their tool. After identifying a number of competing platforms to which their application may be ported, the project manager is unsure of how to determine whether switching platforms is the correct move, and, subsequently, which platform is the best platform to chose for their needs. It may be the case that switching to a different platform opens new implementation opportunities, or perhaps lowers the cost of continued development. However, additional investigation needs to be pursued in order to quantify these benefits.

### 1.1.2 Research Approach

Building on the work previously done in [38], this research seeks to develop quantitative methods for providing strategic decision making support when faced with strategic software platform adoption. Strategic decision making support has been developed in the form of a mathematical model that describes the software development process under different conditions, including, but also in absence of, strategic software platform adoption. This theoretical model was used to generate a stochastic model used in Monte Carlo simulation to evaluate the cost and value of engaging in software development. We have completed multiple experiments for each of the scenarios listed in the previous section, and have compared their results to determine, in each scenario, which option presents the best development strategy, given the parameters and context for the experiment.

### 1.2 Research Contribution

### 1.2.1 Contributions

The contribution of this work is a generalized model for a software development process and life cycle that addresses not only the technical facets of software development, but also the business aspects of software development. Our model, agnostic of any particular applied software development process, provides points of variation which can be tailored and customized for variable granularity in analysis, as is the case with stochastic methods. The scenarios described in the Section 1.1.1 provide a foundation for analysis using this model, while the model has provided applicable decision support to the scenarios described within this work.

### 1.2.2 Thesis Statement

Real options valuation can be applied to strategic decision making surrounding the decision of whether to join a software ecosystem and adopt that ecosystem's software platform. Options valuation mechanisms such as the Black- Scholes model can be adapted and used to provide support for this type of decision making under different scenarios and development contexts.

### 1.3 Dissertation Organization

In this introduction, we have introduced the concept of software ecosystems and, through two example scenarios, some of the complexities associated with choosing between two software platforms to use as the basis for a product. We have briefly described a research approach to modeling and simulating software development within the context of developing using a software platform. In Chapter 2 we will provide the necessary background on software ecosystems to help the reader understand the context described in the introduction. Chapter 2 continues, describing financial options, their valuation and the Black-Scholes model of options valuation, which serves as the foundation for our model of software development and our decision making process. Chapter 3 provides a conceptual model for applying options valuation to the software development life cycle (SDLC), which forms our stochastic model that will be simulated using Monte Carlo simulation. Chapter 4 covers our design of our experimental simulations, sources of stochasticity in our experimental simulation, as well as a brief summary of existing research by the author in applying Black-Scholes options value
to software development.
In Chapter 5 we present the results of multiple experimental simulations on each of the scenarios presented in Section 1.1.1, while Chapter 6 covers the work done to verify our theoretical and stochastic model. Chapter 7 covers related work in the field of software economics as well as existing options valuation techniques. Chapter 8 discusses future improvements that can be made to our model and simulations, which would yield in more realistic results. Finally Chapter 9 provides concluding remarks.

There are a number supplemental reading materials found in the appendices. Appendix A provides additional discussion of the scenarios presented in Section 1.1.1. Appendix B provides a more in-depth discussion of the authors existing work in utilizing options valuation for software development in the context of software product lines and the valuation of variation points. Appendix C provides a brief overview of the Rational Unified Process (RUP), which is necessary for the experiment described in Section 5.3. Appendix D contains the data resulting from the experiments in Chapter 5 and Appendix E contains the data from the verification and validation experiments performed and discussed in Chapter 6.

## Chapter 2

## Background

### 2.1 Software Ecosystems

Software ecosystems are a recent concept in software development that helps understand the collaborative and competitive nature of software development in a distributed, multi-organization context. Software ecosystems rely on the biological notion of an ecosystem to motivate the view of a particular market for software.

In biological ecosystems, traditionally the ecosystem is thought of as the environment in which a set of different organisms live and the interactions and relationships among those organisms. More precisely, Eugene and Barry Odum, commonly referred to as the fathers of modern ecology, described biological ecosystems as follows:
"[The] community includes all populations occupying a given area. [...] The community and the non-living environment function together as an ecological system or ecosystem" [44].

Software Engineers have co-opted the Odum brothers' notion of ecosystems to describe the collaborative and competitive nature of development in a multi-product and multi-organization system.

Several authors have proposed definitions for software ecosystems. Messerschmitt and Szyperski proposed the first definitions of software ecosystems in their seminal 2005 work:
"Traditionally, a software ecosystem refers to a collection of software products that have some given degree of symbiotic relationships" [42].

Their definition lays a simple foundation for software ecosystems, describing a set of products that complement each other in a potentially reciprocating manner. However, their definition exclusively defines the ecosystem as the software, ignoring the people and organizations that use, develop and extend the software. Other authors have proposed more extensive definitions that address more than just the software. Slinger Jansen provides a more comprehensive definition, highlighting the notion that organizations, in addition to software, are elements within the ecosystem:
"We define a software ecosystem as a set of businesses functioning as a unit and interacting with a shared market for software and services, together with the relationships among them. These relationships are frequently underpinned by a common technological platform or market and operate through the exchange of information, resources and artifacts" [33].

Jansen tacitly extends the definition to include the notion of a 'landscape,' namely the shared software market. Additionally, he provides the basis for the notion of a 'platform' on which a set of complementing software products are built. Jan Bosch proposes a different definition, that a software ecosystem is simply a set of [software] solutions that enable, support and automate relevant business and social ecosystems:
"A software ecosystem consists of the set of software solutions that enable, support and automate the activities and transactions by the actors in the associated social or business ecosystem and the organizations that provide these solutions" [10].

However, Bosch later revised his work to provide a reflection of the people that are involved in the development of solutions, rather than simply the solutions themselves:
"A software ecosystem consists of a software platform, a set of internal and external developers and a community of domain experts in service to a community of users that compose relevant solution elements to satisfy their needs" $[11,12]$.

More importantly, Bosch also provides some basis that the development of a platform may be both internal and external to an organization that controls the platform's development. Additionally, Bosch subtly implies that the platform alone may not contain discrete functionality, but rather that the end user's functionality is added by domain experts, providing purpose to the composition of technical assets within the ecosystem. Yet another definition, provided by Mircea Lungu takes a more simplified approach to software ecosystems:
"A software ecosystem is a collection of software projects which are developed and evolve together in the same environment" [37].

None of the above definitions is right or wrong; rather, they focus on specific elements contained within software ecosystems. The choice of a particular definition provides the context for which analysis occurs. Rather than try to solve the different problem of a unified definition for software ecosystems, we instead provide a notional idea of software ecosystems through a descriptive scenario in the next section.

### 2.1.1 Explaining Software Ecosystems

While a precise definition of software ecosystems that is also accepted by the research community is still emerging, there is an agreement on the general description of what constitutes the entities, relationships and activities that compose a software ecosystem.

At the heart of any software ecosystem exists a software platform. This platform may take the form of a library, an externally visible API, a software framework, a software application or even a software standard. In addition to this software platform, there is an organization, or perhaps a consortium of organizations, commonly referred to as the keystone organization [30], that is considered to be the controlling organization or owner of this platform. This organization serves as a governing body for the ecosystem and steers the development and direction of the platform as well as provides some level of technical management for the platform, e.g., repository support, web hosting, database support and more.

Upstream from the keystone organization is the software platform's supply chain: the set of organizations that produce software assets which are utilized in development of the platform. Supplied assets might include standards, input formats, and software artifacts. They may be open source, closed source, open license closed source or restricted in other ways through software licensing.

Downstream from the keystone organization exist two classes of organizations: platform users and product users. Platform users describe organizations that adopt the platform, thereby joining the ecosystem, with the intention of using the platform for their own development or extending the platform to include additional features and functionalities. Traditionally these are thought of as niche players [30], or perhaps as value-added resellers (VARs).

The platform itself is developed by multiple stakeholder groups, including developers from
the platform owner as well as platform users that utilize the platform to develop software products. The exact details of who carries out the implementation varies from ecosystem to ecosystem: in some ecosystems, the platform organization provides few, if any, developers, and the platform is developed from partner organizations within the ecosystem. In others, the platform owner contributes the majority of the development to the platform, while platform users submit patches and feature requests.

On the other hand, Product users describe organizations that use products which are made on the platform software. They are end-users or consumers and are generally seen as noncontributors to the ecosystem, but members of the ecosystem nonetheless.

The generalized activity that happens within a software ecosystem can be described as follows: The keystone organization releases a software platform to the public, often built with a small set of software products, built using the platform. Niche organizations use the platform as a basis for product development, either by developing a new product or by filling the gap created by an absent feature in an existing product. This creation of new functionalities attracts new product users to the ecosystem. These product users, both new and previously existing, contribute feedback and requirements back to the platform users and platform owner; however, product users that encounter a missing feature or unfulfilled niche may in turn become to platform users that contribute back to the ecosystem through software development and extension.

### 2.1.2 A Brief Aside About Software Reuse

There is a considerable overlap between the fields of software ecosystems and software reuse: the primary mode of collaboration within a software ecosystem is the continual use (or perhaps reuse) of assets, notably a software platform, provided by the ecosystem to numerous organizations and numerous projects. However, subtle differences exist between traditional notions of reuse and the asset usage and acquisition within the context of software ecosystems.

Software reuse has primarily been concerned with the development of assets for a software project, and the manner in which they were reused in other projects and by other development teams or organizations. The study of software reuse was two-fold, encompassing both the initial cost of developing reusable assets and the decreased cost of using those reusable assets across multiple projects. Software reuse was also motivated by increased component quality, in the form of cheaper execution [13] and fewer errors [22, 13], as well as decreased software maintenance costs [13]. Most of
the motivation for software reuse in the literature was from an economic perspective: if the increased cost of developing reusable assets resulted in a net savings over the total use of those reusable assets, then reuse was a profitable proposition $[2,25,24]$.

This proposal approaches platform adoption with the same economic perspective provided by the existing reuse literature, with caveats for the different scenarios described in Section 1.1.1: if adoption of a particular platform proves to defray the cost or improve the value of software developed when compared to another platform or another method of development, adopting a platform-based software engineering process, given that particular platform, provides an economic benefit over the alternative.

However, there are some key differences between the motivations and work in reuse literature and the work proposed here. Notably, the reuse literature approaches the economic viability of reuse from two perspectives: first, the defrayed cost of reusing software development assets as well as the increased cost of developing reusable software development assets and, second, the duration required to amortize the cost of the development. While these considerations are relevant in a platform-based software engineering context, particularly given software ecosystems, the perspective of the platform user, who uses the platform as a basis for their project, is less concerned with the increased cost of developing reusable assets and more concerned with the cost-savings accrued from acquiring assets provided by the software platform and ecosystem. This is due in large part to the assumption that platform users and application developers are downstream product consumers, while the platform owners, producing the software platform, are upstream software suppliers.

Additionally, the process of acquisition and subsequent integration of assets differs fundamentally when the comparison is drawn between traditional software reuse and platform-based software engineering. In the traditional setting, reusable assets are acquired by a team, either externally or internally, and then learned and tailored for integration into the larger product being developed by the team. In this case, the acquired assets represent a small portion of the functionality of the overall product. In the platform-based software development context, the platform user and application developer may acquire reusable assets; however, the platform user is less so acquiring and integrating these assets into their code, but rather more so acquiring the software platform, and some assets it provides, and integrating their code into the platform. Thus, their applications and extensions join the greater catalog of software within the ecosystem. This represents a constrained and context-dependent form of reuse that is fundamentally different from the forms
of reuse encountered in software reuse literature.

### 2.2 Options and Their Value

Decision making is a fundamental aspect of software development, and within every decision lies an option. Should we use Java, C ++ or a third option for our development language? Should we use Agile or waterfall for our development style? Do we engage in rapid release or scheduled release? Within every option are two or more fundamental decisions, including deferring decision making. Options provide the flexibility to make a decision at a given point in time or at a later date. Deferring decision making reduces risk and uncertainty until a more complete set of knowledge about the decision is available.

### 2.2.1 An Illustrative Scenario

John is interested in purchasing some land for commercial development from Eric. At the same time, local government is deliberating on zoning and taxation of said land, introducing risk and uncertainty: if the land is rezoned, commercial development may not be possible, and higher taxes would be an additional burden that may render the commercial development business plan unviable. So in order to reduce the uncertainty in this transaction, John offers Eric a fixed sum of $\$ 5,000$ for the right, without obligation, to buy Eric's land at $\$ 35,000$ on or before November 11th, 2015. Eric benefits by the added money that John has offered, while John has time to evaluate his options with respect to new zoning and taxation developments. John has reduced his risk and defrayed the uncertainty in the decision by deferring to a later date. Additionally, the negotiated price protects John from price increases that may result from future events, like the aforementioned government deliberation.

### 2.2.2 Financial Options: Terms and Definitions

While many types of options exist, the above scenario describes the core notion of an option. An option is a financial instrument which represents a contractual agreement between two parties that negotiates the sale of an asset by or on a later date. To formalize our definition of options, it is necessary to define a number of terms:

- Option - The option is an asset in and of itself, and represents the right, without obligation, of the option holder to exercise the option on or before the option's date of expiration.
- Option Owner - The option owner is the party who has acquired the option.
- Option Seller - The option seller is the party who sold the option to the option owner.
- Premium - The premium is the price that the option owner paid to the option seller to purchase the option.
- Expiration Date - The expiration date is the date by which the option owner must decide on whether or not to exercise the option. This is sometimes referred to as the exercise date or the maturity date.
- Underlying Asset - The underlying asset is the asset, goods or services on which the option is specifying the terms for sale.
- Strike Price - The price of the underlying asset at the time at which the option on that asset is acquired.
- Spot Price - The negotiated price of the sale of the underlying asset should the option be exercised.
- Riskless Discount Rate - The theoretical rate of return on an investment with no risk associated with it. The purpose of this parameters is to discount the future value of money back to its present value.

In our above scenario, the option represents John's right to buy Eric's land at a fixed price over a duration of time. John is the owner or buyer of the option, Eric is the seller of the option. The premium was the $\$ 5,000$ that John paid to Eric. The maturity date was the date November 11th, 2015, the date by which John must make the decision to purchase Eric's land. John's decision to purchase the land is referred to as exercising the option. The underlying asset is the land, and the strike price is the negotiated price of the land, $\$ 35,000$. Numerous different configurations of options exist and are being developed every day within the financial world; however, for the purposes of this research, financial options can be described by three properties: type of option, the option's underlying asset, and the exercise style of the option.

### 2.2.3 Option Types

The type of option specifies the nature of the relationship between the option owner and the option seller and the asset owner and the asset buyer. There are two principle types of options: call options, in which the option buyer and the asset buyer are the same stakeholder, and put options, in which the option buyer and the asset seller are the same stakeholder. Call options are more pertinent to this proposal, given that the option owner decides to expend money to acquire assets, mirroring the project managers decision to engage in development, thus spending money, to produce software assets. Consequently, this proposal will focus on call options.

A call option provides the option owner the right, but not the obligation, to purchase the underlying asset at a specified price on or before a specified date. The buyer of the option pays a premium to the asset owner to secure this deal. Typically this is done when the option buyer thinks the price or market value of the underlying asset will increase prior to the specified date. The optin premium is an upfront fee that allows the option buyer to reduce the risk in the purchase, and potentially gain a substantial pay-off should the market value of the asset increase.

From the sellers perspective, the negotiated option price is based on speculation that the market value of the underlying asset will not rise. The cost of the option, in theory, compensates for any increased profits that may be realized from holding the asset should the price or market value of the asset increase, while protecting the owner if the price of asset decreases.

### 2.2.4 Option Assets

The second core component of an option is the underlying asset whose sale or transaction is specified by the option. Within finance, there are many different kinds of assets on which an option can be specified: equity, bond, future, index, commodity, just to name a few. However, the type of asset which this paper will focus on is the real option. A real option is the right, but not the obligation, to commit to a particular decision, typically regarding business strategy and tactics. In this sense, a real option is an option in which the underlying asset is not a physical good or equity, but rather behaviors, strategies and activities engaged in within a business context. One possible example from within software development:

A firm is engaged in the development of a software product. The development process is behind schedule. One possible decision is to abandon the project or to continue with development,
while another is to increase funding, hire more personnel, or any other of myriad possibilities.
This scenario presents several real options:

- Abandon the project, or continue with development.
- Increase funding, or keep funding at the same rate.
- Hire more personnel, or keep staffing the same.

Because the risky asset is a non-physical good, there is a subtle difference between the option, as an asset, and the underlying asset within the option. That is to say, being able to make the decision between abandoning the project or continuing development, for instance, is an asset in itself, where the option is examining the resulting cost and value from abandoning the projects, which is to say the other decisions can be viewed in the same manner.

### 2.2.5 Exercise Style

The third element to an option is the exercise style, referring to the rules that define when an option can be exercised by the option owner. Just as with the underlying asset, researchers and investment bankers have been formulating new ways to specify the exercise style of options since the inception of options as investments. The most well known exercise styles are named after the markets which trade them: European, American, Asian, as well as Bermudan, binary and barrier. This paper will focus on American and European options.

A European style option specifies that the option owner may exercise the option on the specified expiration date, but prohibits exercising the option before the expiration date.

An American style option specifies that the option owner may exercise the option during a period of time up to and including the maturity or expiration date. This option style allows greater flexibility for the option holder, but significantly complicates the calculations involved in options valuation.

### 2.3 Formalizing Options Valuation and Black-Scholes Option Value

Options valuation refers to analysis of options cost and value in order to aid in strategic decision making. Multiple methods exist for options valuation. These methods provide structured frameworks for analyzing the cost and value of a particular option given the type and exercise style of the option.

### 2.3.1 Black-Scholes

The Black-Scholes option valuation method, sometimes called Black-Scholes-Merton, was pioneered by Fischer Black and Myron Scholes in order to produce a closed-form model that approximated a European options theoretical price [4]. Their model was later expanded by Robert Merton and Myron Scholes and their work earned a Nobel Prize in economics [41].

### 2.3.2 Black-Scholes: Assumptions

Black-Scholes utilizes a number of different assumptions on both the assets at hand and the market in which they are traded in order to constrain the options valuation problem.

- The option contains one risky asset, usually the 'stock' or asset being traded. Risky means subject to fluctuations and change due to uncertainty which could result in a negative or positive outcome given an initial frame of reference and perspective.
- The option contains one risk-free asset, usually money. A risk-free asset is an asset that appreciates at a risk-free interest rate, e.g., the rate of return on an investment with no financial loss.
- Fluctuations in the price or value of the asset approximate an infinitesimally random walk. The fluctuations in the asset price are not random and over a long enough period of time represent a zero net-change.
- The asset does not pay dividends.

It is important to note that other assets could be substituted in place of money for the risk-free asset, provided those assets can be mapped to a monetary value. One example might
be replacing money with the work of a full- time employee over a specified duration. Additionally, fluctuations in the price or value of the asset are usually modeled after a geometric Brownian motion. The market specifies the context and environment in which the valuation of the option takes place. Black-Scholes provides the following assumptions and constraints on the market:

- There is no arbitrage within the market. That is, there is no risk-free way to generate profit.
- It is possible to borrow and lend any amount of money, even fractional, at the risk-free interest rate.
- It is possible to buy and sell any amount of asset, even fractional amounts.
- The market is frictionless: Borrowing and lending money and buying and selling asset do not incur any fees or costs.


### 2.4 Black-Scholes Formula: Call Options

At its heart, Black-Scholes is a cost-value model, consisting of two terms, the negotiated cost of the risky asset in question and the value of that risky asset. The basic Black-Scholes equation for modeling a call option has the following form:

$$
\begin{equation*}
C(S, t)=N\left(d_{1}\right) S-N\left(d_{2}\right) K e^{-r(T-t)} \tag{2.1}
\end{equation*}
$$

The equation computes the difference between $S$, the spot price of the underlying asset, the price of the asset on a specified date, and $K$ is the strike price, or rather the negotiated price of acquiring the underlying asset. The function $N(d)$ is the cumulative normal distribution function over a standard normal distribution of the spot price, $N\left(d_{1}\right)$, and strike price, $N\left(d_{2}\right)$, respectively, with respect to the model's volatility. $r$ is the risk-free discount rate applied over the time-difference between $T$, the expiration date of the option, and $t$ the current time. The risk-free discount rate serves to account for the increase in the value of the monetary value of assets over time if exercising were deferred or declined. The cumulative normal distribution functions over a standard distribution take the following form:

$$
\begin{gather*}
N(x)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{x} e^{\frac{1}{2} x^{2}} z d z  \tag{2.2}\\
d_{1}=\frac{1}{\sigma \sqrt{T-t}}\left[\ln \left(\frac{S}{K}\right)+\left(r+\frac{\sigma^{2}}{2}(T-t)\right)\right]  \tag{2.3}\\
d_{2}=\frac{1}{\sigma \sqrt{T-t}}\left[\ln \left(\frac{S}{K}\right)+\left(r-\frac{\sigma^{2}}{2}(T-t)\right)\right] \tag{2.4}
\end{gather*}
$$

In $d_{1}$ and $d_{2}$, the parameters are the same as above, with $\sigma$ representing the volatility of returns on the underlying asset. Synthesizing the equations, we get:

$$
\begin{equation*}
C(S, t)=\left(\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{d_{1}} e^{\frac{1}{2} d_{1}^{2}} z d z\right) S-\left(\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{d_{2}} e^{\frac{1}{2} d_{2}^{2}} z d z\right) K e^{-r(T-t)} \tag{2.5}
\end{equation*}
$$

The first term of the subtraction represents the spot price of the underlying asset $(S)$, discounted by the risk-free discount rate $(r)$, accounting for the volatility $(\sigma)$ between now $(t)$ and the option expiration date $(T)$. The second term is the strike price of the underlying asset $(K)$, discounted by the risk-free discount rate $(r)$, accounting for the volatility $(\sigma)$ between now $(t)$ and the option expiration date $(T)$.

### 2.5 Parameterization of Black-Scholes

The majority of the parameters of Black-Scholes are straightforward and typically set through negotiation between the parties buying and selling the option. However, two parameters, notably $r$, the risk-free discount rate, and $\sigma$, the volatility of the asset, are context dependent, and, in a theoretical context, parameterizable by the modeler or analyst.

Intuition tells us that additional risks incur additional reward and that the lower the risk of a decision or investment, the lower the payout must be. The risk-free discount rate, $r$, represents the theoretical rate of return on an investment that has no risk of negative outcome or financial loss. Given that one of the assumptions of Black-Scholes is the absence of arbitrage, covered in Section 2.3.2, the risk-free discount rate approximates the rate of return on the investment with the least risk. When reducing asset values to money, the risk-free discount rate is typically modeled
after the inflation of the currency, i.e., the least risky investment is holding the investment in liquid capital and letting its value rise and fall with the inflation of currency. By parameterizing $r$ this way, the value of money in the future is discounted back to present day value of money, accounting for inflation, interest or the return on investment if the option had not been exercised.

The volatility, $\sigma$, represents the the potential change over time in both the strike price and spot price of the asset in question. This volatility is usually represented as a 'jiggle' in the values of the asset, representing that its value will not be constant over time. This 'jiggle' is typically modeled through a infinitesimal random walk with a zero net change. That is to say, at any given time, the spot price of the risky asset may be higher or lower than its measurement at a previous point in time; however, given a long enough interval, the net change in the strike or spot price is negligible. Typically this is modeled as Brownian motion, the random motion of particles suspended in fluid.

## Chapter 3

## Model Development

As mentioned in Section 1.2.1, the purpose of this research is to provide support for strategic decision making within the context of software ecosystems. More specifically, the goal of this paper is to provide support for addressing decision making in platform adoption under the scenarios specified in Section 1.1.

In this section, we propose the synthesis of a mathematical model that helps describe and quantify decision making within the context of platform adoption. This will be a two part model which addresses both cost and value: the cost of developing software from an investment standpoint, and the earned value derived, typically in dollars, from engaging in platform-based software development. In this treatment, we have provided examples of apply this model to multiple styles of software development, including big upfront design and iterative-incremental development. The model will be integrated with the Black-Scholes formulation for options value, and provide flexibility for customizing and tailoring to specific contexts. This model will be applied to each of the scenarios proposed in Section 1.1.

### 3.1 Period-Based Software Development

Because the crux of options investment analysis is normalizing the value of assets with respect to the present date and a future date, the first step in this model is to develop a concept of time that lays the foundation for our model.

A period is defined as a fixed amount of time during which development work occurs.

Parameterization of the duration of a period is left to the modeler. Given the definition of a period, let $t, \tau$, and $T$ represent variables of unit period, where $t$ represents the current period of development (e.g., the 7th 30-day period of development), $\tau$ represents the period of completion of a particular phase of development (e.g., requirements will be completed by the third 30-day period of development) and $T$ represents the time-horizon of development (e.g., how far in the future we can foresee development). In practice, the time horizon may represents the period corresponding to the date at which the software is released, an arbitrary date after release during maintenance phases of development or the date of retirement or decomissioning of the system. $t, \tau$, and $T$ are related such that $t \leq \tau \leq T$

A phase of software development is a specific development phase or activity, such as requirements ellicitation or testing, that occurs within one or more periods of development and culminates upon the completion of the final period of development for that phase of the SDLC. Transition from a period $t$ to a period $t+1$ is instantaneous. For example, assume that a period is measured as a single month. The requirements phase of software project might take three months or three periods as represented in our model. At the end of the third period, the requirements phase of development has completed. Furthermore, each period, occuring within a phase of the software development life cycle, has a cost that is incurred instantaneously upon beginning development in that period. This cost represents the cost of the act of engaging in the development during that period. If a phase of the SDLC spans multiple periods, the cost of that phase of the software development life cycle may be divided equally among the periods that compose the phase of the software development life cycle, subject to variations arising from parameterization. Finally, each period of development results in a revenue from the investment made to engage in development in that period. As with the cost, the revenue can be divided equally among the periods in which that phase of development occurs. Again, the return on investment is subject to parameterization by the modeler.

The pay-off from engaging in development during a particular period and phase in the SDLC is valued based on the work products produced. However, the common unit of measurement for both the cost and the value or pay-off is in Dollars in order to keep the mathematics simple and relational. This model for the value of software is reasonable because for any work completed on a task, there exists a work-product output whose value can be assessed. While the value of the work-product may be zero, that is a detail for simulation, rather than our model. Additionally, valuing the work-products of a phase of software development is reasonable due to extant modes
of contracting and subcontracting wherein a prime contractor may contract a phase or portion of a software project to a sub-contractor. That scenario describes a buyer-seller relationship between the contractor and sub-contractor, with the contractor paying money for assets produced by the sub-contractor. This contracting mode of development is quite frequent in military or government software acquisition. This contractor / sub-contractor relationship mirrors the mode of production in which an organization that develops software operates; namely, that the organization produces software, and, in exchange for the labor expense of developing the software, they receive assets as a result of that development. Finally, yet another equally valid view of value is that the value derived from software development represents 'earned value,' which provides a perspective on project performance and progress [23].

### 3.1.1 Options Treatment of Waterfall Model

The Waterfall model, popularized in the 1960s and 1970s, represents the most traditional form of software development, mirroring the design and development of physical goods. Traditional models of waterfall development divides development into 5 phases: requirements, design, implementation, verification and validation, and maintenance. Each phase of development is entered in its sequential order for a duration of time. In the most rigid models of waterfall development, the completion of a phase of development results in work products that are set-in-stone and cannot be changed, or rather, previous phases of development cannot be revisited. While the waterfall model of software development has numerous shortcomings, detailed by a myriad of publications decrying it and its effectiveness, it serves as a good pedagogical example and starting point for applying to investment options valuation. Figure 3.1 provides a graphical depiction of the waterfall method of software development.

When applying investment options analysis, each phase represents one or more periods of investment. Each phase has an initial investment, equivalent to the cost of the option, plus the some fractional portion of the development cost, up to and including $100 \%$. At the end of that period, work products delivered represents the spot-price of the development. Each phase of the software development life cycle, ocurring in one or more development periods, incurs a cost at the beginning of the period and derives a value or earned value or pay-off at the end of the phase.


Figure 3.1: Waterfall Method of Development

### 3.1.2 Options Treatment of Iterative-Incremental Model

Iterative-incremental models of software development provide a more modern alternative to the now pedagogical waterfall model. In iterative-incremental development, the entire duration of development is split among a number of iterations. Each iteration represents a full cycle of each of the traditional phases of the software development life cycle or variations on it. During each iteration, a single increment of the software project is under development. An increment might represent a feature, module or subsystem of the software product. Figure 3.2 provides a visual aid in understanding an iterative-incremental software devleopment process.

Applying options value analysis to iterative-incremental styles of development begins with selecting a period that is compatible with the development pace and duration of each iteration. While iterations are subject to variable time extension, use of timeboxing, or strictly fixed length iterations, as a development mechanism makes iterations function as a standard unit of time. Through utilization of timeboxing, it seems appropriate to set the period of investment to the same duration as an iteration; however, a single increment may span more than one iteration. Similar to the waterfall model, each iteration of software development incurs a cost at the beginning of development and affords a payout at the end of each iteration.

However, this coarse grained examination of the iterative-incremental development process represents only a single view of the options value treatment. Since each iteration is considered a full


Figure 3.2: Iterative-Incremental Method of Development
cycle of the SDLC, iterations could be further subdivided into each phase of the SDLC within an iteration. A more fine grained view accurately represents that at any given stage in development within an iteration, the option to cease development always exists; however, this may not be an improvement over the coarse grained model described above. By taking a micro-level view of each phase of the SDLC within each iteration that develops each increment results in significantly smaller periods, or the use of fractional periods within the options value formulations. Furthermore, the time duration of a phase of the SDLC within the iterative-incremental model of software development may not be equal across all phases, e.g., requirements may last longer than testing. Finally, the possibility exists that a particular increment may last longer than a single iteration; however, multiperiod increments are less troublesome than fractional phases of development within a given period as each period represents yet another point where decision making can be made and the model can be re-evaluated.

### 3.2 Modeling Cost and Value

Software development within the context of software ecosystems takes place utilizing the software platform of the ecosystem as a development asset. The platform itself obviates the requirement of engineering every single module and part of a software product, instead providing community backed assets for common functionalities. Adopting a software platform and subsequently joining an ecosystem results in reaping the rewards of a common set of assets available for the developers that are utilizing the platform. On the other hand, joining an ecosystem and utilizing a software platform for development also provides assets and context which can both positively and negatively impact the value of a software product in development. These impacts are largely determined by the ecosystem, its characteristics and its inhabitants. The next two subsections will provide a general mathematical overview on how to integrate software development utilizing a software platform into a mathematical model specifying the cost and the value of software development as a financial option.

### 3.2.1 Modeling Software Development Costs

By adopting a platform, developers reap the cost savings provided by the assets delivered from the ecosystem. The cost savings are dictated by the number of assets provided by the ecosystem and their supporting materials, such as documentation, process and requirements. We begin to
formalize this notion by providing each phase of software development with a base cost, $C_{i}$, denoting the cost incurred during the development phase $i$, where $i$ denotes a phase within the software development life cycle. The base cost is parameterized by software cost-estimation techniques.

In each phase of software development, the ecosystem either obviates the development of or delivers a percentage of the assets that would be produced by that phase of the SDLC. To provide a basic example, consider the development of a basic GUI application for calculating software cost and value utilizing a waterfall model of development. The development of the GUI represents $25 \%$ of the software. By joining an ecosystem and engaging in platform software development, assets are acquired for creating a GUI from the platform. These assets include software requirements, the architecture of the GUI and its implementing source code, along with unit tests for common functionalities. When developing, analyzing and specifying requirements for the GUI, a certain percentage of those requirements are satisfied by the platform. These assets reduce the number of requirements that need to be developed, analyzed and specified, reducing the overall cost of the requirements phase of development.

In consideration of that scenario, let $C_{\text {Reqs }}$ represent cost of the requirements phase without utilizing platform development. $C_{\text {Reqs }}$ represents the cost of the requirements phase if this application was being built from the ground up. We also define $D_{i}$, the reduction in development cost as a result of asset acquisition from the platform in SDLC phase $i$. Continuing, $C_{\text {Reqs }}^{\prime}$ is the cost of the requirements phase, utilizing the assets provided by the software platform:

$$
\begin{equation*}
C_{\text {Reqs }}^{\prime}=C_{\text {Reqs }} *\left(1-D_{\text {Reqs }}\right) \tag{3.1}
\end{equation*}
$$

Continuing our scenario, assume $C_{\text {Reqs }}=\$ 4,000$. The GUI accounts for $25 \%$ of the requirements, roughly $\$ 1,000$. The platform provides $50 \%$ of the assets required to develop the GUI, or rather, $12.5 \%$ of the requirements and reduces the cost of the requirements phase of the SDLC by $\$ 500$.

$$
\begin{equation*}
C_{\text {Reqs }}^{\prime}=\$ 4,000 *(1-0.125)=\$ 3,500 \tag{3.2}
\end{equation*}
$$

Expanding this concept over the duration of the SDLC, each phase has an associated base cost, a cost reduction as a result of platform asset utilization,

Assuming a basic model of the SDLC, there is a cost reduction that exists for each phase of the software development life cycle. The cost reductions result from the acquisition of assets from the platform and the ecosystem, and these assets in turn reduce the cost of engaging in that phase of development.

- Requirements - The platform and ecosystem provide a portion of requirements documentation for any assets that will be utilized during the implementation phase. This documentation forms a part of the requirements specification and analysis. Additionally, developing with a platform may introduce design constraints that reduce the amount of requirements elicitation that occurs within this phase.
- Design - The platform and the ecosystem provide a foundation to the software being developed. This foundation includes the architecture of any code or infrastructural assets that are utilized in the software project. The use of the platform, and subsequently the architecture, may result in less specification and fewer decisions being made during architectural design.
- Implementation - The platform and the ecosystem provide a common base of code level assets to the software being developed. Utilization of such assets obviates the need to develop custom implementations. However, while utilization of platform code assets may result in less code writing, it may result in more code integration.
- Verification \& Validation - In a perfect world, every asset comes with test artifacts. HoweverFor example, code artifacts should come with unit tests that would have to be written for a custom implemented module. However, that is not always the case. Code assets without test artifacts may require additional development investment to write tests, or may impact or influence a managers decision of whether to adopt a specific platform or not.
- Maintenance - The portions of the software that are composed from assets acquired from the platform are maintained by the members of the ecosystem that contribute to the platform.

There may be an additional cost incurred as a results of the utilization of ecosystem assets. Utilization of third party assets requires education and understanding the assets being utilized.

Utilization of certain types of assets may reduce the extent to which developers engage in certain activities within one phase of software development, while increasing the need for other activities in that or a different phase. For example, acquisition of code modules may reduce the amount of code being written, but does not reduce and may increase the time spent during code integration.

### 3.2.2 Modeling Software Development Value

Just as each phase of the SDLC has an associated cost with engaging in software development, there is an associated value that is the return on that development investment. At the end of each phase of the SDLC, work products and development artifacts have, in theory, been created that represent the output of that phase. Just as with the cost, consider $V_{i}$, the base value derived in phase $i$. Let $A_{i}$ represent the added value contributed to the work products and artifacts as a result of utilizing platform assets. Finally, $V_{i}^{\prime}$ represents value of the work products produced, given platform utilization, which is the sum of the base value and the added value from platform development:

$$
\begin{equation*}
V_{\text {Reqs }}^{\prime}=V_{\text {Reqs }} *\left(1+A_{\text {Reqs }}\right) \tag{3.3}
\end{equation*}
$$

### 3.2.3 Synthesizing Cost and Value

Having defined our timescale for our model (Section 3.1), as well as a conceptual model of software development cost (Section 3.2.1) as well as software development value (Section 3.2.2), we turn our attention to synthesizing these two models for cost and value into a single computational model. Consider the base form of the Black-Scholes in Equation 2.1:

$$
\begin{equation*}
C(S, t)=N\left(d_{1}\right) S-N\left(d_{2}\right) K e^{-r(T-t)} \tag{3.4}
\end{equation*}
$$

Recalling our terms from Section 2.2.2, $S$ represents the spot price, or the market value of the
underlying asset, at time of valuation, subject to volatility modeled by $N\left(d_{1}\right)$, the cumulative normal distribution function over a standard normal distribution, $K$ represents the strike price, or the negotiated price of sale of the underlying asset, subject to the volatility modeled by $N\left(d_{2}\right)$, a cumulative normal distribution function over a standard normal distribution. Because $K$ represents a price paid in the future, we discount $K$ back to the current value of money utilizing the riskless discount factor, $K e^{-r(T-t)}$. We can substitute Equation 3.1 for K , the strike price of an option in software development:

$$
\begin{equation*}
C(t, i)=N\left(d_{1}\right) S-N\left(d_{2}\right)\left(C_{i} *\left(1-D_{i}\right)\right) e^{-r(T-t)} \tag{3.5}
\end{equation*}
$$

where $C_{i}$ represents the cost incurred for pursuing development in period $i$ and $D_{i}$ represents the fractional discount in the strike price, incurred from using a platform engineering approach. Continuing, we substitute Equation 3.3 for S , the sale price, in Equation 3.5, yielding the following equation:

$$
\begin{equation*}
C(t, i)=N\left(d_{1}\right)\left(V_{i} *\left(1+A_{i}\right)\right)-N\left(d_{2}\right)\left(C_{i} *\left(1-D_{i}\right)\right) e^{-r(T-t)} \tag{3.6}
\end{equation*}
$$

where $i$ represents a particular period of development, $V_{i}$ represents the value derived from successfully completing development period $i$ and $A_{i}$ represents the added value from using a platform engineering approach. The resulting equation provides us with 'profit' from engaging in software development period $i$ at time $t$, whose future value is discounted from the future time $T$ back to the present day value.

However, this model only allows us to reason about a single period of software development, $i$, at time $t$, rather than successive periods of software development in the future. Expanding the equation to provide computation for future engagements in software development involves converting our difference to the summation of differences over the lifetime of a software project. We augment

Equation 3.6 as follows:

$$
\begin{equation*}
C(t, i)=\sum_{t=i}^{T} N\left(d_{1}\right)\left(V_{i} *\left(1+A_{i}\right)\right)-\sum_{t=i}^{T} N\left(d_{2}\right)\left(C_{i} *\left(1-D_{i}\right)\right) e^{-r(\tau-t)} \tag{3.7}
\end{equation*}
$$

Because we are now summing the payoff from future investments in engaging in development, it is necessary to redefine the riskless discount term, $e^{-r(T-t)}$ to account for the time difference between today and the period of development, $\tau$, and applying that discount to the investment we are adding to our summation. In simulation, this subtraction is really $\max (0, \tau-t$, as to prevent the occurence of asset appreciation from negative time differences, as asset appreciation is already modeled by the functions for volatility. Furthermore, because we are counting multiple periods of development, $i$ has changed to be an index into a vector of software development periods. Since we are now counting the value accrued from future engagements in development, it becomes necessary to apply the riskless discount rate to the value term as well:

$$
\begin{equation*}
C(t, i)=\sum_{t=i}^{T} N\left(d_{1}\right)\left(V_{i} *\left(1+A_{i}\right)\right) e^{-r(\tau-t)}-\sum_{t=i}^{T} N\left(d_{2}\right)\left(C_{i}-D_{i}\right) e^{-r(\tau-t)} \tag{3.8}
\end{equation*}
$$

Finally, we convert this equation into a probabilistic equation, taking into account random perturbations in both the cost and the value, as well as $P_{i, t}$, the probability of successful completion of development period $i$ during time period $t$. This probability serves as an indicator variable on the uncertain outcome of development. In stochastic simulation, it will be used to determine if a period of software development is succesful when engaging in that period of software development. Unlike financial options, which are either exercised or not, the outcome of a real option is subject to the success of the endeavor being considered. $P_{i, t}$ serves to model the probability that the outcome of software development is executed successfully. In the event of success, the project accrues the value corresponding to the spot price in the model; in the event of failure, some discount to the value accrued from development is applied. Because this equation will be used as the basis for our Monte Carlo simulation, it becomes prudent to compute the expected value of both the strike and
spot price, as follows:

$$
\begin{equation*}
C(S, t, i)=\sum_{\tau=t}^{T} E\left[N\left(d_{1}\right)\left(V_{i} *\left(1+A_{i}\right)\right) e^{-r(\tau-t)}\right]-P_{i, t} E\left[\sum_{\tau=t}^{T} N\left(d_{2}\right)\left(C_{i} *\left(1-D_{i}\right)\right) e^{-r(\tau-t)}\right] \tag{3.9}
\end{equation*}
$$

In order to simplify this equation ${ }^{1}$ by using non-negative units for both the value and the cost of development, we may rewrite this equation in the following form:

$$
\begin{equation*}
C(S, t, i)=-1 *\left(E\left[\sum_{\tau=t}^{T} N\left(d_{1}\right)\left(V_{i} *\left(1+A_{i}\right)\right) e^{-r(\tau-t)}-P_{i, t} E\left[\sum_{\tau=t}^{T} N\left(d_{2}\right)\left(C_{i} *\left(1-D_{i}\right)\right) e^{-r(\tau-t)}\right]\right)\right. \tag{3.10}
\end{equation*}
$$

### 3.3 Experimental Approach

In this section, we lay down a more structured approach to our experimental researching following the simulation road map presented in [17]:

1. Choose a research question.
2. Identify a simple theory that describes the underlying phenomenon with which the research question is concerned.
3. Select a simulation approach.
4. Create a computational representation of the research problem.
5. Verify the computational representation of the research problem.
6. Experiment to build novel theory.

[^1]7. Validate with empirical data.

We begin by more rigorously defining our research question presented in Section 1.1.2. The proposed research question should be intriguing and and contain a tension between juxtaposed concepts [17]. Our research is more explicitly stated as follows:

How do the assets and non-software benefits provided in platform adoption impact the cost of development and value derived from developing a software product based on the adopted software platform?

Following our research question, our simple theory is two fold, 1) that platform adoption delivers additional assets which reduce the cost of developing a software product, and 2) that platform adoption delivers qualities to the products developed from the software platform that increase its value.

For the purpose of exploring the research question, we will be utilizing a stochastic simulation approach. Stochastic approaches to simulation based research are particularly useful for this kind of research for several reasons [17]. Data on the cost and value derived from software development is often kept secret by commercial organizations, making the data difficult to collect. While there exist structured models of software development, development differs from one organization to another. Furthermore, stochastic methods provide flexibility when dealing with non-structured approaches by allowing for variation in the stochastic variables which feed into the simulation.

We were confident in our ability to complete this research for several reasons. The authors have existing experience in conducting stochastic simulation research, as evidenced by previous stochastic simulation research in the field of physical chemistry [55, 18], more relevant work presented in [38], and previous work reviewing research utilizing stochastic methods for the African Journal of Agricultural Research.

In this section, we have laid the foundation for our computational model that integrates the quantification of cost and value within software development with the Black-Scholes OptionsValue equation. We discuss parameters and sources of stochasticity, Section 3.4. We continue to elaborate on our computational model, proposing sources of data for our parameters and sources of stochasticity, as well as variance in our stochastic sources in Section 4, as well as providing a mapping that verifies the mapping between concepts described in Section 2 and the model proposed
in this section. Finally, we present our results in Section 5 and cover the validation of our model in Section 6.

### 3.4 Sources of Stochasticity

Given our treatment of options valuation and the Black-Scholes formulation, we have identified a number of theoretical constructs which act as sources of stochasticity within our simulation approach. The following list details these variables and their units of measure:

- Riskless Discount Rate - Percentage
- Asset Volatility - Percentage
- Cost Reduction Factors - Dollars, but also traditional software size metrics.
- Value Derivation Factors - Dollars, but also traditional software use metrics.
- Probability of Success - Percentage


### 3.5 Summary of Existing Work

In our initial work, presented in [38], we explored a similar research question to the one found in this proposal, namely an exploration in quantifying the value derived from a variation point in a software product line. Extending the work presented in [49], we adapted Shishko and Ebbeler's Black-Scholes formulation to apply to N variation points utilized over M products. The computational model was then converted to a stochastic simulation. We instantiated a product line of 9 products which were composed of 11 unique variation points. Each variation point was developed over a discrete number of development periods of time, with the total cost of the variation point divided equally amongst the development periods. Each product, composed of 2 or more variation points, was assigned a value which was accrued in totality upon release of the product, namely the period following the completion of development of the last variation point of that product.

We structured the simulation scenario by providing different schedules for development, including situations where a variation point was developed from start to finish over a contiguous set of development periods, as well as situations wherein development was started, stopped for a number
of periods, and was later finished. Stochasticity was varied in the simulation through a probabilistic variable, $p_{i, T}$, the probability of successful implementation of variation point $i$ in period $T$. In one scenario, the $p_{i, T}$ was held constant for each period of development, while in another scenario $p_{i, T}$ decreased over each period of development, representing the increasing uncertainty of software development as modelers look further into the future.

The two scenarios, with constant and decreasing $p_{i, T}$, were simulated as a Monte Carlo simulation, performing 200,000 trials per simulation. The results were verified using checks for face validity and internal validity, as well as sensitivity analysis provided by the variance in $p_{i, T}$. The work was well received and later republished as a side-bar in [3]. For a more detailed account of this work, see Appendix B.

## Chapter 4

## Experimental Design and

## Execution

In this section, we detail the sources stochasticity that occur within our computational model, as well as the ways of parameterizing each of these variables.

### 4.1 Period Length and Project Duration

The granularity of our simulation is inherently tied to the choice of period duration within our model, represented by the variables $t, \tau, T$. Periods are used to define the duration of software development phases, in addition to their use in calculating the present-value of future costs and values. A phase of software development occurs in one or more periods. For example, the requirements phase of software development spans the duration of 3 periods of time.

In order to ensure consistency in our model, we will apply three constraints to the duration of periods. First, each period must be the same duration (e.g., each period is 3 months long). This constraint is necessary because time is a factor in applying the riskless discount rate to discount the future value of money back. to the present date If period length is not uniform across our model, the riskless discount rate will not be consistently applied in simulation.

The second constraint is that the period duration should be chosen so as to ensure that each phase of software development occurs in a whole, non-negative (e.g., natural) number of periods.

This constraint is necessary because $t, \tau$ and $T$ are used as counting variables in the two summations within the model.

The third constraint is that an appropriately long (or short) period duration should be chosen as to prevent the occurrence of more than one phase of software development in a single period For example, the duration of a period is six months, and both the implementation and testing phases of software development will take three months each. The reasoning for this constraint is that the Black-Scholes equation is only designed to compute the value of a single of asset which is being traded, or, in this case, a single decision being made.

Choosing an appropriate period duration will also depend on how development is structured. In a big upfront designed context, a suitable period duration in (in days or months) for periods can be found such that each phase of software development occurs in a whole number of periods. In iterative-incremental development settings, period-length can be dictated by iteration or sprint length.

One additional minor constraint of periodicic simulation is that all events occur simultaneously upon entering a time period. This can be mitigated by choosing shorter periods, providing a more fine-grained simulation. However, more fine grained simulations runs the risk of failing to provide more interesting results, as period length becomes shorter than an appreciable amount of time (e.g., 1 day).

### 4.2 Software Development Cost

Since options value formulations typically reduce to fundamental units of value, such as man hours or dollars, it makes sense that the cost of software development, $C_{i}$, will use the same units. The cost of a software project is based on the duration of the development and the number of personnel involved with development in addition to other miscellaneous and one time added costs, such as licensing fees or switching costs for standards and platforms. These factors have been used used to provide realistic values for the cost of each phase of software development.

One possibility for parameterizing the cost of software development is to utilize existing cost estimation methods, such as Boehm's work in $[53,6,8,31]$. In utilizing these methods, we might speculate on the size and complexity of the software being developed and utilize the estimation methods to provide a base cost for the project. Alternative models of software cost, such as those
proposed in [16] might also assist in guiding the parameterization of software development costs.
Alternatively, parameterization of the cost can be guided by historical data on the cost of software projects relative to the size, complexity and duration of the project in order to generate realistic values for the cost of each phase of development.

### 4.3 Software Development Cost Reduction

Cost reduction and savings, $D_{i}$, manifests itself in several ways during the software development life cycle, each manifestation differing based on that phase of software development. It is assumed that in each phase of software development, the platform provides some development assets which defray the cost of software development. For example, an organization adopting a software platform may receive requirements during the requirements engineering phase of the SDLC, while receiving lines of code as the contribution of cost reduction during the implementation phase. Examples for each phase of software development are described below:

- Requirements Engineering - Functional requirements, non-functional requirements.
- Architecture Design - Architectural specifications for systems and subsystems, interfaces.
- Implementation - Lines of code, functions, function points, libraries.
- Verification and Validation - Tests, test plans, required coverage levels.
- Maintenance - Bug fixes and maintenance to the platform.

If parameterized in this fashion, the differing contributions in each phase of the SDLC will need to be reduced to the common unit of measurement, i.e., dollars and/or man hours. For example adopting the Eclipse platform provides us with a set of requirements that no longer need to be specified by our development team resulting in reducing the costs of requirements gathering by $\$ 3,500$ or 115 man-hours.

An alternative way to parameterize cost reduction is through a percentage- based representation. For example, adopting a given software platform provides $13 \%$ of architectural assets necessary in developing a software product. This approach provides a simpler mechanism that is unit-agnostic and obviates the need for converting to a common unit of man-hours or dollars when determining contribution of cost-reduction. This is a strategy of quantifying software development is
similar to approaches taken in software reuse, where a percentage of development can be obviated by reusable software artifacts, presented in the papers discussed in Section 2.1.2. The percentage-based representation of cost reduction was chosen for the experiments presented in this research.

### 4.4 Software Development Value

The value of software development, $V_{i}$, is determined by the purpose of the software being developed. Some software is developed explicitly to be sold to an outside user, through the typical license agreement relationship. However, some software is developed for internal use, usually with the aims of increasing productivity, improving quality. In the former case, measuring value in the form of dollars and cents is appropriate; however, when software is developed for internal use, the return on investment is often realized in increased productivity rather than new inward cash-flows. A prudent way of modeling that increase in productivity is in man-hours saved from increases in efficiency resulting from the software product developed. While man-hours can certainly be converted into a dollars-and-cents value compatible with the cost side of the equation, there is a subtle transformation in the model, wherein the return on investment is quantified not by new cash flows, but rather by an increase in production output as a result of the increases in efficiency. In this scenario, while the result of our model is distilled into a single quantified value (for instance, $\$ 56,000$ ), interpreting the meaning of that value has changed slightly.

While there are trade-offs between these two perspectives on the context and resulting value from software development, part of this research relies on the assumption that there is an open market for trading the assets created through the software development process.

However, yet another perspective perspective on value exists which can serve to allay any concerns on the value of work-products produced during development, namely, 'earned value.' Earned value, or earned value management, is a project management technique that investigates the cost of a project in relation to its potential value and the earned value, or value accrued from development, completed at the time of measurement [23].

### 4.5 Value Addition

Value addition refers to extra value accrued when computing the return on development investment as a result of adopting a specific platform, represented in our model by the variable $A_{i}$. Unfortunately, this is the least well known of parameters within our model. While there exists research into the positive benefits of software ecosystem strategies and platform-based software development, the majority of the research focuses on the benefits that are afforded to the owner of the platform, rather than the developers that extend the platform.

While parameterization of value addition is still an ongoing effort in the context of the work proposed here, one known possibility is the decreased time to market experienced by users that embrace platform-based software engineering. Faster time to market can impact the sales and user adoption of a software product, introducing higher sales revenue into our model, particularly if the resulting product is the initial entrant into the market. However, faster time to market introduces other subtleties to different parts of the model, including the amount of time spent in each software development period.

Increased software quality is another benefit of platform-based software engineering, both from the software ecosystems research perspective [57,10] as well as the perspective of researchers in software reuse [13]. However, the link between software quality and our concrete notion of value is not well understood, and proves difficult to integrate into our model (e.g., how many dollars of revenue are lost on a bug?).

Another possible source for additional value may arise from the development of features that would otherwise be unfeasible outside of platform development. If the platform enables additional features within the software product, the software product may attract additional users that would not otherwise use the software. The result is additional sales, increasing the value derived from software development. Yet another perspective that the additional features increase the sale price of the software, which is particularly applicable in buyer-seller situations featuring shrink wrapped or a la carte, as opposed to custom built, software.

### 4.6 Riskless Discount Rate

Perhaps the easiest source of stochasticity to parameterize, the riskless discount rate, $r$, is a financial instrument used to represent inflation within the economy. The riskless discount rate
could be varied in simulation to give a rough approximation of the health of the economy in which the software is being developed with respect to inflation rates.

The riskless discount rate could be parameterized in several ways. The most simple and straight forward way is to use the common method of matching the riskless discount rate to the interest rate on government-backed bonds. One possibility is to use existing historical data from the Federal Reserve on variance in inflation rates. While potentially beyond the scope of this research, the riskless discount rate could also be varied from period to period in order to simulate macroeconomic events to see how they affect the valuation of software development within that context.

### 4.7 Volatility

The volatility variable, $\sigma$, within the computational representation of our model provides a level of uncertainty that affects both the cost and the value within the calculation. Typically this is modeled as an infinitesimal random walk with a net change of zero. An random walk is a mathematical model of a path made from a succession in random steps. In financial options, the random walk is used to 'jiggle' the value or spot price of the risky asset, as to prevent its value from being constant over time, adding volatility, typically with net effect of zero change. The net effect of zero change means that, while at any given time point the spot price of the risky asset may be higher or lower than a previous time point, over a long enough period of time there is no net change to the spot price of the asset.

Typically, a Wiener process, or Brownian motion, is used to model volatility in financial options. Brownian motion is a random walk model that describes particles suspended in fluid. While a Wiener process would suffice for modeling the volatility of the cost and value of the risky development assets, other possibilities exist. When options valuation techniques are used in a nonmonetary or decision-making context, one way of parameterizing volatility is through the notion of a perfectly matched twin asset. This twin asset is a financial security, which can be easily reduced to a monetary quantity, that has a volatility that approximates the volatility of the non-monetary asset on which options valuation is being performed.

### 4.8 Probability of Successful Implementation

The probability of successful implementation, $p_{i}$ more explicitly refers to the probability that a period of software development will complete on-time. Probability could be parameterized in a number of different ways. In [49], the authors utilized project manager expert opinion for parameterizing their probability of successful completion of a project technology by the readiness date for future applications. In absence of expert opinion, in [38], the authors chose a somewhat liberal probability ( $p=0.85$ ) for their successful implementation of a variation, modeling stochasticity by reducing the probability in future periods of development. A similar approach could be used, utilizing different schemes for reducing probability in future periods, such as linear decline, quadratic decline and so forth.

Additionally, external sources of information provide information on success rates in software development projects. The well known CHAOS Report by the Standish Group is a long running project that examines success and failurealso, rates in IT and software development rated projects. The 2012 CHAOS Report demonstrated that $31.1 \%$ of projects were not completed and canceled while underway, while $52.7 \%$ of projects were completed with cost or schedule overruns and only $16.2 \%$ of projects were completed successfully within the allotted schedule and budget [32]. While there are critics of the Standish Group and their CHAOS Reports [21], these provide suggestive data on the parameterization of the probability of successful implementation.

### 4.9 Discussion

In this section, we have discussed and described the basic building blocks of our theoretical model and how they will be used in our stochastic model and Monte Carlo simulations. For each variable, described in each of the previous subsections, we have described their meaning and significance, how they will vary stochastically and how they will be parameterized in simulation, as well as sources for real world data that will guide our parameterization. In the next section, we will describe the construction of each of the scenarios and their constitutent experiments, as well as data and models that were used for each of the experiments, as well as the results that were generated by each of the experiments. The results for the set of experiments, which are defined by each scenario, are presented and analyzed in an effort to answer the question of which software development decision to make, in each scenario.

## Chapter 5

## Experimental Results

### 5.1 Expected Results and Outputs

The expected result of this research is theory that arises from the analysis of each set of simulations for the scenarios described in Section 1.1.1. Each of these scenarios describes an option with two possible choices, namely adopt one software platform or an alternative software platform, in scenario 1, continue using an existing platform or port code to a different software platform, as seen in scenario 2 . Due to the nature of the Black-Scholes, only a single option can be considered at a time (e.g., adopt a platform). More explicitly, when utilizing the Black-Scholes equation for options analysis, the equation can only compute the cost-value analysis on a single risky asset (i.e., purchasing land or engaging in software development). By itself, the Black- Scholes formulation cannot evaluate the differences between two options (i.e., purchasing this parcel of land or purchasing another parcel of land). Thus, in a situation where multiple investments (decisions) are possible, it is necessary to evaluate each option (buy this parcel of land or do not, buy that parcel of land or do not) and compare the results.

For each scenario, we have performed a number of experiments, investigating the two (or more) decisions proposed in the scenario. Each experiment is composed of a single Monte Carlo simulation, run for 50,000 or more iterations. The results are averaged, computing the standard deviation as well, and summarized in a number of three-dimensional surface plots which describe the metrics outputted by the simulation.

The results of these simulations were analyzed and used to modify the input parameters of


Figure 5.1: Experimental Process
the underlying model(Equation 3.10) iteratively and further refine the scenarios under simulation. Figure 5.1 provides a diagram illustration of this process.

The end result was a more detailed model describing the underlying cost and value of software development, as well as an answer to the contrived questions posed in the scenarios described in Section 1.1.1. Through iteration, were able to determine variation points in the model which could be parameterized with data, or, in the case of a lack of data, models which describe the behavior of development under different conditions.

### 5.2 Simulation Structure and Computation

### 5.2.1 Experimental Simulation

The implementation of our experimental simulation varies slightly from the model presented in Equation 3.10. Equation 3.10 computes the total net value of utilizing a software platform to engaging in software development. While the end result is useful, that result only provides a partial view on the development process, namely that end result. In simulation we have 'unrolled' the computation, recording the results of each period of development with respect to time, as demonstrated in the pseudocode below.

```
for each time period t
    for each development period i
        compute options value of dev. period i wrt time period t
```

The result of this algorithm is a 2-dimensional matrix of values that represent the expected profit (revenue less costs) of each development period measured with respect to time. The resulting matrix provides more flexibility in understanding the rhythm, flow and expected value resulting from development, not only over the course of the development schedule, but also from a more fine grained perspective.

### 5.2.2 Probability and Failure

As noted in Section 4.1, each scenario is constructed from a number of sequential periods. In our simulation, we compute the net value of each period. A random number is generated and checked against the probability of successful completion. In the event of success (i.e., random number $<$ probability of success), the net value is equal to the difference between the value, adjusted by the added value coefficient and the riskless discount factor, and the cost, adjusted by the cost reduction coefficient and the riskless discount factor.

Table 5.1: 4-Bucket Stratified Failure Model

| Severity | Failure Coefficient | Probability of Occurrence |
| :---: | :---: | :---: |
| Minor | $25 \%$ | $50 \%$ |
| Major | $50 \%$ | $25 \%$ |
| Critical | $75 \%$ | $15 \%$ |
| Catastrophic | $90 \%$ | $10 \%$ |

In the event of a failure (i.e., the random number is greater than or equal to probability of success), the value derived is discounted by the failure coefficient captured within our failure model. The failure model is defined by a number of different failure strata, as shown in Table 5.1. An additional random number is generated, which determines the severity of failure in the period of development and the failure coefficient. The value derived from the period of is development then discounted based on the failure coefficient:

$$
\begin{equation*}
\text { finalValue }=\text { simulationValue } *(1.0-\text { failureCoefficient }) \tag{5.1}
\end{equation*}
$$

The value not accrued as a result of failure (the finalValue less the adjustedValue) is then carried over into the next period and added into the potential value that can be accrued for that period. Likewise, an associated cost for that value is carried over into the next period as well, defined
by the failure model:

$$
\begin{align*}
& \text { valueCarryOut }=\text { finalValue }- \text { adjustedValue }  \tag{5.2}\\
& \text { costCarryOut }=\text { simulationCost }- \text { adjustedCost } \tag{5.3}
\end{align*}
$$

In subsequent periods, the probability of successful completion is discounted in relation to the carried over cost:

$$
\begin{equation*}
\text { AdjustedProbability }=\text { Probability } * \frac{(\text { carryAdjustedCost }-\operatorname{costCarryIn})}{\text { carryAdjustedCost }} \tag{5.4}
\end{equation*}
$$

After initial investigations which helped identify these variation points, we focused on four different metrics:

- Net value per period - The earned value derived from each period of development with respect to time.
- Sum net value per period - The running sum of the earned value for each period of development with respect to time.
- Value carry out - The value not realized from a period of development as a result of failure in the development process. This value is carried out from a period of development into the next period of development. The value carry out is proportional to the amount of work not completed during a particular period of development with respect to time.
- Cost carry out - The cost, in Dollars, associated with the value carry out. This is the amount of money or development effort that would be required to finish work that was not completed due to failure in development. The cost carry out value is carried out from one period of development into the next period of development, and is proportional to the value carry out.

These metrics provide an objective point of comparison between the results of each of the experiments performed for each of the scenarios. For each experiment, we have created 3-dimensional plots of each of these metrics with respect to development period and elapsed time. From these plots, the
choice of whether to halt development or to continue on to the next phase of development, based on the projected future income from the project, is supported by the data. Given two development options, the data can be used to determine the optimal strategy for proceeding with development.

### 5.3 Scenario 1: Choice of Two Platforms

Recall the first scenario from Section 1.1.1:

An organization wishes to create a tool to aid in software development. Creating a new workbench from scratch is reinventing the wheel, and the project manager would like to explore open source platforms to serve as the basis for their tool. After evaluating the platform landscape, the project manager identifies two candidate platforms which meet the needs of their project: Eclipse and NetBeans. Considering that this tool may undergo extensive modification and extension, the project manager wishes to evaluate both platforms to determine which is the better fit for the project, given its perceived evolution over time. However, the project manager is unsure how to compare each of these platforms for use. Given that profit margins are low, saving money on engineering costs are a priority.

In this scenario, the manager is interested in investigating which of two platforms will provide a better cost savings and added value. For this descriptive scenario, we have generated a baseline project using the Constructive Cost Estimation Model II (COCOMO II) in order to parameterize the costs and values of development. The generated baseline project was a 100 KLOC project with no significant sources of software reuse or legacy code adaptation. In the context of COCOMO II, all software cost drivers, as well as personnel, platform and project qualities were nominal. And the project generated did not include maintenance phases. We parameterized the cost of personnel with the national average salary of a software engineer ( $\$ 90,374$ ) [26] and determined the cost per month of effort (\$7,531.16).

The generated project provided us with a 4-phase breakdown of the project according to the Rational Unified Process (RUP) [36]. For a brief introduction to the RUP development model, see Appendix C. Each of the phases had a duration in months and average staffing, measured in human personnel, as well as the effort required in that phase. In order to adapt this generated

Table 5.2: Scenario 1 Project Constants

| Parameter | Value |
| :--- | :--- |
| Riskless Discount Rate | 0.05 |
| Probability of Successful Imple- <br> mentation | 0.80 |
| Probability Variation | -0.02 Per Period |
| Cost Per Man-Month | $\$ 7,531.16$ |
| Period Duration | 1 Month |

Table 5.3: Scenario 1 RUP Project Timeline

| Phase | Effort (Man- <br> Months) | Schedule <br> $($ Months $)$ | Total Cost <br> $(\$)$ | Period Cost <br> $(\$)$ | Total Value <br> $(\$)$ | Period <br> Value $(\$)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inception | 27.9 | 4 | $\$ 210,119$ | $\$ 52,529.84$ | $\$ 231,131.30$ | $\$ 52,529.84$ |
| Elaboration | 111.7 | 10.7 | $\$ 841,231$ | $\$ 84,123.06$ | $\$ 925,353.63$ | $\$ 84,123.06$ |
| Construction | 353.6 | 20.3 | $\$ 2,663,018$ | $\$ 156,648.13$ | $\$ 2,929,319.99$ | $\$ 156,648.13$ |
| Transition | 55.8 | 16 | $\$ 420,239$ | $\$ 105,059.68$ | $\$ 462,262.60$ | $\$ 105,059.68$ |
| Sum | 549 | 55 | $\$ 4,134,606.84$ | $\$ 398,360.71$ | $\$ 4,548,067.52$ | $\$ 398,360.71$ |

project to our model, we decomposed the project into 1-month periods, with each period belonging to a different phase of development within the RUP model. Each period's cost was parameterized in a manner proportional to the total cost of the phase with respect to the number of periods in that phase. For example, an inception phase lasting 4 months and costing $\$ 32,000$ was decomposed into 4 periods, each costing $\$ 8,000$. The value of each period was set to the cost $+10 \%$. We have set the probability of successful implementation to $80 \%$, discounting the probability of future developments by $2 \%$ per period. The riskless discount rate was set to $5 \%$. This data is summarized in Table 5.2 and 5.3.

In simulating this scenario, we engineered three experiments. For the sake of completeness and exploration, we began by simulating a bespoke scenario in which a platform was not utilized. This is a rational decision from the perspective of the manager in the scenario, as there is a dearth of hard data suggesting that utilizing a software platform reduces expenses or increases value. It may be the case that utilizing either of these platforms is a suboptimal strategy compared to developing the software product from purely in-house assets. This initial experiment could be considered the 'control' group in an experimental sense. In this scenario, the cost reduction coefficient and added value coefficient are both set to $0 \%$ for each period of development. In the second experiment, we parameterized the simulation for a platform that provides significant architectural assets to
the development organization. In this simulation, we set the cost reduction coefficient to $10 \%$ in inception and elaboration phases, the phases in RUP which correspond to architecture development. In our final experiment, we set the cost reduction coefficient to $5 \%$ during the construction phases, representing a platform that provides significant code level and testing assets. These numbers are reflective of real world improvements as a result of software development process, as shown in [43, 40]. While the cost benefits described in $[43,40]$ are a result of software process, the the savings and outcomes derived from improvement in software process are a modest, but realistic, source of data to parameterize our experiments.

In each experiment, the simulation was ran for 50,000 iterations. For each iteration, the period net value, sum net value and their standard deviations were computed, as well as the cost carry in, cost carry out, value carry in and value carry out for each period. These results were aggregated by taking the average of all iterations. Because the model determines these statistics with respect to both development period and time, three-dimensional surface plots were used to visualize the data. Each plot contains a plane where the x -axis shows the development period, the $y$-axis shows the value, whether its per-period value, sum value, carry out, etc., and the $z$-axis shows the elapsed time. Thus an $(x, y, z)$ coordinate refers to the $y$ value of development period $x$ at elapsed time z. For each experiment, we have included the surface plot of net value per period (that is, value after cost), sum net value per period (running sum of value per period), cost carry out and value carry out.

### 5.3.1 Bespoke Platform

The results for the bespoke platform experiment were largely predictable, as to be expected within this experiment. Figure 5.2 shows the plot of development period vs. net value per period vs. elapsed time. In the figure, there are four distinct plateaus of value, corresponding to each of the four phases in the RUP development model: period [0...3] for inception, [4..13] for elaboration, [14...30] for construction and [31...34] for transition. The plateaus are a result of the fact that the value derived from a given period of development is proportionate to its cost. In the RUP model, fewer personnel and staff are needed for the inception than the elaboration phase, and fewer people are needed for the elaboration phase than the construction phase. Thus, as the project transitions between the RUP development phases, there's an abrupt jump in the net value derived from the periods of development.


Figure 5.2: Bespoke Development: Net Value per Period

Along the z-axis, moving from the origin to the edge of the graph, the surface slopes upward in the y direction. This is result of two factors: the riskless discount rate and the probability model used. As the time difference between the point of measurement $(\mathrm{t})$ and the period of completion for a period of development $(\mathrm{T})$ decreases, the effects of the riskless discount are diminished. Additionally, as that time difference decreases, so does the effect of the probability variation model. As mentioned above, the probability of successful implementation $P_{i}, T$ is discounted by $0.02 *(T-t)$. The closer the point of measurement gets to the actual time of implementation, successful implementation becomes more likely. Eventually the surface stops sloping and plateaus, representing the point where $t \geq T$, thus negating both the effect of the riskless discount rate as well as variation in probability due to our probability model.

Figure 5.3 describes the sum net value per period of development, or rather, the running sum of net value over all periods of development. The $y$-value of the plane slopes upward with respect to both the x -axis and the z -axis. The increasing slope with respect to the x -axis is a result of later periods of development, namely those in elaboration and construction phases, yielding a larger pay out than periods in inception phases. The increasing slope in the z-axis is a result of the aforementioned diminished effect of the riskless discount factor and the probability variation model.

Returning to the initial model described in Table 5.3, the total value of the project was


Figure 5.3: Bespoke Development: Sum Net Value per Period
estimated at $\$ 4,544,857.52$, while the total cost was estimated to be $\$ 4,179,746.60$, yielding a net value of $\$ 365,108.79$. Examining the peak of the surface, the expected payout from development during the final period of development is $\$ 267,312.86,73.21 \%$ of the maximum possible net value after costs, or rather $6.39 \%$ return on investment. Given that the maximum ROI was $8.73 \%$, this is a reasonable, but less than optimal, outcome for this project.

Figures 5.4 and 5.5 show the cost carry outs per period and the value carry out per period, respectively, as a result of failure in development. As expected, the graphs are strikingly similar, as the cost carry out and value carry out of a given period are proportional to each other. Again, we see four distinct plateaus, corresponding to the four phases of development in the RUP model. Surprisingly, both the cost carry out and value carry out tend to slope upwards with respect to the z-axis, which was somewhat unexpected. As $T-t$ approaches 0 , the impact of the probability variation model decreases, leading to fewer failures, which suggests that the cost and value carry out should decrease over time. However, simultaneously, the impact of the riskless discount rate is diminished as well, leading to greater costs and values, which results in greater carry outs, as the carry outs are proportional to the cost and the value with respect to the severity of failure. Intuitively, it seems like the decreased chance of failure (increased chance of success) should reduce the value of the carry outs more than the diminished impact of riskless discount rate increases those


Figure 5.4: Bespoke Development: Cost Carry Out Per Period
values; however, this is not the case. The probability variation model is a linear function, while the riskless discount factor $\left(\exp ^{-r(T-t)}\right)$ is a quadratic function. The impact of the probability variation model is linear across all time periods, while the impact of the riskless discount factor shrinks at a faster rate as $T-t$ approaches 0 .


Figure 5.5: Bespoke Development: Value Carry Out Per Period

### 5.3.2 Architecture Centric Platform

The second experiment describes engaging in development while utilizing what we describe as an 'architecture-centric' software platform, one that reduces the amount of architectural work and expertise required to engineer the software product. In this experiment, architecture-centricity has been manifested through a non-zero cost reduction coefficient applied to the planning stages in our RUP model, namely the inception and elaboration. We have applied a $10 \%$ cost reduction coefficient to each of the periods that compose inception and elaboration, which is in line with the cost savings incurred from utilizing the Software Engineering Institute's Architecture Centric Engineering (ACE) process, as evidenced in [43, 39]. While ACE refers to a process and practice, the use of $10 \%$ as a cost reduction coefficient is grounded in reality. Other real examples of architecture centric platforms may include the CORBA reference architecture [27], which provides guidelines for realizing the CORBA specification in code, and the Eclipse platform [9], which provides a strict plug-in and rich client platform architecture.

Figure 5.6 shows the plot of development period vs. net value vs. elapsed time. The plot is similar to the respective plot in the previous section, with the net value displaying a step function for each of the four phases in the RUP development model. The most distinct difference between these two plots is the marked increase in earned value during periods in the inception and


Figure 5.6: Architecture Centric Platform: Net Value per Period
elaboration phases. Previously periods in the construction phase constituted the periods in which the most earned value was accrued; however, the cost reduction coefficient reduces the cost of engaging in software development during the inception and elaboration phases. The overall affect is that the elaboration phases are the most profitable phases of development, regardless of elapsed time. Additionally, until the mid-point of the project (roughly elapsed time $=15$ periods), the earned value derived from inception periods of development surpasses the earned value derived from construction periods. This is due to two factors. The first factor is the cost reduction coefficient, which reduces the cost of engaging in inception-phase development. The second is a result of the timing of the construction phases of development, which only begin during the 16 th period of development. Prior to beginning construction, the affect of the riskless discount factor and the probability variation model have a negative effect on the earned value of the construction phase periods, pushing their profitability below that of inception phase periods. This behavior makes sense, as the certainty of being able to successfully complete coding, testing and other construction phase activities hinges on the architecture.

Figure 5.7 shows the plot of development period vs. sum net value vs. elapsed time. The shape of surface is the same as the respective plot in the previous section, shown in Figure 5.3. Because the total value of the project has not changed (i.e., the value addition coefficient is $0 \%$ ),


Figure 5.7: Architecture Centric Platform: Sum Net Value per Period
the total value of the project was estimated at $\$ 4,544,857.39$, while the total cost of the project, including the cost reduction coefficient, was estimated at $\$ 4,074,848.76$, yielding a total possible net profit of $\$ 470,008.63$. Examining the peak of the surface, the expected payout from development during the final period of development is $\$ 377,143.45,80.24 \%$ of the maximum possible net value after costs, or rather $9.26 \%$ return on investment. Given that the maximum ROI was $11.53 \%$, this is a less than optimal outcome for this project; however, it represents an $45 \%$ increased ROI over the bespoke experiment.

Figures 5.8 and 5.9 show the plots of the cost carry out and the value carry out. They are both similar to their respective graphs in the previous section, Figures 5.4 and 5.5 , mirroring each other due to the fact that the cost carry out is proportionate to the value carry out. As a result of the cost reduction coefficient, the cost carry out during inception and elaboration phases is roughly $10 \%$ lower in inception and elaboration periods than the respective periods in the bespoke experiment, while the value carry over remains largely the same. The overall effect is that later periods of development during construction and transition have slightly smaller carryouts than in the bespoke experiment as a result of less work being carried over from period to period (i.e., the carry overs of early periods have an increasing effect to the carry outs of later periods).


Figure 5.8: Architecture Centric Platform: Cost Carry Out Per Period


Figure 5.9: Architecture Centric Platform: Value Carry Out Per Period


Figure 5.10: Code Centric Platform: Net Value per Period

### 5.3.3 Code Centric Platform

The final experiment in this scenario reflects engaging in software product development while utilizing a software platform that reduces the cost during construction-phase periods of development. A real world example of such a platform might be numpy or scipy [35], scientific Python libraries which provide a significant amount of functionality, but little architectural or early stage assets. We have represented this platform by setting the cost reduction coefficient during periods in the construction phase to $5 \%$. This represents both a reduction in the number of lines necessary to code, as well as a reduction in the amount of testing performed. The rationale is that the platform provides assets (data structures, algorithms, classes and interfaces) which have already been subjected to verification and validation techniques. This is in line with the results reported in [39].

Figure 5.10 shows the plot of development period vs. net value vs. elapsed time. Again, it is similar to the same plots in the previous two experiments, showing a step function, where the different steps in the surface correspond to the different periods of development. The primary difference between this plot and the previous two is the magnitude of the third step, which corresponds to the construction phase. The large gain in net value during construction phases is due to the combination of the cost reduction coefficient as well as the large costs associated with construction (coding, testing, etc.).


Figure 5.11: Code Centric Platform: Sum Net Value per Period

Figure 5.11 shows the plot of development period vs. sum net value vs. elapsed time. The shape of surface is the same as the respective plots in the previous sections, shown in Figure 5.3 and 5.6. Because the total value of the project has not changed (i.e., the value addition coefficient is $0 \%$ ), the total value of the project was estimated at $\$ 4,548,067.52$, while the total cost of the project, including the cost reduction coefficient, was estimated at $\$ 4,049,644.03$, yielding a total possible net profit of $\$ 498,423.49, \$ 24,194.73$ more than the 'architecture-centric' platform. Examining the peak of the surface, the expected payout from development during the final period of development is $\$ 403,558.83,81 \%$ of the maximum possible net value after costs, or rather $10 \%$ return on investment. Given that the maximum ROI was $12.3 \%$, this is a positive outcome for this project, and represents a $56 \%$ increase in ROI compared to the bespoke experiment and a $7.6 \%$ increase in ROI compared to the architecture-centric platform.

Figures 5.8 and 5.9 show the plots of the cost carry out and the value carry out. They are both similar to their respective graphs in the previous section, Figures 5.4, 5.5, 5.8, and 5.9. The cost carry out during periods in the construction phase is roughly $5 \%$ lower than the respective periods in the bespoke experiment, while the value carry over remains largely the same.


Figure 5.12: Code Centric Platform: Cost Carry Out Per Period


Figure 5.13: Code Centric Platform: Value Carry Out Per Period

### 5.3.4 Analysis

The results from this experiment were interesting. From the outset, choosing between a platform that provides assets during architecture versus a platform that provides reusable assets during the coding phases of a software project is not trivial. Given the model, our scenario and the sources of data, the simulation and model gave a decisive answer that suggested that the code centric platform was the optimal choice. Given that the shape of RUP is a linear software development process with an emphasis on code-level asset construction, it makes sense that construction is the biggest monetary sink during development, and yields significant gains in the face of a platform that reduces the cost of construction over the cost of requirements (inception) or architectural construction (elaboration). Different software development processes may produce different results, however. A model-driven development (MDD) development process emphasizes iterative model development and partial code generation prior to any hands-on coding. Software platforms that reduce the cost of architecture and requirements, or aid in model development, might yield additional savings from lowered costs.

### 5.4 Scenario 2: Switching Platforms

The second scenario targets platform switching. In this situation a platform was used to develop a software product; however, it is no longer the case that this platform is the best platform to use. Recall the scenario from Section 1.1.1:

An organization has developed a software application on an existing open source platform. As the application and the platform have aged, it is no longer clear that this platform is the best platform for their tool. After identifying a number of competing platforms to which their application may be ported, the project manager is unsure of how to determine whether switching platforms is the correct move, and, subsequently, which platform is the best platform to chose for their needs. It may be the case that switching to a different platform opens new implementation opportunities, or perhaps lowers the cost of continued development. However, additional investigation needs to be pursued in order to quantify these benefits.

In order to satisfy this scenario, we have expanded the scenario, using the using the real world experience of the author in funding a mobile app for education in an R1 university in the United States. For background on this scenario, see the expanded description in Appendix A.2.

A large university has developed an app to assist students in their day to day needs on campus. The current implementation has mostly static features, including a queryable student, staff and faculty directory, contact information for university services, sports schedules, transportation routes and scheduling, cafeteria menus and more. The current app utilizes a native web app platform, allowing the app itself to be delivered as a webpage wrapped in a thin mobile app.

The university wishes to release a premium app which expands the functionality of the existing app to include functionality for acquiring sporting events tickets, course registration and management, tuition payment, real time transportation information, payroll information and more. However, because the existing app (and its platform) are a glorified webpage, many necessary components to implement these features (such as push notifications and location services) are not available to the app on its the existing platform. In order to add these features, the app must be developed on a specific mobile platform. Two candidate platforms have been identified: a proprietary mobile platform, which a large majority of the potential user base utilizes, and an open source mobile platform which is used by a smaller portion of potential users. The project manager believes that both platforms will reduce the cost of development, but the open source platform provides large cost reductions due to its open source nature.

Given that the mission of the app is to provide better university engagement with users, for a premium, the best solution is to develop the app for both platforms, but funding is limited. While funding has been acquired to see through the initial 6 month development period, the future is uncertain. However, an arrangement was negotiated wherein any funds generated from the sales of the initial app would pay for the development of the app on the second platform.

Given that future funding is uncertain, selecting which platform to develop on first is a strategic decision with implications for the remainder of the project. On one hand, developing for the open source platform first may reduce the cost of development, reducing the user adoption requirements to maintain solvency. However, the pay off from developing for the proprietary platform first, with its greater number of users, may provide enough income to support continued development for the other platform.

Table 5.4: Scenario 2 Project Constants

| Parameter | Value |
| :--- | :--- |
| Riskless Discount Rate | 0.05 |
| Probability of Successful Imple- <br> mentation | 0.80 |
| Probability Variation | -0.02 Per Period |
| Cost Per Man-Month | $\$ 4,513.33$ |
| Cost Per Period | $\$ 27,080.00$ |
| Period Duration | 1 Month |

Table 5.5: Scenario 2 Construction Parameters

| Platform | User Share | Potential Users | Est. Adopters | Value / User | Total Est. Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Native Web | $100 \%$ | 100,000 | 50,000 | $\$ 2.50$ | $\$ 125,000.00$ |
| Open Source | $30 \%$ | 30,000 | 24,000 | $\$ 5.00$ | $\$ 120,000.00$ |
| Proprietary | $70 \%$ | 70,000 | 56,000 | $\$ 5.00$ | $\$ 280,000.00$ |

### 5.4.1 Scenario Construction

The development team consists of six members and the project is estimated to take six months. Each developer has an employment cost of $\$ 4,513.33$ per period, for a per-period development cost of $\$ 27,080.00$, for a estimated total development cost of $\$ 162,480.00$. This salary is based off of Clemson University's historical pay for programmers and development [15]. We have set the probability of successful implementation to $80 \%$, discounting the probability of future developments by $2 \%$ per period. The riskless discount rate was set to $5 \%$. This data is summarized in Table 5.4.

The university estimates that $70 \%$ of the university population (including students, faculty and staff) uses the proprietary platform, while $30 \%$ uses the open source platform. It is estimated that $100 \%$ of the university will be able to use a native web app platform. In each case, they assume that $80 \%$ of the potential users for the open source and proprietary platform will use the app for their platform, while only $50 \%$ of the potential users for the native web platform will use the app, due to the lack of features available on the native web app platforms. We have parameterized the size of the university population at 100,000 total users. In the case of the proprietary and open source platforms, the revenue of each user is estimated to be $\$ 5.00$, while the native web is $\$ 2.50$. The logic behind this decision is that the university cannot charge as much for an app with fewer features. Table 5.5 gives a break down of potential users and revenue for each platform.

Table 5.6: Scenario 2 Cost Reduction and Added Value Coefficients

| Platform | Cost Reduction Coefficient | Added Value Coefficient |
| :---: | :---: | :---: |
| Native Web | 0.0 | 0.0 |
| Open Source | 0.3 | -0.04 |
| Proprietary | 0.1 | 1.24 |

The cost reduction coefficient for the native web app platform was set to 0.0 , as it is seen as the baseline implementation, similar to the bespoke platform in the previous scenario. The proprietary platform was given a $10 \%$ cost reduction coefficient due to the resources provided by the platform owner to developers. The open source platform was given a $30 \%$ cost reduction coefficient, greater than the proprietary platform, due the fact that the code is open and available, which allows greater access to the code and educational materials on development for that platform.

In this scenario, we are viewing the value as a function of a closed market of users (i.e., there cannot be more than the maximum total users for each platform). As a result, the added value coefficient for each platform was computed in relation to the native web app platform, such that:

$$
\begin{equation*}
\text { AddedValueCof ficient }_{\text {Platform }}=1-\frac{\text { Platform }_{\text {TotalEstimatedValue }}}{\text { NativePlatform } \text { TotalEstimatedValue }} \tag{5.5}
\end{equation*}
$$

Table 5.6 summarizes these parameters.
As stated above, an additional motivation is that developing for both platforms represents full success of the project. Given that the revenue generated from this project is fed back into the funding is available for the project, we have made a slight tweak to the construction of each of the experiments for this scenario. The first period of development represents a funding period, with no cost that yields the estimated funding for the project ( $\$ 162,480$ ). Given that a large part of analysis will be determining whether or not there is funding to continue with development, as well as how much funding, this construction was done to simplify the analysis. Additionally, in each experiment, we have stipulated that the app is released three periods into development, accruing a percentage of its total estimated users each period following the third period.

In the following subsections, we present three experiments and their results for utilizing

Table 5.7: Scenario 2 Parameters: Native Web Platform

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 |
| 2 | 27080 | 0 | 0 | 0 |
| 3 | 27080 | 0 | 0 | 0 |
| 4 | 27080 | 0 | 0 | 0 |
| 5 | 27080 | 50000 | 0 | 0 |
| 6 | 27080 | 30000 | 0 | 0 |
| 7 | 27080 | 20000 | 0 | 0 |

each of the three platforms, as well as two additional experiments wherein both the open source and proprietary platforms are used for development. In these final two experiments, we have permuted the order of development (open source before proprietary, proprietary before open source) in order to analyze which platform yields greater value both throughout development and at the completion of development. The data for each of the experiments described below can be found in Appendix D.2.

### 5.4.2 Native Web Platform

In the first experiment, the manager is investigating whether the native web platform is the best platform for the premium app. The motivation behind this experiment was two-fold. First, we need a baseline scenario with which to compare our results for the open source and proprietary platform. While the model and simulation parameters above can provide an at-a-glance analysis that suggests developing for the proprietary platform is optimal, both the cost and value will vary slightly in simulation due to the probability model, failure model and variance in simulation. The second is that there is not a clear indication that developing for the open-source platform over the native web platform is the optimal decision: the profit margins are thin and its uncertain if, in the face of development hurdles, the open source platform will yield more revenue. Table 5.7 provides a per period break down on cost and value as well as the cost reduction and added value coefficients.

Figure 5.14 describes the net value per period of development when utilizing the native web platform. The net value per period starts high, as a result of the initial seed funding for the project, before plummeting during periods of development where no income is being accrued. There is an increase in the net value at the beginning of the fourth period, corresponding to the new income from new users as the product is released, which diminishes in the following periods as adoption


Figure 5.14: Native Web Platform: Net Value Per Period
rates slow down.
Figure 5.15 shows the sum net value per period of development when utilizing the native web platform. We can see a steady decrease in the running sum of revenue until the fourth period, when it increases due to the release of the app and new users. From period four through period six, the value sees a slight increase followed by a slight decrease as the adoption rate for the app slows down. Ultimately, the app is estimated to generate $\$ 96,423.57$ before funding costs, which is enough to sustain development for an additional 3.5 months.

### 5.4.3 Open Source Platform

As noted above, the open source platform provides a modest cost reduction coefficient, trading off some potential revenue. Table 5.8 provides an overview of the cost and value structure of the experiment. The first period models the period of initial funding, while subsequent periods model the actual development of the app. The $30 \%$ cost reduction coefficient is applied to each of the periods of development, while the $-4 \%$ value addition coefficient is only applied to the 4th, 5 th and 6 th periods due to the $\$ 0$ value derived from the first three and simulation, a $0 \%$ or $-4 \%$ value addition on a period with a $\$ 0$ value has periods. This is largely an inconsequential modeling decision; in both theory the same result.


Figure 5.15: Native Web Platform: Sum Net Value Per Period

Table 5.8: Scenario 2 Parameters: Open Source

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 |
| 2 | 27080 | 0 | 0.3 | 0 |
| 3 | 27080 | 0 | 0.3 | 0 |
| 4 | 27080 | 0 | 0.3 | 0 |
| 5 | 27080 | 50000 | 0.3 | -0.04 |
| 6 | 27080 | 30000 | 0.3 | -0.04 |
| 7 | 27080 | 20000 | 0.3 | -0.04 |



Figure 5.16: Open Source Platform: Net Value Per Period

Figure 5.16 shows the plot of development period vs. net value vs. elapsed time. The surface has a similar shape to the same plot for the native web app, skewed by the initial seed funding. Periods 2, 3 and 4 operate at a net loss, though lower than the native web app platform due to cost reduction coefficient. Periods 5, 6 and 7 see a rising and falling net value from the release of the app and the decline in adoption over time.

Figure 5.17 shows the plot of development period vs. sum net value vs. elapsed time. Initially, the running sum is $\$ 162,480$ and steadily declines over the first three periods, reaching approximately $\$ 105,000$ at its lowest points before accruing revenue from the release of the app. At the end of development, the running sum is approximately $\$ 142,740.64$, which can support an additional 5 months of development.

### 5.4.4 Proprietary Platform

The proprietary platform provides a small cost reduction coefficient, but a significant value addition coefficient due to the pervasive use of this platform. Table 5.9 provides an overview of the cost and value structure of the experiment. As with the open source platform, the first period represents the initial funding with $\$ 0$ cost. The cost reduction coefficient is applied to each of the remaining development periods, while the added value coefficient is only applied to periods with a


Figure 5.17: Open Source Platform: Sum Net Value Per Period

Table 5.9: Scenario 2 Parameters: Proprietary Platform

| Period | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 |
| 2 | 27080 | 0 | 0.1 | 0 |
| 3 | 27080 | 0 | 0.1 | 0 |
| 4 | 27080 | 0 | 0.1 | 0 |
| 5 | 27080 | 50000 | 0.1 | 1.24 |
| 6 | 27080 | 30000 | 0.1 | 1.24 |
| 7 | 27080 | 20000 | 0.1 | 1.24 |

non-zero value, namely periods after the release of the product, for the same reasons noted above.
Figure 5.18 shows the plot of development period vs. net value vs. elapsed time. The surface has a similar shape as the previous two experiments. The valleys between initial funding and the release of the app are deeper than the open source platform due to the smaller cost reduction coefficient; however, the peaks in net value after the release of the app are higher than both the native web app platform and the open source platform due to the large value addition coefficient.

Figure 5.19 shows the plot of development period vs. sum net value vs. elapsed time. While the shape of the surface is similar to both the native web platform and the open source platform, there are some notable differences. First, the valley in the running net value, roughly $\$ 88,396.34$, is


Figure 5.18: Proprietary Platform: Net Value Per Period
deeper than that of the open source platform due to the smaller cost reduction coefficient. This valley represents a smaller amount of liquidity in funding of the project towards the middle of development. However, following the release of the app, the running net value significantly increases, peaking at roughly $\$ 241,850.59$, nearly $169 \%$ and $248 \%$ of the sum net value derived from the open source and native web platforms, respectively. With the funding acquired from the app, development can persist for 8 more months given the current staffing profile.

### 5.4.5 Open Source Before Proprietary

The next experiment investigates the viability of implementing the app on the open source platform and continuing development to implement the app on the proprietary platform following the release of the app on the open source a concatenation of the the open source experiment and proprietary experiment. We have platform. We have constructed this experiment by structuring the simulation as omitted the initial funding period in the proprietary platform. The cost reduction and value addition coefficients were applied to this experiment in the same manner as the previous experiments. Table 5.10 shows the cost and value structure for this experiment.

Figure 5.20 shows the plot of development period vs. net value vs. elapsed time. As expected, the surface shows two valleys, referring to the periods of development prior to the release


Figure 5.19: Proprietary Platform: Sum Net Value Per Period

Table 5.10: Scenario 2 Parameters: Open Source Before Proprietary

| Period | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 |
| 2 | 27080 | 0 | 0.3 | 0 |
| 3 | 27080 | 0 | 0.3 | 0 |
| 4 | 27080 | 0 | 0.3 | 0 |
| 5 | 27080 | 50000 | 0.3 | -0.04 |
| 6 | 27080 | 30000 | 0.3 | -0.04 |
| 7 | 27080 | 20000 | 0.3 | -0.04 |
| 8 | 27080 | 0 | 0.1 | 0 |
| 9 | 27080 | 0 | 0.1 | 0 |
| 10 | 27080 | 0 | 0.1 | 0 |
| 11 | 27080 | 50000 | 0.1 | 1.24 |
| 12 | 27080 | 30000 | 0.1 | 1.24 |
| 13 | 27080 | 20000 | 0.1 | 1.24 |



Figure 5.20: Open Source Before Proprietary: Net Value Per Period
of either app, and two peaks which describe the incoming revenue from the release of the app. A notable difference between this graph and the previous two is that there is a noticeable increase in the net value with respect to the elapsed time (z-axis). This increase was absent in the previous experiments due to the short duration of the projects, mitigating both the affect of the riskless discount factor and the probability variation model.

Figure 5.21 shows the plot of development period vs. sum net value vs. elapsed time. The surface slopes downward from the origin as development consumes the initial funding. In the fourth period, the surface begins sloping upward as incoming revenue is generated from the release of the app on the open source platform. The surface begins dipping again as development on the proprietary app begins.

### 5.4.6 Proprietary Before Open Source

The final experiment investigates the viability of implementing the app using the proprietary platform and continuing app development with the open source platform. The experiment was constructed using the same method as the previous experiment. Table 5.11 shows the cost and value structure for the simulation.


Figure 5.21: Open Source Before Proprietary: Sum Net Value Per Period

Table 5.11: Scenario 2 Parameters: Proprietary Before Open Source

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 |
| 2 | 27080 | 0 | 0.1 | 0 |
| 3 | 27080 | 0 | 0.1 | 0 |
| 4 | 27080 | 0 | 0.1 | 0 |
| 5 | 27080 | 50000 | 0.1 | 1.24 |
| 6 | 27080 | 30000 | 0.1 | 1.24 |
| 7 | 27080 | 20000 | 0.1 | 1.24 |
| 8 | 27080 | 0 | 0.3 | 0 |
| 9 | 27080 | 0 | 0.3 | 0 |
| 10 | 27080 | 0 | 0.3 | 0 |
| 11 | 27080 | 50000 | 0.3 | -0.04 |
| 12 | 27080 | 30000 | 0.3 | -0.04 |
| 13 | 27080 | 20000 | 0.3 | -0.04 |



Figure 5.22: Proprietary Before Open Source: Net Value Per Period

Figure 5.22 shows the plot of development period vs. net value vs. elapsed time. Similar to the previous experiment, the graph depicts two valleys, corresponding to the development periods prior to the release of the app, and two peaks, corresponding to the release of each app and the income generated by the release.

Figure 5.23 shows the plot of development period vs. sum net value vs. elapsed time. As with the previous experiment, there is an immediate decline in the running net value, reaching a valley before the release of the app developed using the proprietary platform, peaking in the sixth period. The running sum of the net value declines after the release of the app on the proprietary platform, during the development of the app on the open source platform. In the eleventh period, the value increases again with the release of the app developed for the open source platform.

### 5.4.7 Scenario 2 Analysis

The experiments produced excellent results that support decision making analysis from two perspectives:

- If we can only develop for a single platform, which platform should we use?
- Is developing for multiple platforms a viable development strategy? If so, which order of


Figure 5.23: Proprietary Before Open Source: Sum Net Value Per Period
development provides the best outcome?

We begin the analysis by investigating the first question, which outcome produces the most revenue. Each of the platforms provides trade-offs which are hard to judge at a glance. The native web app can easily reach the largest group of students for the development time committed, but provides little reduction in cost. The open source platform provides cost savings on development from its supporting materials and assets, as well as the open source nature of the platform, but the limited adoption of that platform by users suggests that fewer total downloads will occur. Finally, the proprietary platform provides some reduction in cost during development, but targets more than twice the population of the open source platform. This problem is further confounded by the difference in price between apps developed for the native web platform and the others. It may be the case that multiple platforms are viable, providing motivation for this kind of analysis.

The native web platform seems to be the worst of the three, operating at a loss in the final period of development. Cumulatively, the app is only valued at roughly $\$ 96,423.57$ before costs, suggesting a net loss after funding (i.e., the project was funded for $\$ 162,480$ and only brought in $\$ 96,423.57$ in revenue). Given that the many of the desired features could not be implemented due to lack of functionality in platform, the reduction in cost, even in spite of the largest potential
customer base, suggests that the native web platform is neither a viable nor an optimal choice for development.

The open source platform fares much better than the native web platform, operating at a loss only during the initial development, which is expected given the parameterization of the simulation. However, the profit margins during the final period of development are thin, ranging in estimation from $-\$ 46.84$ to $\$ 139.62$ in the final period of development due to the decrease in adoption. However, the project comes much closer to 'breaking even' on development costs, yielding a final revenue ranging in estimation between $\$ 139,268.67$ and $\$ 142,740.64$ before costs. While this still results in a negative value proposition after including costs, revenue is increased by nearly $50 \%$ from the native web app. Additionally, there is a higher level of liquidity from that revenue, which could be used to subsist development for up to five months if development hurdles outside the scope of this analysis occurred.

The proprietary platform seems to be the best option of the three. While the costs are slightly higher than the open source platform, the funding required to develop the app is dwarfed by the revenue derived from the release of the app. The net value for each period of development following the release of the app is positive by a five-figure margin, compared to the apps on the native web app and the open source platform which can only achieve that level of revenue during their initial release. Unlike the previous two platforms, the proprietary platforms provides a positive value proposition, yielding a final revenue ranging in estimation between $\$ 216,171.07$ and $\$ 241,850.59$. The only area in which the proprietary platform does not appear to outperform the others is in the running liquidity during development. Prior to the release of the app, the experiment where the app is developed on the open source platform spends less money per development period, yielding a large amount of funding available. However, the difference in price in each development period prior to the release of the app ranges from $\$ 5,000$ to $\$ 12,000$, suggesting this is a relatively small benefit. All of these factors provide strong support for adopting the proprietary platform for the development of the app.

The second point of analysis is a two part question. The first part is to determine whether developing for multiple platforms is a viable strategy. For this analysis, we define viable as being financially solvent, which is to say the revenue generated from the release of the app is sufficient to continue funding development for the next app. Given that the development time prior to release was set at three periods of development, and the cost of development per period was $\$ 27,080$,
the sum net value of development at any given time must be greater than $\$ 81,240$ in order to continue development. In the open-source- first experiment, funding is steady during the first nine periods: in period 9 , the sum net value derived from the development varies between $\$ 93,557.64$ and $\$ 106,203.24$. However, in the 10th period, just before the release of the proprietary app, the funding dips to between $\$ 68,684.77$ and $\$ 90,283.69$, just below the amount necessary to sustain development for the remaining 3 periods. In the face of uncertainty, it may be the case that the best option is to discontinue development on the proprietary app until further funding can be acquired. On the other hand, if revenue projections for the release of the proprietary app are accurate, funding will be available to subsist development once the app is released. However, this represents a risk, given the amount of available funding in period 10. In the event of unexpected development hurdles, the project may fail before the release of the second app, which represents the lion's share of revenue derived from the project.

In the proprietary-first experiment, we see a significantly increased amount of liquidity in the project. The lowest point in liquidity, as measured by the sum net value, is during period 4 just prior to the release of the app developed on the proprietary platform. Following period four, the running sum net value never falls below $\$ 178,000$, just prior to the release of the app built on the open source platform, which represents funding for more than 6 months of continued development time. From these two analyses, it seems that both methods are viable, in that it is expected that the revenue generated from the release of the apps will enable continued development through the lifetime of the project; however, it seems that the open-source-first strategy has an increased risk of funding short falls associated with it.

Another way to analyze these for viability is to determine the point of fiscal solvency for each project. Fiscal solvency, in this case, is defined as having enough funding, as defined by the running sum net value, in a given period to cover the outlay of development costs for the remainder of the project. To determine fiscal solvency, we have used the following equation:

$$
\begin{equation*}
\text { FundingOutlay }_{i, j}=\text { SumNetValue }_{i, j}-((\text { maxPeriod }-i) * \text { CostPerPeriod })^{\text {}} \text { ) } \tag{5.6}
\end{equation*}
$$



Figure 5.24: Open Source Before Proprietary: Fiscal Solvency Outlay
where $i$ refers to a period of development, $j$ refers to the elapsed time at that period of development, maxPeriod refers to the final period of development or the time horizon of the project, and CostPerPeriod refers to the total cost per period of development ( $\$ 27,080$ in this example). The result is a table in which each cell contains the difference between the current amount of funding and the total cost for the remainder of the project with respect to development period and elapsed time.

Figure 5.24 shows the outlay of fiscal solvency for the open-source first experiment. The x -axis represents the development period, the z -axis represents elapsed time and the y -axis is the value as computed by Equation 5.6. The open-source before proprietary development plan runs insolvent until the eleventh period, wherein it has generated enough revenue (with the release of the second app) to subsist development for the remainder of the project development plan.

Figure 5.25 shows the outlay of fiscal solvency for the proprietary first experiment. The x -axis represents the development period, the z -axis represents elapsed time and the y -axis is the value as computed by Equation 5.6. This development reaches fiscal solvency just prior to beginning the development of the open source app, in the sixth period. Following that period, the project remains fiscally solvent for the remainder of the development plan.

While both strategies are viable, the open source first plan seems significantly riskier than the proprietary first development plan. This is due solely to the fact that the liquidity and running funding of the open source first development plan is significantly smaller than the proprietary first


Figure 5.25: Proprietary Before Open Source: Fiscal Solvency Outlay
plan, often by an order of magnitude or more.

### 5.5 Summary of Results

From our experimentation, we have derived a number of generalized results. In Scenario 1, we found that the code-centric platform was the better choice of platform for the reason that the RUP model for software development emphasizes the construction phase. Given that construction phase was the longest and most cost intensive phase of development, a platform the reduced the cost or increased the value of assets derived in construction phases provided greater revenue and cost savings over platforms that emphasized savings in other phases of development. Generally, we can say that the platform chosen as the basis for a software product should complement the development process used to develop that product.

For example, in a model-driven development setting, significant effort is spent in early phase development, utilizing requirements to develop a model of the software product being built. Code is generated from the model, and, rather than revising the code, the model is revised as requirements become less uncertain and more concrete, generating revised code from the revised model. Utilizing a platform that reduces the cost or increases the value of early phase artifacts, such as requirements specification, software models or software architecture, would benefit a project using model driven development over a platform that reudces the cost or increases the value of the code written. On the other hand, Agile and iterative-incremental processes tend to eschew architectural development and
specification; utilizing an architecture-centric platform in these settings would provide cost reduction and value addition to the least utilized assets during those development processes.

In Scenario 2, we saw a far more nuanced set of results. In the first three experiments, which investigated utilizing a single platform, the cost savings of the open-source platform provided a more stable running budget, but a lower overall revenue; however, the proprietary platform had significantly increased revenue but a distinct point of low funding where the occurence of development hurdles could threaten the viability of the project. While this result in itself does not provide a clear indication of which platform to use, the results provide a different perspective on the development schedule in this project. In one case, we have a more stable, robust development schedule that is less susceptible to delays and termination as a result of lack of funding; in the other case, a significantly higher revenue stream. These reuslts would provide the product manager additional information that was not apparent in the initial consideration of platforms. The combined development schedules exhibited an even more nuanced choice. The difference between resulting profit from both opensource first and proprietary first development was not significantly large; however, the state of funding during development varied wildly. These results helped exemplify that the outcome of development was not the only point of consideration in choosing a software platform as the basis for a product; rather, a manager needs to be aware of pitfalls and points of weakness during development. With the aid of the fiscal solvency analysis, the implications of that decision are a little clearer, and can be decided with additional motivations from the project manager.

However, both of these results are ultimately less important than high level results of our simulation and modeling. While the model may not have produced firm, concrete findings on which platform to use in every case, the model and simulations provided additional information which helps aid in strategic decision making. By tailoring, extending and exploiting the flexibility of this model, we have laid a foundation for future investigations into software development under the auspices of software platform adoption, and this model will help to answer more complex, interesting and difficult questions in the future.

## Chapter 6

## Verification and Validation

In a traditional experimental setting, validation would be fairly straightforward. After conducting our experiments, the experimental results would be compared to a set of theoretical results and the percentage of error would be calculated in relation to the theoretical results. However, the work presented in this dissertation presents both a theoretical model for value analysis and derivation as well as the experimental means for providing simulations of this model. Given that both the experimental model and the theoretical process arise from the same source, namely the author, calculating the percentage of error between the experimental results and theoretical calculations is not a sound method for providing validation of the experimental results.

Another validation tactic is to compare the current experimental results to other experimental results generated by other researchers, as was done in [55, 18]. Performing this comparison provides a 'sanity' check to ensure that the results generated from experimentation are consistent with the results of other researchers. In some cases, this comparison can also illuminate scenarios wherein the experimental simulations of a model are in good agreement with each other, but not the underlying model, suggesting previously unknown flaws in the model, as was the case in [55]. However, stochastic approaches provide significant flexibility in parameterizing the model under simulation. Without a common, well-understood model that provides the foundation for experimental simulation, the results of two researchers, simulating the same phenomenon, may have subtle differences, resulting in potentially large differences in their experimental results.

Given the lack of real world data with which to compare our we results, several different methods were utilized for validating the experimental results, including sensitivity analysis,

## internal validity and extreme conditions tests.

### 6.1 Sensitivity Analysis

Sensitivity analysis consists of changing values on parameters and sources of stochasticity within the experimental simulation. Varying these parameters helps illuminate the relationship between the model and its behavior in relation the parameters of the model. If the model is overly sensitive to a particular parameter or input, additional steps should be taken to ensure the accuracy of those parameters [47]. Sensitivity was analyzed in this work as part of the experimental technique in varying the sources of stochasticity in multiple simulation runs. Utilizing multiple Monte Carlo simulation runs will ensure that results are consistent between simulations, while varying the sources of stochasticity will reveal parameters whose value may have a disproportionate effect on the simulations.

To ensure the validity of our model, we performed a number of sensitivity tests. The goal was to show that change in key parameters in the model resulted in a proportional or 'smooth' change in the results, i.e., the model was not overly sensitive to particular values for these parameters.

To ensure the stability of the model, we parameterized a default scenario, shown below in Table 6.1. In each of our tests, one parameter of the scenario was isolated, while other parameters were held constant. A number of experiments were run in which the isolated parameter was varied. Each experiment ran for 50,000 iterations. In each experiment, the final period of development across all time-points was analyzed, using the parameter varied for the sensitivity test. The data of each time-point was analyzed using linear regression analysis on both the net value as well as the sum net value for the final development period.

### 6.1.1 Cost Reduction

In order to verify that the model is stable with respect to the cost reduction coefficient, we ran experiments using the baseline scenario described in Table 6.1. To perform the sensitivity analysis, we used the default failure and carryover models; the value addition was held constant at 0.0 and the riskless discount rate held constant at 0.05 . Ten experiments were then conducted, each for 50,000 iterations, wherein the cost reduction coefficient was varied by 0.10 , from 0.0 to 0.9 , resulting in a total of 10 experiments.

Table 6.1: Baseline Scenario for Sensitivity Analysis

| Period of Completion | Cost $(\$)$ | Value $(\$)$ | Riskless Discount (\%) |
| :--- | :--- | :--- | :--- |
| 1 | 42251 | 95371.58 | 0.05 |
| 2 | 42251 | 95371.58 | 0.05 |
| 3 | 42251 | 95371.58 | 0.05 |
| 4 | 55900 | 95371.58 | 0.05 |
| 5 | 55900 | 95371.58 | 0.05 |
| 6 | 55900 | 95371.58 | 0.05 |
| 7 | 55900 | 95371.58 | 0.05 |
| 8 | 55900 | 95371.58 | 0.05 |
| 9 | 55900 | 95371.58 | 0.05 |
| 10 | 55900 | 95371.58 | 0.05 |
| 11 | 55900 | 95371.58 | 0.05 |
| 12 | 55900 | 95371.58 | 0.05 |
| 13 | 102050 | 95371.58 | 0.05 |
| 14 | 102050 | 95371.58 | 0.05 |
| 15 | 102050 | 95371.58 | 0.05 |
| 16 | 102050 | 95371.58 | 0.05 |
| 17 | 102050 | 95371.58 | 0.05 |
| 19 | 102050 | 95371.58 | 0.05 |
| 19 | 102050 | 95371.58 | 0.05 |
| 20 | 102050 | 95371.58 | 0.05 |
| 21 | 102050 | 95371.58 | 0.05 |
| 22 | 102050 | 95371.58 | 0.05 |
| 23 | 102050 | 95371.58 | 0.05 |
| 24 | 102050 | 95371.58 | 0.05 |
| 25 | 102050 | 95371.58 | 0.05 |
| 26 | 102050 | 95371.58 | 0.05 |
| 27 | 102050 | 95371.58 | 0.05 |
| 28 | 65000 | 95371.58 | 0.05 |
| 29 | 65000 | 95371.58 | 0.05 |
| 30 | 65000 | 95371.58 | 0.05 |
|  |  |  |  |



Figure 6.1: Sensitivity Analysis: Cost Reduction vs. Net Value vs. Elapsed Time

Figure 6.1 shows the surface plot of the net value of each of the 10 experiments. The x axis displays the cost reduction coefficient, while the $y$-axis is the net value for the final period of development and the z -axis describes the elapsed time. Thus, an ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) tuple represents the y -net value, parameterized with probability x as measured at time z. Figure 6.2 shows the same plot as Figure 6.1, but using the sum net value of all previous periods instead.

From these plots, it appears that both the net value and sum net value increase linearly with respect to the cost reduction coefficient. In order to verify these claims, we used linear regression to calculate the line of best fit for each series. The results for the sensitivity test against net value are shown in Table 6.2, and the results of the regression test for sum net value are shown in Table 6.3. The raw data from which these results were derived is shown in Tables 27 and 28 in Appendix E.1.1. For both tables, the regression of each series shows a strong fit ( $R^{2} \geq 0.99$ and 1.00 , respectively), suggesting that nearly $100 \%$ of the variation in the derived value of the development in each time period can be attributed to the change in the cost reduction coefficient. The low standard error on the fit suggests that there was not a significant number of outliers in the results.

Table 6.2: Cost Reduction Regression Analysis for Net Value

| Time Period | Intercept | Slope | $R^{2}$ | StdDev |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 5403.03 | 15613.46 | 1.00 | 25.88 |
| 2 | 5729.52 | 16433.30 | 1.00 | 27.07 |
| 3 | 6081.53 | 17281.33 | 1.00 | 29.08 |
| 4 | 6450.62 | 18180.74 | 1.00 | 29.45 |
| 5 | 6840.35 | 19132.57 | 1.00 | 38.45 |
| 6 | 7259.41 | 20116.57 | 1.00 | 37.03 |
| 7 | 7702.73 | 21155.07 | 1.00 | 44.45 |
| 8 | 8169.18 | 22244.74 | 1.00 | 45.68 |
| 9 | 8664.56 | 23406.62 | 1.00 | 51.52 |
| 10 | 9187.58 | 24619.24 | 1.00 | 50.13 |
| 11 | 9746.14 | 25879.06 | 1.00 | 52.13 |
| 12 | 10335.68 | 27216.51 | 1.00 | 60.83 |
| 13 | 10948.91 | 28635.65 | 1.00 | 63.15 |
| 14 | 11607.45 | 30121.98 | 1.00 | 61.85 |
| 15 | 12322.15 | 31631.01 | 1.00 | 74.79 |
| 16 | 13048.54 | 33279.95 | 1.00 | 73.25 |
| 17 | 13818.45 | 35008.30 | 1.00 | 79.16 |
| 18 | 14654.11 | 36791.56 | 1.00 | 81.92 |
| 19 | 15521.29 | 38683.48 | 1.00 | 90.37 |
| 20 | 16436.93 | 40675.45 | 1.00 | 90.40 |
| 21 | 17406.49 | 42759.34 | 1.00 | 100.44 |
| 22 | 18453.39 | 44916.08 | 1.00 | 117.00 |
| 23 | 19510.69 | 47229.45 | 1.00 | 105.06 |
| 24 | 20682.10 | 49601.19 | 1.00 | 117.80 |
| 25 | 21892.93 | 52118.66 | 1.00 | 107.87 |
| 26 | 23151.02 | 54798.33 | 1.00 | 123.45 |
| 27 | 24527.74 | 57491.24 | 1.00 | 127.12 |
| 28 | 25940.58 | 60377.85 | 1.00 | 130.73 |
| 29 | 27419.22 | 63449.21 | 1.00 | 122.69 |
| 30 | 29052.12 | 66682.84 | 1.00 | 146.43 |

Table 6.3: Cost Reduction Regression Analysis for Sum Net Value

| Time Period | Intercept | Slope | $R^{2}$ | StdDev |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 328364.88 | 1163180.88 | 1.00 | 2304.65 |
| 2 | 345700.41 | 1220542.18 | 1.00 | 2406.89 |
| 3 | 361143.54 | 1278723.04 | 1.00 | 2543.77 |
| 4 | 374734.69 | 1337552.28 | 1.00 | 2654.56 |
| 5 | 386898.91 | 1396580.05 | 1.00 | 2814.35 |
| 6 | 397666.17 | 1455628.41 | 1.00 | 2929.74 |
| 7 | 407010.10 | 1514688.04 | 1.00 | 3056.20 |
| 8 | 414845.34 | 1573808.29 | 1.00 | 3187.06 |
| 9 | 420939.66 | 1633113.93 | 1.00 | 3337.37 |
| 10 | 425402.28 | 1692397.46 | 1.00 | 3449.47 |
| 11 | 428119.26 | 1751680.53 | 1.00 | 3547.36 |
| 12 | 428932.75 | 1811045.09 | 1.00 | 3675.66 |
| 13 | 427799.56 | 1870412.36 | 1.00 | 3742.59 |
| 14 | 426951.17 | 1927478.49 | 1.00 | 3866.78 |
| 15 | 426288.11 | 1982238.05 | 1.00 | 3998.05 |
| 16 | 426033.91 | 2034315.63 | 1.00 | 4043.61 |
| 17 | 426090.63 | 2083717.92 | 1.00 | 4165.89 |
| 18 | 426523.43 | 2130215.55 | 1.00 | 4241.89 |
| 19 | 427274.22 | 2173814.30 | 1.00 | 4340.83 |
| 20 | 428370.81 | 2214272.66 | 1.00 | 4367.12 |
| 21 | 429918.66 | 2251346.42 | 1.00 | 4393.99 |
| 22 | 431843.18 | 2285037.69 | 1.00 | 4475.35 |
| 23 | 434001.69 | 2315322.13 | 1.00 | 4530.88 |
| 24 | 436842.15 | 2341408.05 | 1.00 | 4541.19 |
| 25 | 439887.38 | 2363866.27 | 1.00 | 4603.02 |
| 26 | 443579.13 | 2381858.57 | 1.00 | 4607.65 |
| 27 | 447507.43 | 2395769.95 | 1.00 | 4648.20 |
| 28 | 452083.68 | 2404863.64 | 1.00 | 4656.04 |
| 29 | 455020.14 | 2411399.54 | 1.00 | 4705.15 |
| 30 | 456777.06 | 2414456.29 | 1.00 | 4677.75 |



Figure 6.2: Sensitivity Analysis: Cost Reduction vs. Sum Net Value vs. Elapsed Time

### 6.2 Value Addition

To verify the sensitivity of the value addition coefficient, the same experiment, described in the previous section, was used with the baseline scenario described in Table 6.1. A total of 10 experiments were run, varying the value addition coefficient 0.1 in each simulation, ranging between 0.0 and 0.9. The cost reduction coefficient was held constant at 0.0 , and the riskless discount rate was held constant at 0.05 for each simulation. Each simulation was run for 50,000 iterations, much as the previous sensitivity experiment had been run.

As with the previous experiment, we investigated the net value and sum net value of the last period of development across all time periods. Figures 6.3 shows the plot of value addition coefficient vs. net value vs. elapsed time. Again, the ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) tuple refers to the y -value derived with $x$-value addition coefficient calculated at time-point $z$. The plot suggests that the net value derived from a single period of development at a given time-point increases linearly with the increase in the cost addition coefficient.

Figure 6.4 shows the plot of sum net value vs cost addition coefficient for each of the 10 experiments we ran. As with Figure 6.1, each series represents the sum net value, i.e., the sum of the net value for all preceding periods, of the 30th period of development at a given time-point. Again, the plot suggests that the sum net value derived from development at a given time-point increases


Figure 6.3: Sensitivity Analysis: Value Addition vs. Net Value vs. Elapsed Time
linearly with the increase in the cost addition coefficient.
In order to better verify the linear relationship between the value addition coefficient and net value and sum net value, regression analysis was performed on the data. The results are shown in Tables 6.4 and 6.5. As with regression analysis performed in Section 6.1.1, the results were excellent, with a high $R^{2}$ value ( $R^{2} \geq 0.99$ and 1.0), suggesting that nearly $100 \%$ of the variation in the earned value from the period of development can be attributed to the change in cost addition coefficient. The relatively low standard error shows a minimal variation in these results. Again, the raw data from which the regressions were calculated can be found in Tables 29 and 30 in Appendix E.1.2.

### 6.2.1 Probability of Successful Implementation

In order to test the sensitivity of the probability of successful implementation, we again ran a series of experiments, this time varying the probability of successful implementation stochastically. We used the same scenario shown in Table 6.1, but ran two sets of experiments: one utilizing the stratified failure model, described in Section 5.2, Table 5.1 and another wherein a binary failure model was used. In the binary failure scenario, failure of successful implementation results in a $100 \%$ loss of revenue from a period of development, with a $100 \%$ cost and value carry-over into the next period. In both cases, 11 experiments were run, varying the probability of successful implementation between 0.0 and 1.00, at 50,000 iterations each run. In both experiments, we locked

Table 6.4: Value Addition Regression Analysis for Net Value

| Time Period | Intercept | Slope | $R^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 5306.05 | 21752.38 | 1.00 | 37.66 |
| 2 | 5631.28 | 22956.21 | 1.00 | 38.66 |
| 3 | 5972.58 | 24219.55 | 1.00 | 44.97 |
| 4 | 6329.19 | 25566.46 | 1.00 | 54.46 |
| 5 | 6719.63 | 26963.37 | 1.00 | 50.15 |
| 6 | 7128.31 | 28444.43 | 1.00 | 52.85 |
| 7 | 7557.57 | 30035.25 | 1.00 | 56.44 |
| 8 | 8014.50 | 31697.87 | 1.00 | 66.34 |
| 9 | 8498.43 | 33409.36 | 1.00 | 66.94 |
| 10 | 9006.45 | 35256.98 | 1.00 | 67.92 |
| 11 | 9548.10 | 37209.70 | 1.00 | 76.92 |
| 12 | 10111.65 | 39269.70 | 1.00 | 86.61 |
| 13 | 10722.22 | 41392.83 | 1.00 | 97.77 |
| 14 | 11379.68 | 43640.25 | 1.00 | 87.50 |
| 15 | 12043.12 | 46030.97 | 1.00 | 100.30 |
| 16 | 12751.25 | 48579.05 | 1.00 | 123.02 |
| 17 | 13534.43 | 51162.73 | 1.00 | 119.63 |
| 18 | 14325.49 | 53960.31 | 1.00 | 108.36 |
| 19 | 15202.66 | 56810.74 | 1.00 | 133.40 |
| 20 | 16099.28 | 59888.18 | 1.00 | 129.33 |
| 21 | 17042.79 | 63147.07 | 1.00 | 148.39 |
| 22 | 18081.59 | 66425.00 | 1.00 | 148.38 |
| 23 | 19143.35 | 69919.47 | 1.00 | 144.65 |
| 24 | 20236.95 | 73733.52 | 1.00 | 178.75 |
| 25 | 21476.31 | 77518.79 | 1.00 | 171.80 |
| 26 | 22717.17 | 81549.18 | 1.00 | 188.72 |
| 27 | 24045.52 | 85798.60 | 1.00 | 175.43 |
| 28 | 25489.73 | 90123.15 | 1.00 | 208.44 |
| 29 | 26951.67 | 94772.97 | 1.00 | 206.85 |
| 30 | 28536.98 | 99833.25 | 1.00 | 189.39 |

Table 6.5: Value Addition Regression Analysis for Sum Net Value

| Time Period | Intercept | Slope | $R^{2}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 320491.67 | 1552800.70 | 1.00 | 3050.80 |
| 2 | 337403.75 | 1630698.12 | 1.00 | 3214.02 |
| 3 | 352392.60 | 1707527.31 | 1.00 | 3371.65 |
| 4 | 365514.07 | 1782999.65 | 1.00 | 3517.21 |
| 5 | 377296.18 | 1857098.38 | 1.00 | 3647.41 |
| 6 | 387717.52 | 1929894.02 | 1.00 | 3816.05 |
| 7 | 396653.47 | 2001118.61 | 1.00 | 3927.00 |
| 8 | 404064.66 | 2070804.82 | 1.00 | 4050.43 |
| 9 | 409804.69 | 2138979.44 | 1.00 | 4208.09 |
| 10 | 414013.51 | 2205148.53 | 1.00 | 4346.32 |
| 11 | 416338.77 | 2269562.82 | 1.00 | 4396.48 |
| 12 | 416754.81 | 2332305.10 | 1.00 | 4609.57 |
| 13 | 415290.57 | 2392656.35 | 1.00 | 4665.83 |
| 14 | 414069.10 | 2451037.58 | 1.00 | 4783.23 |
| 15 | 413202.44 | 2507140.49 | 1.00 | 4844.23 |
| 16 | 412594.72 | 2560928.00 | 1.00 | 5015.89 |
| 17 | 412422.17 | 2612121.32 | 1.00 | 5046.33 |
| 18 | 412556.65 | 2660822.60 | 1.00 | 5164.73 |
| 19 | 413114.49 | 2706599.10 | 1.00 | 5171.20 |
| 20 | 414106.71 | 2749456.08 | 1.00 | 5343.89 |
| 21 | 415358.44 | 2789420.41 | 1.00 | 5380.57 |
| 22 | 417041.82 | 2826178.67 | 1.00 | 5411.39 |
| 23 | 419219.81 | 2859478.27 | 1.00 | 5435.37 |
| 24 | 421801.04 | 2889338.06 | 1.00 | 5482.78 |
| 25 | 424895.61 | 2915439.40 | 1.00 | 5519.21 |
| 26 | 428366.63 | 2937718.74 | 1.00 | 5566.61 |
| 27 | 432381.25 | 2955846.23 | 1.00 | 5553.06 |
| 28 | 436864.92 | 2969929.68 | 1.00 | 5560.64 |
| 29 | 439715.75 | 2979950.74 | 1.00 | 5563.33 |
| 30 | 4414477.77 | 2984762.58 | 1.00 | 5592.90 |



Figure 6.4: Sensitivity Analysis: Value Addition vs. Sum Net Value vs. Elapsed Time
the probability variability model, which varies the probability of successful implementation based on the difference between the elapsed time and the completion of a period of development, with the delta value 0.0 , i.e., the probability of successful implementation was not modified with respect to time. We analyzed the net value and sum net value for the final period of development and performed regression analysis on the results.

## Stratified Failure Model

When analyzing the probability for successful implementation, we utilized the net value and sum net value aggregate statistics to insure that the profit derived from a period of development was not overly sensitive to the probability term. Figure 6.5 shows the net value in the final period of development, across all time periods, with varying probability. The x-axis represents the varied probability, while the y -axis shows the net value for the final period of development and the z -axis shows the elapsed time (i.e., the time-point at which the value of the final period of development was computed). The surface shows what appears to be a linear slope with respect to value, the $y$-axis. In order to confirm this slope, we performed regression analysis on each of the 30 series that compose the surface graph. The results are shown in Table 6.6. With high $R^{2}$ values ( $R^{2} \geq 0.9999$ ), the results were quite clear, showing that nearly $100 \%$ of the variance in the net value can be attributed to the changing probability. Additionally, the low standard error on the regression suggests that there were few, if any, outliers.


Figure 6.5: Probability vs. Net Value vs. Elapsed Time with Stratified Failure

Figure 6.6 shows the sum net value, i.e., the sum value of each development previous, in the final period of development. As with the previous figure, the x -axis refers to the probability, the y -axis is the sum net value of all periods of development, and the z -axis refers to the elapsed time. Again, we see what is a surface with a linear slope with respect to sum netvalue, the y-axis. We performed regression analysis, with the results shown in Table 6.7. Each of the series that compose the surface was fit to a linear regression, with excellent results. With $R^{2} \geq 0.9999$, we can surmise that nearly $100 \%$ of the variance in the sum net value can be attributed to the varying probability. Again, the low standard error on the fit suggests that the data is relatively devoid of outliers.

It is important to point out that in the case where there is a low or even $0 \%$ probability of successful implementation, development still yields a profit. This profitability is a result of the stratified failure model that was utilized in this simulation, which penalizes a minimum of $25 \%$ earned value and a maximum of $90 \%$ earned value in the event of a failure. Even in the worst case scenario, development can still yield small returns on investment. The raw data from which the regressions were calculated can be found in Tables 31 and 32 in Appendix E.1.3.

## Binary Failure Model

For the sake of completeness, we performed a second set of experiments wherein we modified the failure model. In the previous experiment, we utilized a stratified failure model. This probabilistic model contained failures of varying severity, based on a likelihood of occurring in the event

Table 6.6: Probability Regression Analysis for Net Value with Stratified Failure Model

| Time Period | Intercept | Slope | RSQ | StdErr |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 4899.60 | 2227.79 | 1.00 | 3.72 |
| 2 | 5159.62 | 2330.08 | 1.00 | 3.51 |
| 3 | 5429.80 | 2444.43 | 1.00 | 2.69 |
| 4 | 5724.43 | 2551.19 | 1.00 | 3.49 |
| 5 | 6020.96 | 2681.13 | 1.00 | 8.10 |
| 6 | 6342.32 | 2807.60 | 1.00 | 5.31 |
| 7 | 6678.90 | 2938.77 | 1.00 | 9.78 |
| 8 | 7035.85 | 3075.80 | 1.00 | 5.20 |
| 9 | 7409.84 | 3221.09 | 1.00 | 7.20 |
| 10 | 7805.33 | 3373.51 | 1.00 | 6.44 |
| 11 | 8221.94 | 3525.12 | 1.00 | 5.64 |
| 12 | 8666.48 | 3686.57 | 1.00 | 8.48 |
| 13 | 9128.37 | 3848.95 | 1.00 | 5.08 |
| 14 | 9621.70 | 4023.77 | 1.00 | 6.47 |
| 15 | 10150.63 | 4196.41 | 1.00 | 7.52 |
| 16 | 10692.49 | 4391.23 | 1.00 | 9.75 |
| 17 | 11273.24 | 4582.85 | 1.00 | 10.05 |
| 18 | 11885.26 | 4780.43 | 1.00 | 7.47 |
| 19 | 12549.13 | 4973.45 | 1.00 | 16.61 |
| 20 | 13234.58 | 5186.97 | 1.00 | 11.43 |
| 21 | 13962.27 | 5405.72 | 1.00 | 11.38 |
| 22 | 14734.16 | 5631.84 | 1.00 | 5.84 |
| 23 | 15554.41 | 5850.87 | 1.00 | 10.71 |
| 24 | 16427.18 | 6076.52 | 1.00 | 9.43 |
| 25 | 17336.28 | 6312.19 | 1.00 | 11.65 |
| 26 | 18321.85 | 6551.89 | 1.00 | 16.30 |
| 27 | 19359.73 | 6796.80 | 1.00 | 29.31 |
| 28 | 20462.59 | 7019.54 | 1.00 | 23.26 |
| 29 | 21601.62 | 7296.89 | 1.00 | 16.54 |
| 30 | 22710.71 | 7685.68 | 1.00 | 22.20 |

Table 6.7: Probability Regression Analysis for Sum Net Value with Stratified Failure Model

| Time Period | Intercept | Slope | RSQ | StdErr |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 250980.65 | 135975.88 | 1.00 | 35.25 |
| 2 | 262721.74 | 141407.22 | 1.00 | 51.77 |
| 3 | 272550.43 | 146822.00 | 1.00 | 36.72 |
| 4 | 280244.35 | 152460.66 | 1.00 | 41.94 |
| 5 | 286373.72 | 158335.29 | 1.00 | 54.37 |
| 6 | 291045.47 | 164228.98 | 1.00 | 41.86 |
| 7 | 294152.21 | 170232.73 | 1.00 | 75.47 |
| 8 | 295540.04 | 176365.30 | 1.00 | 38.47 |
| 9 | 295212.93 | 182616.04 | 1.00 | 40.61 |
| 10 | 292984.06 | 189083.91 | 1.00 | 78.79 |
| 11 | 288743.51 | 195591.96 | 1.00 | 73.55 |
| 12 | 282392.58 | 202419.00 | 1.00 | 67.56 |
| 13 | 273749.27 | 209619.93 | 1.00 | 55.52 |
| 14 | 265306.23 | 216682.53 | 1.00 | 86.14 |
| 15 | 257462.74 | 223665.83 | 1.00 | 88.79 |
| 16 | 250152.95 | 230193.50 | 1.00 | 114.78 |
| 17 | 243533.78 | 236434.91 | 1.00 | 104.30 |
| 18 | 237470.90 | 242501.65 | 1.00 | 109.97 |
| 19 | 232190.66 | 247965.21 | 1.00 | 92.85 |
| 20 | 227563.29 | 253312.35 | 1.00 | 68.18 |
| 21 | 223669.21 | 258156.31 | 1.00 | 169.48 |
| 22 | 220840.58 | 262342.31 | 1.00 | 111.08 |
| 23 | 218729.36 | 266267.90 | 1.00 | 180.21 |
| 24 | 217464.06 | 269852.54 | 1.00 | 89.93 |
| 25 | 217284.32 | 272749.13 | 1.00 | 113.48 |
| 26 | 218156.12 | 275112.99 | 1.00 | 117.26 |
| 27 | 220104.27 | 276885.01 | 1.00 | 118.10 |
| 28 | 223148.20 | 277991.76 | 1.00 | 119.46 |
| 29 | 225462.18 | 278641.01 | 1.00 | 106.19 |
| 30 | 226503.40 | 279168.25 | 1.00 | 118.32 |



Figure 6.6: Probability vs. Sum Net Value vs. Elapsed Time with Stratified Failure
of a failure. In order to ensure that the previous results were not a result of the failure model, we utilized a different failure model in the second experiment, a binary failure model. In this model, any failure is considered a complete failure, which results in 0 value derived from that period of development; subsequently, $100 \%$ of the value and the cost is carried over into the following period.

Figure 6.7 shows the plot of probability vs. net value vs. elapsed time. Immediately noticeable, the net value for the final period of development plummets nearly $\$-2,500,000$ at $0 \%$ chance of successful implementation. This loss of value is expected, given our binary failure model: with $0 \%$ chance of successful implementation, none of the periods of development will complete successfully, and each period's full cost (and earned value) will carry over to the next period. In the final period, the net value is the summation of the cost of each preceding period. Thus, it is expected that as the probability of successful implementation approaches 0 , the net value of the final period of development approaches the negative sum of the cost of each preceding period. The sensitivity of the model to extreme changes in the probability of successful implementation is expected and reflective of the real world: in a situation where successful implementation is not possible, not only do we continue to incur costs (rather than reap profits), but also our costs increase as we try to get more and more work done within a single period. In order to insure that there were not any more subtle sensitivity points, regression analysis was performed on the data, shown in Table 6.8. As with the previous experiments, the was data fit to a linear regression, with excellent results $R^{2} \geq 0.9999$;


Figure 6.7: Probability vs. Net Value vs. Elapsed Time with Binary Failure
however, due to the rapid and extreme results observed as the probability approaches 0 , we see an increased standard error in our linear regression when compared to the previous experiments. However, the standard error is still small in relation to the data which defines it, suggesting that while there are outliers (experiments wherein probability was very low), there were not an overly significant number of outliers.

Figure 6.9 shows the plot of probability vs. sum net value vs. elapsed time. As with the previous figure, the x -axis refers to the probability of successful implementation, the y -value displays value and the $z$-axis shows elapsed time. Again, as the probability approaches zero, we see significant negative sum net value. For the same reasons noted above, the correlation between probability of successful implementation and sum net value is expected. However, the reader may notice that the sum net value for the final period of development is significantly smaller than the net value for the final period of development. This difference is due to the fact that in the event of a failed period, we have incurred $100 \%$ of the cost of that period in addition to carrying over some percent of the cost into the next period (in this case, $100 \%$ ). Given a 0 or low probability of successful implementation and a binary failure model, the net value of the $i$ th period of development can be approximated by
the following summation:

$$
\begin{equation*}
\text { NetValue }_{i}=\sum_{i}^{T *} C_{i} \tag{6.1}
\end{equation*}
$$

Where $\mathrm{T}^{*}$ is the time horizon of computation. However, the the sum net value of the $i$ th period of development is approximated by the following summation:

$$
\begin{equation*}
\text { SumNetValue }_{i}=\sum_{i}^{T *} \sum_{j=0}^{i} C_{j} \tag{6.2}
\end{equation*}
$$

Where $C_{j}$ is the cost of development in period $j$.
Additionally, we performed linear regression on the data. The results were good, with $R^{2} \geq 0.9999$, meaning the vast majority of variance in value was a result of changes to the probability of successful implementation. As with the net value, the standard error of regression was higher due to the outlying nature of the results at extremely low probabilities. But these values are still small when compared to the raw data. The raw data from these tests can be found in Tables 33 and 34 in Appendix E.1.3.

### 6.3 Internal Validity

Internal validity refers to the consistency of a stochastic model under simulation. In stochastic and probabilistic simulations, the results from two separate simulation runs may vary due to the random number generator used or other factors such as the operating system, compiler or simulation computer's hardware. If the variance between the results is too high, the efficacy and validity of both the model and the results of experimentation are questionable. Internal validity can be more accurately measured through multiple simulation runs in order to ensure that variation in experimental results is a consequence of known systemic issues (e.g., floating point error propagation, choice of compiler, random number generator) rather than flaws and faults within the model.

In order to ensure internal validity, we performed a test comparing the difference between two

Table 6.8: Probability Regression Analysis for Sum Net Value with Binary Failure Model

| Time Period | Intercept | Slope | RSQ | StdErr |
| :--- | :--- | :--- | :--- | :--- |
| 1 | -20607.01 | 27720.90 | 1.00 | 49.72 |
| 2 | -22087.01 | 29568.30 | 1.00 | 50.47 |
| 3 | -23699.02 | 31539.46 | 1.00 | 48.17 |
| 4 | -25522.50 | 33781.88 | 1.00 | 56.46 |
| 5 | -27507.90 | 36198.34 | 1.00 | 64.25 |
| 6 | -29727.60 | 38844.37 | 1.00 | 44.89 |
| 7 | -32264.43 | 41847.51 | 1.00 | 96.27 |
| 8 | -35006.35 | 45045.08 | 1.00 | 53.27 |
| 9 | -38241.27 | 48792.01 | 1.00 | 123.02 |
| 10 | -41896.06 | 53029.58 | 1.00 | 105.85 |
| 11 | -46115.11 | 57745.03 | 1.00 | 105.60 |
| 12 | -51262.45 | 63498.48 | 1.00 | 144.32 |
| 13 | -57396.07 | 70188.93 | 1.00 | 198.41 |
| 14 | -65219.48 | 78544.49 | 1.00 | 268.81 |
| 15 | -75394.56 | 89504.07 | 1.00 | 303.50 |
| 16 | -89425.36 | 104030.60 | 1.00 | 450.98 |
| 17 | -109465.03 | 124566.05 | 1.00 | 588.60 |
| 18 | -138885.00 | 154411.51 | 1.00 | 758.59 |
| 19 | -183083.96 | 199229.27 | 1.00 | 940.14 |
| 20 | -248559.39 | 265137.39 | 1.00 | 1285.22 |
| 21 | -343265.11 | 360316.89 | 1.00 | 1447.35 |
| 22 | -476887.09 | 493890.83 | 1.00 | 2439.10 |
| 23 | -655928.99 | 673194.54 | 1.00 | 2449.75 |
| 24 | -883225.80 | 901249.80 | 1.00 | 2491.92 |
| 25 | -1152860.80 | 1170576.83 | 1.00 | 3469.86 |
| 26 | -1454582.98 | 1473801.73 | 1.00 | 3676.61 |
| 27 | -1759731.18 | 1780207.09 | 1.00 | 3710.15 |
| 28 | -2033169.13 | 2053189.88 | 1.00 | 3455.36 |
| 29 | -2240422.25 | 2259042.40 | 1.00 | 5249.73 |
| 30 | -2361411.22 | 2383047.37 | 1.00 | 5678.15 |

Table 6.9: Probability Regression Analysis for Sum Net Value with Binary Failure Model

| Time Period | Intercept | Slope | RSQ | StdErr |
| :--- | :--- | :--- | :--- | :--- |
| 1 | -2658727.55 | 3058103.19 | 1.00 | 8178.32 |
| 2 | -3009745.67 | 3426305.10 | 1.00 | 9624.39 |
| 3 | -3418805.72 | 3853396.37 | 1.00 | 12219.50 |
| 4 | -3891368.42 | 4340649.75 | 1.00 | 12507.52 |
| 5 | -4419509.14 | 4885489.24 | 1.00 | 15630.53 |
| 6 | -5010046.75 | 5492259.51 | 1.00 | 18567.78 |
| 7 | -5662510.67 | 6158190.17 | 1.00 | 24382.97 |
| 8 | -6390063.07 | 6893302.39 | 1.00 | 24234.94 |
| 9 | -7174727.41 | 7690096.59 | 1.00 | 28114.96 |
| 10 | -8036416.13 | 8557528.25 | 1.00 | 31699.72 |
| 11 | -8982527.69 | 9510626.61 | 1.00 | 30918.24 |
| 12 | -10002306.20 | 10537238.34 | 1.00 | 37340.00 |
| 13 | -11122349.17 | 11670437.60 | 1.00 | 44196.62 |
| 14 | -12341669.19 | 12891193.08 | 1.00 | 44023.06 |
| 15 | -13636183.40 | 14180577.06 | 1.00 | 49150.15 |
| 16 | -15038938.23 | 15586953.53 | 1.00 | 50134.59 |
| 17 | -16533776.13 | 17085987.71 | 1.00 | 46678.98 |
| 18 | -18089119.55 | 18635128.06 | 1.00 | 50766.44 |
| 19 | -19712557.00 | 20266736.81 | 1.00 | 55261.43 |
| 20 | -21404602.43 | 21959151.02 | 1.00 | 55645.38 |
| 21 | -23079810.91 | 23619643.96 | 1.00 | 63044.19 |
| 22 | -24795459.22 | 25340345.98 | 1.00 | 43339.58 |
| 23 | -26444101.94 | 26989202.55 | 1.00 | 59106.13 |
| 24 | -27972285.54 | 28521672.92 | 1.00 | 54092.86 |
| 25 | -29313006.68 | 29846233.90 | 1.00 | 64074.70 |
| 26 | -30486692.31 | 31047745.96 | 1.00 | 52456.70 |
| 27 | -31366986.96 | 31918419.63 | 1.00 | 59905.21 |
| 28 | -31940459.59 | 32484572.01 | 1.00 | 70479.49 |
| 29 | -32230317.47 | 32763179.61 | 1.00 | 73434.12 |
| 30 | -32403655.32 | 32965655.77 | 1.00 | 58184.40 |



Figure 6.8: Probability vs. Sum Net Value vs. Elapsed Time with Binary Failure
separate simulation runs of the same scenario. For this test, we utilized the base scenario described in Table 6.1. The cost reduction coefficient and value addition coefficients were held constant at $0 \%$, as well as the riskless discount rate at $5 \%$. The scenario utilized the stratified failure model, and a linearly declining probability of successful implementation with a base value of 0.8 and a delta value of 0.03 per period forecast into the future. The simulations ran for 50,000 iterations per period.

The strategy for determining whether or not our model and simulation had internal validity was to ensure that the difference between two simulation runs were not significantly large, i.e., the difference between results fell within a single standard deviation of both simulation results. After generating the simulation results, the difference between net value per period of the two results was calculated. The absolute value of the difference was then taken to simplify the analysis, such that:

$$
\begin{equation*}
\text { NetValueDifference }_{i, j}=\mid \text { NetV alue }_{i, j}^{1}-\text { NetValue }_{i, j}^{2} \mid \tag{6.3}
\end{equation*}
$$

where NetValue $_{i, j}^{1}$ refers to the net value of the $i t h$ development period, after $j$ th elapsed period of time in the first simulation, and $N e t V a l u e_{i, j}^{2}$ refers to the net value of the $i t h$ development period after $j$ th elapsed period of time in the second simulation. Following that, the difference between the two results result was compared to the standard deviation of the respective period in


Figure 6.9: Internal Validity: Development Period vs. Net Value Difference vs. Elapsed Time
each simulation. An indicator function was used to flag the difference as being 'good', i.e., less than the standard deviation of the respective periods in both simulations, or 'bad', i.e., greater than the standard deviation of the corresponding period in one or both of the simulations, shown below:

$$
\text { InternalValidity }(i, j)=\left\{\begin{array}{l}
\text { Good if NetValueDifference }_{i, j}<\text { StdDev }_{i, j}^{1}  \tag{6.4}\\
\quad \text { andNetValueDifference } \\
i, j
\end{array}<\text { StdDev }_{i, j}^{2} .\right.
$$

The results of this test indicated that the net value difference for each period being less than the standard deviation of its respective periods in both simulations. However, this method of analysis does not provide an indication of the magnitude of the results. In order to visualize this, Figure 6.9 shows a three-dimensional plot of net value difference vs. development period vs. elapsed time. For each period, the net value difference ranges between $\$ 0$ and $\$ 100$, while the net value for each simulation are on the order of $\$ 10,000$.

When running the internal validity tests, it was not necessary to run multiple tests wherein the sources of stochasticity were varied. This decision is supported by the fact that the previous sensitivity tests, covered in Section 6.1, have shown that the results scale linearly and are not overly
sensitive to variations in the parameters. It follows logically then that using the same method to determine internal validity would produce similar results: if the results of both simulations scale linearly, then the difference between net value in two periods would be roughly the same or scale linearly with the variation in the parameters, as would the standard deviation. The result is that the difference would still be less than the standard deviation of the respective periods, or rather, fall within one standard deviation of the difference between the two periods. The raw data for these tests can be found in Tables 35, 36, 37, 38 and 39 in Appendix E. 2

### 6.4 Extreme Conditions Validity

Extreme conditions tests ensure that the model's output is valid even under extreme or unrealistic conditions and settings. Many of the sensitivity tests also functioned as extreme value tests. For instance, in testing the sensitivity of the cost reduction coefficient, we examined values ranging from $0 \%$ to $90 \%$; while $90 \%$ is reasonable from a mechanical standpoint, $90 \%$ or more cost reduction as a result of utilizing a software platform is not a realistic value. The sensitivity tests have shown that the model and simulation will continue to scale beyond $90 \%$; however, a scenario in which a software development platform provides a $100 \%$ or greater cost reduction is a scenario in which the development organization is still expending effort in development, and ergo money, as well as generating revenue, greater than or equal to $100 \%$ of the cost of development. This scenario represents more an exercise in accounting than in the benefits of platform adoptions.

While we examined highly unrealistic cost reduction coefficients, we also examined the added value coefficient, scaling from $0 \%$ to $90 \%$. The results of the sensitivity tests showed that the net value and sum net value for development scale well and produce results that are reflective of the parameters in those tests.

In our sensitivity tests, we examined the probabiilty of successful implementation. In Section 6.2.1, we scaled the probability of successful implementation from $0 \%$ to $100 \%$ using binary failure model. Both are unrealistic in nature: a $100 \%$ chance of failure represents something impossible to achieve. While it may be possible that the amount of work projected for a given development period is unrealistic, a $0 \%$ chance of success is similarly unrealistic. At the same time, the failure model wherein failure means no meaningful output was produced, yet $100 \%$ of the cost was expended, is hardly realistic. However, even under these draconian and unrealistic settings, the model
produced valid results that could be easily explained.
In order to provide a more complete set of extreme value tests beyond the overlapping sensitivity tests, we examined the riskless discount rate. While the Black-Scholes formulation is already validated for the riskless discount rate, it is important to ensure the validity of not only our model, but also the simulation implementing the model. For the purposes of this section, we describe the $r$ as the riskless discount rate, as was described in Section 2.5. The riskless discount factor refers to the multiplicative factor $e^{-r(T-t)}$, also described in Section 2.5.

In performing extreme value verification, we ran three tests, varying the riskless discount rate. We utilized our basic verification and validation scenario described in Table 6.1. In the scenarios, the riskless discount rate was parameterized at $0.05,0.5$ and 1.0. Recalling that the riskless discount is an approximation of the rate of inflation on currency, or rather, the expected return from a riskless investment, usually based on the interest rate of government backed securities and bonds, $r=0.05$ represents a realistic value. Considering that the highest historical interest rates in the United States peaked in the early 1980s at $16 \%-19 \%$, riskless discount rates greater than 0.1 represent an economy in turmoil [46]. As the riskless discount rate grows, the riskless discount factor, which we multiplicatively apply to both cost and value, quickly approaches zero, signifying that the future value of any investment quickly heads towards zero as well. This behavior is demonstrated in Table 6.10 which shows the riskless discount coefficient for varying riskless discount rates and values of $T-t$.

|  | Time Difference ( $\mathrm{T}-\mathrm{t}$ ) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | 0.05 | 1.0000 | 0.9512 | 0.9048 | 0.8607 | 0.8187 | 0.7788 | 0.7408 | 0.7047 | 0.6703 | 0.6376 | 0.6065 |
| $\underset{0}{E}$ | 0.1 | 1.0000 | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |
| $\underset{\sim}{2}$ | 0.2 | 1.0000 | 0.8187 | 0.6703 | 0.5488 | 0.4493 | 0.3679 | 0.3012 | 0.2466 | 0.2019 | 0.1653 | 0.1353 |
| 路 | 0.3 | 1.0000 | 0.7408 | 0.5488 | 0.4066 | 0.3012 | 0.2231 | 0.1653 | 0.1225 | 0.0907 | 0.0672 | 0.0498 |
| $\stackrel{a n}{2}$ | 0.4 | 1.0000 | 0.6703 | 0.4493 | 0.3012 | 0.2019 | 0.1353 | 0.0907 | 0.0608 | 0.0408 | 0.0273 | 0.0183 |
| $\begin{aligned} & \stackrel{y}{0} \\ & \stackrel{y}{y} \end{aligned}$ | 0.5 | 1.0000 | 0.6065 | 0.3679 | 0.2231 | 0.1353 | 0.0821 | 0.0498 | 0.0302 | 0.0183 | 0.0111 | 0.0067 |
|  | 0.6 | 1.0000 | 0.5488 | 0.3012 | 0.1653 | 0.0907 | 0.0498 | 0.0273 | 0.0150 | 0.0082 | 0.0045 | 0.0025 |
|  | 0.7 | 1.0000 | 0.4966 | 0.2466 | 0.1225 | 0.0608 | 0.0302 | 0.0150 | 0.0074 | 0.0037 | 0.0018 | 0.0009 |
|  | 0.8 | 1.0000 | 0.4493 | 0.2019 | 0.0907 | 0.0408 | 0.0183 | 0.0082 | 0.0037 | 0.0017 | 0.0007 | 0.0003 |
|  | 0.9 | 1.0000 | 0.4066 | 0.1653 | 0.0672 | 0.0273 | 0.0111 | 0.0045 | 0.0018 | 0.0007 | 0.0003 | 0.0001 |
|  | 1.0 | 1.0000 | 0.3679 | 0.1353 | 0.0498 | 0.0183 | 0.0067 | 0.0025 | 0.0009 | 0.0003 | 0.0001 | 0.0000 |

Table 6.10: Riskless Discount Coefficient for T-t $=[0 \ldots 10]$


Figure 6.10: Development Period vs. Period Net Value vs. Elapsed Time with r $=0.05$

Figure 6.10 shows the graph of period net value for a riskless discount rate of 0.05 . The x -axis represents the development period, while the y -axis is the net value and the z -axis measures the elapsed time. There are four distinct net-value regions, referring to the four major phases of software development in this scenarios (inception, elaboration, construction and transition). The net-value for each period within a phase increases with respect to the z -axis. This is a result of the diminished impact of the riskless discount factor as more time has elapsed (i.e., the speculative value of a period of development is discounted less as the time between point of measurement and when the period of development occurs decreases).

Figure 6.11 shows the same surface plot, but for the scenario in which the riskless discount rate was set to 0.50 . While the surfaces have largely the same shape, the biggest difference is the value of periods where little time has elapsed. Because of the significant riskless discount rate, the net value of periods of development happening even a short time in the future rapidly approach zero. Referring back to Table 6.10, we see that the riskless discount factor for a period of development occurring five periods in the future is 0.0067 , which is rapidly approaching virtual zero.

Finally, Figure 6.12 describes the same surface in the scenario with a 1.0 riskless discount rate. The results are largely the same as those in the previous scenario, but more pronounced, due to the increased riskless discount rate. This difference is especially apparent when comparing the white-space in the bottom left quadrant of each plot. The raw data for the extreme value tests can


Figure 6.11: Development Period vs. Period Net Value vs. Elapsed Time with r $=0.5$


Figure 6.12: Development Period vs. Period Net Value vs. Elapsed Time with r $=1.0$
be found in Tables 40, 41 and 42 in Appendix E.3.

## Chapter 7

## Related Work

This research blends together theory from two different domains, drawing its theoretical foundations from software ecosystems, while utilizing real options valuation and stochastic methods, namely Monte Carlo simulation, as its experimental method. Within the realm of software ecosystems, very little work has been done on determining the value of software platforms, and even less in performing quantified analysis.

In [10], Bosch notes several benefits from engaging in a software ecosystem strategy, including "Increase[d] value of the core offering to existing users," "increase[d] attractiveness for new users," "increase[d] 'stickiness' of the application platform," "share[d] costs of innovation" among others; however, these qualities of the ecosystem strategy are only realized from the perspective of the keystone organization or platform owner. In [19], the authors provide a comparison between Linux and Windows as software platforms in the form of a case study, as well as providing an extension to the model proposed in [20] that examines the decision between utilizing a proprietary platform or an open- source platform. The results of both papers suggest different strategies for application development, primarily contingent on the reputation effects from the platform that is adopted. Furthermore, they do not include experimentation or simulation into their methods.

The work presented in [14] examines how partnership within a platform ecosystem creates value among partners, particularly independent software vendors (ISVs). The authors performed analysis on ISVs between the years of 1996 and 2004, finding that becoming a partner in a platform ecosystem cocreates value among members of the ecosystem through increased sales, increased chance at an initial public offering (IPO), increased participation in the ecosystem, provided that the ISV is
protected by intellectual property rights. While the study provides strong evidence to suggest that platform adoption is beneficial to the value proposition of ISVs, the study is limited to the SAP ecosystem and does not generalize well to situations that do not use a commercial platform-based business model or organizations whose success is not measured by IPOs or similar metrics.

In [54], van den Berk, Jansen and Luinenberg provide a qualitative model for analyzing the strategy of a platform owner in a software ecosystem. While their model is sufficiently articulate, it only presents ecosystem strategy and assessment from the perspective of the platform owner, rather than a platform user or organization seeking to adopt the software platform. Furthermore, it provides little analysis on the value proposition that a platform user derives from an ecosystem, given the ecosystem's broader strategy.

While the existing ecosystem literature does not sufficiently address the cost and value of engaging in platform-based software engineering, there is a catalogue of literature that suggests real options valuation, both with and without Monte Carlo methods, is an appropriate method for providing support for strategic decision-making.

Gull [28], utilizes real options valuation techniques to analyze software license agreements. The authors examine several different types of options, ultimately determining that software licensing agreements are best modeled by capped call options, representing a long call option with a lower strike price in conjunction with a short call option with a higher strike price (i.e., paying a lower license fee for a longer license term or paying a higher license fee for a short term license). Ultimately, the author found that real options valuation techniques are appropriate for supporting decision making when examining discounted licensing agreements.

In [48], the authors propose using real options valuation as to support decision making in the process of paying down technical debt. The authors discuss the Net Options Value (NOV) technique $[1,51]$ for deciding whether or not to make improvements to a particular model. However, the authors only provide a preliminary discussion on the appropriateness of the NOV technique and other methods, as opposed to experimentation with the methods they discuss.

In [50], Sullivan et al. present a view of software as a portfolio of assets, utilizing real options valuation, net present value and event trees as a quantitative method for evaluating strategic decision making under uncertainty in a software development context. In their work, the authors examine several example scenarios for decision making, including the trade-off between evolvability and market share with an option for restructuring a software product as well as restructuring a
legacy product. The authors apply this financially driven interpretation of software modularity and information hiding [45], Boehm's spiral mode of software development [7] and optimal timing strategies for decision making under uncertainty.

In [51], the authors extend the work presented in [50] and utilize design structure matrices (DSMs) and the design theory presented in [1] as a foundation for structural design of software modules. They then utilize real options value techniques to evaluate the decision to substitute a module for a technically superior module within a design. These two theories are woven together and used to evaluate the classic problem of modularity and information hiding in systems presented in Parnas' seminal work [45].

The work in [52] most closely resembles the work presented in this proposal. The authors utilize two methods of valuation, Net Present Value (NPV) as well as a real options valuation (ROV) method based on European call options, for providing support for the decision to upgrade from the SAP R/2 platform to the SAP R/3 platform in a constructed scenario. When examining NPV, they developed two methods, a simple method which accounted for the $N P V_{1}$ from the decision to upgrade at time $t=0$, and a second NPV method, $N P V_{2}$, which utilized a decision-tree method, allowing for delaying the choice of upgrading. In their analysis, $N P V_{1}$ produced somewhat unrealistic outcomes, being unable to account for uncertainty when upgrading, resulting in $N P V_{1}$ only providing downward risk or worst-case scenario forecasting. $N P V_{2}$ and the ROV method resulted in better upper bounds on the return on investment of upgrading, as a result of accounting for the decision to delay upgrading until more information was available. From their analyses, they remarked that both methods were suitable for supporting this type of decision-making, with the aforementioned caveats noted.

Also closely related to the work presented in this proposal, the authors in [34] propose using the Black-Scholes options valuation method for evaluating the development and implementation of an electronic data warehouse composed of fifteen data marts. In their paper, the authors provided three scenarios of development wherein the cost, savings and revenue of the data marts were varied based on the rate at which they were developed. The authors calculated the option values for each strategy, utilizing a binomial model as well as the Black-Scholes option model. Risk and volatility were then accounted for utilizing a stochastic Monte Carlo approach, comparing the underlying net present values and volatilities between the three strategies. In their analysis, the authors noted that while a multi-phase deployment of data mart offered more options, the result was a trade-off
between deferring management decisions and delaying the cost saving benefits from deployment. Nonetheless, the authors found that the combination of NPV and ROV methods provided decision making support for IT projects where considerations for multi-phase versus single-phase deployment were applicable in the face of project risk.

## Chapter 8

## Future work

The goal of this research was to develop a model that provided decision making support for software platform adoption in the context of software development. While we have achieved a model that provides consistent decision making support, there are many avenues for improvement available for this model and research, namely in the scope and realism of the model.

The model presented in this paper mostly accurately represents a zeroth or perhaps first order approximation. Zeroth or first order approximations describe experiments which provide an educated guess at an answer, often with simplifying assumptions made as necessary to make the model more complete. The most obvious improvements increase both the accuracy of predictions within this model and the level of realism achieved in this model.

### 8.1 Probability of Successful Completion

The probability of successful implementation is a key parameter within the model, and provides our foundations for stochastic simulation. In our experiments, however, the probability of successful implementation was not significantly varied between experiments. This is in part due to the linear sensitivity of this variable, as shown in Section 6.2.1. This parameter could be improved by devising better methods of parameterization. One better method of parameterization could be using historical data from previous projects, while another method could be personnel-centric, wherein a manager examines the personnel on a project and estimates the probability of success of different periods of development and different activities based on the developers assigned to a project (i.e.,
our team is very good at coding, but not very good at architecture work). This could also lead to more sophisticated and accurate models which describe the variance with respect to time of the probability of succesful implementation.

### 8.2 Failure Model

In both sets of experiments, numerous sources of real world data were used in parameterization of the model. However, there is a surprising lack of data on software development cost analysis, both with respect to software platform adoption and the software development process itself. In situations where data was sparse or unobtainable, theoretical models were used to patch the 'holes' in the data. One such example was the creation of the failure model. As was described in Section 5.2, we created a theoretical model which models the severeity of failures within the software velopment process. The model in question was a stratified model, in which different types of failures (e.g., minor, major, etc) were defined with probability of occurrence (e.g., $50 \%, 25 \%$, etc) and a resulting impact (e.g., $25 \%$ reduction in value). A better model could be defined if data were available that accurately described the chance of a particular failure occurring as well as the impact of that failure. Additionally, a stratified model with more strata would yield more accurate results. While we have used this theoretical model to plug that 'hole' in the data, it is conceivable that an organization whose primary mission is software development would have historical data on the success and failure of projects, as well as the severity of failure encountered and the resulting effects of such failure.

### 8.3 Cost and Value Carry Over Model

Defined in tandem with the stratified failure model, we encountered difficulty determining what happens when a failure actually occurs. At first glance, it would seem that failure is simply not completing the work allocated to a particular period of development. However, what happens to the work that is not completed? In our model, we stipulated that in the event of failure, a certain amount of work (associated with cost) is carried over into the next period of development, and a certain amount of value (proporational to that cost) is carried over into the next period of development as well. While this simple model accurately models one perspective on failure, other possible perspectives may exist.

### 8.4 Cost Modeling

While we engineered the cost to be extensible, the model can be made more accurate by utilizing more accurate model of cost. The level of detail in the cost variable is largely up to the modeler: the cost could represent personnel costs, or could also include other costs such as licensing, tooling, education, facilities and equipment. With more sophisticated cost modeling comes more sophisticated cost-handling. In some cases, the cost reduction coefficient may not apply to some types of cost.

### 8.5 Cost and Value Theoretic

One of the central assumptions of this work, and similar software economics research, is that universal value of 'work'. We assume that if a developer spends a certain amount of hours working, they produce work products that have an intrinsic value proportional to the time spent creating them. This perspective rejects the notion that a person spending some amount of time working could produce nothing of value. However, even in the bleakest of scenarios, there is an intrinsic value, in the form of knowledge, which comes from working on a problem, task or project. Additionally, there is a long-term return on investment from working on a project, in the form of a more experienced and seasoned developer who is more capable. Modeling these factors, if possible, will result in a more accurate model.

### 8.6 Scope of Model

The model, computation and simulation presented in this work primarily represent a static development process. From the outset of modeling, the development process is constrained to a discrete number of periods which does not change throughout the simulation and modeling. While this is representative of certain techniques in software development, such as timeboxing, it is not as representative of the real world as possible. The main solution to this problem is to add dynamic elements of the development process, namely that a project development process can vary in the number of periods that define it.

Consider, for instance, a five period development process. In the fifth period of development, half of the work to be completed is left unfinished. In our current model, $50 \%$ of the cost is carried
over into the next period; however, with no additional periods of development left in our scenario, the work is simply left unfinished. With a more dynamic and modeling and simulation approach, the project development would be extended into a sixth period of development, and so on, as long as there is still work to be completed. The result of a Monte Carlo simulation with dynamic project duration would be an average duration of project. Likewise, the cost reduction coefficient reduces the work required in a period of development. A consequence in the reduction of work required in a period development is less time spent doing work, speeding up the pace of development on the project, which leads to a faster time to market.

## Chapter 9

## Conclusion

This dissertation describes research that was completed to address the problem of informed decision making in the face of strategic software platform adoption. We have presented a quantified model for modeling the cost and value of software development using the Black-Scholes formulation for real options value. Our model accounts for the future value of money, discounting it back to the present, as well as incorporating notions of uncertainty to the model. We have used stochastics to evaluate the relationships between the forces that influence this model, utilizing a Monte Carlo approach. We have performed verification and validation on the model, performing sensitivity analysis techniques on the parameters using stoachastic variation. We have performed extreme value validation to ensure the model's consistency when presented unrealistic scenarios, and we have ensured internal validity between mutliple simulation runs of the same experiment. We have provided two different scenarios to examine using our model and simulation methods that examine the interplay of value and cost in decision making in reference to adopting a software platform. The results of the experiments confirmed our initial hypothesis, showing that in each case our model could be used to provide decision making support in the context of software platform adoption. Moving forward, we will begin investigating using more realistic parameters as well as more realistic and refined modules to defray uncertainty and lack of data. Additionally, as noted in the future work section, we will be pursuing simulating dynamic software development processes that vary in their structure and duration, following the pattern of iterative improvement that was used to create the model in the first place.

Appendices

## Appendix A Expanded Problem Statement Scenarios

## A. 1 Scenario 1: Clean Slate Platform Adoption

An organization wishes to create a tool to aid in software development. The project manager is interested in investigating different methods of accomplishing this project, with several candidates available: developing an in-house proprietary workbench, contracting a developer to build a custom workbench, adopting an open platform that fulfills these needs. However, the manager is unsure how to proceed in determining which is the best option.

Intuition says that reinventing the wheel by creating a custom workbench in-house seems foolish at a time when so many platforms for tool development exist; not to mention the added costs of developing, extending, documenting and maintaining an additional project while simultaneously training engineers how to use are even more hidden costs in custom development.

At the same time, hiring an outside contractor to build a workbench to specification may save money on the development end; but, then assurances for quality, delivery schedule, payment and an external development process will have to be made. In the event of emergent bugs and requirements for additional features, the organization will be hamstrung by relying on an outside organization to provide support.

Given the above two options, the project manager believes performing in-house development on top of an open-source integrated development environment (IDE) platform is the optimal solution. But when confronted with the myriad choices of IDE platforms on top of which to build their workbench tool on top of, the manager is uncertain how to examine each platform and what criteria to use when rating them:

- What are the hidden costs of adoption for each candidate platform?
- What are the total costs of adoption for each candidate platform?
- Which platform best meets our functional needs?
- Which platform provides the best support?
- How to evaluate differences in these criteria and relate them to each other?

After evaluating the landscape, the project manager has identified several platforms which functionally meet the needs of the project. Considering that this tool may have a long life cycle
and deployment, the project manager wishes to evaluate these platforms to find which of the two is the better fit for the project. However, the project manager now needs criteria by which they can evaluate the competing platforms.

Expanded descriptions and analysis of the scenarios presented in Section 1.1.1 will be provided as we proceed forward and gain insights into these scenarios.

## A. 2 Scenario 2: Platform Switching

The second scenario describes a situation in which a software product already exists, developed using a software platform. However, over time, the platform has aged and the strategic directions of the developing firm has changed in such a way that has caused the product manager to consider migrating to a new software platform. We have expanded this scenario to reflect the real world experiences [5], [56], [29], of the author in supporting a project to develop a mobile app for a R1 university.

The University originally developed a mobile app for an open-source and proprietary mobile operating systems in Fiscal Year (FY) 2012. The original app was a static app containing information including phone numbers, campus locations and email addresses for notable points of contacts. While the app served its purpose well, to provided an aggregate source of contact information, it was not engineered in such a way to facilitate extension and the addition of new features. In FY2013, $\$ 150,000$ in start-up funds were allocated to hire a software developer and a UI designer to begin development on a new app. Without data on platform usage of the userbase, the app was engineered to be a mobile web app. Mobile web apps are essentially web pages that allow users to browses both static and dynamic data with limited interaction between the software that presents the data and the hardware the app is running on. This provided the developers with the ability to launch the app for both the open-source and proprietary platforms without significant effort porting the app between the two platforms.

This was accomplished using a mobile web app platform, Kuali Mobility, which provides functionality for developing mobile web apps using HTML5 and wrapping them in thin platformspecific app wrappers for iOS, Android, Blackberry and more. The university adopted the Kuali Mobility platform, joining their ecosystem and contributing 1 full-time equivalent employee to working on the platform.

Development proceeded for a year and the app was released in FY2014. Following the release
of the app, the university continued funding the project at the same rate for 12 months, totaling $\$ 150,000$, but discontinued its single FTE support of Kuali. Six months following the release of the app, usage rates were low, showing that only $20 \%$, about 5,000 users, of the university population utilized the app. The developers thought the low usage was a result of utilizing a mobile web app for two reasons. The first was that using a mobile web app as the platfrom constrained the features that could be added. Features utilizing location services, device information and push notifications could not be integrated using a web-only interface. The second was that by not targeting a specific platform, the app was less attractive to users on all platforms (for example, a proprietary OS user wants to use a native app to the platform over a webpage wrapped in an proprietary platform app wrapper).

After surveying the campus population, the development organization determined that $70 \%$ of users, about 18,000 , utilize mobile devices on the proprietary OS platform, and that porting the app from Kuali to the proprietary platform would enable several key features requested by the user base, potentially increasing the number of users of the app. However, any developments done using the proprietary platform as an explicit platform for the app would not applicable to open source or mobile web app implementations of the software.

Given that one of the primary drivers of the project is to get as many users downloading and using the app as possible, the project manager would like to investigate how to proceed. One option is to port the mobile web app to the proprietary mobile platform, while another possibility includes porting it to open source platform. The third possibility exists, which is to do nothing; unsure of how to proceed, the project manager is interested in investigating these choices from an options value perspective.

## Appendix B Software Product Lines: A Quantified Model for Valuing Variation Points

The purpose of the work in [38] was to determine the value of a variation point within a software product line. We start by defining a generalized function that describes the value, $v$, of a non-income producing option that pays off $\mathrm{W}(\mathrm{T})$ at future time T :

$$
\begin{equation*}
v(t, T)=e^{-r(T-t)} E[\max (0, W(T))] \tag{1}
\end{equation*}
$$

In this generalized equation, $E[\max (0, W(T)]$ represents the expected value of $W(T)$, the pay-off from a non-income producing option at time $T$. The term $e^{-r(T-t)}$ defines the riskless discount coefficient. In it, r is the riskless discount rate, which was sufficiently covered in Section 2.4, while $t$ represents the current time, and $T$ represents the future date at which the option will pay-off. The term $e^{-r(T-t)}$ provides us a coefficient that, when multiplied by $W(T)$, discounts the amount of money back to its present day value. Next, we expand this equation to reflect the pay-off from an asset that is utilized in one or more products. For the sake of clarity aiding the understanding of the reader, the equation has been broken down into smaller parts than was originally presented in [38].

$$
\begin{equation*}
v_{i}(t, T)=\max (0,-E[\operatorname{Strike}(i, t, T)]+E[\operatorname{Spot}(i, t, T)]) \tag{2}
\end{equation*}
$$

This equation is a cost value analysis of developing an asset $i$, given current period $t$ and expected date of maturity $T$. The equation $\operatorname{Strike}(i, t, T)$ encodes the cost of developing the asset, given the asset, the current time and the maturity date of that asset, while the $\operatorname{Spot}(i, t, T)$ function encodes the value derived from that asset based on the number of products it is used in within the product line. In both equations, the future value of costs incurred and revenue generated are discounted back to the present day value of money. The equation $\operatorname{Strike}(i, t, T)$ provides cost to
engage in development for a given asset, $i$, and is described as follows:

$$
\begin{equation*}
\operatorname{Strike}(i, t, T)=\sum_{\tau=t}^{T} C_{i}(\tau) e^{-r(\tau-t)} \tag{3}
\end{equation*}
$$

$C_{i}(\tau)$ is our generic cost function for describing the cost of developing asset $i$ in period $\tau$ with an expected maturity date of $T$, which allows for the incremental development of asset $i$. The $\tau$ variable is used to compute the difference in time between a later period of development and the current period of development, which is a parameter in discounting the future cost of development back to the value of present-day dollars, while $r$ describes the riskless discount rate of the option. The term $e^{-r(\tau-t)}$ encodes the discount coefficient, which when multiplied by a future amount of money, discounts the value of the money to its present-day value. The use of $\tau$ in conjunction with the discount coefficient is important, as essentially we are summing the cost incurred by developing this asset over the total time it will be in development. Since we have divided our development into periods, the present value of a future cost depends on how far in the future that development cost is incurred.

$$
\begin{equation*}
\operatorname{Spot}(i, t, T)=p_{i, T} E\left[\sum_{k} \max \left(0, \sum_{\tau=T}^{T *} X_{i, k}(\tau) e^{-r(\tau-t)}\right)\right] \tag{4}
\end{equation*}
$$

The value side of the equation is similar to the cost equation shown in equation 3, but with addition of two new parameters: $p_{i, T}$ and $T *$. $p_{i, T}$ encodes the probability of successful implementation of asset $i$ by exercise date T , while $T *$ describes the time horizon of the software product line. The time horizon refers to the time at which all products under development have been released; as products are released, value is accrued. However analysis could be extended to include further versions of products within the product line. Finally $X_{i, k}(\tau)$ is the net marginal value of the variation point $i$ in product $k$ in period $\tau$, given the successful implementation of variation point $i$.

Essentially, $X_{i, k}(\tau)$ is defined as follows:

$$
\begin{equation*}
X_{i, k}(\tau)=V M P_{i, k}(\tau)-M C_{i, k}(\tau) \tag{5}
\end{equation*}
$$

Where $\left.V M P_{i, k}(\tau)\right)$ is the marginal value of the contribution of variation point $i$ in product $k$, given the successful implementation of variation point $i$, and $M C i, k(\tau)$ is the marginal cost of "productization" of variation point $i$. In the initial product, there may be a cost associated with tailoring the asset to fit the exact needs of the product. This marginal cost is the same as the $C_{\text {Reuse }}$ [16] in successive products after the first use of the variation point. $C_{\text {Reuse }}$ is a cost function used to model the cost of reusing an asset, which was made reusable, in the implementation of a product within a product line according to the Structured and Intuitive Model for Product Line Economics (SIMPLE) [16].

Combining the parts together, we are left with the following equation:

$$
\begin{equation*}
v_{i}(t, T)=\max \left(0,-E\left[\sum_{\tau=t}^{T} C_{i}(\tau) e^{-r(\tau-t)}\right]+p_{i, T} E\left[\sum_{k} \max \left(0, \sum_{\tau=T}^{T *} X_{i, k}(\tau) e^{-r(\tau-t)}\right)\right]\right) \tag{6}
\end{equation*}
$$

Given a variation point, $i$, a current time, $t$, and a maturity date, $T$, for that asset, this deterministic model computes the difference between the expected value of the cost of developing this asset and the expected value of the revenue generated from this asset, given a set of products in which this asset will be used in. The future value of both the expected cost and the expected revenue are discounted back to the value of money at the current time $t$.

## B. 1 Monte Carlo Simulation and Experiment

In order to apply and test our model, we constructed an illustrative scenario to drive an experiment with this equation, quoted below from the original article:

A product line manager is planning a new software product line. We have a product line of 9 products that are scheduled to be released, one every 6 months (every period), beginning
in the 12 th month, covering the next five years. We will consider that the products are identified as products $0-8$ and will be released in numerical order. There will be 11 core assets. The cost of each asset, which includes the cost of the variation point, is estimated for each time period up to the time the asset is released [...] Individual product managers have already made revenue estimates as part of the business case for each product. The value of an asset is computed by allocating the total estimated revenue of each product to the assets used in that product. The value of the asset is the total of all its revenue allocations [38].

Using this scenario, we converted the deterministic form of Equation 6 to a stochastic model. We parameterized Equation 6 with the following values:

- $r=0.05 /$ year
- $P_{i, T}=0.85$.
- $i=$ designation for a variation point. Initially we assumed 1 variation point per asset.
- $k=$ designation for a product such that $1<k<10$. In our initial experiments, we produced one product every six months, but our formulation allows for products to be released in any period. The total value of the asset or product accrues immediately upon release.
- $t=$ current time, initially 0 .
- $T=$ exercise date. This value varies from 0 to $T^{*}$; initially in steps of six months, this is the date by which a variation point is inserted into the asset or not.
- $T *=$ time horizon. 5 years, ten periods of 6 months.

Each of the parameters in the model was represented by a random variable with a normal distribution. We applied two Monte Carlo simulations, performing 200,000 trials per simulation. In the first simulation, we held the value $P_{i, T}$ constant at 0.85 , while in the second simulation, we held $P_{i, T}$ constant at 0.85 for the first two years of development, and then decreased its value by 0.05 in each time period afterwards. The variation in $P_{i, T}$ simulates that estimates are less accurate the further into the future an event is.

Consider Tables 1, 2 and 3. Table 1 provides a product-asset mapping, showing which assets are used in which products. Each column encodes a product in the product line, while each row

Table 1: Asset-Product Mapping

| Asset / Product | $\begin{aligned} & 0 \\ & \text { O} \\ & \text { U } \\ & \text { O} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { H } \\ & \text { U } \\ & \text { U } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { U } \\ & \text { UZ } \\ & 0 \\ & 0 . \end{aligned}$ |  | $\begin{aligned} & H \\ & \text { H } \\ & \text { U } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 0 <br>  <br>  <br> 0 <br> 0 | $\begin{aligned} & \text { N } \\ & \text { U } \\ & \text { Z } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \text { 烒 } \\ & \text { Z } \\ & 0 \\ & 0 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Asset 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Asset 1 | 1 | 1 | 1 |  |  | 1 |  |  |  |
| Asset 2 | 1 |  |  | 1 |  | 1 | 1 | 1 | 1 |
| Asset 3 |  |  |  | 1 | 1 |  |  | 1 | 1 |
| Asset 4 |  | 1 | 1 |  | 1 |  | 1 |  | 1 |
| Asset 5 | 1 | 1 |  |  | 1 | 1 |  | 1 |  |
| Asset 6 |  |  |  |  |  | 1 | 1 | 1 | 1 |
| Asset 7 | 1 | 1 | 1 | 1 |  | 1 | 1 |  |  |
| Asset 8 |  | 1 | 1 |  | 1 |  | 1 | 1 |  |
| Asset 9 |  |  |  | 1 |  | 1 | 1 | 1 |  |
| Asset 10 |  |  | 1 |  | 1 |  | 1 |  | 1 |

encodes an asset. If a given cell $i, j$ is valued 1 , that means that $i t h$ asset is included in the $j t h$ product, while no value indicates that the $i t h$ asset is not included in the $j t h$ product.

Table 2 shows the cost of developing each asset during each period of development, with columns indicating periods of development and rows indicating assets. A non-zero value in cell $i, j$ indicates the cost associated with developing asset $i$ in period $j$. A zero-value in cell $i, j$ indicates that no development took place in the $j t h$ period. An asset is considered matured and ready for use when time has passed the last period in which development occurs for a given asset. In some cases, notably Assets 6, 9 and 10, development was prematurely stopped, and then restarted at a later date.

Table 3 describes the revenue generated from each product once it has matured. Once each of the assets that composed a product was completed, the product was "released" in the following of development. Its revenue was divided among each of the remaining periods until reaching the time horizon.

Table 2: Asset Development Cost Per Period

| Asset / <br> Period | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{O}{0} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \stackrel{0}{0} \\ & 0.0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { H } \\ & \text { O } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 10 \\ & 0.0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \sim \\ & \text { N } \\ & \text { OU } \\ & 0 \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \text { O } \\ & \text { OU } \\ & 0 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Asset 0 | 100 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 1 | 150 | 200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 2 | 100 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 3 | 100 | 100 | 100 | 50 | 50 | 0 | 0 | 0 | 0 | 0 |
| Asset 4 | 300 | 200 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 5 | 100 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 6 | 0 | 50 | 50 | 0 | 100 | 300 | 150 | 0 | 0 | 0 |
| Asset 7 | 150 | 200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 8 | 100 | 200 | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Asset 9 | 200 | 0 | 0 | 150 | 100 | 500 | 0 | 0 | 0 | 0 |
| Asset 10 | 50 | 100 | 150 | 0 | 250 | 0 | 0 | 0 | 0 | 0 |

Table 3: Revenue per Product

| Product | Revenue |
| :--- | :--- |
| Product 0 | 1200 |
| Product 1 | 1500 |
| Product 2 | 900 |
| Product 3 | 1000 |
| Product 4 | 1600 |
| Product 5 | 800 |
| Product 6 | 2100 |
| Product 7 | 1400 |
| Product 8 | 2800 |

Table 4: Simulation Results

| Asset | Number of <br> Products | Cost | Value with <br> constant P | Value with <br> decreasing P |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 9 | 298 | 6960 | 6955 |
| 1 | 4 | 415 | 3252 | 3253 |
| 2 | 5 | 298 | 4076 | 4074 |
| 3 | 4 | 525 | 2673 | 2313 |
| 4 | 5 | 785 | 3761 | 3772 |
| 5 | 5 | 298 | 4061 | 4077 |
| 6 | 4 | 999 | 1596 | 1089 |
| 7 | 6 | 415 | 4519 | 4515 |
| 8 | 5 | 716 | 3314 | 3322 |
| 9 | 4 | 1340 | 1031 | 806 |
| 10 | 4 | 765 | 2602 | 2420 |

## Appendix C Rational Unified Process

Pioneered in the early 1990s by Phillippe Kruchten [36], the Rational Unified Process (RUP) is an iterative, high level software development process developed by Rational Software, a subsidiary of IBM. Rather than a single process, RUP is a high level framework for software development which is adaptable to the organization and development context RUP is being applied to.

RUP has styled itself around a four phase project life cycle consisting of phases for inception, elaboration, construction and transition. This four-phase life cycle provides a high level view of software development similar to a waterfall style of development. However, the top-down high-level view of RUP masks its iterative style of development, with each phase completing a key objective and milestone, and each iteration within the phase completing key objectives for that phase. The phases occur sequentially, though sometimes the phases overlap, with different personal accomplishing different tasks within different phases in tandem. Figure 1 provides one high level view of the RUP process. Below, each of the four phases is described in terms of the milestones being accomplished and the activities being done.


Figure 1: Sample RUP Process

## C.0.1 Inception Phase

The inception phase is when the early viability and planning activities take place. The purpose of this phase is to complete the life cycle objective milestone. This begins with scoping the
project, validating initial costing and budgets, establishing a business case and financial forecasts. Additionally, this is where the traditional requirements elicitation, analysis and specification would occur, as well as the development of an architectural prototype. Interestingly enough, this is the phase where the analysis being presented in this dissertation would occur.

## C.0.2 Elaboration Phase

The elaboration phase is where the begins to mature and take shape. The primary objective of the elaboration phase is the creation of an architecture and the identification and mitigation of risks to the project from analysis completed in this phase. The outcome from this phase includes a nearly complete use-case model, a description and specification of a software architecture, an executable software architecture that realizes significant and core use cases, a revised business plan and risk assessment plan, as well as development plan for the overall architecture. In a traditional waterfall model, this phase is analogous to architectural design, as well as some elements of the detailed design.

## C.0.3 Construction Phase

The construction phase is when the software system is built. The focus of this phase is on development of assets that constitute the software system, including code, tests, components and modules, data formats and additional dependent systems. The outcome of this phase is the first external release of the software, marked by an initial operational capability. In a traditional waterfall style development setting, the construction phase corresponds to the implementation and a portion of the testing phases of development.

## C.0.4 Transition Phase

The primary goal of the transition phase is evolving the software system from a development system to a production system. This includes the application of delivery processes and patterns, as well as making the system available to the user and educating the user on its use, as well as beta testing and validating the produced system against the end users' expectations and validating the quality against benchmarks in the inception phase. If all of these objectives are met, the product release milestone is met and the software development is completed. In the traditional waterfall
model, this phase corresponds to a portion of the testing phase (notably, system testing) and the release of the software.

## Appendix D Experimental Data

## D. 1 Scenario 1

## D.1.1 Scenario Cost and Value

| Period of Completion | Cost | Value (\$) | Riskless Discount |
| :---: | :---: | :---: | :---: |
| 1 | 60074.86 | 66082.34 | 0.05 |
| 2 | 60074.86 | 66082.34 | 0.05 |
| 3 | 60074.86 | 66082.34 | 0.05 |
| 4 | 60074.86 | 66082.34 | 0.05 |
| 5 | 80869.9 | 88956.89 | 0.05 |
| 6 | 80869.9 | 88956.89 | 0.05 |
| 7 | 80869.9 | 88956.89 | 0.05 |
| 8 | 80869.9 | 88956.89 | 0.05 |
| 9 | 80869.9 | 88956.89 | 0.05 |
| 10 | 80869.9 | 88956.89 | 0.05 |
| 11 | 80869.9 | 88956.89 | 0.05 |
| 12 | 80869.9 | 88956.89 | 0.05 |
| 13 | 80869.9 | 88956.89 | 0.05 |
| 14 | 80869.9 | 88956.89 | 0.05 |
| 15 | 153064.2 | 168370.61 | 0.05 |
| 16 | 153064.2 | 168370.61 | 0.05 |
| 17 | 153064.2 | 168370.61 | 0.05 |
| 18 | 153064.2 | 168370.61 | 0.05 |
| 19 | 153064.2 | 168370.61 | 0.05 |
| 20 | 153064.2 | 168370.61 | 0.05 |
| 21 | 153064.2 | 168370.61 | 0.05 |
| 22 | 153064.2 | 168370.61 | 0.05 |
| 23 | 153064.2 | 168370.61 | 0.05 |
| 24 | 153064.2 | 168370.61 | 0.05 |
| 25 | 153064.2 | 168370.61 | 0.05 |
| 26 | 153064.2 | 168370.61 | 0.05 |
| 27 | 153064.2 | 168370.61 | 0.05 |
| 28 | 153064.2 | 168370.61 | 0.05 |
| 29 | 153064.2 | 168370.61 | 0.05 |
| 30 | 153064.2 | 168370.61 | 0.05 |
| 31 | 153064.2 | 168370.61 | 0.05 |
| 32 | 132164.69 | 132164.69 | 0.05 |
| 33 | 132164.69 | 132164.69 | 0.05 |
| 34 | 132164.69 | 132164.69 | 0.05 |
| 35 | 132164.69 | 132164.69 | 0.05 |

Table 5: Scenario 1 Cost and Value Structure

## D.1.2 Scenario 1 Bespoke Platform



Table 6: Scenario 1 Bespoke Platform Net Value per Period


Table 7: Scenario 1 Bespoke Platform Sum Net Value Per Period

## D.1.3 Scenario 1 Code-Centric Platform



Table 8: Scenario 1 Code-Centric Platform Net Value Per Period


Table 9: Scenario 1 Code-Centric Platform Net Value Per Period

## D.1.4 Scenario 1 Architecture-Centric Platform

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1063,51 | ${ }^{10162.30}$ | 9612.06 | 9028.62 | 11328.08 | 10698.25 | 10051.35 | 942.27 | 884.08 | ${ }^{8233.24}$ | ${ }^{7784.46}$ | ${ }^{277.19}$ | 6812.55 | ${ }^{6374.00}$ | ${ }^{335.19}$ | 2980.83 | 2663.20 | 2364.12 | 2123.31 | 1846.60 | ${ }^{1623.26}$ | 1424.94 | ${ }^{1241.80}$ | ${ }^{1062.32}$ | ${ }^{923.25}$ | ${ }^{73.76}$ | 64.13 | 518.90 | 390.52 | 288.30 | ${ }^{199.65}$ | 246.38 | 2439.16 | ${ }^{2376.85}$ |  |
|  | 0629.00 | 1086.2 | 1023.63 | 9588.76 | ${ }^{12036}$ | ${ }^{1388.11}$ | 10703.9 | 10036.82 | 9434.98 | 8817.59 | 283.90 | ${ }^{775.97}$ | ${ }^{7283,75}$ | ${ }^{6828.77}$ | ${ }^{3785.24}$ | 3298.73 | ${ }^{2973.64}$ | 2666.04 | 2354.39 | 2124 | 1840.6 | ${ }^{1647.51}$ | 1422.9 | 1245. | 1070. | ${ }^{939.11}$ | 763.1 | ${ }^{645.9}$ | 515. | ${ }^{400.03}$ | 299 | ${ }^{2526.21}$ | 2504.13 | 2437.8 |  |
|  | 10638.34 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 10642.63 | 10870.98 | 10887.74 | 10888.14 | 13683.45 | 12915.91 | ${ }^{12133.26}$ | 11428.80 | 1069.22 | 10044.94 | 9446.02 | 8850.05 | ${ }_{827.85}$ | ${ }^{7776.22}$ | 4656.54 | 4064.48 | 364.47 | 3325.69 | 2960.20 | 2683.54 | 2359.02 | 2094.19 | 1854.81 | 1665.49 | 1456.21 | 1246.53 | 1070.25 | 938.80 | 768.82 | ${ }_{615.48}$ | 507.75 | 265.41 | 2643.90 | -2576.47 |  |
|  | 10632. | 10832.38 | 10876: |  | 145 | 13772.27 | 129 |  | 1139 | 1069 | 1004 | 9427.81 | 888 | 827 | 5080 | 442.92 | 3984.24 |  | 3286.30 | ${ }^{2950.15}$ | 2668.34 | 2382.27 | 2080.81 | 1862.19 | 1624 | 1436.12 | 1251.18 |  |  |  |  |  |  |  |  |
|  | 1062.1 .16 | ${ }^{1085}$ | 10866.90 |  | 14563.10 | ${ }^{14640.27}$ | 13726.73 | 12935.41 |  | ${ }^{11368.57}$ | 10678.10 | 10079.06 | ${ }_{9423.89}$ | 8819.77 | ${ }_{556.72}$ | 18884.48 | 4450.13 | 4017.08 | 3676.72 | 3309.42 | 3009.50 | 2661.21 | ${ }^{2332.06}$ | 2095.38 | 1856.6 | 1639.76 | 1482.71 | ${ }^{1233,18}$ | 1060.93 | ${ }^{913.47}$ | 769.27 | 2794.63 | 2792.08 |  |  |
|  | 10618.38 | 10849.95 | 10901.06 | 10862.21 | 14453.57 | 14633.28 | 14641.1 | 13770.2 | 12925.79 | 12153.61 | ${ }_{1142.84}$ | 10697.05 | 10030.20 | 943.82 | 6138.40 | 5428.30 | 4930.08 | 445.44 | 4010.34 | 3612.84 | 3288.68 | 2999.87 | 2621.88 | 2386.06 | 2108.13 | 1882.40 | 1625.24 | 1437.06 | 1246.8 | 1080.0 | 90.24 | 286.01 | 2856.9 | 2782.4 |  |
|  | 10620.3 | 1084.5 |  |  | 14540. |  | 4627.8 |  | ${ }^{13788.24}$ | ${ }^{12885.05}$ |  |  |  |  | ${ }^{659.50}$ |  |  |  |  |  | 3658.48 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 10627.30 | 10861.33 | 10861.50 | 10880.66 | 14590.71 | 464.70 | 14667.3 | 1474.03 | 14643.27 | ${ }^{13755.08}$ | ${ }^{2955.67}$ | 12139.18 | 11410.2 | 10723.3 | ${ }^{7277.37}$ | ${ }^{6641.57}$ | 5925.97 | 5885.21 | 493.42 | 477.89 | 1023.97 | 3654.64 | 3220.86 | 2954.19 | 2659.9 | ${ }^{2378.31}$ | 2104.53 | 1881.50 | 1620.87 | 1452.37 | ${ }^{1232.65}$ | 2973.18 | 2991.2 | 2910.35 |  |
| 10 | 10613.56 | ${ }^{10872.38}$ | ${ }^{10887.87}$ | 10899.41 | 14566.00 | ${ }^{14621.06}$ | 1464.4 | 14631.94 | ${ }^{14628.65}$ | 14656.17 | ${ }^{13734.86}$ | 12337.48 | ${ }^{12199.18}$ | ${ }_{11226.66}$ | ${ }_{7956.32}$ | 7107.41 | ${ }_{6421.41}$ | 5847.75 | 5369.61 | 4900.51 | 1412.1 | 3993.21 | 3655.14 | 3301.28 | 2977.8 | 263723 | 2379.08 |  |  |  |  |  |  |  |  |
|  | 1066 | 10875.0. | 10866.21 | 10870.83 | 14562.78 | 14661.61 | 1467. | 4688.52 | 14672.11 | 14631.84 | ${ }^{14658.95}$ |  |  | ${ }^{2155.56}$ | 8631.40 | ${ }^{672.62}$ |  | ${ }^{646}$ | 585.75 |  |  |  | 1037.06 | 3650 |  |  |  |  |  |  |  |  |  |  |  |
|  | 1065.30 | 10836.58 | 10877.80 | 10877.4 | 14565.4 | 14629.0 | 1666.8 | 4654.29 | 14623.98 | 4646.12 | 162.44 | 16655.70 | 13758.55 | 2997.15 | ${ }^{9336.96}$ | 8881.38 | 7696.70 | 7880.10 | ${ }^{6521.62}$ | ${ }^{5911.16}$ | 5411.25 | 1889.00 | 4432.1 | 4031.4 | 3661. | 3295.74 | 2970.23 | 2669.20 | 2399 |  | 1831.2 | 3132.6 |  |  |  |
|  | 10650.70 | ${ }^{10862.31}$ | ${ }^{10861.36}$ | 10883.66 | ${ }^{14605.85}$ | ${ }^{14639.44}$ | 14678.5 | 14676.8 | 14636.04 | 14670.19 | 14648.07 | 14661.87 | 14617.71 | 13751.90 | 10171.53 | 9162.45 | ${ }^{806.84}$ | 774.85 | 7055.47 | ${ }^{677.53}$ | 598.51 | ${ }^{5332.63}$ | 491.77 | 4430.74 | 4017.3 | 3695.38 | 3273.72 | 2964.74 | 2649.62 | 2349.6 | 2104.79 | 3181.4 | 3225. | ${ }^{3186}$ |  |
| , | ${ }^{10627.5}$ | ${ }^{10853.04}$ | 10 | 10902.85 | 14535.50 |  | 14667.7 | 4676.8 | 14619 | ${ }_{14681.55}$ | 14663.48 | ${ }^{14652.81}$ | 14659.61 | ${ }^{14622.25}$ | 11014.64 | ${ }^{993.56}$ | ${ }^{9184.97}$ | 8382.06 | 7655.08 | 7079.71 | ${ }^{6387}$ | ${ }^{5876.40}$ | ${ }^{531.86}$ | 4882.82 | ${ }^{439.69}$ | 4030.07 | 3663.51 | ${ }^{3329.15}$ | 294.61 | 2659.40 | ${ }^{2346.87}$ | 3241.6 | 3307. |  |  |
|  | 1063.40 | ${ }^{10836.24}$ | 10880.64 | 10871.96 | 14569.58 | 466733 | 14628.11 | 4685.60 | ${ }^{14667.78}$ | 14659.03 | ${ }_{14660.21}$ | ${ }^{14649.36}$ | 4639.41 | 14644.17 |  | 10736.4 | 9910.45 | 9075.70 |  |  | 7038,72 |  |  | 540201 |  |  |  |  |  | ${ }^{2958.25}$ |  |  |  |  |  |
| ${ }^{16}$ | 10610.06 | ${ }^{10856.43}$ | 10869.96 | 1088.52 | 14568.03 | ${ }^{14688.10}$ | 14637.61 | 14665.8 | 14644.4 | 14665. | 14655.57 | 14633.42 | 1467.67 | ${ }^{1465}$ | 11955.05 | 11703.2 | 10826.98 | 988.31 | 913433 | 8877.57 | 7719.48 | 7076.07 | ${ }_{6460.57}$ | ${ }_{5926.19}$ | 5389,74 | 1905.14 | 1453.11 | 1083.12 | 3666.83 | 3299.53 | 2942.43 | ${ }^{3307}$ | 3397. |  |  |
| 17 | 062 | ${ }^{0875.78}$ | 10885 | 0873.2. | 14591. | 1699.3 | 14661 | ${ }^{4663}$ | ${ }_{4613.3}$ | 4648 | (658.02 | ${ }^{14656}$ | 4628 | 4646 | 1969 |  |  | 10835 |  | 170 | ${ }^{4055}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 18 | 10615,47 | ${ }^{10870.86}$ | 10911.00 | 10880.5 | 14580.51 | 464.104 | 14677.7 |  | 1.4638 .61 | ${ }_{14654.73}$ | ${ }_{14650.49}$ | 1466.3 .3 | 14653.91 | 14670.07 | 1946. |  | $1169.3{ }^{\text {a }}$ |  | 10759.1 | 9968.72 | 9160.13 | 8413.58 | 7703.45 | 7032.84 | ${ }^{6422.22}$ | 5937.71 | ${ }^{536.00}$ |  | 1442.51 | 403, 15 | 3604.15 | ${ }^{3353}$ | ${ }^{3482}$ |  |  |
| 19 | 10605.8 | 10861.45 | 10887. | ${ }^{10857}$ | 14557 | 4669 | 14634.7 | ${ }^{4653.77}$ | ${ }^{14665}$ | ${ }_{14646}$ | 14663.1 | 14652 | 14888 | ${ }^{1463}$ | ${ }^{1193}$ | 11758. | 1176 | 11715.3 | 11737.74 | 10791 | 0986.43 | ${ }^{1129.69}$ | ${ }_{425.82}$ | ${ }^{7763.95}$ | 704.73 | ${ }^{6481.00}$ | 5914.1 | 5334.84 | 1886.17 | 404.89 | 4099.71 | 3412.6 | 3523.2 |  |  |
| 20 | 10646.8. | 0862. | 0870.9 |  | 14552 | 4622. |  |  |  | ${ }^{14657}$ | 14661 | ${ }^{4618}$ |  | ${ }^{14625}$ | 1966 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 21 | 10620.23 | 10867.78 | ${ }^{0092,61}$ | 10868.15 | ${ }^{14599.14}$ | ${ }_{14666.40}$ | 14667.6 | 1424.57 | 4598. 18 | 1467.13 | ${ }^{14678.81}$ | 14666.66 | 14650.90 | ${ }^{14658.75}$ | 1958.29 |  |  |  | 1717.8 | 1689 |  | 0767.26 |  |  | 418. |  |  | ${ }_{4998.63}$ |  | ${ }_{\text {5367.95 }}$ |  |  |  |  |  |
| 22 | 10633.21 | ${ }^{10868.89}$ | 10893.6 | 10872.22 | 14556.7 | 4632.17 | 1466 | 4685 | 4613. | 14667.0 | ${ }^{14677.30}$ | 14688.3 | 14628.1 | 14674.8 | 11910.6 | 1174.4 | 11702. | 11677.1 | 11 | 11226 | 11730.1. | 1169.3 | 10857 | 9998 | ${ }^{117}$ |  | 7661 | 7053.8 | 6525.2 | 5901.2 |  | 3410 |  |  |  |
|  | 10654.07 | 10833.00 |  |  | 14527.75 | 4667.5 |  |  |  |  | ${ }^{14690.13}$ | 14683.8 | 4664 | ${ }^{14661.17}$ |  |  |  |  |  |  |  |  |  |  | 9937. |  |  |  |  |  |  |  |  |  |  |
| 24 | 10611.1 | 10855. | 10876.70 | 10890.80 | ${ }^{14547.69}$ | ${ }^{4632.80}$ | 4649.9 | 4673.3. | 14642.66 | ${ }^{14618.95}$ | ${ }^{14657}$ | 1465 | 4663.4 | ${ }^{14668}$ | 1910.2 | 11726 | 1163.4 | 1716.8 | 1728. |  | 1175. |  |  | 11678. | 1841 | 9919.03 | 918 | ${ }^{838}$ | 7704.54 | 7105.98 | ${ }^{6475.07}$ | 3413.4 | ${ }^{3638}$ |  |  |
| 25 | 10624.46 | 10840.23 | 10890.0 | 10888.4 | 14529.3 | 16619.8 | 4673 | 4664. | 14658. | 14337. | 14659.73 | 146693. | 4699.1 | 1467.88 | 1994.4 |  | 11888. | 1174 | ${ }^{11648}$ |  |  |  | 1167 | 1167 |  |  |  |  |  |  |  |  |  |  |  |
| 26 | 106 |  |  |  | 14554.1 |  |  |  |  |  |  |  |  | 4633.87 |  |  |  |  |  |  |  |  |  | 1678 |  |  |  |  |  |  |  |  |  |  |  |
| 27 | 0616 | ${ }^{10866.51}$ | ${ }^{10887}$ | 10888.9 | 14577. | 14641.6 | 14883.7 | 4666.8 | 14628 | ${ }^{14671}$ | 14675.9 | ${ }_{12637}$ | ${ }^{14650}$ | ${ }^{14643}$ | ${ }_{11920}$ | 116 | 11.680 .6 | 11704.3 | 11749.8 | ${ }^{1175}$ | ${ }^{1173}$ | ${ }^{1722}$ | 11680 | 1172 | 1172 | 11708 | 11766 | 1082 | 9908.33 | ${ }^{9171.188}$ | ${ }^{8379.16}$ | ${ }^{3268.72}$ | -3599 |  |  |
|  | 10633.22 | 10872.28 | 10897.7] | 10851.7 |  |  |  |  |  |  | 1864.5 | 464.2 | 61.0 | 14637.3 | 1886.68 |  |  | 1703. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 29 | 10631.8 | ${ }^{10876}$ | ${ }^{10883}$ | ${ }^{10887}$ | 14579.76 | 14633.2 | 14638.12 | 1466 | S | ${ }^{14649}$ | 14661.7 | ${ }_{14660}$ | ${ }^{14648}$ | ${ }^{14623 .}$ | 1984.4 | 11724.2 | 11682.72 | 11732.2 | 11699 | 11701.8 | 11726.7 | 11675.4 | 1771. | 1752.1. | 11660 | ${ }^{11228 .}$ | 11703: | ${ }^{11683.4}$ | 17719 | 10797. | ${ }^{9934.84}$ | ${ }^{3154 .}$ | -3514 |  |  |
| \% | 10648. | 10838.1 | 0894.57 | 0878. | 14576.92 |  |  |  | 1651 | ${ }^{1465}$ | ${ }^{1657.93}$ | 649. | , 655 | ${ }^{14640}$ | 11929 | 1175 | 11771.0 | ${ }^{1776}$ | ${ }^{1163}$ | 1775 | 1171 | ${ }^{1167}$ | 171 | 11660 | ${ }^{11765}$ |  |  | 11712.1 | 11693 |  | 10813.7 |  |  |  |  |
| ${ }^{31}$ | 10614.95 | ${ }_{1086.43}$ | 10894,78 | 10883.71 | 14555.87 | 14336.83 | 4642.4 | 4644.9 | 14663.28 | 14651.07 | ${ }^{14653.96}$ | ${ }^{1468}$ | 14637.52 | 1636. | 2019.2 | 11887 |  | 1169. | 11682.3 | 11885 |  |  |  | ${ }^{1733}$ | ${ }^{1678}$ |  | 1738 | ${ }^{1763}$ |  |  |  |  |  |  |  |
| 32 | 1063 | 10855.13 | ${ }^{1086} 1$ | 10868.27 | 14601.16 | 4656.56 | 14670.7 | 4643.5 | 14676.66 | ${ }^{14632}$ | ${ }^{14630}$ | 14670 | 14665.36 | ${ }^{1466}$ | 1189 | 11720.8 | 11718.3 | 11705.1 | 1173.6 | 11745 | 11690. | 11814.08 | 11778.2 | 11671.9 | 11764. | 1780. |  | 11705.2 | 11794 | 116 | 17706.91 | 2822. | -3230. |  |  |
| 33 | 1063 | ${ }_{10860.94}$ | 10863.08 |  |  |  |  |  |  |  | 14699.90 | ${ }^{14664.2}$ |  |  | 1986.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 34 | ${ }^{10638}$ | ${ }^{10861}$ | ${ }^{1089}$ |  | 14567.4 | ${ }^{14677}$ | 1462 |  | 61.87 | 11469 9 45 | 14639.02 | 14655.72 |  | ${ }^{14638}$ |  |  |  |  |  |  |  | 1676.76 | 111889. | 1703.03 |  |  |  |  |  | 1757.2 |  |  |  |  |  |
|  | 10663. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 10: Scenario 1 Architecture-Centric Platform Net Value Per Period

|  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 18 | 19 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1063. | 202 | 0007. ${ }^{\text {a }}$ | ${ }^{39366}$ | ${ }^{176}$ | 1462 | 51. | so90. | ${ }^{97881.52}$ | 98074.7 | 10:5s9. | 13136.4 | 19998.97 | ${ }^{12332296}$ | 29758.15 | 7238 | 332 | F76. | 869 | 716 | 143339.48 | ${ }^{147864.42}$ | ${ }^{1.6006 .23}$ | 47068.55 | 901. | ${ }^{148765 .}$ | 406. | 202. | ${ }^{116}$ | 80. | sol | ${ }^{331}$. | s92 | 515.94 | 141207 |
| $2{ }^{10629.0}$ | 21485.22 | ${ }^{31718}$ | A1307.6 | 5334.47 | ${ }_{473260} 7$ | 7\%ד36.49 | ${ }_{\text {s5773,32 }}$ | 94908.30 | $10^{10372.55}$ | 112009 | ${ }^{119765.71}$ | 1277094.771 | ${ }^{133878823}{ }^{1}$ | ${ }^{137663.4}$ | ${ }^{1209822.21}$ | ${ }_{123335.55}^{1}$ | ${ }^{14658.189} 1$ | ${ }_{\text {148386.28 }}$ | ${ }_{\text {151061.20 }}$ | ${ }_{152901.84}$ | 115659.36 | ${ }_{15 \text { 2572.31 }}$ | 157218.30 | 158888.36 | $1{ }^{15927.45}$ | ${ }_{159991.37}$ | ${ }_{16663733}$ | ${ }_{16152,48}$ | 161552.51 | ${ }^{161852.19}$ | ${ }^{1593}$ | ${ }^{1568}$ |  |  |
| $3^{100683}$ | ${ }^{21481.156}$ | ${ }^{32350}$, | ${ }^{12590.3}$ |  |  | 891.93 | 89825.47 | 99664.09 | 88.37 | 117955.59 | ${ }^{1228277.80} 1$ | ${ }_{13398.54 .}$ | ${ }^{142377.99}$ | 145950.al | 129908.5 | 152405.7 | ${ }_{15334.17}$ | ${ }^{158041.56}$ | 1 160416.12 | 112850.961 | 11613929 | ${ }_{1}^{166077,81}$ | 126778.59 | 128878.52 | 116791.37 | ${ }^{\text {ITOEs5.31 }}$ | ${ }^{171466.13}$ | ${ }_{122100}$ | ${ }^{12259}$ | ${ }^{178301.20}$ | 1704 | 1678 | 16533.58 |  |
| 10612.63 | ${ }^{21513.613}$ | ${ }^{32392.35}$ | 13280.49 | 6693.94 | 68879.95 8 | ${ }^{82013.10}$ | ${ }^{9341.90}$ | 10141.12 | 86.07 | ${ }^{123832.09}$, | ${ }^{32182} 2.14$ | 14076.1991 | ${ }^{14553821} 1$ |  | ${ }^{157255}$ | $1{ }^{160093.74}$ | ${ }^{168229.3}$ |  | ${ }^{168873.13}$ |  |  | ${ }^{1786181.15}$ | ${ }^{178846.64}$ |  | ${ }^{18055}$ | ${ }^{181619.66}$ |  | ${ }_{18337.28}$ |  |  |  |  |  |  |
| 51.106329 | ${ }^{21,665.373}$ | 3234.85 | 43234.99 |  | - |  | 96619.43 | 108012.11 | ${ }^{188706.05}$ | ${ }^{12877^{2} .03}$, | 28179.8. | ${ }^{147726.13}$. 1 | ${ }_{15}^{15350.933} 1$ | 168386.0 | 12.6889 .97 | 128894.201 | ${ }_{1722220.21}{ }^{1}$ | ${ }_{\text {173706. } 50}$ | 178565.65 | 18.132 .99 | ${ }^{18370727}$ | ${ }^{1858785.08}$ | ${ }_{187850.27}$ | 18987.4 .97 | ${ }^{19071}$ | ${ }^{19996226}$ | ${ }^{19304}$ | 193956.28 | ${ }_{197726.833^{1}}$ | ${ }_{195363.36}$ | ${ }^{1926}$ | 1899 | 18877.4 .55 |  |
| ${ }^{6}$-1022 |  |  |  |  |  |  | 99840.01 |  |  |  | ${ }^{123385.75}{ }^{1}$ |  |  |  |  | ${ }^{17665}$ | ${ }_{\text {180337. } 8^{3}}$ |  |  | ${ }^{19053}$ |  |  |  |  | 2011 |  | ${ }^{213388}$ | ${ }^{20484}$ |  |  |  |  |  |  |
| ${ }^{10618.3}$ | ${ }^{21468 .}$ | ${ }^{32369}$ | ${ }_{\text {\|2323.6 }}{ }^{\text {a }}$ | 7772 | 22004.4 | 87045.6 | ${ }^{100815.90}$ | ${ }^{113741.69}$ | ${ }^{128595.30}$ | ${ }^{137316.44}$ | ${ }^{148013.19}$ | ${ }^{1588033.39}$ | ${ }^{167848.211^{1}}$ | ${ }^{173621.6}$ | 127099,93 | 183380.0 | ${ }^{1888325.511}$ | ${ }_{1}^{19242} .85$ | ${ }^{196085.68}$ | 193974.5 | ${ }^{202324.2}$ | 291946.12 | ${ }^{207332.17}$ | 20944.33 | ${ }^{21132273}$ | ${ }^{212947.97}$ | 214385.0.3 | 215631.92 | 216711 | ${ }^{217612.1}$ | ${ }^{21474}$ | 2118 |  |  |
| ${ }^{5} 10622$ |  |  |  |  |  | ${ }^{7085} 5.53$ | - |  |  | 120657.45 |  |  |  | 1793 | 1853 |  | ${ }^{195332}$ |  | 204021.51 | 207 |  | ${ }^{213912.23}$ |  | 218928 |  | ${ }^{2228}$ | ${ }^{224562.35}$ |  | ${ }^{27272} .3 .312$ |  |  |  |  |  |
| ${ }^{9.10627 .3}$ | , | ${ }^{32350.7}$ | ${ }^{133231.60}$ | ${ }^{\text {7772 }}$ | 2416.01 | ${ }^{\text {s7003 } 3.85}$ | ${ }^{101757.41}$ | ${ }^{16600.67}$ | ${ }^{130155.75}$ | 113313.4 | ${ }^{152525.61}{ }^{1}$ | 16662.8 | ${ }_{7 \text { 7386.22 }}$ | 84663.5. | ${ }^{191125.16}$ | ${ }^{197950.133^{2}}$ | ${ }^{202436.3}$ | ${ }^{207371.7}$ | 211882 | 21586 | ${ }^{219521}$ | ${ }^{222812.12}$ | ${ }^{25566.31}{ }^{2}$ |  |  |  |  |  |  |  |  |  |  |  |
| ${ }^{10} 10613.5$ |  | 32373.4. | ${ }_{13273.22}$ |  | ${ }^{72146} 298$ | $8{ }^{87104.70}$ | 101736.63 | ${ }^{116355.28}$ | ${ }^{131021.4}$ | A776.3. |  | $69882.277^{1}$ | ${ }_{181296.63}{ }^{1}$ | ${ }^{189225.9}$ | ${ }^{198333.3}$ | 202754.72 | 208 | 213972 | 218872 | ${ }^{23228}$ | ${ }^{227277 .}$ | 23022 | ${ }^{234224.388}$ | ${ }^{23720}$ | ${ }^{2398}$ | ${ }^{21221}$ | ${ }^{241433}$ | ${ }^{2462020}$ | ${ }^{247858}$ | ${ }^{212928}$ | ${ }^{21622}$ | ${ }^{243198.412}$ |  |  |
| 11.1066 .12 |  | ${ }^{32347}$ |  |  |  |  | 101767.28 | ${ }^{11643}$ | . 23 | ${ }^{1.5733}$ |  |  | 184601 |  | 200095.97 | 207951.4 |  | ${ }^{220275}$ |  | ${ }^{23052}$ |  | ${ }^{2393}$ |  | ${ }^{245965}$ | ${ }^{218500}$ | 22.1588 | 223975 | 256072 |  |  |  |  |  |  |
| $22^{10665.3}$ | ${ }^{21994.88}$ | ${ }^{33272}$ | ${ }^{13250}$ | 7815.5 | 244.598 | 8711 | ${ }^{101765.71}$ | ${ }^{11638}$ | ${ }_{131135.81}$ | ${ }^{145689} .2$ | ${ }_{160353,921}$ | 177112.4. | ${ }^{187709.6}$ | ${ }^{1963}$ | 207837.96 | ${ }^{212343.64}$ | 21951.78 | ${ }^{226036}$ | ${ }^{231997 .}$ | ${ }^{237355}$ | ${ }^{21225}$ | ${ }^{26688}$ | ${ }^{206721}$ | ${ }^{253382}$ | ${ }^{256788}$ | 226068. | 223318 | 2657172.26 | 2678 | ${ }^{268555.97}$ | ${ }^{2665}$ | ${ }^{263319.54}$ | 260205.80 |  |
| ${ }^{13} 12055$ |  |  |  |  |  |  |  |  |  | 14.5813 .02 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $26492.31{ }^{2}$ |  |  |  |  |  |  |  |  |
| ${ }^{14} 10.1087 .577^{2}$ | ${ }^{11480.613}$ | ${ }^{32386}$. | ${ }^{13329}$ | 7825. | ${ }^{2246.95} 8$ | ${ }^{87322.5}$ | ${ }^{1018809}$ | ${ }^{116428.74}$ | ${ }^{3111029}$ | ,773. | 16012.5 | 5086,2 | 189788. | r23.0 | ${ }^{210 \text { cse6.6. }}$ | ${ }^{219877.61}$ | ${ }^{228235.6}$ | ${ }^{235098}$ | ${ }^{222788}$ | 29876. | 255232. | 26060.4 .3 |  | ${ }^{265888.56}$. | 273916.33 | 27 27560.4. 2 | 288889.592 | 283835.202 |  |  |  |  |  |  |
| ${ }^{1510038 .}$ | 21474.653 | ${ }^{32355}$. | \|3327.2 | ${ }^{7796}$ | ${ }^{22464.168}$ | ${ }^{87992} 26$ | ${ }_{1017774.85}$ | ${ }^{11644563}$ | 10466 | ${ }^{145764.4}$ | 60014.2. ${ }^{1}$ | ${ }^{153.6}$ | ${ }^{189697}$ | $201622^{14}$ | ${ }^{123785.612}$ | ${ }^{222289}$ | ${ }^{231344}$ | ${ }^{239777}$ | 227460 | ${ }^{254999.55}$ | ${ }^{260977}$ | ${ }^{268890}$ | 272292.17 | ${ }^{277169.13}$ | ${ }^{2816}$ | ${ }^{2356}$ | ${ }^{2893}$ |  | ${ }^{2955}$ | ${ }^{2932}$ |  |  |  |  |
| 16.10610 |  | ${ }^{32336}$ | 1332 | 779 |  | 8879.7 .11 | ${ }^{101745.57}$ | 116330.04 | .96 | ${ }^{14571.5}$ | .60359.55 | 1750 | 189671.9 | 201627.02 | ${ }^{213330.48}$ | 7.47 | 23405. | 243180.1 | ${ }^{251557}$ | 2592 | 266333 | ${ }^{27281}$ | ${ }^{2887}$ | 284129.73 | 288034 | ${ }^{293487}$ | 297571 | ${ }^{301237}$ | ${ }^{304353}$ |  |  |  |  |  |
| ${ }^{17}{ }^{10629.15}$ | 21504.93 | ${ }^{3230}$ | ${ }^{13220}$ | 7855 | 2505.22 | ${ }^{8766.54} 1$ | ${ }^{101183.46}$ | ${ }^{116773,82}$ | 412222 | 145870.2 | (136.50 | 17 17006.1. | 18871.7 .7 | 20.188 .6 | ${ }^{213836.3}$ | ${ }^{22503,6}$ | 23586. | ${ }^{265791.54}$ | 25192 | 223367 | 27107 | 278063 | ${ }^{284555}$ | O4. | ${ }^{295820}$ | ${ }^{300698}$ | 335113 | ${ }^{30915}$ | ${ }^{312792}$ | ${ }^{31604}$ | ${ }^{312685}$ | ${ }^{30923}$ |  |  |
| $1{ }^{18} 1.10615 .4$ | ${ }^{11486.343}$ |  |  |  |  |  | 1018 | 116652.22 |  | ${ }^{145757.4}$ | ${ }^{160421.81}$ |  | ${ }^{189744}$ | 20.692 .64 | 2134 | ${ }^{22513.15 .5}$ | 238661.4.4 | ${ }^{247620.60}$ | 257 |  |  | 22828 |  |  | ${ }^{30253}$ | ${ }^{30761}$ | ${ }^{312535}$ | ${ }^{31697}$ |  |  |  |  |  |  |
| ${ }^{19} 120605$. |  | ${ }^{32365}$ | ${ }^{13322.9}$ |  |  |  |  | 116039.91 | S0. |  |  | 904.e | s970.9 | [622.13 | 213400.6 | ${ }^{225165.21}$ | 238588.60 | ${ }^{2146818.3}$ | , |  | ${ }^{278525}$ |  | ${ }^{29471}$ | ${ }^{30176}$ | ${ }^{3882} 2$ | ${ }^{31455}$ | ${ }^{31955}$ | ${ }^{32444}$ | 3288 |  |  |  |  |  |
| 20.10646 .81 | 21.509 .323 | ${ }^{32380}$. | 18394.0 | 7866.4 | ${ }^{2468.878} 8$ | ${ }_{87128.29}$ | ${ }^{10178}$ | 116838.69 | 1095.9 | 757.1. | 375 | ${ }^{175034.24}$ | 859.3 | 225.3 | ${ }^{213380.3}$ | ${ }^{225337.74 / 2}$ | 236782 | ${ }^{28853776}$ | 266334 | ${ }^{7112}$ | 28.1063 | 2915 | ${ }^{298547.285}$ | 33625 | ${ }^{13139}$ | ${ }^{31975}$ | ${ }^{32566}$ | 331011 | ${ }^{335876}$ | 0322 | 687 |  |  |  |
| $22^{12062}$ | 11488.013 | 3230.62 | 13325.7 |  |  | ${ }^{7616.961}$ | 1017 | 1116384.72 | $2^{13135050.04}$ | 14.573 .45 | \|6040.51. 1 | 1750 |  | ${ }^{201699.46}$ |  | ${ }^{220072.5 .5}$ | 2367 | ${ }^{24850.78}$ | 2001 |  |  | ${ }^{2926}$ |  |  |  | ${ }^{32488}$ | ${ }^{33138}$ | ${ }^{337302}$ | 1287 | 3753 |  |  |  |  |
| 2.106 |  | ${ }^{23929.78}$ |  |  |  |  | ${ }^{101800}$ |  | ${ }_{31081}$ | 4572s |  | 78039.0 | g971: | . 624 | 213368.9 | ${ }^{225071.4}$ | ${ }^{26874}$ | ${ }^{2184787.72}$ | 26020276 | ${ }^{271932 .}$ | 283627. | ${ }^{29488}$ | ${ }^{304483.25}$ | ${ }^{313600}$ | ${ }^{321999}$ | 329660.93 | ${ }^{36871}$ | ${ }^{333240}$ |  |  |  | 377502 |  |  |
| ${ }^{23}{ }^{106}$ | ${ }^{21507.07}{ }^{3}$ | ${ }_{32146.76}$ | ${ }^{1322}$ | 7822.22 | 2492.808 | ${ }_{87158.30} 1$ | 101817.12 | 116 | ${ }^{131162288}$ | ${ }^{115771.41}{ }^{1}$ | 16045.261 | ${ }^{175119.62}{ }^{11}$ | gryal | 20.649 .02 2 | ${ }^{213354.5}$ | ${ }^{225064.3}$ | ${ }^{236727.29}$ | 218810.37 | ${ }^{260121.33}$ | ${ }^{271811.1}$ | ${ }^{283522.2}$ |  | ${ }^{305885} 5$ | $\underbrace{3525}_{315922}$ | ${ }^{325077.26} 3$ | ${ }^{333050540}$ | ${ }^{31109758.58}$ |  | ${ }^{355994}$ | 36098 | ${ }^{335705.7}$ | ${ }_{3}^{35339}$ |  |  |
| ${ }^{24} 1.10611 .1$ |  | ${ }^{3232}$ | 13233,72 |  |  | 87068.11 | ${ }^{101737.44}$ | ${ }^{11638}$ | ${ }^{130999.55}$ | ${ }^{145656 .}$ | ${ }^{160312.8}$ | 17 17976.3. | 18964.4 | - | ${ }^{213281.5}$ | 22987.1 .3 | 23669.1. | ${ }^{244419,8}$ | ${ }^{260146}$ | 271902 | 223305 | 295388 | 307017. | ${ }^{317855}$ | ${ }^{327777}$ | 33693.3: | 318350 | 33005. | 330161 | ${ }^{36663}$ |  |  |  |  |
| 2251.1062 .4 | ${ }^{21464.69} 3$ | 32354.7 | 13237.15 | ${ }^{7} 766.5$ | ${ }^{6.3}$ | 87600.33 | ${ }^{101724.46}$ | ${ }^{116882 .}$ | ${ }^{31120272}$ | ${ }_{145858.4}$ | 160339.8. | 1779989. | 189674.7 | 201620.2 | ${ }^{12334.072}$ | ${ }^{22503233^{2}}$ | 238677.3 | ${ }^{218825.585}$ | 260134. | 271833 | 283548. | 29521. | 306892. | ${ }^{318820}$ | ${ }^{329683,7}$ | ${ }^{339432.4}$ | 34852 | 356952. | 386472 | 327770.1 | 388399 | 386744 | ${ }^{36107728}$ |  |
| 26100 |  | ${ }^{23255.10}$ |  |  |  | ${ }_{711.3721}$ | 1017 | 116446.55 |  | ${ }^{115777.6}$ |  | 175017.97 |  | 201612.912 | [20.372 | ${ }^{2250393.35}$ | ${ }^{236884.6}$ | ${ }^{2185077.65}$ | 2 |  |  |  |  |  |  | ${ }^{3126}$ | ${ }_{351228}$ |  |  |  |  |  |  |  |
| ${ }^{1061}$ | $11482.17{ }^{3}$ | ${ }_{32369.82}$ | ${ }^{13258}$ | ssa |  | ${ }^{8731.98}$, | ${ }^{101798}$ | ${ }^{426}$ | ${ }^{31098.07}$ | 45774. | mi | 062.0 | 189760.0. | 11262.20 | ${ }^{213326.1 .1}$ | ${ }^{25506.8}$ | ${ }^{236711}$ | ${ }^{2188661.10}$ | 26021.25 | 27192 | 283669 | 295349 | 307778.8. | 318879 | ${ }^{335058 .} 3$ | 322254.9.9 | ${ }^{333079.3 .3}$ | ${ }^{362987}$ | ${ }_{37215}$ | 380388.7 | ${ }^{3727270 .}$ | ${ }^{37367}$ | 70096.913 ${ }^{3}$ |  |
| 28.106 | ${ }^{21502.56 .3}$ | 3200 | ${ }^{13352}$ |  | ${ }^{22506.19} 8$ | 88713.611 |  |  | 122.55 |  |  | ${ }^{195.3}$ | ${ }^{1897327}$ |  |  | ${ }^{225014.755^{2}}$ | ${ }_{236785.5}^{20}$ | ${ }^{2128824.32}$ | ${ }^{260125.584}$ | ${ }^{2711873.3}$ | ${ }^{2338538.2}$ | ${ }^{\text {255279.2. }}$ | ${ }^{30650.29}$ | 318777,67 | ${ }^{330655.88}$ | ${ }^{33215}$ | ${ }^{3338861.68}$ | ${ }^{36659}$ | ${ }^{374608}$ | ${ }^{38376}$ | ${ }^{33053}$ |  |  |  |
| ${ }^{29} 10031.18$ | , | ${ }^{32391}$ | ${ }^{437278.85}$ | 7858.6 | ${ }^{2001.89}$ | 87400.0. | ${ }^{101809.46}$ | 11640.14 | ${ }^{1319090.03}$ | ${ }^{145751.8}$ | 126011.8 | 17 17500.5 | 189683.9 | ${ }^{2011832.3}$ | ${ }^{213356.66}$ | ${ }^{2253393.3882}$ | ${ }^{238671.6}$ | ${ }^{218877.37}$ | 220173.2 | 271899 | ${ }^{283575}$ | ${ }^{2952566.1}$ | 30733s. ${ }^{\text {a }}$ | 318899. | ${ }^{33012822}$ | ${ }^{3212131.5}$ | ${ }^{3588814 .}$ | ${ }^{365334}$ | ${ }^{376332}$ | 388267. | ${ }^{33311}$ | ${ }^{379599}$ |  |  |
| ${ }^{30} 10688.3$ | 21486.4 | ${ }^{32381}$ | ${ }^{132559.4}$ |  | ${ }^{21667.41}$ | s7128. | ${ }^{101798.14}$ | 11649.31 | ${ }^{1311068.84}$ | 145864.7 | 160014.3 | ${ }^{175072.8 .8}$ | ${ }^{189713.1}$ | 201682.5 | ${ }^{2113393.41}$ | ${ }^{225164.45}{ }^{2}$ | ${ }^{236292.3}$ | ${ }^{218566.36}$ | 26032.4 | $22^{27234}$ | 283712: | 295830.04 | 307790.2. | 318855..9. | ${ }^{330555.8}$. | 342292.71 | 35400. | 356898 | 37731..4. | ${ }^{388245.66}$ | 8519 | ${ }_{38176.581}{ }^{\text {a }}$ |  |  |
| ${ }^{31} 10614.9$ | ${ }^{1488.373}$ | ${ }_{32376 .}$ | [3252, ${ }^{\text {a }}$ | ${ }^{7815}$ | 52. 55 | 87095.02 | 101739.93 | 11600321 | ${ }^{1310512.28}$ | ${ }^{145788.2}$ | 6033, 8 | ${ }^{\text {F2031.3 }}$ | 189668.21 | 201687.4 | ${ }^{213477.432}$ | ${ }^{225177.67}{ }^{2}$ | ${ }^{236872.002}$ | ${ }^{218555.43}$ | 220220.13 | 771959.1 | ${ }^{238223.7}$ | ${ }^{295330.8}$ | ${ }^{307773.95}$ | ${ }^{318752}$ | ${ }^{330122.23} 3$ | ${ }^{321260.83}$ | ${ }^{33392.0 .05}$ | 366699.55 | ${ }^{\text {377373.36 }}$ | 389107.69 | 38612 | ${ }^{38278}$ |  |  |
|  | 1493.86 3, |  | 13322.4. |  |  |  | 101789.11 | 11672.78 | 13110.7 | 14775. | comes. | Sori |  |  | 213348.70 |  |  | 18505.71 |  |  | 283756.14 |  |  | s97.35 | ${ }^{30672.74}$ | ${ }^{12226.34}$ |  |  |  | 281.4 |  |  |  |  |
| $3^{106}$ | 21500.473 | ${ }^{32363.55}$ | ${ }^{13272.02}$ |  |  | ${ }^{87378.82}$ 1 |  | 116425.87 | $7{ }^{1312094.59}$ | ${ }^{14579.49}$ | coms. | ${ }^{175132.51}{ }^{12}$ | ${ }^{1897788.866}$ 2 | 20173.44 | ${ }^{121339.38 .82}$ | 488 | ${ }^{236993}$ | ${ }_{2}^{218875558.8}$ | ${ }^{260472}$ | ${ }_{721}^{721}$ | 238585899 | ${ }^{295565}$ | 30290 | ${ }^{318979}$ | ${ }^{330}$ | ${ }^{323835.57}$ | 35412 | ${ }^{36588}$ | ${ }^{3773}$ | ${ }^{3389}$ |  | 383477.99 3 | 38024.15 3 |  |
| 12063 |  |  | ${ }_{1328,82}$ | 57850.31 | 72299.308 | 87.12 .471 | 1017 |  |  |  | 66375.69 |  | ${ }_{129682633}{ }^{2}$ | 201662.96 |  |  | , |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 11: Scenario 1 Architecture-Centric Platform Sum Net Value Per Period

## D. 2 Scenario 2

D.2.1 Native Web Platform Cost and Value

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) | Riskless Discount (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 | 0.05 |
| 2 | 27080 | 0 | 0 | 0 | 0.05 |
| 3 | 27080 | 0 | 0 | 0 | 0.05 |
| 4 | 27080 | 0 | 0 | 0 | 0.05 |
| 5 | 27080 | 50000 | 0 | 0 | 0.05 |
| 6 | 27080 | 30000 | 0 | 0 | 0.05 |
| 7 | 27080 | 20000 |  | 0.05 |  |

Table 12: Scenario 2 Native Web Platform Cost and Value

|  | Elapsed Time |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 . \ddot{0} \\ & 0 \\ & \dot{0} \\ & 0 \end{aligned}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 1 | 159013.49 | -22537.08 | -24981.24 | -23889.80 | 16897.81 | 1894.17 | -5728.15 |
|  | 2 | 159003.09 | -23674.74 | -26234.34 | -25091.00 | 17892.04 | 2031.20 | -5988.47 |
|  | 3 | 159013.16 | -23687.25 | -27578.60 | -26354.24 | 18915.64 | 2192.07 | -6261.76 |
|  | 4 | 159068.24 | -23740.53 | -27575.07 | -27696.16 | 20006.41 | 2340.85 | -6538.79 |
|  | 5 | 159094.73 | -23766.83 | -27573.48 | -27712.40 | 21148.09 | 2516.95 | -6844.27 |
|  | 6 | 159063.05 | -23736.45 | -27583.68 | -27705.33 | 21139.96 | 2710.60 | -7151.29 |
|  | 7 | 159024.21 | -23697.77 | -27576.24 | -27712.38 | 21157.84 | 2702.34 | -7474.43 |

Table 13: Scenario 2 Native Web Platform Net Value Per Period

|  | Elapsed Time |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \vec{\rightharpoonup} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 1 | 159013.49 | 136476.41 | 111495.17 | 87605.37 | 104503.17 | 106397.34 | 100669.18 |
|  | 2 | 159003.09 | 135328.35 | 109094.00 | 84003.00 | 101895.04 | 103926.24 | 97937.77 |
|  | 3 | 159013.16 | 135325.92 | 107747.32 | 81393.08 | 100308.71 | 102500.78 | 96239.03 |
|  | 4 | 159068.24 | 135327.71 | 107752.64 | 80056.48 | 100062.89 | 102403.73 | 95864.94 |
|  | 5 | 159094.73 | 135327.90 | 107754.41 | 80042.01 | 101190.10 | 103707.05 | 96862.78 |
|  | 6 | 159063.05 | 135326.60 | 107742.92 | 80037.59 | 101177.56 | 103888.15 | 96736.86 |
|  | 7 | 159024.21 | 135326.45 | 107750.21 | 80037.82 | 101195.66 | 103898.01 | 96423.57 |

Table 14: Scenario 2 Native Web Platform Sum Net Value Per Period

## D.2.2 Open Source Platform Cost and Value

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) | Riskless Discount (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 | 0.05 |
| 2 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 3 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 4 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 5 | 27080 | 50000 | 0.3 | -0.04 | 0.05 |
| 6 | 27080 | 30000 | 0.3 | -0.04 | 0.05 |
| 7 | 27080 | 20000 | 0.3 | -0.04 | 0.05 |

Table 15: Scenario 2 Open Source Platform Cost and Value

|  | Elapsed Time |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\rightharpoonup}{\circ}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
|  | 1 | 159076.21 | -14866.21 | -17351.32 | -16595.91 | 22296.85 | 7531.86 | -46.84 |  |
|  | 2 | 159075.39 | -15624.01 | -18226.50 | -17430.25 | 23531.75 | 7960.31 | -18.66 |  |
|  | 3 | 159051.18 | -15601.92 | -19160.73 | -18316.32 | 24843.51 | 8401.52 | 5.43 |  |
|  | 4 | 159064.35 | -15613.29 | -19164.98 | -19254.41 | 26218.82 | 8871.51 | 38.34 |  |
|  | 5 | 159021.45 | -15569.01 | -19163.97 | -19255.95 | 27667.51 | 9357.32 | 67.29 |  |
|  | 6 | 159060.28 | -15607.69 | -19164.65 | -19257.63 | 27664.23 | 9909.18 | 92.84 |  |
|  | 7 | 159098.63 | -15645.56 | -19165.91 | -19254.74 | 27658.01 | 9910.58 | 139.62 |  |

Table 16: Scenario 2 Open Source Platform Net Value Per Period

|  | Elapsed Time |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 1 | 159076.21 | 144210.00 | 126858.68 | 110262.77 | 132559.62 | 140091.47 | 140044.63 |
|  | 2 | 159075.39 | 143451.38 | 125224.88 | 107794.63 | 131326.38 | 139286.69 | 139268.03 |
|  | 3 | 159051.18 | 143449.27 | 124288.54 | 105972.22 | 130815.73 | 139217.25 | 139222.67 |
|  | 4 | 159064.35 | 143451.05 | 124286.07 | 105031.66 | 131250.48 | 140121.98 | 140160.33 |
|  | 5 | 159021.45 | 143452.44 | 124288.47 | 105032.52 | 132700.03 | 142057.35 | 142124.64 |
|  | 6 | 159060.28 | 143452.60 | 124287.94 | 105030.32 | 132694.55 | 142603.73 | 142696.57 |
|  | 7 | 159098.63 | 143453.07 | 124287.16 | 105032.42 | 132690.43 | 142601.02 | 142740.64 |

Table 17: Scenario 2 Open Source Platform Sum Net Value Per Period

## D.2.3 Proprietary Platform Cost and Value

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) | Riskless Discount (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 | 0.05 |
| 2 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 3 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 4 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 5 | 27080 | 50000 | 0.1 | 1.24 | 0.05 |
| 6 | 27080 | 30000 | 0.1 | 1.24 | 0.05 |
| 7 | 27080 | 20000 | 0.1 | 1.24 | 0.05 |

Table 18: Scenario 2 Proprietary Platform Cost and Value

|  | Elapsed Time |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & 0 \end{aligned}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 1 | 159092.62 | -20034.89 | -22419.79 | -21442.34 | 68434.30 | 35952.98 | 16588.18 |
|  | 2 | 159025.84 | -20989.63 | -23555.26 | -22532.37 | 72154.39 | 37811.43 | 17511.17 |
|  | 3 | 159100.58 | -21063.22 | -24768.98 | -23665.39 | 76086.63 | 39748.73 | 18472.10 |
|  | 4 | 159112.76 | -21072.84 | -24757.30 | -24884.84 | 80248.11 | 41763.14 | 19397.23 |
|  | 5 | 159083.19 | -21047.05 | -24759.52 | -24878.31 | 84551.78 | 43981.92 | 20332.67 |
|  | 6 | 159001.30 | -20966.44 | -24758.49 | -24873.89 | 84583.45 | 46324.23 | 21358.68 |
|  | 7 | 159006.67 | -20971.26 | -24761.16 | -24877.91 | 84624.78 | 46261.44 | 22568.04 |

Table 19: Scenario 2 Proprietary Platform Net Value Per Period

|  | Elapsed Time |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\rightharpoonup}{\circ} \cdot \stackrel{2}{0}$ |  | 1 | 3 | 4 | 5 | 6 | 7 |  |  |
|  | 1 | 159092.62 | 139057.73 | 116637.94 | 95195.60 | 163629.90 | 199582.89 | 216171.07 |  |
|  | 2 | 159025.84 | 138036.21 | 114480.95 | 91948.58 | 164102.96 | 201914.40 | 219425.57 |  |
|  | 3 | 159100.58 | 138037.36 | 113268.38 | 89602.99 | 165689.62 | 205438.34 | 223910.45 |  |
|  | 4 | 159112.76 | 138039.92 | 113282.62 | 88397.78 | 168645.89 | 210409.03 | 229806.26 |  |
|  | 5 | 159083.19 | 138036.14 | 113276.63 | 88398.32 | 172950.09 | 216932.01 | 237264.68 |  |
|  | 6 | 159001.30 | 138034.87 | 113276.38 | 88402.49 | 172985.94 | 219310.17 | 240668.84 |  |
|  | 7 | 159006.67 | 138035.40 | 113274.25 | 88396.34 | 173021.11 | 219282.55 | 241850.59 |  |

Table 20: Scenario 2 Proprietary Platform Sum Net Value Per Period

## D.2.4 Open Source Before Proprietary Platform Cost and Value

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) | Riskless Discount (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 | 0.05 |
| 2 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 3 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 4 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 5 | 27080 | 50000 | 0.3 | -0.04 | 0.05 |
| 6 | 27080 | 30000 | 0.3 | -0.04 | 0.05 |
| 7 | 27080 | 20000 | 0.3 | -0.04 | 0.05 |
| 8 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 9 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 10 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 11 | 27080 | 50000 | 0.1 | 1.24 | 0.05 |
| 12 | 27080 | 30000 | 0.1 | 1.24 | 0.05 |
| 13 | 27080 | 20000 | 0.1 | 1.24 | 0.05 |

Table 21: Scenario 2 Open Source Before Proprietary Platform Cost and Value


Table 22: Scenario 2 Open Source Before Proprietary Platform Net Value Per Period

|  | Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|  | 1 | 159040.79 | 144205.63 | 126856.09 | 110267.45 | 132578.06 | 140108.35 | 140064.81 | 122922.82 | 106203.24 | 90283.69 | 140081.58 | 166337.87 | 178335.22 |
|  | 2 | 159105.62 | 143454.22 | 125224.02 | 107789.27 | 131323.42 | 139287.20 | 139255.26 | 121235.43 | 103661.06 | 86922.72 | 139439.05 | 167109.76 | 179799.62 |
|  | 3 | 159152.90 | 143454.21 | 124289.40 | 105977.99 | 130805.39 | 139214.47 | 139224.72 | 120280.16 | 101811.44 | 84222.36 | 139582.28 | 168787.90 | 182175.16 |
|  | 4 | 159050.70 | 143451.52 | 124287.00 | 105034.02 | 131265.32 | 140132.31 | 140166.66 | 120250.92 | 100841.58 | 82353.04 | 140688.75 | 171500.07 | 185662.61 |
|  | 5 | 159054.60 | 143450.43 | 124286.21 | 105026.48 | 132716.51 | 142076.85 | 142137.58 | 121199.63 | 100804.15 | 81383.99 | 142897.88 | 175370.92 | 190310.16 |
|  | 6 | 159079.46 | 143452.80 | 124287.14 | 105037.94 | 132715.15 | 142620.38 | 142708.84 | 120697.74 | 99268.06 | 78855.49 | 143757.73 | 177908.01 | 193649.00 |
|  | 7 | 159052.97 | 143451.90 | 124289.57 | 105040.03 | 132721.57 | 142630.17 | 142766.68 | 119625.59 | 97122.32 | 75679.55 | 144115.33 | 180036.50 | 196654.68 |
|  | 8 | 159058.01 | 143450.19 | 124286.22 | 105028.13 | 132700.62 | 142610.68 | 142747.18 | 118420.89 | 94773.41 | 72245.29 | 144427.04 | 182239.50 | 199700.89 |
|  | 9 | 159071.66 | 143455.24 | 124290.76 | 105042.87 | 132726.56 | 142628.00 | 142762.08 | 118435.92 | 93577.18 | 69906.81 | 145998.72 | 185769.93 | 204171.05 |
|  | 10 | 159069.38 | 143451.60 | 124286.98 | 105038.17 | 132714.58 | 142610.28 | 142752.49 | 118426.44 | 93571.64 | 68692.02 | 148946.07 | 190729.59 | 210084.00 |
|  | 11 | 159042.25 | 143451.82 | 124283.55 | 105028.43 | 132707.90 | 142603.40 | 142741.41 | 118415.26 | 93557.64 | 68678.54 | 153291.21 | 197149.94 | 217480.03 |
|  | 12 | 159031.85 | 143452.50 | 124286.21 | 105030.07 | 132694.77 | 142597.70 | 142737.59 | 118411.50 | 93560.17 | 68684.77 | 153282.74 | 199555.38 | 220950.04 |
|  | 13 | 159050.86 | 143453.01 | 124289.68 | 105036.42 | 132709.67 | 142618.65 | 142750.69 | 118425.05 | 93566.92 | 68685.72 | 153291.97 | 199573.32 | 222121.61 |

Table 23: Scenario 2 Open Source Before Proprietary Platform Sum Net Value Per Period

## D.2.5 Proprietary Before Proprietary Platform

| Period of Completion | Cost | Value (\$) | Cost Reduction (\%) | Added Value (\%) | Riskless Discount (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 162480 | 0 | 0 | 0.05 |
| 2 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 3 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 4 | 27080 | 0 | 0.1 | 0 | 0.05 |
| 5 | 27080 | 50000 | 0.1 | 1.24 | 0.05 |
| 6 | 27080 | 30000 | 0.1 | 1.24 | 0.05 |
| 7 | 27080 | 20000 | 0.1 | 1.24 | 0.05 |
| 8 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 9 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 10 | 27080 | 0 | 0.3 | 0 | 0.05 |
| 11 | 27080 | 50000 | 0.3 | -0.04 | 0.05 |
| 12 | 27080 | 30000 | 0.3 | -0.04 | 0.05 |
| 13 | 27080 | 20000 | 0.3 | -0.04 | 0.05 |

Table 24: Scenario 2 Proprietary Before Open Source Platform Cost and Value

|  |  | Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|  |  | 1 | 159041.27 | -19987.37 | -22420.39 | -21444.04 | 68439.74 | 35942.07 | 16611.61 | -12805.73 | -12925.26 | -12314.83 | 16137.67 | 5378.72 | -165.76 |
|  |  | 2 | 159059.63 | -21023.52 | -23554.76 | -22532.66 | 72140.74 | 37837.91 | 17506.23 | -13456.32 | -13584.32 | -12943.78 | 17034.25 | 5701.44 | -154.09 |
|  |  | 3 | 159110.33 | -21072.38 | -24764.48 | -23659.44 | 76110.23 | 39741.69 | 18395.07 | -14151.36 | -14279.65 | -13605.91 | 17968.71 | 6028.95 | -134.20 |
|  | - | 4 | 159051.35 | -21014.85 | -24757.02 | -24877.07 | 80246.91 | 41792.06 | 19345.97 | -14897.21 | -15006.88 | -14300.44 | 18971.78 | 6371.95 | -115.72 |
|  |  | 5 | 159036.40 | -21000.96 | -24763.83 | -24881.98 | 84598.62 | 43899.22 | 20324.60 | -15687.79 | -15771.41 | -15029.79 | 20003.06 | 6752.95 | -93.12 |
|  | Ea | 6 | 159095.38 | -21056.43 | -24762.33 | -24873.52 | 84594.35 | 46306.00 | 21356.68 | -16500.41 | -16574.45 | -15795.69 | 21108.30 | 7135.71 | -72.44 |
|  | $\stackrel{\square}{0}$ | 7 | 159081.08 | -21042.38 | -24762.62 | -24871.05 | 84621.54 | 46290.04 | 22468.25 | -17375.53 | -17409.99 | -16592.69 | 22314.39 | 7519.25 | -41.29 |
|  | - | 8 | 159038.02 | -21003.07 | -24759.19 | -24873.59 | 84575.30 | 46297.07 | 22626.72 | -18253.60 | -18291.44 | -17433.82 | 23517.02 | 7967.71 | -16.51 |
| A |  | 9 | 159066.13 | -21028.41 | -24758.25 | -24876.60 | 84629.44 | 46217.67 | 22600.95 | -18269.66 | -19233.92 | -18316.27 | 24827.42 | 8413.72 | 12.53 |
|  |  | 10 | 159054.11 | -21016.89 | -24766.25 | -24874.60 | 84582.83 | 46328.73 | 22572.04 | -18266.28 | -19233.43 | -19255.74 | 26226.90 | 8870.10 | 37.84 |
|  |  | 11 | 159081.08 | -21042.58 | -24763.70 | -24874.97 | 84539.70 | 46377.83 | 22593.73 | -18261.76 | -19231.02 | -19255.99 | 27667.67 | 9371.10 | 56.49 |
|  |  | 12 | 159053.30 | -21016.17 | -24765.23 | -24869.29 | 84607.64 | 46269.94 | 22611.67 | -18264.34 | -19233.98 | -19253.30 | 27670.95 | 9901.48 | 88.76 |
|  |  | 13 | 159083.19 | -21047.32 | -24763.32 | -24870.17 | 84566.21 | 46337.28 | 22584.29 | -18270.86 | -19230.14 | -19253.47 | 27669.30 | 9915.66 | 135.95 |

Table 25: Scenario 2 Proprietary Before Open Source Platform Net Value Per Period

|  |  | Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|  |  | 1 | 159041.27 | 139053.90 | 116633.51 | 95189.47 | 163629.21 | 199571.28 | 216182.89 | 203377.17 | 190451.91 | 178137.08 | 194274.75 | 199653.47 | 199487.71 |
|  |  | 2 | 159059.63 | 138036.11 | 114481.35 | 91948.68 | 164089.42 | 201927.33 | 219433.56 | 205977.24 | 192392.93 | 179449.15 | 196483.39 | 202184.84 | 202030.75 |
|  |  | 3 | 159110.33 | 138037.95 | 113273.47 | 89614.03 | 165724.26 | 205465.95 | 223861.02 | 209709.67 | 195430.02 | 181824.11 | 199792.82 | 205821.76 | 205687.56 |
|  | . 0 | 4 | 159051.35 | 138036.50 | 113279.48 | 88402.41 | 168649.33 | 210441.38 | 229787.35 | 214890.14 | 199883.26 | 185582.82 | 204554.60 | 210926.55 | 210810.83 |
|  | $\sim$ | 5 | 159036.40 | 138035.44 | 113271.60 | 88389.62 | 172988.25 | 216887.47 | 237212.07 | 221524.29 | 205752.88 | 190723.09 | 210726.14 | 217479.09 | 217385.97 |
|  |  | 6 | 159095.38 | 138038.95 | 113276.62 | 88403.10 | 172997.44 | 219303.45 | 240660.13 | 224159.71 | 207585.26 | 191789.58 | 212897.87 | 220033.58 | 219961.14 |
|  | $\stackrel{\rightharpoonup}{0}$ | 7 | 159081.08 | 138038.70 | 113276.08 | 88405.04 | 173026.57 | 219316.61 | 241784.86 | 224409.33 | 206999.33 | 190406.64 | 212721.02 | 220240.27 | 220198.98 |
|  | $\bigcirc$ | 8 | 159038.02 | 138034.95 | 113275.75 | 88402.16 | 172977.46 | 219274.53 | 241901.25 | 223647.65 | 205356.21 | 187922.39 | 211439.41 | 219407.12 | 219390.61 |
|  |  | 9 | 159066.13 | 138037.72 | 113279.47 | 88402.88 | 173032.32 | 219249.99 | 241850.94 | 223581.28 | 204347.36 | 186031.09 | 210858.51 | 219272.24 | 219284.77 |
| $\stackrel{\leftrightarrow}{\circ}$ |  | 10 | 159054.11 | 138037.22 | 113270.97 | 88396.37 | 172979.20 | 219307.93 | 241879.97 | 223613.69 | 204380.27 | 185124.53 | 211351.43 | 220221.53 | 220259.37 |
|  |  | 11 | 159081.08 | 138038.50 | 113274.80 | 88399.83 | 172939.53 | 219317.36 | 241911.10 | 223649.33 | 204418.31 | 185162.32 | 212829.99 | 222201.09 | 222257.58 |
|  |  | 12 | 159053.30 | 138037.13 | 113271.90 | 88402.61 | 173010.25 | 219280.19 | 241891.86 | 223627.52 | 204393.54 | 185140.24 | 212811.19 | 222712.66 | 222801.42 |
|  |  | 13 | 159083.19 | 138035.88 | 113272.55 | 88402.38 | 172968.59 | 219305.87 | 241890.16 | 223619.30 | 204389.16 | 185135.70 | 212805.00 | 222720.65 | 222856.60 |

Table 26: Scenario 2 Proprietary Before Open Source Platform Sum Net Value Per Period

## Appendix E Verification and Validation Data

## E. 1 Sensitivity Analysis

E.1.1 Cost Reduction

|  | Cost Reduction Coefficient |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|  | 1 | 5365.57 | 6945.27 | 8537.97 | 10113.24 | 11669.84 | 13231.45 | 14791.15 | 16334.68 | 17882.49 | 19419.20 |
|  | 2 | 5689.92 | 7359.65 | 9021.58 | 10680.12 | 12328.97 | 13974.35 | 15610.07 | 17235.69 | 18865.79 | 20478.89 |
|  | 3 | 6037.16 | 7795.86 | 9545.15 | 11286.91 | 13030.76 | 14746.98 | 16471.24 | 18181.90 | 19886.76 | 21598.57 |
|  | 4 | 6409.80 | 8248.31 | 10102.24 | 11922.66 | 13744.91 | 15578.87 | 17381.27 | 19176.48 | 20983.49 | 22771.46 |
|  | 5 | 6780.87 | 8738.14 | 10677.12 | 12609.49 | 14529.91 | 16442.51 | 18345.86 | 20247.12 | 22125.60 | 24003.48 |
|  | 6 | 7207.95 | 9252.05 | 11285.80 | 13325.19 | 15341.93 | 17359.23 | 19346.49 | 21352.20 | 23342.43 | 25305.46 |
|  | 7 | 7634.15 | 9795.07 | 11953.61 | 14079.73 | 16211.28 | 18322.82 | 20422.67 | 22517.69 | 24611.00 | 26677.07 |
|  | 8 | 8106.20 | 10369.60 | 12622.63 | 14875.50 | 17122.89 | 19326.02 | 21549.65 | 23758.96 | 25940.64 | 28121.05 |
|  | 9 | 8585.34 | 10992.77 | 13344.70 | 15728.31 | 18086.71 | 20420.01 | 22727.29 | 25056.95 | 27382.35 | 29650.96 |
|  | 10 | 9109.13 | 11630.67 | 14130.95 | 16602.05 | 19077.37 | 21557.21 | 24002.94 | 26420.19 | 28850.63 | 31281.24 |
|  | 11 | 9673.99 | 12303.82 | 14933.61 | 17542.55 | 20150.94 | 22746.70 | 25316.35 | 27862.92 | 30421.48 | 32964.79 |
|  | 12 | 10250.71 | 13018.40 | 15807.27 | 18528.33 | 21288.43 | 24007.69 | 26716.01 | 29386.40 | 32085.44 | 34742.39 |
|  | 13 | 10851.24 | 13783.38 | 16687.02 | 19605.68 | 22475.44 | 25316.08 | 28153.23 | 31008.96 | 33838.39 | 36630.15 |
|  | 14 | 11511.85 | 14599.63 | 17634.02 | 20701.98 | 23715.34 | 26730.83 | 29718.64 | 32710.48 | 35668.35 | 38632.29 |
|  | 15 | 12216.41 | 15451.19 | 18655.61 | 21855.97 | 25065.93 | 28207.97 | 31358.91 | 34479.65 | 37588.87 | 40680.51 |
|  | 16 | 12944.45 | 16353.99 | 19707.45 | 23071.32 | 26430.06 | 29765.13 | 33084.64 | 36372.09 | 39622.62 | 42893.38 |
|  | 17 | 13705.00 | 17280.97 | 20827.91 | 24397.87 | 27883.07 | 31406.55 | 34882.09 | 38331.63 | 41803.54 | 45203.26 |
|  | 18 | 14535.89 | 18298.39 | 22030.06 | 25740.25 | 29447.11 | 33133.31 | 36789.95 | 40447.44 | 44038.04 | 47642.67 |
|  | 19 | 15371.80 | 19365.50 | 23283.35 | 27203.16 | 31070.73 | 34943.93 | 38794.06 | 42625.36 | 46425.05 | 50205.66 |
|  | 20 | 16302.92 | 20461.93 | 24587.50 | 28708.24 | 32803.92 | 36868.28 | 40902.18 | 44929.15 | 48926.11 | 52918.63 |
|  | 21 | 17255.97 | 21643.97 | 25981.06 | 30302.61 | 34619.73 | 38856.99 | 43139.42 | 47362.58 | 51595.35 | 55724.21 |
|  | 22 | 18283.48 | 22901.24 | 27433.76 | 32019.28 | 36541.19 | 41056.68 | 45466.74 | 49906.10 | 54337.37 | 58710.45 |
|  | 23 | 19356.06 | 24178.53 | 28989.58 | 33744.30 | 38524.86 | 43216.68 | 47940.67 | 52582.39 | 57230.94 | 61875.40 |
|  | 24 | 20512.27 | 25571.80 | 30640.41 | 35643.72 | 40661.89 | 45576.32 | 50546.24 | 55414.15 | 60297.12 | 65162.47 |
|  | 25 | 21735.41 | 27040.19 | 32353.09 | 37620.69 | 42860.81 | 48043.41 | 53221.10 | 58372.09 | 63584.68 | 68631.81 |
|  | 26 | 22957.65 | 28605.42 | 34129.18 | 39659.29 | 45185.35 | 50683.72 | 56130.39 | 61546.18 | 66908.68 | 72296.87 |
|  | 27 | 24327.36 | 30210.20 | 36083.84 | 41882.92 | 47654.56 | 53362.27 | 59127.66 | 64787.71 | 70450.74 | 76100.74 |
|  | 28 | 25732.13 | 31918.89 | 38082.11 | 44149.33 | 50200.29 | 56267.01 | 62233.43 | 68258.21 | 74163.02 | 80101.69 |
|  | 29 | 27225.56 | 33723.25 | 40124.20 | 46570.98 | 52916.31 | 59245.70 | 65583.45 | 71867.32 | 78094.64 | 84362.21 |
|  | 30 | 28819.25 | 35652.04 | 42446.14 | 49193.18 | 55849.25 | 62522.44 | 69151.79 | 75772.84 | 82330.89 | 88856.18 |

Table 27: Period Net Value for Cost Reduction Coefficient Sensitivity Test

|  | Cost Reduction Coefficient |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \ddot{\partial} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|  | 1 | 324929.00 | 443549.83 | 561615.52 | 679078.30 | 795829.08 | 912249.53 | 1027952.96 | 1143119.54 | 1257810.49 | 1371828.49 |
|  | 2 | 342108.42 | 466558.30 | 590456.29 | 713713.69 | 836255.84 | 958300.16 | 1079808.23 | 1200627.15 | 1320947.61 | 1440668.25 |
|  | 3 | 357367.96 | 487712.61 | 617591.70 | 746694.75 | 875116.19 | 1003013.49 | 1130206.07 | 1256851.98 | 1382855.68 | 1508278.63 |
|  | 4 | 370763.75 | 507198.57 | 642962.88 | 777984.02 | 912332.56 | 1046153.02 | 1179209.11 | 1311603.90 | 1443471.87 | 1574652.49 |
|  | 5 | 382640.25 | 525249.80 | 666949.30 | 808010.26 | 948310.67 | 1087954.28 | 1226838.22 | 1365141.77 | 1502775.56 | 1639729.20 |
|  | 6 | 393204.59 | 541868.57 | 689622.48 | 836579.99 | 982810.22 | 1128285.48 | 1273138.28 | 1417286.60 | 1560710.52 | 1703482.87 |
|  | 7 | 402424.35 | 556962.58 | 710875.88 | 863674.72 | 1015812.68 | 1167405.98 | 1318020.88 | 1468012.66 | 1617236.38 | 1765771.05 |
|  | 8 | 410016.21 | 570675.36 | 730547.87 | 889428.20 | 1047520.10 | 1204791.75 | 1361442.05 | 1517199.71 | 1672293.81 | 1826675.68 |
|  | 9 | 415927.51 | 582626.63 | 748422.56 | 913477.50 | 1077458.38 | 1240810.16 | 1403153.45 | 1564860.29 | 1725754.88 | 1885917.93 |
|  | 10 | 420226.41 | 592984.42 | 764817.21 | 935663.16 | 1105801.17 | 1274960.11 | 1443369.93 | 1610838.33 | 1777626.85 | 1943523.78 |
|  | 11 | 422819.85 | 601475.48 | 779484.05 | 956300.69 | 1132319.47 | 1307444.96 | 1481637.19 | 1655085.18 | 1827699.17 | 1999489.00 |
|  | 12 | 423419.93 | 608219.41 | 792188.79 | 974966.10 | 1157035.68 | 1338054.56 | 1518200.22 | 1697463.00 | 1875938.81 | 2053543.92 |
|  | 13 | 422127.29 | 613129.69 | 802800.23 | 991788.29 | 1179709.50 | 1366674.35 | 1552726.86 | 1737876.37 | 1922235.95 | 2105782.72 |
|  | 14 | 421128.96 | 617882.72 | 813436.78 | 1008101.57 | 1201774.20 | 1394502.57 | 1586235.44 | 1777041.07 | 1966985.36 | 2156076.22 |
|  | 15 | 420317.88 | 622549.53 | 823703.32 | 1024118.72 | 1223113.79 | 1421321.69 | 1618496.62 | 1814749.92 | 2010083.57 | 2204497.32 |
|  | 16 | 419941.82 | 627580.44 | 833916.39 | 1039376.45 | 1243779.51 | 1447127.86 | 1649588.22 | 1850948.74 | 2051449.10 | 2251050.93 |
|  | 17 | 419910.89 | 632343.56 | 843940.51 | 1054326.18 | 1263767.24 | 1472060.68 | 1679286.24 | 1885651.53 | 2090965.74 | 2295384.40 |
|  | 18 | 420106.51 | 637567.09 | 853646.26 | 1068850.44 | 1282801.17 | 1495803.96 | 1707699.86 | 1918568.92 | 2128560.21 | 2337599.88 |
|  | 19 | 420576.93 | 642782.75 | 863194.06 | 1082777.55 | 1301055.49 | 1518375.05 | 1734644.98 | 1949901.46 | 2164117.90 | 2377480.38 |
|  | 20 | 421717.09 | 647811.57 | 872393.73 | 1095951.39 | 1318420.34 | 1539779.59 | 1760027.27 | 1979332.07 | 2197581.34 | 2414920.63 |
|  | 21 | 423319.51 | 652949.35 | 881317.01 | 1108675.97 | 1334800.88 | 1559878.79 | 1783906.23 | 2006871.94 | 2228769.34 | 2449756.47 |
|  | 22 | 425102.66 | 658211.34 | 890007.21 | 1120797.08 | 1350287.51 | 1578734.89 | 1806072.39 | 2032356.99 | 2257639.51 | 2481891.84 |
|  | 23 | 427070.14 | 663538.43 | 898272.29 | 1131951.37 | 1364665.83 | 1596101.55 | 1826409.10 | 2055730.71 | 2283964.57 | 2511262.51 |
|  | 24 | 430006.87 | 668785.04 | 906300.62 | 1142789.80 | 1377961.32 | 1611913.26 | 1844933.06 | 2076827.00 | 2307695.06 | 2537545.66 |
|  | 25 | 432988.99 | 673952.68 | 913911.69 | 1152665.01 | 1390003.67 | 1626288.78 | 1861477.54 | 2095559.99 | 2328706.96 | 2560716.68 |
|  | 26 | 436547.66 | 679660.02 | 921214.56 | 1161640.86 | 1400893.27 | 1638961.20 | 1875985.78 | 2111896.01 | 2346746.77 | 2580608.68 |
|  | 27 | 440447.13 | 684920.17 | 927891.79 | 1169815.24 | 1410455.03 | 1649890.40 | 1888269.41 | 2125561.76 | 2361785.78 | 2597002.36 |
|  | 28 | 445094.03 | 690229.06 | 934383.53 | 1177123.71 | 1418643.03 | 1659066.72 | 1898289.22 | 2136526.13 | 2373621.48 | 2609746.23 |
|  | 29 | 447829.01 | 693973.84 | 938632.78 | 1182063.38 | 1424260.94 | 1665247.61 | 1905178.40 | 2144024.10 | 2381760.43 | 2618528.78 |
|  | 30 | 449672.78 | 695957.45 | 941103.30 | 1184612.65 | 1427197.49 | 1668552.83 | 1908768.67 | 2147919.02 | 2385986.61 | 2623053.08 |

Table 28: Period Net Value for Cost Reduction Coefficient Sensitivity Test
E.1.2 Value Addition

|  | Value Addition Coefficient |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|  | 1 | 5360.47 | 7497.03 | 9648.51 | 11801.41 | 13980.17 | 16144.25 | 18335.85 | 20516.60 | 22713.73 | 24948.20 |
|  | 2 | 5683.80 | 7957.49 | 10205.24 | 12486.32 | 14778.58 | 17071.48 | 19380.60 | 21688.29 | 24016.10 | 26347.83 |
|  | 3 | 6044.96 | 8407.44 | 10795.91 | 13218.38 | 15615.14 | 18048.04 | 20457.89 | 22917.83 | 25377.95 | 27830.28 |
|  | 4 | 6403.58 | 8913.74 | 11436.69 | 13966.74 | 16498.17 | 19045.68 | 21623.31 | 24230.25 | 26807.53 | 29415.24 |
|  | 5 | 6790.79 | 9436.33 | 12112.42 | 14777.62 | 17436.49 | 20157.75 | 22859.68 | 25588.24 | 28314.60 | 31057.58 |
|  | 6 | 7205.23 | 10003.28 | 12807.16 | 15607.56 | 18459.19 | 21296.93 | 24161.54 | 27024.19 | 29921.73 | 32796.24 |
|  | 7 | 7637.07 | 10584.00 | 13566.22 | 16520.33 | 19523.60 | 22506.16 | 25534.15 | 28587.54 | 31602.44 | 34672.83 |
|  | 8 | 8115.10 | 11206.70 | 14346.38 | 17466.60 | 20641.87 | 23789.67 | 26996.31 | 30170.95 | 33414.25 | 36637.56 |
|  | 9 | 8602.22 | 11862.56 | 15168.21 | 18471.18 | 21798.69 | 25136.18 | 28484.99 | 31878.36 | 35266.75 | 38657.27 |
|  | 10 | 9093.69 | 12571.19 | 16056.43 | 19530.74 | 23050.53 | 26554.54 | 30108.96 | 33675.94 | 37232.12 | 40846.74 |
|  | 11 | 9668.75 | 13300.00 | 16970.37 | 20653.77 | 24343.65 | 28088.23 | 31808.08 | 35590.80 | 39369.81 | 43131.16 |
|  | 12 | 10237.23 | 14088.43 | 17941.99 | 21814.44 | 25752.92 | 29650.40 | 33608.12 | 37585.34 | 41576.37 | 45574.92 |
|  | 13 | 10860.22 | 14903.54 | 18989.76 | 23069.90 | 27184.29 | 31322.96 | 35495.05 | 39676.32 | 43844.92 | 48143.01 |
|  | 14 | 11510.72 | 15796.51 | 20071.46 | 24395.92 | 28758.48 | 33110.12 | 37516.56 | 41907.10 | 46327.68 | 50783.41 |
|  | 15 | 12184.38 | 16698.11 | 21223.80 | 25782.91 | 30372.01 | 34956.17 | 39548.46 | 44257.93 | 48957.81 | 53589.01 |
|  | 16 | 12938.72 | 17661.12 | 22433.04 | 27227.34 | 32077.08 | 36926.13 | 41803.18 | 46701.50 | 51719.71 | 56630.43 |
|  | 17 | 13688.21 | 18734.93 | 23721.92 | 28821.86 | 33892.20 | 38976.73 | 44148.28 | 49288.70 | 54553.89 | 59749.84 |
|  | 18 | 14483.33 | 19789.58 | 25074.29 | 30441.85 | 35776.83 | 41195.12 | 46655.96 | 52061.05 | 57578.49 | 63019.80 |
|  | 19 | 15409.95 | 20934.13 | 26521.91 | 32167.12 | 37800.73 | 43460.68 | 49175.76 | 54959.22 | 60737.94 | 66507.51 |
|  | 20 | 16288.37 | 22151.31 | 28037.05 | 33990.50 | 39936.23 | 45877.72 | 51924.72 | 58021.27 | 64108.82 | 70153.61 |
|  | 21 | 17264.49 | 23412.29 | 29647.37 | 35893.32 | 42135.48 | 48484.45 | 54833.43 | 61193.74 | 67617.26 | 74107.89 |
|  | 22 | 18290.87 | 24784.01 | 31342.46 | 37895.90 | 44533.11 | 51170.83 | 57823.88 | 64531.94 | 71224.46 | 78130.90 |
|  | 23 | 19333.07 | 26220.40 | 33117.65 | 40001.56 | 46984.95 | 53952.25 | 60969.78 | 68050.70 | 75146.23 | 82294.54 |
|  | 24 | 20492.12 | 27688.17 | 34943.63 | 42248.87 | 49554.15 | 56968.41 | 64282.12 | 71826.55 | 79287.98 | 86878.33 |
|  | 25 | 21698.33 | 29345.99 | 36943.84 | 44601.64 | 52308.85 | 60091.26 | 67859.24 | 75682.92 | 83537.02 | 91528.57 |
|  | 26 | 22977.40 | 30958.35 | 39004.66 | 47083.53 | 55126.20 | 63284.92 | 71531.98 | 79710.37 | 88078.38 | 96387.27 |
|  | 27 | 24330.87 | 32696.24 | 41136.41 | 49641.40 | 58195.89 | 66796.75 | 75382.08 | 84076.07 | 92815.66 | 101477.51 |
|  | 28 | 25767.60 | 34613.24 | 43461.10 | 52394.11 | 61392.57 | 70350.18 | 79370.97 | 88483.65 | 97665.09 | 106952.94 |
|  | 29 | 27261.57 | 36547.63 | 45823.39 | 55219.29 | 64674.85 | 74103.08 | 83711.46 | 93260.71 | 102843.73 | 112549.33 |
|  | 30 | 28827.98 | 38620.19 | 48432.79 | 58340.27 | 68281.92 | 78222.88 | 88324.18 | 98422.76 | 108559.33 | 118587.09 |

Table 29: Period Net Value for Value Addition Coefficient Sensitivity Test

|  | Value Addition Coefficient |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.0 \\ & \dot{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|  | 1 | 324969.53 | 477254.37 | 630354.65 | 784064.84 | 938697.03 | 1093893.02 | 1249852.85 | 1406602.69 | 1564280.33 | 1722550.59 |
|  | 2 | 342095.54 | 502086.02 | 662778.51 | 824330.04 | 986467.82 | 1149542.30 | 1313444.09 | 1478125.82 | 1643476.05 | 1809832.82 |
|  | 3 | 357366.59 | 524801.56 | 693063.70 | 862210.66 | 1032045.69 | 1202882.54 | 1374330.86 | 1546858.83 | 1720104.33 | 1894134.16 |
|  | 4 | 370709.68 | 545558.15 | 721236.12 | 897803.69 | 1075283.91 | 1253517.98 | 1432676.94 | 1612803.40 | 1793672.60 | 1975376.67 |
|  | 5 | 382699.49 | 564785.84 | 747785.06 | 931767.00 | 1116567.22 | 1302236.31 | 1488843.69 | 1676329.15 | 1864858.62 | 2054032.17 |
|  | 6 | 393310.04 | 582573.34 | 772848.50 | 963904.00 | 1155889.90 | 1348882.46 | 1542807.88 | 1737663.53 | 1933545.90 | 2130272.71 |
|  | 7 | 402484.79 | 598692.45 | 795841.53 | 994079.30 | 1193292.73 | 1393354.39 | 1594435.59 | 1796450.84 | 1999470.46 | 2203466.34 |
|  | 8 | 410076.75 | 613072.60 | 817269.51 | 1022300.38 | 1228402.29 | 1435486.97 | 1643568.04 | 1852665.04 | 2062613.78 | 2273812.97 |
|  | 9 | 416009.63 | 625728.36 | 836630.94 | 1048402.46 | 1261271.69 | 1475177.05 | 1690010.75 | 1906031.55 | 2123057.34 | 2341134.58 |
|  | 10 | 420381.90 | 636726.22 | 853988.72 | 1072336.18 | 1291803.11 | 1512333.98 | 1733867.59 | 1956437.90 | 2180375.09 | 2405052.76 |
|  | 11 | 422821.11 | 645462.33 | 869215.63 | 1093929.28 | 1319835.24 | 1546738.18 | 1774877.00 | 2003951.69 | 2234161.69 | 2465428.29 |
|  | 12 | 423461.56 | 652260.86 | 882236.01 | 1113043.25 | 1345191.19 | 1578382.44 | 1812622.25 | 2048147.36 | 2284827.77 | 2522748.30 |
|  | 13 | 422162.07 | 656764.28 | 892818.24 | 1129688.19 | 1367762.37 | 1606948.64 | 1847393.47 | 2089019.73 | 2331696.02 | 2575606.29 |
|  | 14 | 421079.84 | 661545.73 | 903198.50 | 1145827.49 | 1389778.86 | 1634818.47 | 1881141.88 | 2128649.33 | 2377229.28 | 2627090.75 |
|  | 15 | 420291.69 | 666364.08 | 913466.99 | 1161787.24 | 1411295.05 | 1661939.37 | 1913930.95 | 2166965.80 | 2421308.57 | 2676806.93 |
|  | 16 | 419986.47 | 671173.35 | 923585.18 | 1177147.66 | 1431987.07 | 1688103.74 | 1945501.98 | 2203978.87 | 2463837.39 | 2724821.54 |
|  | 17 | 419836.84 | 676065.59 | 933712.12 | 1192412.11 | 1452305.19 | 1713453.12 | 1975900.53 | 2239606.98 | 2504685.83 | 2770789.37 |
|  | 18 | 420150.50 | 681193.27 | 943429.40 | 1207096.65 | 1471814.28 | 1737844.44 | 2005200.98 | 2273788.31 | 2543805.50 | 2814944.89 |
|  | 19 | 420756.51 | 686299.48 | 953147.10 | 1221342.35 | 1490661.77 | 1761312.73 | 2033178.72 | 2306484.19 | 2580988.84 | 2856669.19 |
|  | 20 | 421988.71 | 691671.04 | 962665.18 | 1235012.27 | 1508733.06 | 1783584.99 | 2059782.94 | 2337303.47 | 2616317.49 | 2896560.34 |
|  | 21 | 423330.33 | 696885.53 | 971948.44 | 1248232.23 | 1525826.61 | 1804786.31 | 2084987.42 | 2366652.00 | 2649540.51 | 2933786.87 |
|  | 22 | 425026.46 | 702274.96 | 981022.57 | 1260874.23 | 1542248.98 | 1824777.10 | 2108764.75 | 2393923.93 | 2680689.47 | 2968619.78 |
|  | 23 | 427228.11 | 707829.60 | 989828.80 | 1273014.92 | 1557720.64 | 1843624.97 | 2130845.30 | 2419482.64 | 2709445.63 | 3000829.72 |
|  | 24 | 429799.16 | 713519.02 | 998385.81 | 1284566.28 | 1572081.21 | 1861167.46 | 2151234.27 | 2442956.26 | 2735969.56 | 3030352.68 |
|  | 25 | 432915.41 | 719299.42 | 1006591.78 | 1295542.06 | 1585664.61 | 1877185.34 | 2170045.92 | 2464229.14 | 2759885.87 | 3057073.84 |
|  | 26 | 436463.70 | 725016.81 | 1014593.58 | 1305572.33 | 1597951.09 | 1891714.91 | 2186861.05 | 2483370.01 | 2781288.17 | 3080568.99 |
|  | 27 | 440541.99 | 730742.83 | 1022209.23 | 1314995.10 | 1609332.98 | 1904777.45 | 2201743.14 | 2500112.11 | 2799826.99 | 3100838.72 |
|  | 28 | 445008.53 | 736615.70 | 1029544.37 | 1323826.01 | 1619339.16 | 1916339.64 | 2214597.27 | 2514407.60 | 2815647.26 | 3118007.25 |
|  | 29 | 447865.27 | 740379.66 | 1034527.53 | 1329618.22 | 1626254.16 | 1924193.30 | 2223517.60 | 2524227.05 | 2826339.99 | 3130013.05 |
|  | 30 | 449573.99 | 742874.99 | 1037149.18 | 1332762.08 | 1629882.42 | 1928273.39 | 2228152.33 | 2529418.39 | 2832068.76 | 3136053.82 |

Table 30: Sum Net Value of Final Period for Value Addition Coefficient Sensitivity Test

## E.1.3 Probability with Stratified Failure Model

|  | Probability of Successful Completion |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|  | 1 | 4656.69 | 4864.11 | 5083.45 | 5291.41 | 5504.03 | 5722.39 | 5927.86 | 6138.87 | 6351.00 | 6562.81 | 6776.82 |
|  | 2 | 4903.43 | 5128.85 | 5338.67 | 5571.49 | 5790.04 | 6021.96 | 6237.63 | 6460.02 | 6682.50 | 6900.34 | 7124.27 |
|  | 3 | 5161.18 | 5394.79 | 5626.22 | 5855.68 | 6079.20 | 6334.77 | 6562.61 | 6794.72 | 7025.38 | 7255.54 | 7489.54 |
|  | 4 | 5431.33 | 5674.62 | 5928.90 | 6169.17 | 6406.67 | 6653.90 | 6895.77 | 7144.09 | 7385.83 | 7623.28 | 7873.54 |
|  | 5 | 5719.21 | 5975.23 | 6227.78 | 6491.28 | 6745.57 | 7005.10 | 7259.42 | 7506.97 | 7761.61 | 8023.99 | 8277.22 |
|  | 6 | 6021.52 | 6293.97 | 6564.56 | 6834.69 | 7091.77 | 7370.19 | 7637.93 | 7892.60 | 8158.86 | 8431.40 | 8701.60 |
|  | 7 | 6342.59 | 6626.97 | 6908.31 | 7189.34 | 7464.84 | 7744.49 | 8020.78 | 8305.73 | 8588.29 | 8866.65 | 9147.74 |
|  | 8 | 6679.57 | 6978.80 | 7274.30 | 7564.99 | 7847.71 | 8144.79 | 8448.19 | 8743.43 | 9032.34 | 9317.62 | 9616.76 |
|  | 9 | 7033.70 | 7352.06 | 7649.41 | 7962.09 | 8269.11 | 8572.98 | 8880.48 | 9191.15 | 9485.43 | 9812.11 | 10109.82 |
|  | 10 | 7411.99 | 7728.24 | 8053.39 | 8369.82 | 8692.14 | 9018.55 | 9359.42 | 9666.64 | 9985.61 | 10306.80 | 10628.16 |
|  | 11 | 7806.48 | 8158.14 | 8490.03 | 8820.40 | 9155.53 | 9483.29 | 9831.15 | 10179.22 | 10505.99 | 10832.36 | 11173.08 |
|  | 12 | 8224.87 | 8567.50 | 8932.59 | 9281.14 | 9625.90 | 9995.99 | 10342.16 | 10689.49 | 11041.77 | 11394.78 | 11745.94 |
|  | 13 | 8663.61 | 9036.49 | 9401.32 | 9772.39 | 10134.74 | 10516.74 | 10863.69 | 11240.55 | 11607.33 | 11975.63 | 12348.16 |
|  | 14 | 9132.58 | 9526.00 | 9909.21 | 10285.56 | 10663.66 | 11049.55 | 11439.30 | 11818.27 | 12208.35 | 12584.95 | 12981.27 |
|  | 15 | 9626.20 | 10028.88 | 10441.18 | 10831.35 | 11247.66 | 11646.11 | 12037.26 | 12434.54 | 12852.01 | 13240.27 | 13646.83 |
|  | 16 | 10146.89 | 10561.31 | 10988.70 | 11394.32 | 11829.61 | 12239.75 | 12666.29 | 13096.04 | 13523.18 | 13917.03 | 14346.52 |
|  | 17 | 10696.25 | 11129.10 | 11584.38 | 12040.70 | 12447.90 | 12877.56 | 13309.48 | 13764.24 | 14216.53 | 14644.16 | 15082.08 |
|  | 18 | 11281.10 | 11739.10 | 12189.28 | 12646.29 | 13086.39 | 13572.44 | 14036.43 | 14477.40 | 14959.14 | 15399.21 | 15855.36 |
|  | 19 | 11888.41 | 12358.62 | 12851.58 | 13321.47 | 13819.92 | 14272.25 | 14766.29 | 15218.99 | 15721.39 | 16190.61 | 16668.28 |
|  | 20 | 12544.84 | 13052.12 | 13539.36 | 14044.83 | 14565.55 | 15038.52 | 15529.44 | 16037.97 | 16524.63 | 17046.25 | 17522.88 |
|  | 21 | 13236.32 | 13762.38 | 14275.16 | 14803.40 | 15324.35 | 15819.61 | 16353.42 | 16873.42 | 17379.08 | 17899.90 | 18421.29 |
|  | 22 | 13962.24 | 14518.80 | 15057.51 | 15618.59 | 16129.93 | 16655.65 | 17202.08 | 17747.51 | 18294.30 | 18838.28 | 19365.77 |
|  | 23 | 14736.17 | 15292.92 | 15866.66 | 16430.23 | 16978.11 | 17545.79 | 18125.77 | 18650.71 | 19235.03 | 19777.16 | 20358.68 |
|  | 24 | 15548.99 | 16161.01 | 16732.60 | 17333.61 | 17917.73 | 18457.12 | 19080.00 | 19665.01 | 20217.66 | 20819.78 | 21402.49 |
|  | 25 | 16425.08 | 17020.88 | 17664.77 | 18253.70 | 18851.48 | 19472.40 | 20081.87 | 20699.68 | 21252.67 | 21899.76 | 22499.82 |
|  | 26 | 17355.08 | 17980.08 | 18623.10 | 19232.28 | 19901.18 | 20481.06 | 21111.83 | 21779.30 | 22377.57 | 23011.90 | 23653.41 |
|  | 27 | 18335.89 | 18964.26 | 19680.80 | 20301.45 | 20925.89 | 21579.51 | 22228.96 | 22914.83 | 23579.77 | 24220.23 | 24866.15 |
|  | 28 | 19372.05 | 20045.37 | 20729.94 | 21390.84 | 22069.57 | 22766.61 | 23457.50 | 24144.42 | 24786.33 | 25458.14 | 26141.06 |
|  | 29 | 20482.30 | 21185.55 | 21882.55 | 22558.91 | 23345.99 | 23980.68 | 24649.77 | 25413.09 | 26084.53 | 26762.44 | 27481.34 |
|  | 30 | 21625.45 | 22331.65 | 23071.11 | 23808.45 | 24523.62 | 25275.33 | 25959.56 | 26736.41 | 27407.72 | 28170.00 | 28890.34 |

Table 31: Period Net Value for Probability Sensitivity Test with Stratified Failure Model

|  | Probability of Successful Completion |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{\Delta} \\ & 0 \\ & \stackrel{\rightharpoonup}{E} \\ & \ddot{B} \\ & \stackrel{0}{0} \\ & \stackrel{0}{0} \\ & 0 \end{aligned}$ |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|  | 1 | 255596.50 | 269447.32 | 283289.85 | 297121.95 | 310883.78 | 324653.84 | 338456.91 | 352289.82 | 366002.48 | 379895.07 | 393755.15 |
|  | 2 | 267628.53 | 282063.03 | 296365.41 | 310748.93 | 325095.62 | 339454.05 | 353852.85 | 368175.64 | 382491.36 | 396914.70 | 411219.86 |
|  | 3 | 277656.40 | 292629.68 | 307635.10 | 322431.74 | 337430.16 | 352322.96 | 367312.25 | 382046.66 | 397067.62 | 411951.03 | 426856.45 |
|  | 4 | 285656.02 | 301030.47 | 316573.65 | 332195.16 | 347608.64 | 363134.36 | 378659.29 | 394138.54 | 409604.19 | 424995.42 | 440571.19 |
|  | 5 | 292049.32 | 308142.94 | 324255.40 | 340355.14 | 356451.21 | 372614.11 | 388668.43 | 404793.23 | 420805.83 | 436802.15 | 452965.36 |
|  | 6 | 297041.22 | 313738.59 | 330479.37 | 347167.57 | 363736.21 | 380535.59 | 397172.34 | 413915.16 | 430525.69 | 447289.39 | 463971.24 |
|  | 7 | 300468.68 | 317760.37 | 335116.46 | 352300.56 | 369673.78 | 386944.08 | 404245.20 | 421706.80 | 438961.75 | 456217.76 | 473517.65 |
|  | 8 | 302245.73 | 320258.76 | 338019.83 | 355991.76 | 373894.85 | 391867.62 | 409732.21 | 427747.58 | 445808.73 | 463668.29 | 481529.76 |
|  | 9 | 302250.74 | 320817.54 | 339389.72 | 357907.66 | 376590.73 | 394917.89 | 413574.51 | 432237.89 | 450795.99 | 469415.77 | 487928.91 |
|  | 10 | 300403.25 | 319668.81 | 338951.88 | 358085.09 | 377353.43 | 396378.74 | 415866.72 | 434881.83 | 454184.26 | 473381.54 | 492632.40 |
|  | 11 | 296574.32 | 316503.59 | 336408.73 | 356101.00 | 376065.14 | 395979.84 | 415998.30 | 436044.27 | 455755.15 | 475624.69 | 495553.30 |
|  | 12 | 290609.96 | 311198.66 | 331679.15 | 352403.21 | 372976.73 | 393508.81 | 414227.87 | 434767.87 | 455460.24 | 475978.35 | 496600.20 |
|  | 13 | 282417.80 | 303749.70 | 325157.96 | 346471.99 | 367950.46 | 389004.33 | 410466.74 | 431644.19 | 453071.24 | 474299.43 | 495677.02 |
|  | 14 | 274450.41 | 296708.87 | 318700.78 | 340549.58 | 362766.53 | 384889.64 | 406807.19 | 428947.77 | 450861.97 | 473081.39 | 495048.93 |
|  | 15 | 267061.94 | 289911.98 | 312611.00 | 335238.14 | 358244.57 | 380909.98 | 403832.84 | 426577.30 | 449267.39 | 471922.86 | 494731.03 |
|  | 16 | 260280.23 | 283651.76 | 307216.10 | 330612.55 | 353992.77 | 377517.07 | 400954.78 | 424352.55 | 447943.46 | 471319.60 | 494739.26 |
|  | 17 | 254226.82 | 278366.28 | 302476.00 | 326458.73 | 350614.71 | 374709.50 | 398735.10 | 422650.24 | 447007.49 | 470961.39 | 495090.31 |
|  | 18 | 248791.10 | 273422.39 | 298169.21 | 322678.15 | 347515.56 | 372098.89 | 396888.25 | 421544.78 | 446425.52 | 471084.99 | 495801.77 |
|  | 19 | 244084.16 | 269378.90 | 294639.43 | 320026.21 | 345277.18 | 370651.52 | 395662.57 | 420838.08 | 446329.46 | 471623.62 | 496892.12 |
|  | 20 | 240083.12 | 265930.76 | 291832.22 | 317606.53 | 343464.13 | 369005.20 | 395089.56 | 421051.18 | 446761.34 | 472471.04 | 498380.78 |
|  | 21 | 237046.30 | 263375.25 | 289828.75 | 315999.06 | 342180.42 | 368608.84 | 395180.18 | 421128.57 | 447648.96 | 473982.81 | 500288.17 |
|  | 22 | 234807.40 | 261666.72 | 288389.03 | 315158.13 | 341875.76 | 368564.19 | 395616.96 | 422024.22 | 449151.97 | 475809.25 | 502635.77 |
|  | 23 | 233432.26 | 260787.29 | 287719.94 | 315130.60 | 342182.71 | 369385.55 | 396688.06 | 423905.34 | 450991.61 | 478246.84 | 505446.15 |
|  | 24 | 233073.29 | 260551.25 | 288254.34 | 315885.08 | 343460.23 | 370703.54 | 398331.62 | 425897.39 | 453695.50 | 481206.51 | 508743.02 |
|  | 25 | 233593.74 | 261603.38 | 289424.60 | 317345.89 | 345285.81 | 373268.27 | 400929.60 | 428966.30 | 456702.12 | 484710.19 | 512551.34 |
|  | 26 | 235472.78 | 263579.53 | 291647.80 | 319915.90 | 348069.61 | 376362.17 | 404188.01 | 432486.91 | 460695.11 | 488834.30 | 516897.33 |
|  | 27 | 238394.86 | 266591.80 | 295003.94 | 323371.27 | 351705.61 | 379878.06 | 408485.44 | 436781.87 | 465204.65 | 493539.29 | 521808.55 |
|  | 28 | 242469.47 | 271061.60 | 299476.88 | 328106.37 | 356423.42 | 384790.37 | 413408.53 | 441818.21 | 470306.66 | 498772.32 | 527313.98 |
|  | 29 | 245868.25 | 274442.38 | 303044.60 | 331584.81 | 360440.98 | 388781.82 | 417114.57 | 445827.45 | 474324.37 | 502998.56 | 531544.50 |
|  | 30 | 248123.43 | 276766.71 | 305565.88 | 333927.89 | 362683.02 | 391338.87 | 420004.12 | 448310.90 | 477079.77 | 505733.72 | 534434.74 |

Table 32: Sum Net Value for Probability Sensitivity Test with Stratified Failure Model

## E.1.4 Probability of Successful Completion

|  | Cost Reduction Coefficient |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|  | 1 | -19093.59 | -16533.32 | -13928.05 | -11355.07 | -8785.26 | -6142.40 | -3630.11 | -956.39 | 1572.34 | 4235.41 | 6776.82 |
|  | 2 | -20420.11 | -17668.61 | -14952.07 | -12230.17 | -9357.08 | -6579.35 | -4059.27 | -1121.71 | 1640.26 | 4328.80 | 7124.27 |
|  | 3 | -21868.34 | -18968.44 | -16031.87 | -13125.63 | -10113.94 | -7202.67 | -4289.26 | -1240.00 | 1644.01 | 4618.00 | 7489.54 |
|  | 4 | -23454.60 | -20279.75 | -17160.82 | -14011.90 | -10851.88 | -7819.76 | -4700.61 | -1485.56 | 1516.37 | 4747.73 | 7873.54 |
|  | 5 | -25198.47 | -21928.28 | -18556.41 | -15298.18 | -11847.71 | -8450.74 | -5151.48 | -1758.28 | 1604.85 | 4996.25 | 8277.22 |
|  | 6 | -27123.70 | -23540.62 | -19981.94 | -16274.45 | -12731.15 | -9279.99 | -5583.96 | -2137.38 | 1511.85 | 5172.81 | 8701.60 |
|  | 7 | -29259.48 | -25416.17 | -21586.20 | -17855.37 | -13954.89 | -9949.25 | -6414.66 | -2356.35 | 1442.28 | 5296.97 | 9147.74 |
|  | 8 | -31642.30 | -27554.53 | -23469.85 | -19203.01 | -15066.73 | -10985.62 | -6930.49 | -2767.00 | 1400.08 | 5483.68 | 9616.76 |
|  | 9 | -34318.86 | -29942.52 | -25440.77 | -21077.78 | -16622.48 | -12084.89 | -7499.21 | -3347.92 | 1255.52 | 5675.15 | 10109.82 |
|  | 10 | -37350.65 | -32444.16 | -27798.85 | -23000.37 | -18255.31 | -13262.39 | -8605.45 | -3779.26 | 1071.71 | 5909.16 | 10628.16 |
|  | 11 | -40821.99 | -35678.44 | -30436.68 | -25236.99 | -20033.47 | -14879.50 | -9583.57 | -4612.40 | 581.99 | 5957.11 | 11173.08 |
|  | 12 | -44854.51 | -39245.50 | -33687.90 | -28117.63 | -22212.03 | -16494.82 | -11035.30 | -5236.27 | 394.47 | 6057.22 | 11745.94 |
|  | 13 | -49633.97 | -43607.44 | -37438.99 | -30954.92 | -24999.75 | -18978.48 | -12624.92 | -6281.33 | -23.04 | 6182.17 | 12348.16 |
|  | 14 | -55459.38 | -48875.05 | -41916.10 | -35082.02 | -28362.05 | -21393.47 | -14422.91 | -7487.96 | -973.25 | 6139.20 | 12981.27 |
|  | 15 | -62830.41 | -55409.21 | -48034.12 | -39929.49 | -32394.36 | -25104.22 | -17254.42 | -9348.36 | -1505.69 | 5961.42 | 13646.83 |
|  | 16 | -72594.64 | -63947.56 | -55485.25 | -46717.99 | -38364.09 | -29395.12 | -20739.78 | -11986.13 | -3247.95 | 5619.35 | 14346.52 |
|  | 17 | -86177.11 | -76335.62 | -66543.29 | -56608.06 | -46258.07 | -36143.98 | -25828.28 | -15396.08 | -5502.78 | 4904.88 | 15082.08 |
|  | 18 | -105903.18 | -94032.63 | -82534.22 | -70798.70 | -58464.97 | -46310.12 | -33875.66 | -21169.57 | -9085.83 | 3565.25 | 15855.36 |
|  | 19 | -135390.54 | -120555.72 | -106071.48 | -91390.94 | -75645.44 | -60640.81 | -45492.67 | -30031.01 | -14159.49 | 979.57 | 16668.28 |
|  | 20 | -179920.06 | -160979.56 | -141858.81 | -122237.45 | -102786.47 | -83212.79 | -62893.50 | -43465.89 | -22783.85 | -2803.18 | 17522.88 |
|  | 21 | -246602.40 | -220905.04 | -196012.44 | -169470.51 | -142965.31 | -115823.42 | -90358.22 | -63329.39 | -36113.50 | -8622.86 | 18421.29 |
|  | 22 | -344067.64 | -308538.28 | -272102.21 | -238662.13 | -202151.24 | -165877.38 | -130003.98 | -91785.79 | $-55385.23$ | -18309.40 | 19365.77 |
|  | 23 | -481376.93 | -432297.83 | -382869.15 | -332970.28 | -285052.75 | -236304.32 | -183388.25 | -133960.64 | -83201.81 | -31446.45 | 20358.68 |
|  | 24 | -665962.33 | -597359.01 | -530890.15 | -463431.67 | -395050.57 | -327567.66 | -260532.54 | -188090.29 | -120837.85 | -48221.88 | 21402.49 |
|  | 25 | -900693.28 | -810568.77 | -719822.75 | -626353.24 | -536578.86 | -443844.80 | -352428.06 | -259863.46 | -164290.35 | -73169.28 | 22499.82 |
|  | 26 | -1180615.27 | -1062627.79 | -944443.14 | -828360.94 | -706342.74 | -586895.72 | -465585.51 | -345649.65 | -221270.70 | -100700.25 | 23653.41 |
|  | 27 | -1490364.51 | -1340828.45 | -1186945.80 | -1042057.59 | -892013.24 | -743051.54 | -587018.13 | -437522.00 | -284513.75 | -129703.46 | 24866.15 |
|  | 28 | -1803488.51 | -1620954.84 | -1440889.36 | -1256495.60 | -1081902.65 | -894452.01 | -708983.42 | -527774.26 | -346262.45 | -163056.30 | 26141.06 |
|  | 29 | -2086300.72 | -1875904.07 | $-1667243.64$ | -1459080.21 | -1242687.59 | -1038581.08 | -829675.49 | -611242.65 | -400943.95 | -186034.38 | 27481.34 |
|  | 30 | -2302548.80 | -2071100.35 | -1843442.34 | -1617846.80 | -1378287.06 | -1148254.15 | -912966.77 | -683087.56 | -443780.27 | -207874.00 | 28890.34 |

Table 33: Period Net Value for Probability Sensitivity Test with Binary Value Model

|  | Probability of Successful Completion |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{0}{0} \\ & 0 \\ & \text { \# } \\ & \ddot{B} \\ & \ddot{B} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|  | 1 | -2690834.92 | -2372892.95 | -2057695.93 | -1748150.42 | -1442755.67 | -1132708.65 | -819655.04 | -518399.02 | -210193.48 | 93331.78 | 393755.15 |
|  | 2 | -3047529.90 | -2692931.50 | -2339418.56 | -1987215.07 | -1637105.43 | -1292710.35 | -954089.26 | -608052.77 | -267967.40 | 73579.26 | 411219.86 |
|  | 3 | -3460649.37 | -3060751.63 | -2660308.93 | -2268126.49 | -1880511.93 | -1492708.22 | -1103133.75 | -718748.09 | -337328.02 | 49145.42 | 426856.45 |
|  | 4 | -3934225.09 | -3485828.73 | -3036043.62 | -2595417.03 | -2147989.63 | -1713162.22 | -1283215.78 | -849618.32 | -415955.56 | 9517.25 | 440571.19 |
|  | 5 | -4469248.20 | -3958853.08 | -3451367.30 | -2966130.17 | -2465683.03 | -1980410.89 | -1487483.73 | -991849.10 | -510886.22 | -24405.62 | 452965.36 |
|  | 6 | -5066983.89 | -4490755.39 | -3931211.52 | -3370366.37 | -2809883.26 | -2251404.28 | -1710585.46 | -1157370.71 | -611918.48 | -75075.26 | 463971.24 |
|  | 7 | -5729065.49 | -5083080.95 | -4449337.90 | -3823248.20 | -3203618.82 | -2570830.38 | -1960795.43 | -1347512.87 | -738844.71 | -129485.06 | 473517.65 |
|  | 8 | -6457627.71 | -5746714.36 | -5035155.55 | -4321427.26 | -3617682.37 | -2951335.66 | -2244642.79 | -1550540.34 | -877328.30 | -190316.21 | 481529.76 |
|  | 9 | -7255498.16 | -6461553.95 | -5658058.74 | -4873521.22 | -4080741.22 | -3314094.86 | -2545370.32 | -1787602.12 | -1021266.23 | -265015.97 | 487928.91 |
|  | 10 | -8126477.02 | -7237459.44 | -6326179.85 | -5473802.03 | -4595030.14 | -3742306.56 | -2872921.18 | -2020762.78 | -1189694.88 | -338871.00 | 492632.40 |
|  | 11 | -9075752.64 | -8086438.81 | -7106626.40 | -6133559.91 | -5159980.23 | -4202969.64 | -3262887.55 | -2314491.25 | $-1376978.70$ | -431996.85 | 495553.30 |
|  | 12 | -10110530.03 | -9006207.44 | -7921216.41 | -6834812.26 | -5759667.93 | -4713898.41 | -3650176.87 | -2600749.08 | -1561776.27 | -535299.50 | 496600.20 |
|  | 13 | -11240999.51 | -10039073.25 | -8802932.34 | -7631754.15 | -6430393.46 | -5259304.77 | -4103667.33 | -2944307.33 | -1789859.95 | -654751.92 | 495677.02 |
|  | 14 | -12466148.93 | -11137381.49 | -9795960.27 | -8493921.22 | -7170322.42 | -5895327.58 | -4565805.62 | -3304555.92 | -2018402.08 | -782139.75 | 495048.93 |
|  | 15 | -13784615.53 | -12290609.80 | -10860145.86 | -9418335.87 | -7924390.68 | -6526578.42 | -5103717.52 | -3700336.43 | -2278666.16 | -893006.39 | 494731.03 |
|  | 16 | -15194460.01 | -13572938.47 | -11957142.06 | -10369993.82 | -8815375.74 | -7228508.36 | -5674254.73 | -4123402.61 | -2572170.74 | -1035486.67 | 494739.26 |
|  | 17 | -16692774.05 | -14909966.56 | -13152863.27 | -11404726.56 | -9708678.97 | -7949708.78 | -6273224.44 | -4583147.56 | -2885650.04 | -1175055.53 | 495090.31 |
|  | 18 | -18275025.86 | -16359380.47 | -14434983.47 | -12572581.29 | -10637043.03 | -8789490.52 | -6880665.00 | -5065882.88 | -3167143.59 | -1342815.58 | 495801.77 |
|  | 19 | -19934033.55 | -17843409.05 | -15775697.83 | -13673357.20 | -11640755.27 | -9602475.02 | -7546227.21 | -5525680.12 | -3520510.87 | -1525883.97 | 496892.12 |
|  | 20 | -21658477.61 | -19398324.97 | -17112689.88 | -14877954.65 | -12654752.08 | -10440092.96 | -8244097.54 | -6063527.04 | -3836857.35 | -1666476.60 | 498380.78 |
|  | 21 | -23430958.12 | -20998907.37 | -18545876.48 | -16170136.75 | -13682352.55 | -11340542.96 | -8952244.21 | -6583263.68 | -4198901.30 | -1864644.53 | 500288.17 |
|  | 22 | -25225798.73 | -22601202.30 | -19943231.67 | -17426408.55 | -14835628.47 | -12217539.13 | -9712178.63 | -7131189.53 | $-4606457.40$ | -2033217.02 | 502635.77 |
|  | 23 | -27007098.11 | -24200368.72 | -21408024.34 | -18606426.97 | -15926352.59 | -13160998.48 | -10356198.19 | -7682939.70 | $-4961787.10$ | -2209316.81 | 505446.15 |
|  | 24 | -28727868.57 | -25695996.56 | -22758213.81 | -19909107.05 | -16895110.99 | -14011323.89 | -11121080.52 | -8137697.53 | -5328027.74 | -2369303.76 | 508743.02 |
|  | 25 | -30331335.75 | -27180329.68 | -24107451.30 | -20933836.13 | -17914062.29 | -14730274.14 | -11728845.97 | -8667756.47 | -5584251.99 | -2552701.64 | 512551.34 |
|  | 26 | -31755404.25 | -28493532.91 | -25224937.85 | -22002135.72 | -18779560.38 | -15564950.47 | -12322044.00 | -9074371.80 | -5878494.84 | -2725239.96 | 516897.33 |
|  | 27 | -32940745.58 | -29552133.52 | -26109359.84 | -22809460.58 | -19475587.16 | -16114125.07 | -12718850.78 | -9472166.65 | -6122908.55 | -2832386.21 | 521808.55 |
|  | 28 | -33841913.19 | -30344567.80 | -26895346.54 | -23383994.00 | -20031877.79 | -16500205.34 | -13130493.73 | -9674818.69 | -6322578.54 | -2935509.17 | 527313.98 |
|  | 29 | -34446803.61 | -30882243.35 | -27378103.12 | -23859284.83 | -20277292.86 | -16870298.79 | -13368211.50 | -9866358.10 | -6420143.47 | -2924315.20 | 531544.50 |
|  | 30 | -34777935.80 | -31186150.69 | -27656837.45 | -24189388.65 | -20540235.99 | -17015676.26 | -13540779.17 | -9997949.91 | -6481583.04 | -2963507.14 | 534434.74 |

Table 34: Sum Net Value for Probability Sensitivity Test with Binary Value Model

## E. 2 Internal Validity

|  | Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 |  |  | 7 | 8 |  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | ${ }^{2}$ | 21 | 22 | 23 | 24 | 25 | 6 |  |  |  |  |
|  | 51134.94 | 192 | 46775.22 | 272 | 30661.09 | 28947.83 | 2747 | 25942.88 | 2452 | 23240.03 | 2195 | 20767.60 | . 31 | -6133.59 | -5970.23 | 5782.84 | -566.80 | -5375.16 | 6.9 | -5020.73 | 885.2 | ${ }_{4672.84}$ | 25.8 | 4358.08 | 55.5 | -4052.58 | 3914.36 | 5807.98 | 5678.50 | ${ }_{5} 364.26$ | 5052.39 |
|  | 5112.53 | 520 | 49405.13 | 0.58 | 323 | 30627.61 | 289 | 2741 | 25961.66 | 24532.42 | 23223.45 | 21987.28 | -580.52 | -6358.16 | -6144.79 | -5962.59 | -5771.10 | -5576.00 | -5372.56 | -5213.74 | -5037.95 | -4874.37 | -4696.75 | -4529.75 | -4370.66 | -4219.29 | 4075.27 | 6158.35 | ${ }^{6024.80}$ | 5692.62 | 5365.98 |
|  | 51114.63 | 52029.12 | 52119.82 | 36470.38 | ${ }^{34167.20}$ | 32315.73 | ${ }^{3063}$ | 290 | 27367.49 | 25660.7 | 24 | 232 | -5963.25 | -656.93 | 0.67 | ${ }^{-6208.57}$ | -5975.31 | -5784.21 | 558 | -5389.12 | -519 | -5026.15 | -4851.73 | 1691 | -4521.25 | -4371.65 | -4217.47 | 6552.25 | ${ }^{6404.52}$ | 6043.85 | 5687.03 |
|  | 51129.60 | 52054.98 | 52069.99 | 38538.69 | 36996.40 | 34193.25 | 32331.52 | 3068 | 28956.47 | 27461.77 | 25945.08 | 24551.99 | -6137.84 | ${ }^{-67}$ | -6574.45 | -6374.59 | -6167.72 | -5967.23 | 5766.62 | -5580.42 | -5407.39 | -5209.86 | -5029.13 | 4858.98 | -4681.79 | -4519.00 | ${ }^{4366.87}$ | 6924.43 | 6798.80 | 6405.76 | 6032.20 |
|  | 5131.41 | 52026.03 | 52113.61 | ${ }^{38506.21}$ | 38152.10 | 36105.47 | 341 | 32396 | 305 | 28976.21 | 27448.99 | 259 | -6304.46 | ${ }^{-6958}$ | -6769.85 | -6568.50 | -6374.00 | -6172.15 | -5943.95 | -5765.80 | -5575.82 | -5399.39 | -5214.42 | 5025.00 | -4873.68 | -4685.5 | 4536.30 | 7332.98 | 7205.30 | 6794.55 | ${ }^{6396.41}$ |
|  | 51140.00 | 52006.70 | 52138.12 | 38517.13 | 38127.44 | 38175.95 | 361 | 341 | 32342.92 | 30612.27 | 290 | 27445.14 | -6511.09 | -7177.01 | -7015.58 | -684.53 | -6578.72 | -6349,49 | -6170.56 | -5958.68 | -5779.34 | -5584.5 | -5388.61 | -5209.00 | -5014.92 | -4849.30 | -4681.55 | 7773.73 | 7641.49 | 7194.82 |  |
|  | ¢1106.1 | 51987.69 | 52159.53 | 38519.29 | 38138.61 | 38170.15 | 38166.73 | 36112.50 | 34184.50 | 32333.39 | 30 | 28997.95 | -6699.84 | -7408.04 | 7229.84 | -6968.18 | -6789.22 | -6600.91 | -6362.4 | -6167.43 | -5969.48 | -5768. | 5570.88 | -5393.18 | -5215.97 | -5043 | 4853.30 | 8232.13 | 8094.11 | 7648.58 |  |
|  | 5115.11 | 52049.01 | 52134.49 | ${ }^{38520.29}$ | 38144.83 | 38157.72 | 38157.54 | 38173.05 | ${ }^{36097.11}$ | 34164.54 | ${ }^{32379.57}$ | 305 | 6880.41 | -7633.04 | 7462.89 | . 04 | -6993.76 | -6788.27 | -6591.01 | ${ }^{6374.55}$ | -6169.23 | -5953.34 | -5767.31 | -5574.98 | -5396.41 | -5214.98 | -5038.02 | 8732.04 | 8587.88 | 8115.07 | 7630.12 |
|  | 51120.64 | 52060.22 | 52134.22 | 38543.01 | 38139.81 | 38163.70 | 38178.19 | 38124.41 | 38175.47 | 36998.63 | 34193.75 | 32345.54 | -7064.55 | -7883.80 | -7675.47 | .7458.14 | -7218.56 | -7015.22 | -6800.61 | -6577.26 | -6364.91 | -6152.33 | -5958.45 | -5766.36 | -5571.11 | -5405.37 | 5202.75 | 9258.39 | 9103.17 | 8588.47 | 8092.67 |
|  | 50, 5119.87 | 52013.11 | 52109.89 | ${ }^{38522.60}$ | 38155.26 | 38164.84 | 38174.64 | 38148.51 | 38177.38 | 38114.96 | ${ }^{36126.80}$ | 34165.67 | . 7272.30 | -8080.39 | -7890.29 | -7685.91 | 7454.94 | -7236.54 | -7017.01 | -6791.32 | -6572.26 | ${ }^{6350.59}$ | 6164.42 | -597. 32 | -5760.32 | -5577.68 | -5394.29 | 9817.46 | 9666.32 | 9106.62 | 8596 |
|  | 1 51114.06 | 52031.07 | 52129.57 | 38518.42 | 38150.48 | 38132.66 | 38163.52 | 38172.20 | 38123.96 | 38161.13 | 38174.92 | 36075.00 | ${ }^{7474.76}$ | -8307.06 | -8131.97 | 7890.86 | -7665.08 | 7447.05 | -7221.39 | 7009.59 | -6792.19 | -655.31 | ${ }^{6371.72}$ | 6159.19 | -5967.57 | -5787.95 | -5570.18 | 10383.96 | 10235.43 | 9651.19 | 9102 |
|  | 2 51114.34 | 52027.74 | 52119.87 | 38491.45 | 38180.70 | 38137.31 | 38198.07 | 3815.80 | 38147.09 | 38153.22 | ${ }^{38166.65}$ | 38196.53 | -7773.82 | -8518.61 | -8375.16 | 8139.56 | -7881.35 | -7672.72 | -7437.17 | 7236.08 | 7031.77 | ${ }^{6763.12}$ | 6568.44 | 6335.07 | 6174.14 | -5949.19 | 5775.45 | 10996.16 | 10870.33 | 10242.35 | 9650 |
|  | 31152.59 | 52009.99 | 52138.50 | 38509.21 | 38153.56 | 38177.15 | 38179.09 | 381 | 38134.98 | 38184.84 | ${ }^{38152.62}$ | 38149.57 | -7926.36 | -8779.03 | 8559.89 | 8378.15 | -8121.61 | -7916.44 | 7675.18 | 7435.81 | -7209.79 | -6998.68 | -6780.58 | ${ }^{6597.36}$ | 6378.21 | -6184.02 | 6000.74 | 11694.62 | 11499.83 | 10872.08 |  |
|  | ${ }^{51107.57}$ | 52081.89 | 52069.35 | 38545.37 | 38125.20 | 38175.89 | 38142.04 | 38185.88 | 38157.71 | 38194.86 | 38175.37 | 38170.13 | -7915.96 | -9070.09 | -8821.78 | 8660.07 | -8390.16 | -8150.41 | -7903.32 | 77007.8 | 7490.06 | 7225.44 | 7027.07 | -6784.28 | 6559.61 | ${ }^{-6366.67}$ | -6177.74 | 12369.74 | 12190.99 | ${ }^{11510.24}$ |  |
|  | 551149.44 | 52027.00 | 52080.97 | 38470.85 | 38159.76 | 38188.52 | 38166.46 | 38159.00 | 38128.71 | 38146.66 | 38187.25 | 38118.35 | -7885.04 | -9059.59 | -900.85 | 8806.69 | -8617.48 | -8382.99 | 8153.30 | 7895.49 | 7669.25 | 7420.04 | 7200.98 | 7018.07 | 6781.36 | -6576.64 | -6395.05 | 13101.80 | 12933.97 | 12198.95 |  |
|  | ${ }^{6} 51142.67$ | 52034.59 | 52076.86 | 38554.35 | 38145.36 | 38127.03 | 38156.87 | 38179.88 | 38166.47 | 38159.15 | 38177.91 | 38153.50 | 7909.74 | 8977.07 | -9080.06 | 9075.43 | -8812.35 | -8592. 18 | -8357.04 | 8131.34 | 7851.88 | 7664.37 | ${ }^{7422.61}$ | 7221.82 | 7016.23 | -6798.43 | 6588.97 | 13930.63 | 13690.06 | 12955.52 |  |
|  | 51104.23 | 52039.12 | 52127.39 | 38499.92 | 38156.52 | 38157.99 | 38166.66 | 38161.82 | 38189.01 | 38146.60 | 38166.93 | 38153.29 | 7900.79 | -9020.77 | -9085.34 | 9095.19 | -9076.94 | -8823.06 | 8618.67 | 8339.52 | -8132.46 | 7875.40 | -7656.26 | 7463.15 | 7244.16 | -7023.49 | 6780.44 | 14718.20 | 14517.92 | 13698.9 |  |
|  | 1851103.38 | 52035.24 | 52142.46 | 38529.80 | 38165.78 | 38146.34 | 38158.83 | 38193.15 | 38176.38 | 38133.21 | 38132.43 | 38180.63 | -7903.10 | -9004.47 | -9076. 13 | 9071.55 | -9063.41 | -9034.14 | -8817.54 | 8604.76 | -8345.63 | -8123.5 | 7994.08 | 7679.72 | ${ }^{7445.65}$ | -7233.52 | -6987.77 | 15585.44 | 15388.12 | 14501.38 |  |
| 19 | . 51121.78 | 52008.59 | 52100.13 | 38534.76 | 38167.50 | 38133.71 | 38157.41 | 38174.58 | 38147.93 | 38163.50 | 38152.24 | 38155.62 | -7999.82 | -9024.47 | -9033.45 | 9043.99 | -9073.28 | -9669.61 | -9075.25 | 8809.0 | -8887.95 | 8330.78 | 8131.18 | ${ }^{7945.30}$ | 7699.26 | -7444.24 | 7225.70 | 16558.29 | 16301.08 | 15399.27 |  |
| 20 | 5) 51130.46 | 51978.56 | 52107.22 | 38546.75 | 38161.62 | 88128.09 | 38186.30 | 38162.17 | 38155.29 | 38155.20 | ${ }^{38159.65}$ | 38171.38 | -7930.91 | -9024.81 | -9057.88 | -9993.03 | -9072.47 | -9057.12 | -9059.75 | 9059.85 | -8793.72 | -8605.08 | 8377.69 | 8135.47 | ${ }^{7921.34}$ | -7676.62 | -7473.3 | 17502.46 | 17264.84 | 16290.8 |  |
|  | 151138.47 | 2.68 | 52079.23 | 38542.77 | 38180.12 | 8150.03 | 38135.03 | 38147.36 | 38189.20 | 381138.20 | 38 | 38159.20 | -7901.85 | -903 | -9059.38 | 9960 | -9057.50 | -994 | -9044.04 | 9993.79 | 9088 | 880 | -862 | 8339.15 | -8139.46 | -7920.99 | -766.87 | 18551.58 | 18271.88 | 17271.57 |  |
|  | 22 51097.18 | 52000.65 | 52152.26 | 38479.00 | 38131.41 | 8170.45 | 38144.07 | 38148.93 | 38158.39 | 38169.84 | 38152.84 | 38144.73 | -7908.83 | -9017 | -9070.23 | 9963 | -9954.17 | 907 | -9082.18 | -9060. 15 | 9058 | -9062.88 | 881 | 8565.93 | -835 | -816 | 7921.9 | 19677.98 | 19355.31 | 18270.8 |  |
|  | 531105.19 | 52011.25 | 52132.65 | 38511.66 | 38129.84 | 38188.95 | 38148.60 | 38150.59 | 38184.83 | 38141.83 | 38176.14 | 38167.77 | ${ }^{7925.55}$ | -9022.76 | -9072.85 | 0991.66 | -9075.52 | -9074.7 | -9059.23 | O40 | 9087.46 | -9075.88 | 9066 | 8843. | -8602 | -837 | -815 | 20838.52 | 20492.2 | 19371.2 |  |
| 24 | 4 51133.89 | 51971.56 | 52179.86 | 1.87 | 38185.98 | 38165.40 | 38169.98 | 38148.40 | 38177.81 | 181.8 | ${ }^{8143.5}$ | 8.1 | 7913. | -9091.61 | 9078 | 9045.65 | 9045. | -9030.15 | 9067 | 9965 | -9043.20 | 9064 | 9074.5 | 9049 | -8819.1 | -860 | -8370 | 22092 | 21682 | 20521. |  |
| 25 | 25 51119.68 | 52051.99 | 52116.23 | 38512.60 | 38180.33 | 38183.14 | 38134.51 | 38154.24 | 38174.27 | S189.2 | ${ }^{38137.74}$ | 8150.38 | 30. | -9028.36 | 9045. | 9093.40 | 905 | 9101.66 | 0988 | 9025. | -9071.1 | 9089 | -9993.15 | 9068 | -9062.8 | -8813 | -8587 | 23451 | 22996 | 21716. |  |
|  | 651107.57 | 52033.36 | 52103.76 | 38550.04 | 38143.12 | 38187.69 | 139.25 | 38174.65 | 38201.31 | 8142.5 | 38179.43 | 8143.02 | -7918.7 | -9035.68 | -9066.68 | 9055.40 | 909 | 9082.50 | 056. | 9073 | -9995.7 | 9062 | -9078.34 | 088 | -9994.6 | -9073. | -8845 | ${ }^{2483}$ | 242 | 2300 |  |
|  | 51134.66 | 0.5 .57 | 52141.86 | 38500.25 | 38183.42 | 8189.94 | 38155.54 | 38172.22 | 38170.39 | 38162.6 | ${ }^{38135.76}$ | 38135.42 | 7906.73 | -9022.21 | 9081.50 | -9061.66 | 9087.02 | -9072.70 | 9075.02 | 9952 | -9072.9. | 9106 | -9667.3 | 9074 | -9074.1 | -9071. | -9067 | 26327 | 25750 | 24300 | 22972 |
|  | 8 51138.09 | 52051.07 | 52105.53 | 38545.96 | 38117.27 | 38174.47 | 3820 | 38109.52 | 38144.70 | 38159. | ${ }^{38174.81}$ | 38178.11 | 7917. | -9015.74 | 9966 | -9089.48 | -9078 | -9075.1 | 9075 | -9097 | -9106 | -9072 | -9058 | 9084 | -9661 | -9077 | -9077 | 2789 | 27252.4 | 2576 | 24 |
|  | 51092.22 | 52038.56 | 52103.13 | 56 | 38181.17 | 38169.97 | 38188.97 | 3813 | 38141.34 | 38164.91 | 38148.20 | 381 | -7916.97 | -9052.32 | -9068.96 | 9075.09 | 908 | 9063 | 907 | 9102 | -9117 | -9089.80 | -9121 | 9095 | -9057 | -906 | -9037. | 278 | 28815.8 | 27261.36 | 25737. |
|  | 5145.1 | 52023.2 | 521 | 38483. | 38169 | 38123.73 | 38197.31 | 38131.38 | 38136 | 3 | 38168 | 38170.04 |  | -9051.35 | -9081.67 | -9089.20 | -9079.82 | -9081.93 | -9997.50 | -9086.17 | -9070.71 | --983.92 | -9113.71 | -9087 |  | -9051.23 | -9062.07 | ${ }^{27882.65}$ |  | 28828.36 |  |

Table 35: Internal Validity: Experiment 1 Net Value Period Period


Table 36: Internal Validity: Experiment 1 Net Value Period Period Standard Deviation


Table 37: Internal Validity: Experiment 2 Net Value Period Period


Table 38: Internal Validity: Experiment 2 Net Value Period Period Standard Deviation

|  | Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|  | 1 | 10.59 | 4.25 | 42.9 | 27.86 | 22.49 | 29.57 | 29.72 | 1.02 | 4.75 | 3.25 | 6.58 | 19.58 | 6.61 | 25.15 | 5.88 | 6.20 | 1.30 | 2.39 | 2.41 | 9.67 | 2.14 | 5.26 | 9.24 | 11.25 | 6.86 | 0.82 | 0.64 | 3.07 | 5.06 | 2.92 |
|  | 2 | 16.6 | 10.5 | 19. | 17. | 21.8 | 16.84 | 8.25 | 5.86 | 58.30 | 19.92 | 2.09 | 9.78 | . 69 | 22.67 | 3.89 | 18.49 | 22.04 | 9.56 | 10.99 | 16.60 | 12.93 | 7.64 | 12.88 | 1.07 | 4.72 | 4.80 | 72 | 0.26 | 1.55 | 2.89 |
|  | 3 | 2.10 | 32.2 | 6.64 | 41. | 11.8 | 42.1 | 26. | 39.9 | 8.40 | 3.49 | 7.72 | 4.22 | 18.84 | 7.20 | 01 | 20.03 | . 66 | 9.56 | 17.24 | 20.69 | 6.81 | 11.46 | 0.56 | 1.66 | 9.00 | 7.12 | 9.46 | 12.79 | 10.12 | 8.80 |
|  | 4 | 14.5 | 10.7 | 16.2 | 2.99 | 0.66 | 51.5 | 21. | 13. | 61.3 | 33.46 | 32.67 | 39.34 | 0.48 | 33.37 | 6.96 | 3.54 | 0.47 | 1.26 | 9.69 | 4.38 | 6.77 | 10.70 | 14.7 | 5.54 | 7.85 | 2.4 | 31.67 | 3.15 | 4.10 | 20.66 |
|  | 5 | 20.89 | 9.41 | 17.69 | 11.67 | 13.63 | 3.68 | 28.40 | 0.19 | 10.00 | 29.33 | 5.70 | 20.61 | 12.94 | 6.48 | 0.49 | 1.29 | 36.41 | 14.43 | 15.20 | 24.0 | 3.96 | 20.5 | 11.6 | 16. | 10.59 | 3.89 | 2.57 | 9.47 | 6.89 | 15.50 |
|  | 6 | 10.30 | 25.1 | 26. | 5.70 | 39.05 | 49. | 6.28 | 7.02 | 22.45 | 46.36 | 64. | 24.57 | 22. | 1.05 | 3.89 | 1.62 | 12. | 28.41 | 15.5 | 0.92 | 19. | 0.12 | 3.60 | 5.97 | 18.25 | 6.38 | 13.32 | 12. | 8.81 | 10.20 |
|  | 7 | 10.97 | 19.2 | 3.82 | 37.77 | 5.79 | 11.6 | 59. | 18.20 | 1.2 | 8.75 | 24. | 38.25 | 10.17 | 10.74 | 16.34 | 12.89 | 16.55 | 1.17 | 7.40 | 11.88 | 25.41 | 5.18 | 9.26 | 6.17 | 2.60 | 25.31 | 13.78 | 11. | 21.97 | 10.22 |
|  | 8 | 0.48 | 8.1 | 6.81 | 14. | 54.19 | 41.97 | 8.69 | 2.13 | 15.15 | 27.27 | 40.20 | 1.98 | 4.99 | 45.03 | 9.02 | 0.94 | 0.89 | 32.11 | 2.46 | 10.7 | 25.9 | 12.26 | 10.43 | 6.30 | 25.8 | 3.35 | 7.18 | 29.00 | 3.88 | 7.81 |
|  | 9 | 22.51 | 46.4 | 33.24 | 8.40 | 20.08 | 39.91 | 16.6 | 55.1 | 17.97 | 26.78 | 17. | 12.71 | 27.4 | . 53 | 1.60 | 16.21 | 46.27 | 32.2 | 4.23 | 3.93 | 20.3 | 28.6 | 0.45 | 17.69 | 6.56 | 35.46 | 7.65 | 5.76 | 6.85 | 7.58 |
|  | 10 | 10.49 | 4.78 | 0.13 | 39.04 | 6.5 | 19.6 | 46.36 | 87.74 | 0.50 | 0.08 | 25. | 4.97 | 73.36 | 21.0 | 17.3 | 32. | 28.53 | 23.98 | 16. | 4.55 | 9.53 | 3.36 | 9.84 | 3.57 | 22.69 | 7.61 | 19.85 | 12.63 | 6.06 | 7.41 |
|  | 11 | 60.8 | 36.62 | 4.75 | 7.10 | 26.66 | 15.75 | 14.8 | 16.53 | 24.96 | 29.15 | 2.05 | 13.16 | 9.86 | 16.35 | 20.97 | 30.9 | 27.98 | 33.62 | 28.38 | 23.05 | 29.8 | 31.82 | 1.12 | 3.42 | 12.7 | 13.0 | 2.20 | 7.96 | 4.56 |  |
|  | 12 | 8.49 | 31.4 | 5.2 | 58.58 | 34.78 | 37.2 | 9.82 | 3.21 | 33.7 | 14.65 | 2.9 | 9.45 | 34. | 2.07 | 8.76 | 10.6 | 29.92 | 25.64 | 15.08 | 16.0 | 5.2 | 23.59 | 37. | 24.2 | 10.6 | 20.56 | 12.10 | 2.51 | 6.18 | 12.14 |
|  | 13 | 2.19 | 29.09 | 31.1 | 22.79 | 9.26 | 42.7 | 11.55 | 3.3 | 15.54 | 21.31 | 2.4 | 42.88 | 48.7 | 24.6 | 17.6 | 24.46 | 2.02 | 14.55 | 1.65 | 4.10 | 57.0 | 2.19 | 3.62 | 19.51 | 13.36 | 0.52 | 22.78 | 16.79 | 15.39 | 3.27 |
|  | 14 | 33.19 | 33.75 | 2.43 | 18.05 | 0.60 | 21.81 | 9.50 | 30.58 | 26. | 7.10 | 9.07 | 5.76 | 3.38 | 3.70 | 20 | 11.24 | 0.22 | 4.05 | 83 | 59.1 | 13. | 13.14 | 3.73 | 69.62 | 26.59 | 2.31 | 2.6 | 14.79 | 3.12 |  |
|  | 15 | 39 | 2.22 | 19. | 26. | 36.93 | 24.4 | 1.33 | 33.14 | 0.39 | 24.4 | 4.53 | 7.64 | 9.05 | 24.39 | 6.18 | 19. | 22.78 | 4.99 | 46.92 | 12. | 21.5 | 0.37 | 9.16 | 26.4 | 13.3 | 26.8 | 4.08 | 11.70 | 21 | 11.3 |
| $\stackrel{\circ}{\circ}$ | 16 | 3.43 | 28. | 35.39 | 2.85 | 45.41 | 52.1 | 14.57 | 3.45 | 1.34 | 44. | 49.02 | 13.56 | 5.83 | 49.58 | 4.05 | 20.06 | 13.69 | 44.43 | 0.58 | 16.74 | 38.39 | 2.46 | 20.34 | 25.65 | 27.41 | 9.04 | 12.72 | 19.74 | 14.26 |  |
|  | 17 | 14.7 | 5.70 | 19.43 | 7.67 | 5.53 | 7.42 | 24.30 | 18.11 | 12. | 32.02 | 18.20 | 13.58 | 5.06 | 18.56 | 10.6 | 43.77 | 21.13 | 16.68 | 45.70 | 22.52 | 6.31 | 0.90 | 0.86 | 13. | 3.90 | 3.48 | 16.31 | 9.59 | 6.07 | 11.72 |
|  | 18 | 31.00 | 33.37 | 1.10 | 20.64 | 10.10 | 33.44 | 40 | 74 | 50.38 | 70.00 | 44.26 | 44.39 | 6.13 | 31.02 | 34.41 | 7.2 | 8.89 | 8.47 | 17.83 | 18.99 | 1.2 | 40.3 | 13.02 | 60.79 | 29.2 | 9.97 | 20.93 | 25.3 | 0.3 |  |
|  | 19 | 39. | 12. | 12.8 | 49.5 | 9.14 | 33. | 38.03 | 10.11 | 56 | 33. | 13.1 | 29. | 7.84 | 17.58 | 13.72 | 32. | 2.87 | 53.46 | 38.44 | 18.15 | 31.5 | 2.14 | 6.85 | 10.89 | 42.00 | 24.60 | 41.29 | 17.25 | 34.52 | 27.65 |
|  | 20 | 14.6 | 14.78 | 31.4 | 37.3 | 15. | 41.13 | 42.84 | . 3 | 3 | 27.48 | 6.80 | 19.45 | 43.76 | 5.96 | 29.66 | 17.95 | 25.12 | 0.83 | 7.61 | 0.21 | 37.30 | 21.3 | 2.24 | 41.99 | 3.28 | 7.24 | 11.62 | 2.25 | 6.31 | 9.43 |
|  | 21 | 46.0 | 53.9 | 26.8 | 0.07 | 17.30 | 11.5 | 14.53 | 35.0 | 25 | 44.77 | 42.2 | 20.9 | 13.2 | 7.51 | 6. 27 | 23.67 | 31. | 16.83 | 32.43 | 36.05 | 56. | 41.0 | 50.9 | 5.89 | 10.3 | 29.0 | 4.9 | 45.03 | 3.97 | 10. |
|  | 22 | 18.4 | 0.75 | 13.02 | 11.07 | 65.74 | 21.55 | 8.82 | 39.91 | 36.84 | 25.00 | 36 | 12.22 | 31.56 | 18. | 12.20 | 5.48 | 55.16 | 12.08 | 49.63 | 6.73 | 46.1 | 66. | 34. | 17.5 | 72.4 | 14.05 | 28.1 | 5.93 | 13.20 |  |
|  | 23 | 1.72 | 49.98 | 16.08 | 5.38 | 34.24 | 12.01 | 3.69 | 4.93 | 9.50 | 17.87 | 42 | 23. | 3.65 | 22.23 | 14.26 | 22. | 24.1 | 54.29 | 14.67 | 32.51 | 25. | 21. | 7.18 | 4.82 | 37.50 | 5.94 | 19.44 | 22.69 | 42.73 | 8.93 |
|  | ${ }^{24}$ | 48.64 | 8.5 | 1.8 | 51. | 28.72 | 4.91 | 33.6 | 9.3 | 32.3 | 1.59 | 62.5 | 10.02 | 17.70 | 1.81 | 32.81 | 0.16 | 0.0 | 15.19 | 60.2 | 3.97 | 30.1 | 3.60 | 15.90 | 74.3 | 8.29 | 5.65 | 6.00 | 4.57 | 7.82 | 6.27 |
|  | 25 | 2.38 | 0.13 | 53.29 | 37.16 | 1.72 | 11.39 | 24.71 | 7.12 | 38.91 | 35.35 | 3.24 | 29.86 | 8.02 | 60.11 | 44.68 | 2.68 | 22.93 | 2.09 | 11.8 | 2.40 | 68.2 | 4.0 | 22.25 | 16.73 | 2.58 | 18.5 | 8.86 | 38.4 | 4.0 | 4.96 |
|  | 26 | 50.8 | 30.25 | 5.94 | 39.41 | 17.81 | 17.48 | 10.65 | 37.07 | 7.43 | 12.69 | 21. | 11. | 18.07 | 24.33 | 25 | 19.55 | 13.54 | 21.50 | 43.1 | 1.35 | 21.0 | 43.45 | 20. | 24.5 | 26.99 | 13.49 | 19.14 | 22 | 8.12 | 63.76 |
|  | 27 | 28.5 | 40.04 | 47. | 14.27 | 2.53 | 6.41 | 5.29 | 9.24 | 9.29 | 23.79 | 12.8 | 23. | 36.9 | 3.03 | 47. | 5.07 | 0.03 | 3.96 | 33.96 | 4.65 | 80.12 | 46.69 | 36.81 | 7.56 | 21.15 | 2.5 | 12.18 | 18.01 | 5.62 | 7.11 |
|  | 28 | 20.12 | 22.32 | 20 | 0.85 | 17.41 | 25.73 | 49.80 | 39.12 | 9.90 | 15.77 | 17.18 | 24.21 | 42.24 | 12.90 | 6.01 | 13.09 | 15.70 | 15.74 | 42.93 | 5.39 | 37.73 | 0.2 | 18. | 17.46 | 71.0 | 32.4 | 4.94 | 30.36 | 21.7 | 35.28 |
|  | ${ }^{29}$ | 72 | 9.94 | 10.28 | 37.27 | 21.89 | 8.48 | 10.15 | 26.43 | 8.26 | 42.40 | 18.86 | 9.02 | 59.02 | 47.43 | 46.12 | 36.19 | 27.10 | 27.10 | 38 | 18.3 | 21.8 | 12.08 | 11.11 | 16.75 | 2.01 | 35. | 9.06 | 4.74 | 19.66 | 12.3 |
|  | 30 | 41.77 | 63.29 | 35.14 | 59.25 | 27.30 | 9.86 | 29.16 | 61.79 | 48.02 | 24.08 | 16.93 | 10.11 | 29.24 | 13.45 | 3.14 | 14.00 | 25.03 | 10.77 | 23.18 | 20.71 | 17.01 | 2.19 | 14.36 | 39.77 | 22.19 | 6.44 | 11.08 | 22.45 | 23.09 | 51.71 |

Table 39: Internal Validity: Experiment 2 - Experiment 1 Absolute Value

## E. 3 Extreme Value


#### Abstract

                            


Table 40: Extreme Value Test: Net Value Per Period with $r=0.05$

| Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | ${ }^{25}$ | 26 | 27 | 28 | 29 |  |
|  | ${ }^{1} 78679.11$ | 48865.88 | 29347.27 | 14870.23 | 8927.94 | 5372.76 | 3241.71 | 1957.17 | 1180.61 | 713.45 | ${ }^{430.84}$ | 260.17 | 54.10 | 32.57 | 19.45 | ${ }^{11.66}$ | 6.98 | 4.20 | 2.51 | 1.50 | 0.90 | 0.54 | 0.32 | 0.19 | 0.12 | 0.07 | 0.04 | 0.07 | ${ }_{0} 0.04$ |  |
|  | 278690.55 | 80 | 490 | 24 | 14 | 89 | 5377.40 | 3243.95 | 1958.48 | 1181.35 | 713.58 | 430.80 | 90.41 | 54.20 | 32.47 | 19.51 | 11.70 | 7.01 | 4.19 | 2.51 | 1.51 | 0.90 | 0.54 | 0.32 | 0.19 | 0.12 | 0.07 | 0.12 | 0.07 | 0.04 |
|  | 78686.98 | 808 | 810 | 41609.5 | 2476 | 148 | 8930.67 | 5373.85 | 3245.58 | 1957.00 | 1181.49 | 713.53 | 151.06 | 90.54 | 54.27 | 32.48 | 19.47 | ${ }^{11.67}$ | 6.98 | 4.18 | 2.52 | 1.51 | 0.90 | 0.54 | 0.32 | 0.19 | 0.12 | 0.19 | 0.12 |  |
|  | 787 | 80878 | 81011. | 68831 | 41427.91 | 247 | 148 | 892 | 53 | 3242.10 | 1956.39 | 1181.70 | 252.58 | 150.80 | 90.43 | 54.21 | 32.59 | 19.50 | 11.69 | 7.00 | 4.19 | 2.51 | 1.51 | 0.90 | 0.54 | 0.32 | 0.19 | 0.32 | 0.19 | 0.12 |
|  | ${ }^{78660.27}$ | 80817.1 | 811 | 68857.42 | 68525.8 | 41382 | 247 | 14853.98 | 8930.61 | 7.84 | 3244.62 | 96.84 | 420.38 | 97 | 150.63 | 90.40 | 54.22 | 32.52 | 19.52 | 11.67 | 6.99 | 4.19 | 2.52 | 1.51 | 0.90 | 0.54 | 0.32 | 0.53 | ${ }^{0.32}$ |  |
|  | 78686.8 | 808 | 811 | 68847.52 | 68497.7. | 685 | 414 | 24 | 14848.60 | 8925.21 | 7.07 | ${ }^{3243.20}$ | 703.19 | 419.33 | 251.42 | 150.71 | 90.44 | 54.28 | 2.53 | 19.45 | 11.69 | 6.97 | 4.21 | 2.51 | 1.50 | 0.90 | 0.54 | 0.88 | ${ }^{0.53}$ | 0.32 |
|  | 78678.7 | 80838.52 | 81042.93 | 68900.0 | 68489.05 | 68539.41 | 68571.96 | 41416.15 | 24779.4 | 14844.08 | 3.32 | 5380.31 | 1175.28 | 700.96 | 420.15 | 250.86 | 150.99 | 90.30 | 54.12 | 32.49 | 19.52 | ${ }^{11.69}$ | ${ }^{7} .00$ | 4.20 | 2.52 | 1.50 | 0.90 | 1.46 | ${ }^{0.88}$ | 0.53 |
|  | 78679.47 | 80822.30 | 81041.8 | 68892.60 | 68523.55 | 68560.07 | 68525.21 | 68562.03 | 41428.31 | 24762.36 | 14845.79 | 8929.2 | 1970.50 | 1.63 | 700.08 | 420.10 | 251.50 | ${ }^{150.76}$ | 90.37 | 54.18 |  | 19.50 |  | 7.00 | ${ }^{4.20}$ | 2.51 | 1.50 | ${ }^{42}$ | 1.46 | 0.88 |
|  | 786 | 808 | 810 | 68909.71 | 6850 | 68555 | 685 | 68525.2 | 685 | 414 | 24 | 14845.94 | 3317.52 | 1955.11 | 1171.26 | 701.02 | 419.52 | 251.01 | 151.20 | 90.30 | 54.24 | 32.42 | 19.5 | ${ }^{11.65}$ | ${ }^{7} .00$ | 4.19 | 2.51 | 4.00 | ${ }^{2} .41$ |  |
|  | 78665.76 | 80838. | 81107.29 | 68846 | 68495 | 68564.67 | 68512.28 | 68609.54 | 68557 | 68526.91 | 41419.28 | 24767.02 | 5597.67 | 3279.15 | 1956.97 | 1168.76 | 702.07 | 19.61 | 251.45 | 150.67 | 0.49 |  |  | 19.50 | 11.6 | 6.99 | 19 |  |  |  |
|  | 786 | 8882 | 81112.27 | 68797.66 | 6850 | 685 | 68554.46 | 68535.85 | 685 | 68544.57 | 68587.04 | ${ }^{41391.36}$ | 9512.3 | 5497.67 | 3277.12 | 1956.23 | 169.6 | 699.49 | 19.17 | 251.30 | ${ }^{151.16}$ | 90.36 | ${ }^{5} 4.22$ |  | 19.48 | 1.6 | . 01 |  |  |  |
| 12 | 278668.74 | 80826.02 | 81109.51 | 68842.42 | 68496.54 | 68511.46 | 68567.95 | 68855.70 | 68542.53 | 65572.24 | 68519.25 | ${ }_{68606.38}$ | 16298.73 | 9271.20 | 5505.23 | 3277.77 | 1955.06 | 1170.90 | ${ }^{699.14}$ | 19.86 | 251.46 | 151.01 |  | 54.16 |  | 19.5 |  |  |  |  |
| 13 | -378674.22 | 80863.28 | 81036.56 | 68875.45 | 68551.44 | 68533.90 | 68563.11 | 68516.29 | 68566.71 | 68553.85 | 68505.13 | 68573.24 | 27124.79 | 15712.65 | 9270.77 | 5496.88 | 3277.81 | 1950.56 | 1172.01 | 701.75 | 420.14 | 251.83 | 150.69 | 90.51 | 54.2 | 32.48 | ${ }^{9.43}$ |  |  |  |
| 14 | ${ }^{4} 78723.82$ | 80793.53 | 81030.79 | 68906.39 | 68464.74 | 68612.06 | 68856.14 | 68833.00 | 68551.70 | 68561.43 | 68564.17 | 68526.70 | 27114.23 | 26107.50 | 15704.08 | ${ }^{9273.38}$ | 548.40 | 3273.28 | 1961.09 | 1169.85 | 700.97 | 419.34 | 251.17 | 150.74 | 90.49 | 54.29 | 32.53 |  | 30.11 |  |
| 15 | ${ }^{5} 78884.71$ | 30859.91 | 80998.60 | 68882.02 | 68994.33 | 68565.60 | 68547.05 | 68599.19 | 68537.74 | 68559.91 | 68560.26 | 68 | 27101.06 | 26136.66 | 26132.54 | 15709.45 | 9261.01 | 5496.70 | 3274.59 | 1957.20 | 1169.55 | 699.94 | 419.57 | 252.00 | ${ }^{150.61}$ | ${ }^{90.33}$ | ${ }_{5} 5.28$ | 82.64 |  |  |
| 16 | ${ }^{178698.42}$ | 80831.81 | 81052.83 | 68885.21 | 68481.60 | 68 | 68552.97 | 68551.99 | 68599.85 | 68550.30 | 68531.36 |  | 27070.85 | 26159.22 | 26150.56 | 26128.49 | 15703.89 | 9284.27 | 5487.32 | 3274.89 | 1955.91 | 1168.95 | 701.18 | ${ }^{420.30}$ | 251.85 | 150.89 | 90.42 | 136.86 |  |  |
| 17 | 7 | 80863.79 | 81049.83 | 68851.08 | 68553.72 | 68 | 68556.06 | 68532.80 | 68 | 68580.77 | 68554.30 | 68541.06 | 27090.03 | 26117.06 | 26150.96 | 26177.29 | 26172.32 | 15695.51 | 9278.76 | 5499.98 | ${ }^{3275.53}$ | 1956.76 | 1169.69 | ${ }^{699.90}$ | 418.72 | 251.36 | ${ }^{150.73}$ | 226.61 | ${ }^{136.96}$ |  |
| 18 | 878682.57 | 80855.79 | 81041.3 | 68871.9 | 68553.68 | 688 | 68556.90 | 68508.03 | 68 | 68514.60 | 68523.47 | ${ }^{68577.66}$ | 27098.10 | 26110.25 | 26157.07 | 26177.28 | 26164.07 | 26154.47 | 15703.82 | 9275.48 | 5498.8 | 275.2 | 1952.77 | 1170.01 | 700.08 | 19.63 | 251.43 | 375.65 | 22.60 |  |
| 19 | ${ }^{9} 78836.31$ | 80853.13 | 81078.3 | 68872.62 | 68512.48 | 68 | 68523.74 | 68 | 68 | 68573.8 | 68513.27 | ${ }^{68545.59}$ | 27129.31 | 26100.18 | 26098.09 | 26192.84 | 26169.9 | 26177.6 | 26115.3 | 157 | 9266 | 5484.45 | 3274.66 | 1957.62 | 167 | 700.64 | 419.68 | 621.79 | 375.74 | 226.58 |
| 20 | 20 | 88839.22 | 81 | 688 | 685 | 685 | 683 | 68525.22 | 68566.87 | 68576.55 | 68 | 68514.34 | 27064.27 | 26130.24 | 26 | 26159.4 | 26179.44 | 26092.92 | 26129.32 | 26142 | 15742 | 9262.01 | 503.49 | 3272.68 | 957 | 168. | . 63 | 1030.85 | 621.95 | 375.58 |
| 1 | 2178692.10 | 80777.29 | 81129.43 |  | 68532.99 |  | 68569.41 | 68560.13 |  | 6 | 68522.29 | 68 | 27075.48 | 26168.23 | 26114.93 | 26117.05 | 26176.78 | 26112.29 | 26155.78 | 26164 | 26176 | 15707 | 9274.63 | 5495.14 | 277.22. | 1959. | 168.1 | 1706.97 | 83 | ${ }^{622.41}$ |
| 22 | 2278699.02 | 7.92 | 810 | 68819.49 | 68 | 688 | 68570.74 | 68581.80 | 68500.12 | 68552.61 | 68563.64 | 68553.39 | 27114.51 | 26144.0 | 26109.72 | 26163.85 | 26126.0 | 26165.18 | 26115.20 | 26206 | 26144.28 | 26142.26 | 15691.7 | 9272.31 | 5509.52 | 3278 | 1953. | 2830.51 | 1708.06 | 103 |
| 23 | 23 | 80893.97 |  |  |  | 68569.70 |  | 33 | 68563.35 | 68592.56 | 68523.56 | 68555.70 | 27078.42 | 26127.22 | 26150.64 | 26143.13 | 26119.4 | 26155.66 | 26135.97 | 26148 | 26183.71 | 26142.64 | 26174. | 15679.52 | 9263.44 | 550 | 3272.33 | 4691.02 | 2832.74 | 1707.87 |
| 24 | 24786 | 80873.21 | 81 | 68821.94 | 68 | 68524.75 | 68529.35 | ${ }^{68617.40}$ | 68500.94 | 68617.40 | 68526.45 | 68530.73 | 27101.21 | 26143.31 | 26144.29 | 26154.36 | 26142.9 | 26129.74 | 26181.60 | 26162 | 26148.14 | 26123.3 | 26134 | 26168.43 | 15701.85 | 9289. | 5492.68 | 7783. | 4707.14 | 283 |
| 25 | 25 | 80854.22 | 81 | 68850.65 | 6 | 6 | 68542.61 | ${ }^{68566.53}$ | 685 | 68542.8 | $68:$ | 68533.40 | 2705 | 2615 | 26147.80 | 26174.3 | 26124.9 | 26175.5 | ${ }^{2616}$ | ${ }^{2612}$ | 26172.24 | 26135 | ${ }^{2613}$ | 26169 | 261 | ${ }^{15718}$ | ${ }^{9273}$ | ${ }^{1292}$ | 781 | 4702.39 |
| 26 | 26 | 80804.65 | 810 | 68915.1 | 68 | 684 | 68887.00 | ${ }^{68548.91}$ | 68569.7 | 68560.7 | 68510.57 | 68573.27 | 27135.71 | 26112.87 | 26155.41 | 26152.60 | 26159.40 | 26166.84 | 26123.58 | 26159 | 26155.76 | 26158.41 | 2614 | 2615 | 26118.93 | 2618 | 15717 | 21517 | 1300 | 781 |
| 27 | 27 | 808 | 810 | 688 | 68474.09 | 686 | 68515.06 | 68820.44 | 68551.0 | 68574.93 | 68580.36 | 68546.53 | 2.92 | 26159.62 | 26147.15 | 261 | 26142.99 | 26141.8 | 26150 | 26199.1 | 26125.90 | 26162.40 | 26124 | 26115 | 26207.43 | ${ }^{2612}$ | ${ }^{2612}$ | 35861 | 217 | ${ }^{130}$ |
| 28 | 28.78868 .74 | 80838.60 | 810 | 68843.83 | 68556.2 | 68519.91 | 68883.16 | ${ }_{68530.45}$ | 68542.07 | 68534.16 | 68542.27 | 68855.16 | . 39 | 26108.98 | 26165.04 | 26119.15 | 26202.42 | 26098.19 | 26200.63 | 26172.54 | 26142.60 | 26110.01 | 26180.19 | 26125.96 | 26162.20 | 26148 | 26143 | 59386 | 3639 | 21713.42 |
| 29 | ${ }^{67}$ | 80805.23 | 81056.05 | 68875.66 | 68535.10 | 68561.37 | 68568.54 | ${ }^{68550.27}$ | 68520.52 | 68506.72 | 68598.15 | 68509.15 | 1.78 | 26129.50 | 26141.41 | 26139.62 | 26121.71 | 26149 | 26163 | 26156.57 | 26175.54 | 26150.95 | 26129 | 26150.68 | 26095.13 | 26117.71 | 26157.26 | 5445 | 6021 |  |
| 30 | 3078724.1 | 80 | ${ }_{81015.69}$ | 68 | 68 | ${ }^{6}$ | 6 | 68499.67 | ${ }^{6}$ | 68535.22 | 68527.94 | 2 |  |  | 9 | 26118.04 | 26164.99 | 8 |  | 26147.66 | 26138.52 | 26133.39 | 1 |  |  | 26180.10 | 26151.03 | 59346.83 | 60248.49 |  |

Table 41: Extreme Value Test: Net Value Per Period with $\mathrm{r}=0.5$

| Elapsed Time |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | ${ }^{23}$ | ${ }^{24}$ | ${ }^{25}$ | 26 | 27 | 28 | 29 | ${ }^{30}$ |
|  | 78695.20 | 29651.94 | 10711.83 | 3285.94 | 1199.89 | 439.42 | 160.89 | 58.98 | 21.60 | 7.91 | 2.90 | 1.06 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | . 00 | 0.00 | 0.00 | 0.00 | 8.00 | 0.00 | . 00 |
| 2 | 78685.19 | 80871.54 | 29711.00 | 9049.44 | ${ }^{3285.30}$ | 1199.11 | 439.33 | 161.01 | 58.97 | ${ }^{21.61}$ | 7.92 | 2.90 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | ${ }^{0.00}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | . 00 | 0.00 | 0.00 |
|  | 78882.2 | 808 | 810 | 5252. | 9033.01 | ${ }^{3285.27}$ | 1200 | 439.43 | 160.89 | 58.95 | 21.61 | 7.91 | 1.01 | ${ }^{0.37}$ | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|  | 78687.93 | 80836.53 | 81025.45 | 68872.86 | 25120.45 | 9034.11 | 3283.46 | 1199.64 | 439.07 | 160.95 | 58.97 | ${ }^{21.60}$ | 2.79 | 1.02 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | ${ }^{0.00}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|  | 78672.91 | 80824.70 | 81052.25 | 68890.26 | 68545.94 | ${ }^{25113.73}$ | 9099.65 | 3283.55 | 1200.58 | ${ }^{439.26}$ | 160.89 | 58.96 | 7.67 | 2.79 | ${ }^{1.02}$ | 0.37 | 13 | 0.05 | . 02 | 0.01 | 0.00 | ${ }_{0}^{0.00}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | ${ }^{0.00}$ | 0.00 |
|  | 78685.07 | 80816.35 | 810 | 68831.53 | 68494.93 | 6 | 4.49 | ${ }^{9033.56}$ | 3284.33 | 1200.85 | 9.47 | 60.92 | . 10 | 7.67 | 2.79 | 1.02 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|  | 78696.28 | 80794.218 | 810 | 6 | 68520.73 | 68574.82 | 68567.03 | 25118.86 | 9037.57 | 5.64 | 99.66 | 9.34 | . 94 | 21.09 | 7.66 | 2.79 | 1.02 | 0.37 | 0.13 | ${ }^{0.05}$ | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | ${ }^{0.00}$ | 0.00 | 0.00 |
| 8 | 78713.68 | 80796.95 | 81069.01 | 68898.76 | 68499.10 | ${ }^{6}$ | 68524.33 | 68544.63 | 25 | 9032.30 | 3285.78 | 1200.03 | 9.63 | S. 00 | . 08 | 7.65 | 2.79 | 1.02 | 0.37 | ${ }^{0.13}$ | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|  | 78721.67 | 80754.65 | 810 | 68 | 65548.05 | 68547.19 | 68330.56 | 68555.61 | 68551.04 | 25122.50 | 9038.99 | 3282.41 | 439.81 | 159.22 | 57.99 | ${ }^{21.07}$ | 7.68 | 2.79 | 1.01 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10 | 78706.05 | 80851.11 | 81022.27 | 68857.14 | 68544.73 | 6 | 68495.90 | 68866.58 | 68529.89 | 68556.25 | 25122. | 9034.18 | 16.65 | 38.45 | 59.73 | 57.90 | 21.08 | 7.67 | 2.79 | 1.02 | ${ }^{0.37}$ | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | ${ }^{0.00}$ | 0.00 | 0.00 |
| 11 | 78658.72 | 808 | 8109 | 68911. | 6 | 6 | 68569.47 | 68548.95 | 68582.80 | 68523.9 | 68553.9 | 25104 | ${ }^{421.90}$ | 10.72 | ${ }^{38.45}$ | 158.92 | 58.03 | ${ }^{21.08}$ | 7.66 | 2.80 | 1.02 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| 12 | 786 | 8080 | ${ }^{8103}$ | 68871.64 | 685 | 6855 | 6856 | 6849 | 68569 | 68570.5 | 68523 | 68575 | 86.89 | 3363.25 | 1208.21 | 8.89 | . 36 | . 91 | 21.08 | ${ }^{7.65}$ | ${ }^{2.79}$ | 1.02 | 0.37 | 0.13 | 0.05 | 0.02 | 0.01 | 0.01 | 0.00 | 0.00 |
| ${ }^{13}$ | 78698.18 | 80792.82 | 810 | 688 | 685 | 68599.63 | 6854 | ${ }^{68517.17}$ | 68571.99 | 68559.4 | 68534.8 | 6855.0 | 27124.09 | 9523.15 | 3368.72 | 1211.15 | ${ }^{438.35}$ | 159.39 | 57.96 | 21.08 | 7.67 | 2.79 | 1.02 | ${ }^{0.37}$ | 0.13 | 0.05 | ${ }^{0.02}$ | ${ }^{0.02}$ | 0.01 | 0.00 |
|  | 7873.86 | 808 | 8104 | 68822.3 | 68 | 68568.29 | 68591.67 | 68534.07 | 68563.20 | 68486.88 | 68615.6 | 68529.5 | 27146.98 | 26110.09 | 9517.70 | 3366.08 | 1210.54 | 438.33 | 159.47 | 58.05 | 21.08 | 7.69 | 2.79 | 1.02 | 0.37 | 0.13 | 0.05 | 0.05 | 0.02 | 0.01 |
| 15 | 78 | 808 | 8105 | 688 | 68 | 684 | ${ }^{68585.52}$ | 68553.92 | 68557.34 | 68568.27 | 6855.6 | 68522.7 | 27088.17 | 26126.57 | 26150.10 | 9533.50 | 3360.32 | 1211.11 | 438.54 | 159.22 | 57.85 | 21.06 | 7.66 | 2.79 | 1.02 | 0.37 | ${ }^{0.13}$ | 0.12 | 0.05 | 0.02 |
| ${ }_{1}$ | 78737.29 | 80761.55 | 810 | 689 | 688 | 685 | 68540.42 | 68616.66 | 68517.90 | 68563.51 | 68584 | 68493 | 0.44 | 26127.31 | 26156.11 | 26145.24 | 9541.09 | 3362.28 | 1208.14 | 439.00 | 159.15 | 57.85 | 21.10 | 7.66 | 2.79 | 1.02 | 0.37 | 0.34 | 0.12 | 0.05 |
|  | 7867.98 | 808 | 81046.51 | csis | 68 | 6 | 685 | 68991.06 | 68603.83 | 68615.2 | 68519 | 6853 | 8.5 | 26112.48 | 26125.90 | 26169.50 | 26149.90 | 954.07 | 3369.46 | 1212.58 | 438.20 | 159.33 | 57.81 | 21.12 | 7.67 | 2.79 | 1.01 | ${ }^{0.93}$ | 0.34 | 0.12 |
| 18 | 78672.55 | 808 | 81 | 687 | 68 | 685 | 68563.09 | 68883.80 | 68541.68 | 688 | 6849 | 6859 | 27148.11 | 2615.29 | 26168.32 | 26109.68 | 261 | 26165.14 | 9517.16 | 3368.02 | 1210.17 | 438. | 159.19 | 57.87 | 21.07 | 7.67 | 2.79 | 2.53 | 0.93 | 0.34 |
| 19 | 786 | 808 | 810 | 688 | 68 | 68 | 68616.04 | 68507.85 | 68551.90 | 688 | 68570 | 6855: | 27077.12 | 26126.01 | 26127.39 | 26175.27 | 261 | 26143 | 26137 | 9524.38 | 3372 | 1211.16 | 437.64 | 159.44 | 57.87 | 21.10 | ${ }^{7.6}$ | 6.91 | 2.53 | 0.93 |
| 20 | 78 | 80872.27 | 81 | 688 | 684 | 68 | 68883.08 | 68539.94 | 688882.24 | 685 | 68540 | 6856 | 27061.47 | 26152.94 | 26157.98 | 26153.18 | 2611 | 261 | 261 | 2614 | 9530 | 3367.73 | 1212 | 438.65 | 159.37 | 58.06 | 21.0 | 18.88 | 6.91 | 2.53 |
| 21 | 78661.82 | 80848.46 | 810 | 68869.60 | 68530.36 | 68553.95 | 68563.25 | 68561.72 | 68499.53 | 68600.21 | ${ }^{68564.43}$ | 68512.61 | 27108.63 | 26100.68 | 26176.40 | 26148.72 | 26173.94 | 26089.57 | 26207.42 | 26125.32 | 2614.82 | 9529 | 3370.92 | 1211.25 | 438. | 159 | 57.94 | 51.50 | 18.87 | 6.91 |
| 22 | 78735.38 | 807 | 81065.15 | 68867.79 | ${ }^{68470.67}$ | 68572.71 | 68542.48 | 68584.67 | 68520.56 | 68595.65 | ${ }_{68527.65}$ | 68530.71 | 27109.54 | 26142.87 | 26141.38 | 26155.18 | 26187.75 | 26079.1 | 26168.8 | 26141.83 | 26179.01 | 26126.85 | 9519 | 3369.31 | 1210.46 | 438. | 159 | 140.59 | 51.46 | 18.86 |
| 23 | 78712.97 | 80805.07 | ${ }^{81094.40}$ | 68842.47 | 68530.78 | 68562.57 | 68543.63 | 68547.81 | 68568.98 | 68528.20 | ${ }^{68533.56}$ | 68561.61 | 27100.04 | 26146.91 | 26197.03 | 26133.43 | 26161.14 | 26143.19 | 26170.45 | 26146.31 | 26148.34 | 26157.68 | 26164. | 9535.66 | 3362.50 | ${ }^{1208.50}$ | 438. | 383.98 | 140.72 | 1.56 |
| 24 | 78708.44 | 8083.908 | 81034.99 | 68869.17 | 65547.88 | ${ }^{68582.62}$ | 65540.12 | 68510.49 | 68520.25 | ${ }^{68556.56}$ | ${ }^{68805.07}$ | 68481.3 | 27175.40 | 26148.76 | 26138.54 | 26133.36 | 26150.21 | 26181.4 | 26174.8 | 26129.17 | 26200.24 | 26132.50 | 26115.76 | 26170 | 9523.50 | 362.62 | 208 | 1048.56 | 83.79 | 140.54 |
| 25 | 78709.27 | 80769.21 | ${ }^{81072.16}$ | 68905.38 | 68504.69 | 68580.56 | 68541.78 | 68524.00 | 68569.83 | 68588.85 | ${ }^{68513.60}$ | 68545.74 | 27109.31 | 26087.46 | 26161.63 | 26159.68 | 26133.38 | 26151.59 | 26147.19 | 26115.70 | 26195.38 | 26139.90 | 26111.90 | 26137.50 | 26151.74 | 9528.55 | 3365.24 | 269.94 | ${ }^{0550.03}$ | 383.97 |
| 26 | 78708.91 | 8079.94 | 81040.17 | 68893.82 | 68517.11 | 68557.3 | 68564.37 | 68516.26 | 68553.31 | 68576.02 | ${ }^{68565.86}$ | 68557 | 27082.25 | 26185.79 | 2616.06 | 26886.84 | 26202.31 | 26134 | 26154.31 | 26142.37 | 2616.97 | 26123.31 2 | 26148.05 | 26124.15 | 26196.50 | 26123.42 | 9527 | 39 | 2876.58 | 1050.16 |
| ${ }^{27}$ | 78675.06 | 8089 | 81027.25 | 68815.64 | 68528.19 | 68600.67 | 68536.33 | 68507.48 | 68598.80 | 68473.99 | ${ }_{68576.07}$ | 68629 | 27061.78 | 26162.66 | 26154.35 | 26188.34 | 26122.50 | 26181 | 26114.76 | 26174.42 | 26136.67 | 26163.15 | 26134.10 | 26148.53 | 26119.50 | 26181.18 | 2616.51 | 04 | 7920.22 | 2875.22 |
| 28 | 78628.21 | 80862.73 | ${ }^{81071.81}$ | 68843.55 | 68542.58 | 68542.10 | 68563.76 | 68560.61 | 68509.23 | 68553.36 | 68835.45 | 6850 | 27125.05 | 26996.41 | 26101.49 | 26154.05 | 26168.08 | 26192.25 | 26104.03 | 26132.2 | 2618.75 | 26108.312 | 26140.56 | 26176.17 | 26169.61 | 26156.64 | 26187.87 | 59339 | 22072.99 | 7917 |
| 29 | 78705.46 | 80797.83 | 8119.34 | 68814.36 | 68539.10 | 68518.70 | 68542.97 | 68557.49 | 68527.63 | 68598.02 | 68600.51 | 68507.77 | 27103.74 | 26120.75 | 2615.00 | 26124.36 | 26136.02 | 26154.69 | 26138.43 | 26209.57 | 2613.81 | 26154.482 | 26126.00 | 26155.88 | 26180.20 | 26157.94 | 26103.98 | 59428 | 60224.80 | 220 |
| 30 | 78644.54 | 808 | 105 | 68849.17 | 68569.1 | 68523.18 | 68568.86 | 68529.18 | 68523.30 | 6857 | 6858 | 68540.31 | 27090.01 | 26156.93 | 26166.44 | 26679.59 | 26179.10 | 26142.53 | 26188.53 | 26154 | 26143.71 | 26135.892 | 26154.77 | 20 | 35 | 26179.13 | 609 | 59384.16 | 60244.58 |  |

Table 42: Extreme Value Test: Net Value Per Period with $\mathrm{r}=1.0$

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[^0]:    Title Page

[^1]:    ${ }^{1}$ While this seems like an unnecessary step, in financial accounting, the spot price is typically regarded as a positive number, due to the fact that the spot price represents a inbound cash flow. Conversely, the strike price is typically regarded as a negative number, representing a outbound cash flow. The transformation of Equation 3.10 is a resulting of swapping the minuend and the subtrahend in the subtraction term and multiplying the result by negative 1 to preserve the correctness of the computation. The result is that non-negative numbers can be used for both the strike and spot price in the computation.

