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PHASOR MEASUREMENT UNIT DEPLOYMENT APPROACH FOR MAXIMUM OBSERVABILITY CONSIDERING VULNERABILITY ANALYSIS

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PHASOR MEASUREMENT UNIT DEPLOYMENT APPROACH FOR MAXIMUM
OBSERVABILITY CONSIDERING VULNERABILITY ANALYSIS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Electrical and Computer Engineering

by
Jyoti Paudel
December 2015

Accepted by:
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ABSTRACT

In recent years, there has been growing interest of synchrophasor measurements like Phasor Measurement Units (PMUs) Power systems are now being gradually populated by PMU since they provide significant phasor information for the protection and control of power systems during normal and abnormal situations. There are several applications of PMUs, out of which state estimation is a widely used. To improve the robustness of state estimation, different approaches for placement of PMUs have been studied.

This thesis introduces an approach for deployment the PMUs considering its vulnerability. Two different analysis have been considered to solve the problem of locating PMUs in the systems. The first analysis shows that using a very limited number of PMUs, maximum bus observability can be obtained when considering the potential loss of PMUs. This analysis have been done considering with and without conventional measurements like zero injections and branch flow measurements. The second analysis is based on selection of critical buses with PMUs. The algorithm in latter is specifically used for the system which has existing PMUs and the scenario where new locations for new PMUs has to be planned. The need for implementing this study is highlighted based on attack threads on PMUs to minimize the system observability. Both the analysis are carried out using Binary Integer Programming (BIP). Detail procedure has been explained using flow charts and effectiveness of the proposed method is testified on several IEEE test systems.

DEDICATION

To my beloved parents and my family for whom my whole life pertains.

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CHAPTER ONE

INTRODUCTION

Power Systems has become more and more complex due to rapid increasing demand for electricity and are mostly being operated in stressed condition [1]. This situation has been one of the most responsible cause for high cost blackouts. To overcome such problem, a real-time wide area monitoring, protection and control system (WAMS) for proper management of the resources of power system is a necessary. WAMS allows the operators to ensure system security and smooth operation. An important tool for Energy Management System (EMS) is state estimation. Based on measurements taken throughout the network, state estimation gives an estimation of the state variables of the power system while checking that these estimates are consistent with the measurements. Traditionally, input measurements have been provided by the SCADA system (Supervisory Control and Data Acquisition). A disadvantage is that the measurements are not synchronized, which means that state estimation is not very precise during dynamic phenomena in the network.

With the advent of real-time Phasor Measurement Units (PMU's), synchronized phasor measurements are possible which allows monitoring of dynamic phenomena. Among the various application of PMUs, one of the most significantly affected area is state estimator. To enhance the state estimation, PMUs are the key users and most suitable devices for WAMS accomplishment [2]. PMU is a device which measures positive sequence voltage and current utilizing the Global Positioning System (GPS) to synchronize them to a common time frame [3].

1.1 Historical Overview

Phase angles of voltage phasors of power system buses have always been a keen interest of power engineers. According to theoretical prospective, the active (real) power flow in a distribution line is proportional to the sine of the angle difference between voltages at the two terminals of the line. Angle difference is treated as a basic parameter to measure the condition of power network.

In early 1980s, the modern equipment for direct measurement of phase angle difference was introduced [4, 5]. The method used for synchronizing the clock was via LORAN-C signal, GOES satellite transmission and HBG radio transmissions (in Europe). Later, researcher used positive going-zero crossing of a phase voltage to estimate the local phase angle with respect to the time reference. The phase angle difference between voltages at two buses was established utilizing the difference of measured angles to common reference, for both locations. However, the measurement accuracies were order of $40\mu\text{s}$ and was still insufficient to capture the harmonics in voltage waveform. Therefore, these were not preferable for wide-area phasor measurement systems. Later when GPS satellite were being deployed significantly in number, it was realized that GPS time signal can be utilized as an input to sampling clock. The GPS provides timing, ranging from 1 nanosecond to 10 nanoseconds [6]. At the same time, the GPS receiver can supply a unique pulse signal in one-second intervals, which is known as 1 pulse per second (PPS).

Hence, embedding such a high precision system in a measuring device, it was made clear that this system offered the most effective way of synchronizing power system

measurements over great distance. Eventually, PMU using GPS were built commercially and its deployment began to carry out on power systems worldwide.

1.2 Phasor Measurement Unit

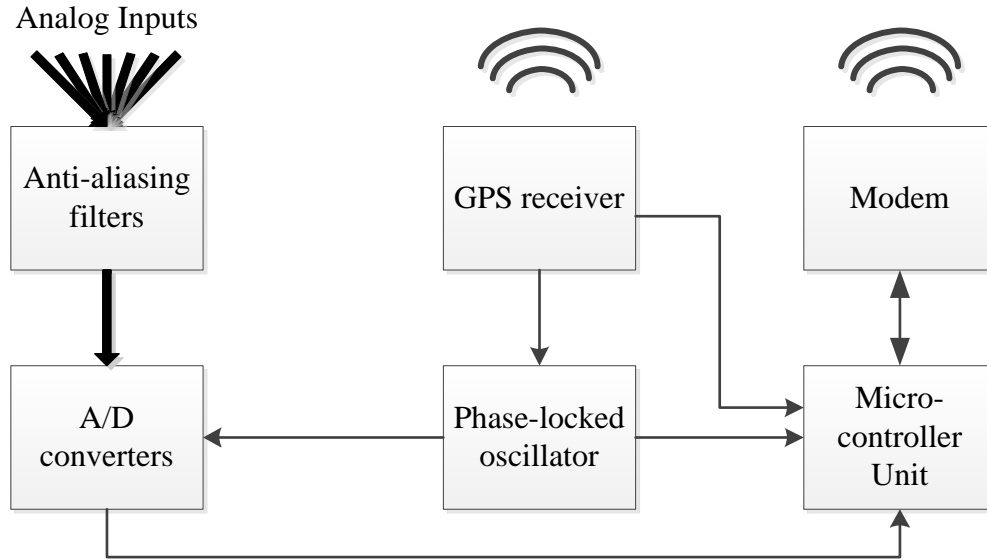


Figure 1.1 Simplified Phasor Measurement Unit block diagram [7]

PMUs were introduced in 1988 by Dr. Arun G. Phadke and Dr. James S. Thorp at Virginia Tech. A block diagram of PMU is shown in Figure 1.1. PMU can measure 50/60 Hz waveforms (voltages and currents) typically at a rate of 48 samples per cycle (2880 samples per second). At first the anti-aliasing filters are present in the input to the PMU. These anti-aliasing filters produce some delay which is the function of signal frequency. The delay occurs due to filter characteristics. So PMU has to compensate the occurred delay since the sample data are taken after anti-aliasing delay is introduced by the filter. The analog AC waveforms are digitized by an Analog to Digital converter for each phase. A phase-lock oscillator with a Global Positioning System (GPS) reference source provides the needed

high-speed synchronized sampling with 1 microsecond accuracy. The captured phasors are to be time-tagged based on the time of the UTC Time Reference.

Compared with traditional measurements received from Supervisory Control and Data Acquisition (SCADA) system, these synchronized PMUs are hundred times faster in capturing data and greater in measurement accuracy. SCADA systems are based on quasi-steady state and therefore are not capable of measuring transient phenomena. SCADA systems usually consists of Remote Terminal Units (RTUs) and are interfaced with sensors that can measure only magnitude not the phasors. Integration of PMUs into the system provides faster processing for state estimation due to the relationship between PMUs and state variables. A power system is said to be observable if the measurements deployed on it allows to determine the bus voltage magnitude and angle at every buses of the system. The system observability can be estimated by considering the topology of the network, the types and location of the measurements. Installing a PMU in a bus gives direct observability because the phasor voltage of that bus is measured by PMU directly. Also knowing the fact that PMU gives the current phasor of the branch or line which is interconnected to the bus where PMU is installed. This feature of PMU typically makes the adjacent bus indirectly observable because once the current phasors are available; voltage phasors can be estimated or calculated using line parameters.

1.3 Literature Review on PMU placement

The primary goal of state estimator is to find the optimal estimates of bus voltage phasors based on the available measurements in the system as well as the system network topology. Generally, the measurements are provided by Remote Terminal Units (RTUs) at

the substation and include real/reactive power flows, power injections, magnitudes of bus voltages and branch currents. Now, since commercial PMUs are widely available in the market, some of the utilities or power industries have started deploying in their network, whereas many of them intend to install in the grid/system in near future. At the end, the most concerned question lies in the cost. As the cost of PMUs and their installation are relatively very high, planning engineers are facing problem on planning the best location for PMUs' placement. Planning can either be for initial stage or for additional new PMUs where the system network already contains some sets of PMUs.

As discussed in earlier section, observability of a system highly depends on number of installed PMUs. If there is an occurrence of any unexpected outage in a system or in a PMU itself, it will affect the system observability and may cause a serious problem [8-10]. Due to the critical nature of power systems, complete observability of all nodes at all times is required.

The PMUs placement in strategic locations has been the vital research topic for PMU application. Even though there are numerous applications of PMUs, this research and discussion on PMU placement are strictly limited to state estimation application only. Power engineers have introduced various methodologies all across the world [11]. The researchers have approached the PMUs placement problem using two methods: (i) Heuristic approach (ii) Mathematical approach.

1.3.1 Heuristic approach

Heuristic approach has been widely adopted in this area. Simulated annealing is used in [12] to find the placement location based on desired depth of unobservability. This thesis

discussed the impact of depth of unobservability on the number of PMUs and was based on network topology. A similar approach which utilizes the stochastic models to capture dynamic state estimation uncertainties was also introduced in [13]. A sensitivity constraint optimal PMU placement is presented in [14]. Location for placement of PMUs were identified based on the buses with higher sensitivity and buses with more outlets. The simulated annealing algorithm is used in the model to get full system observability. Reference [15] solves the PMU placement problem using recursive Tabu search. Though the algorithm used for this approach give satisfactory results for larger bus systems but no robust contingency is considered. A new parallel Tabu search method for solving PMU placement problem was presented in [16]. The model used in this literature has considered the system with and without communication constraint. However the developed process execution time is high even for lesser bus systems. An optimal deployment of PMUs using differential evolution concept was presented in [17] for normal operating condition. Literature [18] addresses on $N-1$ PMU failure and solves the PMU placement problem using the same differential evolution process. Genetic algorithm in [19] solves the PMU optimization problem with an objective to get full system observability with higher measurement redundancy. Immunity genetic algorithm is proposed in [20]. The approach used in [20] is relatively time consuming and is not preferable for large systems. Binary Particle Swam Optimization (PSO) is another optimization approach that is enormously used in this field. In [21], a simple PMU placement has been implemented using BPSO but the algorithm does not consider details regarding PMU vulnerability. Since all the

techniques discussed in heuristic approach, being iterative in nature, requires time for convergence and also the convergence fully depends on the initial guess.

1.3.2 Mathematical approach.

Mathematical approach has been gaining popularity from recent years. They are easy to apply in the situation where a definite solution is required. They are based on formulas derived from mathematical calculations. Integer linear programming is a common approach as presented in [22], in which a general formulation for PMU placement using conventional and without conventional measurement is taken into consideration. Conventional measurements refers to zero injection measurements and branch flow measurements. Using a similar concept, a unified approach is presented in [23] using binary integer linear programming. The mathematical formulation described in this literature considers the single PMU losses using zero injection and flow measurements separately. Contingency constrained optimal PMU placement using exhaustive search approach is proposed in [24]. This literature has taken several zero-injection buses in account for PMU placement considering single PMU loss and measurement channel limitation. Mixed Integer Linear Programming is used in [25] which considers zero injection and branch flow measurements in order to maximize the measurement redundancy and reduce the number of PMUs. However, the approach in [25] requires almost twice the amount of PMUs to obtain full system observability under contingency operation than at normal operating conditions.

1.4 Scope of the work and Objectives

To the best of my knowledge, all the literatures on PMU placement approaches mentioned so far is limited to single PMU loss. However, PMU placement based on vulnerability analysis is scant. Vulnerability can arise from various aspects like equipment failure. Although PMUs are highly accurate enough to provide reliable data, there is an unavailable possibility of PMU which may be caused by communication failures or line outages. Furthermore, the networked PMUs might be rendered out of service by natural disasters such as hurricanes or PMUs can be intentionally taken down by malicious attacks. PMUs are also prone to cyber-attack. Since PMUs rely on GPS signal, there is a threat to GPS spoofing which gradually result in false reading or loss of measurements [26]. Such practical scenarios are likely to occur which are vulnerable to PMUs. Therefore while placing the PMUs in the system for state estimation; its vulnerability should also be accounted to study its impact on system observability. Hence, our objective of work is to propose PMU placement approach considering vulnerability analysis.

The objective of this research work are as follows:

1. Deploying a fixed number of PMUs in the system considering the vulnerability of PMU, in absence of conventional measurements.
2. Deploying a fixed number of PMUs in the system considering the vulnerability of PMU, in presence of conventional measurements like zero injection and branch flow measurements.
3. A strategy for placing additional PMUs in the system considering the resiliency of measurement systems.

1.5 Outline of Thesis

This thesis is composed of five chapters. Chapter 1 gives a background on PMU, their features and their importance in state estimation. Apart from that, literature reviews on PMU placement methodologies and the objectives are also presented. A detail explanation of the model used in PMU deployment approach considering with and without conventional measurements is presented in Chapter 2. Their differences are elaborated with simple examples. PMU placement considering its potential loss is also presented in chapter 3. The influence of considering conventional measurements is also discussed. A flow chart with consecutive steps for describing the approach is well presented. Cases study and results obtained from the approach are also discussed. Chapter 4 explains the need to select critical buses and the reason for prioritizing such buses when planning for PMU deployment. A strategy for placing additional PMUs in the system with pre-existing PMUs is formulated and the effectiveness of the approach is testified on IEEE test systems. Finally chapter 5 summaries the work and presents the recommendation for future research.

CHAPTER TWO

INTEGER LINEAR PROGRAMMING FORMULATION FOR OPTIMAL PMU PLACEMENT

Over the last 10 years, Integer Linear Programming (ILP) or Integer programming (IP) has been gaining a practical interest in variety of applications like planning, scheduling, telecommunication network and more. This could possibly because of the enormous grown in fast computing and improved algorithm. It is said that Integer Programming is the foundation for much of analytical decision-making problems [27]. It contains three main bodies: *variables*, *constraints* and *objective function*. *Variables* are the decision makers with integer values, *constraints* are used to restrict the values to a feasible region. In Integer Programming formulation, constraints must always be linear. It can be linear equality or linear inequality or both. *Objective function* then defines whether to maximize or minimize to get the optimal solution, depending upon the problem. The *objective function* should also be linear in nature.

Using ILP for finding optimal PMU placement is currently on trend since it saves the CPU computation time so greatly. This thesis work is done using Binary Integer Programming (BIP). The difference between LIP and BIP is that the decision variables are binary values (0, 1) in BIP. This chapter will give a review of general ILP formulation for optimal PMU placement and is derived from [22]. The mathematical formulation will be categorized into two different approaches: (i) Without conventional measurement (ii) With conventional measurement.

2.1 Without Conventional Measurement

The approach in this section is applicable where the conventional measurements like branch flow or any injection measurements are not considered.

2.1.1 Mathematical Formulation The basic PMU placement problem for full observability is formulated as follows:

$$\text{Min} \sum_{k=1}^N Cx_k \quad (2.1.1)$$

Subject to

$$\mathbf{A}\mathbf{X} \geq \mathbf{B}_{\text{PMU}} \quad (2.1.2)$$

$$\mathbf{C} = [1 \ 1 \ \cdots \ 1]_{1 \times N} \quad (2.1.3)$$

$$\mathbf{X} = [x_1 \ x_2 \ \cdots \ x_N]^T \quad (2.1.4)$$

$$x_k \in \{0,1\}$$

where

x_k binary decision variable for PMU location;

$$x_k = \begin{cases} 1 & \text{if PMU is present at bus } k \\ 0 & \text{otherwise} \end{cases} \quad (2.1.5)$$

C the cost function

N number of bus nodes of the system.

A bus connectivity matrix for N bus system and is defined as

$$A_{i,j} = \begin{cases} 1 & \text{if bus } i \text{ and } j \text{ are adjacent or } i = j \\ 0 & \text{otherwise} \end{cases} \quad (2.1.6)$$

B_{PMU} matrix with $N \times 1$ column vector whose entities are all ones.

The objective function in (2.1.1) defines the minimum PMU required to get full system observability. Since the cost function for all PMUs is unity, it means the cost for each PMUs are assumed to be equal. Inequality constraint (2.1.2) defines each bus in the system should be observed at least by one PMU. The matrix A is the system admittance matrix, which is transformed into binary form. This constraint guarantees the full observability. The number of required constraints is $N \times N$. The solution of this optimization problem gives the minimum optimal number of PMUs and their corresponding locations.

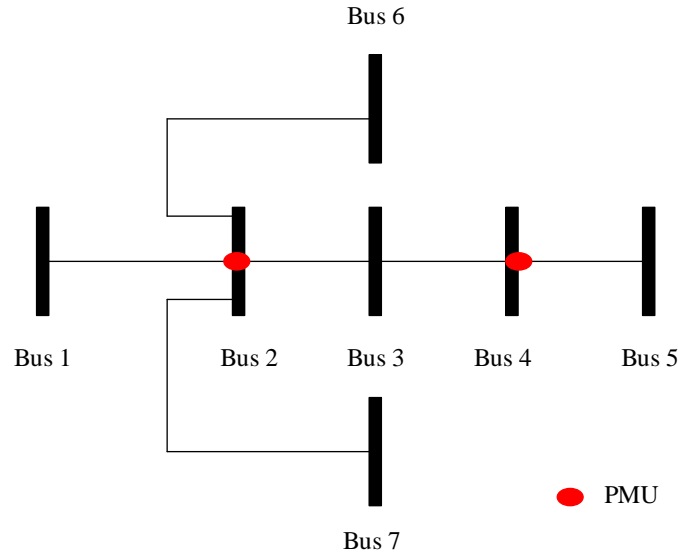


Figure 2.1 Seven bus system

2.1.2 Example Illustration

To illustrate the above problem, a seven bus system is used as shown in Figure 2.1.

The connectivity matrix in binary form is:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now multiplying the connectivity matrix with the decision variables, for each bus the constraints are as follows:

$$A_i \mathbf{X} = \begin{cases} A_1 \mathbf{X} = x_1 + x_2 & \geq 1 \\ A_2 \mathbf{X} = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \\ A_3 \mathbf{X} = x_2 + x_3 + x_4 & \geq 1 \\ A_4 \mathbf{X} = x_3 + x_4 + x_5 & \geq 1 \\ A_5 \mathbf{X} = x_4 + x_5 & \geq 1 \\ A_6 \mathbf{X} = x_2 + x_6 & \geq 1 \\ A_7 \mathbf{X} = x_2 + x_7 & \geq 1 \end{cases}$$

The operator sign “+” serves as logical “OR” and all the ones at the right hand side of inequality constraint is the observability constraint. This means at least one variable of each row containing summation the variables must be one or greater. For example, the very first row indicates the bus 1 and according to constraint, PMU should be placed either at bus 1 or bus2 to make bus 1 observable. Similarly the PMU should be placed either at bus 1, 2, 3, 6, or bus 7 in order to make bus 2 observable and so on. The resultant optimal number

of PMU for this system is two and the optimal locations are at bus 2 and bus 4. Hence the placement gives full observability of the buses.

2.2 With Conventional Measurement

The most economical way of reducing the number of optimal PMUs to place in the system is to consider conventional measurements. As mentioned earlier, conventional measurements in this thesis refers to measurements like zero-injection and branch flow measurements. Branch flow measurements are measurements between any two buses and are already available in practical existing systems. Such measurements are already there in the systems and the installation cost for branch measurements are very less compared to PMUs. Using branch flow measurement voltage angle of any one bus can be determined. Zero injection measurements are found in zero-injection buses. Any bus which does not have generation or load is considered as zero-injection nodes. These zero-injection buses need no metering and considered as accurate measurement for state estimation. They are also regarded as pseudo measurements. When the system contains zero-injection buses there are some rules associated with system observability [28].

- i. *The first rule implies:* in zero-injection cluster (all the buses adjacent to zero-injection bus and itself), if the zero-injection bus is observable and its adjacent buses are all observable except one bus then the non-observable bus will eventually become observable by applying KCL equation at zero-injection bus.
- ii. *The second rule is:* within the zero-injection cluster if all the buses are observable except the zero-injection bus, then that particular zero-injection bus can be identified as observable by using nodal equations.

Combining these two rules simplifies that a zero-injection cluster is observable when it has at most one unobservable bus.

2.2.1 An Observability Model

The observability model considers a sample six bus system as shown in Figure. 2.2 to explain the relationship between the PMUs and conventional measurements based on bus observability. To illustrate the model let us define a vector $G = AX$. The element $g_i = A_{i,j} x_i$ of G indicates the measurement redundancy. Measurement redundancy means the number of bus i is reached by a PMU. X is the PMU placement column matrix and x_i is the i th element of X and also regarded as decision variable; A is the bus connectivity matrix and $A_{i,j}$ is the i th row, j th column of A . The representation of matrix A and decision variable matrix X is same as defined in earlier section 2.1.

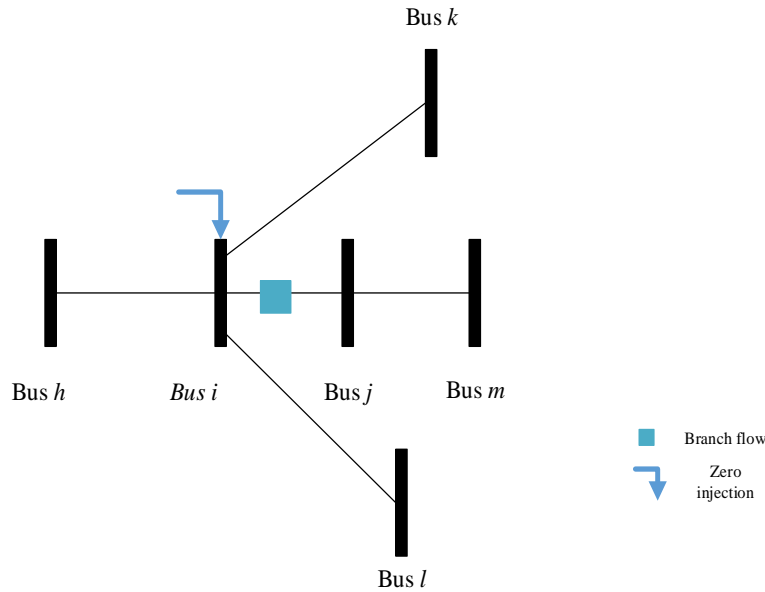


Figure 2.2 A sample system

To demonstrate the resultant observability criteria to be fulfilled due to the presence of conventional measurement, following three cases need to be analyzed.

- i. *Branch Flow Measurement.* As seen from Figure 2.2, either bus i or j can be made observable by this measurement whereas, the other bus must be observed by the PMU.

$$g_i + g_j \geq 1 \quad (2.2.1.1)$$

- ii. *Zero-injection Measurement.* Considering the zero injection measurement rules as mentioned earlier, out of five buses in zero-injection cluster, minimum four buses should be observed by PMU.

$$g_h + g_i + g_j + g_k + g_l \geq 4 \quad (2.2.1.2)$$

- iii. *Hybrid Measurement.* Combination of branch flow measurement and zero-injection measurement is referred in this paper as hybrid measurement (bus i as shown in Figure 2.2). Excluding the conventional measurement buses, the remaining buses except one bus in the zero-injection cluster should be observed by PMU. That one bus takes the merit of hybrid measurement so it can be made observable.

$$g_h + g_k + g_l \geq 2 \quad (2.2.1.3)$$

The right hand side of the equation (2.2.1.1-2.2.1.3) indicates the number of buses that should be observed by PMU itself.

2.2.2 Mathematical Formulation

From the observability model in section 2.2.1, it is obvious that the system with presence of conventional measurement can be categorized into two different types of buses; the bus associated with conventional measurement and the bus which is not associated with any conventional measurements. Therefore while formulating the constraint for optimal

PMU placement, the categorized buses should be kept in an order such that the bus not associated with conventional measurement is in first order, then the buses with associated conventional measurement is placed.

Now, the mathematical formulation when considering conventional measurement is given below:

$$\text{Min} \sum_{k=1}^N Cx_k \quad (2.2.2.1)$$

Subject to

$$\begin{bmatrix} I_{M \times M} & 0 \\ 0 & T_{meas} \end{bmatrix} (\mathbf{P}\mathbf{G}) = \mathbf{T}_{con} \mathbf{P}(\mathbf{A}\mathbf{X}) \geq \mathbf{b}_{con} \quad (2.2.2.2)$$

where T_{meas} and \mathbf{b}_{con} are summation of buses associated with conventional measurements and a constant number for those buses' observability respectively. These are interpreted same as described in above three cases; mentioned in section 2.2.1. M is the number of buses not associated with conventional measurements and \mathbf{P} is a permutation matrix.

2.2.3 Example Illustration

Let us consider a seven bus system considering a zero injection measurement and a branch flow measurement. In the Figure.2.3, bus 2 is zero injection bus and branch flow measurement is in between bus 2 and bus 3. According to above formulation, we have,

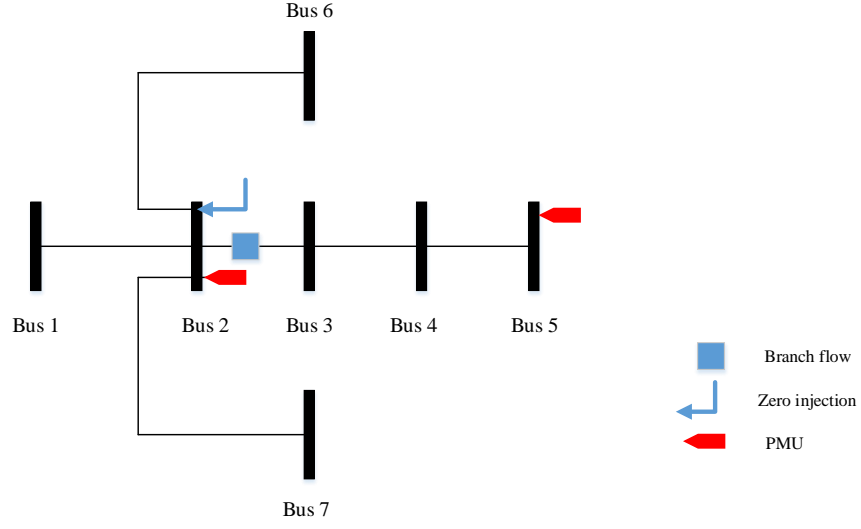


Figure 2.3 A sample system with conventional measurements

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Bus 4 and 5 are not associated to these two conventional measurements. The two equality constraints corresponding to two conventional measurements are $g_2 + g_3 \geq 1$ and $g_1 + g_6 + g_7 \geq 2$.

$$T_{meas} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} \text{Branch meas. 2 - 3} \\ \text{Injection meas. 2} \end{matrix}$$

Each column in T_{meas} refers to buses 1, 2, 3, 6, 7 respectively.

$$\mathbf{T}_{\text{con}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{T}_{\text{con}} \times \mathbf{P} \times \mathbf{A} \times \mathbf{X} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

The resultant optimal number of PMU is two and the strategic location of PMU is at bus 2 and bus 5. This example shows that conventional measurements are utilized in PMU placement. Since a very small system was taken into consideration, the optimal number of PMU considering with and without conventional measurement turned out to be equal. However the placement location is different. The efficiency of this model is significant for larger bus systems which will be discussed in next chapter.

CHAPTER THREE

PMU DEPLOYMENT APPROACH FOR MAXIMUM OBSERVABILITY CONSIDERING ITS POTENTIAL LOSS

This chapter deals with the PMU placement approach which is implemented to get maximum buses observability by addressing the PMU vulnerability. It is determined by analyzing the PMU loss and their impact on system bus observability. In order to implement this method, it requires more number of PMUs than normal operating condition. Since PMUs are expensive, the approach is equally flexible even for increasing the PMU requirement to just one additional than what is required for normal operating condition for each tested systems. Unlike the above discussed literatures mentioned in chapter one, possible loss of each PMUs that were supposed to be installed in the system are taken into consideration. Loss of each PMUs are evaluated periodically in a way that only one PMU loss is taken into consideration in each period. This is because the probability of single PMU loss is more than the two PMU losses. The optimization model is further divided into two parts. This chapter first explains the methodology without using conventional measurement and secondly analyzes using conventional measurements.

3.1 Optimization Model without Conventional Measurement

3.1.1 Mathematical Formulation

The PMU deployment optimization problem proposed in this part is used to analyze the vulnerability of PMU. This problem consists of two parts. The first is the increasing of the N_{PMUs} by one as illustrated in Figure 3.1. N_{PMUs} refers to the optimal number of PMUs required in the system to get full observability as discussed in chapter two. Then consider the loss of $(N_{PMUs} + 1)$ one at a time and solve the problem in (3.1.1.1) to (3.1.1.4). The main

objective of this problem is to locate the available number of PMUs in such a way observable buses is satisfied.

$$Max(\sum_{i=1}^N x_i + \sum_{i=N+1}^n x'_i) \quad (3.1.1.1)$$

Subject to

$$\mathbf{A} \mathbf{X} \geq \mathbf{B} \quad (3.1.1.2)$$

$$x'_i \times P_{\min,i} \leq b'_i \leq x'_i \times P_{\max,i} \quad (3.1.1.3)$$

$$\sum \mathbf{X} = m \quad (3.1.1.4)$$

$$\mathbf{X} = [x_1 \quad x_2 \quad x_i \quad x_N]^T \quad (3.1.1.5)$$

where

x_i decision variable for PMU placement as described in chapter two,

x'_i binary decision variable which represents the buses observed by PMUs only,

N number of buses in the system,

n number of nonzero elements of connectivity matrix \mathbf{A} ,

\mathbf{A} bus connectivity matrix as represented in chapter two,

\mathbf{B} observability constraint, column vector with all ones

$$\mathbf{B} = [b_1 \quad b_2 \quad \cdots \quad b_n]_{n \times 1}$$

$P_{\min,i}$ minimum number of nonzero elements of i^{th} bus, which is chosen as 1,

$P_{\max,i}$ total number of nonzero elements of connectivity matrix \mathbf{A} corresponding to i^{th} bus,

b'_i the product of \mathbf{A} and \mathbf{X} after removing one of the PMUs,

m the number PMUs available for the system which is greater than the requirement for normal operating condition.

Equation (5) implies

$$x_i' = \begin{cases} 0 & \text{if } b_i' > 0 \\ 1 & \text{if } b_i' = 0 \end{cases} \quad (3.1.1.6)$$

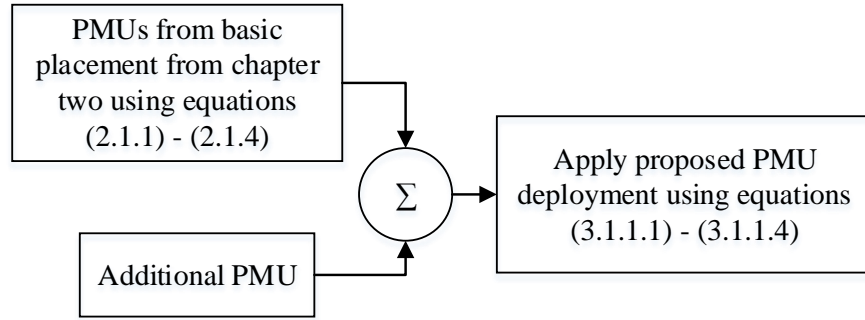


Figure 3.1 Relationship of optimization under normal condition and proposed model

Before the description of above formulation, it is important to understand the term “measurement redundancy”. Theoretically, “Measurement redundancy” of a bus means the number of times the bus is being observed by PMUs. In practice, measurement redundancy of each bus can be determined from the product of matrix **A** and **X**. Likewise, the system’s measurement redundancy can be calculated by summing the measurement redundancy of each buses.

This model tries to place the PMUs in such a way that the system’s measurement redundancy is increased. The objective function (3.1.1.1) gives the location of available number of PMUs and the maximum number of non-zero elements of **A** matrix that are particularly being observed by PMUs respectively. The inequality constraint (3.1.1.2) indicates that each buses should be observed by at least one PMU. Another inequality

constraints in (3.1.1.3) is an important and necessary condition to judge the strategic location of PMU such that the measurement redundancy of the system is maximized even when there is a single loss of PMU. The equality constraint in (3.1.1.4) denotes the number of PMUs to be deployed in the system.

3.1.2 Deployment Approach

The flowchart for the proposed method is shown in Figure. 3.2. The algorithm is described in following steps.

Step 1: Calculate the bus connectivity matrix \mathbf{A} of the bus system in terms of binary elements (1 and 0).

Step 2: Set the available number of PMUs m to be deployed in the system

Step 3: Optimize the PMU location considering constraints (3.1.1.2) and (3.1.1.4) as the only constraint from section 3.1.1.

Step 4: Once the PMU locations are globally optimized, remove one of the installed PMUs from the system to analyze the vulnerability. This process is carried out by removing i th column from connectivity matrix which is equivalent to removing a PMU at i th bus. Follow the objective function and all the constraint presented in equation (3.1.1.1)-(3.1.1.4) from section III.

Step 5: After the optimization, the number of observed nodes (covered by PMUs) are obtained. Repeat step 4 for other remaining PMUs. In this proposed model only one PMU is removed at a time.

Step 6: Evaluate whether the inequality constraint (3.1.1.3) is satisfied or not to indicate the maximum observability.

Step 7: If maximum observability is obtained this ends the process. Else update the PMU location again and follow steps 4 to 6 until the program terminates with optimal result.

When analyzing the effect of removing a PMU from a bus i , only the buses that are connected to it are assessed instead of all the buses. In doing so, the number of variables and constraint equations are reduced by an enormous amount especially, when the size of the system is large. Essentially, the numbers of additional variables are $N + n$ and number of additional constraint equations is $N + 2n$.

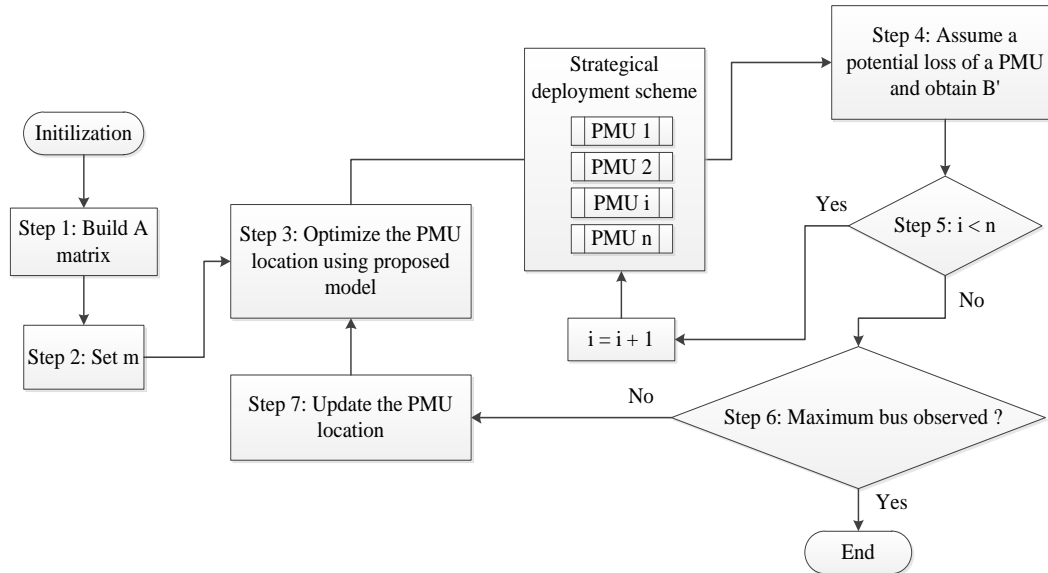


Figure 3.2 Flow chart of proposed algorithm

3.1.3 Case Study

The proposed method is tested on the IEEE 14, 30, 57, 118 and 300 bus test systems [29]. The single line diagram of the test systems can be obtained from [29, 30]. The

optimization is executed in Matlab environment using the binary linear programming toolbox.

3.1.3.1 PMU Placement Locations

The optimal number of PMUs (N_{PMUs}) and the corresponding locations are shown in the second and third columns of Table 3.1, respectively. Whereas the PMU deployment for maximizing the number of observable buses (measurable buses by PMUs) is summarized in columns four and five of Table 3.1. The resultant PMU locations for IEEE 14-bus system are buses 2, 6, 7, 9 and 14. In this particular system, all the PMU locations are same as that of basic placement case (under normal operation). The extra PMU is located at bus 14. Though it seems that the optimization is carried out locally but it is not. Coincidentally it happens to be optimal locations. In 57-bus system, only 6 buses are commonly placed by both of them. For larger systems like 118 and 300 bus systems, large number of PMUs is identically placed.

3.1.3.2 Effect on System Observability

The optimization problem tested in this work assumes the criticality of each PMU. It is not known earlier that which PMU measurement will be lost. Therefore, each installed PMU is assumed to have equal possibility of unavailability. Table 3.2 shows the detailed observability study of the 14-bus system. In this Table, the number of PMUs is 5 in total and the unobservable buses (while considering loss of each of the single assigned PMU) are presented. As expected from the proposed approach, small number of buses is unobserved. The redundancies of the respective buses are also shown for the system. As it can be seen

Table 3.1 PMU installed location for the test systems

Test system	Basic PMU placement		Proposed PMU deployment	
	Optimal no. of PMUs (N_{PMUs})	Optimal PMU locations at buses	m	PMU location at buses
14 bus	4	2 6 7 9	5	2 6 7 9 14
30 bus	10	1 7 9 10 12 18 24 25 27 28	11	1 7 10 11 12 19 24 25 27 28 29
57 bus	17	1 2 6 13 19 22 25 27 29 32 36 39 41 45 47 51 54	18	1 6 9 15 19 22 24 28 31 32 35 38 41 47 50 53 56 57
118 bus	32	3 7 9 11 12 17 21 25 28 34 37 41 45 49 53 56 62 63 68 70 71 76 79 85 86 89 92 96 100 105 110 114	33	3 5 10 11 12 17 21 25 28 34 37 40 45 49 53 56 62 63 68 70 71 77 80 85 86 90 92 96 100 105 110 114 118
300 bus	87	1 2 3 11 12 15 17 22 23 25 26 27 33 37 38 43 48 49 53 54 55 58 59 60 62 64 65 68 71 73 79 83 85 86 88 92 93 98 99 101 109 111 112 113 116 118 119 128 132 135 138 139 143 145 152 157 163 167 173 183 187 188 189 190 193 196 202 204 208 210 211 213 216 217 219 222 226 228 267 268 269 270 272 273 274 276 294	88	1 2 3 11 12 15 20 22 23 25 27 33 37 38 43 48 49 53 54 55 58 59 62 64 68 69 71 73 79 83 85 86 88 89 93 98 99 101 103 109 111 112 113 116 118 119 122 132 133 138 143 145 152 157 160 163 173 177 183 187 189 190 193 196 200 204 208 210 211 213 216 217 219 222 224 228 238 252 267 268 269 270 272 273 274 276 294 300

from Table 3.2, with the PMU at bus 14 is lost, the system still manage to get full observability. Tables 3.3, 3.4 and 3.5 summarizes the observability of the IEEE 30, 57 and 118 bus systems respectively. The redundancy of each bus is not shown but the numbers of buses which are observed by more than one PMU are clearly mentioned.

Table 3.2 Observability results for the IEEE 14 bus system

When PMU at bus 2 is lost														
Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy of each bus	0	0	0	2	1	1	2	1	3	1	1	1	2	2
Unobservable buses	1,2,3													
When PMU at bus 6 is lost														
Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy of each bus	1	1	1	3	1	0	2	1	3	1	0	0	1	2
Unobservable buses	6,11,12													
When PMU at bus 7 is lost														
Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy of each bus	1	1	1	2	2	1	1	0	2	1	1	1	2	2
Unobservable buses	8													
When PMU at bus 9 is lost														
Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy of each bus	1	1	1	2	2	1	1	1	2	0	1	1	2	1
Unobservable buses	10													
When PMU at bus 14 is lost														
Buses	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Redundancy of each bus	1	1	1	3	2	1	2	1	2	1	1	1	1	1
Unobservable buses	null													

Table 3.3 Observability results for the IEEE 30 bus system

Lost PMUs	1	7	10	11	12	
Unobservable buses	1,2,3	5,7	10,17,21	11	4,12,13,14,15,16	
Number of buses observed by more than one PMU	10	10	7	9	10	
Lost PMUs	19	24	25	27	28	29
Unobservable buses	18,19	23	26	null	8	null
Number of buses observed by more than one PMU	8	9	9	7	9	8

Table 3.4 Observability results for the IEEE 57 bus system

Lost PMUs	1	6	9	15	19	22
Unobservable buses	2,16,17	4,5,6,7	9,10,12,55	3,14,45	18,19,20	21
Number of buses observed by more than one PMU	15	16	14	14	17	14
Lost PMUs	24	28	31	32	35	38
Unobservable buses	24,25,26	27,28,29	30	33	35,36	37,44
Number of buses observed by more than one PMU	16	17	15	14	16	13
Lost PMUs	41	47	50	53	56	57
Unobservable buses	43	46,47	50,51	52,53,54	40	39
Number of buses observed by more than one PMU	14	16	16	17	14	16

Table 3.5 Observability results for the IEEE 118 bus system

Lost PMUs	3	5	10	11	12	17
Unobservable buses	1	6,8	9,10	14	2,7,14,117	15,17,18,30,31,113
No. of redundant buses	40	39	40	38	39	39
Lost PMUs	21	25	28	34	37	40
Unobservable buses	20,21,22	23,25,26	28,29	19,36,43	33,35,38	41
No. of redundant buses	40	39	39	39	36	37
Lost PMUs	45	49	53	56	62	
Unobservable buses	44,46	47,48,50,51	52,53	55,56,57,58	60,61,62,67	
No. of redundant buses	38	36	40	39	39	
Lost PMUs	63	68	70	71	77	80
Unobservable buses	63,64	65,68,116	24,74	72,73	78	79
No. of redundant buses	39	39	38	38	37	34
Lost PMUs	85	86	90	92	96	100
Unobservable buses	83,84,88	87	90	93,102	95	101
No. of redundant buses	38	38	39	37	37	34
Lost PMUs	105	110	114	118		
Unobservable buses	105,107,108	109,110,111,112	32,114,115	118	–	
No. of redundant buses	38	40	40	39	–	

From these Tables, it can be noticed that the impact of PMU loss on the observability is more for the larger systems. Some of the PMUs are very critical, therefore losing those particular PMUs will result in greater number of unobservable buses. The most critical PMU for 118 bus system is at bus 17. When the PMU placed at bus 17 is lost, almost 6 buses become unobservable. Apart from that, the implemented algorithm locates the available PMUs in such a way that the maximum number of observable buses are obtained when any of single PMU is unavailable. For the 300 test system, the large number of uncovered buses is 8 which corresponding to missing the PMU at bus 268. This system is found fully observable if the PMU at bus 300 lost.

3.2 Optimization Model with Conventional Measurement

In this section, we will only discuss the mathematical formulation and the simulated results because all other approaches are same as that of model discussed in section 3.1. The only difference lies is the consideration of conventional measurements.

3.2.1 Mathematical formulation

The objective of PMU placement problem is to analyze the vulnerability of PMUs by deploying a limited number of PMUs m corresponding to the cost associated with it. For simplicity, the cost for all the PMUs is assumed as unity and all the PMUs have sufficient channels to observe adjacent buses' current phasors.

Mathematically,

$$Max(\sum_{i=1}^N c_i x_i + \sum_{i=N+1}^n x_i') \quad (3.2.1.1)$$

Subject to

$$\mathbf{T}_{\text{con}}\mathbf{P}(\mathbf{A}\mathbf{X}) \geq \mathbf{b}_{\text{con}} \quad (3.2.1.2)$$

$$x_i' \times b_{\text{con},i} \leq b_i' \leq x_i' \times P_{\text{max},i} \quad (3.2.1.3)$$

$$\sum \mathbf{X} = m \quad (3.2.1.4)$$

$$\mathbf{X} = [x_1 \quad x_2 \quad x_i \quad x_N]^T \quad (3.2.1.5)$$

where,

c is the cost function defined by row matrix containing all ones as $[1 \quad 1 \quad 1]_{1 \times N}$.

Remaining all other matrix vectors, variables are described earlier in section 3.1 and 2.2.2 (chapter two). The only difference between in the above formulation between the proposed placement techniques considering conventional measurement and without conventional measurement is constraint (3.2.1.2) and (3.2.1.3). The detail explanation for (3.2.1.2) was already discussed in chapter two. When compared to mathematical formulation used in section 3.1 of this chapter when considering system without conventional measurement and here is that the observability constraint or all buses are not same or unity. Due to advantage of zero injection and flow measurement, it is not necessary that each of the buses should be observed by at least a PMU. Depending upon the buses associated with and without zero injection and flow measurement, the value of observability constraint for some buses can be 0 or 1 or 2 or so on.

3.2.2 Case Study

The proposed method is tested on IEEE test system buses 14, 30,118 and 2383 western polish. The single line diagram of test systems can be obtained from [29]. The measurement bus specification of conventional measurements are shown in Table 3.6 for

all four test systems. Two different cases has been designed to analyze the result. Case A is for PMU placement without using conventional measurement and case B is for placement based on conventional measurements. Information given in Table 3.6 is designated only for case B. For the system with 2383 bus system, the location of zero injection is shown in appendix A.

Table 3.6 Test system specifications

IEEE Test System		14		30	118	2383
No. of Zero-injection Buses		1		6	10	552
Location of Zero-injection Buses		7		6, 9, 22, 25, 27, 28	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	-
Number of Branch Flow Measurement		2		1	1	1
Branch Flow Measurement Buses	From	6	7	2	15	8
	To	12	8	6	33	18

Table 3.7 Comparisons of PMU deployment schemes in two cases

Test System	Case A			Case B		
	N_{min}	m	PMU Locations	N_{min}	m	PMU Locations
IEEE bus 14	4	5	2, 6, 7, 9, 14	3	4	2, 4, 6, 9
IEEE bus 30	10	11	1, 7, 10, 11, 12, 19, 24, 25, 27, 28, 29	7	8	1, 7, 10, 12, 19, 24, 27, 28
IEEE bus 118	32	33	3, 5, 10, 11, 12, 17, 21, 25, 28, 34, 37, 40, 45, 49, 53, 56, 62, 63, 68, 70, 71, 77, 80, 85, 86, 90, 92, 96, 100, 105, 110, 114, 118	28	29	3, 9, 12, 15, 19, 22, 27, 31, 32, 34, 42, 45, 49, 53, 56, 62, 65, 70, 75, 79, 85, 86, 90, 92, 96, 100, 105, 110, 118
2383 polish	746	747	-	561	562	-

3.2.3.1 PMU Placement Locations

The optimal number of PMU for the full observability of the test system, under normal operating conditions (N_{min}) is achieved. Depending upon that, an additional PMU is made available to deploy into the system. All the total number (m) PMUs globally optimized and are placed at specific locations as shown in Table 3.7. In order to maintain the measurement accuracy, the number of PMUs is considered to be greater than or equal to conventional measurements. Since conventional measurements are used in case B, the requirement for number of PMUs is decreased by enormous amount. Comparing the PMU placement location between two cases, it can be inferred that the maximum number of buses happens be to commonly placed. As seen for 14 bus system, in case A, there are 5 PMUs and out of them, the location chosen for almost three PMUs i.e. 2, 6 and 9 are at the same location in case B, as in case A. The common location is seen more as the system buses increases or for the large system like 118 bus system and 2383 bus system. For 2383 polish system, the PMU locations are shown in appendix A.

While proposing optimization process, no constraint was imposed such that the PMU will not be placed at zero injection buses. Due to this obvious reason, proposed optimized method results in deploying PMU even at zero-injection buses. For example, in IEEE 30 bus system, bus-27 is referred as zero-injection bus and one of the optimal location of PMU is bus-27 itself.

3.2.3.2 Effect of System Observability

The optimization problem tested in this work assumes the criticality of each PMU. It is not known earlier that which PMU measurement will be lost. Therefore, each installed

PMU is assumed to have equal possibility of unavailability. The result shown in Table 3.7 does not guarantee to give full observability of the system. Figure 3.4 and Figure 3.5 shows the detailed observability study of IEEE 14 and 30 bus test system respectively. Each of those two figures illustrates measurement redundancy ($A_i x_i$) of i^{th} bus when each of the single PMU are lost. As it can be seen from the Fig. 3.4, when PMU at bus 4 is lost, all the buses have redundancy of at least one except for bus 8. Since bus 7 is zero-injection bus therefore using zero-injection rules, bus 8 can be made observable. Hence it can be said that even if the PMU at bus 4 is lost, the remaining PMUs manage to give full observability of the system. Such scenario might not be possible when other PMUs are out of service.

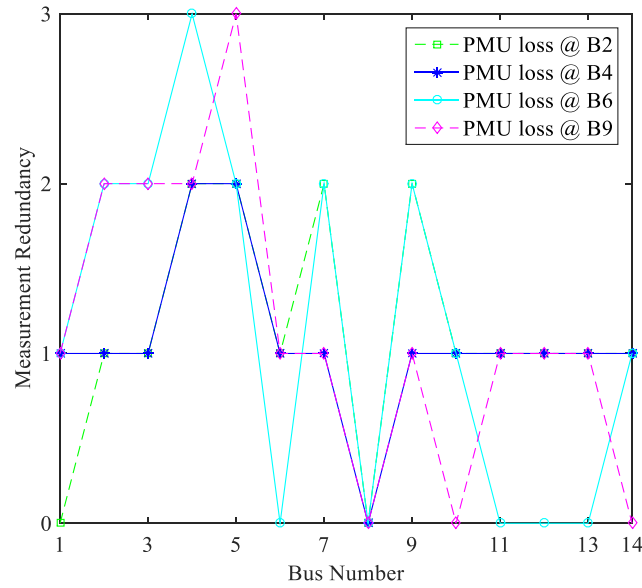


Figure 3.3 Measurement redundancy of IEEE 14 bus system

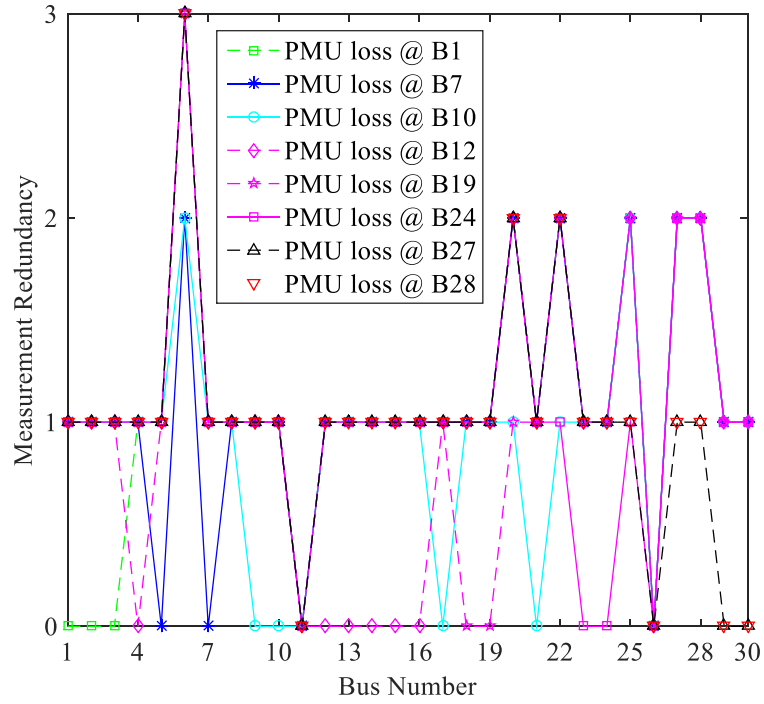


Figure 3.4 Measurement redundancy of IEEE 30 bus system

Nevertheless, the maximum number of buses will be observable. Figure 3.3 and 3.4, also illustrates the criticality of the placed PMUs. To clarify further, from the redundancy chart, it can be known that losing which of the PMU results in greater number of unobservable buses. For example for IEEE 30 bus system, the Figure 3.4 shows that when PMU at bus 12 is unavailable, in total eight buses will be unobserved directly. PMU at bus 12 can be treated as critical one. The redundancy chart for remaining test system are not shown here but can be easily obtained with the help of PMU placed location resulted in Table 3.7.

3.3 Conclusion

This chapter discussed the PMU placement strategy with very limited amount of PMUs to maximize the system observability. All the deployed PMUs were considered to have an equal probability of failure. Therefore the obtained PMU location ensure that even if any one of the PMU falls out, the remaining deployed PMUs still makes the maximum buses observable. An additional maximum observability constraint was proposed. The proposed method is also significant to analyze which of the PMUs is more critical. Binary Integer Programming was used to obtain deterministic solution and the optimization was rather done globally. This approach can be efficiently used to study the system observability for random installation of the PMUs in sub-transmission system.

CHAPTER FOUR

A STRATEGY FOR PMU PLACEMENT CONSIDERING THE RESILIENCY OF MEASUREMENT SYSTEM

This chapter presents an approach to find strategic locations for additional Phasor Measurement Units (PMUs) installation while considering resiliency of existing PMU measurement system. Due to the critical nature of power systems, complete observability of all nodes at all times is required. However, the networked PMUs might be rendered out of service by natural disasters such as hurricanes or PMUs can be intentionally taken down by malicious attacks. Enough attention should be given to PMU vulnerability while placing PMUs in the system. The concept of economically deploying PMUs considering resiliency of existing system post attack is missing in the above literatures mentioned in chapter one. Hence, this chapter highlights a considerable interest in improving PMU redundancy at minimum cost. In order to ascertain a subset of nodes which are most likely to be attacked, a virtual attack agent is modeled. The aim of the virtual attack agent is to reduce system observability to a minimum while carrying out a coordinated attack on a subset of PMU installation nodes. This virtual attack is used by the operator agent to identify a set of critical nodes whose redundancy needs to be increased. The planner agent then finds strategic locations to place additional PMUs in order to increase redundancy of critical nodes while minimizing incurred cost.

4.1 Agent Based PMU Placement Framework

An uncertainty constraint PMU placement problem can be expressed in three different agent based stages:

- *Attacker*: A virtual attack agent is introduced whose goal is to take down a set of installed

PMUs to reduce system observability. Uncertain events like intentional attacks is an important aspect that needs to be considered while making PMU placement decision. Due to geographical span of interconnected power systems planning a coordinated attack on all of the installed PMUs is improbable. Hence, the virtual attack agent will carry out coordinated attacks on a subset of installed PMUs that are deemed critical. Here, the set of critical PMUs are the ones when taken out of service minimizes system observability. Cardinality of the critical set is assumed to vary depending on the resources available to virtual attack agent.

- *Operator*: At this stage, the operator has to take corrective measures to mitigate the possible damage caused by the attacker. The operator agent identifies a set of critical nodes based on virtual attack agents attack plan. The operator agent then relays the corrective measure, which in this case is to increase the redundancy of critical nodes, to the planner agent.
- *Planner*: The task of planner is to deploy additional PMUs to increase redundancy of critical nodes at minimum cost.

Schematic representation of the three cyclic stages is shown in Figure 4.1. The schematic is cyclic in nature because of the nature of the problem, where the virtual attack agent comes up with strategies to minimize system observability given a set of PMU locations. The operator and planner agents then mitigate the effect of virtual attack agent by placing additional PMUs at strategic locations. The virtual attack agent then starts a new cycle with the new set of PMU installation locations.

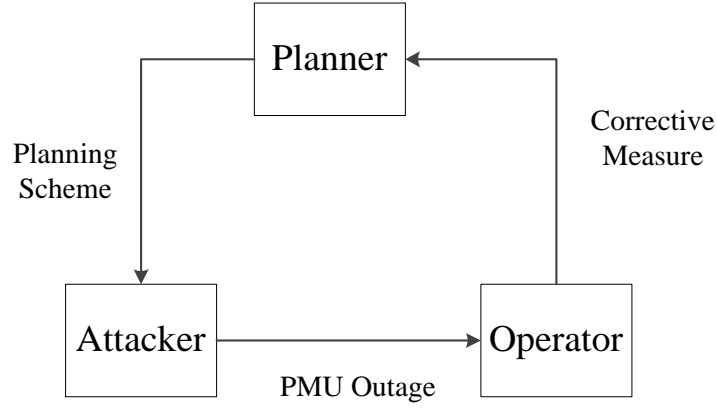


Figure 4.1 Relationship between three agents in PMU placement

Each undesired PMU outage caused by the virtual attack agent is an optimization scenario for the operator. These undesired outages can be single, double or multiple based on virtual attack agent's resources. Let P be the number of PMUs deployed into the system and Ψ be the scenario which corresponds to the number of PMUs to be attacked by the attacker. The total scenario can be represented as combinatorial number ${}_P C_\Psi$ as:

$${}_P C_\Psi = \frac{P!}{\Psi!(P-\Psi)!} \quad (4.1)$$

Since there are hundreds of thousands of possible attack scenarios, it is impossible to enumerate all scenarios for large systems due to computational burden. Instead, by adopting the approach in (4.2) a worst case scenario can be obtained.

$$\Psi = \eta\% \times P \quad (4.2)$$

where $\eta \in [0, 100]$ – representing the percentage of installed PMUs that are attacked. As a worst-case scenario, an assumption has been made that the attacker can attack up to 50% of the total deployed PMUs. Depending upon η value, a set of attacked PMUs $\Psi = [\Psi_1, \Psi_2, \dots, \Psi_z]$ is obtained from the optimization problem and this set is named as critical

PMUs. The programming framework for the agent based PMU placement is shown in Figure 4.2.

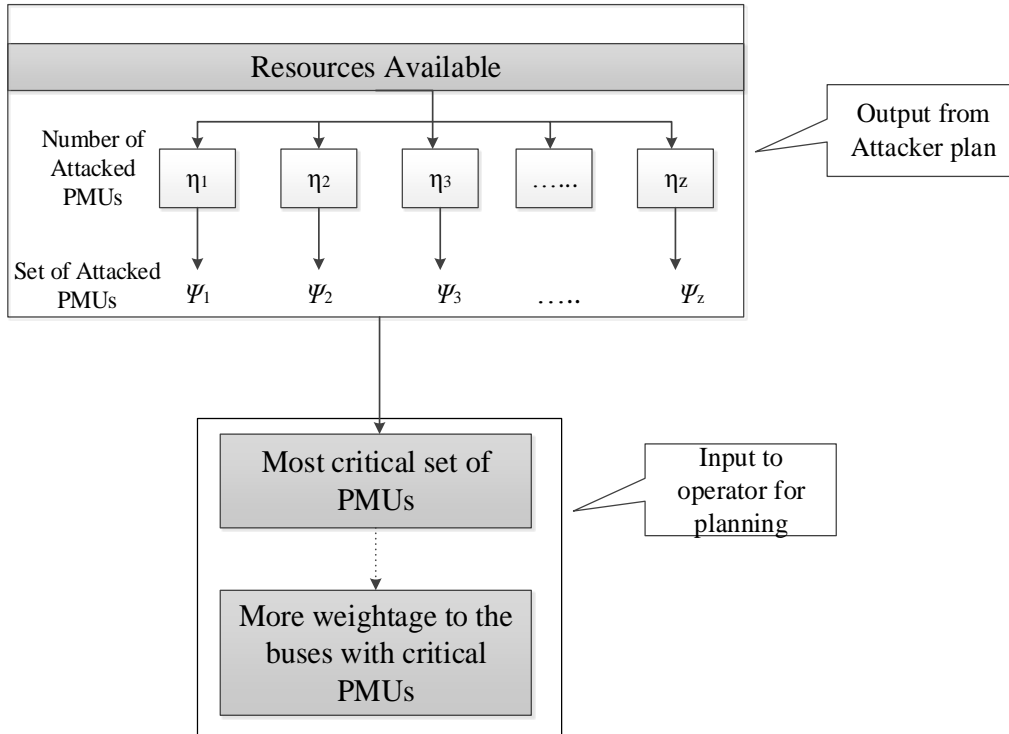


Figure 4.2 PMU placement Framework

4.2 Mathematical Formulation

Development of agent models as an optimization problem is discussed in this section. The initial deployment locations for PMUs, which act as the starting point for the proposed agent based framework are obtained using optimal PMU placement algorithm from [22].

4.2.1 Virtual Attack Agent

The objective of virtual attack agent is to attack a subset of installed PMUs in the system such that the system bus observability is minimized. The attack agent is modeled using binary integer programming.

The mathematical formulations for attacker's objective is as follows:

$$\min \sum_{k=l+1}^m \xi_k \quad (4.2.1.1)$$

Subject to

$$\left(\sum A_i \right) \xi_k \geq A_i x_i \quad (4.2.1.2)$$

$$\sum_{p=1}^l x(p) = \left(\sum x \right) - \psi_z \quad (4.2.1.3)$$

$$\xi_k \in \{0,1\} \quad \text{and} \quad x_p, x_i \in \{0,1\} \quad (4.2.1.4)$$

The objective function (4.2.1.1) ξ_k is the decision variable that tends to give the observability of each bus in terms of binary variable. If the bus is observable by PMUs remaining in the system after the coordinated attack by virtual attack agent then ξ_k will take the value of 1 and if the bus is not observable by any of the PMUs then ξ_k will take the value '0'. In general, observability of a bus can be 0 in which case the bus is not observable or

observability can be a positive number which means the bus is observable.

$$\xi_i = \begin{cases} 1 & \text{if } A_i \cdot x_p > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.2.1.5)$$

Since the available PMUs were placed based on system network topology, it becomes necessary to define a network connectivity matrix A.

Elements in matrix A are defined as follows:

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \text{ or } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases} \quad (4.2.1.6)$$

In constraint (4.2.1.2), x_i is an auxiliary binary variable of PMU placement. If the PMU is present at the i^{th} bus then x_i is regarded as 1 otherwise 0. Before the attack, the observability of the i^{th} bus denoted by left-hand side of (4.2.1.2) should be equal to the product of connectivity matrix of bus i and PMU placement variable x_i . Since the attacker already know the exact location of the PMUs, the attacker agent tries to enumerate all the possibilities to destroy or damage the PMU which are critical. This procedure is presented in (4.2.1.3). The word ‘critical’ defines those set of PMUs whose installation in the system increases the system observability. Post attack the variable x_i is zero for the disabled or attacked PMU. In this case, the constraint (4.2.1.2) will act as inequality constraint because the observability of the bus at left hand side will be greater than right hand side. The connectivity matrix is always fixed as long as all the transmission lines in the system are in service. The variable x_p is the PMU placement variable post attack. Depending upon the auxiliary variable x_p , the attacker performs all combinatorial number and checks the observability of each bus one by one. Those combination sets where the observability of bus shows the maximum number, the attacker tries to attack on those particular sets of

PMUs. Constraint (4.2.1.2) helps the attacker to judge the most attractive set of PMUs to act on.

Computational complexity of this optimization model increases substantially when dealing with large number of system buses. From (4.2.1.2), the total number of inequality constraints is equal to the number of system buses N and the equality constraint (4.2.1.3) is split into two sections, one for the set of the buses where PMUs were installed and other for the set of buses where PMUs were not installed. Therefore the total number of constraints is $N+1+1$. Similarly the total numbers of variables are twice the number of system buses M . This is because the first half $M/2$ denotes the auxiliary variable of PMU placement post attack and the other half $M/2$ denotes the bus observability.

4.2.2 Operator Agent

The responsibility of the operator is to identify vulnerable nodes based on the behavior of virtual attack agent. Vulnerable nodes in this context are a set of critical buses whose observability is compromised by the virtual attack agent. Critical buses are the buses include critical PMU installation buses and buses that are observable by critical PMUs.

The number of PMUs attacked by virtual attack agent is a percentage of the total number of installed PMUs. Since, larger systems have larger number of installed PMUs, the number of critical buses also tends to increase with system size. Since various sets of PMUs were obtained depending upon the availability of attacker's resources. Now, with the concern of PMU's and their installation cost, from those several sets of classified critical PMUs, the planner has to choose only the most repeated PMUs among all sets of critical PMUs. To obtain this, following formulation is used.

$$R = \psi_1 \cup \psi_2 \quad (4.2.1.1)$$

$$S = R \cap \psi_3 \cap \dots \cap \psi_z \quad (4.2.2.2)$$

where $S=[s_1, s_2, \dots, s_w]$, denotes set of critical PMUs in (4.2.2.2).

The critical buses are those buses that are observable from the set of critical PMUs.

$$W_c^B = \theta(A_i | s_w) \quad (4.2.2.3)$$

$$W_c^B = \{w_{c,1}^B, w_{c,2}^B, \dots, w_{c,f}^B\} \quad (4.2.2.4)$$

where W_c^B is represented for set of critical buses obtained from each critical PMUs s_w and θ is the index of buses which are adjacent to critical PMU located buses.

4.2.3 Planner Agent

The objective of the planner agent is to install additional PMUs in strategic locations to mitigate the vulnerability posed by virtual attack agent. The optimal PMU placement considering the critical PMUs is as follows:

$$\min \sum_{i=1}^N c_i x_i' \quad (4.2.3.1)$$

$$(A_i | \overline{w_{c,f}^B}) x_i' \geq b_i \quad (4.2.3.2)$$

$$(A_i | w_{c,f}^B) x_i' \geq b_i' \quad (4.2.3.3)$$

$$A_{eq} X' = \sum_{i=1}^N x_i \quad (4.2.3.4)$$

$$c_i = [1 \quad 1 \quad \dots \quad 1]_{1 \times N} \quad (4.2.3.5)$$

$$x_i' \in \{0, 1\}, \quad \forall i = \{1, 2, \dots, N\} \quad (4.2.3.6)$$

The objective function (4.2.3.1) implies that minimum number of PMU is placed in the system and x'_i is the new decision variable for PMU placement for this particular model. It is defined same as x_i as described earlier in attacker's model. In this model, b_i is observability constraint for non-critical buses and is considered equivalent to one. Whereas for critical buses, the observability constraint b'_i is considered as two. Therefore constraints (4.2.3.2) and (4.2.3.3) describes that each non-critical bus $\overline{w_c^B}$ and critical buses w_c^B must be observable by at least one PMU and two PMUs respectively. Equality constraint (4.2.3.4) represents that original PMUs has to be placed in the same location. Thus, under any uncertain events or attacks, all the buses are still observable and with higher redundancy with additional number of PMUs in the system. There are N number of variables and $2N$ number of constraints. In this proposed model, the restrictions on number of additional PMUs is not implemented. However, this optimization model has the potential to optimize the fixed amount of additional PMUs just by adding a new constraint such that the summation of decision variable is equal to a constant number.

4.3 Case study

The performance of proposed model is tested on 14, 30, 57 and 118 IEEE test bus systems including large power system 2383 bus Western Polish system [29, 30]. All the testified cases are implemented on 1.70 GHz processor with 6 GB of RAM using CPLEX12.6.2 Solver. The optimization is executed in MATLAB environment.

4.3.1 Critical PMUs

The number of critical PMUs depends upon the size of the system and the system topology. The set of PMUs that poses a higher influence in increasing the system bus

observability are shown in the Table 4.1. The critical PMUs are obtained based upon the resources available to the attacker. The percentage shown in the Table 4.1 indicates that the attacker has ability to damage certain percentage of the total deployed PMUs in the system. For a small system like 14 bus system, only 4 PMUs are needed in the system for full observability before attack. 10% of 4 PMUs being a negligible number, 20% and 50% of total placed PMUs is considered for execution. N_{min} is the number of attacked PMUs. Similarly, Table 4.1 demonstrates all the critical PMUs for different IEEE systems.

To further analyze strictly critical PMUs, only one set of PMUs per system is evaluated. The PMUs that happens to be critical for more than twice among the differentiated level of resources availability are only considered as most critical PMUs. Figure 4.1 shows all such single set of most critical PMUs for 14, 30, 57 and 118 IEEE bus systems only. The model was further tested for larger power systems like IEEE 300 and 2383 Western Polish system. For the larger system, the most critical PMU buses are shown in Table 4.2. The critical PMUs for larger systems are selected based on 10% of total installed PMUs. Since the numbers of PMUs installed in IEEE 300 and 2383 Western Polish system outnumbered to smaller system, PMU installed buses are not shown in the described Table 4.2.

Table 4.1 Critical PMUs for several IEEE systems

IEEE System	PMU location	Resources available to the attacker									
		10%		20%		30%		40%		50%	
		N_{min}	ψ_1	N_{min}	ψ_2	N_{min}	ψ_3	N_{min}	ψ_4	N_{min}	ψ_5
14	2, 7, 10, 13	–	–	1	2	–	–	–	–	2	2, 13
30	1, 2, 6, 10, 11, 12, 15, 19, 25, 29	1	1 0	2	6, 1 0	3	6, 10, 25	4	6, 10, 12, 15	5	6, 10, 12, 15, 19
57	2, 6, 12, 19, 22, 25, 27, 32, 36, 39, 41, 45, 46, 49, 51, 52, 55	2	6, 4 1	3	6, 32, 41	5	6, 22, 32, 41, 46	7	6, 12, 22, 32, 41, 49, 55	9	6, 12, 22, 32, 36, 39, 41, 49, 55
118	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 77, 80, 85, 87, 91, 94, 101, 105, 110, 114	3	5 6, 1 0 5, 1 1 0	6	49, 56, 80, 85, 10 5, 1 10	10	5, 12, 17, 49, 56, 80, 85, 10 5, 11 0	13	5, 12, 15, 17, 34, 37, 40, 49, 56, 80, 85, 10 5, 11 0	16	5, 12, 15, 17, 34, 37, 40, 45, 49, 56, 62, 80, 85, 94, 105, 110

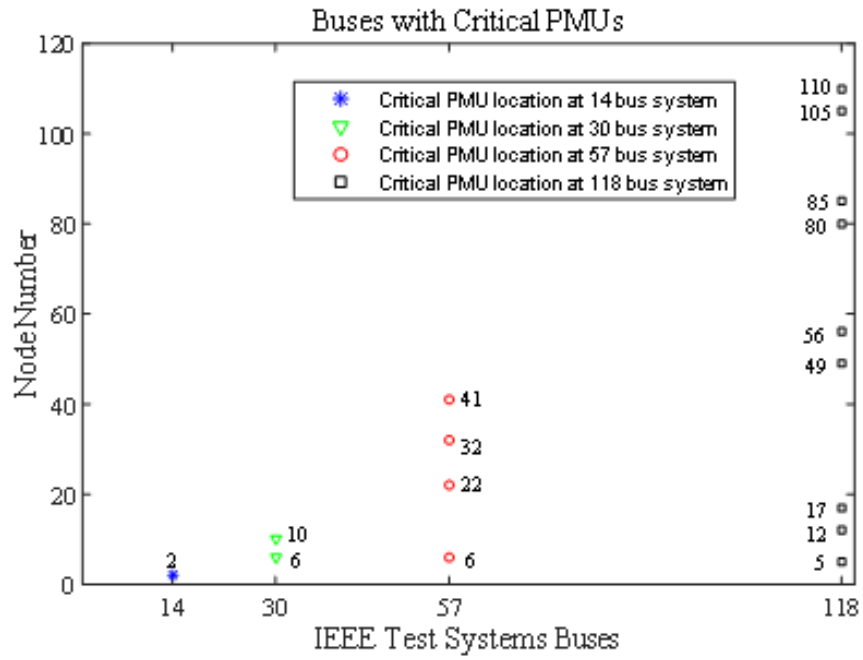


Figure 4.3 Critical PMUs in different test systems

Table 4.2 Critical buses with PMUs on larger systems

IEEE Test System	Total installed PMUs	Selected critical PMU buses								
300 bus system	87	3	15	109	112	190	268	269	270	272
2383 polish	746	6	18	29	133	246	309	310		
		321	322	353	354	361	365	366		
		374	425	456	494	511	525	526		
		527	546	556	613	644	645	679		
		694	717	750	754	755	796	797		
		870	923	944	978	979	1050	1096		
		1120	1138	1190	1201	1212	1213	1216		
		1217	1245	1483	1504	1524	1647	1664		
		1669	1680	1761	1822	1882	1883	1885		
		1919	1920	2112	2113	2166	2195	2196		
		2235	2258	2261	2274	2323				

4.3.2 Planning Scheme for PMU Placement

The PMU placement planning scheme is presented in this section. The goal of the planning scheme is to place additional PMUs in order to mitigate the loss of observability in the event of an attack. From the previous section, the set of critical buses with respect to loss of observability was obtained. The planner agent uses this information to obtain PMU placement scheme for installing additional PMUs with least cost to increase redundancy of critical nodes. For the most critical buses as shown in Table 4.3, the measurement redundancy was set to 2 i.e. the most critical buses must be observable by at least 2 PMUs. The resultant optimal numbers of PMUs are shown in Table 4.4. For IEEE 14 bus system, two additional PMUs are required to increase redundancy of five critical buses. Similarly for IEEE 30 bus system four additional PMUs are required to increase redundancy of 12 critical buses. Since the original PMU deployment was shown in Table 4.1, the following Table 4.4 shows PMU locations only for additional PMUs. Due to space limitation, location of additional PMUs for larger systems are not tabulated but are rather summarized as follows. For IEEE 300 bus system, 30 additional PMUs are required. While the 2383 bus polish system required 252 PMUs in addition to the originally placed 746 PMUs to obtain full bus system observability and increased redundancy at critical buses.

Table 4.3 Critical buses for different test systems

IEEE Test System	Critical Buses															
14	1 2 3 4 5															
30	2 4 6 7 8 9 10 17 20 21 22 28															
57	4 5 6 7 8 11 21 22 23 31 32 33 34 38 41 42 43 56															
118	2 3 4 5 6 7 8 11 12 14 15 16 17 18 30 31 42 45 47 48 49 50 51 54 55 56 57 58 59 66 69 77 79 80 81 83 84 85 86 88 89 96 97 98 99 103 104 105 106 107 108 109 110 111 112 113 117															

Table 4.4 Comparison of No. of optimal PMUs under normal and abnormal condition

IEEE Test System	No. of Optimal PMUs		% of Additional PMUs compared with original placement	Additional PMU Placement Location considering critical buses
	Normal Operating Condition	More weightage to critical PMUs		
14	4	6	50%	1 4
30	10	14	40%	5 8 12 16 22
57	17	26	53%	4 7 11 21 23 30 33 34 42
118	32	51	59%	4 6 8 18 32 46 54 57 58 78 83 88 96 100 106 108 111 112 117

4.4 Conclusion

This chapter proposes a planning approach for optimal PMU placement making the system more resilient to PMU failure. The likelihood of undesired events are analyzed by creating a virtual attack agent which intends to damage some of the critical PMUs in the system. Operator agent is used to obtain a subset of buses that are critical based on the attack pattern of virtual attack agent. Simulation results illustrate the ability of the planner agent to place additional PMUs at strategic locations to increase the redundancy of critical buses. The developed framework was tested on several test systems including a 2383 bus western polish system and optimal results were obtained in all cases.

APPENDICES

APPENDIX A

2383 Western Polish System

The following tables are exclusively for 2383 polish system

Table A 1 2383 Polish system bus specification

Number of Zero- injection Buses	Location of Zero- injection Buses								
552	1	2	3	4	5	6	7	8	9
	11	12	13	14	15	19	20	21	22
	23	24	25	26	27	28	32	33	34
	35	36	37	38	39	40	46	47	48
	49	50	51	52	53	54	55	56	59
	60	61	62	66	68	69	70	71	72
	73	74	75	76	77	78	79	80	81
	82	88	89	90	91	92	94	96	98
	99	100	101	102	106	107	108	112	113
	114	115	116	117	118	119	120	121	122
	128	129	130	133	134	135	136	137	138
	141	142	143	144	145	146	147	148	149
	150	151	152	153	154	155	156	157	158
	159	160	161	162	163	164	165	166	167
	168	169	170	171	172	173	174	175	178
	179	194	204	243	280	309	310	312	321
	322	332	336	355	361	362	374	375	449
	450	470	513	516	517	518	519	539	546
	564	565	568	569	576	587	614	617	634
	644	645	662	663	682	726	727	734	751
	777	786	789	791	797	799	800	801	806
	807	812	817	819	821	822	825	826	829
	833	836	840	844	848	854	855	856	863
	864	869	874	876	879	880	881	885	893
	898	903	915	916	921	923	924	926	927
	928	931	932	933	934	936	937	938	940
	941	946	953	956	966	967	969	971	972
	977	981	984	985	986	987	989	990	991
	1003	1006	1007	1008	1013	1017	1018	1019	1021
	1023	1025	1031	1040	1043	1044	1047	1048	1049
	1050	1052	1057	1062	1063	1065	1069	1070	1081
	1085	1092	1093	1095	1096	1098	1101	1108	1111
	1112	1114	1115	1119	1120	1121	1123	1127	1129

	1131	1135	1151	1152	1153	1157	1158	1159	1161
	1162	1164	1166	1167	1178	1180	1188	1189	1196
	1199	1200	1208	1209	1212	1213	1219	1222	1225
	1226	1227	1230	1231	1239	1243	1248	1252	1253
	1256	1257	1261	1264	1265	1271	1274	1275	1279
	1281	1288	1298	1299	1300	1305	1306	1307	1315
	1318	1319	1321	1323	1334	1335	1341	1343	1344
	1350	1353	1362	1365	1370	1371	1376	1378	1382
	1383	1384	1392	1394	1396	1397	1400	1401	1407
	1412	1413	1423	1431	1432	1439	1440	1443	1448
	1449	1455	1456	1459	1460	1465	1466	1468	1470
	1484	1491	1494	1498	1499	1502	1503	1511	1520
	1522	1523	1524	1531	1532	1539	1548	1560	1561
	1562	1563	1564	1569	1570	1577	1581	1583	1589
	1591	1592	1601	1618	1621	1631	1632	1636	1647
	1648	1649	1650	1665	1670	1681	1695	1713	1732
	1736	1740	1747	1748	1762	1774	1775	1777	1780
	1783	1790	1797	1802	1811	1812	1814	1822	1823
	1835	1840	1846	1851	1865	1872	1877	1878	1880
	1881	1885	1887	1896	1899	1902	1903	1906	1907
	1914	1919	1920	1922	1944	1948	1956	1958	1972
	1982	1987	2009	2014	2017	2019	2025	2031	2059
	2060	2061	2062	2063	2064	2065	2066	2067	2068
	2069	2070	2071	2072	2073	2074	2075	2076	2077
	2078	2079	2080	2081	2089	2090	2092	2109	2111
	2112	2113	2133	2134	2157	2158	2163	2165	2166
	2169	2170	2181	2186	2187	2188	2227	2257	2258
	2261	2280	2314	2325	2340	2357	2362	2365	2366
	2367	2377	2378						

From the Table A 1, it can be noticed that there are altogether 552 buses which are indicated as zero injection buses. The above mentioned buses do not have generation or the load. Since these buses are large in quantity it is probable that the requirement of PMUs will significantly decrease. To prove this statement, the below Table A 2 shows the effectiveness.

Table A 2 Comparisons of PMU deployment schemes for 2383 polish system

Case A						Case B					
PMU Location						PMU location					
6	22	23	28	29	35	12	15	18	29	35	39
42	43	55	58	61	63	41	50	61	69	80	84
64	71	73	74	76	81	86	88	105	126	129	130
90	105	106	110	119	121	133	150	151	154	158	160
126	133	136	138	143	145	162	172	175	178	179	189
152	163	168	174	176	179	190	192	195	199	202	209
187	188	189	192	194	196	213	214	215	217	219	220
200	208	213	215	217	219	222	229	235	240	244	245
224	229	230	233	236	240	246	247	249	259	265	275
246	247	249	251	257	259	281	282	287	292	293	295
263	265	275	277	281	282	299	307	308	309	310	314
284	285	287	295	299	303	316	319	324	326	337	339
307	309	310	314	319	321	341	347	348	352	353	354
322	324	330	334	335	336	356	360	363	365	366	367
338	339	347	348	352	353	369	371	372	380	383	384
354	355	360	361	365	366	389	393	394	404	408	411
368	372	373	375	377	378	418	425	428	434	446	448
379	380	382	383	391	404	456	460	464	466	468	475
409	413	417	425	438	439	476	481	485	493	494	497
444	445	446	448	459	463	502	511	512	515	516	523
468	476	479	480	481	486	525	526	527	529	530	540
493	494	498	500	502	510	542	545	550	551	556	563
511	512	515	516	517	519	568	575	579	583	584	586
522	523	525	526	527	529	591	592	601	603	609	613
535	540	545	547	548	550	615	618	622	635	643	644
551	553	556	557	560	564	649	650	651	655	657	658
565	568	569	571	582	583	660	663	675	676	679	685
584	590	591	593	594	595	687	689	690	692	694	696
603	613	615	618	622	626	698	703	705	707	710	713
635	638	644	645	646	649	714	716	717	722	723	733
651	661	662	666	679	680	740	750	754	755	763	768
682	685	690	692	694	696	769	771	773	775	778	785
703	704	708	709	710	712	794	796	797	802	808	814
715	716	717	721	722	724	815	817	821	826	832	833
726	727	735	740	741	747	834	838	839	852	857	858
748	750	754	758	763	765	859	861	870	871	876	883
768	778	784	785	796	797	892	895	899	900	906	908
798	799	806	812	817	819	912	914	918	919	920	943
821	822	824	825	829	831	947	961	967	968	973	978
834	839	854	855	858	859	979	981	985	990	992	993

863	869	870	871	873	876	996	997	1024	1050	1054	1057
885	892	898	902	910	912	1059	1063	1069	1070	1076	1082
915	916	919	923	926	928	1095	1096	1097	1104	1105	1106
932	936	937	944	953	960	1120	1134	1138	1140	1141	1149
966	971	972	973	978	979	1155	1168	1175	1182	1184	1190
981	985	986	989	990	997	1191	1192	1195	1201	1202	1213
1002	1006	1007	1017	1018	1019	1216	1217	1222	1231	1232	1233
1024	1025	1041	1042	1049	1050	1240	1245	1250	1251	1256	1261
1051	1054	1057	1059	1063	1065	1269	1275	1283	1284	1285	1289
1069	1070	1081	1082	1087	1092	1295	1309	1311	1320	1328	1330
1093	1095	1096	1101	1106	1108	1335	1337	1342	1343	1345	1346
1111	1112	1114	1119	1120	1123	1351	1354	1365	1371	1372	1375
1127	1129	1130	1138	1140	1141	1381	1384	1415	1416	1417	1418
1151	1153	1154	1157	1161	1162	1422	1426	1427	1439	1460	1468
1164	1167	1169	1178	1182	1183	1469	1476	1483	1489	1490	1500
1184	1186	1189	1190	1192	1193	1503	1505	1506	1507	1514	1518
1198	1199	1201	1203	1204	1207	1520	1522	1524	1531	1533	1534
1209	1212	1213	1216	1217	1218	1536	1538	1539	1540	1552	1553
1221	1225	1226	1228	1229	1230	1556	1560	1564	1565	1576	1580
1231	1233	1234	1238	1240	1243	1584	1591	1594	1595	1601	1603
1245	1246	1250	1253	1254	1257	1605	1607	1610	1616	1618	1619
1258	1259	1261	1265	1270	1273	1623	1624	1625	1642	1643	1647
1274	1277	1278	1284	1285	1288	1650	1652	1655	1656	1658	1659
1297	1306	1307	1309	1311	1314	1662	1667	1668	1669	1673	1680
1317	1318	1320	1321	1324	1328	1683	1684	1686	1687	1688	1689
1330	1333	1334	1337	1338	1345	1691	1697	1701	1716	1717	1723
1346	1350	1353	1356	1359	1361	1726	1728	1729	1730	1734	1735
1365	1366	1367	1371	1375	1378	1744	1745	1752	1755	1760	1761
1384	1387	1392	1393	1395	1400	1772	1786	1793	1796	1805	1810
1407	1410	1413	1414	1415	1416	1814	1829	1831	1841	1844	1845
1421	1426	1427	1430	1434	1438	1862	1867	1869	1871	1873	1879
1439	1441	1445	1448	1449	1454	1881	1882	1883	1884	1885	1886
1457	1460	1462	1465	1466	1468	1889	1892	1894	1897	1900	1904
1469	1470	1471	1472	1475	1476	1906	1916	1918	1919	1920	1921
1483	1484	1486	1489	1490	1492	1923	1924	1926	1929	1930	1933
1494	1498	1499	1502	1504	1505	1945	1949	1951	1953	1963	1974
1507	1511	1512	1514	1516	1517	1989	1993	1996	1998	2001	2002
1518	1520	1522	1523	1524	1531	2003	2006	2007	2020	2021	2037
1532	1533	1539	1540	1543	1545	2038	2039	2042	2044	2045	2047
1547	1550	1552	1556	1561	1569	2048	2050	2052	2054	2056	2060
1570	1574	1575	1576	1580	1582	2065	2087	2088	2091	2099	2101
1585	1589	1592	1603	1604	1607	2102	2105	2107	2113	2119	2121
1610	1619	1620	1624	1632	1635	2122	2124	2127	2134	2135	2137
1647	1652	1656	1659	1660	1662	2140	2143	2145	2146	2154	2157

1664	1667	1669	1673	1674	1680	2161	2167	2168	2170	2172	2173
1681	1682	1684	1686	1687	1690	2174	2175	2190	2191	2194	2195
1692	1694	1696	1712	1716	1717	2196	2202	2203	2209	2210	2217
1722	1723	1728	1729	1734	1735	2218	2221	2223	2224	2229	2232
1740	1745	1747	1748	1751	1755	2233	2235	2236	2244	2245	2247
1756	1757	1760	1761	1766	1771	2251	2255	2260	2261	2265	2270
1772	1774	1776	1783	1793	1795	2274	2279	2281	2283	2286	2289
1797	1802	1806	1811	1812	1814	2291	2300	2306	2309	2311	2313
1822	1825	1829	1840	1845	1858	2323	2330	2342	2345	2350	2360
1859	1862	1864	1866	1867	1871	2372	2374	2379	2380		
1873	1874	1875	1882	1883	1884						
1885	1889	1892	1895	1898	1901						
1902	1903	1905	1906	1907	1910						
1912	1914	1916	1918	1919	1920						
1926	1932	1933	1937	1939	1940						
1943	1944	1947	1951	1955	1957						
1961	1963	1965	1974	1977	1978						
1980	1982	1991	1992	1999	2000						
2001	2003	2006	2007	2019	2020						
2024	2025	2027	2031	2032	2034						
2035	2039	2040	2044	2047	2052						
2053	2054	2056	2059	2060	2061						
2062	2063	2067	2068	2069	2071						
2074	2076	2078	2079	2080	2085						
2086	2087	2090	2091	2092	2093						
2098	2102	2105	2106	2108	2112						
2113	2121	2122	2128	2133	2134						
2135	2137	2140	2144	2146	2157						
2159	2166	2167	2168	2169	2170						
2172	2173	2174	2176	2184	2185						
2187	2191	2193	2194	2195	2196						
2202	2203	2204	2209	2217	2218						
2223	2224	2229	2235	2236	2244						
2245	2249	2251	2252	2255	2258						
2259	2261	2264	2265	2274	2277						
2280	2281	2283	2285	2289	2290						
2292	2293	2298	2300	2303	2305						
2306	2309	2313	2323	2336	2339						
2340	2342	2345	2350	2352	2360						
2361	2364	2369	2373	2374	2379						
2380	2381	2383									

In Table A 2, case A indicates the simulation result for PMU placement approach without considering conventional measurements and case B is designated for PMU placement approach when conventional measurements are considered in the optimization model. Both the cases are solved for the potential loss of one PMU in order to get maximum observability of the buses with limited number of PMUs. From the Table A 2, it can be observed that the PMU requirement is relatively less in case B than in case A with the difference of 185 number of PMUs. Economically, this number makes a huge difference in the budget. Therefore keeping this in mind, the planning engineers always prefers to consider conventional measurements when analyzing the PMU placement. The western polish system is a large system, therefore the most of the buses selected for PMU placement happens to be common on both cases.

APPENDIX B

MATLAB CODE

Without Conventional Measurements

MATLAB Code for PMU placement under normal condition

```
function res=pmu_placement(casename)

% get bus connectivity matrix in binary form (Ybus)
ps=runpf(casename);
Ybus=abs(ps.Ybus);
Ybus(Ybus>0)=1;
Ybus(Ybus<0)=1;

% inequality constraint for PMU observability
A=Ybus;
nBus=size(Ybus,1);
f=ones(nBus,1);
b=ones(nBus,1);

% get optimal number of PMUs and their locations
tic
res.x=cplexbilp(f,-A,-b);
res.Aineq.x=A;
toc
```

MATLAB code for PMU placement considering vulnerability analysis

```
function res=pmu_vulnerability(casename)
clc

% get optimal placement
res_optimal=pmu_placement(casename);

% get Ybus
ps=runpf(casename);
Ybus=abs(ps.Ybus);
Ybus(Ybus>0)=1;
Ybus(Ybus<0)=1;
```

```

nBus=size(Ybus,1);

% find connectivity
count=0;
for n=1:nBus
    [ind]=find(Ybus(:,n)); % locates all nonzero elements of Ybus, and returns the linear
    indices of those elements in vector ind.
    Ind{n}=ind; % gives nodes numbers in column form which are connected to n
    bus
    count=count+length(ind);
end

ind_place=1:nBus;
ind_beta=length(ind_place)+1:length(ind_place)+count;
nVars=length(ind_place)+length(ind_beta);

ind_cons_place=1:nBus;
ind_cons_vul_ub=length(ind_cons_place)+1:length(ind_cons_place)+count;
ind_cons_vul_lb=length(ind_cons_place)+length(ind_cons_vul_ub)+1:length(ind_cons_pl
ace)+length(ind_cons_vul_ub)+count;

nCons=length(ind_cons_place)+length(ind_cons_vul_ub)+length(ind_cons_vul_lb);

A=sparse(nCons,nVars); % squeezing out any zero element
b=sparse(nCons,1);

% constraints
A(ind_cons_place,ind_place)=-Ybus;
b(ind_cons_place)=-1; %  $ax \geq 1$  i.e.  $-ax \leq -1$ 
b(ind_cons_vul_ub)=0; %  $b' - \beta * P_{max}$ 
b(ind_cons_vul_lb)=0; %  $\beta * P_{min} - b'$ 
P_max=sum(Ybus,2); % Compute the sum of the elements in each column.
P_min=1;

count=0;
for n=1:nBus
    for m=1:length(Ind{n})
        temp=Ind{n};
        count=count+1;
        A(ind_cons_vul_ub(m),ind_place)=Ybus(temp(m),:); %  $b'$ 
        A(ind_cons_vul_lb(m),ind_place)=-Ybus(temp(m),:); %  $-b'$ 

        A(ind_cons_vul_ub(m),ind_beta(count))=-P_max(n); %  $b' - \beta * P_{max}$ 
    end
end

```

```

        A(ind_cons_vul_lb(m),ind_beta(count))=P_min; % beta*Pmin-b'
    end
    A(ind_cons_vul_ub(count-length(temp)+1:count),n)=0; % remove nth column i.e.
    remove pmu at nth bus
end

Aeq=sparse(1,nVars);
Aeq(1,ind_place)=1;
beq(1)=nnz(res_optimal.x)+1; % change the extra number of pmus here as +n, where n is
the excess number of pmus ; nnz= number of non zero element

f=zeros(nVars,1);
f(ind_beta)=-1; % max beta

% optimal result
tic
[res.vul.x,res.vul.fmax,res.vul.status]=cplexbilp(f,A,b,Aeq,beq);
toc

res.opt.x=res_optimal.x;

```

With Conventional Measurements

MATLAB Code for PMU placement under normal condition

Main program:

```

function resc=BIP_convmeasurement(casename)

clc
% get Tcon and permuation matrix
conm_matrix=Tconventional(casename);
P=permutation(casename);
meas_obs=observability_constraint(casename);
b_c=meas_obs.b_c;
% lower bound
lb=zeros(size(P.G,1),1);

% upper bound
ub=ones(size(P.G,1),1);
T=-(conm_matrix.conm_matrix*P.PM*P.G);

```

```

% define observability constraints
T=sparse(T);
b_no_m=ones(length(conm_matrix.NCB),1);
b=[b_no_m;b_c];

% define objective function
f=ones(size(P.G,1),1);

% get optimal result
[resc.x,resc.fval]=cplexbilp(f,T,b,[],[],lb,ub);
resc.T=T;
resc.conm_matrix.NCB=conm_matrix.NCB;
function b_obs=observability_constraint(casename)

%----- code for observability constraint 'b'-----

clc
FM_index=[1 2 3 5 6 7 8 12]; % flow measurement buses for 14 bus system
FM_index=[2 6]; % flow measurement buses for 30 bus system
FM_index=[49 50 52 53]; % flow measurement buses for 57 bus system
FM_index=[15 33]; % flow measurement buses for 118 bus system
FM_index=[8 18]; % flow measurement bus for 2383 bus system
P=permutation(casename);
Tbus=1:length(P.G);

NCB=setdiff(Tbus,P.TCM); % buses those are not connected to any kind of measurements
b_c=zeros(size(P.WT,1),1);
for i=1:length(P.WT)
    if P.WT(i)==2
        b_c(i)=1;
    else if P.WT(i)==1
        if ismember(P.WT(i,2),FM_index)==1
            X=P.G(P.WT(i,2),:); % adjacent buses to each nodes
            X(:,union(NCB,FM_index))=[]; % gives adjacent buses which are associated
with conventional measurement buses
            X_new=sum(X)-1;
            b_c(i)=X_new;
        else
            X=P.G(P.WT(i,2),:);
            X(:,union(NCB,FM_index))=[];
            X_new=sum(X)-1;
            b_c(i)=X_new;
        end
    end
end

```



```

        end
    end
end
b_obs.b_c=b_c;

% -----Permutation-----
function conm=permutation(casename)

% get Ybus
ps=runpf(casename);
Ybus=abs(ps.Ybus);
Ybus(Ybus>0)=1;
Ybus(Ybus<0)=1;
G= full(Ybus);

%-----data for 14 bus system-----
M=5;
Type 1= injection
Type 2= flow
WT=[2 1 5;
    2 2 3;
    2 6 12;
    2 7 8;
    1 7 0]; % Type; Node from bus; Node to bus

%-----data for 30 bus system-----
M=7;
WT=[2 2 6;
    2 9 11;
    2 10 17;
    2 14 12;
    2 24 22;
    1 6 0;
    1 9 0;
    1 22 0;
    1 25 0;
    1 27 0;
    1 28 0];

%-----data for 57 bus system-----
M=17;
WT=[2 49 50
    2 52 53
    1 4 0

```

```

1 7 0
1 11 0
1 21 0
1 22 0
1 24 0
1 26 0
1 34 0
1 36 0
1 37 0
1 39 0
1 40 0
1 45 0
1 46 0
1 48 0];

%-----data for 118 bus system-----
M=11;
WT=[2 15 33
    1 5 0
    1 9 0
    1 30 0
    1 37 0
    1 38 0
    1 63 0
    1 64 0
    1 68 0
    1 71 0
    1 81 0];

%-----data for 2383 bus system-----
M=553;
result=matrixforzero injection;
ww=result.ww;
WT=[2 8 18;
    ww];

j=0;
u=0;
N=size(G,1);
PM=zeros(N,N);
for i=1:size(WT,1)
    if WT(i,1)==1 % injection measurement
        XT=G(WT(i,2),:); % XT will give the ith row of G bus
        % to distinguish the connected bus and not connected bus with the

```

```

% ith node measurement(1 means connected,0 means not connected bus)
for kk=1:length(XT)
    if XT(kk)==1

        CB1(j+1)=kk; % CB1 gives the injection measurement connected bus node
number
        j=j+1;
    end
end
elseif WT(i,1)==2 % flow measurement
    CB2(1,u+1:u+2)=WT(i,2:3); % CB2 gives the flow measurement connected bus
    u=u+2;
end
end
TCM= union(CB1,CB2); % This gives the total measurement connected bus node

L=length(TCM);
z=0;

for dd=1:N
    X = find(dd==TCM);
    T = isempty(X);
    if T==1
        PM(z+1,dd)=1; % Permutation matrix for not connected bus with measurements
        z = z +1;
    end
end

Q=N-L;
for dd=1:N
    X = find(dd==TCM);
    T = isempty(X);
    if T~=1
        PM(Q+1,dd)=1; % Permutation matrix for not connected bus with measurements
        Q = Q +1;
    end
end
P=PM; % Permutation matrix for 14-by-14 bus system

[a1,b1]=size(CB2);
[c1,d1]=size(TCM);
[e1,f1]=size(WT);
[g1,h1]=size(G);
[i1,j1]=size(PM);

```

```

field1 = 'P'; value1 = zeros(N,N);
field2 = 'CB2'; value2 = zeros(a1,b1);
field3 = 'M'; value3 = 0;
field4 = 'TCM'; value4 = zeros(c1,d1);
field5 = 'WT'; value5 = zeros(e1,f1);
field6 = 'G'; value6 = zeros(g1,h1);
field7 = 'PM'; value7 = zeros(i1,j1);

conm=
struct(field1,value1,field2,value2,field3,value3,field4,value4,field5,value5,field6,value6,fi
eld7,value7);
conm.P=P;
conm.CB2=CB2;
conm.M=M;
conm.TCM=TCM;
conm.WT=WT;
conm.G=G;
conm.PM=PM;

```

```

%-----Tconventional-----

```

```

function conm=Tconventional(casename)
clc
% Code for Tcon matrix, i.e. Tcon=[I 0;0 Tmeas]
% get permutation
P=permutation(casename);
P.PM;
Tmeas=zeros(P.M,length(P.TCM));
B=length(P.TCM); % total number of measurement connected buses
I=eye(length(P.G)-length(P.TCM)); % Identity matrix with diagonal= no. of
measurement not associated with conventional measurement
O1 =zeros(size(I,1),size(Tmeas,2));
O2 =zeros(P.M,size(I,2));

k=0;
for i=1:size(P.WT,1)
    if P.WT(i)==2
        for j=2:3
            for v=1:B
                if P.WT(i,j)==P.TCM(:,v)

```

```

        Tmeas(k+1,v)=1;
    end
end
end
k=k+1;
end

end

TBus=1:length(P.G);    % total number of buses
P.TCM;                % total measurement connected buses
NCB=setdiff(TBus,P.TCM); % buses those are not connected to any kind of measurements

for y=1:size(Tmeas,1)
    if P.WT(y)==1
        ZGnew=P.WT(y,2);
        ROZ=P.G(ZGnew,:);
        ROZ(:,NCB)=[];
        ROZ;
        if Tmeas(y,:)==0
            Tmeas(y,:)=ROZ;
            Tmeas(y,:)=Tmeas(y,:)-Tmeas(k,:);
            Tmeas(Tmeas<0)=0;
        end
    end
end
end
conm.Tconn=[I O1;O2 Tmeas];
conm_matrix=conm.Tconn;
[a1,b1]=size(NCB);
[c1,d1]=size(conm_matrix);

field1 = 'NCB'; value1 = zeros(a1,b1);
field2 = 'conm_matrix'; value2 = zeros(c1,d1);
conm = struct(field1,value1,field2,value2);
conm.NCB=NCB;
conm.conm_matrix=conm_matrix;

```

MATLAB Code for PMU placement considering vulnerability analysis

```
meas_obs=observability_constraint(casename);
b_c=meas_obs.b_c;
resc_optimal=BIP_convmeasurement(casename);
T=resc_optimal.T;
conm_matrix.NCB=resc_optimal.conm_matrix.NCB;
T=sparse(T);
count=0;
for n=1:size(-T,2)
    [ind]=find(T(:,n)); % locates all nonzero elements of T, and returns the linear indices
    % of those elements in vector ind.
    Ind{n}=ind; % gives nodes numbers in column form which are connected to n
    % bus
    count=count+length(ind);
end

ind_place=1:size(-T,2);
ind_beta=length(ind_place)+1:length(ind_place)+count;
nVars=length(ind_place)+length(ind_beta);

ind_cons_place=1:size(-T,1);
ind_cons_vul_ub=length(ind_cons_place)+1:length(ind_cons_place)+count;
ind_cons_vul_lb=length(ind_cons_place)+length(ind_cons_vul_ub)+1:length(ind_cons_pl
ace)+length(ind_cons_vul_ub)+count;

nCons=length(ind_cons_place)+length(ind_cons_vul_ub)+length(ind_cons_vul_lb);

A=sparse(nCons,nVars); % squeezing out any zero element
b=sparse(nCons,1);

% define constraints
A(ind_cons_place,ind_place)=T;
b_no_m=ones(length(conm_matrix.NCB),1);
b(ind_cons_place)=-[b_no_m;b_c];
b(ind_cons_vul_ub)=0; % b'-beta*Pmax
b(ind_cons_vul_lb)=0; % beta*Pmin-b'
P_max=sum(-T,2); %Compute the sum of the elements in each row.
P_min=[b_no_m;b_c];

count=0;
k=0;
for n=1:size(-T,2)
```

```

for m=1:length(Ind{n})
    temp=Ind{n};
    count=count+1;
    A(ind_cons_vul_ub(m),ind_place)=-T(temp(m,:)); % b'
    A(ind_cons_vul_lb(m),ind_place)=T(temp(m,:)); % -b'

    A(ind_cons_vul_ub(m),ind_beta(count))=-P_max(temp(m)); % b'-beta*Pmax
    A(ind_cons_vul_lb(m),ind_beta(count))=P_min(temp(m)); % beta*Pmin-b'
end
A(ind_cons_vul_ub(count-length(temp)+1:count,n)=0; % remove nth column i.e.
remove pmu at nth bus
end

Aeq=sparse(1,nVars);
Aeq(1,ind_place)=1;
beq(1)=nnz(resc_optimal.x)+1; % change the extra number of pmus here as +n, where n is
the excess number of pmus ; nnz= number of non zero element

f=zeros(nVars,1);
f(ind_beta)=-1; % max beta

tic
[resc.vul.x,resc.vul.fmax,res.vul.status]=cplexbilp(f,A,b,Aeq,beq);
toc

resc.opt.x=resc_optimal.x;

```

MATLAB code for PMU placement considering the resiliency of measurement system

```
clc
close all
clear all

% Main Attack Agent Code

Run=1;
while Run > 0
casename=input('casename=');

res_basicopt=pmu_placement(casename);
PMUplaced=find(res_basicopt.x)
howmanyPMUs=length(PMUplaced)
attack_resources=round([0.1 0.2 0.3 0.4 0.5]*howmanyPMUs);
for i=1:length(attack_resources)
    %nAttack=input('nAttack=');
    nAttack(i)=attack_resources(i)
    res_attack(i)=pmu_attack(casename,nAttack(i));
    AttackPMUs =res_attack(i).attackNodes
    howmanyattack(i)=length(AttackPMUs)
end
% planners prospective

nBus=res_basicopt.nBus; % get the row size of connectivity matrix from basic optimal
placement problem
if nBus< 300
temp1=union(res_attack(1).attackNodes,res_attack(2).attackNodes);
temp2=union(temp1,res_attack(3).attackNodes);
temp3=intersect(temp2,res_attack(4).attackNodes);
critical_PMU=intersect(temp3,res_attack(5).attackNodes)'; % most critical PMU bus

% only for larger systems -- 300 and 2383 bus
else
critical_PMU1=intersect(res_attack(1).attackNodes,res_attack(2).attackNodes);
critical_PMU2=intersect(critical_PMU1,res_attack(3).attackNodes);
critical_PMU=intersect(critical_PMU2,res_attack(5).attackNodes)';
end

% PMU placement inequality constraint

Aineq=res_basicopt.Aineq; % get a part of the constraint matrix from basic optimal
placement problem
```



```
nBus=res_basicopt.nBus; % get the row size of connectivity matrix from basic optimal placement problem
```

```
sizeofCP = length(critical_PMU) % length of critical PMUs
```

```
A_in = [0];
```

```
for i = 1: sizeofCP
```

```
    A_in = [A_in find(Aineq(critical_PMU(i,:),:))];
```

```
end
```

```
critical_bus = A_in(2:end)
```

```
critical_bus_unique=unique(critical_bus); % critical buses are obtained
```

```
for k=1:nBus
```

```
    if ismember(k,[critical_bus_unique])==1
```

```
        B(k,1)=2; % obsservability for critical buses only
```

```
    else
```

```
        B(k,1)=1; % observability for non-critical buses only
```

```
    end
```

```
end
```

```
% PMU placement equality constraint
```

```
% to restrict PMUs in the same pre-located buses
```

```
pre_PMU_loc=find(res_basicopt.x==1); % find pre-located PMUs in the system before attack
```

```
Aeq_new=zeros(1,size(Aineq,2)); % because number of variables is the same as Aineq
```

```
Aeq_new(1,pre_PMU_loc)=1;
```

```
Beq_new(1,1)=length(pre_PMU_loc);
```

```
% optimization function
```

```
func=ones(nBus,1);
```

```
tic
```

```
[x,fval,exitflag,output] = cplexbilp(func, -Aineq, -B,Aeq_new,Beq_new)
```

```
toc
```

```
disp('Do you want to run another case?  ')
```

```
Run = input('Press "Any Number" for Yes and "0 or Enter" for No.: ');
```

```
end
```

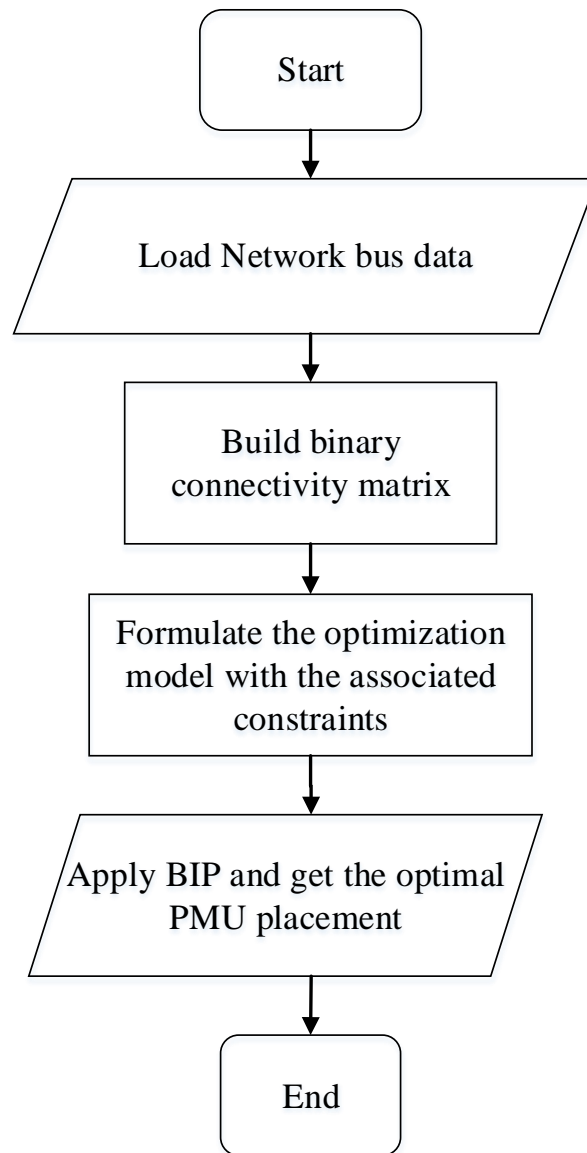


Figure B 1 General Flow chart for PMU placement approach executed in MATLAB

The above Figure B 1 is a general procedure followed while coding the program for PMU placement. The detail approach depending upon the objectives of this thesis is already shown in respective chapters.

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